

Organisation of the course

Introduction to ML

Linear models

SL principles

'rees and neural networks

lisk optimisation



Given by: Tony Bonnaire (+ Pablo Mas for the project)



Format: Lectures + hands-on sessions then data challenge (in chemistry)



Exam: Paper analysis (50%) + oral presentation for the challenge (50%)



Aim: Introduce you to the basics of ML principles and carry out a project



Some references:

- Deep Learning: Foundations and Concepts, Bishop & Bishop, 2023,
- Deep Learning, Goodfellow et al., 2016,
- <u>Deep Learning with Python</u>, Chollet, 2016,
- Learning Theory from First Principles, Francis Bach, 2024,
- https://challengedata.ens.fr: a bank of data science challenges to apply all the things we will learn in this
 course

Contact: tony.bonnaire@ens.fr

Material on Github: github.com/tbonnair/

Intelligent systems

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Risk optimisatior

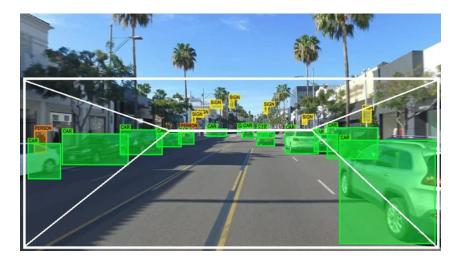
AI goal

Design **systems** capable of performing complex tasks requiring *intelligence* (i.e. using reasoning, perception or language) **to take decisions** and **make predictions**.

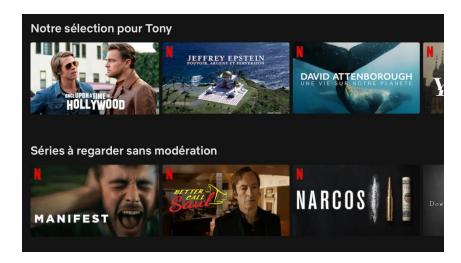








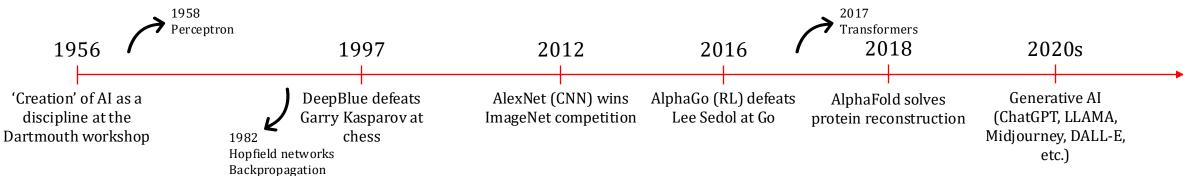




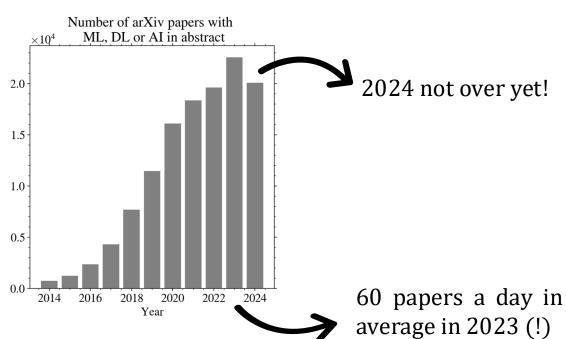
Al revolution

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Some (selected) AI breakthroughs



AI in science



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Some scientific applications

Healthcare

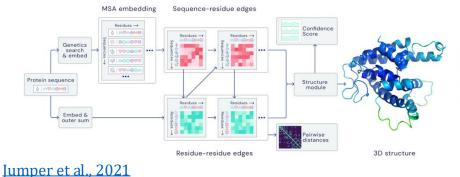
- Drug discovery
- Protein structure reconstruction

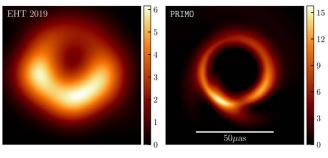
Astrophysics and cosmology

- Galaxy deblending
- Image restoration
- Source separation

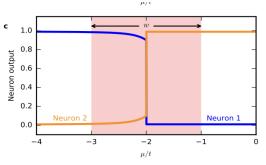
Theoretical physics

- Study phase transitions
- Discover experiments and equations





Medeiros et al., 2023



Van Nieuwenburg et al., 2017

••• And many more (climate forecast, fraud detection in cybersecurity, binding energies in quantum chemistry)

What is "learning"?

