Nonparametric Bayesian Methods: Models, Algorithms, and Applications

Tamara Broderick
ITT Career Development Assistant Professor
Electrical Engineering & Computer Science
MIT

Nonparametric Bayes

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"Wikipedia phenomenon"

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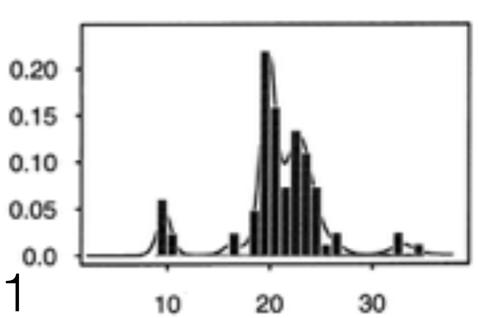

[Escobar, West 1995; Ghosal et al 1999]

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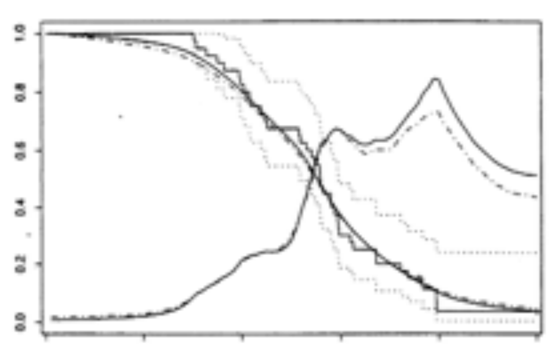

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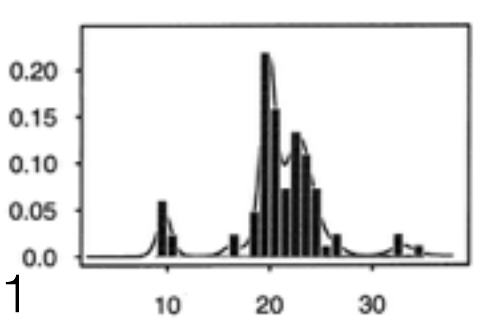

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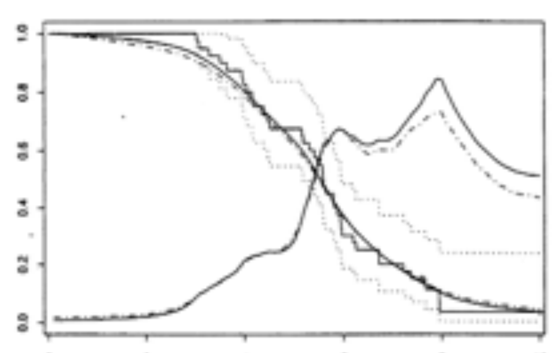

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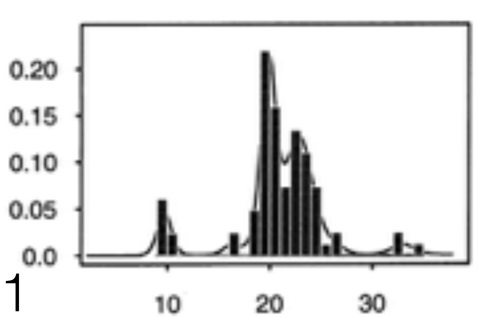

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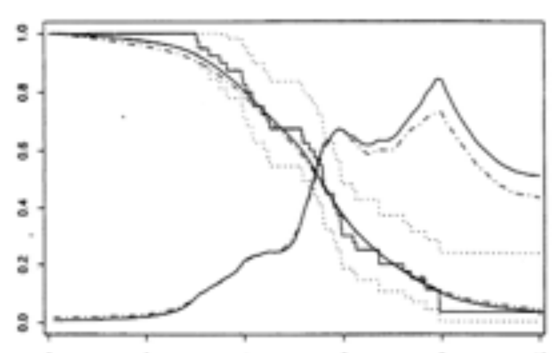

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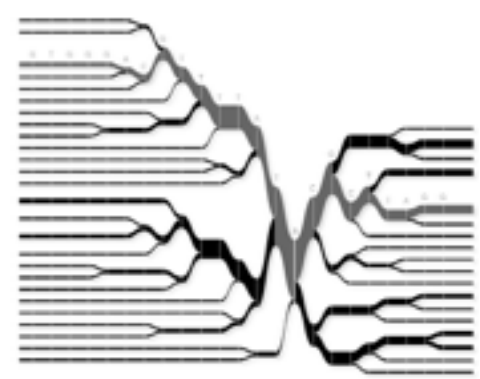
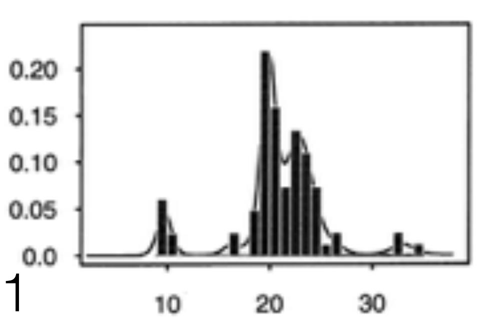


[Ewens, 1972; Hartl, Clark 2003]



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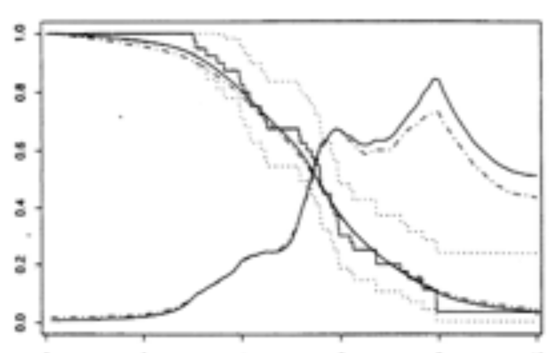

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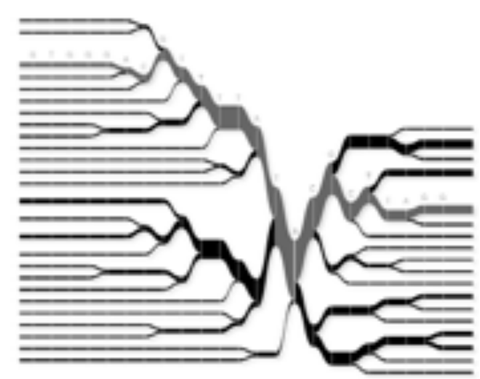

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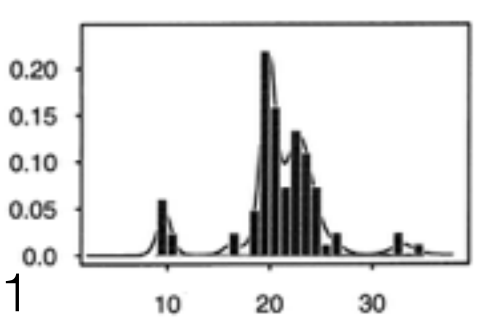

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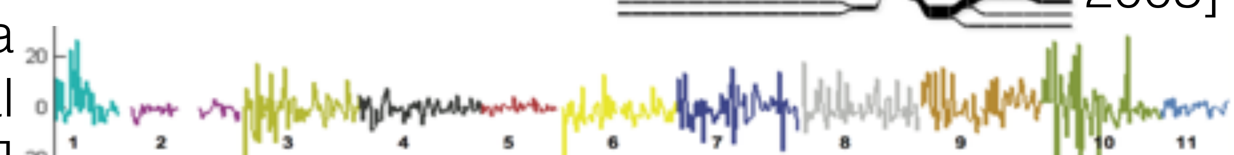

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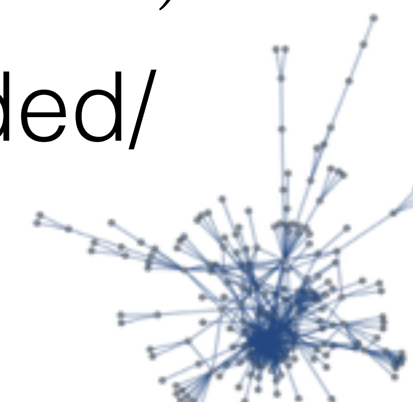

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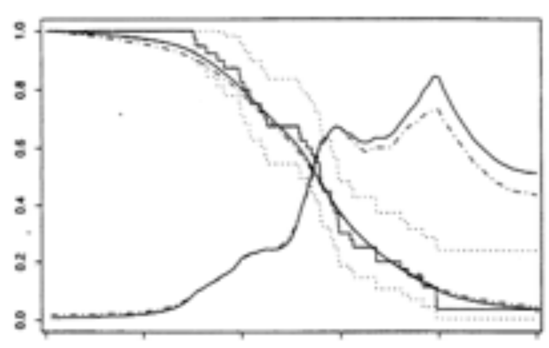

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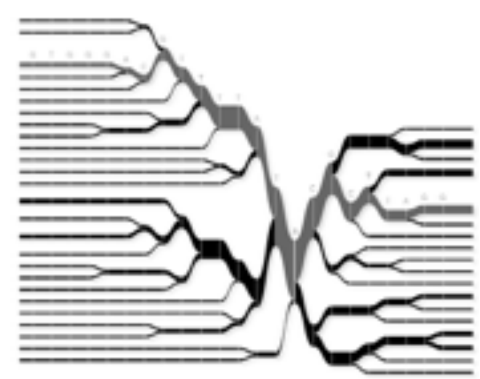

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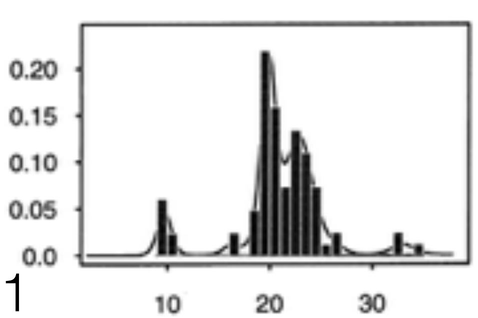

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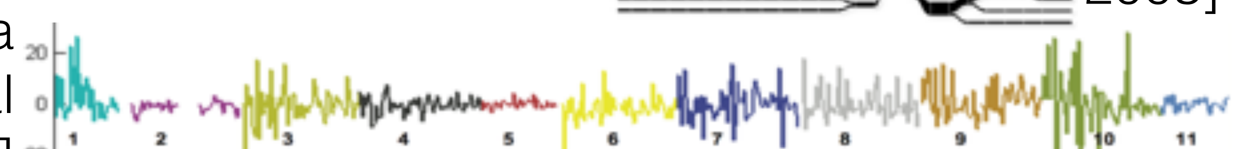

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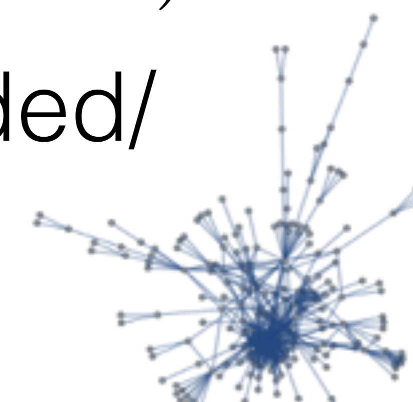

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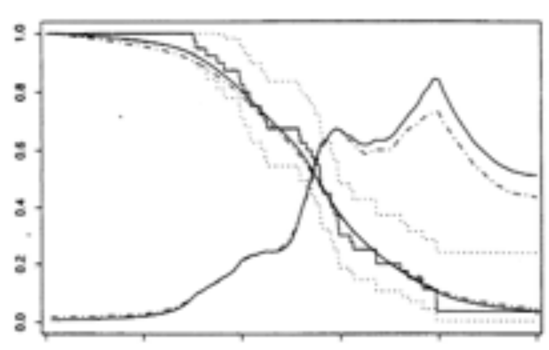

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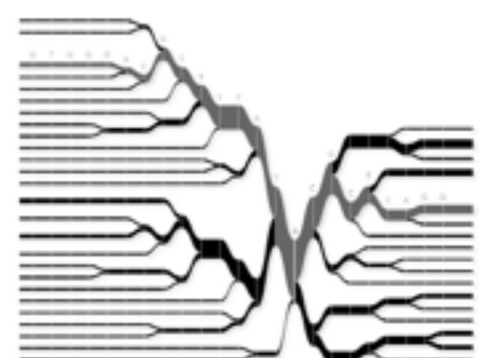

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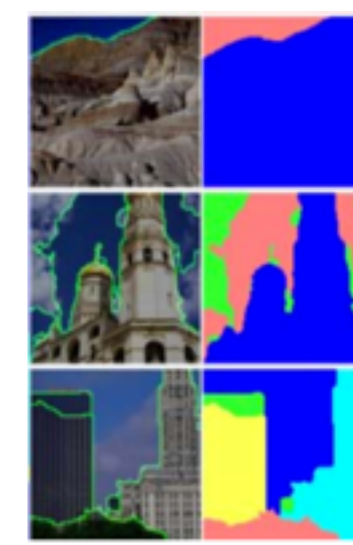

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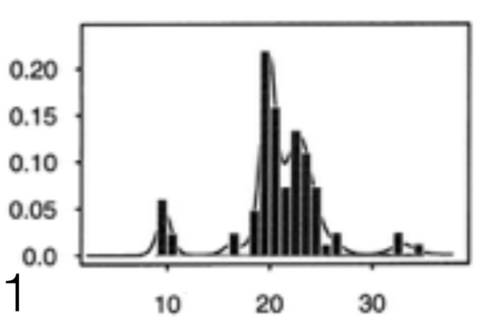

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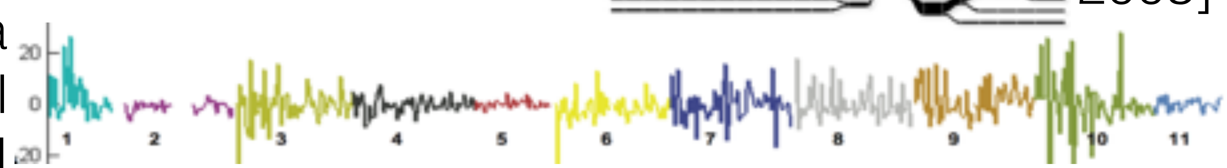

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    - Why is NPBayes challenging but practical?

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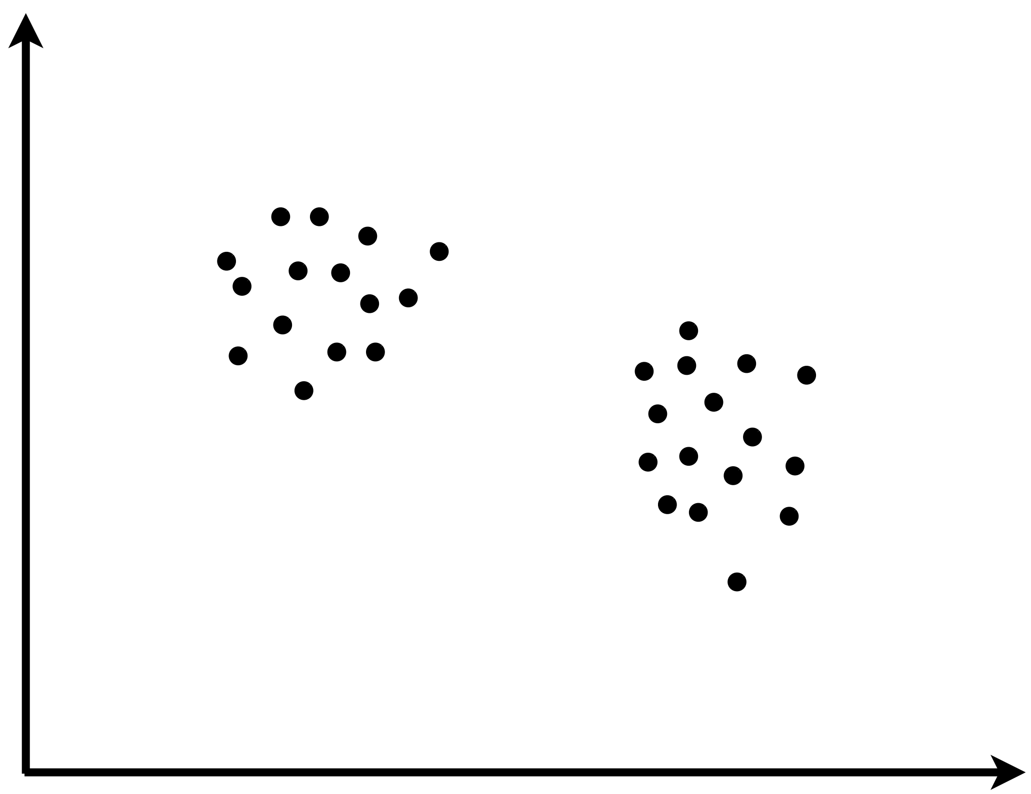

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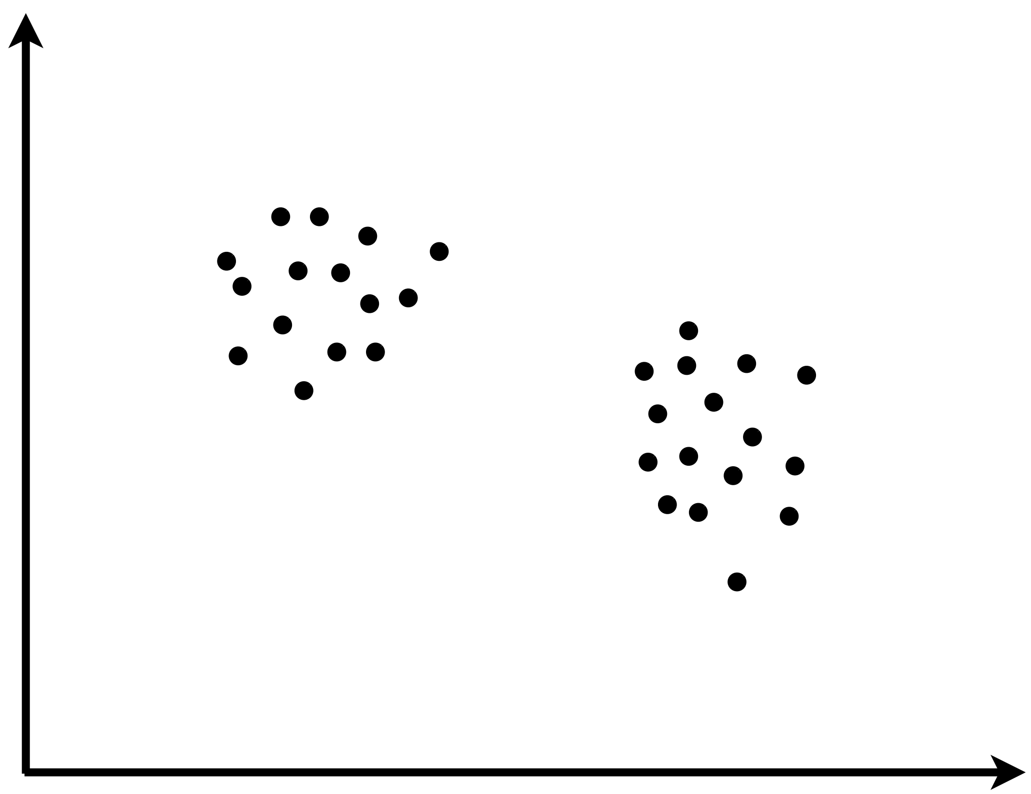

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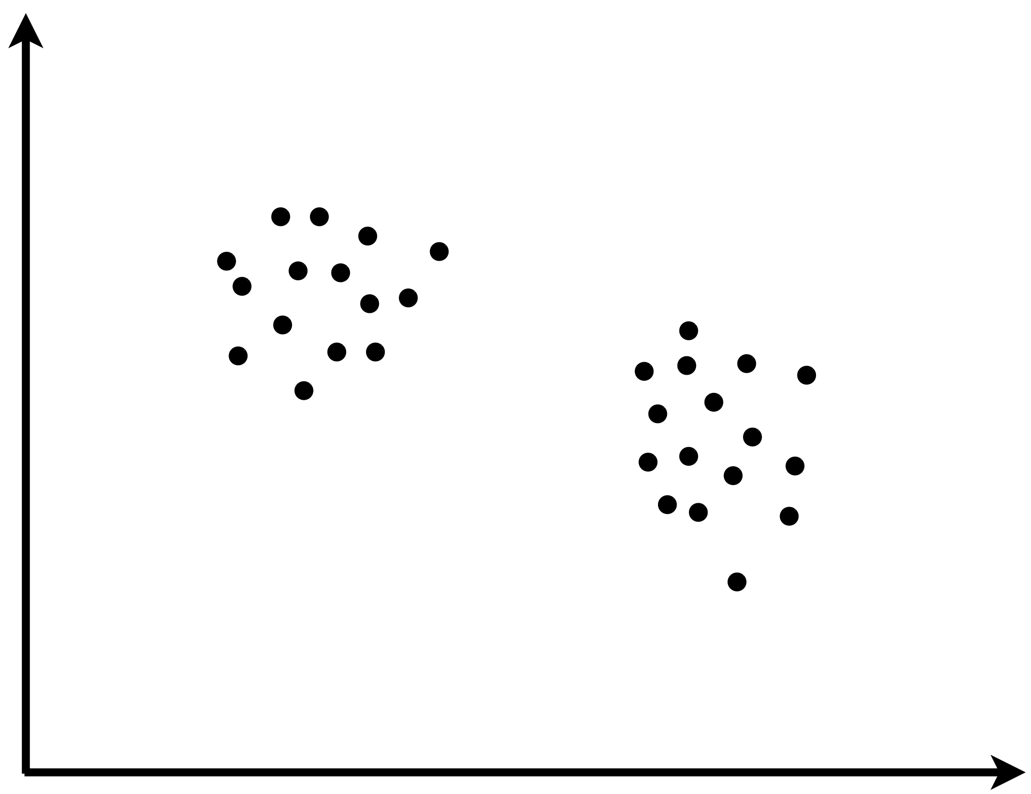

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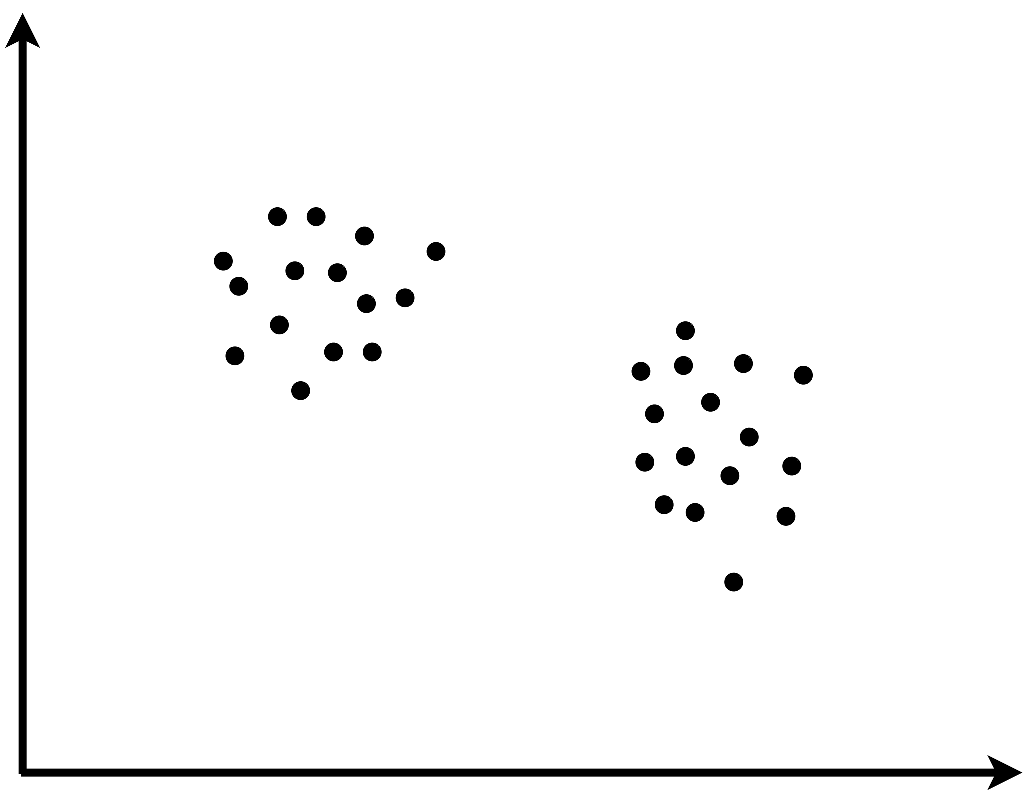

