Response to the referee

We thank the referee for their critical assessment of our work. In the following we address their concerns point by point. All changes made to the manuscript have been boldfaced.

Referee Point 1) — Since neither set of models using the two different opacity tables provide a good match to the properties of the Jao gap, the possibility that the convective kissing instability is not the correct explanation for the gap has to be considered.

The first paper to offer an explanation for the existence of the gap (MacDonald & Gizis 2018) states 'Convective mixing is treated as a diffusion process with the diffusion coefficient determined from mixing length theory.' and 'The fully implicit nature of our code also prevents the convective kissing instability discovered and described by van Saders & Pinsonneault (2012).' The treatment of convective mixing as a diffusion process is probably the reason why MacDonald & Gizis find a single episode of convection zone merger (for models in which merger occurs) and hence a single dip in the luminosity function, which seems to be the case for the Jao gap (inferred from figure 1 of this paper).

In contrast, the use of instantaneous mixing leads to multiple episodes of convection zone merger and this could be the reason why the authors find two dips in the luminosity function (inferred from their figures 7 and 8). The authors need to give in the paper a convincing argument as to why the instantaneous mixing approximation is valid, particularly as using the diffusion approach seems to more physically consistent with current understanding of turbulent mixing. This should involve estimates using mixing length theory of the mixing time scale at all stages of the merger, with special attention to mixing time scales within one mixing length on either side of the point of contact between the merging convection zones. This region is where the mixing time scale is most likely to be the longest.

The mixing time scale then needs to compared to other relevant time scales including the time scale for deuterium to come into equilibrium and the time scale at which the helium-3 abundance is modified by nuclear reactions.

If it turns out that the instantaneous mixing approximation is not valid, then there needs to be discussion of alternative approaches and their consequences. The thrust of the paper could be changed to show that the CKI is not the correct explanation of the gap.

Reply: We respectfully disagree with the referee that a diffusive model of convective mixing should be preferred over an instantaneous model of convective mixing. Much of the following has been included as text in the manuscript.

Chabrier & Baraffe 1997 identify that an instantaneous mixing model of convection may have a strong suppressing effect on a models luminosity. The root cause of this effect is the build up of a deuterium gradient in a models core when the mixing timescale is significantly longer than the deuterium burning timescale. Chabrier & Baraffe estimate the mixing timescale for a fully convective star ($\sim 0.1 M_{\odot}$) is 10^8. However, these estimates relied on the mixing length approximation of convection and make use of assumptions only valid in super-adiabatic regions (estimating the velocity of turbulent eddies as $(\nabla - \nabla_{adb})^{1/2}$). Whereas, in 1997 these estimates were the best available, currently hydrodynamical approximations, which are more physically motivated and more accurate, should be preferred. These hydrodynamical approximations of the mixing timescale are an order of magnitude shorter than those Chabrier and Baraffe estimate (10^7s compared to 10^8s). Moreover, Chabrier and Baraffe make use of the mixing time over the entire radius of the star (the overturn time). However, when judging the efficacy of an instantaneous mixing approximation the more relevant timescale is the mixing time over one shell. For the configuration we evolve models using (with 5000 shells), the mixing time over one shell will be of order 10^4s. This is now much shorter than the deuterium lifetime against proton capture. Therefore, the deuterium gradient predicted in Chabrier & Baraffe — which MacDonald & Gizis 2018 address through use of a diffusive model of convection — will not build up and need not be accounted for.

In addition to these timescale arguments, we direct the referee to recent work conducted by the MESA collaboration (Jermyn et al. 2022 Figure 2 & private communication with Aaron Dotter - Figures). MESA has recently implemented time dependent convection (TDC) in addition to standard mixing length theory (MLT) making use of instantaneous mixing. The models presented in Figures 1 and 2 are for stars within the Jao Gap (identified in MESA in Mansfield & Kroupa 2021). Note the extreme similarity between the instantaneous approximation and the time dependent convective models. Additionally, in the lower panel of Figure 2 note that both the MLT and TDC models show periodic dips in the core ^{3}He mass fraction, as is predicted by CKI.

