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① $d_T = 2.2$ km

$C_{Cl,min} = 1.2$ mg/L

$K = -1.23 \frac{1}{hr}$

$C_{Cl,0} = 5$ mg/L

$r = -k\, C_{Cl}$

a)

$$V \frac{dC_{Cl}}{dt} = \cancel{in}^{0} - \cancel{out}^{0} + \cancel{acc}^{0} + deg$$

$$\cancel{V} \frac{dC_{Cl}}{dt} = rV = -kC_{Cl}\cancel{V}$$

$$\int_{5}^{1.2} \frac{dC_{Cl}}{C_{Cl}} = \int_{0}^{t} -k\, dt$$

$$\ln C_{Cl} \Big]_{5}^{1.2} = -kt \Big]_{0}^{t}$$

$$\ln(1.2) - \ln(5) = -1.427 = -kt$$

$$t = \frac{-1.427}{-1.23 \frac{1}{hr}}$$

$$\boxed{t = 1.16 \text{ hrs}}$$

b) $A_{pipe} = \pi (1)^2 = 3.14\ m^2$

$V_{pipe} = (3.14\ m^2)(2200\ m) = 6911.5\ m^3$

$v = \left(37 \frac{m^3}{min}\right)\left(\frac{1}{3.14\ m^2}\right) = 11.78 \frac{m}{min}$

↳ if C_{Cl} takes 1.16 hrs = 69.6 mins to degrade from 5 mg/L to 1.2 mg/L, then,

$$d = \left(11.78 \frac{m}{min}\right)(69.6 \text{ mins}) = 819.88\ m$$

is how far the water travels before needing to be boosted.

$$\# \text{ of booster stations} = \frac{d_T}{d} = \frac{2200\ m}{819.9\ m} = 2.68 = 2$$

$$\boxed{\begin{array}{l} d_{booster\ \#1} = 819\ m \\ d_{booster\ \#2} = 1638\ m \end{array}}$$

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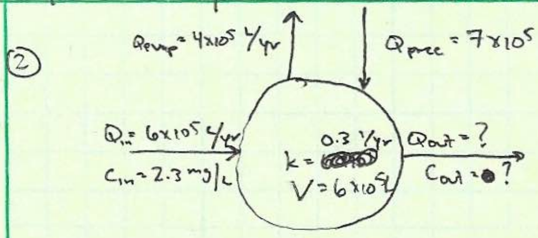
(2)



Assumptions
- steady state
- liquid is incompressible
- lake is well mixed

I) find Q_{out} & C_{out}

$$\cancel{\frac{dV}{dt}}^{0} = Q_{in} + Q_{prec} - Q_{evap} - Q_{out}$$

$$Q_{out} = Q_{in} + Q_{prec} - Q_{evap}$$
$$= 6\times10^5 \tfrac{L}{yr} + 7\times10^5 \tfrac{L}{yr} - 4\times10^5 \tfrac{L}{yr}$$

$$\boxed{Q_{out} = 9\times10^5 \tfrac{L}{yr}}$$

$$\cancel{V\frac{dc}{dt}}^{0} = Q_{in}C_{in} - Q_{out}C_{out} + rV \qquad r = -\left(0.3\tfrac{1}{yr}\right)C_{out}$$

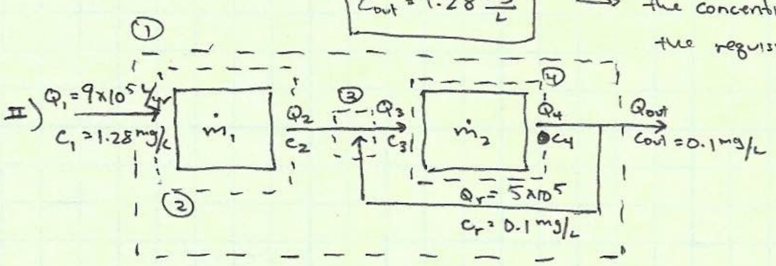
$$= Q_{in}C_{in} - Q_{out}C_{out} - 0.3\tfrac{1}{yr}C_{out}V$$
$$= \left(6\times10^5\tfrac{L}{yr}\right)\left(2.3\tfrac{mg}{L}\right) - \left(9\times10^5\tfrac{L}{yr}\right)C_{out} - \left(0.3\tfrac{1}{yr}\right)\left(6\times10^5 L\right)C_{out}$$

$$\left(1.08\times10^6\tfrac{L}{yr}\right)C_{out} = 1.38\times10^6\tfrac{mg}{yr}$$

$$C_{out} = \frac{\left(1.38\times10^6\tfrac{mg}{yr}\right)}{\left(1.08\times10^6\tfrac{L}{yr}\right)}$$

$$\boxed{C_{out} = 1.28\tfrac{mg}{L}}$$ → the concentration is not being lowered to the requisite 0.1 $\frac{mg}{L}$

