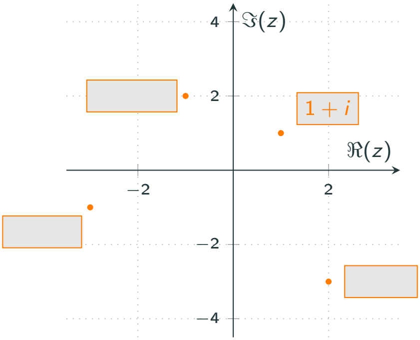
AL FM Pure

The Argand Diagram, (TeX)

May 18, 2021

Argand Diagram

Argand Diagrams are a way of geometrically representing complex numbers.



Definition

$$z = x + iy.$$

The x-axis is the real component.

The y-axis is the imaginary component.

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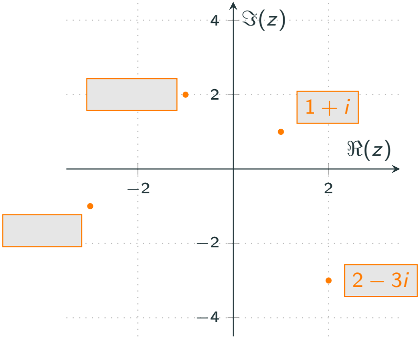

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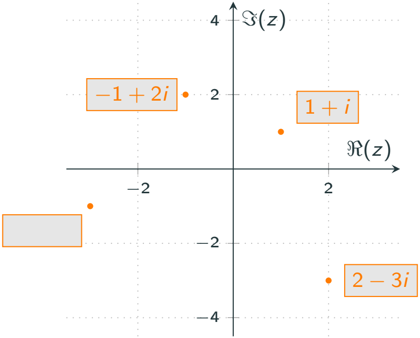

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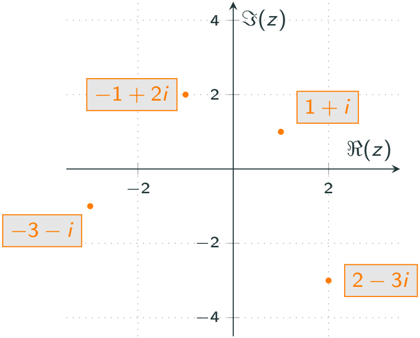

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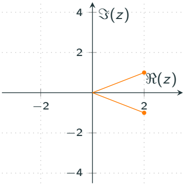
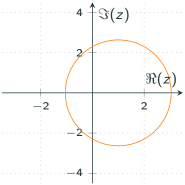
Why Visualise Complex Numbers?

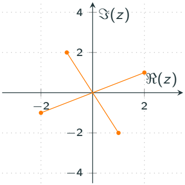
Just as with standard 2D coordinates, Argand diagrams help us interpret the relationship between complex numbers in a geometric way:



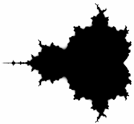
Recall that complex roots of a polynomial come in conjugates $a \pm bi$. That means when plotted on an Argand diagram, the real axis is a line of symmetry for solutions of polynomial equations.



Sketch $|z-1| = 2$. Later in this chapter we will see how to represent the locus of points that satisfy a given equation or inequality.



Solve $z^4 = 1 + i$. When you find the nth roots of a complex number, the solutions are the same distance from the origin

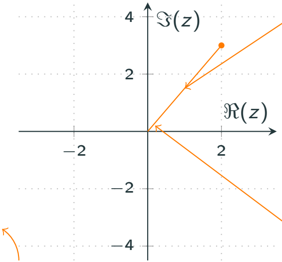


and equally spaced.

You may recognise images like the ones above. They are Mandelbrot sets, and are plotted on an Argand diagram.

Argument and Modulus

Rather than writing complex numbers in Cartesian form (where we write the complex number as the sum of a real and a complex part) we can also write complex number in modulus-argument form.



The modulus of a complex number, z, is the distance of the point representing that complex number from the origin on an Argand diagram. It is denoted $|z|$.

