EQUATIONS FOR BITANGENTS TO SYMMETRIC QUARTICS

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Abstract. This is supplementary data to *Bitangents to symmetric quartics*, by the authors.

0. About this file

For each of the 12 types of symmetric nonsingular planar quartics, an example quartic is provided with this exact automorphism group. Numerical estimates for each of the 28 bitangents are provided, grouped into orbits under the automorphism group.

The code used to compute bitangents given an input quartic equation was modified from the supplementary code to [PSV11], and is included here in Appendix A.

Contents

1. Type I quartics

1.1. **Equation.** : $x^3y + y^3z + z^3y$

1.2. **Lines.** : (all in a single orbit)

```
x + 0.30798*y + 1.55496*z,
x + y + z,
x + 5.04892*y + 3.24698*z,
x + 0.6431*y + 0.19806*z,
x + (0.40097 + 0.5028*I)*y + (-0.04407 - 0.1931*I)*z,
x + (0.40097 - 0.5028*I)*y + (-0.04407 + 0.1931*I)*z,
x + (3.14795 + 3.9474*I)*y + (-0.72252 - 3.16557*I)*z,
x + (3.14795 - 3.9474*I)*y + (-0.72252 + 3.16557*I)*z,
x + (0.62349 + 0.78183*I)*y + (-0.22252 - 0.97493*I)*z,
x + (0.62349 - 0.78183*I)*y + (-0.22252 + 0.97493*I)*z,
x + (-0.57942 + 0.27903*I)*y + (0.12349 + 0.15485*I)*z,
x + (-0.57942 - 0.27903*I)*y + (0.12349 - 0.15485*I)*z,
x + (-4.54892 + 2.19064*I)*y + (2.02446 + 2.53859*I)*z,
x + (-4.54892 - 2.19064*I)*y + (2.02446 - 2.53859*I)*z,
x + (-0.90097 + 0.43388*I)*y + (0.62349 + 0.78183*I)*z,
x + (-0.90097 - 0.43388*I)*y + (0.62349 - 0.78183*I)*z,
x + (0.19202 + 0.24079*I)*y + (-0.34601 - 1.51597*I)*z,
x + (0.19202 - 0.24079*I)*y + (-0.34601 + 1.51597*I)*z,
x + (-0.27748 + 0.13363*I)*y + (0.9695 + 1.21572*I)*z,
x + (-0.27748 - 0.13363*I)*y + (0.9695 - 1.21572*I)*z,
x + (-0.06853 - 0.30026*I)*y + (-1.40097 - 0.67467*I)*z,
x + (-0.06853 + 0.30026*I)*y + (-1.40097 + 0.67467*I)*z,
x + (-0.22252 - 0.97493*I)*y + (-0.90097 - 0.43388*I)*z,
x + (-0.22252 + 0.97493*I)*y + (-0.90097 + 0.43388*I)*z,
x + (-1.12349 - 4.92233*I)*y + (-2.92543 - 1.40881*I)*z,
x + (-1.12349 + 4.92233*I)*y + (-2.92543 + 1.40881*I)*z,
x + (-0.1431 - 0.62698*I)*y + (-0.17845 - 0.08594*I)*z,
x + (-0.1431 + 0.62698*I)*y + (-0.17845 + 0.08594*I)*z]
```

2. Type II Quartics

2.1. **Equation.** $x^4 + y^4 + z^4$

2.2. **Lines.** With C_8 isotropy:

```
y + (-0.70710678 - 0.70710678*I)*z
y + (-0.70710678 + 0.70710678*I)*z
y + (0.70710678 + 0.70710678*I)*z
y + (0.70710678 - 0.70710678*I)*z
x + (-0.70710678 - 0.70710678*I)*y
x + (-0.70710678 + 0.70710678*I)*y
x + (0.70710678 + 0.70710678*I)*y
x + (0.70710678 - 0.70710678*I)*y
x + (-0.70710678 - 0.70710678*I)*z
x + (-0.70710678 + 0.70710678*I)*z
x + (0.70710678 + 0.70710678*I)*z
x + (0.70710678 - 0.70710678*I)*z
```

