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Internet Transport Economics: Model and Analysis

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***Abstract*— With the rise of video streaming and cloud services, the Internet has evolved into a content-centric service platform. Due to the best-effort service model of the Internet, the quality of service (QoS) of Internet services however cannot be guaranteed. Furthermore, characterizing QoS is challenging since it depends on the autonomous business decisions such as capacity planning, routing strategies and peering agreements of network providers. To quantify the QoS for Internet-based services, we regard the Internet infrastructure as a transport system for data packets and study the Internet ecosystem and the economics of transport services collectively provided by the autonomous network providers. In contrast to the traditional transport economics that studies the movement of people and goods over space and time, our focus in the *Internet transport economics* is the movement of streams of data packets that create information services. In particular, we model the supply of network capacities and demands of throughput driven by network protocols and establish a macroscopic network equilibrium under which both the end-to-end delays and drop rates of Internet routes can be derived. We show that this equilibrium solution always exists and its uniqueness can be guaranteed under various realistic scenarios. We analyze the impacts of user demands and resource capacities on the network equilibrium and provide implications of Netflix-Comcast type of peering on the QoS of users. We demonstrate that our framework can be used as a building block to understand the routing strategies under a Wardrop equilibrium and to enable further studies such as Internet peering and in-network caching.**

***Index Terms*— Internet transport, quality of service, network economics, fixed-point equilibrium.**

I. Introduction

THE Internet is the underpinning of today's digital economy. As a packet switching network, it provides transport/delivery services for data packets, through which innovative applications and online services across the globle are enabled. For example, Netflix [12] streams videos to residential homes over the Internet and now accounts for up to over one-third of peak U.S. downstream traffic. Such content services require low delay and high throughput and are quite sensitive to quality of service (QoS); however, the causes and effects of quality degradation are often difficult to diagnose and predict if occurs. In January 2014, the download speed of Netflix users behind the access provider Comcast was found to be dropped by 25%. Netflix blamed Comcast [9] for throttling traffic, but Comcast claimed that Netflix was sending high rates of traffic to intentionally congest the peering links. Although this peering dispute was resolved by Netflix paying Comcast to reach a premium peering agreement, this paid prioritization practice had raised concerns about net neutrality [39], whose impacts on the Internet and its evolution are largely unknown. Consequently, the U.S. Federal Communications Commission (FCC) decided to closely monitor but exempt these non-neutral practices from its recent Open Internet Order [1], because it lacked background "in the Internet traffic exchange context." Both the debate of net neutrality and peering disputes are issues of network economics, which boil down to understanding the policy implications on the QoS for Internet applications. Besides peering, QoS also influences many decisions of autonomous networks such as routing and in-network caching; and thus, plays a crucial role in understanding the formation of the current Internet and projecting its future evolution.

However, characterizing the QoS of an application is highly challenging, as it is influenced not only by the link capacities along the routes used for content delivery, but also by the amount of competing traffic across these links. The former can be regarded as the supply of network resources driven by the capacity plannings of *network providers* (NPs); while the latter can be regarded as the demand of data traffic influenced by the routing strategies of *content providers* (CPs). Furthermore, a comprehensive QoS model also need to capture the impacts of the inter-dependent interactions such as the peering agreements made among these autonomous business entities.

In this work, we take an economics approach to study the Internet ecosystem and its transport service. *By developing a holistic model with an analytical framework, we enable the analyses of network economics and performance such as the impacts of autonomous inter-dependent business decisions on the resulting QoS.* Unlike the traditional transport economics [34] that studies the movement of people and goods over space and time, Internet transport economics studies the movement of streams of data packets that creates information services. In particular, we consider the QoS metrics of end-to-end delay and drop rate of transport services and model their supply and demand based on the characteristics of network resources deployed by NPs and network protocols used by applications, respectively. We formulate and study the network equilibrium under which the QoS metrics are determined in a steady state. Our framework and fixed-point analysis generalize prior work [11], [15] that model the average rate of TCP flows, where neither the existence nor the uniqueness of solution has been established. Our analytical contributions include the following.

- We prove the existence of a general network equilibrium and derive the conditions for its uniqueness (Theorem 1).

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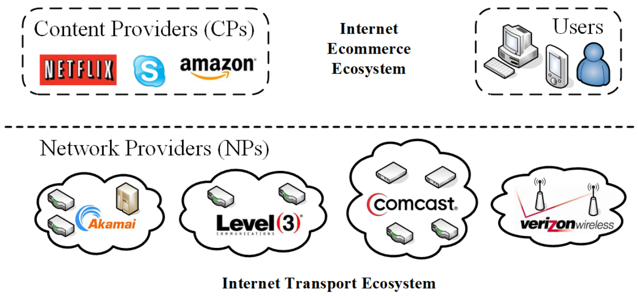

Fig. 1. A macroscopic model of the Internet transport ecosystem.

- We prove the uniqueness of network equilibrium for any linear topology or lossless network (Theorem 2 and 4).
- We analyze the impact of user population and network capacity on the network equilibrium (Theorem 3 and 5).
- We show that the equilibrium solution can be used as a building block to understand the delay-optimal routing strategies under a Wardrop equilibrium (Theorem 6).

Our analytical results provide new implications on the peering relationship between Netflix and Comcast on users' QoS too.

- Even if Comcast dedicates isolated resources for Netflix, the QoS will degrade as Netflix attracts more users.
- Under any fixed number of users, if Netflix increases capacities, it will increase its aggregate throughput but make downstream resources at Comcast more congested.
- Under a general lossless network, the increase of any resource capacity might not always improve the QoS on any route due to the inter-dependency among the routes.

We foresee that the fundamental physical model of the Internet transport ecosystem and the equilibrium solution proposed in this work will further enable broader analyses of the strategic routing, peering and pricing decisions of the autonomous networks and their impacts on the QoS of Internet services.

II. Internet Transport: Scope and Context

Originated as a communication network, today's Internet serves as an online platform that connects end-users to content providers (CPs)[1] forming the Internet ecommerce ecosystem shown at the top of Figure 1. Although the CPs that provide similar services compete for users in various geographical regions using regular business strategies such as service differentiation and pricing, a unique characteristic is that the delivery of online services is achieved via the *transport service* provided by the underlying Internet infrastructure, shown as the *Internet transport ecosystem* at the bottom of Figure 1. Since the transport service fundamentally determines the QoS of Internet applications and the quality of experiences (QoE) of users, which further affect the competitions in the Internet ecommerce ecosystem, in this work, we focus on the Internet transport ecosystem and aims to characterize the QoS of transport service provided. Although the Internet ecommerce ecosystem[2] may also affect the transport ecosystem, as itself needs sophisticated economic analyses on the interactions among end users, CPs and other service providers such as network pricing [5], [18], [22], [36] and service differentiation [17], [40], our study assumes that such factors are given exogenously and leaves the holistic analysis of both ecosystems as a direction of future extensions.

The Internet transport service is not provided or controlled by a single administrative domain, but a collection of *network providers* (NPs), including 1) access providers that connects to residential users, e.g., broadband and mobile providers like Comcast and Verizon, respectively, 2) transit providers that provide transit services for other networks, e.g., Level 3, and 3) content delivery networks (CDNs) that help CPs distribute contents to end-users. We define the NPs to be any autonomous entities that own network resources and control their resources for transporting data packets of digital contents. Thus, besides the autonomous systems (ASes) that join inter-domain routing via BGP, NPs also include parts of the major CPs who deploy infrastructures for content delivery such as Netflix.

Within the Internet transport ecosystem, the interconnection decisions of the NPs form the network topology of the Internet; the bilateral peering agreements between an NP and a CP or another NP determine which routing paths are available to any CP for content delivery; and the CPs' routing decisions determine how packets are routed towards end-users. Although all the interconnection, peering and routing decisions influence the QoS for traffic flows, we focus on the QoS under any joint routing strategies of the CPs using any fixed set of routes over any potential Internet topology, and we characterize QoS in terms of performance metrics such as delay, loss and throughput. In particular, our model includes a set $\mathcal{I}$ of CPs, and a set $\mathcal{K}$ of network resources provided by the NPs which forms a set $\mathcal{L}$ of routes, and we will derive the steady-state end-to-end delay D_l, gain G_l and input rate φ_l along each route $l \in \mathcal{L}$ and the corresponding delay d_k, gain g_k and input rate ϕ_k, i.e., the aggregated input from the routes that traverse the resource, at each resource $k \in \mathcal{K}$. Notice that loss rates can be directly derived from the gains and throughputs can be derived by multiplying the gains with the corresponding input rates. Table I summaries the main notation used in this paper.

III. Internet Transport: Supply and Demand

The Internet interconnects billions of hosts or end-systems by a network of communication links and packet switches. To understand the QoS of traffic flows, we take an economic perspective and study the Internet's steady-state under which the supply and demand of its transport service is in balance. In typical market economics, commodities comprise goods and services and steady-state is prescribed by a market equilibrium that determines the market price and transaction volume of the commodities produced and consumed. Instead of producing physical goods, the Internet provides transport services for data packets, whose demand and supply are driven by the QoS metrics of delay and loss rate, regarded as "congestion prices", resulting in equilibrium rates of traffic flows sent by CPs.

In this section, we first try to understand the commodities produced and consumed from the Internet transport ecosystem.

[1]We use the term content providers (CPs) in a broader sense to indicate any entity that provides online services by generating data traffic to end-users.

[2]For example, CPs' pricing may affect user demand, which further influences the capacity planning of network providers in the transport ecosystem.

