Semi-supervised Learning with Deep Generative Models

Kingma et. al. (2014)

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CS 7180

Motivating Question

How can we model data of increasing size when obtaining label information is difficult?

High-level Answer

We can estimate missing label information by using a probabilistic model.

Specifying the Probabilistic Model for Missing Labels

- Data appears as pairs $(\mathbf{X}, \mathbf{Y}) = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)\}$ with the i-th observation $x_i \in \mathbb{R}^D$ and a corresponding class label $y_i \in \{1, ..., L\}$
  - Each pair of observations (x_i, y_i) has a corresponding latent variable z_i
  - Empirical distribution over the labelled and unabelled subsets is referred to as $\tilde{p}_l(\mathbf{x}, y)$ and $\tilde{p}_u(\mathbf{x})$
- We can estimate y_i for x_i in distribution $\tilde{p}_u(\mathbf{x})$ by finding the maximum probability of $p(y_i)$ by using a set of features related to z_i and a predictive model
  1. **Latent-feature discriminative model (M1)**
  2. **Generative semi-supervised model (M2)**
  3. **Stacked generative semi-supervised model (M1+M2)**

Bayes Rule is used when specifying M1 & M2

$$
\begin{aligned}
p(x, y) &= p(x)p(y|x) \\
&= p(y)p(x|y) \\
p(x|y) &= \frac{p(x)p(y|x)}{p(y)}
\end{aligned}
$$

for models M1 [1], $p(z|x)$, and M2 [2]; $p(y|x)$

[1] Kingma et. al. (2014) equation (1)
[2] Kingma et. al. (2014) equation (2)

(M1) Latent-feature discriminative model

$$y \Leftarrow p(z|x) = \frac{p(z)p(x|z)}{p(x)}$$

where

$$\begin{aligned} p(z) &= \mathcal{N}(z|0, I) && \text{Gaussian distribution of } z \text{ given a missing label } y \\ p(x|z) &= f(x; z, \theta) && \text{likelihood function, parameters } \theta \text{ of a set of } z \\ p(x) &= \tilde{p_u}(x) && \text{unlabelled subset of } x_i \in \mathbb{R}^D \end{aligned}$$

Kingma et. al. (2014) eq. (1)

(M1) Predicting Class Labels y

Approximate samples from the posterior distribution over the latent variables $p(z|x)$ are used as features to train a classifier that predicts class labels y

- (transductive) SVM
- multinomial regression

TODO: Add pictures or simulation here

(M2) Generative semi-supervised model

$$p(y|\mathbf{x}) = \frac{p(\mathbf{x}|y)p(y)}{p(\mathbf{x})} \approx \frac{p_\theta(\mathbf{x}|y,\mathbf{z})p(y)}{p(\mathbf{x})}$$

where

$p(y) = Cat(y\|\pi)$	multinomial distribution, y can be latent
$p(\mathbf{z}) = \mathcal{N}(\mathbf{z}\|\mathbf{0},\mathbf{I})$	Gaussian distribution of z when missing y
$p_\theta(\mathbf{x}\|y,z) = f(\mathbf{x};y,\mathbf{z},\theta)$	likelihood function, nonlinear parameters
$p(x)$	all x in dist. of real numbers; $x \in \mathbb{R}^D$

Stacked generative semi-supervised model (M1 + M2)

Combine M1 and M2

1. Learn a new latent representation z_1 from M1
2. Use embeddings from z_1 instead of raw data x, to create a generative semi-supervised model M2

TODO: Add a picture or something here

Scaling Up: Lower Bound Objective

Lower Bound Objective[3]: computation of the exact posterior distribution is intractable for models M1 and M2

$$\text{M1:}\ \ q_\phi(z|x) = \mathcal{N}(z|u_\phi(x), \mathrm{diag}(\sigma^2_\phi(\mathbf{x}))) \tag{3}$$

$$\begin{aligned} \text{M2:}\ \ q_\phi(\mathbf{z}|y,\mathbf{x}) &= \mathcal{N}(\mathbf{z}|\mu_\phi(y,\mathbf{x}), \mathrm{diag}(\sigma^2_\phi(\mathbf{x}))); \\ q_\phi(y|\mathbf{z}) &= \mathrm{Cat}(y|\pi_\phi(x)), \end{aligned} \tag{4}$$

where

$\sigma_\phi(x)$	vector of standard deviations
$\pi_\phi(x)$	probability vector
$\mu_\phi(x), \sigma_\phi(x), \pi_\sigma(x)$	Maximum likelihood Priors (MLPs)

[3]Kingma et. al. (2014) equations (3), (4)

Scaling Up: M1 Model Objective

$$\log p_\theta(x) \geq \mathbb{E}$$

Scaling Up: M2 Model Objective