Spring 2022 Directed Reading Program Final Presentations

Thomas Brazelton and Marielle Ong

Please join us for the Directed Reading Program's Spring 2022 final presentations. A tentative schedule is attached!

Time	DRL A1	DRL A4
4:00—4:20	*Arrive, chat, say hi*	
4:20—4:30	*Opening remarks*	
4:30—4:45	Kevin You	Richard Fried
4:45—5:00	Brian Lee	Ling Xu
5:00—5:15	David Kogan	Ekaterina Skorniakova
5:15—5:30	Quincy Alston	Zhaobo Han
5:30—5:45	Mason Larkin	Shriya Karam
5:45—6:00	Carolina Mora	Zakaria Sines
6:00—7:00	*Dinner*	
7:00—7:15	Nick Pilotti	Zhong Zhang
7:15—7:30	Angel Munoz	Cianán Conefrey-Shinozaki
7:30—7:45	Iris Horng	Alexander Kassouni
7:45—8:00	Patrik Farkas	Yiyang Liu
8:00—8:15	Mike Zhou	Justin Lipitz & Kevin Li*
8:15—8:30		

* = joint talk

Titles and abstracts

Differential Geometry: Extrinsic and Intrinsic Geometries of Surfaces

Ling Xu

Mentor: Hunter Stufflebeam

Abstract: In this talk, we will explore the differences between intrinsic and extrinsic geometries of surfaces and how they interact. We will define and use mechanisms and properties like fundamental forms, Gaussian curvature, and mean curvature to discuss problems in differential geometry. In particular, time permitting, we want to introduce the Weyl embedding problem of manifolds.

The Set-Theoretic Constructions of Mathematics

Kevin You

Mentor: Jin Wei

Abstract: In this presentation, we will go over the ZFC axioms towards constructing regular mathematical objects. We argue that all regular mathematics can be done in an extremely small set that we define recursively. That is, the natural numbers, the rationals, the reals, and the complex numbers and all possible algebraic operations structures on these sets belong to this set.

Stochastic Models in Queuing Theory

Shriya Karam

Mentor: Léonardo Guihoto

Abstract: Queuing models are composed of arrivals that enter a system and departures that are serviced by (a) server(s) within the system. In this presentation, we will introduce stochastic arrival and departure processes in queuing models and discuss steady state behavior under scenarios with a finite number of servers. We will also discuss applications of queuing models in traffic flow theory in modeling capacity and delays.

Topological Data Analysis and Music

Iris Horng

Mentor: Jacob Van Hook

Abstract: The goal of this project is to learn about topological data analysis and how it can be applied to analyzing musical data. Using Vidit Nanda's "Computational Algebraic Topology Lecture Notes," we will learn about simplicial complexes and filtrations in order to study persistent homology and barcodes. Referencing the "Topology of Musical Data" by William Sethares and Ryan Budney, we will then explore topological features in music like the Circle of Fifths, as well as how to achieve and analyze barcodes using selections from musical pieces.

The Fundamental Group and Homology

Angel Munoz

Mentor: Yi Wang

Abstract: We will be introducing the fundamental group and its utility towards informing topology, such as brief sketches of proof of Brouwer's fixed point theorem and Borzuk-Ulom. Conversely, we will also have topology informing algebra by demonstrating the increased accessibility of free group representation as consequences of connected graph homotopy. Time permitting, discussion of Van Kampen's theorem will be included.

Type Theory and the Calculus of Constructions

Alexander Kassouni

Mentor: Alvaro Pintado

Abstract: Type theories are formal systems that appear frequently in computer science. The Curry-Howard Isomorphism provides a correspondence between type theory and logic, which is the basis for modern proof assistants. This talk will discuss the Calculus of Constructions, a type theory which is powerful enough to express mathematical proof. Some attention will be paid to its practical application in Lean.

Stochastic Calculus in Bond Pricing

Brian Lee

Mentor: Artur Bicalho Saturnino

Abstract: We will use examples from arbitrage-free bond pricing to motivate an introduction to Brownian motion and Ito calculus. After modeling the spot rate as a stochastic process, we will develop the idea of a yield curve. Then we will find a PDE that describes the fair prices of bonds with different maturities based on the assumption that arbitrage opportunities cannot exist in efficient markets.

Consistency Proofs in Set Theory

Zakaria Sines

Mentor: Krishan Canzius

Abstract: I will be providing a basic introduction of logical consistency proofs in the context of set theory. The audience will be introduced to notions of model theory and the common axioms of ZF. A finitistic model of ZF can be constructed which is consistent relative to ZF, and this example will be highlighted to illuminate the idea of a basic consistency proof.

Combinatorial Games and the Surreal Numbers

Richard Fried

Mentor: Joanne Beckford

Abstract: We have studied how combinatorial games such as hackenbush inspire a whole new set of numbers called the surreal numbers. These numbers have various interesting properties as well as applications to playing and solving games.

