Modeling an Epidemic

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The Problem

How does an infection spread through a population?

How many people should we vaccinate to stop the spread?

Our Infection

- People who have never had the infection can get it.
  - Unless they have received the vaccine.

- Once infected, people either recover or die.
  - If they recover, they become immune.
  - If they die, they are removed from the population.

This is similar to Measles, Mumps, and Rubella.

Types of People

- Susceptible: Haven't had the infection yet.
- Infected: Currently have the infection.
- Recovered: Used to have the infection (now immune).
- **Dead**: The infection killed them.

Rates

- How quickly susceptible people get infected.
  - Susceptible to Infected.
- How quickly infected people recover.
  - Infected to Recovered.
- How quickly infected people die.
  - Infected to **Dead**.

Variables

- The number of people in each category are represented by S, I, R, and **D**. Total population is N = S + I + R + **D.**
- The number of Susceptible people at time t is represented by S(t). Same for I(t), R(t), and **D**(t). N is constant.
- Whenever we go from time t to t+1, the population levels change.

Change Per Day

Susceptible

- The number of "interactions" between Susceptible and Infected is S(t) * I(t).
- StoI represents the probability an interaction will infect a susceptible person.
- So, the Susceptible group changes at rate:

$$dS/dt = -[StoI * S(t) * I(t)]$$

Change Per Day

Infected

- Enter from Susceptible
- Leave to Recovered and **Dead**.
  - There are no "interactions"
- So, the rate of change of Infected is:

dI/dt = [StoI * S(t) * I(t)] - [I(t) * ItoR] - [I(t) * Ito**D**]

Change Per Day

Recovered

- Come in from Infected.
- No one leaves.
- So, the rate of change of Recovered is:

$$dR/dt = I(t) * ItoR$$

Change Per Day

Dead

- Come in from Infected.
- No zombies.
- So, the rate for the **Dead** is:

$$d\mathbf{D}/dt = I(t) * I\text{to}\mathbf{D}$$

What It All Means

- Suppose we're at time t.
- We know all of the current population levels.
- We want to estimate t+1's levels.
- So, we calculate the change over time for each group (dS/dt, dI/dt, dR/dt, and d**D**/dt).
- Then multiply that change by our time step and add it to the current population level.
- For instance,

$$S(t+1) = (dS/dt)*tstep + S(t)$$

Vaccination

- If a Susceptible person receives the vaccine, they become immune.

- We can represent this by removing those people from Susceptible and adding them to Recovered.

- If an infection spreads on its own, how many people should we vaccinate to stop it?

Do We Have An Epidemic?

- If the Infected population is increasing at the beginning, there will be an outbreak.
- In other words, is dI/dt positive or negative?

dI/dt = [StoI * S(t) * I(t)] - [I(t) * ItoR] - [I(t) * Ito**D**]

R_0 = (StoI * N) / (ItoR + Ito**D**)

- If $R_0 > 1$, we have an epidemic.
- If $R_0 < 1$, the infection is contained.

R_0 and Vaccination

- Imagine we vaccinate a proportion *p.*
- Then, $R_{0\mathbf{p}}=(1-p)*R_0$. Or, R_0 after vaccination.
- We want $R_{0\mathbf{p}}$ to be < 1. So, call the proportion we need to vaccinate p_c. Then,

$$1 = (1-p_c)*R_0$$

$$p_c = 1 - 1/R_0$$

- So, the number of people we need to vaccinate is $N * p_c$.