# EQUATIONS FOR BITANGENTS TO SYMMETRIC QUARTICS

CANDACE BETHEA AND THOMAS BRAZELTON

Abstract. This is supplementary data to *Bitangents to symmetric quartics*, by the authors.

## 0. About this file

For each of the 12 types of symmetric nonsingular planar quartics, an example quartic is provided with this exact automorphism group. Numerical estimates for each of the 28 bitangents are provided, grouped into orbits under the automorphism group.

The code used to compute bitangents given an input quartic equation was modified from the supplementary code to [PSV11], and is included here in Appendix A.

## Contents



## 1. Type I quartics

1.1. **Equation.** : $x^3y + y^3z + z^3y$

### 1.2. **Lines.** : (all in a single orbit)

```
x + 0.30798*y + 1.55496*z,
x + y + z,
x + 5.04892*y + 3.24698*z,
x + 0.6431*y + 0.19806*z,
x + (0.40097 + 0.5028*I)*y + (-0.04407 - 0.1931*I)*z,
x + (0.40097 - 0.5028*I)*y + (-0.04407 + 0.1931*I)*z,
x + (3.14795 + 3.9474*I)*y + (-0.72252 - 3.16557*I)*z,
x + (3.14795 - 3.9474*I)*y + (-0.72252 + 3.16557*I)*z,
x + (0.62349 + 0.78183*I)*y + (-0.22252 - 0.97493*I)*z,
x + (0.62349 - 0.78183*I)*y + (-0.22252 + 0.97493*I)*z,
x + (-0.57942 + 0.27903*I)*y + (0.12349 + 0.15485*I)*z,
x + (-0.57942 - 0.27903*I)*y + (0.12349 - 0.15485*I)*z,
x + (-4.54892 + 2.19064*I)*y + (2.02446 + 2.53859*I)*z,
x + (-4.54892 - 2.19064*I)*y + (2.02446 - 2.53859*I)*z,
x + (-0.90097 + 0.43388*I)*y + (0.62349 + 0.78183*I)*z,
x + (-0.90097 - 0.43388*I)*y + (0.62349 - 0.78183*I)*z,
x + (0.19202 + 0.24079*I)*y + (-0.34601 - 1.51597*I)*z,
x + (0.19202 - 0.24079*I)*y + (-0.34601 + 1.51597*I)*z,
x + (-0.27748 + 0.13363*I)*y + (0.9695 + 1.21572*I)*z,
x + (-0.27748 - 0.13363*I)*y + (0.9695 - 1.21572*I)*z,
x + (-0.06853 - 0.30026*I)*y + (-1.40097 - 0.67467*I)*z,
x + (-0.06853 + 0.30026*I)*y + (-1.40097 + 0.67467*I)*z,
x + (-0.22252 - 0.97493*I)*y + (-0.90097 - 0.43388*I)*z,
x + (-0.22252 + 0.97493*I)*y + (-0.90097 + 0.43388*I)*z,
x + (-1.12349 - 4.92233*I)*y + (-2.92543 - 1.40881*I)*z,
x + (-1.12349 + 4.92233*I)*y + (-2.92543 + 1.40881*I)*z,
x + (-0.1431 - 0.62698*I)*y + (-0.17845 - 0.08594*I)*z,
x + (-0.1431 + 0.62698*I)*y + (-0.17845 + 0.08594*I)*z]
```

## 2. Type II Quartics

### 2.1. **Equation.** $x^4 + y^4 + z^4$

### 2.2. **Lines.** With $C_8$ isotropy:

```
y + (-0.70710678 - 0.70710678*I)*z
y + (-0.70710678 + 0.70710678*I)*z
y + (0.70710678 + 0.70710678*I)*z
y + (0.70710678 - 0.70710678*I)*z
x + (-0.70710678 - 0.70710678*I)*y
x + (-0.70710678 + 0.70710678*I)*y
x + (0.70710678 + 0.70710678*I)*y
x + (0.70710678 - 0.70710678*I)*y
x + (-0.70710678 - 0.70710678*I)*z
x + (-0.70710678 + 0.70710678*I)*z
x + (0.70710678 + 0.70710678*I)*z
x + (0.70710678 - 0.70710678*I)*z
```

