

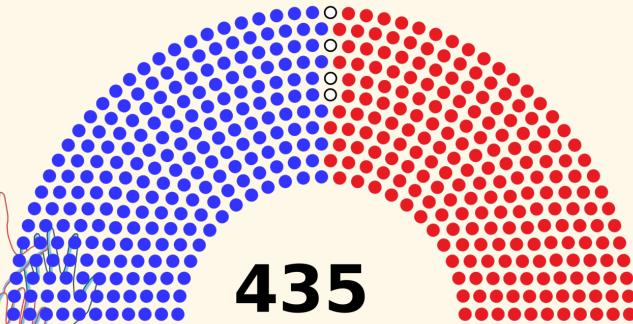
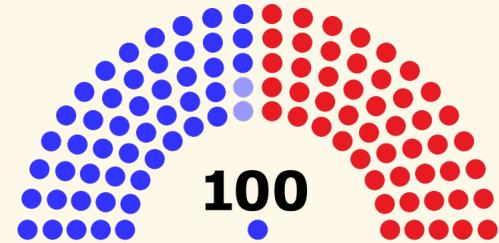
8. Gerrymandering



Basic definitions

The United States Congress is divided into two houses, the Senate and the House of Representatives.

Each state gets two Senators, regardless of how big the population is, so there are 100 Senators:



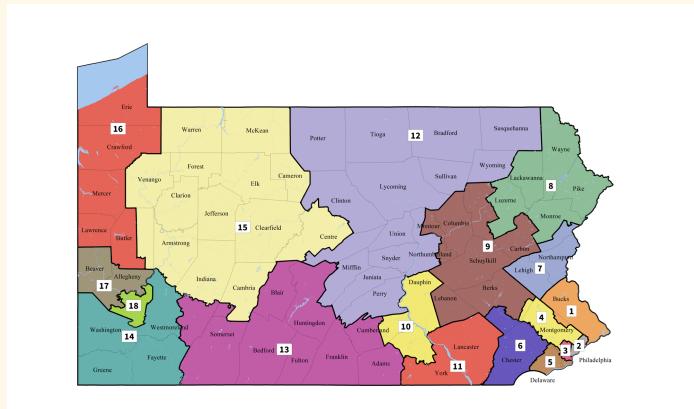
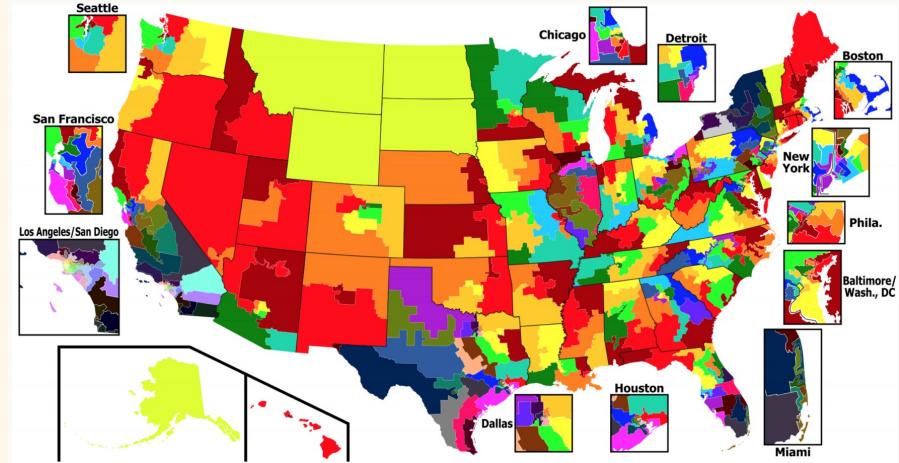
The other body of Congress is called the *House of Representatives*. Each state is subdivided into a number of voting districts depending on the population of the state.

US Congressional Districts

Here are the Congressional districts.

We see states with low populations like Montana or Wyoming are all one big district (so they only have one Congressperson). States with more massive populations have more representatives. For example California has 53 districts and gets 53 representatives.

Pennsylvania has 18 congressional districts right now:



US Congressional Districts

Q:

How do we determine how many districts each state gets?

A:

We have to regularly recount the population of each state!

Every 10 years the government does a *census*, which means they ask for every single person in the country to fill out information about their household.

This is mandated by the Constitution, and the first census took place in 1790.

The census is important, not only because it dictates how many congressional districts each state will get, but also how much money the government should allocate to each state for federal programs like Medicare, federally subsidized transportation, etc.



US Congressional Districts

The most recent census took place in 2020. The population of Pennsylvania, for example, changed from 12,702,379 to 13,002,700 in the past ten years. The US population, on the other hand, rose from 308M to 331M.

Pennsylvania will drop from 18 districts down to 17 districts, starting in the 2022 midterm elections.

When states have to change their number of districts (like Pennsylvania), a process occurs called ***redistricting***.

As we have seen, districts determine the balance of power in congress, and can have a huge effect on how individuals are represented.

In order to have a fair democracy, it is important that districts are drawn *fairly*.



Districting example

Suppose there are 50 voters, voting between two parties, the *orange party*, and the *green party*. Here are the voters and where they live:

Let's suppose we want to draw ***districts*** of equal size on this map. Each district will elect a representative by majority rules.

