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continuation on page 231

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Multiple Attribute Decision Making
Methods and Applications
A State-of-the-Art Survey

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PREFACE

This monograph is intended for an advanced undergraduate or graduate course as well as for the researchers who want a compilation of developments in this rapidly growing field of operations research. This is a sequel to our previous work entitled "Multiple Objective Decision Making--Methods and Applications: A State-of-the-Art Survey," (No. 164 of the Lecture Notes).

The literature on methods and applications of Multiple Attribute Decision Making (MADM) has been reviewed and classified systematically. This study provides readers with a capsule look into the existing methods, their characteristics, and applicability to analysis of MADM problems.

The basic MADM concepts are defined and a standard notation is introduced in Part II. Also introduced are foundations such as models for MADM, transformation of attributes, fuzzy decision rules, and methods for assessing weight.

A system of classifying seventeen major MADM methods is presented. These methods have been proposed by researchers in diversified disciplines; half of them are classical ones, but the other half have appeared recently. The basic concept, the computational procedure, and the characteristics of each of these methods are presented concisely in Part III. The computational procedure of each method is illustrated by solving a simple numerical example.

Part IV of the survey deals with the applications of these MADM methods. The literature has been classified into selection of commodity, site, people, project, public facility, etc. A summary of each reference on applications is given.

A choice rule for MADM methods, a unified approach to MADM problems, and proposed future study are presented in Part V.

An updated bibliographical listing of twenty-five books, monographs or conference proceedings, and about 500 selected papers, reports or theses is presented.

We are indebted to the outstanding pioneering survey of this field done by Dr. Kenneth R. MacCrimmon in 1968 and 1973; and to Professors Doris Grosh, William Schenck-Hamlin and P. L. Yu for their various comments and suggestions. The first draft was used in the first author's Spring 1980 class of "Advanced Topics in Operations Research." Dale G. Finkner, M. H. Lee, M. J. Lin, Cynthia S. McCahon, K. S. Raju, and Larry M. Strecker have tested and critically evaluated many methods. Special thanks are due to Merla Oppy for typing and Jean Burnham for editing.

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TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. BASIC CONCEPTS AND FOUNDATIONS	16
1. Definitions	16
1.1 Terms for MCDM Environment	16
1.2 MCDM Solutions	18
2. Models for MADM	24
2.1 Noncompensatory Model	24
2.2 Compensatory Model	25
3. Transformation of Attributes	26
3.1 Quantification of Fuzzy Attributes	27
3.2 Normalization	29
4. Fuzzy Decision Rules	32
4.1 Definition of Fuzzy Set	33
4.2 Some Basic Operations of Fuzzy Sets	35
5. Methods for Assessing Weight	41
5.1 Eigenvector Method	41
5.2 Weighted Least Square Method	48
5.3 Entropy Method	52
5.4 LINMAP	57
III. METHODS FOR MULTIPLE ATTRIBUTE DECISION MAKING	58
1. Methods for No Preference Information Given	58
1.1.1 Dominance	58
1.1.2 Maximin	61
1.1.3 Maximax	64

