Variational Bayes and beyond: Foundations of scalable Bayesian inference

Tamara Broderick
Associate Professor
MIT

http://tamarabroderick.com/tutorial_2021_ssc.html

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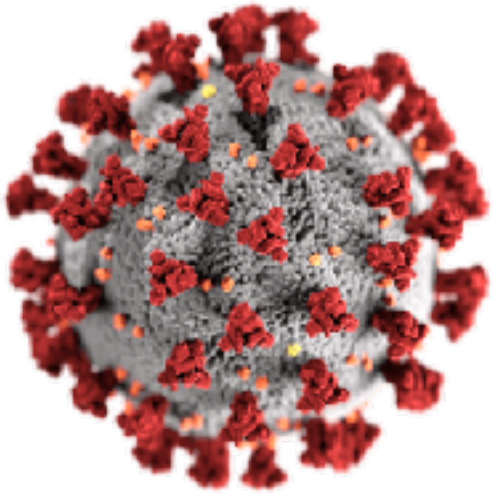
Rough schedule:	Part I: 11 am Eastern Time
	Break: 12 noon
	Part II: 12:30 pm
	Break: 1:30 pm
	Part III after the Break
	Finish: by 3:00 pm ET

Bayesian inference

Bayesian inference



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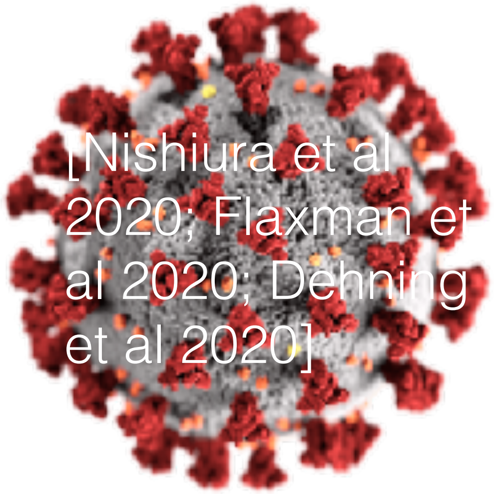

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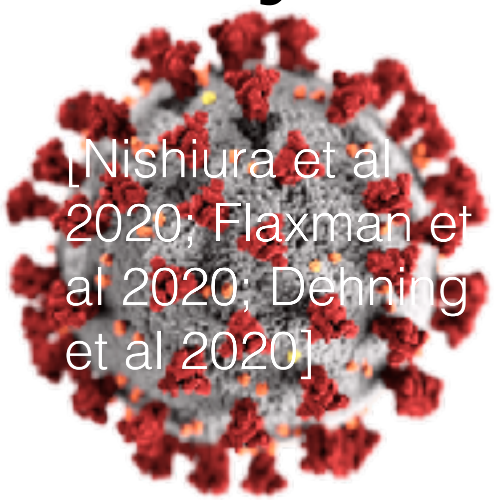

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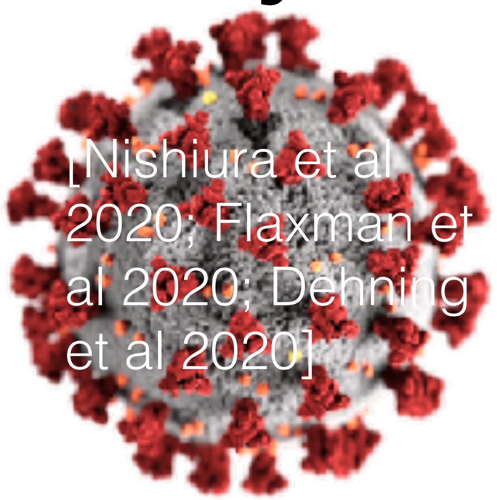

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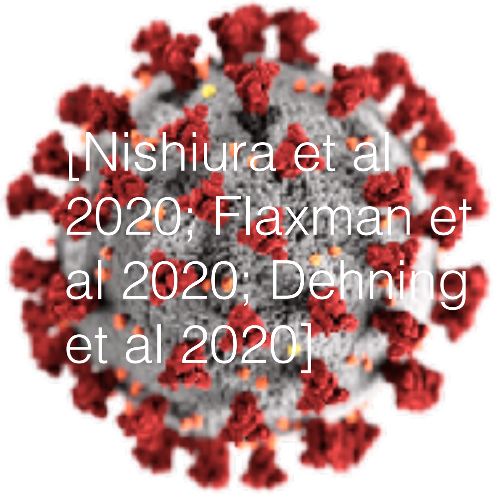
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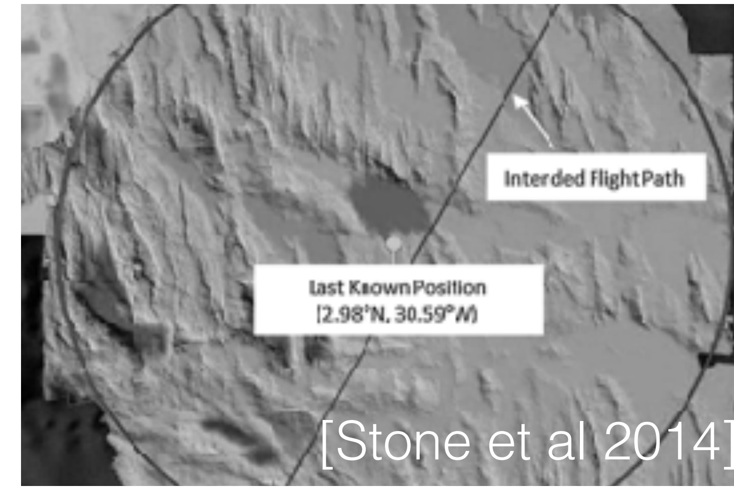

[Stone et al 2014]

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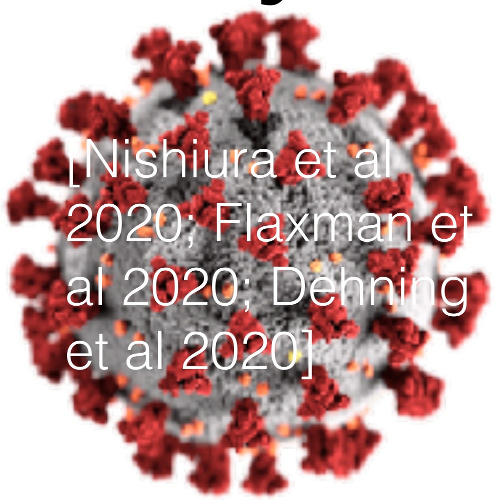

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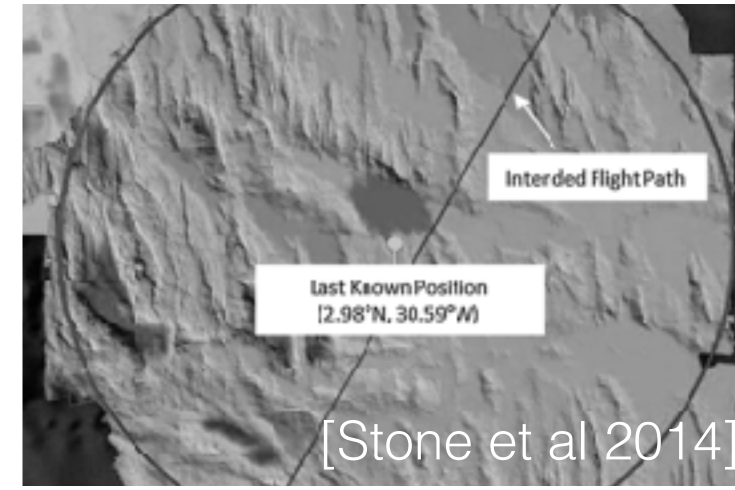

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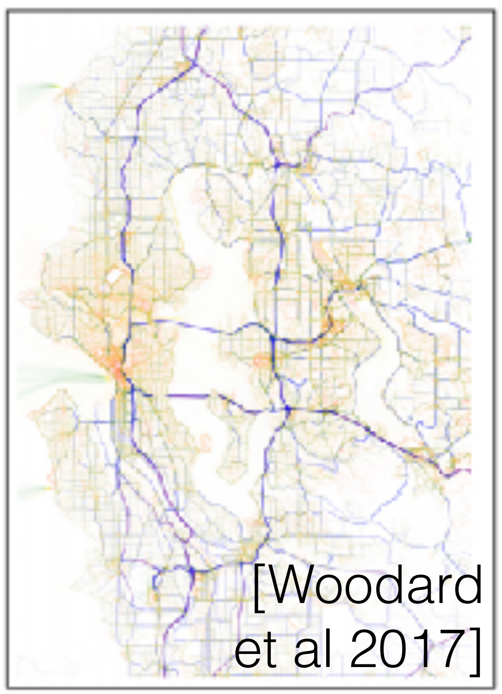

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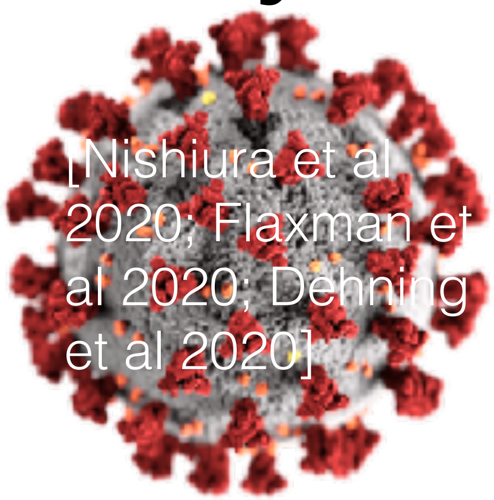

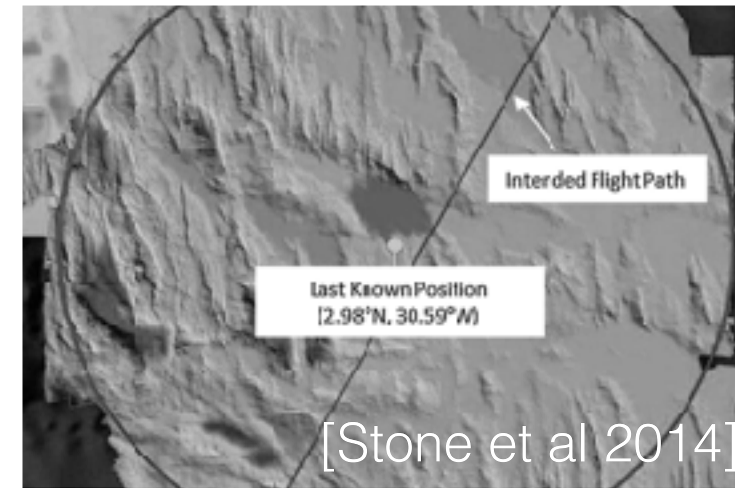

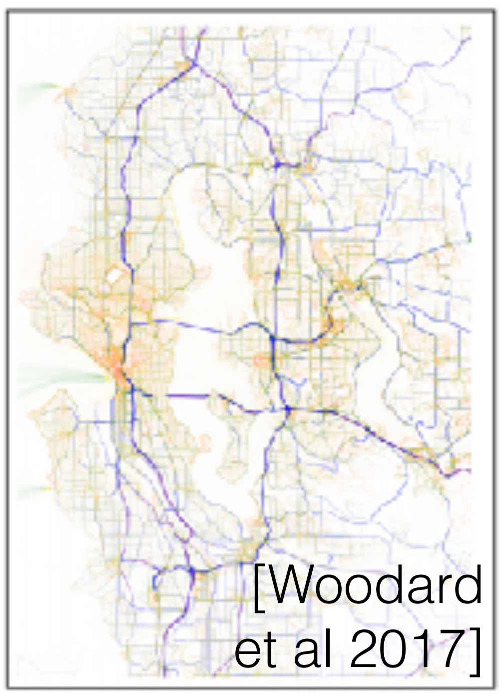

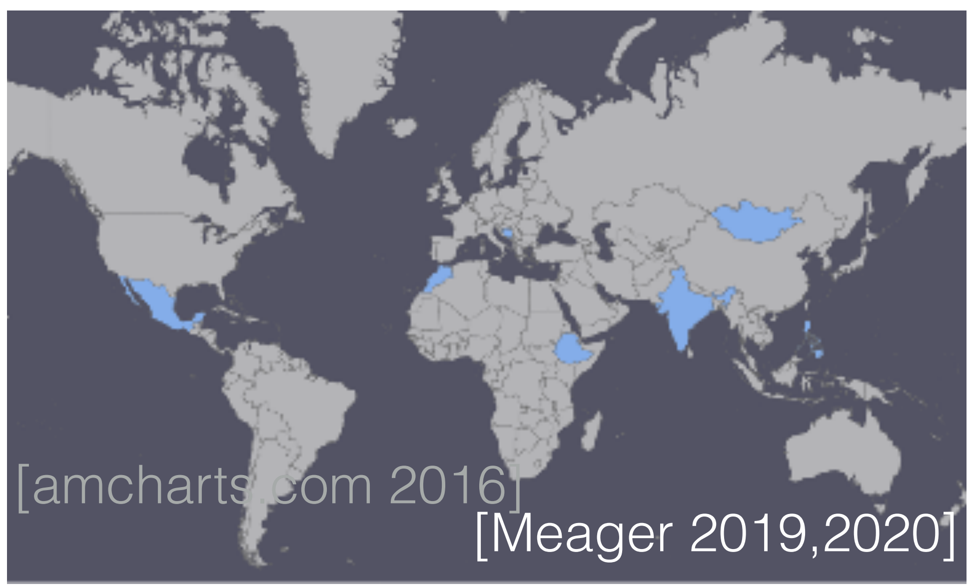

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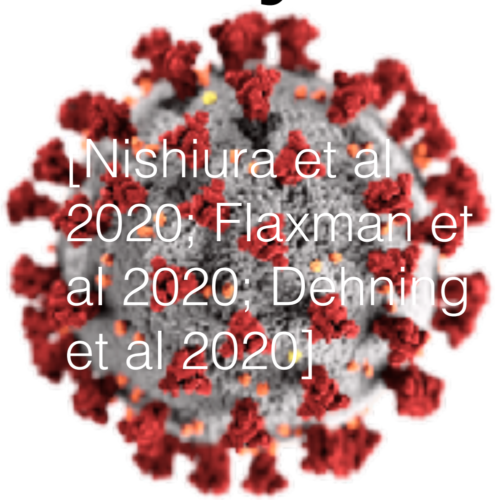

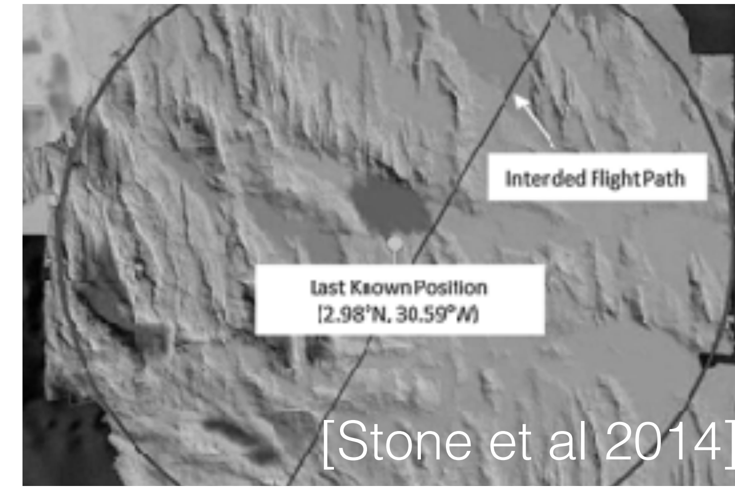

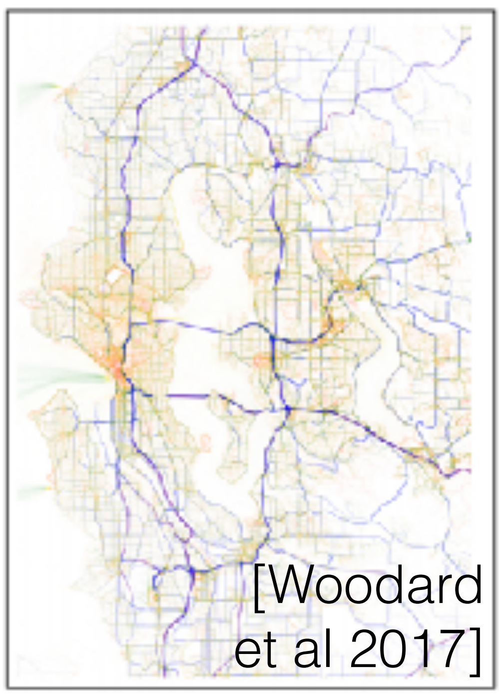

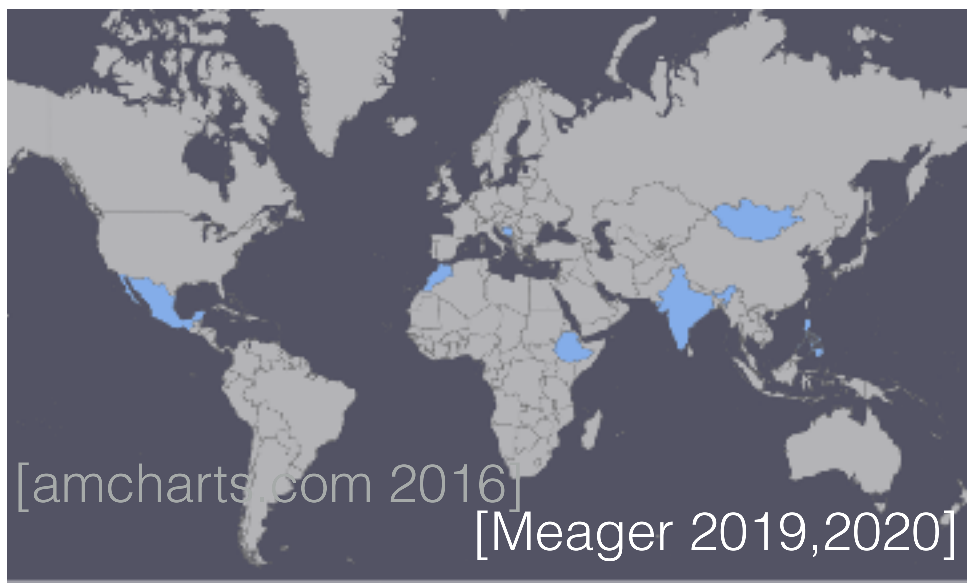

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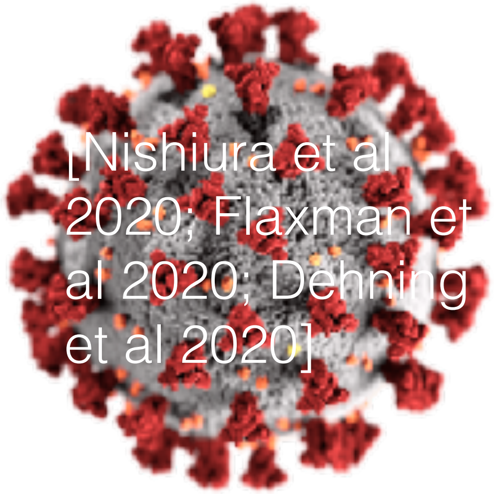

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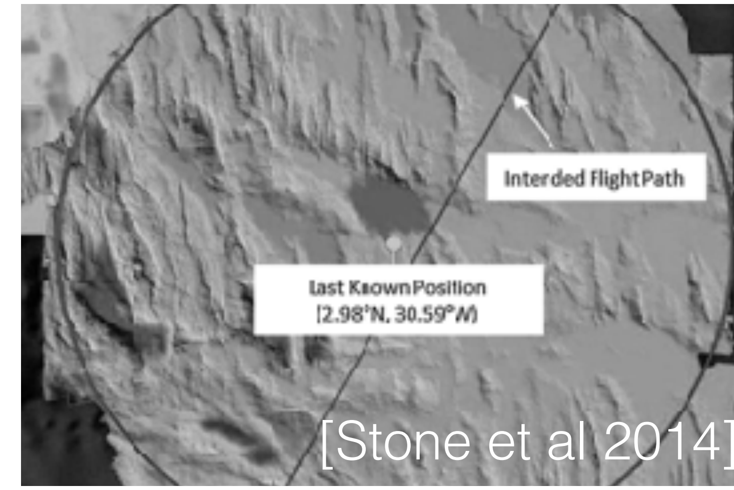

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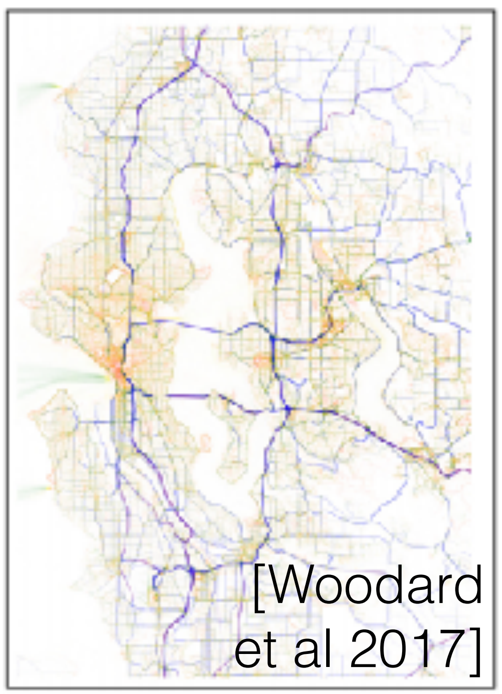

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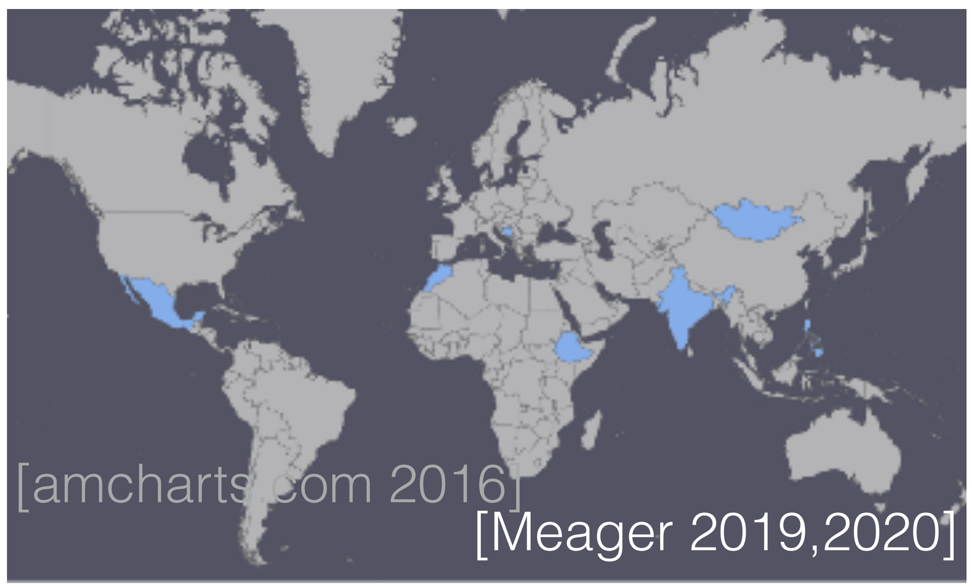

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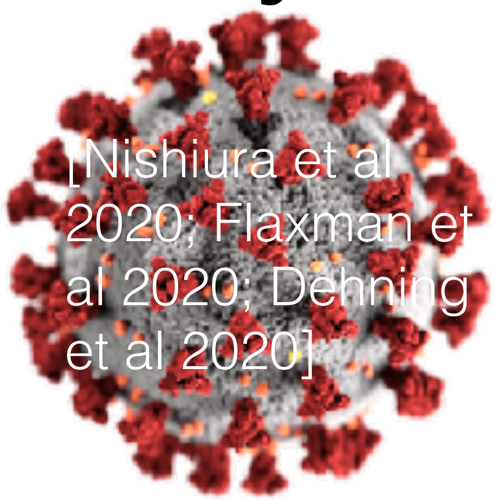
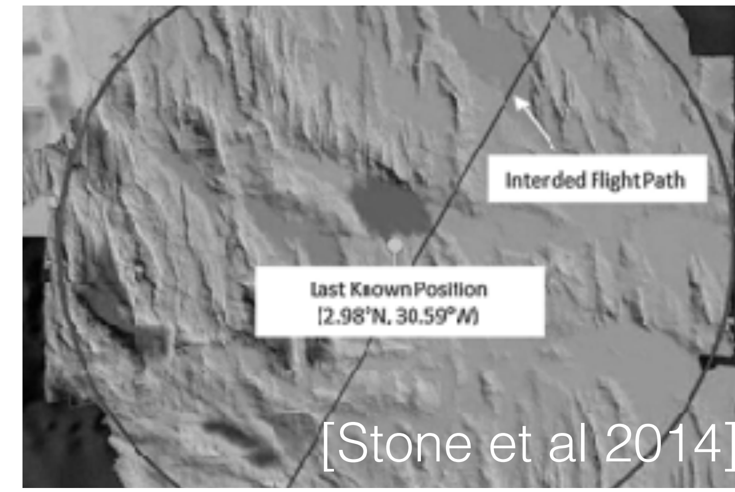
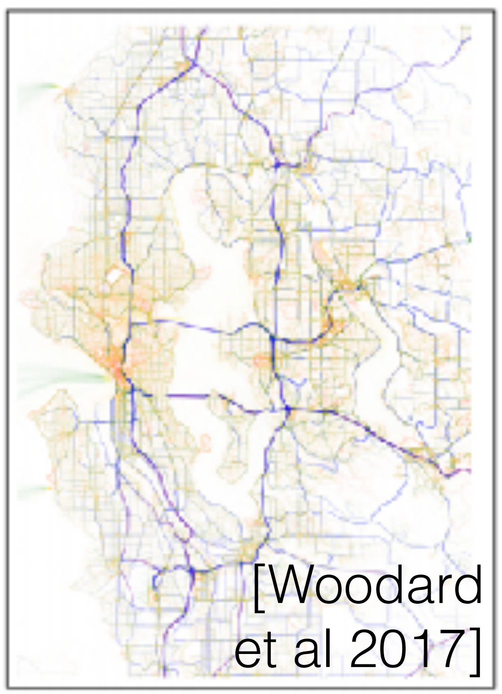
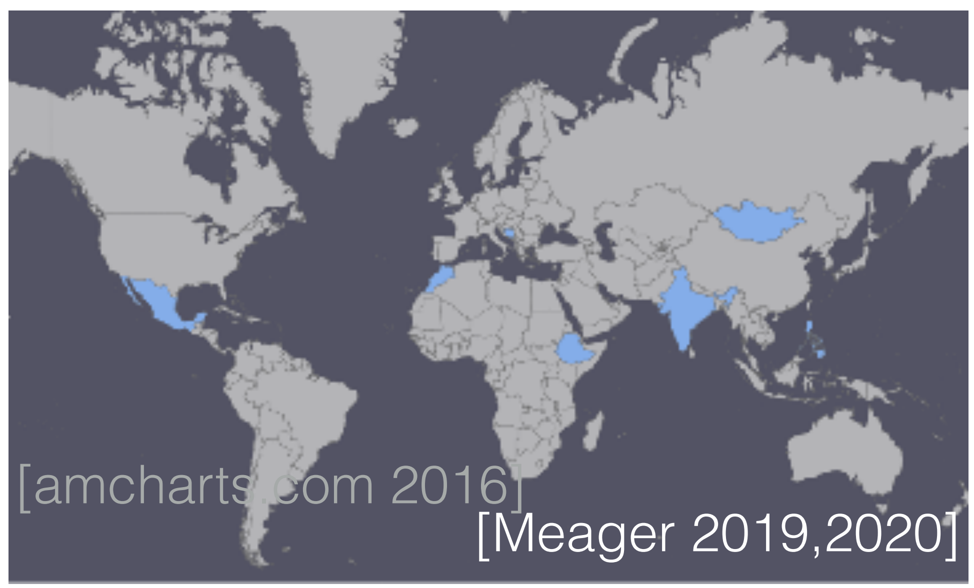


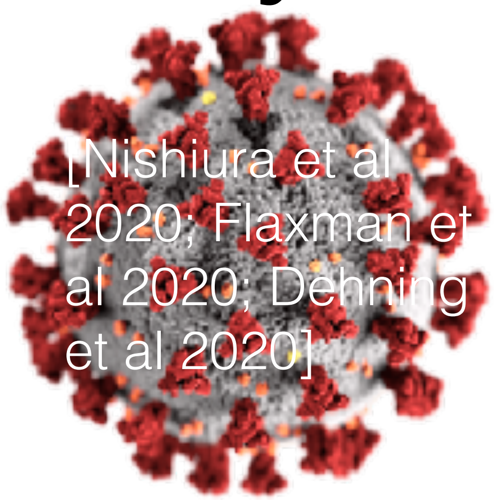






- Goals: good point estimates, uncertainty estimates

Bayesian inference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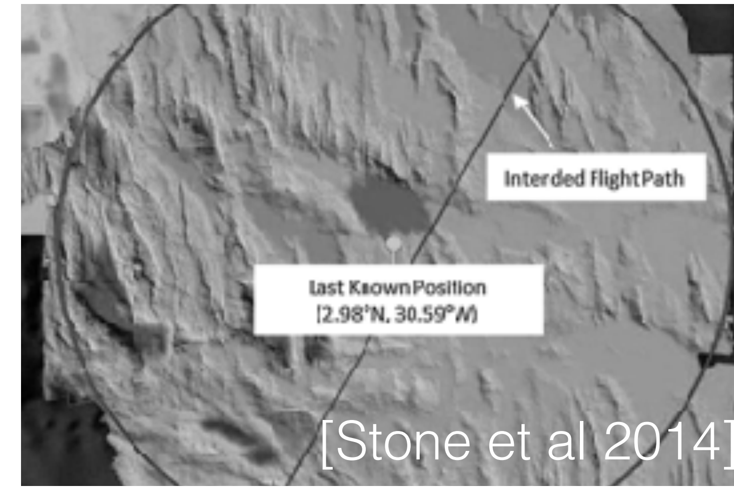