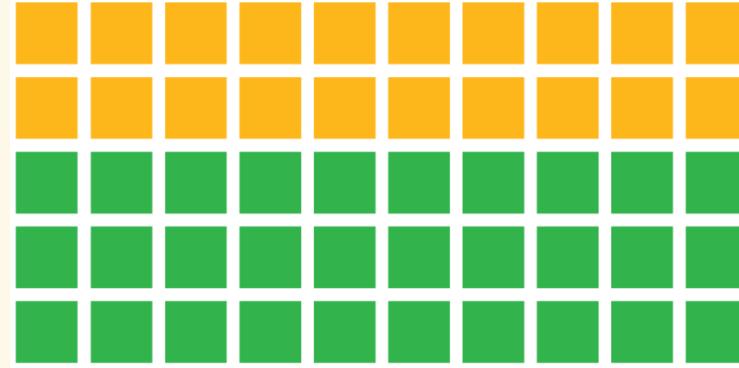
Q:

How many representatives does each party get under this districting?

A:

Orange wins districts 1 and 2, getting two representatives. Green gets three representatives.

So green has more power than orange.

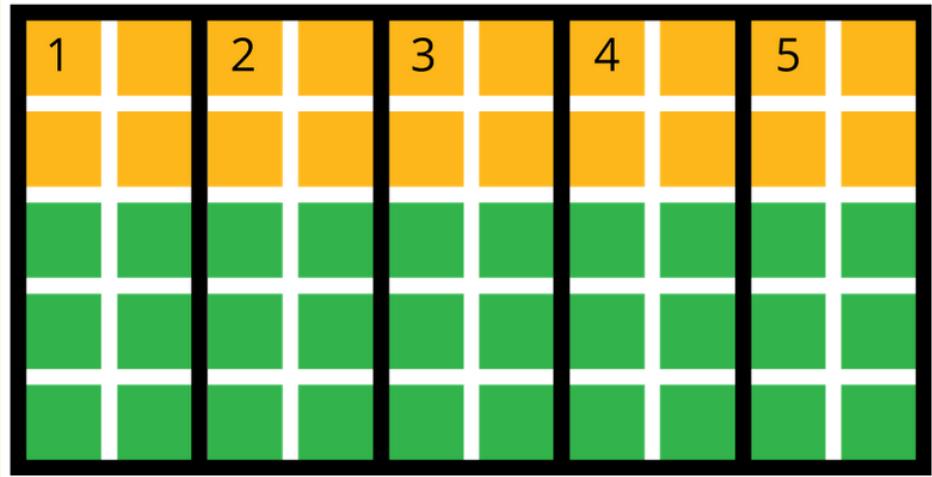


Districting example

Suppose the green party is in charge of drawing the districts, and they do it like this:

Q:

How many representatives does each party get under this districting?



A:

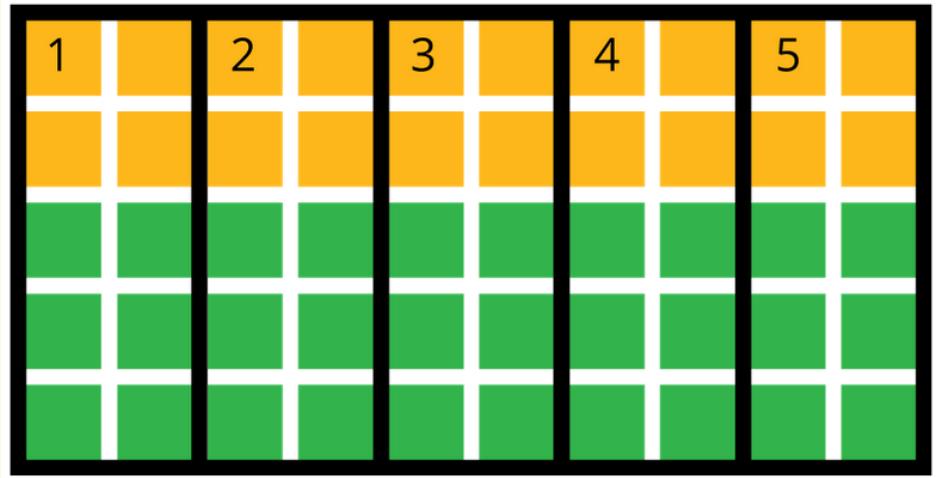
Green wins every district, so they have 5 representatives and orange has 0 representatives.

Discuss:

How is it possible that 40% of voters like orange, but 0% of representatives are from the orange party?

Districting example

The orange block of support was split up into a ton of districts, none of which they had a majority in. This is a form of ***gerrymandering***, where the party drawing the districts does it in such a way that they have more representation and their opponents have less representation.