With $C_6$ isotropy:

```
x + 1.0*I*y - z
x + 1.0*I*y + (-1.0*I)*z
x + (-1.0*I)*y - z
x + 1.0*I*y + z
x + 1.0*I*y + 1.0*I*z
x - y - z
x + (-1.0*I)*y + (-1.0*I)*z
x + (-1.0*I)*y + z
x - y + (-1.0*I)*z
x + (-1.0*I)*y + 1.0*I*z
x + y - z
x - y + z
x - y + 1.0*I*z
x + y + (-1.0*I)*z
x + y + z
x + y + 1.0*I*z
```

## 3. Type III Quartics

3.1. **Equation.** $x^4 + y^4 + z^4 + (4\zeta_3 + 2)x^2 + y^2$

3.2. **Lines.** With $C_2$ isotropy:

```
x + (-0.50000084 + 0.50000314*I)*z
x + (-0.50000314 - 0.50000084*I)*z
x - y + (0.5 - 0.86603*I)*z
x - y + (-0.5 + 0.86603*I)*z
x - y + (-0.86603 - 0.5*I)*z
x - y + (0.86603 + 0.5*I)*z
x + (0.50000314 + 0.50000084*I)*z
x + (0.50000084 - 0.50000314*I)*z
y + (0.50000314 + 0.50000084*I)*z
y + (-0.50000084 + 0.50000314*I)*z
y + (0.50000084 - 0.50000314*I)*z
y + (-0.50000314 - 0.50000084*I)*z
x + 1.0*I*y + (0.5 + 0.86603*I)*z
x + 1.0*I*y + (0.86603 - 0.5*I)*z
x + 1.0*I*y + (-0.86603 + 0.5*I)*z
x + 1.0*I*y + (-0.5 - 0.86603*I)*z
x + (-1.0*I)*y + (0.5 + 0.86603*I)*z
x + (-1.0*I)*y + (0.86603 - 0.5*I)*z
x + (-1.0*I)*y + (-0.86603 + 0.5*I)*z
x + (-1.0*I)*y + (-0.5 - 0.86603*I)*z
x + y + (0.5 - 0.86603*I)*z
x + y + (-0.86603 - 0.5*I)*z
x + y + (-0.5 + 0.86603*I)*z
x + y + (0.86603 + 0.5*I)*z
```

With $C_{12}$ isotropy:

```
x + (-0.36602540 - 0.36602540*I)*y
```

```
x + (0.36602540 + 0.36602540*I)*y
x + (-1.3660254 + 1.3660254*I)*y
x + (1.3660254 - 1.3660254*I)*y
```

## 4. Type IV Quartics

**4.1. Equation.** $x^4 + y^4 + z^4 - 3(x^2y^2 + y^2z^2 + x^2z^2)$

**4.2. Lines.** With even $C_2$ isotropy:

```
y + 2.4142136*I*z
y + (-2.4142136*I)*z
y + (-0.41421356*I)*z
y + 0.41421356*I*z
x + 2.4142136*I*y
x + 2.4142136*I*z
x + (-2.4142136*I)*z
x + (-2.4142136*I)*y
x + 0.41421356*I*y
x + (-0.41421356*I)*y
x + (-0.41421356*I)*z
x + 0.41421356*I*z
```

With odd $C_2$ isotropy:

```
x + (-1.41421)*y - z
x + 1.41421*y - z
x + (-1.41421)*y + z
x + y + (-1.41421)*z
x + 1.41421*y + z
x + y + 1.41421*z
x + 0.70711*y + (-0.70711)*z
x + (-0.70711)*y + (-0.70711)*z
x + 0.70711*y + 0.70711*z
x + (-0.70711)*y + 0.70711*z
x - y + (-1.41421)*z
x - y + 1.41421*z
```

