1
X

$$\chi(X) := \sum_d (-1)^d \dim H^d(X;k)$$

k
X
c_d
d

$$\chi(X) = \sum_d (-1)^d c_d.$$

k
V
$k V \otimes V^\vee \tau V^\vee \otimes V k$

1
$\in$
$V \simeq$
$V \otimes$
$V^\vee$
$w \otimes$
$v \mapsto$
$w(v)$
X
du-
al-
iz-
able[1]
$X^\vee$

$$: 1 \to X \otimes X^\vee, : X^\vee \otimes X \to 1,$$

$$X \otimes X \otimes X^\vee \otimes X \otimes X$$

$$X^\vee \otimes X^\vee \otimes X \otimes X^\vee \otimes X^\vee$$

$X^\vee$
$A \mapsto$
$(A \otimes$
$X, 1)$
$(X^\vee, ,)$
$X^\vee$
X
$X \to$
Y
$Y^\vee \to$
$X^\vee$
X
$1 X \otimes X^\vee \tau X^\vee \otimes X 1$

(1)
τ
(A)
A
k
$k \neq$
2
$\langle a_1, \ldots, a_n \rangle$
hy-
per-
bolic
plane
k^2
$\langle 1, -1 \rangle$
V
k
B
B
$V \simeq$
$V_0 \oplus$
$H \oplus$
W
B
V_0
H
W
$B(w, w) \neq$

They
are
called
strongly
du-
al-
iz-
able
in
the

0
$w \in$
W
B_1, B_2, B_3
$B_1 \oplus$
$B_2 \simeq$
$B_1 \oplus$
B_3
$B_2 \simeq$
B_3
$M(k)$
k
(k)
$M(k) \to$
(k)
$() =$
$() =$
$\oplus$
(k)
$Witt$
$ring^2$
(k)
(k)
k
$(k) =$
$\ker(\dim :$
$(k) \to$
$/2)$
$n =$
1
$\det :$
$(k)/(k)^2 \to$
$k^\times/(k^\times)^2$
(k)
$\langle a \rangle$
$a \in$
$k^\times$
$\langle a \rangle =$
$\langle ab^2 \rangle$
$\langle a \rangle \langle b \rangle =$
$\langle ab \rangle$
$\langle a \rangle +$
$\langle b \rangle =$
$\langle a+$
$b \rangle +$
$\langle ab(a+$
$b) \rangle$
$a+$
$b \neq$
0
$\langle a \rangle +$
$\langle -a \rangle =$
$\langle 1 \rangle +$
$\langle -1 \rangle$
X
k
∞
X
$(_X)$
X
X
$(_X)$
1
1
X
$X \to$
Y
k
$f_*, f_! : _X \to _Y$
$f^*, f^! : _Y \to _X$
$f^* \dashv$
f_*
$f_! \dashv$
$f^!$
$f_! \to$
f_*
f
f
$f_\sharp : _X \to _Y$
f^*
f
$(f_\sharp, f^*)$
$(f_!, f^!)$