Definition

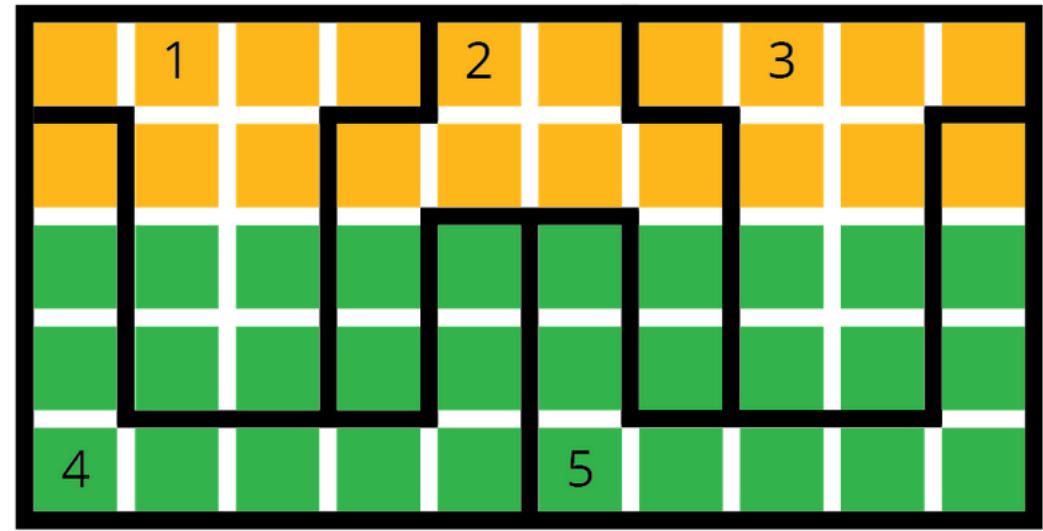
Cracking is a form of gerrymandering where a block of voters is split up into a ton of districts, and in each of these districts they form a minority.



Districting example

Consider this example, where the orange party is in charge of districting. What is the balance of power?

Orange now takes power, with 3 representatives compared to green's 2 representatives. How did they do this?

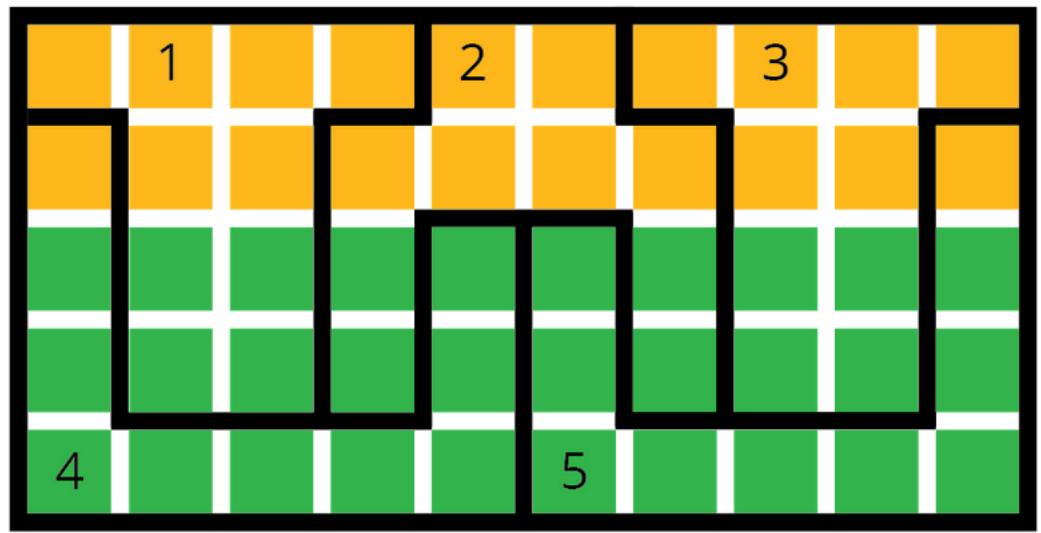


They **packed** 4 and 5 with green voters, giving green only 2 representatives. They then **cracked** the rest of the green support.

Definition

Packing is a form of gerrymandering where a block of similar voters are put into few districts, so they aren't as powerful as they would be if they were spread over many districts.

Gerrymandering



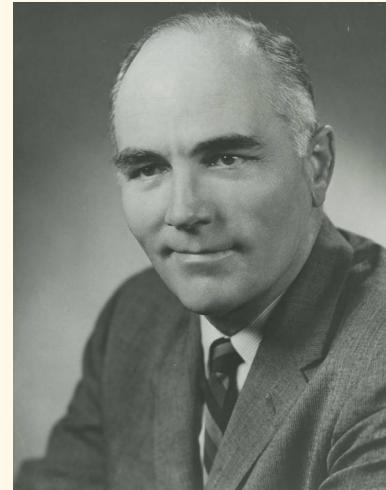
If a party is in charge of districting, they can often sway the balance of power among representatives by *gerrymandering*.



Gerrymandering in Tuskegee

In 1957 in Tuskegee Alabama (during the last decade of Jim Crow), there were 4 times as many black residents as white residents.

