Statistical and computational trade-offs in Bayesian inference

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ITT Career Development
Assistant Professor, MIT

With: Nick Boyd, Trevor Campbell, Ryan Giordano, Joseph Gonzalez, Jonathan H. Huggins, Stefanie Jegelka, Brian Kulis, Michael I. Jordan, Rachael Meager, Xinghao Pan, Andre Wibisono, Ashia C. Wilson

Statistical and computation trade-offs

- Bayesian inference

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  - Complex, modular models

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  - Complex, modular models; posterior distribution

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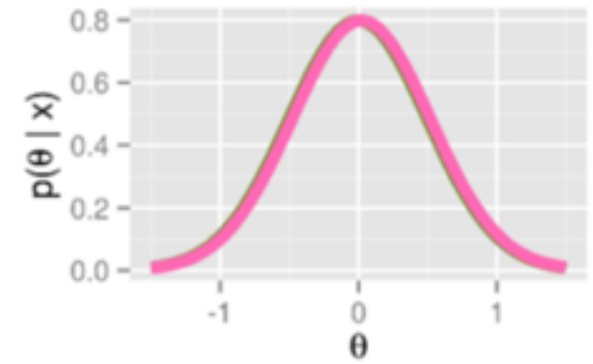

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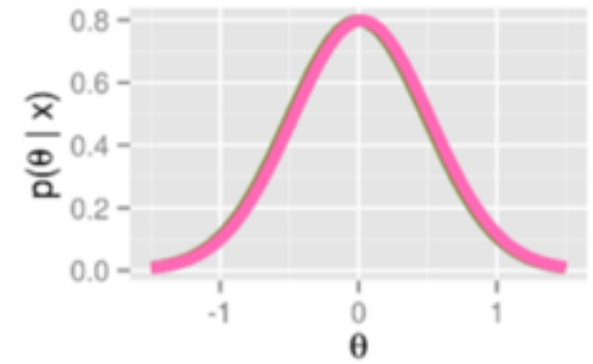

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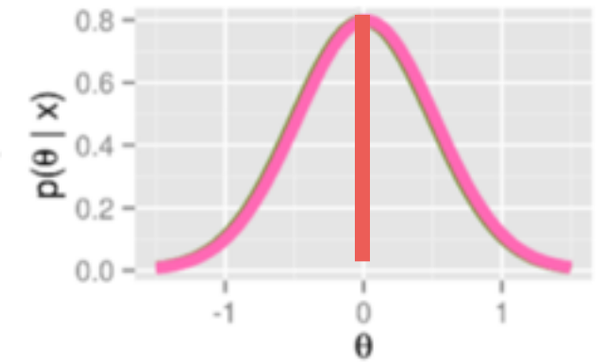

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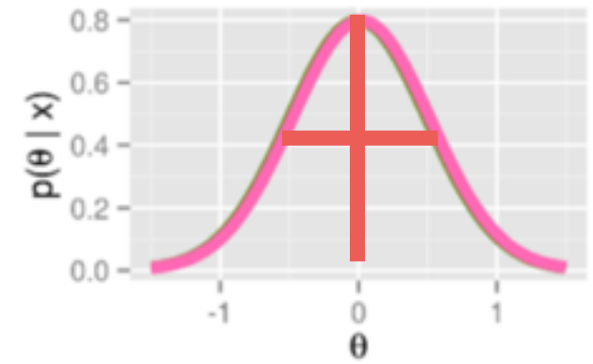

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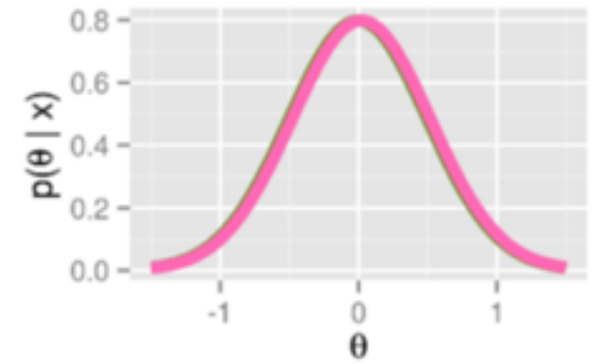

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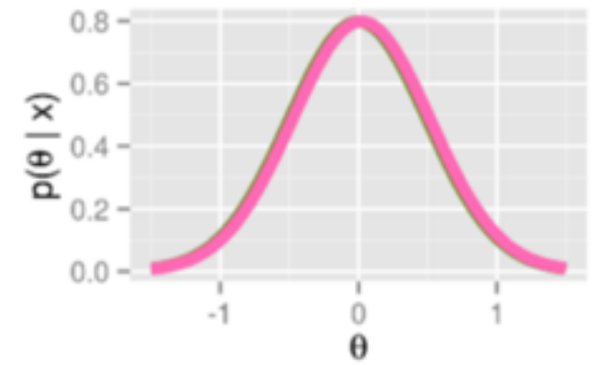

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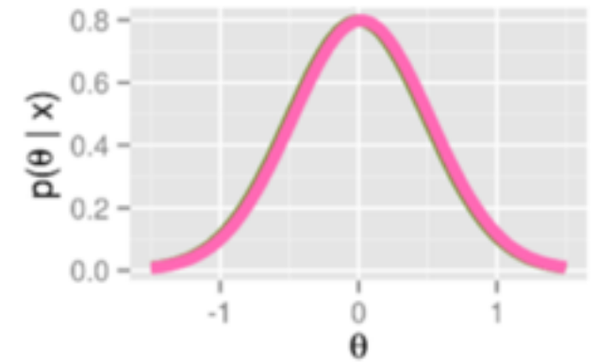

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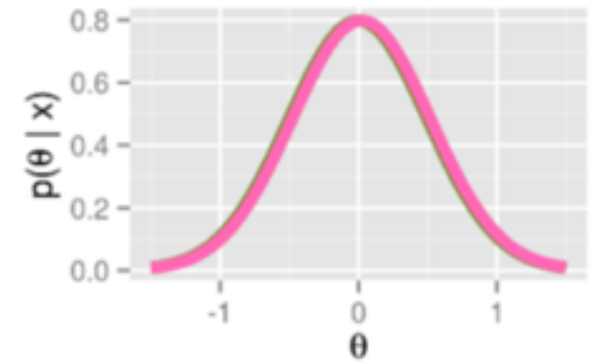

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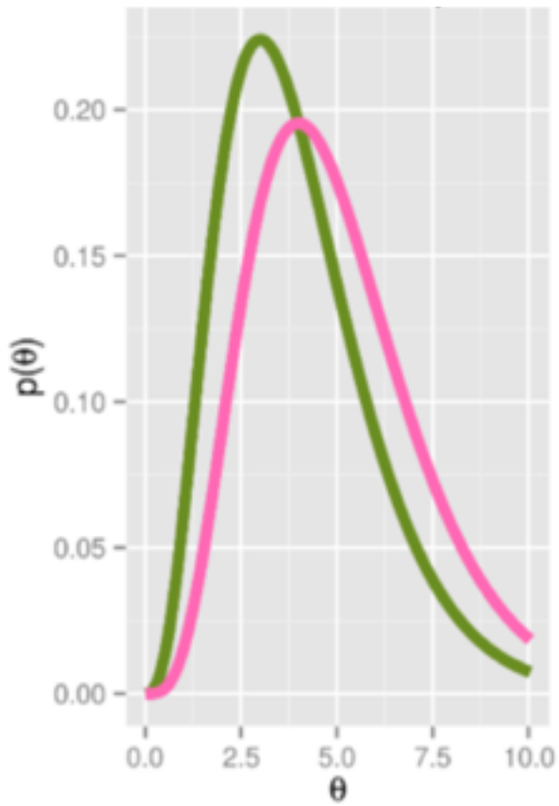

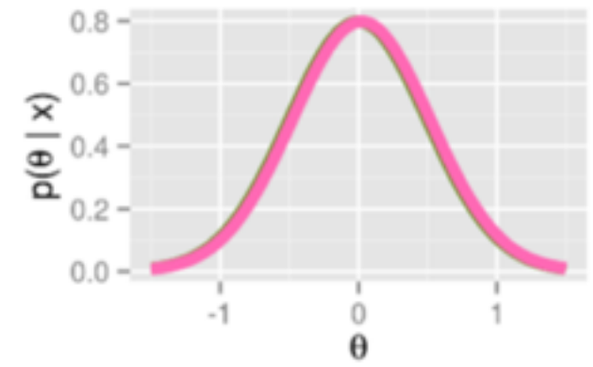

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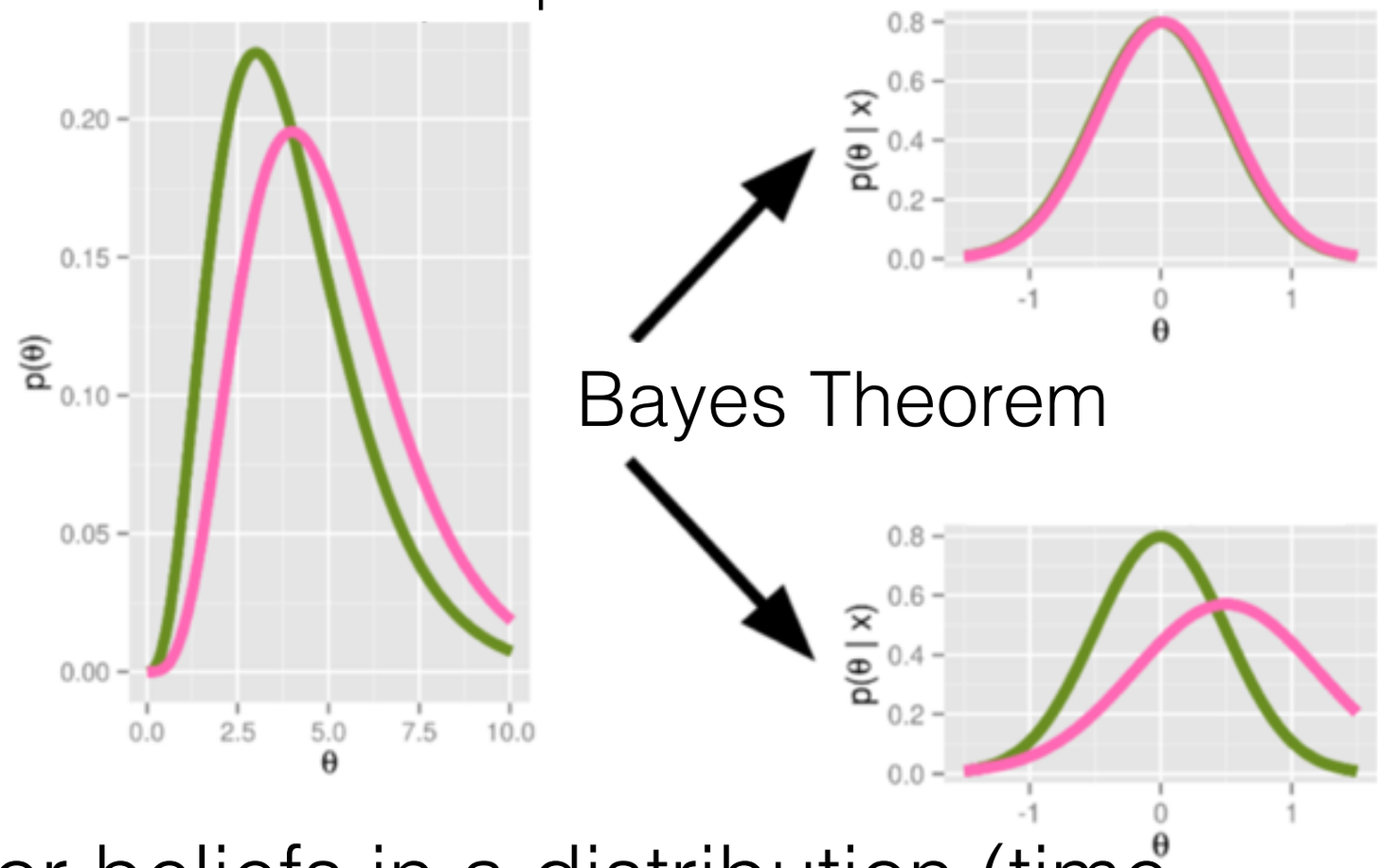

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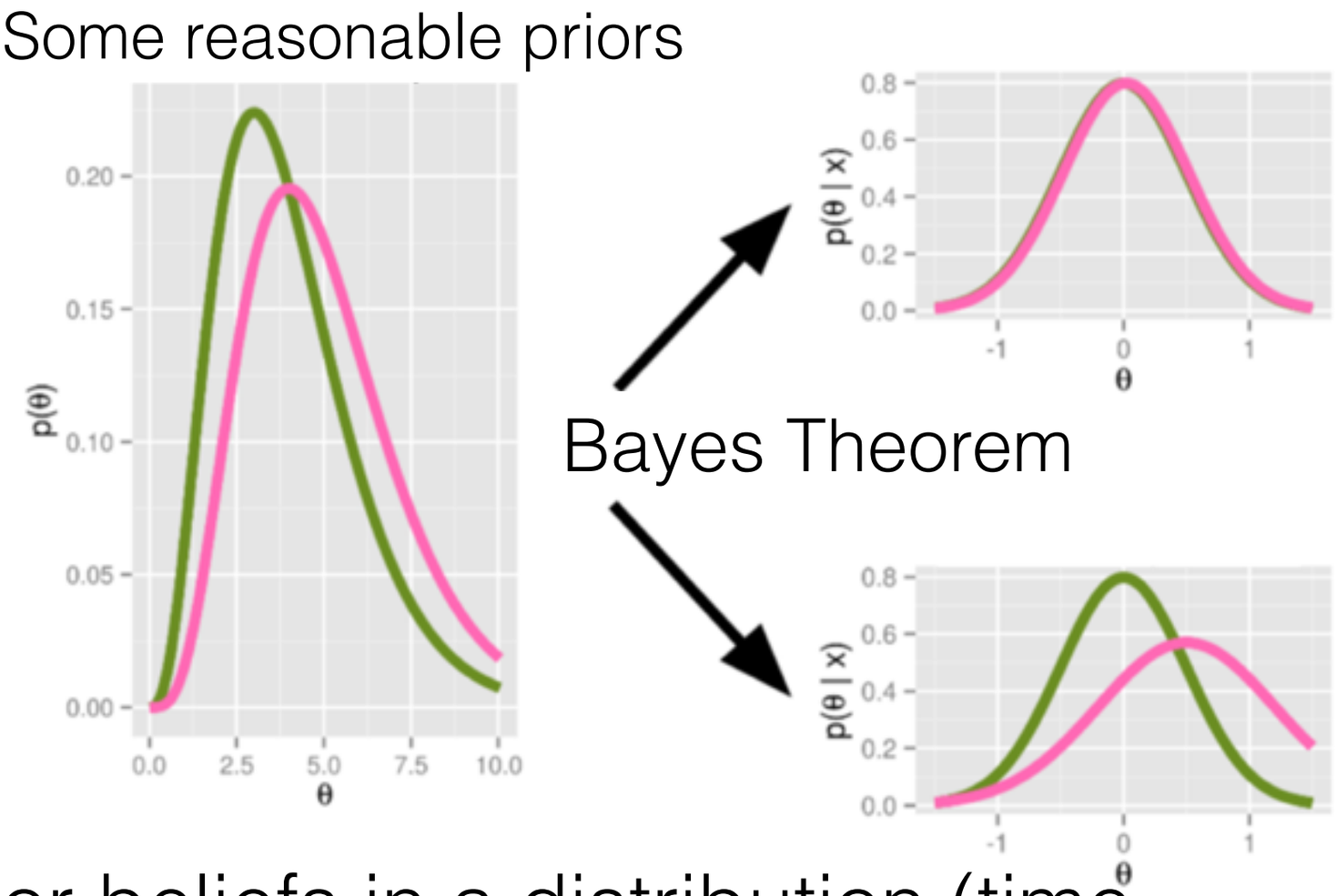

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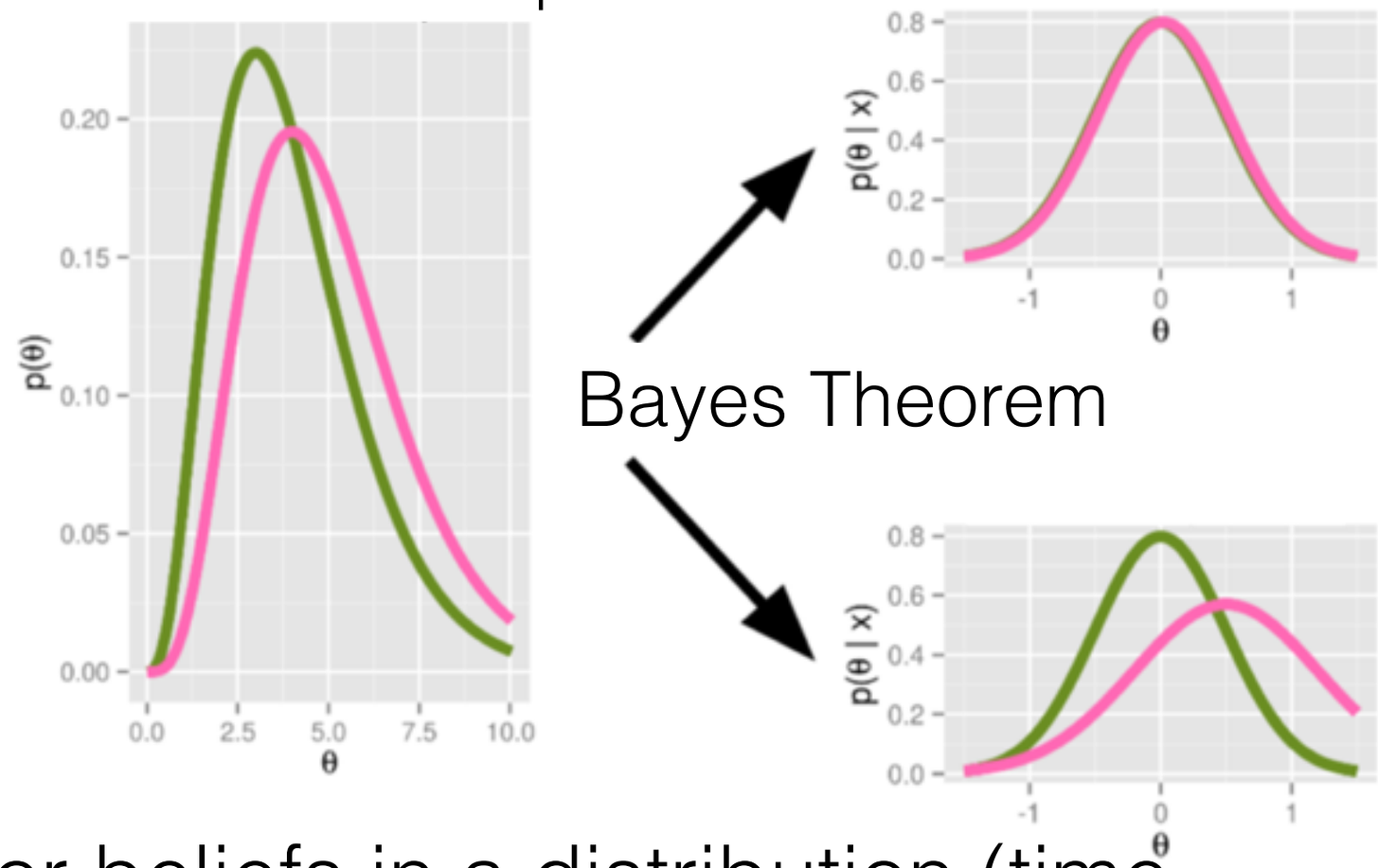

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  - Global vs. local robustness

Roadmap

- Posterior approximation trade-offs
  - Point estimates
  - Uncertainties
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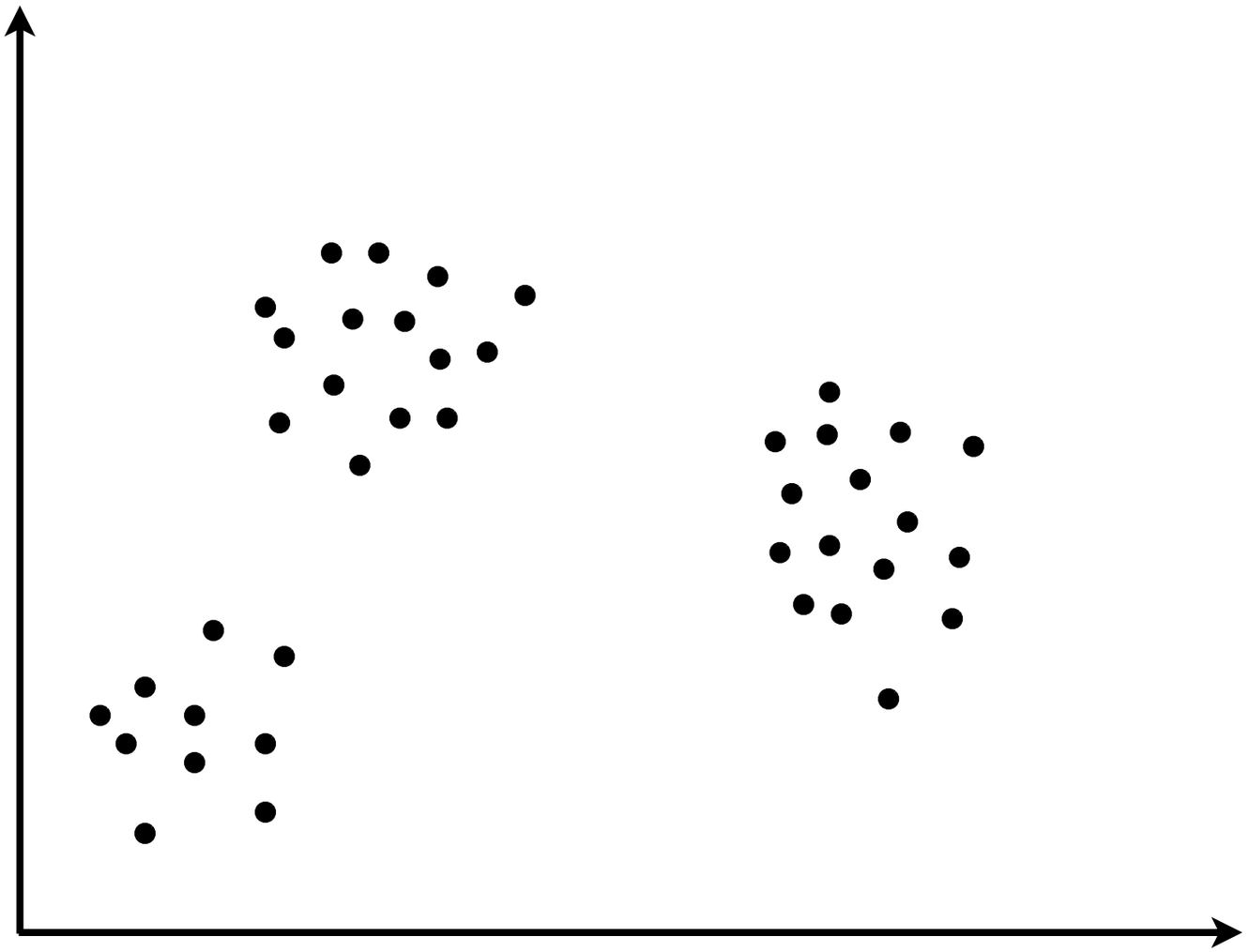

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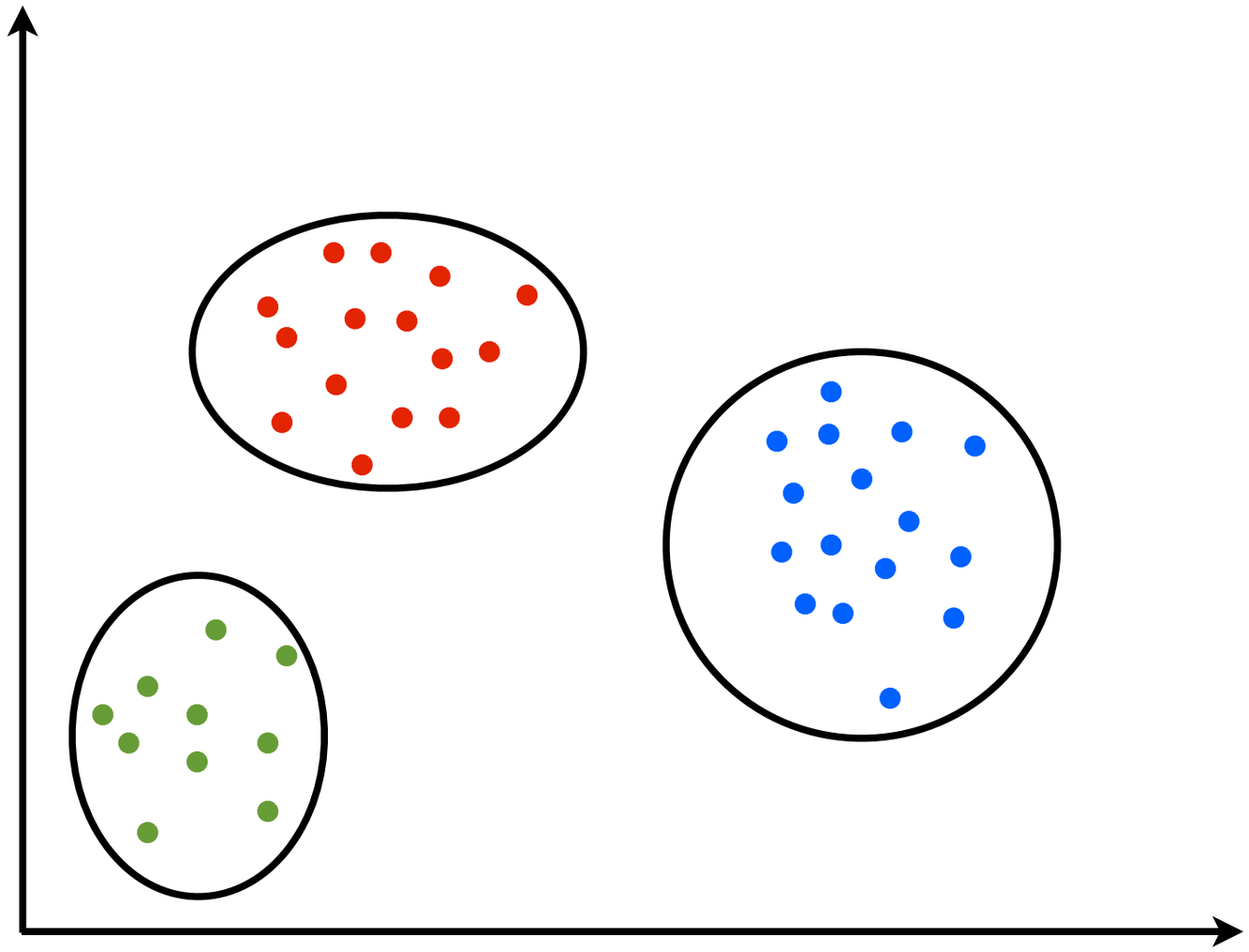

"Clusters"

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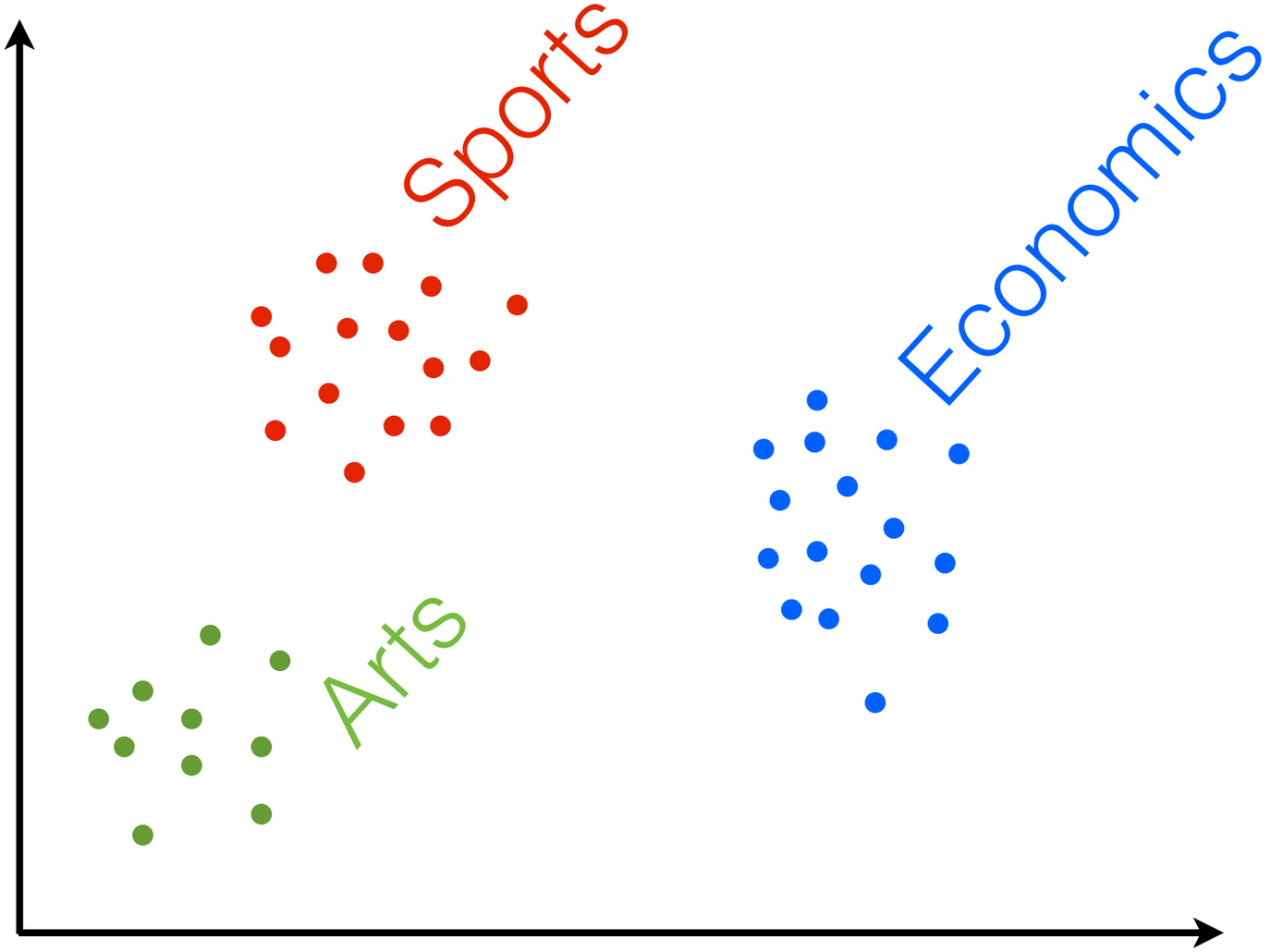

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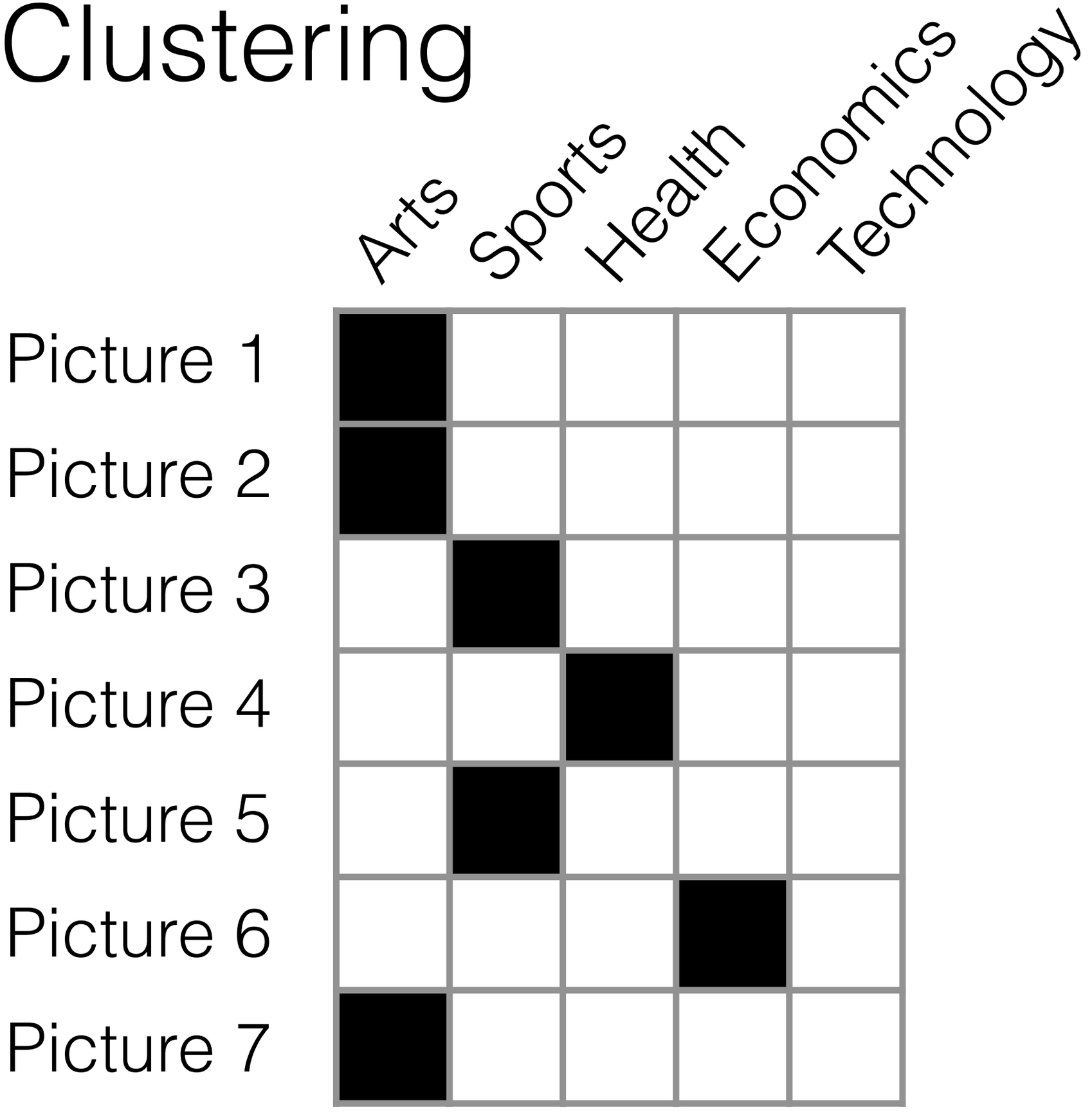
Arts
Sports
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Picture 1
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Picture 4
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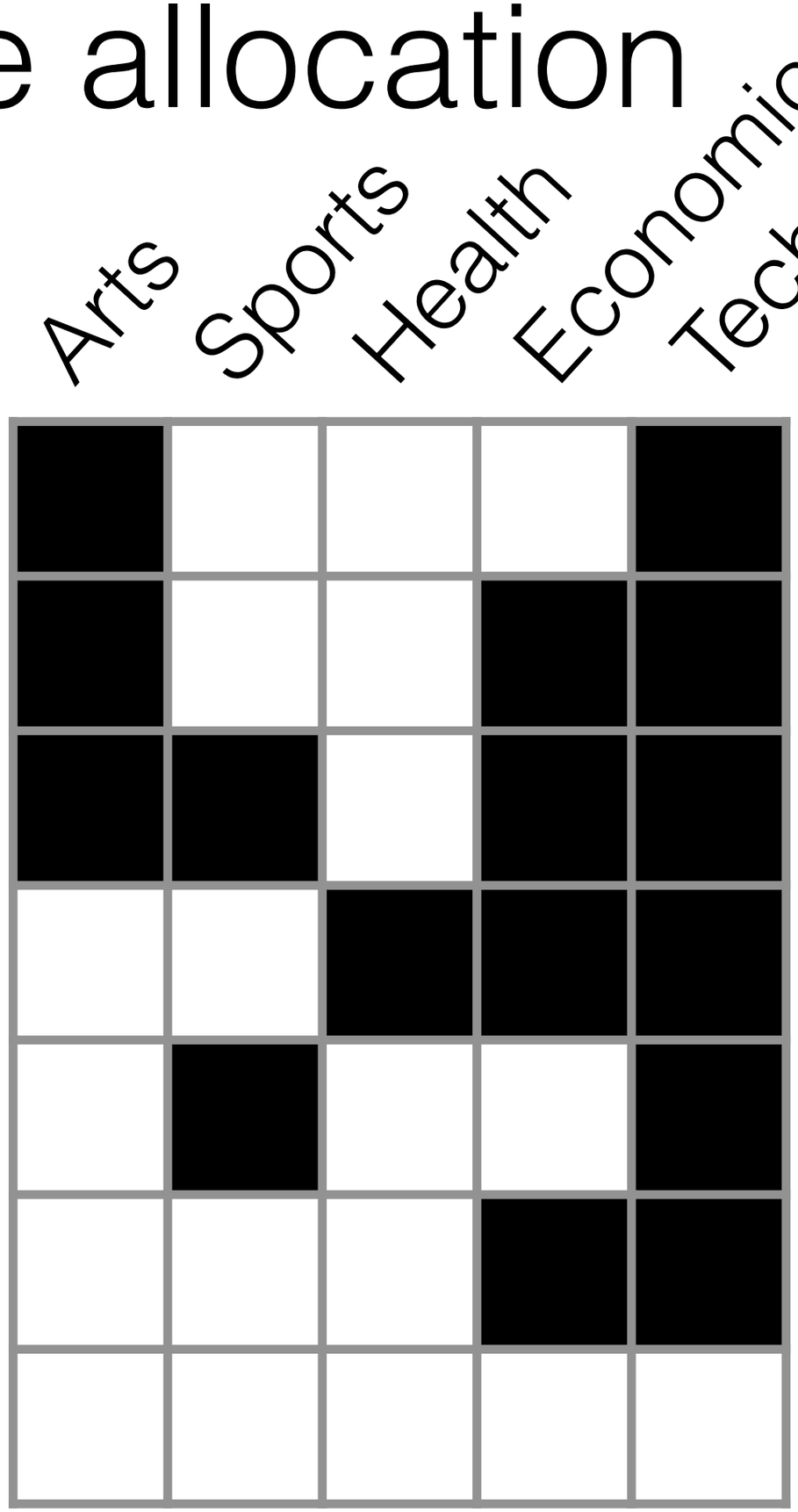
Picture 7

Feature allocation



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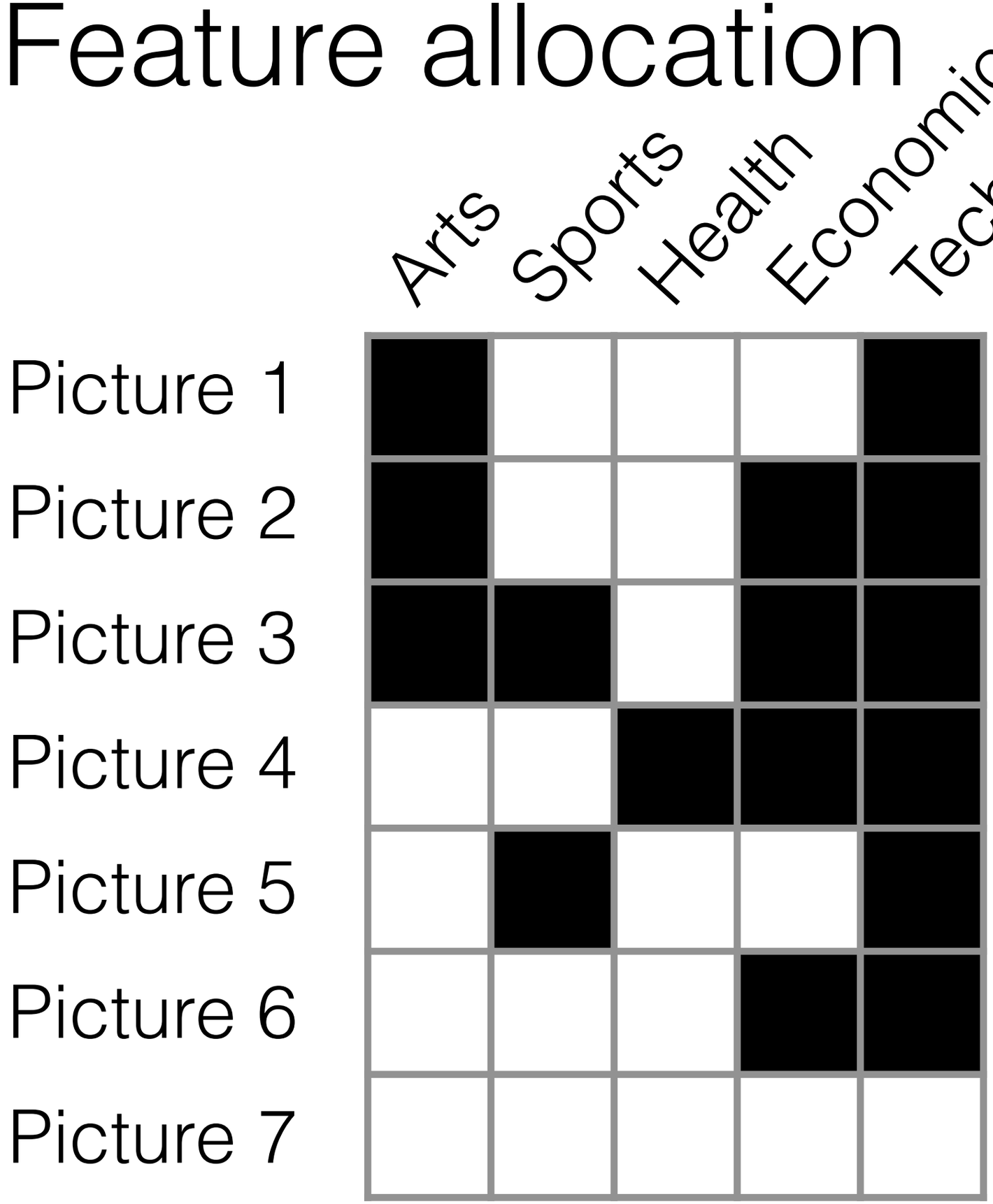

Many other possible latent structures in data

How do we learn latent structure?

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But...
- Nowadays: can have petabytes of data
- Practitioners turn to what runs

MAD-Bayes perspectives

- Bayesian nonparametrics assists the optimization-based inference community

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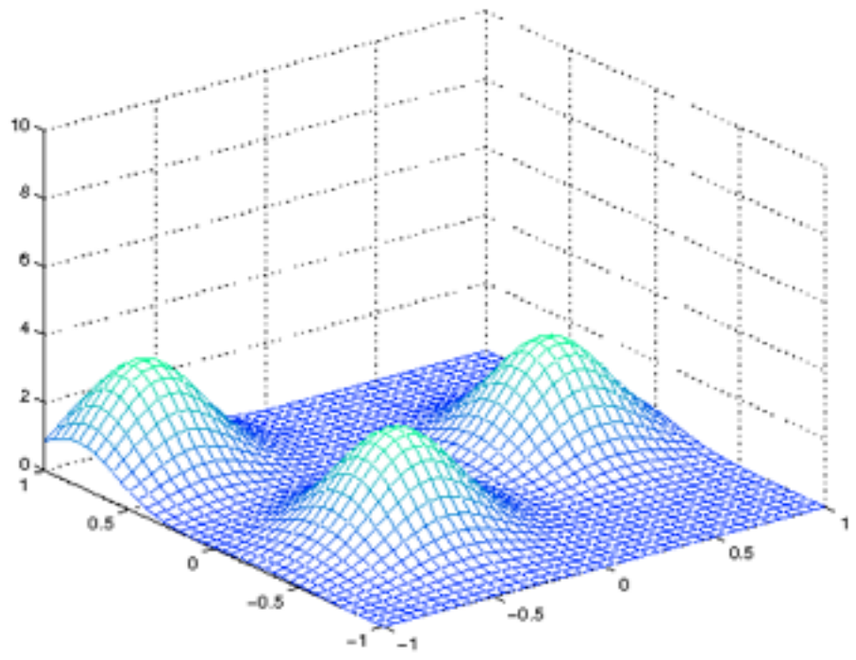

- Inspiration
  - Consider a finite Gaussian mixture model
  - The steps of the EM algorithm limit to the steps of the K-means algorithm as the Gaussian variance is taken to 0

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The MAD-Bayes idea

- Start with nonparametric Bayes model
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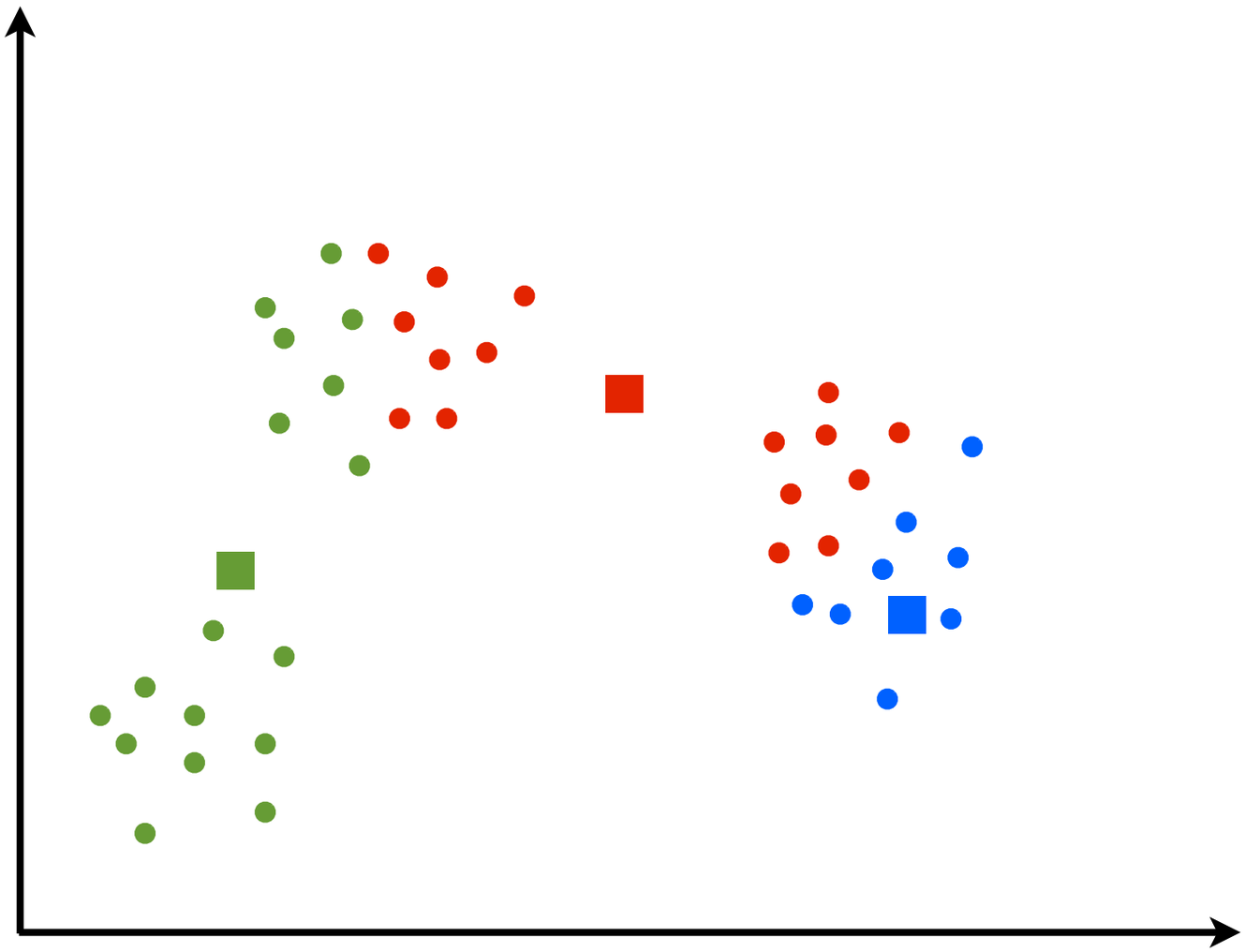

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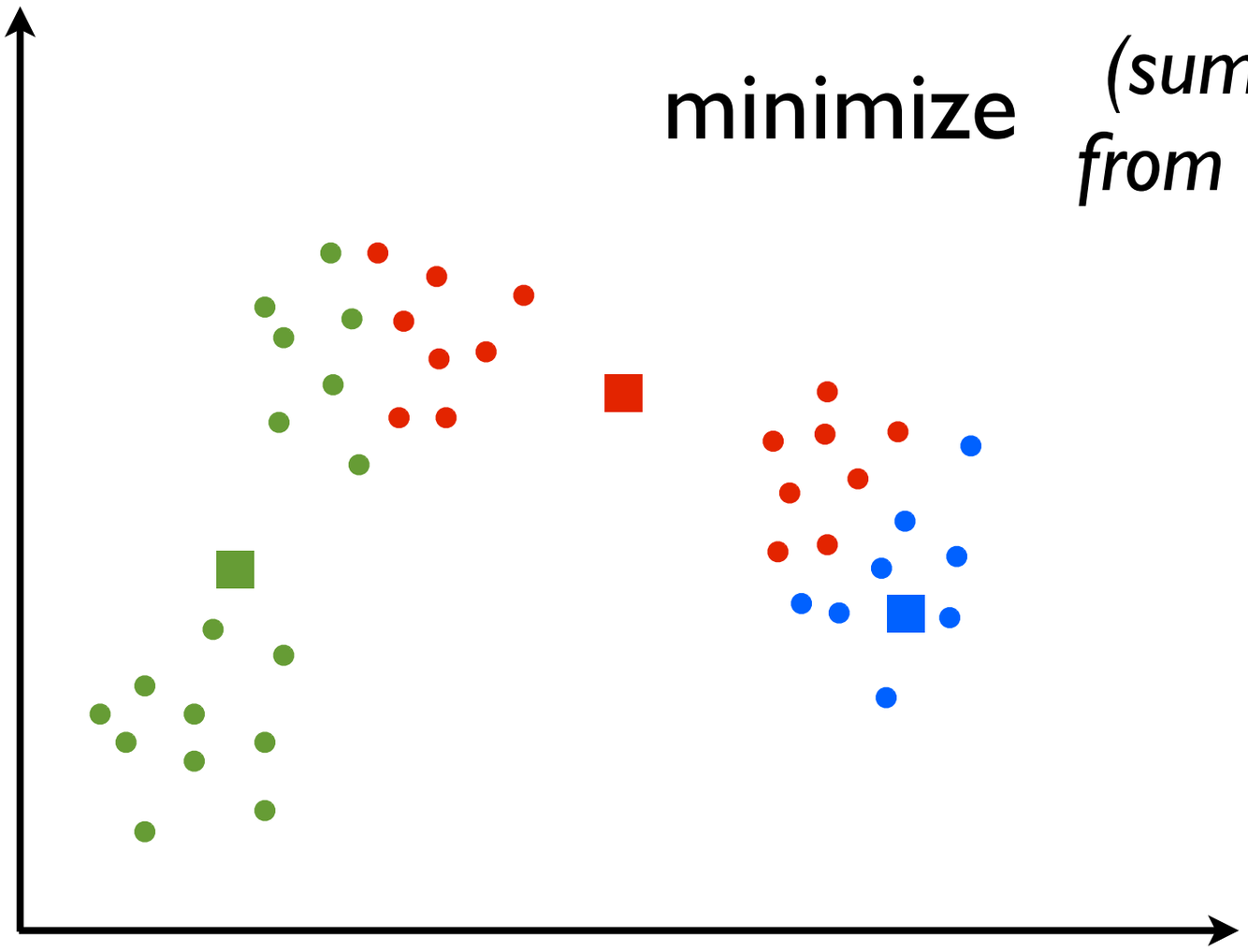