For both of these reasons we content that an instantaneous mixing model of convection is a good approximation in this instance and that the results we present, showing kissing instability, are valid.

Referee Point 2) — The convective mixing method used by MacDonald & Gizis needs to be properly described along with the resulting differences from using instantaneous mixing.

Reply: Additional text has been added to the paper addressing the results of MacDonald & Gizis.

Referee Point 3) — On line 184, the authors say that the OPLIB tables were created to resolve the discrepancy between helioseismic and solar model predictions of chemical abundances in the Sun. I find this to be an odd way to phrase the problem. Presumably the authors are referring to the difficulty of making solar models that match the sound speed profile (or almost equivalently the depth of the surface convection zone) determined by helioseismology given the composition constraints provided by the then recent new measurements of the surface abundances, notably the oxygen abundance. The authors should rephrase the problem and also discuss whether using OPLIB opacities help resolve the problem or not.

Reply: We agree with the referee that the invocation of helioseismic discrepancies is confusing. Because this is not a helioseismic paper we drop from the text, instead just mentioning that the OPLIB tables make use of the most up to date physics, which is the most relevant point to our work. We do note here though that the OPLIB opacities did not serve to resolve any discrepancies.

Referee Point 4) — The distinction between low temperature and high temperature opacity sources made in the paragraph beginning on line 196 is somewhat artificial. Presumably the transition between low temperature and high temperature opacity is chosen to be between $10^{4.3}$ and $10^{4.5}$ K because there is where the Ferguson et al. opacities and OPAL opacities are close to each other. Perhaps the evolution could be affected if a different temperature range for the transition is used. The authors should stress in this section that as far as modeling the gap the main impact of using different opacities is on the radiative zone, and give the temperature and R (or density) ranges that are relevant to the radiative zone(s) so that the reader can see from figure 3 the expected change in opacity, and also if log R = -1.5 is truly a representative value in the radiative zone.

Also, from figure 3, it seems that the OPLIB opacities are lower than the OPAL opacities for temperatures greater than $10^{5.5}$ K and not 10^5 K as stated in the figure caption and also on line 203.