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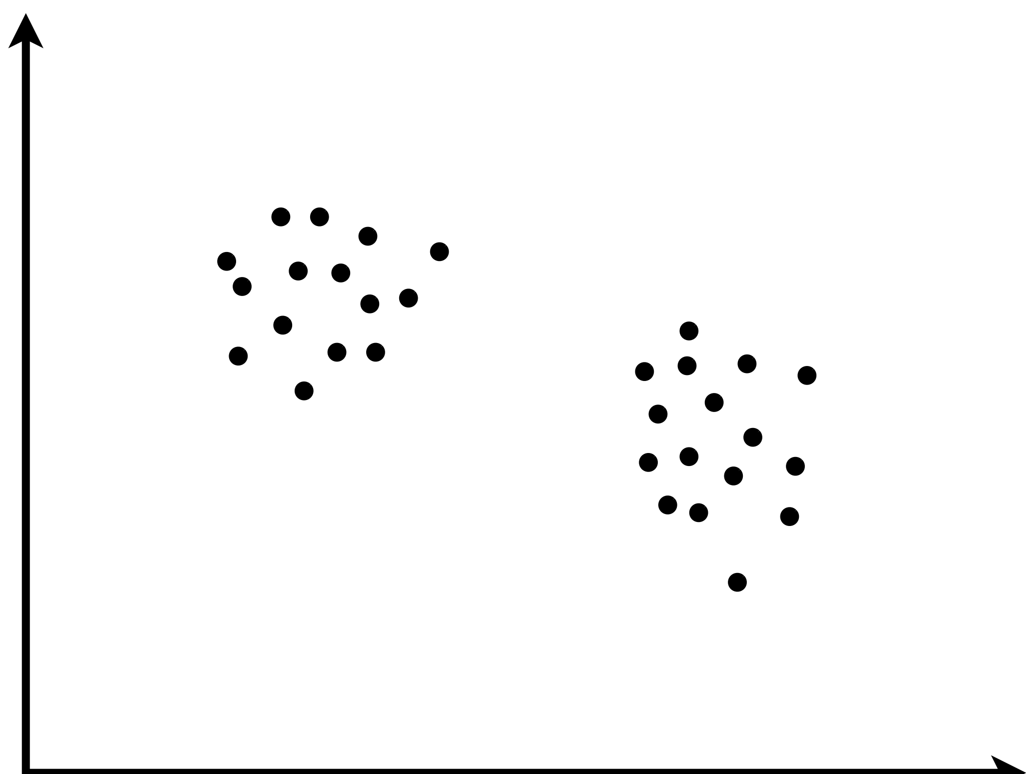

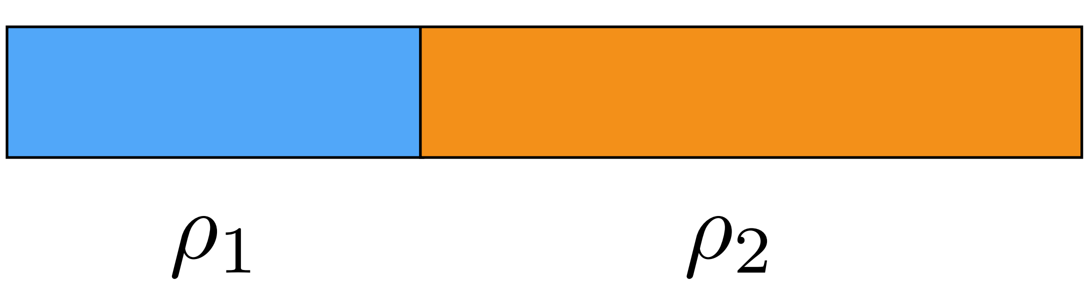

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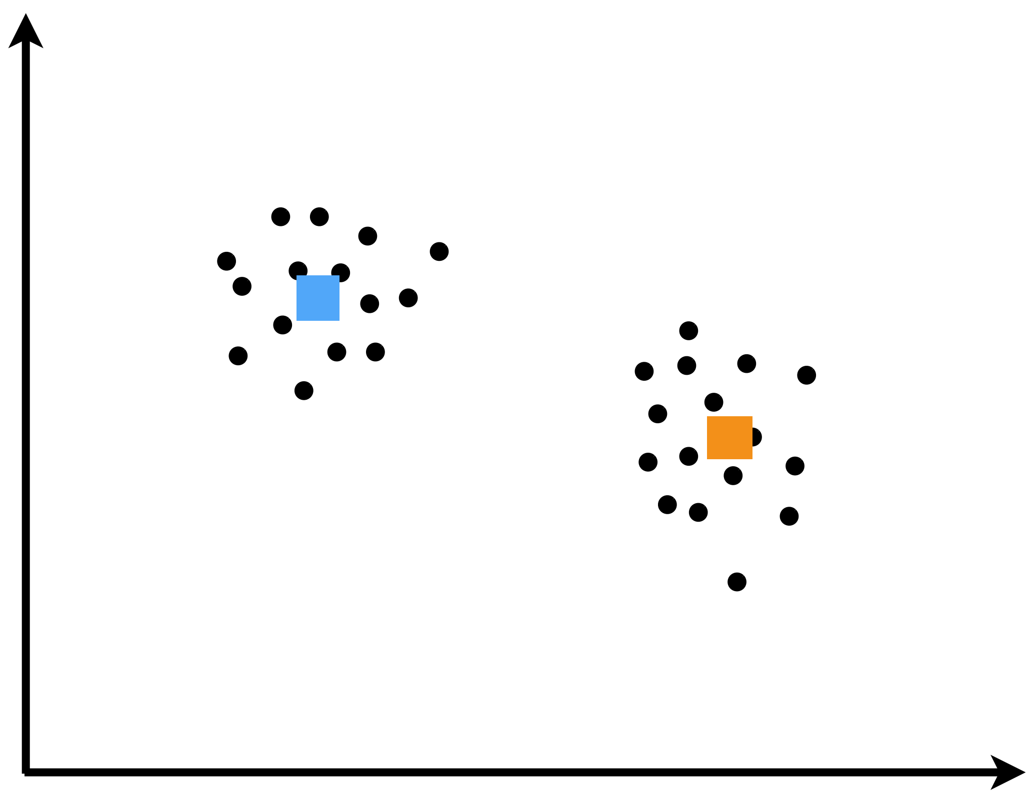

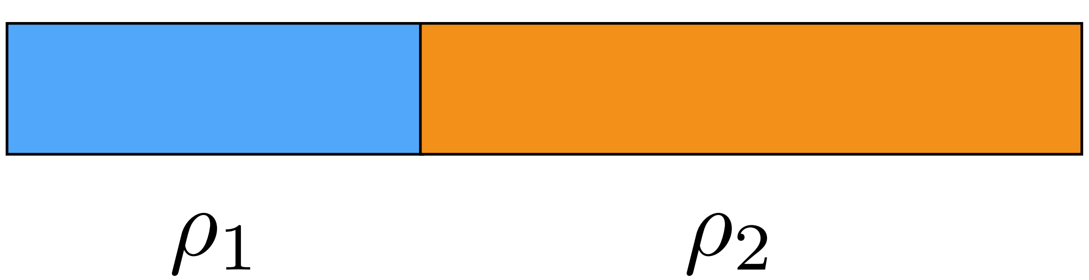

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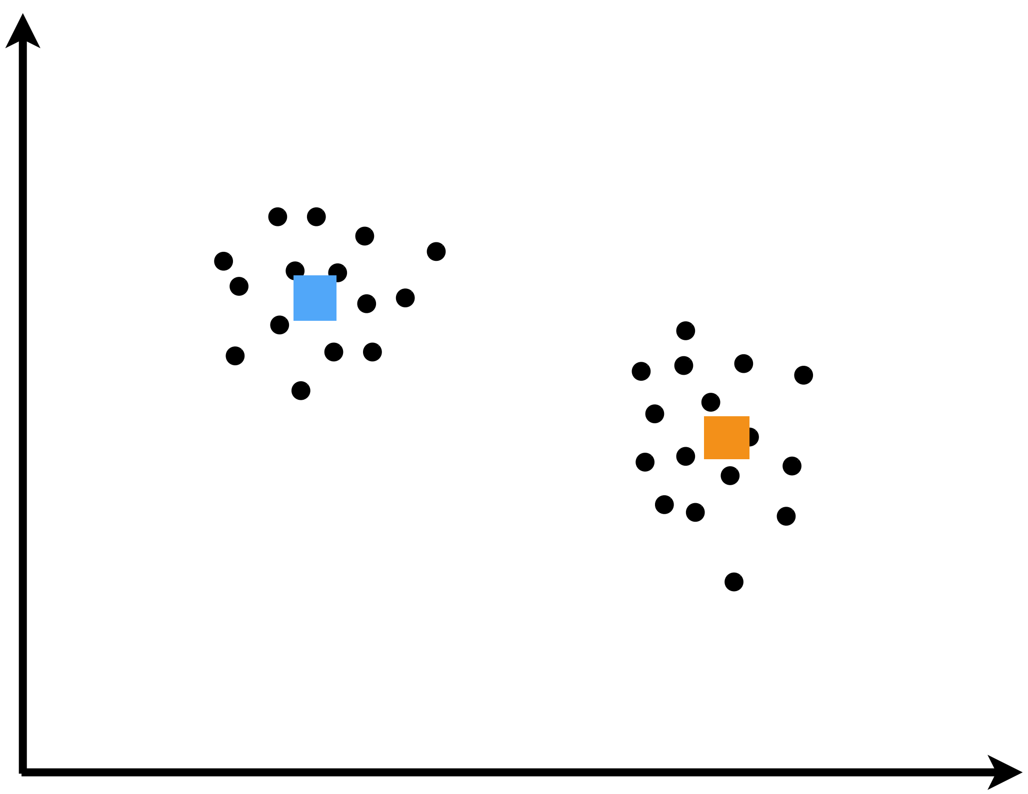

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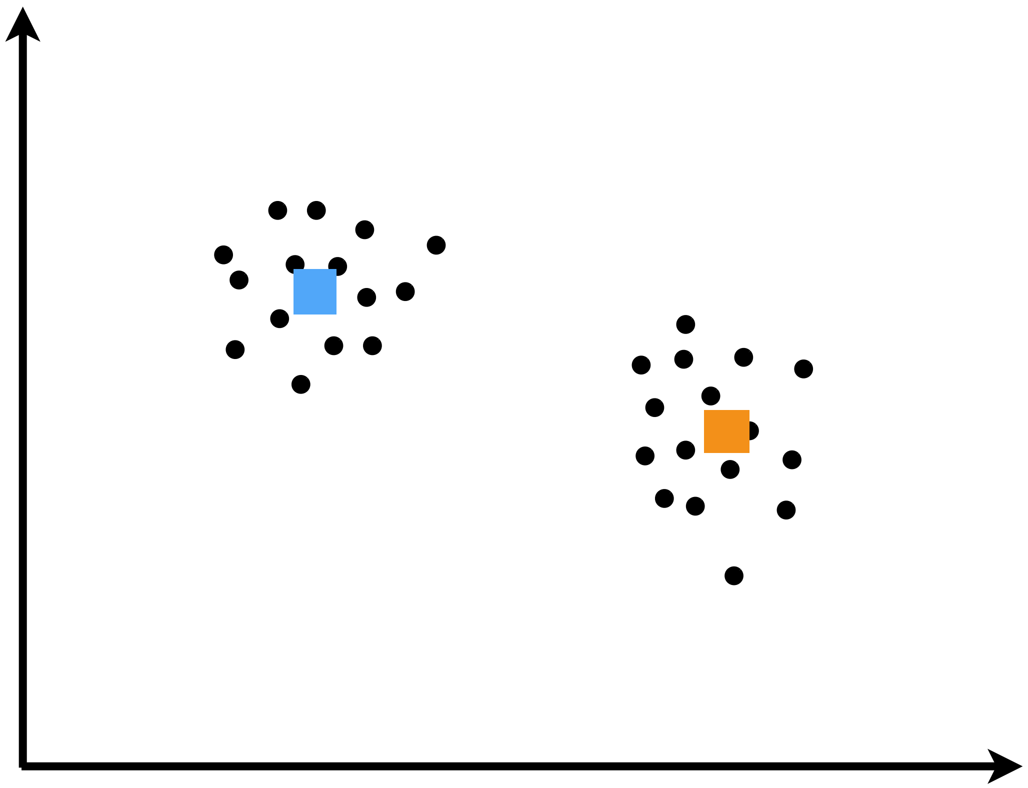

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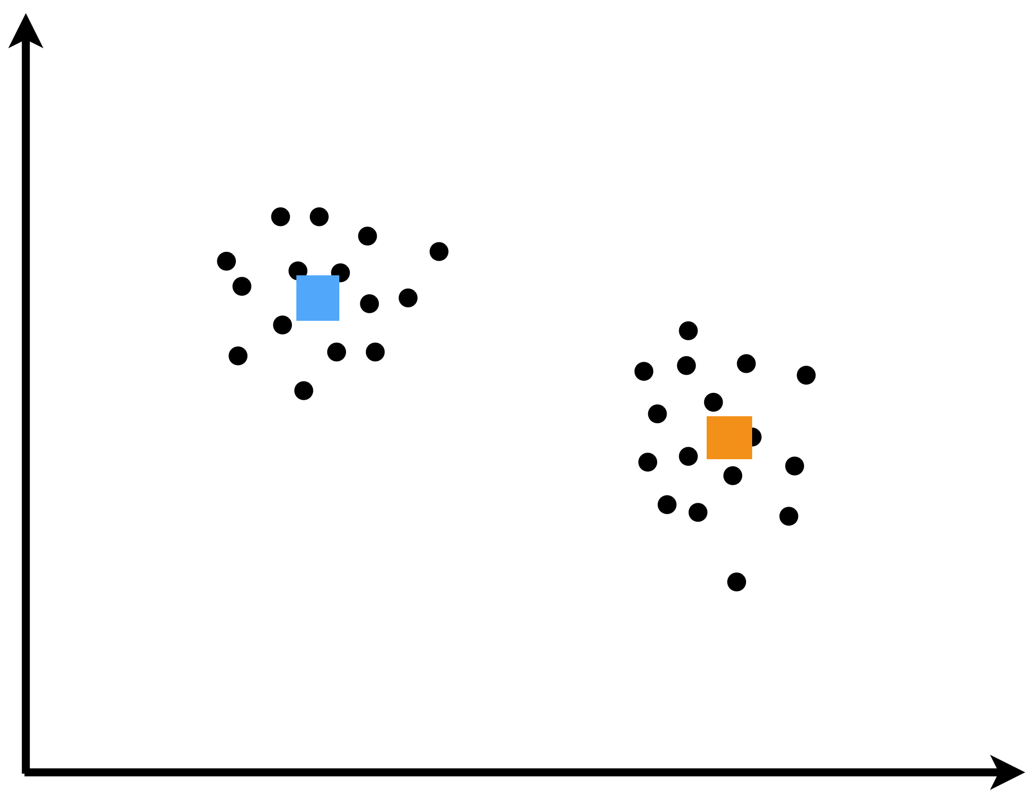

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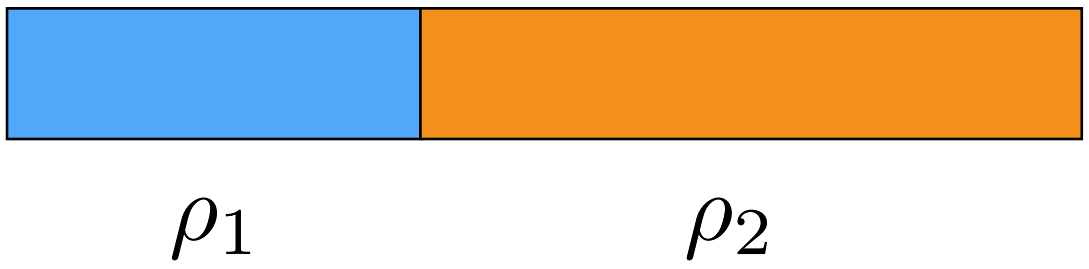

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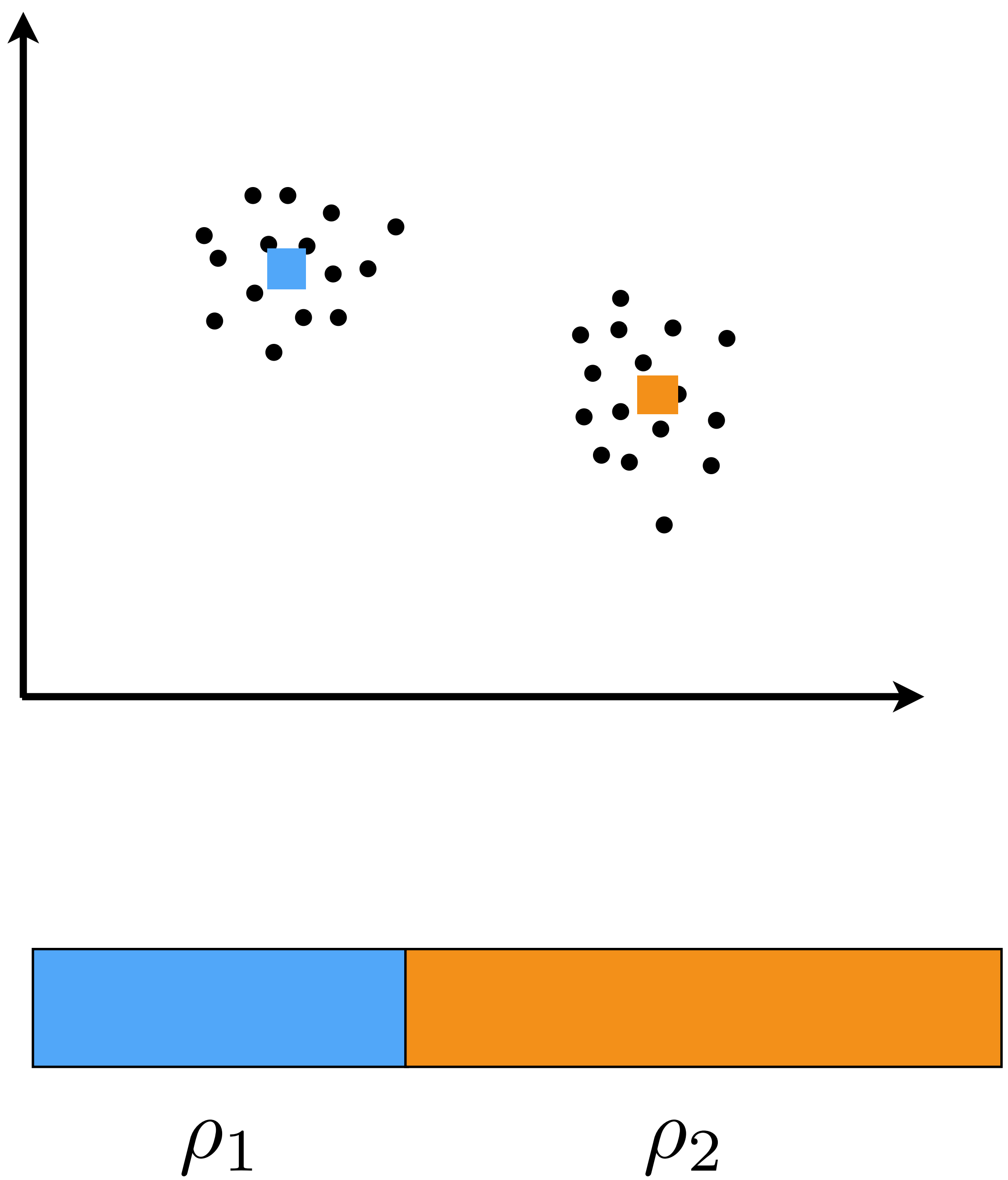

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  $$\rho_1 \sim \text{Beta}(a_1, a_2)$$
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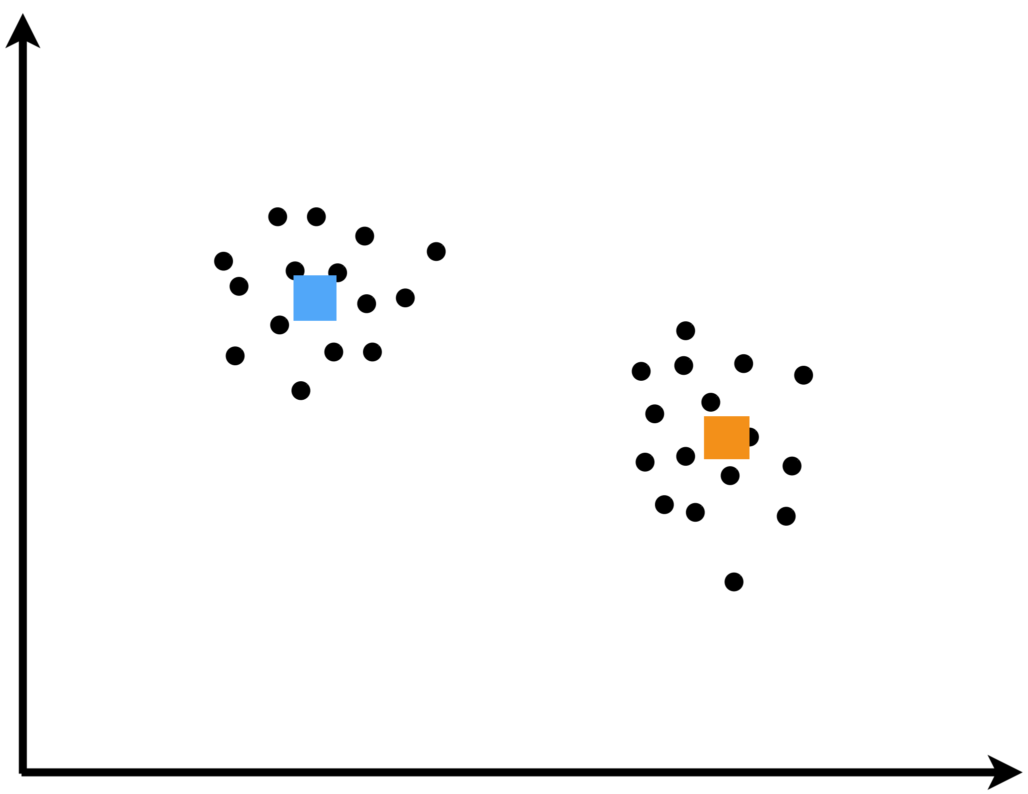

- Finite Gaussian mixture model (K=2 clusters)

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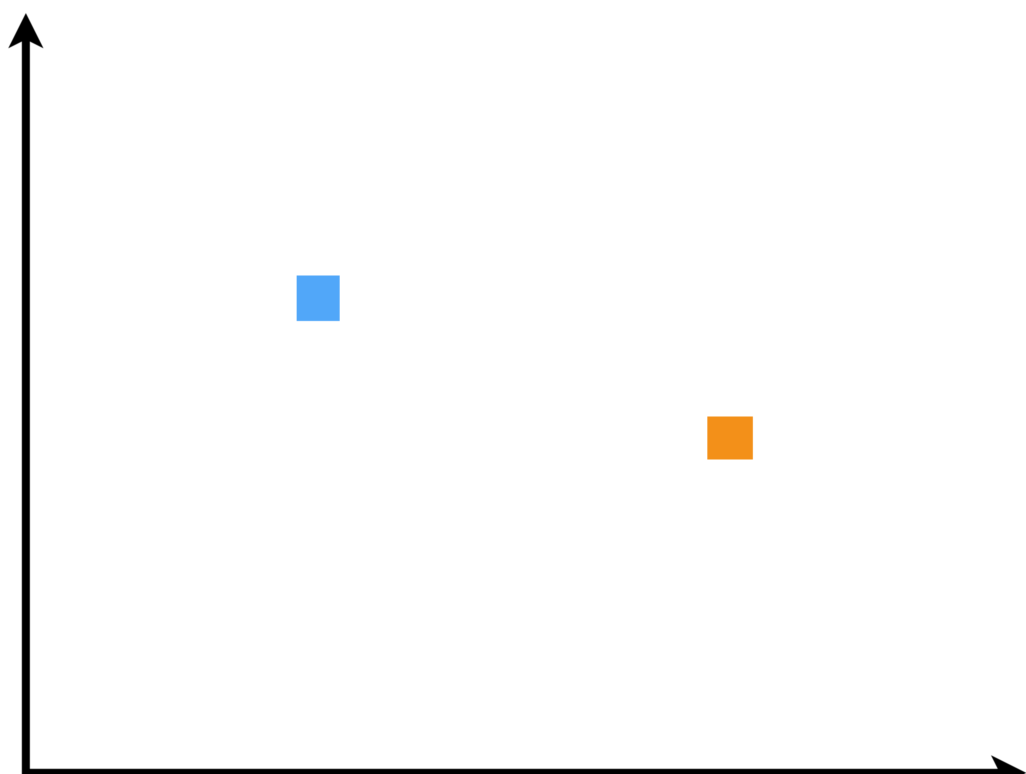