With $S_3$ isotropy

```
x + y - z
x - y + z
x - y - z
x + y + z
```

## 5. Type V Quartics

**5.1. Equation.** $x^4 + y^4 + z^4 - 4x^2y^2$

5.2. **Lines.** With $C_2^{(1)}$ isotropy

```
x + 0.75983569*z
x + (-0.75983569)*z
x + (-0.75983569*I)*z
x + 0.75983569*I*z
y + (-0.75983568)*z
y + 0.75983569*z
y + (-0.75983569*I)*z
y + 0.75983569*I*z
```

With $C_2^{(2)}$ isotropy

```
x + y + (-0.53728 - 0.53728*I)*z
x + y + (0.53728 - 0.53728*I)*z
x + y + (0.53728 + 0.53728*I)*z
x + y + (-0.53728 + 0.53728*I)*z
x - y + (-0.53728 - 0.53728*I)*z
x - y + (0.53728 - 0.53728*I)*z
x - y + (-0.53728 + 0.53728*I)*z
x - y + (0.53728 + 0.53728*I)*z
```

With $C_2^{(3)}$ isotropy

```
x + (-1.0*I)*y + (-0.9306 + 0.9306*I)*z
x + 1.0*I*y + (0.9306 - 0.9306*I)*z
x + 1.0*I*y + (-0.9306 - 0.9306*I)*z
x + (-1.0*I)*y + (0.9306 + 0.9306*I)*z
x + (-1.0*I)*y + (-0.9306 - 0.9306*I)*z
x + 1.0*I*y + (0.9306 + 0.9306*I)*z
x + 1.0*I*y + (-0.9306 + 0.9306*I)*z
x + (-1.0*I)*y + (0.9306 - 0.9306*I)*z
```

With central $C_4$ isotropy:

```
x + 1.9318517*y
x + (-0.51763809)*y
x + 0.51763809*y
x + (-1.9318517)*y
```

## 6. Type VI Quartics

6.1. **Equation.** $x^4 + xy^3 + yz^3$

6.2. **Lines.** With trivial isotropy:

```
x + (-0.59197 + 1.02532*I)*y + (-0.07781 - 0.44126*I)*z,
x + 1.18394*y + (-0.44807)*z,
x + 1.18394*y + (0.22404 - 0.38804*I)*z,
x + 1.18394*y + (0.22404 + 0.38804*I)*z,
x + (-0.59197 - 1.02532*I)*y + (-0.34324 - 0.28802*I)*z,
x + (-0.59197 - 1.02532*I)*y + (-0.07781 + 0.44126*I)*z,
x + (-0.59197 + 1.02532*I)*y + (-0.34324 + 0.28802*I)*z,
```

```
x + (-0.59197 - 1.02532*I)*y + (0.42105 - 0.15325*I)*z,
x + (-0.59197 + 1.02532*I)*y + (0.42105 + 0.15325*I)*z
```

With trivial isotropy:

```
x + (0.48258 - 0.83585*I)*y + (0.22404 + 1.27057*I)*z,
x + (-0.96516)*y + 1.29017*z,
x + (0.48258 + 0.83585*I)*y + (0.22404 - 1.27057*I)*z,
x + (-0.96516)*y + (-0.64509 - 1.11732*I)*z,
x + (0.48258 + 0.83585*I)*y + (-1.21236 + 0.44126*I)*z,
x + (0.48258 + 0.83585*I)*y + (0.98833 + 0.82931*I)*z,
x + (0.48258 - 0.83585*I)*y + (0.98833 - 0.82931*I)*z,
x + (-0.96516)*y + (-0.64509 + 1.11732*I)*z,
x + (0.48258 - 0.83585*I)*y + (-1.21236 - 0.44126*I)*z
```