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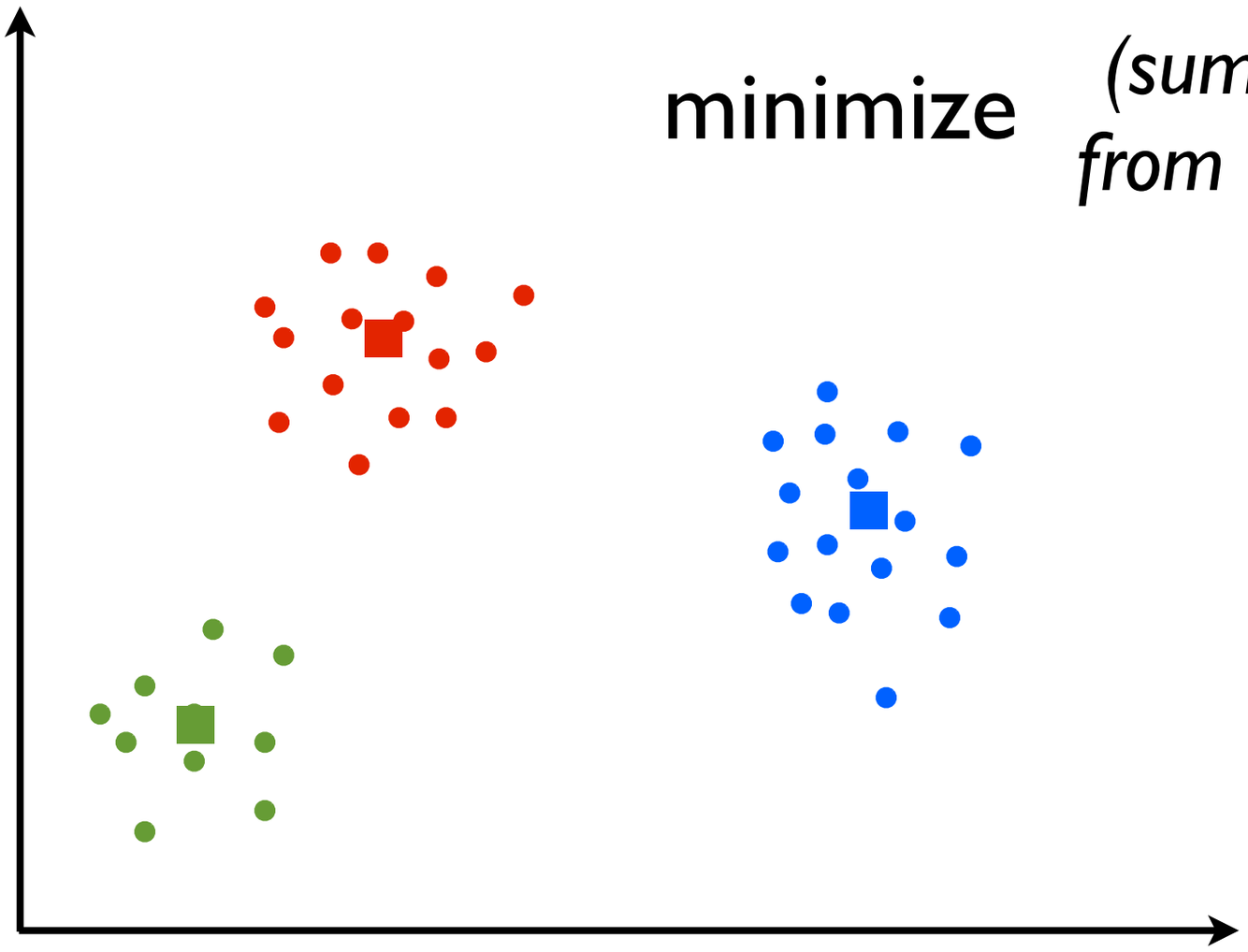

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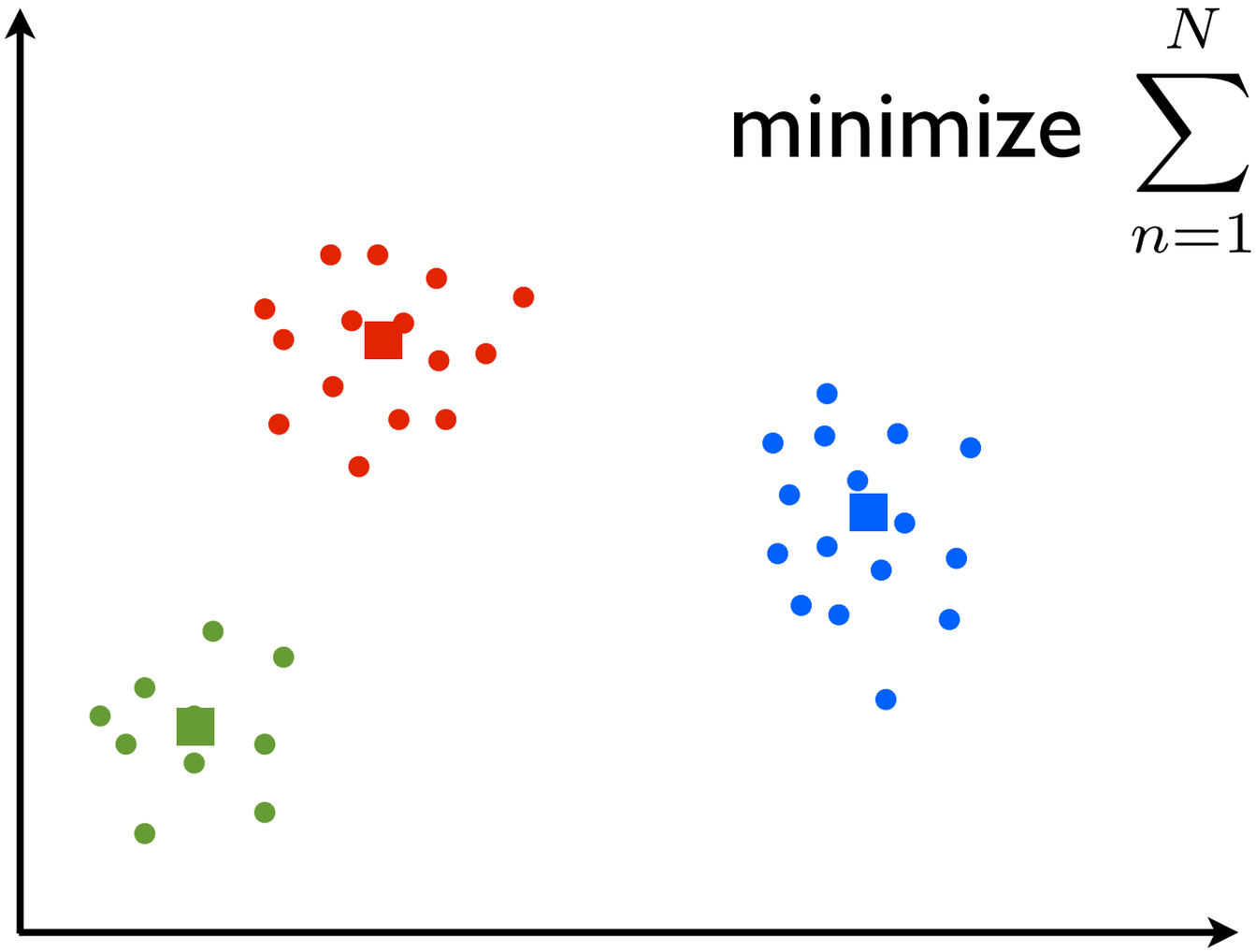

$$\text{minimize} \sum_{n=1}^{N} \|x_n - \text{center}_n\|^2$$

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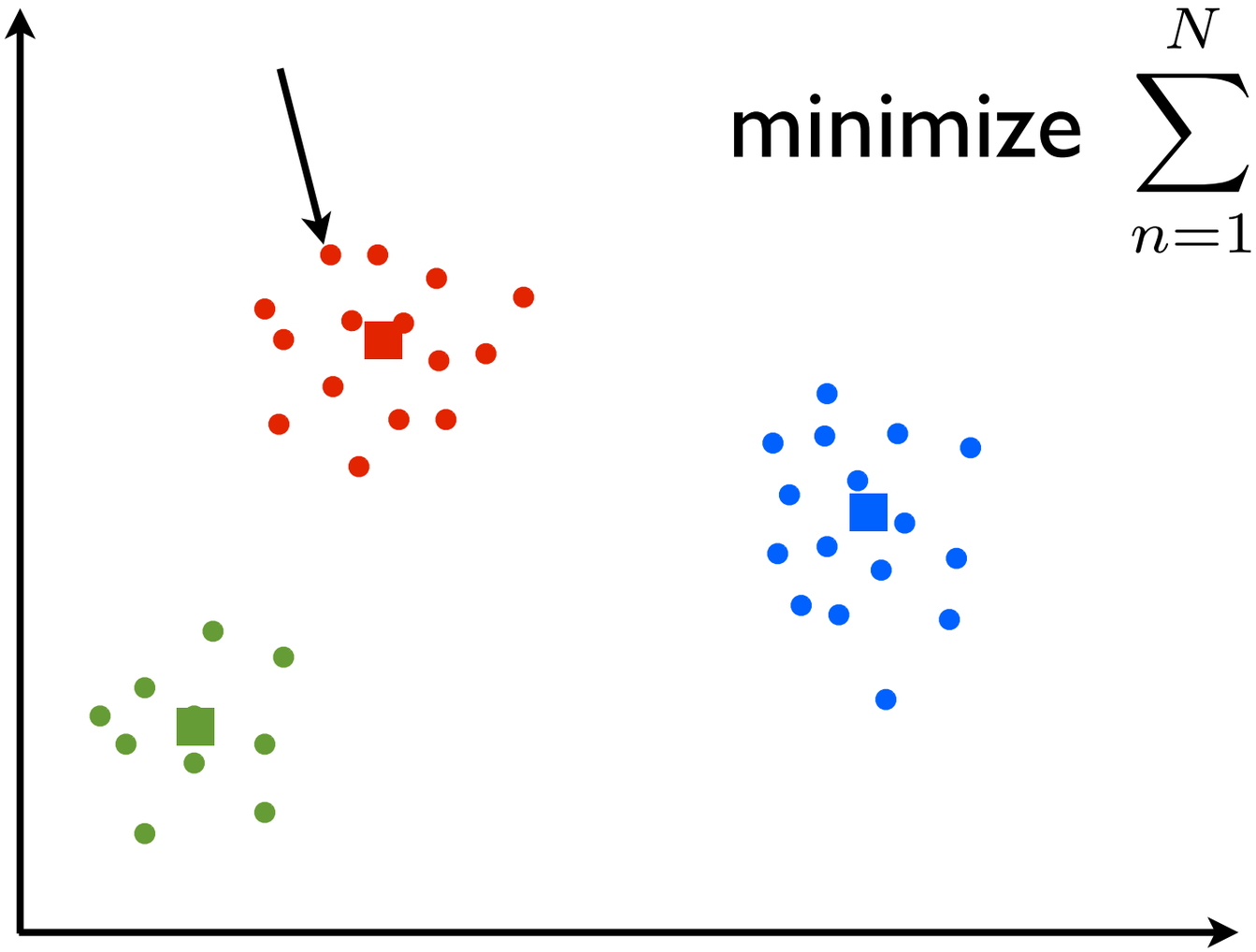

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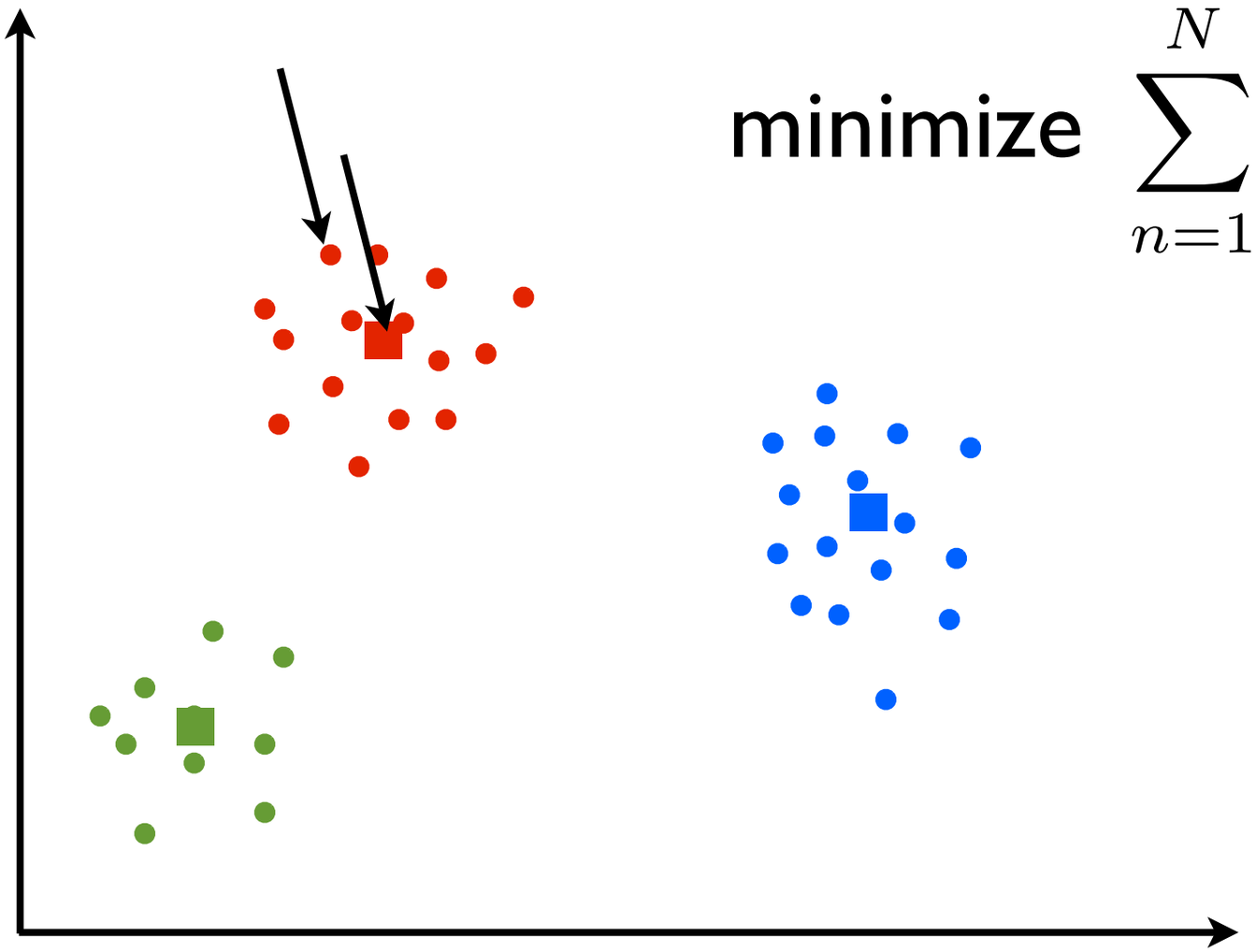

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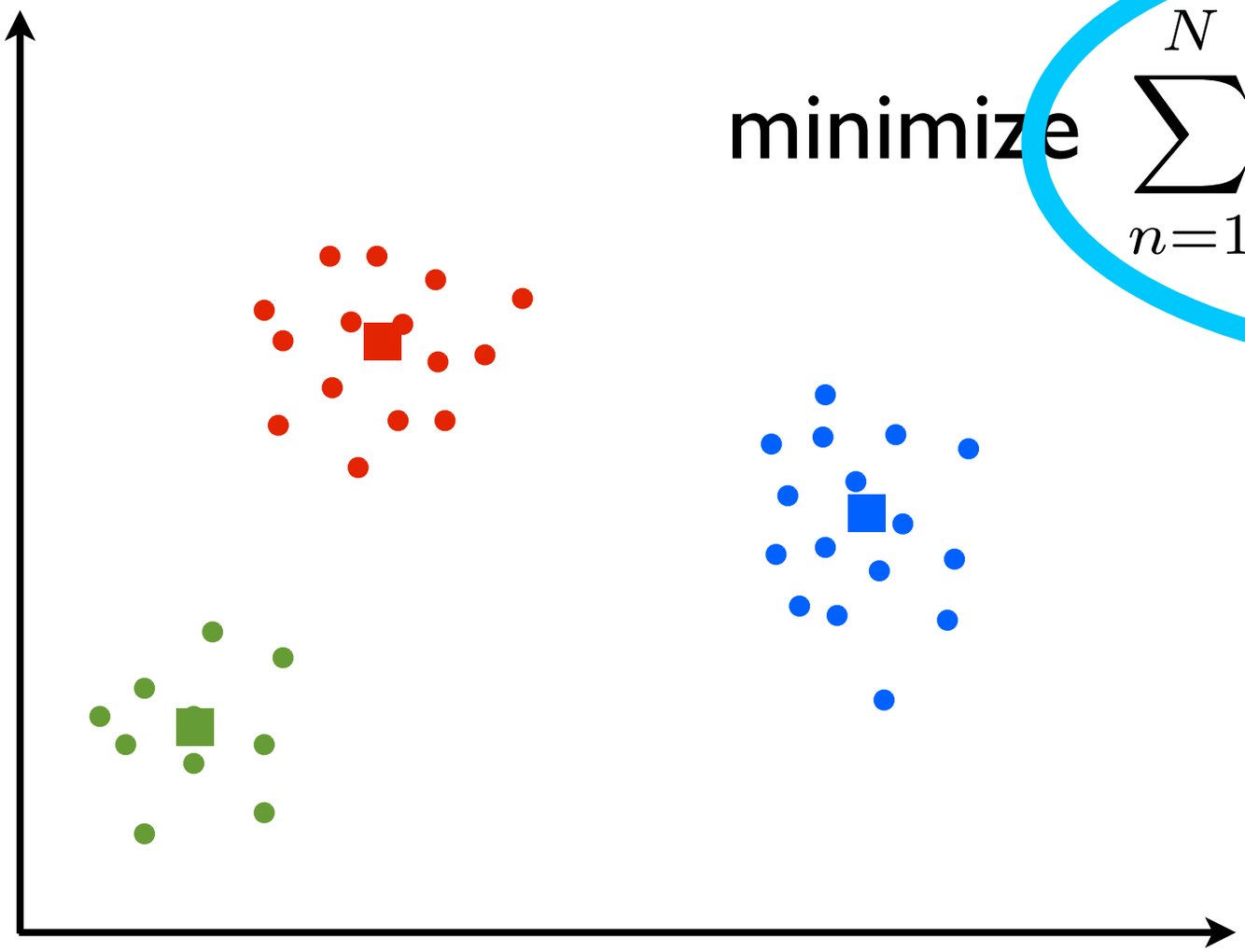
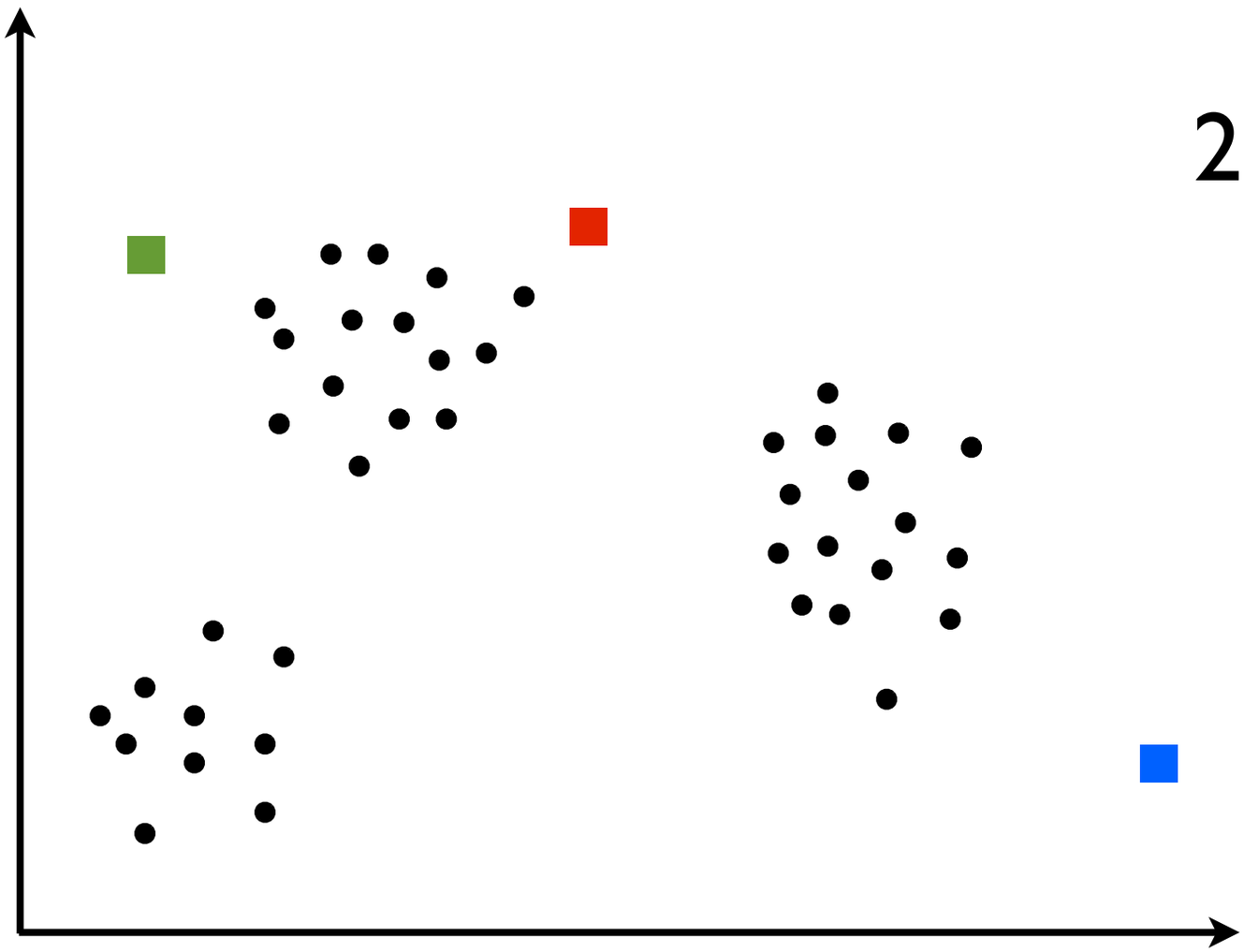


Lloyd's algorithm



Iterate until no changes:

1. For n = 1, ..., N
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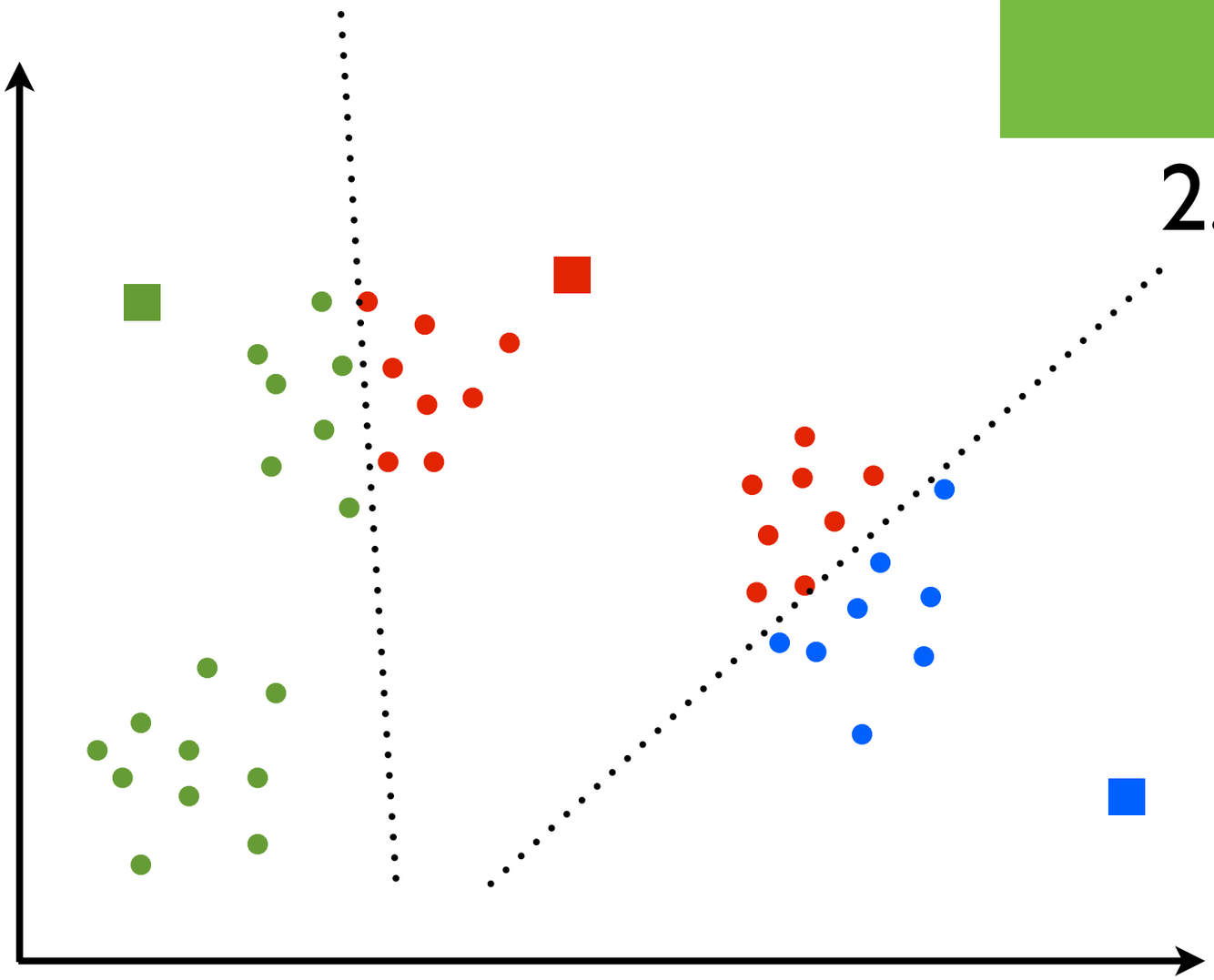

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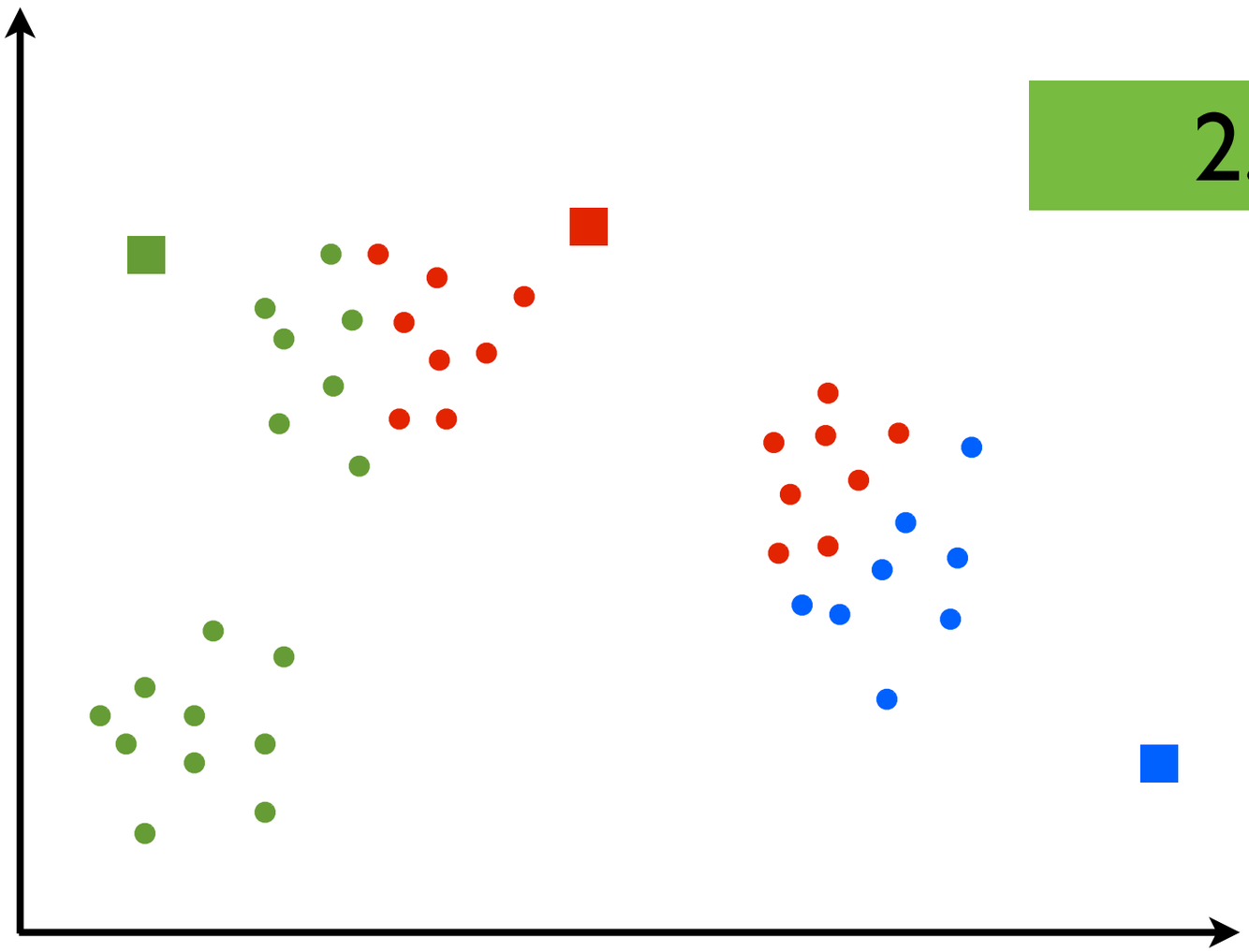

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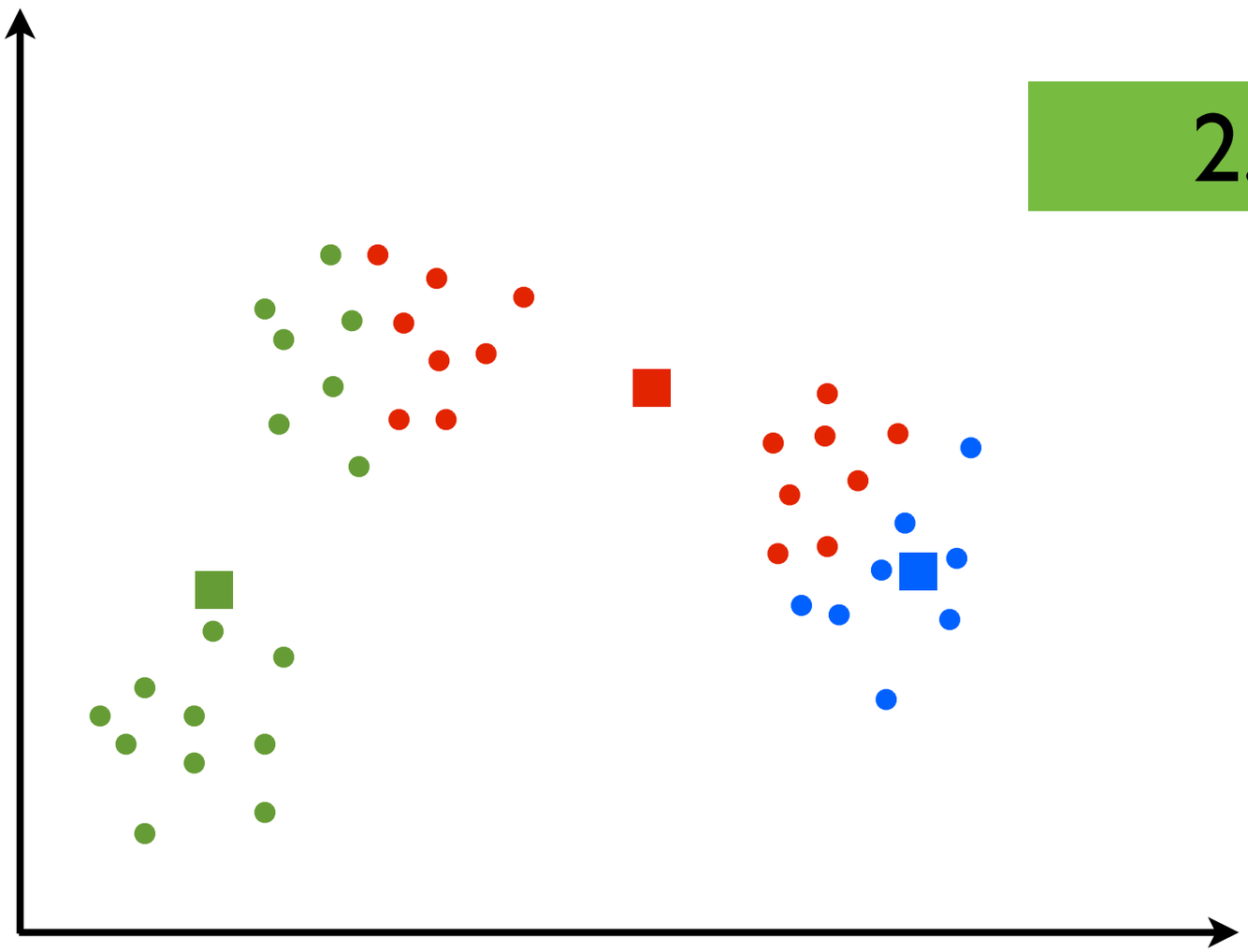

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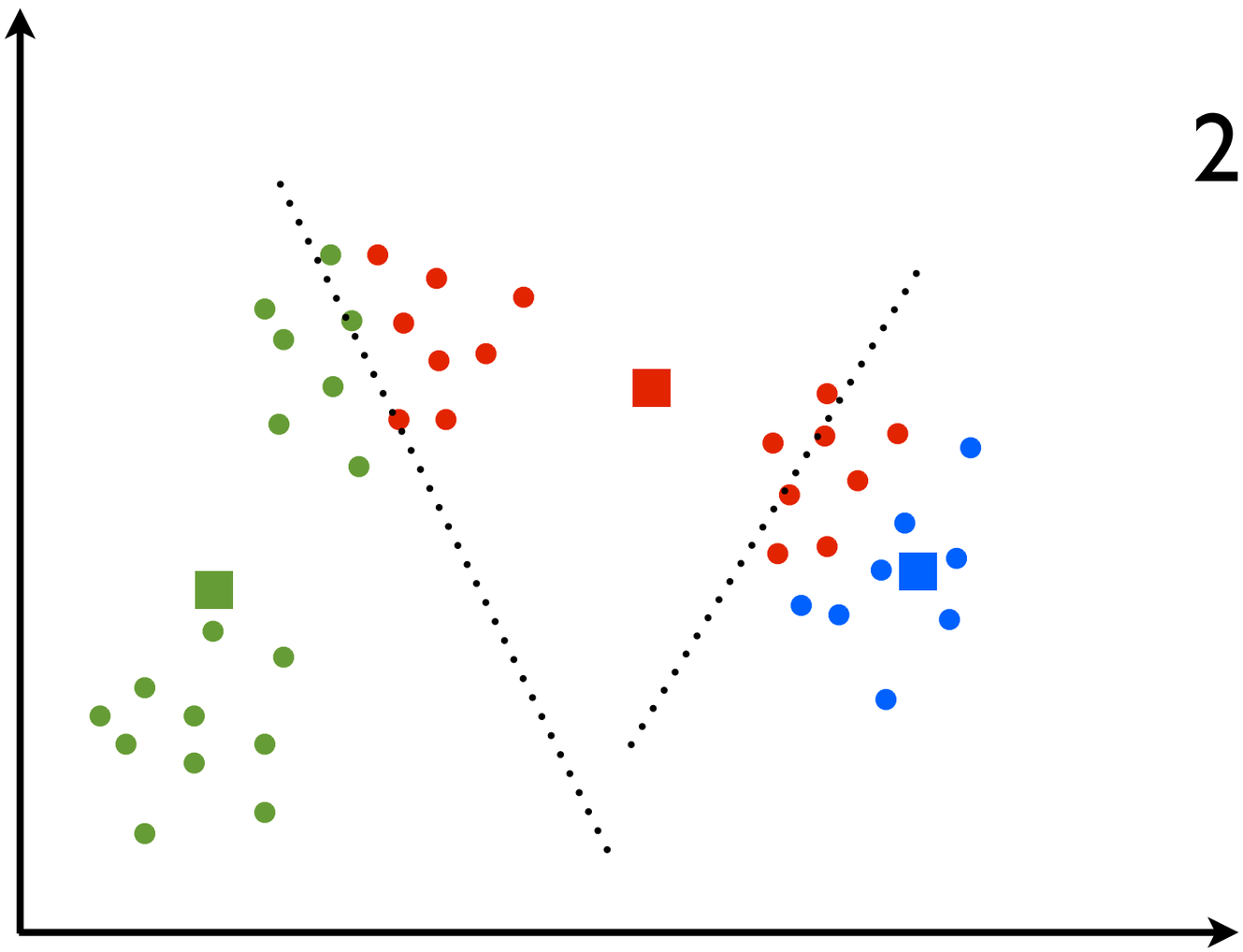

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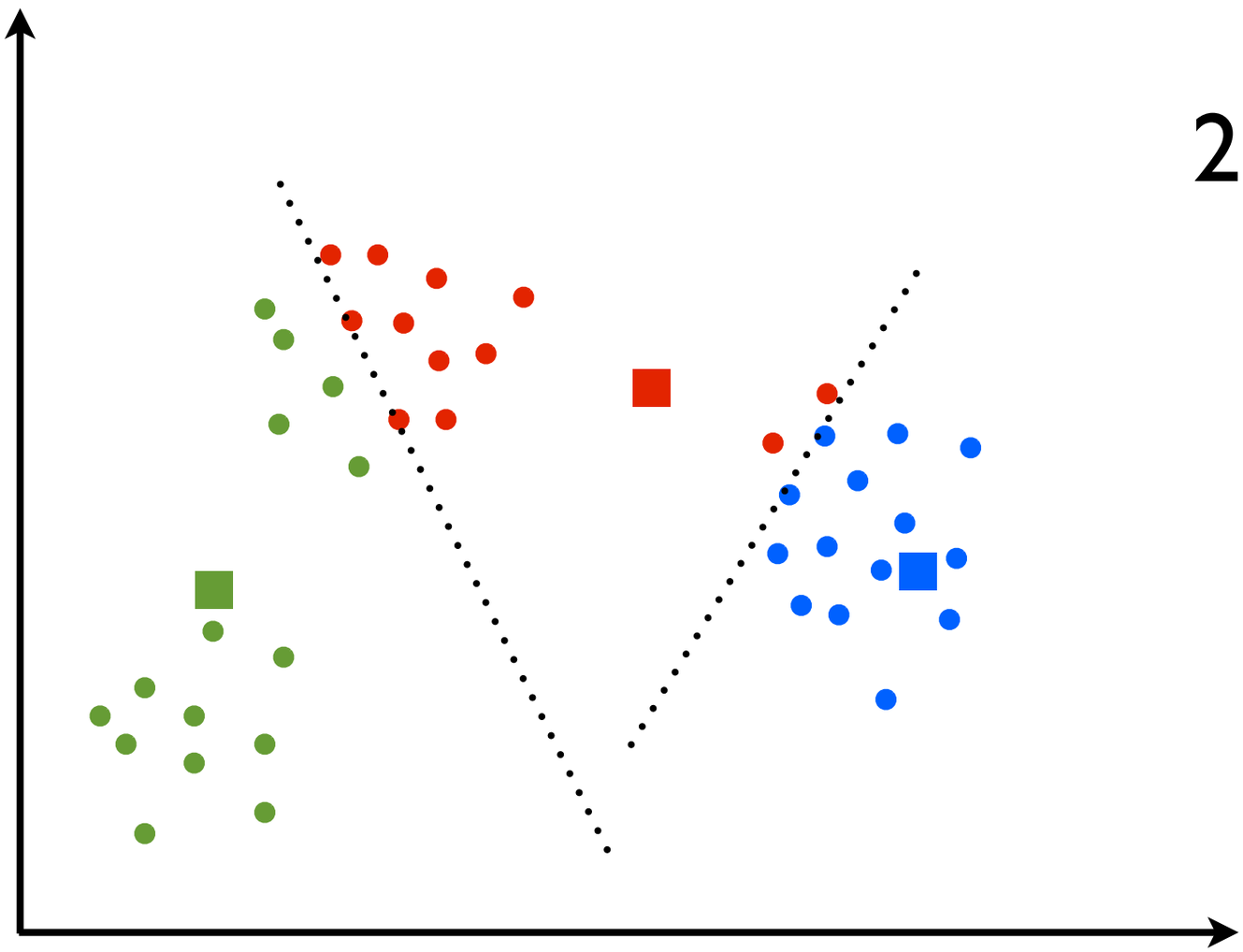

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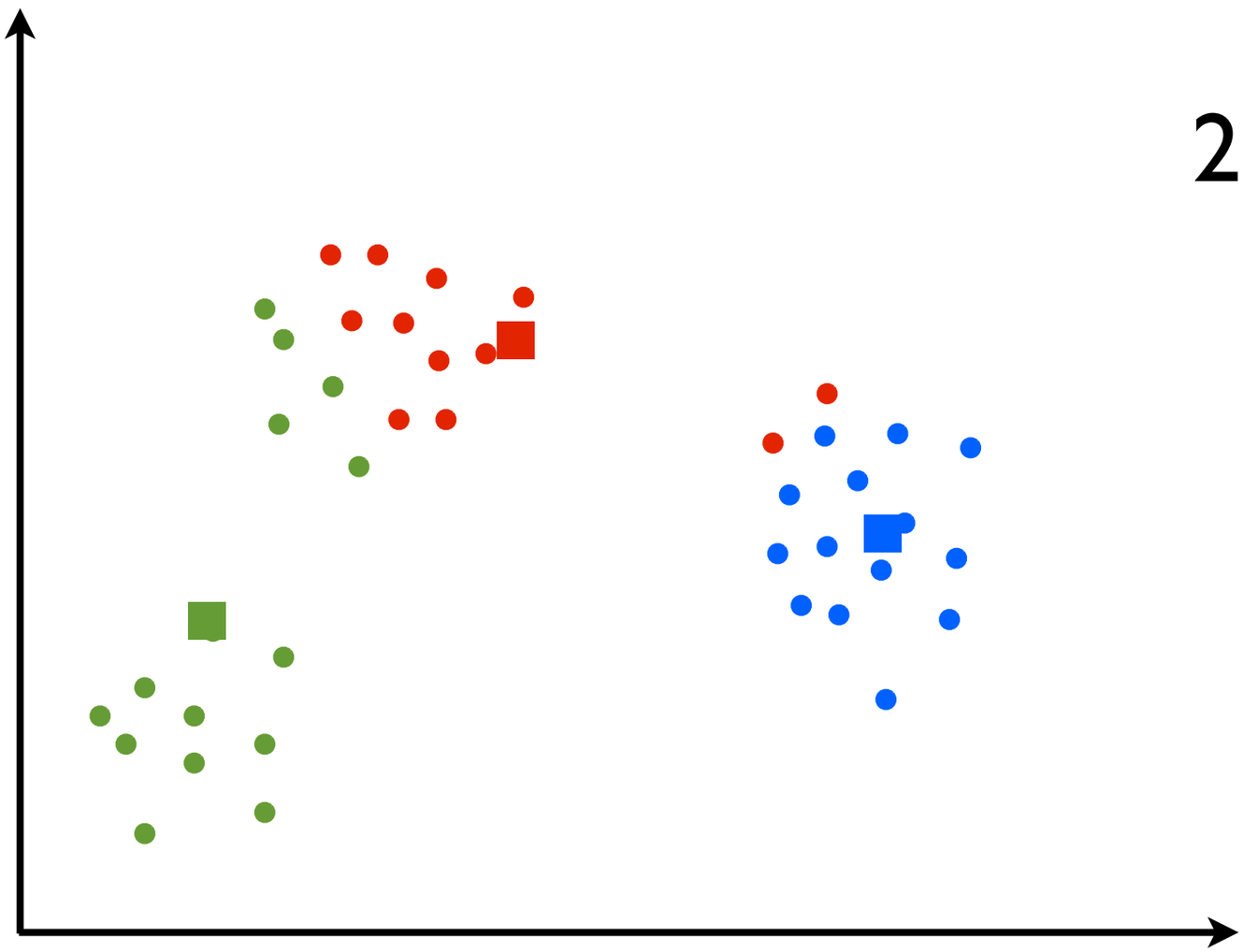

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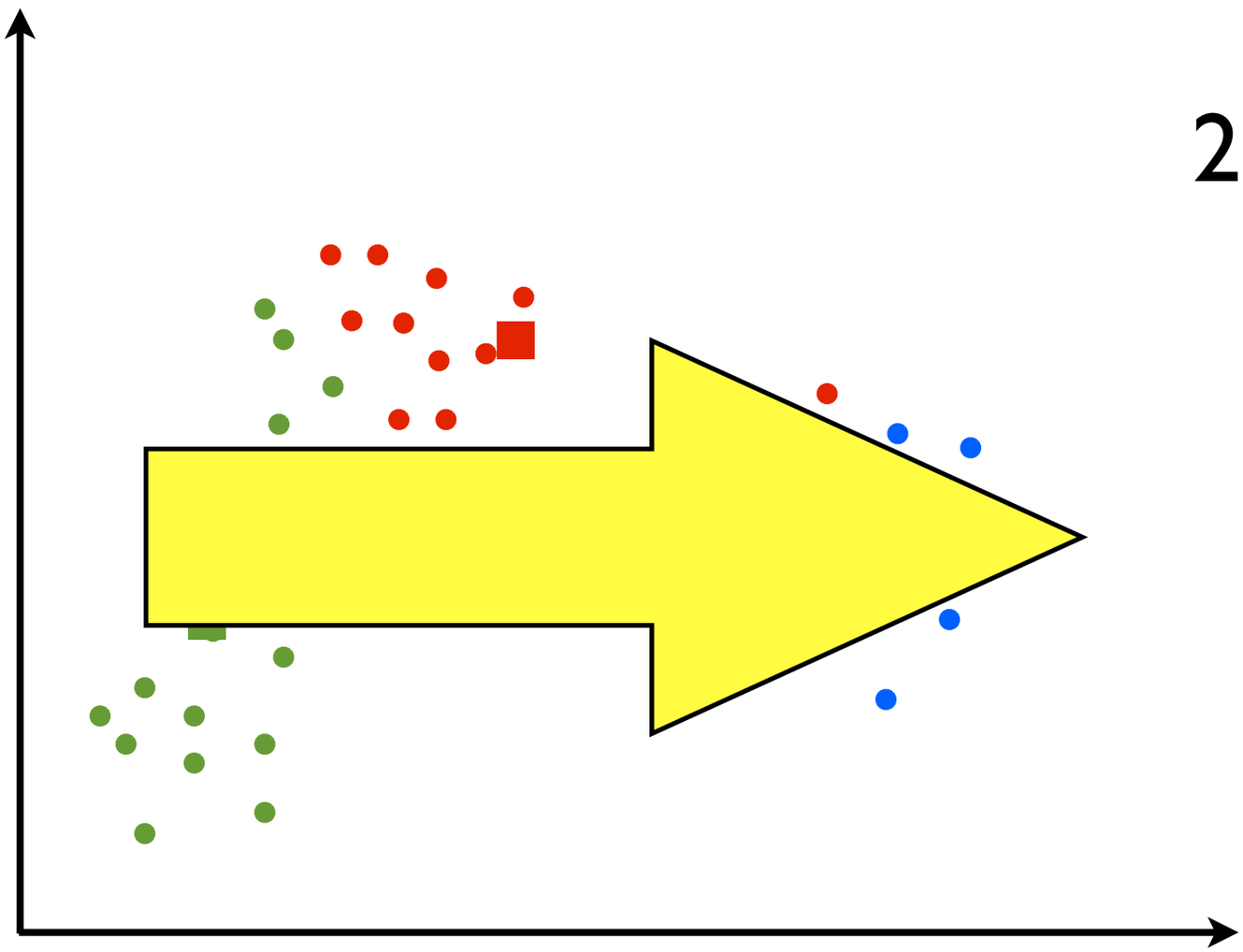

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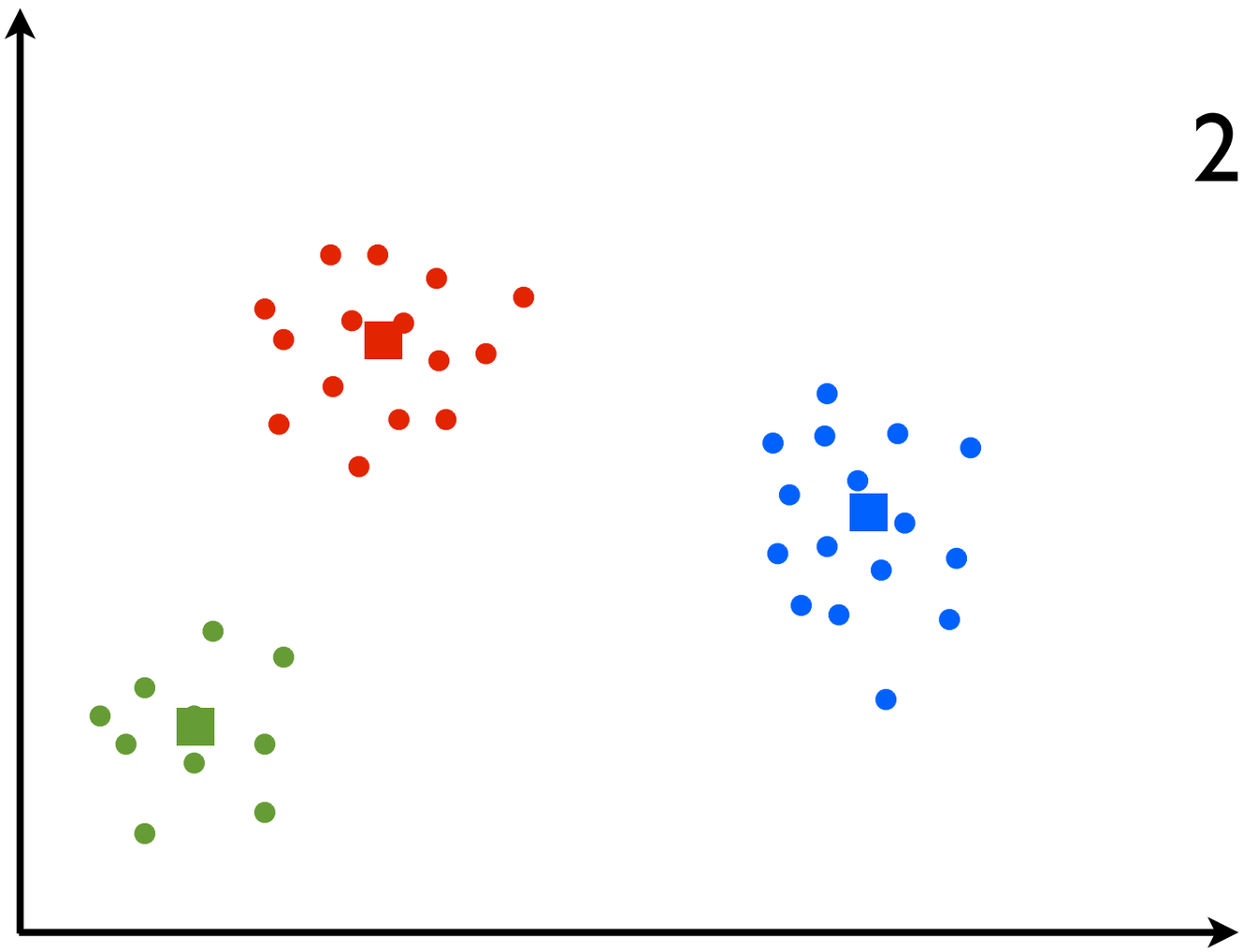