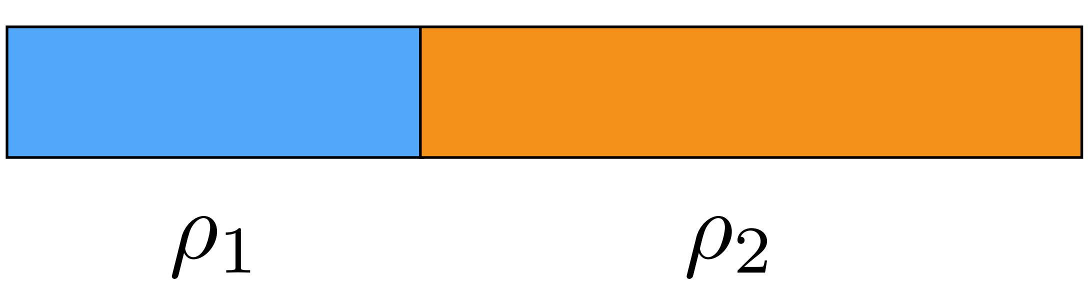

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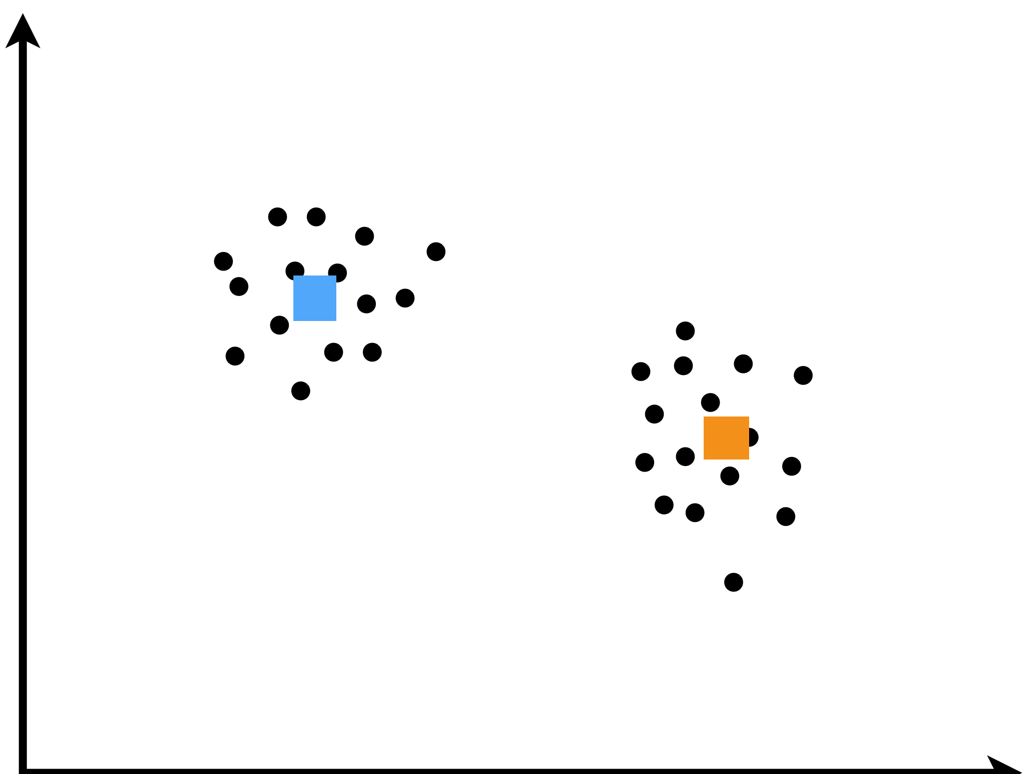

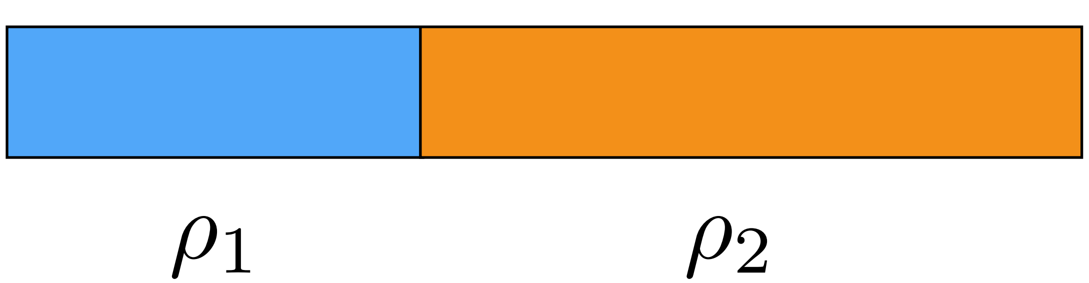

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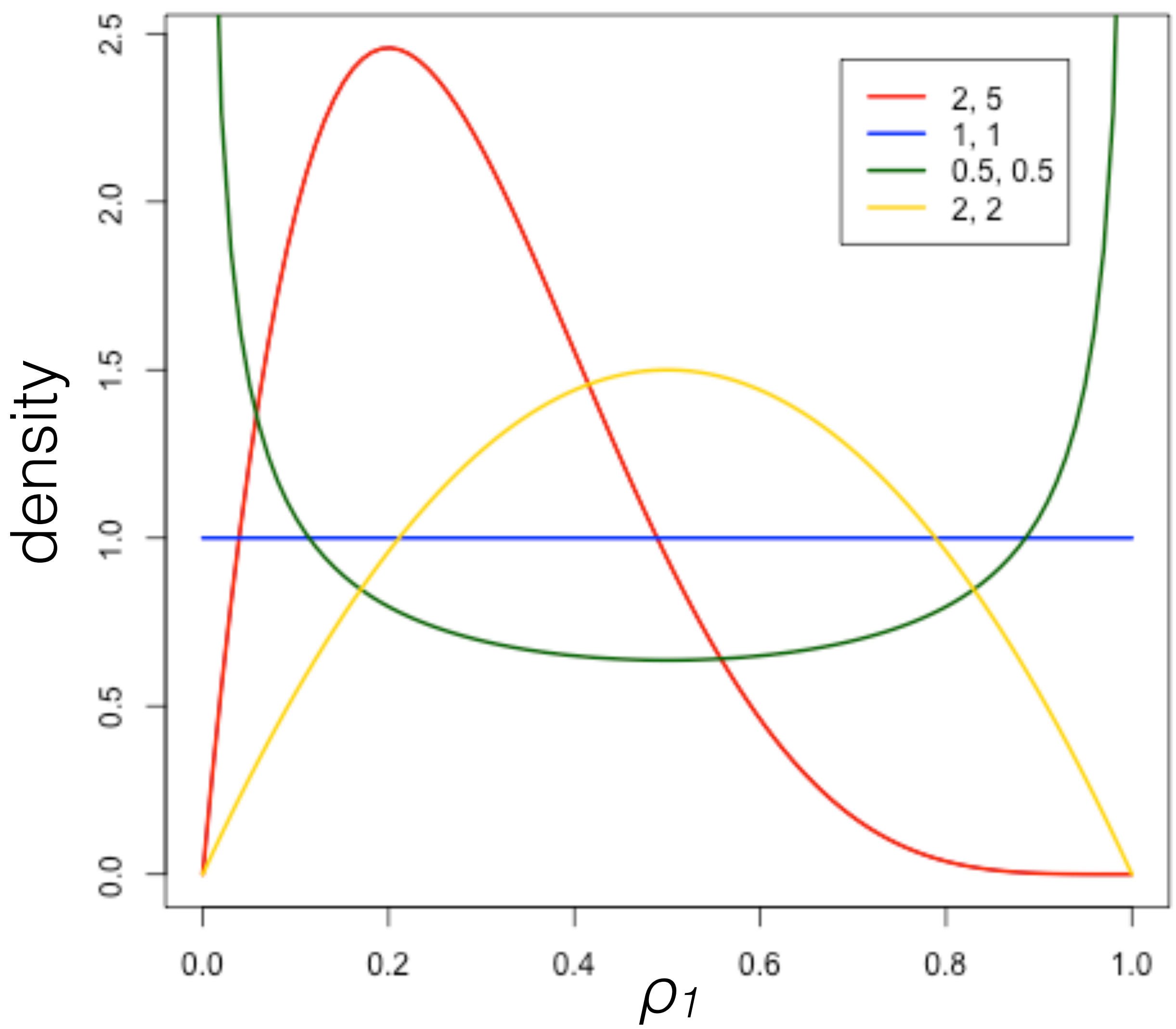

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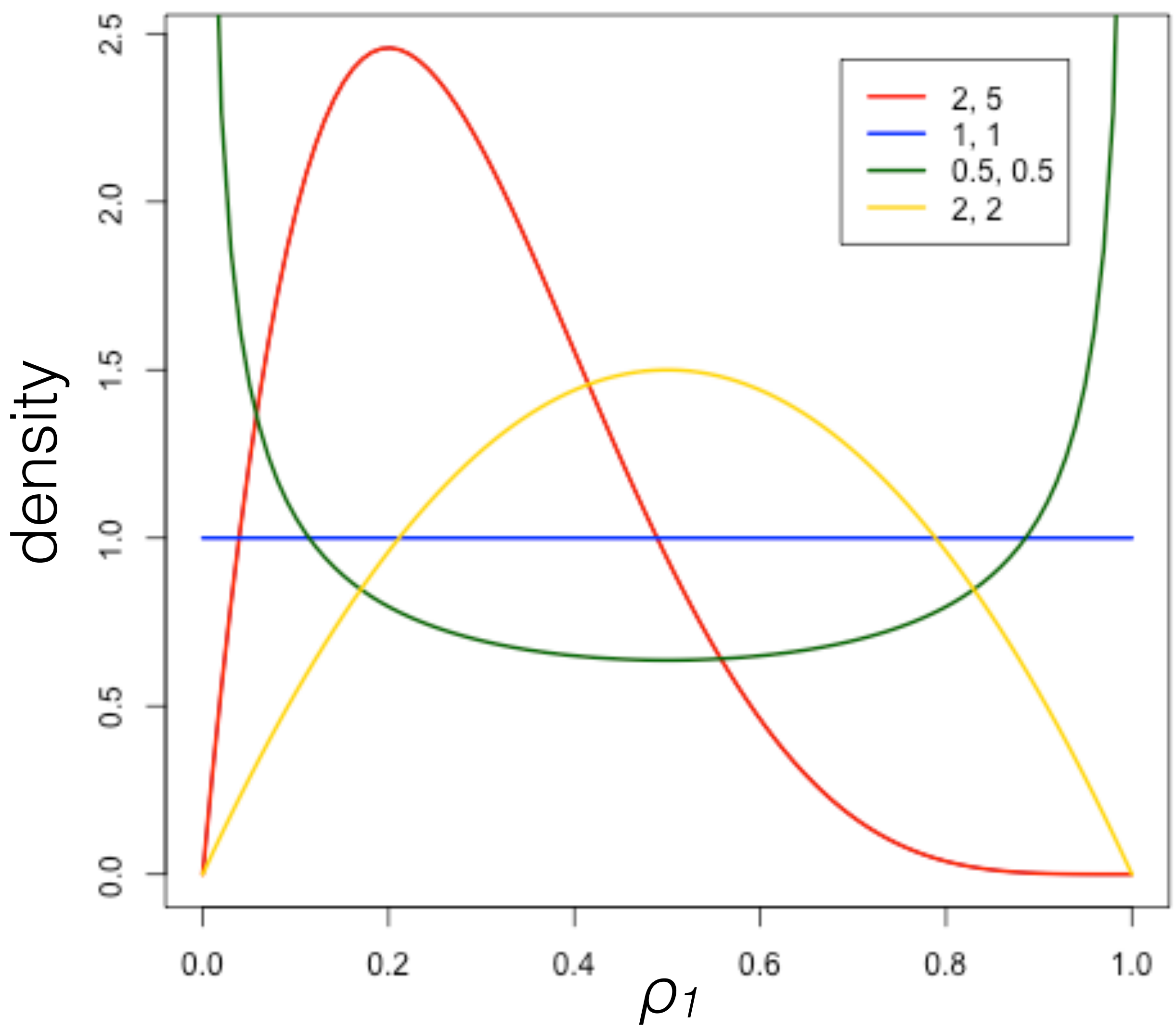

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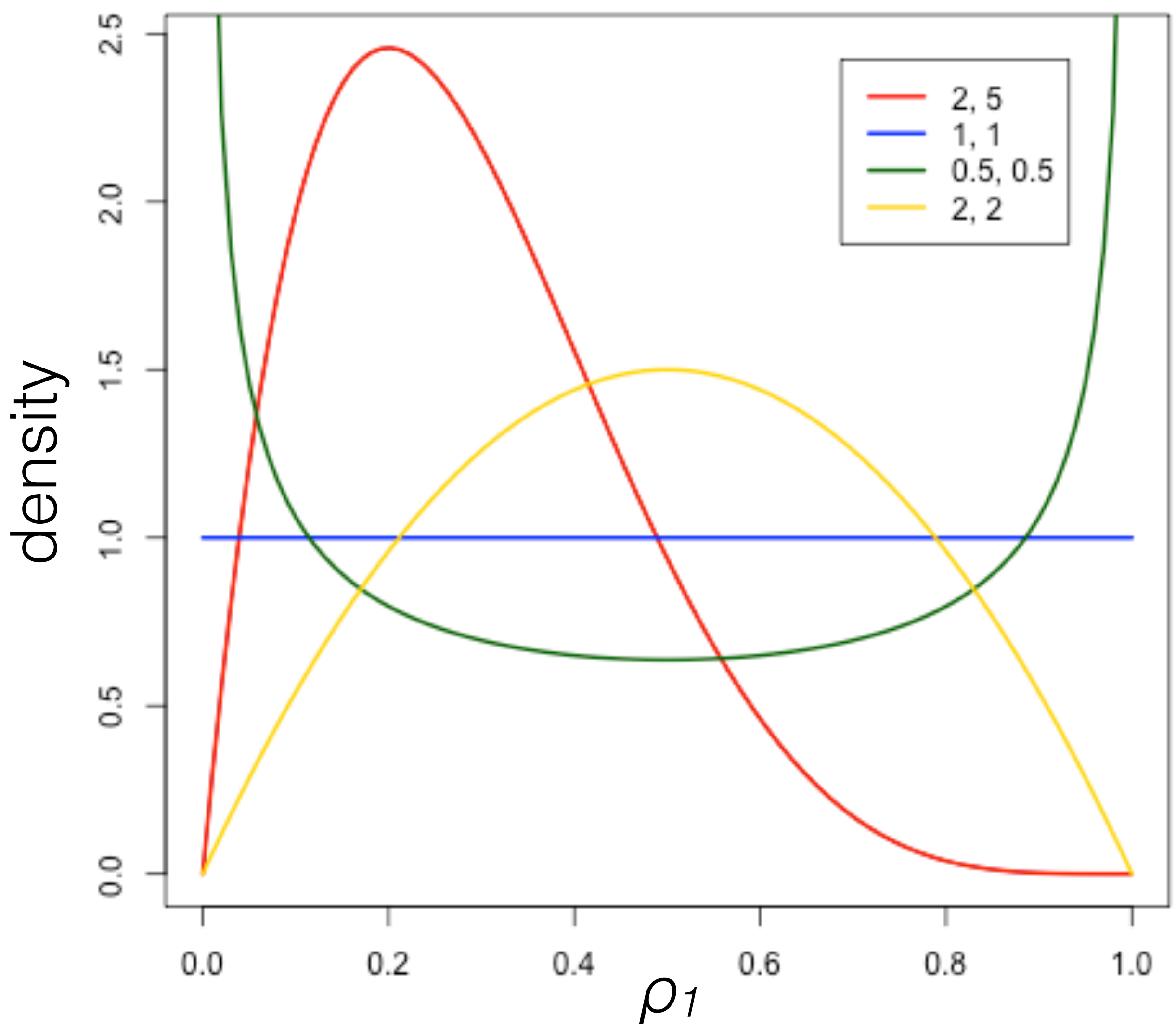

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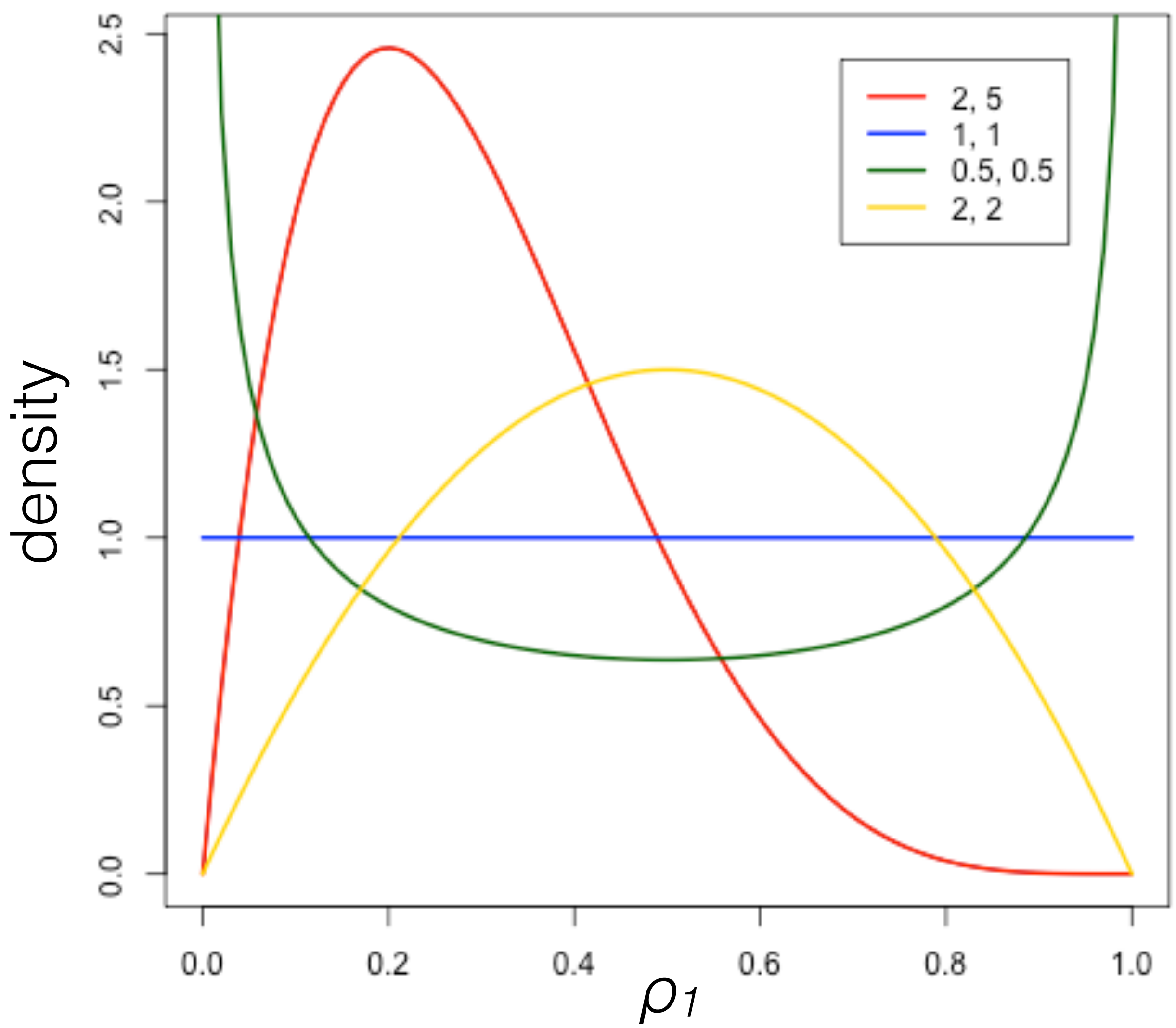

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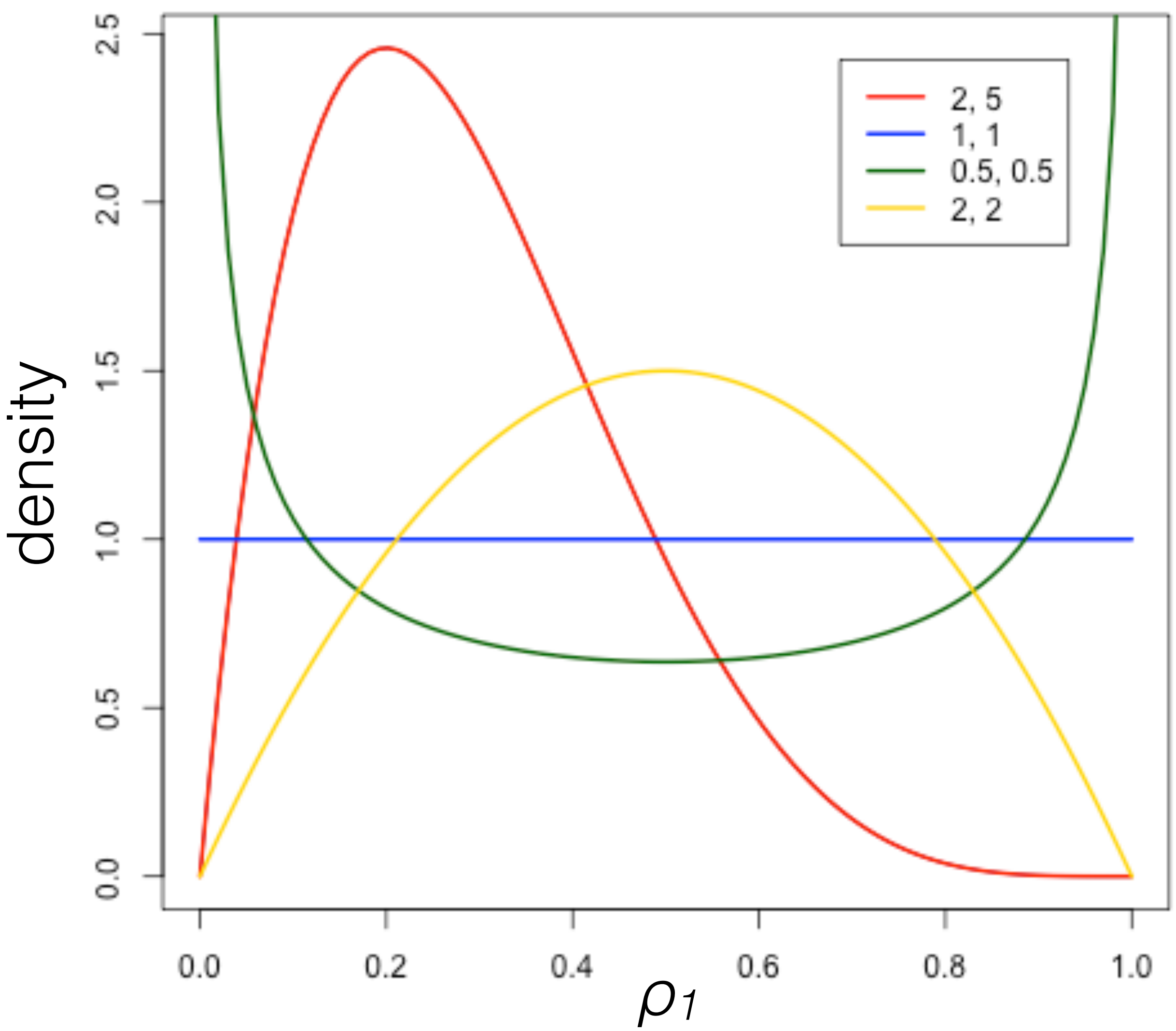

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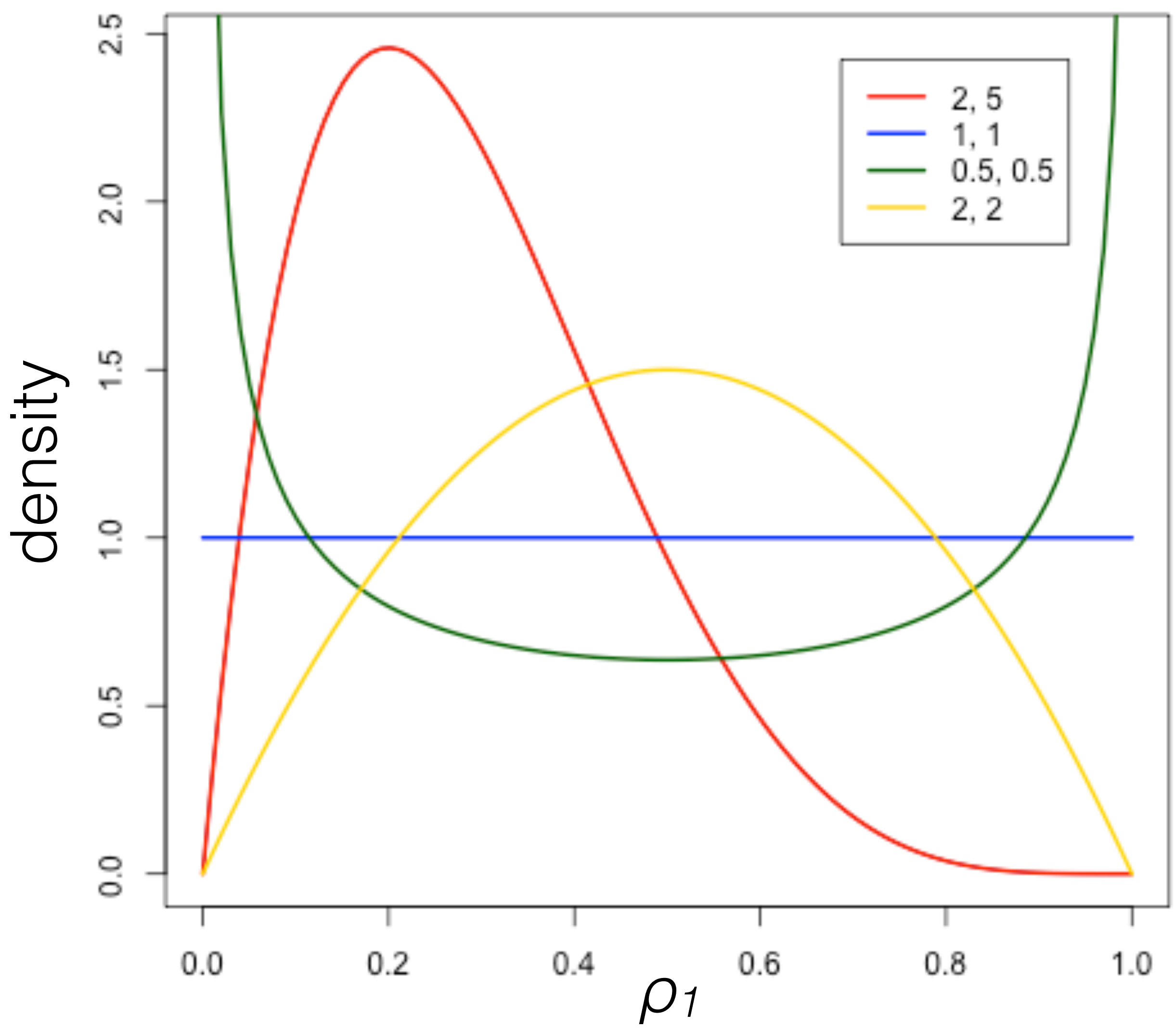

