Chapter 5

PROMETHEE METHODS

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Abstract This paper gives an overview of the PROMETHEE-GAIA methodology for MCDA. It starts with general comments on multicriteria problems, stressing that a multicriteria problem cannot be treated without additional information related to the preferences and the priorities of the decision-makers. The information requested by PROMETHEE and GAIA is particularly clear and easy to define for both decision-makers and analysts. It consists in a preference function associated to each criterion as well as weights describing their relative importance. The PROMETHEE I, the PROMETHEE II complete ranking, as well as the GAIA visual interactive module are then described and commented. The two next sections are devoted to the PROMETHEE VI sensitivity analysis procedure (human brain) and to the PROMETHEE V procedure for multiple selection of alternatives under constraints. An overview of the PROMETHEE GDSS procedure for group decision making is then given. Finally the DECISION LAB software implementation of the PROMETHEE-GAIA methodology is described using a numerical example.

Keywords: MCDA, outranking methods, PROMETHEE-GAIA, DECISION LAB.

1. History

The PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by J.P. Brans and presented for the first time in 1982 at a conference organised by R. Nadeau and M. Landry at the Université Laval, Québec, Canada (L'Ingéniérie de la Décision. Elaboration d'instruments d'Aide à la Décision). The same year several applications using this methodology were already treated by G. Davignon in the field of Heath care.

A few years later J.P. Brans and B. Mareschal developed PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case). The same authors proposed in 1988 the visual interactive module GAIA which is providing a marvellous graphical representation supporting the PROMETHEE methodology.

In 1992 and 1994, J.P. Brans and B. Mareschal further suggested two nice extensions: PROMETHEE V (MCDA including segmentation constraints) and PROMETHEE VI (representation of the human brain).

A considerable number of successful applications has been treated by the PROMETHEE methodology in various fields such as Banking, Industrial Location, Manpower planning, Water resources, Investments, Medicine, Chemistry, Health care, Tourism, Ethics in OR, Dynamic management, ... The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use.

2. Multicriteria Problems

Let us consider the following multicriteria problem:

$$\max\{g_1(a), g_2(a), \ldots, g_j(a), \ldots, g_k(a) | a \in A\}, \tag{5.1}$$

where A is a finite set of possible alternatives $\{a_1, a_2, \ldots a_i, \ldots, a_n\}$ and $\{g_1(\cdot), g_2(\cdot), \ldots, g_j(\cdot), \ldots g_k(\cdot)\}$ a set of evaluation criteria. There is no objection to consider some criteria to be maximised and the others to be minimised. The expectation of the decision-maker is to identify an alternative optimising all the criteria.

Usually this is a *ill-posed mathematical* problem as there exists no alternative optimising all the criteria at the same time. However most (nearly all) human problems have a multicriteria nature. According to our various human aspirations, it makes no sense, and it is often not fair, to select a decision based on one evaluation criterion only. In most of cases at least technological, economical, environmental and social criteria should always be taken into account. Multicriteria problems are therefore extremely important and request an appropriate treatment.

The basic data of a multicriteria problem (5.1) consist of an evaluation table (Table 5.1).

Table 5.1. Evaluation table.

a	$g_1(\cdot)$	$g_2(\cdot)$	$\cdots$	$g_j(\cdot)$	$\cdots$	$g_k(\cdot)$
a_1	$g_1(a_1)$	$g_2(a_1)$	$\cdots$	$g_j(a_1)$	$\cdots$	$g_k(a_1)$
a_2	$g_1(a_2)$	$g_2(a_2)$	$\cdots$	$g_j(a_2)$	$\cdots$	$g_k(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	$g_1(a_i)$	$g_2(a_i)$	$\cdots$	$g_j(a_i)$	$\cdots$	$g_k(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_n	$g_1(a_n)$	$g_2(a_n)$	$\cdots$	$g_j(a_n)$	$\cdots$	$g_k(a_n)$

Let us consider as an example the problem of an individual purchasing a car. Of course the price is important and it should be minimised. However it is clear that in general individuals are not considering only the price. Not everybody is driving the cheapest car! Most people would like to drive a luxury or sports car at the price of an economy car. Indeed they consider many criteria such as price, reputation, comfort, speed, reliability, consumption, ... As there is no car optimising all the criteria at the same time, a *compromise* solution should be selected. Most decision problems have such a multicriteria nature.

The solution of a multicriteria problem depends not only on the basic data included in the evaluation table but also on the decision-maker himself. All individuals do not purchase the same car. There is no absolute best solution! The best compromise solution also depends on the individual *preferences* of each decision-maker, on the *"brain"* of each decision-maker.

Consequently, *additional information* representing these preferences is required to provide the decision maker with useful decision aid.

The natural dominance relation associated to a multicriteria problem of type (5.1) is defined as follows:
For each $(a,b) \in A$:

$$\begin{cases} \forall j : g_j(a) \geq g_j(b) \\ \exists k : g_k(a) > g_k(b) \end{cases} \iff aPb,$$

$$\forall j : g_j(a) = g_j(b) \iff aIb, \tag{5.2}$$

$$\begin{cases} \exists s : g_s(a) > g_s(b) \\ \exists r : g_r(a) < g_r(b) \end{cases} \iff aRb,$$

where *P, I,* and *R* respectively stand for *preference, indifference* and *incomparability.* This definition is quite obvious. An alternative is better than another if it is at least as good as the other on all criteria. If an alternative is better on a criterion s and the other one better on criterion r, it is impossible to decide which is the best one without additional information. Both alternatives are therefore incomparable!

Alternatives which are not dominated by any other are called *efficient solutions.* Given an evaluation table for a particular multicriteria problem, most of the alternatives (often all of them) are usually efficient. The dominance relation is very poor on P and I. When an alternative is better on one criterion, the other is often better on another criterion. Consequently incomparability holds for most pairwise comparisons, so that it is impossible to decide without additional information. This information can for example include:

- Trade-offs between the criteria;
- A value function aggregating all the criteria in a single function in order to obtain a mono-criterion problem for which an optimal solution exists;
- Weights giving the relative importance of the criteria;
- Preferences associated to each pairwise comparison within each criterion;
- Thresholds fixing preference limits;
- ...

Many multicriteria decision aid methods have been proposed. All these methods start from the same evaluation table, but they vary according to the additional information they request. The PROMETHEE methods require very clear additional information, that is easily obtained and understood by both decision-makers and analysts.

The purpose of all multicriteria methods is to enrich the dominance graph, i.e. to reduce the number of incomparabilities *(R).* When a utility function is built, the multicriteria problem is reduced to a single criterion problem for which an optimal solution exists. This seems exaggerated because it relies on quite strong assumptions (do we really make all our decisions based on a utility function defined somewhere in our brains?) and it completely transforms the structure of the decision problem. For this reason B. Roy proposed to build outranking relations including only realistic enrichments of the dominance relation (see [86] and [87]). In that case, not all the incomparabilities are withdrawn but the information is reliable. The PROMETHEE methods belong to the class of outranking methods.