TABLE I
SUMMARY OF NOTATION USED IN THIS ARTICLE

Notation	Semantics
$\mathcal{K}, k$	Set of network resources and a resource $k \in \mathcal{K}$
d_k, g_k	Delay and gain of network resource k
Ψ_k, Ψ_k^g	Delay and gain functions of resource k
λ_k, ϕ_k	Generic and aggregate input rate to resource k
μ_k, τ_k	Capacity and throughput of resource k
$\boldsymbol{d}, \boldsymbol{g}, \boldsymbol{\phi}$	Vectors of delay, gain and input rate of resources
$\mathcal{L}, l$	Set of network routes and a route $l \in \mathcal{L}$
D_l, G_l	End-to-end delay and gain of network route l
φ_l	Aggregate input rate to network route l
Φ_l	Aggregate input rate function of route l
$\boldsymbol{\varphi}, \boldsymbol{D}, \boldsymbol{G}$	Vectors of delay, gain and input rate of routes
$\mathcal{I}, i$	Set of content providers (CPs) and a CP $i \in \mathcal{I}$
s_{il}	Number of users of CP i served via route l
Λ_i	Average per-user sending rate function of CP i

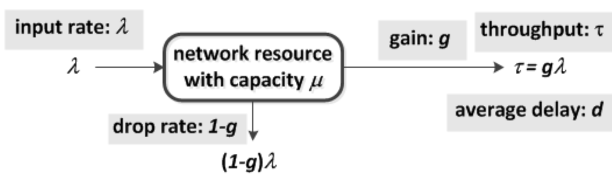

Fig. 2. Illustration of a generic network resource.

We start modeling the physical behaviors and mechanisms of network resources and protocols that fundamentally drive the supply and demand of the Internet transport services.

A. Supply-Side Model of Network Resources

The transportation of packets across the Internet is enabled by switching devices, e.g., routers and switches, and links, e.g., fiber optics. Despite the differences in technical characteristics, we conceptualize *network resource* to be any physical entity that enables the transportation of data packets from one physical location to another. Depending on the modeling granularity, a network resource may represent a physical link, an autonomous system or an ISP in practice.

Figure 2 illustrates the basic behavior of a network resource. Given a capacity μ and an input rate λ, a fraction of packets might be dropped due to congestion and buffer overflow. We denote the *gain* of transmission by g and the drop rate of packets by $1-g$; and therefore, define the rate of successful transmission or *throughput* by $\tau = g\lambda$. Each transmitted packet spends some time going through the network resource. We denote this average sojourn time or *delay* of packets by d.

We consider a generic network that comprises a set $\mathcal{K}$ of network resources. Since both delay and gain depend on the capacity of and the input rate to a network resource, we assume that the delay d_k and gain g_k of each resource $k \in \mathcal{K}$ are functions of its capacity μ_k and input rate λ_k as follows.

Assumption 1 (Delay and Gain Monotonicity): The delay and gain of any resource $k \in \mathcal{K}$ can be expressed as the functions

$$d_k = \Psi_k(\lambda_k, \mu_k) \quad \text{and} \quad g_k = \Psi_k^g(\lambda_k, \mu_k), \tag{1}$$

where $\Psi_k(\lambda_k, \mu_k)$ is non-decreasing in λ_k and non-increasing in μ_k; and $\Psi_k^g(\lambda_k, \mu_k)$ is non-increasing in λ_k and non-decreasing in μ_k. Both $\Psi_k(\lambda_k, \mu_k)$ and $\Psi_k^g(\lambda_k, \mu_k)$ are differentiable.

Assumption 1 states that when the capacities expand or the input rates reduce, network resources become less congested and thus provide lower delays and higher gains, and vice-versa. Notice that here we consider adding the capacity of any single resource rather than adding more separate resources, which will change the set of resource $\mathcal{K}$ accordingly in our model.

Since network elements often consist of buffers with queues, e.g., in the output ports of routers, queueing models can be used to characterize the performance of network resources. We illustrate some examples of such queueing models as follows.

The $M/G/1$ Model: Under a general service time of packets, the delay is specified by the Pollaczek-Khinchine formula as

$$d_k = \Psi_k(\lambda_k, \mu_k) = \frac{1}{\mu_k} + \frac{\mathbb{E}[S^2]}{2} \frac{\lambda_k}{1 - \lambda_k/\mu_k}, \quad \forall \lambda_k < \mu_k.$$

Notice that the capacity μ_k determines the mean service time $\mathbb{E}[S] = \mu_k^{-1}$, which is the lower-bound for delay. Furthermore, under any admissible input rate λ_k, the delay d_k depends on the second moment of service time $\mathbb{E}[S^2]$. Thus, the variability of service time under the queueing model can be used to model network resources that have different delay characteristics.

Given a fixed capacity μ_k and a desirable delay d_k, we can also derive the maximum amount of admissible rate as

$$\lambda_k^{max} = \frac{\left(d_k - \mu_k^{-1}\right)}{\mathbb{E}[S^2]/2 + \left(d_k - \mu_k^{-1}\right)\mu_k^{-1}}, \quad \forall d_k > \mu_k^{-1},$$

where a lower variability of $\mathbb{E}[S^2]$ implies that the network resource can accommodate higher rates under delay constraints.

One limitation of the $M/G/1$ model is that it assumes an infinite queue and cannot characterize the delays and losses under heavily loaded scenarios. A natural extension is the $M/G/1/K$ model that captures the buffer size or the applied active queue management mechanisms. Next, we illustrate an example where the service time follows a gamma distribution.

The $M/\Gamma/1/K$ Model: This queueing model assumes that the system can accommodate at most K packets at any time and the service time distribution is governed by a shape parameter α that determines its mean and second moment as

$$\mathbb{E}[S] = \mu^{-1} \quad \text{and} \quad \mathbb{E}[S^2] = \left(\alpha^{-1} + 1\right)\mu^{-2}.$$

To better illustrate the behaviors of delay and gain of a network resource k under the $M/\Gamma/1/K$ model, we normalize the capacity to be $\mu_k = 1$ without loss of generality.

Figure 3 plots the delay d_k and gain g_k as functions of the input rate λ_k in the left and right sub-figures, respectively. The shape parameter α is fixed to be 1, while $K = 5, 10$ and 20 in the three curves. In general, we observe that the

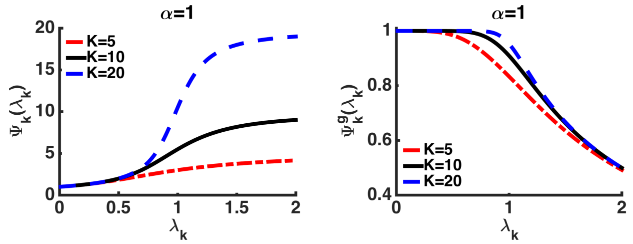

Fig. 3. Delay and gain of a network resource under varying input rates.

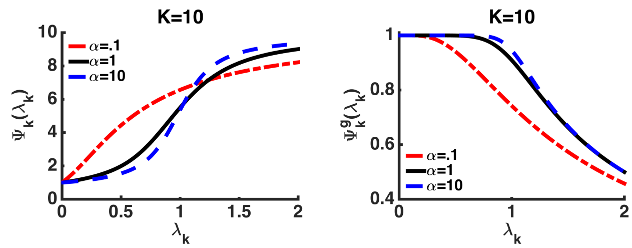

Fig. 4. Delay and gain of a network resource under varying input rates.

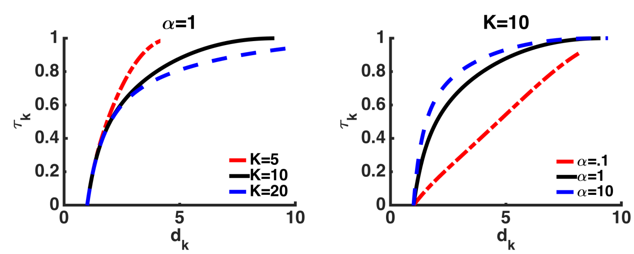

Fig. 5. Maximum achievable throughput of under varying desirable delay.

delay increases but the gain decreases with the input rate, satisfying Assumption 1. In particular, when the buffer size K increases, the network resource achieves a higher gain, as well as a higher throughout, at a cost of higher delay. This reflects the general tradeoff between delay and throughput when active queue management (AQM) mechanisms [10] are employed to limit the length of queue.

Similar to Figure 3, Figure 4 plots the delay d_k and gain g_k under $K = 10$, while $\alpha = 0.1, 1$ and 10 in the three curves. We observe that when α increases, the network resource achieves a higher gain; however, the delay increases with α only under a heavy traffic region where $\lambda_k > \mu_k$. The parameter α can be used to model various elasticity characteristics of network resources, i.e., how the throughput and gain of a resource respond to the changes in the external input rate.

Based on Assumption 1, the supply of service provided by any resource k can be characterized by the quality metrics d_k and g_k. However, both are not exogenous, but endogenously determined by the quantity metric λ_k. Because $\tau_k = g_k\lambda_k$, the realized service can be characterized by a pair (d_k, τ_k), capturing both the quality and quantity of the service.

Figure 5 plots the maximum throughput τ_k resource k can accommodate under a desirable delay d_k that varies along the x-axis. We fix α and vary K in the left sub-figure and fix K and vary α in the right sub-figure. Although a larger K reduces the drop rate in general, we observe in the left sub-figure that it increases the delay when accommodating the same amount of throughput and accommodates less throughput under the same desirable delay. In the right sub-figure, we observe that a larger α reduces the delay when accommodating the same amount of throughput and accommodates more throughput under the same delay. Notice that each curve shows the characteristics of the service provided by the resource, since each point (d_k, τ_k) represents a possible resulting service under certain input rate.

Before we close this subsection, we would like to emphasize that neither is the queueing model the only means to describe network resources, nor is it applied in a conventional manner. For example, the service time distribution in a queueing model is used under our context to capture the characteristics of a network resource in terms of its delay and gain in response to the input rates.

B. Demand-Side Model of Users and Application Protocols

Becuase the Internet transport services are based on the end-to-end routes that consist of multiple network resources, we define a route l to be a non-empty set $l \subseteq \mathcal{K}$ of network resources that are serially connected. Based on the additivity and multiplicative properties of delay and gain, we define the delay D_l and gain G_l of any route l as

$$D_l = \sum_{k \in l} d_k \quad \text{and} \quad G_l = \prod_{k \in l} g_k. \tag{2}$$

We denote the set of content providers (CPs) by $\mathcal{I}$ and the average sending rate of any CP $i \in \mathcal{I}$ to an active user by λ_i, which can be regarded as the user's demand for transport services. This demand is driven by end-users running various application and network protocols and is influenced by the service quality in terms of delay and gain of the routing paths. For example, the PFTK formula [27] characterizes the rate of a single TCP Reno flow as a function of delay and gain as

$$\lambda_i(D, G) \propto (2D)^{-1}\sqrt{\frac{3}{4(1-G)}} + o\left(\sqrt{\frac{1}{1-G}}\right), \tag{3}$$

which states that the flow rate is inversely proportional to the round-trip time, measured by $2D$, and the square root of drop rate, measured by $\sqrt{1-G}$. Under serve network congestion that induces high delay and drop rate, active users might become impatience and stop using the transport service. We define $\eta_i(D, G)$ to be the probability that an active user will still use the service under the delay D and gain G. For any CP $i \in \mathcal{I}$, we define the average per-user sending rate by

$$\Lambda_i(D, G) = \eta_i(D, G)\lambda_i(D, G). \tag{4}$$

Assumption 2 (Demand Rate Monotonicity): The average per-user sending rate $\Lambda_i(D, G)$ of any CP $i \in \mathcal{I}$ is a differentiable function, decreasing in D and non-decreasing in G.