II)



$\dot m_1 = \frac{1}{2}\dot m_2$

$C_4 = C_r = C_{out} = 0.1$ mg/L

Assume: Steady-state, liquid is incompressible

MB #1: $\cancel{\frac{dV}{dt}}^{0} = Q_1 - Q_{out}$ $\quad \boxed{Q_{out} = 9\times10^5\tfrac{L}{yr}}$

MB #2: $\cancel{\frac{dV}{dt}}^{0} = Q_1 - Q_2$ $\quad \boxed{Q_2 = 9\times10^5\tfrac{L}{yr}}$

MB #3: $\cancel{\frac{dV}{dt}}^{0} = Q_2 + Q_r - Q_3$

$Q_3 = Q_2 + Q_r$

$Q_3 = 9\times10^5 + 5\times10^5 \tfrac{L}{yr}$

$\boxed{Q_3 = 14\times10^5\tfrac{L}{yr}}$

$V\frac{dc}{dt} = Q_2C_2 + Q_rC_r - Q_3C_3$

$Q_3C_3 = Q_2C_2 + Q_rC_r$

MB #4: $\cancel{\frac{dV}{dt}}^{0} = Q_3 - Q_4 \Rightarrow \boxed{Q_4 = 14\times10^5\tfrac{L}{yr}}$

$\cancel{V\frac{dc}{dt}}^{0} = Q_1C_1 - Q_2C_2 - \dot m_1$

$Q_2C_2 = Q_1C_1 - \dot m_1$

~~$C_2 = \frac{(9\times10^5\frac{L}{yr})(2.3\frac{mg}{L})}{9\times10^5\frac{L}{yr}}$~~

$\dot m_1 = Q_1C_1 - Q_2C_2$

$\cancel{V\frac{dc}{dt}}^{0} = Q_3C_3 - Q_4C_4 - \dot m_2$

$= Q_2C_2 + Q_rC_r - Q_4C_4 - 2(Q_1C_1 - Q_2C_2)$

$= 3Q_2C_2 + Q_rC_r - Q_4C_4 - 2Q_1C_1$

$3Q_2C_2 = Q_4C_4 + 2Q_1C_1 - Q_rC_r$

$= \left(14\times10^5\tfrac{L}{yr}\right)\left(0.1\tfrac{mg}{L}\right) + 2\left(9\times10^5\tfrac{L}{yr}\right)\left(1.28\tfrac{mg}{L}\right) - \left(5\times10^5\tfrac{L}{yr}\right)\left(0.1\tfrac{mg}{L}\right)$

$C_2 = \frac{\left(2.394\times10^6\ mg/yr\right)}{3\left(9\times10^5\tfrac{L}{yr}\right)}$

$\boxed{C_2 = 0.887\ mg/L}$

$\dot m_1 = Q_1C_1 - Q_2C_2 = \left(9\times10^5\tfrac{L}{yr}\right)\left(1.28\tfrac{mg}{L}\right) - \left(9\times10^5\tfrac{L}{yr}\right)\left(0.887\tfrac{mg}{L}\right)$

$\boxed{\dot m_1 = 3.53\times10^5\tfrac{mg}{yr}}$

$\dot m_2 = 2\dot m = 2\left(3.53\times10^5\tfrac{mg}{yr}\right)$

$\boxed{\dot m_2 = 7.07\times10^5\tfrac{mg}{yr}}$

$\boxed{C_4 = 0.1\tfrac{mg}{L}}$

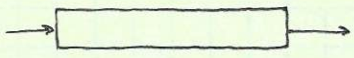
$C_3 = \frac{Q_2C_2 + Q_rC_r}{Q_3}$

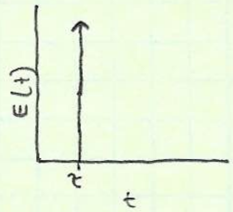
$= \frac{\left(9\times10^5\tfrac{L}{yr}\right)\left(0.887\tfrac{mg}{L}\right) + \left(5\times10^5\tfrac{L}{yr}\right)(0.}{\left(14\times10^5\tfrac{L}{yr}\right)}$