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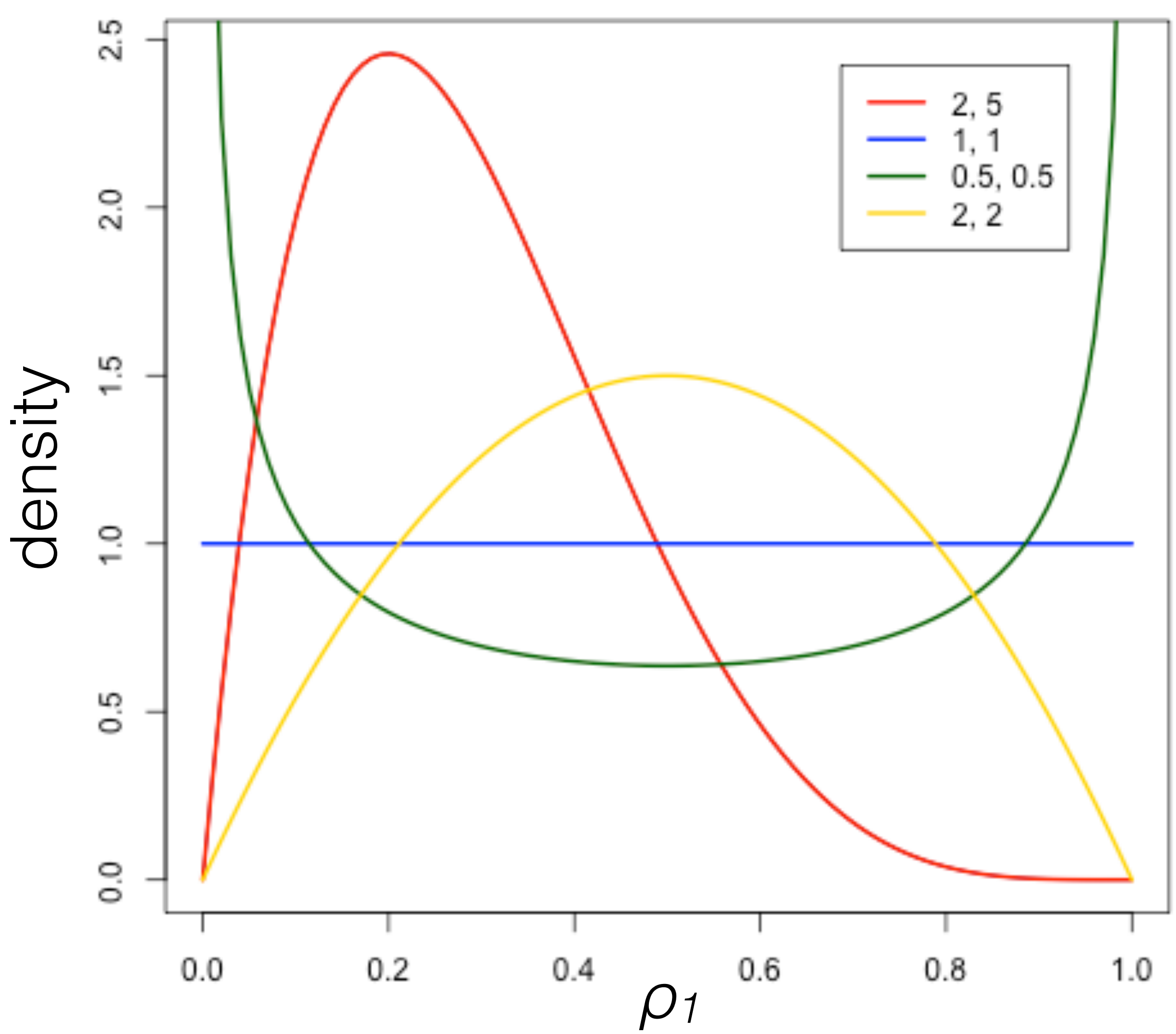

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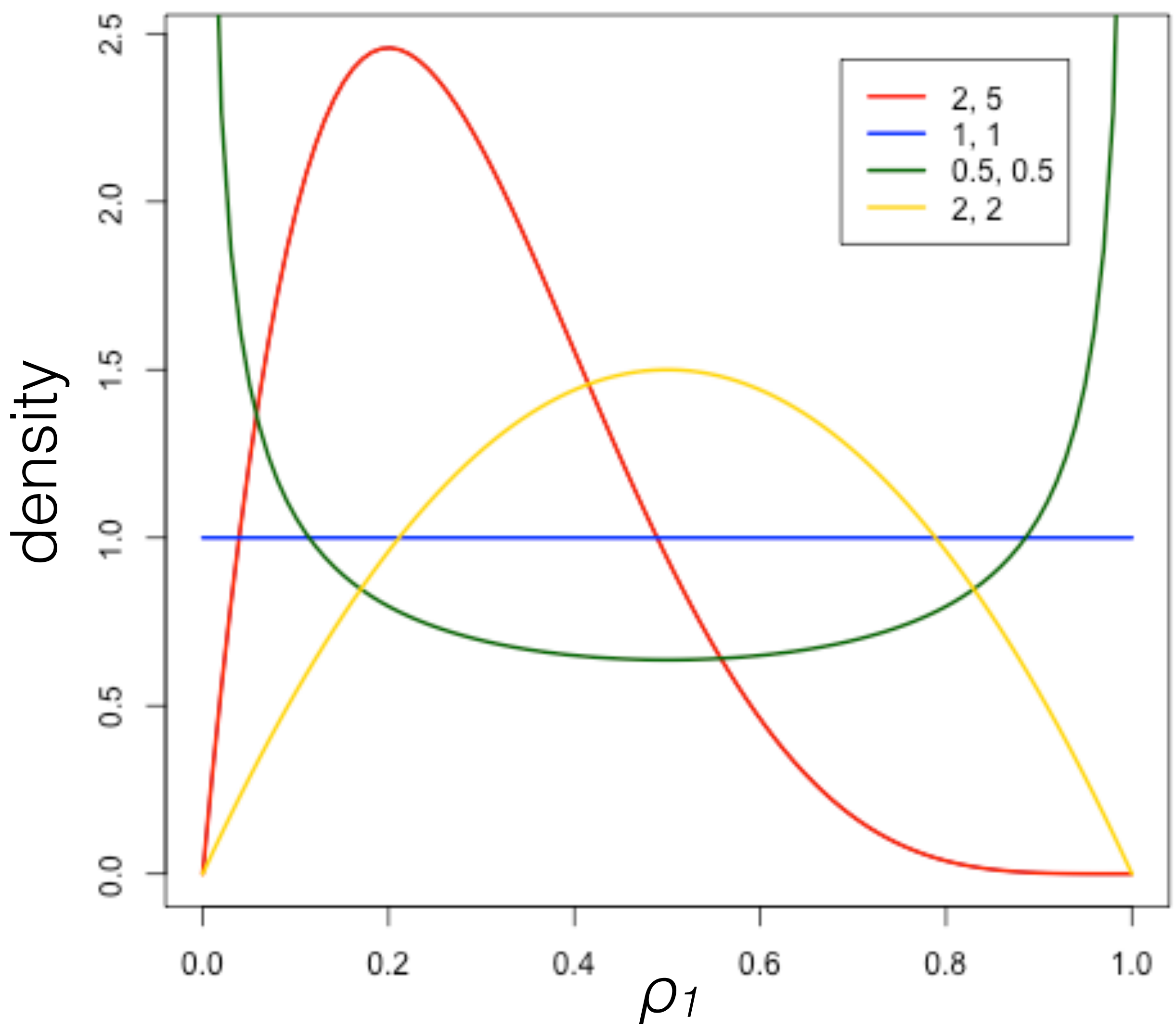

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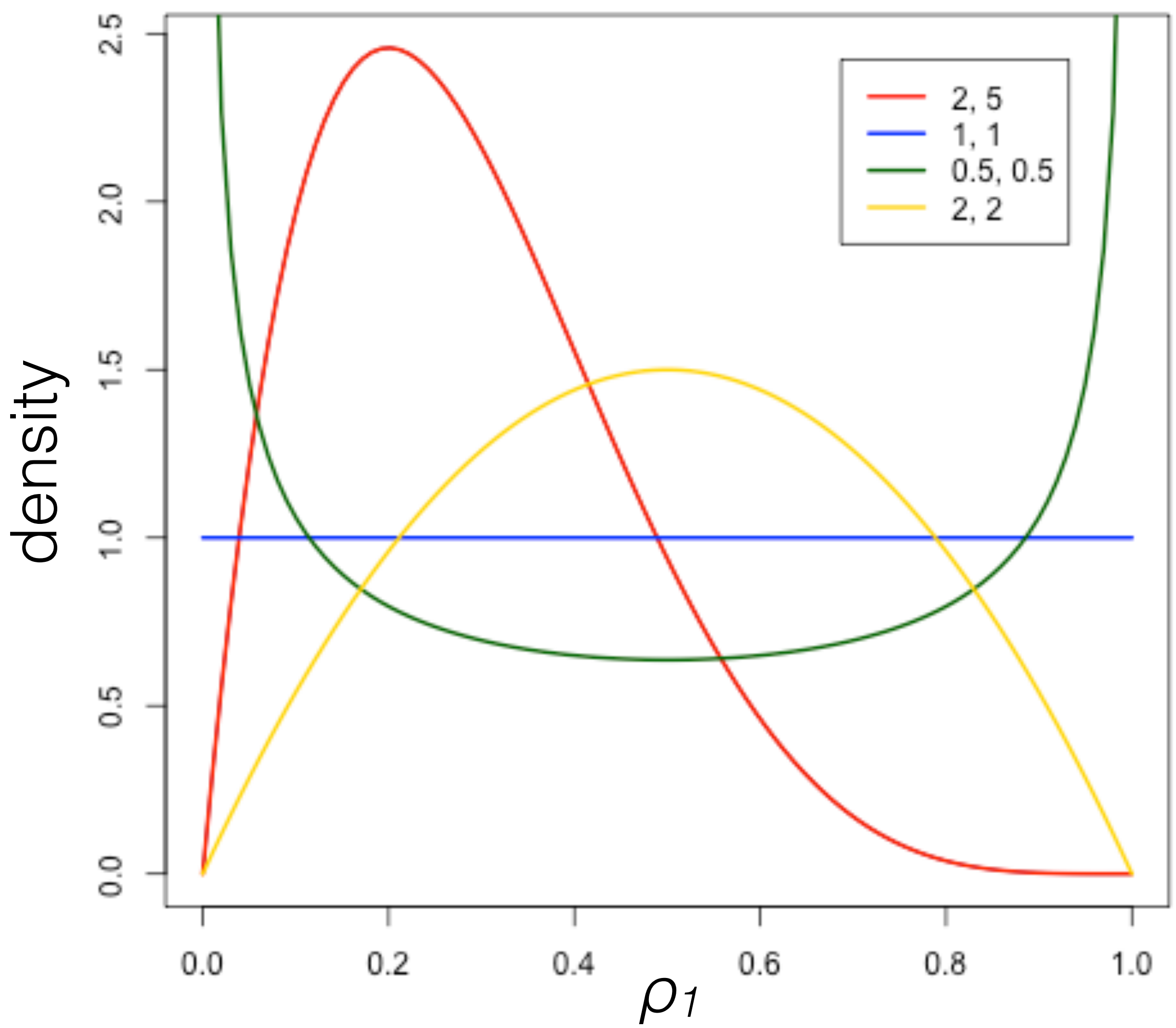

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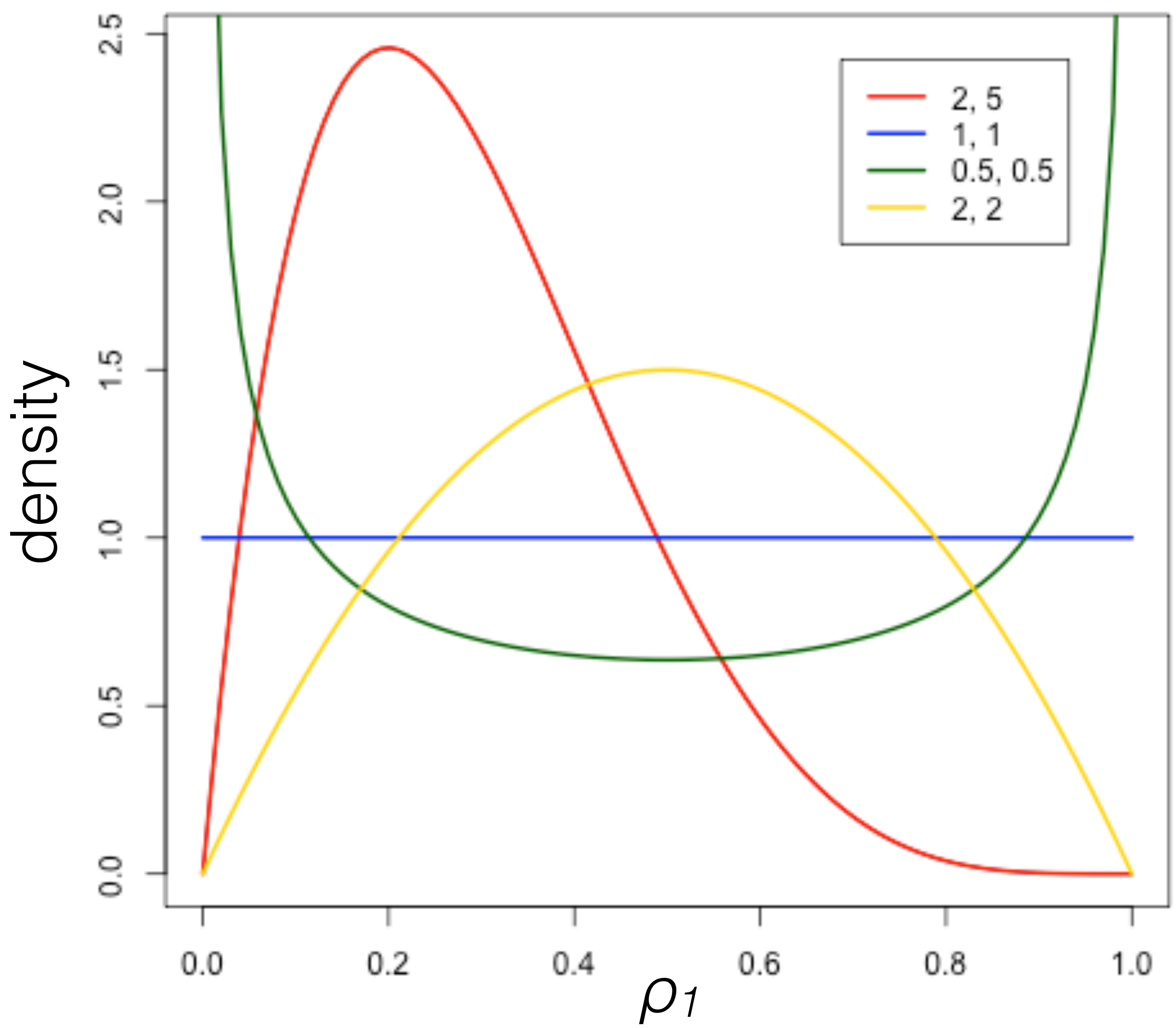

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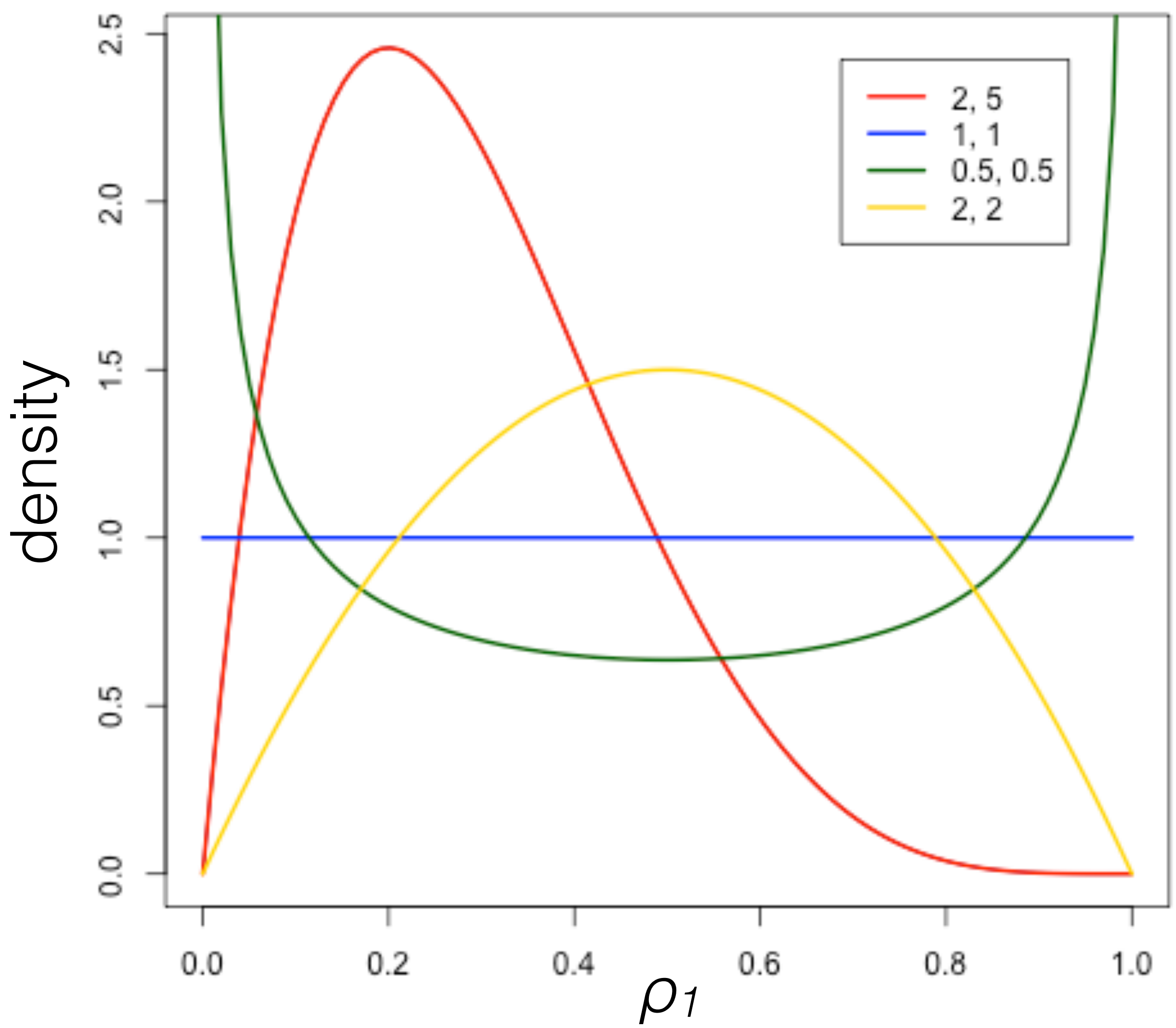

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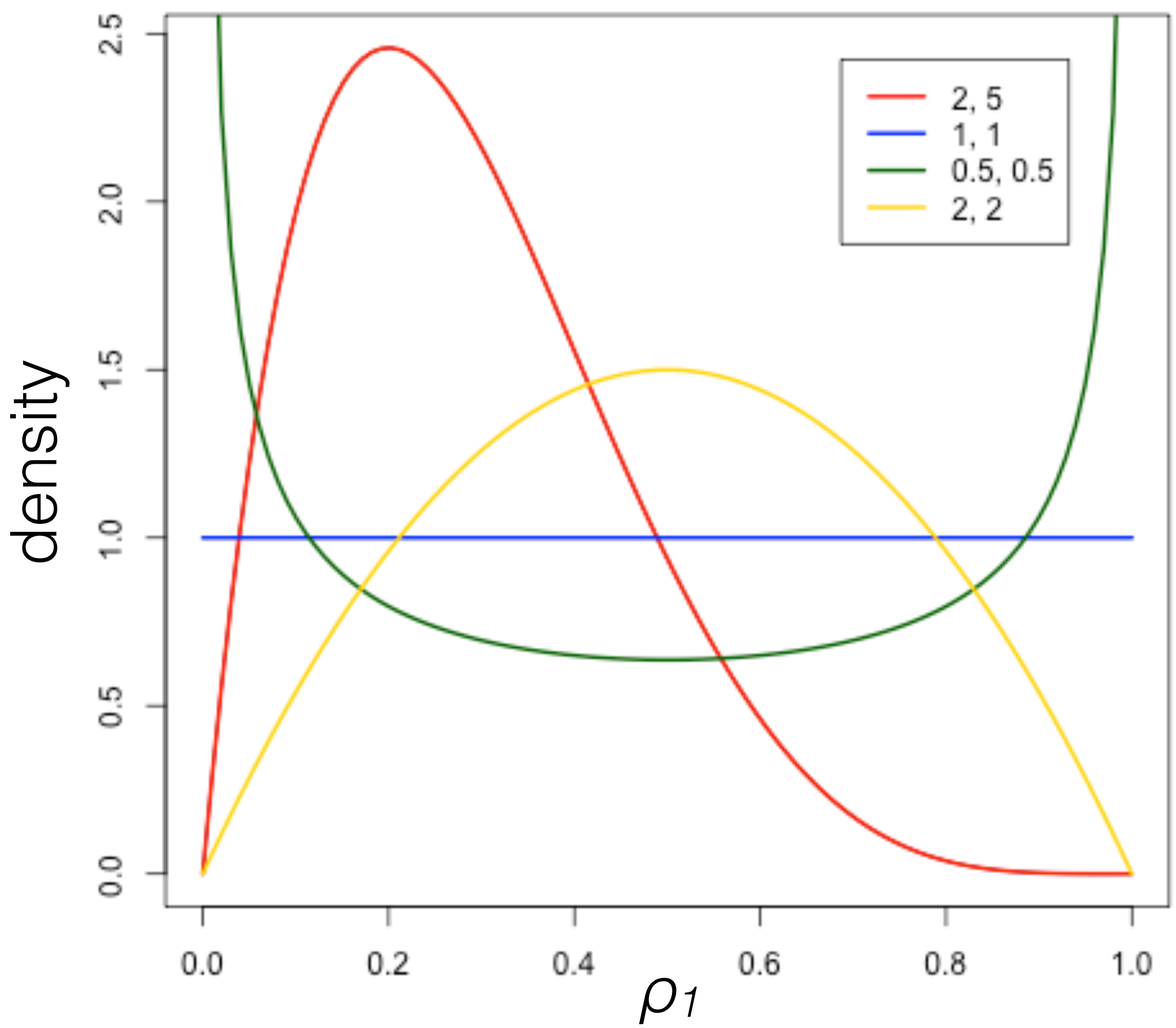