Introduction to ML

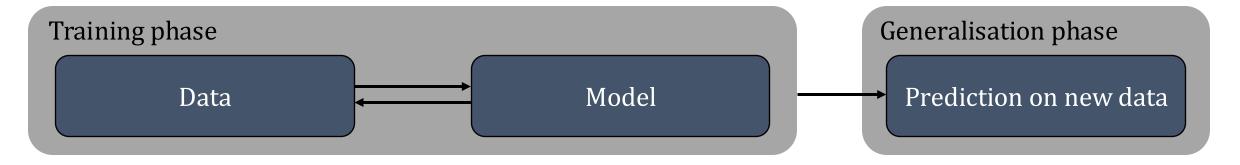
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Machine Learning came as a solution to design intelligent systems, replacing handcrafted decision rules by learnt rules using training data and optimisation of parameterised models.



Images

Linear models

Tabular

Trees and forests

Sound

Neural networks

Text

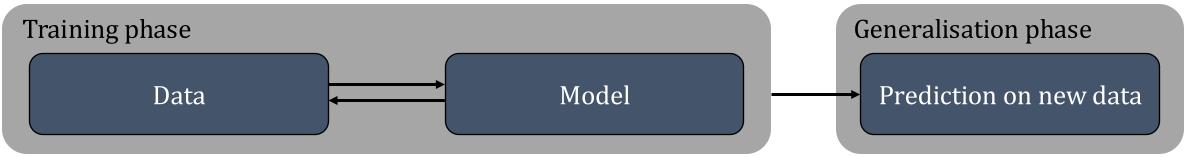
CNN

...

....

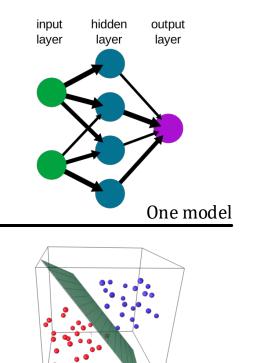
What is "learning"? An example

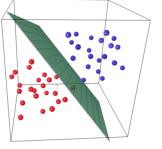
Introduction to ML





Images of a "cat" or "dog"





Another model



"cat" or "dog"?

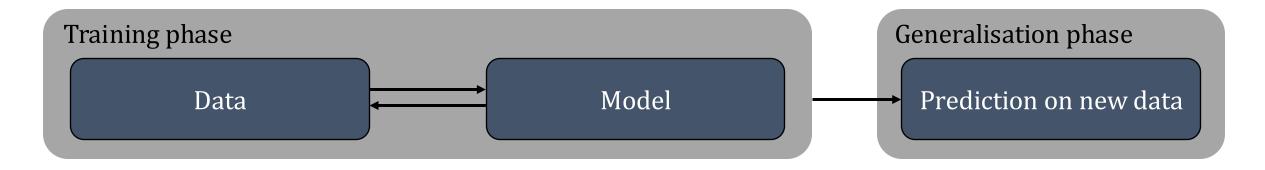
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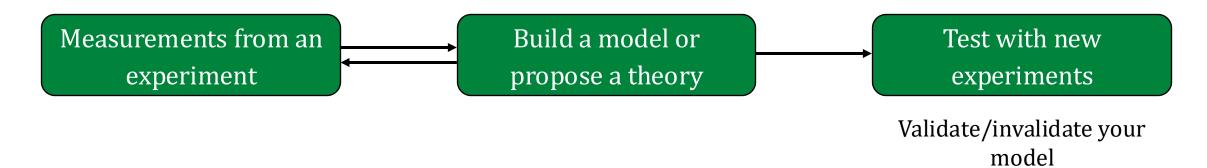
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Risk optimisatior



...In fact, all this is close to what you know!