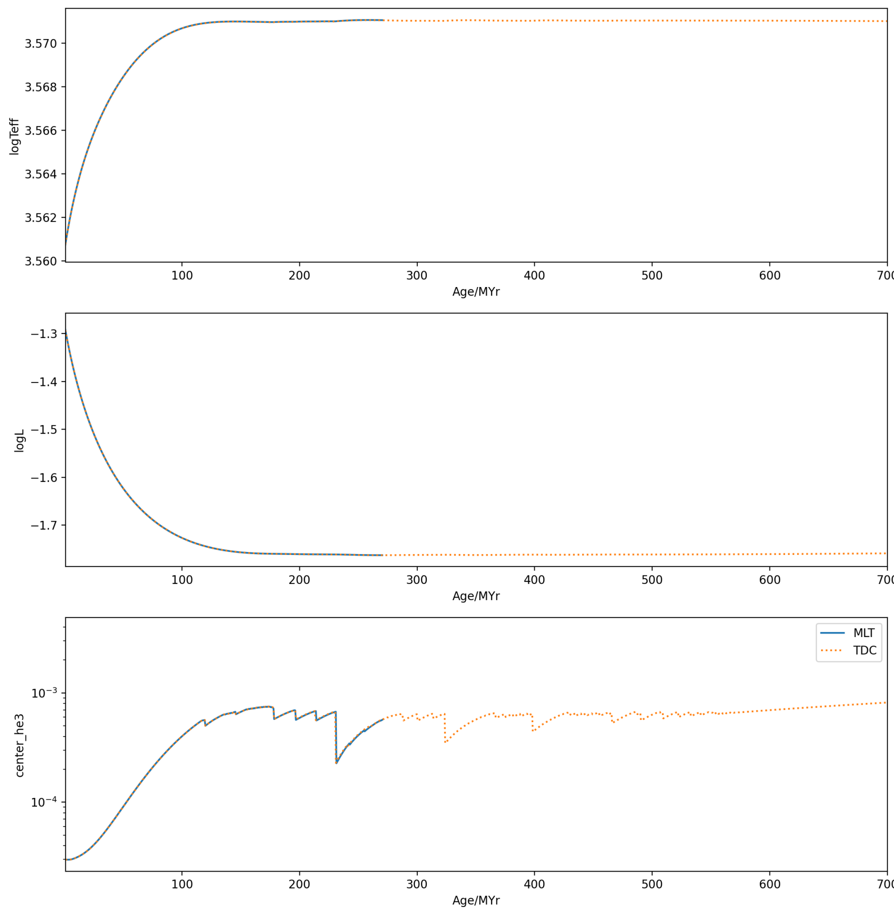

Figure 1: Results aquired through private communication with Aaron Dotter showing MESA models from Mansfield & Kroupa 2021 (a model near the Jao Gap). One model makes use of a standard instantaneous mixing approximation while the other uses the newly introduced Time Dependent Convection. Note the similarity between the models

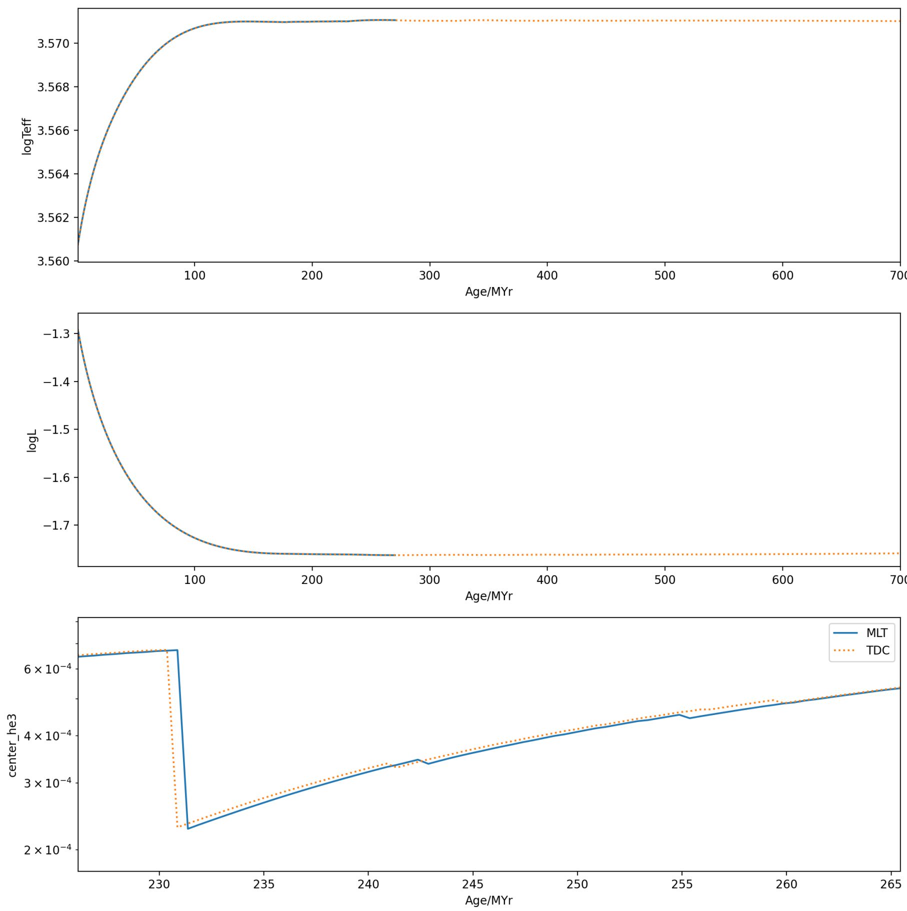

Figure 2: Results aquired through private communication with Aaron Dotter showing MESA models from Mansfield & Kroupa 2021 (a model near the Jao Gap). One model makes use of a standard instantaneous mixing approximation while the other uses the newly introduced Time Dependent Convection. Note the similarity between the models

Reply: The range where DSEP ramps from low temperature to high temperature opacities is determined by the temperature range where molecules can start to form. It is important for the low temperature opacities to be the only opacity source by the time the first molecules start forming. We choose to ramp the opacity source so that there is not a hard discontinuity. This same ramp has been used as standard in all DSEP models since 2008, see Dotter et al. 2008 for further details.

