AL FM Pure

Complex Roots of Equations, (TeX)

May 25, 2021

Roots in Conjugate Pairs

Definition

Complex roots of polynomial equations with real coefficients always occur in complex conjugate pairs. In other words, if $a + bi$ is the root of an equation then so is $a - bi$.

Note that this is only true if the coefficients of the polynomial are real.

Problem

Prove that complex roots of a quadratic, with real coefficients, come in conjugate pairs.

Proof 1: Using the quadratic formula:

Proof 2: Let the roots be α and β. Then $(x-\alpha)(x-\beta) = \boxed{x^2 - (\beta+\alpha)x + \alpha\beta}$. Since the statement says that there are real coefficients, $\alpha + \beta$ and $\alpha\beta$ must be real. Let $\alpha = a_1 + b_1 i$, and $\beta = a_2 + b_2 i$. Then $\alpha + \beta =$ $\boxed{}$ If we require this to be real then $\boxed{}$ and therefore $b_2 = -b_1$. Also $\alpha\beta =$ $\boxed{}$ $\boxed{}$ In order for this to be real, $a_1 b_2 + a_2 b_1 = 0$. But $b_2 = -b_1$ therefore: $-a_1 b_1 + a_2 b_1 = 0$ and $b_1(a_2 - a_1) = 0$ Thus either $b_1 = b_2 = 0$(i.e the roots weren't complex) or $a_2 = a_1$, which combined with $b_2 = -b_1$, means the roots are conjugates.

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Test Your Understanding

Problem

Given that $\alpha = 7 + 2i$ is one of the roots of a monic quadratic equation with real coefficients,

- *state the value of the other root, β.*
- *find the quadratic equation.*

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$\beta = 7 - 2i$

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Solution

Slow way:

$$(z - (7 - 2i))(z - (7 + 2i)) = 0.$$
$$z^2 - (7 + 2i)z - (7 - 2i)z + (7 - 2i)(7 + 2i) = 0.$$
$$z^2 - 7z + 2iz - 7z - 2iz + 7 - 14i + 14i + 4 = 0.$$
$$z^2 - 14z + 53 = 0.$$

Quick (preferred) way:

$$(z - (7 - 2i))(z - (7 + 2i)) = (z - 7 - 2i)(z - 7 + 2i).$$

This is a difference of two squares:

$$= (z - 7)^2 - (2i)^2.$$
$$z^2 - 14z + 47 - (-4).$$
$$= z^2 - 14z + 53.$$

Past Paper Question

Problem

Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

- *write down the other root of the equation,*
- *find the value of p and the value of q.*

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$z_2 = 2 + 4i$

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Solution

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Solution

$(z-2+4i)(z-2-4i)=0$

$\implies z^2-4z+4-(-16)$

$\implies z^2-4z+20=0$

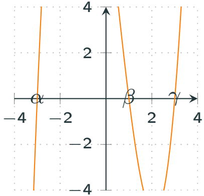
Definition

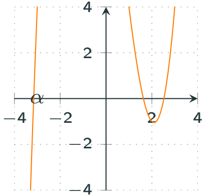
Any cubic (with coefficients of x^3 of 1) can be expressed as:

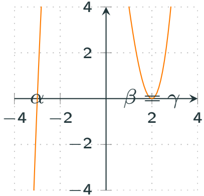
$$y = (x-\alpha)(x-\beta)(x-\gamma)$$

where α, β and γ are the roots.

However, this 3 roots may not necessarily all be real...







All 3 roots are real.

Are there any other possibilities?

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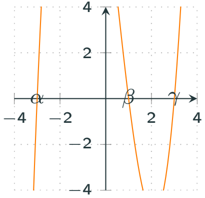

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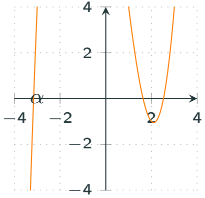
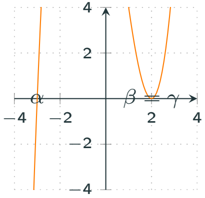


1 real root, 2 complex roots.



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Roots of Cubics

Definition

Any cubic (with coefficients of x^3 of 1) can be expressed as:

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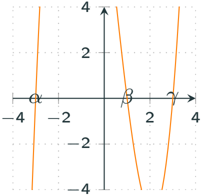

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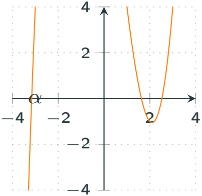

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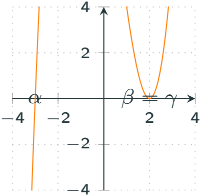

3 real roots (one repeated)

Are there any other possibilities?

Solution

No: since cubics have a range of $-\infty$ to ∞, it must cross the x axis. And it can't cross an even number of times, otherwise the cubic would start and end in the same vertical direction.

Problem

$x = 2$ is one of the roots of the polynomial $x^3 - x - 6$. Find the other two roots.