Iterate until no changes:

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2. Update cluster means

MAD-Bayes

The MAD-Bayes idea

- Start with nonparametric Bayes model
- Take a similar limit to get a **K-means-like objective**

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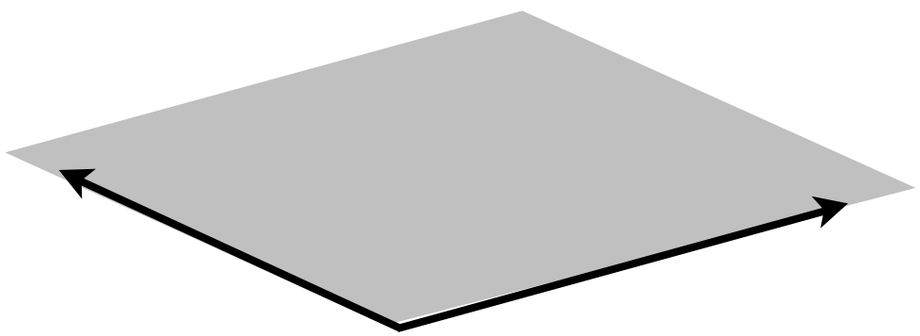
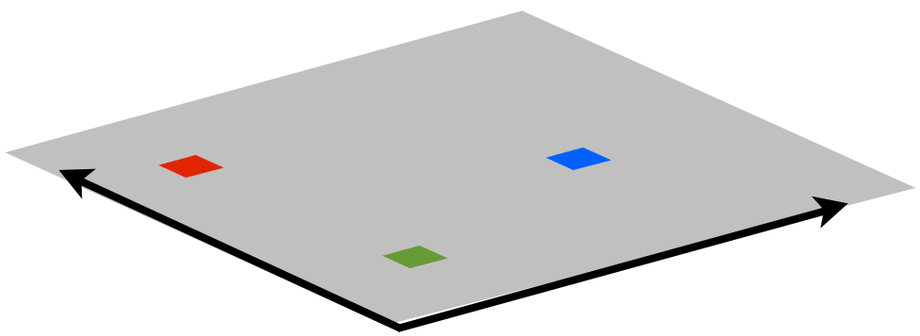
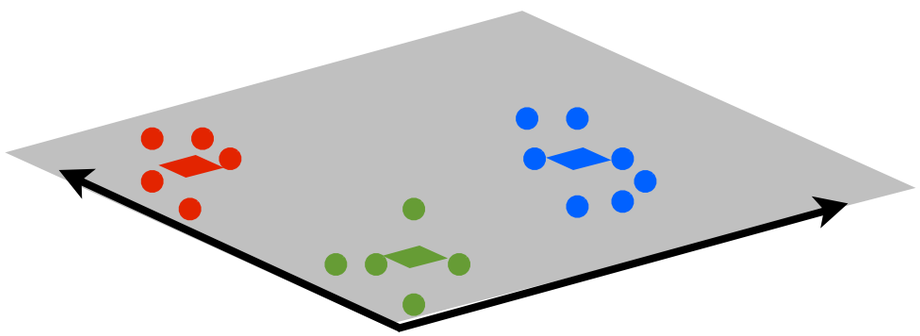
Bayesian model



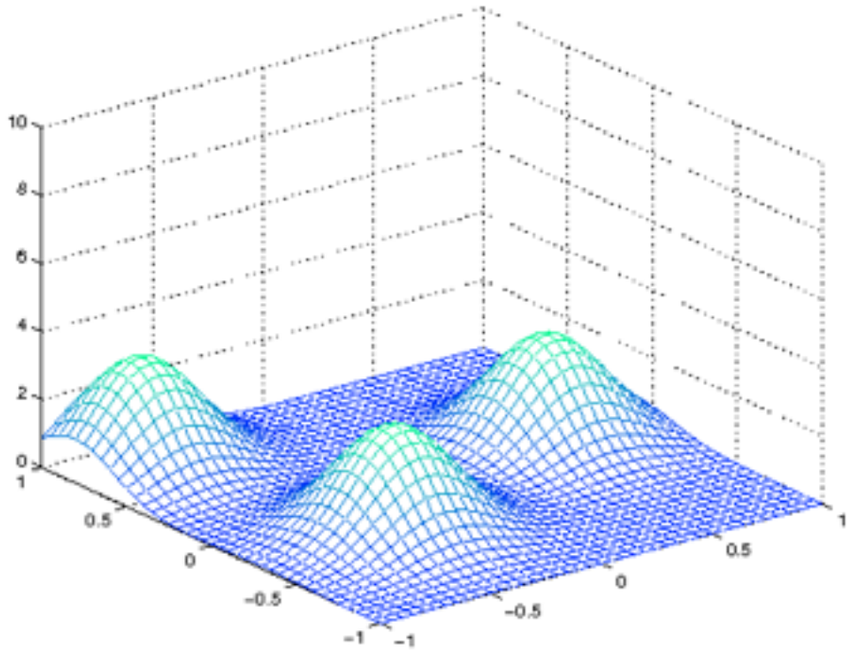
Bayesian model



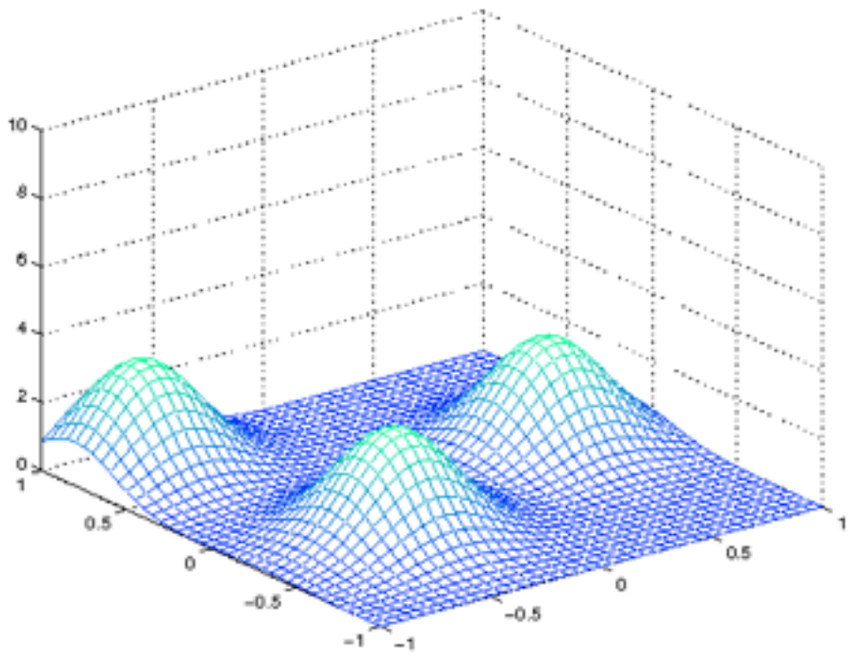
Bayesian model



Bayesian model



Bayesian model



Nonparametric

- number of parameters can grow with the number of data points

MAD-Bayes

The MAD-Bayes idea

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MAD-Bayes

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$$\text{argmax}_{\text{parameters}} \mathbb{P}(\text{parameters}|\text{data})$$

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  - ◇ "Small-variance asymptotics"

MAD-Bayes

Bayesian posterior

K-means-like objectives

MAD-Bayes

Bayesian posterior

K-means-like objectives

Mixture of K Gaussians  K-means

MAD-Bayes

<u>Bayesian posterior</u>

<u>K-means-like objectives</u>

Mixture of K Gaussians → K-means

Dirichlet process mixture → Unbounded number of clusters

MAD-Bayes

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MAD-Bayes

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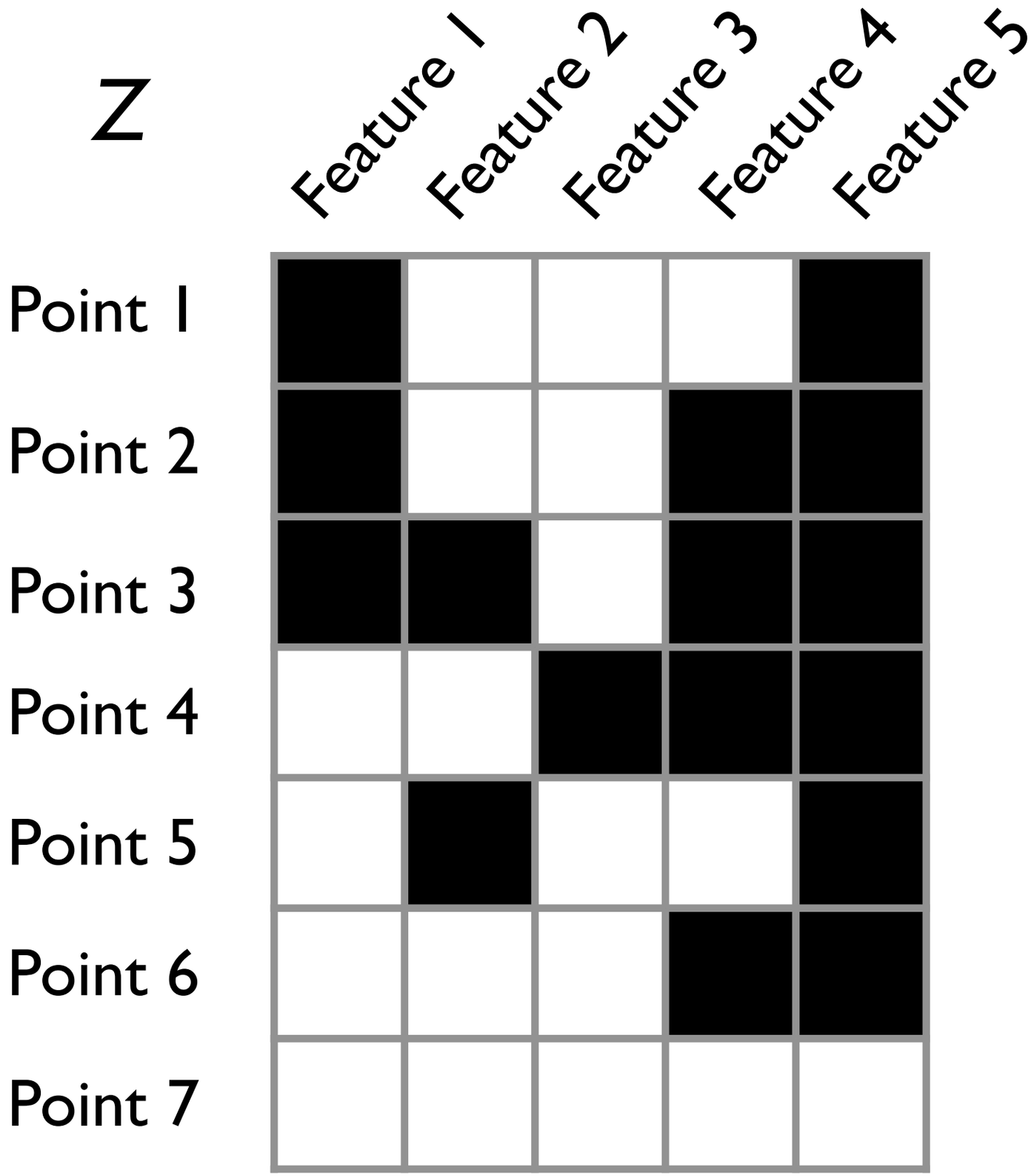
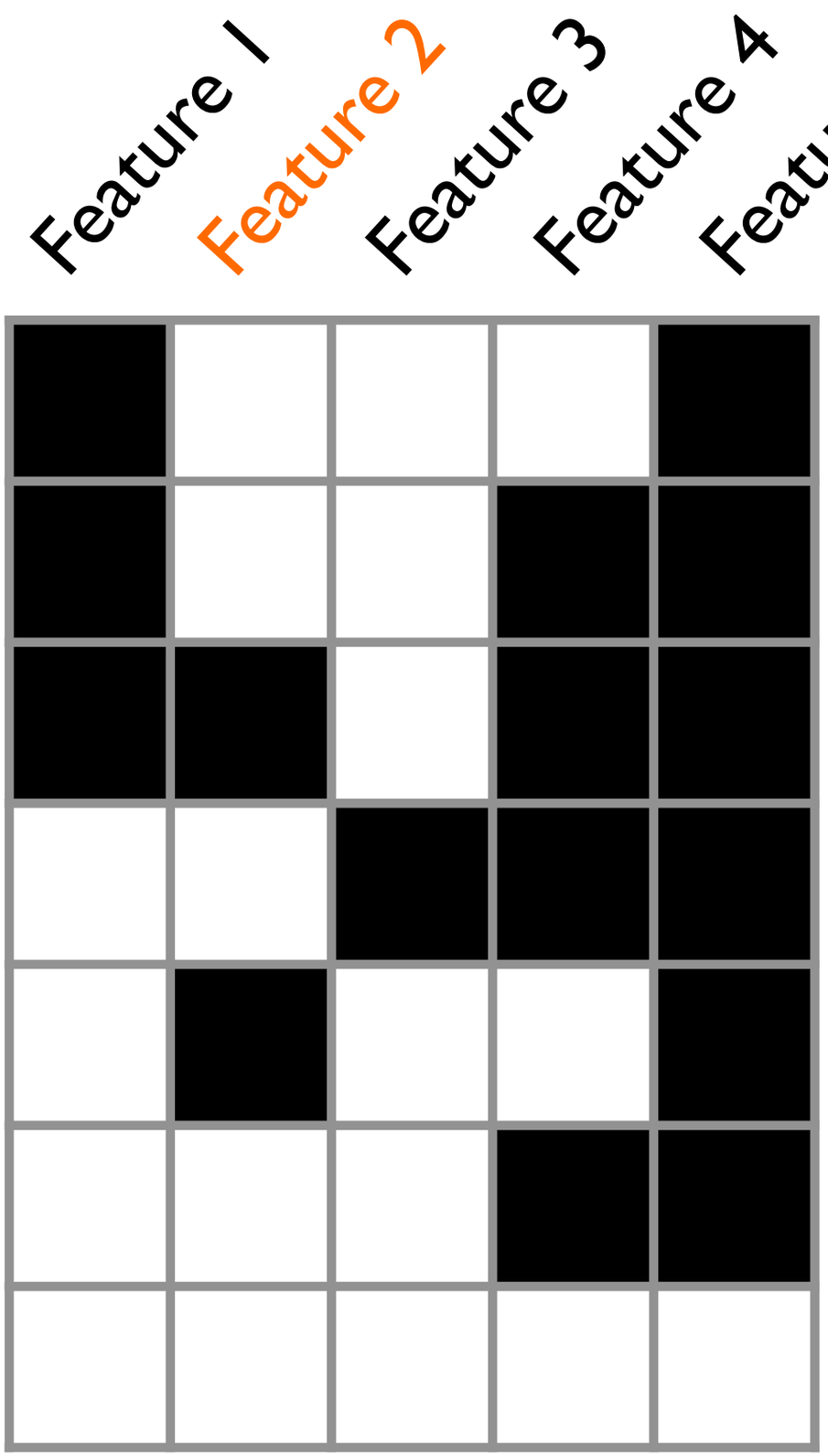
⋮	⋮
Beta process	Features

Features


Z
Feature 1
Feature 2
Feature 3
Feature 4
Feature 5
Point 1
Point 2
Point 3
Point 4
Point 5
Point 6
Point 7

Features

Z


Feature 1
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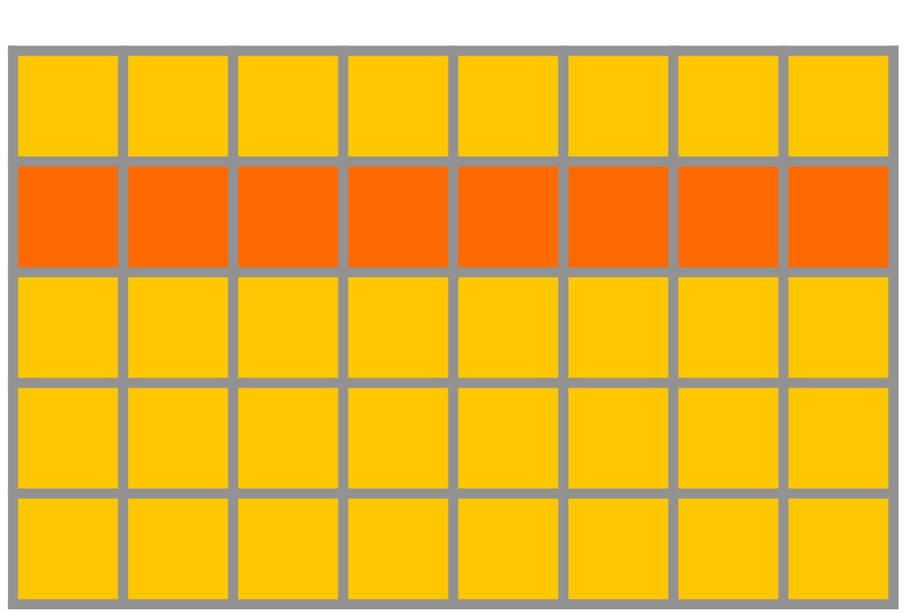
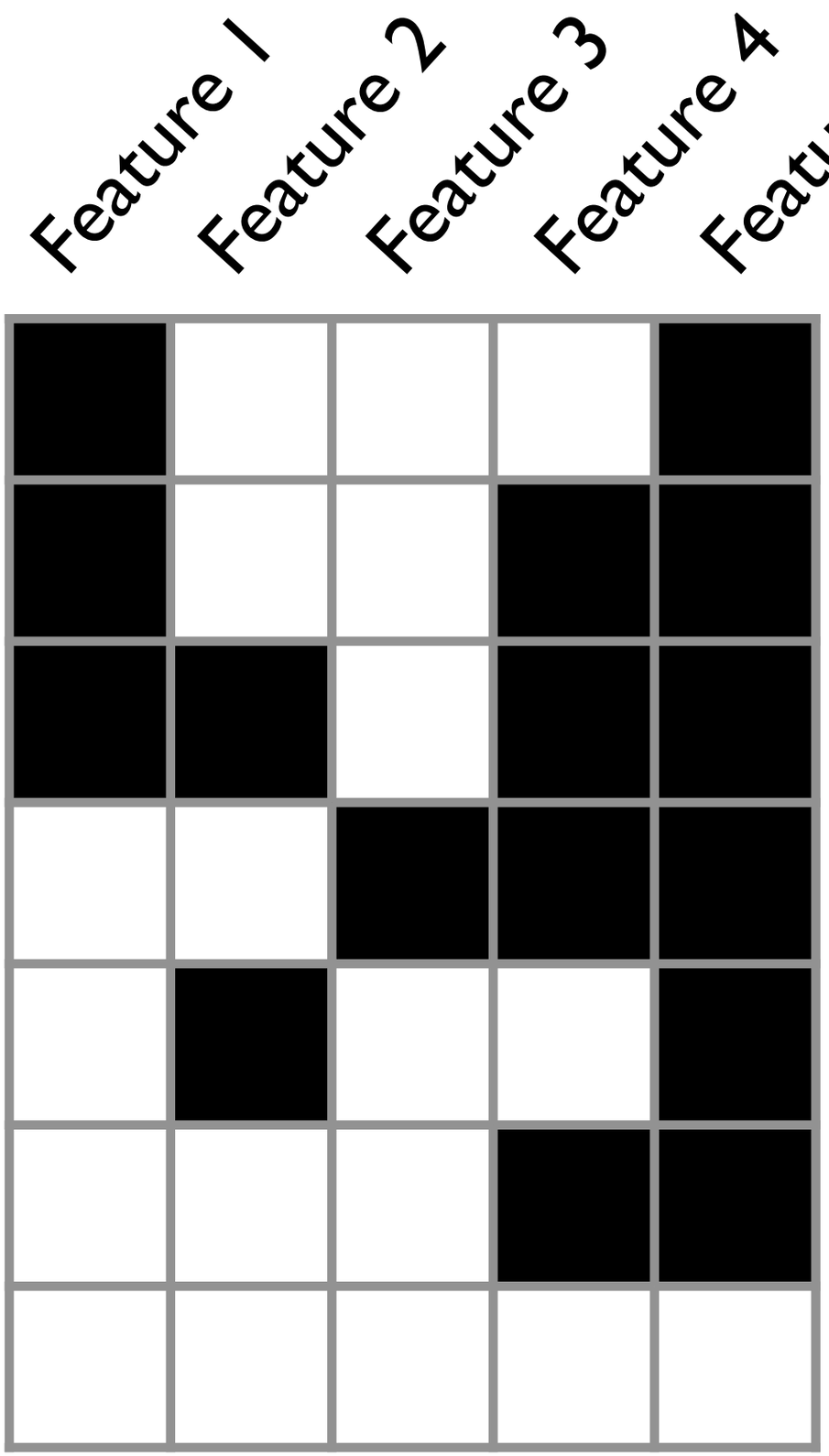
A



Features

Z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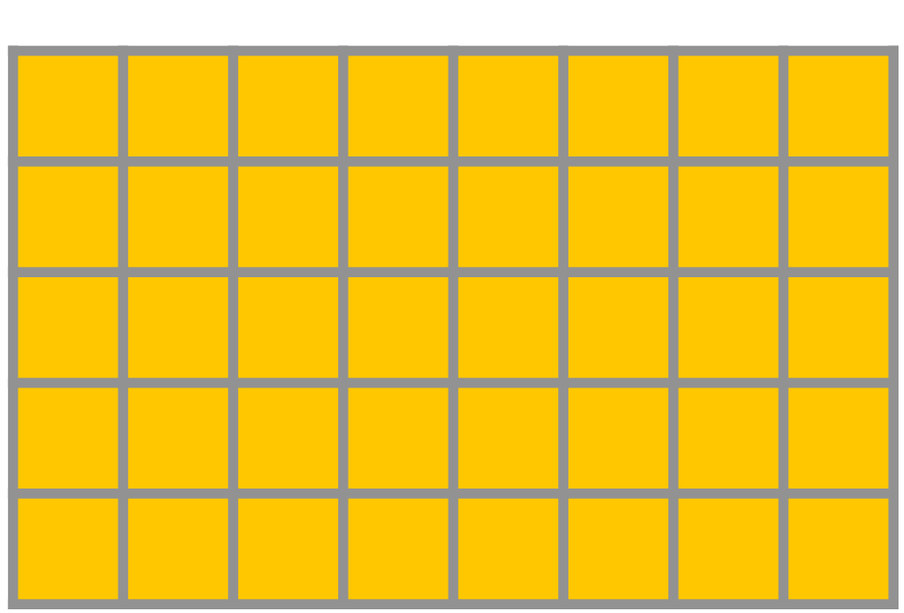

MAD-Bayes

Bayesian posterior

$$\mathbb{P}(Z, A|X)$$

$$\propto \frac{1}{(2\pi\sigma^2)^{ND/2}} \exp\left\{-\frac{1}{2\sigma^2}\mathbf{tr}((X-ZA)'(X-ZA))\right\}$$

$$\cdot \frac{\gamma^{K^+} \exp\left\{-\sum_{n=1}^{N} \frac{\gamma}{n}\right\}}{\prod_{h=1}^{H} \tilde{K}_h!} \prod_{k=1}^{K^+} \frac{(S_{N,k}-1)!(N-S_{N,k})!}{N!}$$

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BP-means objective

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MAD-Bayes

Griffiths & Ghahramani (2006) computer vision problem "tabletop data"

[Griffiths, Ghahramani 2006]

MAD-Bayes

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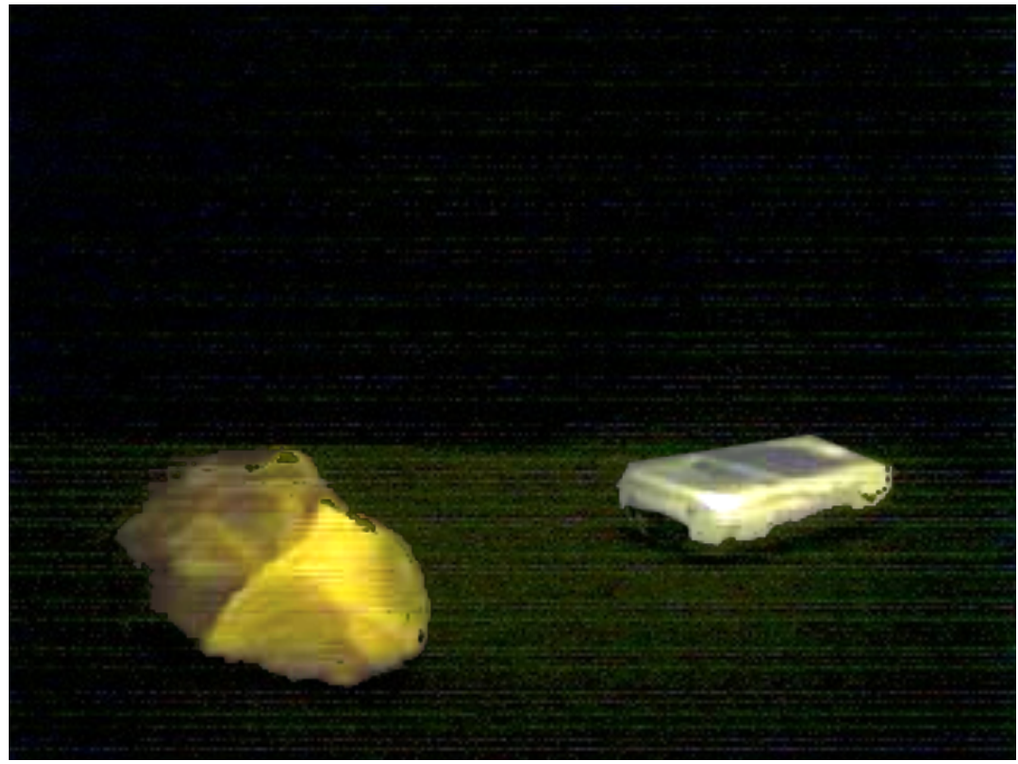
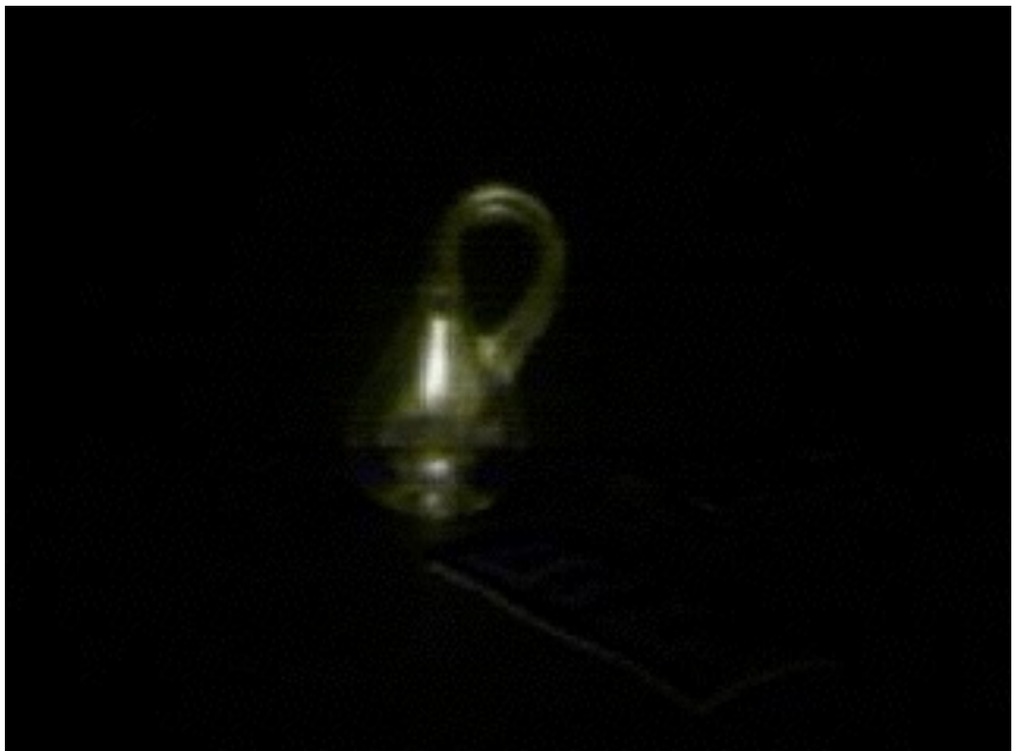


MAD-Bayes

BP-means features: table and four objects






MAD-Bayes

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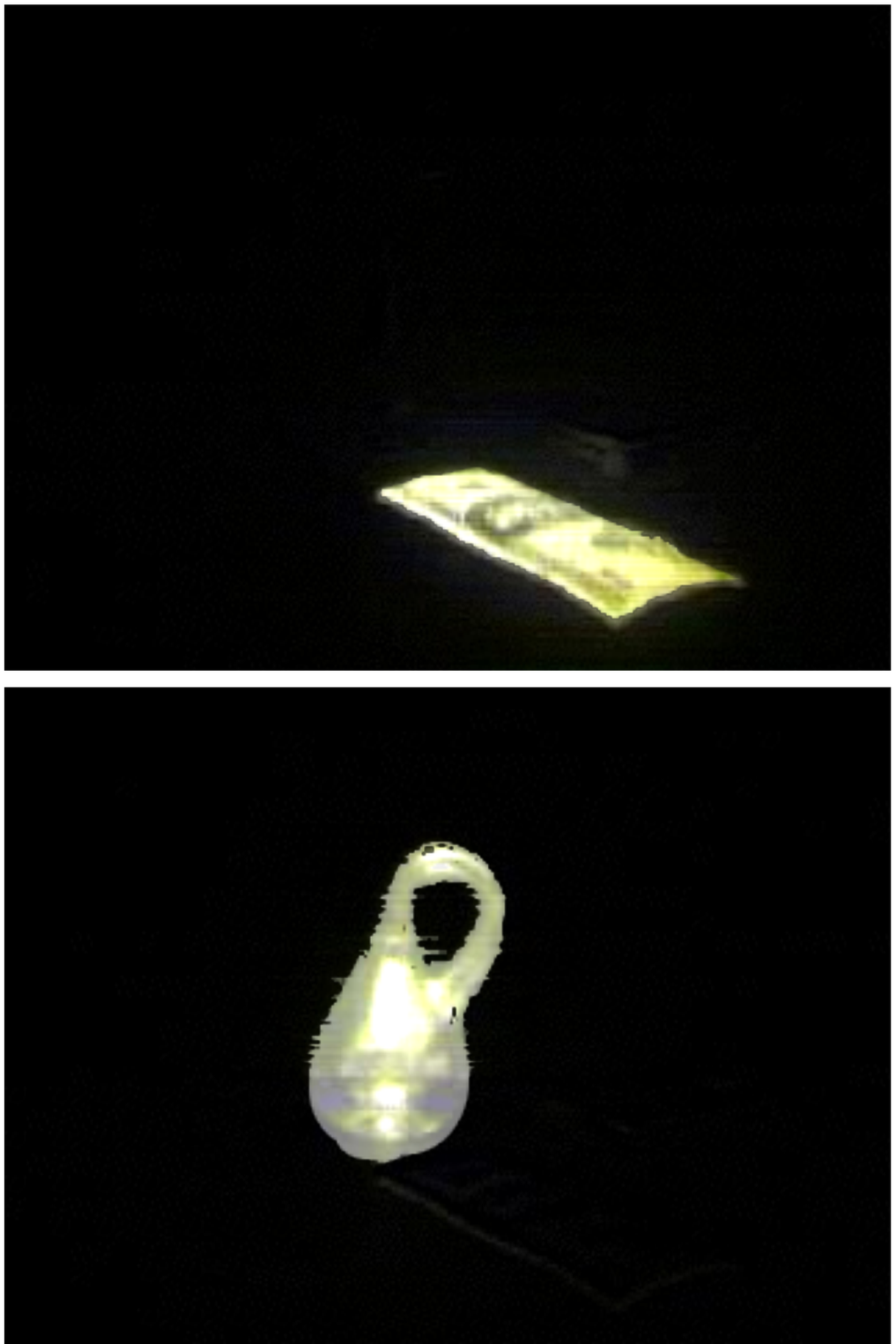

MAD-Bayes

Griffiths & Ghahramani (2006) computer vision problem "tabletop data"

Bayesian posterior
Gibbs sampler

BP-means algorithm

$8.5 * 10^3$ sec

0.36 sec

Still faster by order of magnitude
if restart 1000 times

MAD-Bayes

Parallelism and optimistic concurrency control

	DP-means alg.	BP-means alg.
# data points	**134M**	**8M**
time per iteration	**5.5 min**	**4.3 min**

MAD-Bayes

Bayesian posterior	K-means-like objectives
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MAD-Bayes Conclusions

- We provide new optimization objectives and regularizers
  - In fact, general means of obtaining more
  - Straightforward, fast algorithms

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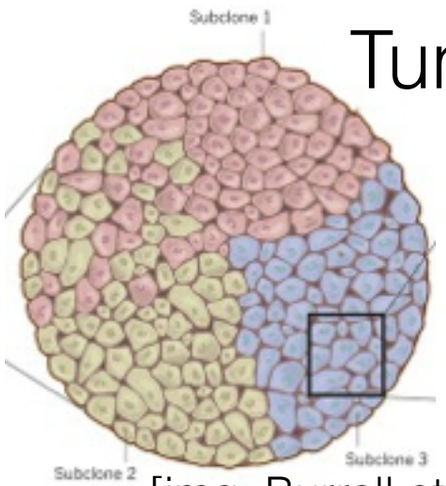
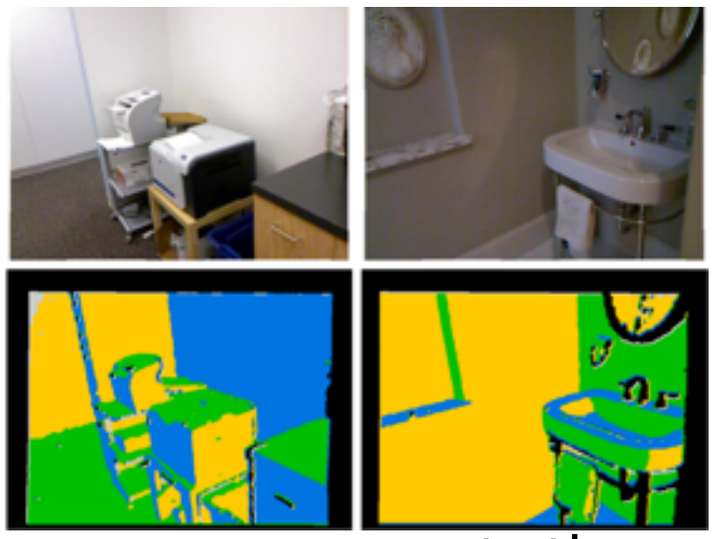


Tumor heterogeneity
[Xu et al 2015]

[img: Burrell et al 2013]



Real-time scene segmentation
[Straub et al 2016]

Protein matching and alignment
[Green 2015]

Change points in disease progression
[Huggins et al 2015]

Roadmap

- Posterior approximation trade-offs
  - Point estimates
  - Uncertainties
- Robustness trade-offs
  - Local robustness quantification
- Theoretical guarantees on quality

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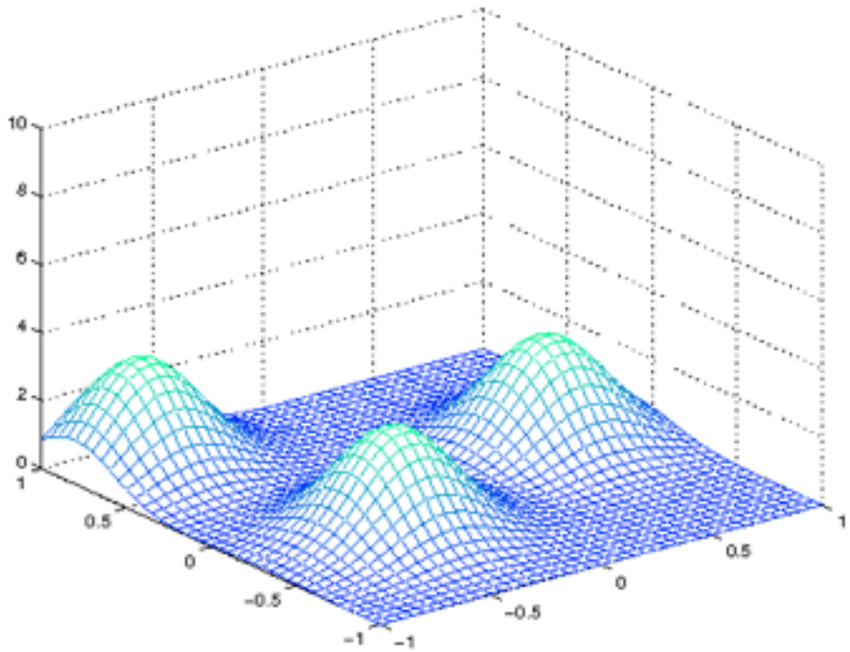
Variational Bayes

Variational Bayes

- Variational Bayes (VB)

Variational Bayes



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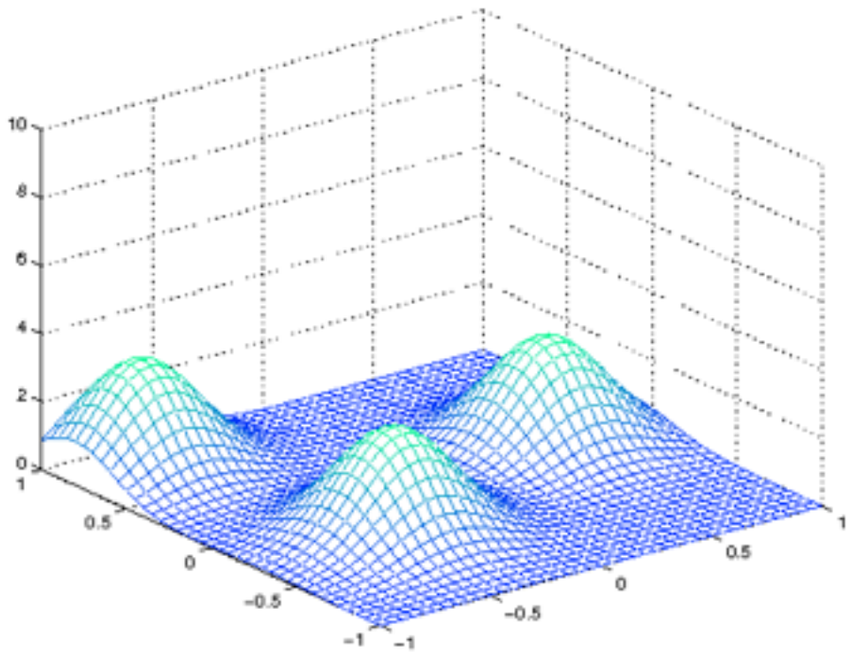
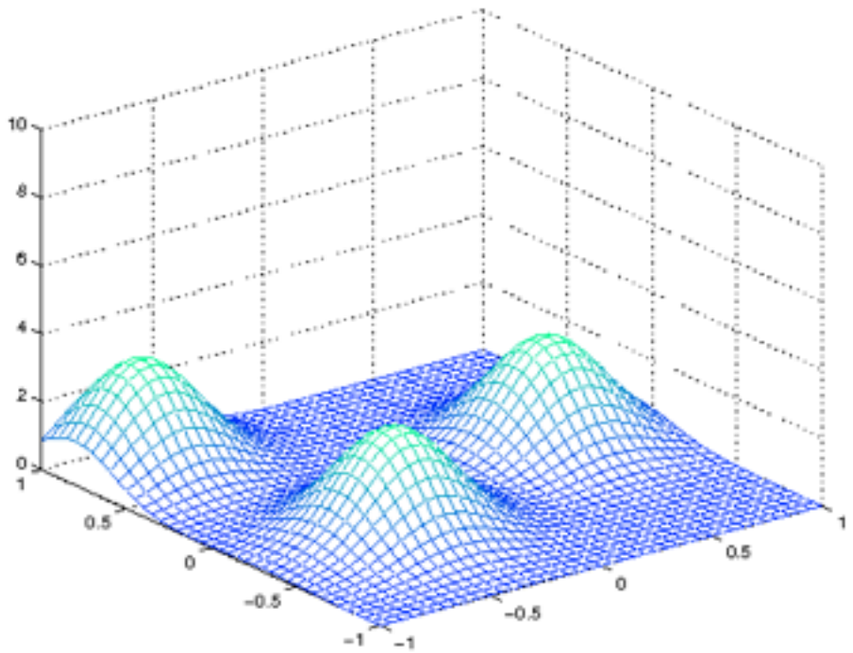


- Variational Bayes (VB)
  - Approximation $q^*(\theta)$ for posterior $p(\theta|x)$

Variational Bayes



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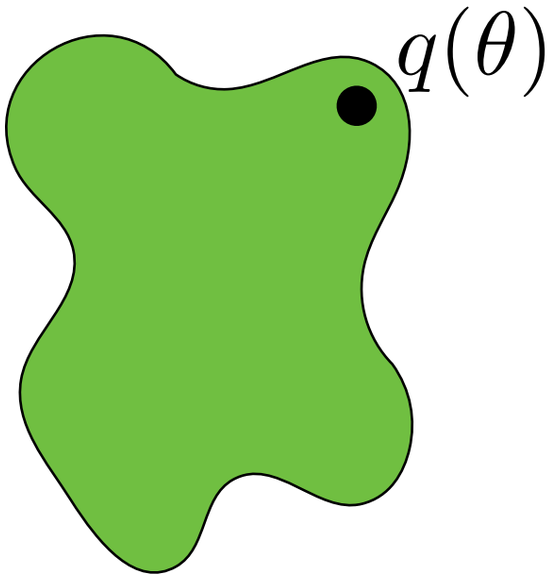

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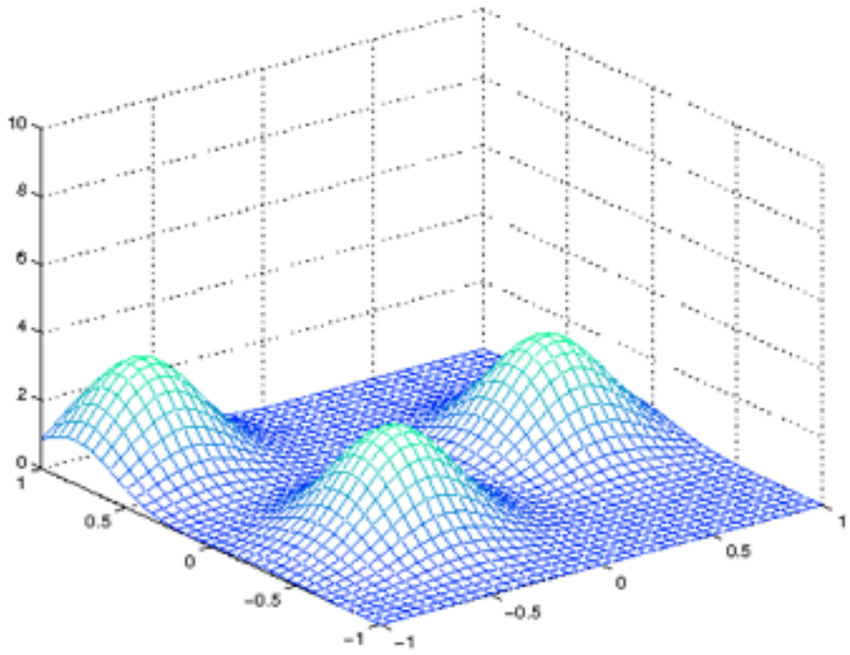

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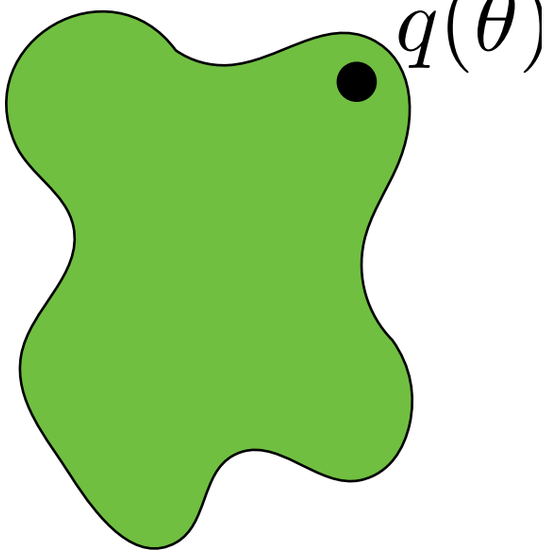

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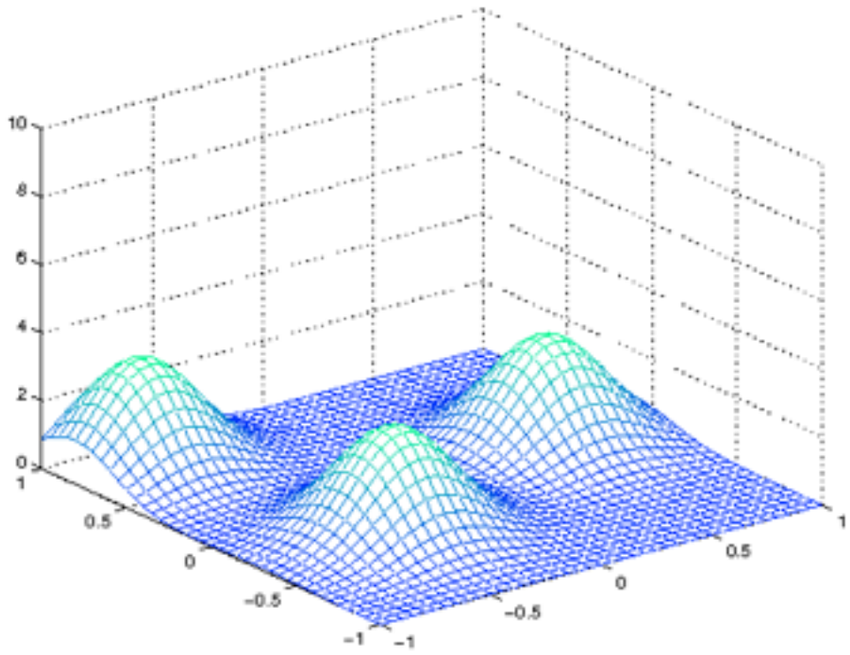

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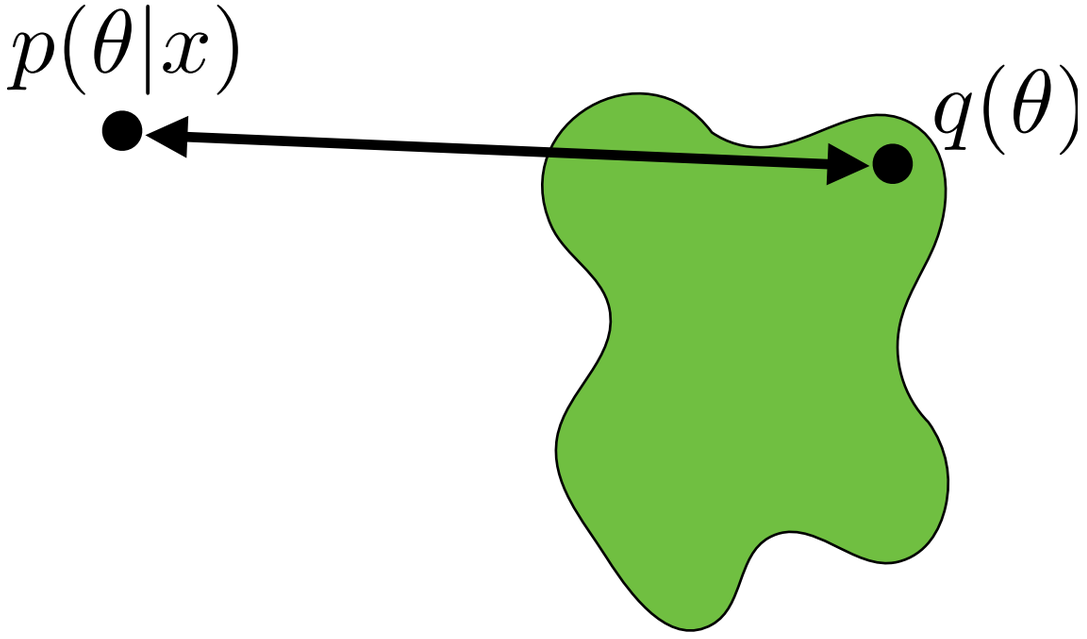

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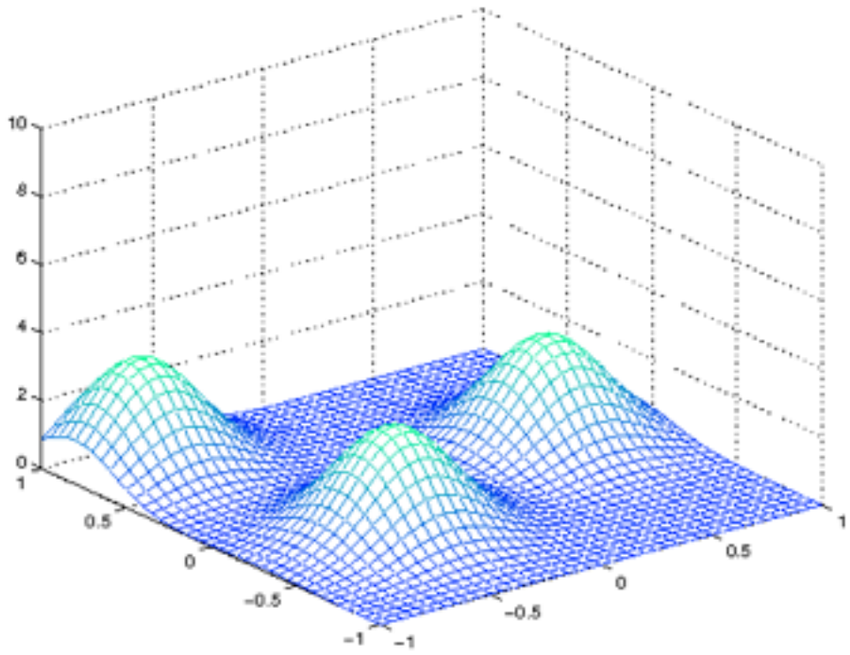

