Tyler Bradley | ENVE 660 | HW #1 | 1/13/18

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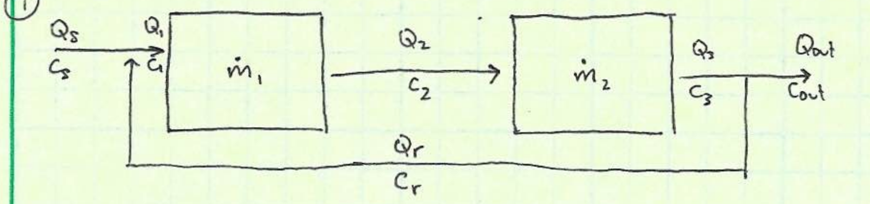


Given:

$\dot{m}_2 = \frac{1}{3}\dot{m}_1$ $\quad C_{out} = C_3 = C_r = 15\ mg/L$

$Q_s = 100\ L/s$

$C_s = 100\ mg/L$

$Q_r = 70\ L/s$

Assume:

• steadystate

a) Find Q_1, Q_2, Q_3, Q_{out}

Q_1: using the connection b/t source and recycled flows as CV

$$\frac{dV}{dt} = 0 = Q_s + Q_r - Q_1$$

$$Q_1 = 100 + 70\ L/s$$

$$\boxed{Q_1 = 170\ L/s}$$

Q_2: using first tank as CV:

$$\frac{dV}{dt} = 0 = Q_1 - Q_2 \Rightarrow \boxed{Q_2 = 170\ L/s}$$

Q_{out}: using entire system as CV

$$\frac{dV}{dt} = Q_s - Q_{out} \Rightarrow \boxed{Q_{out} = 100\ L/s}$$

Q_3: using second tank as CV:

$$\frac{dV}{dt} = 0 = Q_2 - Q_3 \Rightarrow \boxed{Q_3 = 170\ L/s}$$

b) Find C_1, C_2, C_3, C_r

$$\boxed{C_3 = C_r = C_{out} = 15\ mg/L}$$

using connection b/t source and recycle as CV:

$$V\frac{dC}{dt} = Q_s C_s + Q_r C_r - Q_1 C_1 = 0$$

$$Q_1 C_1 = (100\ L/s)(100\ mg/L) + (70\ L/s)(15\ mg/L)$$

$$C_1 = (11050\ mg/s) / (170\ L/s)$$

$$\boxed{C_1 = 65\ mg/L}$$

using 1st tank as CV:

$$V\frac{dC}{dt} = 0 = Q_1 C_1 - Q_2 C_2 - \dot{m}_1$$

$$Q_2 C_2 = Q_1 C_1 - \dot{m}_1$$

$$\dot{m}_1 = Q_1 C_1 - Q_2 C_2$$

using second tank as CV:

$$V\frac{dC}{dt} = 0 = Q_2 C_2 - Q_3 C_3 - \dot{m}_2$$

$$Q_2 C_2 = Q_3 C_3 + \frac{1}{3}\dot{m}_1 = Q_3 C_3 + \frac{1}{3}(Q_1 C_1 - Q_2 C_2)$$

$$\frac{4}{3} Q_2 C_2 = (170\ L/s)(15\ mg/L) + \frac{1}{3}(170\ L/s)(65\ mg/L)$$

$$C_2 = (6233.33\ mg/s) / \left(\frac{4}{3}(170\ \frac{L}{s})\right)$$

$$\boxed{C_2 = 27.5\ mg/L}$$

c) Find $\dot{m}_1$ and $\dot{m}_2$

$$\dot{m}_1 = Q_1 C_1 - Q_2 C_2$$

$$= (170\ L/s)(65\ mg/L) - (170\ L/s)(27.5\ mg/L)$$

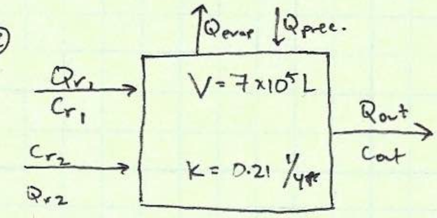
$$\boxed{\dot{m}_1 = 6375\ mg/s}$$

$$\dot{m}_2 = \frac{1}{3}\dot{m}_1 = \frac{1}{3}(6375\ \frac{mg}{s})$$

$$\boxed{\dot{m}_2 = 2125\ mg/s}$$

Tyler Bradley | ENVE 660 | HW#1 | 1/13/18

2



Given:

$Q_{r_1} = 5 \times 10^5$ L/yr
$C_{r_1} = 2.3$ mg/L
$Q_{r_2} = 7. \times 10^5$ L/yr
$C_{r_2} = 3.2$ mg/L
$Q_{evap} = 5.1 \times 10^5$ L/yr
$Q_{prec} = 2.1 \times 10^5$ L/yr

Assumptions
- steady-state
- lake is well mixed
  ↳ $C_{out} = C_{in\ lake}$

a) find Q_{out}:

$$\frac{dV}{dt} = 0 = Q_{in} - Q_{out} = Q_{r_1} + Q_{r_2} + Q_{prec.} - Q_{evap} - Q_{out}$$

$$Q_{out} = 5 \times 10^5 \frac{L}{yr} + 7 \times 10^5 \frac{L}{yr} + 2.1 \times 10^5 \frac{L}{yr} - 5.1 \times 10^5 \frac{L}{yr}$$

$$\boxed{Q_{out} = 9 \times 10^5 \frac{L}{yr}}$$

b) find C_{out}:

$r = -kC_{out}$

$$V\frac{dC}{dt} = 0 = Q_{in}C_{in} - Q_{out}C_{out} + Vr = Q_{r_1}C_{r_1} + Q_{r_2}C_{r_2} - Q_{out}C_{out} - VkC_{out}$$

$$(Q_{out} + Vk)C_{out} = (5 \times 10^5\ L/yr)(2.3\ mg/L) + (7 \times 10^5 \tfrac{L}{yr})(3.2\ mg/L)$$

$$C_{out} = (3.39 \times 10^6\ mg/yr) / (9 \times 10^5\ L/yr + (0.21)\, 7 \times 10^5\ L)$$

$$\boxed{C_{out} = 3.24\ mg/L}$$

c) find time, t, for V=0 if $Q_{r_2} = Q_{prec} = 0$ L/yr

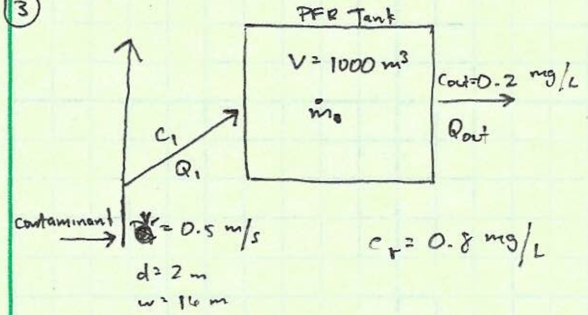
$$\int_{7 \times 10^5 L}^{0 L} dV = \int_0^t (Q_{r_1} - Q_{evap} - Q_{out})$$

$$-7 \times 10^5\ L = (5 \times 10^5\ L/yr - 5.1 \times 10^5\ L/yr - 9 \times 10^5\ L/yr)\, t$$

$$\boxed{t = 0.77\ yr}$$

Tyler Bradley | ENVE 660 | HW #1 | 1/13/18

3



Assumption
- PFR tank is at steady state
- assume river Area is rectangular
  - ↳ probably not the most accurate assumption

find $\dot{m}$:

$$\frac{dV}{dt} = 0 = Q_1 - Q_{out} \qquad Q_1 = Q_{out}$$

$$Q_1 = \frac{1}{10}(Q_r) = (0.1)(v_R A) = 0.1\left(0.5\,\frac{m}{s}\right)(2\,m)(16\,m)$$

