# Critical Thinking about Evidence (Baysesian Belief Network)

Calculation Sheet

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## Evidence

**Evidence 1:** X lent J his car and cell phone for much of the day
**Evidence 2:** X asked the victim for a ride after school (she declined as she was busy)
**Evidence 3:** After the likely time of the murder X smoked marijuana with friends and was observed to have a high level of anxiety when the police called and asked him if he knew the whereabouts of the victim.
**Evidence 4:** J implicated X as the murderer, even though this mean J had to plead guilty to a crime (accessory after the fact) and could have been punished
**Evidence 5:** X's cell phone was used to call a friend known only to him during a time when X says J was alone with the phone.

## Justification

The justification for all of the evidence probablities comes from knowledge obtained through Serial's season 1 podcast by Sarah Koenig in 2014. The podcast delved into the States case against X and provided context to the 5 pieces of evidence used in the Bayseian Belief Network calculations.

The prior probability values were taken from an article published by the U.S. Department of Justice in 2009 on the statistics for "Female Victims of Violence."

### Key

**lent car to J:** X lent his car to J for the day (Evidence 1)
**AFR:** X asked victim for ride (Evidence 2)
**smoked:** X & J smoked marijiuna and X was nervious when police called (Evidence 3)
**AATF:** J confessed that X murdered victim and J helped cover it up (accessory after the fact) (Evidence 4)
**phone call:** X's cell was used to call someone only X knows when X claims J had the phone (Evidence 5)

#### Prior Values

P[X Guilty] - 0.20
P[J Guilty] - 0.10
P[O Guilty] - 0.70

According to the Bureau of Justice's article "Female Victims of Violence:"

> In 2007, 24% of female homicide victims were killed by a spouse or ex-spouse;
> 21% were killed by a boyfriend or girlfriend; and 19% by another family member.
> In an additional 25% of cases in 2007, females were killed by others they knew.
> An estimated 10% of female murder victims were killed by a stranger (Bureau of Justice Statistics 2015).

Based on this, let's set the prior probablity of X being guilty as 0.20 since he was the ex-boyfriend of the victim. Because J knew the victim but wasn't an intimate partner or very close with the victim, let's set his prior probablity to 0.10 and set the prior probablity for anyone else commiting the murder at 0.70.

**Evidence 1**

P[lent car to J| X Guilty] - 0.15
P[lent car to J| J Guilty] - 0.10
P[lent car to J| O Guilty] - 0.05

X said he lent his car to J so J could buy his girlfriend, Stephanie, a birthday present. X and Stephanie were good friends and he wanted her to be happy and have another gift. J maintained that X had lent him the phone and car to pick him up after X had killed the victim. The likelihood that X lent his car to J if X had did it seems possible. The birthday excuse is obviously easy to justify by knowing Stephanie's birthday, though X had said him and J weren't great friends so it seems a little weird to lend him his car. Because of these reasons, let's put the likelihood that X lent J his car is X had done it at 0.15. The probability that J had the and killed the victim seems like it could be higher. It seems like it could be a could way for him to blame X. However, it seems like this would have had to be a last minute decision from J then; he didn't know he would have the car that day. J's motive is also less clear. Let's put this probability at 0.10. Finally, the probability that X lent his car to J if a third party had committed the crimes is simply that of the possibility of him loaning his car to J on any given day. Since they weren't great friends, let's put this probability at 0.05 (Koenig 2014).

**Evidence 2**

P[AFR| X Guilty] - 0.20
P[AFR| J Guilty] - 0.05
P[AFR| O Guilty] - 0.01

The probability that X had asked the victim for a ride if he committed the murder seems likely. They had just broken up and talking might be a good thing and they still maintained their friendship. However the victim would have known that X had a car so it might've seemed off, or X would've known that the victim had to somewhere to be after school. Therefore, the probability that X asked for a ride if he did it would be put at 0.20. Neither X or J deny that J had been lent the car that day. The likelihood of X asking for a ride if J had done it, or consequentially if any other person had done it, would be just the same as any day, say 0.01. However, if we throw any speculation into the idea that X knew anything at all about J's plan, he might have wanted to be with her to try to help her, let's increase it to 0.05 (Koenig 2014).