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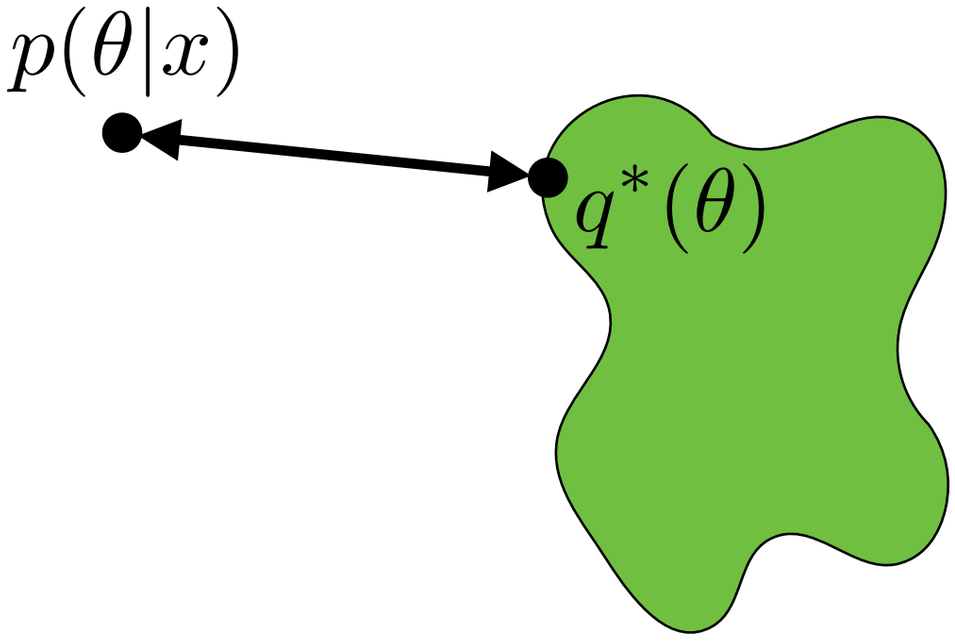

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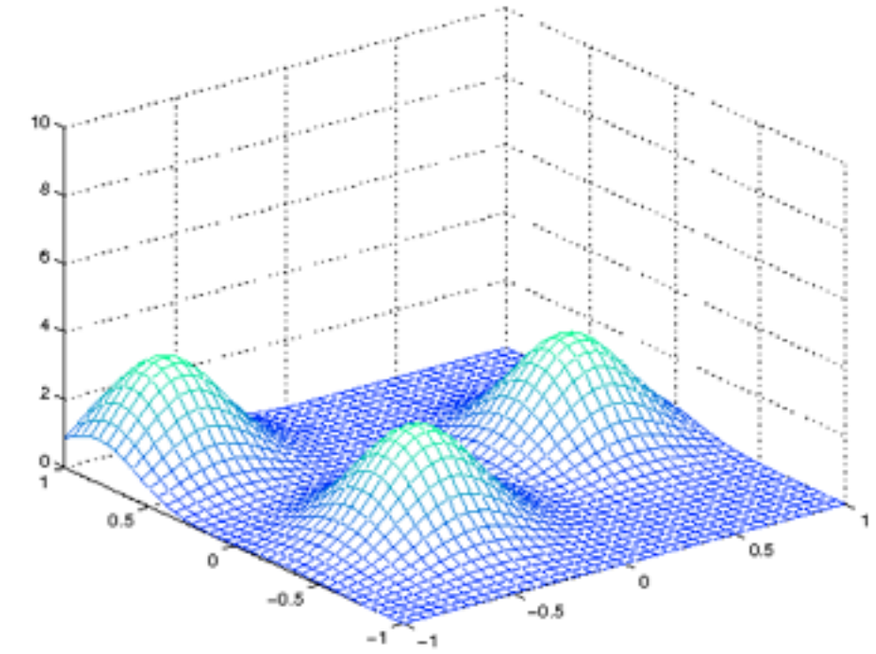

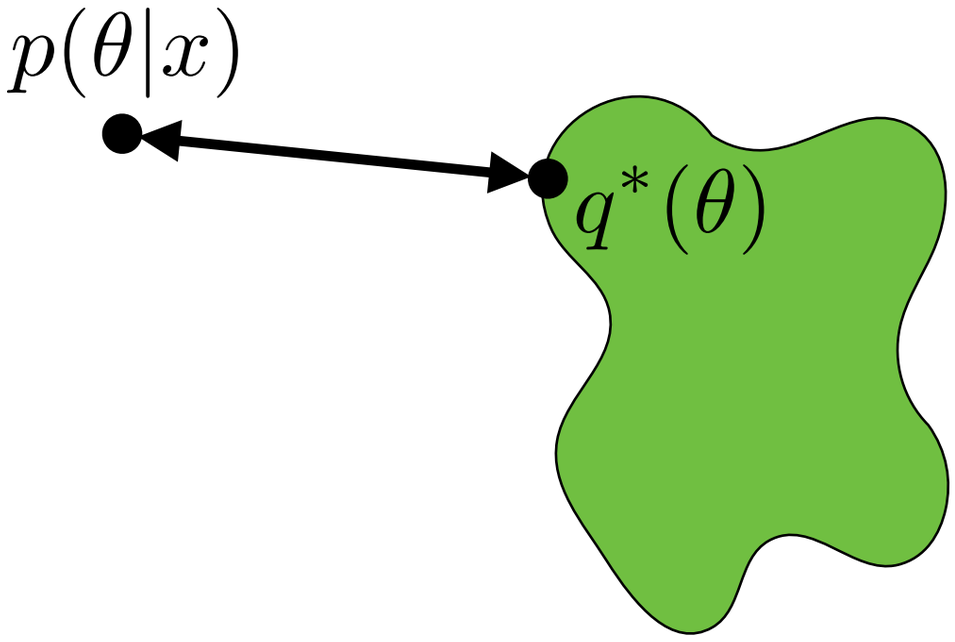

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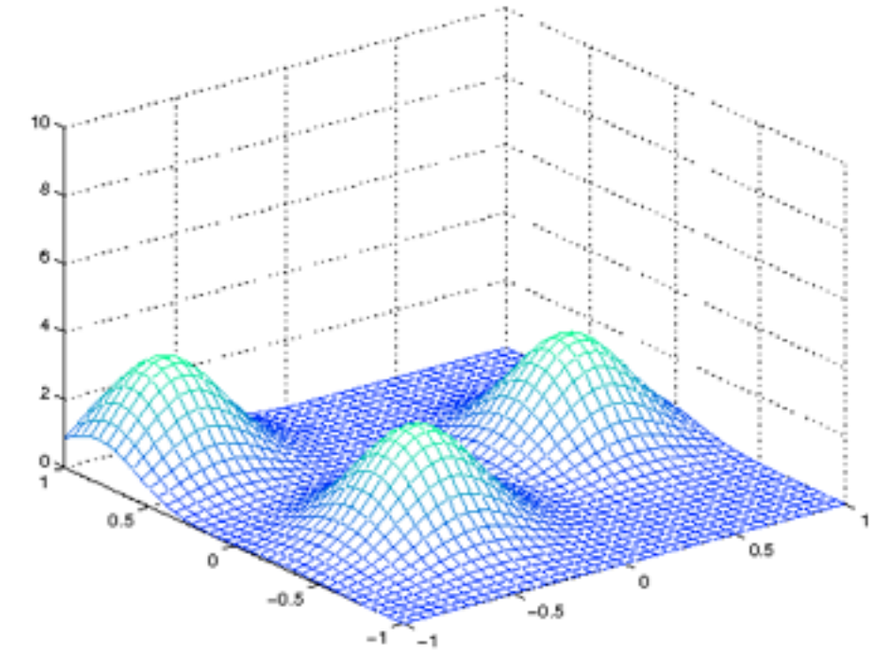

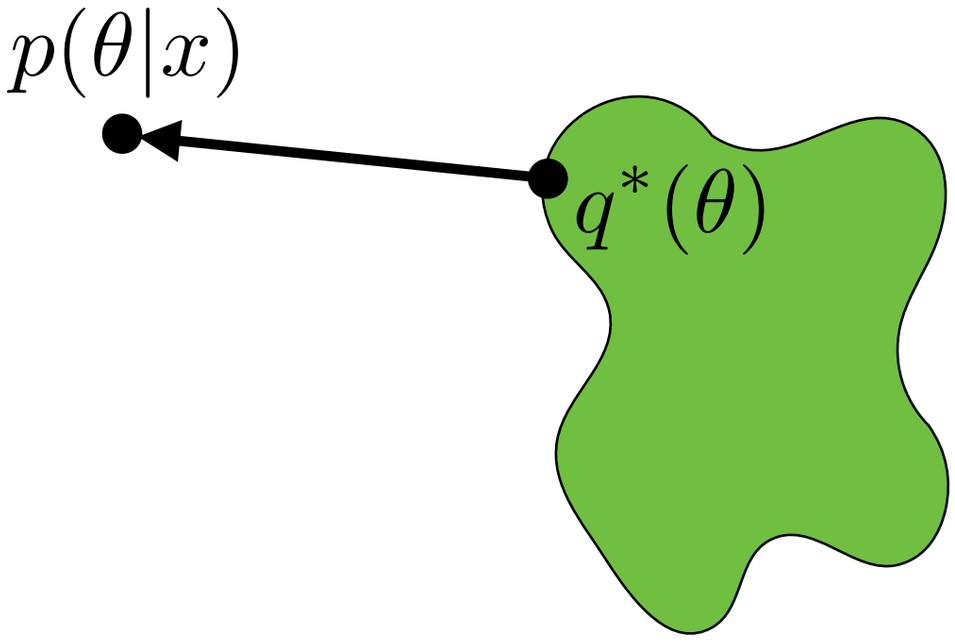

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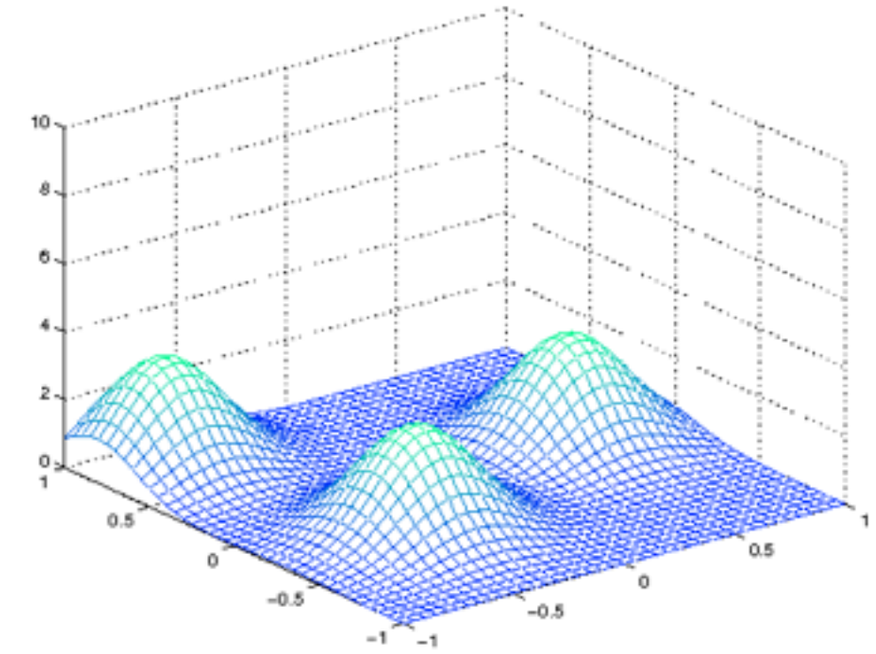

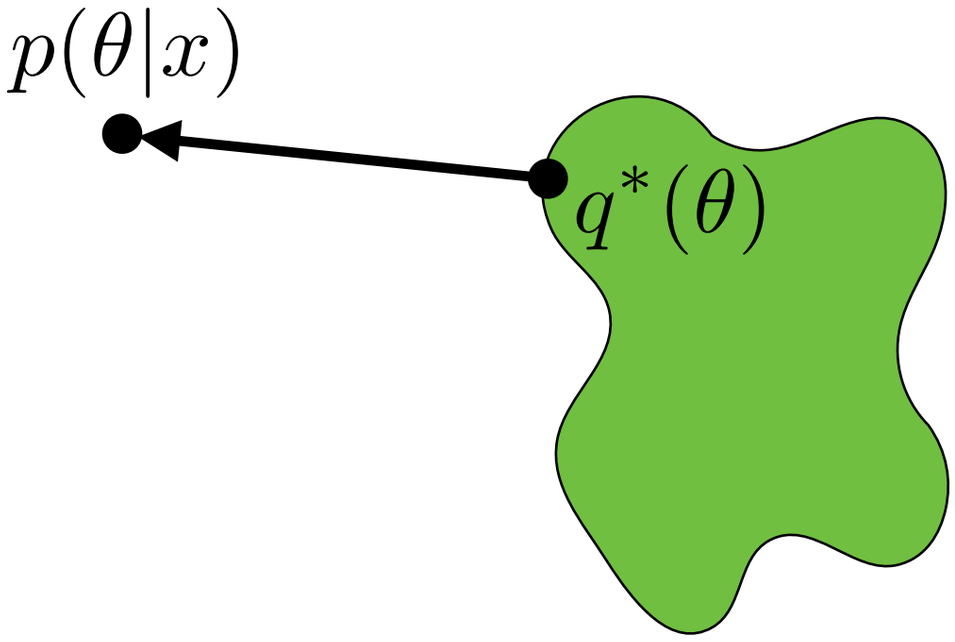

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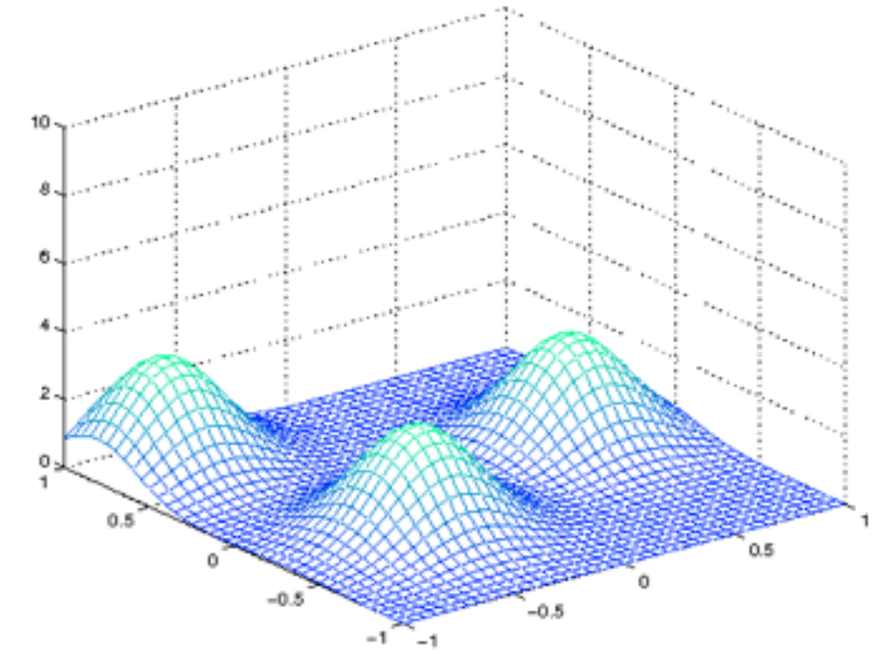

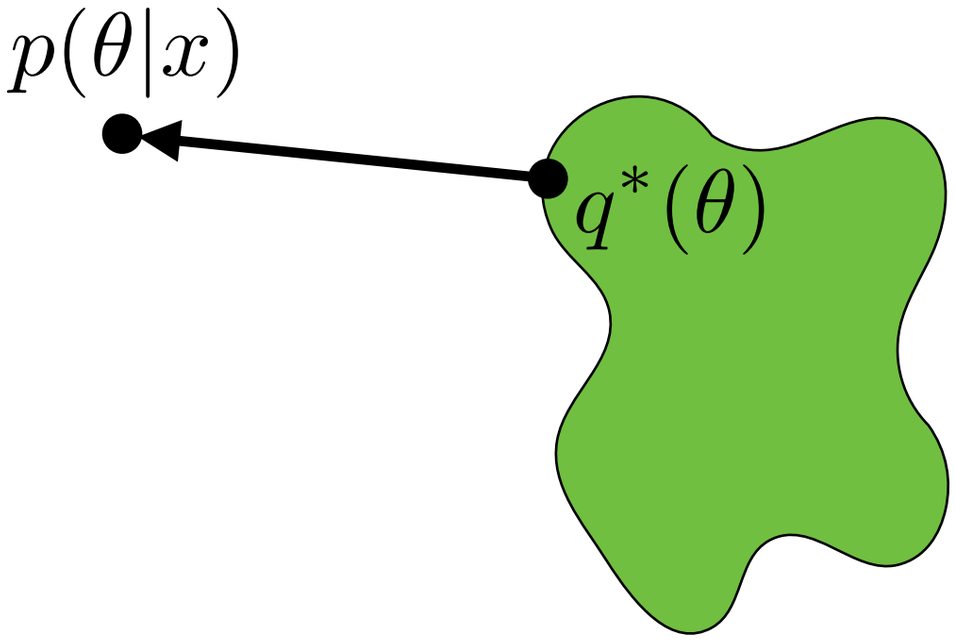

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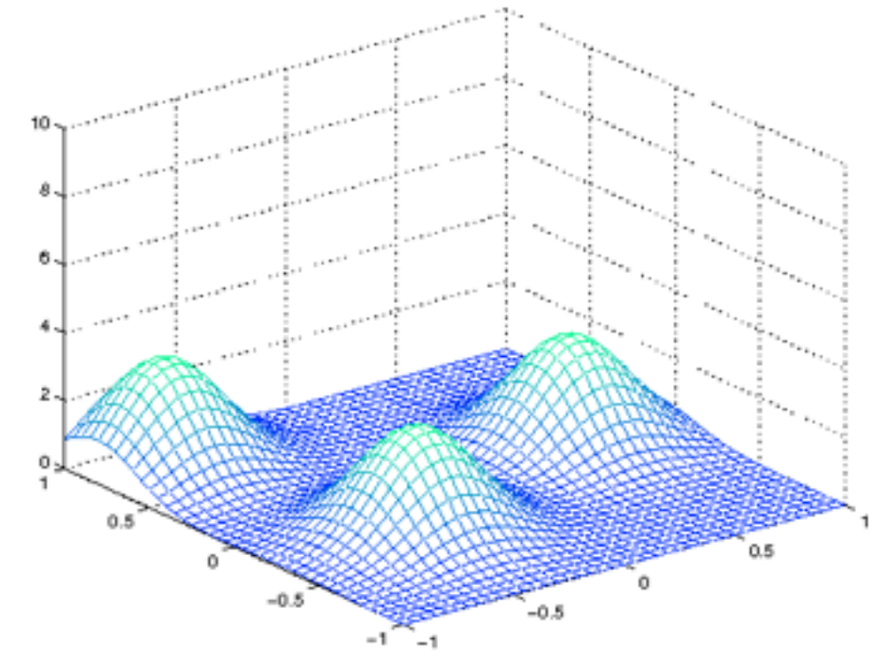

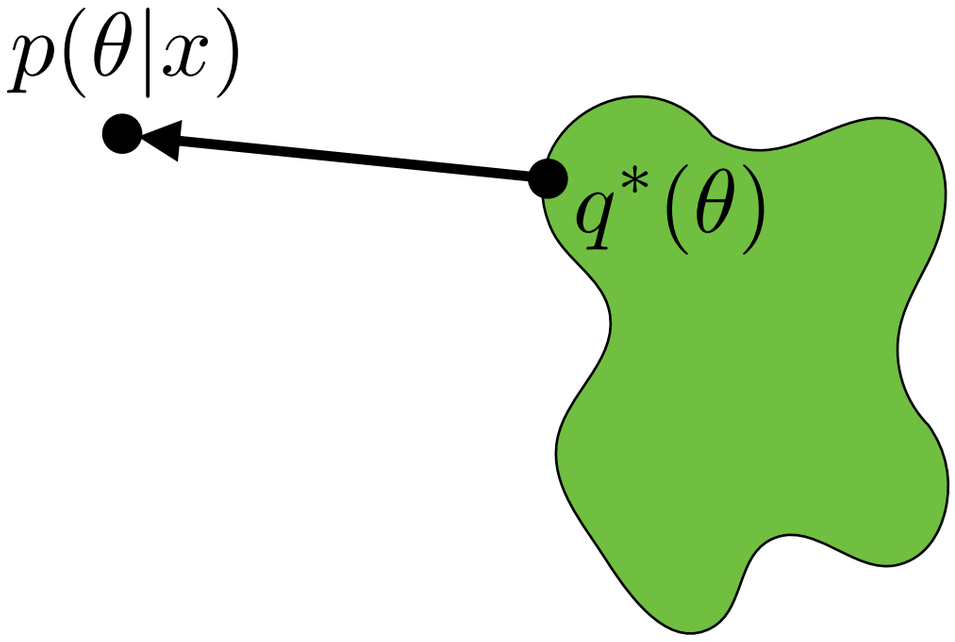

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[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

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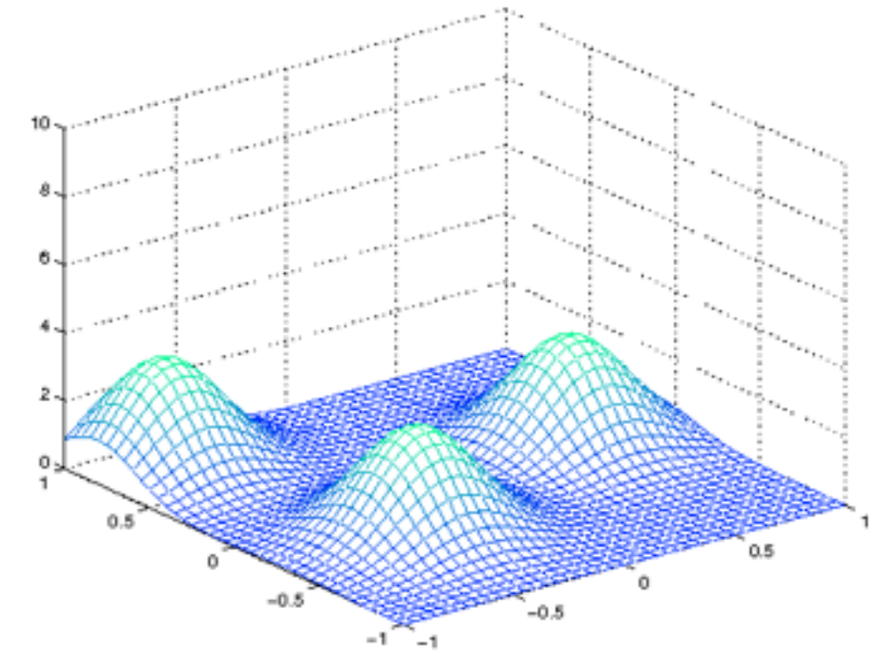

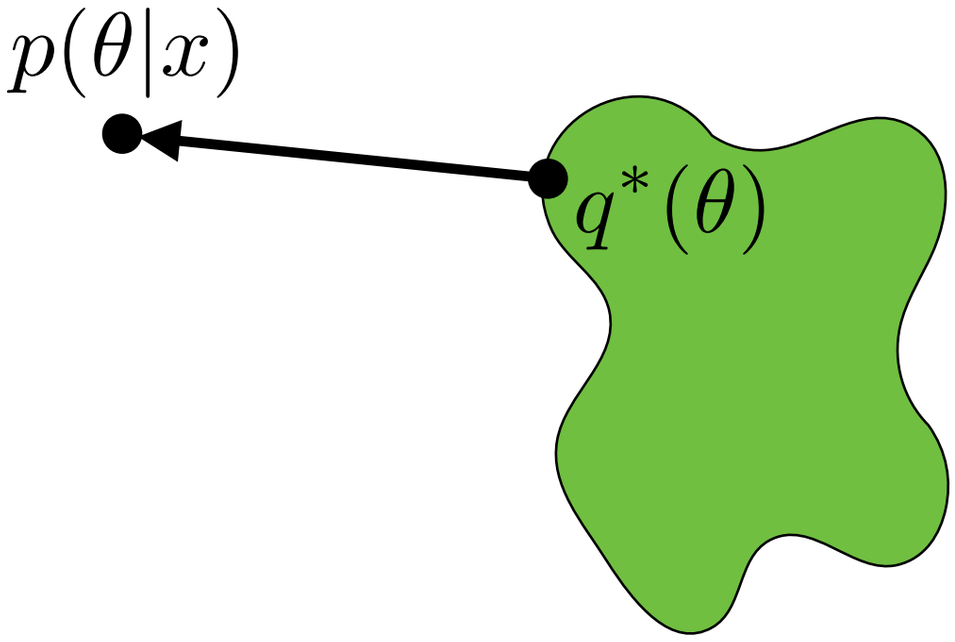

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[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

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- Posterior approximation trade-offs
  - Point estimates (MAD-Bayes)
  - Uncertainties
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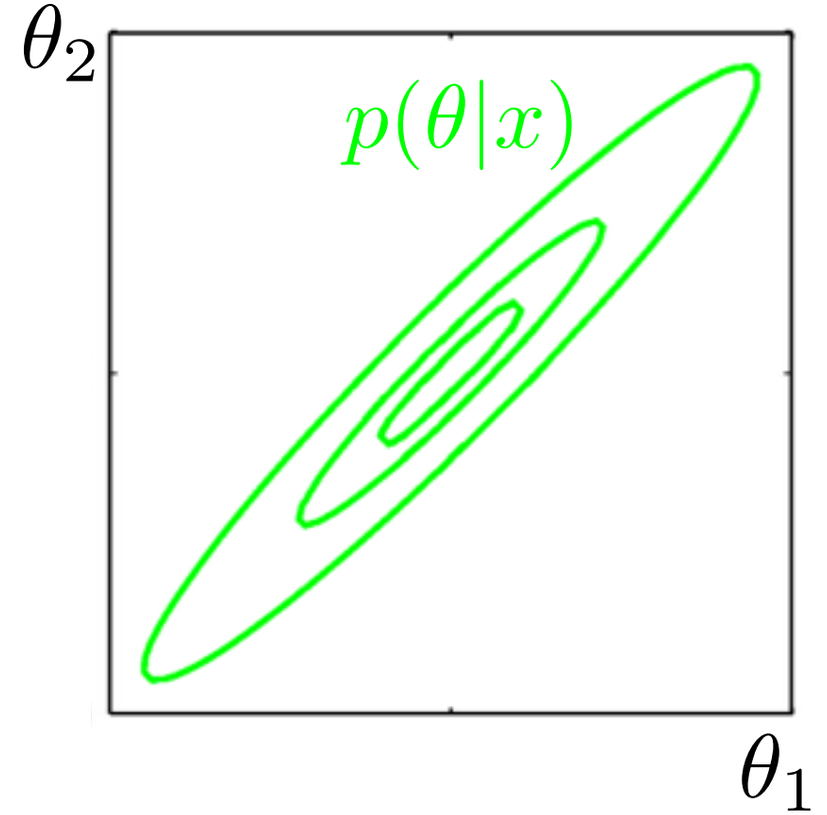

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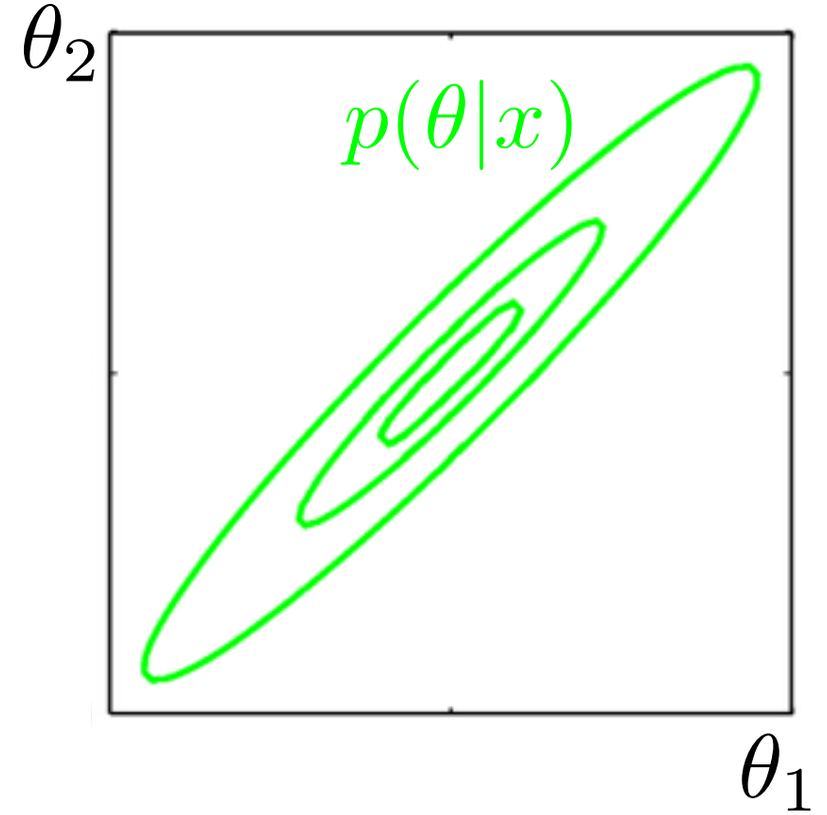

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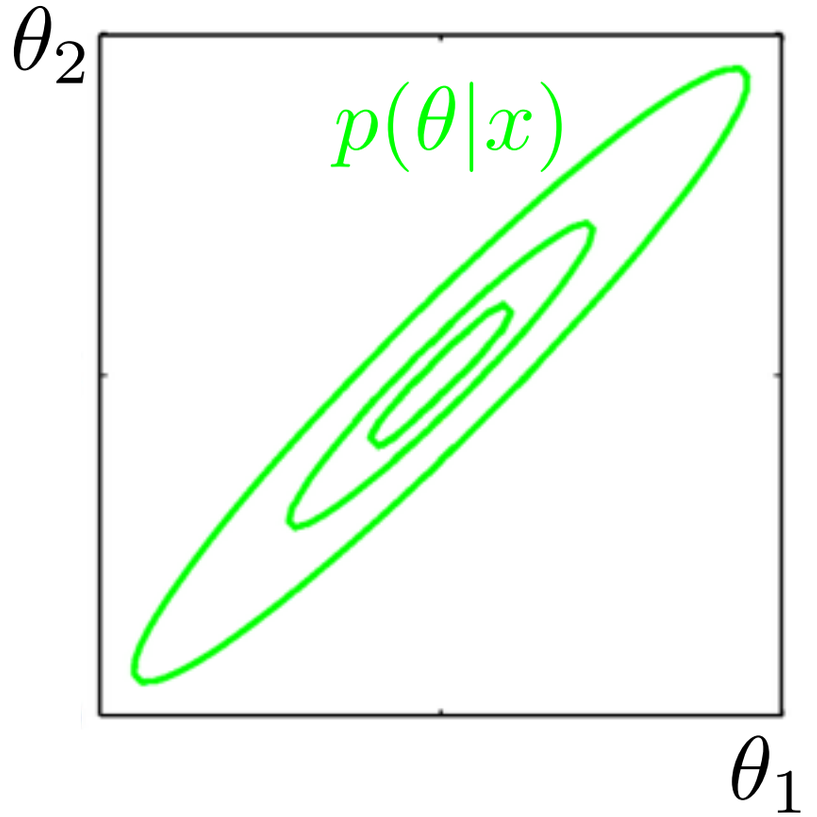

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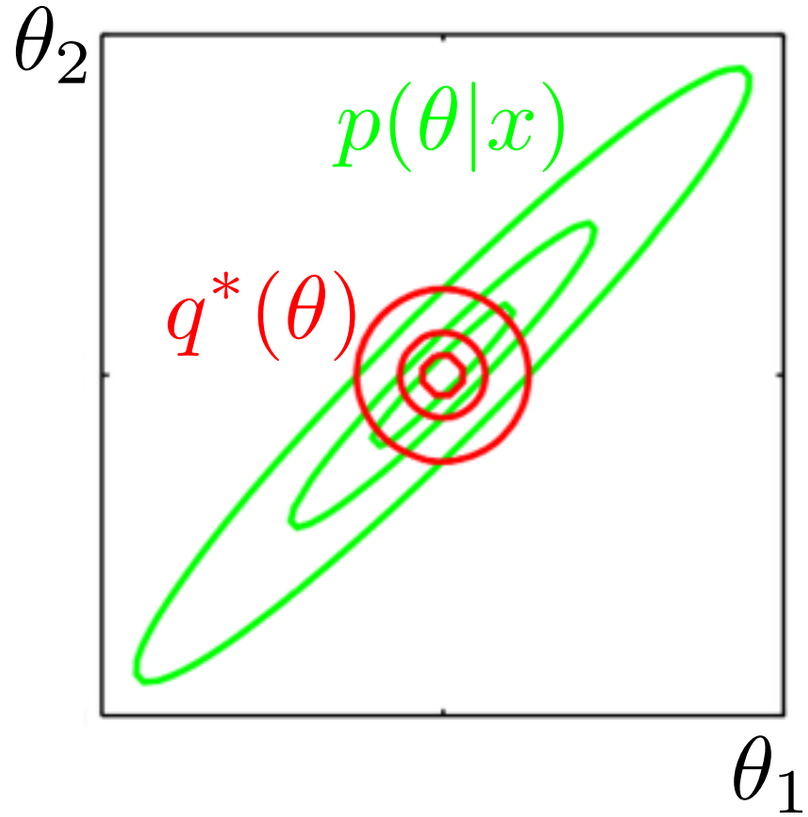

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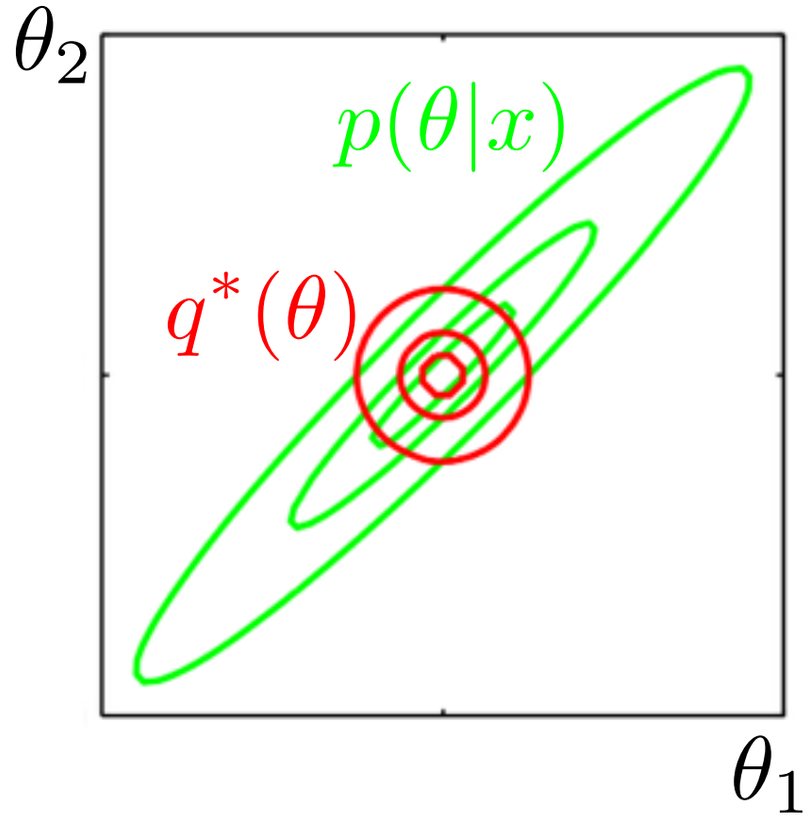

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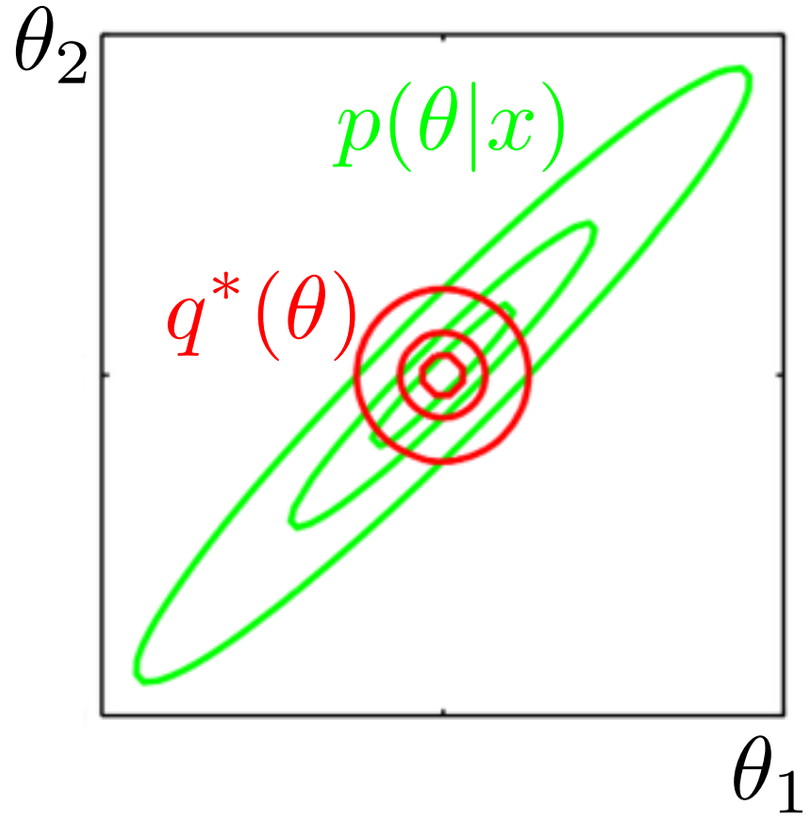

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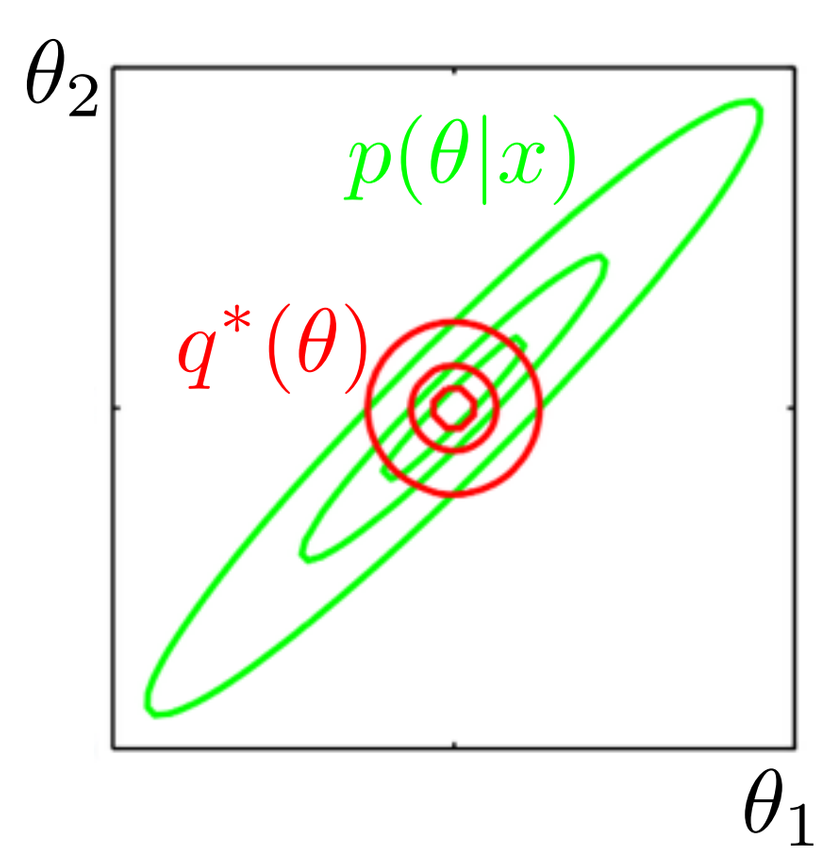

[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]

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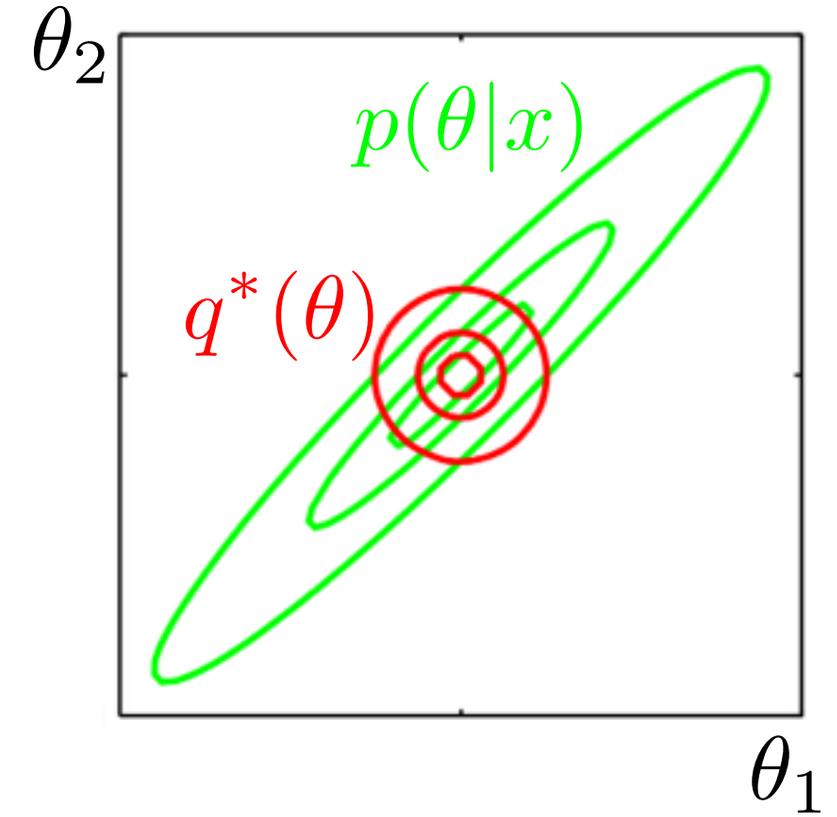

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[MacKay 2003; Bishop 2006; Wang, Titterington 2004; Turner, Sahani 2011]
[Fosdick 2013; Dunson 2014; Bardenet, Doucet, Holmes 2015]

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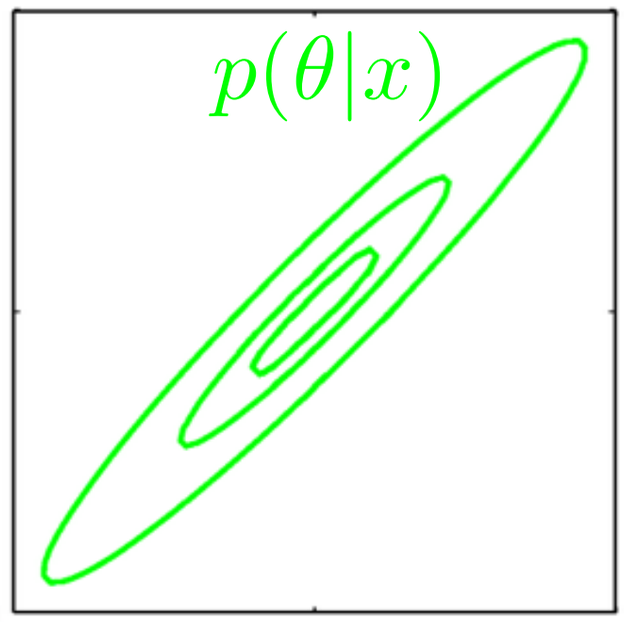

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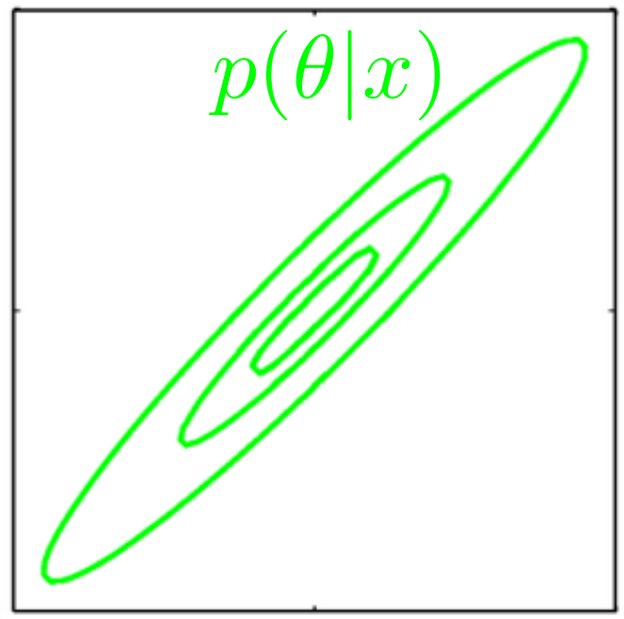

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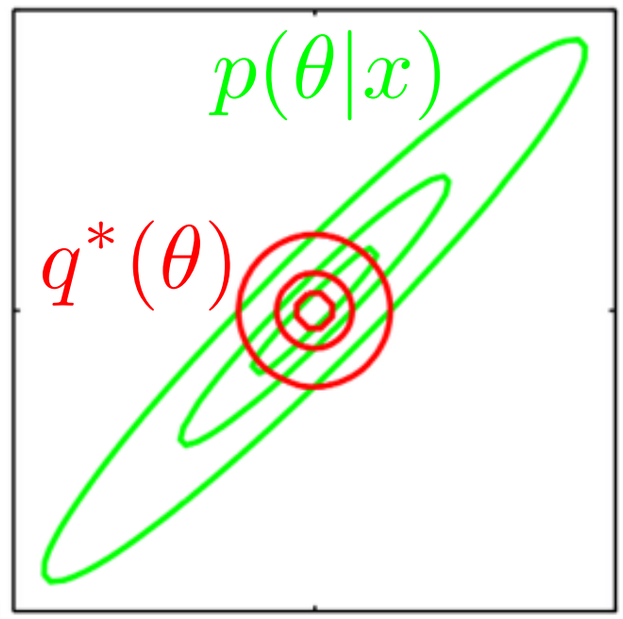

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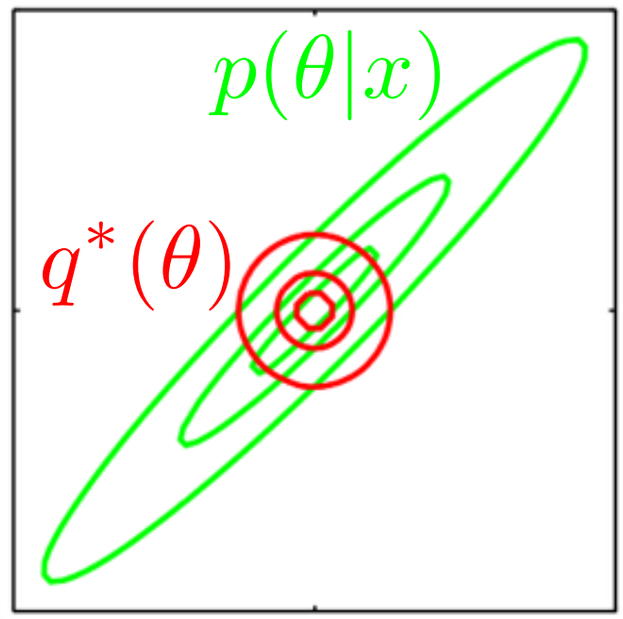

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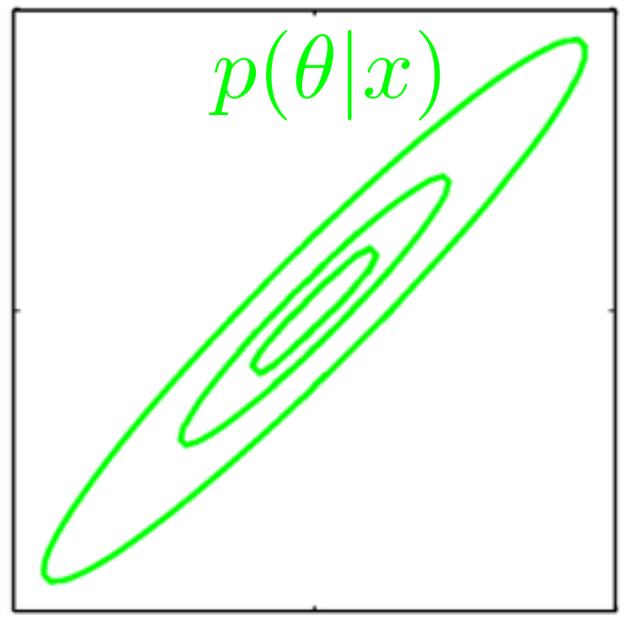

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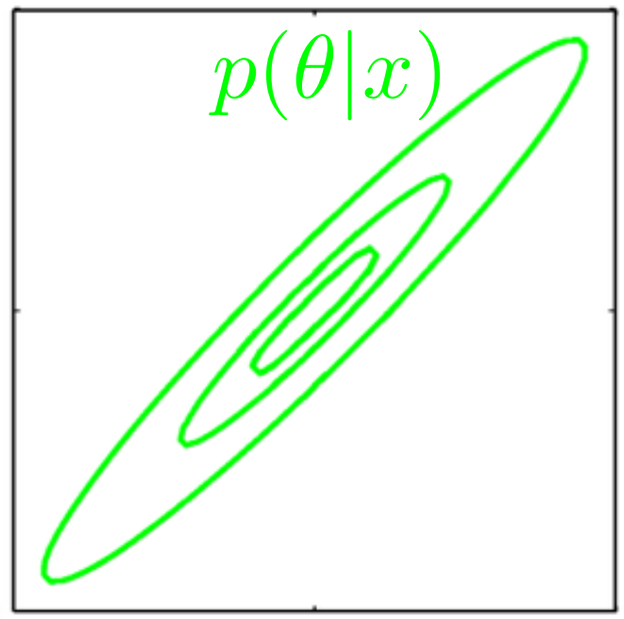

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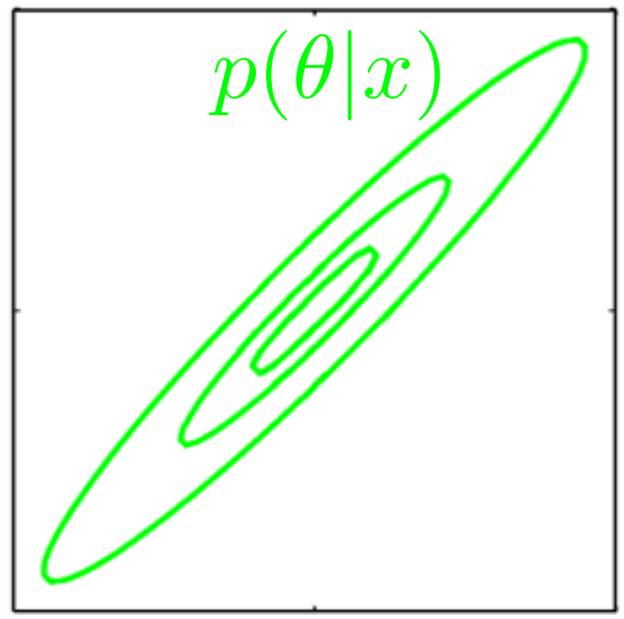

