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① $q = KC^n$

$n = 2$

$K = 3 \times 10^3 \frac{L^2}{mg \cdot g\,Adsorbent\,A}$

$C_0 = 50\ mg/L$

$C = 5\ mg/L$

$\frac{W}{V_L} = ?$

Assume steady state, fresh adsorbent ($q_{init} = 0$)

$0 = V_L(C_0 - C) + W(q_{init} - q)$

$V_L(C_0 - C) = WKC^n$

$\frac{W}{V_L} = \frac{(C_0 - C)}{KC^n} = \frac{\left(50\frac{mg}{L} - 5\frac{mg}{L}\right)}{\left(3 \times 10^3 \frac{L^2}{mg \cdot g}\right)\left(5\frac{mg}{L}\right)^2}$

$\boxed{\frac{W}{V_L} = 6 \times 10^{-4} \frac{g\,A}{L}}$

② $q = q_{max}\frac{KC}{1 + KC}$

$K = 60\ L/mg$

$\Gamma = 7\ mg/m^2$

$\frac{W}{V} = 3 \times 10^{-4} \frac{g\,B}{L}$

$C = 5\ mg/L$

$C_0 = 50\ mg/L$

SSA = ?

Assume: steady state & fresh adsorbent

$q = \Gamma(SSA)$

$V_L(C_0 - C) = Wq$

$q = \frac{V_L}{W}(C_0 - C) = \left(\frac{1}{3 \times 10^{-4} \frac{gB}{L}}\right)\left(50\frac{mg}{L} - 5\frac{mg}{L}\right)$

$q = 1.5 \times 10^5 \frac{mg}{gB}$

$SSA = \frac{q}{\Gamma} = \frac{1.5 \times 10^5 \frac{mg}{gB}}{7\ mg/m^2}$

$\boxed{SSA = 21428.6\ m^2/g\,B}$

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③ a) Batch reactor

MB:

change in total adsorbate mass in system from adsorption from 0 to t = change in dissolved adsorbate mass + change in adsorbed adsorbate mass

Assume: steady state

$$0 = V_L(C_{init} - C_{fin}) + W(q_{init} - q_{fin})$$

$$\boxed{\frac{W}{V_L} = -\frac{C_{init} - C_{fin}}{q_{init} - q_{fin}}}$$

b) $C_{init} = 0.1\ \frac{mmol}{L}$

$C_{fin} = 0.01\ \frac{mmol}{L}$

$\frac{W}{V_L} = 21.5\ \frac{gA}{L}$

$q = kC_{fin}$

Assume: fresh adsorbent ($q_{init} = 0$)

$$\frac{W}{V_L} = -\frac{C_{init} - C_{fin}}{-kC_{fin}}$$

$$k = \frac{V_L}{W}\left(\frac{C_{init} - C_{fin}}{C_{fin}}\right) = \frac{1}{21.5\frac{gA}{L}}\left(\frac{0.1\frac{mmol}{L} - 0.01\frac{mmol}{L}}{0.01\frac{mmol}{L}}\right)$$

$$\boxed{k = 0.419\ \frac{L}{gA}}$$

④ CFSTR

a) MB:

$$V_L\frac{dC}{dt} + W\frac{dq}{dt} = Q(C_{in} - C_{out}) + X(q_{in} - q_{out})$$

$$\boxed{X = \frac{-Q(C_{in} - C_{out})}{(q_{in} - q_{out})}}$$

Assume:
steady state
$Q_{in} = Q_{out}$
$X_{in} = X_{out}$

b) $C_{in} = 0.1\ \frac{mmol}{L}$

$C_{out} = 0.01\ \frac{mmol}{L}$

$X = 4\ \frac{gB}{hr}$

$Q = 1\ \frac{L}{hr}$

$V = 10\ L$

$q = q_{max}\frac{kC_{out}}{1 + kC_{out}}$

$q_{max} = 0.6\ \frac{mmol}{gB}$

$k = ?$

Assume: fresh adsorbent entering system: $q_{in} = 0$

$$q_{out} = \frac{Q(C_{in} - C_{out})}{X} = \frac{1\frac{L}{hr}\left(0.1\frac{mmol}{L} - 0.01\frac{mmol}{L}\right)}{4\frac{gB}{hr}} = 0.0225\ \frac{mmol}{gB}$$