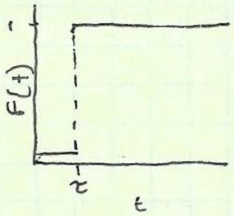
$\boxed{C_3 = 0.606\tfrac{mg}{L}}$

$k\tau = 10$

③ i)







Assume steady-state, no dispersion, constant A

$$\cancel{V\frac{dC}{dt}}^{0} = QC_{in} - QC_{out} + rV$$

$$r = -kC \qquad V = A\Delta x$$

$$QC_{in} - QC_{out} = QC_{in} - Q(C_{in} + \Delta C)$$

$$0 = QC_{in} - Q(C_{in} + \Delta C) - kCA\Delta x$$

$$= -Q\Delta C - kCA\Delta x$$

$$\Delta x \to 0 \quad \Delta x = dx \quad \Delta C = dC$$

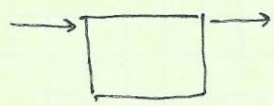
$$\int_{C_{in}}^{C_{out}} \frac{1}{C}\, dC = -k\frac{A}{Q}\int_0^L dx$$

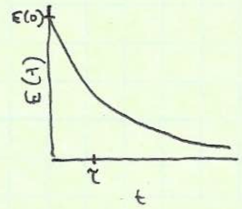
$$\ln\left(\frac{C_{out}}{C_{in}}\right) = -k\frac{A(L)}{Q} = -k\frac{V}{Q} = -k\tau$$

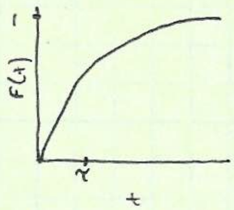
$$\frac{C_{out}}{C_{in}} = e^{-k\tau}$$

$$\boxed{\frac{C_{out}}{C_{in}} = 4.54\times 10^{-5}}$$

ii)







Assume steady state

$$r = -kC_{out}$$

$$\cancel{V\frac{dC}{dt}}^{0} = QC_{in} - QC_{out} + rV$$

$$= Q(C_{in} - C_{out}) - kC_{out}V$$

$$k\frac{V}{Q} = k\tau = \frac{(C_{in} - C_{out})}{C_{out}}$$

$$k\tau = \frac{C_{in}}{C_{out}} - 1$$

$$\frac{C_{in}}{C_{out}} = 1 + k\tau$$

$$\frac{C_{out}}{C_{in}} = \frac{1}{1+k\tau} = \frac{1}{1+10}$$

$$\boxed{\frac{C_{out}}{C_{in}} = 0.091}$$

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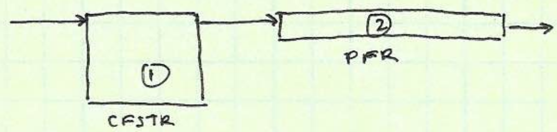
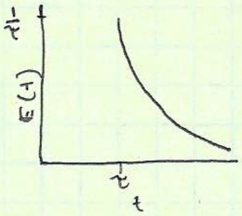
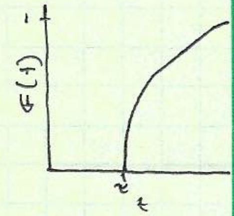
(3) iii)







using $\frac{C_{out}}{C_{in}}$ relationships defined in i & ii for PFR and CFSTR respectively:

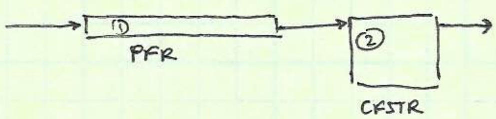
$C_{out,1} = C_{in,2}$ $\qquad C_{out,1} = \frac{C_{in,1}}{1+k\tau}$ $\qquad C_{out,2} = C_{in,2}\, e^{-k\tau}$