With trivial isotropy:

```
x + (-0.21878)*y + 0.68649*z,
x + (0.10939 + 0.18947*I)*y + (0.11921 - 0.67606*I)*z,
x + (0.10939 - 0.18947*I)*y + (0.11921 + 0.67606*I)*z,
x + (0.10939 - 0.18947*I)*y + (-0.64509 - 0.23479*I)*z,
x + (0.10939 + 0.18947*I)*y + (-0.64509 + 0.23479*I)*z,
x + (-0.21878)*y + (-0.34324 + 0.59451*I)*z,
x + (-0.21878)*y + (-0.34324 - 0.59451*I)*z,
x + (0.10939 + 0.18947*I)*y + (0.52588 + 0.44126*I)*z,
x + (0.10939 - 0.18947*I)*y + (0.52588 - 0.44126*I)*z
```

With $C_9$ isotropy:

```
y
```

## 7. Type VII Quartics

**7.1. Equation.** $x^4 + y^4 + z^4 + -3x^2y^2 + xyz^2$

**7.2. Lines.** With trivial isotropy:

```
# Orbit of 0, no isotropy

x + (-0.20871)*y + (-0.9137)*z
x + (-0.20871)*y + 0.9137*z
x + 0.20871*y + 0.9137*I*z
x + 0.20871*y + (-0.9137*I)*z
x + (-4.79129)*y + 4.3778*z
x + (-4.79129)*y + (-4.3778)*z
x + 4.79129*y + (-4.3778*I)*z
x + 4.79129*y + 4.3778*I*z
```

With isotropy $C_2^{(1)}$:

```
x + y + (0.59161 + 0.3873*I)*z
x - y + (-0.3873 + 0.59161*I)*z
x + y + (-0.59161 - 0.3873*I)*z
```

```
x - y + (0.3873 - 0.59161*I)*z
```

With isotropy $C_2^{(1)}$:

```
x + y + (0.59161 - 0.3873*I)*z
x - y + (0.3873 + 0.59161*I)*z
x + y + (-0.59161 + 0.3873*I)*z
x - y + (-0.3873 - 0.59161*I)*z
```

With isotropy $C_2^{(2)}$:

```
x + 1.0*I*y + (-0.86603 + 0.86603*I)*z
x + (-1.0*I)*y + (0.86603 + 0.86603*I)*z
x + 1.0*I*y + (0.86603 - 0.86603*I)*z
x + (-1.0*I)*y + (-0.86603 - 0.86603*I)*z
```

With isotropy $C_2^{(2)}$:

```
x + (-1.0*I)*y + (-1.32288 + 1.32288*I)*z
x + 1.0*I*y + (1.32288 + 1.32288*I)*z
x + (-1.0*I)*y + (1.32288 - 1.32288*I)*z
x + 1.0*I*y + (-1.32288 - 1.32288*I)*z
```

With isotropy $C_2^Z$:

```
x + (-0.58662693)*y
x + (-1.7046609)*y
x + 1.7046609*y
x + 0.58662693*y
```

## 8. Type VIII Quartics

8.1. **Equation.** $x^4 + y^4 - 3x^2y^2 + yz^3$

8.2. **Lines.** With isotropy $C_2$:

```
y + (0.46415889 - 0.80394677*I)*z
y + (0.46415889 + 0.80394677*I)*z
y + (-0.92831777)*z
```

With trivial isotropy:

```
x + 1.85047*y + (0.19641 + 0.3402*I)*z
x + 1.85047*y + (0.19641 - 0.3402*I)*z
x + 1.85047*y - 0.39283*z
x + (-1.85047)*y + (-0.19641 + 0.3402*I)*z
x + (-1.85047)*y + 0.39283*z
x + (-1.85047)*y + (-0.19641 - 0.3402*I)*z
```

With trivial isotropy:

```
x + (-0.56125*I)*y + (-1.18243*I)*z
x + (-0.56125*I)*y + (-1.02401 + 0.59121*I)*z
x + (-0.56125*I)*y + (1.02401 + 0.59121*I)*z
x + 0.56125*I*y + (-1.02401 - 0.59121*I)*z
```

```
x + 0.56125*I*y + (1.02401 - 0.59121*I)*z
x + 0.56125*I*y + 1.18243*I*z
```