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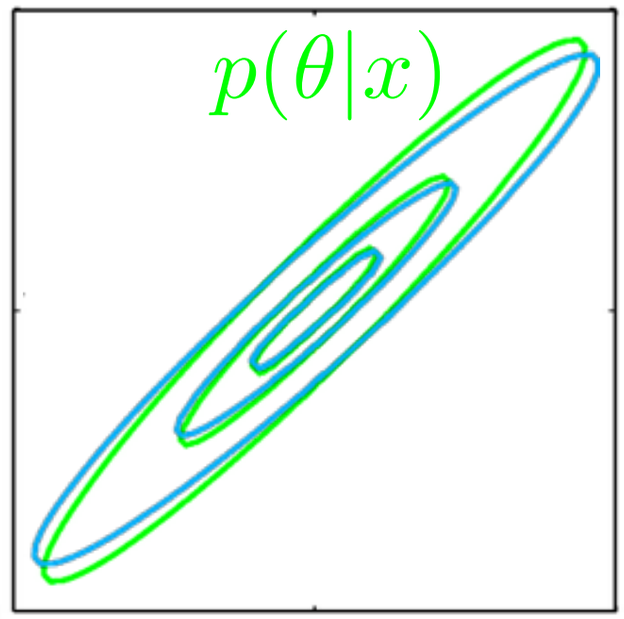

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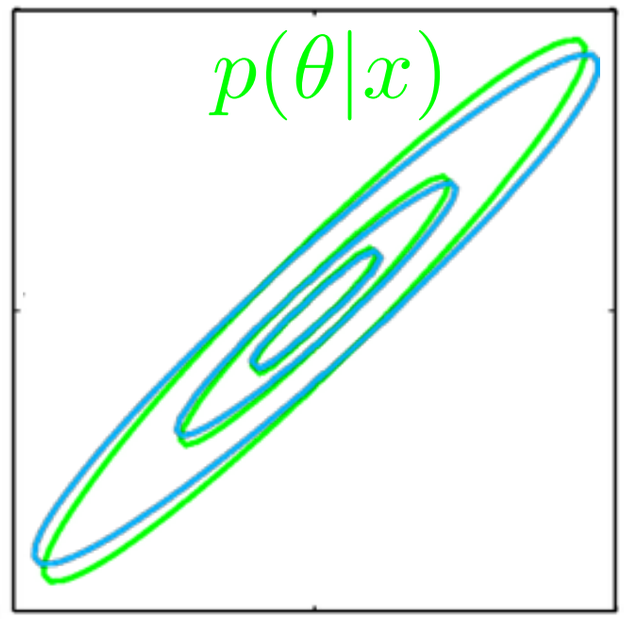

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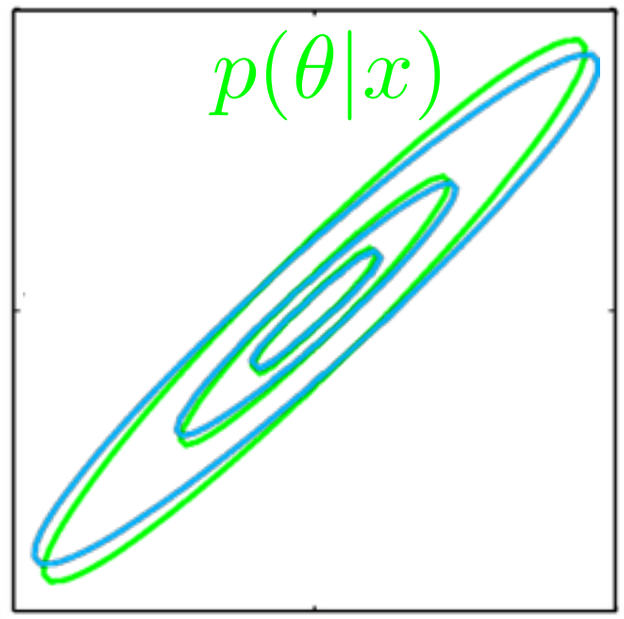

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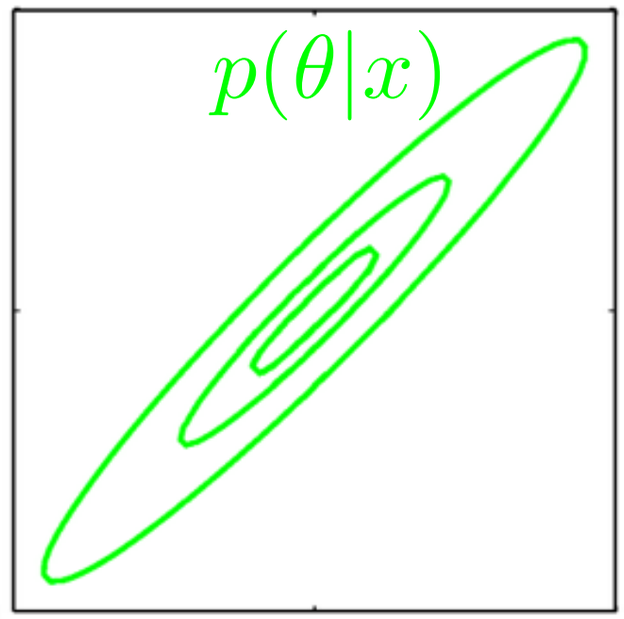

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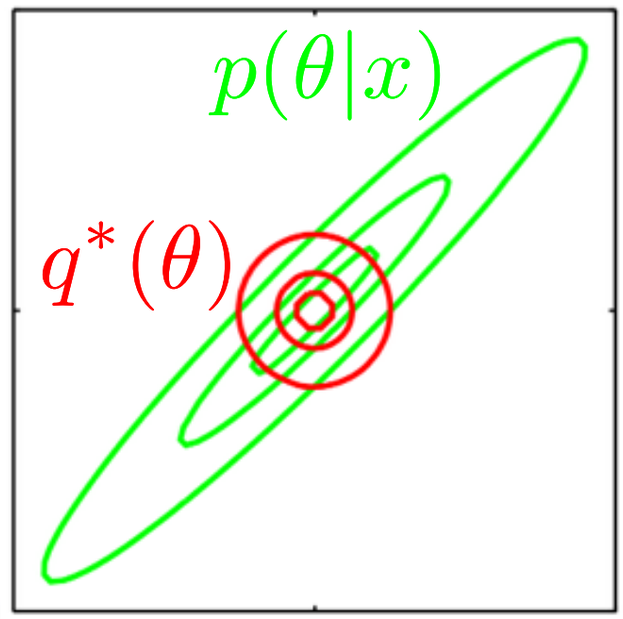

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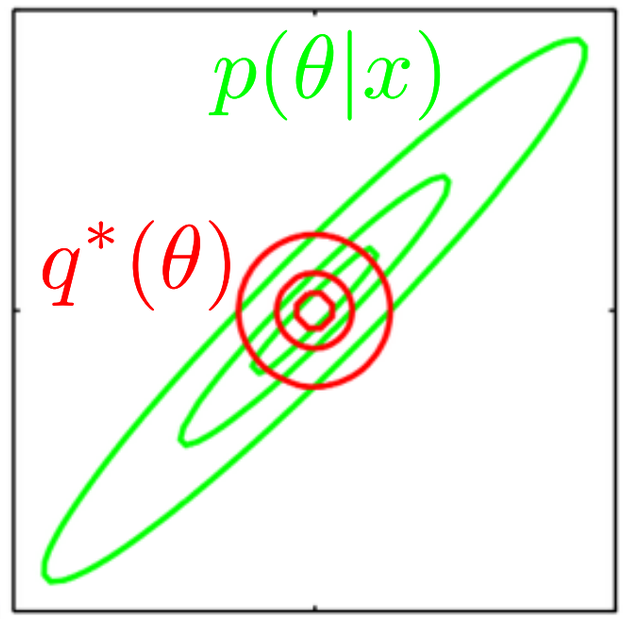

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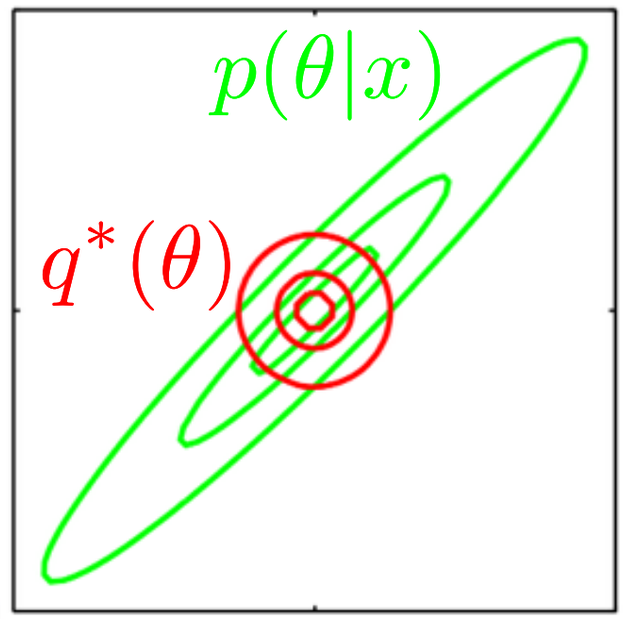

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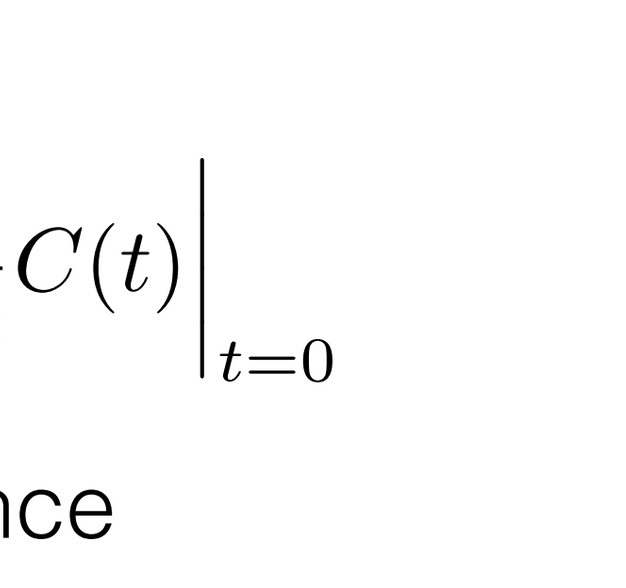

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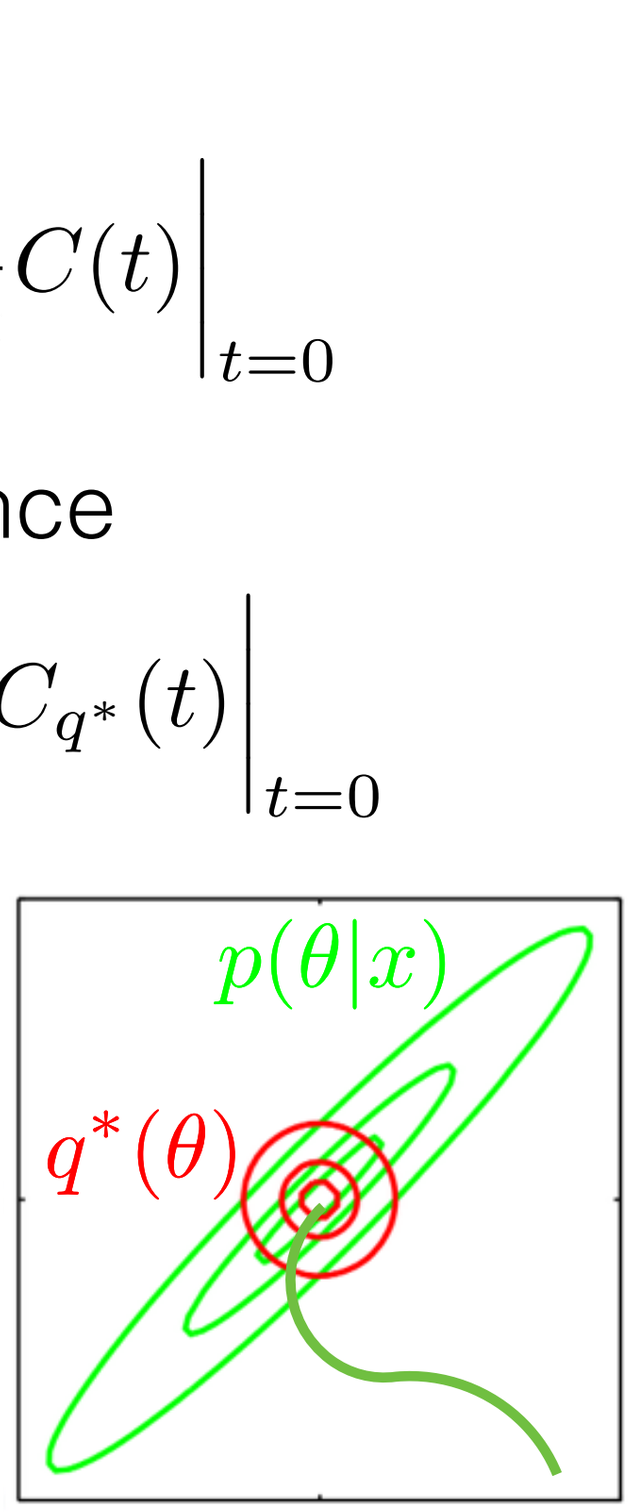

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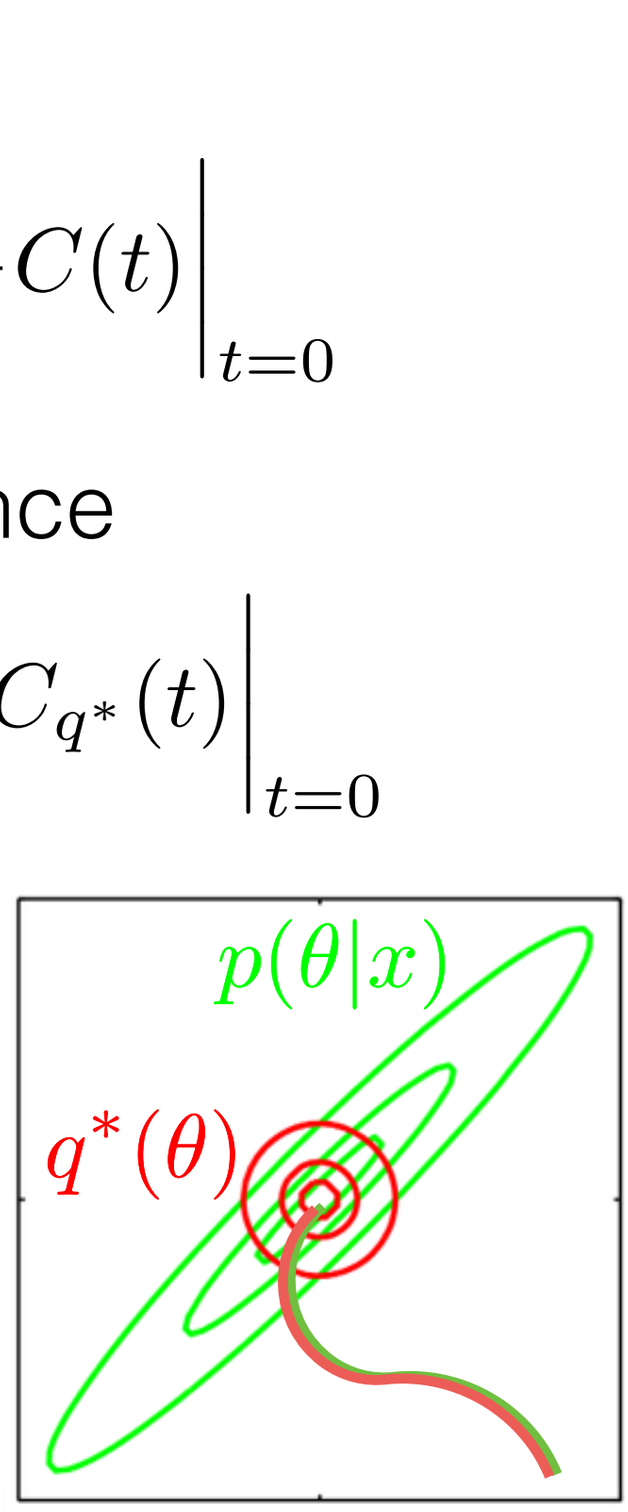

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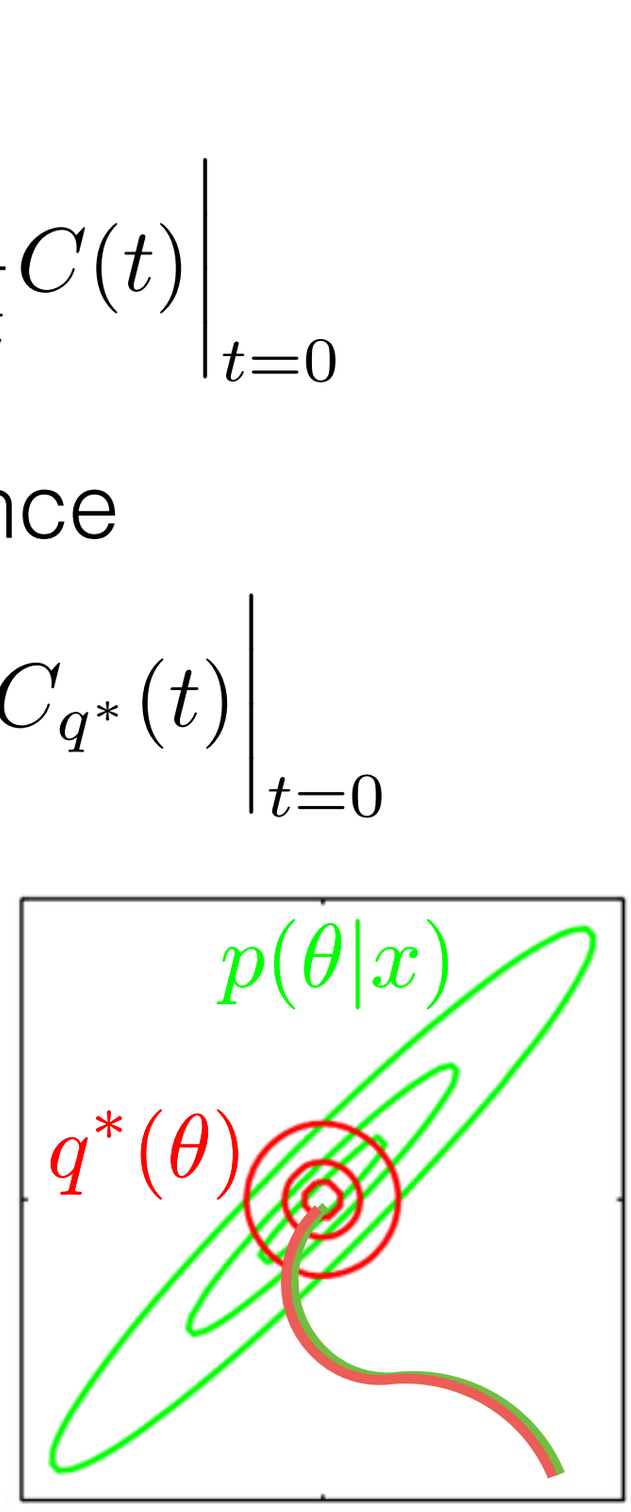

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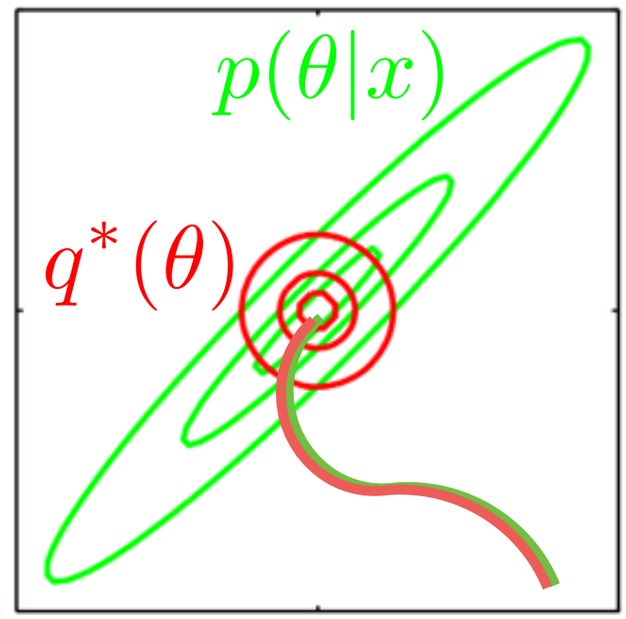

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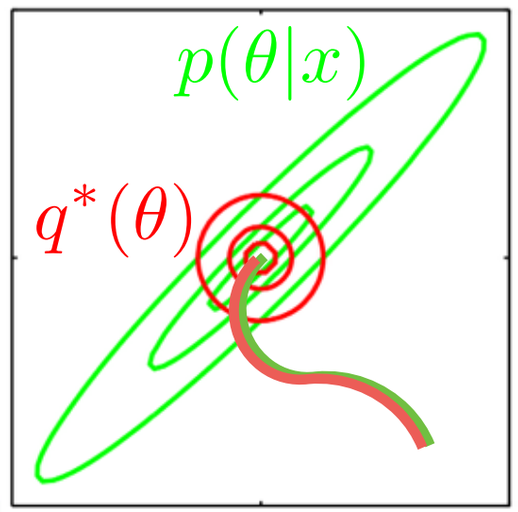

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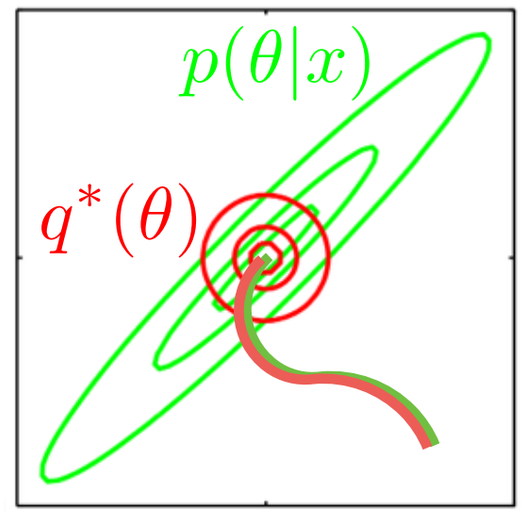

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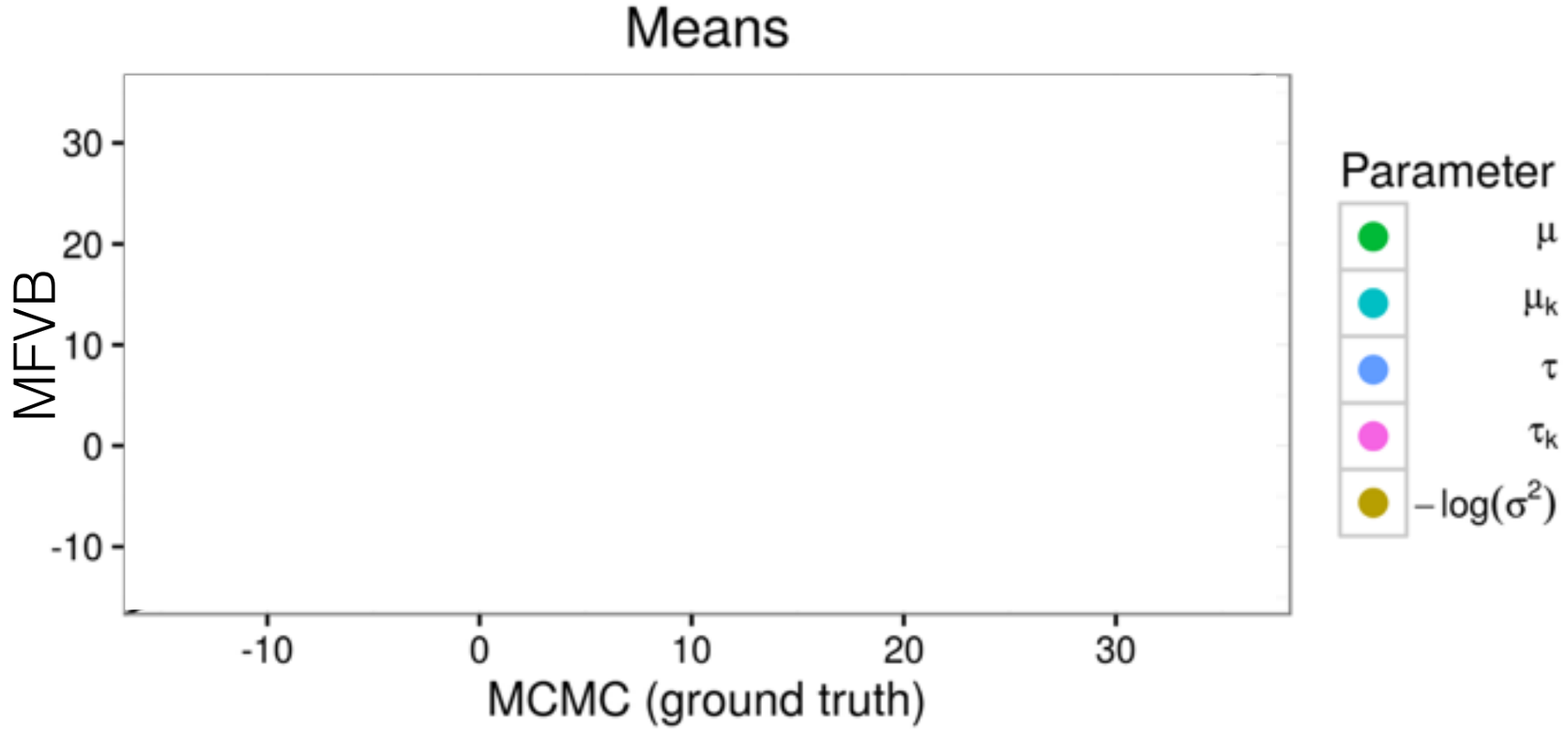

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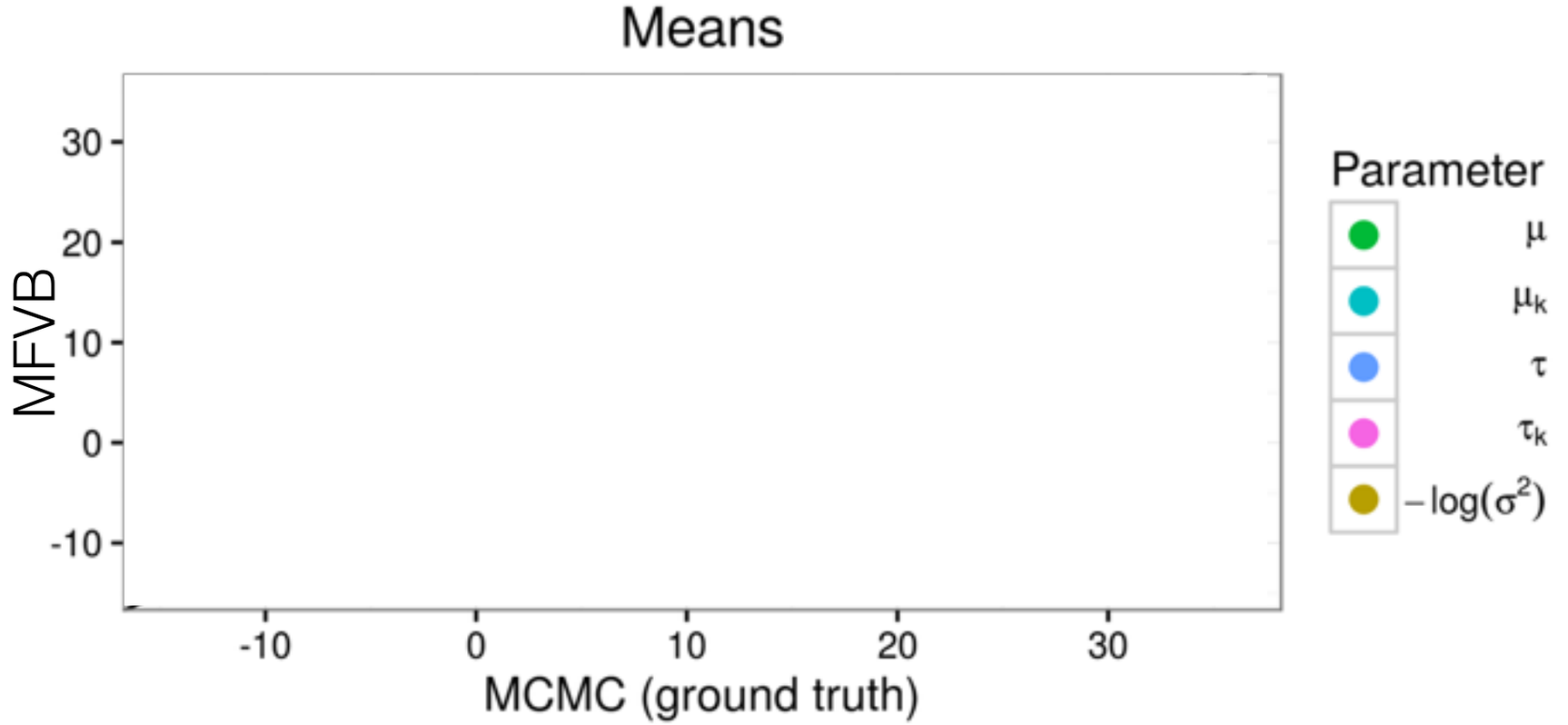

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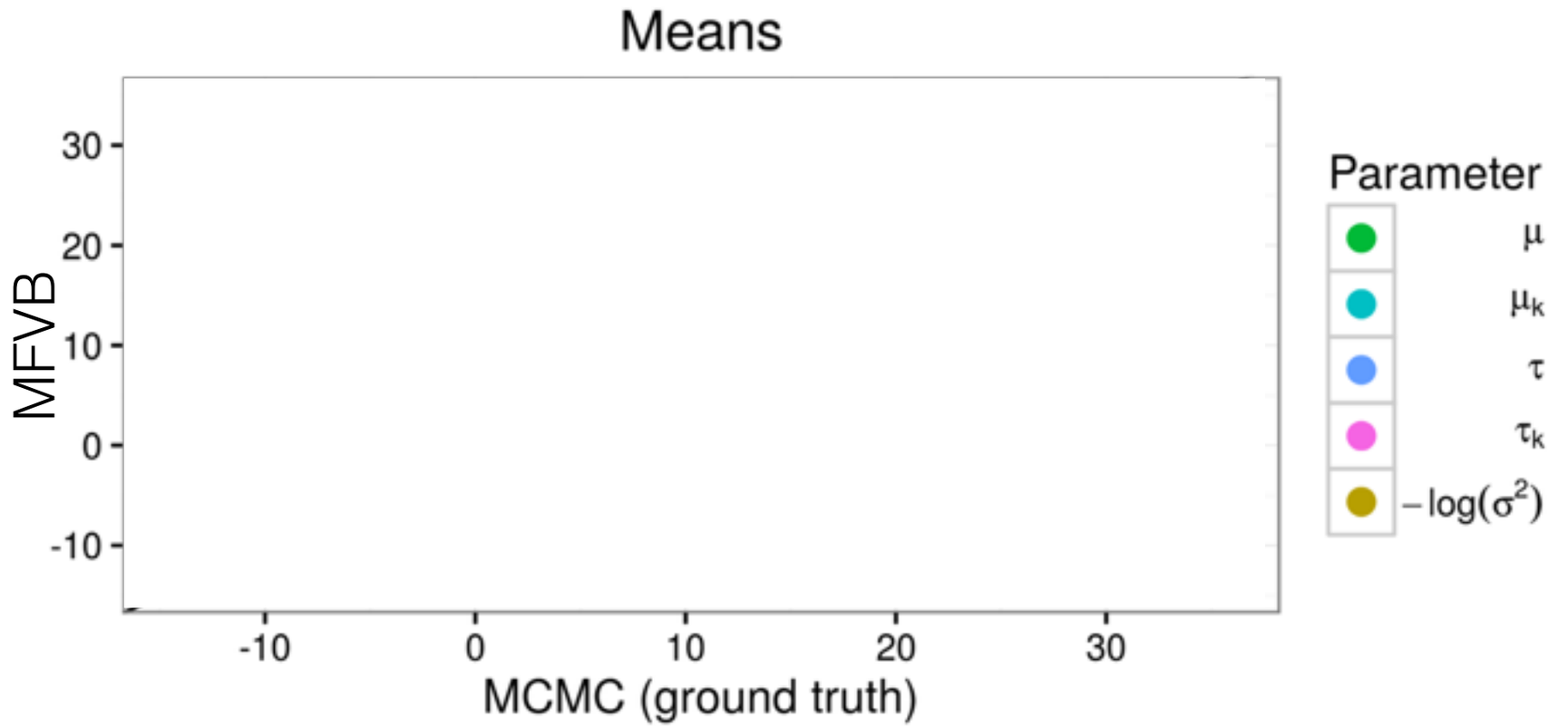

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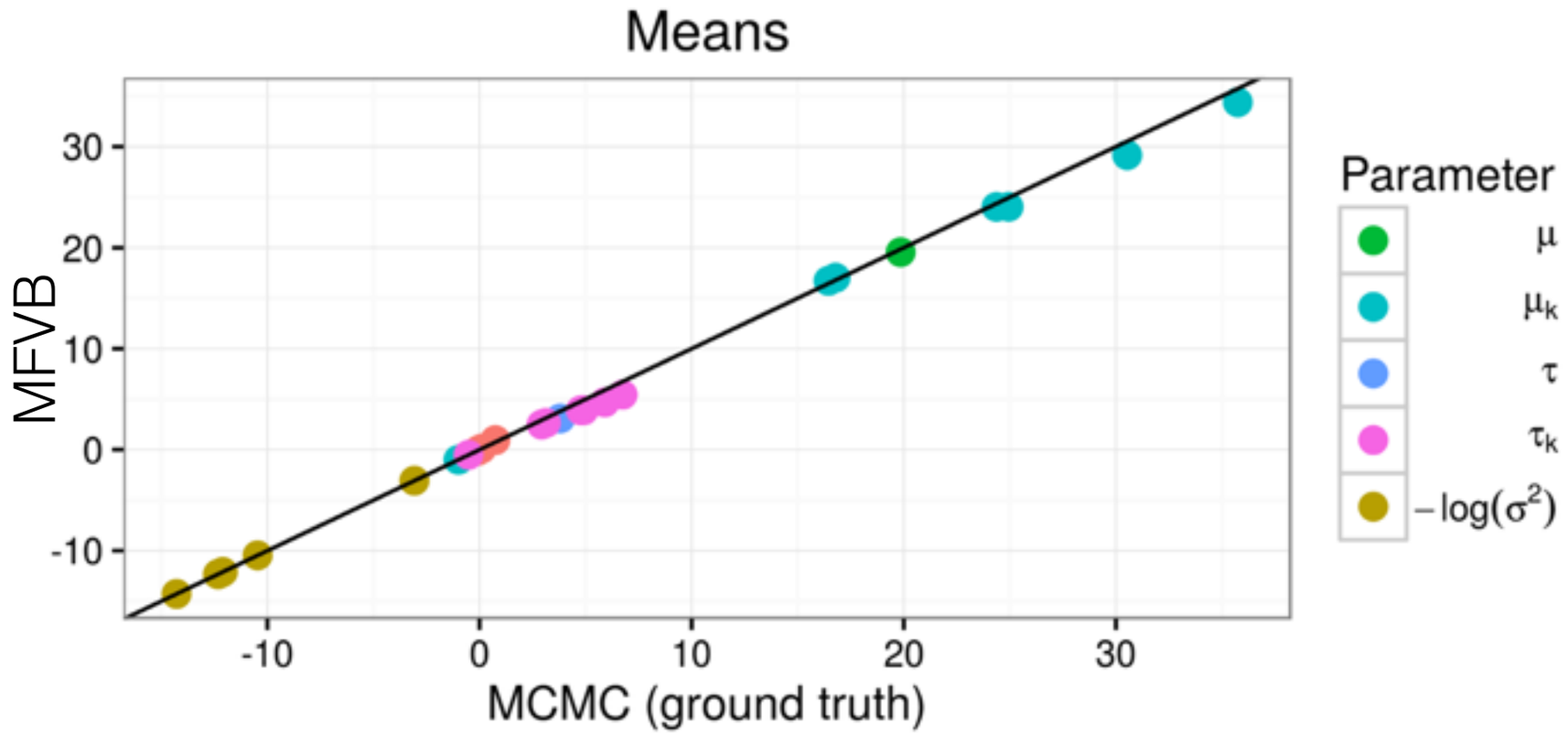

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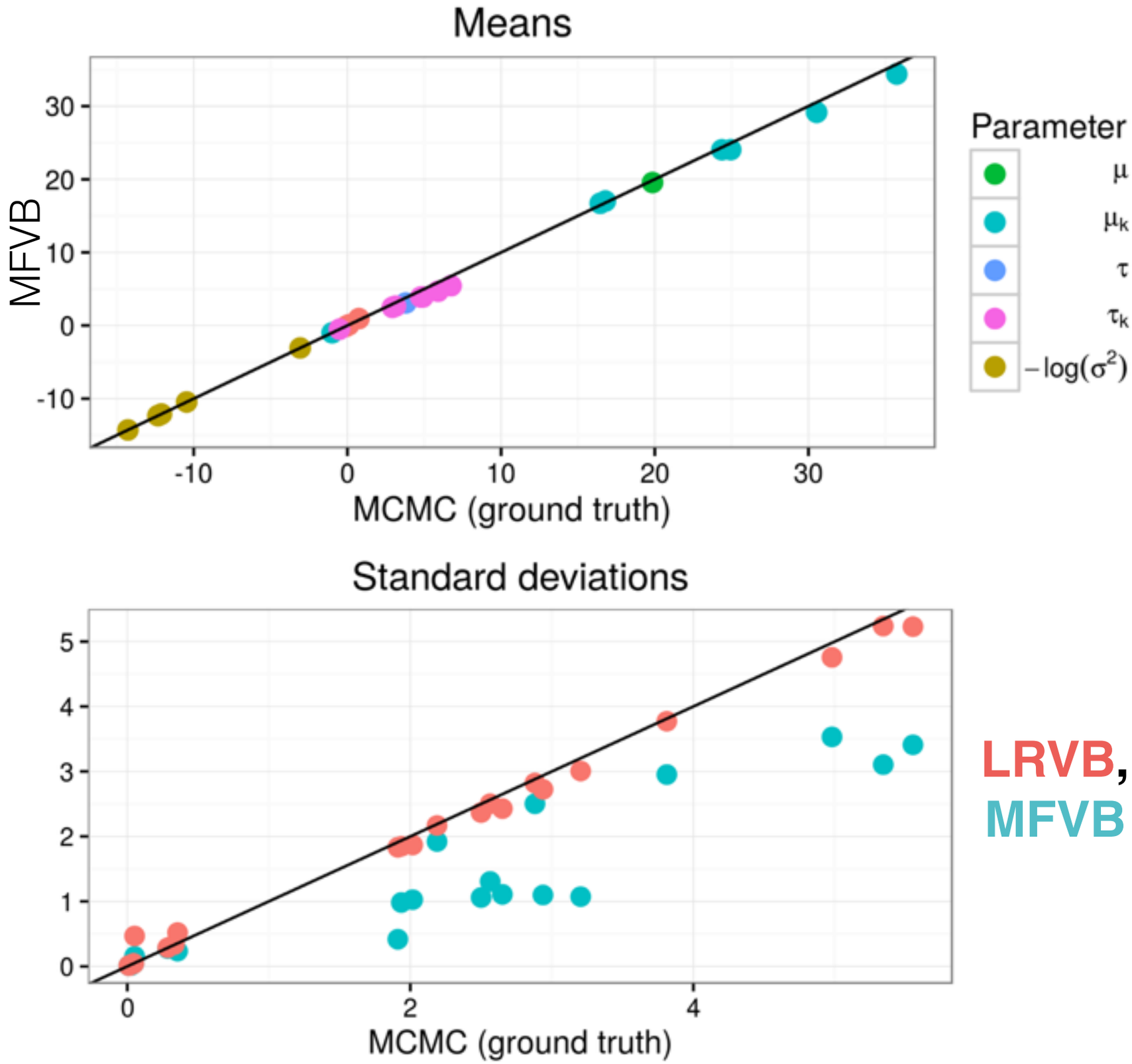

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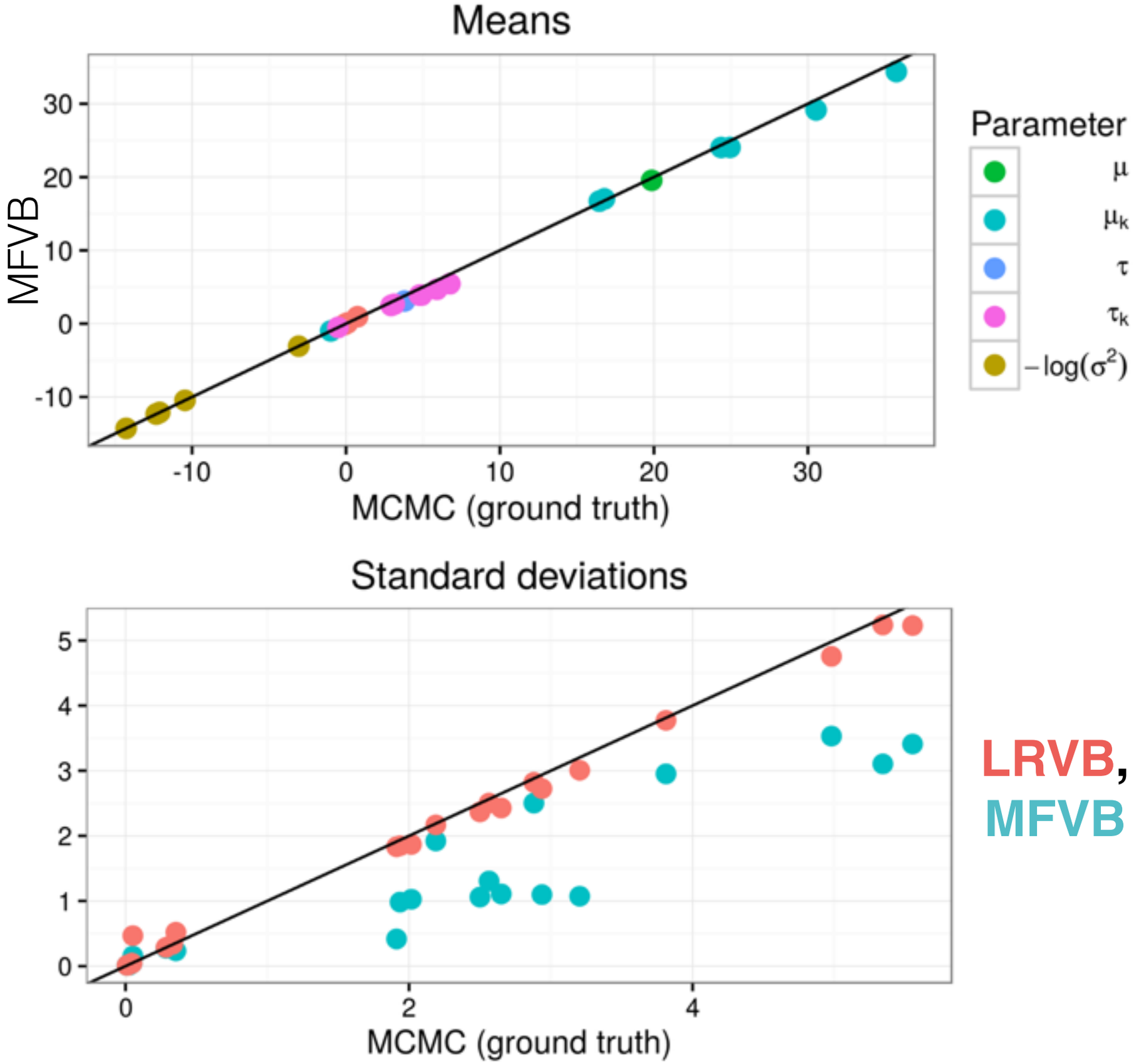

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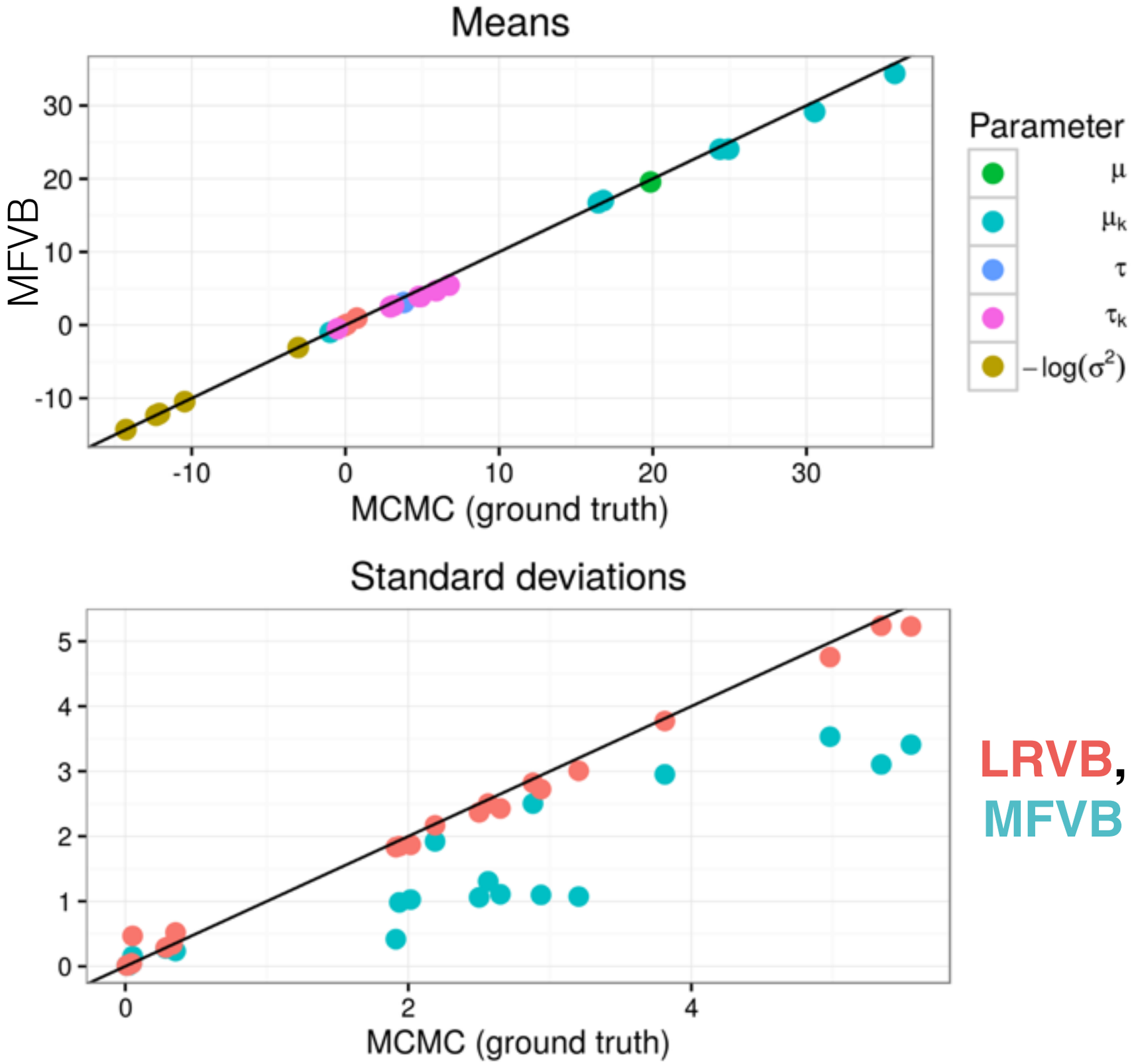

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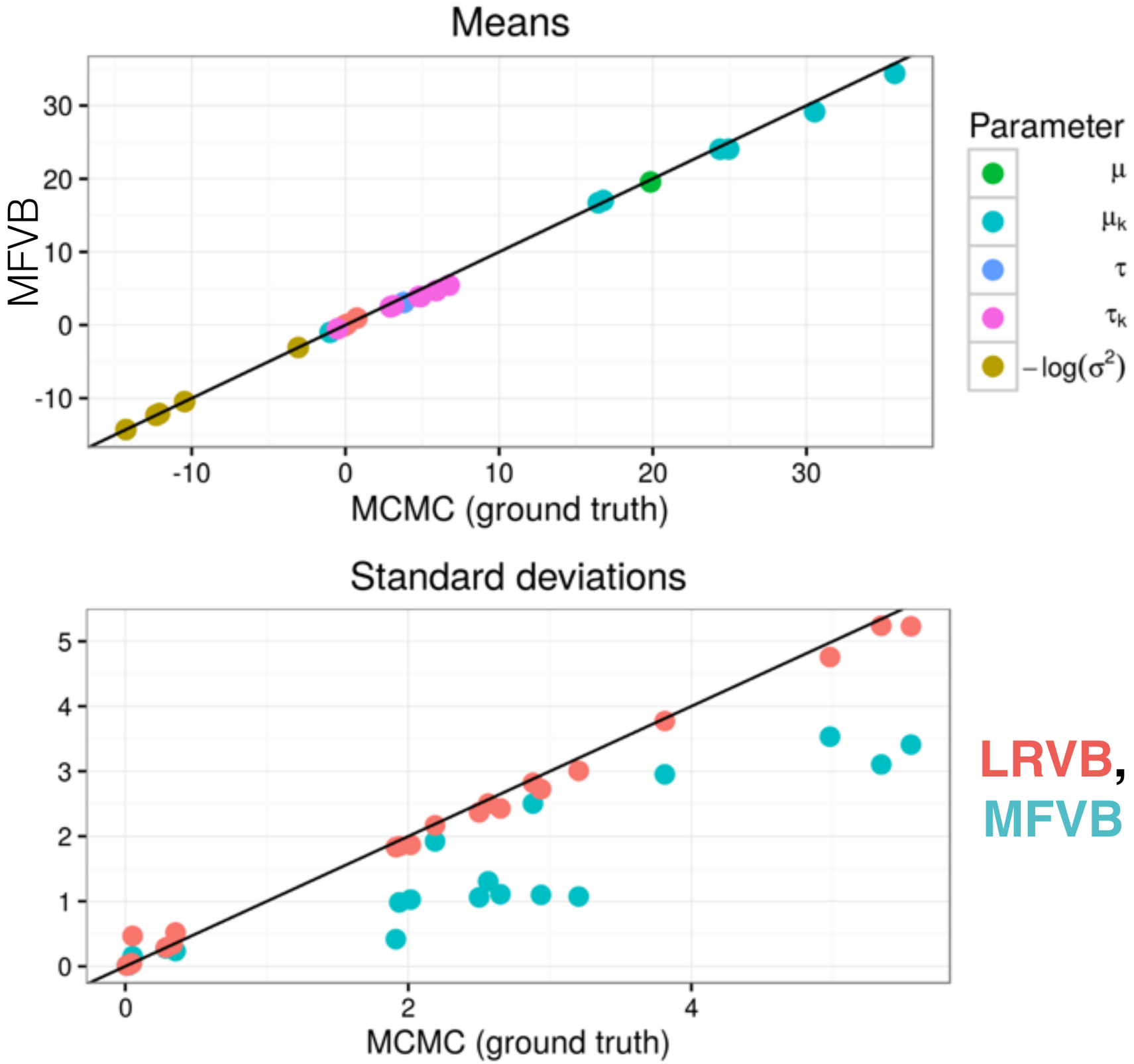
- Mean is 1.68 std dev from 0