Problem

$x = -\frac{1}{2}$ is one of the roots of the polynomial $2x^3 - 5x^2 + 5x + 4$. Find the other two roots.

You should know from the Algebra topic from you're A-level how to factorise a cubic once you've got one root by using your favourite method of polynomial division.

Past Paper Questions

Problem

$x = 2$ is one of the roots of the polynomial $x^3 - x - 6$. Find the other two roots.

Solution

The factor theorem tells us that $(x - 2)$ is a factor. We can then work out the factorisation is $(x - 2)(x^2 + 2x + 3)$. The other roots arise when the second bracket is zero. Using the quadratic formula, we obtain the roots:

$$x = -1 \pm i.$$

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Solution

By the factor theorem, $2x + 1$ must be a factor. We can then work out that the factorisation is $(2x + 1)(x^2 - 3x + 4)$. The other roots occur when the second bracket is equal to zero. Using the quadratic formula, we obtain the roots:

$$x = \frac{3 \pm i}{2}.$$

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Finding Other Roots

Definition

Remember that complex roots of polynomials always come in complex conjugate pairs.

Problem

$-1+2i$ *is one of the roots of the cubic* $z^3 - z^2 - z - 15$.

Find the other two roots.

Problem

$2-i$ *is one of the roots of the cubic* $z^3 - 11z + 20$. *Find the other two roots.*

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Solution

The other complex root is the complex conjugate: $-1-2i$.

Now expand $(z-(-1+2i))(z-(-1-2i)) = z^2+2z+5$.

We can now work out that the factorisation is $(z^2+2z+5)(z-3)$ so the other root is 3.

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Problem

$2 - i$ is one of the roots of the cubic $z^3 - 11z + 20$. Find the other two roots.

Solution

$2 + i$ is the complex conjugate and hence is the other complex root.

$(z - (2 + i))(z - (2 - i)) = z^2 - 4z + 5$

The factorisation is $(z^2 - 4z + 5)(z + 4)$, so the real root is -4.

Test Your Understanding

Problem

Given that $3 + i$ *is a root of the quartic equation,*

$$2z^4 - 2z^3 - 39z^2 + 120z - 50 = 0.$$

Solve the equation completely.

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Solve the equation completely.

Solution

Another root is $3-i$.

So...

$$\begin{aligned}(z-(3+i))(z-(3-i)) &= \\ &= z^2 - (3+3)z + 3^2 + 1^2 \\ &= z^2 - 6z + 10\end{aligned}$$

.

is a factor of $2z^4 - 2z^3 - 39z^2 + 120z - 50$

$$(z^2 - 6z + 10)(...) = 2z^4 - 2z^3 - 39z^2 + 120z - 50.$$

Use (mostly) common sense to determine the other bracket:

- *It must start with* $2z^2$ *in order to get the* $2z^4$ *term in the expansion.*
- *It must end with* -5 *to get the* -50 *term.*
- *So we know the second bracket is of the form* $(2z^2 + az - 5)$.

To work out the a we need to compare either z^3, z^2 *or* z *terms in the expansion, say for example the* $-3z^3$ *term:*

$$a - 12 = -3 \implies a = 9.$$

Past Paper Question

Problem

Given that 2 and $5+2i$ are roots of the equation,

$$x^3 - 12x^2 + cx + d = 0, \quad c, d \in \mathbb{R},$$

- *write down the other complex root of the equation.*
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Solution

$5 - 2i$ *is a root.*

Solution

$(x - (5 + 2i))(x - (5 - 2i)) = x^2 - 10x + 29$

$x^3 - 12x^2 + cx + d = (x^2 - 10x + 29)(x - 2)$

$c = 49, \qquad d = -58$

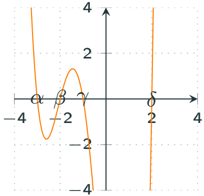
Quartics

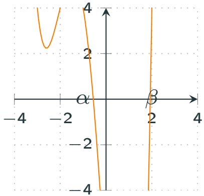
Definition

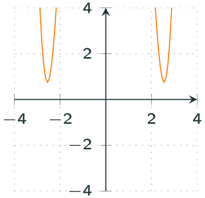
Any quartic (with coefficient of x^4 of 1) can be expressed as:

$$y = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta),$$

where α, β, γ and δ are the roots.







4 real roots (some potentially repeated)

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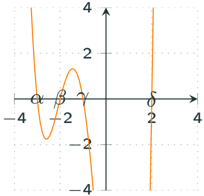

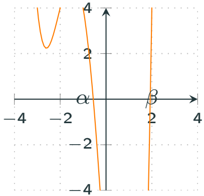

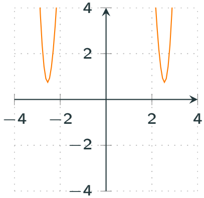

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2 real roots, pair of complex conjugate roots.

