Sample motivic geometry syllabus

Overview

(1) Classical interpolation results
- discuss N_1, N_2, N_3, N_4 over $\mathbb{C}$

(2) Quantum cohomology and Gromov–Witten classes
- Brief overview of stacks
- Define virtual fundamental classes

(3) Quantum cohomology and Gromov–Witten classes II
- Explain the recursive formula for N_d

(4) Interpolation over $\mathbb{R}$
- Discuss the Degtyarev–Kharlamov paper
- Define Welschinger invariants for planar real rational curves

(5) Welschinger invariants
- Define symplectic manifolds and J-holomorphic curves
- Present Welschinger invariants via Welschinger's original work

(6) Welschinger invariants via open Gromov–Witten theory
- Jake Solomon's thesis

(7) Relative orientations
- Define relative orientations a la Okonek–Teleman's intrinsic signs paper
- Explain the connection to Pin structures and symplectic manifolds

(8) $\mathbb{A}^1$-enumerative geometry
- Black box motivic spaces, discuss (quadratic) versions of relative orientations of vector bundles
- Give some examples

(9) Global and local $\mathbb{A}^1$-degrees
- Define a relative orientation of a map $f\colon X \to Y$
- Explain how to compute local and global $\mathbb{A}^1$-Brouwer degrees

(10) Welschinger invariants in $\mathbb{A}^1$-homotopy theory
- Recap norms and traces on Grothendieck–Witt rings
- Give Levine's quadratically enriched version of Welschinger invariants

(11) KLSW, part I ($S = \mathbb{P}^2$ and $D = \mathcal{O}(d)$)
- Carefully define the Konsevich moduli space $\overline{\mathcal{M}}_{0,n}(\mathbb{P}^2, \mathcal{O}(d))$, and the evaluation map. Set $S = \mathbb{P}^2$ and $D = \mathcal{O}(d)$ for simplicity throughout.
- Study the geometry of this moduli space via the singularities of curves, define the relevant loci on the moduli stack

(12) KLSW, part II ($S = \mathbb{P}^2$ and $D = \mathcal{O}(d)$)
- Prove the evaluation map is relatively oriented in characteristic zero and in positive characteristic

(13) KLSW, part III ($S = \mathbb{P}^2$ and $D = \mathcal{O}(d)$)
- Compute the local degree of the evaluation map
- State the main theorem

(14) KSLW, part IV (time pending)
- State the general result for del Pezzo surfaces.

(15) Jaramillo-Puentes–Pauli (time pending)
- Tropical enumeration of rational curves though points on a toric surface