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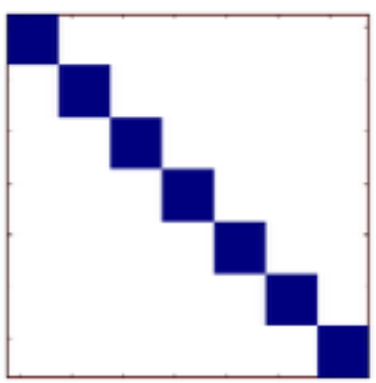
V 

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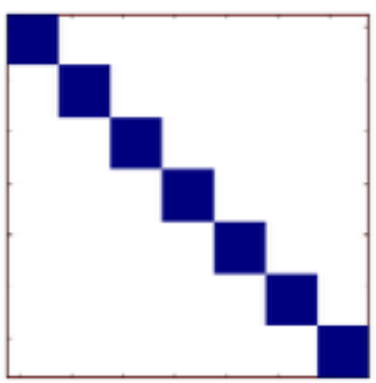
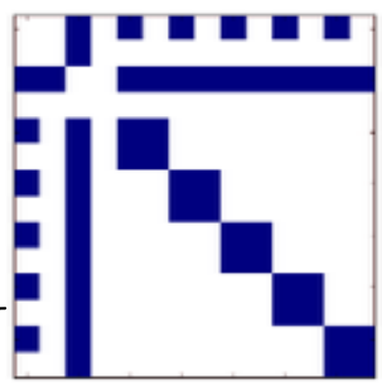
V 

H 

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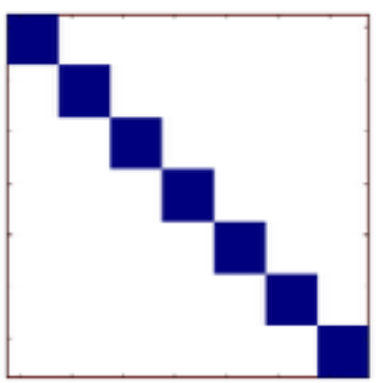
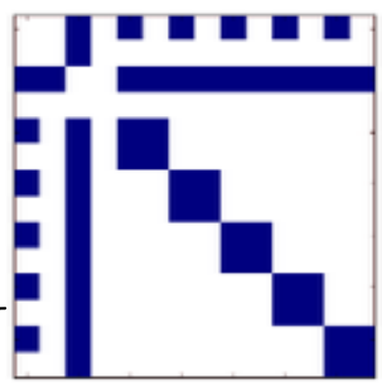
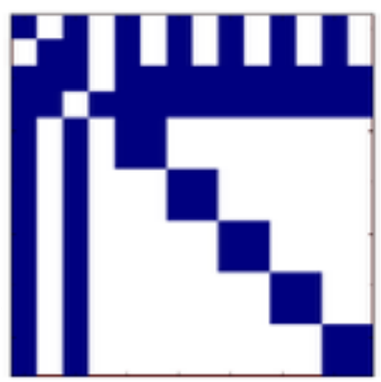
V 

H 

$I - VH$ 

Roadmap

- Posterior approximation trade-offs
  - Point estimates (MAD-Bayes, variational Bayes)
  - Uncertainties
- Robustness trade-offs
  - Local robustness quantification
- Theoretical guarantees on quality

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Robustness quantification

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$$p(\theta|x)$$

$$\propto_\theta p(x|\theta)p(\theta)$$

Robustness quantification

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$$p(\theta|x,\alpha)$$

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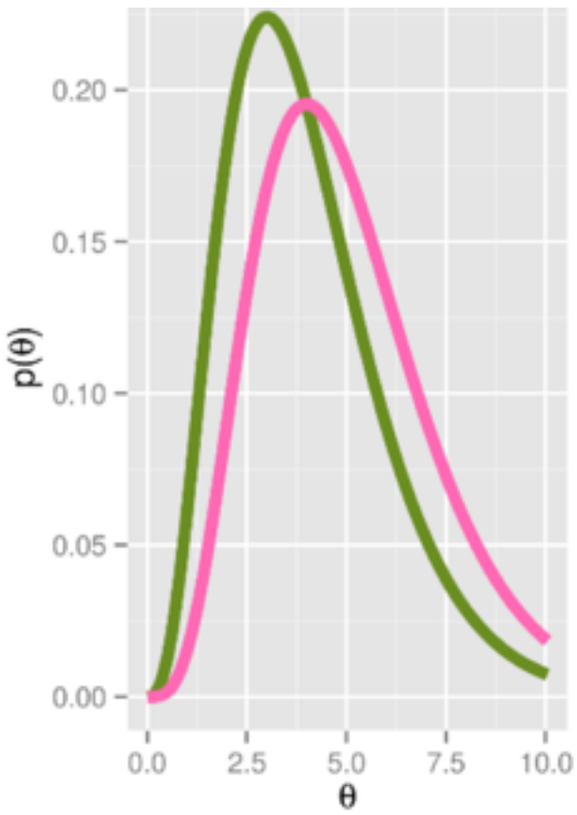

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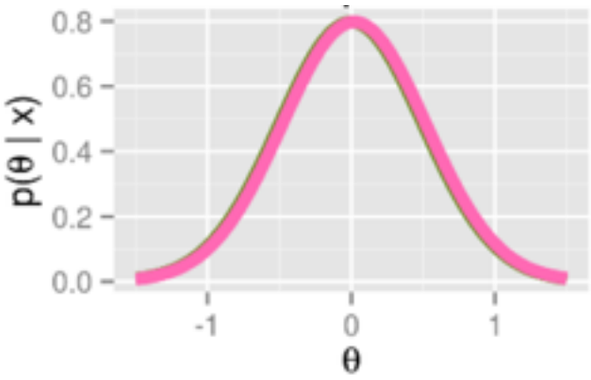

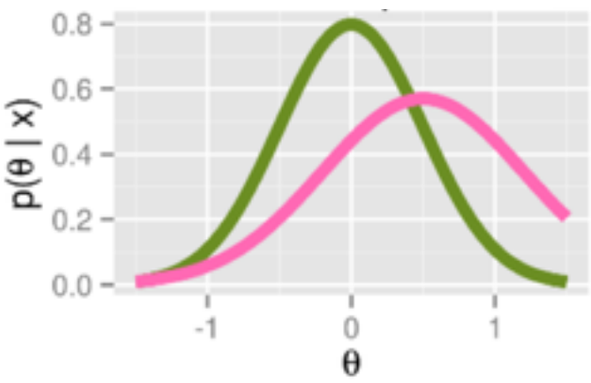

Robustness quantification

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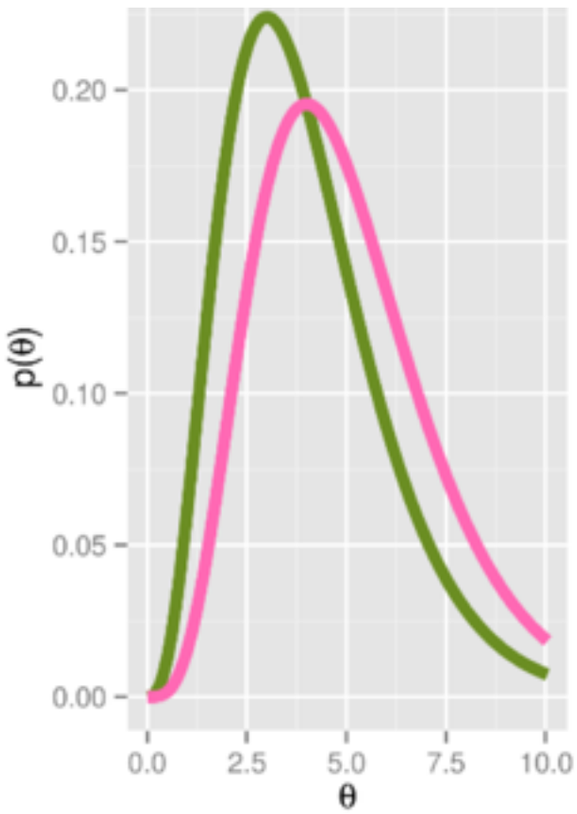

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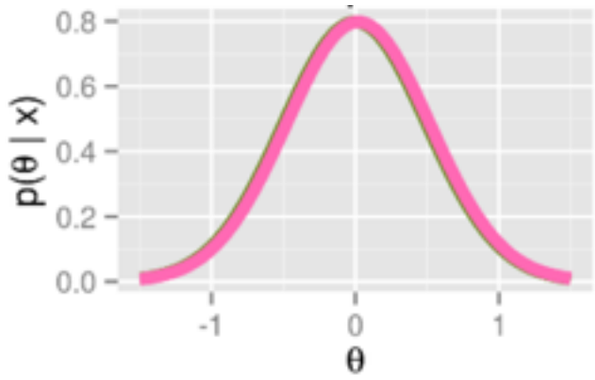

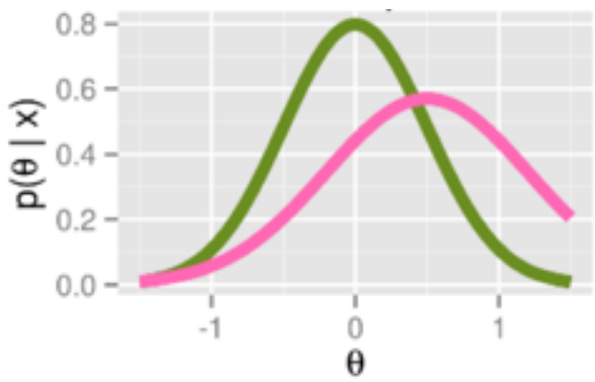

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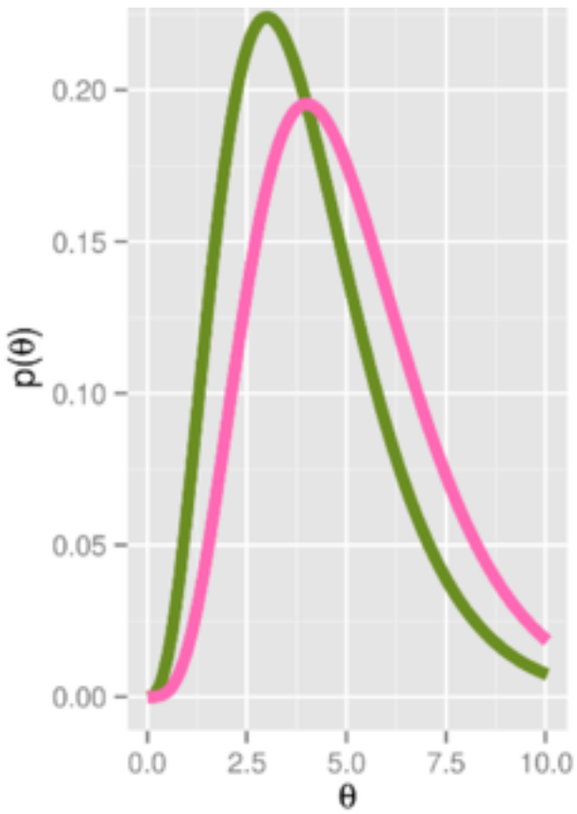

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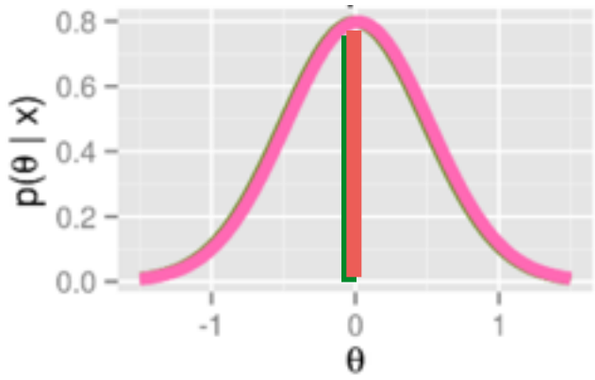

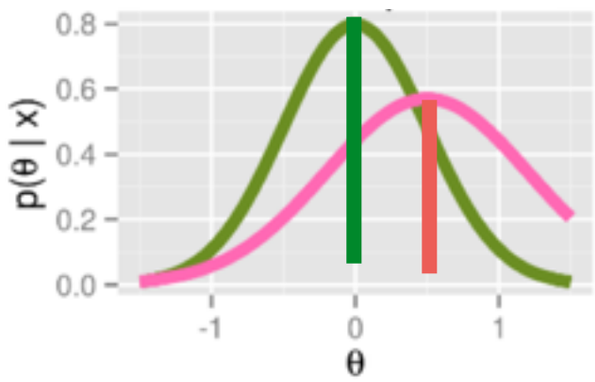

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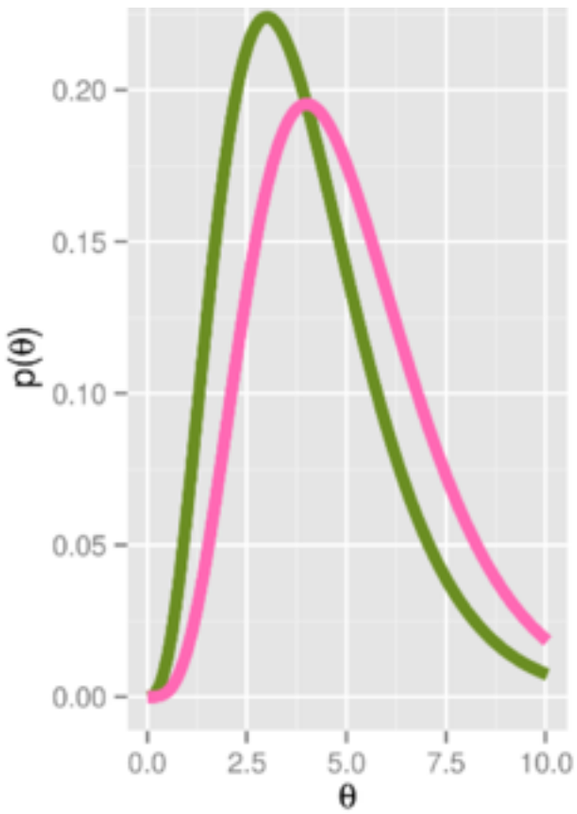

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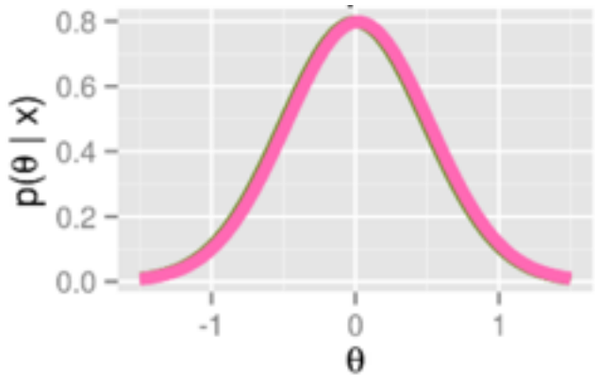

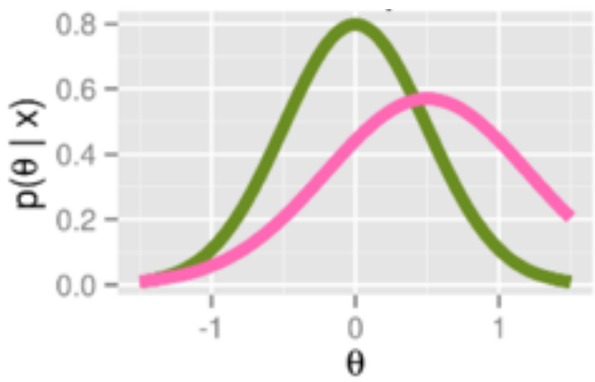

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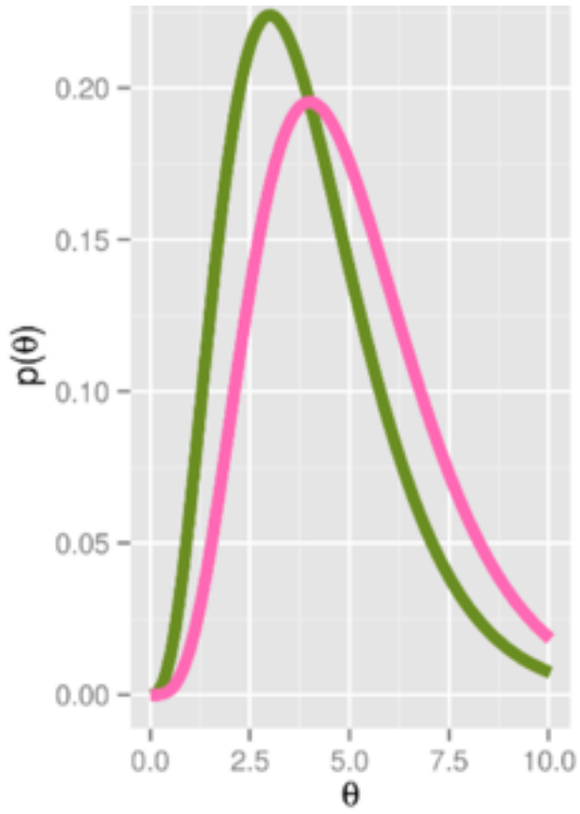

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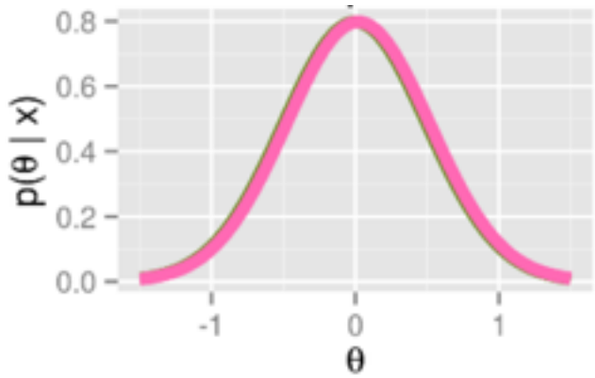

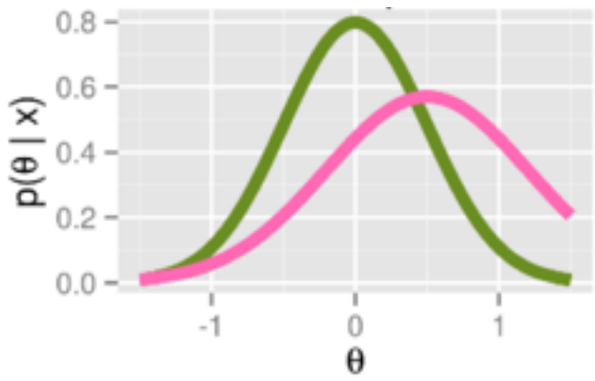

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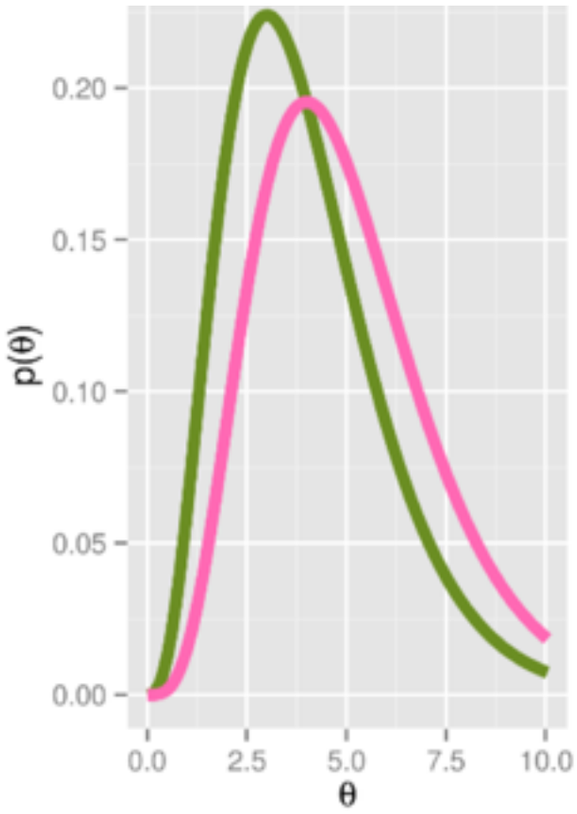

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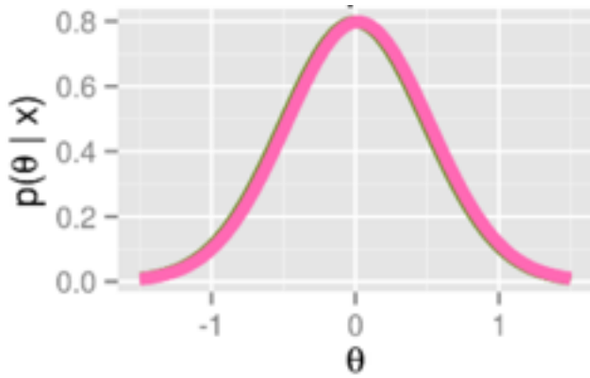

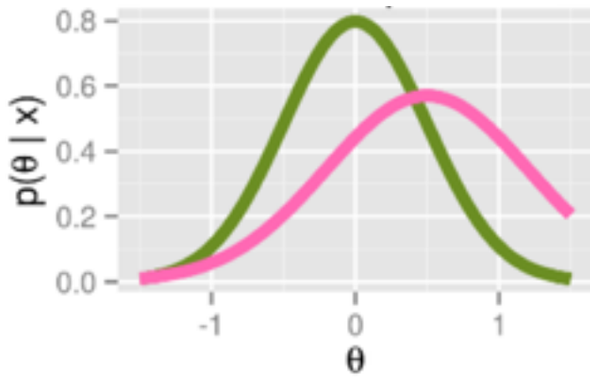

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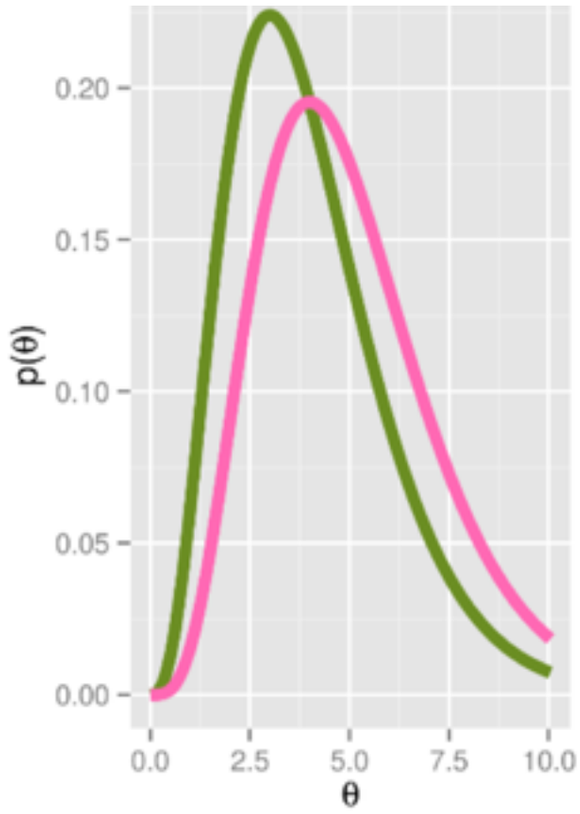

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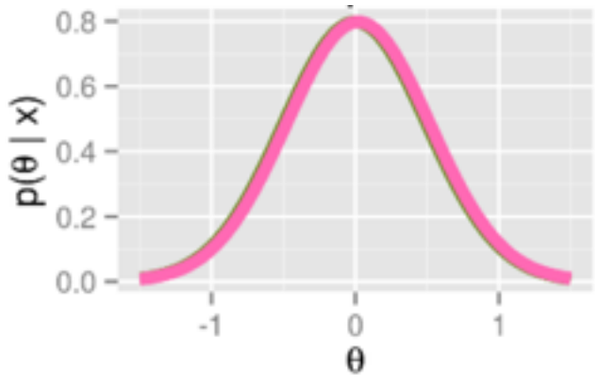

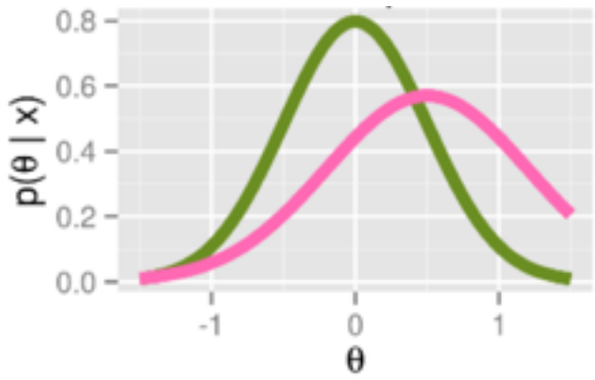

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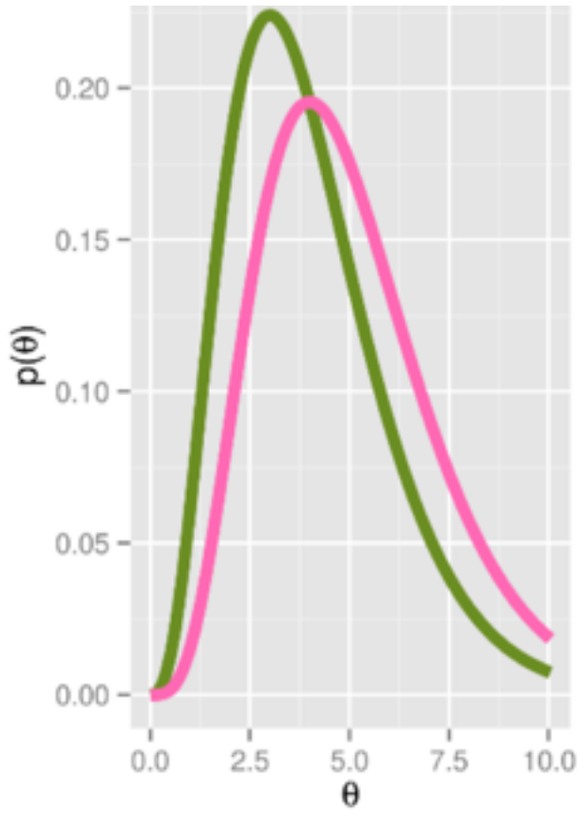

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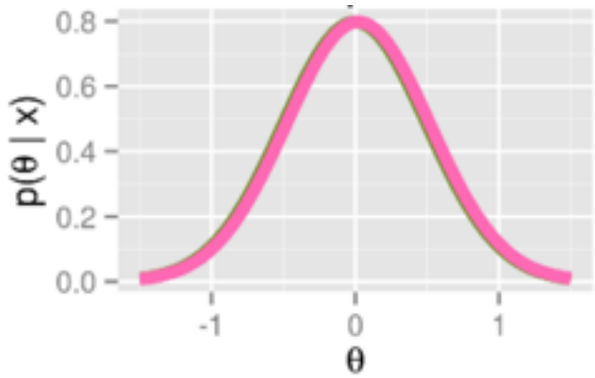

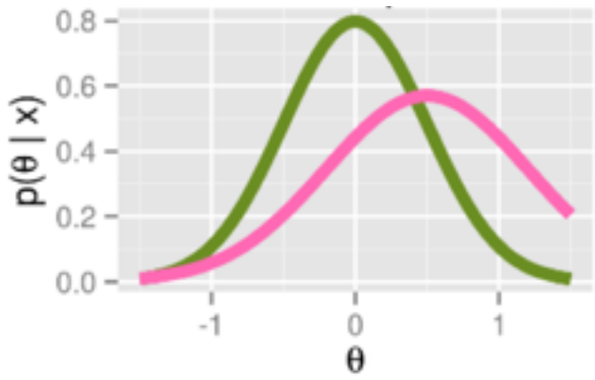

LRVB estimator

Robustness quantification

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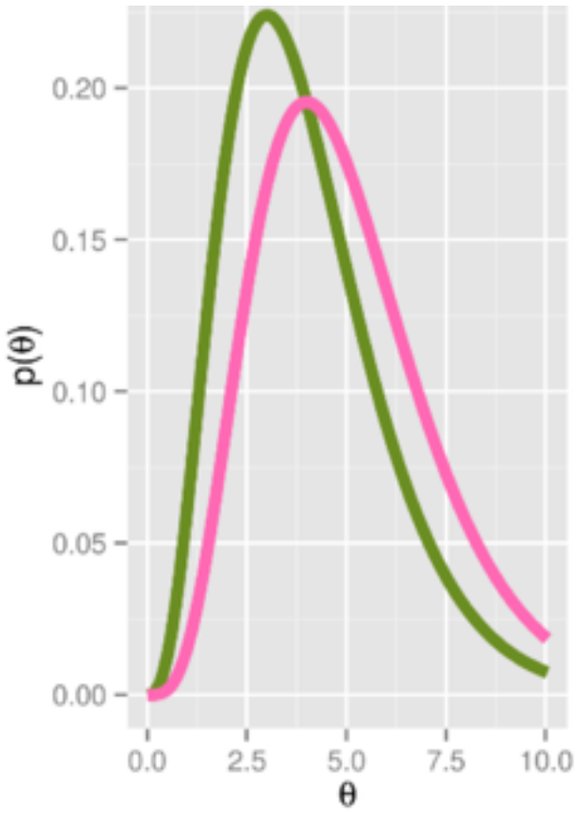

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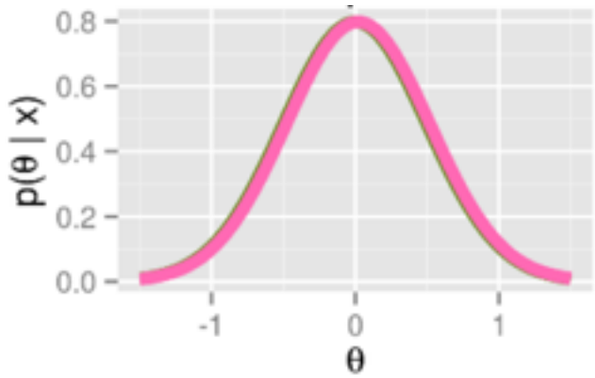

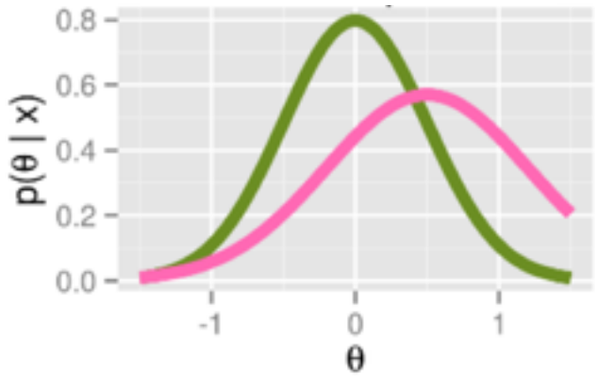

← LRVB estimator

- When q^*_α in exponential family

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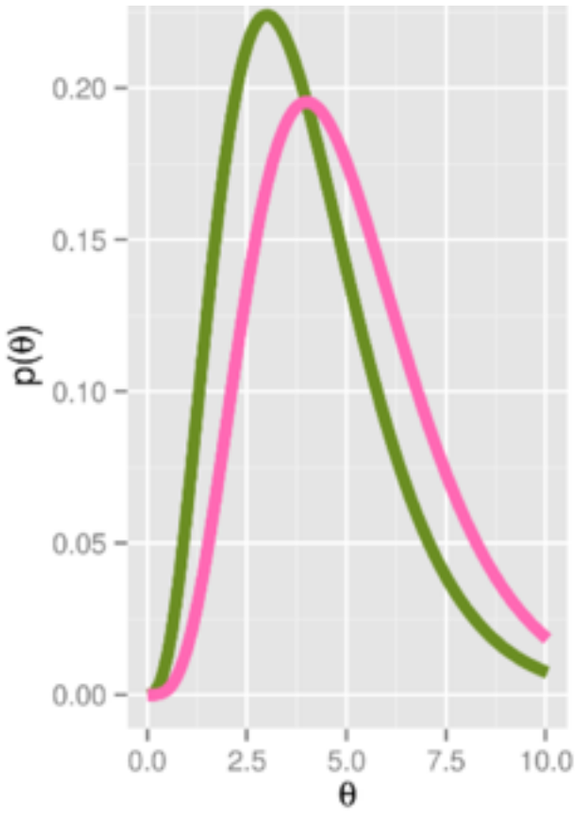
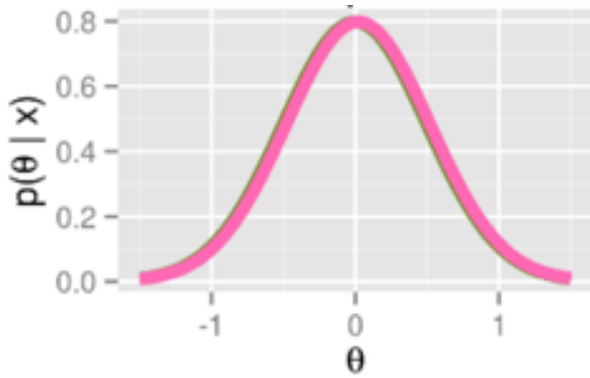
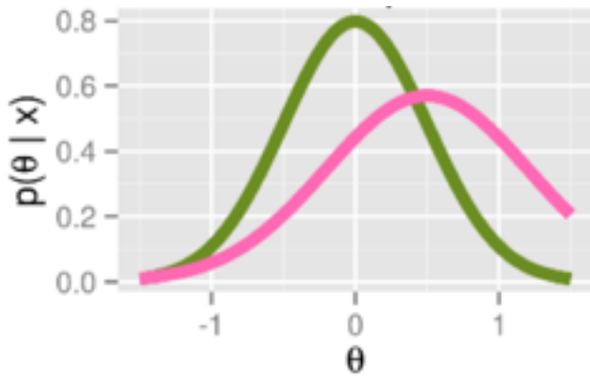


Bayes Theorem





← LRVB estimator

- When q^*_α in exponential family

$$\hat{S} = A\left(\left.\frac{\partial^2 KL}{\partial m \partial m^T}\right|_{m=m^*}\right)^{-1} B$$

Microcredit Experiment

- Simplified from Meager (2015)
- K microcredit trials (Mexico, Mongolia, Bosnia, India, Morocco, Philippines, Ethiopia)
- N_k businesses in kth site (~900 to ~17K)
- Profit of nth business at kth site:

1 if microcredit

profit

$$y_{kn} \overset{indep}{\sim} \mathcal{N}(\mu_k + T_{kn}\tau_k, \sigma_k^2)$$

- Priors and hyperpriors:

$$\begin{pmatrix} \mu_k \\ \tau_k \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu \\ \tau \end{pmatrix}, C\right) \qquad \begin{pmatrix} \mu \\ \tau \end{pmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{pmatrix} \mu_0 \\ \tau_0 \end{pmatrix}, \Lambda^{-1}\right)$$

$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \text{Sep\&LKJ}(\eta, c, d)$$

Microcredit Experiment

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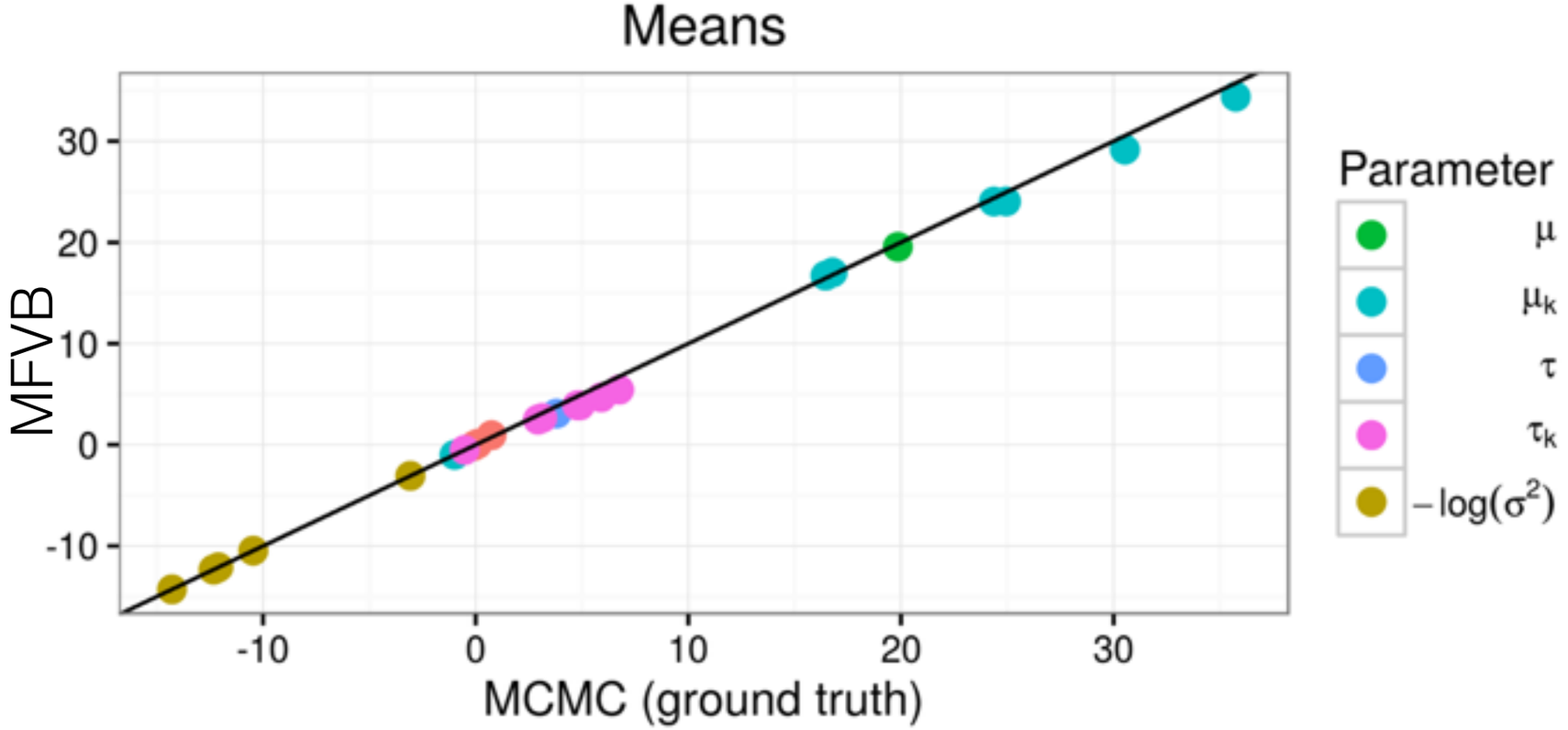
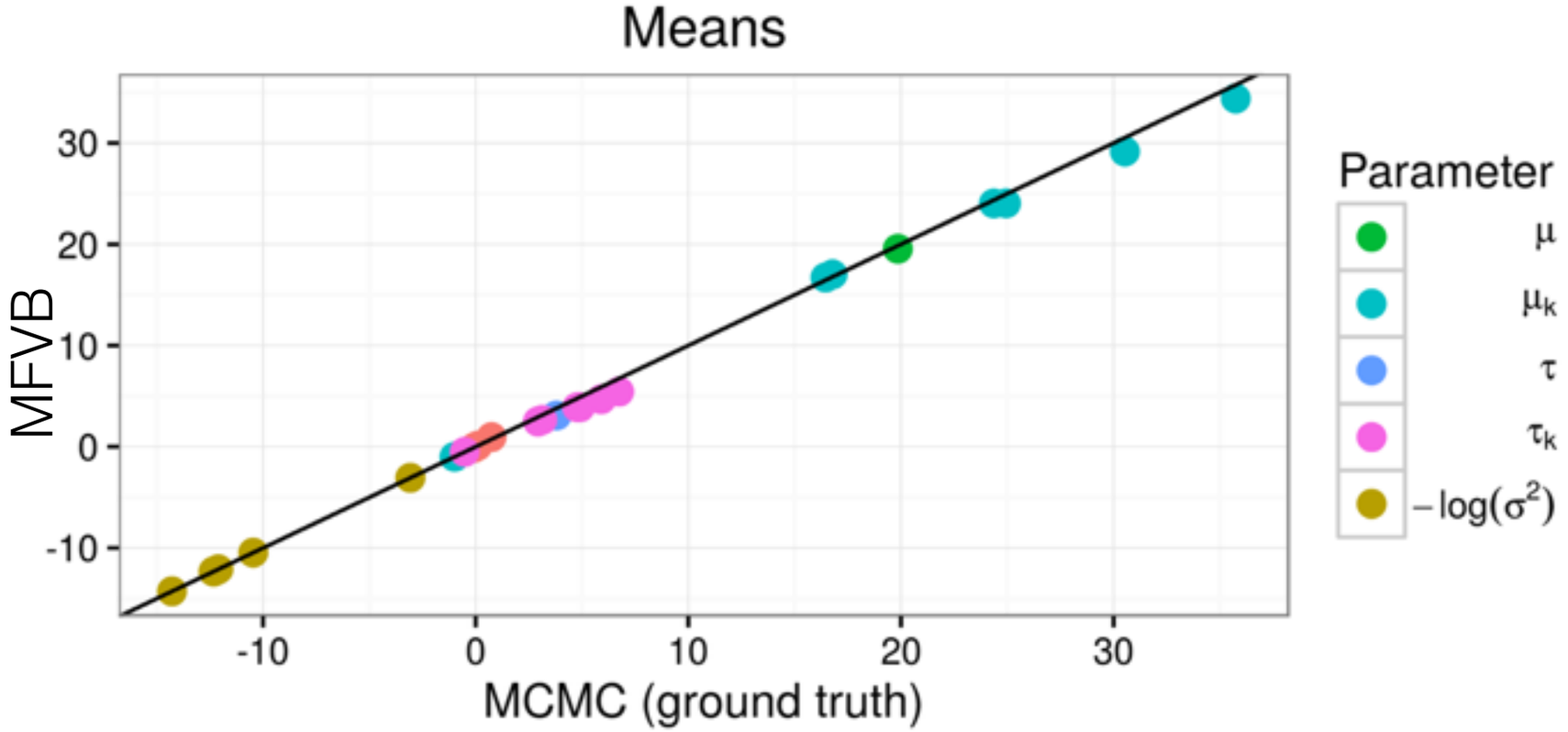


Microcredit Experiment

- Perturb Λ_{11}: 0.03 ➔ 0.04



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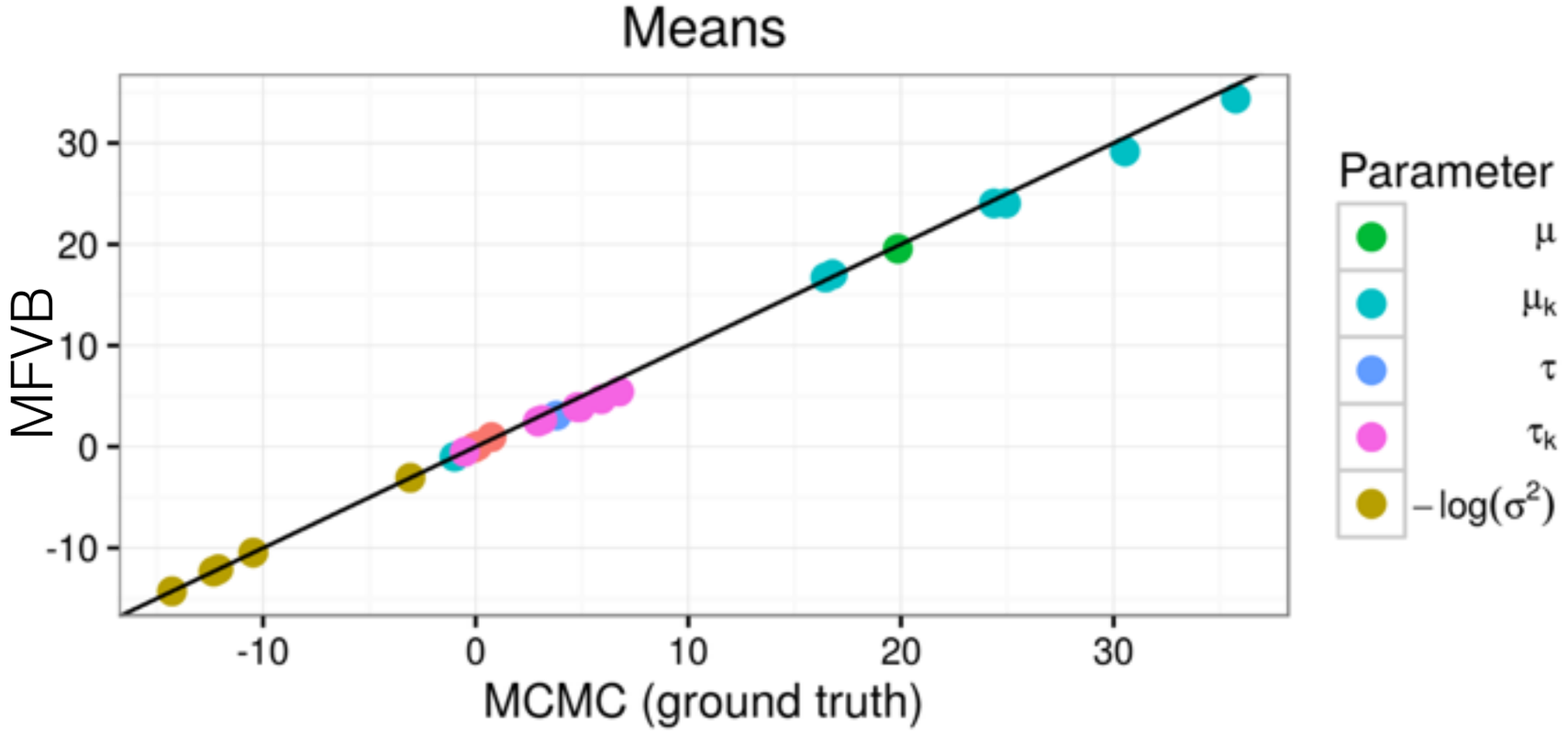

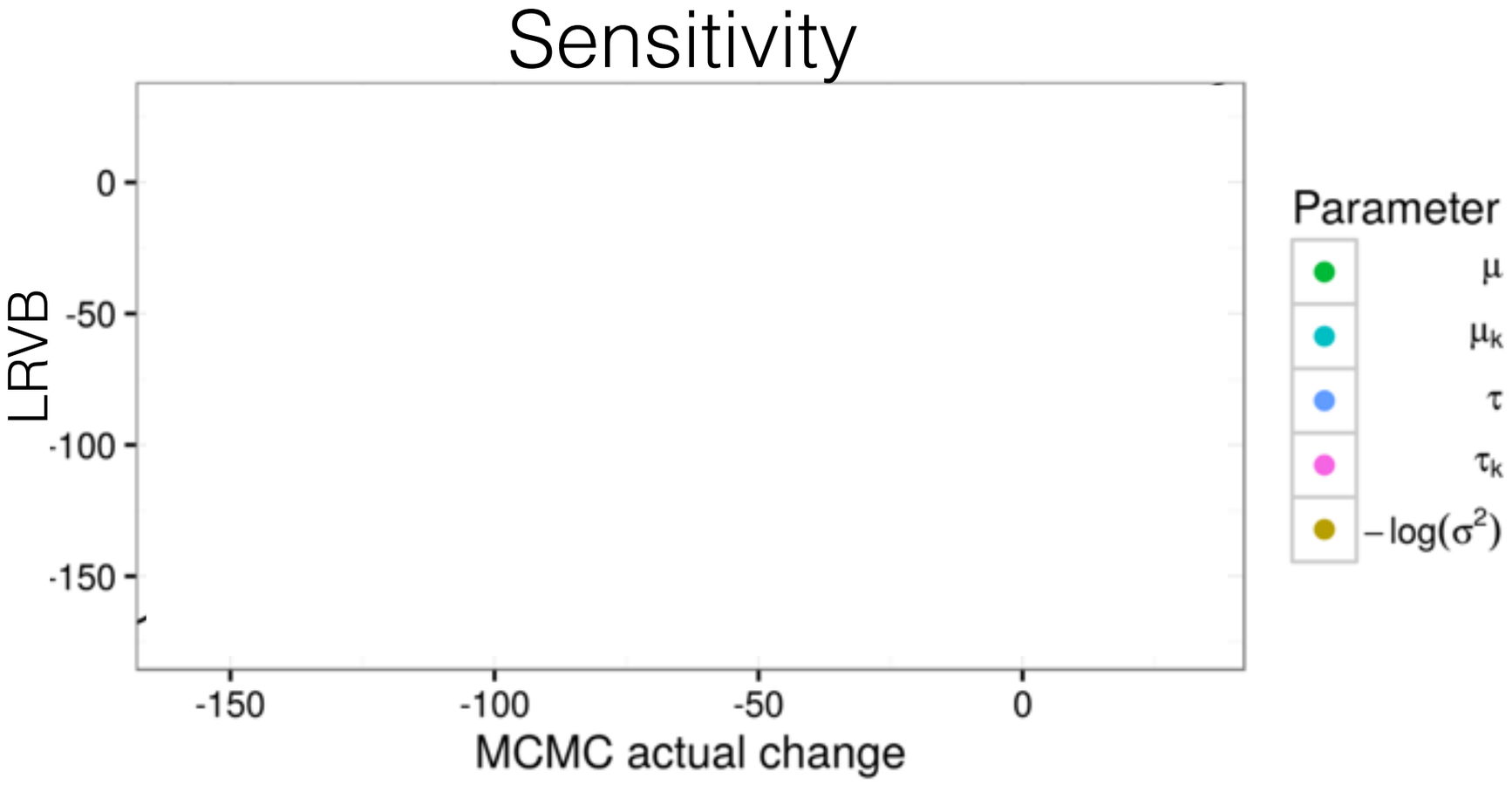

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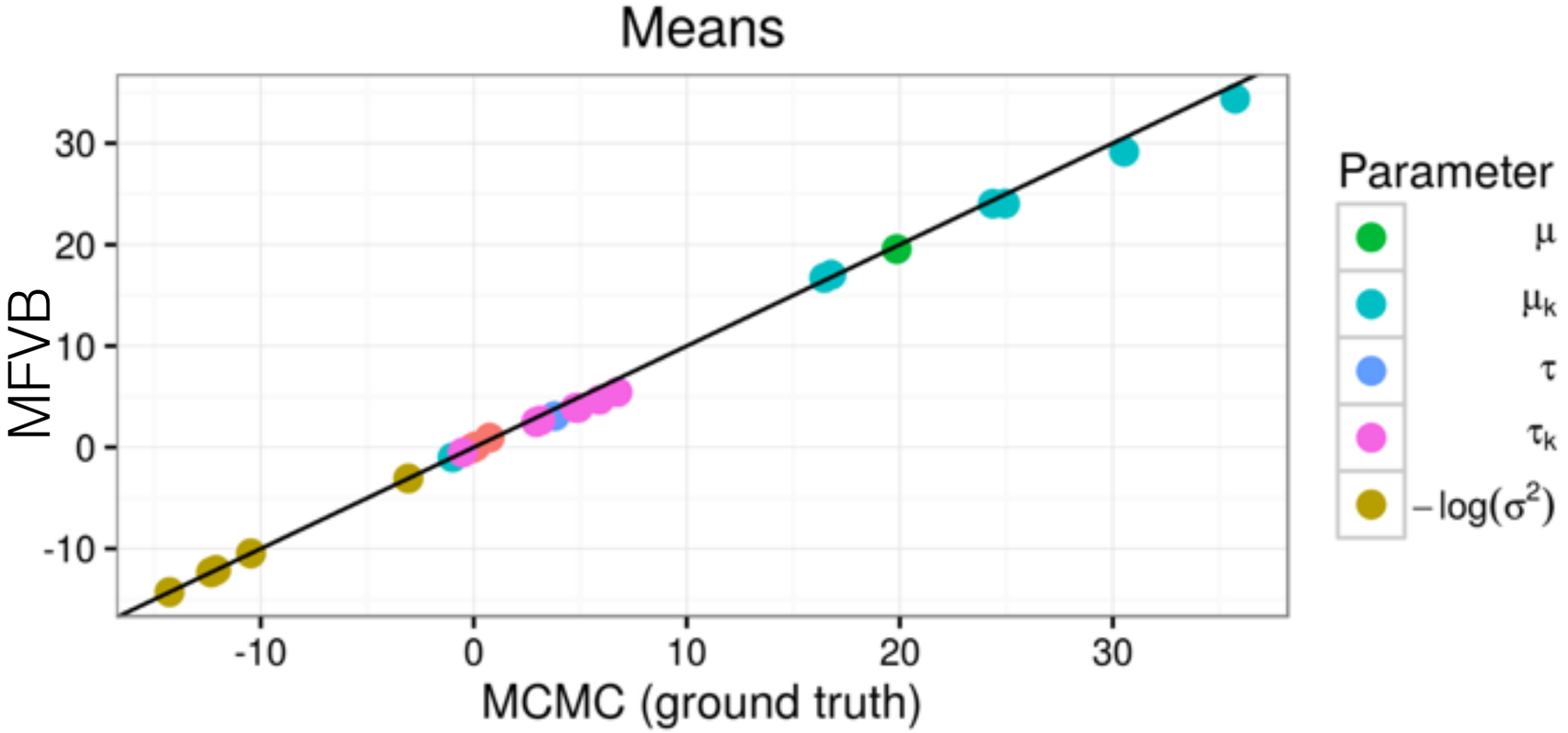