Although Jim Crow laws disenfranchised the black vote, this was at the height of the Civil Rights movement. State Senator and plantation owner Sam Engelhardt was afraid of losing a white majority vote:



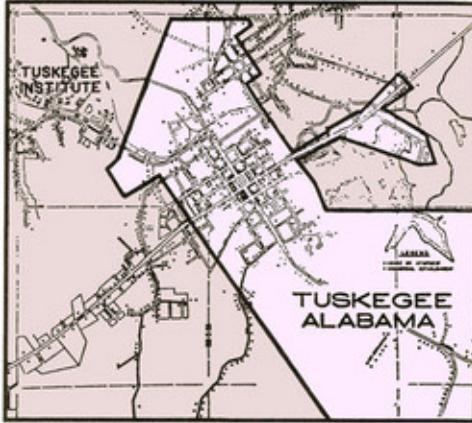
"Civil rights legislation is going to pass the United States Senate either this year or the next and we're going to be prepared for it. We couldn't stand seeing a Negro in the Alabama Legislature"

- Sam Engelhardt, white supremacist and state senator

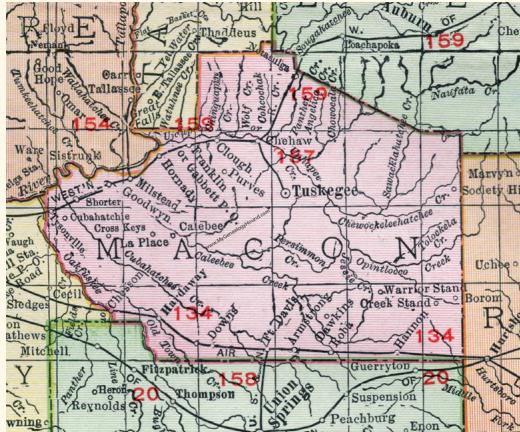


Gerrymandering in Tuskegee

Engelhardt proposed (and passed) Act 140, which changed the boundaries of Tuskegee to exclude African Americans from the city so that they couldn't vote and have a say in the city legislature.



Engelhardt continued to, as he put it, "split Negro political power," proposing and passing a constitutional amendment to "abolish Macon county" which was 84% black.

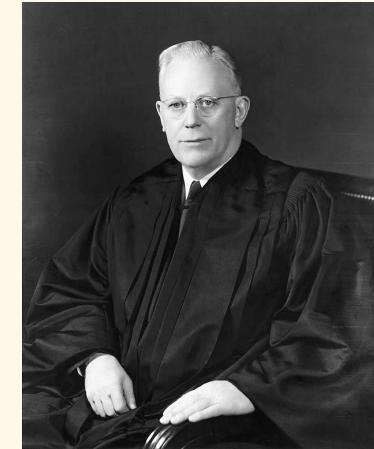


The New York Times reported:

"Advocates of the change said it was necessary to prevent Negro control of the county government in view of Federal civil rights legislation to use national authority to guarantee voting rights." [\[ref\]](#)

Gerrymandering in Tuskegee

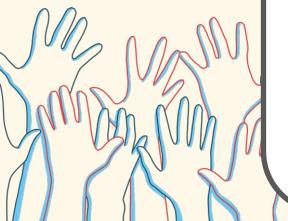
In 1960, the US Supreme Court (the Warren Court) ruled in *Gomillion v. Lightfoot* that the way the Tuskegee district was drawn was a violation of the Fifteenth Amendment, since it was designed to disenfranchise African Americans.



Chief Justice Earl Warren (from 1953 – 1969)

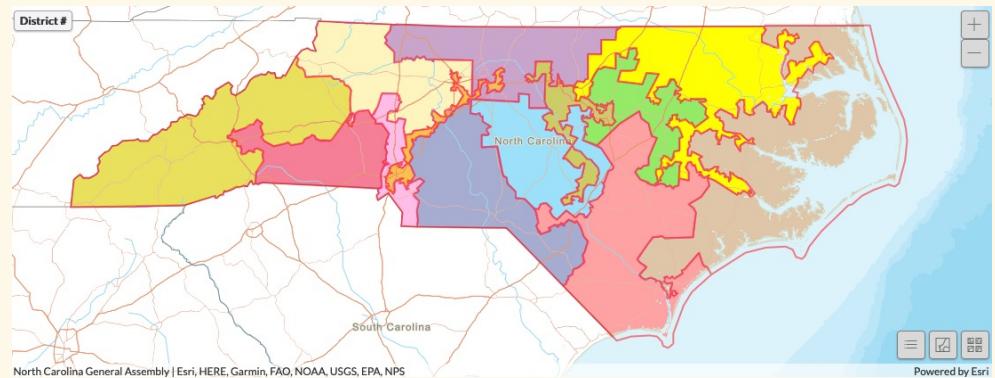
In the Voting Rights Act of 1965, **racial gerrymandering** was declared illegal and a violation of civil rights.

It is often hard to distinguish between racial and partisan gerrymandering, however, as they often take similar shapes.



North Carolina, 2011

In 2011, Republicans in the North Carolina legislature redistricted following the 2010 census: [\[link\]](#)



The windy orange district (NC-12) is intended to pack urban African American neighborhoods.

In NC-1 (bright yellow), around 100,000 African American voters in Durham were packed into a district.