In order to build an appropriate multicriteria method some requisites could be considered:

Requisite 1: The amplitude of the deviations between the evaluations of the alternatives within each criterion should be taken into account:

$$d_j(a,b) = g_j(a) - g_j(b). \tag{5.3}$$

This information can easily be calculated, but is not used in the efficiency theory. When these deviations are negligible the dominance relation can possibly be enriched.

Requisite 2: As the evaluations $g_j(a)$ of each criterion are expressed in their own units, *the scaling effects* should be completely eliminated. It is not acceptable to obtain conclusions depending on the scales in which the evaluations are expressed. Unfortunately not all multicriteria procedures are respecting this requisite!

Requisite 3: In the case of pairwise comparisons, an appropriate multicriteria method should provide the following information:

a is preferred to b;
a and b are indifferent;
a and b are incomparable.

The purpose is of course to reduce as much as possible the number of incomparabilities, but not when it is not realistic. Then the procedure may be considered as fair. When, for a particular procedure, all the incomparabilities are systematically withdrawn the provided information can be more disputable.

Requisite 4: Different multicriteria methods request different additional information and operate different calculation procedures so that the solutions they propose can be different. It is therefore important to develop methods being *understandable* by the decision-makers. "Black box" procedures should be avoided.

Requisite 5: An appropriate procedure should not include technical parameters having no significance for the decision-maker. Such parameters would again induce "Black box" effects.

Requisite 6: An appropriate method should provide information on the *conflicting nature* of the criteria.

Requisite 7: Most of the multicriteria methods are allocating weights of relative importance to the criteria. These weights reflects a major part of the *"brain"* of the decision-maker. It is not easy to fix them. Usually the decision-makers strongly hesitate. An appropriate method should offer *sensitivity tools* to test easily different sets of weights.

The PROMETHEE methods and the associated GAIA visual interactive module are taking all these requisites into account. On the other hand some mathematical properties that multicriteria problems possibly enjoy can also be considered. See for instance [95]. Such properties related to the PROMETHEE methods have been analysed by [7] in a particularly interesting paper.

The next sections describe the PROMETHEE I and II rankings, the GAIA methods, as well as the PROMETHEE V and VI extensions of the methodology. The PROMETHEE III and IV extensions are not discussed here. Additional information can be found in [17]. Several actual applications of the PROMETHEE methodology are also mentioned in the list of references.

3. The PROMETHEE Preference Modelling Information

The PROMETHEE methods were designed to treat multicriteria problems of type (5.1) and their associated evaluation table.

The additional information requested to run PROMETHEE is particularly clear and understandable by both the analysts and the decision-makers. It consists of:

- Information between the criteria;
- Information within each criterion.

3.1 Information between the Criteria

Table 5.2 should be completed, with the understanding that the set $\{w_j, j = 1, 2, \ldots, k\}$ represents weights of relative importance of the different criteria. These weights are non-negative numbers, independent from the measurement

Table 5.2. Weights of relative importance.

$g_1(\cdot)$	$g_2(\cdot)$	$\ldots$	$g_j(\cdot)$	$\ldots$	$g_k(\cdot)$
w_1	w_2	$\ldots$	w_j	$\ldots$	w_k

units of the criteria. The higher the weight, the more important the criterion. There is no objection to consider normed weights, so that:

$$\sum_{j=1}^{k} w_j = 1. \tag{5.4}$$

In the PROMETHEE software PROMCALC and DECISION LAB, the user is allowed to introduce arbitrary numbers for the weights, making it easier to express the relative importance of the criteria. These numbers are then divided by their sum so that the weights are normed automatically.

Assessing weights to the criteria is not straightforward. It involves the priorities and perceptions of the decision-maker. The selection of the weights is his *space of freedom*. PROMCALC and DECISION LAB include several sensitivity tools to experience different set of weights in order to help to fix them.

3.2 Information within the Criteria

PROMETHEE is not allocating an intrinsic absolute utility to each alternative, neither globally, nor on each criterion. We strongly believe that the decision-makers are not proceeding that way. The preference structure of PROMETHEE is based on *pairwise comparisons*. In this case the deviation between the evaluations of two alternatives on a particular criterion is considered. For small deviations, the decision-maker will allocate a small preference to the best alternative and even possibly no preference if he considers that this deviation is negligible. The larger the deviation, the larger the preference. There is no objection to consider that these preferences are real numbers varying between 0 and 1. This means that for each criterion the decision-maker has in mind a function

$$P_j(a,b) = F_j\left[d_j(a,b)\right] \quad \forall a,b \in A, \tag{5.5}$$

where:

$$d_j(a,b) = g_j(a) - g_j(b) \tag{5.6}$$

and for which:

$$0 \le P_j(a,b) \le 1. \tag{5.7}$$

In case of a criterion to be maximised, this function is giving the preference of a over b for observed deviations between their evaluations on criterion $g_j(\cdot)$. It should have the following shape (see Figure 5.1). The preferences equals 0 when the deviations are negative.

The following property holds:

$$P_j(a,b) > 0 \;\Rightarrow\; P_j(b,a) = 0. \tag{5.8}$$

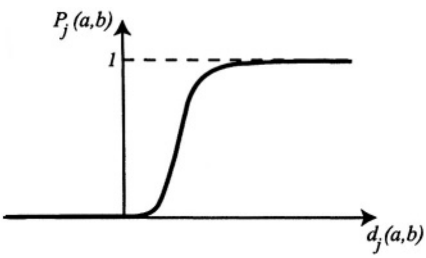


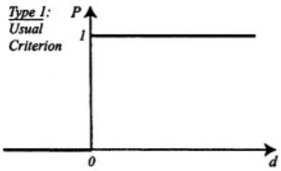
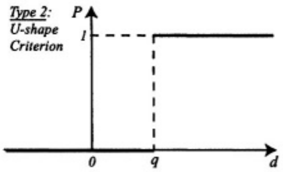
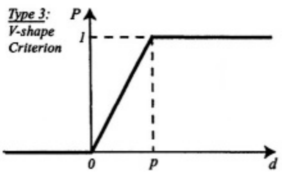
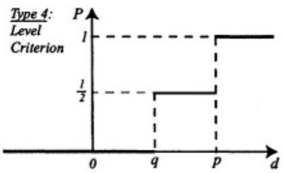
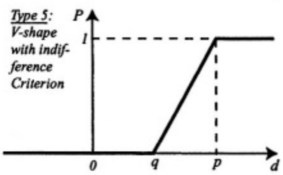
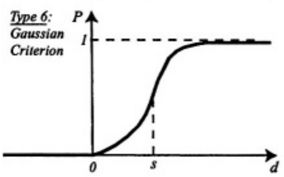
Figure 5.1. Preference function.