**Evidence 3**

P[smoked| X Guilty] - 0.20
P[smoked| J Guilty] - 0.05
P[smoked| O Guilty] - 0.01

X was naturally a first suspect in the case for being a recent ex-boyfriend of the victim. One could asertain, with X being the bright person he is, that he knew this. Additionally, he had been high and certainly talking to the police while high would be a surreal experience, especially when you were trying to not only hide that you were high but also maintain your innocence. Therefore, if X had done it he would of course be nervous, he isn't a trained killer, though again he would've been nervous anyway. Put this at 0.20. Again, the likelihood of him being nervous while J or anyone else had done it would be roughly the same, again only increasing for J if X had known anything about it. X would've been anxious regardless because he was high and was worried about his friend. Say 0.05 and 0.01 again, respectively (Koenig 2014).

**Evidence 4**

P[AATF| X Guilty] - 0.70
P[AATF| J Guilty] - 0.35
P[AATF| O Guilty] - 0.01

J gave testimony that X had killed the victim and that he made him help bury the body. J said he did it for fear X would tell the police about him selling weed and other illegal things. It seems weird that J would be an accessory to murder to not get caught selling weed, yet he was so willing to come to the police with details after the fact. Because of this, the likelihood that J implicated X if X had done it would be the likelihood of J telling the truth while also implicating himself, put this at 0.70. Additionally, when considering J, there doesn't seem to be much motive behind the thought that he had killed the victim. Let's consider that J had killed the victim, he would be making this testimony almost completely up, which X, and potentially other witnesses to their day, would be able to deny or have an alibi for. Considering how risky this is, let's would put this probability rather low, increasing only to account for the idea that he was trying to cover up for himself, say 0.35. The probability that J came up with this whole story is neither him or X were the killers seems completely far fetched. J was mostly likeable and seems out of character to completely fabricate this story to incriminate himself and a friend for something they did not do, put this at 0.01 (Koenig 2014).

**Evidence 5**

P[phone call| X Guilty] - 0.10
P[phone call| J Guilty] - 0.05
P[phone call| O Guilty] - 0.02

Finally, the call made to the number only X knew while J supposedly had the phone could go many different ways. You could subscribe to the belief that X killed her and he really had the phone and made the call. There's also the possibility that J had hit her number on speed dial accidently, either while he was waiting for X for call him when X killed the victim or while he was the killer. Additionally, the receiver of the call had put them both together at the time of the call, though at a location where J worked even though he hadn't started working there until later, meaning she could be misremembering or implicating X. Because of these reasons, put the likelihood that this call was made if X killed the victim at 0.10 (since it goes against the story that J tells), the probability that J had done it seems low since he didn't know her, but her number had been on speedial, as 0.05, and the probability that it was a buttdial again, but with someone else having committed it, very low as the probability of any accidental call, 0.02 (Koenig 2014).

## Equations

The equations that follow are based on Bayes Theorem. It works off of conditional probablity where we are looking at the probablity of each suspects guilt given each piece of evidence happens.

Let's take the equation used for evidence 1 below. The probablity that suspect $i$ is guilty given that X lent his car to J is calculated by mulitping the probablity that X lent his car to J given suspect $i$ is guilty and the prior probablity of suspect $i$ being guilty and dividing that by the sum for each suspect of the probablity that X lent his car to J given suspect $i$ is guilty mulitplied by the prior probablity of suspect $i$ being guilty. This gives you the posterior probablity for each suspects guilt for evidence 1. Since there are 5 pieces of evidence, this is done 5 times with the calculated posterior probablity for evidence 1 used for the prior probablity for evidence 2 and the posterior probablity for evidence 2 is the prior probablity for evidence 3 and so on until you get the posterior probablity for evidence 5 which is the overall probablity of guilt for each suspect given all the evidence (Kammen and Hassenzahl 2001).

The diagnostic benefit equation is a way to evaulate the evidence and its effect on the probablity for the guilt of suspect $i$. It takes the ratio of the probablity of the evidence given $i$ is guilty divided by the probablity of the evidence given $i$ is not guilty (Guiran 2018).