With trivial isotropy:

```
# Orbit of trivial isotropy

x + (0.70756 - 0.23507*I)*y + (-0.52306 - 0.12198*I)*z
x + (0.70756 - 0.23507*I)*y + (0.36717 - 0.39199*I)*z
x + (0.70756 - 0.23507*I)*y + (0.15589 + 0.51397*I)*z
x + (-0.70756 + 0.23507*I)*y + (-0.36717 + 0.39199*I)*z
x + (-0.70756 + 0.23507*I)*y + (0.52306 + 0.12198*I)*z
x + (-0.70756 + 0.23507*I)*y + (-0.15589 - 0.51397*I)*z
```

With trivial isotropy:

```
x + (0.70756 + 0.23507*I)*y + (0.15589 - 0.51397*I)*z
x + (0.70756 + 0.23507*I)*y + (-0.52306 + 0.12198*I)*z
x + (-0.70756 - 0.23507*I)*y + (0.52306 - 0.12198*I)*z
x + (0.70756 + 0.23507*I)*y + (0.36717 + 0.39199*I)*z
x + (-0.70756 - 0.23507*I)*y + (-0.15589 + 0.51397*I)*z
x + (-0.70756 - 0.23507*I)*y + (-0.36717 - 0.39199*I)*z
```

With $C_6$ isotropy:

```
y
```

## 9. Type IX Quartics

**9.1. Equation.** $x^3z + y^3z + x^2y^2 - 25xyz^2 + 10z^4$

**9.2. Lines.** With trivial isotropy:

```
x + 0.16706*y + (-0.60089)*z
x + (-0.08353 - 0.14467*I)*y + (0.30044 - 0.52038*I)*z
x + (-0.08353 + 0.14467*I)*y + (0.30044 + 0.52038*I)*z
x + (-2.99302 + 5.18406*I)*y + (1.79847 + 3.11504*I)*z
x + (-2.99302 - 5.18406*I)*y + (1.79847 - 3.11504*I)*z
x + 5.98604*y + (-3.59694)*z
```

With trivial isotropy:

```
x + (-0.02344 + 0.0406*I)*y + (2.61072 + 4.52191*I)*z
x + (-10.66564 - 18.47342*I)*y + (55.69007 - 96.45804*I)*z
x + (-0.02344 - 0.0406*I)*y + (2.61072 - 4.52191*I)*z
x + (-10.66564 + 18.47342*I)*y + (55.69007 + 96.45804*I)*z
x + 21.33127*y + (-111.38015)*z
x + 0.04688*y + (-5.22145)*z
```

With trivial isotropy:

```
x + 0.28062*y + 0.59766*z
x + (-0.14031 - 0.24302*I)*y + (-0.29883 + 0.51759*I)*z
```

```
x + (-0.14031 + 0.24302*I)*y + (-0.29883 - 0.51759*I)*z
x + (-1.78177 - 3.08611*I)*y + (-1.06489 + 1.84445*I)*z
x + 3.56354*y + 2.12978*z
x + (-1.78177 + 3.08611*I)*y + (-1.06489 - 1.84445*I)*z
```

With $C_2$ isotropy:

```
x + (-0.5 + 0.86603*I)*y + (4.29692 + 7.44248*I)*z
x + (-0.5 - 0.86603*I)*y + (4.29692 - 7.44248*I)*z
x + y + (-8.59384)*z
```

With $C_2$ isotropy:

```
x + (-0.5 - 0.86603*I)*y + (0.69826 + 4.06576*I)*z
x + y + (3.17192 - 2.63759*I)*z
x + (-0.5 + 0.86603*I)*y + (-3.87018 - 1.42817*I)*z
```

With $C_2$ isotropy:

```
x + (-0.5 - 0.86602*I)*y + (-3.87017 + 1.42815*I)*z
x + y + (3.17192 + 2.63759*I)*z
x + (-0.5 + 0.86602*I)*y + (0.69827 - 4.06574*I)*z
```

With $S_3$ isotropy:

```
z
```

## 10. Type X Quartics

10.1. **Equation.** $x^4 + y^4 + z^4 - 9x^2y^2 - 3y^2z^2 - 8x^2z^2$

10.2. **Lines.** With trivial isotropy:

```
x + (-2.49875)*y + (-1.56867)*z
x + (-2.49875)*y + 1.56867*z
x + 2.49875*y + (-1.56867)*z
x + 2.49875*y + 1.56867*z
```