$$Q_1 = Q_2 = 1.6\,\frac{m^3}{s}$$

$$c_r = C_1 = 0.8\ mg/L$$

$$V\frac{dC}{dt} = 0 = Q_1 C_1 - Q_2 C_2 - \dot{m}$$

$$\dot{m} = Q_1 C_1 - Q_2 C_2 = \left(1.6\,\frac{m^3}{s}\right)\left(0.8\ mg/L\right)\left(\frac{1000\ L}{1\ m^3}\right) - \left(1.6\,\frac{m^3}{s}\right)\left(0.2\ mg/L\right)\left(\frac{1000\ L}{1\ m^3}\right)$$

$$\boxed{\dot{m} = 960\ mg/s}$$

Tyler Bradley | ENVE 660 | HW #1 | 1/13/18

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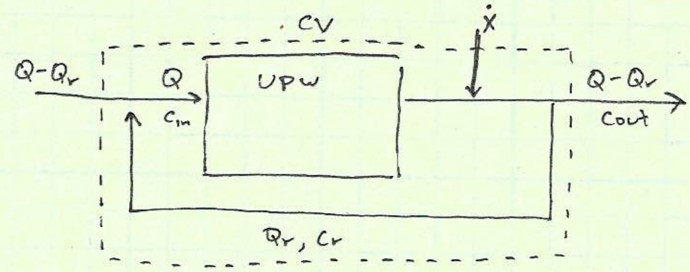
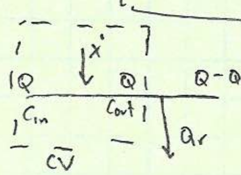


a) what is C_{out} in terms of $\dot{X}$ and flow rates

$$V\frac{dC}{dt} = 0 = \cancel{(Q-Q_r)C_{in}}^{\,0} - (Q-Q_r)C_{out} + \dot{X}$$

$$\boxed{C_{out} = \dot{X}/(Q-Q_r)}$$

b)



$C_{in} = C_{acc}$

$$V\frac{dc}{dt} = QC_{in} - QC_{out} + \dot{X}$$

$$QC_{in} = QC_{out} - \dot{X}$$

$$QC_{in} = Q\left(\frac{\dot{X}}{Q-Q_r}\right) - \dot{X}$$

$$C_{in} = \frac{\dot{X}}{Q-Q_r} - \frac{\dot{X}}{Q} = \frac{Q\dot{X}}{Q(Q-Q_r)} - \frac{(Q-Q_r)\dot{X}}{(Q-Q_r)Q} = \frac{\dot{X}(Q-Q+Q_r)}{Q^2-QQ_r} = \frac{\dot{X}Q_r}{Q^2-QQ_r}$$

$$= \frac{\dot{X}Q_r}{Q^2-QQ_r}\left(\frac{1/Q}{1/Q}\right) = \frac{\dot{X}R}{Q-QR} = \frac{\dot{X}R}{Q(1-R)}$$

$$\boxed{C_{acc} = \frac{\dot{X}R}{Q(1-R)}} \Rightarrow Q(1-R)C_{acc} = \dot{X}R$$

$$\frac{(1-R)}{R} = \frac{\dot{X}}{QC_{acc}}$$

$$\frac{1}{R} - 1 = \frac{\dot{X}}{QC_{acc}}$$

$$\boxed{R = 1 + \frac{QC_{acc}}{\dot{X}}}$$

c) $Q = 2500\ \frac{L}{min}$, $\dot{X} = 300\ mg/min$, $C_{acc} = 0.2\ mg/L$, $Q-Q_r = ?$

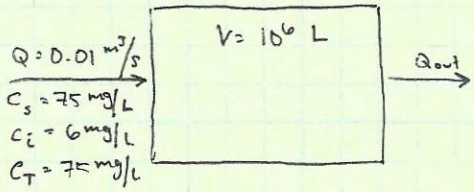
$$QC_{in} = Q\left(\frac{\dot{X}}{Q-Q_r}\right) - \dot{X}$$

$$Q-Q_r = \frac{Q\dot{X}}{QC_{in}+\dot{X}} = \frac{(2500\ L/min)(300\ mg/min)}{(2500\ L/min)(0.2\ mg/L) + 300\ \frac{mg}{min}} = \frac{750{,}000\ \frac{L\cdot mg}{min^2}}{800\ \frac{mg}{min}}$$

$$\boxed{Q-Q_r = 937.5\ L/min}$$

Tyler Bradley | ENVE 660 | HW #1 | 1/14/18

5



Assume
- steady-state
- liquid is incompressible
- all incoming organics are soluble