The scientific method



ML building blocks

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Training phase

$$\mathcal{D} = \{x^{(i)}\}_{i=1}^{n_{\text{train}}} \longrightarrow \text{Embedding } \phi \xrightarrow{\phi(x^{(i)})}$$
Training set



- 1. Data are **unstructured**, sometimes **noisy** and **unprocessed** like pixels of an image or sequence of characters or words.
- 2. The embedding $\phi(x^{(i)})$ is a **structured**, **numerical** vector representation of the data whose elements are **meaningful features** that depends on the data and the purpose. It can be **handcrafted or learnt**.

Finding a good embedding is a central part of ML: it eases the problem by preserving the essential structure of the data that matters for the task but makes it solvable using simple models.

ML building blocks

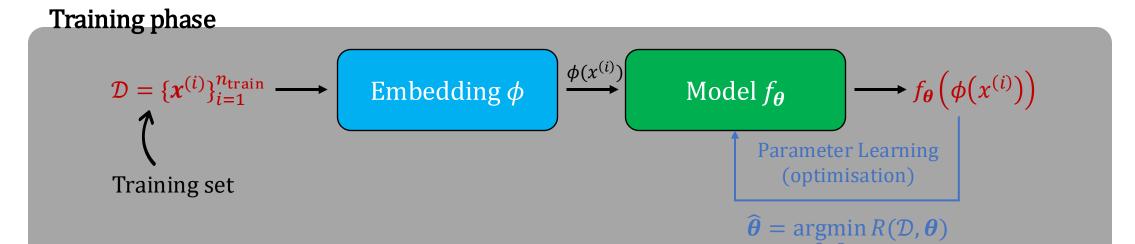
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Some notations and terminologies:

- $x^{(i)} \in \mathbb{R}^d$ is one **training data** (there are n_{train} of them),
- $\phi(x^{(i)}) \in \mathbb{R}^{d'}$ is an embedding of $x^{(i)}$ sometimes called *feature vector*,
- $\theta \in \Theta \subset \mathbb{R}^p$ are the *parameters* of the model,
- $R(\mathcal{D}, \boldsymbol{\theta})$ is the **risk** and measures the error of the model with parameters $\boldsymbol{\theta}$ on data \boldsymbol{X} .

At the end of the training procedure, we have a model $f_{\widehat{\theta}}$ committing an error of $R_{\text{train}} = R(\mathcal{D}, \widehat{\boldsymbol{\theta}})$ on the training set.

ML building blocks

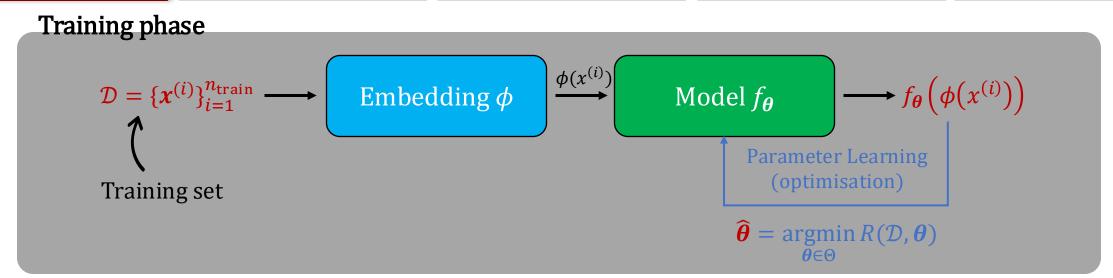
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Generalisation phase

$$\widetilde{\mathcal{D}} = \{\widetilde{\mathbf{x}}^{(i)}\}_{i=1}^{n_{\text{test}}} \longrightarrow \operatorname{Embedding} \phi \xrightarrow{\phi(\mathbf{x}^{(i)})} \operatorname{Model} f_{\widehat{\boldsymbol{\theta}}} \longrightarrow f_{\widehat{\boldsymbol{\theta}}} \left(\phi(\widetilde{\mathbf{x}}^{(i)})\right)$$
Test set

Using the test set, we can evaluate the test error $R_{\text{test}} = R(\widetilde{\mathcal{D}}, \widehat{\boldsymbol{\theta}})$ and compare it to R_{train} to detect **generalisation issues** (**overfitting** or **underfitting**).