Assumption 2 states that the demand for transport service will not increase if the delay or gain of the route deteriorates. Notice that although the demand in terms of users might not necessarily decrease if the taken end-to-end route is critical, e.g., no alternatives to reach the destination, the sending rate of a typical user generally decreases when the perform metrics D and G degrade, which is driven by the network protocols such as TCP that back off in the face of network congestion.

The Iso-Elastic Demand Model: Although Assumption 2 is very general, one neat model for the demand function can be

$$\Lambda_i(D, G) \propto D^{-\beta}(1-G)^{-\gamma},$$

where the parameters β and γ define the elasticity of demand rate Λ_i with respective to delay D and gain G, which capture the demand rate's sensitivity to the round-trip time and drop-rate, respectively. The PFTK formula (3) has shown that the elasticity parameters satisfy $\beta = 1$ and $\gamma = 1/2$ for an active TCP Reno flow. For network protocols that are less sensitive to delay or loss, e.g., UDP, the values of the corresponding parameters will be lower.[3] It is interesting to notice that from an economics point of view, an *elastic demand function* corresponds to high values of β and γ; however from a networking perspective, it models the *inelastic traffic* [33], e.g., live streaming, that is intolerable to delay and packet losses.

IV. Network Equilibrium

In the previous section, we interpret the demand of Internet transport as the sending rates of traffic originated from CPs, responding to the end-to-end route-level delay and loss, similar to the demand of commodities driven by their prices. However, the supply is manifested as the achieved throughput, influenced by the drop rates incurred at the individual resources along the routes and driven endogenously by the capacities of resources. In this section, we first define an equilibrium of a network system, under which both the supply and demand of transport services are balanced in a steady-state, and characterize such an equilibrium solution in terms of existence and uniqueness.

We denote the set of all feasible routes in the system as $\mathcal{L}$, which are determined by the collective interconnection decisions of ISPs with other ISPs and CPs and materialized via the BGP inter-domain routing protocol. Any route $l \in \mathcal{L}$ consists of an ordered sequence of network resources that lead to a number of end-users in a geographical region. For any CP $i \in \mathcal{I}$, we denote the number of end-users it serves via route $l \in \mathcal{L}$ by s_{il}. If a route l is not available to CP i, possibly due to its peering relationships with ISPs, or CP i chooses not to use route l due to performance reasons, s_{il} equals zero by definition without loss of generality. As a result, the aggregate sending rate from all CPs along route l to end-users can be defined as

$$\varphi_l = \Phi_l(D_l, G_l) = \sum_{i \in \mathcal{I}} s_{il}\Lambda_i(D_l, G_l), \tag{5}$$

which is a function of route l's service quality in terms of delay D_l and gain G_l. Given the rate φ_l of any route $l \in \mathcal{L}$, the input rates to any resource $k \in \mathcal{K}$ can be derived by

$$\phi_k = \sum_{l \ni k} \varphi_l \prod_{\kappa \in l(k)} g_\kappa, \tag{6}$$

[3]If the protocol does not respond to a metric, its parameter equals zero.

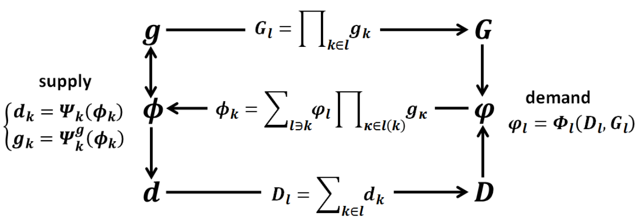

Fig. 6. An illustration of the network equilibrium.

where $l(k)$ defines the set of resources on route l before k. Equation (6) states that the aggregate input rate to any resource k equals the sum over the sending rates φ_l whose route utilizes resource k, i.e., $k \in l$, discounted by the gains of the resources $l(k)$ preceeding k along various routes. In particular, if $l(k)$ is empty, i.e., k is the first resource on route l, the nullary product equals the multiplicative identity 1 by convention.

So far a network system can be defined by a triple $(\mathcal{I}, \mathcal{K}, \mathcal{L})$ that describe the sets of CPs, network resources and routes. Although the capacity of deployed network resource μ_k might be influenced by other economic variants such as NPs' pricing and CPs' peering decisions, we do not model such strategic interactions explicitly and take the capacity planning outcome as exogenously given. Under a steady state, the rates ϕ_k and φ_l over the resources and routes determine and are influenced by the quality metrics (d_k, g_k) and (D_l, G_l) at the resource- and route-level, respectively. We denote $\boldsymbol{\phi}, \boldsymbol{d}$, and $\boldsymbol{g}$ as the vectors of resource-level metrics and $\boldsymbol{\varphi}, \boldsymbol{D}$ and $\boldsymbol{G}$ as the corresponding vectors at the route level. By using Equations (1), (2), (5) and (6), we can define an equilibrium of a network system $(\mathcal{I}, \mathcal{K}, \mathcal{L})$ as follows.

Definition 1 (Network Equilibrium): For any system $(\mathcal{I}, \mathcal{K}, \mathcal{L})$, a tuple $(\boldsymbol{\phi}, \boldsymbol{d}, \boldsymbol{g}, \boldsymbol{\varphi}, \boldsymbol{D}, \boldsymbol{G})$ is an equilibrium if and only if

$$\begin{cases} \varphi_l = \Phi_l(D_l, G_l) = \sum\limits_{i \in \mathcal{I}} s_{il}\Lambda_i(D_l, G_l), & \forall\ l \in \mathcal{L}. \\ (d_k, g_k) = \left(\Psi_k(\phi_k, \mu_k), \Psi^g_k(\phi_k, \mu_k)\right), & \forall\ k \in \mathcal{K}. \\ (D_l, G_l) = \left(\sum\limits_{k \in l} d_k, \prod\limits_{k \in l} g_k\right), & \forall\ l \in \mathcal{L}. \\ \phi_k = \sum\limits_{l \ni k} \varphi_l \prod\limits_{\kappa \in l(k)} g_\kappa, & \forall\ k \in \mathcal{K}. \end{cases} \tag{7}$$

The first two equations of Definition 1 specify the demand of transport services at the route level and the corresponding supply at the resource level, respectively. The last two equations specify the physics of delay, gain and aggregate input rate that links the metrics between the resource and route levels.

Figure 6 visualizes the complex relationship among different metrics under an equilibrium. The right shows that the demand $\boldsymbol{\varphi}$ is driven by the route level metrics $\boldsymbol{D}$ and $\boldsymbol{G}$; the left shows that the endogenously supplied quality metrics $\boldsymbol{d}$ and $\boldsymbol{g}$ are driven by the resource-level demand $\boldsymbol{\phi}$. In between the demand and supply, relationships of the gains, rates and delays at the resource and route levels are shown from top to bottom.

Notice that if the vector $\boldsymbol{\phi}$ under an equilibrium is known, all others can be uniquely determined. To compactly express equilibria in a vector form and characterize them, we define a vector $\Phi(\boldsymbol{\phi})$ of aggregate sending rates along all the routes as a function of $\boldsymbol{\phi}$, where each entry $\Phi_l(\boldsymbol{\phi})$ is defined as

$$\Phi_l(\boldsymbol{\phi}) = \sum_{i\in\mathcal{I}} s_{il}\Lambda_i\left(\sum_{k\in l}\Psi_k(\phi_k,\mu_k),\ \prod_{k\in l}\Psi^g_k(\phi_k,\mu_k)\right). \quad (8)$$

We also define an $|\mathcal{L}|\times|\mathcal{K}|$ matrix $H(\boldsymbol{\phi})$ as functions of $\boldsymbol{\phi}$, where each entry $H_{lk}(\boldsymbol{\phi})$ is the effective gain along route l before arriving resource k under $\boldsymbol{\phi}$, defined as

$$H_{lk}(\boldsymbol{\phi}) = \mathbb{1}_{\{k\in l\}}\prod_{\kappa\in l(k)}\Psi^g_\kappa(\phi_\kappa,\mu_\kappa). \quad (9)$$

The following result characterizes the existence and uniqueness of equilibrium based on the properties of $\Phi(\cdot)$ and $H(\cdot)$.

Theorem 1: Under Assumption 1 and 2, for any system $(\mathcal{I},\mathcal{K},\mathcal{L})$, there always exists an equilibrium that satisfies

$$\boldsymbol{\phi} = H(\boldsymbol{\phi})^T\Phi(\boldsymbol{\phi}). \quad (10)$$

Furthermore, let $\phi^{max}_k = \sum_{l\ni k}\Phi_l(0,1)$ for all $k\in\mathcal{K}$. The equilibrium is unique, if for all $\boldsymbol{\phi}\in\times_{k\in\mathcal{K}}[0,\phi^{max}_k]$

$$|\nabla_{\boldsymbol{\phi}}\Phi(\boldsymbol{\phi})H(\boldsymbol{\phi}) + \nabla_{\boldsymbol{\phi}}H(\boldsymbol{\phi})\Phi(\boldsymbol{\phi}) - I| \neq 0, \quad (11)$$

where ∇ and I are the gradient operator and identity matrix.

Proof of Theorem 1: $\Phi(\boldsymbol{\phi})$ is a vector of functions $\varphi_l(\boldsymbol{\phi})$, each of which is a composite of the first three equations in Definition 1. Therefore, the fourth equation can be written in a vector form as Equation (10) to define an equilibrium.

