Semi-supervised Learning with Deep Generative Models

Kingma et. al. (2014)

Tyler Brown

CS 7180

Motivating Question

How can we model data of increasing size when obtaining label information is difficult?

High-level Answer

We can estimate missing label information by using a probabilistic model.

Specifying the Probabilistic Model for Missing Labels

- Data appears as pairs $(\mathbf{X}, \mathbf{Y}) = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)\}$ with the i-th observation $x_i \in \mathbb{R}^D$ and a corresponding class label $y_i \in \{1, ..., L\}$
  - Each pair of observations (x_i, y_i) has a corresponding latent variable z_i
  - Empirical distribution over the labelled and unabelled subsets is referred to as $\tilde{p}_l(\mathbf{x}, y)$ and $\tilde{p}_u(\mathbf{x})$
- We can estimate y_i for x_i in distribution $\tilde{p}_u(\mathbf{x})$ by finding the maximum probability of $p(y_i)$ by using a set of features related to z_i and a predictive model
  1. **Latent-feature discriminative model (M1)**
  2. **Generative semi-supervised model (M2)**
  3. **Stacked generative semi-supervised model (M1+M2)**

Bayes Rule is used when specifying M1 & M2

$$
\begin{aligned}
p(x, y) &= p(x)p(y|x) \\
&= p(y)p(x|y) \\
p(x|y) &= \frac{p(x)p(y|x)}{p(y)}
\end{aligned}
$$

We need to find an inferred posterior distribution $p_\theta(.)$ for M1 [1]

$$p_\theta(\mathbf{x}|\mathbf{z})$$

and M2 [2]

$$p_\theta(\mathbf{x}|y, \mathbf{z})$$

[1] Kingma et. al. (2014) equation (1)
[2] Kingma et. al. (2014) equation (2)