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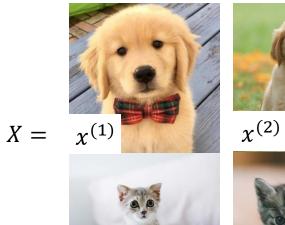
Supervised learning

• Training data are actually *X* and *Y* coming as pairs

 $x^{(i)}$ is the *i*th **data vector** of the training base, and $y^{(i)}$ is called the **target (or predicted) variable**

$$\mathcal{D} = \left\{ \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \right\}_{i=1}^{n_{\text{train}}}, \qquad \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \in \mathbb{X} \times \mathbb{Y}$$

• If Y is continuous, then the task is called regression, and if Y is discrete, then it is a classification problem.



 $\chi^{(3)}$

x⁽²⁾

Example: Determine if an image encodes a cat or a dog (called a **classification** task)

$$Y = \{1, 1, 0, 0\} \qquad f_{\theta}(x^{(i)}) = \widehat{y}^{(i)} \longrightarrow f_{\widehat{\theta}}$$
$$\widehat{\theta} = \underset{\theta}{\operatorname{argmin}} R(\mathcal{D}, \theta)$$

Training data

Model

Optimisation

Learning through empirical risk minimisation

Introduction to ML

Supervised learning

Training data are actually *X* and *Y* coming as pairs

$$\mathcal{D} = \left\{ \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \right\}_{i=1}^{n_{\text{train}}}, \qquad \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \in \mathbb{X} \times \mathbb{Y}$$

- If \mathbb{Y} is continuous, then the task is called regression, and if \mathbb{Y} is discrete, then it is a classification problem.
- Ideally, we would like to minimise the expected risk, i.e. the expected value of a loss function $\ell(y,\hat{y})$

$$R(\mathcal{D}, \boldsymbol{\theta}) = \mathbb{E}_{X,y}[\ell(y, \hat{y})]$$

 $R(\mathcal{D}, \pmb{\theta}) = \mathbb{E}_{X,y}[\ell(y, \hat{y})]$ **Loss function:** measures how bad your model is on a single example



However, we do not know p(X, y) so in practice we rely on the **empirical risk** instead

$$\widehat{R}(\mathcal{D}, \boldsymbol{\theta}) = \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} \ell(y^{(i)}, \widehat{y}^{(i)}).$$

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Training data are actually X and Y coming as pairs

$$\mathcal{D} = \left\{ \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \right\}_{i=1}^{n_{\text{train}}}, \qquad \left(\boldsymbol{x}^{(i)}, \boldsymbol{y}^{(i)} \right) \in \mathbb{X} \times \mathbb{Y}$$

• If \mathbb{Y} is continuous, then the task is called regression, and if \mathbb{Y} is discrete, then it is a classification problem.

Examples of tasks

Classification

Regression

Timeseries prediction

Segmentation

Examples of models

Artificial Neural network

Random forest

Linear regression

Logistic regression

Naïve Bayes

Nearest neighbours

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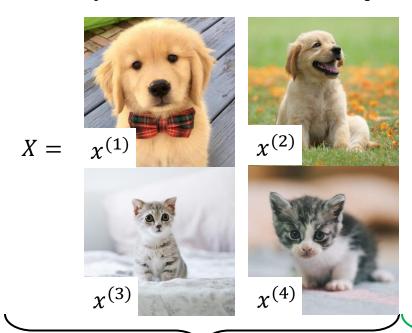
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Unsupervised learning

- Training data are the set of $x^{(i)}$'s only; no known results to predict
- In unsupervised learning, one seeks **patterns or structures** in *X* without prior labels
- Usually boils down to model the probability distribution of the dataset



Example: Generate new images of cats and dogs (called a sampling task)

$$f_{\theta}(x) = p_{\theta}(x)$$
 $f_{\widehat{\theta}}$
$$\widehat{\theta} = \underset{\theta}{\operatorname{argmin}} R(\mathcal{D}, \theta)$$
 such that $p_{\theta}(x) \approx p(x)$

Training data

Model

Optimisation

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Unsupervised learning

- Training data are the set of $x^{(i)}$'s only; no known results to predict
- In unsupervised learning, one seeks **patterns or structures** in *X* without prior labels
- Usually boils down to model the probability distribution of the dataset