$$q_{out} = q_{max}\frac{kC_{out}}{1 + kC_{out}}$$

$$q_{out} + q_{out}kC_{out} - q_{max}kC_{out} = 0$$

$$k = -\frac{q_{out}}{(q_{out}C_{out} - q_{max}C_{out})} = -\frac{0.0225\frac{mmol}{gB}}{\left(0.0225\frac{mmol}{gB}\right)\left(0.01\frac{mmol}{L}\right) - \left(0.6\frac{mmol}{gB}\right)\left(0.01\frac{mmol}{L}\right)}$$

$$\boxed{k = 3.896\ \frac{L}{mmol}}$$

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(5) $V_L = 10\,L$

$C_{init} = 100\,\frac{mmol}{L}$

$C_{out,max} = 70\,\frac{mmol}{L}$

$W_1 = 8\,g$

$q_{init,1} = 0$

$q = kC_{out}$

$k = 2.5\,\frac{L}{g}$

Pass #1 Assume: steady state

$$0 = V_L(C_{init} - C_{fin}) + W(\overset{0}{q_{init}} - q_{fin})$$

$$= V_L C_{init} - V_L C_{fin} - WkC_{fin}$$

$$0 = V_L C_{init} - C_{fin}(V_L + Wk)$$

$$C_{fin} = \frac{V_L C_{init}}{V_L + Wk} = \frac{(10\,L)(100\,\frac{mmol}{L})}{(10\,L) + (8\,g)(2.5\,\frac{L}{g})} = 33.3\,\frac{mmol}{L}$$

$$q_{out,1} = q_{in,2} = kC_{out,1} = (2.5\,\frac{L}{g})(33.3\,\frac{mmol}{L}) = 83.25\,\frac{mmol}{g}$$

Pass #2

$$0 = V_L(C_{in} - C_{fin}) + W(q_{in} - q_{fin})$$

$$= V_L C_{in} - V_L C_{fin} + Wq_{in} - WkC_{fin}$$

$$\frac{V_L C_{in} + Wq_{in}}{V_L + Wk} = C_{fin} = \frac{(10\,L)(100\,\frac{mmol}{L}) + (8\,g)(83.25\,\frac{mmol}{g})}{(10\,L) + (8\,g)(2.5\,\frac{L}{g})}$$

$$C_{fin,2} = 55.53\,\frac{mmol}{L}$$

$$q_{fin,2} = q_{in,3} = (55.53\,\frac{mmol}{L})(2.5\,\frac{L}{g}) = 138.8\,\frac{mmol}{g}$$

Pass #3

$$C_{fin} = \frac{V_L C_{in} + Wq_{in}}{V_L + Wk} = \frac{(10\,L)(100\,\frac{mmol}{L}) + (8\,g)(138.8\,\frac{mmol}{g})}{(10\,L) + (8\,g)(2.5\,\frac{L}{g})}$$

$$C_{fin} = 70.3\,\frac{mmol}{L}$$

$C_{fin,3} > C_{fin,max}$ so 2 passes is the most that can be used

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Case #1

6. $V = 10\ L$
$C_{in} = 100\ \frac{mmol}{L}$
$W = 8\ g$
$q = kC_{fin}$
$k = 2.5\ \frac{L}{g}$

Case #2 → two reactors
$V_1 = V_2 = 5\ L$
$q_{init,1} = q_{init,2} = 0$

Case #1 Assume steady state

$$0 = V_L(C_{in} - C_{fin}) + W(\cancel{q_{init}}^{0} - kC_{fin})$$

$$C_{fin} = \frac{V_L C_{in}}{V_L + Wk} = \frac{(10\ L)(100\ \frac{mmol}{L})}{(10\ L) + (8\ g)(2.5\ \frac{L}{g})} = 33.3\ \frac{mmol}{L}$$