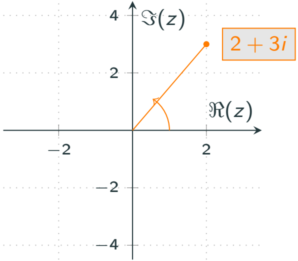
The argument of a complex number ,z, is the angle (measured anticlockwise) of the line to the point representing that complex number from the real axis on an Argand diagram. It is denoted $\arg(z)$.
Measured in radians usually from $[-\pi, \pi)$

Calculating Argument and Modulus

Definition

For a complex number in Cartesion form:

- Pythogoras' Theorem can be used to calculate the modulus.
- Right-angled trigonometry (and specifically arctan) can be used to calculate the argument.



$|2+3i| = \boxed{}$

$\arg(2+3i) = \boxed{}$

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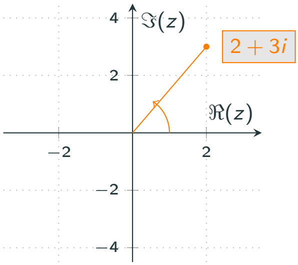

$$|2+3i| = \boxed{\sqrt{2^2+3^2} = \sqrt{13}}$$

$$\arg(2+3i) = \boxed{}$$

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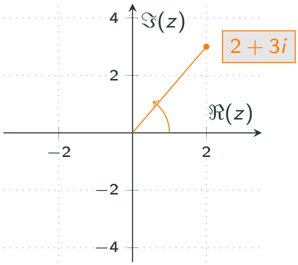

$$|2+3i| = \boxed{\sqrt{2^2+3^2} = \sqrt{13}}$$

$$\arg(2+3i) = \boxed{\arctan\frac{3}{2} = 0.983}$$

Test Your Understanding

Give exact answers where possible, otherwise to 3dp.

z	$\lvert z \rvert$	$\arg(z)$
-1		
i		
$1+i$		
$1+2i$		
$1-2i$		
$-1+2i$		
$-1-2i$		
$3+4i$		
$-5+12i$		
$1-i\sqrt{3}$		

Problem

Given that $\arg(3+a+4i)=\frac{\pi}{3}$ *and that* a *is real, determine* a.

Solution

$\frac{4}{3+a}=\tan\frac{\pi}{3}=\sqrt{3}.$

Thus $a=\frac{4}{\sqrt{3}}-3.$

Problem

Given that $\arg(5+i+ai)=\frac{\pi}{4}$ *and that* a *is real, determine* a.

Test Your Understanding

Give exact answers where possible, otherwise to 3dp.

z	$\lvert z\rvert$	$\arg(z)$
-1	$\boxed{1}$	$\boxed{\pi}$
i	$\boxed{1}$	$\boxed{\frac{\pi}{2}}$
$1+i$	$\boxed{\sqrt{2}}$	$\boxed{\frac{\pi}{4}}$
$1+2i$	$\boxed{\sqrt{5}}$	$\boxed{\arctan \frac{2}{1}}$
$1-2i$	$\boxed{\sqrt{5}}$	$\boxed{-\arctan \frac{2}{1}}$
$-1+2i$	$\boxed{\sqrt{5}}$	$\boxed{\pi - \arctan \frac{2}{1}}$
$-1-2i$	$\boxed{\sqrt{5}}$	$\boxed{-\pi + \arctan \frac{2}{1}}$
$3+4i$	$\boxed{5}$	$\boxed{\arctan \frac{4}{3}}$
$-5+12i$	$\boxed{13}$	$\boxed{\pi - \arctan \frac{12}{5}}$
$1-i\sqrt{3}$	$\boxed{2}$	$\boxed{-\frac{\pi}{3}}$

Problem

Given that $\arg(3+a+4i) = \frac{\pi}{3}$ *and that* a *is real, determine* a.

Solution

$\frac{4}{3+a} = \tan\frac{\pi}{3} = \sqrt{3}$.

Thus $a = \frac{4}{\sqrt{3}} - 3$.

