Spectral functions in NEdyson

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1 Introduction and Overview

The Non-equilibrium spectral function definition used here is given here.

$$\begin{aligned} A(\omega, t_{add}/2) &= -\frac{1}{\pi}\mathrm{Im}\int dt_{rel} e^{i\omega t_{rel}} G^R(t, t') \\ &= -\frac{1}{\pi}\mathrm{Im}\int dt_{rel} e^{i\omega t_{rel}} G^R\left(\frac{t_{add}+t_{rel}}{2}, \frac{t_{add}-t_{rel}}{2}\right) \end{aligned}$$

Here, $t_{add} = t + t'$ and $t_{rel} = t - t'$. We can now use the constraints on the retarded component of the Green's function to find bounds for the integration.

- $t > t' \implies t_{add} + t_{rel} \geq t_{add} - t_{rel} \implies t_{rel} \geq 0$
- $t' \geq 0 \implies t_{add} - t_{rel} \geq 0 \implies t_{rel} \leq t_{add}$
- $t \leq t_{max} \implies t_{add} + t_{rel} \leq 2t_{max} \implies t_{rel} \leq 2t_{max} - t_{add}$

These constraints give us that our evaluation of the spectral function should be

$$A(\omega, t_{add}/2) = -\frac{1}{\pi}\mathrm{Im}\int_0^{\min(t_{add}, 2t_{max}-t_{add})} dt_{rel} e^{i\omega t_{rel}} G^R\left(\frac{t_{add}+t_{rel}}{2}, \frac{t_{add}-t_{rel}}{2}\right)$$

This corresponds to starting the integration on the $t == t'$ line and moving diagonally downwards away from the origin. Doing this, there are two different types of stripes, those which have t_{add} even and those odd. This means we have twice as many spectral time points as we do greens function time points, which makes sense, since $t_{add} \in [0, 2t_{max}]$. We only have values of G^R at values where both arguments are integers, so when t_{add} is even, so must t_{rel}, this means that within the integral, $dt_{rel} = 2dt$ where dt is the timestep when solving for G. This leads us to the calculation that is done in the code, which is

$$A(\omega, t_{add}/2) = -\frac{2dt}{\pi}\mathrm{Im}\sum_{t_r=\mathrm{mod}(t_{add},2)}^{\min(t_{add}, 2t_{max}-t_{add})} e^{i\omega t_r} G^R\left(\frac{t_{add}+t_r}{2}, \frac{t_{add}-t_r}{2}\right) \tag{1}$$

Here we have the issue of a lack of data at both short and long times. The most data is available in the region around $t_{add} = t_{max}$. In order to help remedy this, there is a python script called GRexpand.py in the python folder. This implements the linear prediction code written by Joseph Kleinhenz, description of what this code does can be found in the notes folder.

The linear prediction code takes several parameters. First is –dir which points to the location of the Retarded green's function data the path must not contain the "_GR.dat" portion of the path. -p is a parameter that sets how far back the function looks when calculating the next value, typically around 10. Next is –nfit which is used to say how many of the final data points are used to generate the expansion coefficients. Typically set to around 150. Lastly –is used to tell the solver how far up to solve. for early times, there may be many high energy oscillations in the system that we don't want to use to expand outwards.

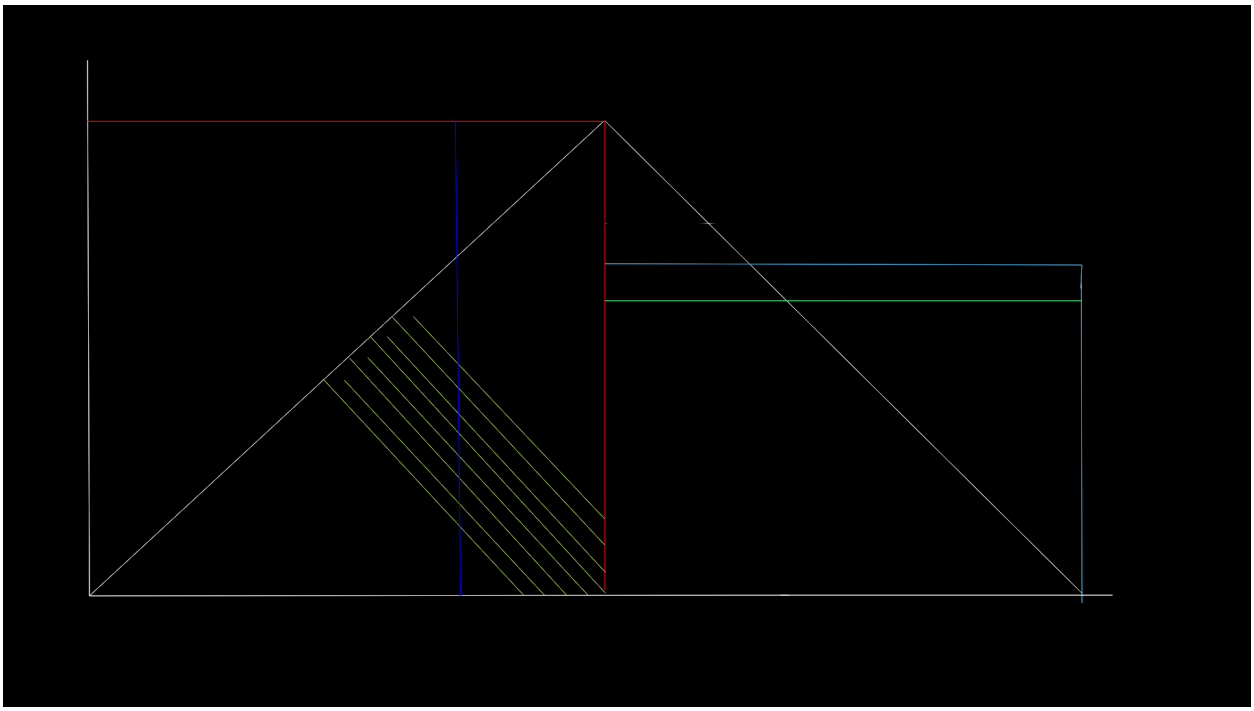

Figure 1: Red lines show t_{max}, green diagonal lines show spectral function calculation without linear prediction. Distance from dark blue to red is nfit, values under/on the diagonal within the light blue box are solved for except if in between green and blue, which is ntp.

To calculate a spectral function with linear prediction data, one can run calcA.cpp with a pointer to the location of the data files. This will check if there is an expansion data set or not and call the corresponding function based on the availability of expanded data.