Page

2. Methods for Information on Attribute Given 67

2.1 Methods for Standard Level of Attribute Given 68

2.1.1 Conjunctive Method (Satisficing Method) 68

2.1.2 Disjunctive Method 71

2.2 Methods for Ordinal Preference of Attribute Given 73

2.2.1 Lexicographic Method 74

2.2.2 Elimination By Aspects 77

2.2.3 Permutation Method 84

2.3 Methods for Cardinal Preference of Attribute Given 92

2.3.1 Linear Assignment Method 93

2.3.2 Simple Additive Weighting Method 99

2.3.3 Hierarchical Additive Weighting Method 104

2.3.4 ELECTRE Method 115

2.3.5 TOPSIS 128

2.4 Methods for Marginal Rate of Substitution of Attributes Given 141

2.4.1 Hierarchical Tradeoffs 146

3. Methods for Information on Alternative Given 153

3.1 Methods for Pairwise Preference Given 153

3.1.1 LINMAP 154

3.1.2 Interactive Simple Additive Weighting Method 168

3.2 Method for Pairwise Proximity Given 176

3.2.1 Multidimensional Scaling with Ideal Point 177

IV. APPLICATIONS 192

1. Commodity Selection 192

2. Facility Location (Siting) Selection 195

3. Personnel Selection 197

Page

4. Project Selection 198

4.1 Environmental Planning 198

4.2 Land Use Planning 200

4.3 R & D Project 201

4.4 Water Resources Planning 202

4.5 Miscellaneous 202

5. Public Facility Selection 204

V. CONCLUDING REMARKS 207

On MADM Methods Classification 207

On Applications of MADM 207

On Multiple Objective Decision Making (MODM) Methods 207

On Multiattribute Utility Theory (MAUT) 208

A Choice Rule for MADM Methods 210

A Unified Approach to MADM 213

On Future Study 222

VI. BIBLIOGRAPHY 226

Books, Monographs, and Conference Proceedings 226

Journal Articles, Technical Reports, and Theses 228

LIST OF FIGURES

Figure	Title	page
1.1	A taxonomy of methods for multiple attribute decision making	9
2.1	Decision variable space representation of the feasible area of Examples 1 and 2	20
2.2	Objective function space representation of the feasible area of Example 1	21
2.3	Objective function space representation of the feasible area of Example 2	22
2.4	Assignment of values for an interval scale	28
2.5	Fuzzy subset B induced by a mapping	39
3.1	Expected number of nondominated alternatives	60
3.2	A graphical representation of aspects in the three-alternative case	80
3.3	Graphical presentation of Table 3.1	91
3.4	A hierarchy for priorities of fighter aircrafts	105
3.5	A hierarchy of overall well-being	112
3.6	Euclidean distances to the ideal and negative-ideal solutions in two-dimensional space	129
3.7	Typical indifference curves observed in TOPSIS	140
3.8	The marginal rate of substitution as a function of X_1 and X_2	142
3.9	A set of indifference curves between MPG and the space of passenger compartment	144
3.10	A set of indifference curves between maximum speed and reliability	148
3.11	A set of indifference curves between maximum payload and ferry range	150
3.12	A set of indifference curves between cost and maneuverability	151
3.13	An ascending curve on the initial configuration showing the value of d_{ij}	180

Figure	Title	page
3.14	A perfect configuration	181
3.15	Two dimensional configuration for 11 car models	184
3.16	Two dimensional configuration by the ALSCAL program for 7 candidates	189
5.1	A taxonomy of methods for multiple objective decision making	209
5.2	MADM method specification chart	211
5.3	A process diagram for unified approach to multiple attribute decision making	215
5.4	The partially ordered set of Table 5.5	223

LIST OF TABLES

Table	Title	page
1.1	MADM vs MODM	4
1.2	Classification of books, monographs and conference proceedings	7
1.3	Classification of references on multiple attribute decision making	10
1.4	List of journals relevant to MADM	13
2.1	A fighter aircraft selection problem	18
2.2	The scale and its description	45
2.3	Wealth comparison matrix	46
2.4	Normalized wealth eigenvector	46
2.5	Comparison of numerical results for wealth comparison matrix	51
3.1	The best order for the different w's	90
3.2	Overall satisfaction with job	110
3.3	Comparison of jobs with respect to six attributes	110
3.4	Rank order of dissimilarities between pairs of car models	185
3.5	Rank order of dissimilarities between pairs of candidates	187
3.6	Multiple regression of each attribute on configuration dimensions	190
4.1	Classification of references on applications	193
5.1	Preference rankings by four MADM methods	218
5.2	Majority vote between alternatives	218
5.3	Preference ranking by four MADM methods	221
5.4	Majority vote between alternatives	221
5.5	Aggregate preference ranking from three techniques	223