For criteria to be minimised, the preference function should be reversed or alternatively given by:

$$P_j(a,b) = F_j\left[-d_j(a,b)\right]. \tag{5.9}$$

We have called the pair $\{g_j(\cdot), P_j(a,b)\}$ the *generalised criterion* associated to criterion $g_j(\cdot)$. Such a generalised criterion has to be defined for each criterion. In order to facilitate the identification six types of particular preference functions have been proposed (see table 5.3).

Table 5.3. Types of generalised criteria ($P(d)$: Preference function).

Generalised criterion	Definition	Parameters to fix
	$P(d) = \begin{cases} 0 & d \leq 0 \\ 1 & d > 0 \end{cases}$	–
	$P(d) = \begin{cases} 0 & d \leq q \\ 1 & d > q \end{cases}$	q
	$P(d) = \begin{cases} 0 & d \leq 0 \\ \frac{d}{p} & 0 \leq d \leq p \\ 1 & d > p \end{cases}$	p
	$P(d) = \begin{cases} 0 & d \leq q \\ \frac{1}{2} & q < d \leq p \\ 1 & d > p \end{cases}$	p, q
	$P(d) = \begin{cases} 0 & d \leq q \\ \frac{d-q}{p-q} & q < d \leq p \\ 1 & d > p \end{cases}$	p, q
	$P(d) = \begin{cases} 0 & d \leq 0 \\ 1 - e^{-\frac{d^2}{2s^2}} & d > 0 \end{cases}$	s

In each case 0, 1 or 2 parameters have to be defined, their significance is clear:

q is a threshold or indifference;
p is a threshold of strict preference;
s is an intermediate value between q and p.

The q indifference threshold is the largest deviation which is considered as negligible by the decision maker, while the p preference threshold is the smallest deviation which is considered as sufficient to generate a full preference.

The identification of a generalised criterion is then limited to the selection of the appropriate parameters. It is an easy task.

The PROMCALC and DECISION LAB software are proposing these six shapes only. As far as we know they have been satisfactory in most real-world applications. However there is no objection to consider additional generalised criteria.

In case of type 5 a threshold of indifference q and a threshold of strict preference p have to be selected.

In case of a Gaussian criterion (type 6) the preference function remains increasing for all deviations and has no discontinuities, neither in its shape, nor in its derivatives. A parameter s has to be selected, it defines the inflection point of the preference function. We then recommend to determine first a q and a p and to fix s in between. If s is close to q the preferences will be reinforced for small deviations, while close to p they will be softened.

As soon as the evaluation table $\{g_j(\cdot)\}$ is given, and the weights w_j and the generalised criteria $\{g_j(\cdot), P_j(a,b)\}$ are defined for $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, k$, the PROMETHEE procedure can be applied.

4. The PROMETHEE I and II Rankings

The PROMETHEE procedure is based on pairwise comparisons (cfr. [8]–[16], [59], [60]). Let us first define aggregated preference indices and outranking flows.

4.1 Aggregated Preference Indices

Let $a, b \in A$, and let:

$$\begin{cases} \pi(a,b) = \sum_{j=1}^{k} P_j(a,b) w_j, \\ \pi(b,a) = \sum_{j=1}^{k} P_j(b,a) w_j. \end{cases} \tag{5.10}$$

$\pi(a,b)$ is expressing with which degree a is preferred to b over all the criteria and $\pi(b,a)$ how b is preferred to a. In most of the cases there are criteria for which a is better than b, and criteria for which b is better than a, consequently $\pi(a,b)$ and $\pi(b,a)$ are usually positive. The following properties hold for all $(a,b) \in A$.

$$\begin{cases} \pi(a,a) = 0, \\ 0 \leq \pi(a,b) \leq 1, \\ 0 \leq \pi(b,a) \leq 1, \\ 0 \leq \pi(a,b) + \pi(b,a) \leq 1. \end{cases} \tag{5.11}$$

It is clear that:

$$\begin{cases} \pi(a,b) \sim 0 \text{ implies a weak global preference of } a \text{ over } b, \\ \pi(a,b) \sim 1 \text{ implies a strong global preference of } a \text{ over } b. \end{cases} \tag{5.12}$$

As soon as $\pi(a,b)$ and $\pi(b,a)$ are computed for each pair of alternatives of A, a complete valued outranking graph, including two arcs between each pair of nodes, is obtained (see Figure 5.2).

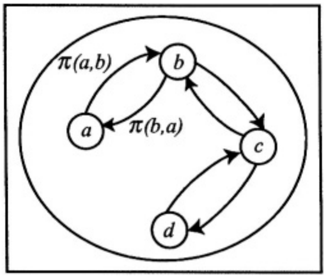


Figure 5.2. Valued outranking graph.

4.2 Outranking Flows

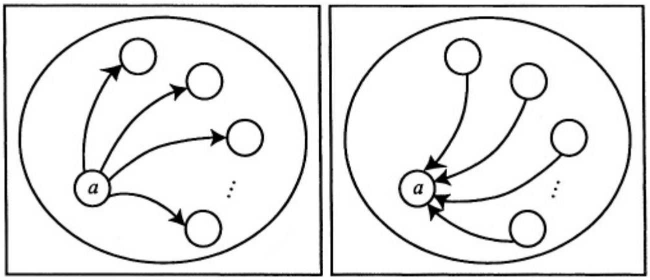
Each alternative a is facing $(n-1)$ other alternatives in A. Let us define the two following outranking flows:

- *the positive outranking flow:*

$$\phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a,x), \tag{5.13}$$

- *the negative outranking flow:*

$$\phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x,a). \tag{5.14}$$



(a) The $\phi^+(a)$ outranking flow. (b) The $\phi^-(a)$ outranking flow.

Figure 5.3. The PROMETHEE outranking flows.

The positive outranking flow expresses how an alternative a is *outranking* all the others. It is its *power, its outranking character.* The higher $\phi^+(a)$, the better the alternative (see Figure 5.3(a)).

The negative outranking flow expresses how an alternative a is *outranked* by all the others. It is its *weakness, its outranked character.* The lower $\phi^-(a)$ the better the alternative (see Figure 5.3(b)).

4.3 The PROMETHEE I Partial Ranking

The PROMETHEE I partial ranking (P^I, I^I, R^I) is obtained from the positive and the negative outranking flows. Both flows do not usually induce the same rankings. PROMETHEE I is their intersection.

$$\begin{cases} aP^Ib & \text{iff} & \begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b); \end{cases} \\ aI^Ib & \text{iff} & \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b); \\ aR^Ib & \text{iff} & \begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) > \phi^-(b), \text{ or} \\ \phi^+(a) < \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b); \end{cases} \end{cases} \tag{5.15}$$

where P^I, I^I, R^I respectively stand for preference, indifference and incomparability.