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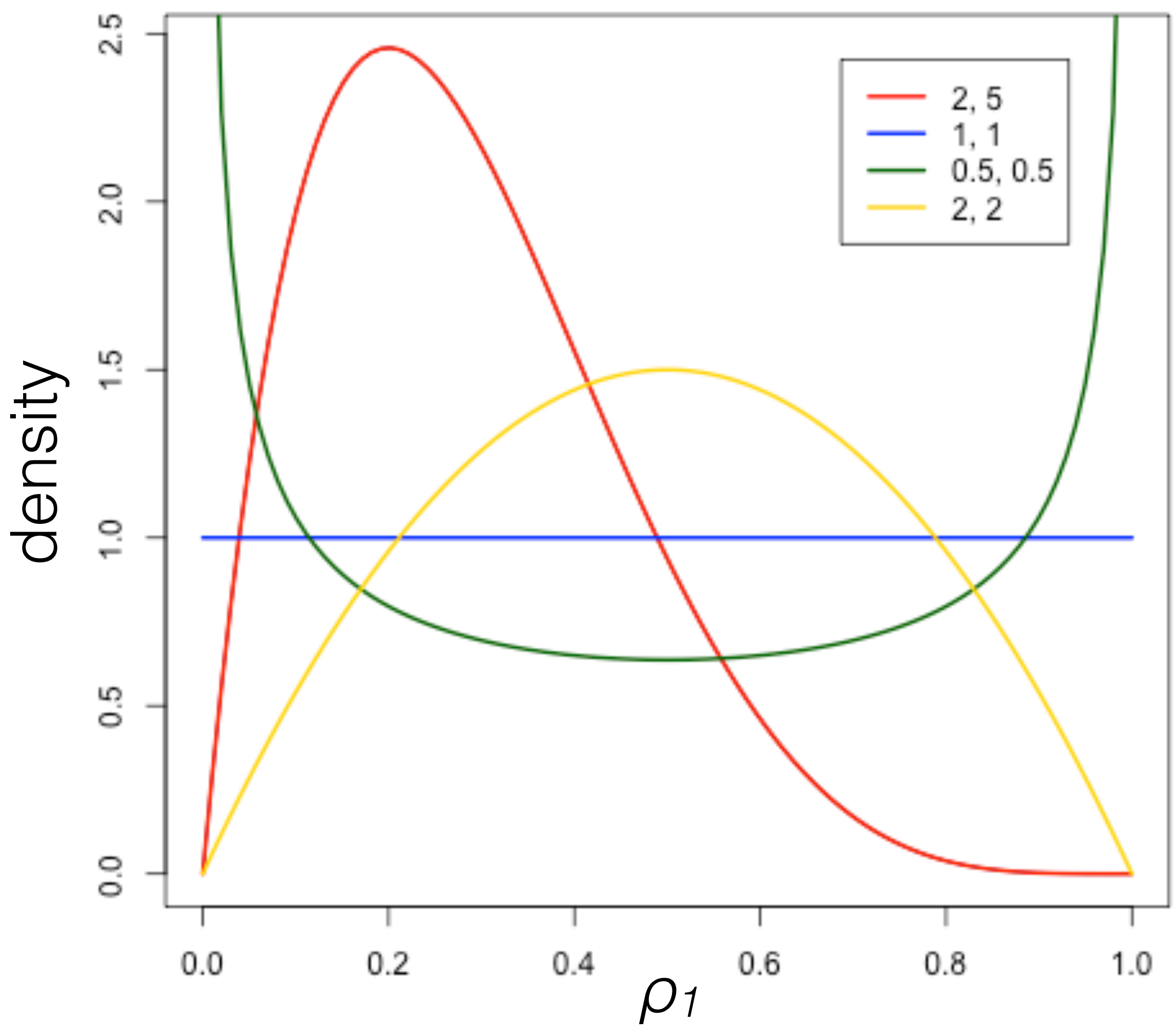

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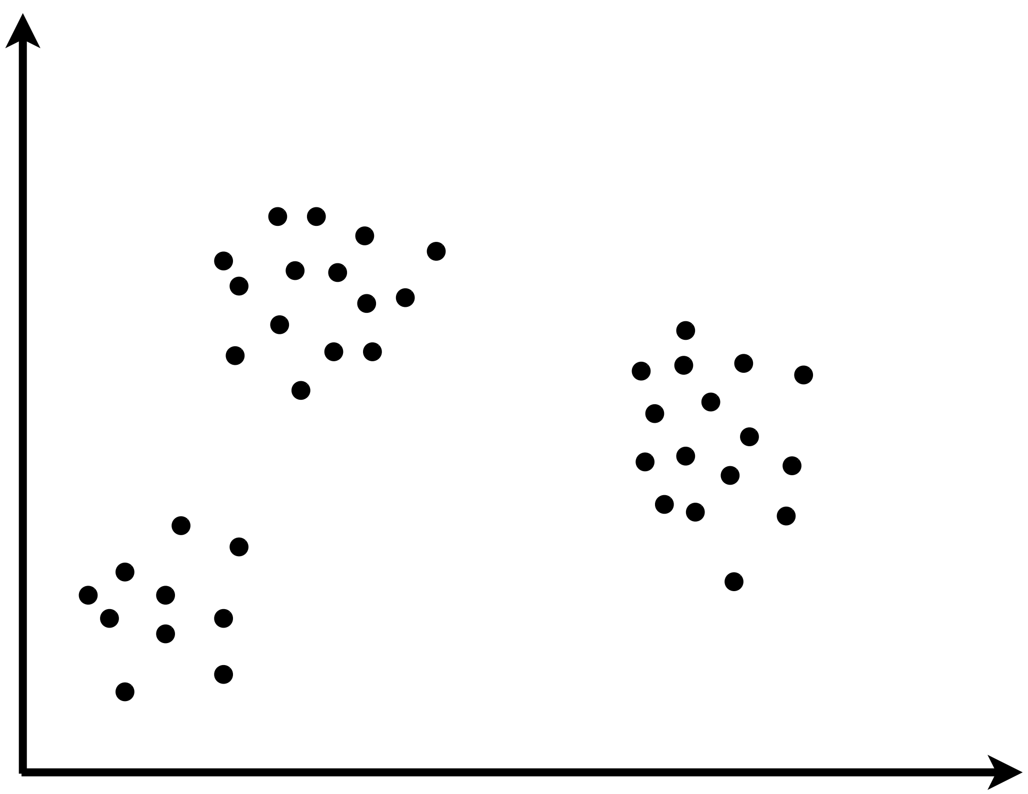

- Finite Gaussian mixture model (*K* clusters)

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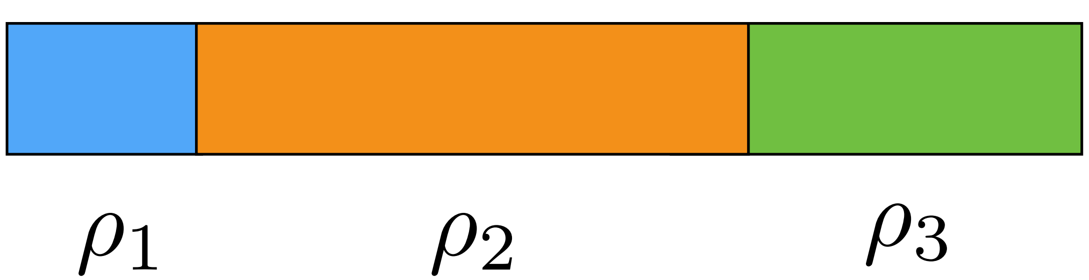

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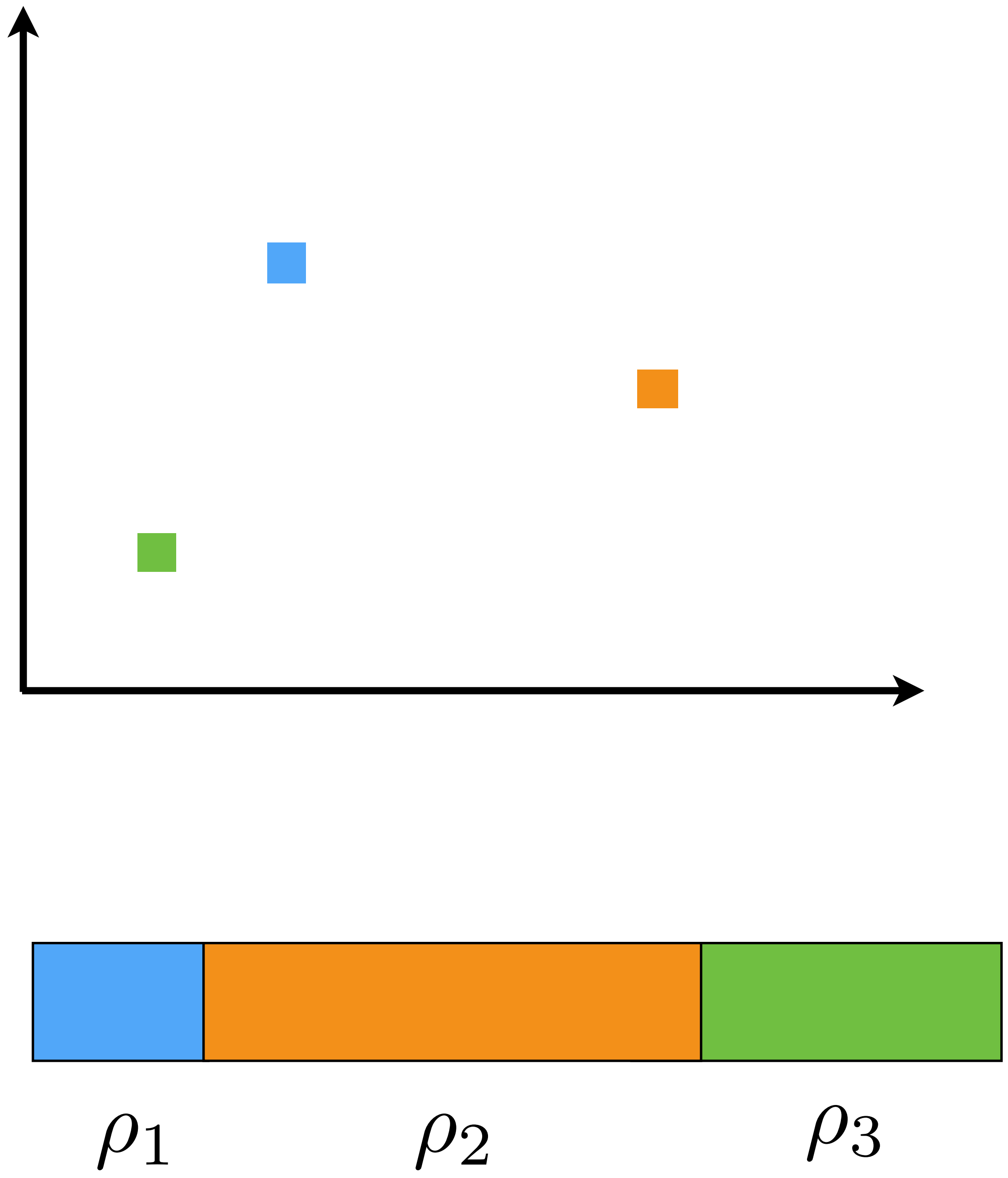

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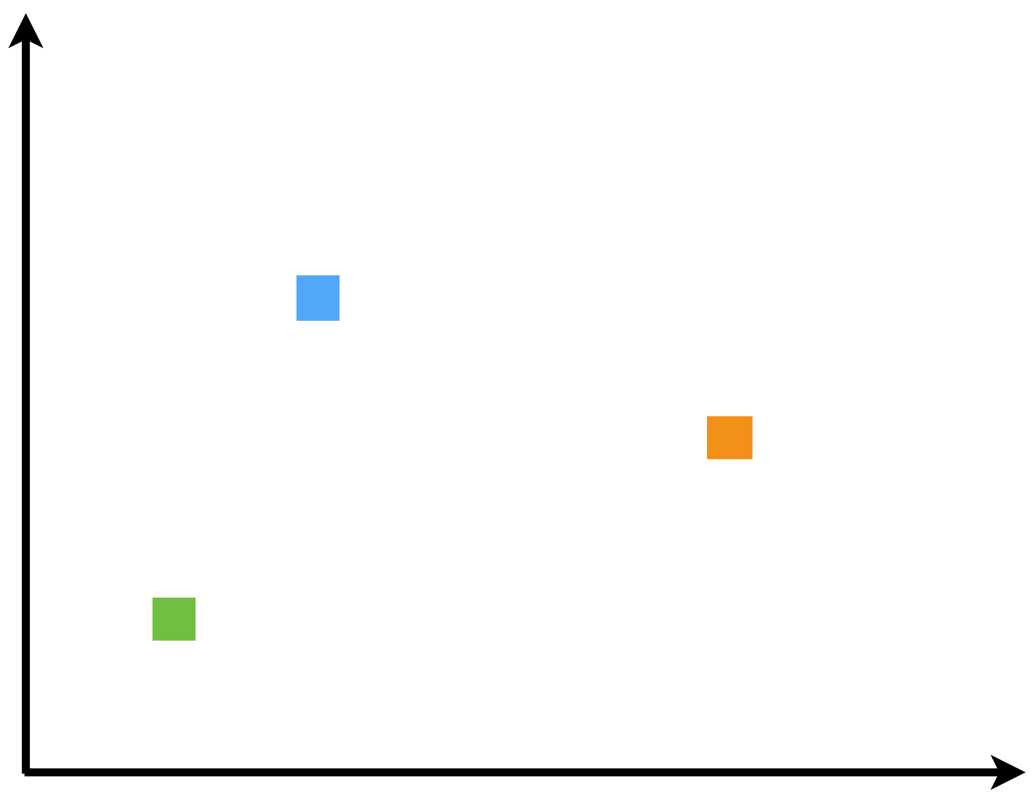

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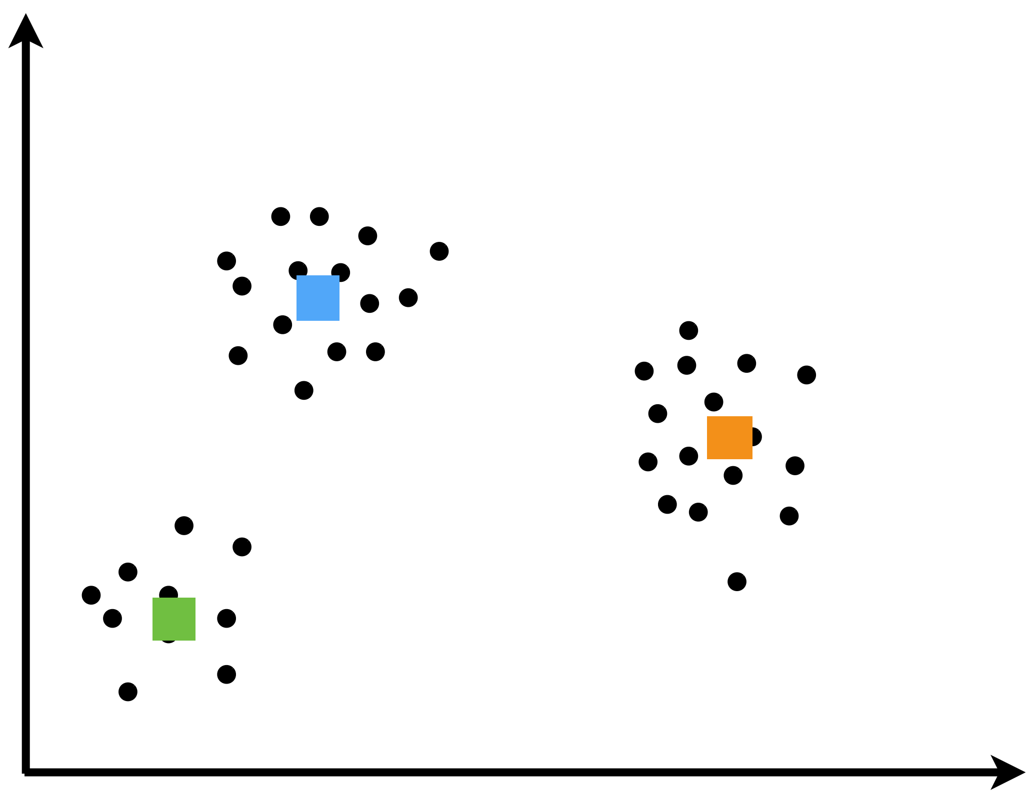

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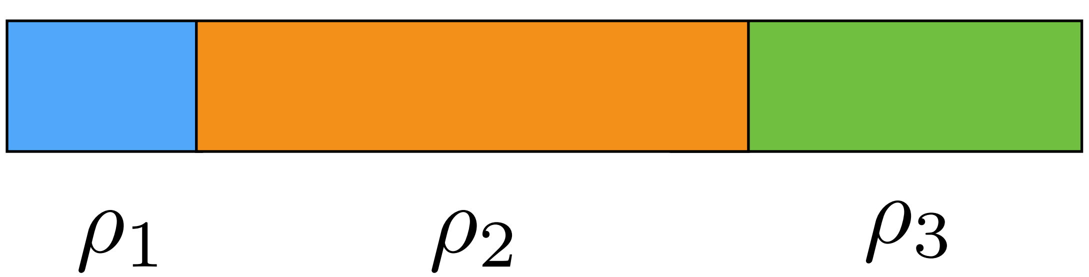
$$x_n \overset{indep}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma)$$



Dirichlet distribution review

$$\mathrm{Dirichlet}(\rho_{1:K}|a_{1:K}) = \frac{\Gamma(\sum_{k=1}^{K} a_k)}{\prod_{k=1}^{K} \Gamma(a_k)} \prod_{k=1}^{K} \rho_k^{a_k - 1} \qquad a_k > 0$$

Dirichlet distribution review

$$a_k > 0$$

$$\rho_k \in (0,1)$$

$$\sum_k \rho_k = 1$$

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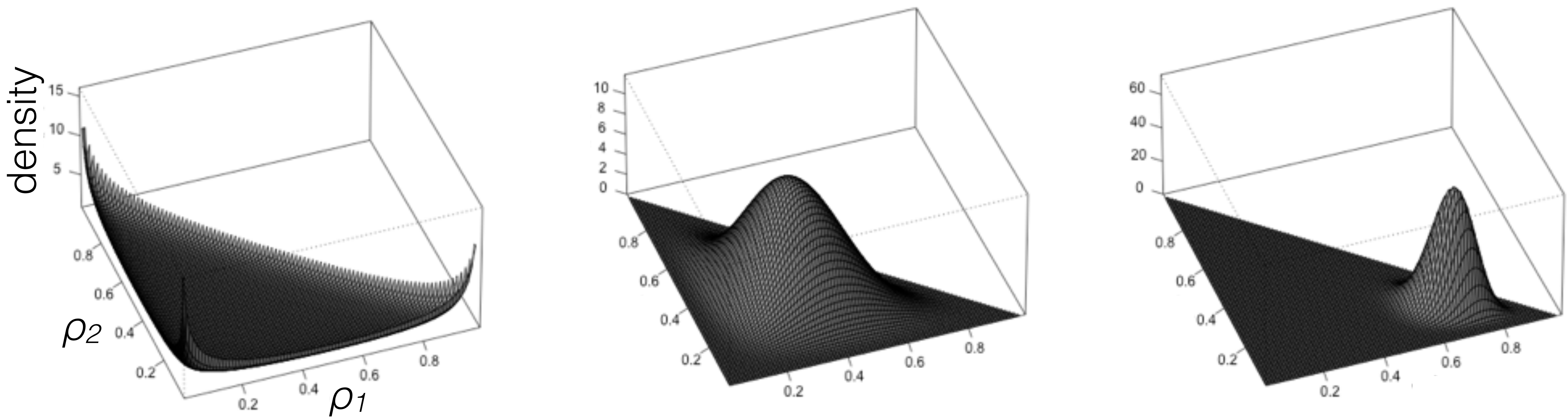
a = (0.5,0.5,0.5) a = (5,5,5) a = (40,10,10)



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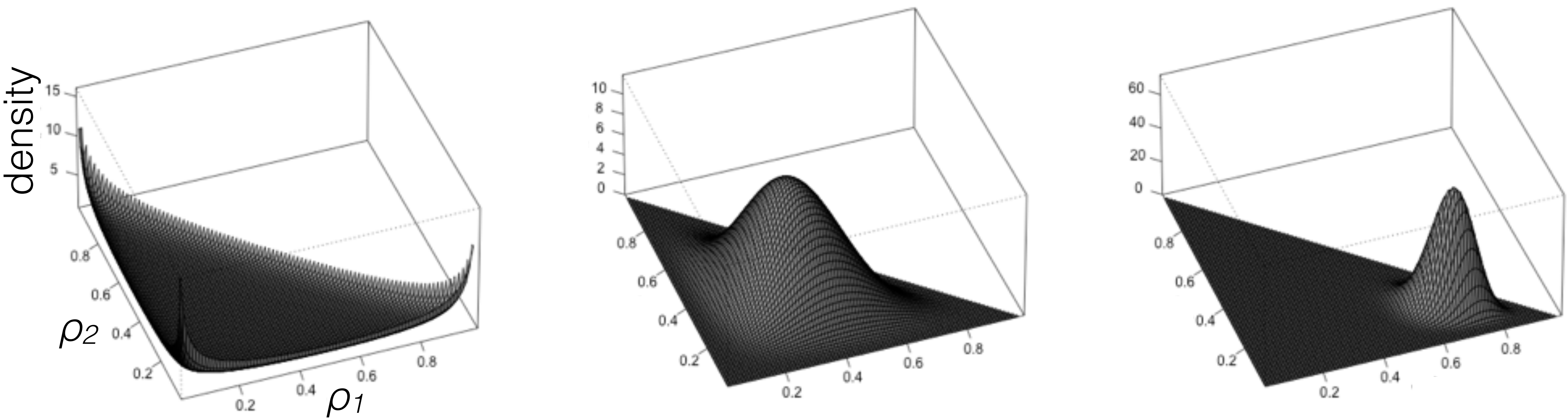

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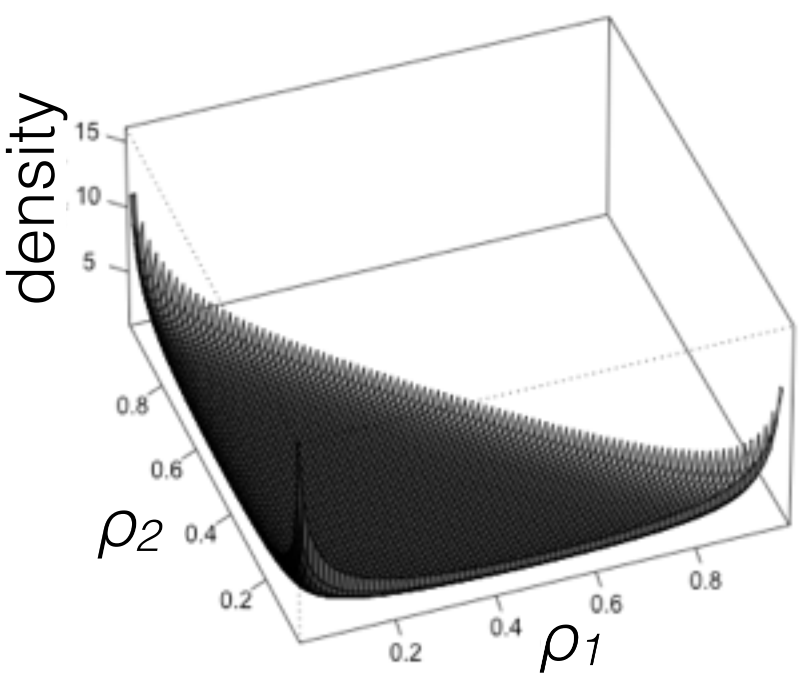
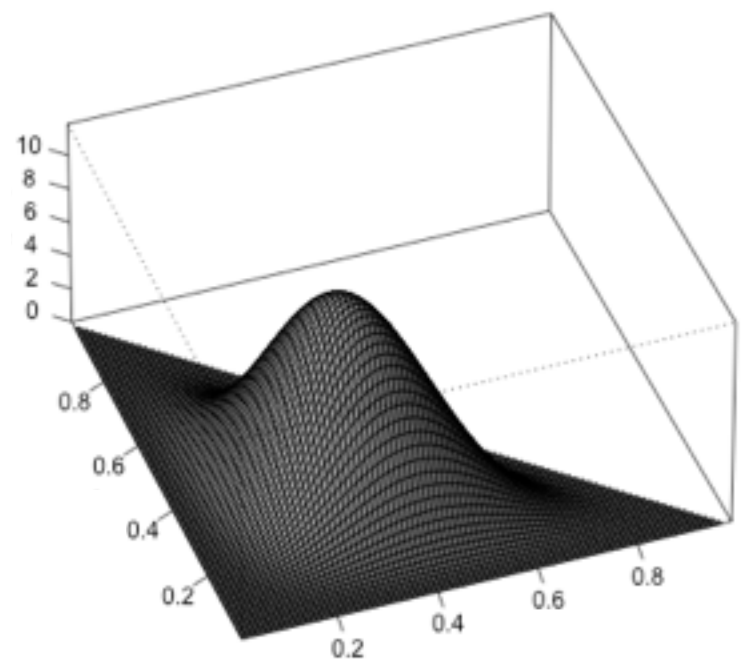


a = (5,5,5)