CASE #2 $\quad C_{fin,1} = C_{init,2}$

tank 1

$$0 = V_L(C_{in} - C_{fin}) + W(\cancel{q_{init}}^{0} - kC_{fin})$$

$$0 = V_L C_{in} - V_L C_{fin} - WkC_{fin} \Rightarrow C_{fin,1} = \frac{V_L C_{in}}{V_L + Wk} = \frac{(5\ L)(100\ \frac{mmol}{L})}{(5\ L) + W(2.5\ \frac{L}{g})}$$

tank #2

$$0 = V_L(C_{in} - C_{fin}) + W(\cancel{q_{init}}^{0} - q_{fin}) = V_L\left(\frac{V_L C_{in,1}}{V_L + Wk} + C_{fin}\right) - WkC_{fin}$$

$$WkC_{fin} = \frac{V_L^2 C_{in,1}}{V_L + Wk} + V_L C_{fin}$$

$$\cancel{WkC_{fin} V_L} + W^2 k^2 C_{fin} = V_L^2 C_{in,1} + V_L^2 C_{fin} + \cancel{WkV_L C_{fin}}$$

$$W^2 = \frac{V_L^2 (C_{in,1} + C_{fin})}{k^2 C_{fin}} = \frac{(5\ L)^2 (100\ \frac{mmol}{L} + 33.3\ \frac{mmol}{L})}{(2.5\ \frac{L}{g})^2 (33.3\ \frac{mmol}{L})}$$

$$W^2 = 16.01\ g^2$$

$$\boxed{W = 4.00\ g}$$

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(7)

$$q = q_{max} \frac{K C_{fin}}{1 + K C_{fin}}$$

$SSA_{clay} = 350 \ \frac{m^2}{g}$

$SSA_2 = 50 \ m^2/g$

$SSA_{sand} = 5 \ m^2/g$

$\Gamma = 5 \ mg/m^2$

$C_{fin} = 20 \ mg/L$

$K_L = 28 \ L/mg$

clay

$$q = \Gamma(SSA) = \left(5 \frac{mg}{m^2}\right)\left(350 \frac{m^2}{g}\right) = 1750 \frac{mg}{g}$$

$$q = q_{max} \frac{K C_{fin}}{1 + K C_{fin}}$$

$$q_{max} = \frac{q(1 + K C_{fin})}{K C_{fin}} = \frac{\left(1750 \frac{mg}{g}\right)\left(1 + \left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)\right)}{\left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)}$$

$$\boxed{q_{max} = 1753.13 \ \frac{mg}{g}}$$

second mixture

$$q = \Gamma(SSA) = \left(5 \frac{mg}{m^2}\right)\left(50 \frac{m^2}{g}\right) = 250 \frac{mg}{g}$$

$$q_{max} = \frac{q(1 + K C_{fin})}{K C_{fin}} = \frac{\left(250 \frac{mg}{g}\right)\left(1 + \left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)\right)}{\left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)}$$

$$\boxed{q_{max} = 250.446 \ \frac{mg}{g}}$$

sand mixture

$$q = \Gamma(SSA) = \left(5 \frac{mg}{m^2}\right)\left(5 \frac{m^2}{g}\right) = 25 \frac{mg}{g}$$

$$q_{max} = \frac{q(1 + K C_{fin})}{K C_{fin}} = \frac{\left(25 \frac{mg}{g}\right)\left(1 + \left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)\right)}{\left(28 \frac{L}{mg}\right)\left(20 \frac{mg}{L}\right)}$$

$$\boxed{q_{max} = 25.045 \ \frac{mg}{g}}$$

↳ SSA and q_{max} are directly proportional

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(8) Quiz question:

A waste stream containing 120 mg/L of contaminant A flows into a reactor that behaves as a CFSTR. If regulatory standards require effluent concentrations of contaminant A to be ~~equal~~ less than 25 mg/L, determine if the reactor will meet regulatory requirements. Assume adsorption follows a linear isotherm where $k = 2 \frac{L}{mg}$. The ~~volume~~ liquid flow rate of the reactor is 15 L/min and 12 g/min of adsorbent is used.

solution:

$C_{in} = 120$ mg/L

$C_{out} = ?$

$Q = 15$ L/min

$X = 12$ g/min

$q = kC_{out}$

$k = 2 \frac{L}{g}$

Assume steady state and $q_{init} = 0$

$$V\frac{dc}{dt} + \cancel{w\frac{dq}{dt}}^{0} = Q(C_{in} - C_{out}) + X(\cancel{q_{in}}^{0} - q_{out})$$

$$0 = QC_{in} - QC_{out} - XkC_{out}$$

$$C_{out} = \frac{QC_{in}}{(Q + Xk)}$$

$$= \frac{(15 \frac{L}{min})(120 \frac{mg}{L})}{(15 \frac{L}{min}) + (12 \frac{g}{min})(2 \frac{L}{g})}$$

$C_{out} = 46$ mg/L

$46 > 25 \rightarrow$ not in regulatory compliance.