With C_6 isotropy:

```
x + 1.0*I*y - z
x + 1.0*I*y + (-1.0*I)*z
x + (-1.0*I)*y - z
x + 1.0*I*y + z
x + 1.0*I*y + 1.0*I*z
x - y - z
x + (-1.0*I)*y + (-1.0*I)*z
x + (-1.0*I)*y + z
x - y + (-1.0*I)*z
x + (-1.0*I)*y + 1.0*I*z
x + y - z
x - y + z
x - y + 1.0*I*z
x + y + (-1.0*I)*z
x + y + z
x + y + 1.0*I*z
```

3. Type III Quartics

3.1. **Equation.** $x^4 + y^4 + z^4 + (4\zeta_3 + 2)x^2 + y^2$

3.2. **Lines.** With C_2 isotropy:

```
x + (-0.50000084 + 0.50000314*I)*z
x + (-0.50000314 - 0.50000084*I)*z
x - y + (0.500000000000000 - 0.866030000000000*I)*z
x - y + (-0.500000000000000 + 0.866030000000000*I)*z
x - y + (-0.866030000000000 - 0.500000000000000*I)*z
x - y + (0.866030000000000 + 0.500000000000000*I)*z
x + (0.50000314 + 0.50000084*I)*z
x + (0.50000084 - 0.50000314*I)*z
y + (0.50000314 + 0.50000084*I)*z
y + (-0.50000084 + 0.50000314*I)*z
y + (0.50000084 - 0.50000314*I)*z
y + (-0.50000314 - 0.50000084*I)*z
x + 1.00000000000000*I*y + (0.500000000000000 + 0.866030000000000*I)*z
x + 1.00000000000000*I*y + (0.866030000000000 - 0.500000000000000*I)*z
x + 1.00000000000000*I*y + (-0.866030000000000 + 0.500000000000000*I)*z
x + 1.00000000000000*I*y + (-0.500000000000000 - 0.866030000000000*I)*z
x + (-1.00000000000000*I)*y + (0.500000000000000 + 0.866030000000000*I)*z
x + (-1.00000000000000*I)*y + (0.866030000000000 - 0.500000000000000*I)*z
x + (-1.00000000000000*I)*y + (-0.866030000000000 + 0.500000000000000*I)*z
x + (-1.00000000000000*I)*y + (-0.500000000000000 - 0.866030000000000*I)*z
x + y + (0.500000000000000 - 0.866030000000000*I)*z
x + y + (-0.866030000000000 - 0.500000000000000*I)*z
x + y + (-0.500000000000000 + 0.866030000000000*I)*z
x + y + (0.866030000000000 + 0.500000000000000*I)*z
```

With C_{12} isotropy:

```
x + (-0.36602540 - 0.36602540*I)*y
```

```
x + (0.36602540 + 0.36602540*I)*y
x + (-1.3660254 + 1.3660254*I)*y
x + (1.3660254 - 1.3660254*I)*y
```

4. Type IV Quartics

4.1. **Equation.** $x^4 + y^4 + z^4 - 3(x^2y^2 + y^2z^2 + x^2z^2)$

4.2. **Lines.** With even C_2 isotropy:

```
y + 2.4142136*I*z
y + (-2.4142136*I)*z
y + (-0.41421356*I)*z
y + 0.41421356*I*z
x + 2.4142136*I*y
x + 2.4142136*I*z
x + (-2.4142136*I)*z
x + (-2.4142136*I)*y
x + 0.41421356*I*y
x + (-0.41421356*I)*y
x + (-0.41421356*I)*z
x + 0.41421356*I*z
```

With odd C_2 isotropy:

```
x + (-1.41421)*y - z
x + 1.41421*y - z
x + (-1.41421)*y + z
x + y + (-1.41421)*z
x + 1.41421*y + z
x + y + 1.41421*z
x + 0.70711*y + (-0.70711)*z
x + (-0.70711)*y + (-0.70711)*z
x + 0.70711*y + 0.70711*z
x + (-0.70711)*y + 0.70711*z
x - y + (-1.41421)*z
x - y + 1.41421*z
```