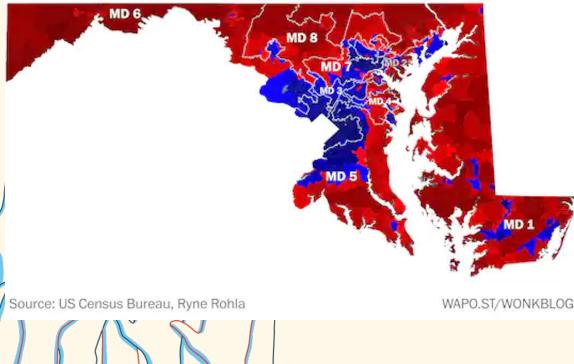
A district court in NC ruled that these districts were racially gerrymandered. This was upheld by the Supreme Court, so North Carolina was forced to redraw their districts. [\[ref\]](#)

Maryland, 2011

In Maryland in 2011, Democrats were in charge of redistricting. Through gerrymandering, Democrats went from 5/8 seats to 7/8 seats.

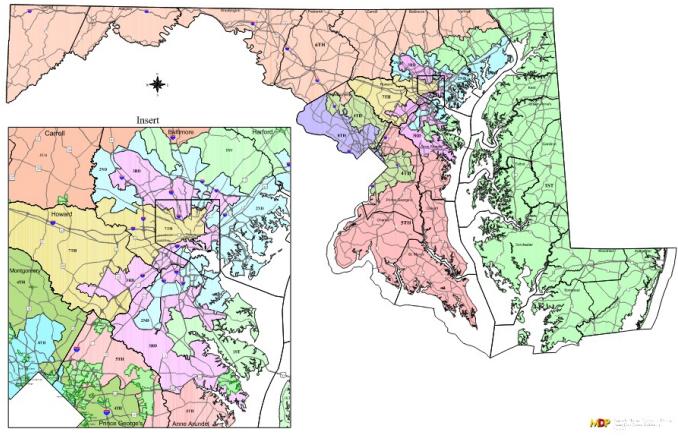
Maryland's gerrymandered congressional map

Congressional district boundaries superimposed on 2016 presidential election results (red = Republican majority, blue = Democratic majority)

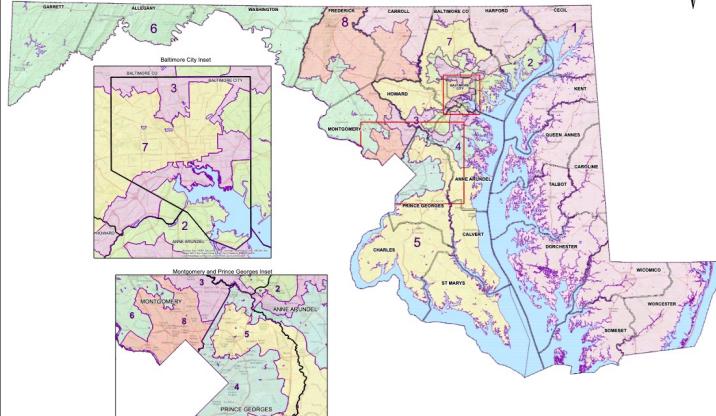


- In MD-6, Democrats **cracked** a Republican base in the north. They added in Montgomery county, which consists of Democratic voters in DC suburbs.
- Similarly in MD-8, they took rural Republican voters in Frederick and tied them in with a solid Democratic block in the north DC suburbs
- In MD-4 Democrats tied eastern DC suburbs in with Republican voters near Annapolis

2002 CONGRESSIONAL DISTRICTING PLAN
Senate Bill 805 May 6th, 2002



Maryland 2011 Congressional Districts
Senate Bill 1, October 20, 2011

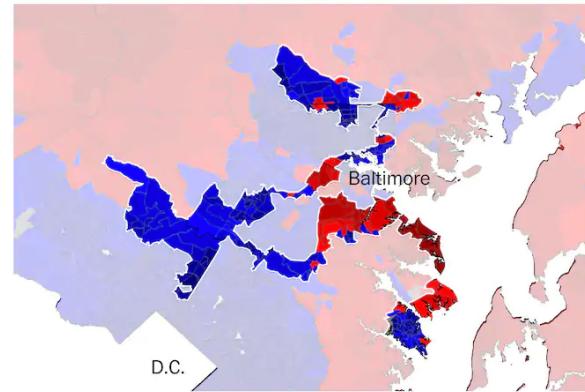


Maryland, 2011

By far the weirdest looking new district was MD-3:

The Supreme Court ruled that this was a ***partisan gerrymander***, and in 2019 the Supreme Court ruled that partisan gerrymanders are nonjusticiable, meaning the Supreme Court can't rule against them.