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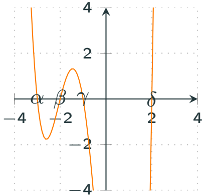

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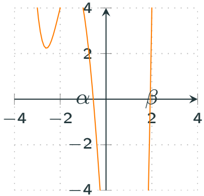
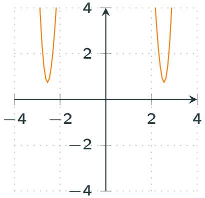


2 real roots, pair of complex conjugate roots.



No real roots. Two pairs of complex conjugate roots.

Test Your Understanding

Problem

Show that $z^2 + 4$ *is a factor of* $z^4 - 2z^3 + 21z^2 - 8z + 68$.

Hence solve the equation $z^4 - 2z^3 + 21z^2 - 8z + 68 = 0$.

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Hence solve the equation $z^4 - 2z^3 + 21z^2 - 8z + 68 = 0$.

Solution

$z^4 - 2z^3 + 21z^2 - 8z + 68 = (z^2 + 4)(z^2 + az + 17)$

Comparing z^3 *terms:*

$$-2 = a.$$
$$\therefore z^4 - 2z^3 + 21^2 - 8z + 68 = (z^2 + 4)(z^2 - 2z + 17).$$

Solving $z^2 + 4 = 0$*:*

$$z^2 = -4.$$
$$z = \pm 2i.$$

Solving $z^2 - 2z + 17 = 0$:

$$z = 1 \pm 4i.$$

* Given that $1+2i$ is one of the roots of a quadratic equation with real coefficients, find the equation.

  $x^2-2x+5=0$

* Given that $a+4i$, where a is real, is one of the roots of a quadratic equation with real coefficients, find the equation.

* Show that $x=3$ is a root of the equation $2x^3-4x^2-5x-3=0$. Hence solve the equation completely.

* Given that $-4+i$ is one of the roots of the equation $x^3+4x^2-15x-68=0$, solve the equation completely.

* Given that $2+3i$ is one of the roots of the equation $x^4+2x^3-x^2+38x+130=0$, solve the equation completely.

* Find the four roots of the equation $x^4-16=0$. Show these roots on an Argand Diargram.

Cubics and Quartics Exercises

- Given that $1 + 2i$ is one of the roots of a quadratic equation with real coefficients, find the equation.

  $\boxed{x^2 - 2x + 5 = 0}$

- Given that $a + 4i$, where a is real, is one of the roots of a quadratic equation with real coefficients, find the equation.

  $\boxed{x^2 - 2ax + a^2 + 16}$

- Show that $x = 3$ is a root of the equation $2x^3 - 4x^2 - 5x - 3 = 0$. Hence solve the equation completely.

- Given that $-4 + i$ is one of the roots of the equation $x^3 + 4x^2 - 15x - 68 = 0$, solve the equation completely.

- Given that $2 + 3i$ is one of the roots of the equation $x^4 + 2x^3 - x^2 + 38x + 130 = 0$, solve the equation completely.

- Find the four roots of the equation $x^4 - 16 = 0$. Show these roots on an Argand Diargram.

Cubics and Quartics Exercises

- Given that $1 + 2i$ is one of the roots of a quadratic equation with real coefficients, find the equation.

  $\boxed{x^2 - 2x + 5 = 0}$

- Given that $a + 4i$, where a is real, is one of the roots of a quadratic equation with real coefficients, find the equation.

  $\boxed{x^2 - 2ax + a^2 + 16}$

- Show that $x = 3$ is a root of the equation $2x^3 - 4x^2 - 5x - 3 = 0$. Hence solve the equation completely.

  $\boxed{\text{Roots are } 3, -\frac{1}{2} + \frac{1}{2}i, -\frac{1}{2} - \frac{1}{2}i.}$

- Given that $-4 + i$ is one of the roots of the equation $x^3 + 4x^2 - 15x - 68 = 0$, solve the equation completely.

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Cubics and Quartics Exercises

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- Show that $x=3$ is a root of the equation $2x^3-4x^2-5x-3=0$. Hence solve the equation completely.

  Roots are $3, -\frac{1}{2}+\frac{1}{2}i, -\frac{1}{2}-\frac{1}{2}i$.

- Given that $-4+i$ is one of the roots of the equation $x^3+4x^2-15x-68=0$, solve the equation completely.

  Roots are 4, $-4+i$ and $-4-i$.

- Given that $2+3i$ is one of the roots of the equation $x^4+2x^3-x^2+38x+130=0$, solve the equation completely.

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Cubics and Quartics Exercises

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- Given that $2 + 3i$ is one of the roots of the equation $x^4 + 2x^3 - x^2 + 38x + 130 = 0$, solve the equation completely.

  $\boxed{\text{Roots are } 2 + 3i, 2 - 3i, -3 + i \text{ and } -3 - i}$

- Find the four roots of the equation $x^4 - 16 = 0$. Show these roots on an Argand Diargram.

Cubics and Quartics Exercises

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- Find the four roots of the equation $x^4 - 16 = 0$. Show these roots on an Argand Diargram.

  $\boxed{\text{Roots are } 2, -2, 2i, -2i.}$