With S_3 isotropy

```
x + y - z
x - y + z
x - y - z
x + y + z
```

5. Type V Quartics

5.1. **Equation.** $x^4 + y^4 + z^4 - 4x^2y^2$

5.2. **Lines.** With $C_2^{(1)}$ isotropy

```
x + 0.75983569*z
x + (-0.75983569)*z
x + (-0.75983569*I)*z
x + 0.75983569*I*z
y + (-0.75983568)*z
y + 0.75983569*z
y + (-0.75983569*I)*z
y + 0.75983569*I*z
```

With $C_2^{(2)}$ isotropy

```
x + y + (-0.53728 - 0.53728*I)*z
x + y + (0.53728 - 0.53728*I)*z
x + y + (0.53728 + 0.53728*I)*z
x + y + (-0.53728 + 0.53728*I)*z
x - y + (-0.53728 - 0.53728*I)*z
x - y + (0.53728 - 0.53728*I)*z
x - y + (-0.53728 + 0.53728*I)*z
x - y + (0.53728 + 0.53728*I)*z
```

With $C_2^{(3)}$ isotropy

```
x + (-1.0*I)*y + (-0.9306 + 0.9306*I)*z
x + 1.0*I*y + (0.9306 - 0.9306*I)*z
x + 1.0*I*y + (-0.9306 - 0.9306*I)*z
x + (-1.0*I)*y + (0.9306 + 0.9306*I)*z
x + (-1.0*I)*y + (-0.9306 - 0.9306*I)*z
x + 1.0*I*y + (0.9306 + 0.9306*I)*z
x + 1.0*I*y + (-0.9306 + 0.9306*I)*z
x + (-1.0*I)*y + (0.9306 - 0.9306*I)*z
```

With central C_4 isotropy:

```
x + 1.9318517*y
x + (-0.51763809)*y
x + 0.51763809*y
x + (-1.9318517)*y
```

6. Type VI Quartics

6.1. **Equation.** $x^4 + xy^3 + yz^3$

6.2. **Lines.** With trivial isotropy:

```
x + (-0.59197 + 1.02532*I)*y + (-0.07781 - 0.44126*I)*z,
x + 1.18394*y + (-0.44807)*z,
x + 1.18394*y + (0.22404 - 0.38804*I)*z,
x + 1.18394*y + (0.22404 + 0.38804*I)*z,
x + (-0.59197 - 1.02532*I)*y + (-0.34324 - 0.28802*I)*z,
x + (-0.59197 - 1.02532*I)*y + (-0.07781 + 0.44126*I)*z,
x + (-0.59197 + 1.02532*I)*y + (-0.34324 + 0.28802*I)*z,
```

```
x + (-0.59197 - 1.02532*I)*y + (0.42105 - 0.15325*I)*z,
x + (-0.59197 + 1.02532*I)*y + (0.42105 + 0.15325*I)*z
```

With trivial isotropy:

```
x + (0.48258 - 0.83585*I)*y + (0.22404 + 1.27057*I)*z,
x + (-0.96516)*y + 1.29017*z,
x + (0.48258 + 0.83585*I)*y + (0.22404 - 1.27057*I)*z,
x + (-0.96516)*y + (-0.64509 - 1.11732*I)*z,
x + (0.48258 + 0.83585*I)*y + (-1.21236 + 0.44126*I)*z,
x + (0.48258 + 0.83585*I)*y + (0.98833 + 0.82931*I)*z,
x + (0.48258 - 0.83585*I)*y + (0.98833 - 0.82931*I)*z,
x + (-0.96516)*y + (-0.64509 + 1.11732*I)*z,
x + (0.48258 - 0.83585*I)*y + (-1.21236 - 0.44126*I)*z
```