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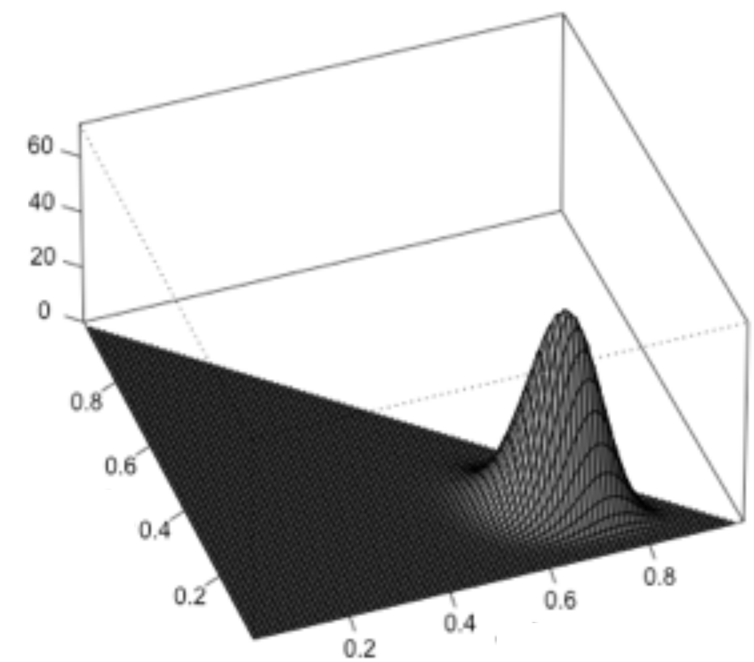

- What happens? $a = a_k = 1$

[demo]

Dirichlet distribution review

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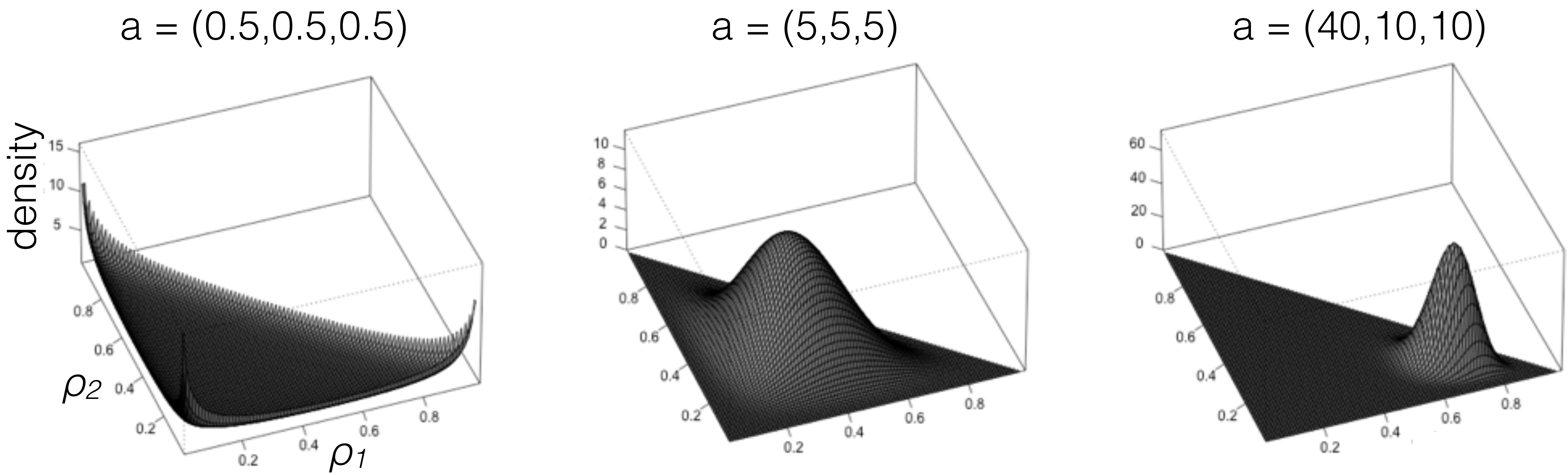

- What happens? $a = a_k = 1$ $\quad a = a_k \to 0$

[demo]

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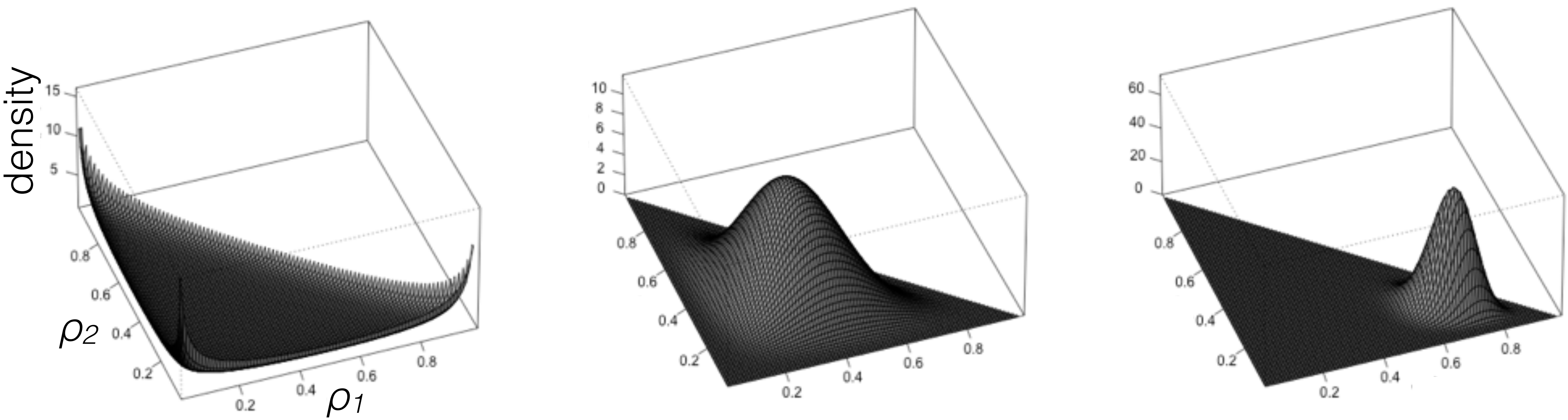

- What happens? $a = a_k = 1$ $a = a_k \to 0$ $a = a_k \to \infty$

[demo]

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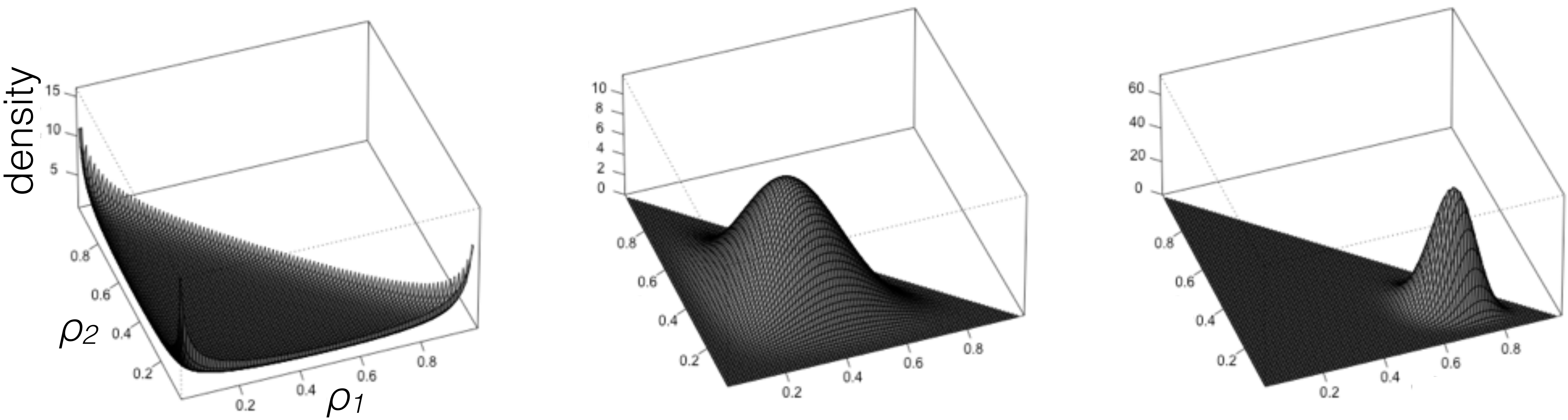

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[demo]

- Dirichlet is conjugate to Categorical

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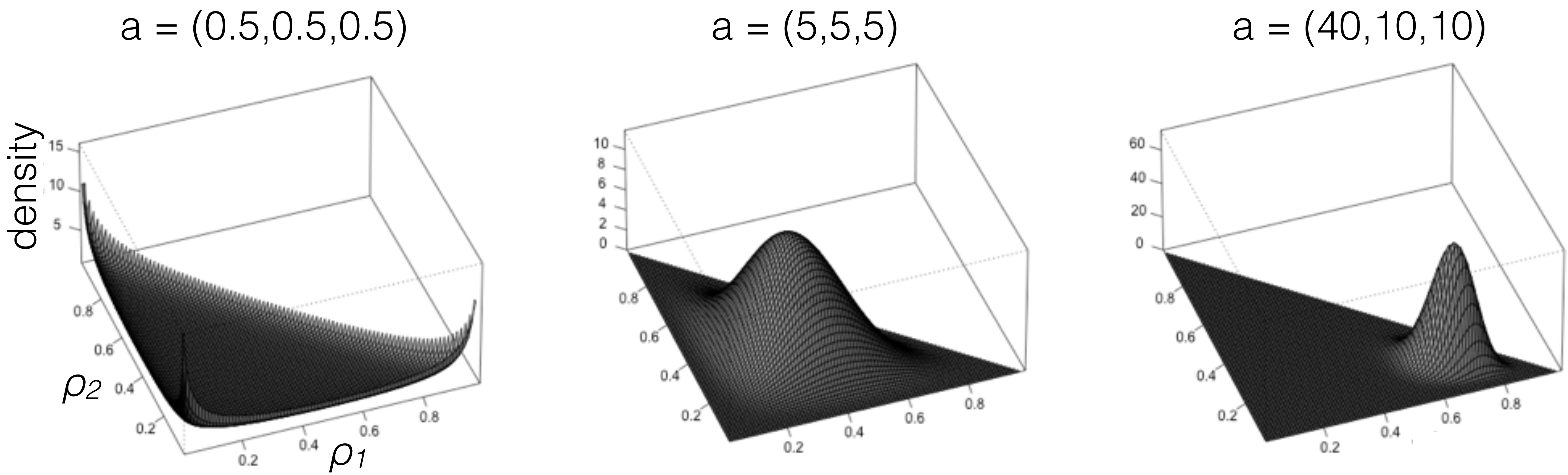

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[demo]

- Dirichlet is conjugate to Categorical

$\rho_{1:K} \sim \mathrm{Dirichlet}(a_{1:K}), z \sim \mathrm{Cat}(\rho_{1:K})$

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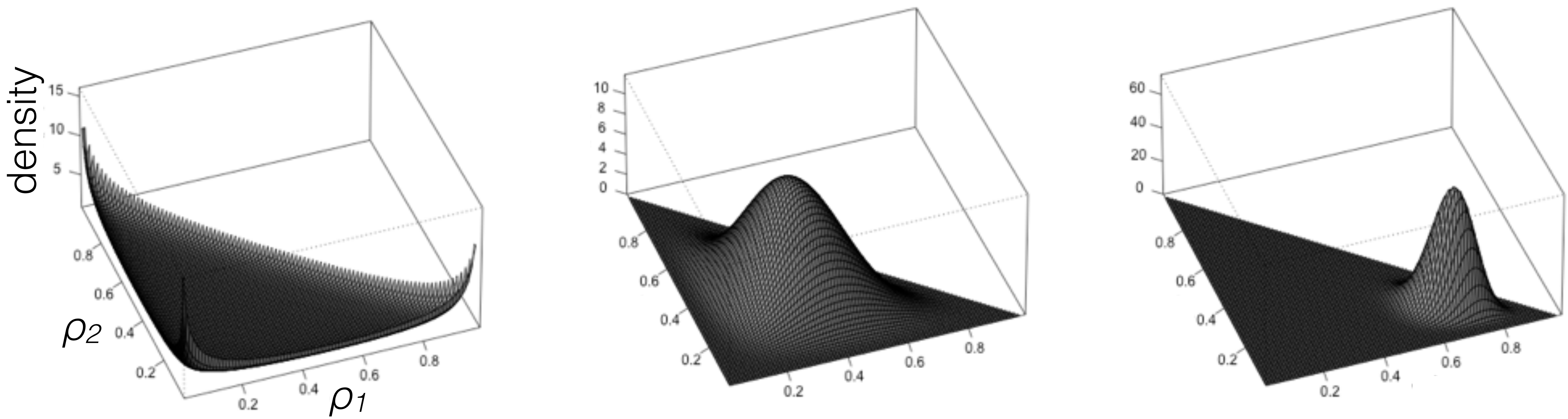

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[demo]

- Dirichlet is conjugate to Categorical

$$\rho_{1:K} \sim \mathrm{Dirichlet}(a_{1:K}), z \sim \mathrm{Cat}(\rho_{1:K})$$
$$\rho_{1:K} | z \stackrel{d}{=} \mathrm{Dirichlet}(a'_{1:K}), a'_k = a_k + \mathbf{1}\{z = k\}$$

What if $K > N$?

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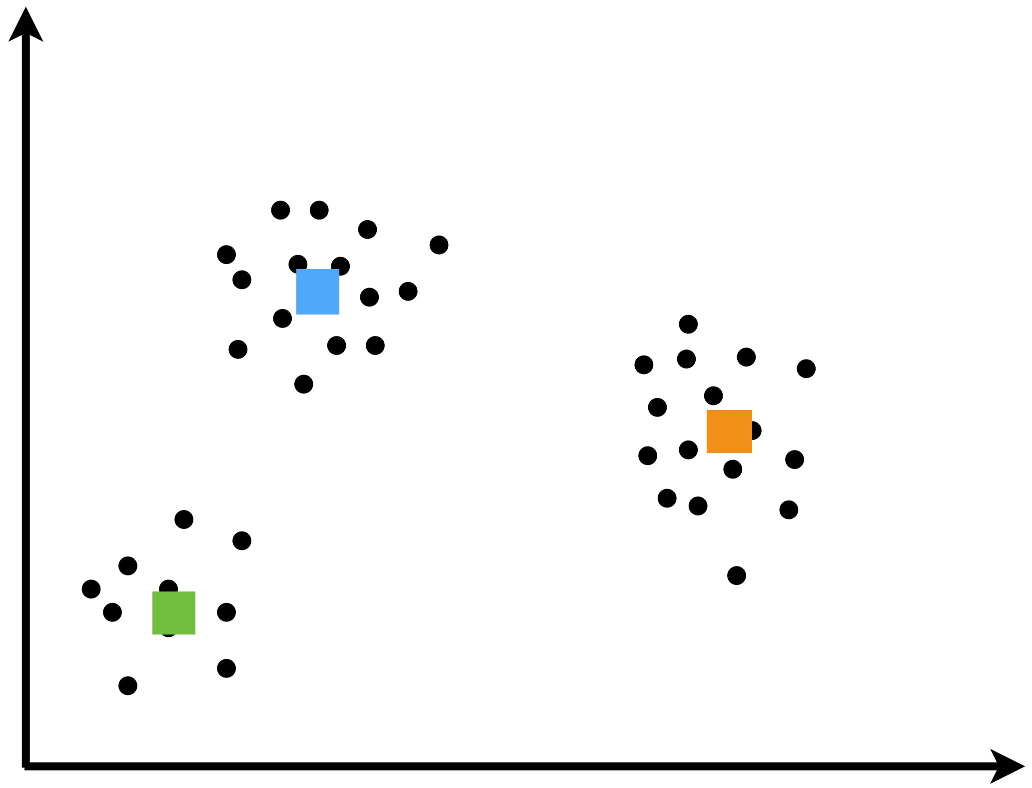

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ρ_1 ρ_2 ρ_3 ρ_{1000}

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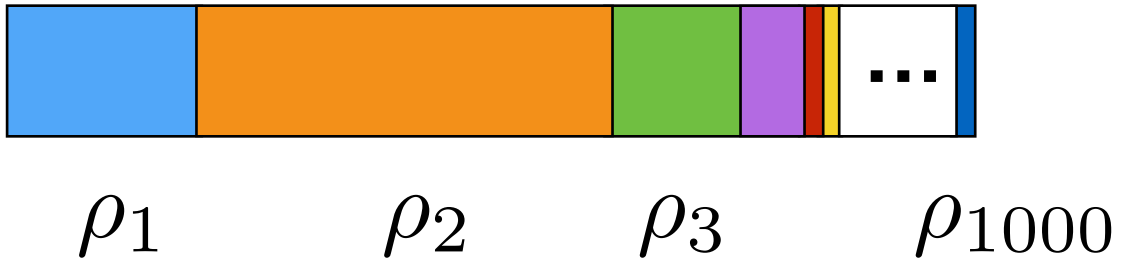
- e.g. species sampling, topic modeling, groups on a social network, etc.



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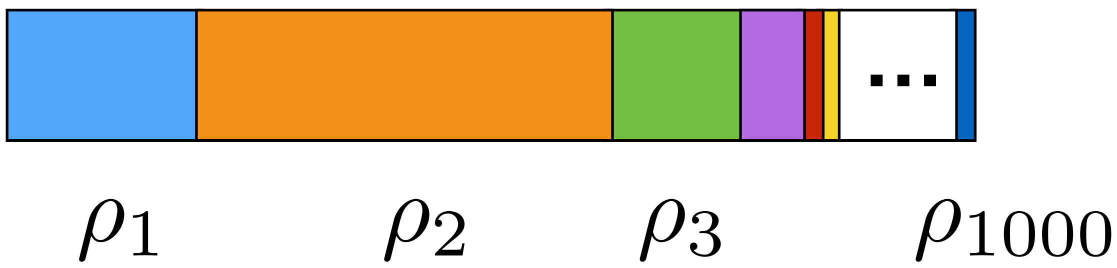

- Components: number of latent groups

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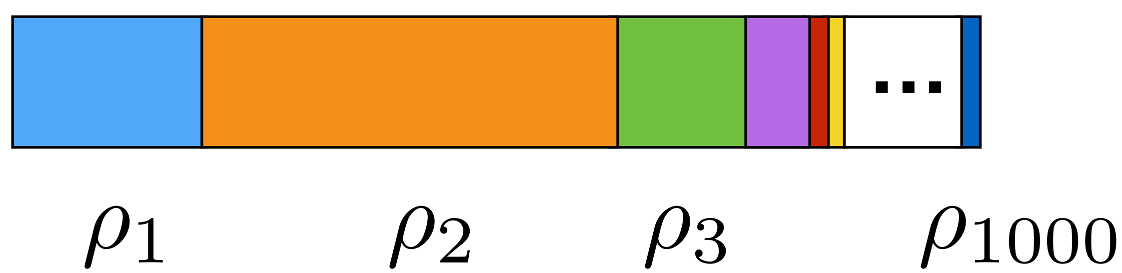

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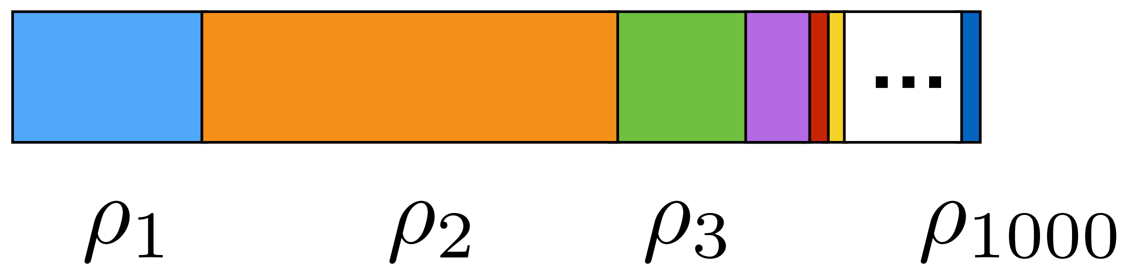

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- [demo 1, demo 2]

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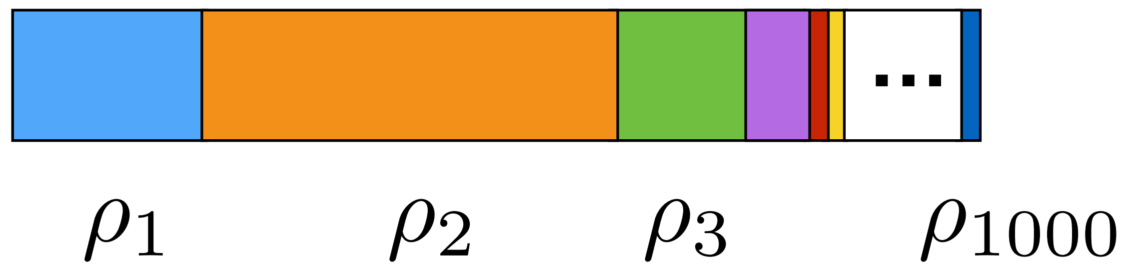

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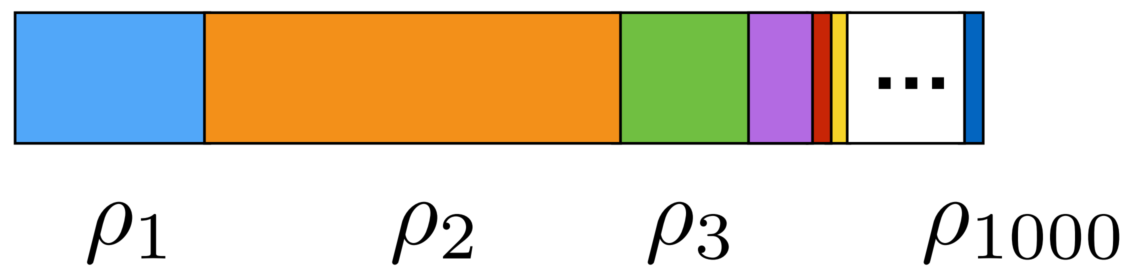

- Components: number of latent groups

- Clusters: number of components represented in the data

- [demo 1, demo 2]

- Number of clusters for N data points is $< K$ and random

- Number of clusters grows with N

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data

Choosing $K = \infty$

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- "Stick breaking"

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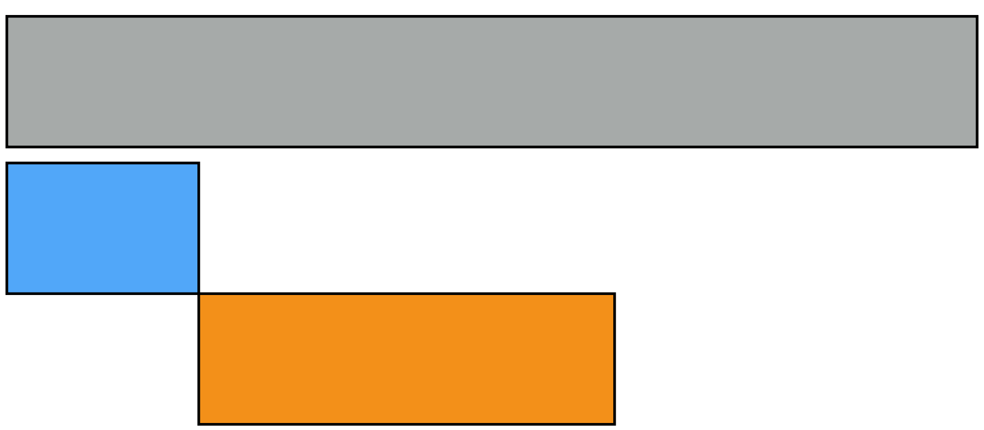

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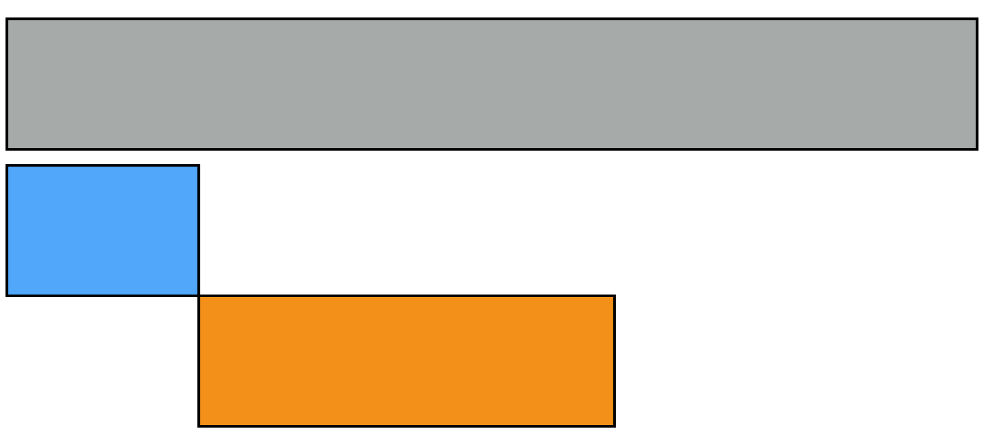