#### Evidence 1

$$P[i|lent\ car\ to\ J] = \frac{P[lent\ car|i] * P[i]}{(P[lent\ car|X] * P[X]) + (P[lent\ car|J] * P[J]) + (P[lent\ car|O] * P[O])}$$

#### Evidence 2

$$P[i|AFR] = \frac{P[AFR|i] * P[i]}{(P[AFR|X] * P[X]) + (P[AFR|J] * P[J]) + (P[AFR|O] * P[O])}$$

#### Evidence 3

$$P[i|smoked] = \frac{P[smoked|i] * P[i]}{(P[smoked|X] * P[X]) + (P[smoked|J] * P[J]) + (P[smoked|O] * P[O])}$$

#### Evidence 4

$$P[i|AATF] = \frac{P[AATF|i] * P[i]}{(P[AATF|X] * P[X]) + (P[AATF|J] * P[J]) + (P[AATF|O] * P[O])}$$

#### Evidence 5

$$P[i|phone\ call] = \frac{P[phone\ call|i] * P[i]}{(P[phone\ call|X] * P[X]) + (P[phone\ call|J] * P[J]) + (P[phone\ call|O] * P[O])}$$

#### Diagnostic Benefit

$$Diagnostic\ Benefit = \frac{P[evidence|i]}{P[evidence|not\ i]}$$

$$P[evidence|not\ i] = \frac{(P[evidence|not\ i_1] * P[evidence\ i_1] + P[evidence|not\ i_2] * P[evidence\ i_2]}{1 - P[i\ guilty]}$$

## Calculations

Tables 1-5 show the input and output for each piece of evidence using Bayes Theorem for the posterior probablity. An explanation of how the calculations were performed is in the previous section called Equations. The probability output from evidence 1 in Table 1 is used as the prior probablity for evidence 2 in Table 2 and so on for the calucaltions in evidence 3-5 in Tables 3-5. The output probablity in Table 5 is the overall probablity that each subject could have committed the murder given the evidence.

The overall calculated probablity that each suspect is guilty is:
P[X Guilty] - 0.99
P[J Guilty] - 0.01
P[O Guilty] - 0.00

Evidence 2 (Table 2) has the greatest diagnostic benefit for the probablity that X committed the crime, while Evidence 1 (Table 1) has the greatest diagnostic benefit for both J and O.

Table 1: Evidence 1 Probablities

| | **Inputs** | | **Outputs** | |
|---|---|---|---|---|
| **Suspect** | **P[i Guilty]** | **P[lent car to J\|i Guilty]** | **P[i\|lent car to J]** | **Diagnostic Benefit** |
| X | 0.20 | 0.15 | 0.40 | 2.67 |
| J | 0.10 | 0.10 | 0.13 | 1.38 |
| O | 0.70 | 0.05 | 0.47 | 0.38 |

Table 2: Evidence 2 Probablities

| | **Inputs** | | **Outputs** | |
|---|---|---|---|---|
| **Suspect** | **P[i Guilty]*** | **P[AFR†\|i Guilty]** | **P[i\|AFR]** | **Diagnostic Benefit** |
| X | 0.40 | 0.20 | 0.88 | 10.59 |
| J | 0.13 | 0.05 | 0.07 | 0.51 |
| O | 0.47 | 0.01 | 0.05 | 0.06 |

* P[i Guilty] for evidence 2 is the output value for P[i|lent car to J] from evidence 1
† AFR = X asked victim for ride

Table 3: Evidence 3 Probablities

| | **Inputs** | | **Outputs** | |
|---|---|---|---|---|
| **Suspect** | **P[i Guilty]*** | **P[smoked†\|i Guilty]** | **P[i\|smoked]** | **Diagnostic Benefit** |
| X | 0.88 | 0.20 | 0.98 | 5.96 |
| J | 0.07 | 0.05 | 0.02 | 0.26 |
| O | 0.05 | 0.01 | 0.00 | 0.05 |

* P[i Guilty] for evidence 3 is the output value for P[i|AFR] from evidence 2
† smoked = X and J smoked marijiuana and X was nervous when police called asking about victim

Table 4: Evidence 4 Probablities

| | Inputs | | Outputs | |
|---|---|---|---|---|
| **Suspect** | **P[i Guilty]*** | **P[AATF†\|i Guilty]** | **P[i\|AATF]** | **Diagnostic Benefit** |
| X | 0.98 | 0.70 | 0.99 | 2.27 |
| J | 0.02 | 0.35 | 0.01 | 0.50 |
| O | 0.00 | 0.01 | 0.00 | 0.01 |