With trivial isotropy:

```
x + (-0.21878)*y + 0.68649*z,
x + (0.10939 + 0.18947*I)*y + (0.11921 - 0.67606*I)*z,
x + (0.10939 - 0.18947*I)*y + (0.11921 + 0.67606*I)*z,
x + (0.10939 - 0.18947*I)*y + (-0.64509 - 0.23479*I)*z,
x + (0.10939 + 0.18947*I)*y + (-0.64509 + 0.23479*I)*z,
x + (-0.21878)*y + (-0.34324 + 0.59451*I)*z,
x + (-0.21878)*y + (-0.34324 - 0.59451*I)*z,
x + (0.10939 + 0.18947*I)*y + (0.52588 + 0.44126*I)*z,
x + (0.10939 - 0.18947*I)*y + (0.52588 - 0.44126*I)*z
```

With C_9 isotropy:

```
y
```

7. Type VII Quartics

7.1. **Equation.** $x^4 + y^4 + z^4 + -3x^2y^2 + xyz^2$

7.2. **Lines.** With trivial isotropy:

```
# Orbit of 0, no isotropy

x + (-0.20871)*y + (-0.9137)*z
x + (-0.20871)*y + 0.9137*z
x + 0.20871*y + 0.9137*I*z
x + 0.20871*y + (-0.9137*I)*z
x + (-4.79129)*y + 4.3778*z
x + (-4.79129)*y + (-4.3778)*z
x + 4.79129*y + (-4.3778*I)*z
x + 4.79129*y + 4.3778*I*z
```

With isotropy $C_2^{(1)}$:

```
x + y + (0.59161 + 0.3873*I)*z
x - y + (-0.3873 + 0.59161*I)*z
x + y + (-0.59161 - 0.3873*I)*z
```

```
x - y + (0.3873 - 0.59161*I)*z
```

With isotropy $C_2^{(1)}$:

```
x + y + (0.59161 - 0.3873*I)*z
x - y + (0.3873 + 0.59161*I)*z
x + y + (-0.59161 + 0.3873*I)*z
x - y + (-0.3873 - 0.59161*I)*z
```

With isotropy $C_2^{(2)}$:

```
x + 1.0*I*y + (-0.86603 + 0.86603*I)*z
x + (-1.0*I)*y + (0.86603 + 0.86603*I)*z
x + 1.0*I*y + (0.86603 - 0.86603*I)*z
x + (-1.0*I)*y + (-0.86603 - 0.86603*I)*z
```

With isotropy $C_2^{(2)}$:

```
x + (-1.0*I)*y + (-1.32288 + 1.32288*I)*z
x + 1.0*I*y + (1.32288 + 1.32288*I)*z
x + (-1.0*I)*y + (1.32288 - 1.32288*I)*z
x + 1.0*I*y + (-1.32288 - 1.32288*I)*z
```

With isotropy C_2^Z:

```
x + (-0.58662693)*y
x + (-1.7046609)*y
x + 1.7046609*y
x + 0.58662693*y
```

8. Type VIII Quartics

8.1. **Equation.** $x^4 + y^4 - 3x^2y^2 + yz^3$

8.2. **Lines.** With isotropy C_2:

```
y + (0.46415889 - 0.80394677*I)*z
y + (0.46415889 + 0.80394677*I)*z
y + (-0.92831777)*z
```

With trivial isotropy:

```
x + 1.85047*y + (0.19641 + 0.3402*I)*z
x + 1.85047*y + (0.19641 - 0.3402*I)*z
x + 1.85047*y - 0.39283*z
x + (-1.85047)*y + (-0.19641 + 0.3402*I)*z
x + (-1.85047)*y + 0.39283*z
x + (-1.85047)*y + (-0.19641 - 0.3402*I)*z
```

With trivial isotropy:

```
x + (-0.56125*I)*y + (-1.18243*I)*z
x + (-0.56125*I)*y + (-1.02401 + 0.59121*I)*z
x + (-0.56125*I)*y + (1.02401 + 0.59121*I)*z
x + 0.56125*I*y + (-1.02401 - 0.59121*I)*z
```

```
x + 0.56125*I*y + (1.02401 - 0.59121*I)*z
x + 0.56125*I*y + 1.18243*I*z
```