Because the maximum input rate to resource k is ϕ^{max}_k, any equilibrium $\boldsymbol{\phi}$ lies in the feasible domain $\times_{k\in\mathcal{K}}[0,\phi^{max}_k]$. To show the existence of equilibrium, let $F(\boldsymbol{\phi}) = H(\boldsymbol{\phi})^T\Phi(\boldsymbol{\phi})$. $F(\cdot)$ is a continuous mapping from the convex compact subset $\times_{k\in\mathcal{K}}[0,\phi^{max}_k]$ of a Euclidean space to itself; and therefore, by Brouwer fixed-point theorem [20], there always exists a fixed point that satisfies $\phi = F(\phi) = H(\boldsymbol{\phi})^T\Phi(\boldsymbol{\phi})$.

For any active resource k, $d_k > 0$ and the maximum rate ϕ^{max}_k cannot be achieved under an equilibrium, because any route $l\ni k$ has a positive delay D_l and by Assumption 2, its demand $\Lambda_i(D_l,G_l)$ will be lower than $\Lambda_i(0,G_l)$, which is lower than its contribution $\Lambda_i(0,1)$ to ϕ^{max}_k. As a result, any boudary value of $\times_{k\in\mathcal{K}}[0,\phi^{max}_k]$ cannot be an equilibrium. By the chain rule, the gradient of $F(\boldsymbol{\phi})$ can be written as $\nabla_{\boldsymbol{\phi}}F(\boldsymbol{\phi}) = \nabla_{\boldsymbol{\phi}}\Phi(\boldsymbol{\phi})H(\boldsymbol{\phi}) + \nabla_{\boldsymbol{\phi}}H(\boldsymbol{\phi})\Phi(\boldsymbol{\phi})$ and Equation (11) guarantees that $\mathbf{1}$ cannot be an eigenvalue of $\nabla_{\boldsymbol{\phi}}F(\boldsymbol{\phi})$. Finally, by using the Kellogg's fixed-point theorem [19], we conclude that the uniquess of equilibrium can be guaranteed by Equation (11) over the feasible domain $\times_{k\in\mathcal{K}}[0,\phi^{max}_k]$. ∎

Theorem 1 shows that any equilibrium in terms of $\boldsymbol{\phi}$ is a fixed-point solution of Equation (10) in a vector form and the existence of such a solution is always guaranteed. Although the uniqueness of equilibrium cannot be guaranteed, it provides a sufficient condition on the function $\Phi(\cdot)$ of the route-level rates and the matrix $H(\cdot)$ of the effective gains in the compact domain of possible input rates to the resources, because each ϕ^{max}_k defines the maximum input rate to resource k when the delays and gains along any route equals 0 and 1, respectively.

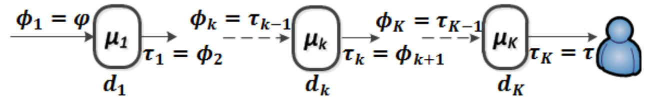

Fig. 7. An illustration of an independent route with a linear topology.

In the next two sections, we will discuss applications of the equilibrium framework to understand the impacts of the strategic behaviors of various parties, e.g., peering and routing decisions of CPs and capacity planning decisions of ISPs.

V. Linear Lossy Networks

In this section, we consider systems that consist of independent routes among which no resource is shared, i.e., $l\cap l' = \emptyset$ for any $l,l'\in\mathcal{L}$. Despite being a special case, it represents important real scenarios of service differentiation and pricing, under which ISPs dedicate isolated network resources to form service classes and CPs choose routes in the form of peering decisions, e.g., premium peering [6], [23], based on pricing.

When routes are independent, we can focus on a typical route which forms a linear topology of K resources from CPs to end-users as shown in Figure 7. We drop the subscript l and denote the number of end-users of CP i by s_i. We also denote the aggregate sending rate and throughput of the route under study by φ and τ. $\Phi(\boldsymbol{\phi})$ becomes a scalar function defined as

$$\Phi(\boldsymbol{\phi}) = \sum_{i\in\mathcal{I}} s_i\Lambda_i\left(\sum_{\kappa=1}^{K}\Psi_\kappa(\phi_\kappa,\mu_\kappa),\prod_{\kappa=1}^{K}\Psi^g_\kappa(\phi_\kappa,\mu_\kappa)\right).$$

We define the output rate of resource k as $\tau_k = \phi_k\Psi^g_k(\phi_k)$. Consequently, $\tau = \tau_K$ and the input rates to the individual resources satisfy $\phi_1 = \varphi$ and $\phi_{k+1} = \tau_k$ for all $k > 1$.

Corollary 1: When $K = 1$, there is always a unique equilibrium. When $K = 2$, the equilibrium is unique if

$$g_1\frac{\partial\Phi}{\partial\phi_1} + \frac{\partial\Psi^g_1}{\partial\phi_1}\Phi(\boldsymbol{\phi}) \neq \left(\frac{\partial\Phi}{\partial\phi_1} - 1\right)\left[g_1 - \left(\frac{\partial\Phi}{\partial\phi_2}\right)^{-1}\right] \quad (12)$$

for all $(\phi_1,\phi_2)\in[0,\phi^{max}_1]\times[0,\phi^{max}_2]$, where $g_1 = \Psi^g_1(\phi_1)$

$$\text{and}\quad \frac{\partial\Phi}{\partial\phi_k} = \sum_{i\in\mathcal{I}} s_i\left[\frac{\partial\Lambda_i}{\partial D}\frac{\partial\Psi_k}{\partial\phi_k} + \frac{\partial\Lambda_i}{\partial G}\frac{G}{g_k}\frac{\partial\Psi^g_k}{\partial\phi_k}\right]. \quad (13)$$

Proof of Corollary 1: For $K = 1$, $H(\boldsymbol{\phi}) = 1$ and $\Phi(\boldsymbol{\phi}) = \Phi(\phi_1) = \sum_{i\in\mathcal{I}} s_i\Lambda_i(\Psi_1(\phi_1,\mu_1),\Psi^g_1(\phi_1,\mu_1))$ and therefore,

$$\nabla_{\boldsymbol{\phi}}\Phi(\boldsymbol{\phi}) = \sum_{i\in\mathcal{I}} s_i\left(\frac{\partial\Lambda_i}{\partial D}\frac{\partial\Psi_1}{\partial\phi_1} + \frac{\partial\Lambda_i}{\partial G}\frac{\partial\Psi^g_1}{\partial\phi_1}\right).$$

By Assumption 1 and 2, $\partial\Lambda_i/\partial D < 0$, $\partial\Lambda_i/\partial G \geq 0$, $\partial\Psi_1/\partial\phi_1 \geq 0$ and $\partial\Psi^g_1/\partial\phi_1 \leq 0$, implying $\nabla_{\boldsymbol{\phi}}\Phi(\boldsymbol{\phi}) \leq 0$. Consequently, condition (11) becomes $|\nabla_{\boldsymbol{\phi}}\Phi(\boldsymbol{\phi}) - 1| \neq 0$, which always holds. For $K = 2$, $H(\boldsymbol{\phi}) = (1,\Psi^g_1(\phi_1,\mu_1)) = (1,g_1)$ and the sufficient condition (11) becomes

$$\left|\begin{bmatrix}\partial\Phi/\partial\phi_1\\ \partial\Phi/\partial\phi_2\end{bmatrix}[1,g_1] + \begin{bmatrix}0 & \partial\Psi^g_1/\partial\phi_1\\ 0 & 0\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right| \neq 0,$$

which can be further simplified to the condition (12). ∎

Corollary 1 states that if a route consists of a single resource, the uniqueness of equilibrium is guaranteed; however, when $K = 2$, the sufficient condition of (11) is reduced to (12). Notice that the left hand side of (12) can be written as $\partial\tau_1/\partial\phi_1$ or $\partial\phi_2/\partial\phi_1$, which captures the changes in the input rate ϕ_2 when ϕ_1 changes. Since $\partial\Phi(\boldsymbol{\phi})/\partial\phi_k$ is negative by Assumption 1 and 2, the right hand side of (12) is always negative; and therefore, condition (12) is satisfied if $\partial\tau_1/\partial\phi_1$ is non-negative. Although not assumed, it is a natural consequence if network resources are controlled in a *work conserving* manner.

Assumption 3 (Work Conservation): Given any required delay d, resource k can accommodate a maximum throughput $T_k(d)$, and $T_k(d)$ is continuous and non-decreasing in d. Under any input rate λ_k, the induced throughput satisfies

$$\tau_k \triangleq \lambda_k \Psi_k^g(\lambda_k, \mu_k) = \min\{\lambda_k, T_k(\Psi_k(\lambda_k, \mu_k))\}.$$

Lemma 1: Under Assumption 3, Assumption 1 implies that throughput $\tau_k = \lambda_k \Psi_k^g(\lambda_k, \mu_k)$ is non-decreasing in λ_k.

Proof of Lemma 1: By Assumption 1, i.e., $\Psi_k(\lambda_k)$ is non-decreasing in λ_k, and by Assumption 3, $T_k(d)$ is non-decreasing in d, we can deduce the composite function $T_k(\Psi_k(\lambda_k))$ is non-decreasing in λ_k. Because under Assumption 3, $\lambda_k \Psi_k^g(\lambda_k) = \min\{\lambda_k, T_k(\Psi_k(\lambda_k))\}$, the throughput $\lambda_k \Psi_k^g(\lambda_k)$ must be non-decreasing in λ_k as well. ∎

Assumption 3 states that if resource k can accommodate a throughput rate $T_k(\Psi_k(\lambda))$ under the induced delay $\Psi_k(\lambda)$, it will fully utilize the capacity to accommodate $T_k(\Psi_k(\lambda))$, unless the demand λ_k is less. Consequently, Lemma 1 shows that under this work conservation assumption, the monotonicity of throughput τ_k can be guaranteed as a result of Assumption 1.

Theorem 2: Under Assumption 1 to 3, there always exists a unique equilibrium for any system with a linear topology.

Proof of Theorem 2: Because any input rate ϕ_k determines the delay d_k, gain g_k and throughput τ_k, and under a linear topology $\phi_1 = \varphi$ and $\phi_k = \tau_{k-1}$ for $k > 1$, the sending rate φ uniquely determines all other parameters in the system. Suppose we have two different rates $\varphi^* > \varphi^\star$ under equilibrium, we prove the uniqueness of equilibrium by contradiction.