Examples of tasks

Clustering Data augmentation Dimensionality reduction Sampling

Examples of models

Autoencoder

Boltzmann Machine

Diffusion models

Gaussian mixture model

Generative Adversarial network

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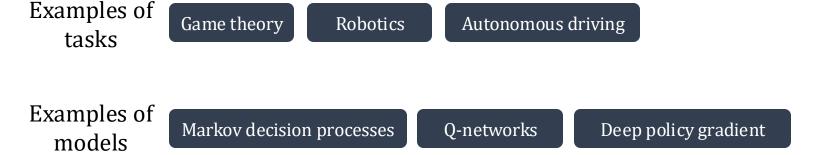
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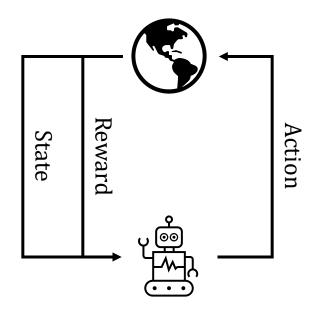
Frees and neural network:

Risk optimisatioi

Reinforcement learning

- The philosophy is different: the model does not try to "imitate" like in supervised learning nor to find patterns but "tries" things
- It is based on an agent interacting with an environment
- The agent tries to find the best possible sequence of states and actions to maximise a reward





Why do we need ML?

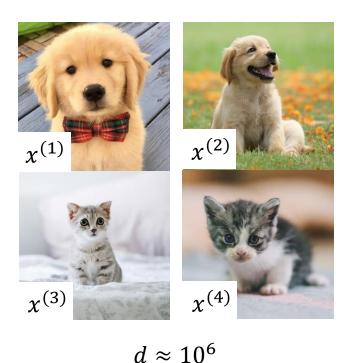
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1-nearest neighbour

 A simple classification rule is for instance associating to a data the label of its closest neighbour in the *d*-dimensional space.

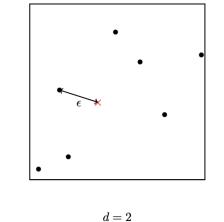
$$\hat{y} = y^{(m)} \text{ with } m = \operatorname{argmin}_i ||x - x^{(i)}||_2^2$$

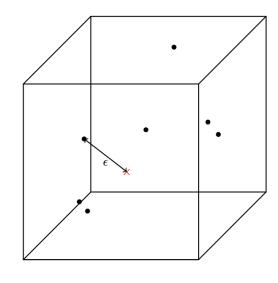
In this case $R_{\text{train}} = 0$ but R_{test} is very large! Why?

Curse of dimensionality

d = 1

- To sample a $[0,1]^d$ space with a shortest distance to a test point at most ϵ , we need $n_{\rm train} \geq \epsilon^{-d} = e^{-d\log \epsilon}$
- $d \approx 80$ requires more samples than the number of atoms in the universe





d = 3

Why do we need ML?

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Traditional methods typically break down in high-dimensional spaces (curse of dimensionality) and it is impossible to design handcrafted decision rules for complex tasks.

The curse of dimensionality is the central problem of machine learning. To fight it, ML relies on prior information about the problem:

- Find an appropriate embedding or feature representation of the data to simplify the problem,
- Exploit **structures** in the data (invariances, sparsity, long-range correlations, etc.) to define the model,
- Penalise complex models leading to poor generalisation performances using regularisation.

Linear models on feature vectors

Contents:

- Linear regression model
- L2-loss for regression and normal equations
- Linear classification model
- Softmax function, cross-entropy loss for classification



Linear regression: the model

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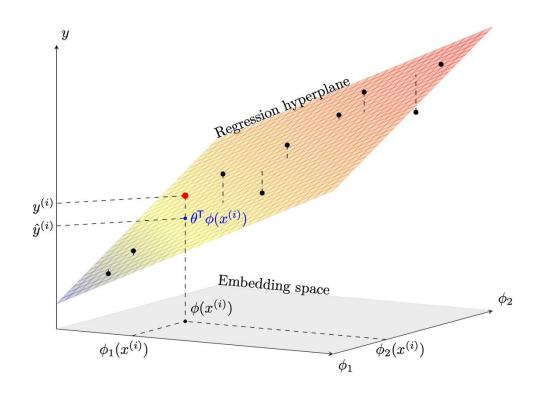
Linear regression