With trivial isotropy:

```
# Orbit of trivial isotropy

x + (0.70756 - 0.23507*I)*y + (-0.52306 - 0.12198*I)*z
x + (0.70756 - 0.23507*I)*y + (0.36717 - 0.39199*I)*z
x + (0.70756 - 0.23507*I)*y + (0.15589 + 0.51397*I)*z
x + (-0.70756 + 0.23507*I)*y + (-0.36717 + 0.39199*I)*z
x + (-0.70756 + 0.23507*I)*y + (0.52306 + 0.12198*I)*z
x + (-0.70756 + 0.23507*I)*y + (-0.15589 - 0.51397*I)*z
```

With trivial isotropy:

```
x + (0.70756 + 0.23507*I)*y + (0.15589 - 0.51397*I)*z
x + (0.70756 + 0.23507*I)*y + (-0.52306 + 0.12198*I)*z
x + (-0.70756 - 0.23507*I)*y + (0.52306 - 0.12198*I)*z
x + (0.70756 + 0.23507*I)*y + (0.36717 + 0.39199*I)*z
x + (-0.70756 - 0.23507*I)*y + (-0.15589 + 0.51397*I)*z
x + (-0.70756 - 0.23507*I)*y + (-0.36717 - 0.39199*I)*z
```

With C_6 isotropy:

```
y
```

9. Type IX Quartics

9.1. **Equation.** $x^3z + y^3z + x^2y^2 - 25xyz^2 + 10z^4$

9.2. **Lines.** With trivial isotropy:

```
x + 0.16706*y + (-0.60089)*z
x + (-0.08353 - 0.14467*I)*y + (0.30044 - 0.52038*I)*z
x + (-0.08353 + 0.14467*I)*y + (0.30044 + 0.52038*I)*z
x + (-2.99302 + 5.18406*I)*y + (1.79847 + 3.11504*I)*z
x + (-2.99302 - 5.18406*I)*y + (1.79847 - 3.11504*I)*z
x + 5.98604*y + (-3.59694)*z
```

With trivial isotropy:

```
x + (-0.02344 + 0.0406*I)*y + (2.61072 + 4.52191*I)*z
x + (-10.66564 - 18.47342*I)*y + (55.69007 - 96.45804*I)*z
x + (-0.02344 - 0.0406*I)*y + (2.61072 - 4.52191*I)*z
x + (-10.66564 + 18.47342*I)*y + (55.69007 + 96.45804*I)*z
x + 21.33127*y + (-111.38015)*z
x + 0.04688*y + (-5.22145)*z
```

With trivial isotropy:

```
x + 0.28062*y + 0.59766*z
x + (-0.14031 - 0.24302*I)*y + (-0.29883 + 0.51759*I)*z
```

```
x + (-0.14031 + 0.24302*I)*y + (-0.29883 - 0.51759*I)*z
x + (-1.78177 - 3.08611*I)*y + (-1.06489 + 1.84445*I)*z
x + 3.56354*y + 2.12978*z
x + (-1.78177 + 3.08611*I)*y + (-1.06489 - 1.84445*I)*z
```

With C_2 isotropy:

```
x + (-0.5 + 0.86603*I)*y + (4.29692 + 7.44248*I)*z
x + (-0.5 - 0.86603*I)*y + (4.29692 - 7.44248*I)*z
x + y + (-8.59384)*z
```

With C_2 isotropy:

```
x + (-0.5 - 0.86603*I)*y + (0.69826 + 4.06576*I)*z
x + y + (3.17192 - 2.63759*I)*z
x + (-0.5 + 0.86603*I)*y + (-3.87018 - 1.42817*I)*z
```

With C_2 isotropy:

```
x + (-0.5 - 0.86602*I)*y + (-3.87017 + 1.42815*I)*z
x + y + (3.17192 + 2.63759*I)*z
x + (-0.5 + 0.86602*I)*y + (0.69827 - 4.06574*I)*z
```