We have updated references in our paper to Log(R)=-1.5 as a good approximation throughout a star. For a GS98 solar composition 1 solar mass model this is a good approximation; however, we calculate Log(R) throughout a 0.356 solar mass model (the same model used to generate Figure 2 in the manuscript) and find that a more representative value of Log(R) is -0.79. It is not surprising that this value is somewhat larger given the lower mass of this model and the commensurately lower pressures and temperatures.

Referee Point 5) — In section 3.2, mention is made of the solar surface Z/X ratio but the actual value is not given. Is it the value recently determined by Magg et al. (2022), Z/X = 0.0225 or some other earlier value? The authors should state the actual Z/X value used.

Also, the authors need to say whether or not they include gravitational settling and element diffusion in their solar modeling, and if they do, say how it is done (e.g. are elements grouped or treated independently). The authors should also include discussion of how well their solar models replicate the sound speed profile determined from helioseismology.

Reply: The calibrated Z/X value has been added into the text in section 3.2 along with clarification that we do include gravitational settling with elements grouped together. We do not include a discussion of sound speed in our manuscript. This paper is not a discussion of seismology so a diversion to that would be distracting for readers and out of place.

Referee Point 6) — Presumably, the authors use their solar calibrated models to set the mixing length ratio and initial abundances for their calculations of the evolution of models of low mass stars. Do they use primordial or present-day solar abundances? Why should the mixing length ratio be the same as the solar calibrated? There is evidence that the mixing length varies with stellar properties (e.g. Trampedach et al. 2014; Joyce & Chaboyer 2018). A better fit to the location of the gap might be obtained by adjusting the mixing length ratio. The authors need to address these questions.

Reply: We use GS98 solar abundances (Grevesse & Sauval 1998) for all models. While, as the referee says, there is substantial evidence of a metallicity dependence for the mixing length parameter this dependence has only been calibrated for higher mass stars. In order to fully address this point we have run an additional grid of models with the mixing length parameter dramatically lowered ($\alpha_{ML} = 1.5$ & $\alpha_{ML} = 1.0$). Results of that grid are shown in Figures 3, 4, and 5.

What is clear from these additional grids is that the Jao Gap location is sensitive to the value of the mixing length parameter with the magnitude of the gap being inversely proportional too the mixing length parameter ($G_{mag} \overset{\propto}{\sim} -0.15\alpha_{ML}$). Moreover, dramatically lowering the mixing length parameter to $\alpha_{ML} = 1.0$ does bring the location of the Jap Gap we model in closer agreement with the empirically measured location of the Gap.

However, there is no a priori reason which we are aware of to expect the mixing length parameter to be so much lower in these low mass stars than it would be in the sun. Much of the work calling into question the use of solar calibrated mixing lengths in higher mass stars has identified a [Fe/H]