- 75% of incoming organics is consumed (DOC)
- 30% of consumed organics is converted to new cellular material
- 70% of consumed organics is released as $CO_2 \rightarrow$ DIC

$$\frac{dV}{dt} = 0 = Q_{in} - Q_{out}$$

$$Q_{out} = Q_{in} = 0.01\ m^3/s$$

$$= (0.01\ m^3/s)\left(\frac{1000\ L}{1\ m^3}\right) = 10\ L/s$$

a)

DOC $\rightarrow C_s$

$\rightarrow$ 75% of incoming dissolved organics are consumed

$$V\cancel{\frac{dc}{dt}}^{0} = Q_{in}C_{in} - Q_{out}C_{out} + \cancel{\text{Generation}}^{0} - \text{consumption}$$

$$= (0.01\ m^3/s)(75\ mg/L)(1000\ L/1\ m^3) - (0.01\ m^3/s)\,C_{s,out}(1000\ L/m^3) - (0.01\ \tfrac{m^3}{s})(75\ mg/L)(0.75)(1000\ m^3/L)$$

$$C_{s,out} = \left[(750\ mg/s) - (562.5\ mg/s)\right] / (10\ L/s)$$

$$\boxed{C_{s,out} = 18.75\ mg/L} \rightarrow \text{DOC}$$

TOC $\rightarrow C_t$

$\rightarrow$ 75% of incoming DOC is consumed but only 70% is converted to DIC

$$V\cancel{\frac{dc}{dt}}^{0} = Q_{in}C_{in} - Q_{out}C_{out} + \cancel{\text{generation}}^{0} - \text{consumption}$$

$$= (10\ L/s)(75\ mg/L) - (10\ L/s)\,C_{t,out} - (10\ L/s)(75\ mg/L)(0.75)(0.7)$$

$$C_{t,out} = (750\ mg/s - 393.75\ mg/s) / (10\ L/s)$$

$$\boxed{C_{t,out} = 35.63\ mg/L}$$

DIC $\rightarrow C_i$

$\rightarrow$ 70% of incoming DOC is converted to DIC

$$V\cancel{\frac{dc}{dt}}^{0} = Q_{in}C_{in} - Q_{out}C_{out} + \text{generation} - \cancel{\text{consumption}}^{0}$$

$$0 = (10\ L/s)(6\ mg/L) - (10\ L/s)\,C_{i,out} + (10\ L/s)(75\ mg/L)(0.75)(0.7)$$

$$C_{i,out} = (60\ mg/s + 393.75\ mg/s) / 10\ L/s$$

$$\boxed{C_{i,out} = 45.38\ mg/L}$$

1 mg biomass = 0.5 mg C

b) How much biomass is created per liter influent

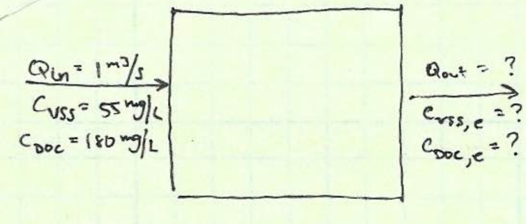
$\hookrightarrow$ 30% of consumed TOC is converted to cellular material

$$V\cancel{\frac{dc}{dt}}^{0} = \cancel{Q_{in}C_{in}}^{0} - \cancel{Q_{out}C_{out}}^{0} + \text{generation} - \cancel{\text{consumption}}^{0}$$

$$= (10\ L/s)(75\ mg/L)(0.75)(0.3)\left(\frac{1\ mg\ \text{biomass}}{0.5\ mg\ C}\right)$$

$$\boxed{= 337.5\ mg\ \text{biomass}/L}$$

ENVE 660 HW #1 1/18/18



Assumptions
- steady-state
- liquid is incompressible

DOC = DOM in this case

- for every gram of DOM that is degraded, a portion is converted to CO_2 and H_2O and another portion is converted to 0.4 g of new biomass

a) $r_{DOC} = r_s = -\dfrac{k_1 S^2 X}{(S^2 + K_s)}$ $\rightarrow$ $S = C_{DOC,e}$