With S_3 isotropy:

```
z
```

10. Type X Quartics

10.1. **Equation.** $x^4 + y^4 + z^4 - 9x^2y^2 - 3y^2z^2 - 8x^2z^2$

10.2. **Lines.** With trivial isotropy:

```
x + (-2.49875)*y + (-1.56867)*z
x + (-2.49875)*y + 1.56867*z
x + 2.49875*y + (-1.56867)*z
x + 2.49875*y + 1.56867*z
```

With trivial isotropy:

```
x + (-0.65498)*y + (-0.63748)*z
x + 0.65498*y + 0.63748*z
x + 0.65498*y + (-0.63748)*z
x + (-0.65498)*y + 0.63748*z
```

With trivial isotropy:

```
x + 0.4002*y + 0.41118*z
x + (-0.4002)*y + (-0.41118)*z
x + 0.4002*y + (-0.41118)*z
x + (-0.4002)*y + 0.41118*z
```

With trivial isotropy:

```
x + 1.52676*y + (-2.43201)*z
x + (-1.52676)*y + 2.43201*z
x + 1.52676*y + 2.43201*z
x + (-1.52676)*y + (-2.43201)*z
```

With trivial isotropy:

```
x + (-0.24805077*I)*z
x + 0.24805077*I*z
1.0000000*x + (-0.24805077*I)*z
1.0000000*x + 0.24805077*I*z
```

With C_2^L isotropy:

```
y + 0.71838335*I*z
y + (-0.71838335*I)*z
```

With C_2^L isotropy:

```
y + 1.2287796*I*z
y + (-1.2287796*I)*z
```

With C_2^R isotropy:

```
x + 1.0273042*I*z
x + (-1.0273042*I)*z
```

With C_2^R isotropy:

```
x + 1.1565916*I*y
x + (-1.1565916*I)*y
```

11. Type XI Quartics

11.1. **Equation.** $x(x-y)(x-2y)(x-3y)+yz^3$

11.2. **Lines.** With trivial isotropy:

```
y - z
y + (0.50000000 - 0.86602540*I)*z
y + (0.50000000 + 0.86602540*I)*z
```

With trivial isotropy:

```
x + 0.21958*y + (-0.40224)*z
x + 0.21958*y + (0.20112 - 0.34835*I)*z
x + 0.21958*y + (0.20112 + 0.34835*I)*z
```

With trivial isotropy:

```
x + (-1.5 - 0.48203*I)*y + (0.98297 + 0.56752*I)*z
x + (-1.5 - 0.48203*I)*y + (-1.13503*I)*z
x + (-1.5 - 0.48203*I)*y + (-0.98297 + 0.56752*I)*z
```

With trivial isotropy:

```
x + (-3.21958)*y + 0.40224*z
x + (-3.21958)*y + (-0.20112 + 0.34835*I)*z
x + (-3.21958)*y + (-0.20112 - 0.34835*I)*z
```

With trivial isotropy:

```
x + (-2.08999 + 0.20908*I)*y + (0.54475 + 0.13927*I)*z
x + (-2.08999 + 0.20908*I)*y + (-0.15177 - 0.5414*I)*z
x + (-2.08999 + 0.20908*I)*y + (-0.39298 + 0.40213*I)*z
```

With trivial isotropy:

```
x + (-2.08999 - 0.20908*I)*y + (0.54475 - 0.13927*I)*z
x + (-2.08999 - 0.20908*I)*y + (-0.15177 + 0.5414*I)*z
x + (-2.08999 - 0.20908*I)*y + (-0.39298 - 0.40213*I)*z
```

With trivial isotropy:

```
x + (-1.5 + 0.48203*I)*y + (0.98297 - 0.56752*I)*z
x + (-1.5 + 0.48203*I)*y + 1.13503*I*z
x + (-1.5 + 0.48203*I)*y + (-0.98297 - 0.56752*I)*z
```

With trivial isotropy:

```
x + (-0.91001 + 0.20908*I)*y + (0.39298 + 0.40213*I)*z
x + (-0.91001 + 0.20908*I)*y + (0.15177 - 0.5414*I)*z
x + (-0.91001 + 0.20908*I)*y + (-0.54475 + 0.13927*I)*z
```