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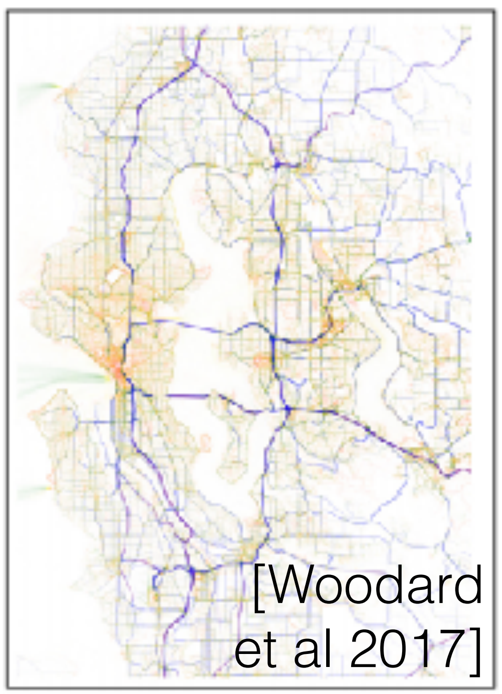
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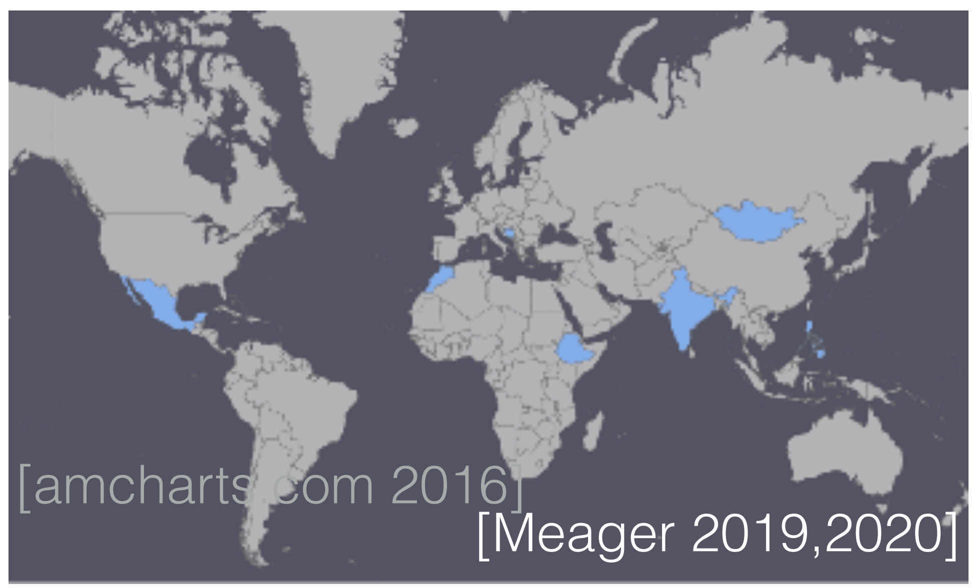
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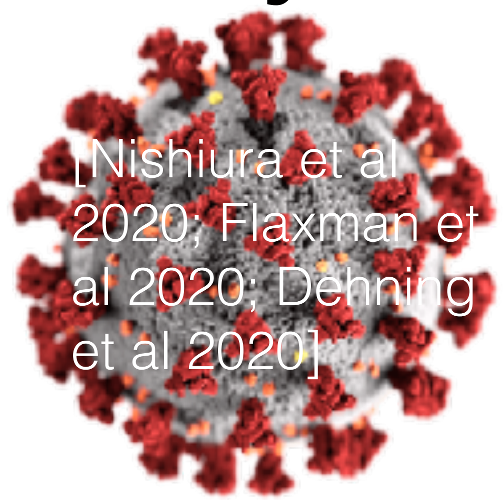
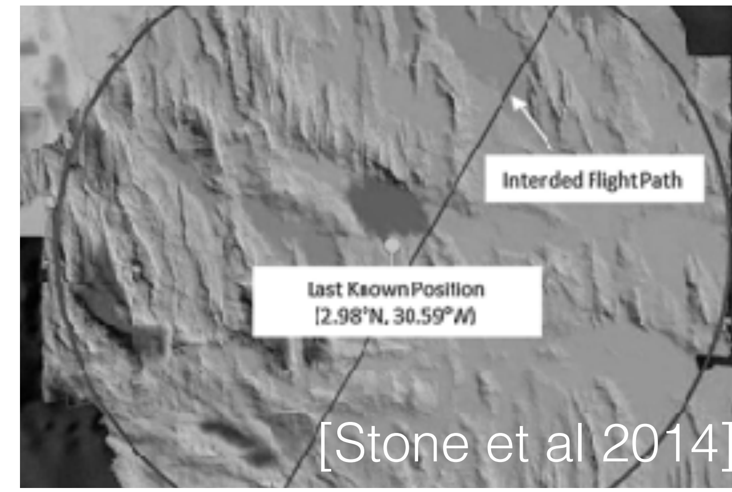
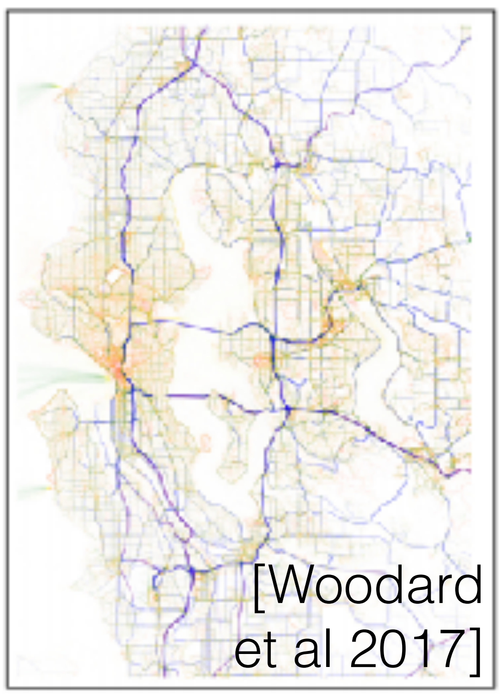
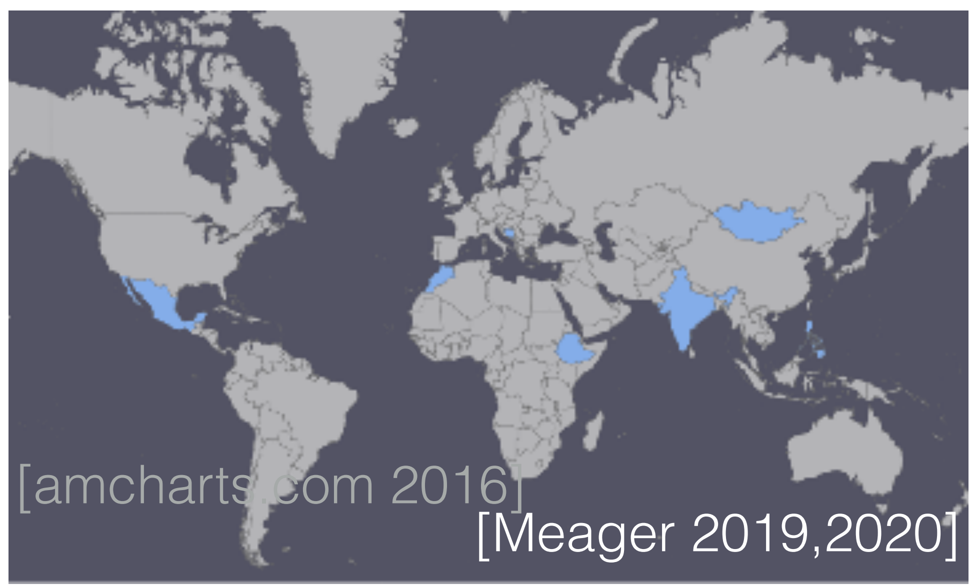
- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info

Bayesian inference









[mc-stan.org]

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  - More: interpretable, modular, expert info

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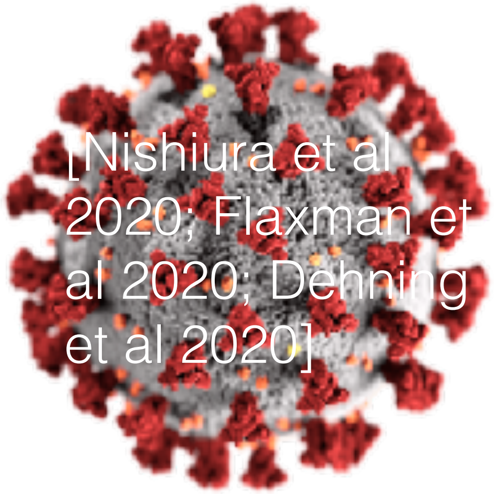

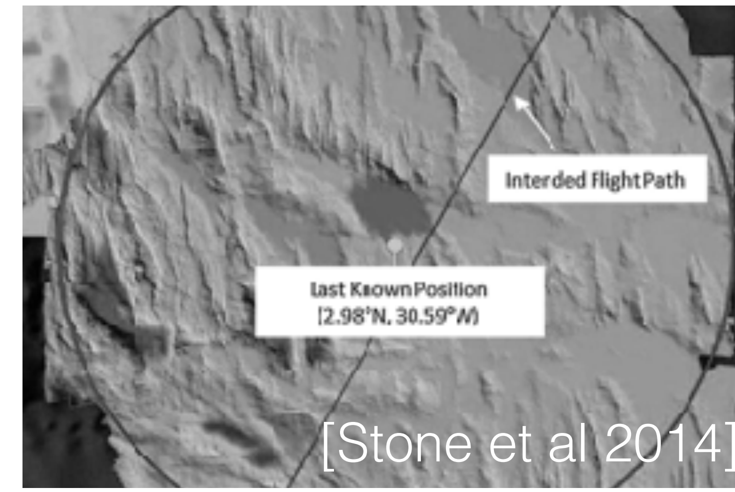

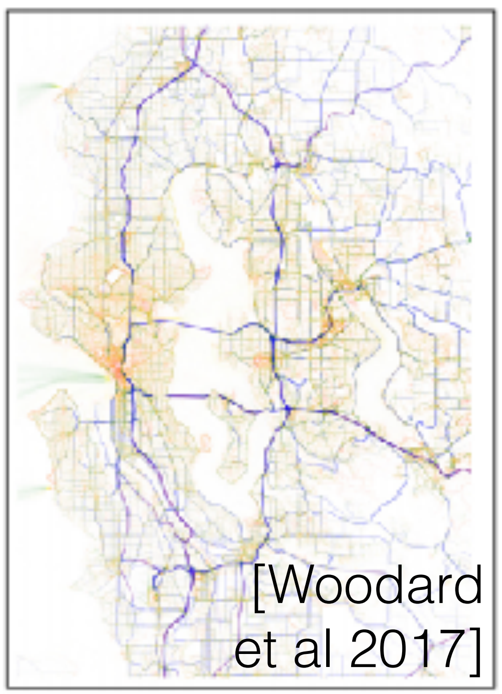

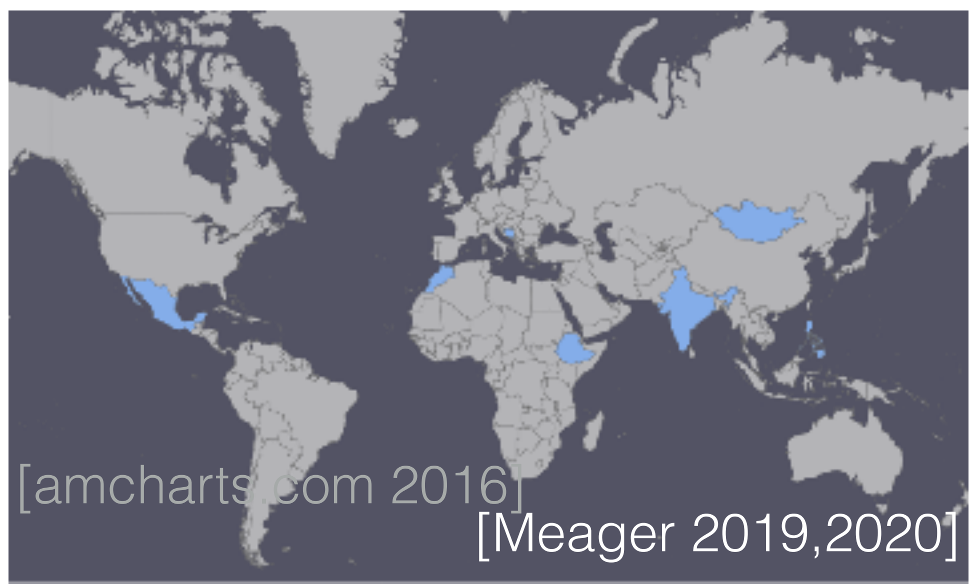

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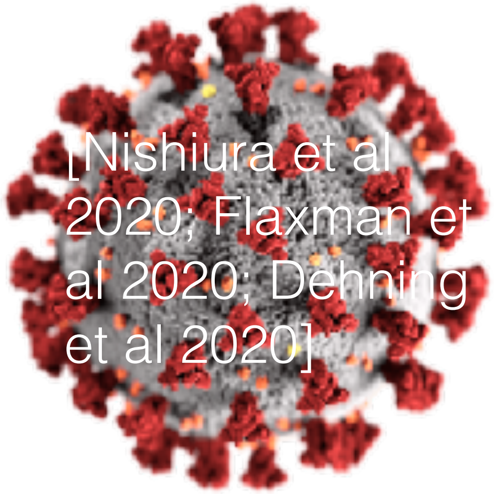
- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info
- Challenge: speed (compute, user), reliable inference

Bayesian inference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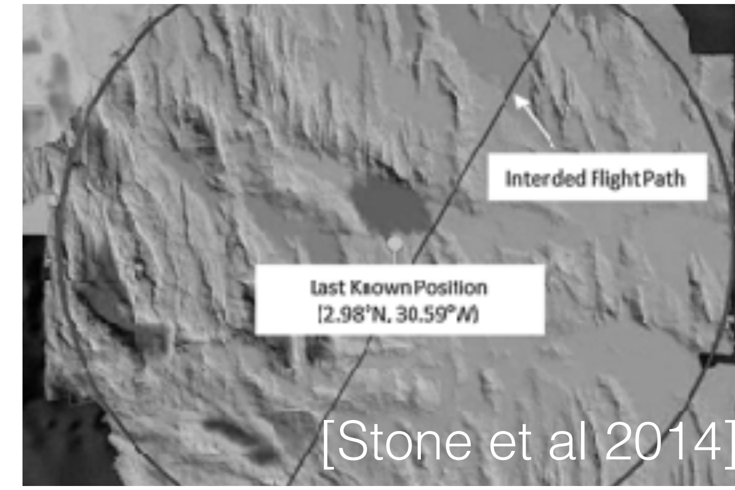

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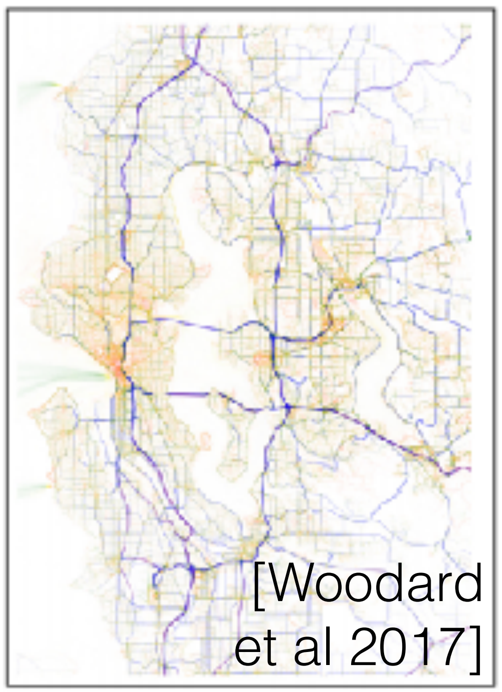
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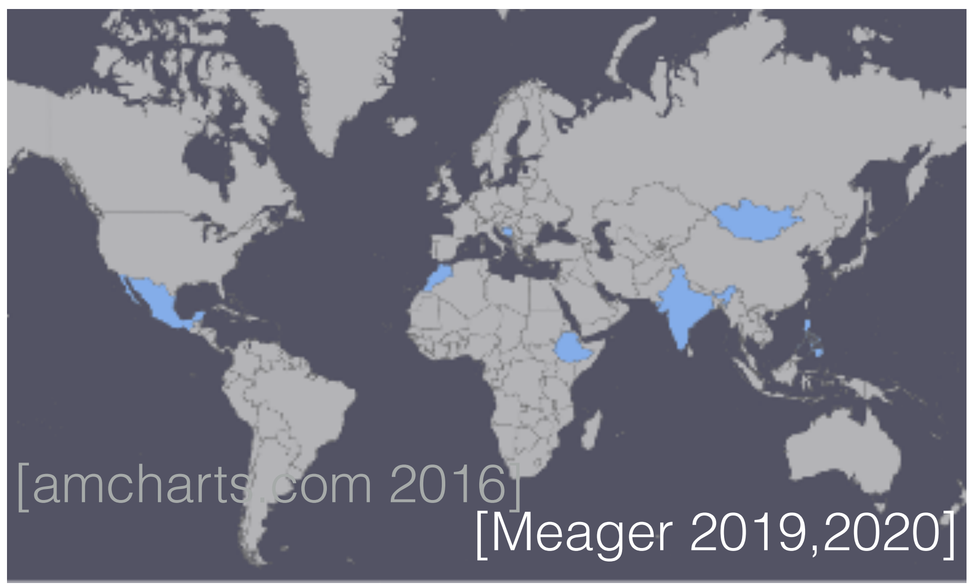
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- Goals: good point estimates, uncertainty estimates
  - More: interpretable, modular, expert info
- Challenge: speed (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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- Modern problems: often large data, large dimensions

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- Variational Bayes can be very fast

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"Arts"	"Budgets"	"Children"	"Education"
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FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
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LOVE	CONGRESS	LIFE	HAITI

[Blei et al 2003]

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.

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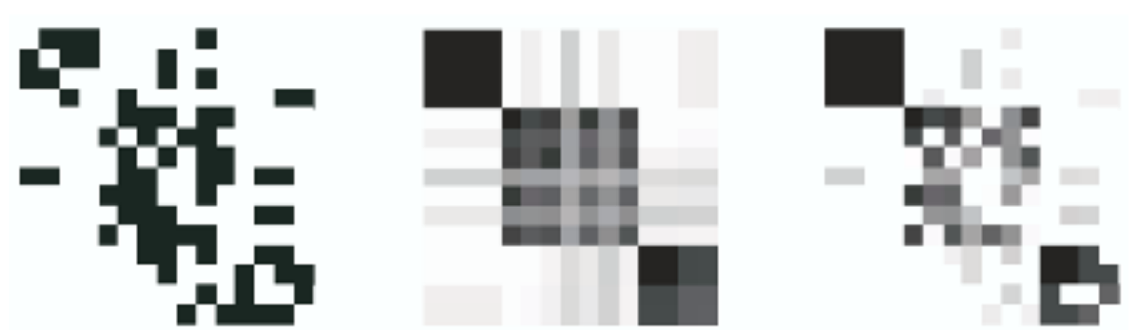

[Airoldi et al 2008]

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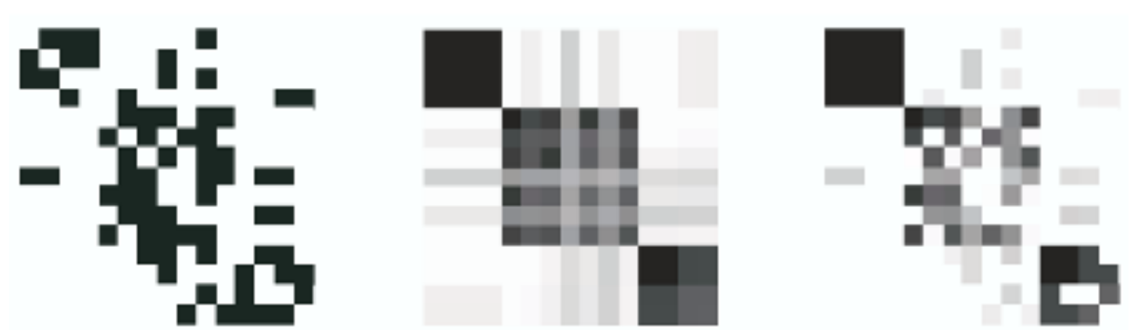

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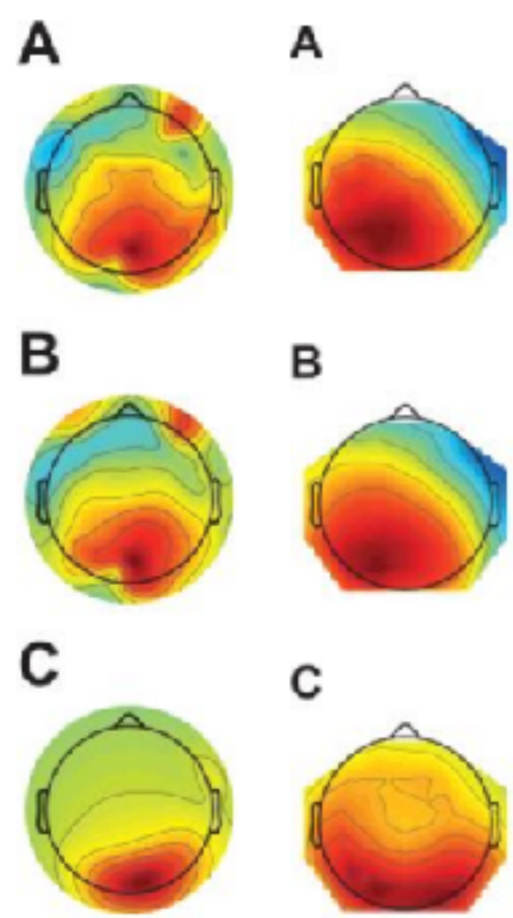

[Gershman et al 2014]

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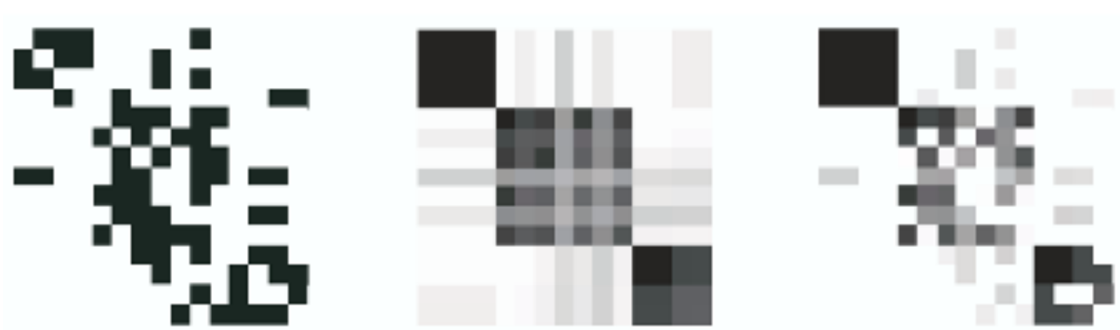

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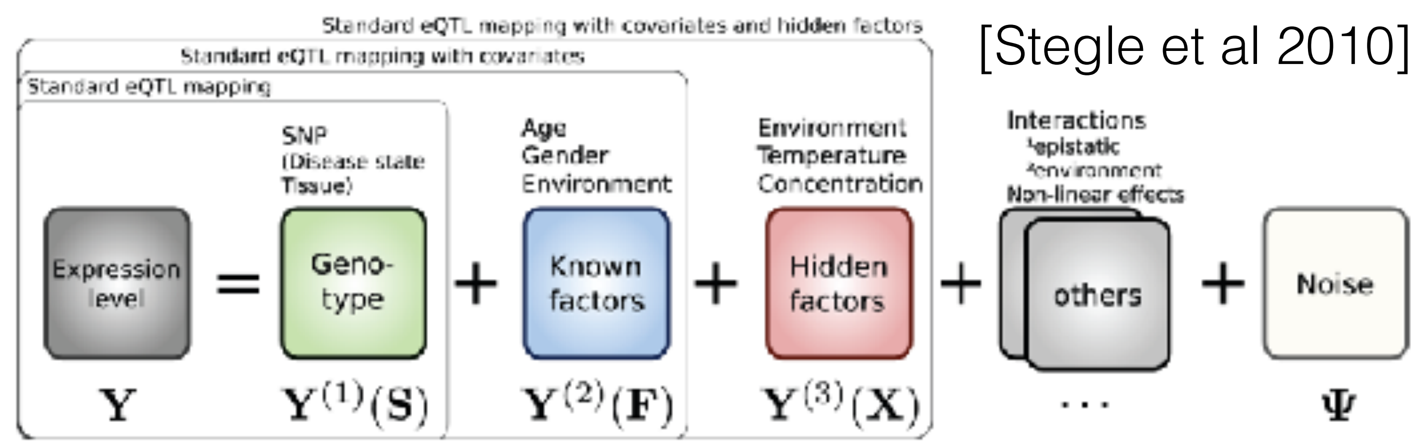
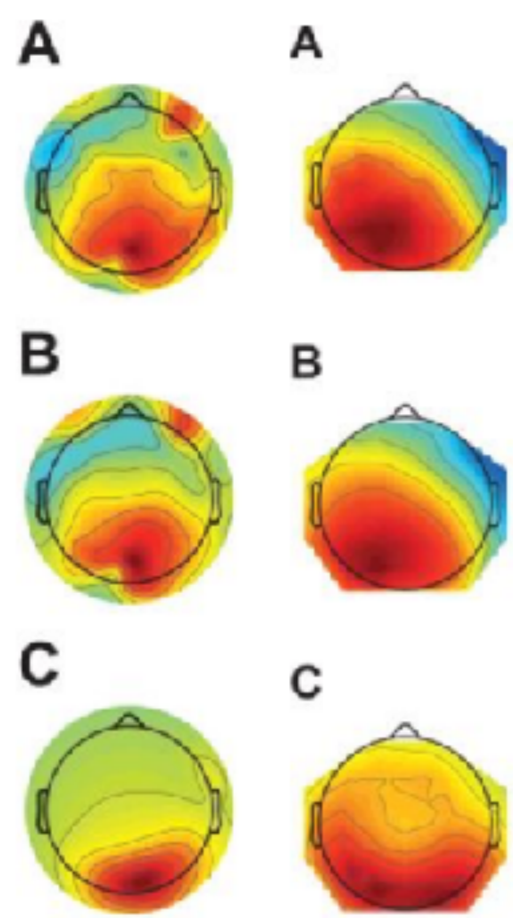
[Stegle et al 2010]





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ACTOR	NEW	SAYS	BENNETT
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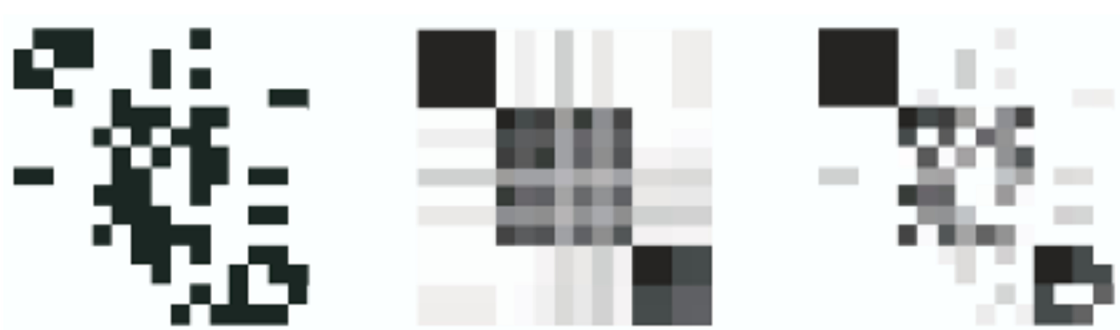
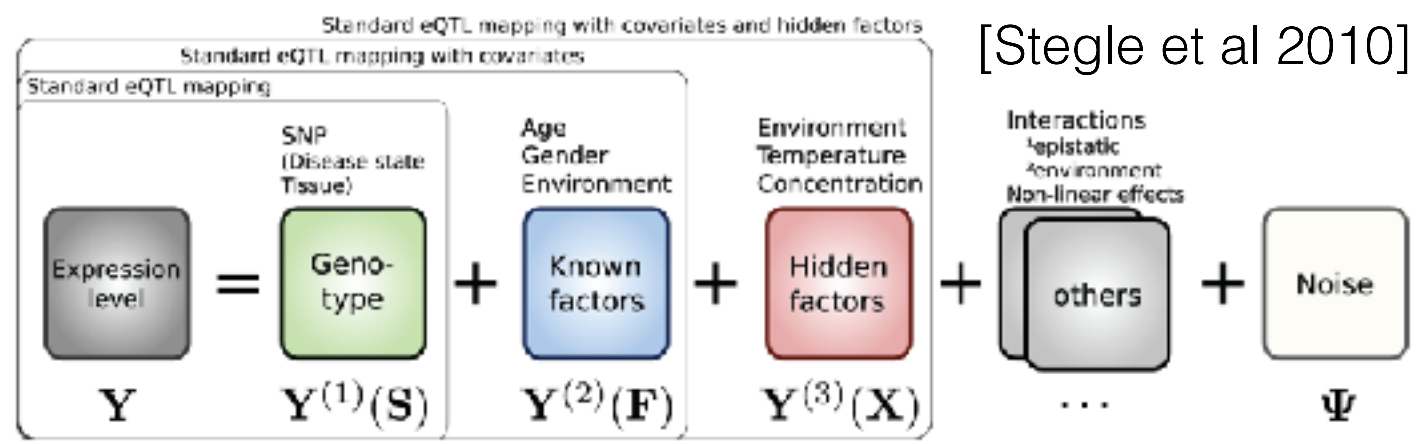
YORK	PLAN	WELFARE	NAMPHY
OPERA	MONEY	MEN	STATE
THEATER	PROGRAMS	PERCENT	PRESIDENT
ACTRESS	GOVERNMENT	CARE	ELEMENTARY
LOVE	CONGRESS	LIFE	HAITI

[Blei et al 2003]

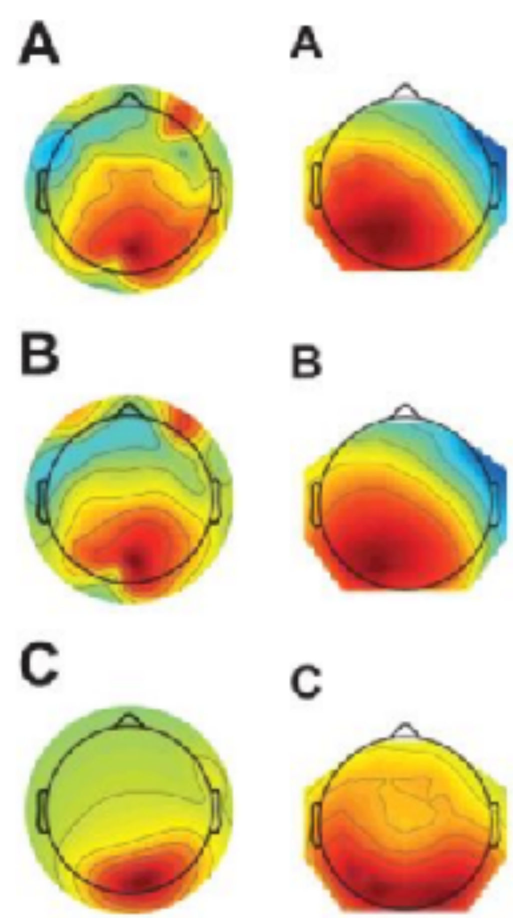
The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. "Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services," Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center's share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.