$k_1 = 8\ \dfrac{mg\ DOC}{mg\ VSS - d}$ $K_s = 100\left(\dfrac{mg\ DOC}{L}\right)^2$ $X = 120\ \dfrac{mg\ VSS}{L} = C_{VSS,e}$

what is $\tau\ (= V/Q)$ to achieve $C_{DOC,e} = 3\ mg\ DOC/L$

$\dfrac{d\cancel{V}^{\,0}}{dt} = Q_{in} - Q_{out}$

$Q_{in} = Q_{out}$

$$V\frac{\cancel{dC_{DOC}}^{\,0}}{dt} = Q_{in}C_{in} - Q_{out}C_{out} + \cancel{\text{generation}}^{\,0} - \text{consumption}$$

$$= Q\left(180\ \overset{\text{mg DOC}}{\frac{mg}{L}}\right) - Q\left(3\ \frac{mg\ DOC}{L}\right) - \frac{k_1 S^2 X}{(S^2 + K_s)}V$$

$$= 177\ \frac{mg\ DOC}{L} - \frac{\left(8\ \frac{mg\ DOC}{mg\ VSS \cdot d}\right)\left(3\ \frac{mg\ DOC}{L}\right)^2\left(120\ \frac{mg\ VSS}{L}\right)}{\left(3\ \frac{mg\ DOC}{L}\right)^2 + 100\ \frac{mg\ DOC^2}{L^2}}\ \frac{V}{Q}$$

$$= 177\ \frac{mg\ DOC}{L} - \left(79.27\ \frac{mg\ DOC}{L \cdot d}\right)\tau$$

$$\tau = \left(177\ \frac{mg\ DOC}{L}\right) \Big/ \left(79.27\ \frac{mg\ DOC}{L \cdot d}\right)$$

$$\boxed{\tau = 2.23\ \text{days}}$$

b) $r_{solids} = -k_d X$ write MB for VSS and compute k_d

$\dfrac{d\cancel{V}^{\,0}}{dt} = Q_{in} - Q_{out}$

$Q_{in} = Q_{out} = 1\ m^3/s = 1000\ L/s$

$V = \left(1000\ \frac{L}{s}\right)(2.33\ d)\left(\frac{86400\ s}{1\ d}\right) = 2.01 \times 10^8\ L$

$0.4\ \dfrac{mg\ VSS}{mg\ DOC}$ is created as DOC is consumed

solids decay at $r_{solids} = -k_d X$

$$V\frac{\cancel{dC_{VSS}}^{\,0}}{dt} = Q_{in}C_{VSS,in} - Q_{out}C_{VSS,e} + \text{generation} - \text{consumption}$$

$$= (1000\ L/s)(55\ mg/L) - (1000\ L/s)(120\ mg\ VSS/L) + \left(0.4\ \frac{mg\ VSS}{mg\ DOC}\right)\frac{\left(\frac{8\ mg\ DOC}{mg\ VSS\ d}\right)\left(3\ \frac{mg\ DOC}{L}\right)^2\left(120\ \frac{mg\ VSS}{L}\right)}{\left(3\ \frac{mg\ DOC}{L}\right)^2 + 100\left(\frac{mg\ DOC}{L}\right)^2}V$$

$$- K_d\left(120\ \frac{mg\ VSS}{s}\right)V$$

$$K_d\left(120\ \frac{mg\ VSS}{L}\right)(2.01 \times 10^8\ L) = 6.37 \times 10^9\ \frac{mg\ VSS}{s}$$

$$K_d = \left(6.37 \times 10^9\ \frac{mg\ VSS}{s}\right) \Big/ 2.41 \times 10^{10}\ mg\ VSS$$

$$\boxed{K_d = 0.26\ 1/s}$$