* P[i Guilty] for evidence 4 is the output value for P[i|smoked] from evidence 3

† AATF = J confessed that X murdered victim and J helped cover it up (accessory after the fact)

Table 5: Evidence 5 Probablities

| | Inputs | | Outputs | |
|---|---|---|---|---|
| **Suspect** | **P[i Guilty]*** | **P[phone call†\|i Guilty]** | **P[i\|phone call]** | **Diagnostic Benefit** |
| X | 0.99 | 0.10 | 0.99 | 2.00 |
| J | 0.01 | 0.05 | 0.01 | 0.50 |
| O | 0.00 | 0.02 | 0.00 | 0.20 |

* P[i Guilty] for evidence 5 is the output value for P[i|AATF] from evidence 4

† phone call = X's cell phone was used to call someone only X knows while X claims J had the phone

## Uncertainty Analysis

As a part of this analysis, an uncertainty analysis is appropriate to see how uncertainty is associated with the probability of guilt for each suspect (X, J, and O) after each piece of evidence was considered. In order to accomplish this, a monte carlo simulation was performed for each of the steps in the bayesian belief network. To perform the monte carlo simulation, random data had to be generated. The random data were generated for the prior probabilities of guilt for each suspect and the likelihood of each evidence being true given a particular suspect was guilty by fitting an empirical CDF to the data. The CDF that was used was a function of the beta distributions, which has the desirable characteristics in this case of being bounded by 0 and 1. The CDF was fit using $5^{th}$, $50^{th}$, and $95^{th}$ percentiles for each probability that were determined by the authors after listening and discussing the Serial podcast (2014). The distributions of the random samples can be seen in Figure 1.

In order to assess the fit, the $5^{th}$, $50^{th}$, and the $95^{th}$ percentiles were taken from each of the simulated distributions and compared to the empirical values (Table 6). All estimated values were within reasonable distance from the empirical values provided.

After assessing the accuracy of the simulations, the monte carlo analysis was completed by calculating the posterior probabilities for each suspect for each of the five pieces of evidence presented (Figure 2). With each subsequent piece of evidence presented, it became more and more likely that X had committed the crime. In addition, the uncertainty in the posterior probabilities decreased for all suspects with each subsequent piece of evidence. For example, the prior probabilty that X was guilty had a 95% confidence interval of (0.107 - 0.323) and the final posterior probabilty that X was guilty, Prob[call to X friend|suspect], had a 95% confidence interval of (0.896 - 1).