District 3 -- Democrat John Sarbanes



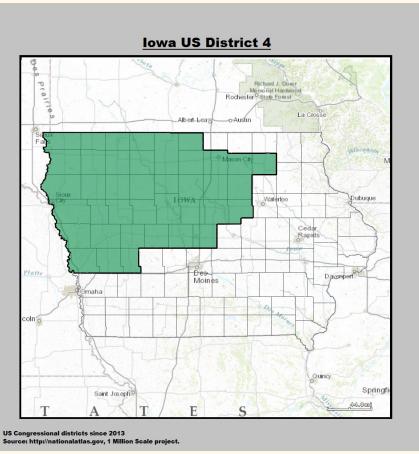
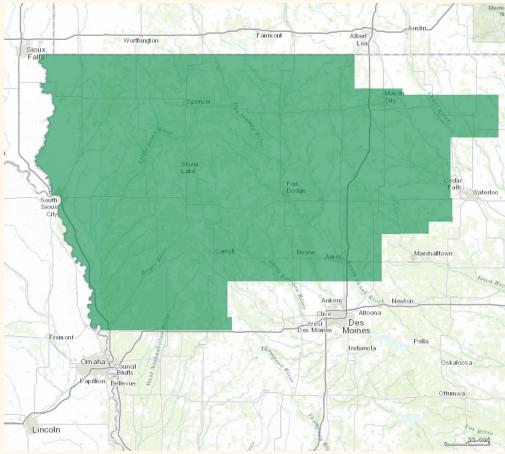
Source: US Census Bureau, Ryne Rohla

WAPO.ST/WONKBLOG

How do we district?

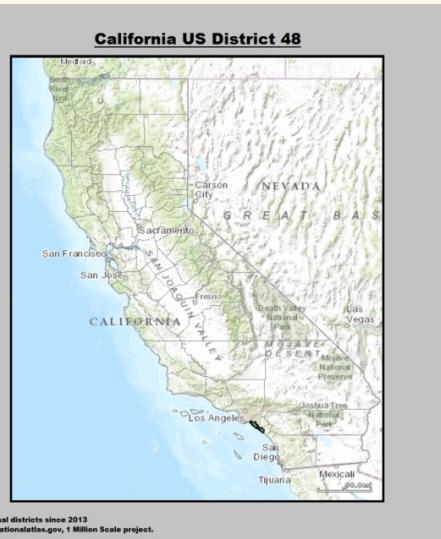
The point of districting is so that similar voters can be represented together.

Iowa's 4th district is predominantly rural, with a lot of farmland. It receives some of the most federal farm subsidies out of all districts in the United States. As a result, it makes sense for this to be a district, as voters will elect a representative who can represent the interest of farmers in Congress.



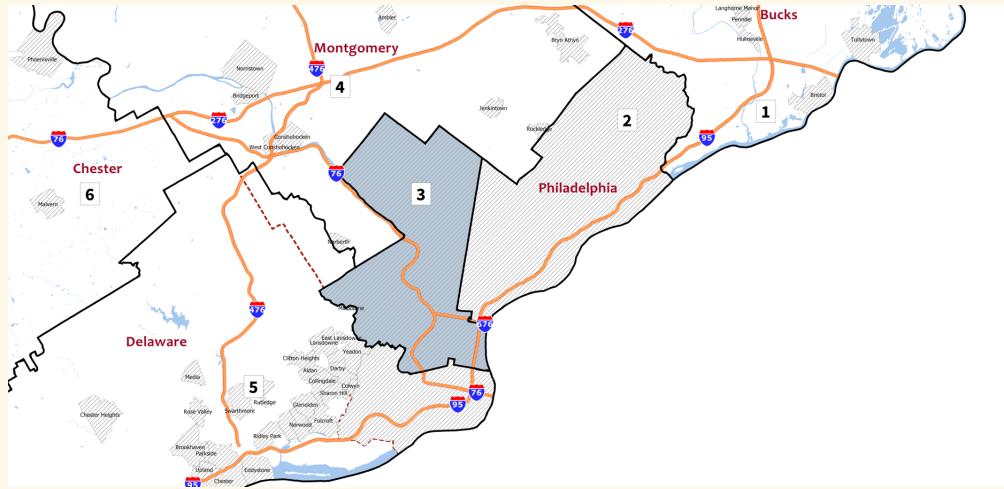
How do we district?

California's 48th district is an urban area running along the coastline. Their constituents likely share similar interests with regards to the tourism economy, flood protection, etc.



How do we district?

Pennsylvania's 3rd district is highly urban, and voters likely share similar concern about public transportation, infrastructure, housing development, etc.



Districting

In general it is hard to draw districts. Some desirable principles that districts tend to abide by are:

- contiguity – districts should be *connected*
- equal population
- preserving county boundaries
- preserving city boundaries

Big Q:

How do you tell if a district is gerrymandered?

A:

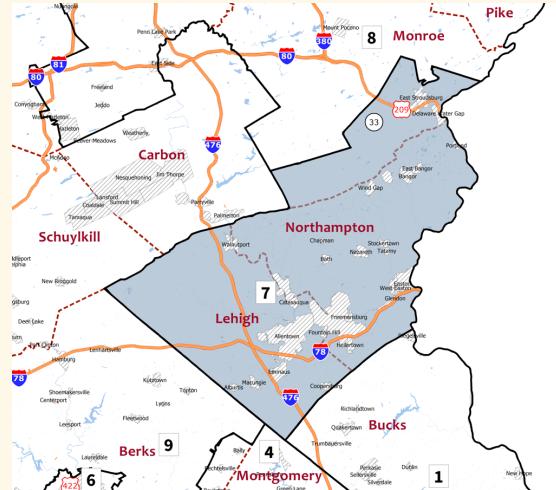
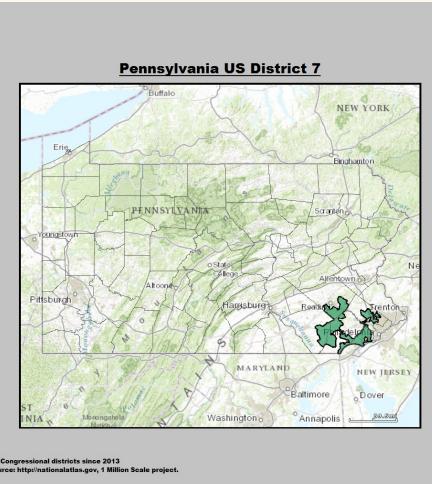
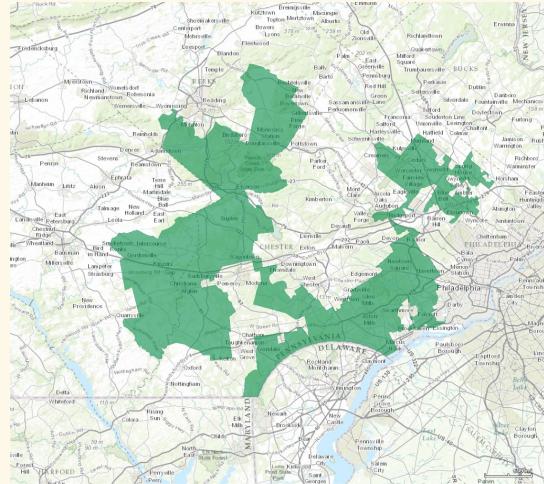
This is a *really hard question*. Mathematicians, statisticians, political scientists, etc., are actively working on this problem.



Detecting gerrymandering

Districts that cause the most concern are ones that look “windy” like a salamander.

Pennsylvania's 7th district (2013-2019) was an example of one of these windy weird shapes. It was declared a gerrymandered district, and after redistricting, it now looks like this:

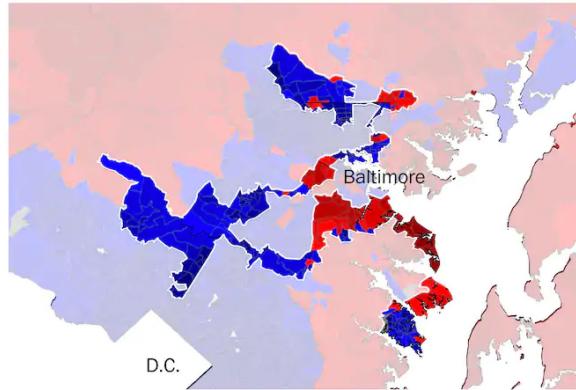


Detecting gerrymandering

There are a lot of geometric ways to quantify how “weird” a district looks. One idea is looking at its shape relative to the area it encloses.

That is, comparing its **area** and its **perimeter**.

District 3 -- Democrat John Sarbanes



Source: US Census Bureau, Ryne Rohla

WAPO.ST/WONKBLOG

For example in MD-3, the area it encloses is pretty small, but the perimeter of the district is massive.

Detecting gerrymandering

Isoperimetric Inequality (1883)

For any closed curve drawn on a sheet of paper, whose perimeter has length P and which encloses a region of area A , we have that:

$$\frac{A}{P^2} \leq \frac{1}{4\pi}$$

A:

A circle has area πr^2 , and perimeter (or circumference) $2\pi r$. So its Polsby-Popper score is:

$$PP = 4\pi \frac{A}{P^2} = 4\pi \frac{(\pi r^2)}{(2\pi r)^2} = 4\pi \cdot \frac{\pi r^2}{4\pi^2 r^2} = 1$$

For any district, we can then look at the quantity

$$PP = 4\pi \frac{A}{P^2}$$

This is called the **Polsby-Popper score**. It always lies between 0 and 1.

Q:

What is the Polsby-Popper score of a circle?

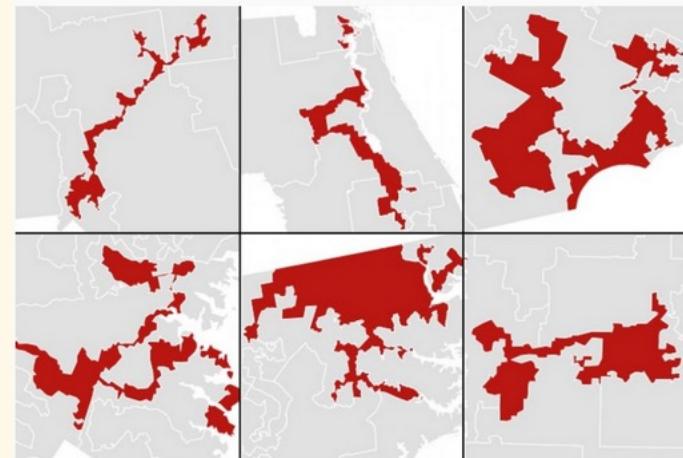
Detecting gerrymandering

Districts with Polsby-Popper scores closer to 1 are ***more like circles***. Districts with low Polsby-Popper scores have a lot of excess perimeter relative to the area they enclose – that is, they are more salamander-shaped.