With trivial isotropy:

```
x + (-0.65498)*y + (-0.63748)*z
x + 0.65498*y + 0.63748*z
x + 0.65498*y + (-0.63748)*z
x + (-0.65498)*y + 0.63748*z
```

With trivial isotropy:

```
x + 0.4002*y + 0.41118*z
x + (-0.4002)*y + (-0.41118)*z
x + 0.4002*y + (-0.41118)*z
x + (-0.4002)*y + 0.41118*z
```

With trivial isotropy:

```
x + 1.52676*y + (-2.43201)*z
x + (-1.52676)*y + 2.43201*z
x + 1.52676*y + 2.43201*z
x + (-1.52676)*y + (-2.43201)*z
```

With trivial isotropy:

```
x + (-0.24805077*I)*z
x + 0.24805077*I*z
1.0000000*x + (-0.24805077*I)*z
1.0000000*x + 0.24805077*I*z
```

With $C_2^L$ isotropy:

```
y + 0.71838335*I*z
y + (-0.71838335*I)*z
```

With $C_2^L$ isotropy:

```
y + 1.2287796*I*z
y + (-1.2287796*I)*z
```

With $C_2^R$ isotropy:

```
x + 1.0273042*I*z
x + (-1.0273042*I)*z
```

With $C_2^R$ isotropy:

```
x + 1.1565916*I*y
x + (-1.1565916*I)*y
```

## 11. Type XI Quartics

11.1. **Equation.** $x(x-y)(x-2y)(x-3y)+yz^3$

11.2. **Lines.** With trivial isotropy:

```
y - z
y + (0.50000000 - 0.86602540*I)*z
y + (0.50000000 + 0.86602540*I)*z
```

With trivial isotropy:

```
x + 0.21958*y + (-0.40224)*z
x + 0.21958*y + (0.20112 - 0.34835*I)*z
x + 0.21958*y + (0.20112 + 0.34835*I)*z
```

With trivial isotropy:

```
x + (-1.5 - 0.48203*I)*y + (0.98297 + 0.56752*I)*z
x + (-1.5 - 0.48203*I)*y + (-1.13503*I)*z
x + (-1.5 - 0.48203*I)*y + (-0.98297 + 0.56752*I)*z
```

With trivial isotropy:

```
x + (-3.21958)*y + 0.40224*z
x + (-3.21958)*y + (-0.20112 + 0.34835*I)*z
x + (-3.21958)*y + (-0.20112 - 0.34835*I)*z
```

With trivial isotropy:

```
x + (-2.08999 + 0.20908*I)*y + (0.54475 + 0.13927*I)*z
x + (-2.08999 + 0.20908*I)*y + (-0.15177 - 0.5414*I)*z
x + (-2.08999 + 0.20908*I)*y + (-0.39298 + 0.40213*I)*z
```

With trivial isotropy:

```
x + (-2.08999 - 0.20908*I)*y + (0.54475 - 0.13927*I)*z
x + (-2.08999 - 0.20908*I)*y + (-0.15177 + 0.5414*I)*z
x + (-2.08999 - 0.20908*I)*y + (-0.39298 - 0.40213*I)*z
```

With trivial isotropy:

```
x + (-1.5 + 0.48203*I)*y + (0.98297 - 0.56752*I)*z
x + (-1.5 + 0.48203*I)*y + 1.13503*I*z
x + (-1.5 + 0.48203*I)*y + (-0.98297 - 0.56752*I)*z
```

With trivial isotropy:

```
x + (-0.91001 + 0.20908*I)*y + (0.39298 + 0.40213*I)*z
x + (-0.91001 + 0.20908*I)*y + (0.15177 - 0.5414*I)*z
x + (-0.91001 + 0.20908*I)*y + (-0.54475 + 0.13927*I)*z
```