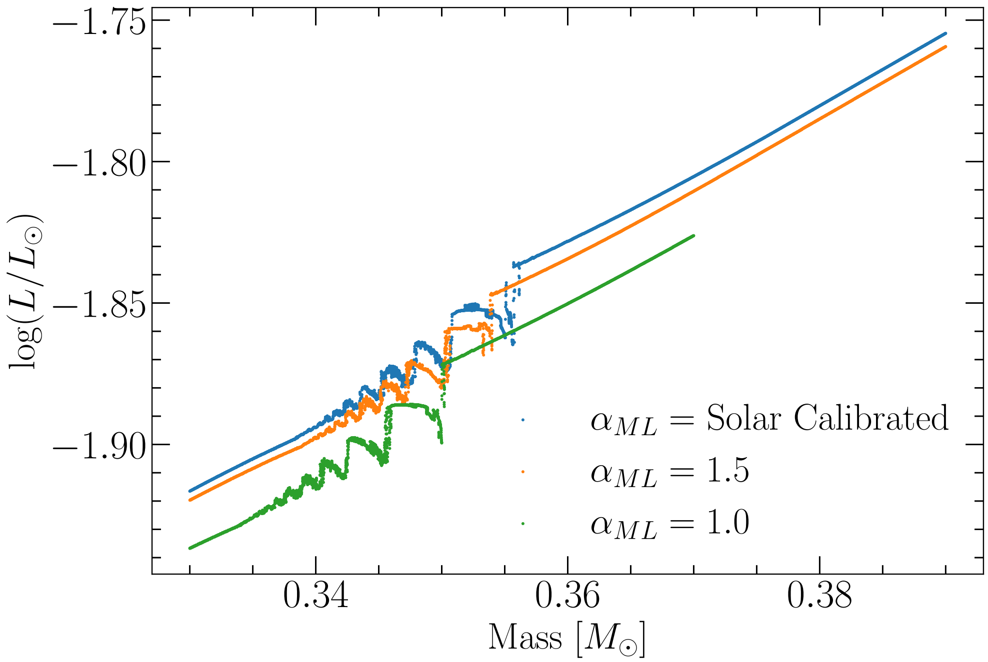

Figure 3: Mass Luminosity relation for mixing length grid. From top to bottom: solar calibrated mixing length, mixing length of 1.5, and mixing length of 1.0

dependence. Given that the population of stars which the Jao Gap has been identified in is relatively similar to the sun in composition extrapolating the empirical calibrations backward would not predict such a large dip in the mixing length.

Therefore, we have modified the manuscript to include a limited discussion of the perils of using solar calibrated mixing length parameters, along with the results of our $\alpha_{ML} = 1.5$ and 1.0 populations; however, we still favor the use of the solar calibrated mixing lengths. Finally, we have added into the conclusions a call for further work to be done studying the Jao Gap location as a potential calibration point for the mixing length of lower mass stars.

We thank the referee for pointing us in this direction.

Referee Point 7) — On trying the web interface for the OPLIB, it seems that the process of interpolating between rho and R is unnecessary. Once T is specified, it is possible to get the same set of R values as for the OPAL tables by specifying the starting value of rho. Then only interpolation in T is needed to get a table in the same form as the OPAL tables. The authors need to clarify why they chose their approach. Also, it would be helpful to include in figure 15 a line plot of fractional difference against log T for log R = -1.5 or a different R value if -1.5 is not found to be representative for the radiative zone (see point 4).

Reply: While given some fixed temperature, T, it is possible to set the density grid on the webform such that R is exactly specified. This is not feasible when using a range of temperatures and densities simultaneously. Specifically, because each temperature within such a range would require a unique density range to be specified in order to match the OPAL table R range. This is not a feature offered by the webform.