Problem

Given that $\arg(5+i+ai) = \frac{\pi}{4}$ *and that* a *is real, determine* a.

Solution

$\frac{a+1}{5} = \tan(\frac{\pi}{4}) = 1$. *So* $a = 4$.

Past Paper Question

Problem

$$z = 2 - 3i.$$

- *Show that* $z^2 = -5 - 12i$.
- *Find, showing your working*
  - *the value of* $|z^2|$,
  - *the value of* $\arg(z^2)$, *giving your answer in radians to 2 dp.*
  - *Show* z *and* z^2 *on a single Argand diagram.*

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Solution

$(2-3i)(2-3i) = 4 - 6i + 9i^2 =$
$4 - 6i - 9 = -5 - 12i$

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Solution

$$= \sqrt{(-5)^2 + (-12)^2} = 13.$$

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Solution

$$\tan\alpha = \frac{12}{5}.$$
$$\arg(z^2) = -(\pi - 1.176) = -1.97.$$

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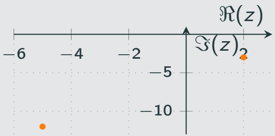

Loci

Definition

The **locus** (plural **loci**) of a particular condition is all the places that condition is true.

The locus of $|z| = 1$ is **all the places where** $|z| = 1$.

The locus of $\arg z = \frac{\pi}{6}$ is **all the places where** $\arg z = \frac{\pi}{6}$.

We represent these loci on an Argand diagram.

For loci, it is helpful to think of the complex numbers as vectors (where the amount across and up is the real and imaginary part of the complex number)

Problem

Draw the locus of $|z| = 1$

Problem

Draw the locus of $\arg z = \frac{\pi}{6}$.

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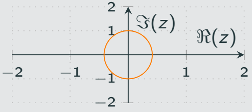

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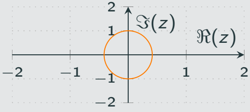

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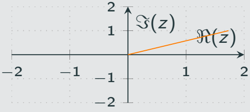

Locus of $\text{Re}(z) = k$ or $\text{Im}(z) = k$

Remember that:

- $\text{Re}(z)$ means the real part of z.
- $\text{Im}(z)$ means the imaginary part of z.

Problem

Draw the locus of $\text{Re}(z) = -1$.

Problem

Draw the locus of $\text{Im}(z) = \pi$.

Definition

The locus of $\text{Re}(z) = k$ is a **vertical line** at k on the real axis.

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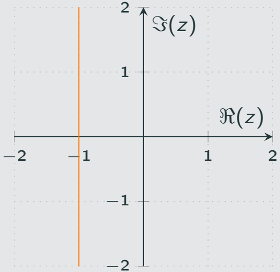

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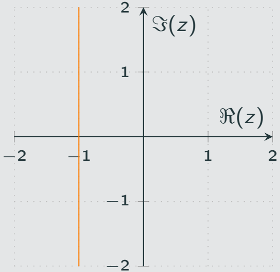

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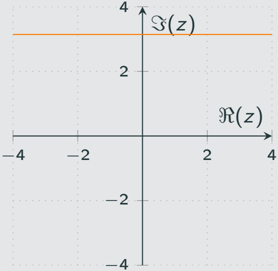
Draw the locus of $\text{Im}(z) = \pi$.

Solution



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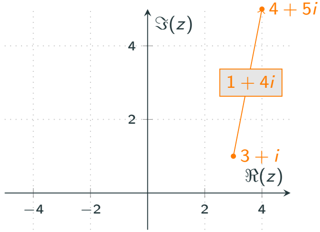
Subtraction on an Argand Diagram

For loci, it is helpful to think of the complex numbers as vectors (where the amount across and up is the real and imaginary part of the complex number)



$4 + 5i - (3 + i) =$

This can be represented on an Argand diagram as ...

Therefore $|4 + 5i - (3 + i)|$ represents the

So the distance between $4 + 5i$ and $3 + i$ is

Definition

For two complex numbers z and w, $|z - w|$ is the **distance** between them.