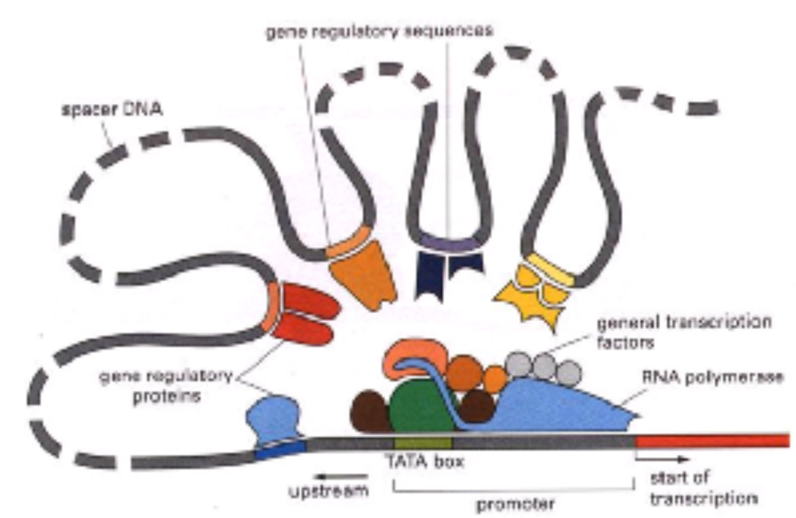
[Airoldi et al 2008]



[Stegle et al 2010]



[Gershman et al 2014]



[Xing et al 2004]
[Xing 2003]

[Blei et al 2018]

Roadmap

- Bayes & Approximate Bayes review

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- What is:
  - Variational Bayes (VB)
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Bayesian inference

Bayesian inference

parameters

θ

Bayesian inference

parameters

$$p(\theta)$$

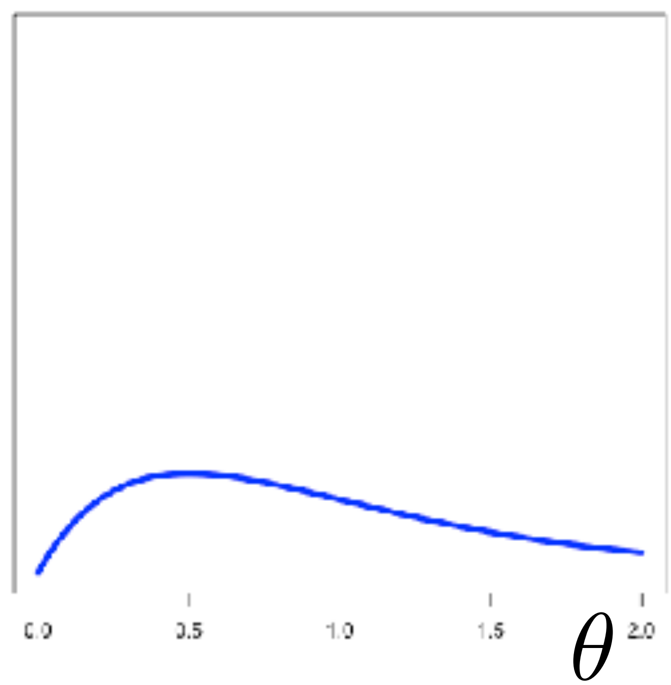
prior

Bayesian inference

parameters

$p(\theta)$

prior

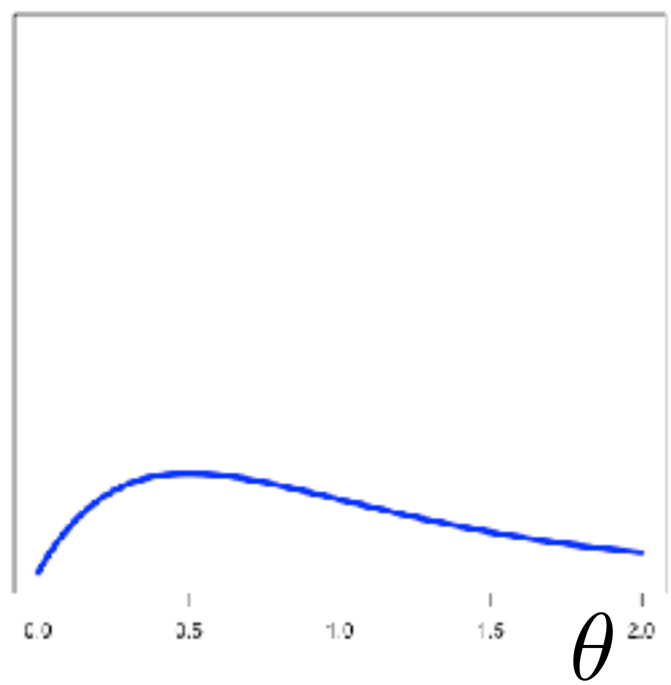


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior



Bayesian inference

data parameters

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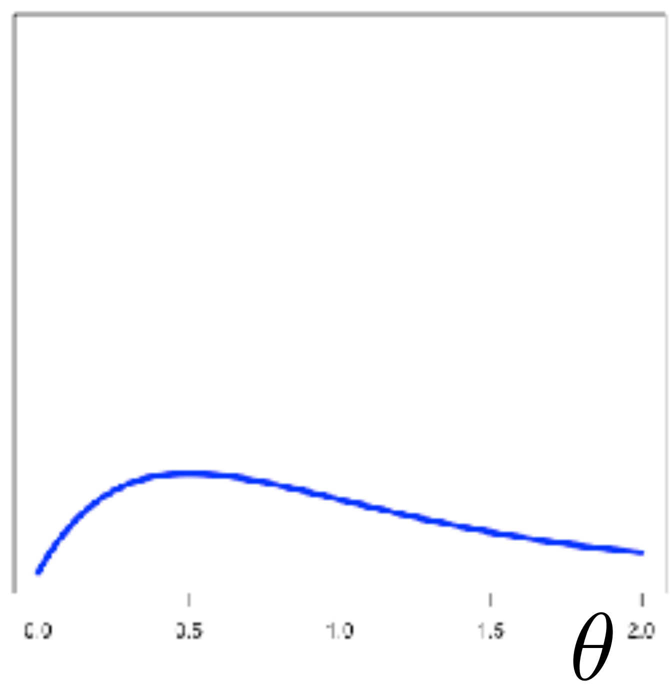

Bayesian inference

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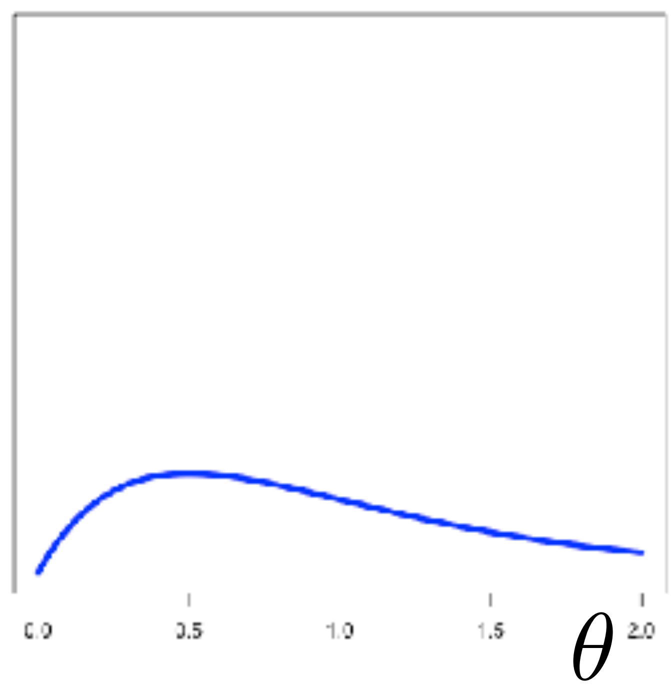
$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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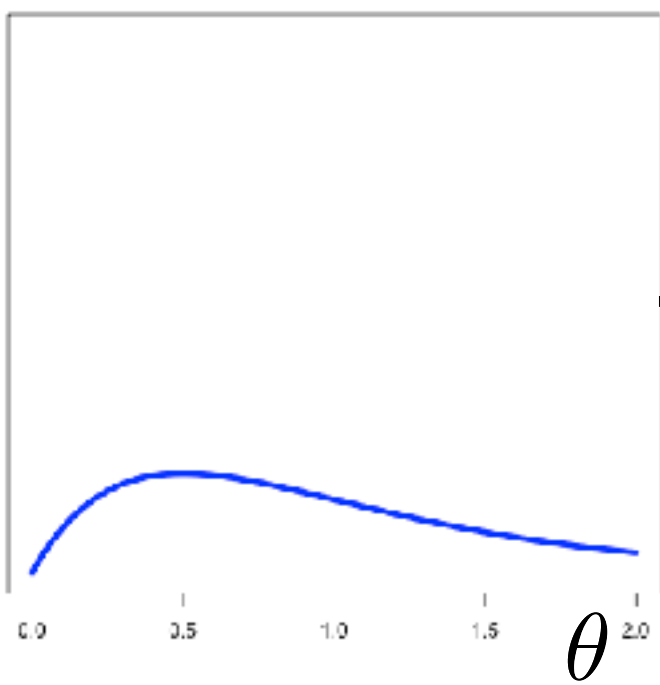

Bayes Theorem

Bayesian inference

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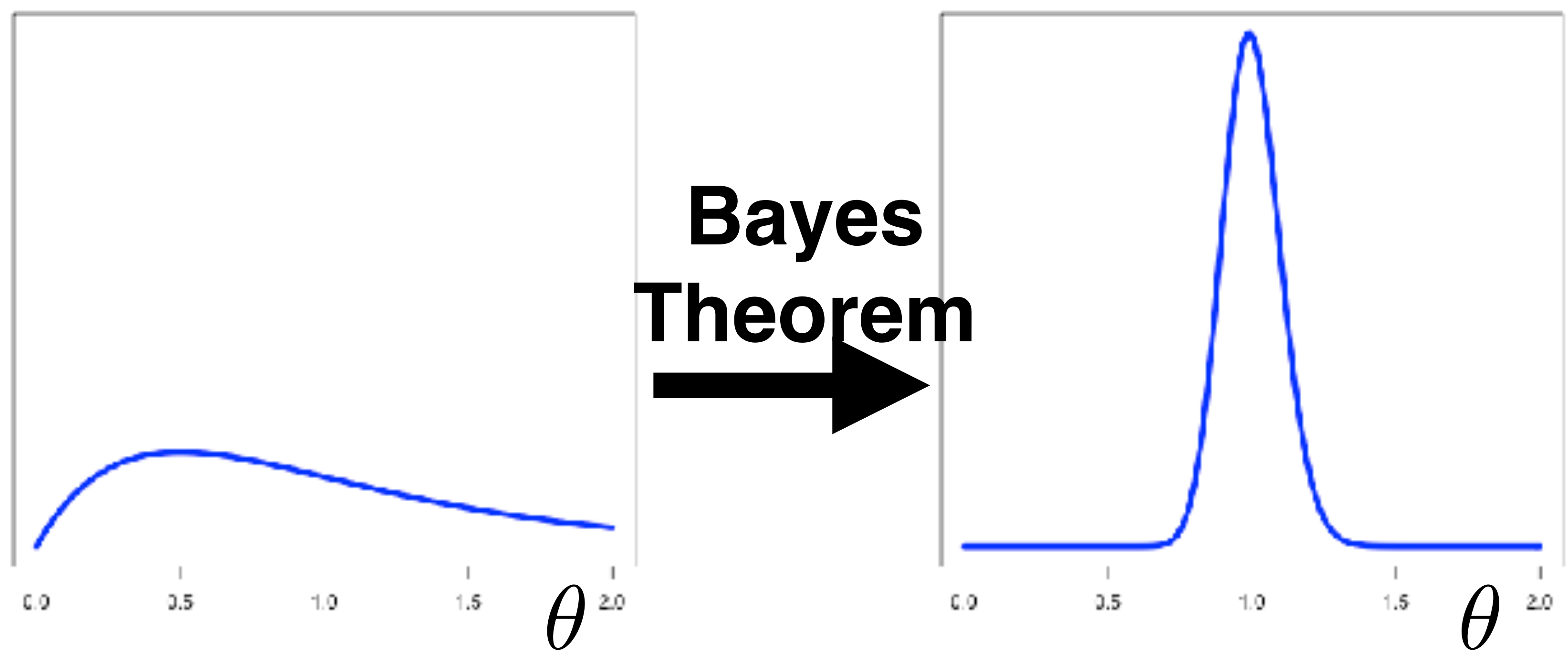

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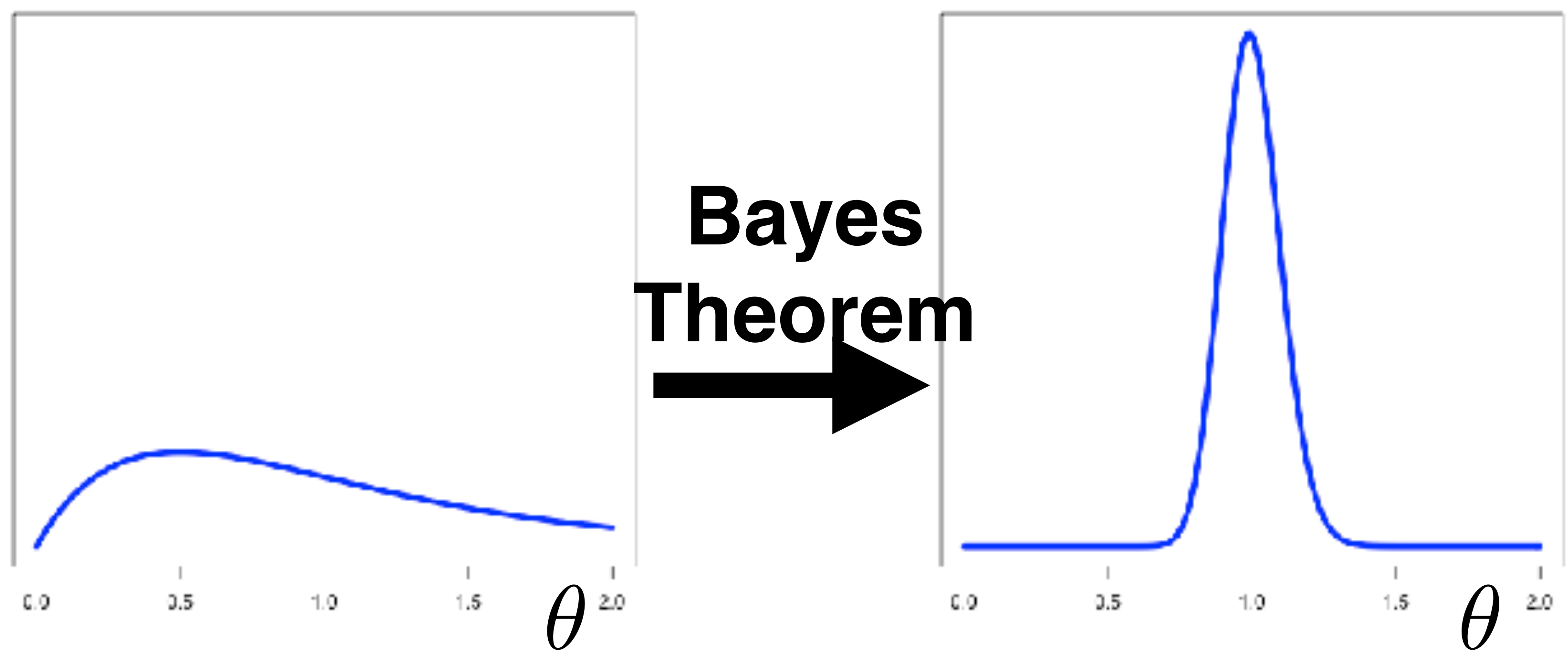

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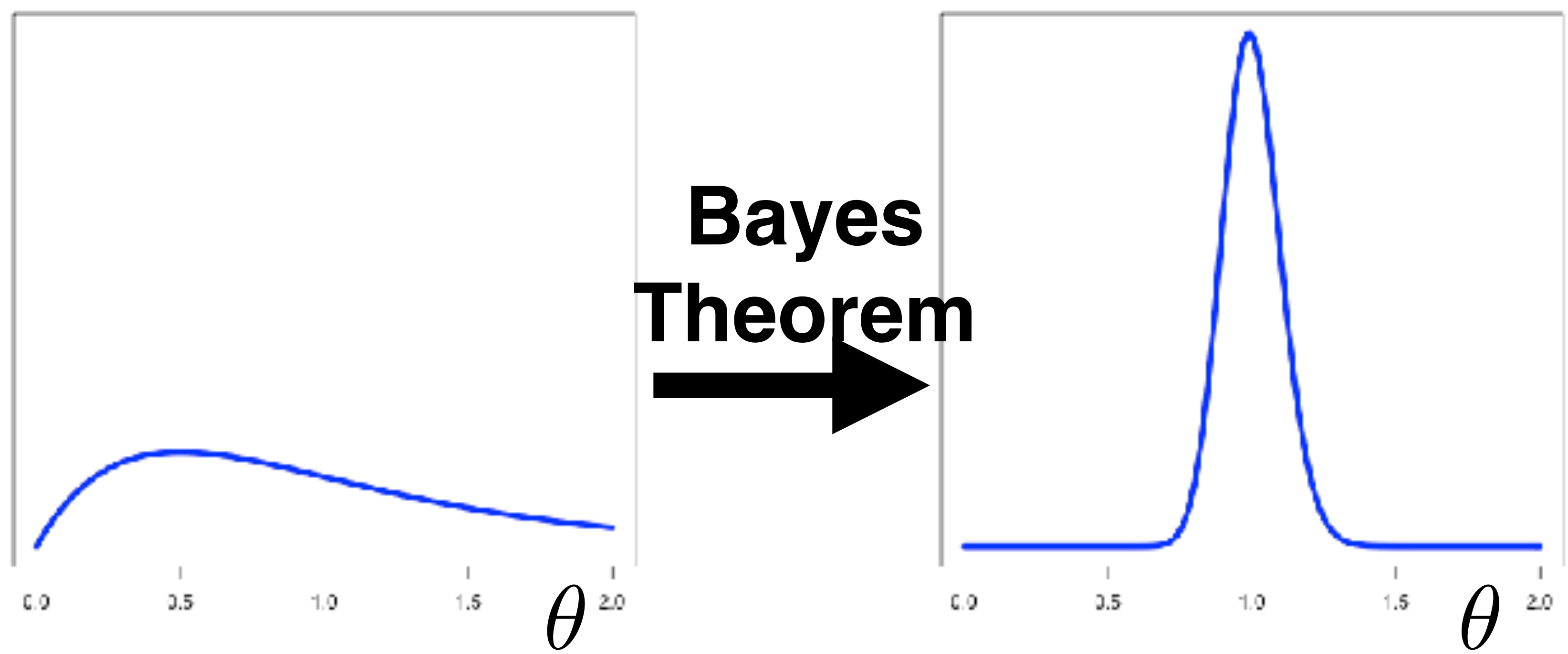

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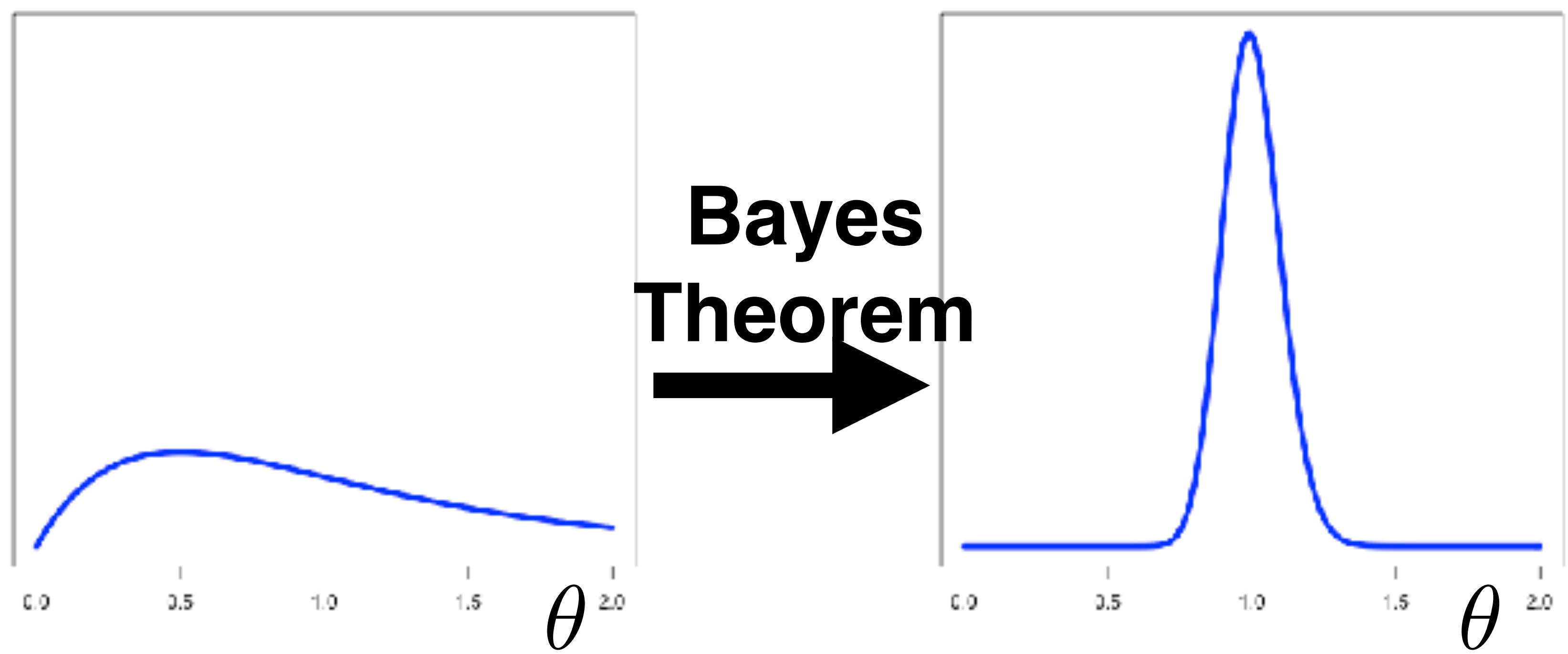

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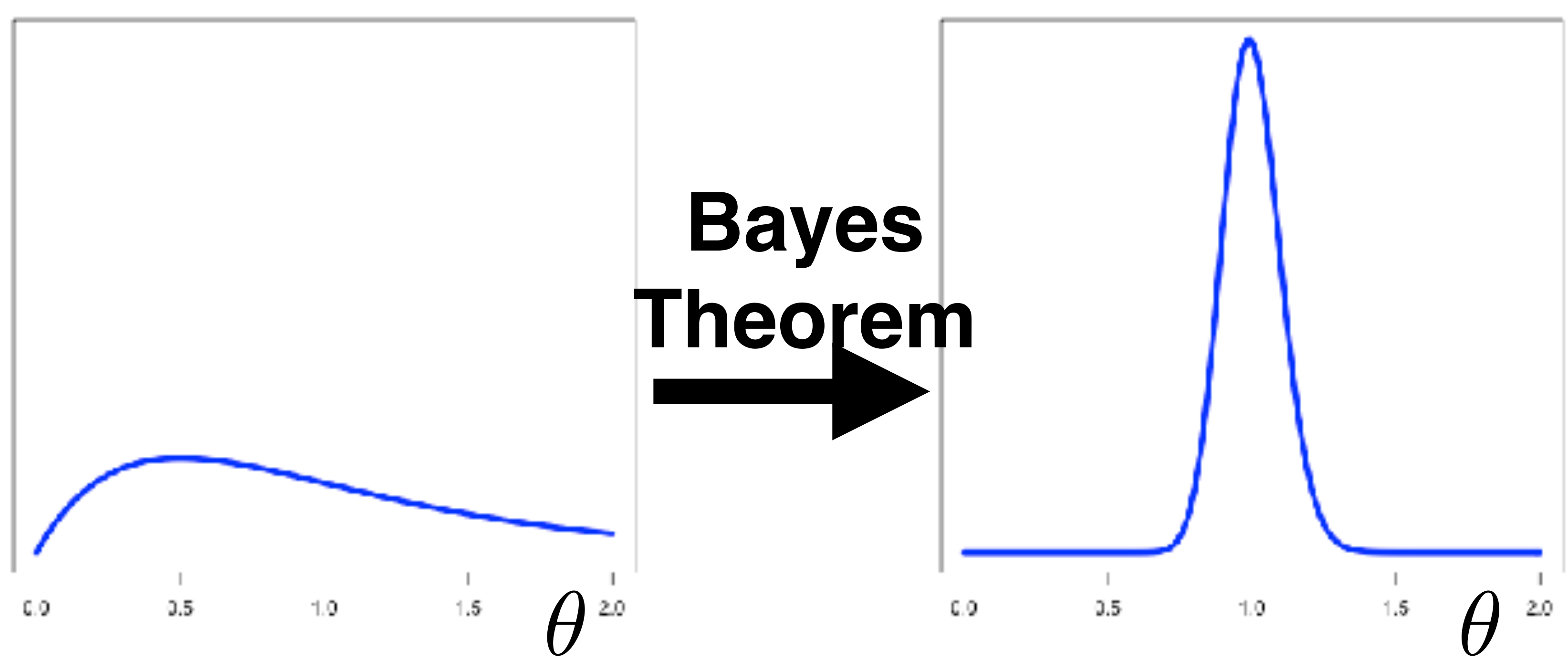