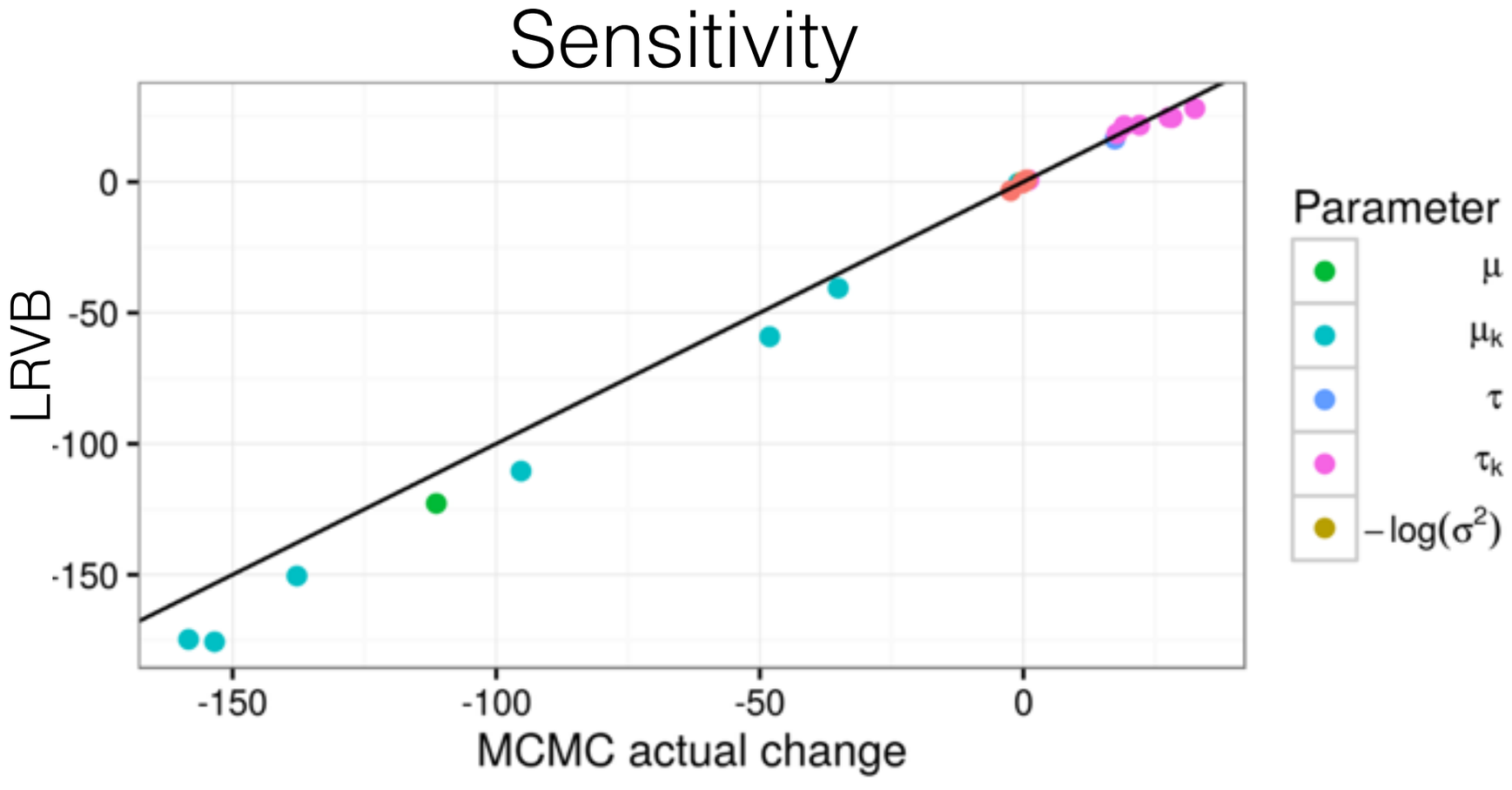

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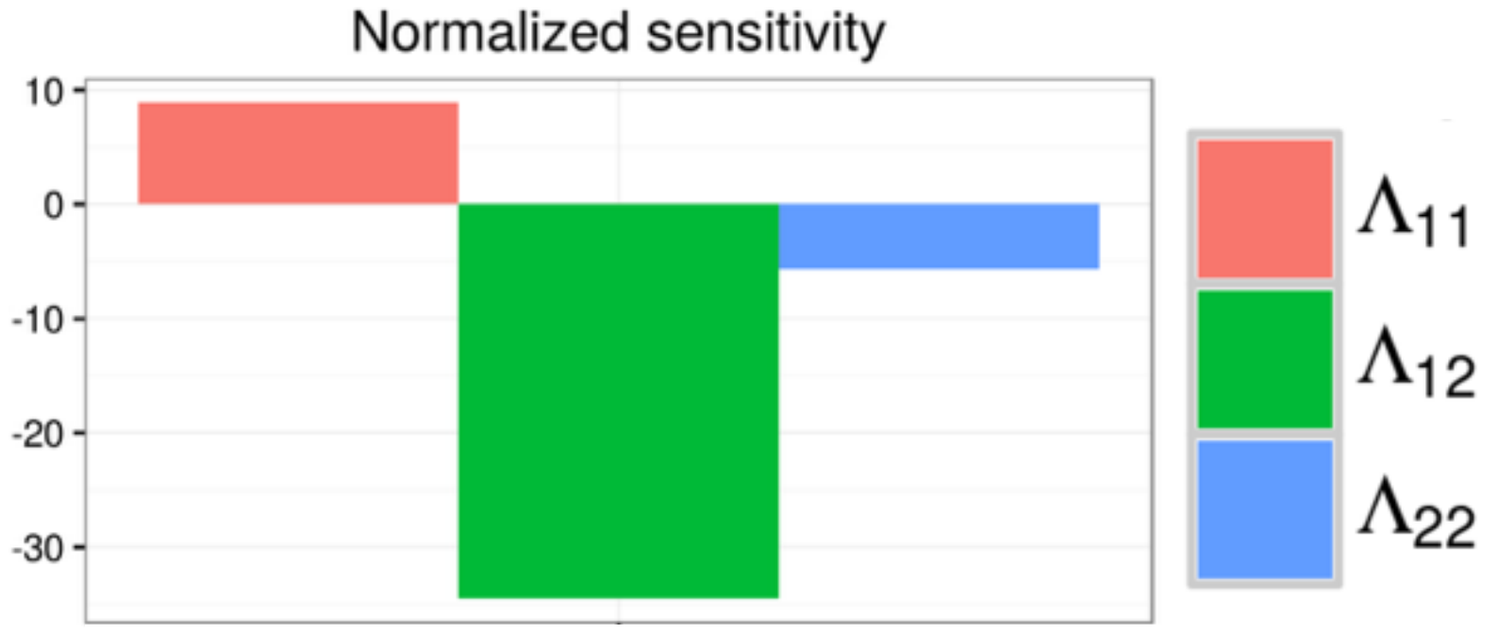

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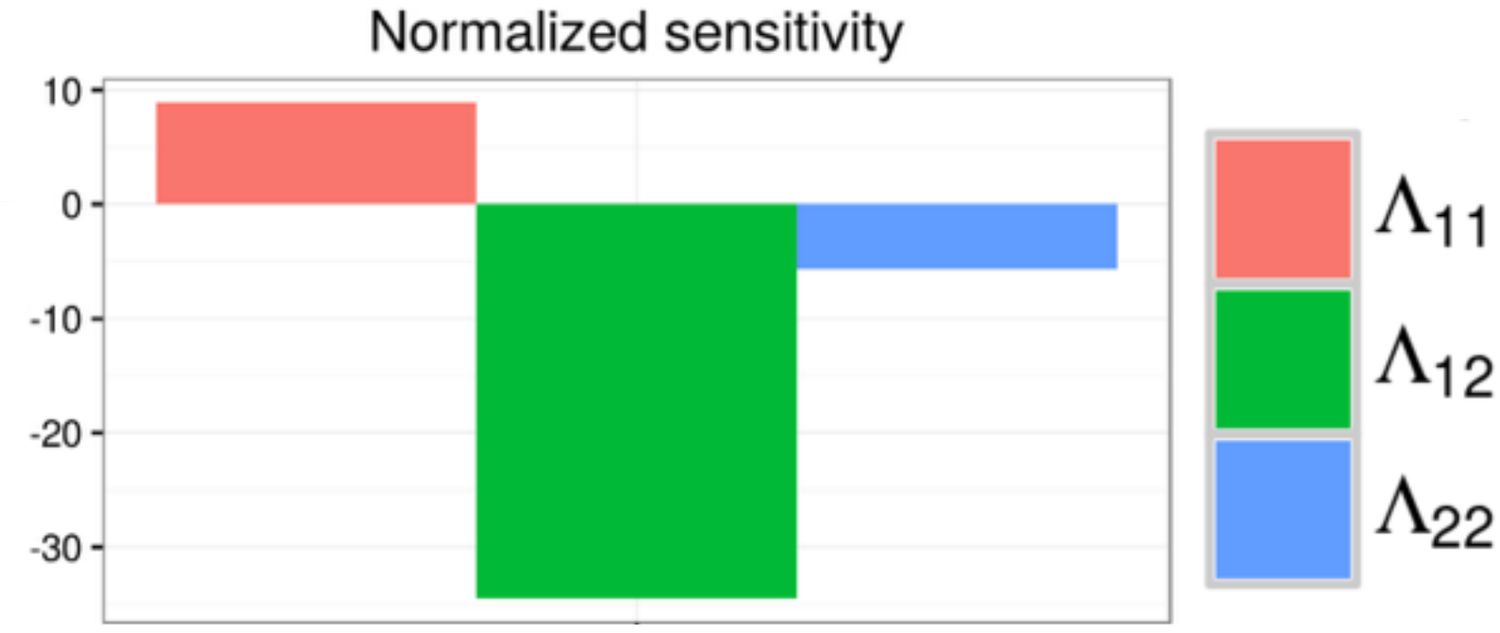

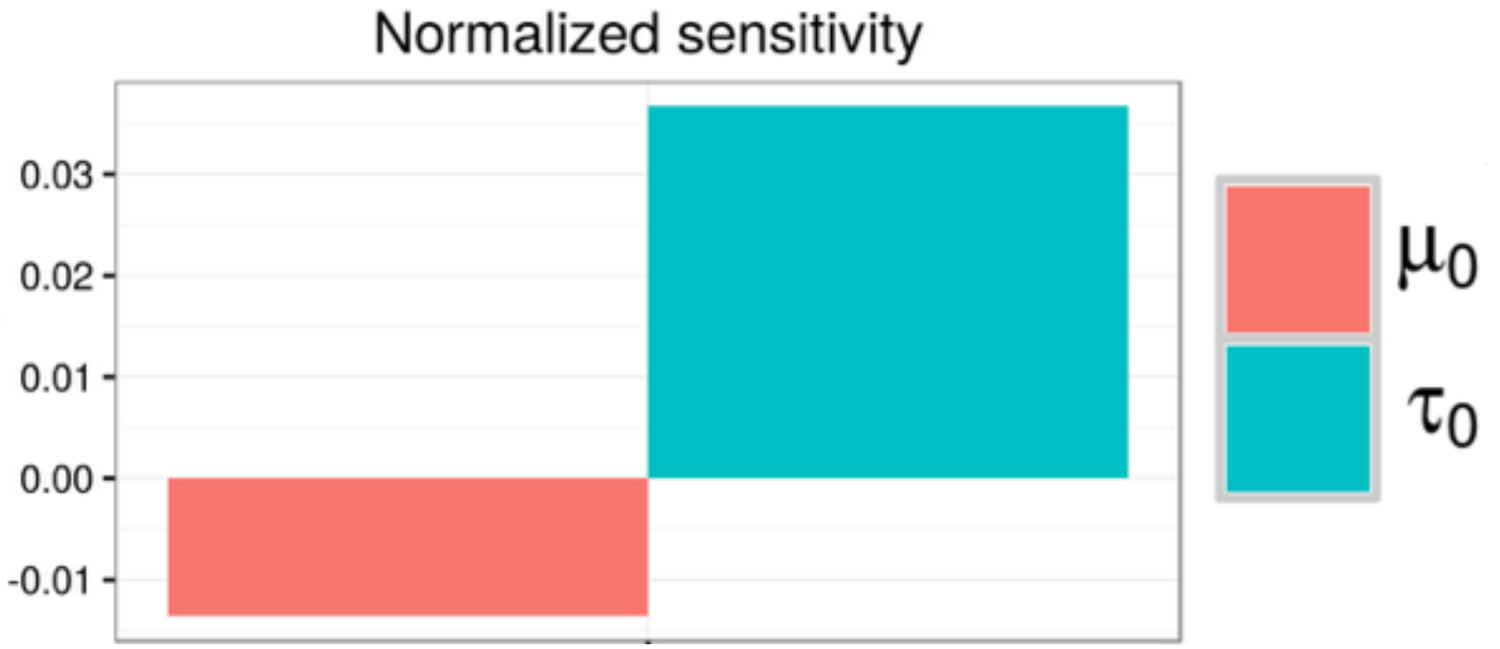

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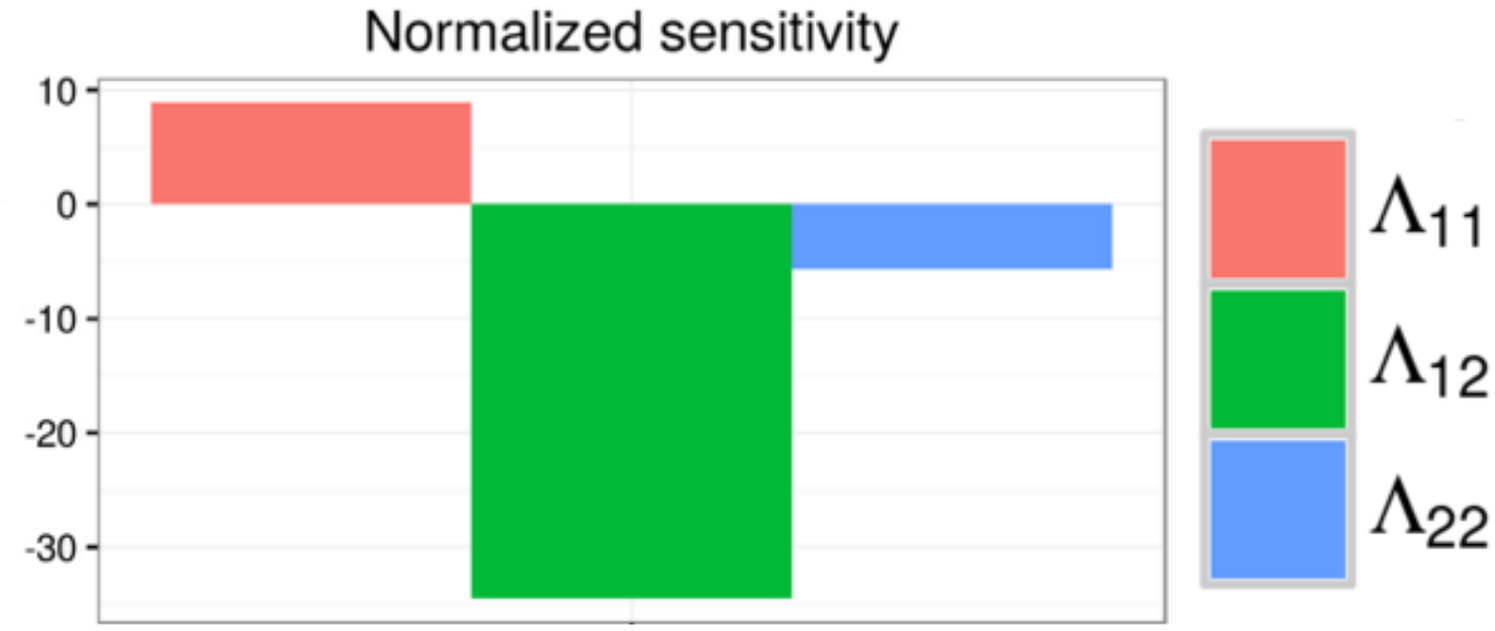

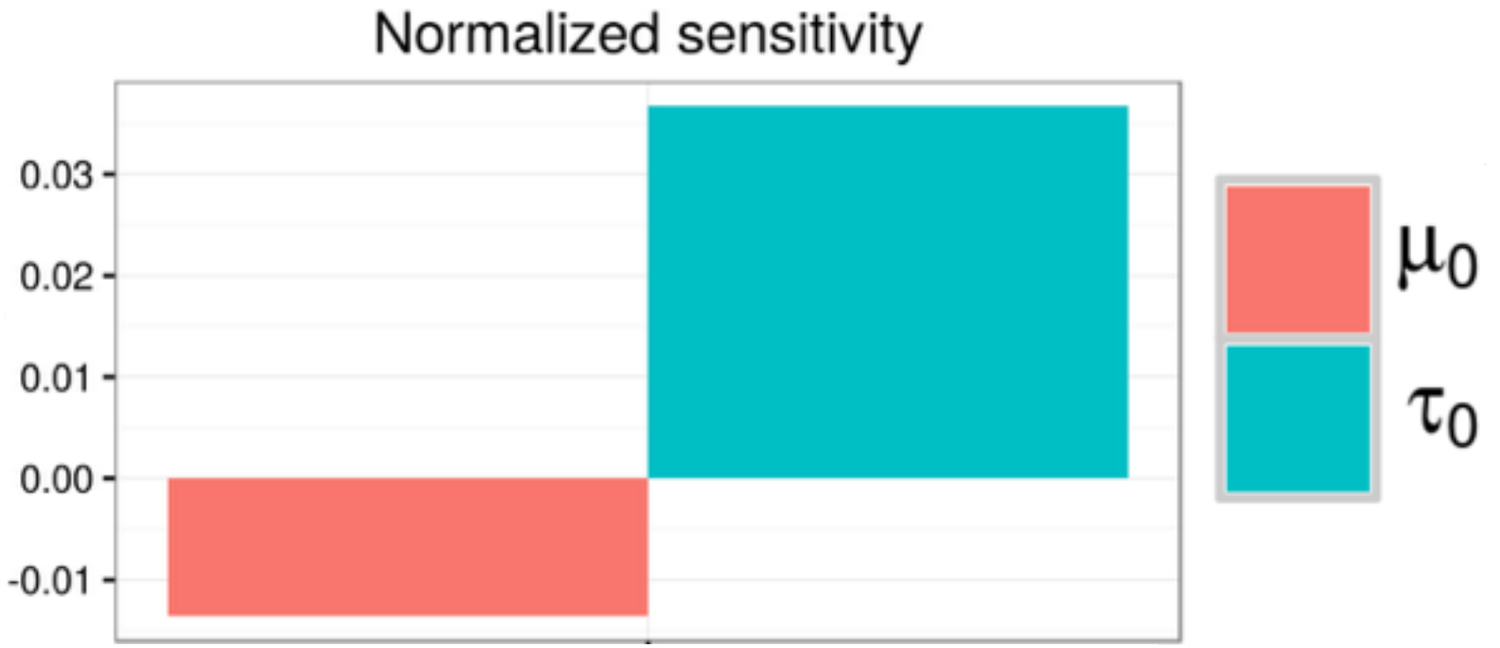

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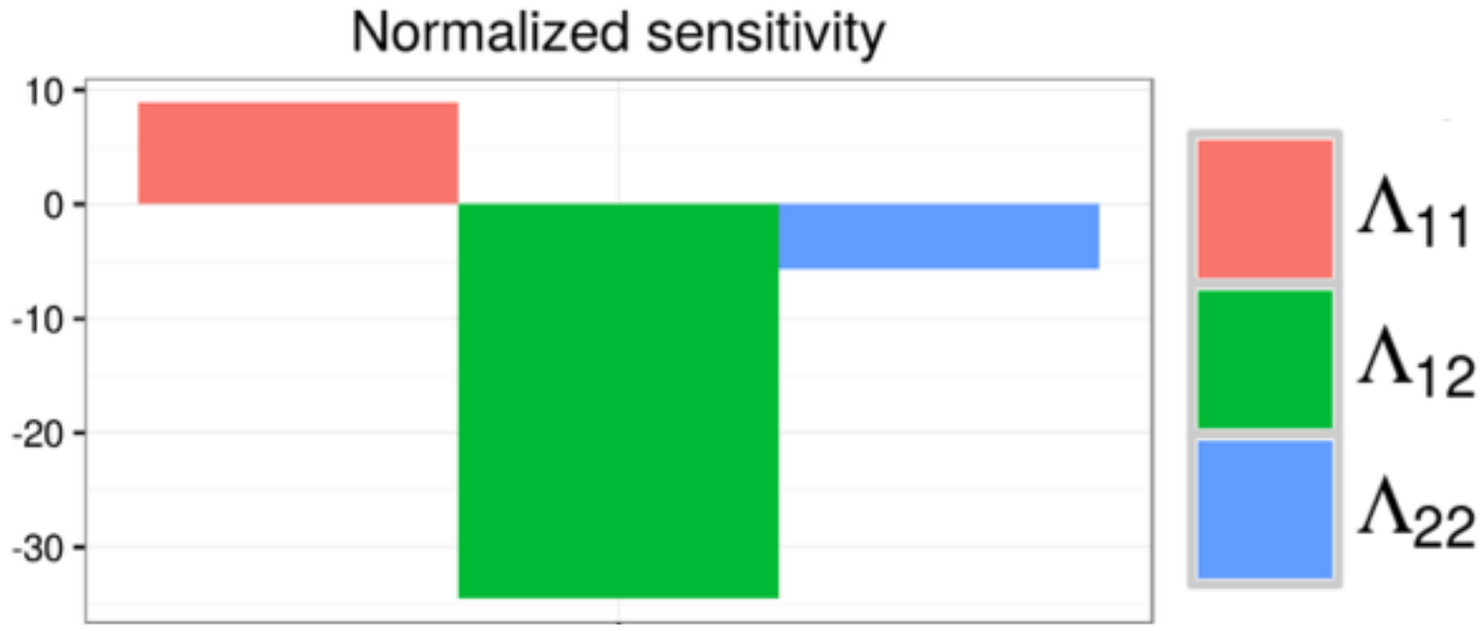

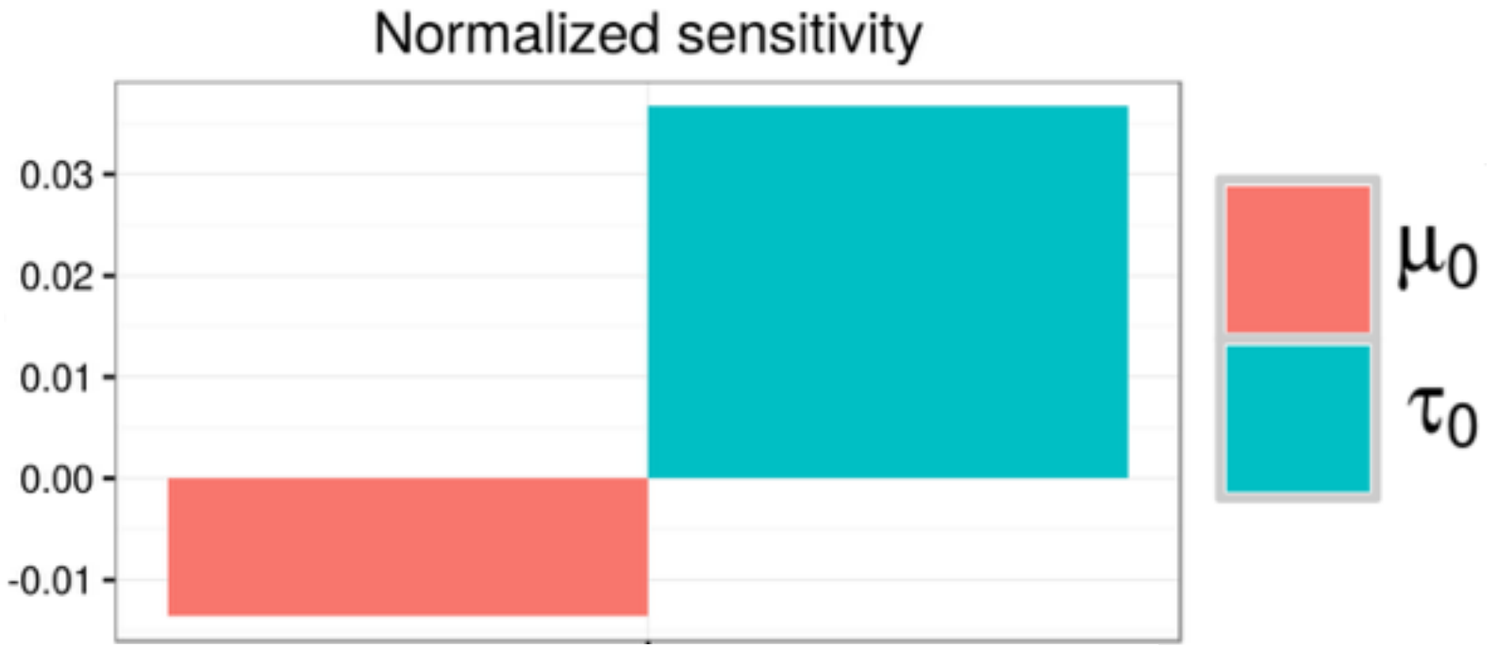

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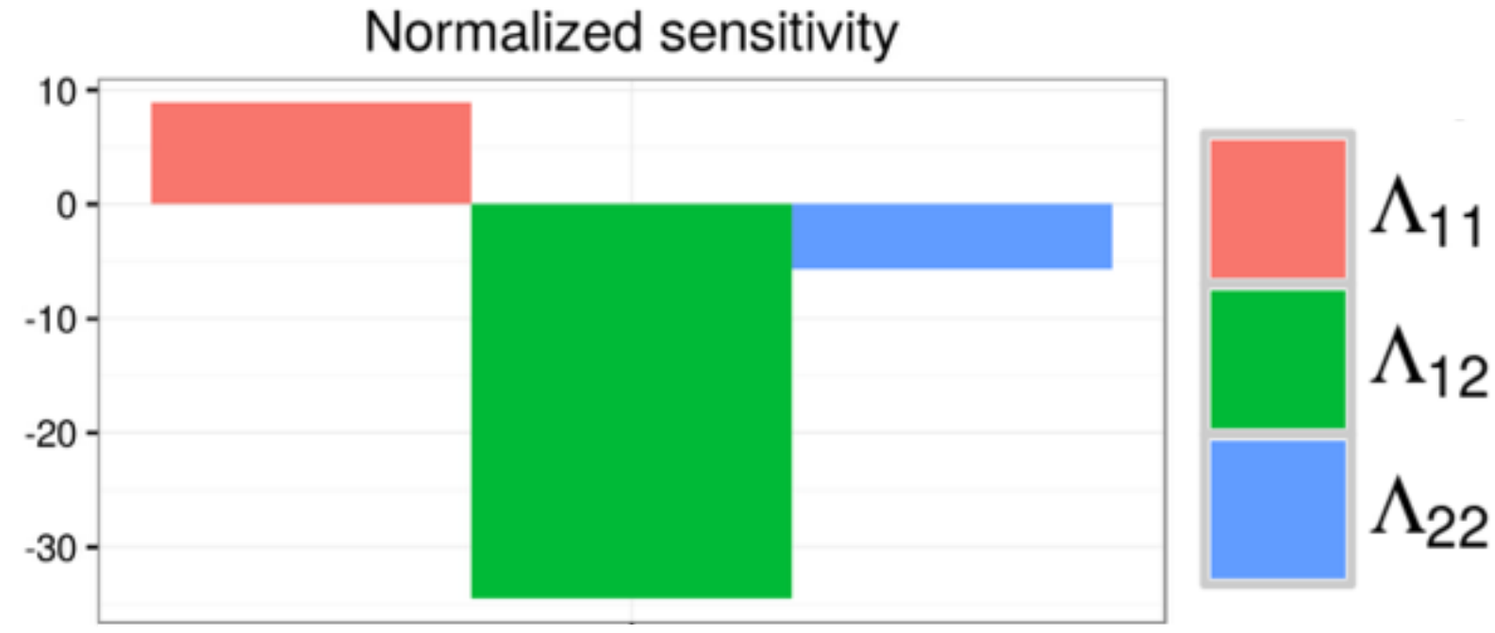
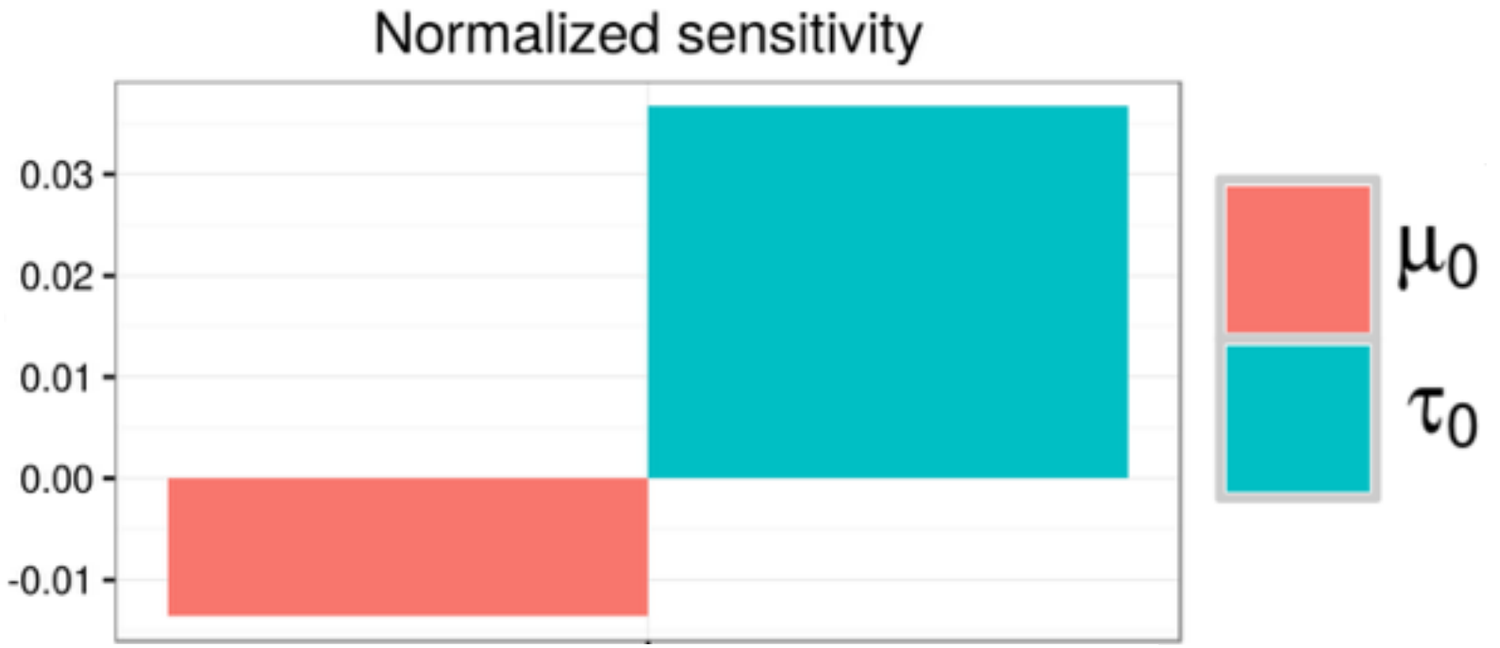
⇒ Mean > 2 std dev





Roadmap

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- Pre-process data to get a smaller, weighted data set

[Agarwal et al 2005, Feldman & Langberg 2011]

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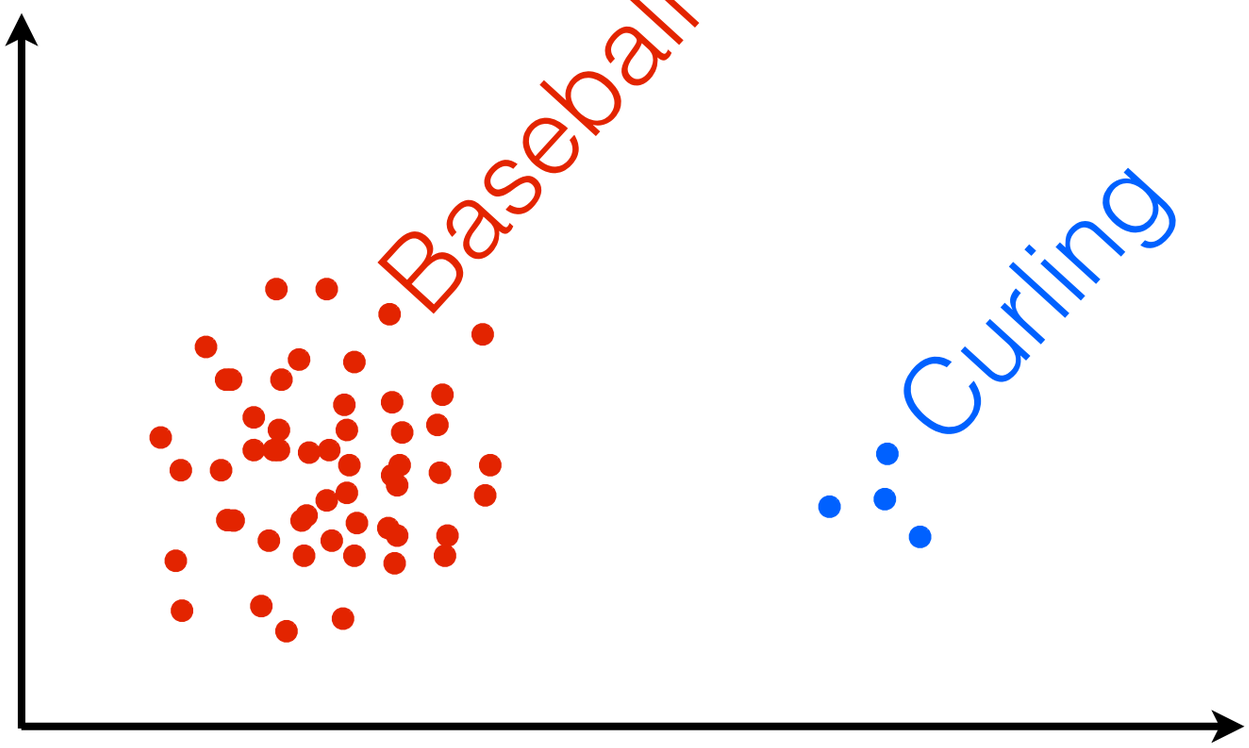

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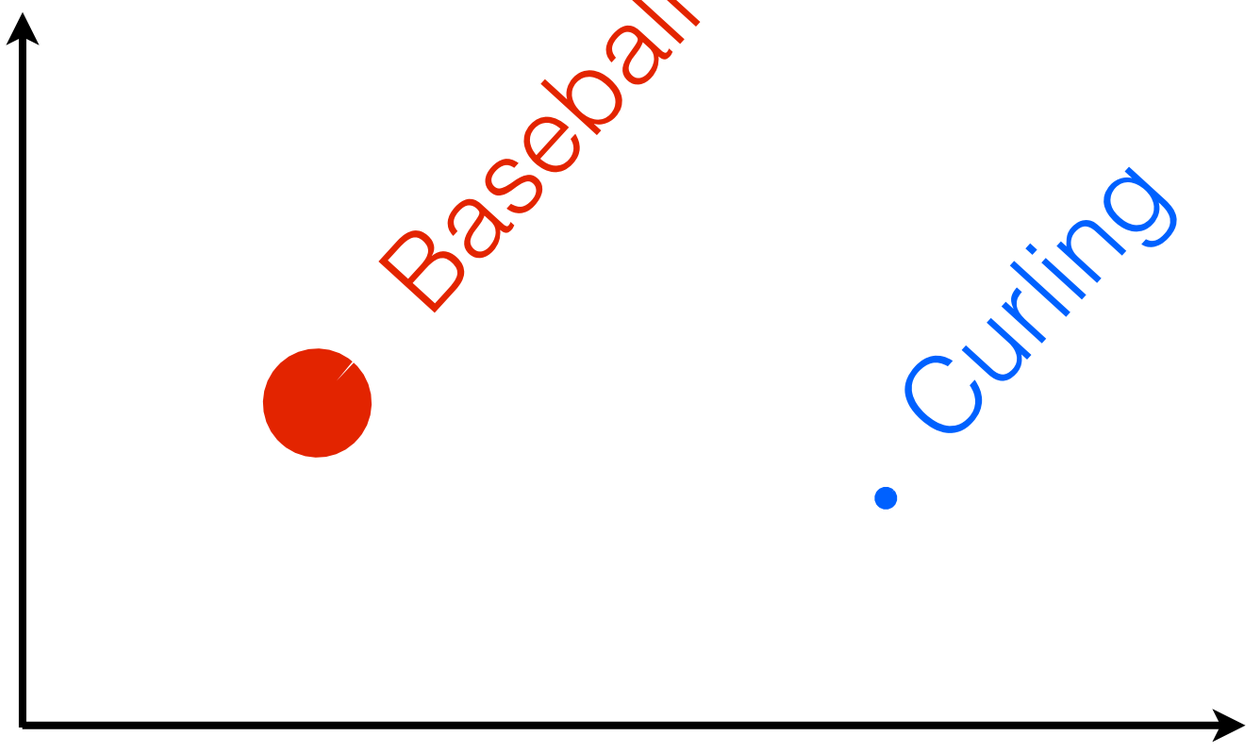

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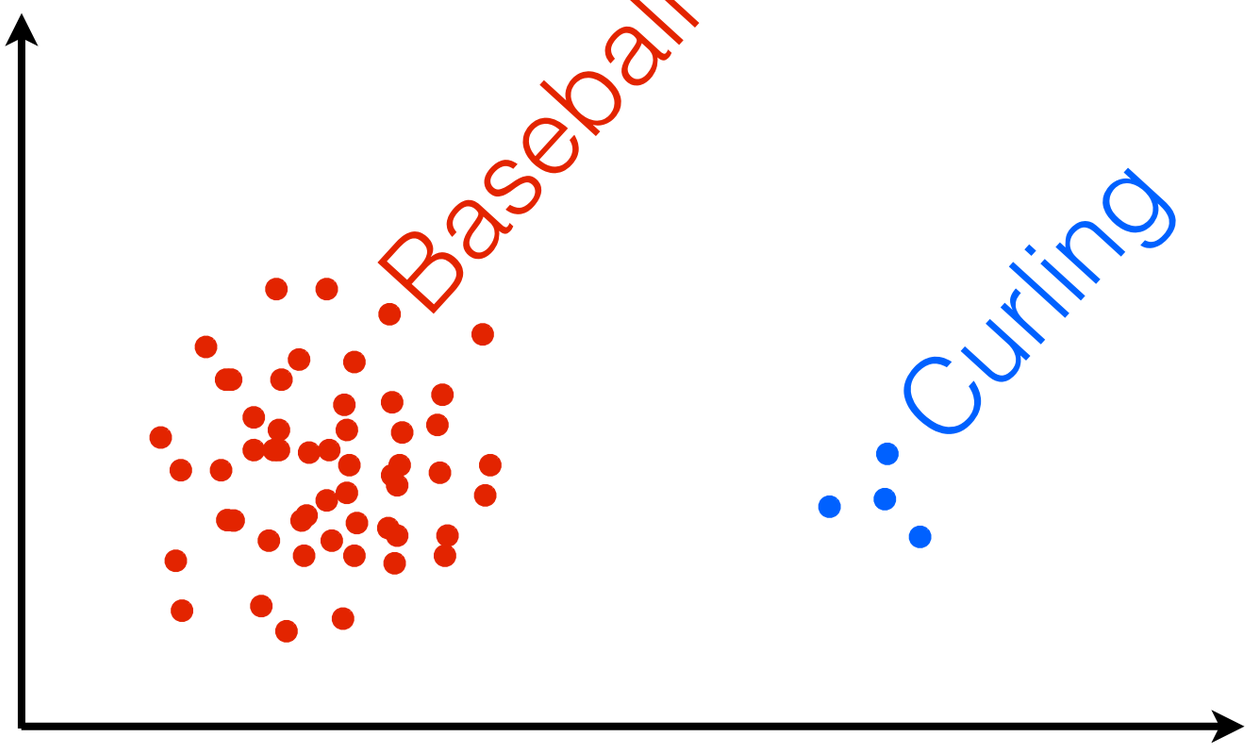

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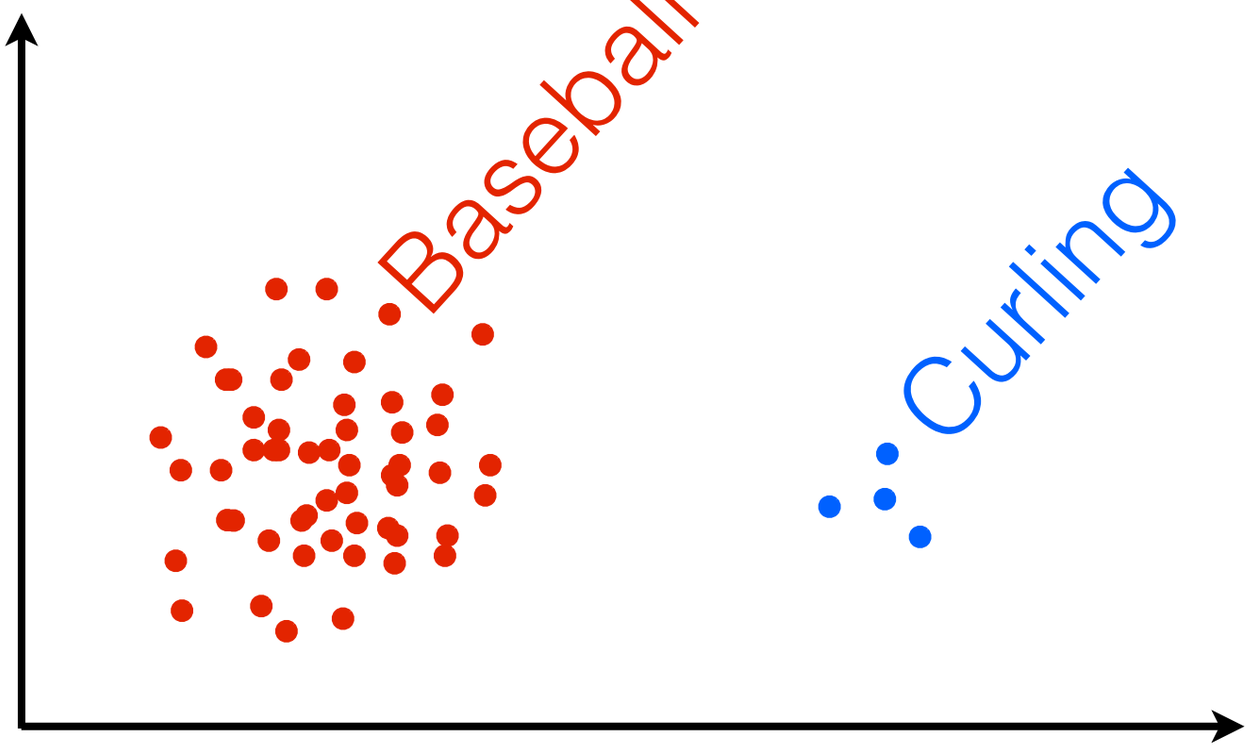

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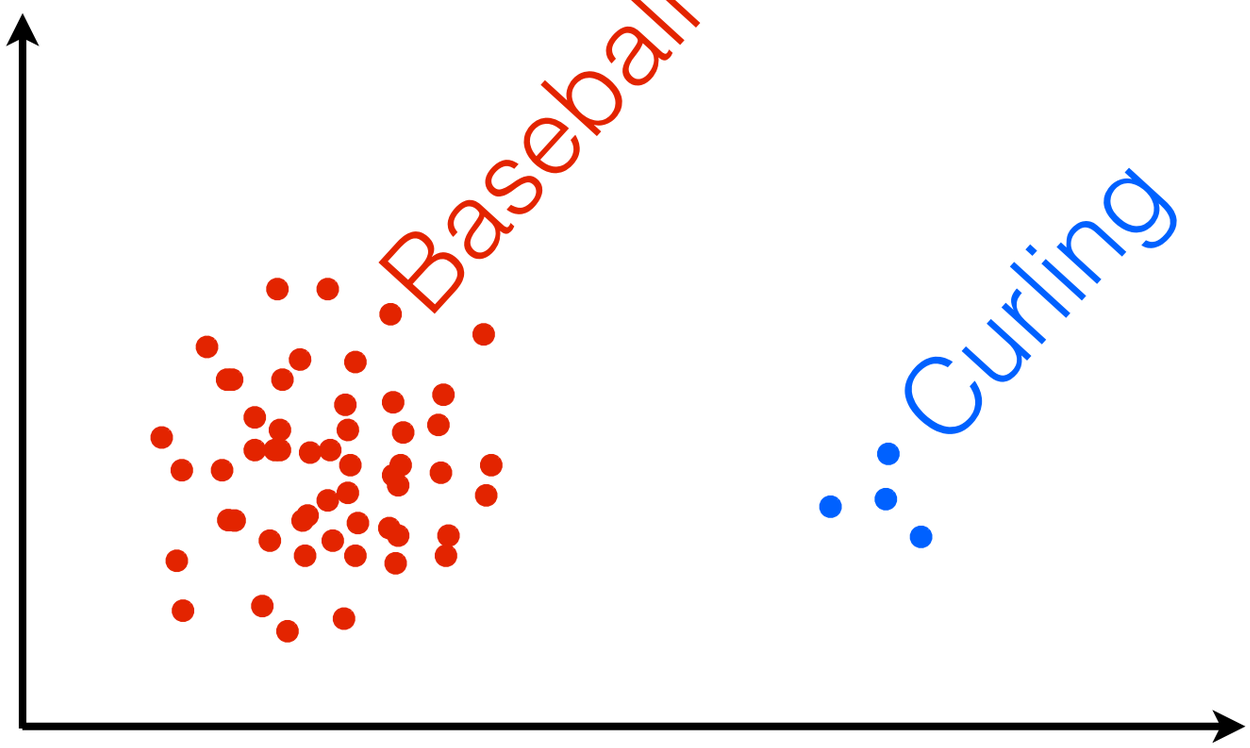

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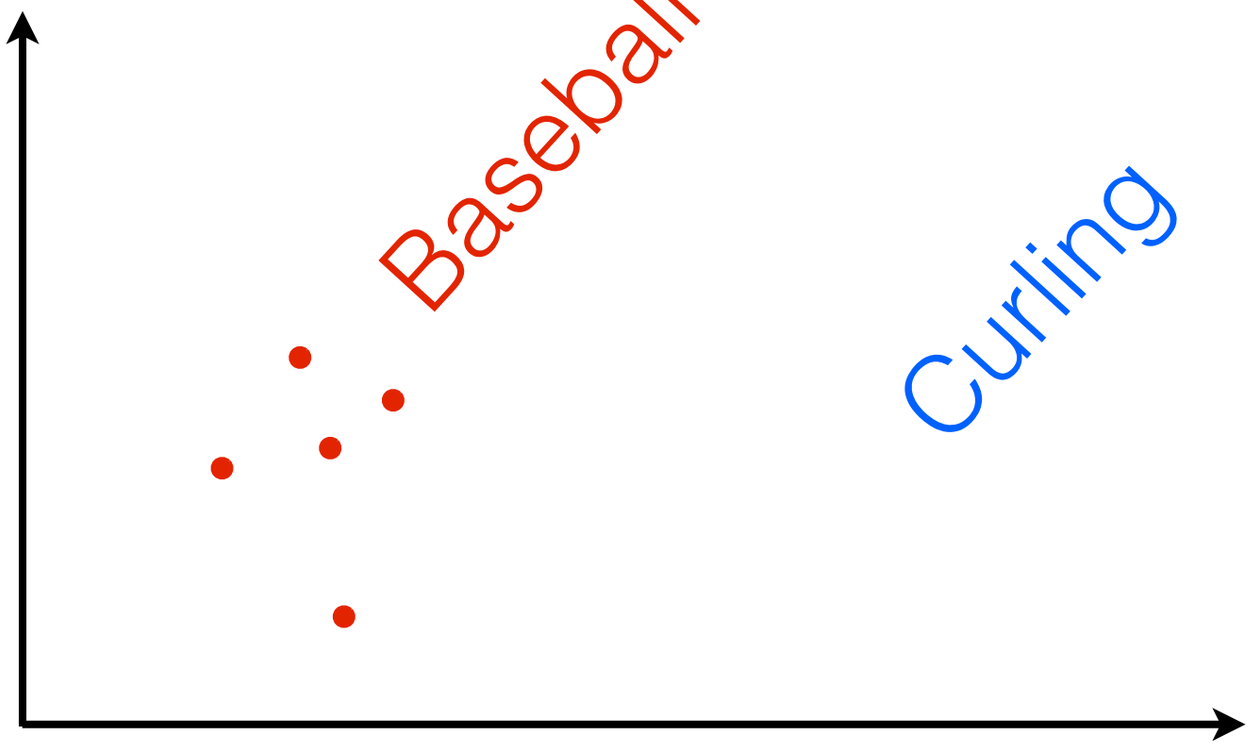