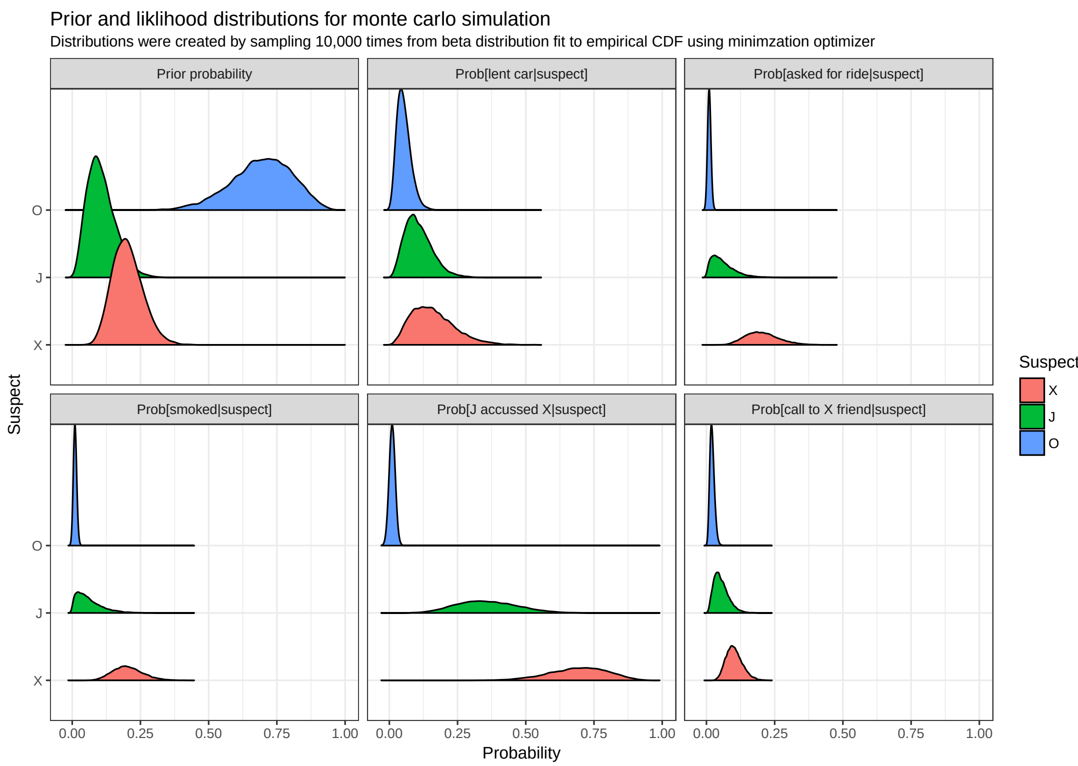


Figure 1: Prior distribution and likelihood distribution from 10,000 draws from fit beta distributions

Table 6: Difference between empirical quantiles and simulated data quantiles

| Suspect | Evidence | 5th | 50th | 95th |
|---|---|---|---|---|
| X | Prior probability | -0.0693922 | 0.0012670 | -0.0001499 |
| X | Prob[lent car\|suspect] | -0.0021287 | -0.0000825 | -0.0060857 |
| X | Prob[asked for ride\|suspect] | -0.0131058 | 0.0008325 | -0.0104906 |
| X | Prob[smoked\|suspect] | -0.0703247 | 0.0001182 | -0.0007147 |
| X | Prob[J accussed X\|suspect] | -0.0063471 | 0.0012754 | -0.0027295 |
| X | Prob[call to X friend\|suspect] | -0.0075225 | 0.0007373 | -0.0055088 |
| J | Prior probability | -0.0264926 | 0.0003354 | 0.0015563 |
| J | Prob[lent car\|suspect] | -0.0279355 | 0.0003750 | 0.0023228 |
| J | Prob[asked for ride\|suspect] | 0.0009214 | -0.0004955 | 0.0067420 |
| J | Prob[smoked\|suspect] | -0.0016626 | 0.0003210 | -0.0078779 |
| J | Prob[J accussed X\|suspect] | 0.0128165 | -0.0017241 | 0.0538433 |
| J | Prob[call to X friend\|suspect] | -0.0077925 | 0.0006813 | -0.0051611 |
| O | Prior probability | 0.0048883 | -0.0005256 | 0.0781862 |
| O | Prob[lent car\|suspect] | -0.0143307 | 0.0004338 | 0.0008652 |
| O | Prob[asked for ride\|suspect] | -0.0000098 | 0.0000598 | 0.0826023 |
| O | Prob[smoked\|suspect] | 0.0000099 | 0.0000688 | 0.0825225 |
| O | Prob[J accussed X\|suspect] | 0.0000077 | -0.0001034 | 0.0822900 |
| O | Prob[call to X friend\|suspect] | 0.0003985 | 0.0000534 | 0.0143430 |