With trivial isotropy:

```
x + (-0.91001 - 0.20908*I)*y + (0.39298 - 0.40213*I)*z
x + (-0.91001 - 0.20908*I)*y + (0.15177 + 0.5414*I)*z
x + (-0.91001 - 0.20908*I)*y + (-0.54475 - 0.13927*I)*z
```

With $C_3$ isotropy:

```
y
```

## 12. Type XII Quartics

12.1. **Equation.** $x^4 + x^2(-y^2 - 4z^2) + y^4 - y^2z^2 - z^4$

12.2. **Lines.** With trivial isotropy:

```
x + (0.96825 + 0.25*I)*y + (-0.99367 - 0.48721*I)*z
x + (-0.96825 - 0.25*I)*y + (0.99367 + 0.48721*I)*z
```

With trivial isotropy:

```
x + (0.96825 - 0.25*I)*y + (-0.99367 + 0.48721*I)*z
x + (-0.96825 + 0.25*I)*y + (0.99367 - 0.48721*I)*z
```

With trivial isotropy:

```
x + (-0.79057 - 0.61237*I)*y + (-0.3978 + 0.81133*I)*z
x + (0.79057 + 0.61237*I)*y + (0.3978 - 0.81133*I)*z
```

With trivial isotropy:

```
x + (-0.79057 + 0.61237*I)*y + (-0.3978 - 0.81133*I)*z
x + (0.79057 - 0.61237*I)*y + (0.3978 + 0.81133*I)*z
```

With trivial isotropy:

```
x + (0.79057 - 0.61237*I)*y + (-0.3978 - 0.81133*I)*z
x + (-0.79057 + 0.61237*I)*y + (0.3978 + 0.81133*I)*z
```

With trivial isotropy:

```
x + (0.79057 + 0.61237*I)*y + (-0.3978 + 0.81133*I)*z
x + (-0.79057 - 0.61237*I)*y + (0.3978 - 0.81133*I)*z
```

With trivial isotropy:

```
x + (-0.96825 - 0.25*I)*y + (-0.99367 - 0.48721*I)*z
x + (0.96825 + 0.25*I)*y + (0.99367 + 0.48721*I)*z
```

With trivial isotropy:

```
x + (-0.96825 + 0.25*I)*y + (-0.99367 + 0.48721*I)*z
x + (0.96825 - 0.25*I)*y + (0.99367 - 0.48721*I)*z
```

With trivial isotropy:

```
x + (-2.5031952)*z
x + 2.5031952*z
```

With trivial isotropy:

```
x + (-0.51573862*I)*z
x + 0.51573862*I*z
```

With trivial isotropy:

```
x + (-0.44721360 - 0.54772256*I)*y
x + (0.44721360 + 0.54772256*I)*y
```

With trivial isotropy:

```
x + (-0.44721360 + 0.54772256*I)*y
x + (0.44721360 - 0.54772256*I)*y
```