As $\varphi^* > \varphi^\star$, $\phi_1^* > \phi_1^\star$ and by Assumption 1, $d_1^* \geq d_1^\star$ and $g_1^* \leq g_1^\star$. Furthermore, with Assumption 3 and Lemma 1, we deduce that $\tau_1^* > \tau_1^\star$. Because $\phi_k = \tau_{k-1}$ for $k > 1$, the same logic can be applied to derive $d_k^* \geq d_k^\star$, $g_k^* \leq g_k^\star$ and $\tau_k^* > \tau_k^\star$ for any resource k. Consequently, we deduce that the aggregate delay and gain satisfy $D^* \geq D^\star$ and $G^* \leq G^\star$. This contradicts Assumption 2, which states that the demand will not increase under a higher delay and a lower gain. ∎

Theorem 2 shows that under an assumption of work conservation, any linear topology obtains a unique equilibrium. Thus, we denote the unique equilibrium by $(\boldsymbol{\phi}^*, \boldsymbol{d}^*, \boldsymbol{g}^*, \varphi^*, D^*, G^*)$. Next, we analyze the impact of ISPs' capacities and CPs' user population on the throughput τ^* of the route and the delays $\boldsymbol{d}^*$ and gains $\boldsymbol{g}^*$ of the individual resource under the equilibrium. In particular, we denote the capacities of resources by $\boldsymbol{\mu}$ and the numbers of end-users of the CPs by $\boldsymbol{s}$, and express the equilibrium as functions of $\boldsymbol{s}$ and $\boldsymbol{\mu}$ such as $\phi_k^*(\boldsymbol{\mu})$ and $\phi_k^*(\boldsymbol{s})$.

Theorem 3: Let $\boldsymbol{s}' = (s_i', \boldsymbol{s}_{-i})$ and $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$ for any vector $\boldsymbol{s}_{-i}$ of end-users of CPs other than i with the condition $s_i' > s_i$. We must have $\varphi^*(\boldsymbol{s}') > \varphi^*(\boldsymbol{s})$ and for any resource k,

$$\tau_k^*(\boldsymbol{s}') \geq \tau_k^*(\boldsymbol{s}), \quad d_k^*(\boldsymbol{s}') \geq d_k^*(\boldsymbol{s}) \quad \text{and} \quad g_k^*(\boldsymbol{s}') \leq g_k^*(\boldsymbol{s}).$$

Let $\boldsymbol{\mu}' = (\mu_k', \boldsymbol{\mu}_{-k})$ and $\boldsymbol{\mu} = (\mu_k, \boldsymbol{\mu}_{-k})$ for any $\boldsymbol{\mu}_{-k}$ and $\mu_k' > \mu_k$. We must have $\tau^*(\boldsymbol{s}') \geq \tau^*(\boldsymbol{s})$ and for any $\kappa > k$,

$$\phi_\kappa^*(\boldsymbol{\mu}') \geq \phi_\kappa^*(\boldsymbol{\mu}), \quad d_\kappa^*(\boldsymbol{\mu}') \geq d_\kappa^*(\boldsymbol{\mu}) \quad \text{and} \quad g_\kappa^*(\boldsymbol{\mu}') \leq g_\kappa^*(\boldsymbol{\mu}).$$

Proof of Theorem 3: We first show that $\varphi^*(\boldsymbol{s}') > \varphi^*(\boldsymbol{s})$ by contradiction. Suppose $\varphi^*(\boldsymbol{s}') \leq \varphi^*(\boldsymbol{s})$, because $\phi_1 = \varphi$ and by Assumption 3, we deduce $d_k^*(\boldsymbol{s}') \leq d_k^*(\boldsymbol{s})$ and $g_k^*(\boldsymbol{s}') \geq g_k^*(\boldsymbol{s})$ for any resource k, which implies $D^*(\boldsymbol{s}') \leq D^*(\boldsymbol{s})$ and $G^*(\boldsymbol{s}') \geq G^*(\boldsymbol{s})$. However, under no worse congestion and by Assumption 2, each rate Λ_i will be non-decreasing under $\boldsymbol{s}'$ and thus $\varphi^*(\boldsymbol{s}') > \varphi^*(\boldsymbol{s})$, which reaches a contradiction. Given $\varphi^*(\boldsymbol{s}') > \varphi^*(\boldsymbol{s})$, by Assumption 1 and Lemma 1, we can further deduce that $d_k^*(\boldsymbol{s}') \geq d_k^*(\boldsymbol{s})$, $g_k^*(\boldsymbol{s}') \leq g_k^*(\boldsymbol{s})$ and $\tau_k^*(\boldsymbol{s}') \geq \tau_k^*(\boldsymbol{s})$ for any resource k.

We then show that $\tau_k^*(\boldsymbol{\mu}') \geq \tau_k^*(\boldsymbol{\mu})$ by contradiction. Suppose $\tau_k^*(\boldsymbol{\mu}') < \tau_k^*(\boldsymbol{\mu})$, by Assumption 3, $d_\kappa^*(\boldsymbol{s}') \leq d_\kappa^*(\boldsymbol{s})$ and $g_\kappa^*(\boldsymbol{s}') \geq g_\kappa^*(\boldsymbol{s})$ for all downstream resources $\kappa > k$. This is also true for resource k and all upstream resources due to the monotonicity of throughput of Lemma 1 and the capacity impact in Assumption 1, which implies that $\phi_\kappa^*(\boldsymbol{\mu}') \leq \phi_\kappa^*(\boldsymbol{\mu})$ for all upstream resources $\kappa < k$. Similarly as before, this implies that $D^*(\boldsymbol{\mu}') \leq D^*(\boldsymbol{\mu})$ and $G^*(\boldsymbol{\mu}') \geq G^*(\boldsymbol{\mu})$, and therefore, $\varphi^*(\boldsymbol{\mu}') > \varphi^*(\boldsymbol{\mu})$. However, since $\phi_1 = \varphi$, this contradicts with $\phi_1^*(\boldsymbol{\mu}') \leq \phi_1^*(\boldsymbol{\mu})$. Given $\tau_k^*(\boldsymbol{\mu}') \geq \tau_k^*(\boldsymbol{\mu})$, by Assumption 1 and Lemma 1, we further deduce $d_\kappa^*(\boldsymbol{\mu}') \geq d_\kappa^*(\boldsymbol{\mu})$, $g_\kappa^*(\boldsymbol{\mu}') \leq g_\kappa^*(\boldsymbol{\mu})$ and $\phi_\kappa^*(\boldsymbol{\mu}') \geq \phi_\kappa^*(\boldsymbol{\mu})$ for all downstream resources $\kappa > k$. ∎

Theorem 3 shows the unilateral impact of population s_i and capacity μ_k on the equilibrium. When s_i increases, the aggregate sending rate φ^* must increase and no resource will induce a higher gain or a lower throughput or delay. This implies the route delay D and gain G will be non-decreasing and non-increasing, respectively, which further implies that the aggregate sending rate of the CPs other than CP i will be non-increasing, but that of CP i will increase. When μ_k increases, the aggregate throughput τ^* will not decrease. In particular, no downstream resource $\kappa > k$ will receive a lower input rate and induce a lower delay or a higher gain. However, if the decrease of delay and increase of gain at resource k cannot compensate the reversed effects at the subsequent downstream resources, the sending rate φ^* might decrease and relieve the congestion at the upstream resources $\kappa < k$ in the new equilibrium.

VI. General Lossless Networks

In this section, we consider general topologies, but focus on lossless networks, i.e., $g_k = 1$ and $\phi_k = \tau_k$ for all $k \in \mathcal{K}$. We refer to both the sending rate and throughput by $\boldsymbol{\phi}$ and $\boldsymbol{\varphi}$. These scenarios model the cases where application protocols and end-users are very sensitive to packet losses and will reduce demand under losses, e.g., adapting to low-resolution for video streaming, such that losses rarely occur under equilibria.

We define a $|\mathcal{K}| \times |\mathcal{L}|$ routing matrix R with each entry $R_{kl} = \mathbb{1}_{\{k \in l\}}$. Notice that $R = H(\mathbf{0})^T$ and determines the network topology, since $\Psi_k^g(0) = 1$ holds naturally for any resource. Consequently, the network equilibrium of Definition 1 can be compactly represented in a vector form as

$$\boldsymbol{d} = \boldsymbol{\Psi}(\boldsymbol{\phi}, \boldsymbol{\mu}), \quad \boldsymbol{\varphi} = \boldsymbol{\Phi}(\boldsymbol{D}, \boldsymbol{s}), \quad \boldsymbol{\phi} = R\boldsymbol{\varphi} \ \text{ and } \ \boldsymbol{D} = R^T \boldsymbol{d},$$

where $\Phi_l(D_l, \boldsymbol{s}_l) = \sum_{i \in \mathcal{I}} s_{il} \Lambda_i(D_l, 1)$ and $\boldsymbol{s}_l$ is the lth column of $\boldsymbol{s}$, which is the $|\mathcal{I}| \times |\mathcal{L}|$ matrix of user population.

Theorem 4: Under Assumption 1 and 2, there always exists a unique equilibrium for any lossless system.