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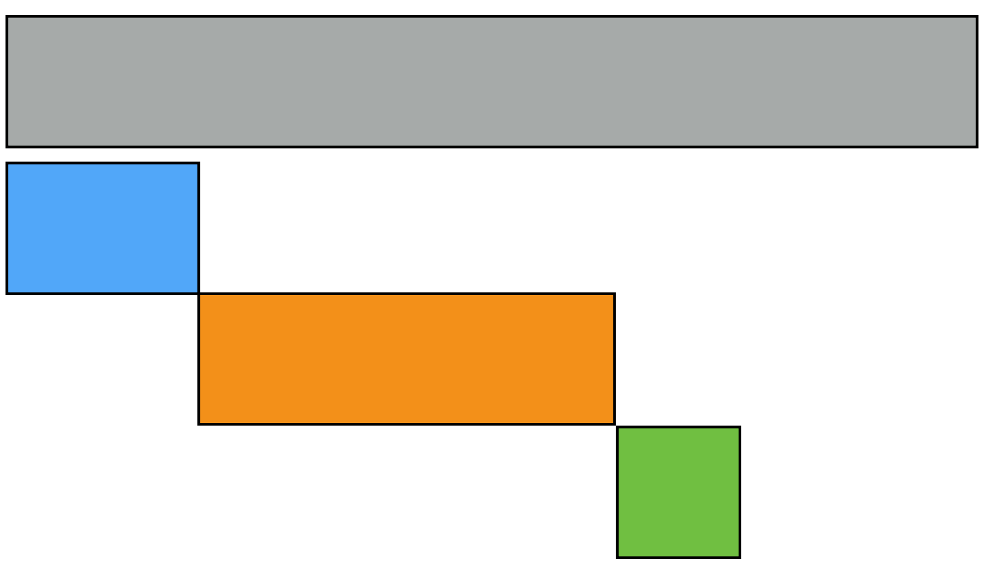

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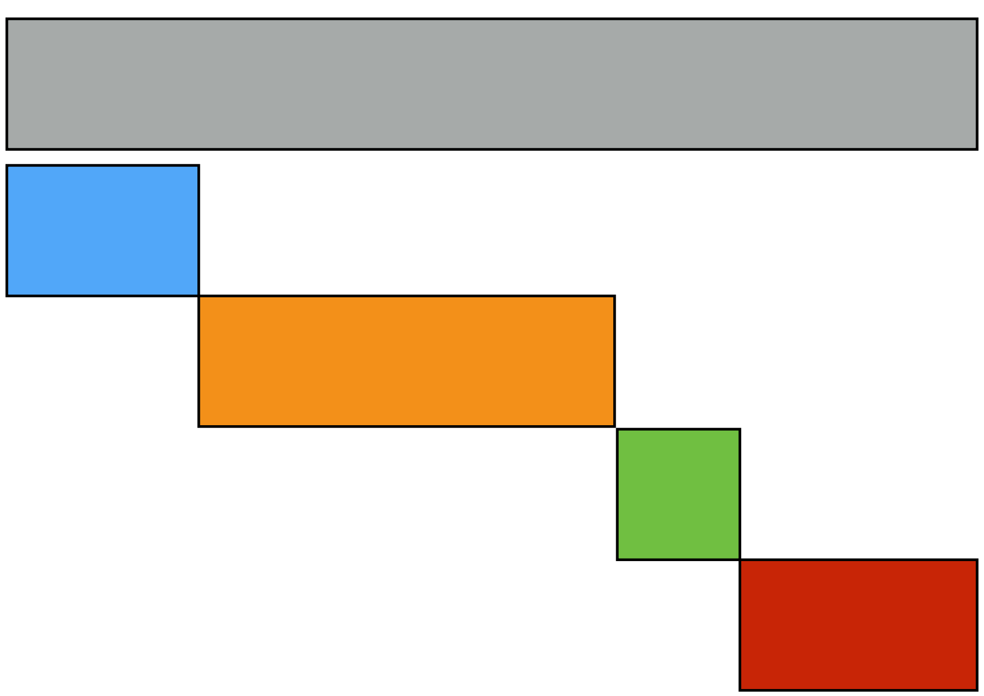

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$$\rho_4 = 1 - \sum_{k=1}^{3} \rho_k$$

Choosing $K = \infty$

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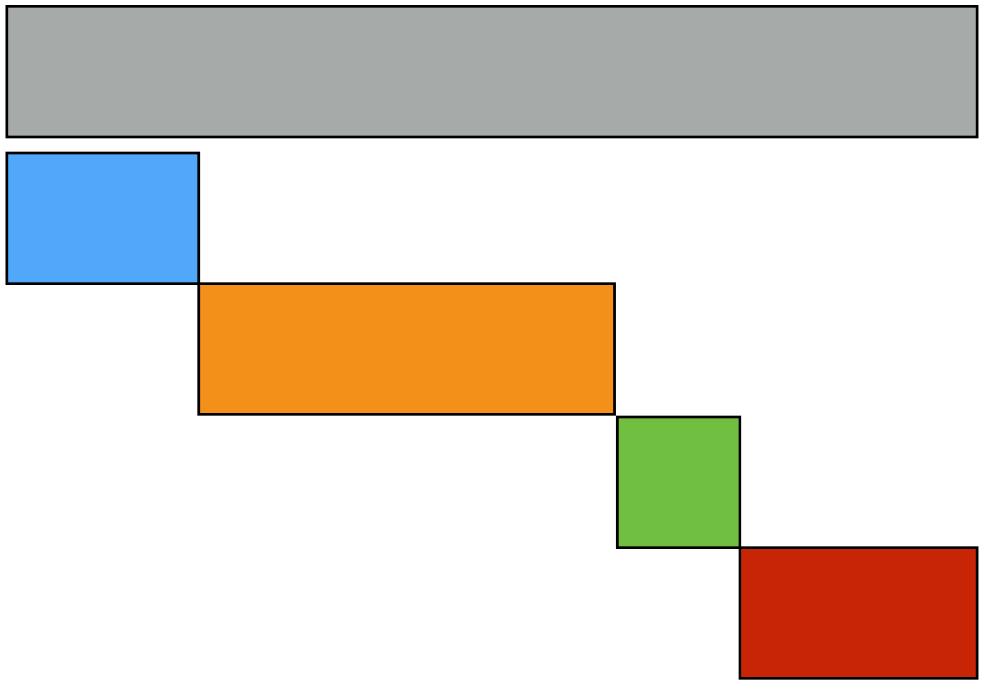

Choosing $K = \infty$

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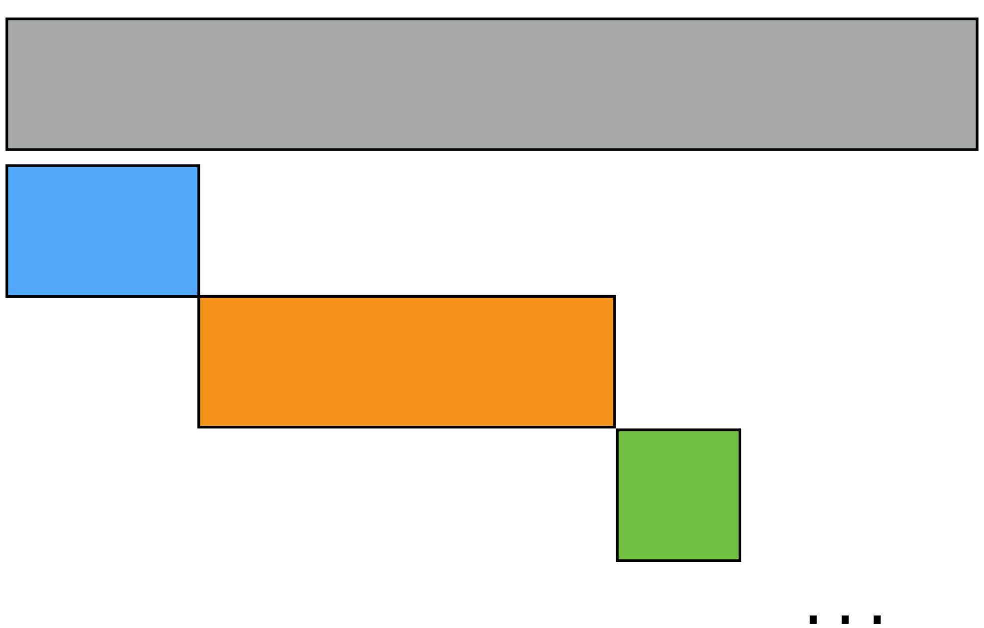

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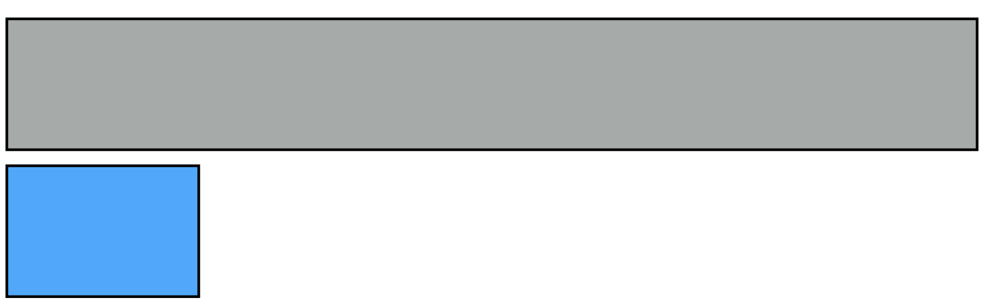

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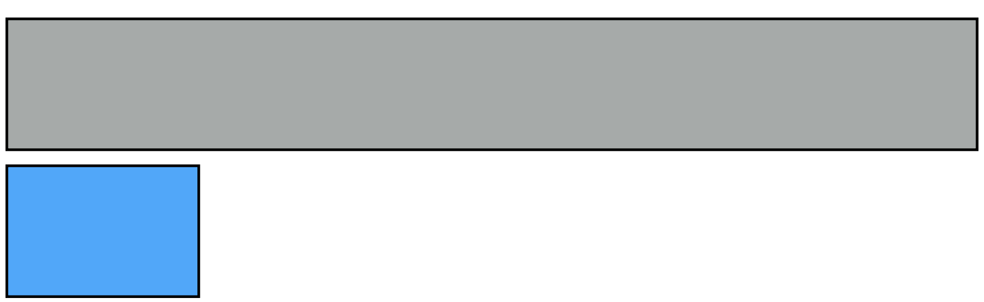

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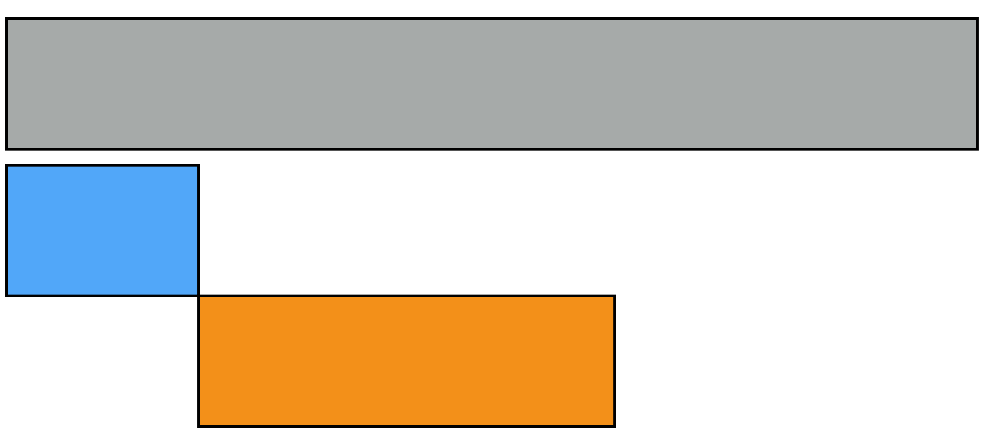

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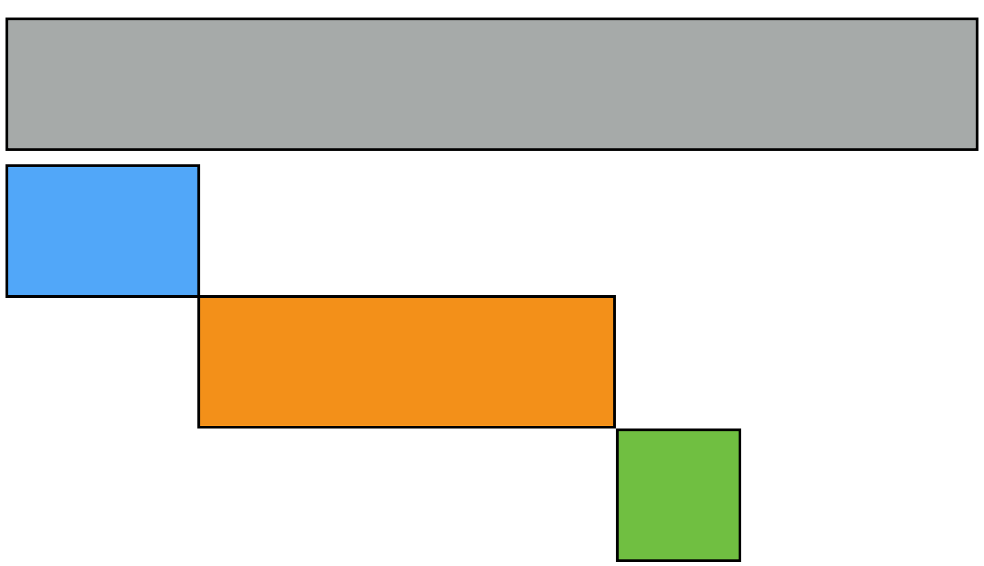

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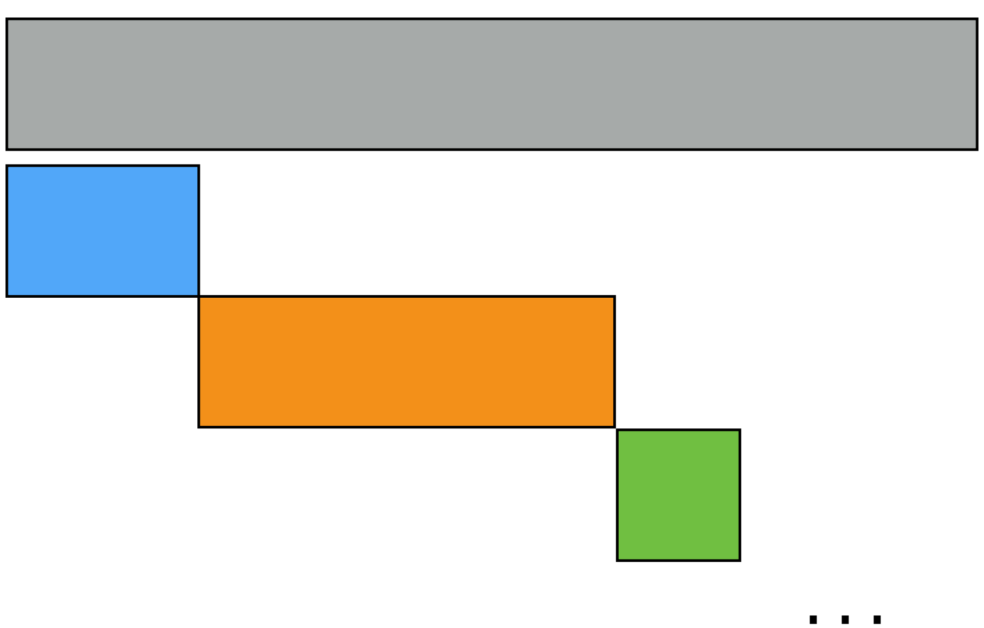

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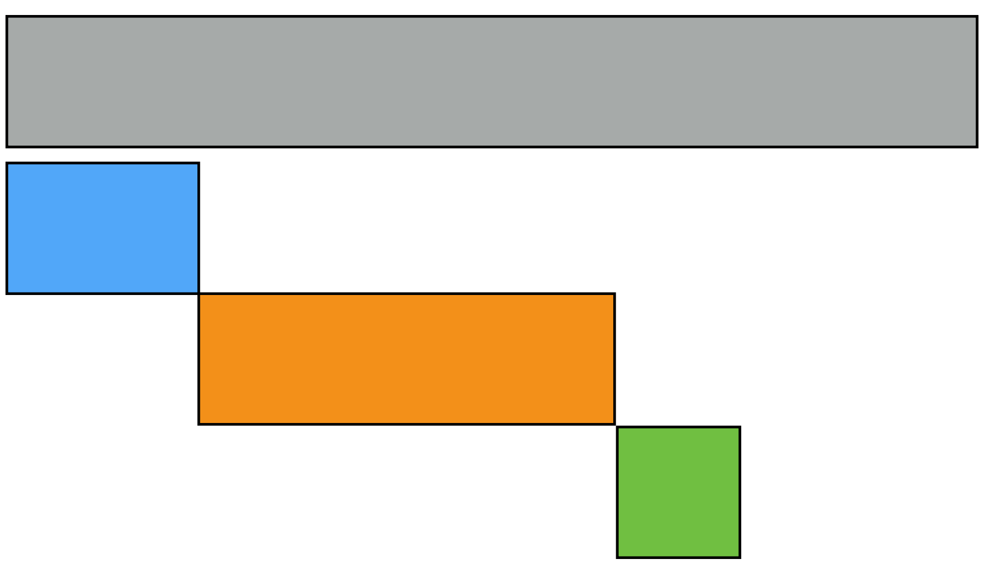

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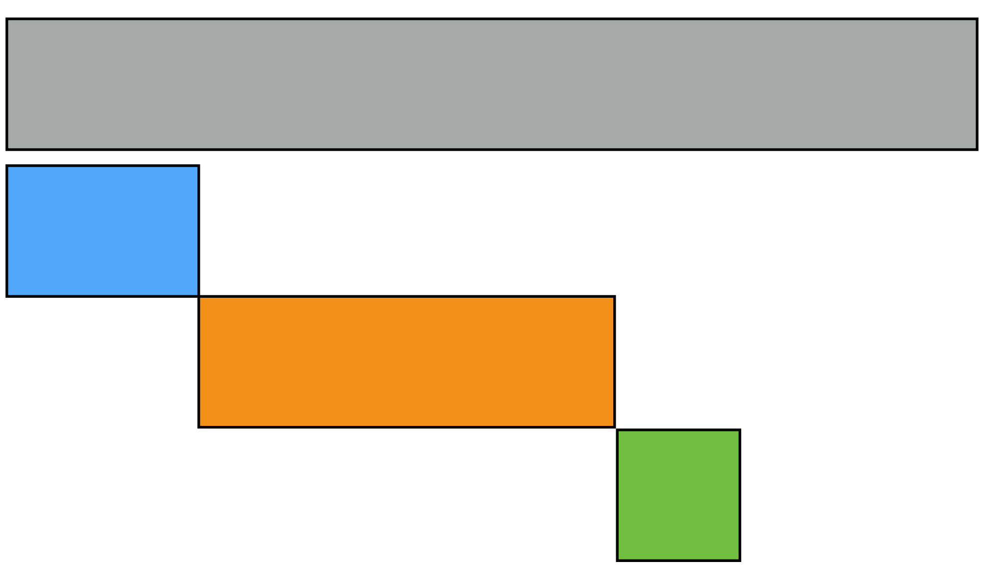

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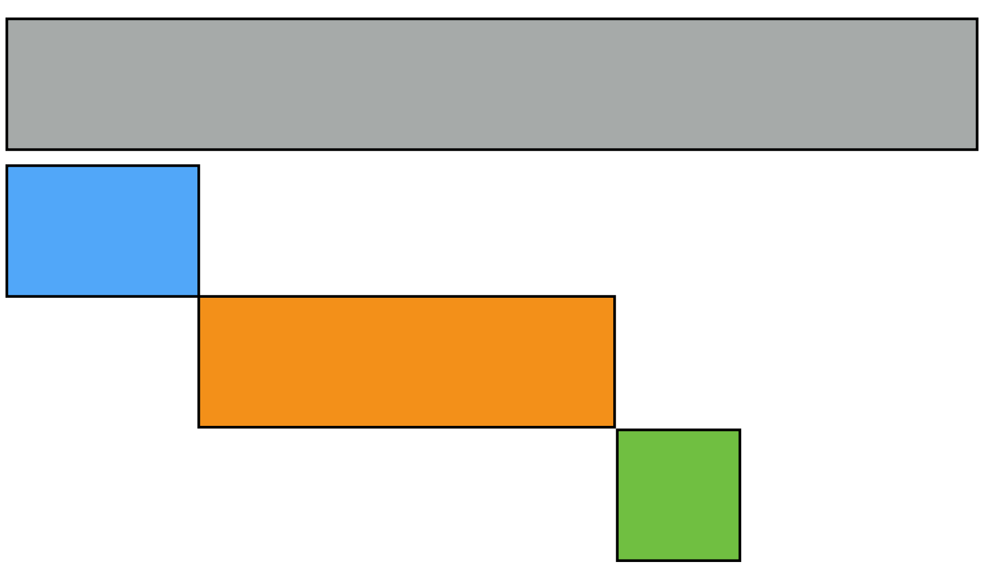

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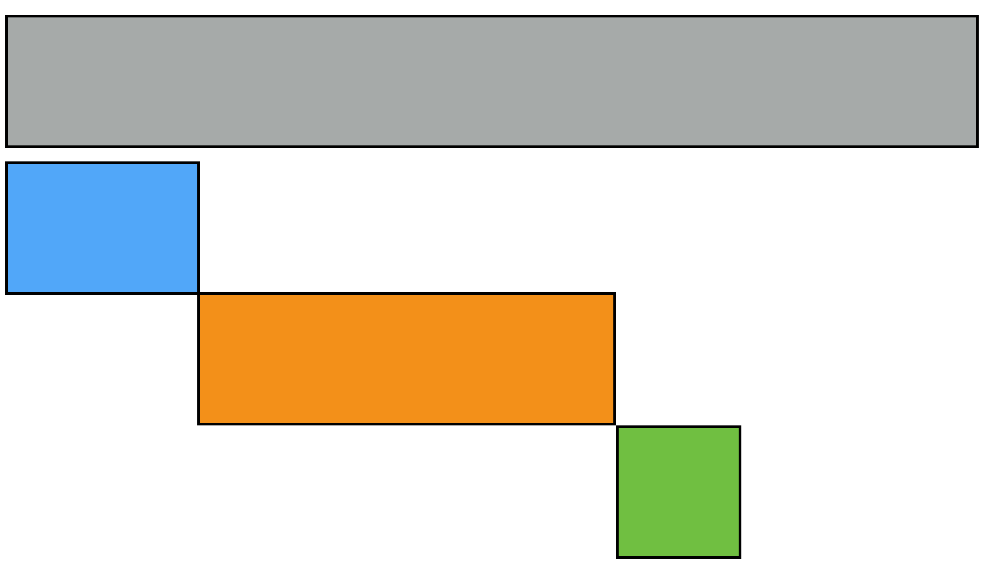

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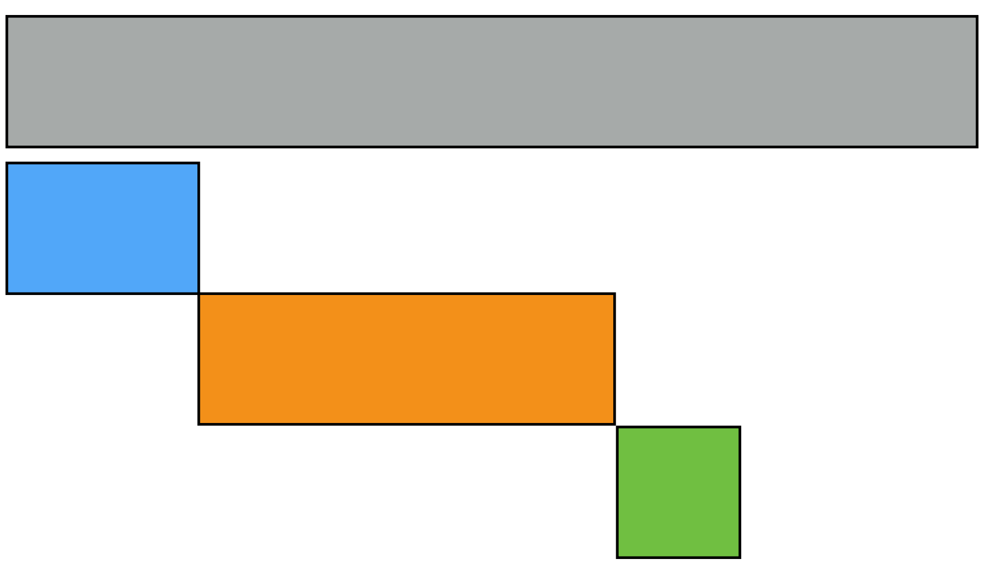