With $C_2$ isotropy:

```
y + (-2.2947737)*z
```

With $C_2$ isotropy:

```
y + 2.2947737*z
```

With $C_2$ isotropy:

```
y + (-1.1251606*I)*z
```

With $C_2$ isotropy:

```
y + 1.1251606*I*z
```

## Appendix A. Code to compute quartics

The following Sage code is adapted from Vinzant's webpage from her research on quartics with Plaumann and Sturmfels. It inputs a quartic polynomial and outputs (among other things) equations for its tritangents.

```
def compute_bitangents(f):
    F=QQ; T.<x,y,z>=PolynomialRing(F)
    # Check whether the quartic is smooth
    Grad=ideal(f,diff(f,x),diff(f,y),diff(f,z))
    # if not Grad.dimension()==0:
    #     sys.exit("Quartic is not smooth!")

    R.<x,y,z,a,b,a0,a1,a2,a3,a4>=PolynomialRing(F)
    f0=f.base_extend(R)
    S.<a,b>=PolynomialRing(F)
    digits=50
    threshold=0.000000000001
    almostzero=threshold

    Line= a*x+b*y+z;
    puresquare=ideal(
        a0*a3^2-a1^2*a4,8*a0^2*a3-4*a0*a1*a2+a1^3,
        8*a1*a4^2-4*a2*a3*a4+a3^3,
        8*a0*a1*a4-4*a0*a2*a3+a1^2*a3,
        8*a0*a3*a4-4*a1*a2*a4+a1*a3^2,
        16*a0^2*a4+2*a0*a1*a3-4*a0*a2^2+a1^2*a2,
        16*a0*a4^2+2*a1*a3*a4-4*a2^2*a4+a2*a3^2
        );
    Res=f0.resultant(Line,z)
    Res=Res.subs(y=1)
    phi=hom(R,S,[
        0,
        0,
        0,
        a,
        b,
        Res.coefficient({x:0}),
        Res.coefficient({x:1}),
        Res.coefficient({x:2}),
        Res.coefficient({x:3}),
        Res.coefficient({x:4})
        ])
    bit1 = phi(puresquare)

    I_ideal=singular.groebner(singular(bit1))
    singular.lib('solve.lib')
    VRing=singular.solve(I_ideal,digits)
    singular.set_ring(VRing)
    B1=singular("SOL")
```

```
nreal1=0
Bitangents=[]
RealBitangents=[]
for k in [1..len(B1)]:
    real=0;
    if ((B1[k][1].impart()).absValue()<threshold) and \
    ((B1[k][2].impart()).absValue()<threshold):
        real=1
        RealBitangents=RealBitangents+[(float(B1[k][1].repart())+ \
        float(B1[k][1].impart())*i)*x+(float(B1[k][2].repart())+ \
        float(B1[k][2].impart())*i)*y+z]
    nreal1=nreal1+real
    Bitangents=Bitangents+[(float(B1[k][1].repart())+ \
    float(B1[k][1].impart())*i)*x+ \
    (float(B1[k][2].repart())+ \
    float(B1[k][2].impart())*i)*y+z]
Line=a*x+y
Res=f0.resultant(Line,y)
Res=Res.subs(z=1)
phi=hom(R,S,[
    0,
    0,
    0,
    a,
    0,
    Res.coefficient({x:0}),
    Res.coefficient({x:1}),
    Res.coefficient({x:2}),
    Res.coefficient({x:3}),
    Res.coefficient({x:4})
])
bit2=phi(puresquare)+ideal(b)

if dimension(bit2)==-1: nreal2=0
else:
      I_ideal=singular.groebner(singular(bit2))
      singular.lib('solve.lib')
      VRing=singular.solve(I_ideal,digits)
      singular.set_ring(VRing)
      B2=singular("SOL")
      nreal2=0
      for k in [1..len(B2)]:
            real=0
            if ((B2[k][1].impart()).absValue()<threshold) \
            and ((B2[k][2].impart()).absValue()<threshold):
                real=1
                RealBitangents=RealBitangents+ \
                [(float(B2[k][1].repart())+ \
                float(B2[k][1].impart())*i)*x+y]
```

```
            nreal2=nreal2+real
            Bitangents=Bitangents+ \
            [(float(B2[k][1].repart())+ \
            float(B2[k][1].impart())*i)*x+y]

Res=f0.resultant(x)
Res=Res.subs(z=1)
phi=hom(R,F,[
    0,
    0,
    0,
    0,
    0,
    Res.coefficient({y:0}),
    Res.coefficient({y:1}),
    Res.coefficient({y:2}),
    Res.coefficient({y:3}),
    Res.coefficient({y:4})
])
bit3=phi(puresquare)
if bit3==ideal(0):
    nreal3=1
    Bitangents=Bitangents+[x]
    RealBitangents=RealBitangents+[x]
else: nreal3=0

NRealBit=nreal1+nreal2+nreal3
if len(Bitangents)!=28:
    return OSError("Less than 28 bitangents computed")

return Bitangents
```

## References

[PSV11] Daniel Plaumann, Bernd Sturmfels, and Cynthia Vinzant, *Quartic curves and their bitangents*, J. Symbolic Comput. **46** (2011), no. 6, 712–733. MR 2781949