When aP^Ib, a higher power of a is associated to a lower weakness of a with regard to b. The information of both outranking flows is consistent and may therefore be considered as sure.

When aI^Ib, both positive and negative flows are equal.

When aR^Ib, a higher power of one alternative is associated to a lower weakness of the other. This often happens when a is good on a set of criteria on which b is weak and reversely b is good on some other criteria on which a is weak. In such a case the information provided by both flows is not consistent. It seems then reasonable to be careful and to consider both alternatives as incomparable. The PROMETHEE I ranking is prudent: it will not decide which action is best in such cases. It is up to the decision-maker to take his responsibility.

4.4 The PROMETHEE II Complete Ranking

PROMETHEE II consists of the (P^{II}, I^{II}) complete ranking. It is often the case that the decision-maker requests a complete ranking. The *net outranking flow* can then be considered.

$$\phi(a) = \phi^+(a) - \phi^-(a). \tag{5.16}$$

It is the balance between the positive and the negative outranking flows. The higher the net flow, the better the alternative, so that:

$$\begin{cases} aP^{II}b & \text{iff} \quad \phi(a) > \phi(b), \\ aI^{II}b & \text{iff} \quad \phi(a) = \phi(b). \end{cases} \tag{5.17}$$

When PROMETHEE II is considered, all the alternatives are comparable. No incomparabilities remain, but the resulting information can be more disputable because more information gets lost by considering the difference (5.16).

The following properties hold:

$$\begin{cases} -1 \leq \phi(a) \leq 1, \\ \sum_{x \in A} \phi(a) = 0. \end{cases} \tag{5.18}$$

When $\phi(a) > 0$, a is more outranking all the alternatives on all the criteria, when $\phi(a) < 0$ it is more outranked.

In real-world applications, we recommend to both the analysts and the decision-makers to consider both PROMETHEE I and PROMETHEE II. The complete ranking is easy to use, but the analysis of the incomparabilities often helps to finalise a proper decision.

As the net flow $\phi(\cdot)$ provides a complete ranking, it may be compared with a utility function. One advantage of $\phi(\cdot)$ is that it is built on clear and simple preference information (weights and preferences functions) and that it does rely on comparative statements rather than absolute statements.

4.5 The Profiles of the Alternatives

According to the definition of the positive and the negative outranking flows (5.13) and (5.14) and of the aggregated indices (5.10), we have:

$$\phi(a) = \phi^+(a) - \phi^-(a) = \frac{1}{n-1} \sum_{j=1}^{k} \sum_{x \in A} \left[P_j(a,x) - P_j(x,a)\right] w_j. \tag{5.19}$$

Consequently,

$$\phi(a) = \sum_{j=1}^{k} \phi_j(a) w_j \tag{5.20}$$

if

$$\phi_j(a) = \frac{1}{n-1} \sum_{x \in A} \left[P_j(a,x) - P_j(x,a) \right]. \tag{5.21}$$

$\phi_j(a)$ is the single criterion net flow obtained when only criterion $g_j(\cdot)$ is considered (100% of the total weight is allocated to that criterion). It expresses how an alternative a is outranking $(\phi_j(a) > 0)$ or outranked $(\phi_j(a) < 0)$ by all the other alternatives on criterion $g_j(\cdot)$.

The profile of an alternative consists of the set of all the single criterion net flows: $\phi_j(a), j = 1, 2, \ldots, k$.

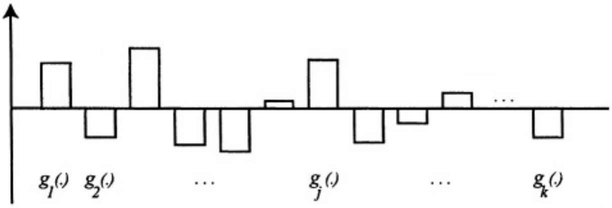


Figure 5.4. Profile of an alternative.

The profiles of the alternatives are particularly useful to appreciate their *"quality"* on the different criteria. It is extensively used by decision-makers to finalise their appreciation.

According to (5.20), we observe that the global net flow of an alternative is the scalar product between the vector of the weights and the profile vector of this alternative. This property will be extensively used when building up the GAIA plane.

5. The GAIA Visual Interactive Module

Let us first consider the matrix $M(n \times k)$ of the single criterion net flows of all the alternatives as defined in (5.21).

5.1 The GAIA Plane

The information included in matrix *M* is more extensive than the one in the evaluation table 5.1, because the degrees of preference given by the generalised criteria are taken into account in *M*. Moreover the $g_j(a_i)$ are expressed on their own scale, while the $\phi_j(a_i)$ are dimensionless. In addition, let us observe, that *M* is not depending on the weights of the criteria.

Table 5.4. Single criterion net flows.

	$\phi_1(\cdot)$	$\phi_2(\cdot)$	$\dots$	$\phi_j(\cdot)$	$\dots$	$\phi_k(\cdot)$
a_1	$\phi_1(a_1)$	$\phi_2(a_1)$	$\dots$	$\phi_j(a_1)$	$\dots$	$\phi_k(a_1)$
a_2	$\phi_1(a_2)$	$\phi_2(a_2)$	$\dots$	$\phi_j(a_2)$	$\dots$	$\phi_k(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	$\phi_1(a_i)$	$\phi_2(a_i)$	$\dots$	$\phi_j(a_i)$	$\dots$	$\phi_k(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_n	$\phi_1(a_n)$	$\phi_2(a_n)$	$\dots$	$\phi_j(a_n)$	$\dots$	$\phi_k(a_n)$

Consequently the set of the n alternatives can be represented as a cloud of n points in a k-dimensional space. According to (5.18) this cloud is centered at the origin. As the number of criteria is usually larger than two, it is impossible to obtain a clear view of the relative position of the points with regard to the criteria. We therefore project the information included in the k-dimensional space on a plane. Let us project not only the points representing the alternatives but also the unit vectors of the coordinate-axes representing the criteria. We then obtain:

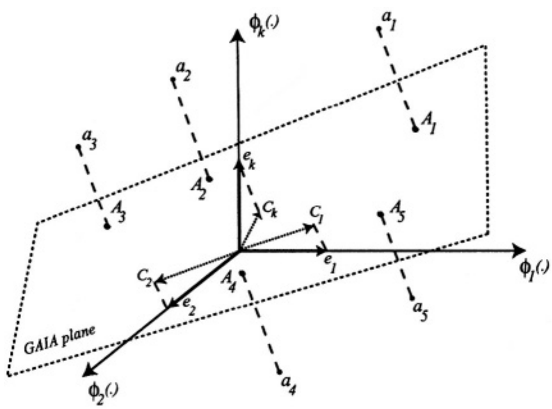


Figure 5.5. Projection on the GAIA plane.