With trivial isotropy:

```
x + (-0.91001 - 0.20908*I)*y + (0.39298 - 0.40213*I)*z
x + (-0.91001 - 0.20908*I)*y + (0.15177 + 0.5414*I)*z
x + (-0.91001 - 0.20908*I)*y + (-0.54475 - 0.13927*I)*z
```

With C_3 isotropy:

```
y
```

12. Type XII Quartics

12.1. **Equation.** $x^4 + x^2(-y^2 - 4z^2) + y^4 - y^2z^2 - z^4$

12.2. **Lines.** With trivial isotropy:

```
x + (0.96825 + 0.25*I)*y + (-0.99367 - 0.48721*I)*z
x + (-0.96825 - 0.25*I)*y + (0.99367 + 0.48721*I)*z
```

With trivial isotropy:

```
x + (0.96825 - 0.25*I)*y + (-0.99367 + 0.48721*I)*z
x + (-0.96825 + 0.25*I)*y + (0.99367 - 0.48721*I)*z
```

With trivial isotropy:

```
x + (-0.79057 - 0.61237*I)*y + (-0.3978 + 0.81133*I)*z
x + (0.79057 + 0.61237*I)*y + (0.3978 - 0.81133*I)*z
```

With trivial isotropy:

```
x + (-0.79057 + 0.61237*I)*y + (-0.3978 - 0.81133*I)*z
x + (0.79057 - 0.61237*I)*y + (0.3978 + 0.81133*I)*z
```

With trivial isotropy:

```
x + (0.79057 - 0.61237*I)*y + (-0.3978 - 0.81133*I)*z
x + (-0.79057 + 0.61237*I)*y + (0.3978 + 0.81133*I)*z
```

With trivial isotropy:

```
x + (0.79057 + 0.61237*I)*y + (-0.3978 + 0.81133*I)*z
x + (-0.79057 - 0.61237*I)*y + (0.3978 - 0.81133*I)*z
```

With trivial isotropy:

```
x + (-0.96825 - 0.25*I)*y + (-0.99367 - 0.48721*I)*z
x + (0.96825 + 0.25*I)*y + (0.99367 + 0.48721*I)*z
```

With trivial isotropy:

```
x + (-0.96825 + 0.25*I)*y + (-0.99367 + 0.48721*I)*z
x + (0.96825 - 0.25*I)*y + (0.99367 - 0.48721*I)*z
```

With trivial isotropy:

```
x + (-2.5031952)*z
x + 2.5031952*z
```

With trivial isotropy:

```
x + (-0.51573862*I)*z
x + 0.51573862*I*z
```

With trivial isotropy:

```
x + (-0.44721360 - 0.54772256*I)*y
x + (0.44721360 + 0.54772256*I)*y
```

With trivial isotropy:

```
x + (-0.44721360 + 0.54772256*I)*y
x + (0.44721360 - 0.54772256*I)*y
```