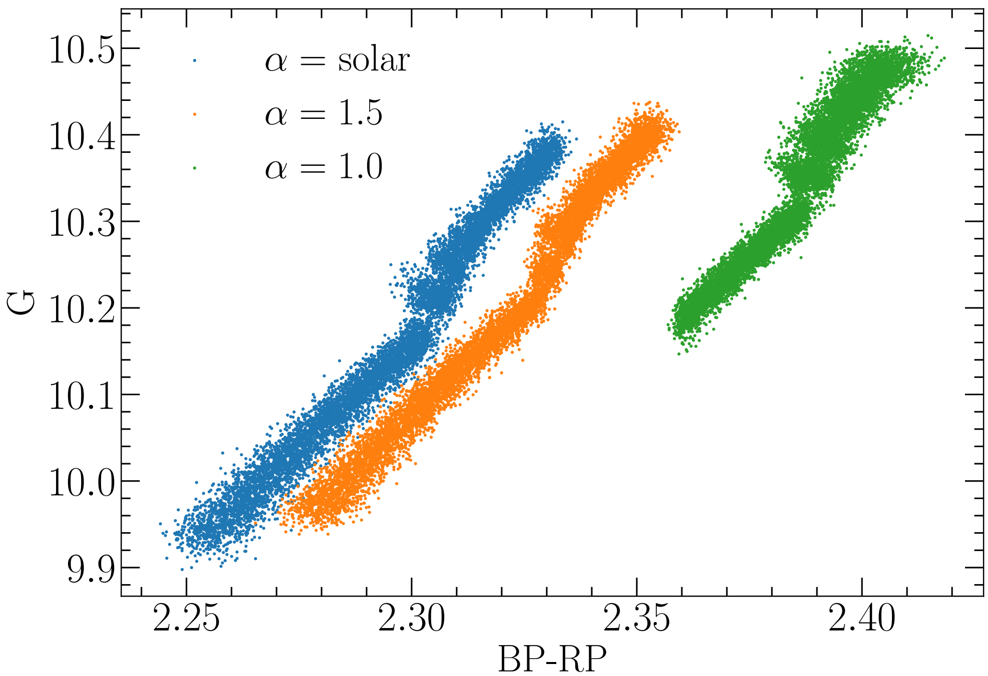

Figure 4: Jao Gap seen in three populations using OPLIB high temperature opacities. From left to right these use a solar calibrated mixing length, a mixing length of 1.5, and a mixing length of 1.0

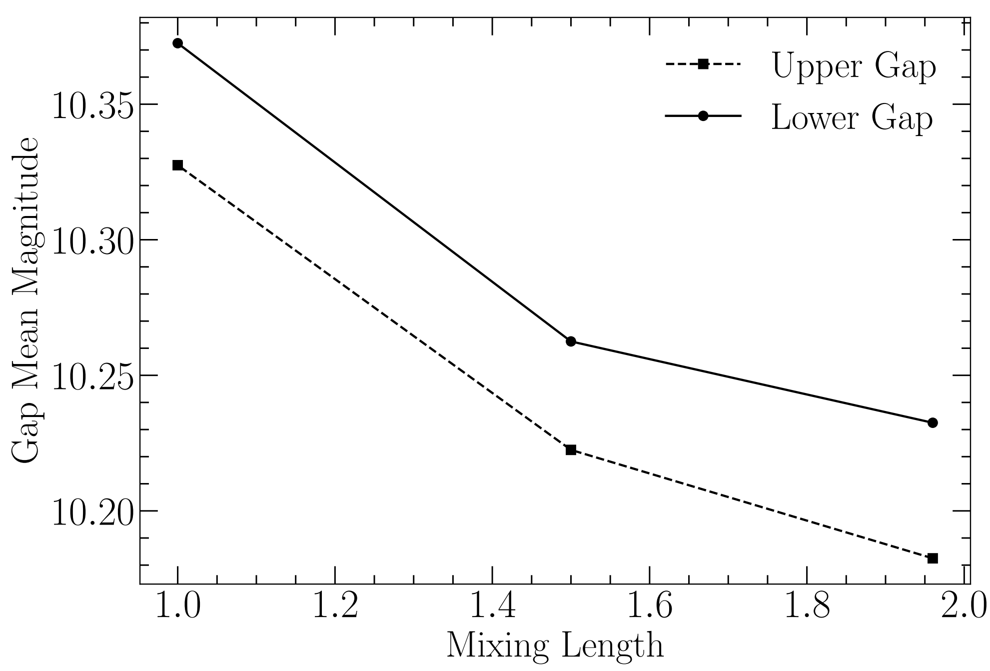

Figure 5: Identified locations of the two density dips we observe as a function of the Mixing Length used in those populations.

The reason we avoid querying a single temperature at a time is that making a separate call to the TOPS webform for not just each composition but also each temperature in that composition would increase the traffic per OPAL table by a factor of 70 (from 126 calls currently to 8820). This is both infeasible from a runtime perspective (as opening a new call is the most time intensive part of the scraper as the JavaScript engine is initialized and would likely push the generation time of a single table from 10 minutes to over 5 hours) and would place a significantly increased load on the Los Alamos servers. We have spoken with the Los Alamos team and have their consent to release this software with only with the current traffic it generates.

A brief description of this has been included in Appendix B.

The vertical line on Figure 13 has been updated to reflect the more accurate approximation of Log(R) = -0.79 throughout the star as opposed to -1.5.