The GAIA *plane* is the plane for which as much information as possible is preserved after projection. According to the *principal components analysis* technique it is defined by the two eigenvectors corresponding to the two largest eigenvalues of the covariance matrix $M'M$ of the single criterion net flows.

Of course some information get lost after projection. The GAIA plane is a *meta model* (a model of a model). Let δ be the quantity of information preserved.

In most applications we have treated so far δ was larger than 60% and in many cases larger than 80%. This means that the information provided by the GAIA plane is rather reliable. This information is quite rich, it helps to understand the structure of a multicriteria problem.

5.2 Graphical Display of the Alternatives and of the Criteria

Let $(A_1, A_2, \ldots, A_i, \ldots, A_n)$ be the projections of the n points representing the alternatives and let $(C_1, C_2, \ldots, C_j, \ldots, C_k)$ be the projections of the k unit vectors of the coordinates axes of $\mathbb{R}^k$ representing the criteria. We then obtain a GAIA plane of the following type:

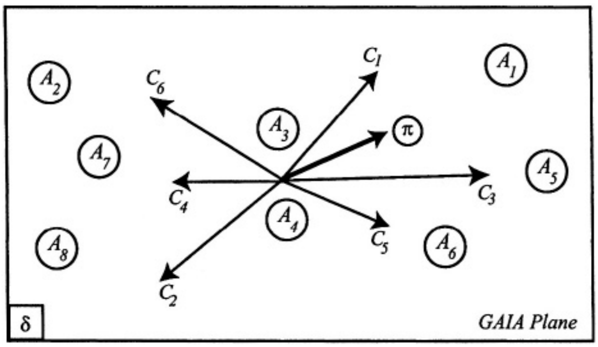


Figure 5.6. Alternatives and criteria in the GAIA plane.

Then the following properties hold (see [59] and [16]) provided that δ is sufficiently high:

P1: The longer a criterion axis in the GAIA plane, the more discriminating this criterion.

P2: Criteria expressing similar preferences are represented by axes oriented in approximatively the same direction.

P3: Criteria expressing conflicting preferences are oriented in opposite directions.

P4: Criteria that are not related to each others in terms of preferences are represented by orthogonal axes.

P5: Similar alternatives are represented by points located close to each other.

P6: Alternatives being good on a particular criterion are represented by points located in the direction of the corresponding criterion axis.

On the example of Figure 5.6, we observe:

- That the criteria $g_1(\cdot)$ and $g_3(\cdot)$ are expressing similar preferences and that the alternatives a_1 and a_5 are rather good on these criteria.
- That the criteria $g_6(\cdot)$ and $g_4(\cdot)$ are also expressing similar preferences and that the alternatives a_2, a_7, and a_8 are rather good on them.
- That the criteria $g_2(\cdot)$ and $g_5(\cdot)$ are rather independent
- That the criteria $g_1(\cdot)$ and $g_3(\cdot)$ are strongly conflicting with the criteria $g_4(\cdot)$ and $g_2(\cdot)$
- That the alternatives a_1, a_5 and a_6 are rather good on the criteria $g_1(\cdot)$, $g_3(\cdot)$ and $g_5(\cdot)$
- That the alternatives a_2, a_7 and a_8 are rather good on the criteria $g_6(\cdot)$, $g_4(\cdot)$ and $g_2(\cdot)$
- That the alternatives a_3 and a_4 are never good, never bad on all the criteria,
- ...

Although the GAIA plane includes only a percentage δ of the total information, it provides a powerful graphical visualisation tool for the analysis of a multicriteria problem. The discriminating power of the criteria, the conflicting aspects, as well as the "quality" of each alternative on the different criteria are becoming particularly clear.

5.3 The PROMETHEE Decision Stick. The PROMETHEE Decision Axis

Let us now introduce the impact of the weights in the GAIA plane. The vector of the weights is obviously also a vector of $\mathbb{R}^k$. According to (5.20), the PROMETHEE net flow of an alternative a_i is the scalar product between the vector of its single criterion net flows and the vector of the weights:

$$\begin{aligned} a_i : \quad & (\phi_1(a_i), \phi_2(a_i), \ldots, \phi_j(a_i), \ldots, \phi_k(a_i)), \\ w : \quad & (w_1, w_2, \ldots, w_j, \ldots, w_k). \end{aligned} \tag{5.22}$$

This also means that the PROMETHEE net flow of a_i is the projection of the vector of its single criterion net flows on w. Consequently, the relative positions of the projections of all the alternatives on w provides the PROMETHEE II ranking.

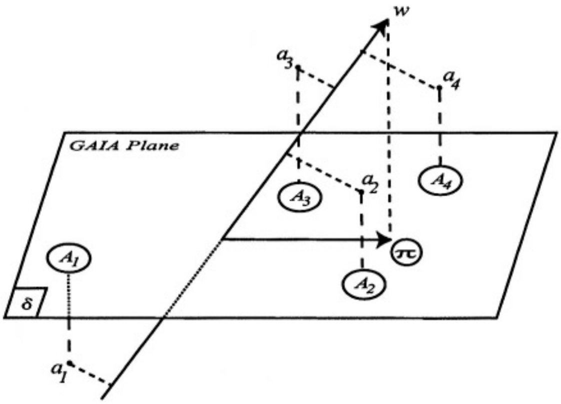


Figure 5.7. PROMETHEE II ranking. PROMETHEE decision axis and stick.

Clearly the vector w plays a crucial role. It can be represented in the GAIA plane by the projection of the unit vector of the weights. Let π be this projection, and let us call π the *PROMETHEE decision axis.*

On the example of Figure 5.7, the PROMETHEE ranking is: $a_4 \succ a_3 \succ a_2 \succ a_1$. A realistic view of this ranking is given in the GAIA plane although some inconsistencies due to the projection can possibly occur.

If all the weights are concentrated on one criterion, it is clear that the PROMETHEE decision axis will coincide with the axis of this criterion in the GAIA plane. Both axes are then the projection of a coordinate unit vector of $\mathbb{R}^k$. When the weights are distributed over all the criteria, the PROMETHEE decision axis appears as a weighted resultant of all the criterion axes $(C_1, C_2, \ldots, C_j, \ldots, C_k)$.

If π is long, the PROMETHEE decision axis has a strong decision power and the decision-maker is invited to select alternatives as far as possible in its direction.

If π is short, the PROMETHEE decision axis has no strong decision power. It means, according to the weights, that the criteria are strongly conflicting and that the selection of a good compromise is a hard problem.