Proof of Theorem 4: Theorem 1 shows the existence of equilibrium. Here, we show the uniqueness by contradiction. Suppose for any fixed $\boldsymbol{\mu}$ and $\boldsymbol{s}$, there exist two equilibria $(\boldsymbol{\phi}', \boldsymbol{d}', \boldsymbol{\varphi}', \boldsymbol{D}') \neq (\boldsymbol{\phi}, \boldsymbol{d}, \boldsymbol{\varphi}, \boldsymbol{D})$. Let us define the difference of the equilibria by

$$(\boldsymbol{\phi}_\delta, \boldsymbol{d}_\delta, \boldsymbol{\varphi}_\delta, \boldsymbol{D}_\delta) = (\boldsymbol{\phi}', \boldsymbol{d}', \boldsymbol{\varphi}', \boldsymbol{D}') - (\boldsymbol{\phi}, \boldsymbol{d}, \boldsymbol{\varphi}, \boldsymbol{D})$$

and without loss of generality, we order the resources and routes such that $\boldsymbol{d}_\delta = (\boldsymbol{d}_\delta^+, \boldsymbol{d}_\delta^-)$ and $\boldsymbol{\varphi}_\delta = (\boldsymbol{\varphi}_\delta^+, \boldsymbol{\varphi}_\delta^-)$, where $\boldsymbol{d}_\delta^+$ and $\boldsymbol{\varphi}_\delta^+$ consist of the positive differences and $\boldsymbol{d}_\delta^-$ and $\boldsymbol{\varphi}_\delta^-$ consist of the non-positive ones. By Assumption 1 and 2, we can write $\boldsymbol{\phi}_\delta = (\boldsymbol{\phi}_\delta^+, \boldsymbol{\phi}_\delta^-)$ and $\boldsymbol{D}_\delta = (\boldsymbol{D}_\delta^-, \boldsymbol{D}_\delta^+)$ correspondingly, and express

$$\boldsymbol{\phi} = R\boldsymbol{\varphi} \quad \text{as} \quad \begin{bmatrix} \boldsymbol{\phi}_\delta^+ \\ \boldsymbol{\phi}_\delta^- \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varphi}_\delta^+ \\ \boldsymbol{\varphi}_\delta^- \end{bmatrix}.$$

By negating the sign of some components, we define

$$\tilde{R} = \begin{bmatrix} R_{11} & -R_{12} \\ R_{21} & -R_{22} \end{bmatrix},$$

$\tilde{\boldsymbol{\varphi}}_\delta = (\boldsymbol{\varphi}_\delta^+, -\boldsymbol{\varphi}_\delta^-)$ and $\tilde{\boldsymbol{D}}_\delta = (-\boldsymbol{D}_\delta^-, \boldsymbol{D}_\delta^+)$. By the linear equations $\boldsymbol{\phi} = R\boldsymbol{\varphi}$ and $\boldsymbol{D} = R^T \boldsymbol{d}$, we deduce

$$\tilde{R}\tilde{\boldsymbol{\varphi}}_\delta = \boldsymbol{\phi}_\delta \quad \text{and} \quad \tilde{\boldsymbol{\varphi}}_\delta \geq 0; \tag{14}$$

$$\tilde{R}^T(-\boldsymbol{d}_\delta) = \tilde{\boldsymbol{D}}_\delta \geq 0 \quad \text{and} \quad \boldsymbol{\phi}_\delta^T(-\boldsymbol{d}_\delta) < 0. \tag{15}$$

However, by using Farkas's Lemma [31], we conclude that the two conditions (14) and (15) cannot be satisfied simultaneously; and therefore, the equilibrium must be unique. ■

Theorem 4 guarantees the uniqueness of network equilibrium for lossless systems. Similarly, we denote the unique equilibrium by $(\boldsymbol{\phi}^*, \boldsymbol{d}^*, \boldsymbol{\varphi}^*, \boldsymbol{D}^*)$ and express the equilibrium as a function of $\boldsymbol{\mu}$ and $\boldsymbol{s}$ such as $\boldsymbol{D}^*(\boldsymbol{s}, \boldsymbol{\mu})$ and $\boldsymbol{\varphi}^*(\boldsymbol{s}, \boldsymbol{\mu})$.

We study the impacts of capacities $\boldsymbol{\mu}$ and user population $\boldsymbol{s}$ on the route-level equilibrium $(\boldsymbol{\varphi}^*, \boldsymbol{D}^*)$ using primitives like $\nabla_{\boldsymbol{D}}\Phi$, $\nabla_{\boldsymbol{\mu}}\Psi$ and $\nabla_{\boldsymbol{\varphi}}D$. $\nabla_{\boldsymbol{D}}\Phi$ and $\nabla_{\boldsymbol{\mu}}\Psi$ are diagnal matrices with diagnal entries to be $\partial\Phi_l/\partial D_l$ and $\partial\Psi_k/\partial\mu_k$, respectively. Becuase $\boldsymbol{D} = R^T\boldsymbol{d} = R^T\Psi(\boldsymbol{\phi}, \boldsymbol{\mu}) = R^T\Psi(R\boldsymbol{\varphi}, \boldsymbol{\mu})$, by the chain rule, we can define $\nabla_{\boldsymbol{\varphi}}D \triangleq R^T\nabla_{\boldsymbol{\phi}}\Psi R$, where $\nabla_{\boldsymbol{\phi}}\Psi$ is a diagnal matrix with the kth diagnal entry being $\partial\Psi_k/\partial\phi_k$. Each entry $[\nabla_{\boldsymbol{\varphi}}D]_{l_1 l_2}$ captures the marginal delay on route l_2 due to the marginal change in the sending rate on route l_1, which equals the aggregate changes in the delays of resources $\sum_{k \in l_1 \cap l_2} \partial\Psi_k/\partial\phi_k$ shared by routes l_1 and l_2.

Theorem 5: The impact of capacities $\boldsymbol{\mu}$ on the route-level equilibrium $(\boldsymbol{\varphi}^*, \boldsymbol{D}^*)$ can be characterized by

$$\begin{cases} \nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* = \left(I - \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi\right)^{-1} R^T\nabla_{\boldsymbol{\mu}}\Psi; \\ \nabla_{\boldsymbol{\mu}}\boldsymbol{\varphi}^* = \left(I - \nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{\varphi}}D\right)^{-1} \nabla_{\boldsymbol{D}}\Phi R^T\nabla_{\boldsymbol{\mu}}\Psi. \end{cases} \tag{16}$$

The impact of users $\boldsymbol{s}$ on $(\boldsymbol{\varphi}^*, \boldsymbol{D}^*)$ can be characterized by

$$\begin{cases} \nabla_{\boldsymbol{s}}\boldsymbol{D}^* = \left(I - \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi\right)^{-1} \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{s}}\Phi; \\ \nabla_{\boldsymbol{s}}\boldsymbol{\varphi}^* = \left(I - \nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{\varphi}}D\right)^{-1} \nabla_{\boldsymbol{s}}\Phi. \end{cases} \tag{17}$$

Proof of Theorem 5: Given $\boldsymbol{D}^*(\boldsymbol{s}, \boldsymbol{\mu})$ is under equilibrium,

$$\boldsymbol{D}^*(\boldsymbol{s}, \boldsymbol{\mu}) = R^T\Psi(R\Phi(\boldsymbol{D}^*(\boldsymbol{s}, \boldsymbol{\mu}), \boldsymbol{s}), \boldsymbol{\mu}).$$

By differentiating $\boldsymbol{\mu}$ and $\boldsymbol{s}$ on both sides, we obtain

$$\begin{aligned} \nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* &= R^T[\nabla_{\boldsymbol{\phi}}\Phi R\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* + \nabla_{\boldsymbol{\mu}}\Phi] \\ &= R^T\nabla_{\boldsymbol{\phi}}\Phi R\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* + R^T\nabla_{\boldsymbol{\mu}}\Phi \\ &= \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* + R^T\nabla_{\boldsymbol{\mu}}\Phi \\ \nabla_{\boldsymbol{s}}\boldsymbol{D}^* &= R^T\nabla_{\boldsymbol{\phi}}\Phi R[\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{s}}\boldsymbol{D}^* + \nabla_{\boldsymbol{s}}\Phi] \\ &= \nabla_{\boldsymbol{\varphi}}D[\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{s}}\boldsymbol{D}^* + \nabla_{\boldsymbol{s}}\Phi] \\ &= \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi\nabla_{\boldsymbol{s}}\boldsymbol{D}^* + \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{s}}\Phi \end{aligned}$$

By rearranging the above, we obtain

$$\begin{cases} (I - \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi)\nabla_{\boldsymbol{\mu}}\boldsymbol{D}^* = R^T\nabla_{\boldsymbol{\mu}}\Phi; \\ (I - \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{D}}\Phi)\nabla_{\boldsymbol{s}}\boldsymbol{D}^* = \nabla_{\boldsymbol{\varphi}}D\nabla_{\boldsymbol{s}}\Phi, \end{cases} \tag{18}$$

which implies the first equations of (16) and (17). Similarly, given $\boldsymbol{\varphi}^*(\boldsymbol{s}, \boldsymbol{\mu})$ is under equilibrium, we have

$$\boldsymbol{\varphi}^*(\boldsymbol{s}, \boldsymbol{\mu}) = \Phi(R^T\Psi(R\boldsymbol{\varphi}^*(\boldsymbol{s}, \boldsymbol{\mu}), \boldsymbol{\mu}), \boldsymbol{s}).$$

By differentiating $\boldsymbol{\mu}$ and $\boldsymbol{s}$ on both sides and using the same logic, we deduce the second equations of (16) and (17). ■

Theorem 5 provides the sensitivity analysis and comparative statics for the route-level equilibrium $(\boldsymbol{\varphi}^*, \boldsymbol{D}^*)$ under varying $\boldsymbol{\mu}$ and $\boldsymbol{s}$, where marginal changes in equilibrium are presented as functions of primitive metrics like the gradient of Ψ and Φ. Although more abundant capacities reduce delays ($\nabla_{\boldsymbol{\mu}}\Psi \leq \mathbf{0}$) and increase throughput ($\nabla_{\boldsymbol{D}}\Phi \leq \mathbf{0}$), Equation (16) shows that this impact on the equilibrium might not be monotonic on every route due to the inter-dependency among the routes, as the inverse of a positive matrix might not be positive definite. Similarly, although more users induce higher throughput ($\nabla_{\boldsymbol{s}}\Phi \geq \mathbf{0}$) and delay ($\nabla_{\boldsymbol{\varphi}}D \geq \mathbf{0}$), this monotonicity might not hold under the equilibrium as shown by Equation (17).

Although the network equilibrium is partially affected by $\boldsymbol{s}$, this user population matrix is ultimately determined by the strategic routing decisions of CPs. Based on our equilibrium framework, we can further study the CPs' routing decisions. To model end-users in different geographical regions, we denote the set of geographical regions by $\mathcal{J}$ and the maximum number of users that are interested in using CP i in region j by m_{ij}. The last mile of each route targets certain region $j \in \mathcal{J}$; and therefore, we denote the set of routes to reach region j by $\mathcal{L}^j$ and define $\mathcal{L} = \cup_{j \in \mathcal{J}}\mathcal{L}^j$ as the set of all possible routes. We define the set of routes available for CP i to route traffic to the end-users in region j by $\mathcal{L}_i^j \subseteq \mathcal{L}^j$ and define $\mathcal{L}_i = \cup_{j \in \mathcal{J}}\mathcal{L}_i^j$ as the set of routes available to SP i.