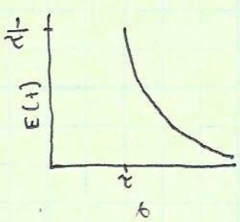
$$C_{out} = \frac{C_{in}}{1+k\tau} e^{-k\tau}$$

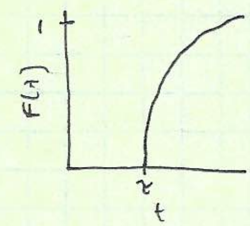
$$\frac{C_{out}}{C_{in}} = \frac{1}{1+k\tau} e^{-k\tau} = \frac{1}{1+10} e^{-10}$$

$$\boxed{\frac{C_{out}}{C_{in}} = 4.127 \times 10^{-6}}$$

iv)







using relationships from i & ii for PFR and CFSTR, respectively:

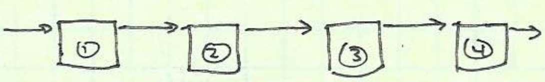
$C_{out,1} = C_{in,2}$ $\qquad C_{out,1} = C_{in,1}\, e^{-k\tau}$ $\qquad C_{out,2} = \frac{C_{in,2}}{1+k\tau} = \frac{C_{out,1}}{1+k\tau}$

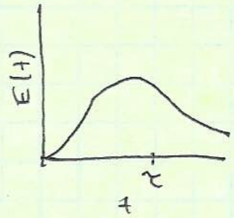
$$C_{out,2} = \frac{C_{in} e^{-k\tau}}{1+k\tau}$$

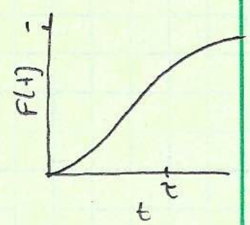
$$\frac{C_{out}}{C_{in}} = \frac{e^{-k\tau}}{1+k\tau} = \frac{e^{-10}}{11}$$

$$\boxed{\frac{C_{out}}{C_{in}} = 4.127 \times 10^{-6}}$$

v)







Using eq. from ii for CFSTR:

$C_{out,1} = C_{in,2}$ $\qquad C_{out,2} = C_{in,3}$ $\qquad C_{out,3} = C_{in,4}$

$$C_{out,1} = \frac{C_{in,1}}{1+k\tau} \;;\; C_{out,2} = \frac{C_{in,2}}{1+k\tau} = \frac{C_{out,1}}{1+k\tau} = \frac{C_{in,1}}{(1+k\tau)(1+k\tau)} = \frac{C_{in,1}}{(1+k\tau)^2}$$

$$C_{out,3} = \frac{C_{in,3}}{1+k\tau} = \frac{C_{out,2}}{1+k\tau} = \frac{C_{in,1}}{(1+k\tau)^3}$$

$$C_{out,4} = \frac{C_{in,4}}{1+k\tau} = \frac{C_{out,3}}{1+k\tau} = \frac{C_{in,1}}{(1+k\tau)^4}$$

$$\frac{C_{out}}{C_{in}} = \frac{1}{(1+k\tau)^4} = \frac{1}{11^4}$$

$$\boxed{\frac{C_{out}}{C_{in}} = 6.83 \times 10^{-5}}$$

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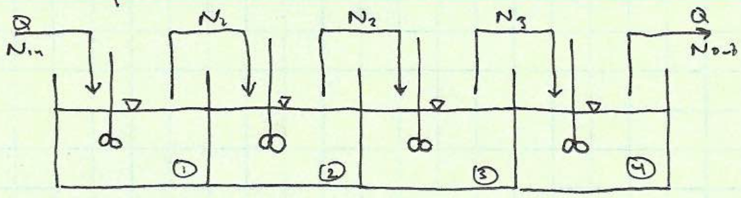
4



$r = -kC^nN$

$N_{in} = 105$ organisms/L

$k = 0.11 \frac{L^{0.82}}{mg^{0.82}\cdot min}$

$C = 5$ mg/L

$n = 0.82$

$\tau_{TOT} = 40$ min

$\tau_i = 40\ min/4 = 10$ min

Assumption:

- steady-state
- $\tau_1 = \tau_2 = \tau_3 = \tau_4$

#1:

$$V\cancel{\frac{dN}{dt}}^{0} = QN_{in} - QN_1 + rV$$

$$= Q(N_{in} - N_1) - kC^nN_1V$$

$$\frac{V}{Q} = \frac{N_{in} - N_1}{kC^nN_1} = \tau_1$$

$$kC^n\tau_1 = \frac{N_{in}}{N_1} - 1$$

$$\frac{N_1}{N_{in}} = \frac{1}{1 + kC^n\tau_1}$$

$$N_1 = \frac{N_{in}}{1 + kC^n\tau_1} = \frac{(105\ org./L)}{1 + (0.11\frac{L^{0.82}}{mg^{0.82}\cdot min})(5\ mg/L)^{0.82}(10\ min)} = \frac{(105\ org/L)}{5.117}$$

$$N_1 = 20.52\ \text{organisms/L}$$

#2: using equation derived above substituting N_{in} with N_1 and N_1 with N_2

$$N_2 = \frac{N_1}{1 + kC^n\tau_2} = \frac{20.52\ org./L}{5.117}$$

$$N_2 = 4.01\ \text{organisms/L}$$

#3:

$$N_3 = \frac{N_2}{1 + kC^n\tau_3} = \frac{4.01\ org./L}{5.117}$$

$$N_3 = 0.784\ org/L$$

#4:

$$N_{out} = \frac{N_3}{1 + kC^n\tau_4} = \frac{0.784\ org./L}{5.117}$$

$$\boxed{N_{out} = 0.153\ \text{organisms/L}}$$

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(5)

Time (min)	C_A (mol/L)
0	167
1	16.1
2	8.5

a) from experimental data what is k?

↳ data was plotted as C_A vs t (zero-order), $\ln(C_A)$ vs t (First-order), and $\frac{1}{C_A}$ vs t (second-order) in R, code on next page. The second order was the closest to linearity. Linear model was fit to $\frac{1}{C_A}$ vs t to find k given the second order reaction equation:

$$\frac{1}{C_A} = \frac{1}{C_{A_0}} + kt$$

k is given by slope of the line

$$\boxed{k = 0.0558 \ \frac{L}{mol \cdot min}}$$

b) would a CFSTR or PFR be desirable to achieve 99% removal?

↳ can assess this by comparing τ required to achieve 99% removal

- Assume steady-state

$C_{in} = 167 \ \frac{mol}{L}$

$$\eta = 0.99 = \frac{C_{in} - C_{out}}{C_{in}}$$

$$C_{out} = C_{in} - 0.99(C_{in}) = 167 \ \frac{mol}{L} - 0.99\left(167 \ \frac{mol}{L}\right)$$

$$C_{out} = 1.67 \ \frac{mol}{L}$$

CFSTR:

↳ for second-order rxn:

$$\tau = \frac{1}{k_2 C_{out}}\left(\frac{C_{in}}{C_{out}} - 1\right) = \frac{1}{\left(0.0558 \frac{L}{mol \cdot min}\right)\left(1.67 \frac{mol}{L}\right)}\left(\frac{167 \frac{mol}{L}}{1.67 \frac{mol}{L}} - 1\right)$$

$$\tau = 1062.39 \ min$$

PFR:

↳ for second-order rxn:

$$\tau = \frac{1}{k_2 C_{in}}\left(\frac{C_{in}}{C_{out}} - 1\right) = \frac{1}{\left(0.0558 \frac{L}{mol \cdot min}\right)\left(167 \frac{mol}{L}\right)}\left(\frac{167 \frac{mol}{L}}{1.67 \frac{mol}{L}} - 1\right)$$

$$\tau = 10.63 \ min$$

$$\tau_{PFR} \ll \tau_{CFSTR}$$

↳ choose to use a PFR to remove compound A

```
#=================================================================
# This script contains work for ENVE 660 Midterm
# Question #5
#
# Tyler Bradley
# 2018-02-09
#=================================================================
```