Homotopy and Poincaré's lemma

Zhaobo Han

Mentor: Andres Mejia

Abstract: We will first introduce the concepts of differential forms and de-Rham cohomology of a smooth manifold, after which we will introduce the definition of pullback maps of differential forms. Ultimately we will put these definitions together to sketch a proof of Poincaré's Lemma, one of whose corollaries reveals the connection between the homotopy type of a manifold and its de-Rham cohomology.

Excess Intersection in Enumerative Geometry

Zhong Zhang

Mentor: Thomas Brazelton

Abstract: We will first briefly introduce the setup like complex projective spaces, homogeneous polynomials, and the existence of extraneous solutions when we count intersections. Then, we will define vector bundles, sections, and the Euler class and explain how this helps us to get the right count. Lastly, we will give a concrete example of counting intersection points of two plane conics that have a line in common.

First Order Logic and Relational Databases

Quincy Alston

Mentor: Eben Blaisdell

Abstract: We will introduce the rules and domain of first order logic. First order logic uses quantifiers, along with free variables and terms. We will explore the essence of why free variables and quantifiers make first order logic useful for proving statements on sets. We will then discuss the practical applications of first order logic in the SQL programming language and relational databases. SQL queries replicate first order sentences and logical operators. We do this by considering tables as logical predicates.

Topological Data Analysis for Prediction Markets

Justin Lipitz and Kevin Li

Mentors: Ryan Brill and Julian Gould

Abstract: We will provide an introduction to the basics of topological data analysis through the lens of prediction markets. Topological data analysis has been shown to be able to predict financial markets crashes. We investigate its applications towards opinion cascade in prediction markets, as well as introduce other methods for securities trading.

Lie Group, Lie Algebras, and Representaions

Nicholas Pilotti

Mentor: Souparna Purohit

Abstract: In the early days of group theory, the most studied examples of groups were permutation group and matrix groups. As the historical process of abstraction progressed, the former became the study of finite groups and the ladder became the study of Lie groups. Studying Lie groups requires background in differential geometry, but this is not necessary for a first pass, which requires nothing more than linear algebra, calculus, and maybe some abstract algebra. In this talk, I will discuss Lie groups, explain their relation to Lie algebras, and review the classification of representations of complex semi-simple Lie algebras. My reference for this talk is Brian Hall's Lie Groups, Lie Algebras, and Representations.

Applications of topological data analysis to the detection of bifurcations

Mason Larkin

Mentor: Miguel Lopez

Abstract: In this presentation, we are applying principles from topological data analysis such as persistent homology to study topological features of dynamical systems. Specifically, we will be analyzing bifurcations in the Duffing oscillator, using Khushboo Mittal and Shalabh Gupta's paper titled "Topological characterization and early detection of bifurcations and chaos in complex systems using persistent homology" as a guide.

Distribution of Primes

Mike Zhou

Mentor: Andrew Kwon

Abstract: Starting from the works of Euler, we will begin by diving into eighteenth-century proofs that opened a new perspective to prime numbers. We will explore topics such as Chebyshev estimates, Mertens estimates, and the analytic tools used to achieve their respective bounds, and end with a brief sketch of the Prime Number Theorem.

Martingales and Applications

David Kogan

Mentor: Jae Ho Choi

Abstract: We will introduce the formal definitions of Conditional Expectation and Martingales. Then we discuss the applications of Martingales to Random Walks, and the Ballot Theorem.

Application of Topology in Robotics

Ekaterina Skorniakova

Mentor: Travis Leadbetter

Abstract: How can we use topology to ensure moving robots do not collide? In this talk, we will be walking through some major concepts of Topology, such as continuity, connectedness, and homeomorphism in the context of a classical robotics engineering problem, collision avoidance and motion planning. We will see how topological concepts can connect to familiar theorems in calculus, and how they can significantly simplify complex problems.

TBD

Carolina Mora

Mentor: Marielle Ong

Abstract: TBD

Geodesic Equation

Yiyang Liu

Mentor: Christopher Bailey

Abstract: Starting with the principle of least action, we would derive the Euler-Lagrange equations, and talk about how it determines geodesic equations. We will in particular look at how it behaves with the Lagrangian that governs Newtonian mechanics, which is just the kinetic energy, and see how it leads to a particular geodesic equation. Time permits, we will talk about some examples of geodesics.

2D Topological Quantum Field Theories

Cianán Conefrey-Shinozaki

Mentor: Maxine Calle

Abstract: 2D Topological Quantum Field Theories (2D TQFT) can be defined in many equivalent ways: Axiomatically, it's a rule which associates closed "space-like" $(n-1)$-manifolds to vector spaces, and "time-like" n-manifolds to linear maps in such a way that it generalizes the time evolution of wave-functions in quantum mechanics. Categorically, it's a symmetric monoidal functor from n-dimensional cobordisms to vector fields. In my talk we will explore the category of cobordisms, look at an example of a 2D TQFT, and understand broadly how one can connect this theory to statements in introductory quantum mechanics.

Fractals

Patrik Farkas

Mentor: Elijah Gunther

Abstract: This talk will be an introduction into fractals along with a discussion of several examples like Sierpinski triangle and Cantor set. We will also discuss some measure theory as it underlies some fractals and their properties.