- What kind of problems one can solve efficiently, even in large dimensions? → Linear systems!
- Let us talk first about regression: the answer is modelled as

$$f_{\boldsymbol{\theta}}\left(\phi_1^{(i)}, \phi_2^{(i)}, \cdots\right) = \boldsymbol{\theta}^{\mathrm{T}} \boldsymbol{\phi}^{(i)} = \hat{y}^{(i)}$$

• It is sometimes convenient to add an affine term (also called **bias** in the neural network literature), which can be absorbed in the feature vector making it of dimension d'+1 where $\boldsymbol{\theta}=[\theta_0,\theta_1,\theta_2,\cdots]^T$, $\boldsymbol{\phi}^{(i)}=\left[1,\phi_1^{(i)},\phi_2^{(i)},\cdots\right]^T$.

• **Geometric interpretation**: projection of an embedding vector onto a **hyperplane** parameterised by θ .



Linear regression: example and ERM

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Linear regression

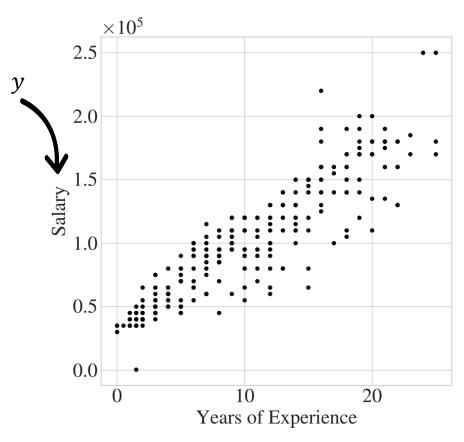
- Example: salary prediction based on the years of experience
- Data are n = 373 couples $(\phi^{(i)}, y^{(i)}) \Rightarrow$ **Supervised learning**
- The target variable $y \in \mathbb{R}$ is continuous \Rightarrow Regression
- The linear model is

$$\hat{y}^{(i)} = \theta_0 + \theta_1 \phi_1^{(i)},$$

where $\phi_1^{(i)}$ is the nb. of years of experience of the $i^{ ext{th}}$ training example

- Now the model is fixed, how to find $\widehat{\boldsymbol{\theta}}$, the best possible parameters for our model and data?
- This is done using empirical risk minimisation (ERM)

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} R(\mathcal{D}, \boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} \ell(\widehat{y}^{(i)}, y^{(i)})$$





Linear regression: solution to ERM

Linear models

Linear regression

A common **choice** of loss for regression is a **squared loss function**

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \frac{1}{n_{\text{train}}} \sum_{i=1}^{n_{\text{train}}} (\widehat{y}^{(i)} - y^{(i)})^2$$

Here, the optimisation problem can be solved analytically in closed-form. Rewriting the risk matricially, we have

$$R(\mathbf{X}, \boldsymbol{\theta}) = \frac{1}{n_{\text{train}}} \|\mathbf{\Phi}\boldsymbol{\theta} - \mathbf{y}\|_{2}^{2}$$

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1^{(1)} & \cdots & \phi_{d\prime}^{(1)} \\ \vdots & \ddots & \vdots \\ \phi_1^{(n_{\text{train}})} & \cdots & \phi_{d\prime}^{(n_{\text{train}})} \end{pmatrix} \in \mathbb{R}^{n_{\text{train}} \times d'} \qquad \mathbf{Target \, vector} \\ \mathbf{y} = \begin{bmatrix} y^{(1)}, \dots, y^{(n_{\text{train}})} \end{bmatrix}^{\text{T}} \in \mathbb{R}^{n_{\text{train}}}$$

$$\mathbf{y} = \left[y^{(1)}, \dots, y^{(n_{\text{train}})} \right]^{\text{T}} \in \mathbb{R}^{n_{\text{train}}}$$

The analytical minimisation of the squared loss in linear regression gives the unique solution (when d' < n)

known as *normal equations*

$$\widehat{\boldsymbol{\theta}} = (\boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{y}$$