Reading in required libraries

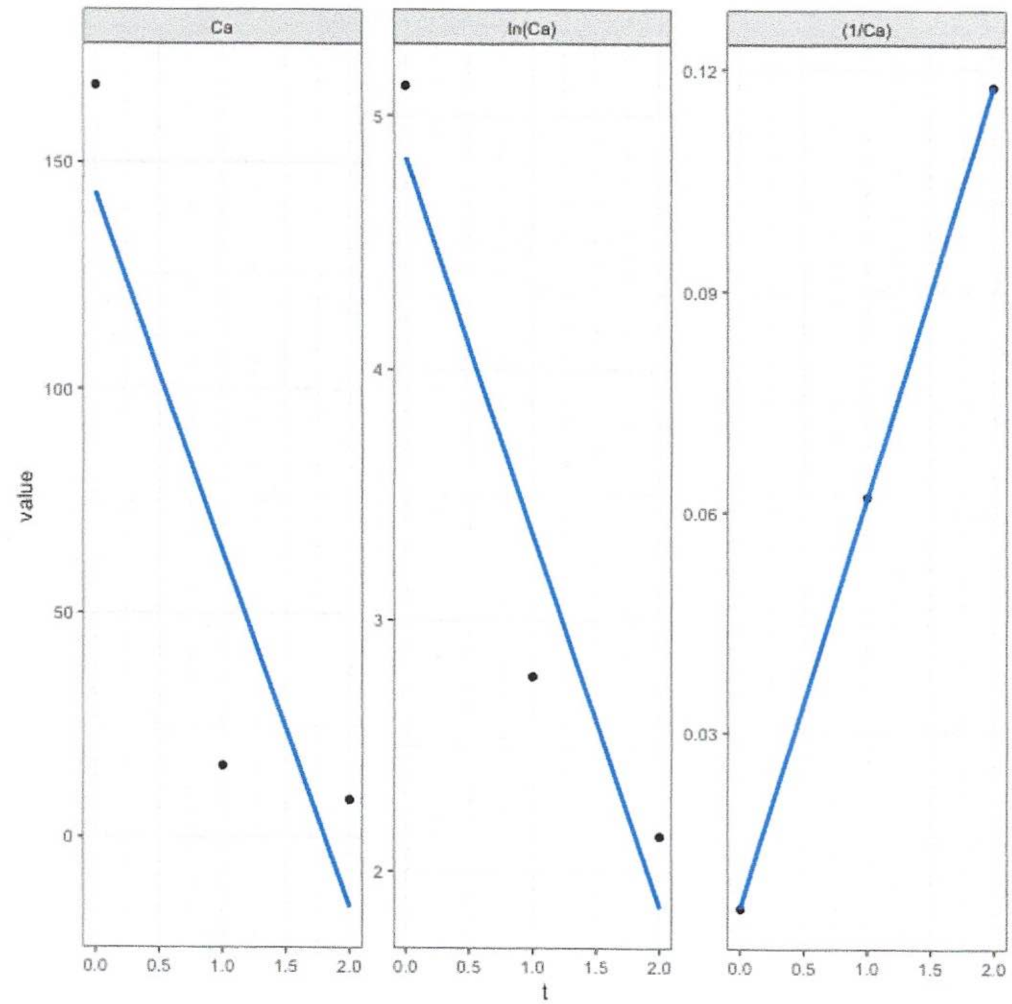
```
library(tidyverse)
```

Writing in experimental data

```
df <-tribble(
  ~t, ~Ca,
  0,  167,
  1,  16.1,
  2, 8.5
)
```

Plotting zero, first, and second order relationships as Ca vs t, ln(Ca) vs t, and (1/Ca) vs t, respectively

```
df %>%
  mutate(
    ln_Ca = log(Ca), # log() function defaults to ln()
    Ca_inv = (1/Ca)
  ) %>%
  gather(key = order, value = value, Ca:Ca_inv) %>%
  mutate(order = factor(order, levels = c("Ca", "ln_Ca", "Ca_inv"),
                        labels = c("Ca", "ln(Ca)", "(1/Ca)"))) %>%
  ggplot(aes(t, value)) +
  facet_wrap(~ order, scales = "free") +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  theme_bw()
```



Linear model fit to (1/Ca) vs t since it has the best fit

```
model <- df %>%
  mutate(Ca_inv = 1/Ca) %>%
  lm(Ca_inv ~ t, data = .)
```

getting the model coefficients k = 0.0558295

```
broom::tidy(model)
```

```
##          term    estimate    std.error statistic     p.value
## 1 (Intercept) 0.006086111 0.0002193283  27.74886 0.022932275
## 2           t 0.055829517 0.0001698910 328.61962 0.001937248
```