With C_2 isotropy:

```
y + (-2.2947737)*z
```

With C_2 isotropy:

```
y + 2.2947737*z
```

With C_2 isotropy:

```
y + (-1.1251606*I)*z
```

With C_2 isotropy:

```
y + 1.1251606*I*z
```

Appendix A. Code to compute quartics

The following Sage code is adapted from Vinzant's webpage from her research on quartics with Plaumann and Sturmfels. It inputs a quartic polynomial and outputs (among other things) equations for its tritangents.

```
def compute_bitangents(f):
    F=QQ; T.<x,y,z>=PolynomialRing(F)
    # Check whether the quartic is smooth
    Grad=ideal(f,diff(f,x),diff(f,y),diff(f,z))
    # if not Grad.dimension()==0:
    #     sys.exit("Quartic is not smooth!")

    R.<x,y,z,a,b,a0,a1,a2,a3,a4>=PolynomialRing(F)
    f0=f.base_extend(R)
    S.<a,b>=PolynomialRing(F)
    digits=50
    threshold=0.000000000001
    almostzero=threshold

    Line= a*x+b*y+z;
    puresquare=ideal(
        a0*a3^2-a1^2*a4,8*a0^2*a3-4*a0*a1*a2+a1^3,
        8*a1*a4^2-4*a2*a3*a4+a3^3,
        8*a0*a1*a4-4*a0*a2*a3+a1^2*a3,
        8*a0*a3*a4-4*a1*a2*a4+a1*a3^2,
        16*a0^2*a4+2*a0*a1*a3-4*a0*a2^2+a1^2*a2,
        16*a0*a4^2+2*a1*a3*a4-4*a2^2*a4+a2*a3^2
        );
    Res=f0.resultant(Line,z)
    Res=Res.subs(y=1)
    phi=hom(R,S,[
        0,
        0,
        0,
        a,
        b,
        Res.coefficient({x:0}),
        Res.coefficient({x:1}),
        Res.coefficient({x:2}),
        Res.coefficient({x:3}),
        Res.coefficient({x:4})
        ])
    bit1 = phi(puresquare)

    I_ideal=singular.groebner(singular(bit1))
    singular.lib('solve.lib')
    VRing=singular.solve(I_ideal,digits)
    singular.set_ring(VRing)
    B1=singular("SOL")
```

```
nreal1=0
Bitangents=[]
RealBitangents=[]
for k in [1..len(B1)]:
    real=0;
    if ((B1[k][1].impart()).absValue()<threshold) and \
    ((B1[k][2].impart()).absValue()<threshold):
        real=1
        RealBitangents=RealBitangents+[(float(B1[k][1].repart())+ \
        float(B1[k][1].impart())*i)*x+(float(B1[k][2].repart())+ \
        float(B1[k][2].impart())*i)*y+z]
    nreal1=nreal1+real
    Bitangents=Bitangents+[(float(B1[k][1].repart())+ \
    float(B1[k][1].impart())*i)*x+ \
    (float(B1[k][2].repart())+ \
    float(B1[k][2].impart())*i)*y+z]
Line=a*x+y
Res=f0.resultant(Line,y)
Res=Res.subs(z=1)
phi=hom(R,S,[
    0,
    0,
    0,
    a,
    0,
    Res.coefficient({x:0}),
    Res.coefficient({x:1}),
    Res.coefficient({x:2}),
    Res.coefficient({x:3}),
    Res.coefficient({x:4})
])
bit2=phi(puresquare)+ideal(b)

if dimension(bit2)==-1: nreal2=0
else:
      I_ideal=singular.groebner(singular(bit2))
      singular.lib('solve.lib')
      VRing=singular.solve(I_ideal,digits)
      singular.set_ring(VRing)
      B2=singular("SOL")
      nreal2=0
      for k in [1..len(B2)]:
            real=0
            if ((B2[k][1].impart()).absValue()<threshold) \
            and ((B2[k][2].impart()).absValue()<threshold):
                real=1
                RealBitangents=RealBitangents+ \
                [(float(B2[k][1].repart())+ \
                float(B2[k][1].impart())*i)*x+y]
```

```
            nreal2=nreal2+real
            Bitangents=Bitangents+ \
            [(float(B2[k][1].repart())+ \
            float(B2[k][1].impart())*i)*x+y]

Res=f0.resultant(x)
Res=Res.subs(z=1)
phi=hom(R,F,[
    0,
    0,
    0,
    0,
    0,
    Res.coefficient({y:0}),
    Res.coefficient({y:1}),
    Res.coefficient({y:2}),
    Res.coefficient({y:3}),
    Res.coefficient({y:4})
])
bit3=phi(puresquare)
if bit3==ideal(0):
    nreal3=1
    Bitangents=Bitangents+[x]
    RealBitangents=RealBitangents+[x]
else: nreal3=0

NRealBit=nreal1+nreal2+nreal3
if len(Bitangents)!=28:
    return OSError("Less than 28 bitangents computed")

return Bitangents
```

References

[PSV11] Daniel Plaumann, Bernd Sturmfels, and Cynthia Vinzant, *Quartic curves and their bitangents*, J. Symbolic Comput. **46** (2011), no. 6, 712–733. MR 2781949