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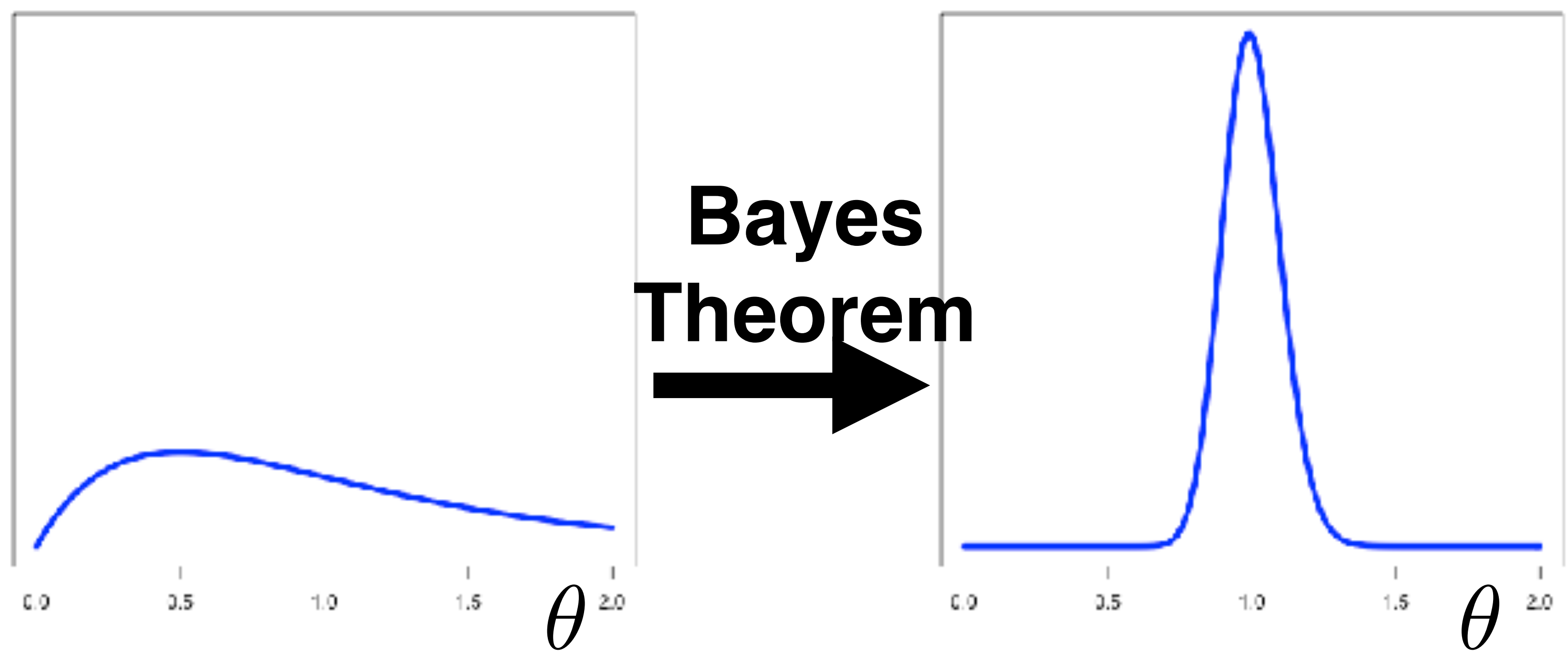

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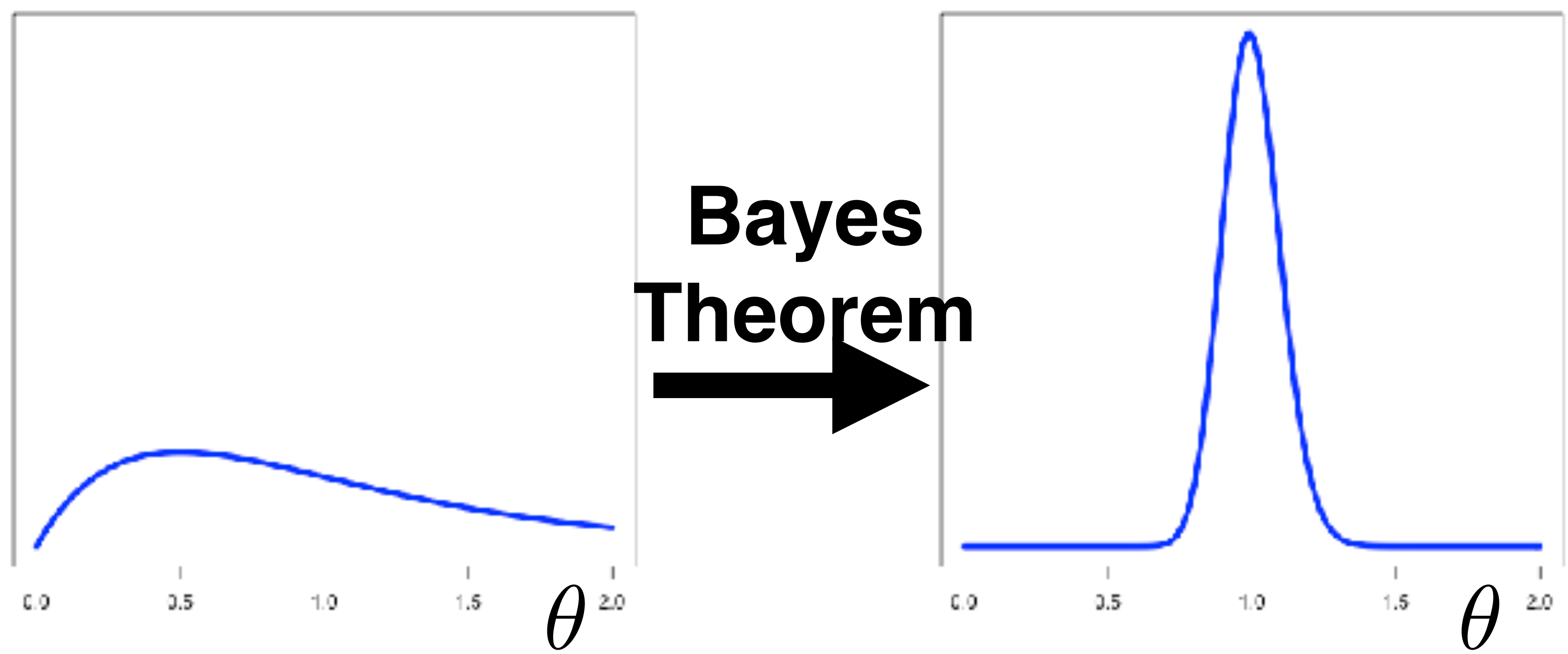

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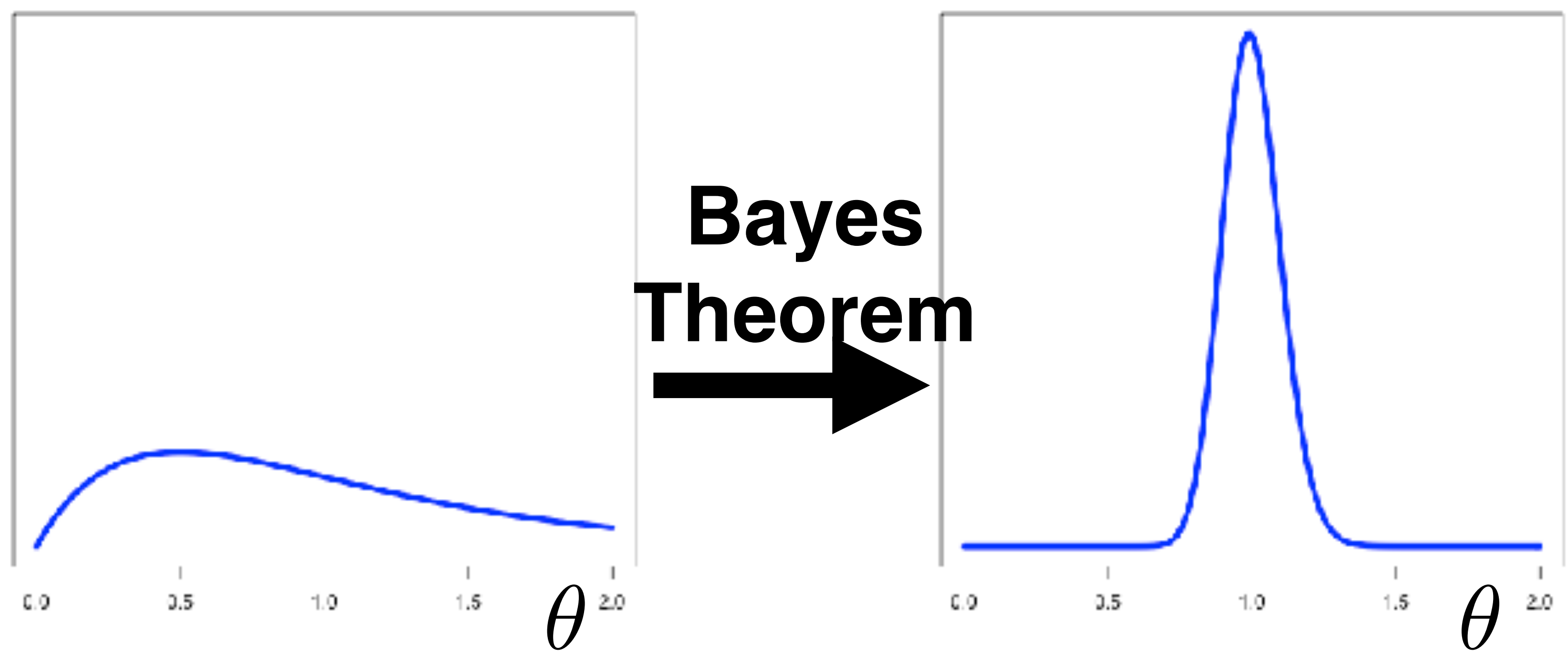

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$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta)/p(y_{1:N})$$

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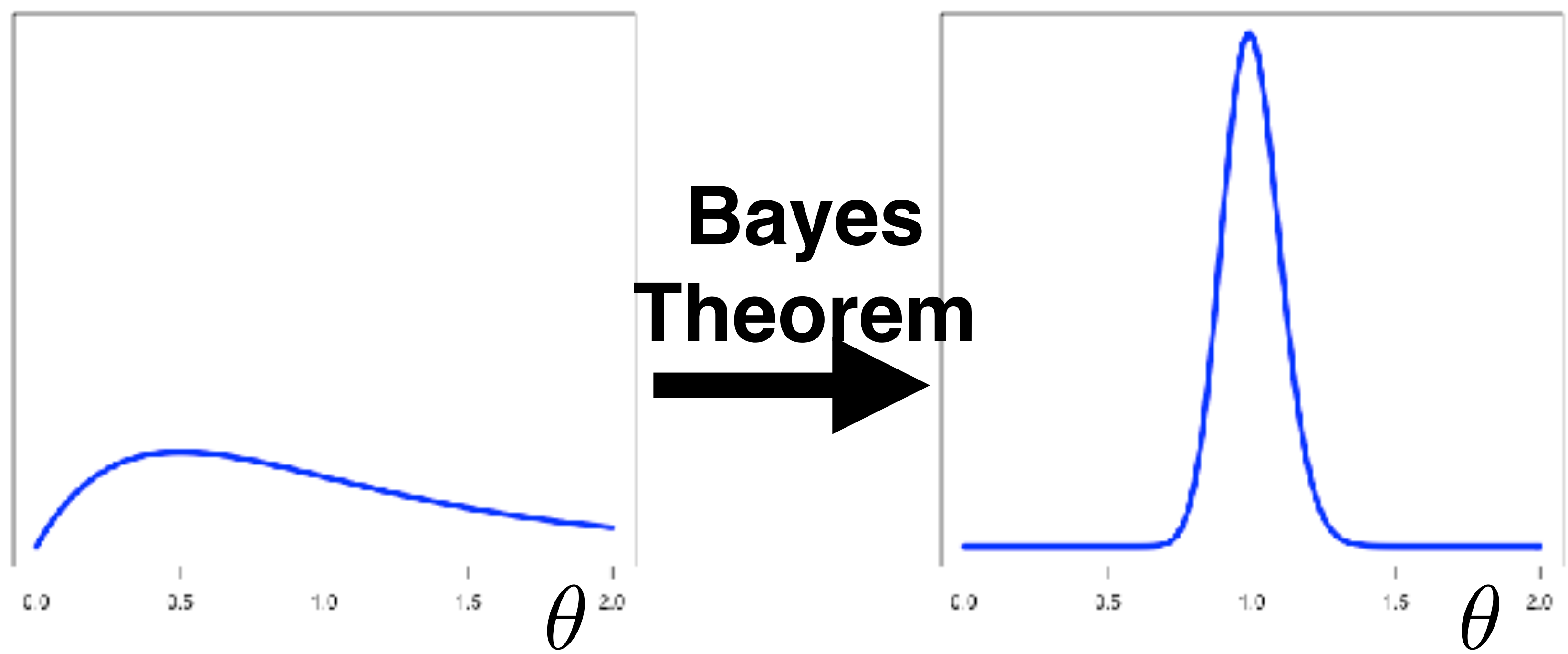

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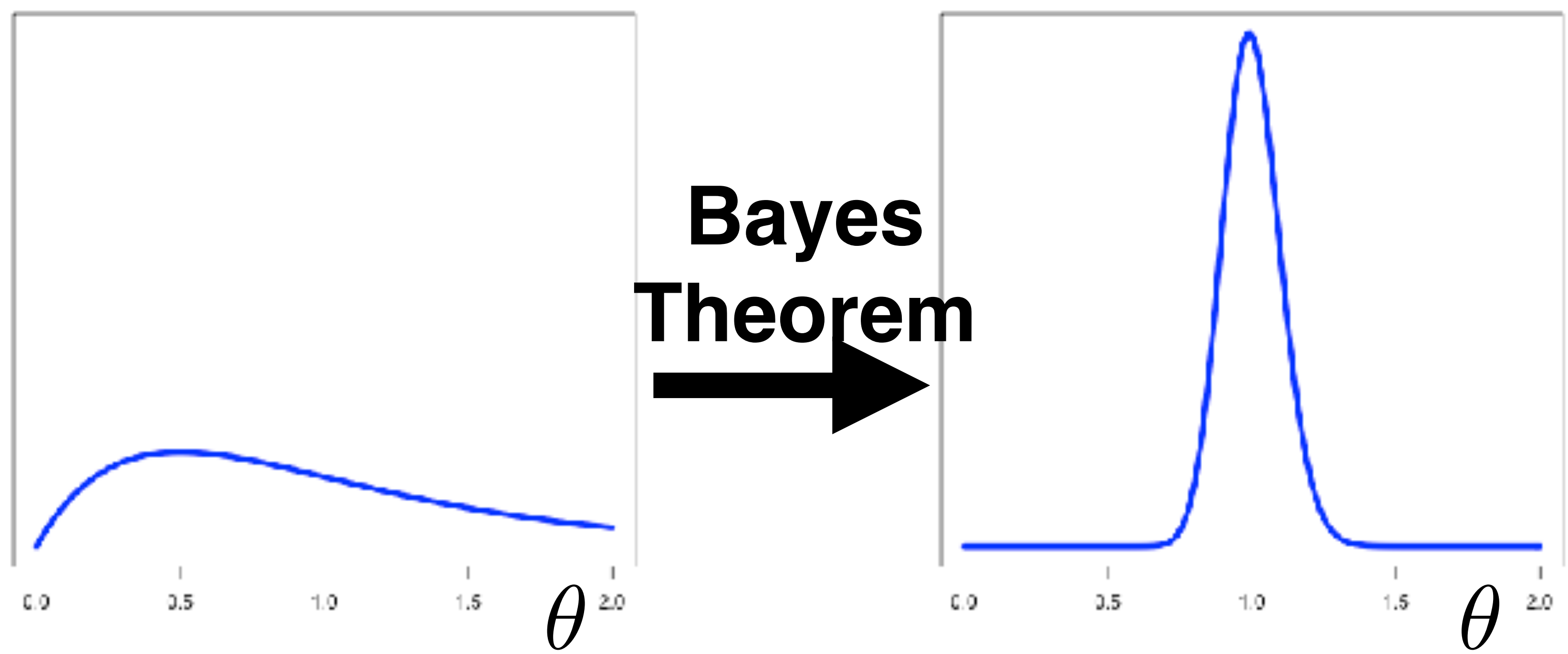

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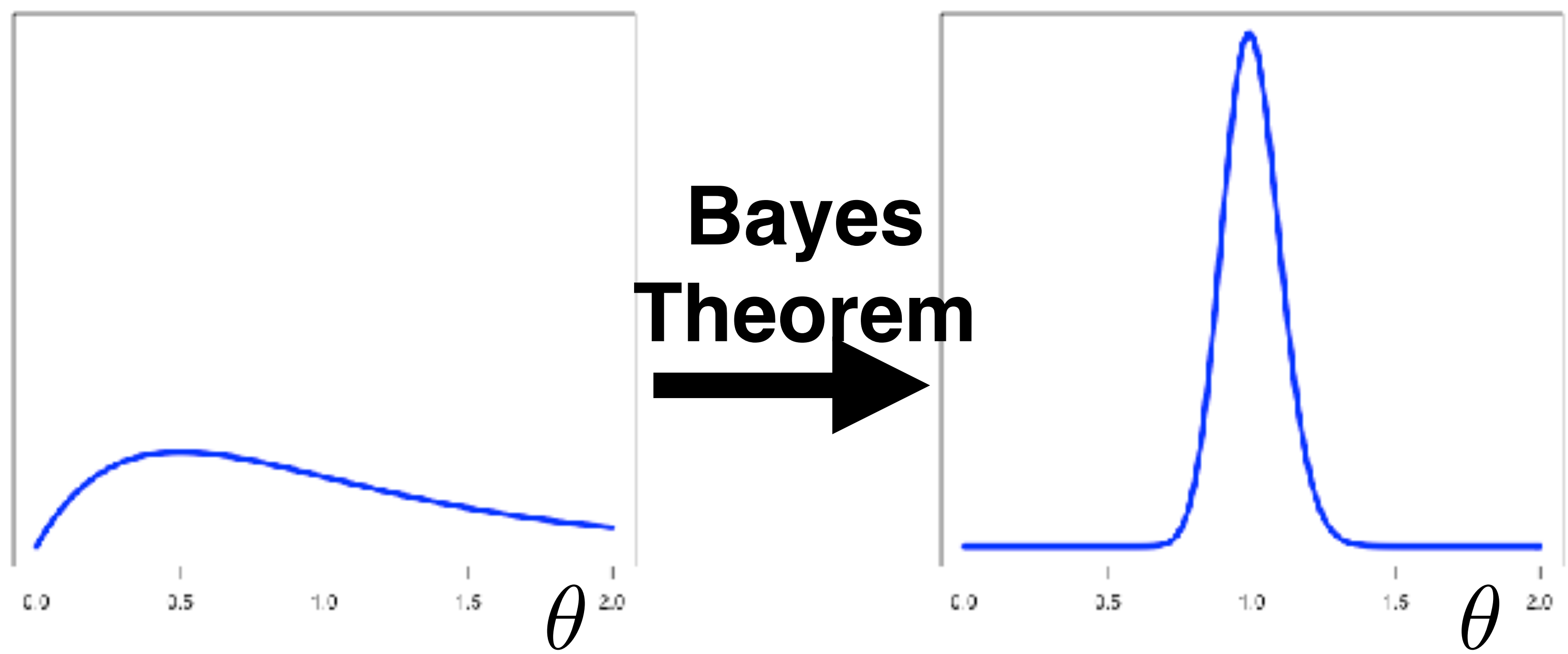

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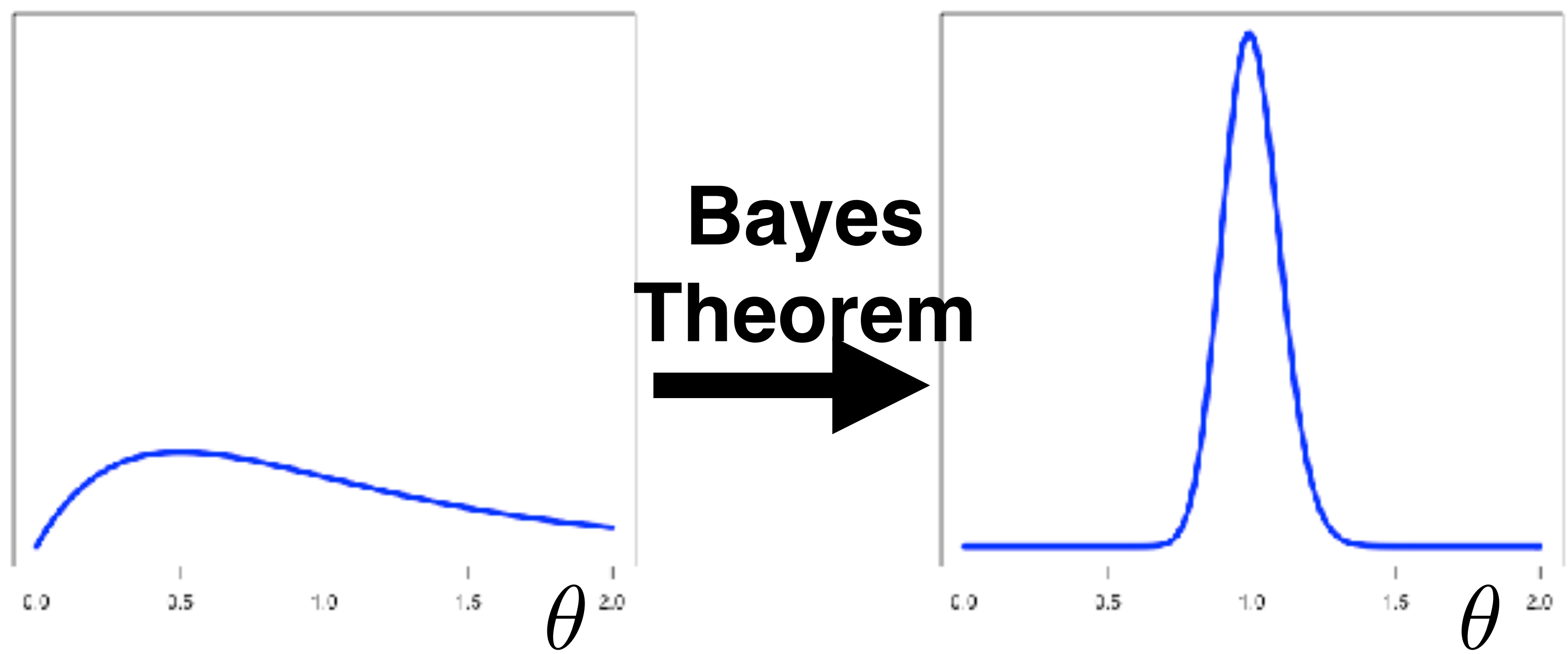
$$p(\theta|y_{1:N}) = p(y_{1:N}|\theta)p(\theta) / \int p(y_{1:N}, \theta)d\theta$$

posterior likelihood prior evidence



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[Bardenet, Doucet, Holmes 2017]

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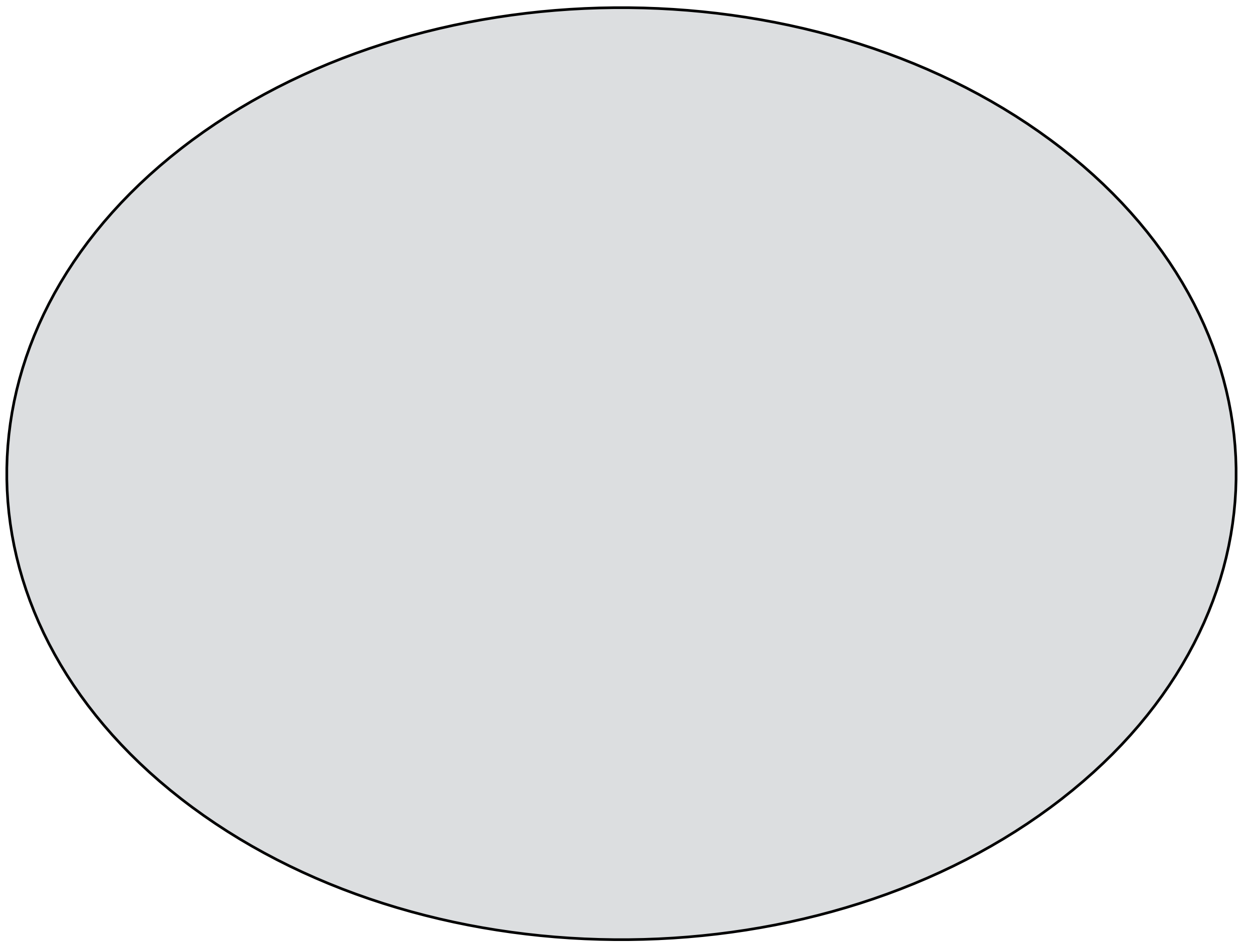

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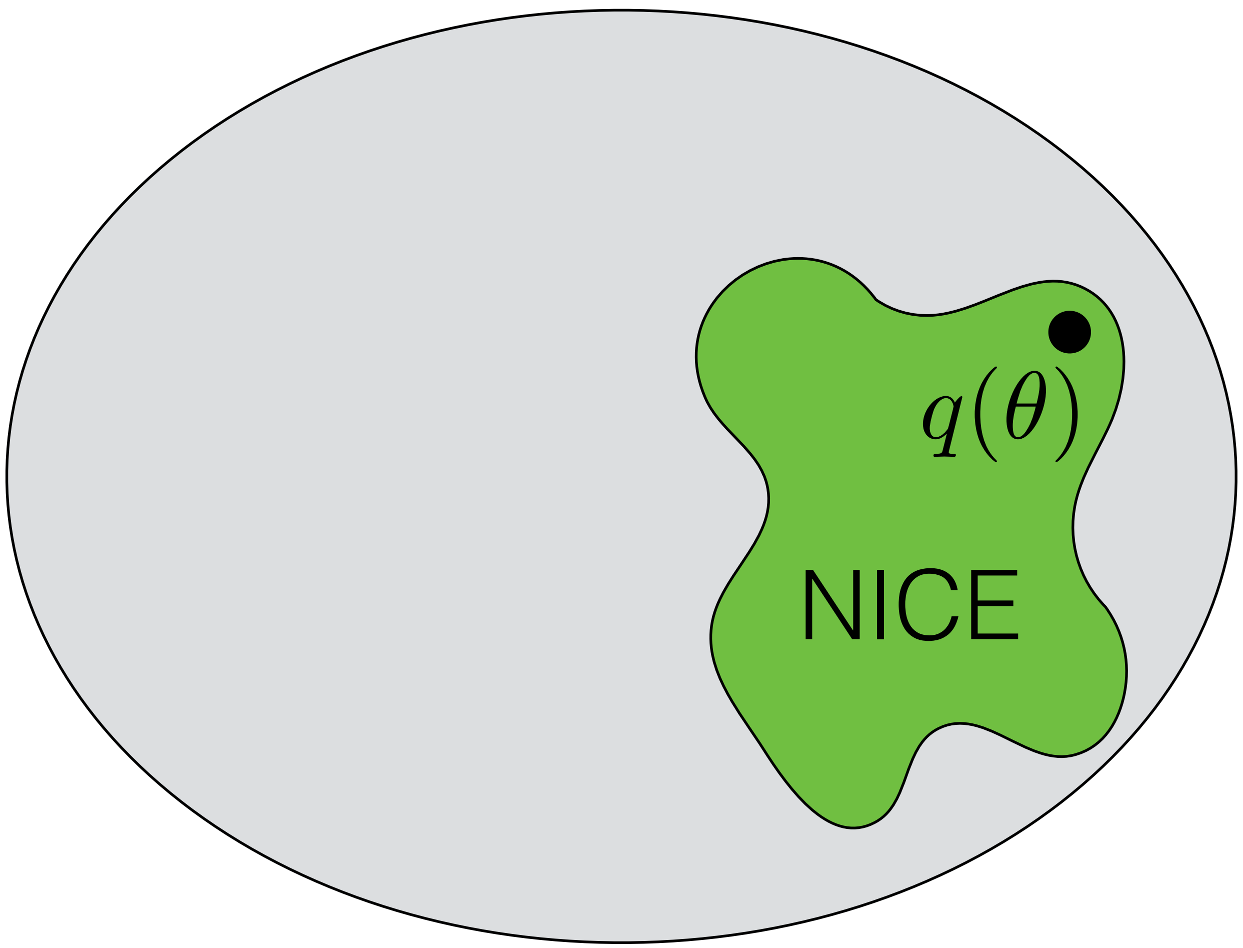