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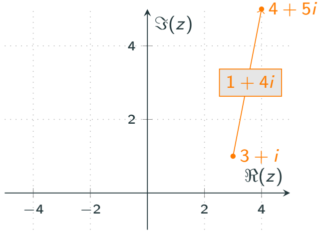

$4+5i-(3+i) = \boxed{1+4i}$

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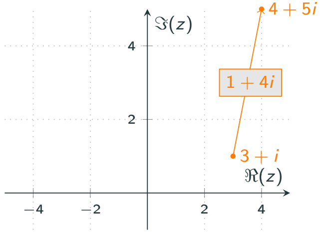

$4 + 5i - (3 + i) = \boxed{1 + 4i}$

This can be represented on an Argand diagram as ...

The vector you need to add to $3 + i$ to get to $4 + 5i$

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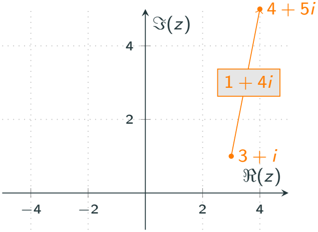

$4+5i-(3+i)=$ $\boxed{1+4i}$

This can be represented on an Argand diagram as ...

The vector you need to add to $3+i$ to get to $4+5i$

Therefore $|4+5i-(3+i)|$ represents the distance between the two points.

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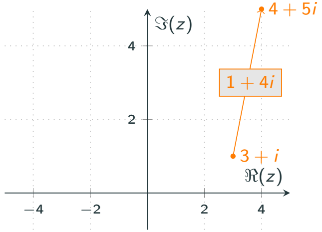


$4+5i-(3+i)=$ $\boxed{1+4i}$

This can be represented on an Argand diagram as ...

$\boxed{\text{The vector you need to add to } 3+i \text{ to get to } 4+5i}$

Therefore $|4+5i-(3+i)|$ represents the $\boxed{\text{distance between the two points.}}$

So the distance between $4+5i$ and $3+i$ is $\boxed{|1+4i|=\sqrt{1^2+4^2}=\sqrt{17}.}$

Definition

For two complex numbers z and w, $|z-w|$ is the **distance** between them.

Locus of $|z-w|=r$

Problem

Draw the locus of $|z-(3+4i)|=2$.

Problem

Draw the locus of $|z+(2+i)|=3$.

Definition

In general if w is a complex number and r is a real number, $|z-w|=r$ will be a circle of radius r with center w.

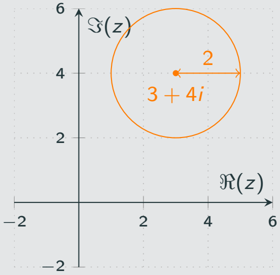
Even though z and w are both letters representing complex numbers
z is a variable and w is a constant in this sentence

Locus of $|z - w| = r$

Problem

Draw the locus of $|z - (3 + 4i)| = 2$.

Solution



Problem

Draw the locus of $|z + (2 + i)| = 3$.

Definition

In general if w is a complex number and r is a real number, $|z - w| = r$ will be a circle of radius r with center w.

Even though z and w are both letters representing complex numbers
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Locus of $|z - w| = r$

Problem

Draw the locus of $|z - (3 + 4i)| = 2$.

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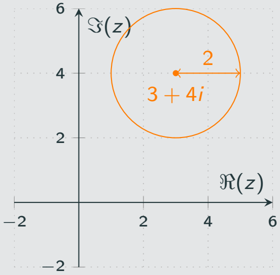

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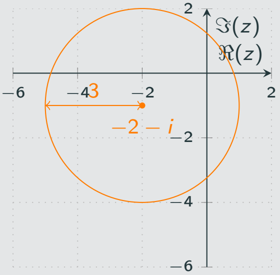

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z is a variable and w is a constant in this sentence

Locus of $|z - w_1| = |z - w_2|$

Problem

Draw the locus of $|z - (4 + 2i)| = |z - (4 + 4i)|$.