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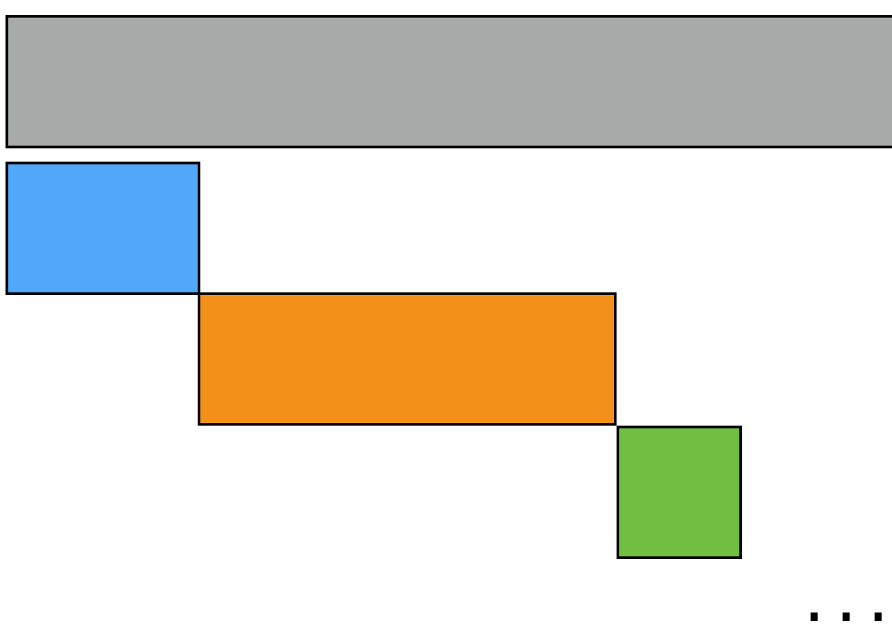

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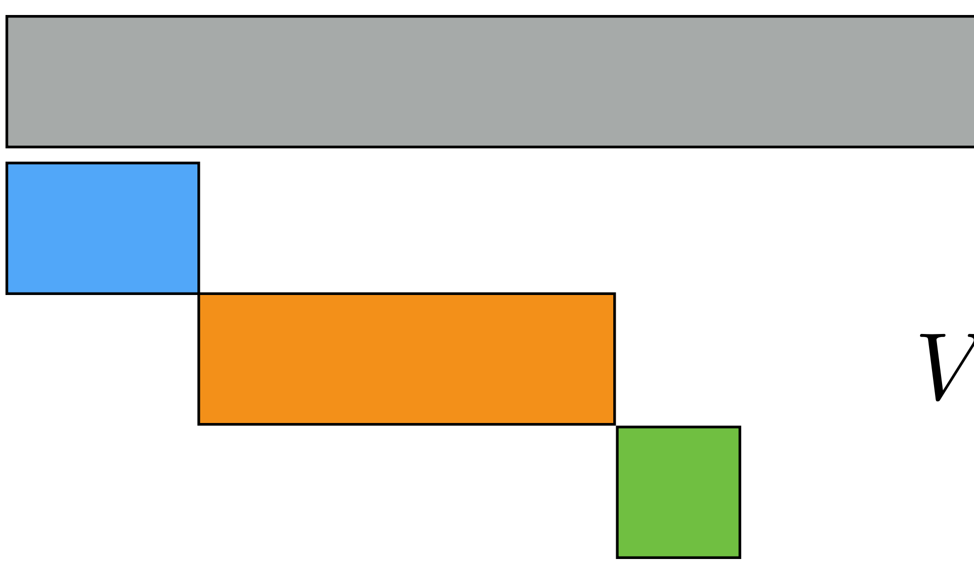

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Distributions

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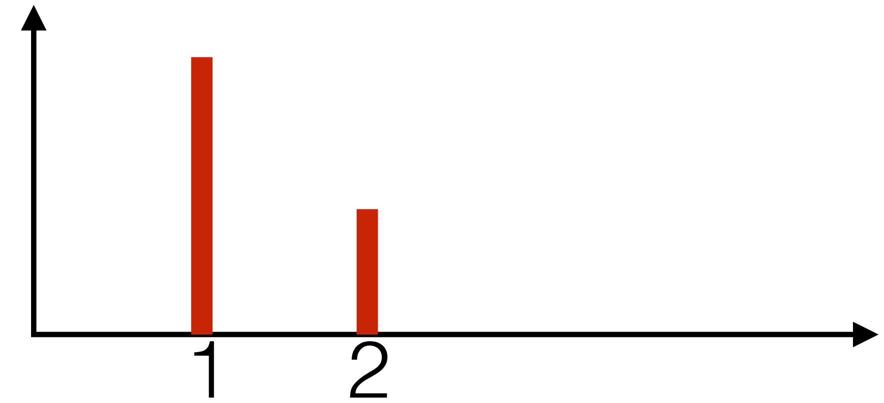

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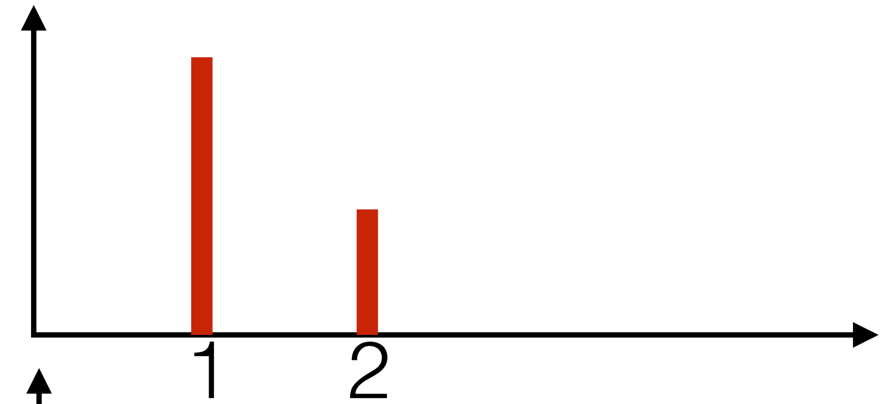

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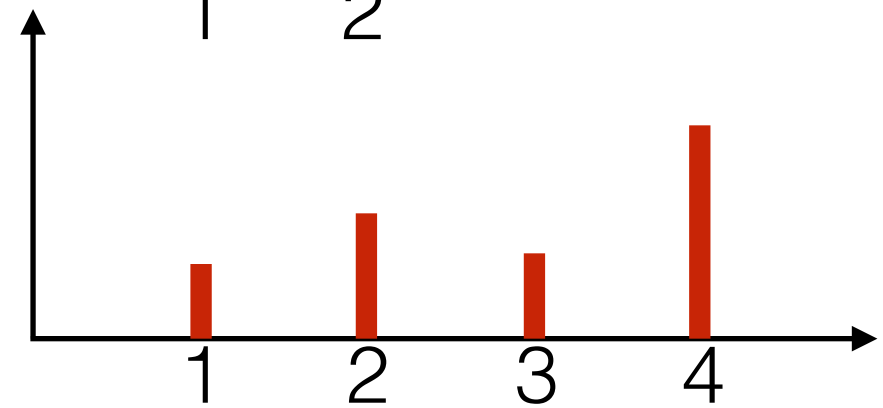

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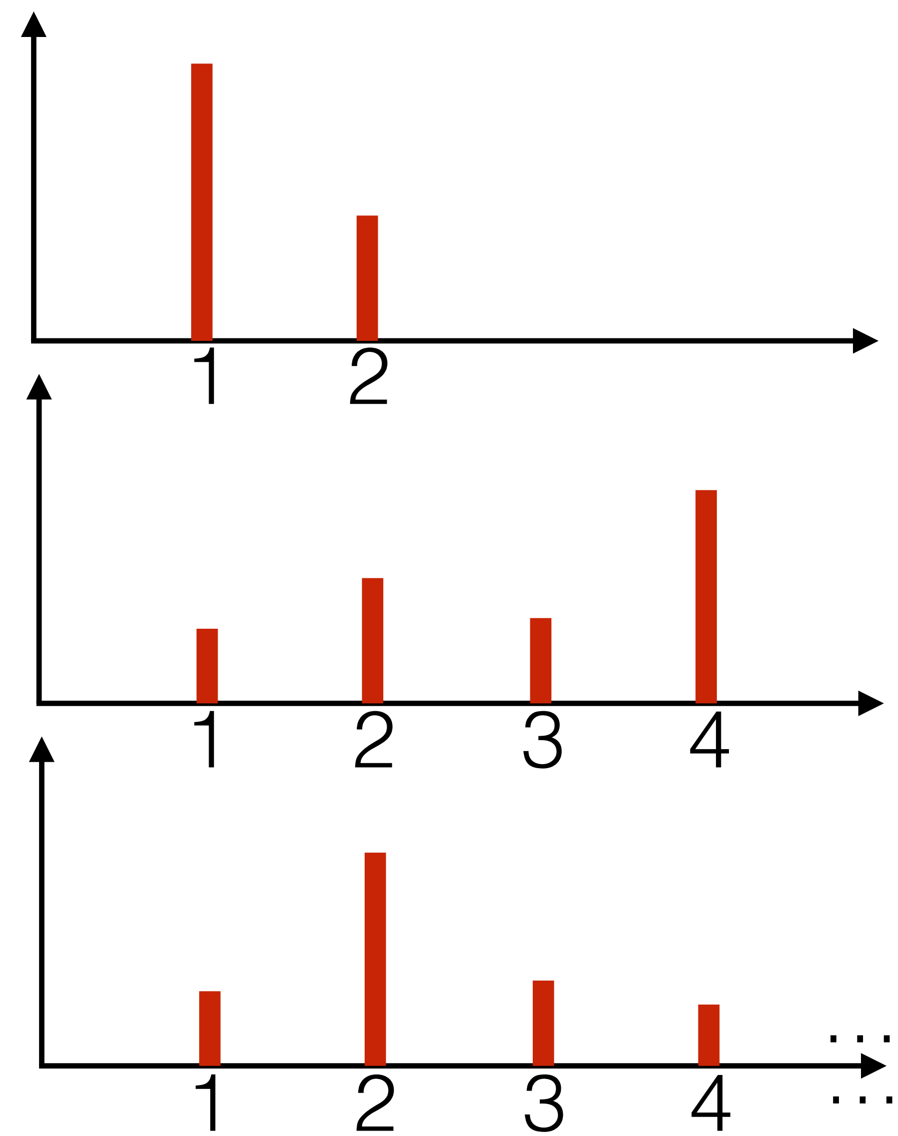

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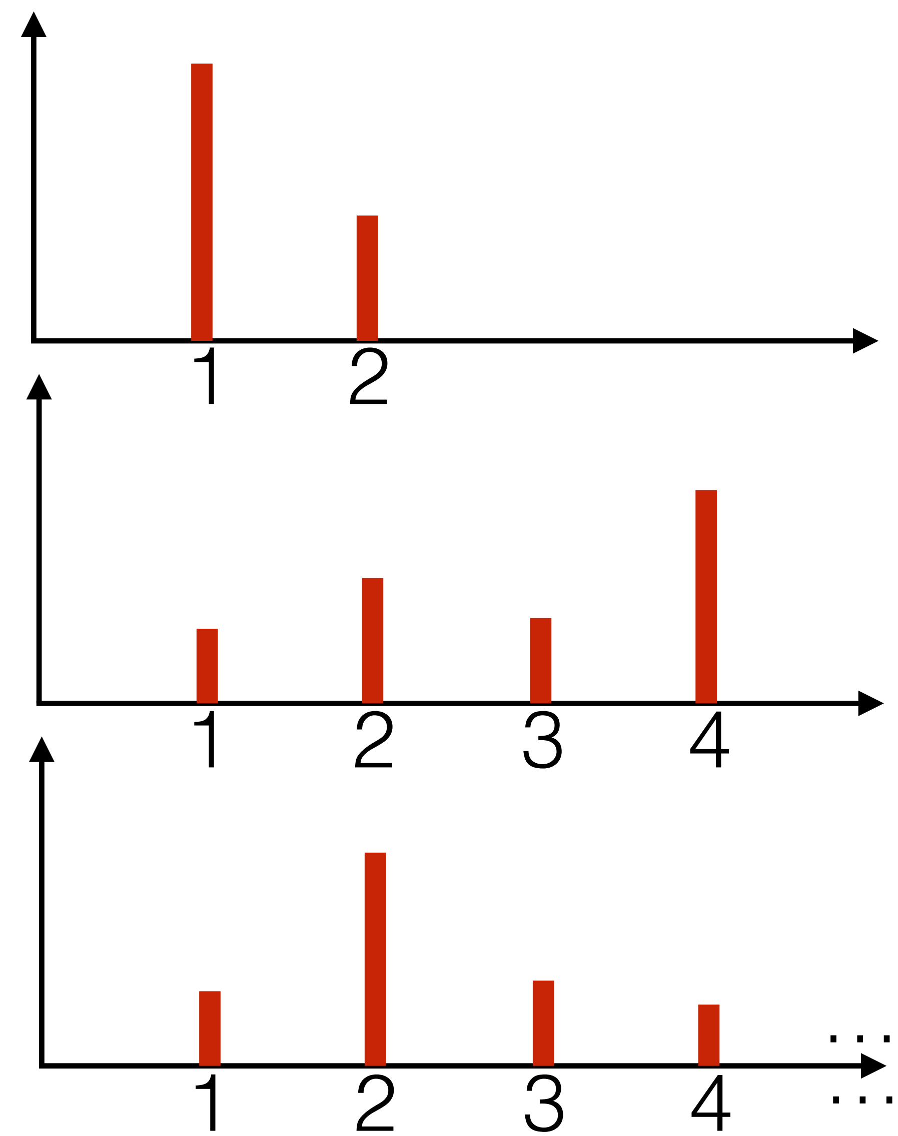

- Infinity of parameters: components
- Growing number of parameters: clusters

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

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  - What does an infinite/growing number of parameters really mean (in NPBayes)?
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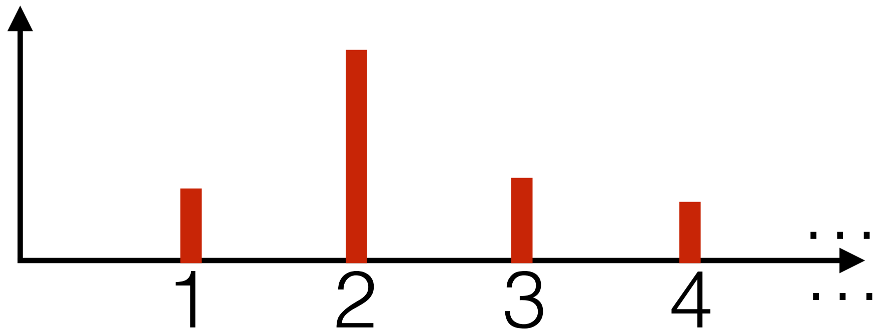
Exercises



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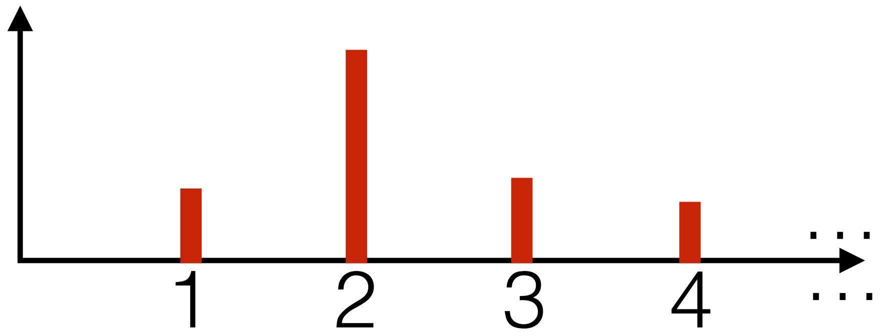
[slides, code: www.tamarabroderick.com/tutorials.html]



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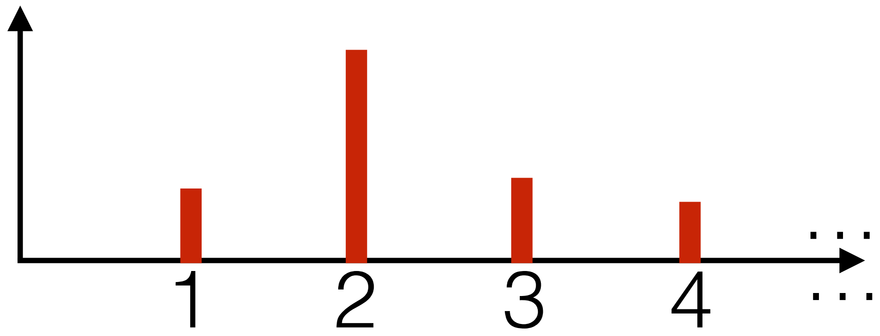

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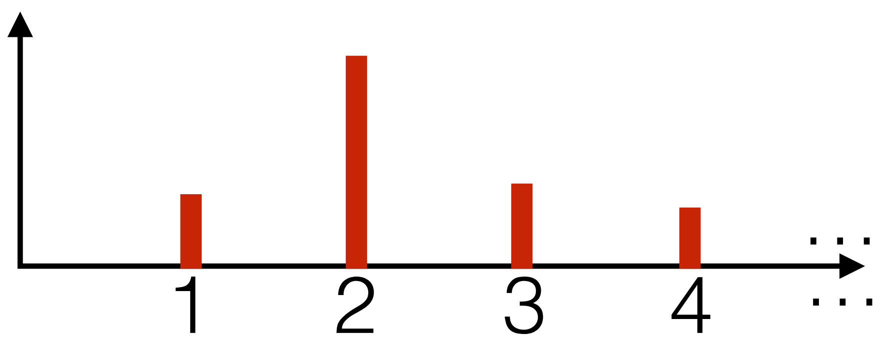

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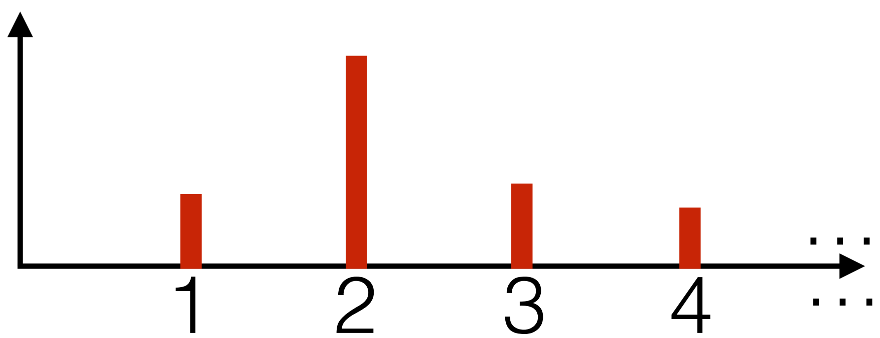

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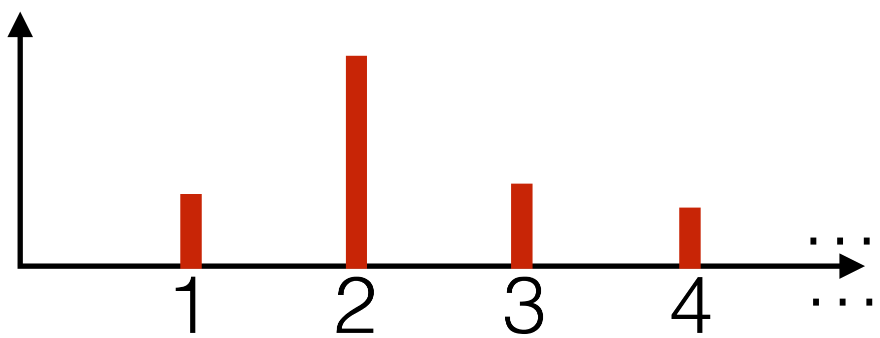

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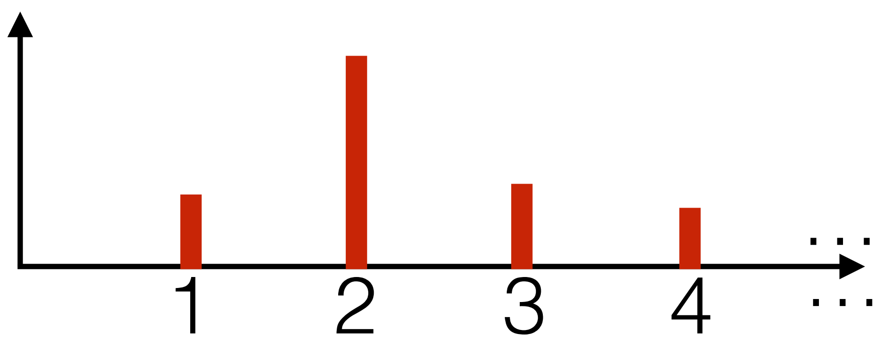

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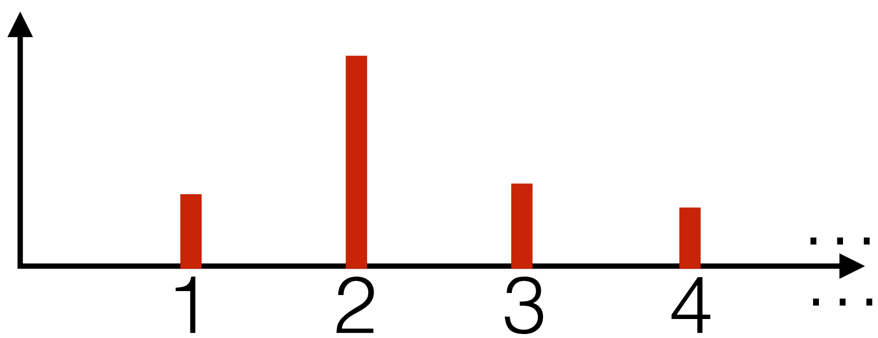

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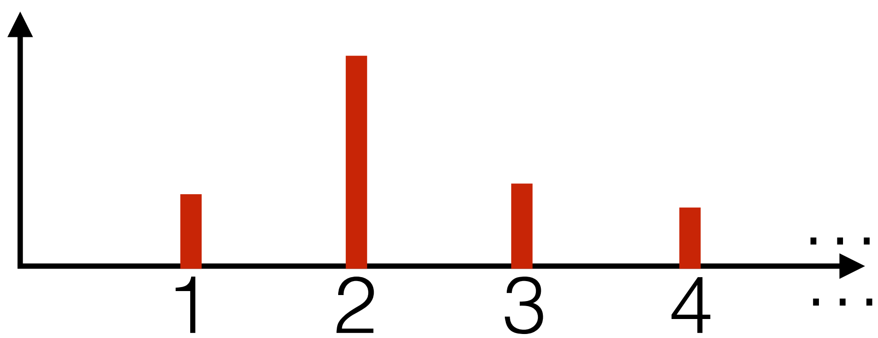

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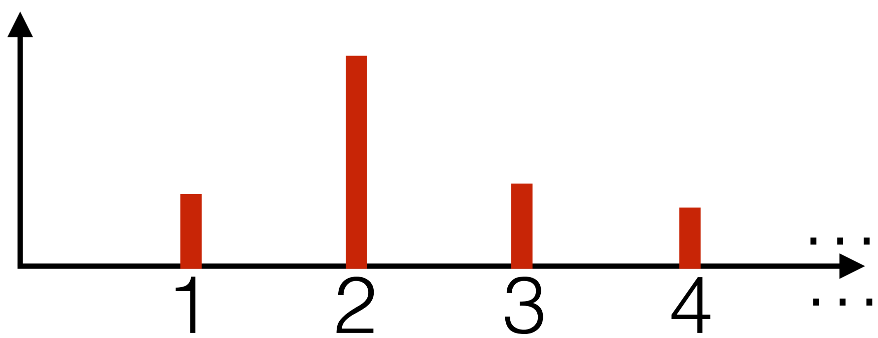

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$$\rho_1 \stackrel{d}{=} \mathrm{Beta}(a_1, \sum_{k=1}^{K} a_k - a_1) \perp\!\!\!\perp \frac{(\rho_2, \ldots, \rho_K)}{1-\rho_1} \stackrel{d}{=} \mathrm{Dirichlet}(a_2, \ldots, a_K)$$

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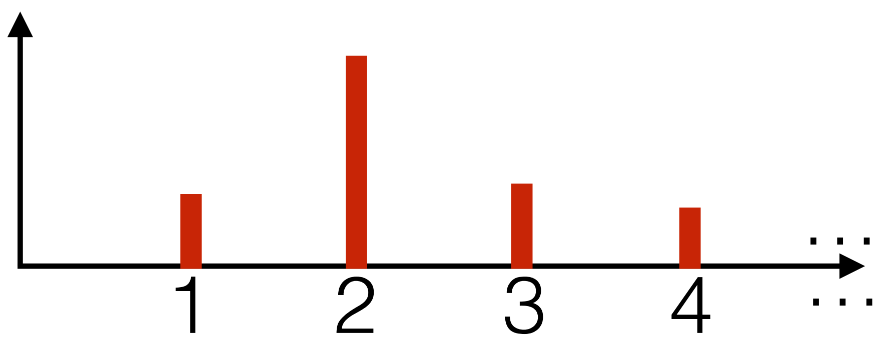

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- For which stick-breaking (a_k, b_k) can you prove $\sum_{k=1}^{\infty} \rho_k = 1$?

References

A full reference list is provided at the end of the “Part III” slides.