Unit 7: Hypothesis Testing

7.1 Example:

t-Test: Paired Two Sample for Means

Store	Con1	Con2
1	141	118
2	184	167
3	132	137
4	161	168
5	176	175
6	196	197
7	169	143
8	199	169
9	150	123
10	218	197

	Con1	*Con2*
Mean	172.6	159.4
Variance	750.27	789.38
Observations	10	10
Pearson Correlation	0.8633	
Hypothesized Mean Difference	0	
df	9	
t Stat	2.8747	
P(T<=t) one-tail	0.0092	
t Critical one-tail	1.8331	
P(T<=t) two-tail	0.0183	
t Critical two-tail	2.2622	
	Difference in Means	13.2

Interpretation:

Diet A shows a higher mean and median weight loss than Diet B, meaning on average, participants lost more weight with Diet A. The IQRs are similar (3.285 vs 3.451), so the spread of the middle 50% of results is roughly the same. For Diet A, Q1 = 3.748, which means at least 75% of participants lost more than about 3.7 kg. For Diet B, Q1 = 1.953, so many people lost less than 2 kg, showing lower effectiveness overall.

Summary:

Diet A produced a greater and more consistent weight loss than Diet B, with most participants losing at least 3.7 kg compared with only about 2 kg for Diet B, indicating that Diet A was the more effective diet.

7.1 Exercise

Recall that in the previous unit exercises, a two-tailed test was undertaken whether the population mean impurity differed between the two filtration agents in Data Set G.

Suppose instead a one-tailed test had been conducted to determine whether Filter Agent 1 was the more effective. What would your conclusions have been?

Batch	**Agent1**	**Agent2**
1	7.7	8.5
2	9.2	9.6
3	6.8	6.4
4	9.5	9.8
5	8.7	9.3
6	6.9	7.6
7	7.5	8.2
8	7.1	7.7
9	8.7	9.4
10	9.4	8.9
11	9.4	9.7
12	8.1	9.1

t-Test: Paired Two Sample for Means

	Agent1	*Agent2*
Mean	8.25	8.6833333
Variance	1.0590909	1.0778788
Observations	12	12
Pearson Correlation	0.9010558	
Hypothesized Mean Difference	0	
df	11	
t Stat	-3.2639386	
P(T<=t) one-tail	0.003773	
t Critical one-tail	1.7958848	
P(T<=t) two-tail	0.007546	
t Critical two-tail	2.2009852	
	Difference in Means	-0.433333

Summary

Because the calculated p = 0.0037 < 0.05, reject H_0. There is strong evidence that Filter Agent 1 produces a lower mean impurity than Agent 2. This indicates that Agent 1 is significantly more effective at reducing impurity.

7.2 Example

F-Test Two-Sample for Variances		
	Variable 1	*Variable 2*
Mean	5.3412	3.70996
Variance	6.429280612	7.66759359
Observations	50	50
df	49	49
F	0.838500442	
P(F<=f) one-tail	0.269951478	
F Critical one-tail	0.622165468	
p2	0.539902956	

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	5.3412	3.70996
Variance	6.429280612	7.66759359
Observations	50	50
Pooled Variance	7.048437101	
Hypothesized Mean Difference	0	
df	98	
t Stat	3.072143179	
P(T<=t) one-tail	0.001375772	
t Critical one-tail	1.660551217	
P(T<=t) two-tail	0.002751544	
t Critical two-tail	1.984467455	
Difference in Means	1.63124	

7.2 Exercise

Consider the bank cardholder data of Data Set C. Open the Excel workbook Exa8.6C.xlsx which contains this data from the Exercises folder.

Assuming the data to be suitably distributed, complete an appropriate test of whether the population mean income for males exceeds that of females and interpret your findings. What assumptions underpin the validity of your analysis, and how could you validate them?

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	*Variable 2*
Mean	52.91333333	44.23333333
Variance	233.1289718	190.1758192
Observations	60	60
Pooled Variance	211.6523955	
Hypothesized Mean Difference	0	
df	118	
t Stat	3.267900001	
P(T<=t) one-tail	0.000709735	
t Critical one-tail	1.657869522	
P(T<=t) two-tail	0.00141947	
t Critical two-tail	1.980272249	

Summary

The data is described as suitably distributed, so equal variances are assumed. An F-test was not required, and a t-test assuming equal variances was used. Because the calculated $p = 0.0007 < 0.05$, reject H_0. There is strong evidence that the mean income for males is higher than that for females.

This indicates that male income is significantly greater at the population level.

Assumptions

The analysis assumes that the data are suitably distributed and variances are equal. This means the data are considered balanced across the two groups. The assumption could be validated using an F-test to check for equality of variances. Because the question states the data are suitably distributed, the F-test was not required in this case.