Definition

In general if w_1 and w_2 are complex numbers, $|z - w_1| = |z - w_2|$ will be the perpendicular bisector of w_1 and w_2.

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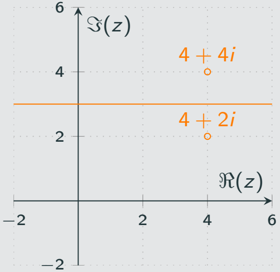

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In general if w_1 and w_2 are complex numbers, $|z - w_1| = |z - w_2|$ will be the perpendicular bisector of w_1 and w_2.

Locus of $\arg(z - w) = \theta$

Say we want to draw the locus of $\arg(z - (2 + 3i)) = \frac{\pi}{4}$. We already know that on the Argand diagram $z - (2 + 3i)$ is the 'vector' between z and $2 + 3i$. If the argument of that is $\frac{\pi}{4}$ it means that the angle between that vector and the real axis is $\frac{\pi}{4}$.

Problem

Draw the locus of $\arg(z - (2 + 3i)) = \frac{\pi}{4}$.

Definition

The locus of $\arg(z - w) = \theta$ is a half-line beginning at w which makes an angle θ with the positive real axis.

Locus of $\arg(z - w) = \theta$

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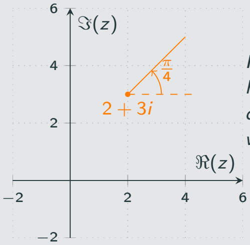
Solution



Remember this is only a half-line. In the other direction the argument would be $\frac{\pi}{4} - \pi = -\frac{3\pi}{4}$.

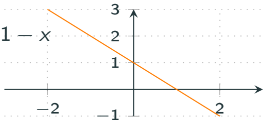
Definition

The locus of $\arg(z - w) = \theta$ is a half-line beginning at w which makes an angle θ with the positive real axis.

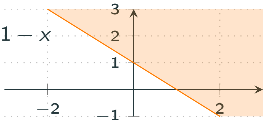
Regions

At GCSE an equation which linked two variables represented a line and an inequality represented a region.

$y = 1 - x$



$y \geq 1 - x$



The same applies in the Argand diagram with equations (as we have seen) or inequalities involving complex numbers.

Problem

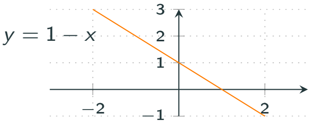
Draw the locus of $|z| \leq 1$.

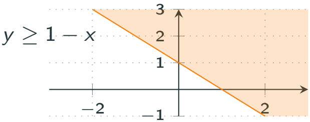
Problem

Draw the locus of $|z| > 2$.

Regions

At GCSE an equation which linked two variables represented a line and an inequality represented a region.





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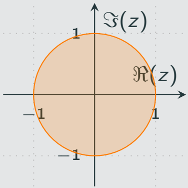
Problem

Draw the locus of $|z| \leq 1$.

Solution



Problem

Draw the locus of $|z| > 2$.

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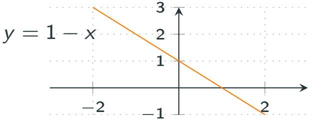

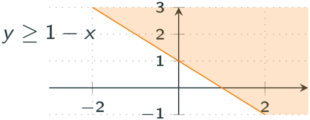

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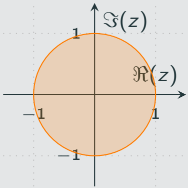

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Draw the locus of $|z| > 2$.

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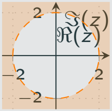

Because it's $>$ *the line is dotted.*

Test Your Understanding

Problem

Draw the region for which $|z-(1+i)|<3$.

Problem

Draw the region for which $|z-i|\geq 1$.

Problem

Draw the region for which $|z+1-i|\leq|z-3+3i|$.

Problem

Draw the region for which $|z|>|z+2i|$.