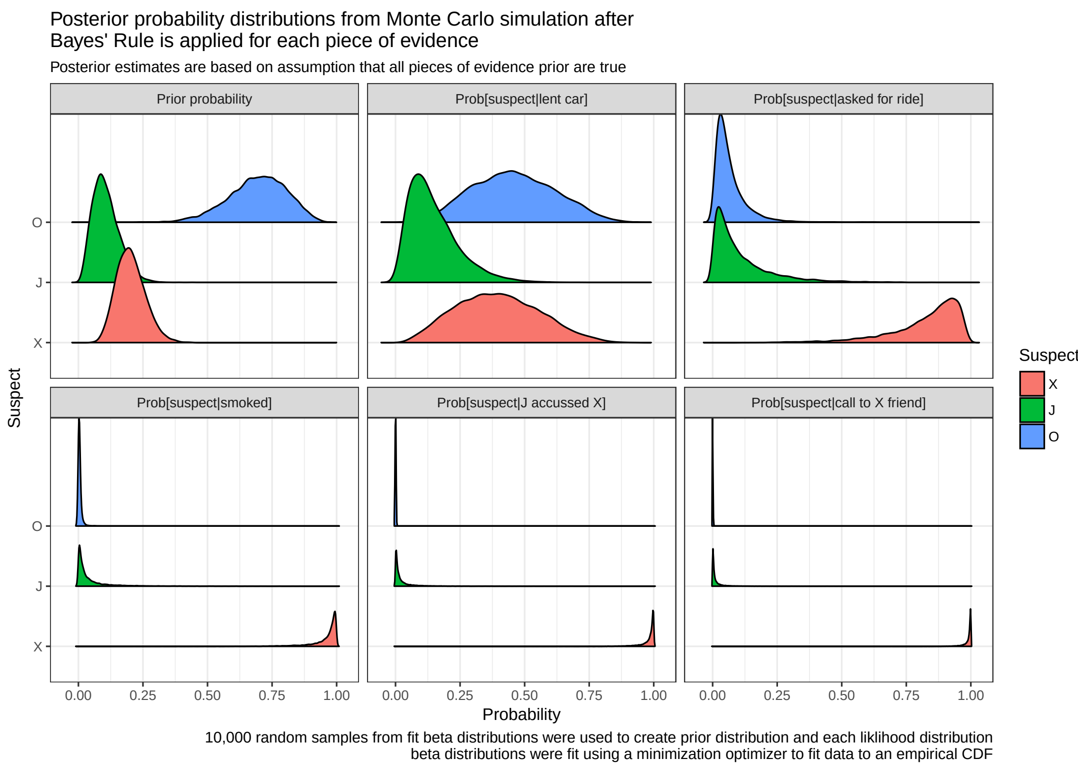


Figure 2: Posterior probability distributions for each suspect's guilt given each subsequent piece of evidence

Table 7: Summary of Uncertainty Analysis

| Suspect | Probability | Mean | Median | Std. Dev. | 95% CI |
|---|---|---|---|---|---|
| X | Prior probability | 0.203 | 0.199 | 0.055 | 0.107 - 0.323 |
| X | Prob[suspect\|lent car] | 0.396 | 0.39 | 0.16 | 0.113 - 0.718 |
| X | Prob[suspect\|asked for ride] | 0.826 | 0.865 | 0.134 | 0.469 - 0.974 |
| X | Prob[suspect\|smoked] | 0.951 | 0.978 | 0.075 | 0.741 - 0.998 |
| X | Prob[suspect\|J accussed X] | 0.975 | 0.991 | 0.05 | 0.844 - 1 |
| X | Prob[suspect\|call to X friend] | 0.985 | 0.996 | 0.036 | 0.896 - 1 |
| J | Prior probability | 0.107 | 0.1 | 0.051 | 0.028 - 0.223 |
| J | Prob[suspect\|lent car] | 0.147 | 0.126 | 0.095 | 0.023 - 0.387 |
| J | Prob[suspect\|asked for ride] | 0.107 | 0.068 | 0.112 | 0.005 - 0.421 |
| J | Prob[suspect\|smoked] | 0.044 | 0.018 | 0.073 | 4.64e-04 - 0.25 |
| J | Prob[suspect\|J accussed X] | 0.025 | 0.009 | 0.05 | 2.16e-04 - 0.156 |
| J | Prob[suspect\|call to X friend] | 0.015 | 0.004 | 0.036 | 8.42e-05 - 0.104 |
| O | Prior probability | 0.693 | 0.701 | 0.112 | 0.45 - 0.886 |
| O | Prob[suspect\|lent car] | 0.457 | 0.453 | 0.159 | 0.162 - 0.767 |
| O | Prob[suspect\|asked for ride] | 0.067 | 0.049 | 0.059 | 0.008 - 0.228 |
| O | Prob[suspect\|smoked] | 0.005 | 0.003 | 0.006 | 3.45e-04 - 0.021 |
| O | Prob[suspect\|J accussed X] | 7.74e-05 | 4.08e-05 | 1.23e-04 | 4.28e-06 - 3.66e-04 |
| O | Prob[suspect\|call to X friend] | 1.77e-05 | 8.28e-06 | 3.38e-05 | 7.06e-07 - 9.41e-05 |

# References


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