When the weights are modified, the positions of the alternatives and of the criteria remain unchanged in the GAIA plane. The weight vector appears as a *decision stick* that the decision-maker can move according to his preferences in favour of particular criteria. When a sensitivity analysis is applied by modifying the weights, the PROMETHEE decision stick (w) and the PROMETHEE decision axis (π) are moving in such a way that the consequences for decision-making are easily observed in the GAIA plane (see Figure 5.8).

Decision-making for multicriteria problems appears, thanks to this methodology, as a piloting problem. Piloting the decision stick over the GAIA plane.

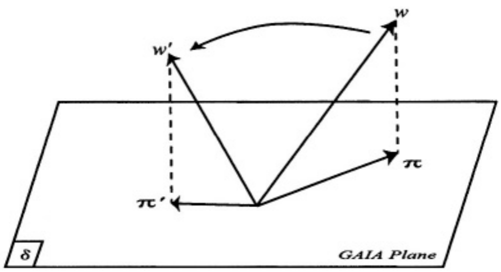


Figure 5.8. Piloting the PROMETHEE decision stick.

The PROMETHEE decision stick and the PROMETHEE decision axis provide a strong sensitivity analysis tool. Before finalising a decision we recommend to the decision-maker to simulate different weight distributions. In each case the situation can easily be appreciated in the GAIA plane, the recommended alternatives are located in the direction of the decision axis. As the alternatives and the criteria remain unchanged when the PROMETHEE decision stick is moving, the sensitivity analysis is particularly easy to manage. Piloting the decision stick is instantaneously operated by the PROMCALC and the DECISION LAB softwares. The process is displayed graphically so that the results are easy to appreciate.

6. The PROMETHEE VI Sensitivity Tool (The "Human Brain")

The PROMETHEE VI module provides the decision-maker with additional information on his own personal view of his multicriteria problem. It allows to appreciate whether the problem is *hard or soft* according to his personal opinion.

It is obvious that the distribution of the weights plays an important role in all multicriteria problems. As soon as the weights are fixed, a final ranking is proposed by PROMETHEE II. In most of the cases the decision-maker is hesitating to allocate immediately precise values of the weights. His hesitation is due to several factors such as *indetermination, imprecision, uncertainty, lack of control, ...* on the real-world situation.

However the decision-maker has usually in mind some order of magnitude on the weights, so that, despite his hesitations, he is able to give some intervals including their correct values. Let these intervals be:

$$w_j^- \leq w_j \leq w_j^+, \; j = 1, \dots, k. \tag{5.23}$$

Let us then consider the set of all the extreme points of the unit vectors associated to all allowable weights. This set is limiting an area on the unit hypersphere in $\mathbb{R}^k$. Let us project this area on the GAIA plane and let us call (*HB*) (*"Human Brain"*) the obtained projection. Obviously (*HB*) is the area including all the extreme points of the PROMETHEE decision axis (π) for all allowable weights.

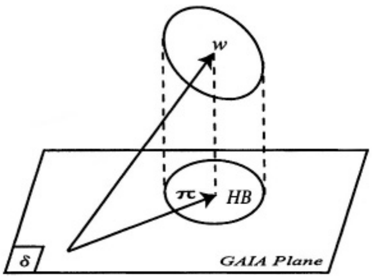
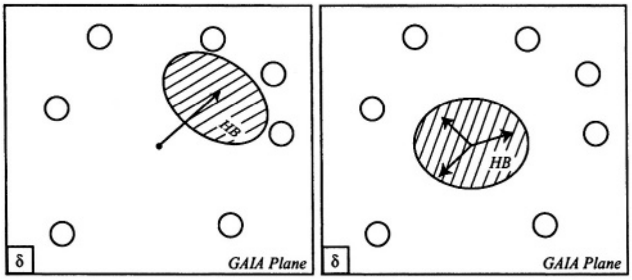


Figure 5.9. "Human Brain".

Two particular situations can occur:

S1: (*HB*) does not include the origin of the GAIA plane. In this case, when the weights are modified, the PROMETHEE decision axis (π) remains globally oriented in the same direction and all alternatives located in this direction are good. The multicriteria problem is rather easy to solve, it is a *soft* problem.



(a) Soft problem. (b) Hard problem.

Figure 5.10. Two types of decision problems.

S2: Reversely if (*HB*) is including the origin, the PROMETHEE decision axis (π) can take any orientation. In this case compromise solutions can be possibly obtained in all directions. It is then actually difficult to make a final decision. According to his preferences and his hesitations, the decision-maker is facing a *hard* problem.

In most of the practical applications treated so far, the problems appeared to be rather soft and not too hard. This means that most multicriteria problems offer at the same time good compromises and bad solutions. PROMETHEE allows to select the good ones.

7. PROMETHEE V: MCDA under Constraints

PROMETHEE I and II are appropriate to select one alternative. However in some applications a subset of alternatives must be identified, given a set of constraints. PROMETHEE V is extending the PROMETHEE methods to that particular case. (see [13]).

Let $\{a_i, i = 1, 2, \ldots, n\}$ be the set of possible alternatives and let us associate the following boolean variables to them:

$$x_i = \begin{cases} 1 & \text{if } a_i \text{ is selected,} \\ 0 & \text{if not.} \end{cases} \tag{5.24}$$

The PROMETHEE V procedure consists of the two following steps:

STEP 1: The multicriteria problem is first considered without constraints. The PROMETHEE II ranking is obtained for which the net flows $\{\phi(a_i), i = 1, 2, \ldots, n\}$ have been computed.

STEP 2: The following {0,1} linear program is then considered in order to take into account the additional constraints.

$$\max \left\{ \sum_{i=1}^{k} \phi(a_i) x_i \right\} \tag{5.25}$$

$$\sum_{i=1}^{n} \lambda_{p,i} x_i \sim \beta_p \quad p = 1, 2, \ldots, P \tag{5.26}$$

$$x_i \in \{0, 1\} \quad i = 1, 2, \ldots, n, \tag{5.27}$$

where ~ holds for =, $\geq$ or $\leq$. The coefficients of the objective function (5.25) are the net outranking flows. The higher the net flow, the better the alternative. The purpose of the {0,1} linear program is to select alternatives collecting as much net flow as possible and taking the constraints into account.

The constraints (5.26) can include cardinality, budget, return, investment, marketing,... constraints. They can be related to all the alternatives or possibly to some clusters.

After having solved the {0,1} linear program, a subset of alternatives satisfying the constraints and providing as much net flow as possible is obtained. Classical 0-1 linear programming procedures may be used.

The PROMCALC software includes this PROMETHEE V procedure.