Consequently, we define $\mathcal{M} \triangleq \{m_{ij} : i \in \mathcal{I}, j \in \mathcal{J}\}$ as the aggregate user demand in the regions and $\mathcal{S}(\mathcal{M}) = \times_{i\in\mathcal{I}}\mathcal{S}_i(\mathcal{M})$ as the set of all feasible routing strategies of the CPs, where each $\mathcal{S}_i(\mathcal{M})$ is defined as

$$\mathcal{S}_i = \left\{\boldsymbol{s}_i : s_{il} \geq 0, \sum_{l\in\mathcal{L}_i^j} s_{il} = \sum_{l\in\mathcal{L}^j} s_{il} = m_{ij}, \forall j \in \mathcal{J}\right\},$$

which is a simplex of strategy space constrained by $s_{il} = 0$ for all $l \notin \mathcal{L}_i$, i.e., CPs cannot use any route they do not own.

Under any feasible routing profile $\boldsymbol{s} \in \mathcal{S}$, the unique network equilibrium determines the end-to-end delay $\boldsymbol{D}^*(\boldsymbol{s})$ along the routes. To optimize users' performance in terms of minimizing their delays, among all the available routes, CPs might always want to route users along the route with the minimum delay. However, CPs' routing strategies are inter-dependent, which leads to the definition of a Wardrop equilibrium [38].

Definition 2 (Wardrop Equilibrium): A feasible routing strategy profile $\boldsymbol{s}^* \in \mathcal{S}(\mathcal{M})$ is a Wardrop routing equilibrium if for any route $l \in \mathcal{L}$ and CP $i \in \mathcal{I}$ with $s^*_{il} > 0$,

$$D^*_l(\boldsymbol{s}^*) \leq D^*_{l'}(\boldsymbol{s}^*), \quad \forall l' \in \mathcal{L}_i^{J(l)},$$

where $J(l)$ denotes the region of destination of route l.

Definition 2 states that under a Wardrop equilibrium, if any route $l \in \mathcal{L}^j$ is used by CP i to serve its users in region j, there does not exist any other feasible route $l' \in \mathcal{L}_i^j$ for CP i to serve users in region j whose end-to-end delay $D^*_{l'}$ is strictly smaller than the delay D^*_l of the original route l. Notice that this definition is built upon the delays $\boldsymbol{D}^*(\boldsymbol{s})$ under the unique equilibrium as a function of the routing strategies of the CPs.

Theorem 6 ([34]): A feasible routing strategy profile $\boldsymbol{s}^* \in \mathcal{S}(\mathcal{M})$ is a Wardrop routing equilibrium if and only if

$$(\boldsymbol{s} - \boldsymbol{s}^*)^T \boldsymbol{D}^*(\boldsymbol{s}^*) \geq \boldsymbol{0}, \quad \forall \boldsymbol{s} \in \mathcal{S}(\mathcal{M}). \tag{19}$$

There always exists such a Wardrop routing equilibrium and the equilibrium is unique if $\boldsymbol{D}^*$ is strictly monotone [7], i.e.,

$$(\boldsymbol{D}^*(\boldsymbol{s}') - \boldsymbol{D}^*(\boldsymbol{s}))(\boldsymbol{s}' - \boldsymbol{s}) > \boldsymbol{0}, \quad \forall \boldsymbol{s}' \neq \boldsymbol{s}, \boldsymbol{s}', \boldsymbol{s} \in \mathcal{S}(\mathcal{M}). \tag{20}$$

Theorem 6 characterizes the Wardrop equilibrium as a form of variational inequality [7] in (19). Since the feasible domain $\mathcal{S}(\mathcal{M})$ is compact, the existence of equilibrium can be guaranteed. Notice that a sufficient condition for the uniqueness of equilibrium, i.e., $\boldsymbol{D}^*$ being strict monotone, is that the gradient matrix $\nabla_{\boldsymbol{s}}\boldsymbol{D}^*$ defined in Equation (17) is positive definite, which implies that the increase in user demand along any route will not reduce the delay along any other route.

VII. A Study of Netflix-Comcast Type of Dispute

In this section, we utilize the proposed equilibrium solution to evaluate the cases of a CP reaching users via a last-mile access provider (AP), emulating the Netflix-Comcast type of disputable peering scenarios. We analyze the impacts of user population, capacity expansion and in-network caching on the QoS in equilibrium and derive implications on potential disputes. We assume that via a private peering, the AP provides dedicated resources to the CP, and therefore, both the CP and AP are modeled as a network resource using the $M/\Gamma/1/K$ model with parameters $K = 10$ and $\alpha = 1$, forming an independent route. We use the per-flow demand $\lambda(D,G) = (2D)^{-1}[4(1-G)/3]^{-0.5}$ packets to emulate the behavior of TCP, and specify the capacity of the CP and AP are μ_1 and μ_2 in terms of per thousand packets. We consider a population of s active users demanding traffic and the network equilibrium is determined by the triple (s, μ_1, μ_2).

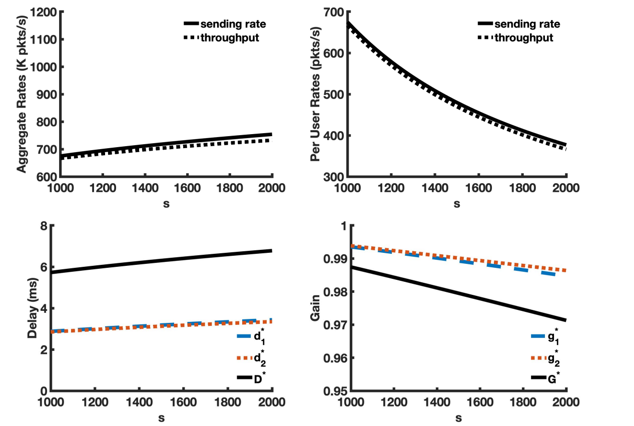

Fig. 8. Rates, delay and gain under varying user size s with $\mu_1 = \mu_2 = 1000$.

Under fixed capacities $\mu_1 = \mu_2 = 1000$, Figure 8 plots the aggregate and per-user rates in the upper subfigures and the corresponding delays and gains in the lower subfigures, as the user size s increases from 1000 to 2000 along the x-axis. We find that although the aggregate sending rate and throughput increase mildly, the per-user rates drop almost by half, caused by the increase in delays and the decrease in gains in both the CP and AP, due to the doubling in user size. This observation shows that even the AP provides isolated resources for the CP, the end-to-end performance will degrade as the CP becomes popular and attracts more users, which partially implying that Comcast might not have intentionally throttled Netflix's traffic.

To improve the QoS for the increased user population, the CP could unilaterally deploy more resources. Figure 9 plots the corresponding QoS metrics for the user size $s = 2000$, as the capacity μ_1 of the CP increases from 1000 to 2000 along the x-axis. In the upper subfigures, we observe that both the aggregate and per-user rates increase; however, the trends exhibit a concave shape of "diminishing return". By observing the delays and gains in the lower subfigures, one could understand that as the increase of μ_1 reduces the delay d_1^* and increases the gain g_1^* of the CP, the input rate into the AP increases, resulting in the increase in its delay d_2^* and decrease in its gain g_2^*. Consequently, although the end-to-end delay D^* decreases, the end-to-end gain G^* also decreases, which limits the achievable rates for users in equilibrium. This observation implies that a unilateral capacity expansion at the CP-side is not sufficient to provide QoS for the increased user population, since the improved QoS at the CP will increase the input rate into the AP, making the downstream resource even more congested. Although the per-user throughput increases, we find that the end-to-end drop rate may still decreases.

Instead of deploying an extra capacity of 1000 (thousand packets per second) entirely at the CP-side, we evaluate the

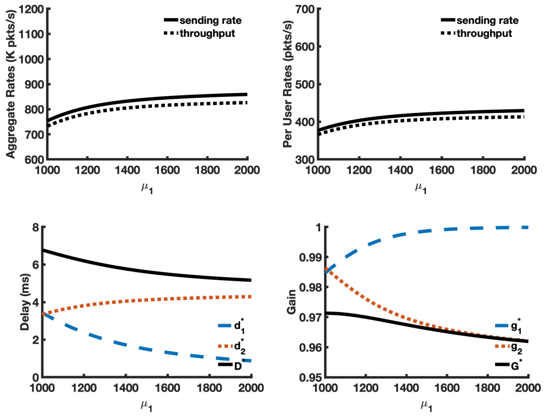

Fig. 9. QoS metrics under varying capacity μ_1 with $s = 2000, \mu_2 = 1000$.

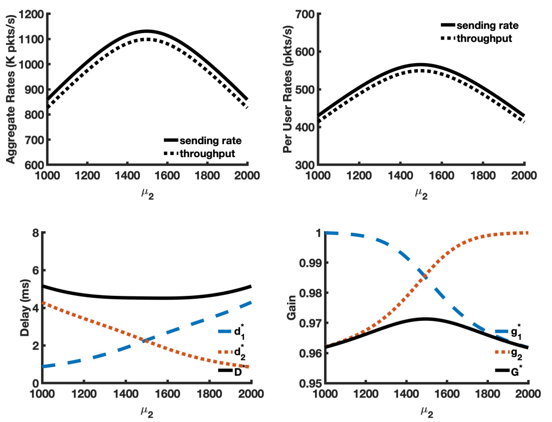

Fig. 10. QoS metrics under various capacities with $s = 2000$ and $\mu_1 + \mu_2 = 3000$.

impacts of distributing capacity resource between the CP and AP such that the total capacity satisfies $\mu_1 + \mu_2 = 3000$. This models the situations where the CP not only expands its own capacity, but also tries to enter a peering contract to purchase more capacity for its traffic in the last-mile. Figure 10 plots the QoS metrics for the user size $s = 2000$, as the capacity μ_2 of the AP increases from 1000 to 2000 along the x-axis. We observe that more resource is distributed to the AP, its delay d_2^* and gain g_2^* improve, at a cost of the falling QoS at the CP. Nonetheless, the end-to-end delay D^* and gain G^* exhibit a single valley/peak pattern, showing that a balanced resource distribution could lead to optimal QoS metrics. This is confirmed by the single-peak patterns in the upper subfigures, showing that under such an optimal resource distribution, the throughput and sending rates are maximized under equilibrium. This observation implies that in order to guarantee the QoS of end-users, CPs need to enter private peering agreements with APs through which APs will expand capacities to relieve the bottleneck at the last-mile. This has been seen as an outcome of the Netflix-Comcast dispute, although the financial settlement was not publicly known.