Test Your Understanding

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Draw the region for which $|z-(1+i)|<3.$

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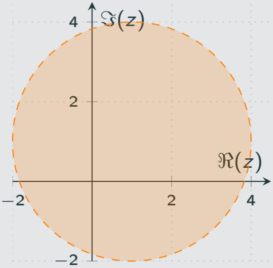

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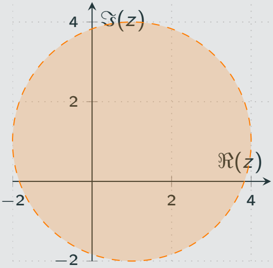

Problem

Draw the region for which $|z+1-i| \leq |z-3+3i|$.

Solution

Problem

Draw the region for which $|z-i| \geq 1$.

Problem

Draw the region for which $|z| > |z+2i|$.

Test Your Understanding

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Draw the region for which $|z-(1+i)| < 3$.

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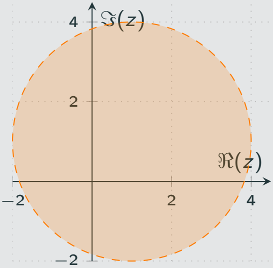

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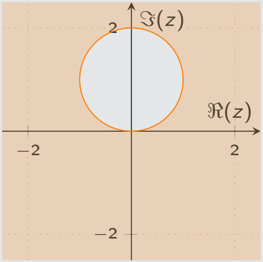

Problem

Draw the region for which $|z| > |z+2i|$.

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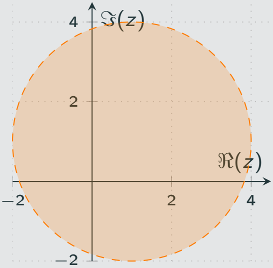

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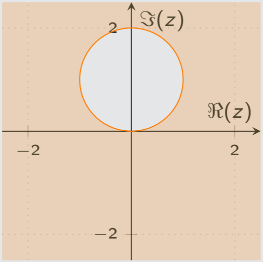

Problem

Draw the region for which $|z|>|z+2i|$.

Solution

Test Your Understanding

Problem

Draw the region for which $\Re(z) \leq 5$.

Problem

Draw the region for which $\Im(z) > \Re(z)$.

Problem

Draw the region for which $-\frac{\pi}{6} < \arg(z + 1 - i) \leq \frac{\pi}{6}$.

Problem

Draw the region for which $|z| < 3$ *and* $\frac{\pi}{6} \leq \arg z \leq \frac{\pi}{4}$.

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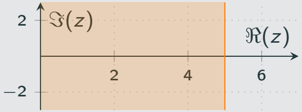

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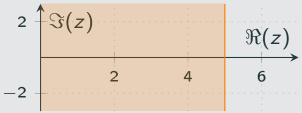

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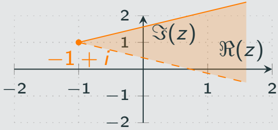

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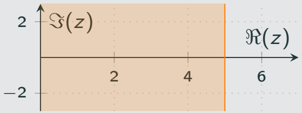

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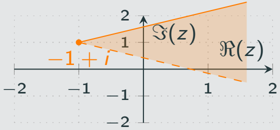

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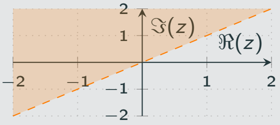

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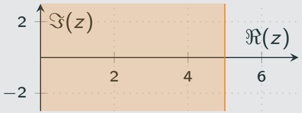

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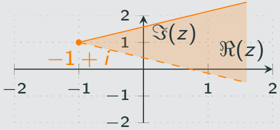

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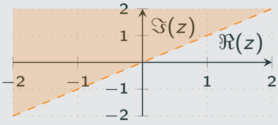

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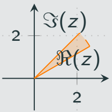

Minimising/Maximising $\arg(z)$ and $|z|$

Problem

A complex number z is represented by the point P. Given that $|z - 5 - 3i| = 3$;

- *Sketch the locus of P.*
- *Find the Cartesian equation of the locus.*
- *Find the maximum value of* $\arg z$ *in the interval* $(-\pi, \pi)$.
- *Find the minimum and maximum value of* $|z|$.