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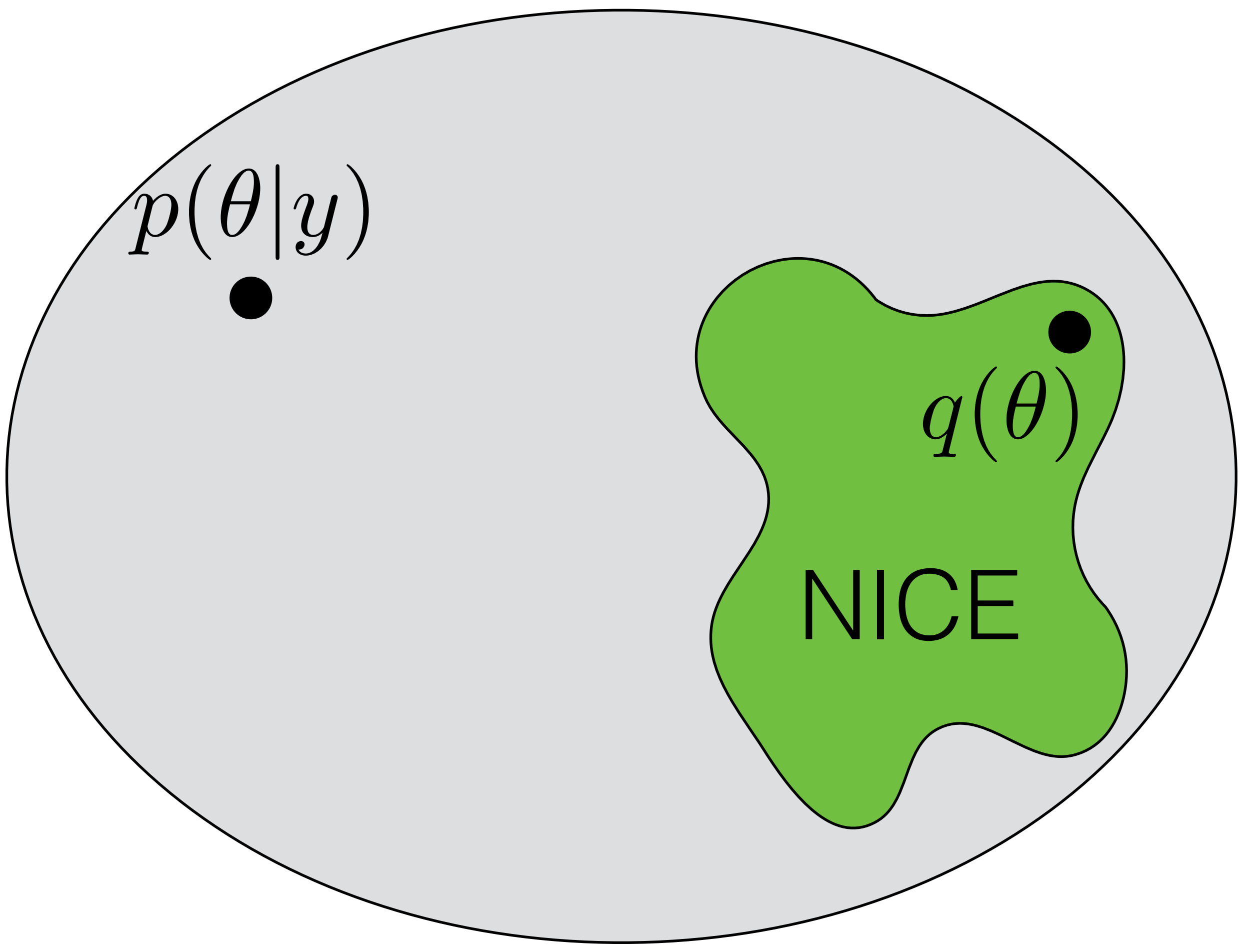

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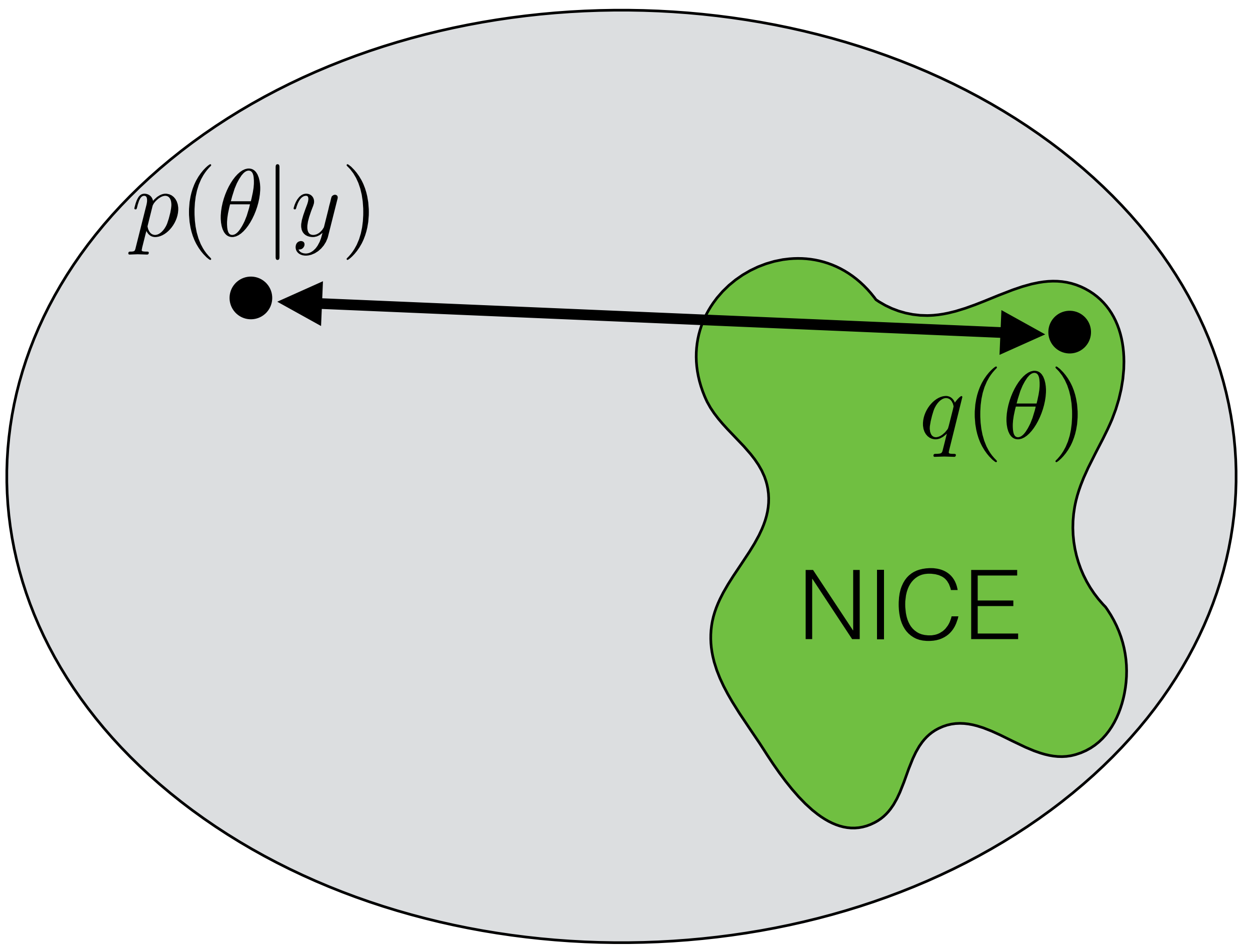

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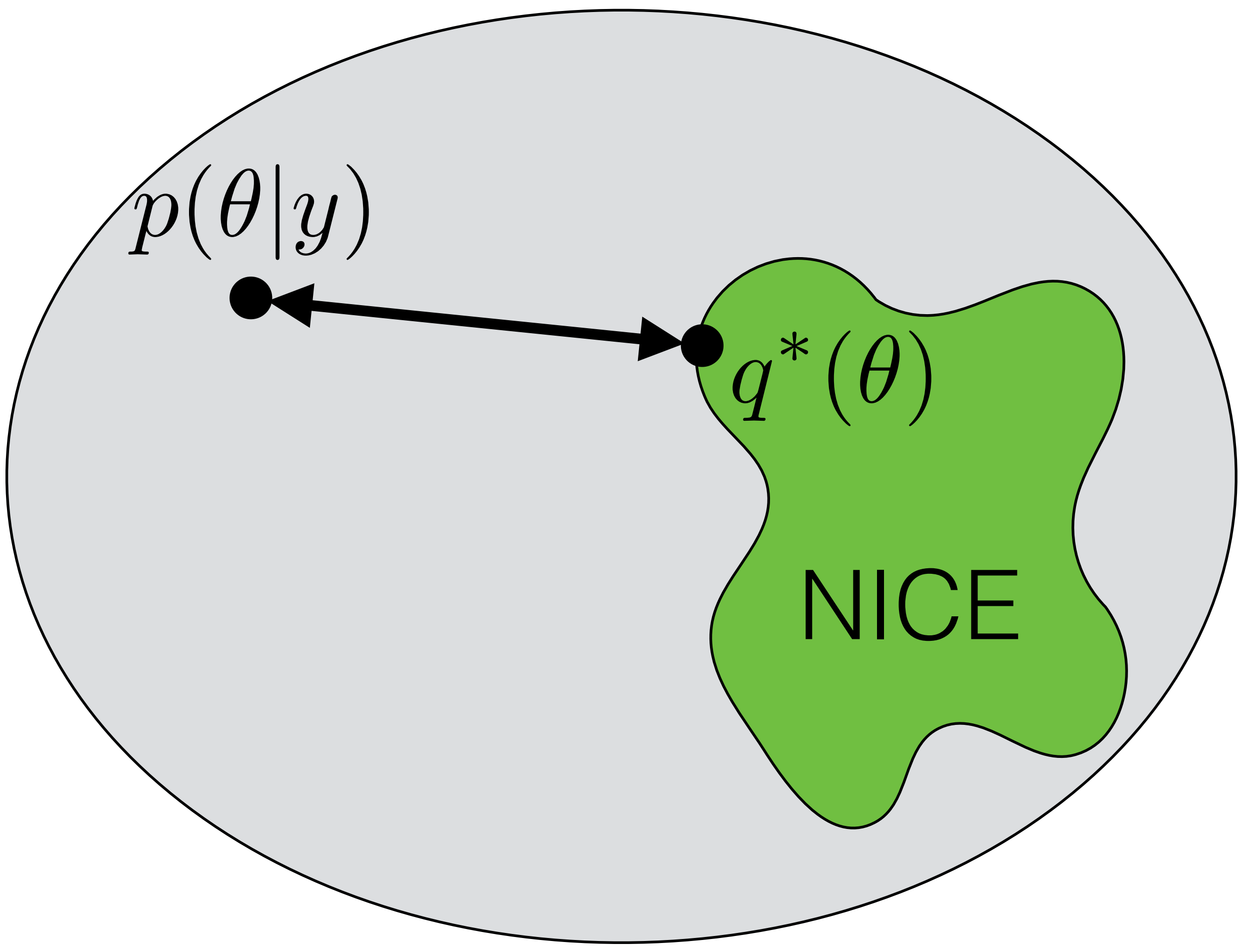

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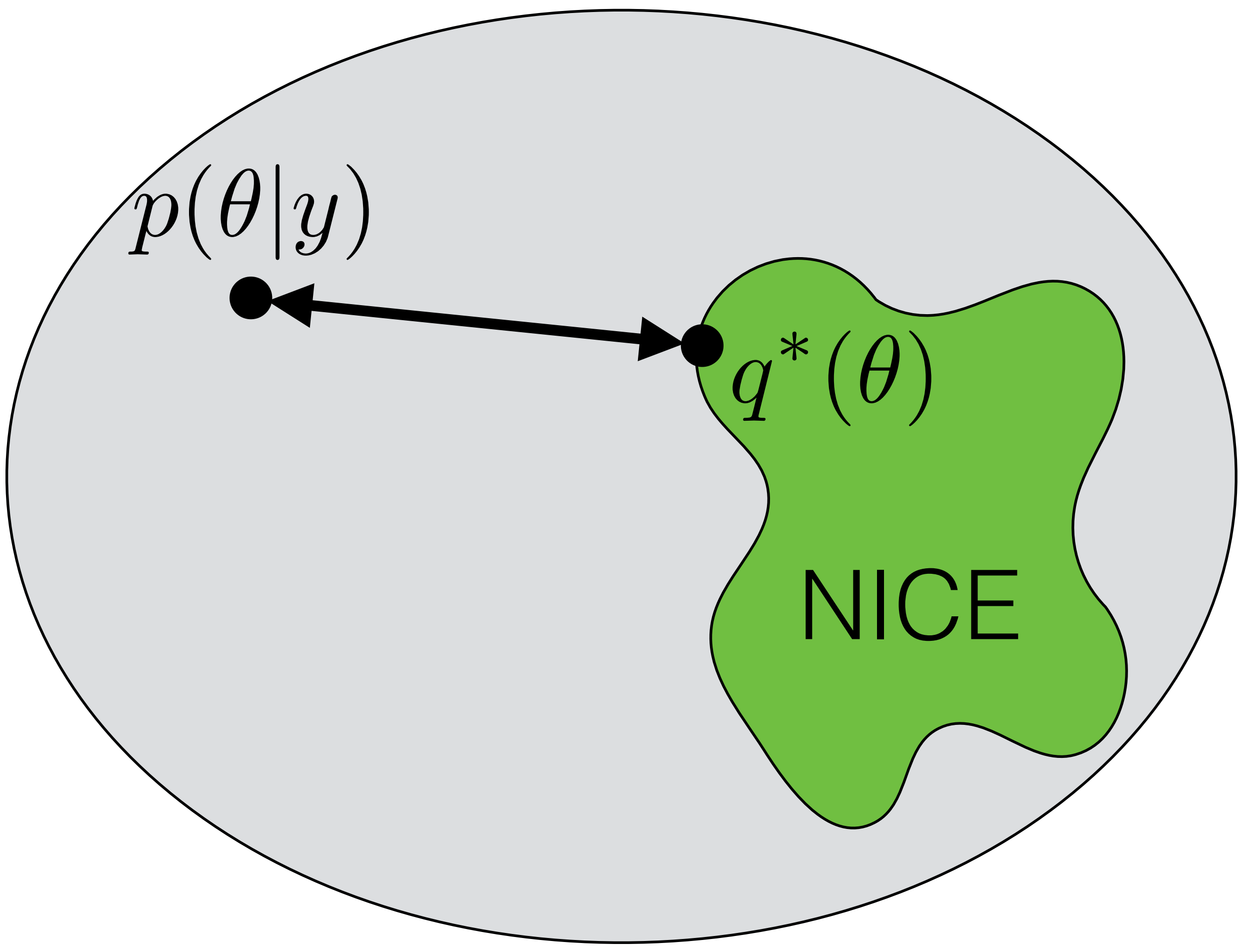

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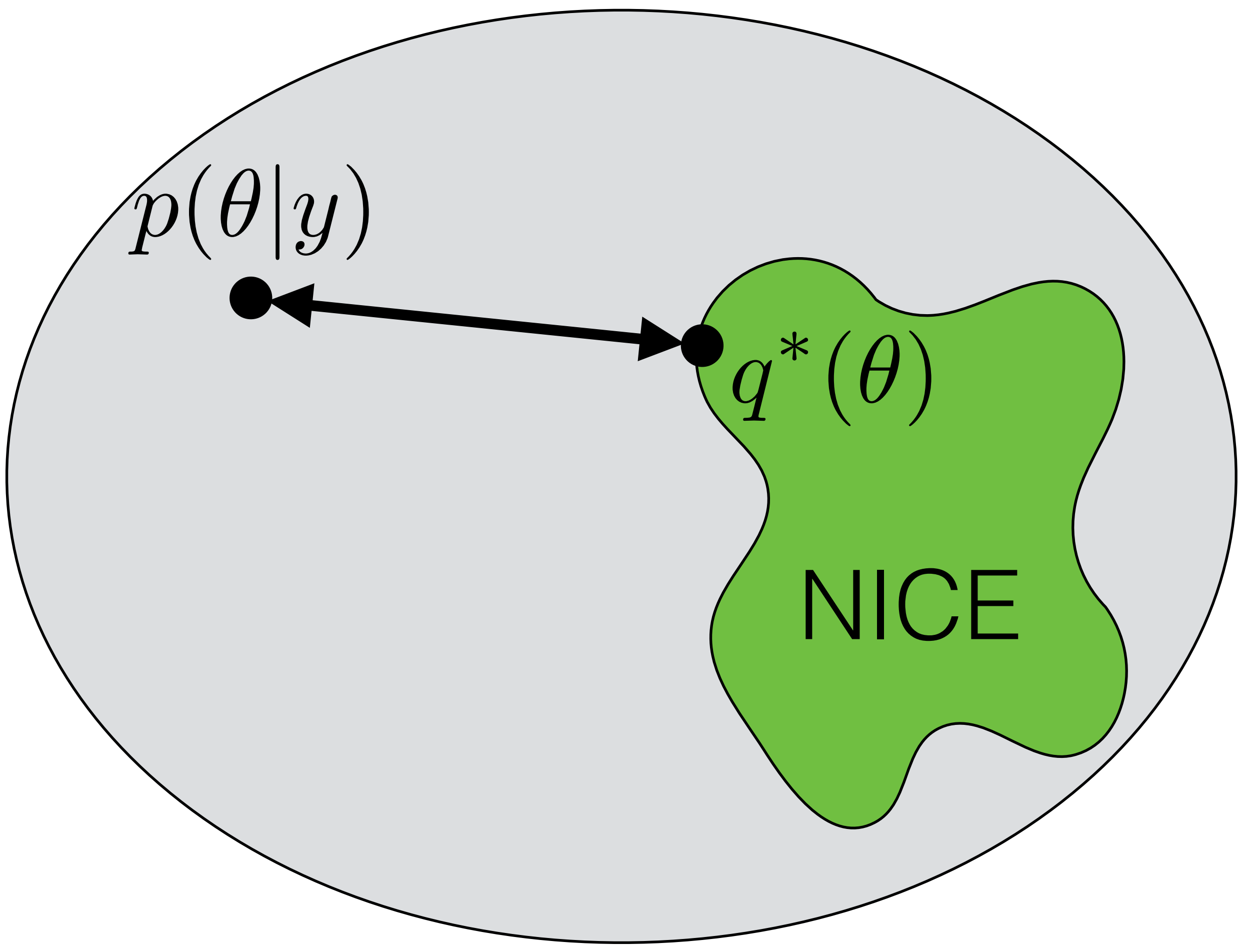

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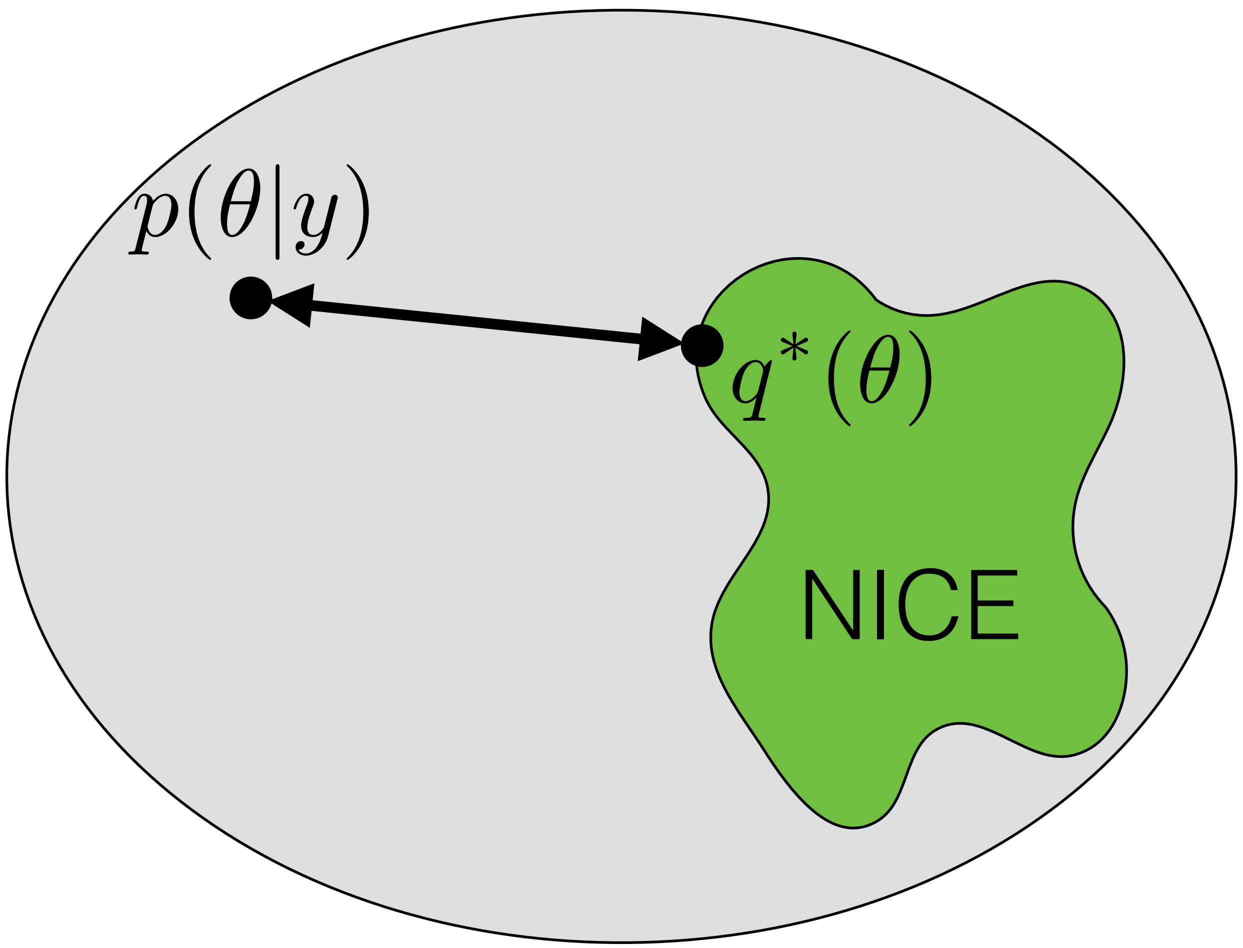

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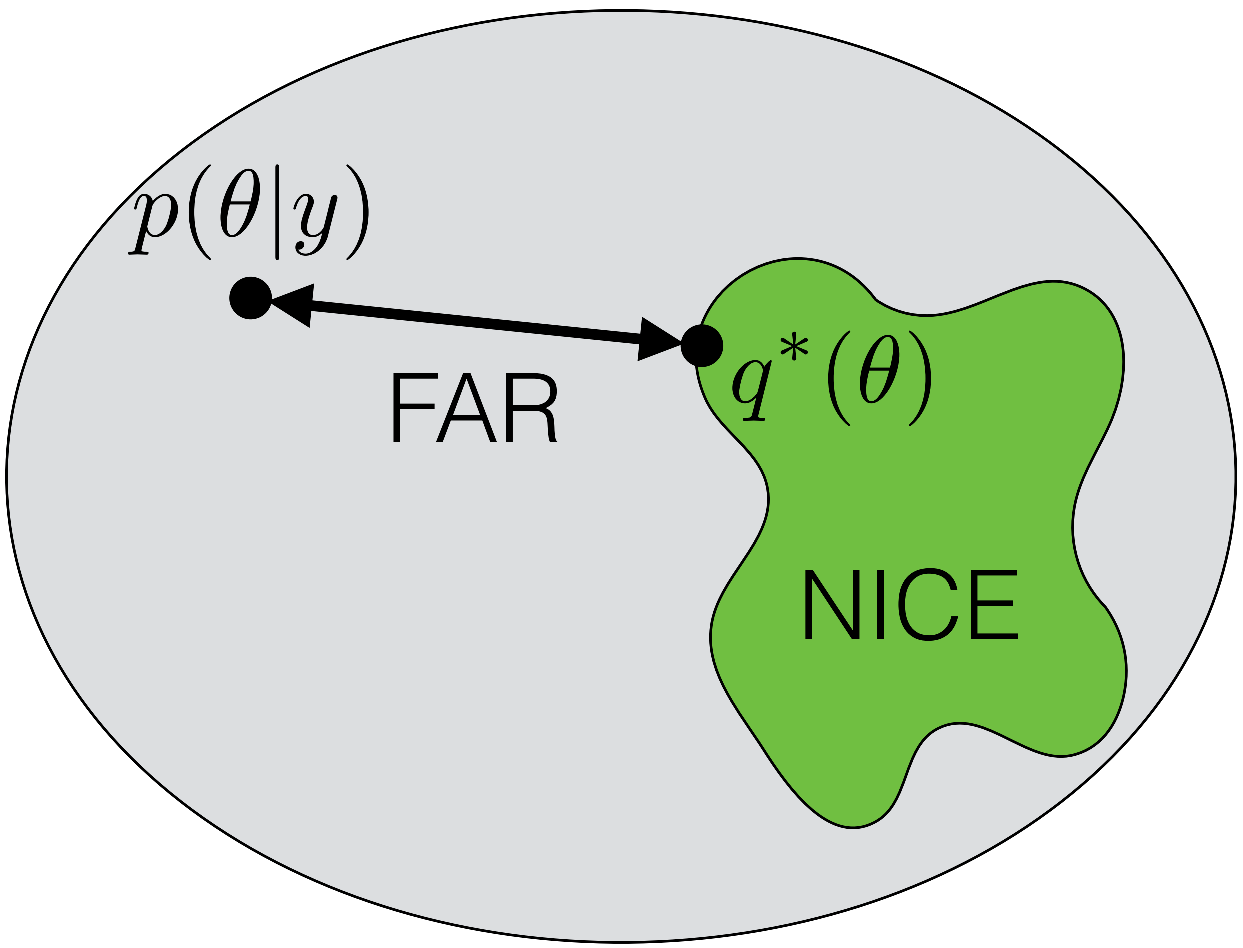

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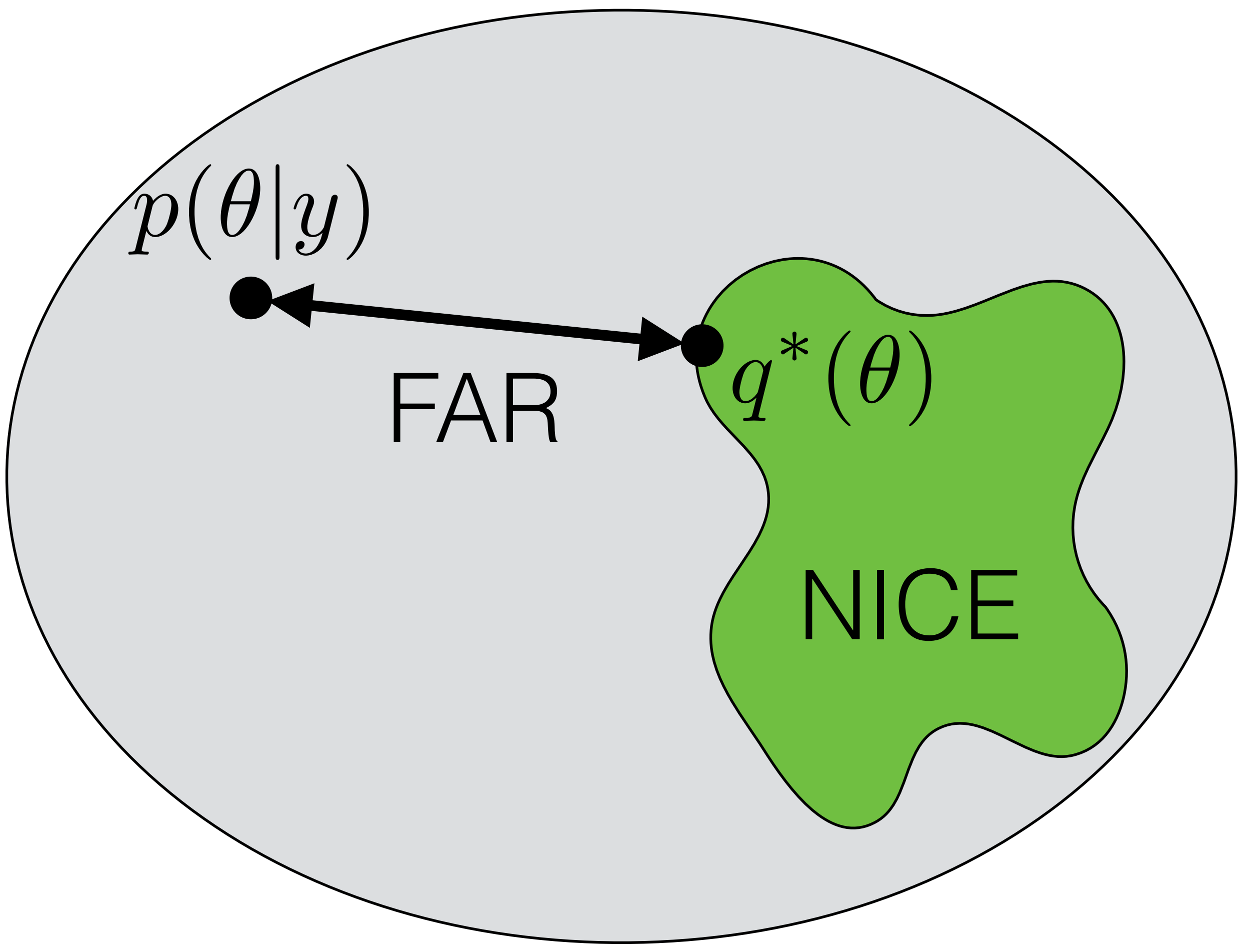