8. The PROMETHEE GDSS Procedure

The PROMETHEE Group Decision Support System has been developed to provide decision aid to a group of decision-makers $(DM_1), (DM_2), \ldots, (DM_r)$, $\ldots (DM_R)$ (see [54]). It has been designed to be used in a GDSS room including a PC, a printer and a video projector for the facilitator, and R working stations for the DM's. Each working station includes room for a DM (and possibly a collaborator), a PC and Tel/Fax so that the DM's can possibly consult their business base. All the PC's are connected to the facilitator through a local network.

There is no objection to use the procedure in the framework of teleconference or video conference systems. It this case the DM's are not gathering in a GDSS room, they directly talk together through the computer network.

One iteration of the PROMETHEE GDSS procedure consists in 11 steps grouped in three phases:

- PHASE I: Generation of alternatives and criteria
- PHASE II: Individual evaluation by each *DM*
- PHASE III: Global evaluation by the group

Feedback is possible after each iteration for conflict resolution until a final consensus is reached.

8.1 PHASE I: Generation of Alternatives and Criteria

STEP 1: First contact Facilitator — DM's
The facilitator meets the DM's together or individually in order to enrich his knowledge of the problem. Usually this step takes place in the business base of each DM prior to the GDSS room session.

STEP 2: Problem description in the GDSS room
The facilitator describes the computer infrastructure, the PROMETHEE methodology, and introduces the problem.

STEP 3: Generation of alternatives
It is a computer step. Each DM implements possible alternatives including their extended description. For instance strategies, investments, locations, production schemes, marketing actions, ... depending on the problem.

STEP 4: Stable set of alternatives
All the proposed alternatives are collected and displayed by the facilitator one by one on the video-screen, anonymously or not. An open discussion takes place, alternatives are canceled, new ones are proposed, combined ones are merged, until a stable set of n alternatives $(a_1, a_2, \ldots, a_i, \ldots, a_n)$ is reached. This brainstorming procedure is extremely useful, it often generates alternatives that were unforeseen at the beginning.

STEP 5: Comments on the alternatives
It is again a computer step. Each DM implements his comments on all the alternatives. All these comments are collected and displayed by the facilitator. Nothing gets lost. Complete minutes can be printed at any time.

STEP 6: Stable set of evaluation criteria
The same procedure as for the alternatives is applied to define a stable set of evaluation criteria $(g_1(\cdot), g_2(\cdot), \ldots, g_j(\cdot), \ldots g_k(\cdot))$. Computer and open discussion activities are alternating. At the end the frame of an evaluation table (Type Table 5.1) is obtained. This frame consists in a $(n \times k)$ matrix. This ends the first phase. Feedbacks are already possible to be sure a stable set of alternatives and criteria is reached.

8.2 PHASE II: Individual Evaluation by each DM

Let us suppose that each DM has a decision power given by a non-negative weight $(\omega_r, r = 1, 2, \ldots, R)$ so that:

$$\sum_{r=1}^{R} \omega_r = 1. \tag{5.28}$$

STEP 7: Individual evaluation tables
The evaluation table $(n \times k)$ has to be completed by each DM. Some evaluation values are introduced in advance by the facilitator if there is an objective agreement on them (prices, volumes, budgets, ...). If not each DM is allowed to introduce his own values.

All the DM's implement the same $(n \times k)$ matrix, if some of them are not interested in particular criteria, they can simply allocate a zero weight to these criteria.

STEP 8: Additional PROMETHEE information
Each DM develops his own PROMETHEE-GAIA analysis. Assistance is given by the facilitator to provide the PROMETHEE additional information on the weights and the generalised criteria.

STEP 9: Individual PROMETHEE-GAIA analysis
The PROMETHEE I and II rankings, the profiles of the alternatives and the GAIA plane as well as the net flow vector $\phi_r(\cdot)$ are instantaneously obtained, so that each DM gets his own clear view of the problem.

8.3 PHASE III: Global Evaluation by the Group

STEP 10: Display of the individual investigations
The rankings and the GAIA plane of each DM are collected and displayed by the facilitator so that the group of all DM'S is informed of the potential conflicts.

STEP 11: Global evaluation
The net flow vectors $\{\phi_r(\cdot), r = 1, 2, \ldots, R\}$ of all the DM's are collected by the facilitator and put in a $(n \times R)$ matrix. It is a rather small matrix which is easy to analyse. Each criterion of this matrix expresses the point of view of a particular DM.

Each of these criteria has a weight ω_r and an associated generalised criterion of Type 3 $(p = 2)$ so that the preferences allocated to the deviations between the $\phi_i^r(\cdot)$ values will be proportional to these deviations.

A global PROMETHEE II ranking and the associated GAIA plane are then computed. As each criterion is representing a DM, the conflicts between them are clearly visualised in the GAIA plane. See for example Figure 5.11 where DM_3 is strongly in conflict with DM_1, DM_2 and DM_4.

The associated PROMETHEE decision axis (π) gives the direction in which to decide according to the weights allocated to the DM's.

If the conflicts are too sensitive the following feedbacks could be considered: Back to the weighting of the DM's. Back to the individual evaluations. Back to the set of criteria. Back to the set of alternatives. Back to the starting phase and to include an additional stakeholder ("DM") such as a social negotiator or a government mediator.

The whole procedure is summarised in the following scheme (Figure 5.12):

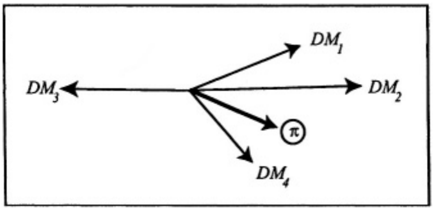


Figure 5.11. Conflict between DM's.

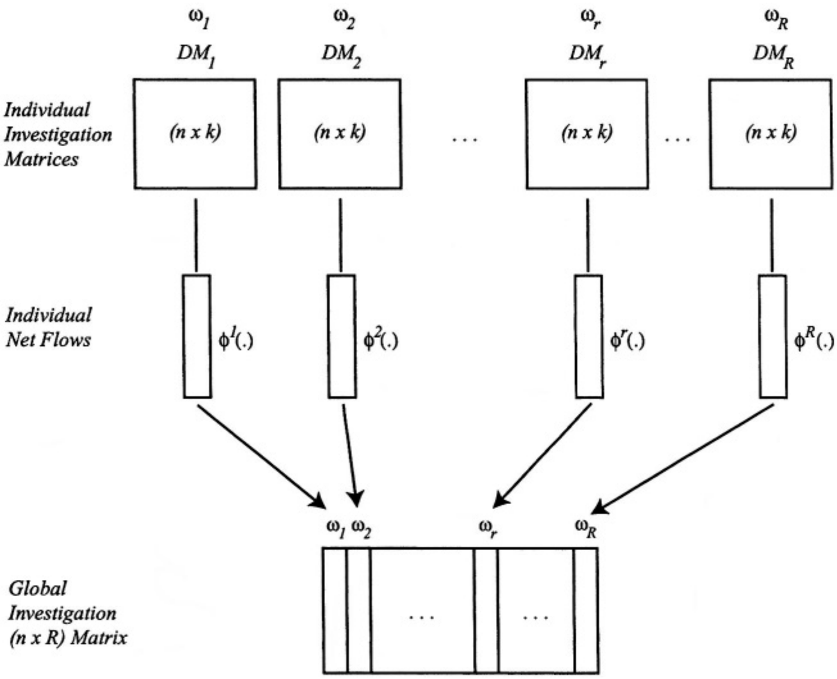


Figure 5.12. Overview PROMETHEE GDSS procedure.