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(6)

time (s)	Tracer Conc. (g/m^3)	$t * C(t)$ ($\frac{g \cdot s}{m^3}$)	$(t - t_{avg})^2 * C(t)$ ($\frac{g \cdot s^2}{m^3}$)
0	0	0	0
100	5	500	2.25 x 10^5
200	13	2600	1.64 x 10^5
300	14	4200	2.12 x 10^3
400	8	3200	6.15 x 10^4
500	5	2500	1.76 x 10^5
600	2	1200	1.66 x 10^5
700	1	700	1.50 x 10^5
800	0	0	0

$$t_{avg} = \frac{\int_0^\infty t \cdot C(t)\,dt}{\int_0^\infty C(t)\,dt}$$

using Simpson's Rule:

$$\int_0^\infty C(t)\,dt = \frac{h}{3}\left[f(a) + 4 \times f\left(\frac{a+b}{2}\right) + f(b)\right] \qquad h = \frac{b-a}{2}$$

applying it every three data points ($h = 100$)

$$\int_0^\infty C(t)dt = \frac{100}{3}(0 + 4(5) + 13) + \frac{100}{3}(13 + 4(14) + 8) + \frac{100}{3}(8 + 4(5) + 2) + \frac{100}{3}(2 + 4(1) + 0)$$

$$= 4866.67 \ g/m^3$$

$$\int_0^\infty t \cdot C(t)\,dt = \frac{100}{3}(0 + 4(500) + 2600) + \frac{100}{3}(2600 + 4(4200) + 3200) + \frac{100}{3}(3200 + 4(2500) + 1200) + \frac{100}{3}(1200 + 4(700) + 0)$$

$$= 1.52 \times 10^6 \ \frac{s \cdot g}{m^3}$$

$$t_{avg} = \frac{1.52 \times 10^6 \ \frac{g \cdot s}{m^3}}{4866.67 \ \frac{g}{m^3}}$$

$$\boxed{t_{avg} = 312.3 \ s}$$

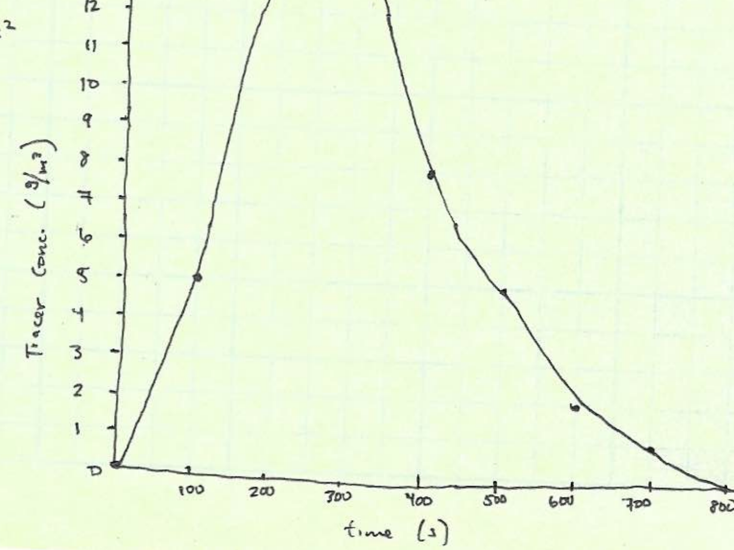
$$\sigma^2 = \frac{\int_0^\infty (t - t_{avg})^2 \cdot C(t)\,dt}{\int_0^\infty C(t)\,dt}$$

$$\int_0^\infty (t-t_{avg})^2 \cdot C(t)\,dt = \frac{100}{3}(0 + 4(2.25 \times 10^5) + 1.64 \times 10^5) + \frac{100}{3}(1.64 \times 10^5 + 4(2.12 \times 10^3) + 6.15 \times 10^4) + \frac{100}{3}(6.15 \times 10^4 + 4(1.76 \times 10^5) + 1.66 \times 10^5) + \frac{100}{3}(1.66 \times 10^5 + 4(1.5 \times 10^5) + 0)$$

$$= 9.98 \times 10^7 \ \frac{g \cdot s^2}{m^3}$$

$$\sigma^2 = \frac{9.98 \times 10^7 \ \frac{g \cdot s^2}{m^3}}{4866.67 \ g/m^3} = 20517 \ s^2$$

$$\boxed{\sigma = 143 \ s}$$



* why is this question worth 8 points?
↳ your dog is 8 years old