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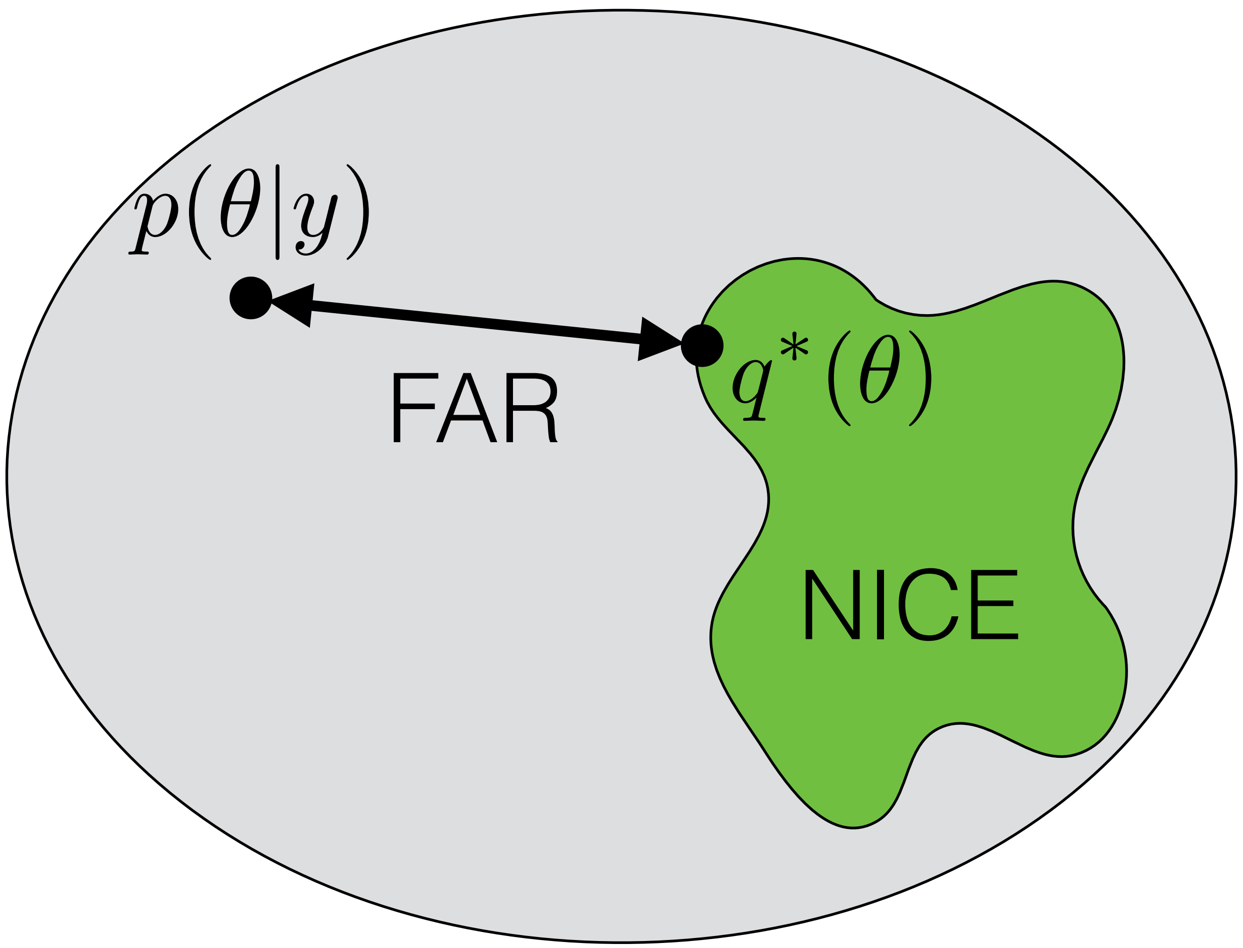

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$$KL(q(\cdot)||p(\cdot|y))$$

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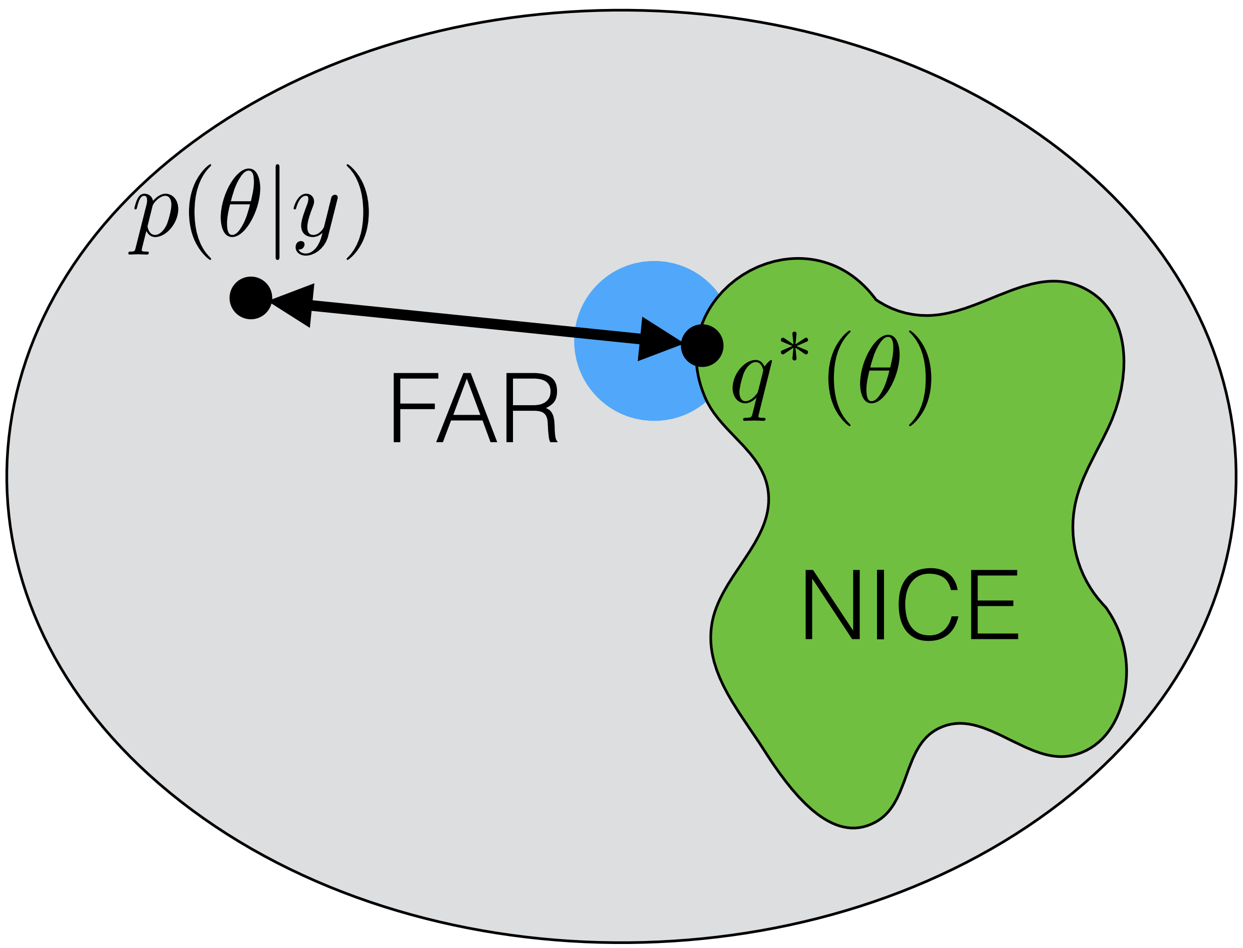

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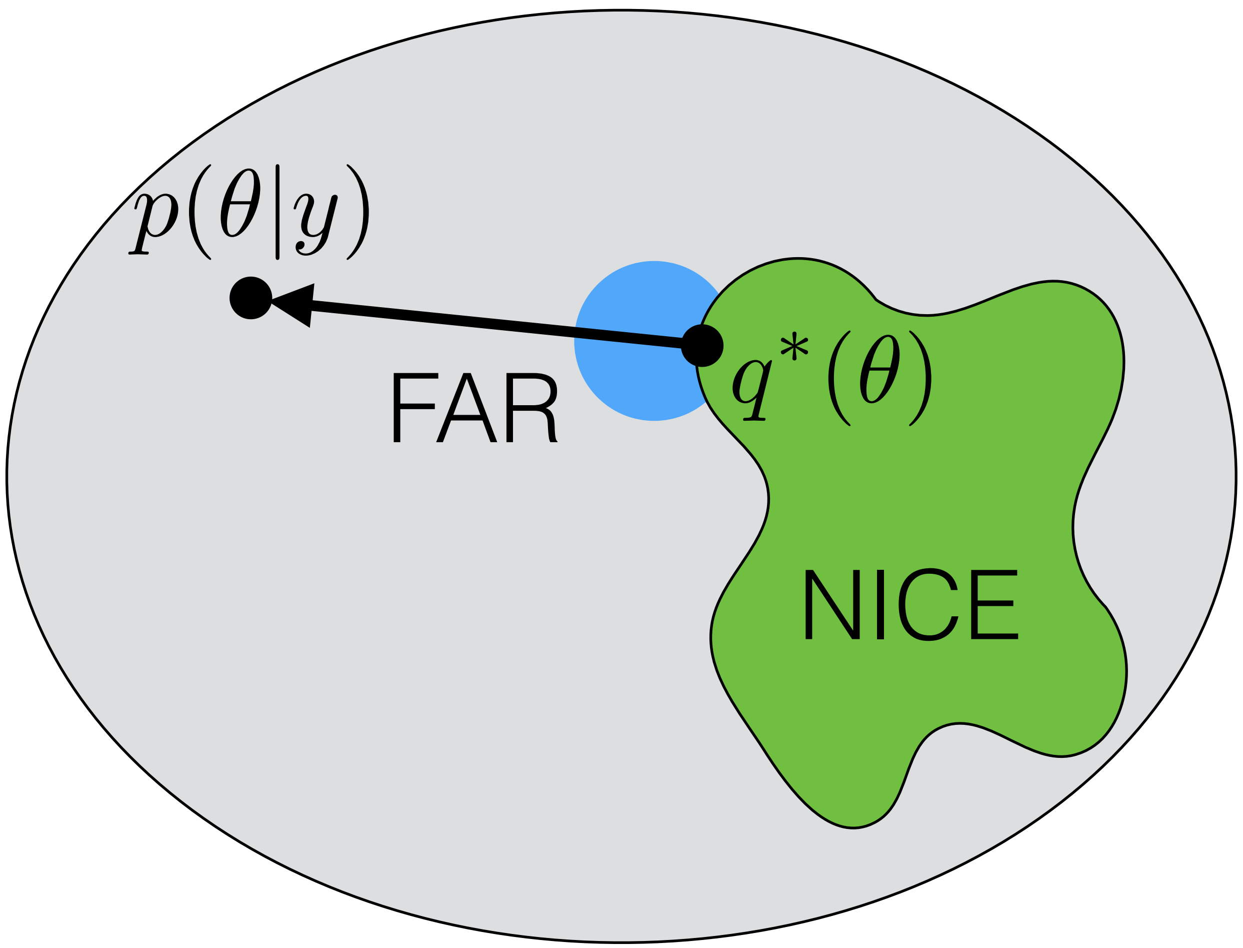

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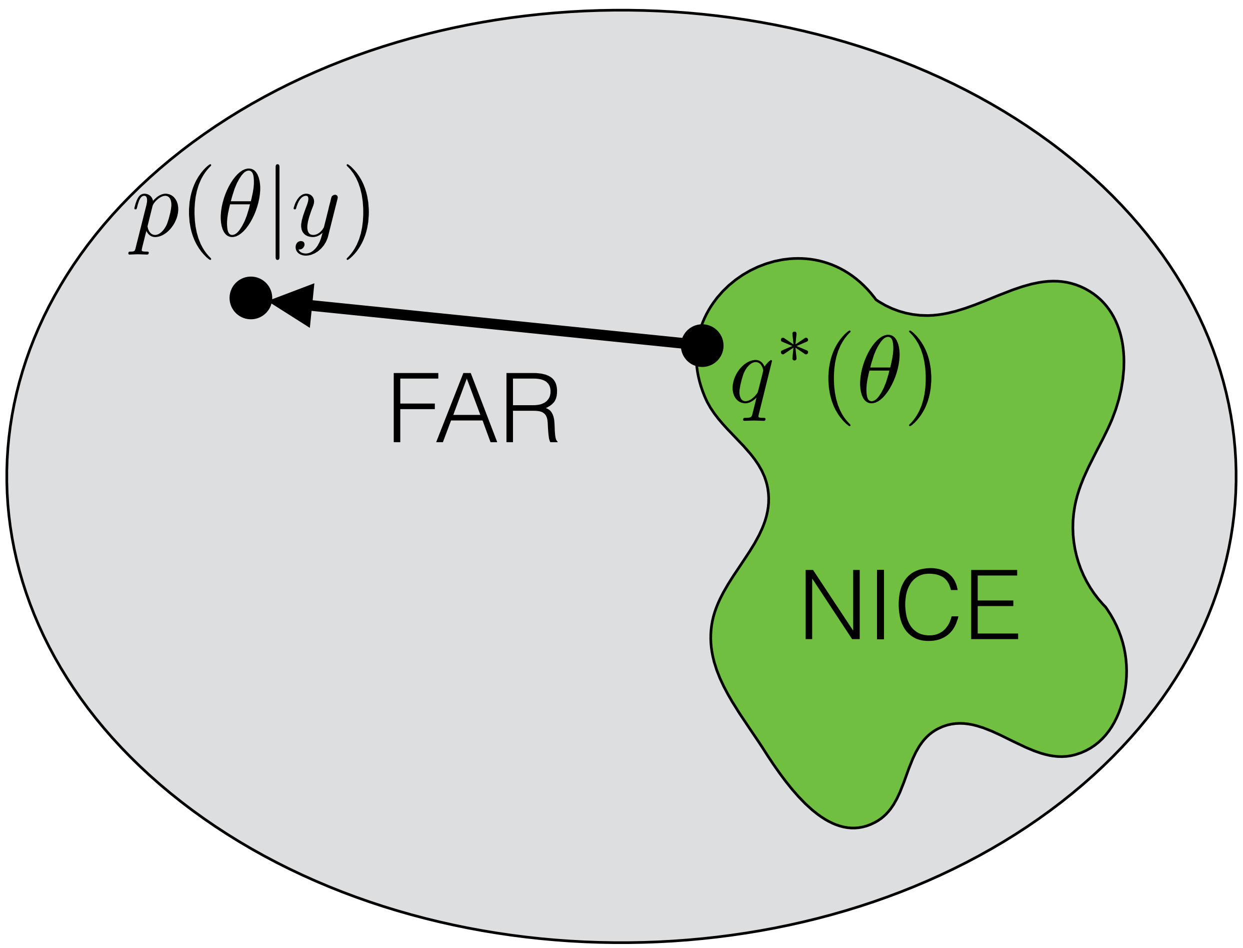

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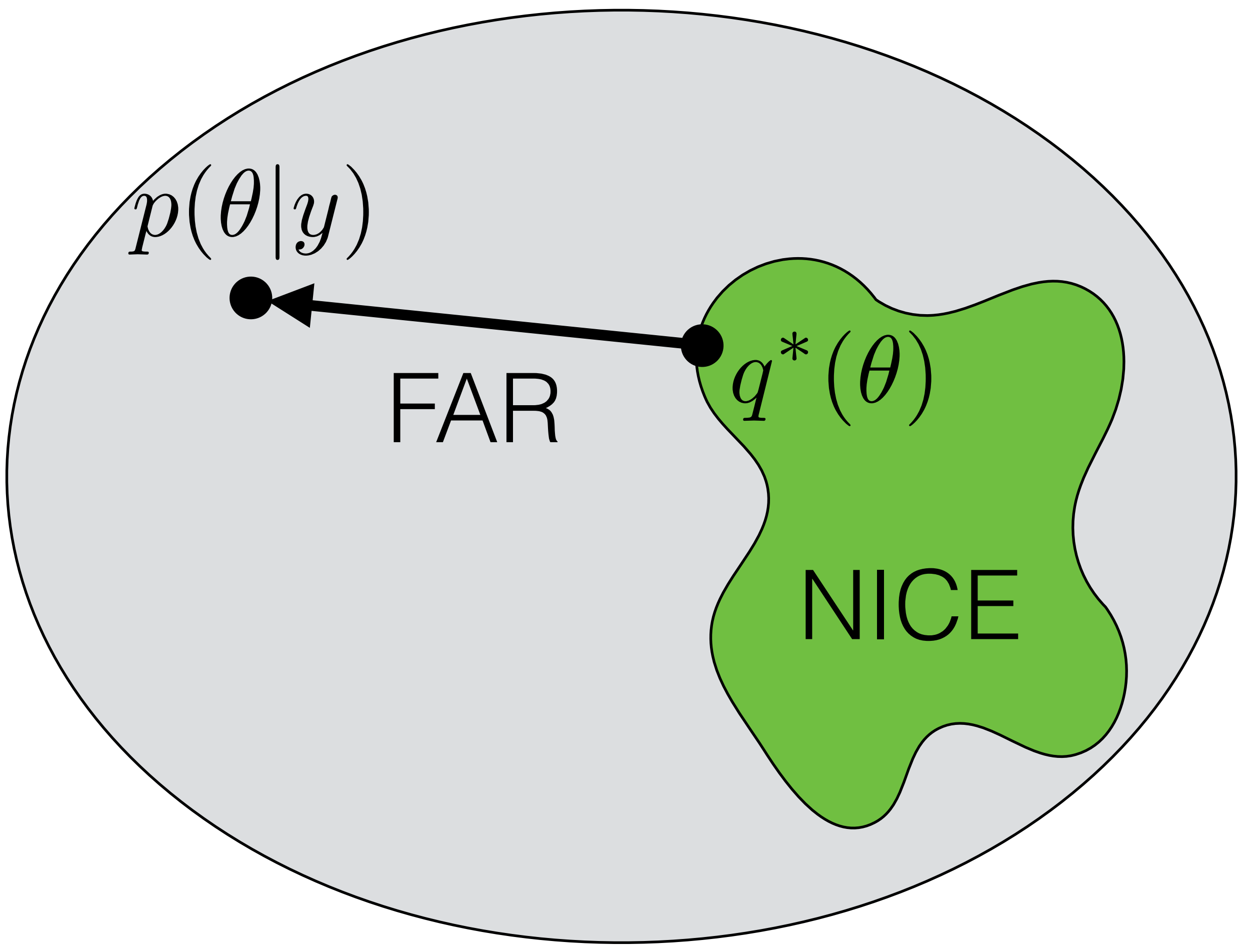

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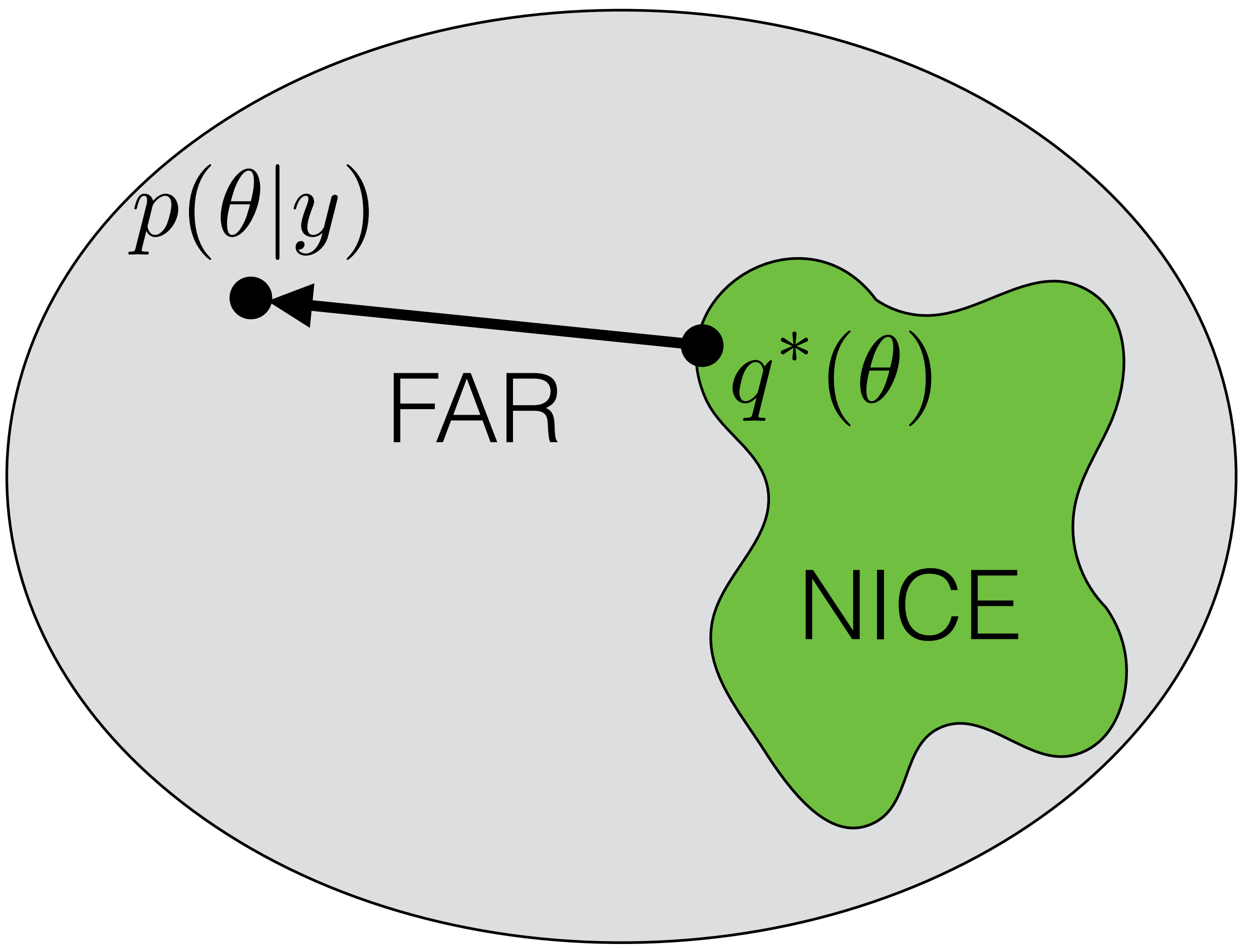

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- VB practical success: point estimates and prediction

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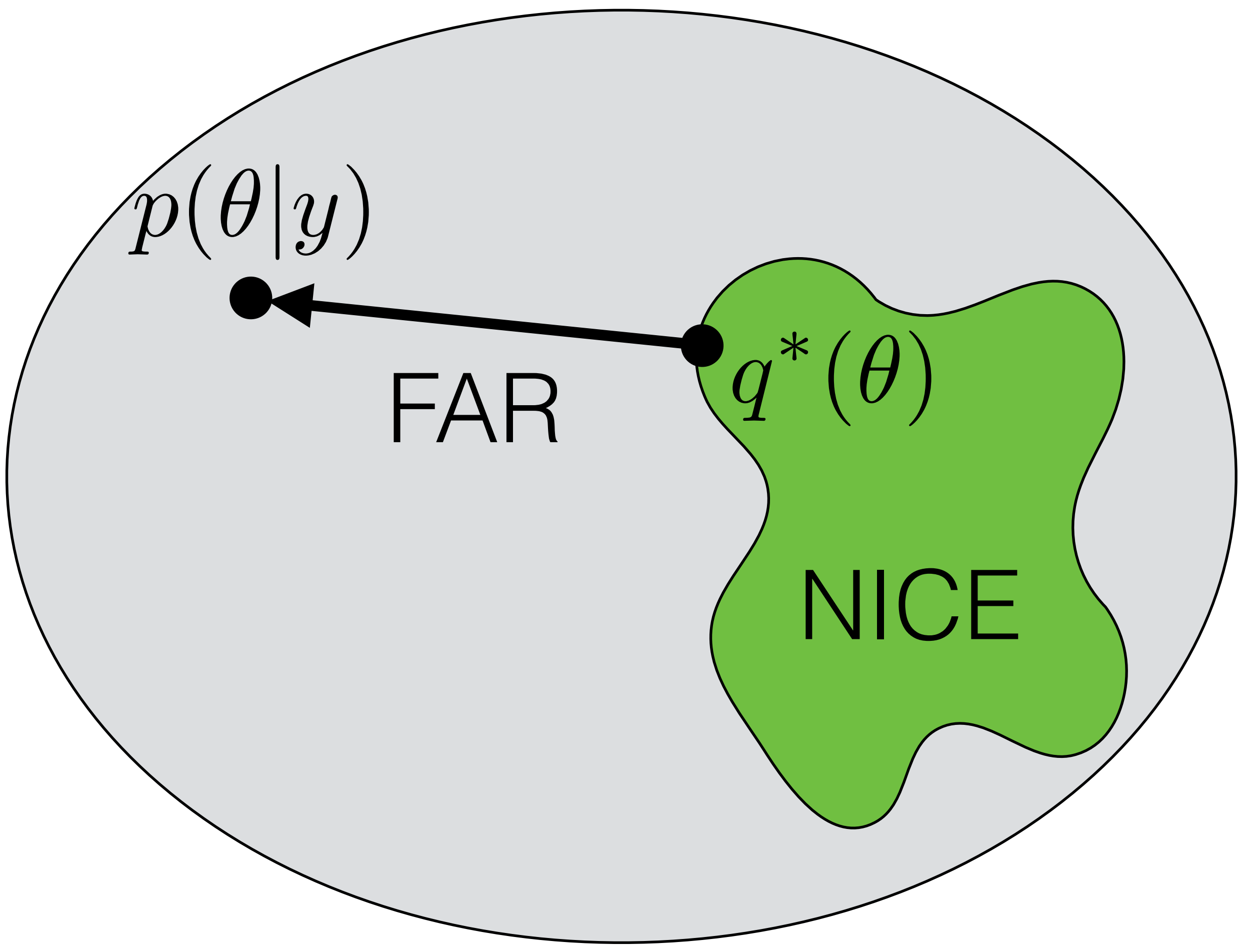

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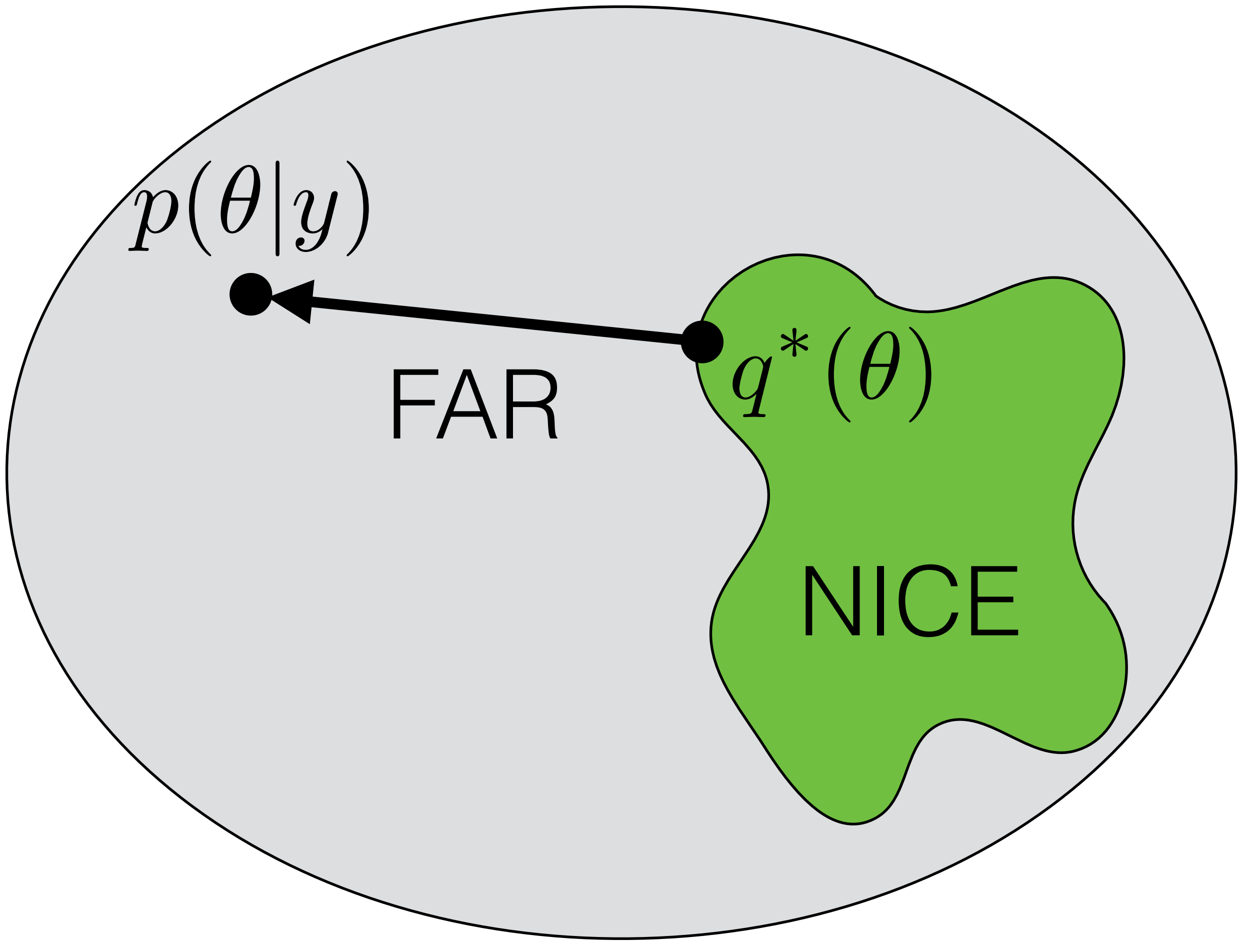

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- VB practical success: point estimates and prediction, fast, streaming, distributed (3.6M Wikipedia, 350K Nature)

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Why KL?