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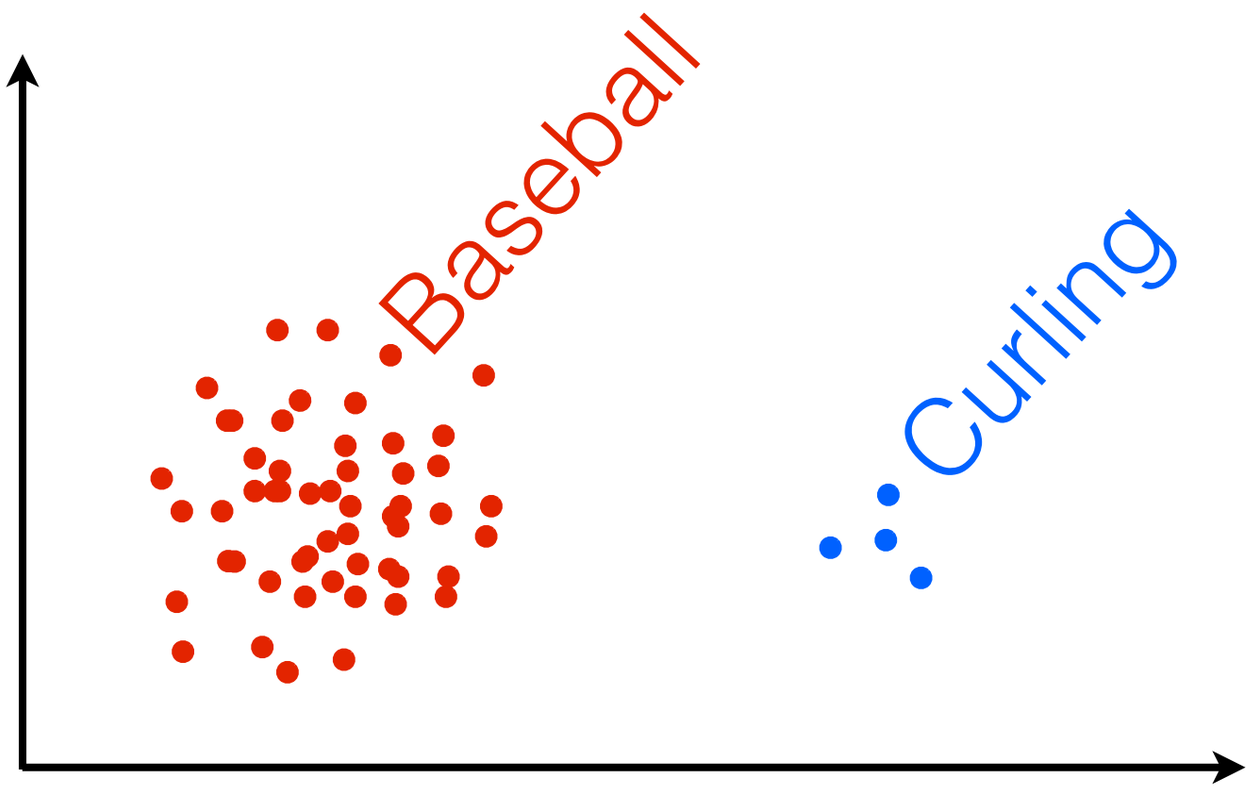

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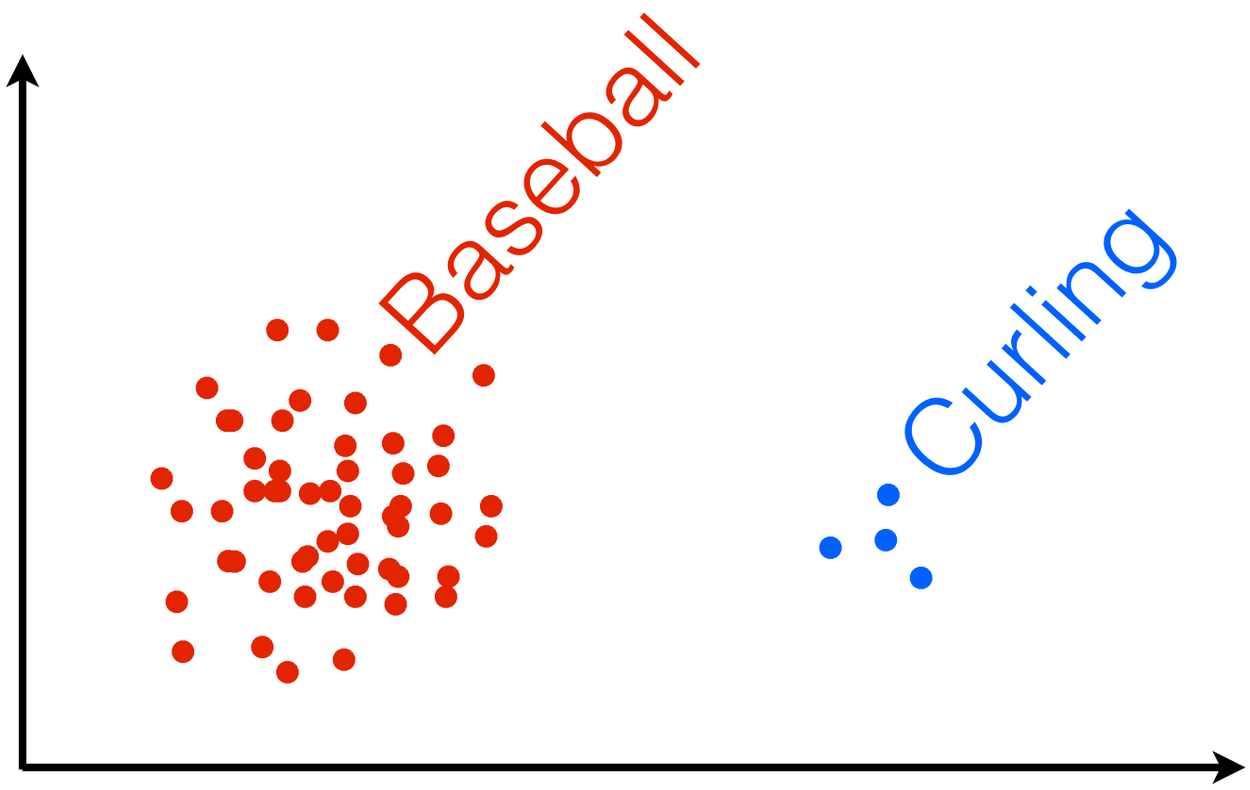

- Cf. data squashing, subsampling
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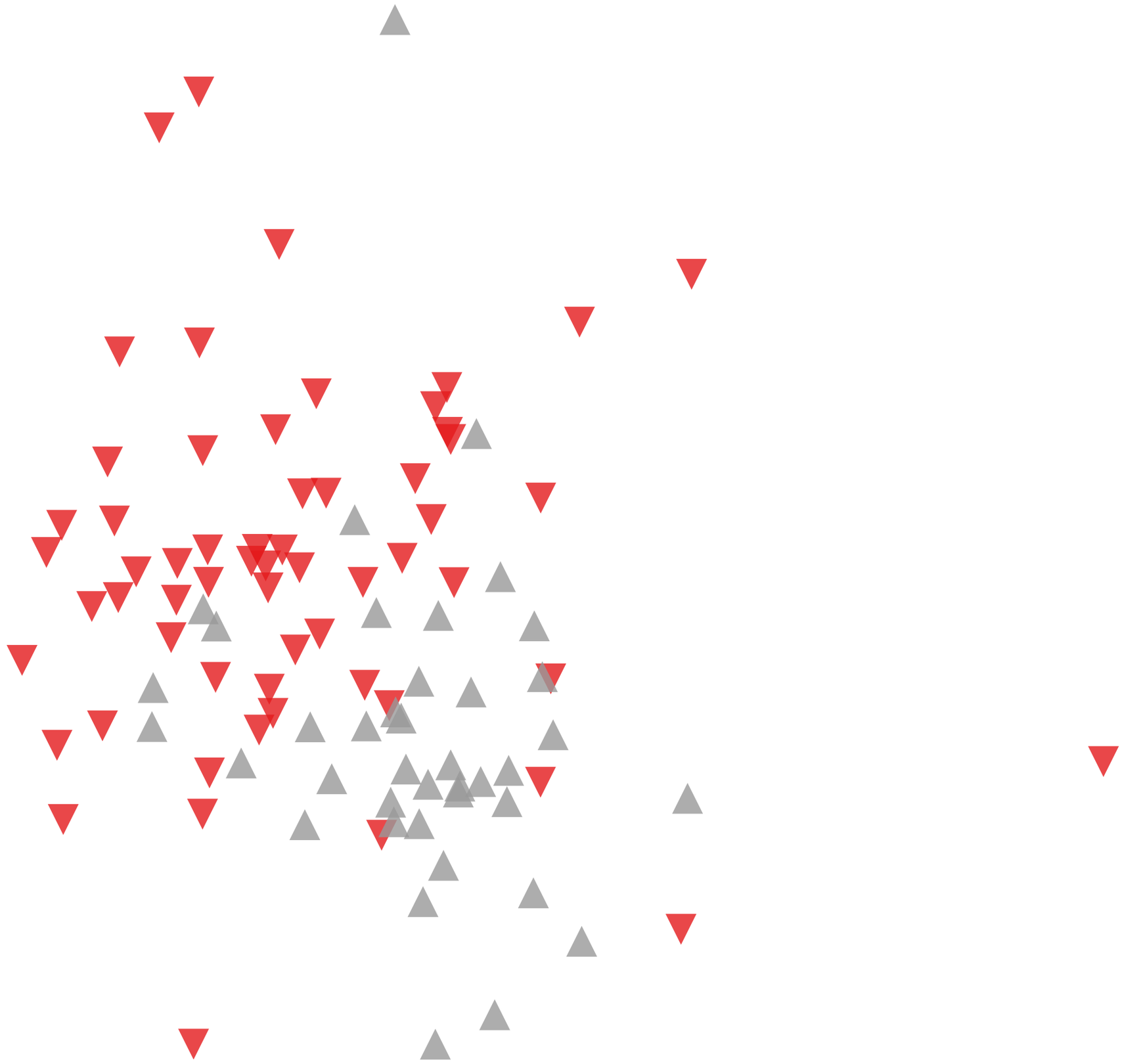
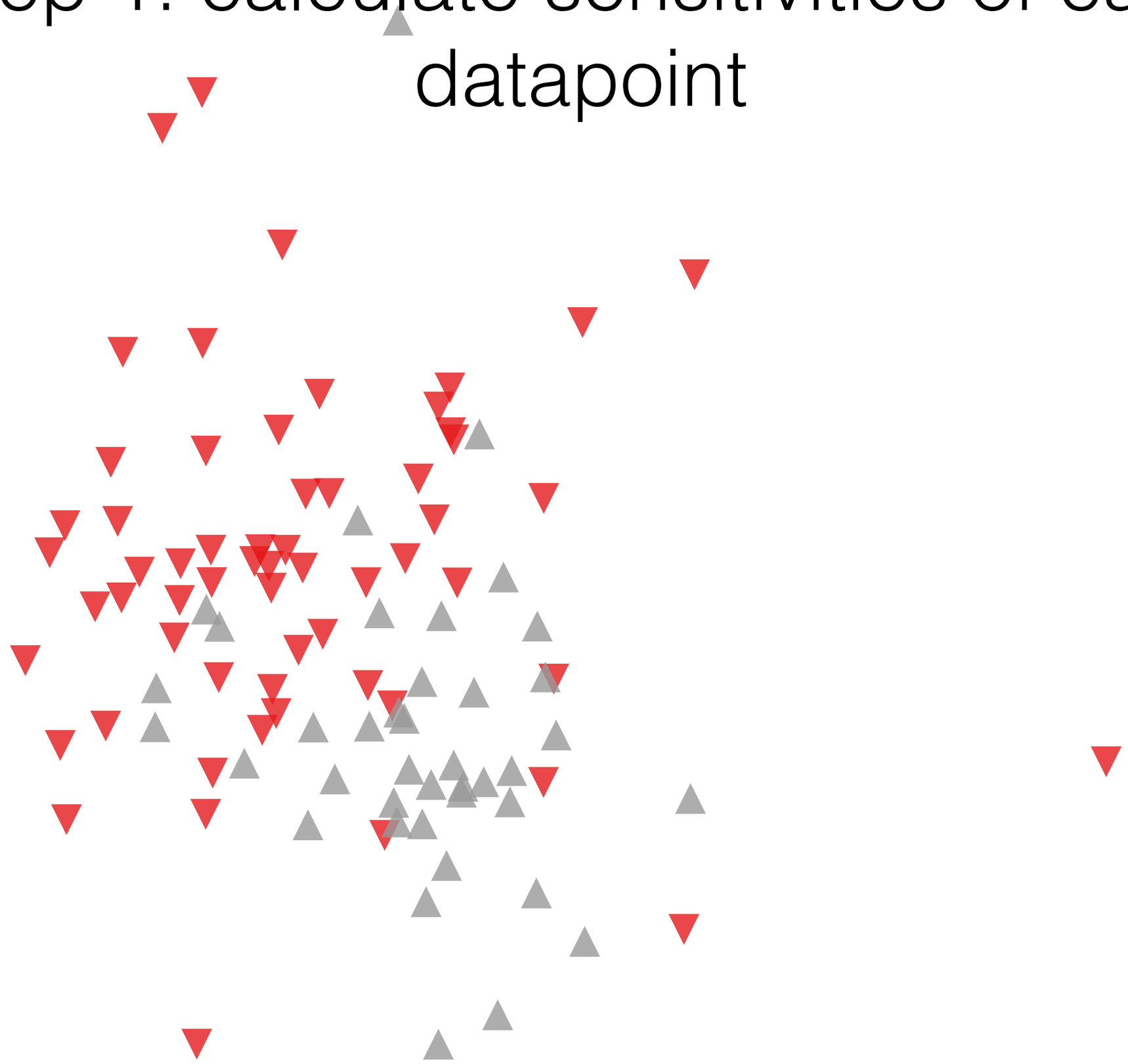


Step 1: calculate sensitivities of each datapoint



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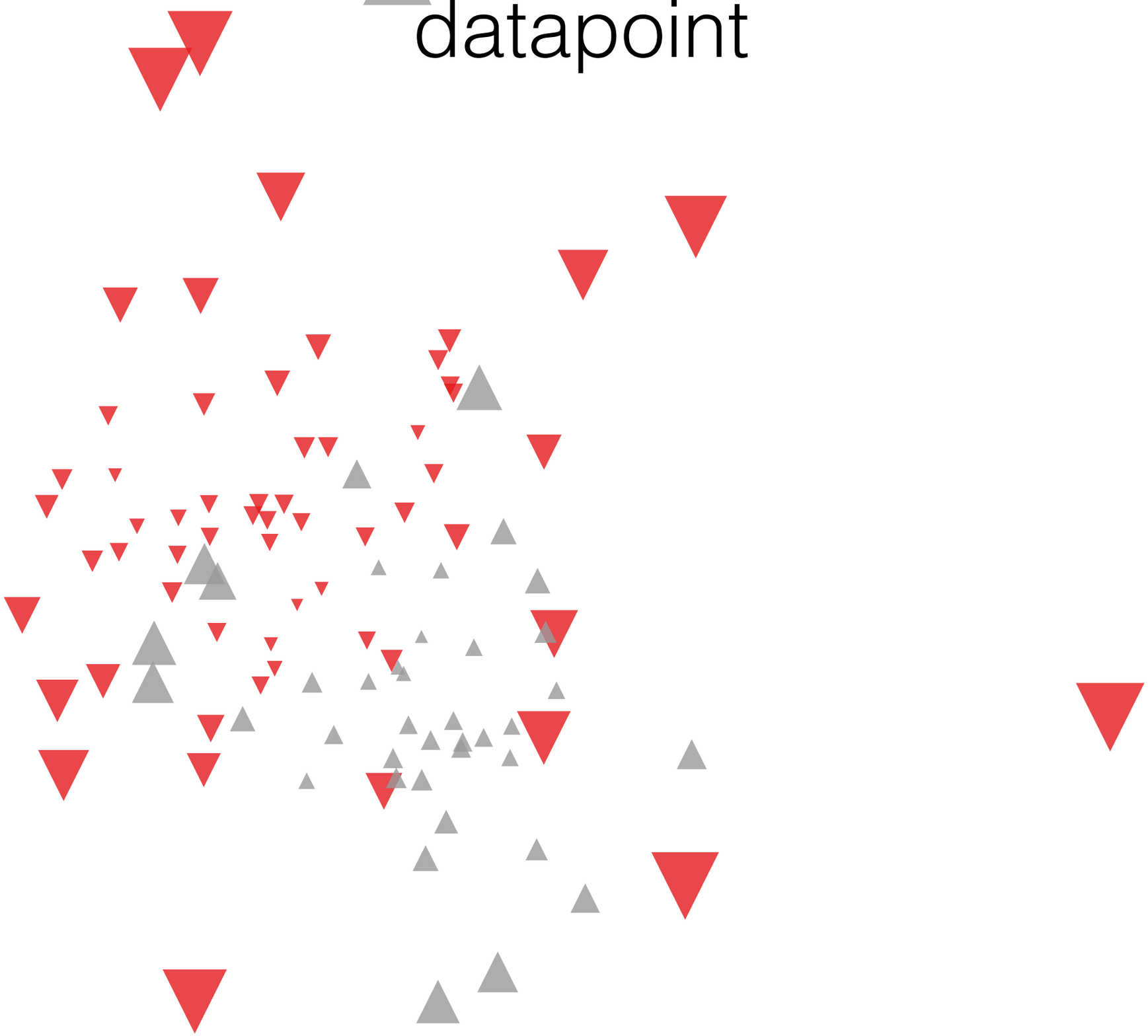

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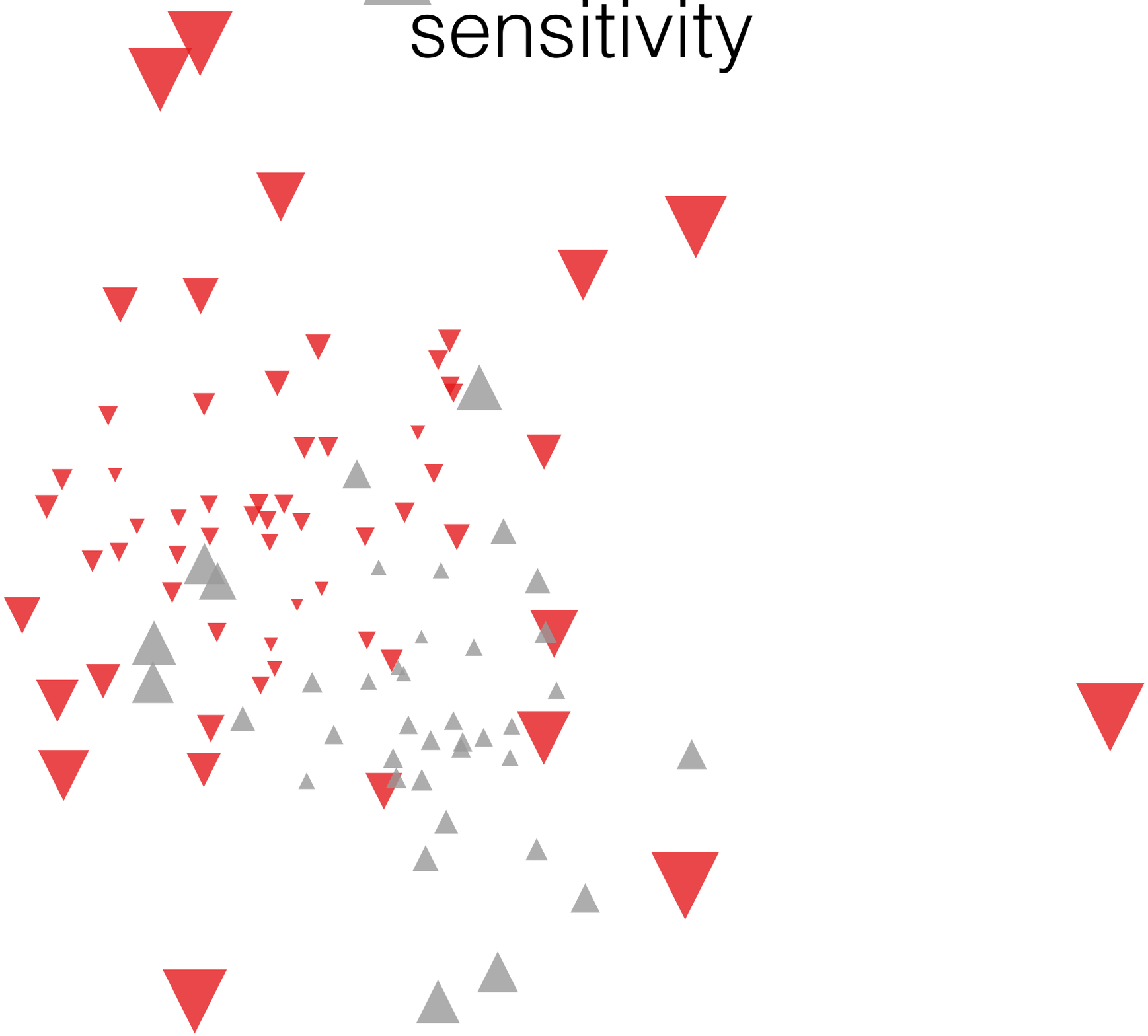

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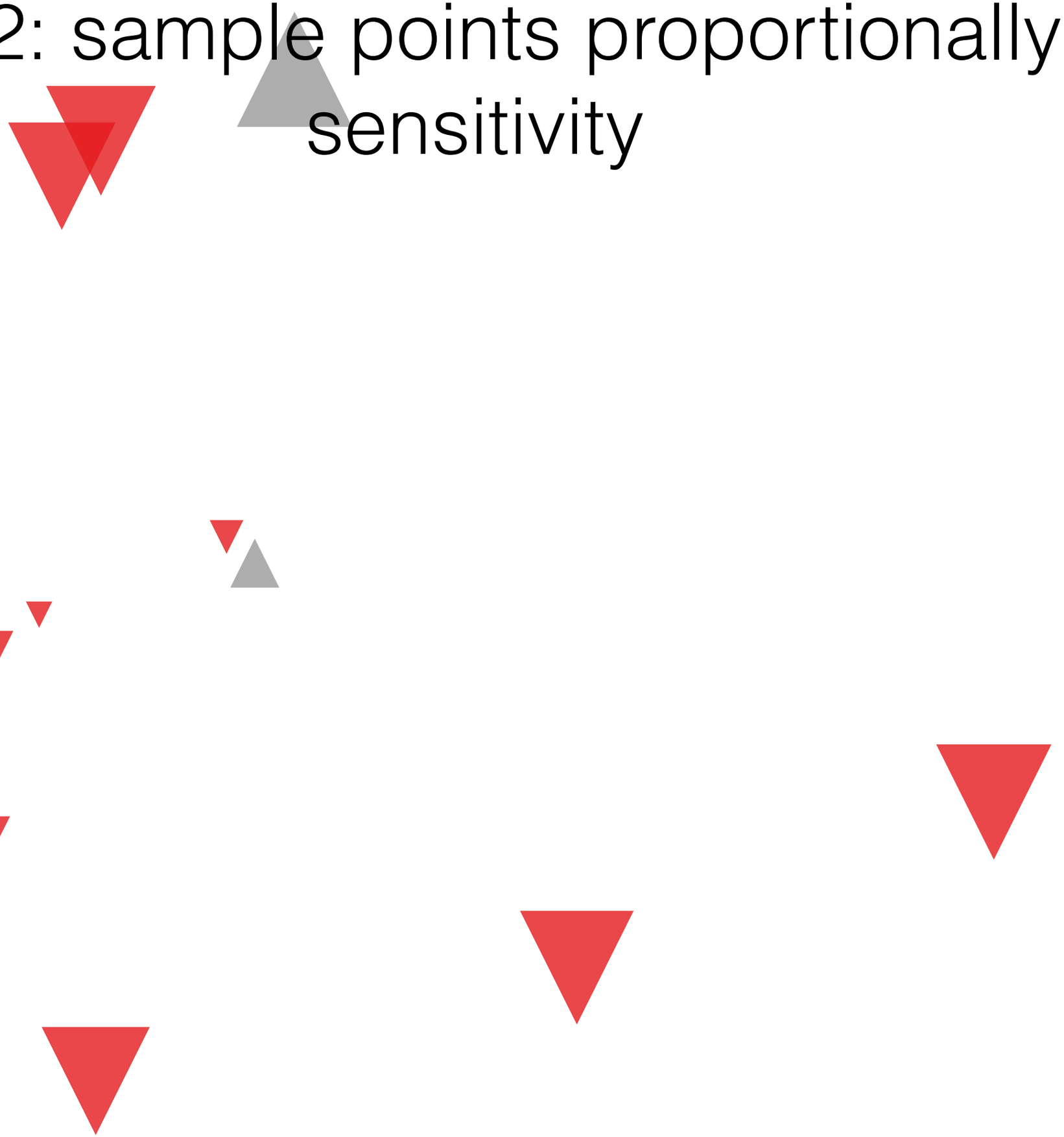

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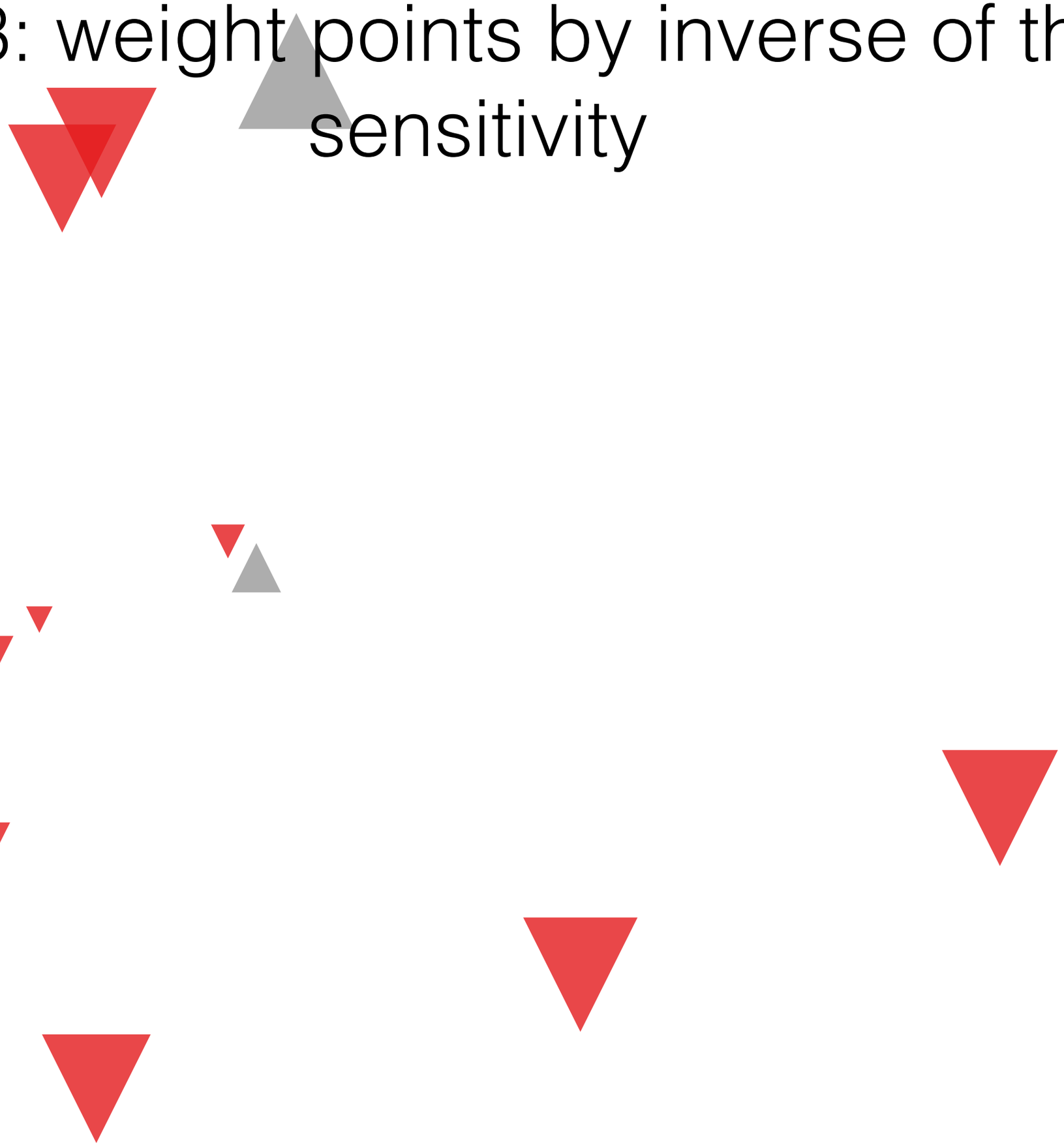

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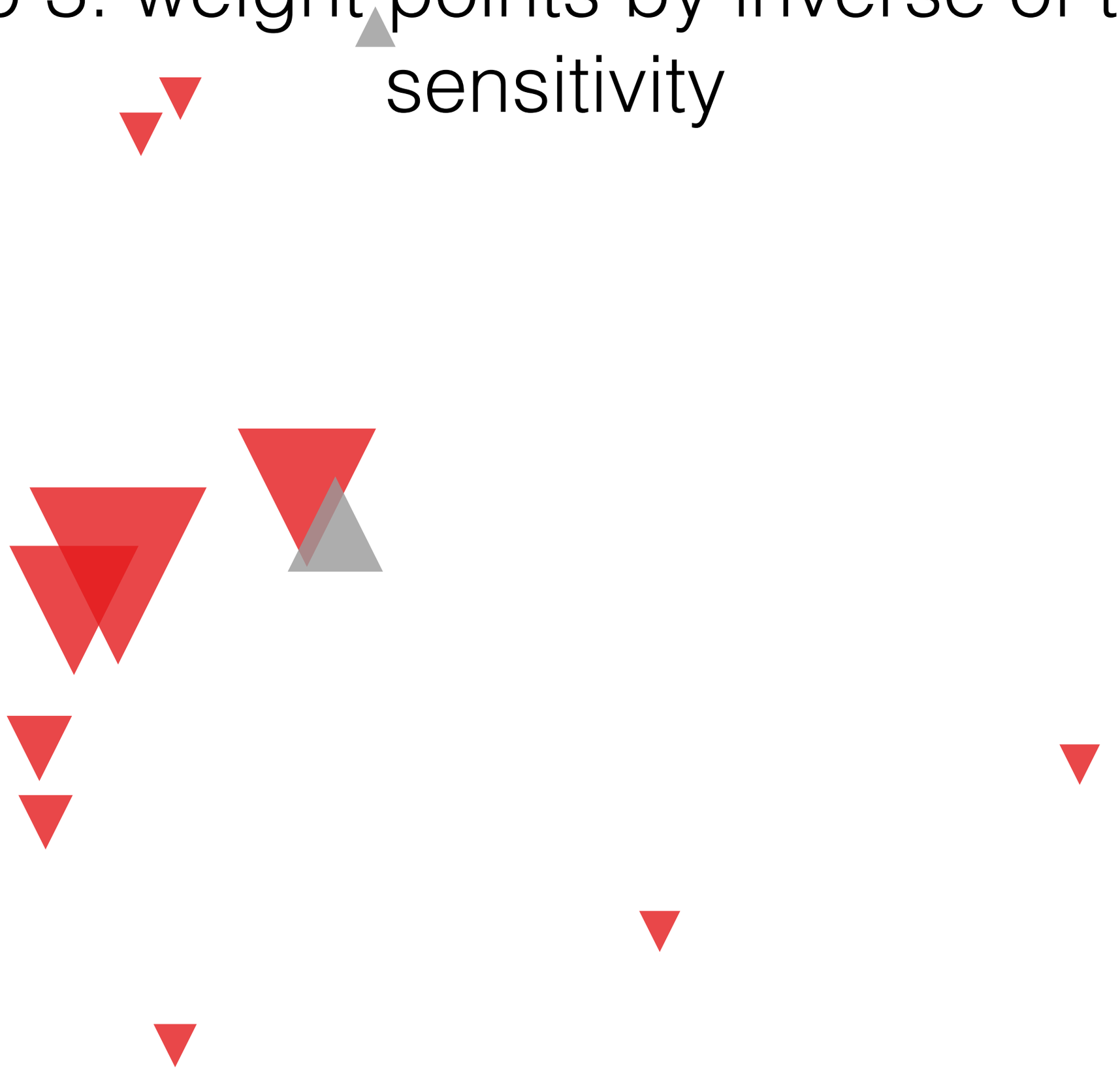

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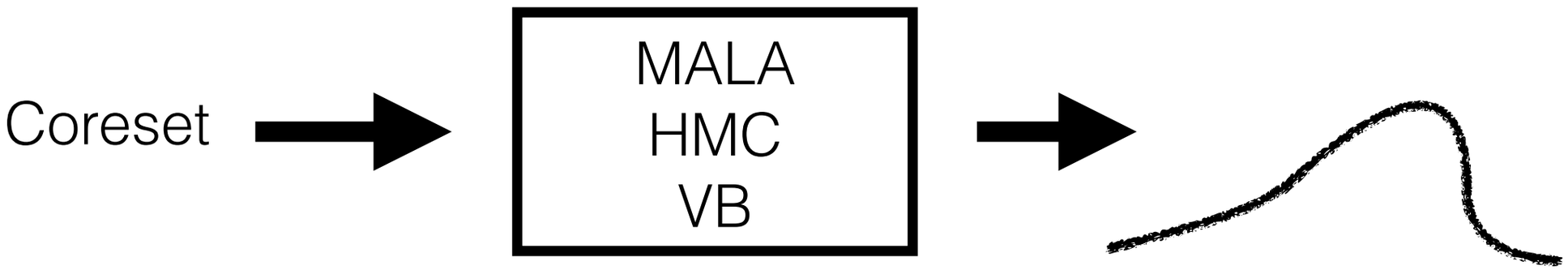

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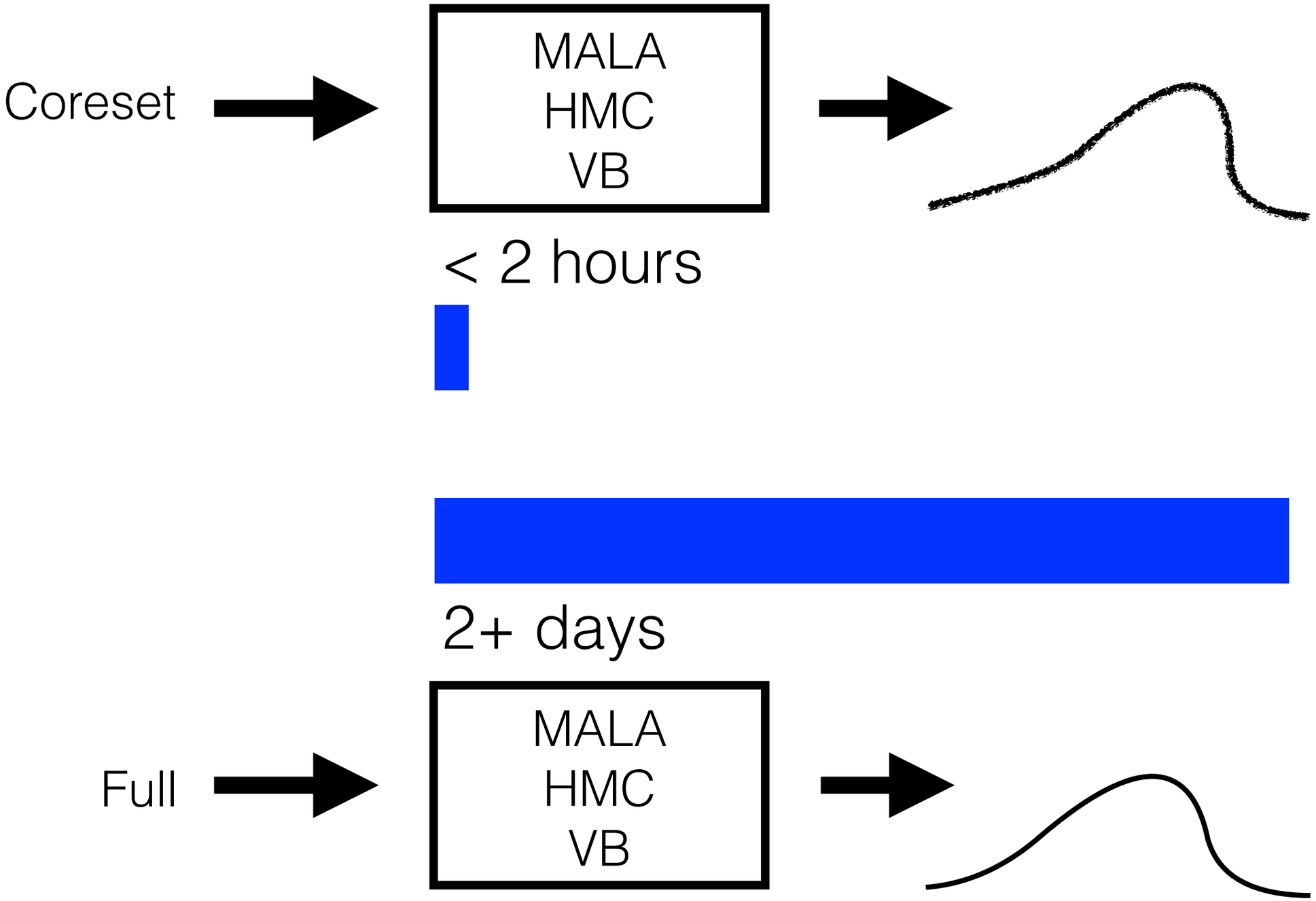

Results

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Thm sketch (HCB). Choose ε > 0, δ ∈ (0,1). Our algorithm runs in *O*(*N*) time and creates $\text{coreset-size} \sim \text{const} \cdot \epsilon^{-2} + \log(1/\delta)$

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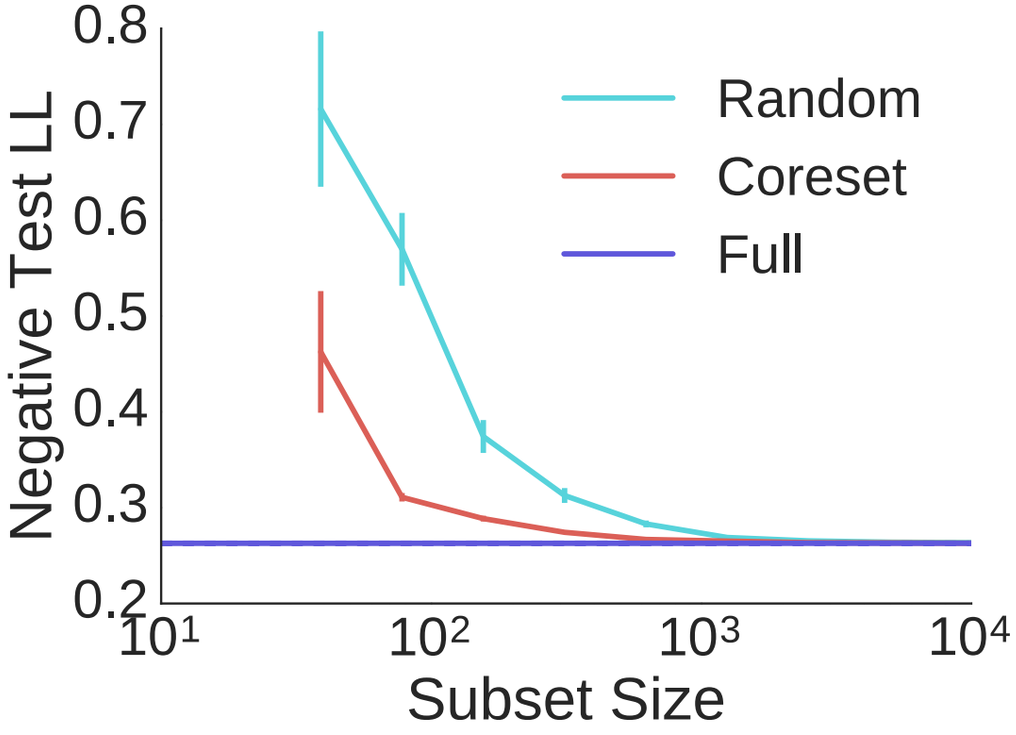
Synthetic Binary10 Data



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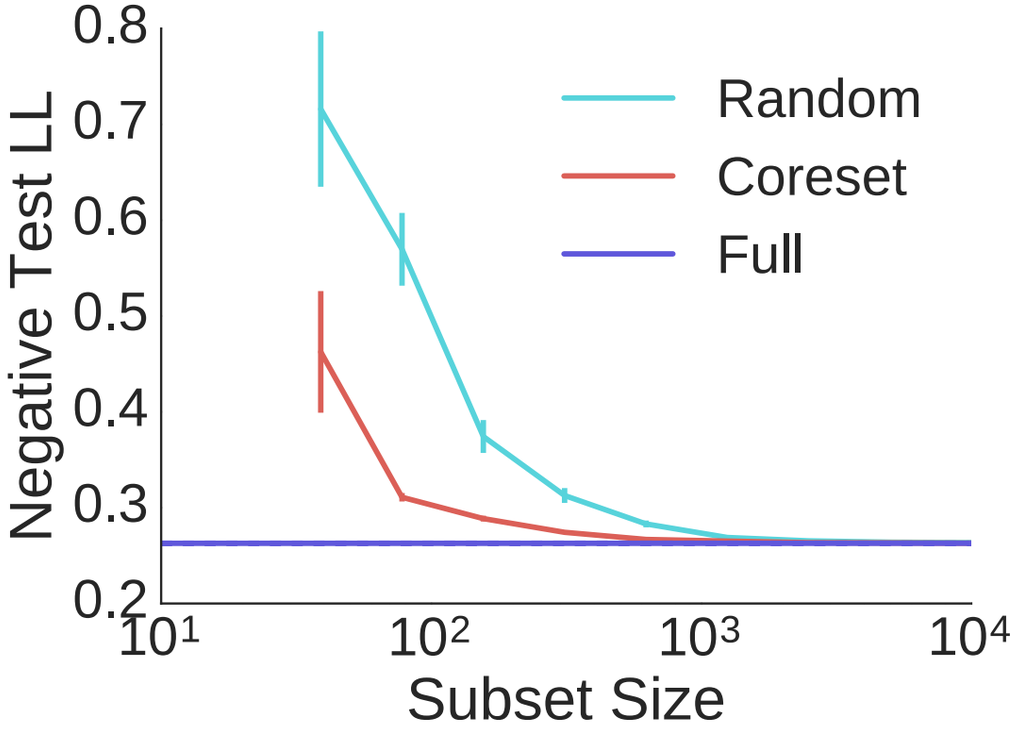
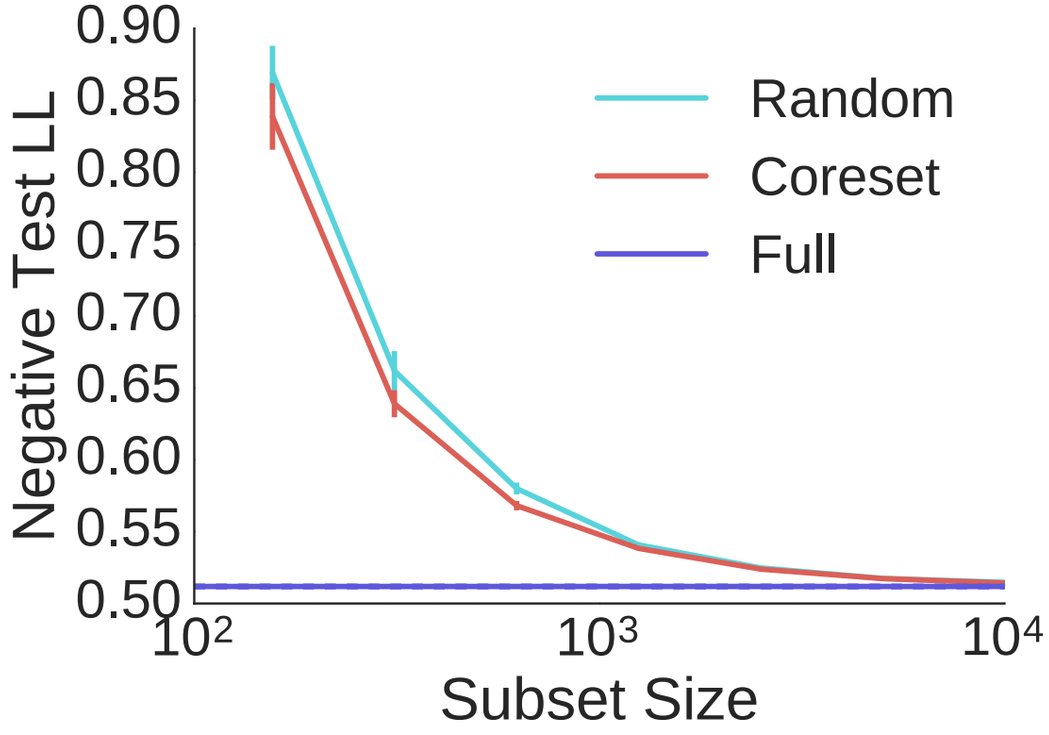


Covertype Data



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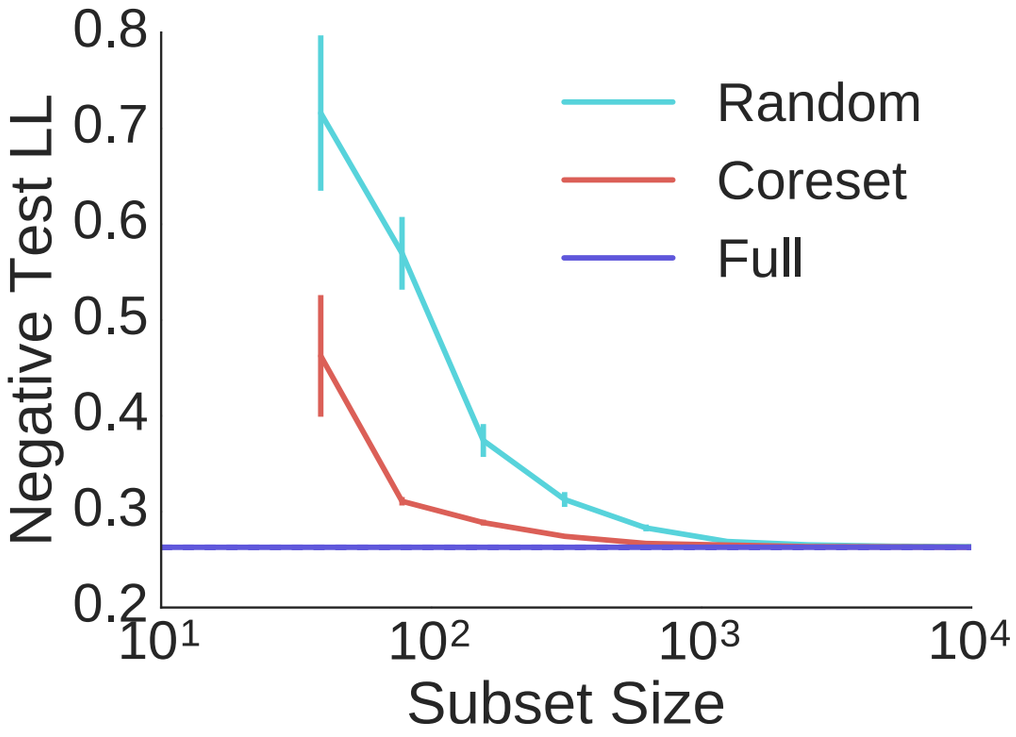

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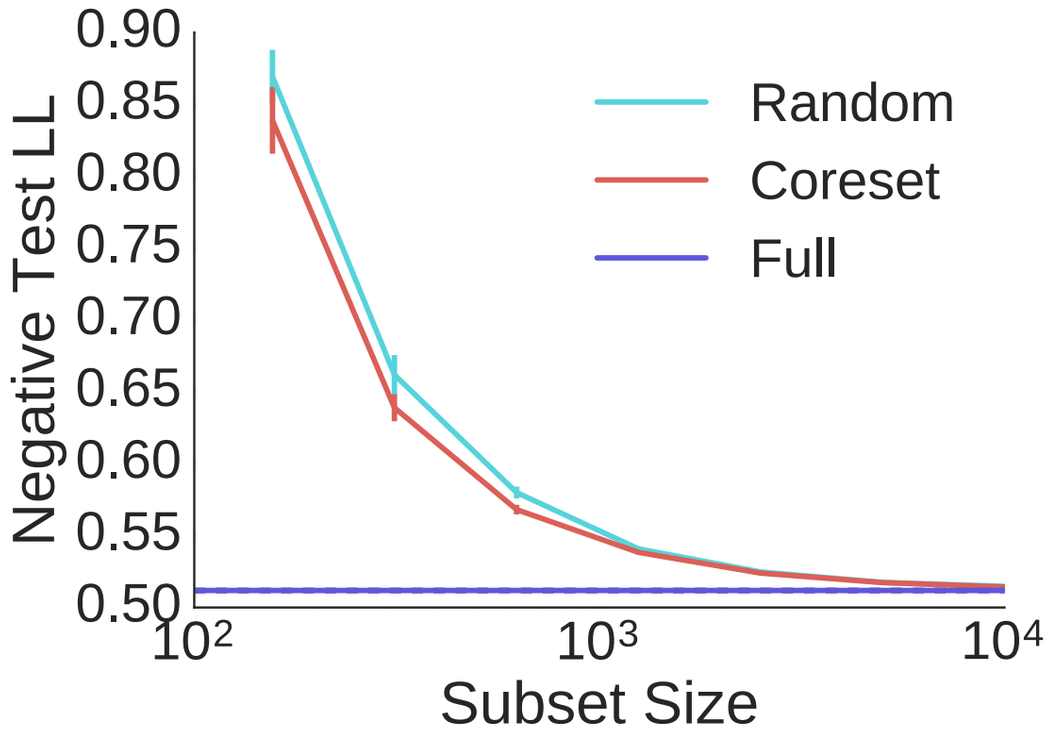

Can quantify the propagation of error in streaming and parallel settings.

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  - Point estimates (MAD-Bayes, variational Bayes)
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- Robustness trade-offs
  - Local robustness quantification (Linear response)
  - Theoretical guarantees on quality (coresets)

Roadmap

- Posterior approximation trade-offs
  - Point estimates (MAD-Bayes, variational Bayes)
  - Uncertainties (Linear response)
- Robustness trade-offs
  - Local robustness quantification (Linear response)
- Theoretical guarantees on quality (coresets)

- Interested in your data and models
  - Point estimates, uncertainty quantification, robustness quantification
  - Bayesian logistic regression; up next: high-dimensional, other models

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