9. The DECISION LAB Software

DECISION LAB is the current software implementation of the PROMETHEE and GAIA methods. It has been developed by the Canadian company Visual Decision, in cooperation with the authors. It replaces the PROMCALC software that the authors had previously developed.

DECISION LAB is a Windows application that uses a typical spreadsheet interface to manage the data of a multicriteria problem (Figure 5.13).

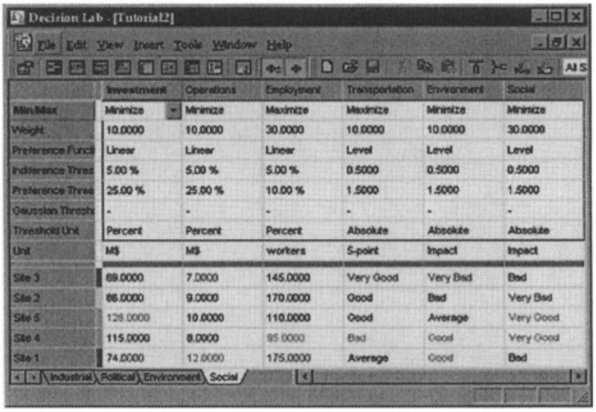


Figure 5.13. Main window.

All the data related to the PROMETHEE methods (evaluations, preference functions, weights, ...) can be easily defined and input by the user. Besides, DECISION LAB provides the user with additional features like the definition of qualitative criteria, the treatment of missing values in the multicriteria table or the definition of percentage (variable) thresholds in the preference functions. Categories of alternatives or criteria can also be defined to better identify subgroups of related items and to facilitate the analysis of the decision problem.

All the PROMETHEE and GAIA computations take place in real-time and any data modification is immediately reflected in the output windows. The PROMETHEE rankings, action profiles and GAIA plane are displayed in separate windows and can easily be compared (Figure 5.14).

Several interactive tools and displays are available for facilitating extensive sensitivity and robustness analyses. It is possible to compute weight stability intervals for individual criteria or categories of criteria. The walking weights display (Figure 5.15) can be used to interactively modify the weights of the criteria and immediately see the impact of the modification on the PROMETHEE II complete ranking and on the position of the decision axis in the GAIA plane. This can particularly useful when the decision-maker has no clear idea of the appropriate weighting of the criteria and wants to explore his space of freedom.

The PROMETHEE GDSS procedure is also integrated in DECISION LAB through the definition of several scenarios for a same decision problem. Scenarios share the same lists of alternatives and criteria but can include different preference functions, different sets of weights and even different evaluations for some criteria. Each scenario can be analysed separately using PROMETHEE and GAIA. But it is also possible to aggregate all the scenarios and to generate

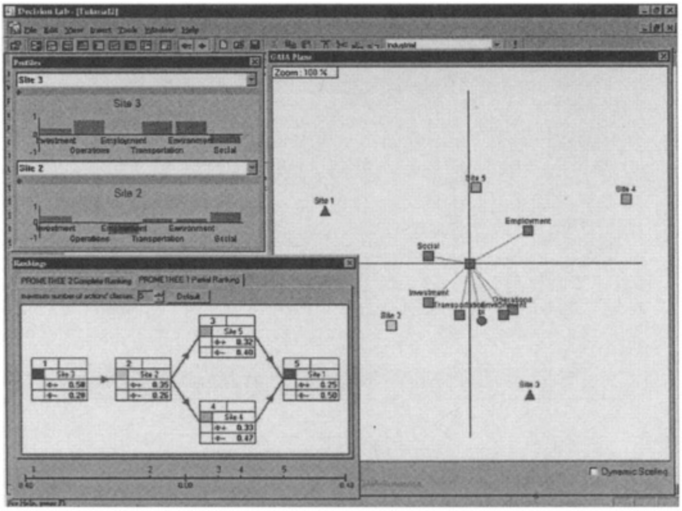


Figure 5.14. PROMETHEE rankings, action profiles, GAIA plane.

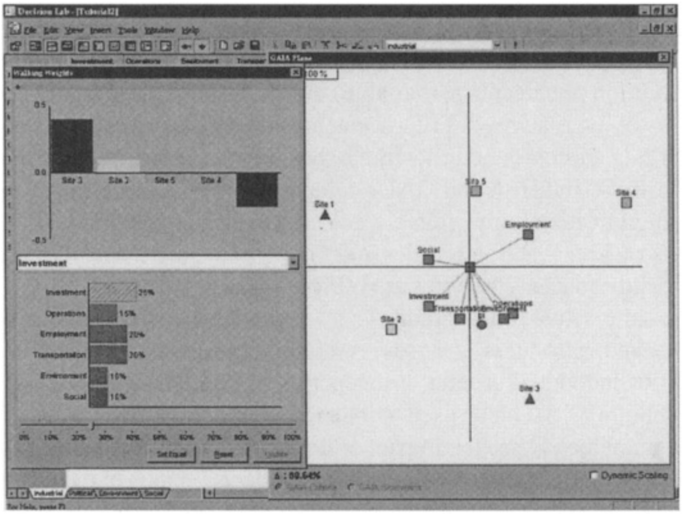


Figure 5.15. Walking weights.

the PROMETHEE group rankings as well as the group GAIA plane. Conflicts between decision-makers can easily be detected and analysed.

At the end of an analysis, the DECISION LAB report generator can produce tailor-made reports including the tables and graphics required by the user. The reports are in the html format so that they can easily be edited in a word processor or published on paper or on the web.

DECISION LAB can easily be interfaced with other programs like for instance databases. Its own interface can also be adapted to specific needs (special menus or displays, additional analysis modules, ...).

The next step in PROMETHEE software is a web-based implementation which is being developed under the Q-E-D name (Quantify-Evaluate-Decide). The Q-E-D demo web site will be launched during the spring 2003 at `http://www.q-e-d.be`.

Additional information on DECISION LAB can also be obtained on the following web sites: `http://www.idm-belgium.com` and `http://www.visualdecision.com`.

Acknowledgments

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