- Variational Bayes

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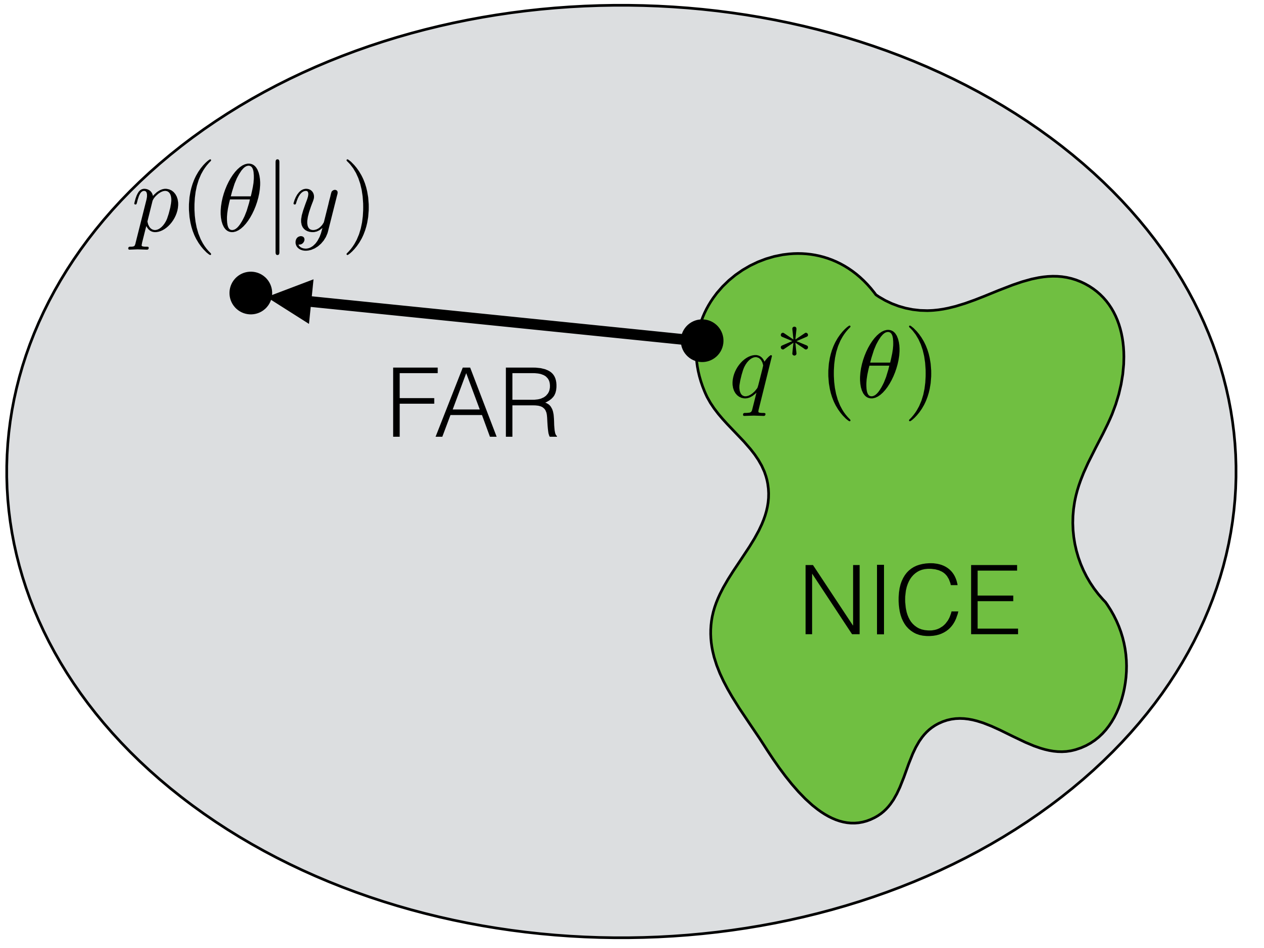

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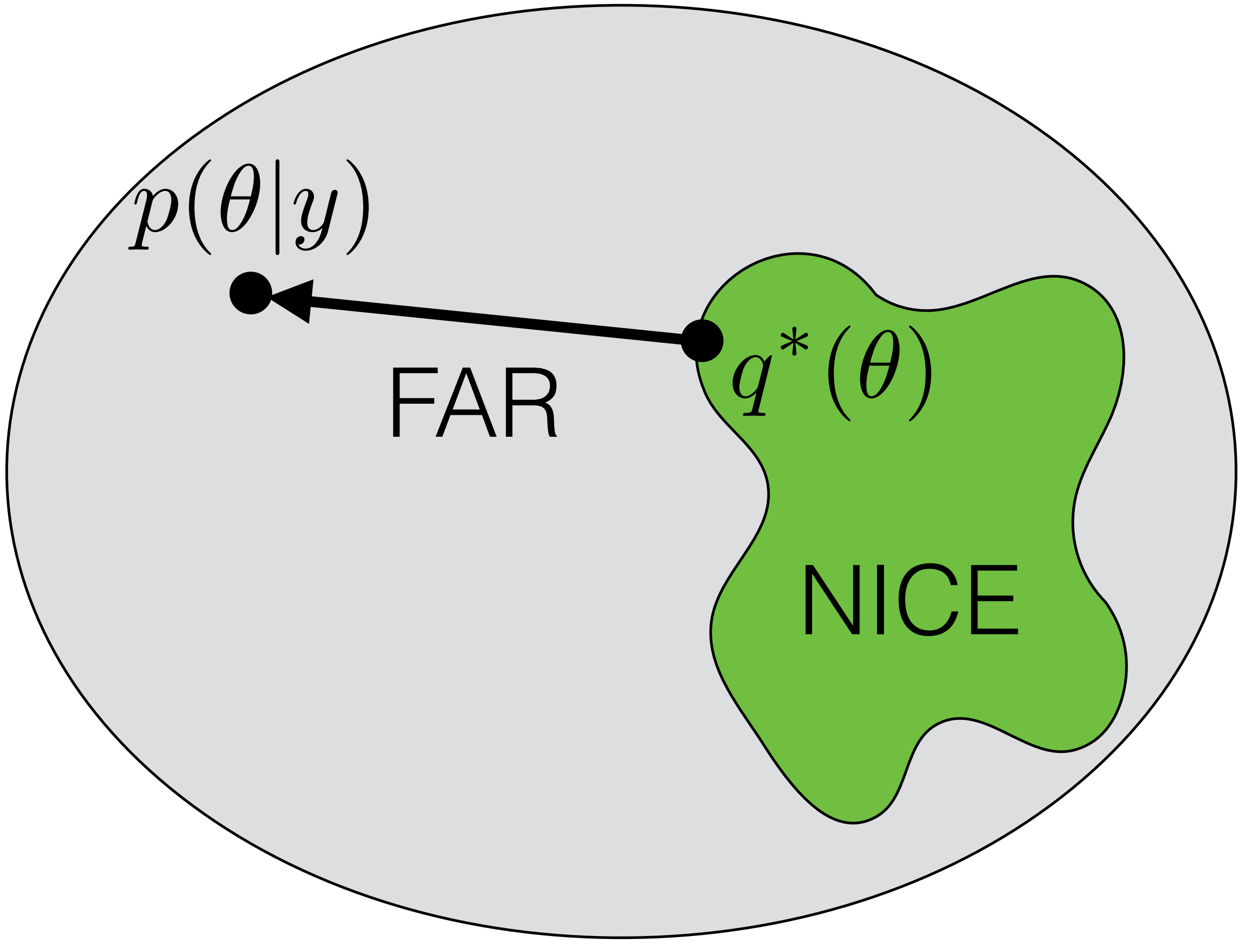

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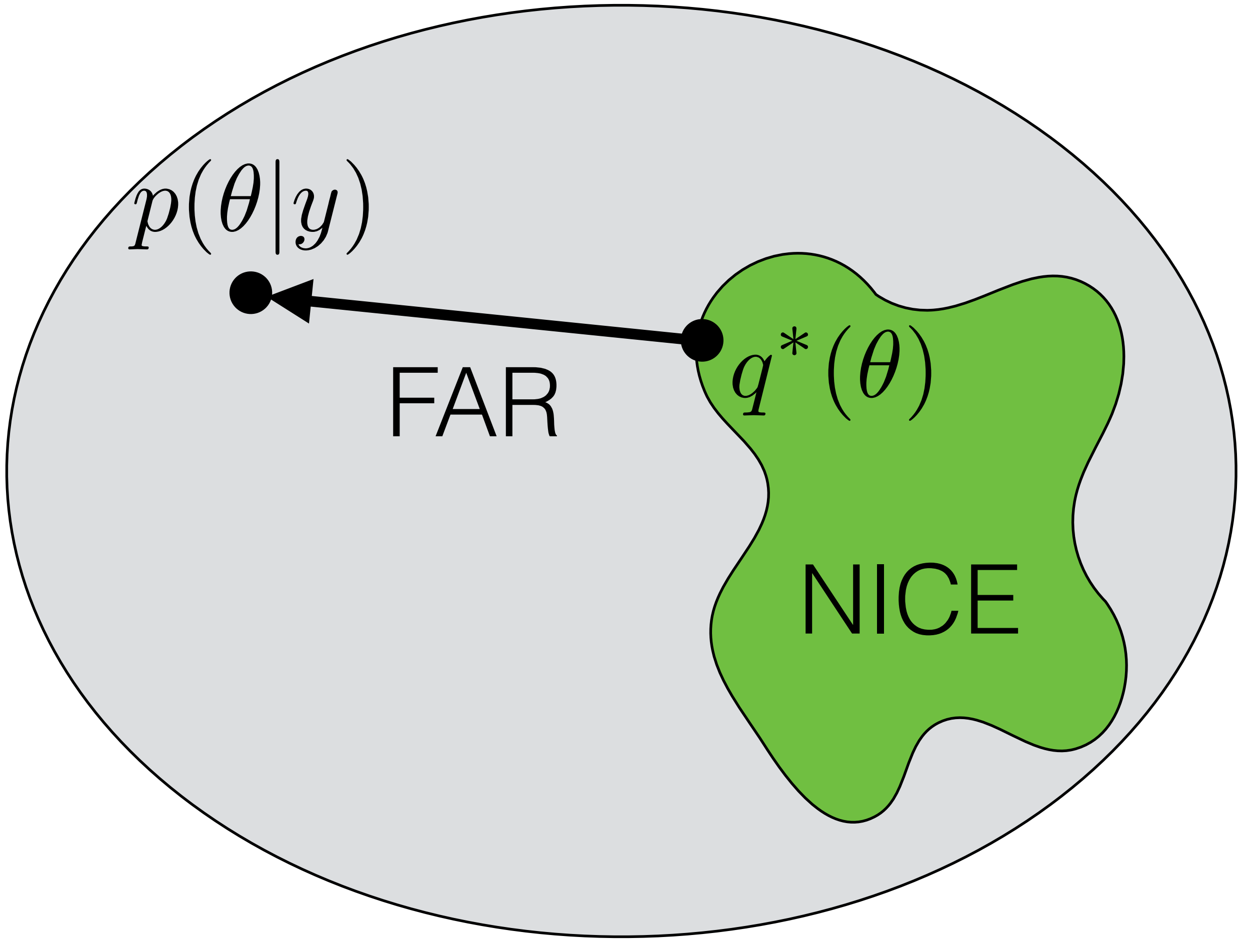

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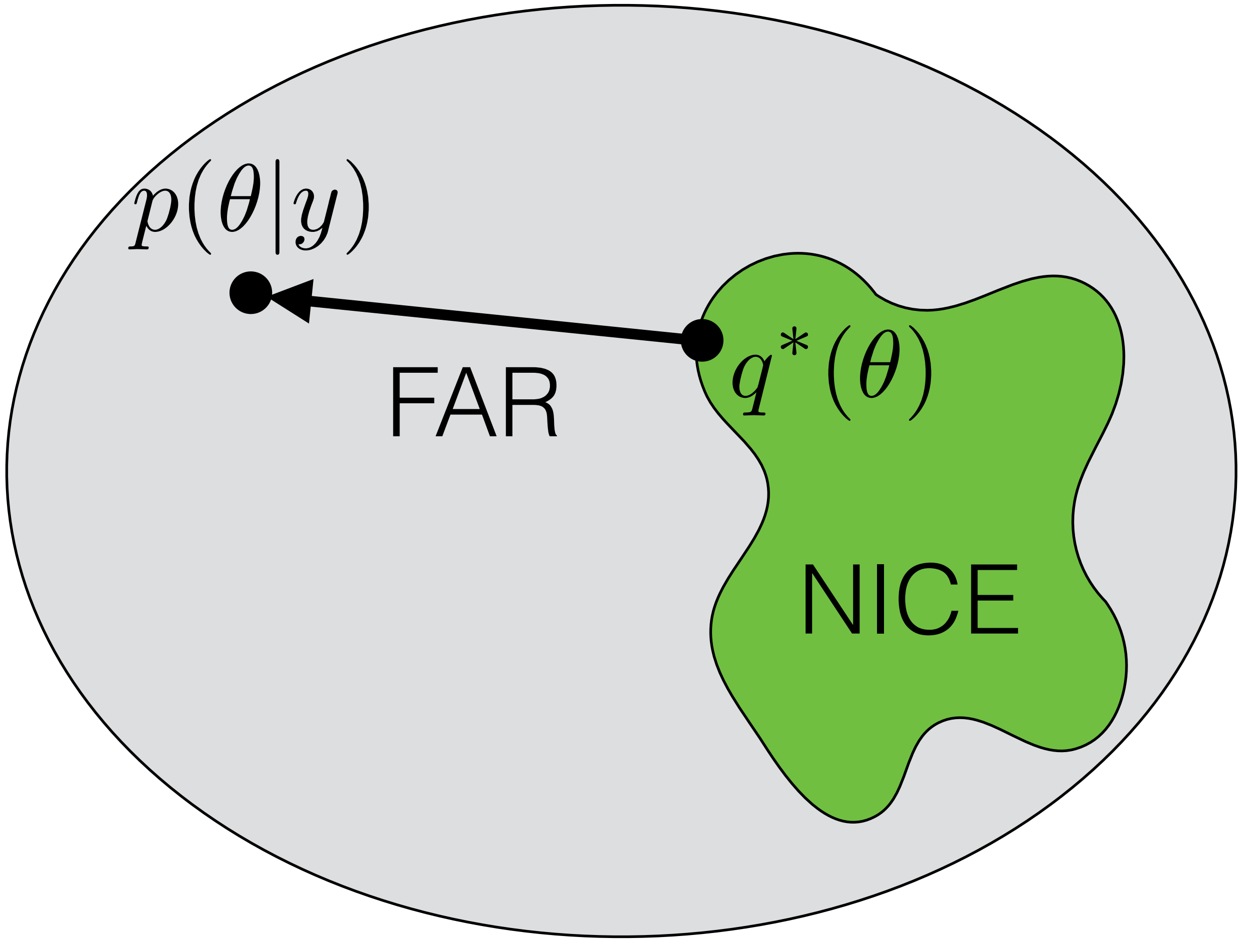

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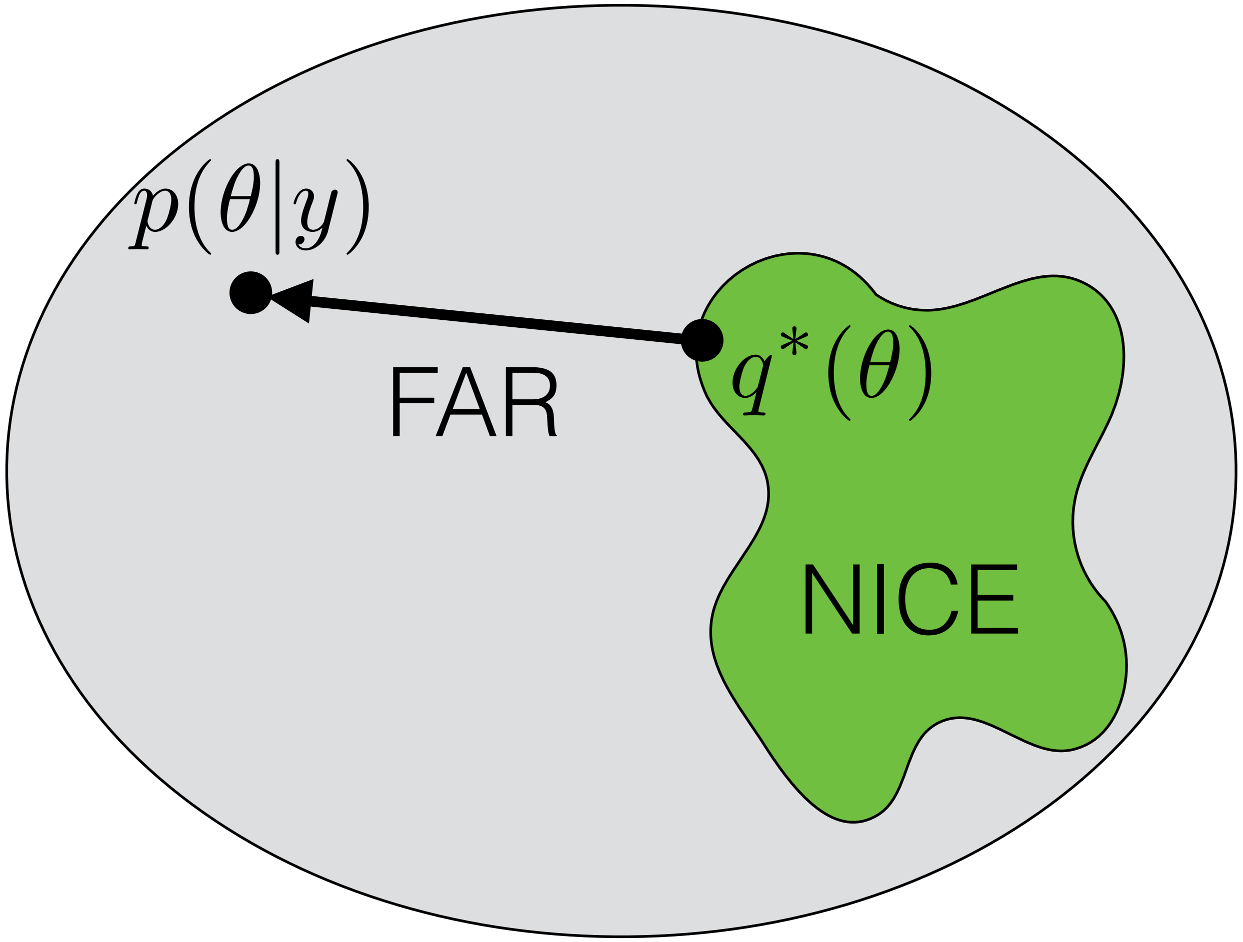

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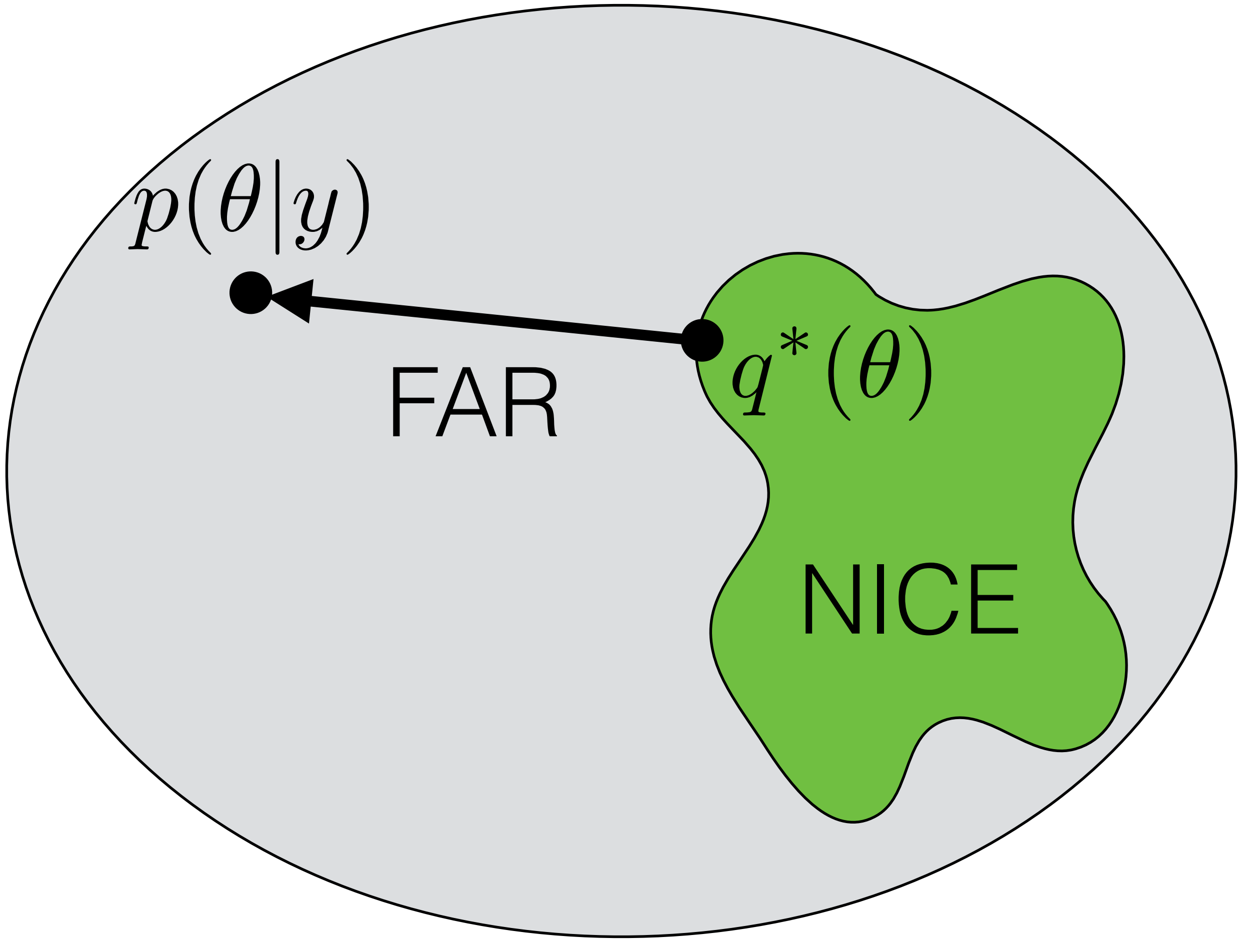

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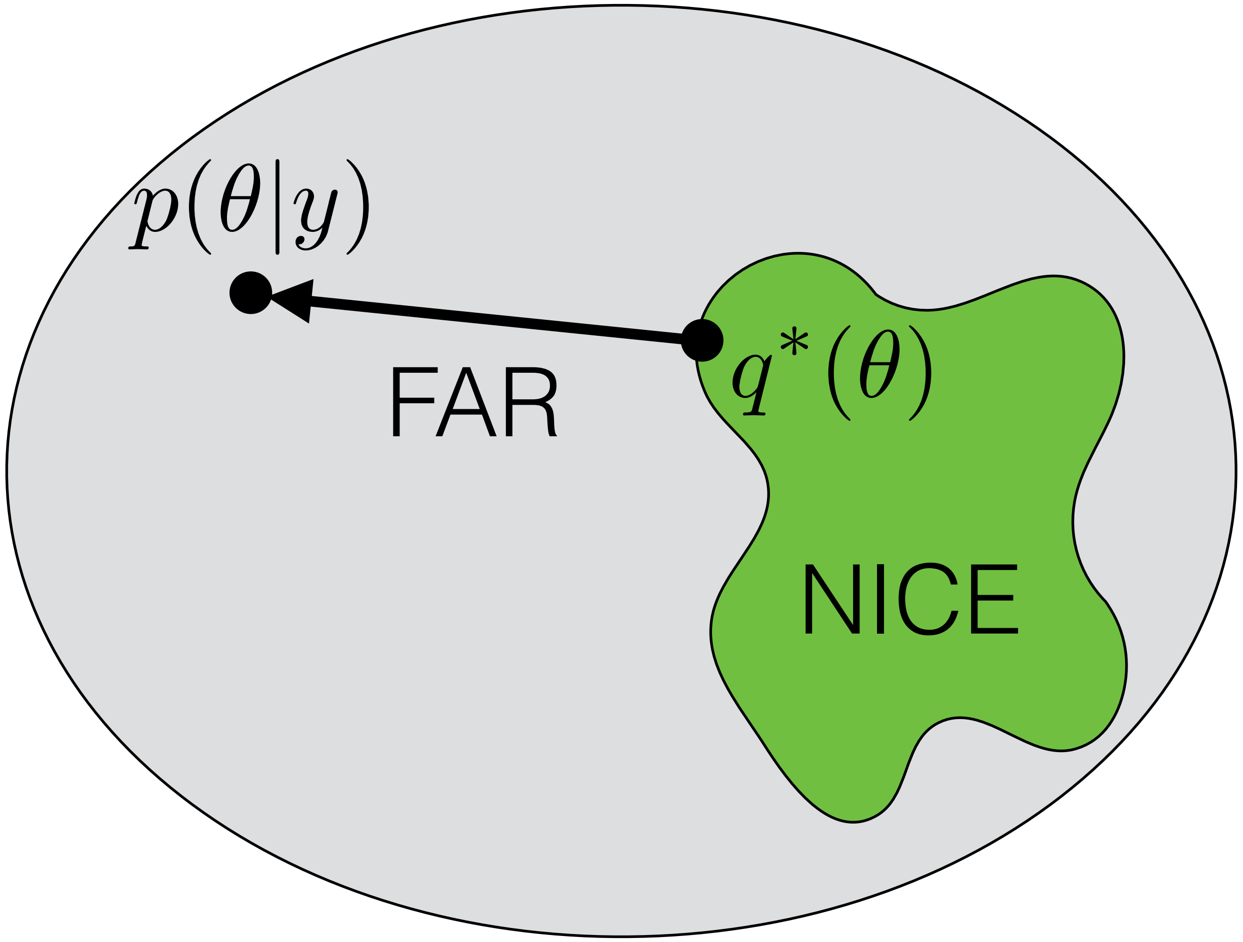