Besides capacity expansion, giant CPs such as Netflix also leverage in-network caching [3] extensively to deploy cachable content inside IXPs and ISPs, making them closer to end users. To evaluate such impacts, we vary the percentage of user size $s = 2000$ that will be served via cache servers inside the AP,

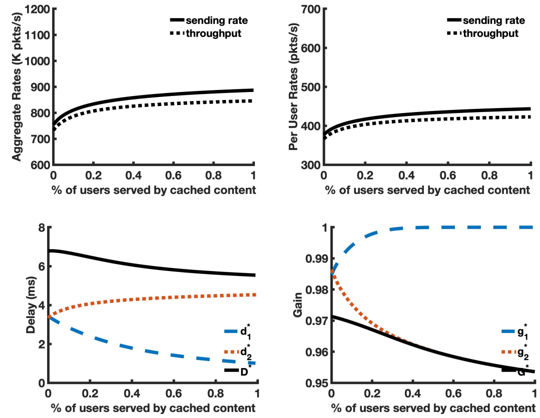

Fig. 11. QoS metrics under various degrees of in-network caching.

and plot the QoS metrics in Figure 11. Under this scenario, user traffic might traverse a shorter route that only consists of the AP's network resource with capacity $\mu_2 = 1000$. We observe that QoS metrics exhibit similar trends as the case of the CP expanding capacity μ_1 in Figure 9, because moving more users traffic downstream towards the users will similarly 1) reduce the delay d_1^* and increases the gain g_1^* of the CP, and 2) increase the input rate into the AP, resulting in the increase in its delay d_2^* and decrease in its gain g_2^*. Although the end-to-end delay D^* decreases as the CP utilizes more in-network caching, as the percentage of user traffic that traverses the end-to-end route also decreases, the increases in the aggregate and per-user rates also exhibit the "diminishing return" that limits the achievable rates for users in equilibrium. This observation illustrates that even without any capacity expansion at the CP or the AP, employing in-network caching could improve the QoS for users. This is an cheaper and more effective solution than the unilateral capacity expansion at the CP, although the CP needs to contract with the AP for deploying cache servers.

In summary, although we only presented simple numerical evaluations on a stylized CP-AP scenario, the results have brought some insights that explain why the real practices of CPs, i.e., contracting and peering with last-mile APs for in-network caching and capacity guarantee, indeed provide the best cost-effective solution for QoS improvement.

VIII. Broader Models and Applications

Although the model and analyses presented so far are based on fixed network topologies and predetermined user populations of the CPs, they can be further extended to include richer dynamics in both the Internet e-commerce ecosystem and the Internet transport ecosystem. In general, the Internet transport ecosystem consists of a set $\mathcal{N}$ of *network providers* (NPs) that includes any type of ISP, e.g., transit and access, CDNs and any physical entity that owns network resources. The set of all network resources can be defined as $\mathcal{K} \triangleq \cup_{n\in\mathcal{N}}\mathcal{K}_n$, where each $\mathcal{K}_n$ defines the subset of resources owned by NP $n \in \mathcal{N}$. A holistic Internet transport ecosystem model can be described by a tuple $(\mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L}, \mathcal{M}, \mathcal{N})$.

Figure 12 illustrates the relationships among the different entities in the macroscopic Internet transport ecosystem model. The top blue rectangle shows the Internet E-commerce

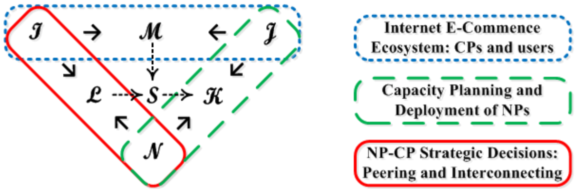

Fig. 12. A broader model of the Internet transport ecosystem.

ecosystem where CPs $\mathcal{I}$ interact with users in various regions $\mathcal{J}$ and influence the demand $\mathcal{M}$ of users, which is not explicitly modeled in this work. This application layer of the Internet is enabled by the underlying physical transport system built by the NPs $\mathcal{N}$. The right green rectangle represents the supply-side where NPs make capacity planning decisions to deploy network resources $\mathcal{K}$ that form the routes $\mathcal{L}^j$ targeted for any region $j \in \mathcal{J}$. The left red rectangle represents the demand-side where NPs negotiate interconnection and peering contracts with CPs and other NPs. The interconnections form the physical Internet topology; while the peering agreements form the effective routes $\mathcal{L}_i$ used by each CP i's traffic. Under this macroscopic model, the user demand $\mathcal{M}$ and available routes $\mathcal{L}$ form the space $\mathcal{S}$ of routing strategies of the CPs, and any particular routing decision will eventually determine the delay and gain metrics on the resources $\mathcal{K}$ as well as the end-to-end delay D_l and gain G_l for each route $l \in \mathcal{L}$.

In two of the previous sections, we demonstrated the use of our equilibrium framework for analyzing the impact of the capacity planning of ISPs and the routing decisions of CPs. Because the network equilibrium solution is built upon physical models of supply and demand of the Internet transport services, it can be used as a building block to understand the consequences of various strategic decisions of ASes. Consequently, higher-layer game-theoretical and optimization models can be established to study the business interactions among the ASes and desirable network protocols and policies for the Internet ecosystem. We briefly discuss some further applications of the macroscopic ecosystem model as follows.

Internet Peering: The effects of peering determine the Internet topology and are reflected via an enriched set $\mathcal{L}$ of available routes. Furthermore, any NP n can control its resources $\mathcal{K}_n$ to create different routes, e.g., public and private peering points, and configure BGP export policies to make specific routes $\mathcal{L}_i$ available to any peering counter-party $i \in \mathcal{I}$ based on peering agreements such as premium peering. Thus, peering can be understood as a result of NPs' strategic controls of network resources and topology in the transport layer.

CDN and In-Network Caching: Although our framework does not explicitly model the storage capacities of the NPs, the use of CDN or caching can be reflected by the changes in the source ASes of contents. Because cached contents will have shorter routes towards end-users, CPs' decisions on cache deployment of CDNs [16] will effectively change the routing of contents towards users.

IX. Related Work

Early studies of network economics focused on the impact of selfish routing [26], [28], [32], [35] on network efficiency. Orda *et al.* [26] studied a routing game under which users split throughput demands among parallel links. Roughgarden and Tardos [32] analyzed the performance degeneration caused by selfish routing. These theoretical works assume fixed demands of users and only considered a single latency metric. We model elastic demands driven by network protocols such as TCP and characterizes both the drop rate and delay metrics. Teixeira *et al.* [35] showed via controlled experiments that intra-domain hot-potato routing causes high delays and slow convergence for inter-domian BGP [29] routes. Based on realistic topologies and traffic demands, Qiu *et al.* [28] studied selfish overlay routing in intra-domain environments via simulations. We focus on the macroscopic ecosystem where CPs make inter-domain routing decisions to enhance the QoS for end-users.

As the Internet topology is driven by the bilateral business relationship [8], [14] between ASes, many recent studies have focused on Internet peering [4], [6], [21], [23], [24], [37]. Gao [14] characterized the *valley-free* property to infer ASes' business relationships based on the BGP routing protocol and data. Castro *et al.* [4] revealed the presence of remote peering, where remote networks peer via a layer-2 provider. Faratin *et al.* [8] and Lodhi *et al.* [21] discussed the complexity of peering and the emergence of new agreements, e.g., premium peering [6], [23], [37], which however raised new peering disputes [9]. To resolve such peering disputes, Ma *et al.* [24] designed a multilateral profit sharing mechanism for ISP settlements. Although CPs can obtain better QoS by using premium peering [6], [23], [37] with access providers, the impact of such peering agreements on the resulting routing behaviors of CPs and the QoS of applications are largely unknown. Our equilibrium model captures the QoS in terms of both drop rates and delays of various routes, on top which Internet peering agreements can be better analyzed.

Extensive research was conducted to understand the QoS of TCP traffic flows under congestion control [13] and Active Queueing Management (AQM) [10] schemes. Mathis *et al.* [25] first proposed a renewal theory model for TCP Reno. Padhye *et al.* [27] derived the PFTK-formula that describes the TCP throughput as a function of loss rate and round trip time; however, both of which need to be known. Firoiu and Borden [10] analyzed the interactions between TCP and a bottleneck RED [13] queue using a fixed-point method. Bu and Towsley [2] extended the fixed-point framework for multiple bottlenecks, and Gibbens *et al.* [15] applied an M/M/1/B link model in a similar framework. The most extensive model was developed in Firoiu *et al.* [11] which consists of seven sets of equations and was evaluated by numerical methods; however, neither the existence nor the uniqueness of solution has been settled. Instead of modeling the detailed AQM and transport protocols, we apply general supply and demand functions to model network capacities and protocols. Consequently, we are able to derive the existence and uniqueness properties of equilibrium for general network topologies.

X. Conclusion

In this paper, we present a macroscopic network equilibrium model for the Internet transport ecosystem, which is built upon the supply of network capacity resources and the throughput demands driven by network protocols for transport services. Under such a network equilibrium, QoS metrics of drop rate and delay can be characterized for all the end-to-end routes. Through fixed-point analyses, we show the existence of equilibrium and its uniqueness under linear topologies or lossless scenarios. Through sensitivity analyses, we show the impacts of user demands and resource capacities on the network equilibrium, which provide implications of Netflix-Comcast type of peering on the QoS of users. Further studies of peering and caching can also be analyzed under our framework.

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