Semi-supervised Learning with Deep Generative Models

Kingma et. al. (2014)

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CS 7180

Motivating Question

How can we model data of increasing size when obtaining label information is difficult?

High-level Answer

We can estimate missing label information by using a probabilistic model.

Specifying the Probabilistic Model for Missing Labels

- Data appears as pairs $(\mathbf{X}, \mathbf{Y}) = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)\}$ with the i-th observation $x_i \in \mathbb{R}^D$ and a corresponding class label $y_i \in \{1, ..., L\}$
  - Each pair of observations (x_i, y_i) has a corresponding latent variable z_i
  - Empirical distribution over the labelled and unabelled subsets is referred to as $\tilde{p}_l(\mathbf{x}, y)$ and $\tilde{p}_u(\mathbf{x})$
- We can estimate y_i for x_i in distribution $\tilde{p}_u(\mathbf{x})$ by finding the maximum probability of $p(y_i)$ by using a set of features related to z_i and a predictive model
  1. **Latent-feature discriminative model (M1)**
  2. **Generative semi-supervised model (M2)**
  3. **Stacked generative semi-supervised model (M1+M2)**

Bayes Rule is used when specifying M1 & M2

TODO: this slide doesn't transition well....

$$
\begin{aligned}
p(x, y) &= p(x)p(y|x) \\
&= p(y)p(x|y) \\
p(x|y) &= \frac{p(x)p(y|x)}{p(y)}
\end{aligned}
$$

We need to find an inferred posterior distribution $p_\theta(.)$ for M1 [1]

$$p_\theta(\mathbf{x}|\mathbf{z})$$

and M2 [2]

$$p_\theta(\mathbf{x}|y, \mathbf{z})$$

[1] Kingma et. al. (2014) equation (1)

[2] Kingma et. al. (2014) equation (2)

(M1) Latent-feature discriminative model

$$y \Leftarrow p(z|x) = \frac{p(z)p(x|z)}{p(x)}$$

where

$p(z) = \mathcal{N}(z|0, I)$ Gaussian distribution of z given a missing label y

$p(x|z) = f(x; z, \theta)$ likelihood function, parameters θ of a set of z

$p(x) = \tilde{p_u}(x)$ unlabelled subset of $x_i \in \mathbb{R}^D$

Kingma et. al. (2014) eq. (1)

(M1) Predicting Class Labels y

Approximate samples from the posterior distribution over the latent variables $p(z|x)$ are used as features to train a classifier that predicts class labels y

- (transductive) SVM
- multinomial regression

TODO: Add pictures or simulation here

(M2) Generative semi-supervised model

$$p(y|\mathbf{x}) = \frac{p(\mathbf{x}|y)p(y)}{p(\mathbf{x})} \approx \frac{p_\theta(\mathbf{x}|y,\mathbf{z})p(y)}{p(\mathbf{x})}$$

where

$p(y) = Cat(y\|\pi)$	multinomial distribution, y can be latent
$p(\mathbf{z}) = \mathcal{N}(\mathbf{z}\|\mathbf{0},\mathbf{I})$	Gaussian distribution of z when missing y
$p_\theta(\mathbf{x}\|y,z) = f(\mathbf{x}; y, \mathbf{z}, \theta)$	likelihood function, nonlinear parameters
$p(x)$	all x in dist. of real numbers; $x \in \mathbb{R}^D$