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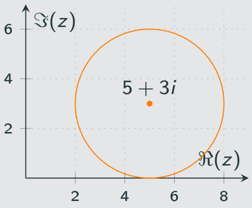

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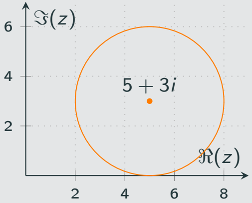

Solution

$(x-5)^2+(y-3)^2=9$

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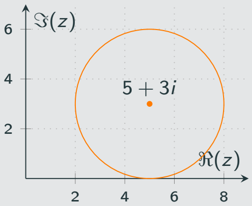

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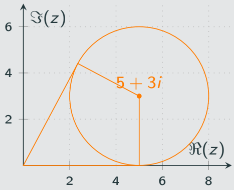

$\arg z = 2\arctan\left(\frac{3}{5}\right).$

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A complex number z is represented by the point P. Given that $|z - 5 - 3i| = 3$;

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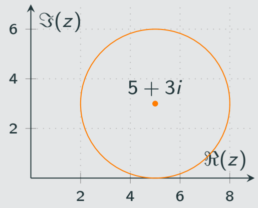

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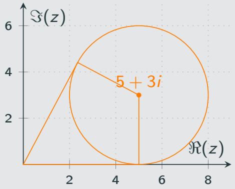


$\arg z = 2\arctan\left(\frac{3}{5}\right)$.

Solution

The minimum and maximum z can be found be drawing a line between the origin and the centre of circle, and seeing where the line intersects the circle. Note that we do not need the actual z themselves!

Min $= \sqrt{34} + 3$ *Max* $= \sqrt{34} - 3$

Test your Understanding

Problem

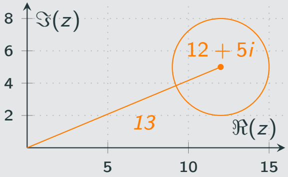
Given that the complex number z satisfies the equation $|z - 12 - 5i| = 3$, find the minimum value of z and the maximum.

Test your Understanding

Problem

Given that the complex number z satisfies the equation $|z - 12 - 5i| = 3$, find the minimum value of z and the maximum.

Solution



Center is 13 away from the origin.

$Min = 13 - 3 = 10.$

$Max = 13 + 3 = 16.$

More Difficult Minimising $|z|$ Question

Problem

- *Find the Cartesian equation of the locus of z if $|z-3| = |z+i|$, and sketch the locus of z on an Argand diagram.*
- *Hence, find the least possible value of $|z|$.*

Write $z = x + iy$, plug it in and simplify to an equation linking x and y.

You could probably use vectors in 2D but the easiest approach is probably to use coordinate geometry

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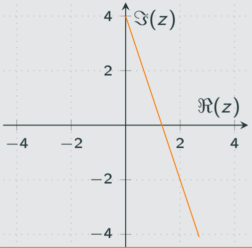
Solution

$|x + iy - 3| = |x + iy + i|$

$|x - 3 + iy| = |x + i(y+1)|$

$(x-3)^2 + y^2 = x^2 + (y+1)^2$

$y = -3x + 4$



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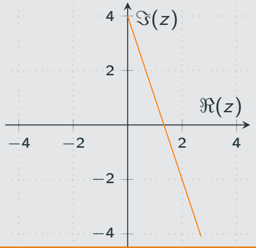

You could probably use vectors in 2D but the easiest approach is probably to use coordinate geometry

Solution

The minimum distance is the perpendicular distance from the origin.

Gradient of loci:-3

Gradient of perpendicular line: $\frac{1}{3}$

Equation of perpendicular line:

$y = \frac{1}{3}x$

$\frac{1}{3}x = -3x + 4$

$\implies x = \frac{6}{5}, y = \frac{2}{5}$

Distance $= \sqrt{\left(\frac{6}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{2\sqrt{10}}{5}$.