This is one mathematical tool used to detect gerrymandered districts!

$$PP = 4\pi \frac{A}{P^2}$$

Here are some districts with very low PP scores:



On the top left we recognize what used to be NC-12, with a PP score of 0.0291.

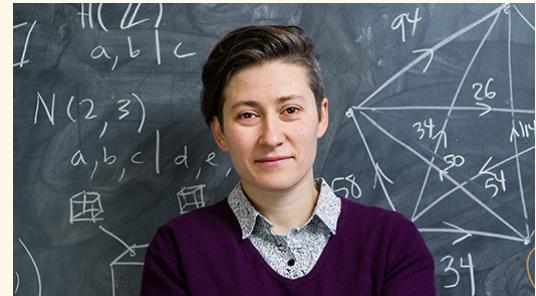


Detecting gerrymandering

The Polsby-Popper score is pretty granular as a metric – it doesn't incorporate information like geography, racial demographics, city and county boundaries, etc.

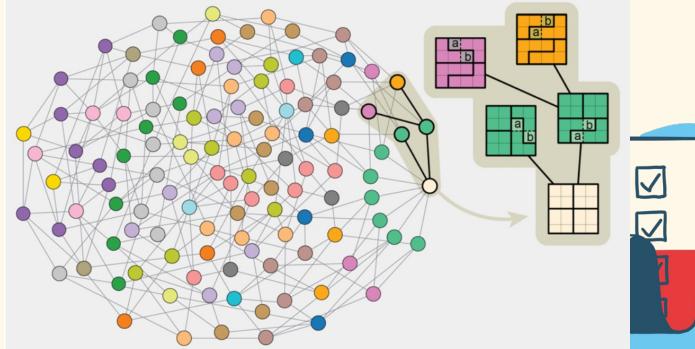
Mathematical tools for gerrymandering can get very complex very quickly.

Mathematicians are actively involved in building mathematical tools to detect gerrymandering, and training mathematicians to testify as expert witnesses in redistricting cases.



Dr. Moon Duchin, PI of the *Metric Geometry and Gerrymandering Group* at Tufts University

We can efficiently explore valid districting plans by traveling randomly around a "metagraph," defined by moves such as the unit swaps pictured. In the highlighted inset, each pattern has squares marked \textcircled{a} and \textcircled{b} whose district assignments are exchanged to arrive at the configuration of the pattern shown. The edges in the network represent these simple swap moves. The metagraph models the space of all valid districting plans and can be used to sample many billions of plans. Geometers are trying to understand the shape and structure of that universe of plans.

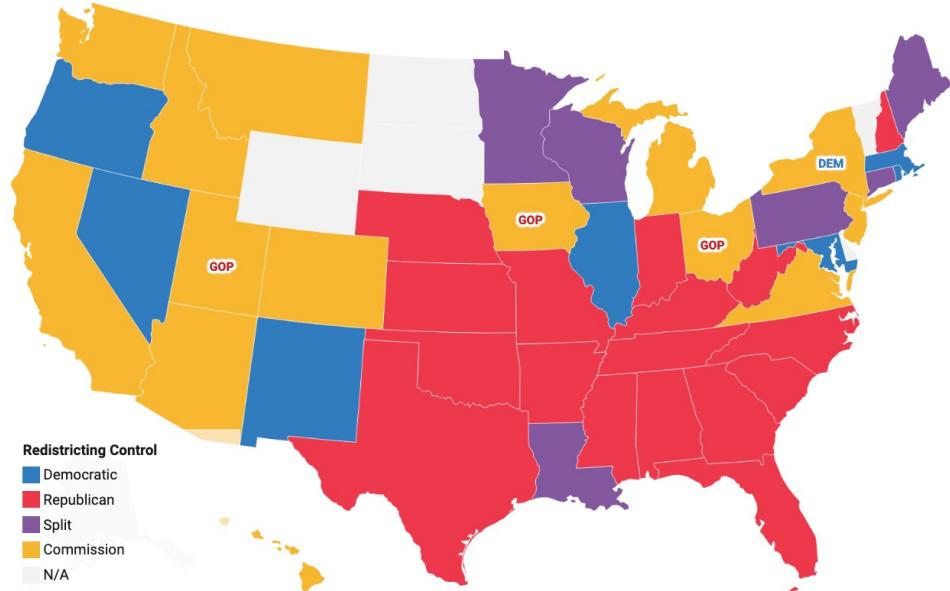


2020 Redistricting

The 2020 census is currently still being tallied. After this finishes, states will begin the process of redistricting. [\[ref\]](#)

Who Controls Redistricting

Commissions that are subject to being overruled by partisan legislatures are denoted with GOP or DEM.



Key Vocab

- Redistricting
- Gerrymandering (racial and partisan)
- Packing
- Cracking
- Polsby-Popper score



Exercises



Exercise 1: For the following group of voters:

1. Draw three connected districts of equal size so that green wins at least two districts.
2. Draw three connected districts of equal size so that orange wins at least two districts.



Exercises



Exercise 2: Compute the Polsby-Popper score of the following objects:

1. A square
2. An equilateral triangle
3. A rectangle

How do your answers compare to the observation that a higher Polsby-Popper score is more similar to a circle?