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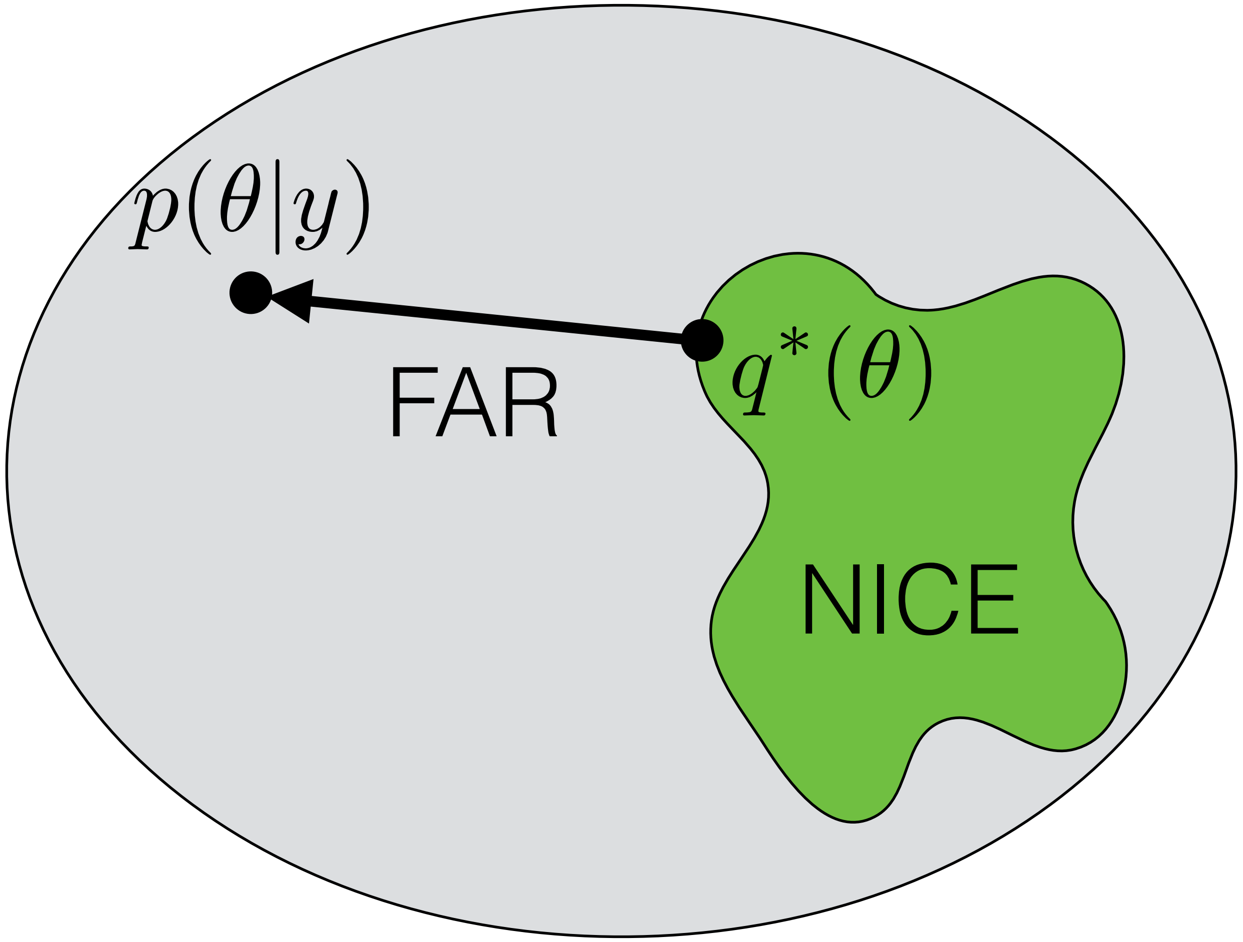

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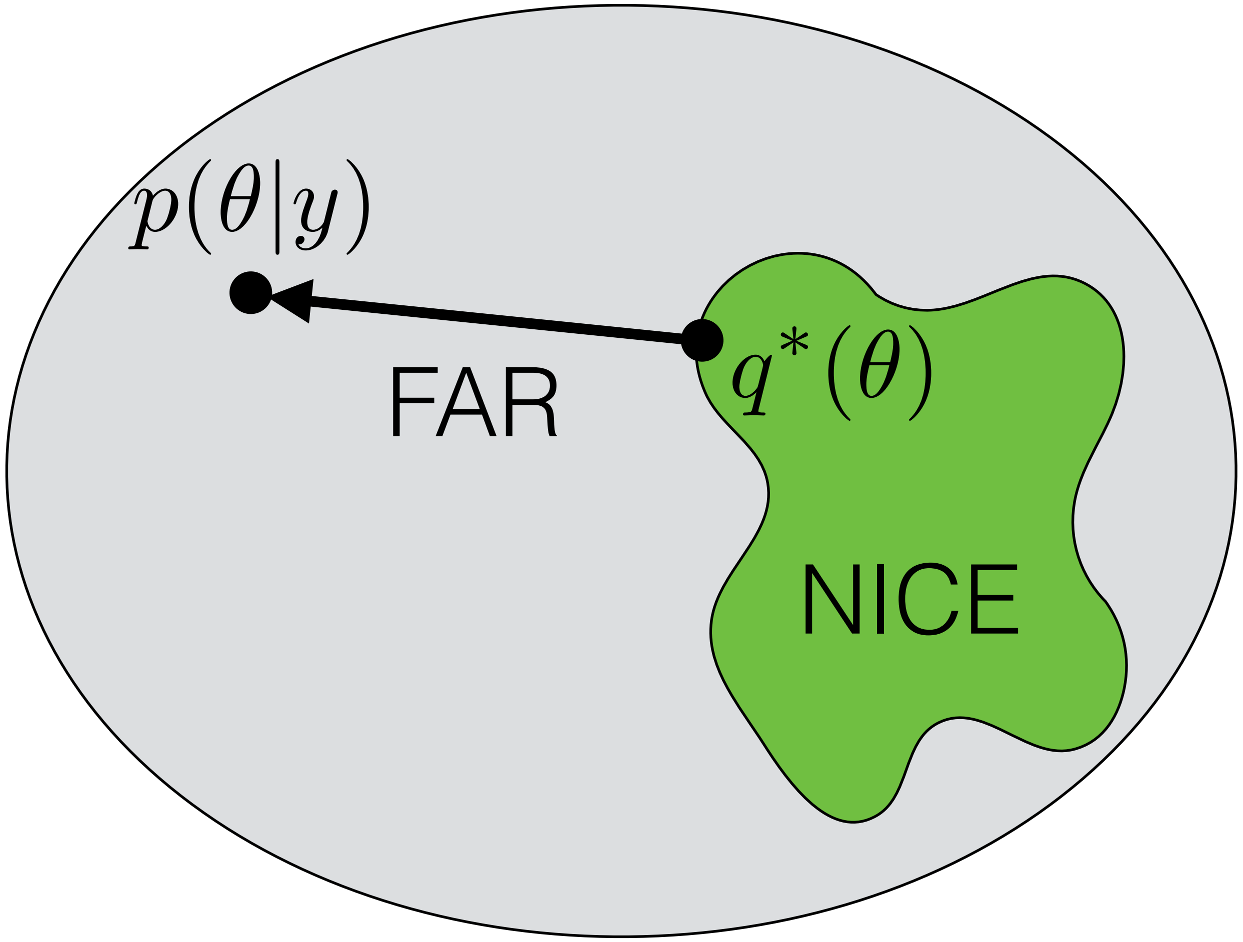

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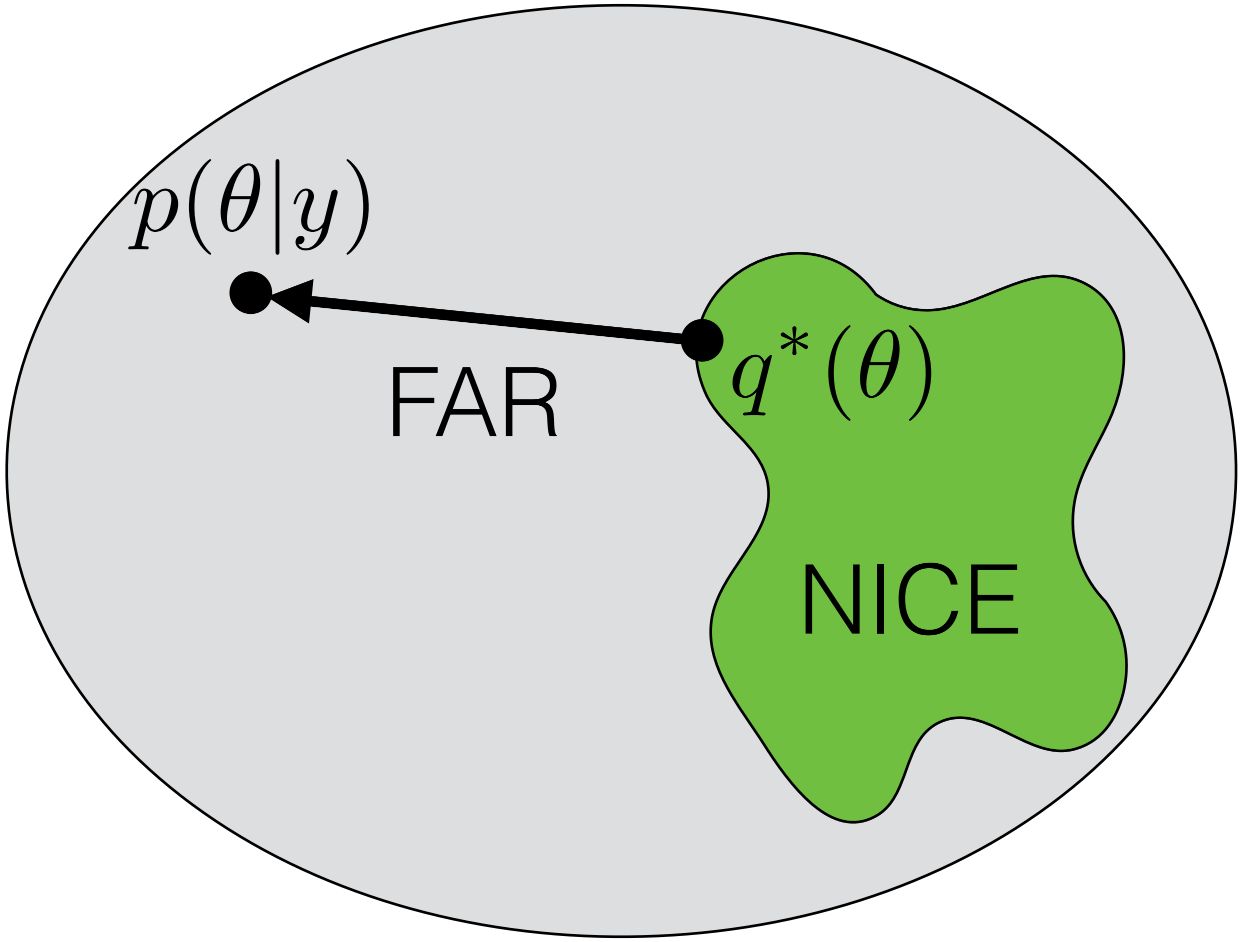

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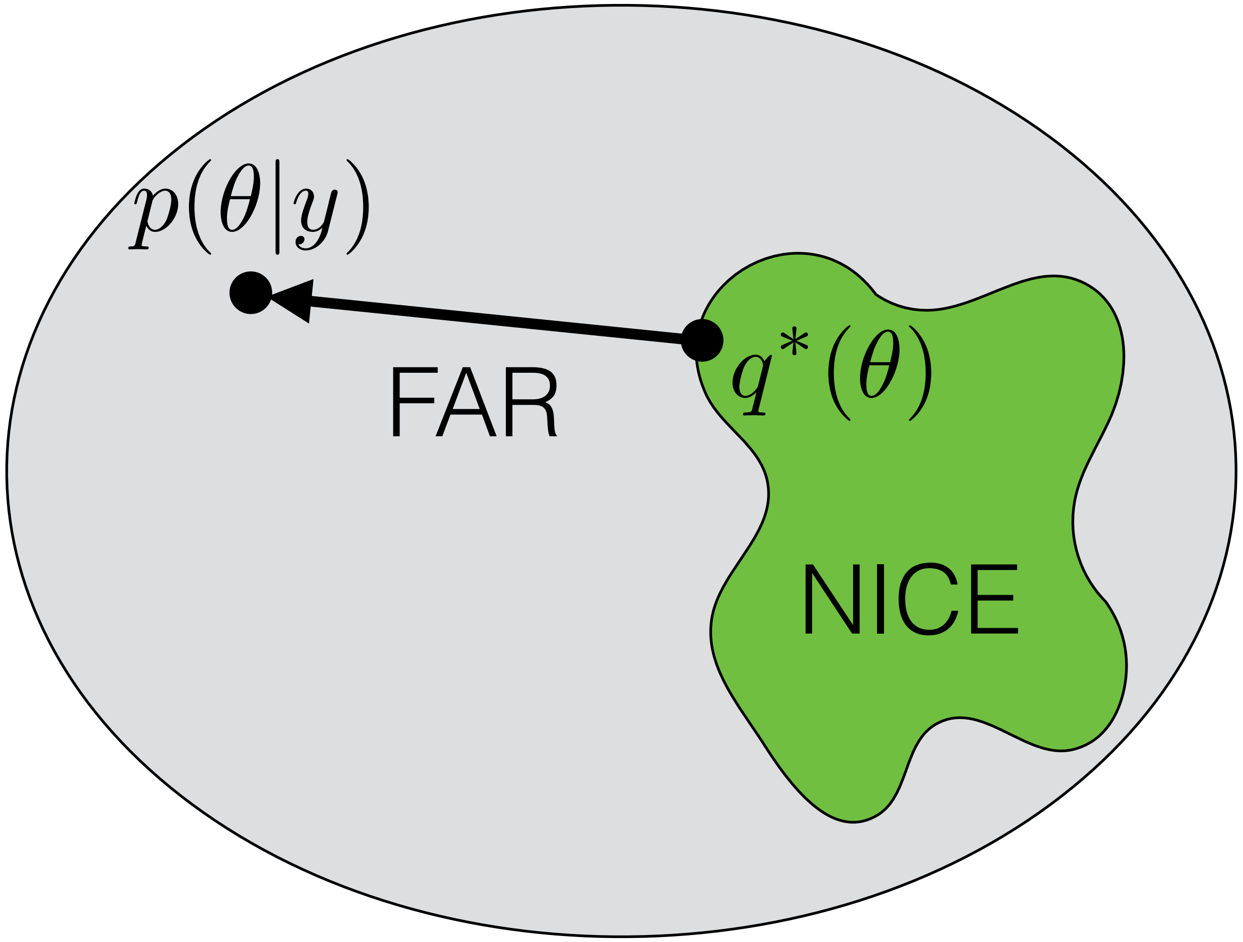

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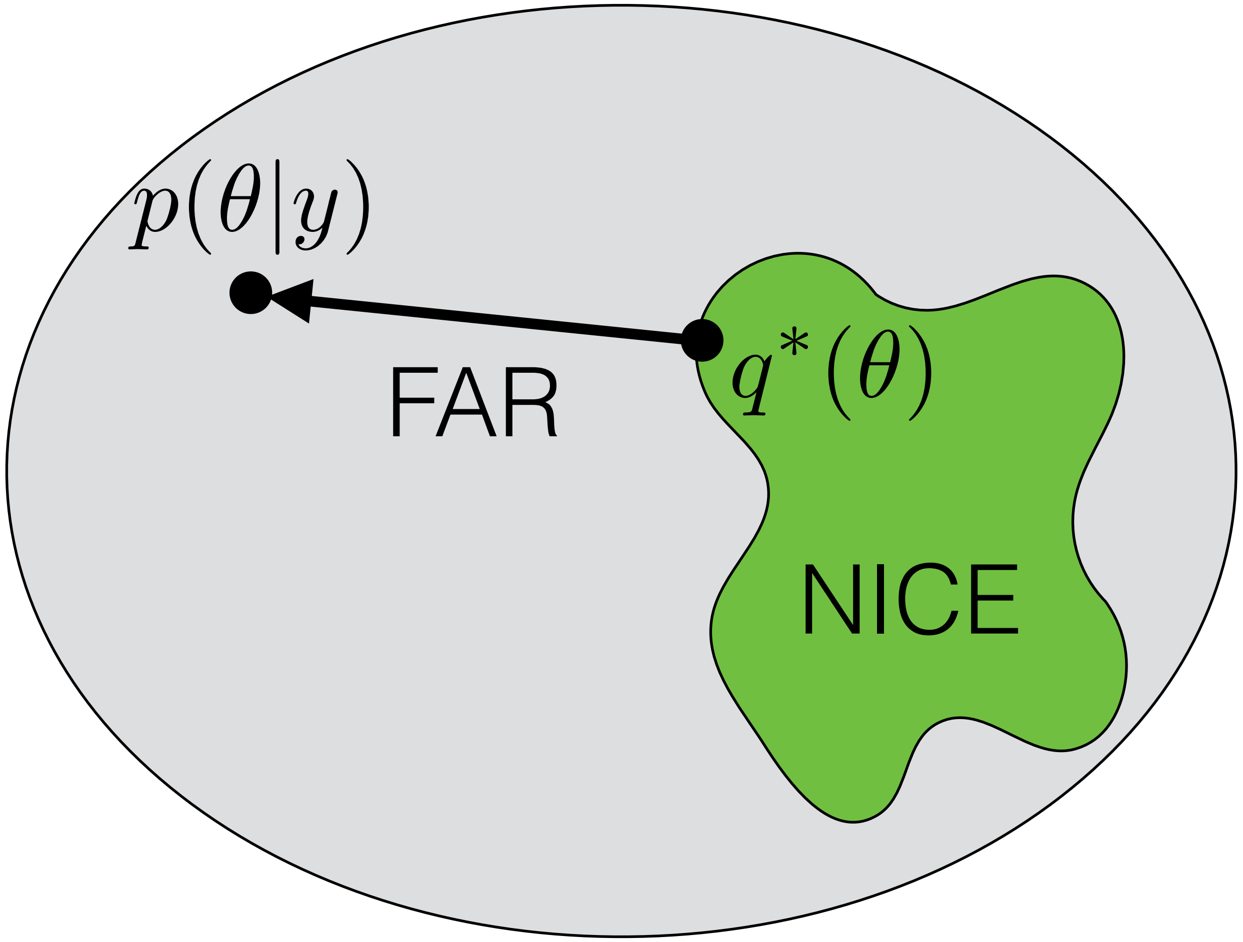

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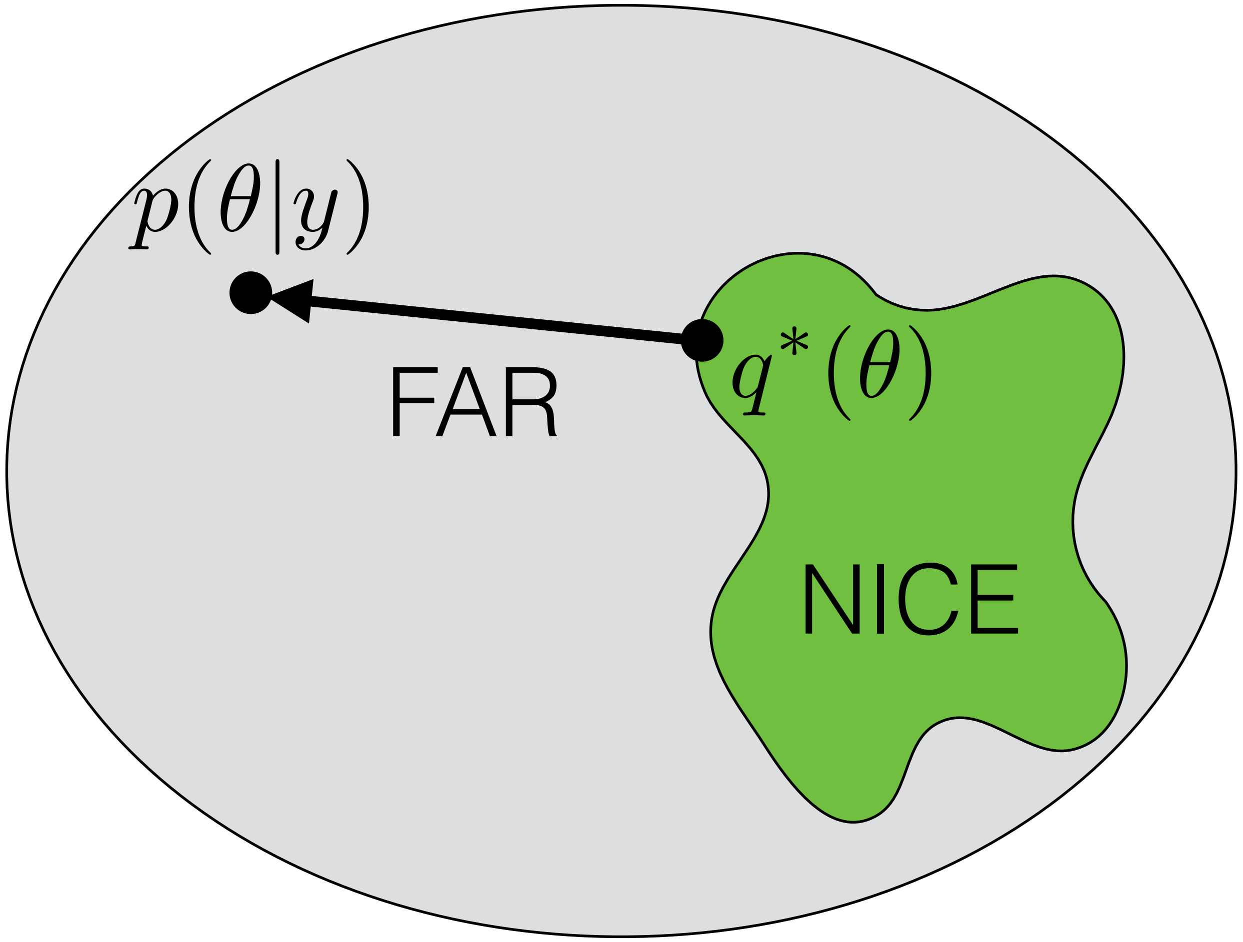

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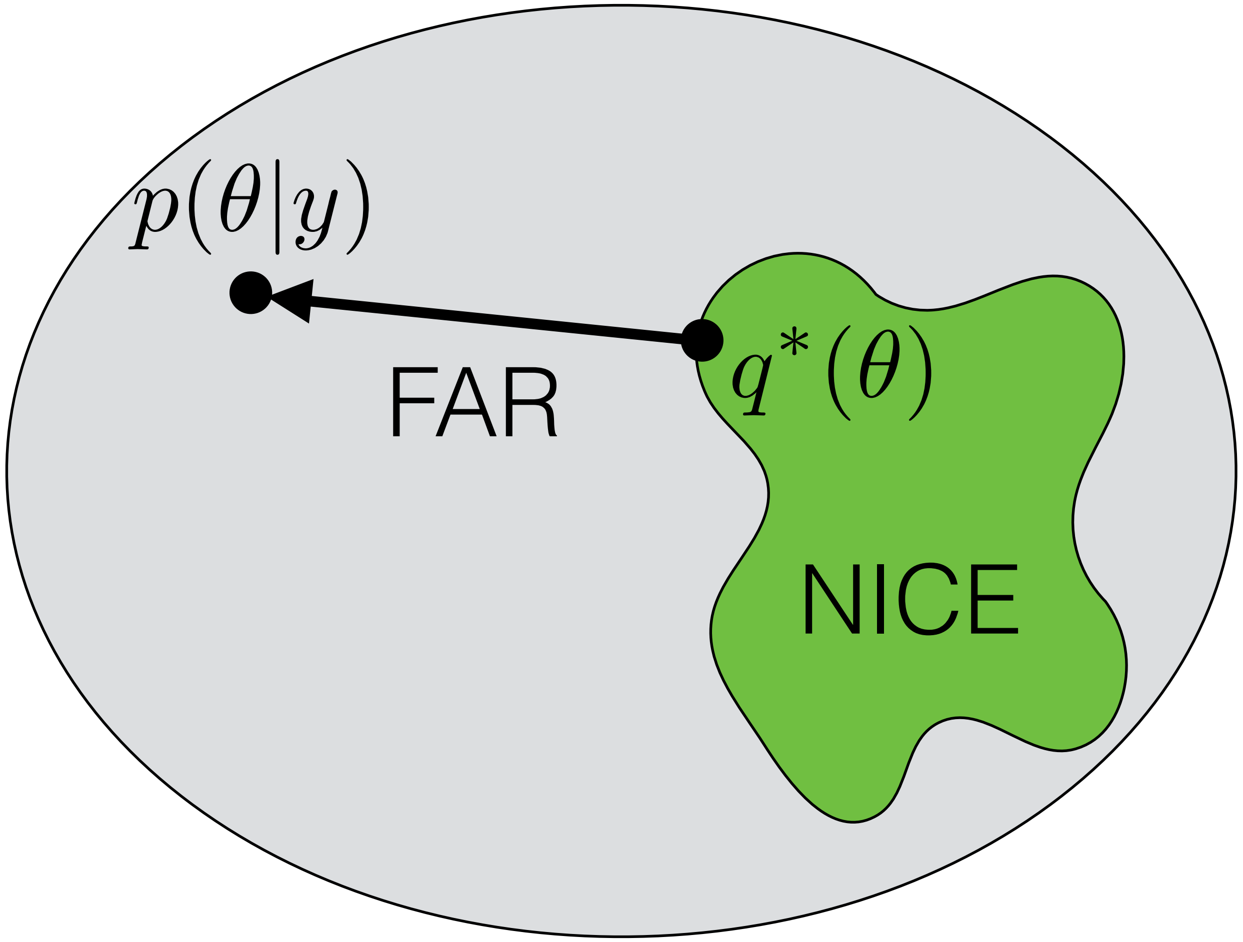

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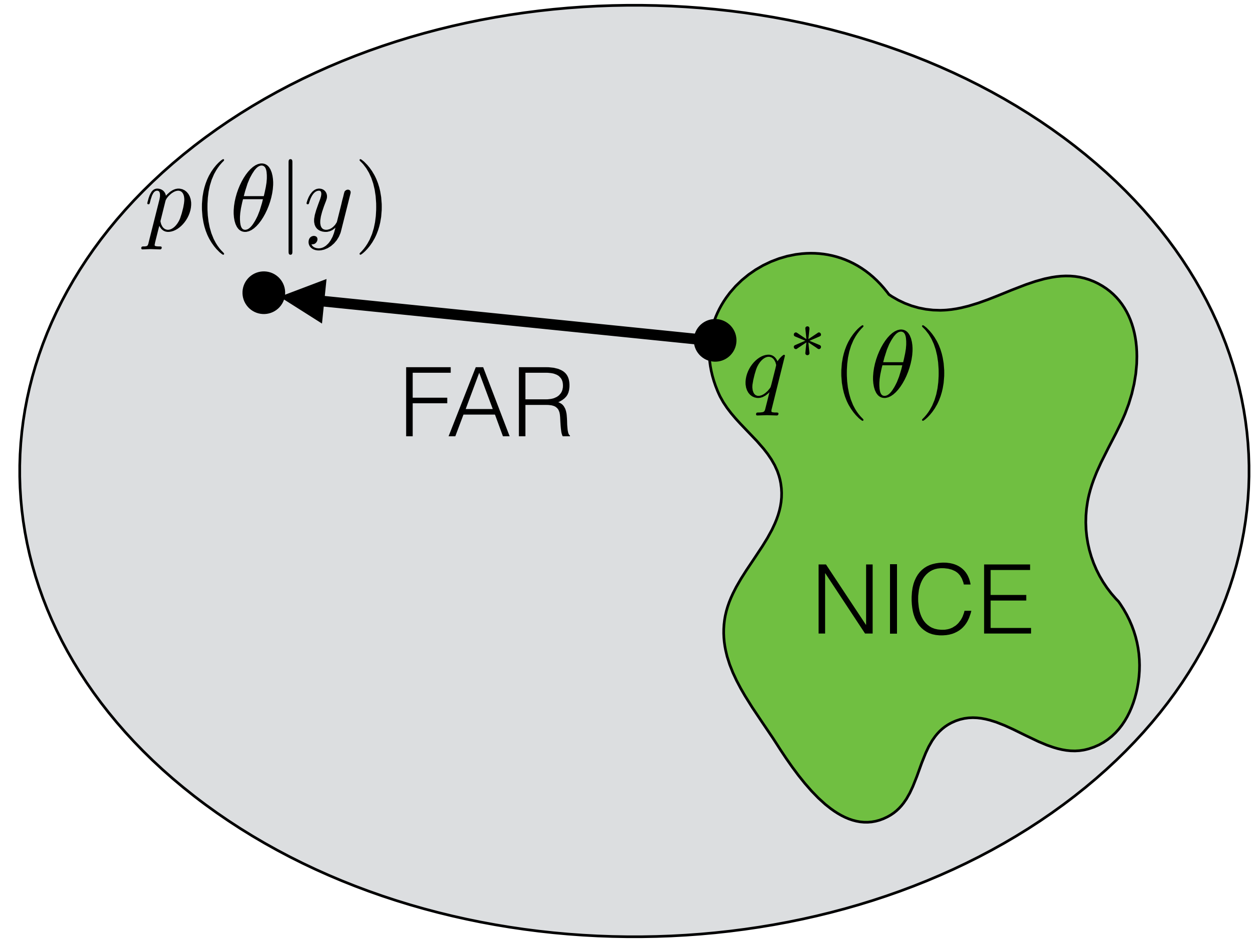

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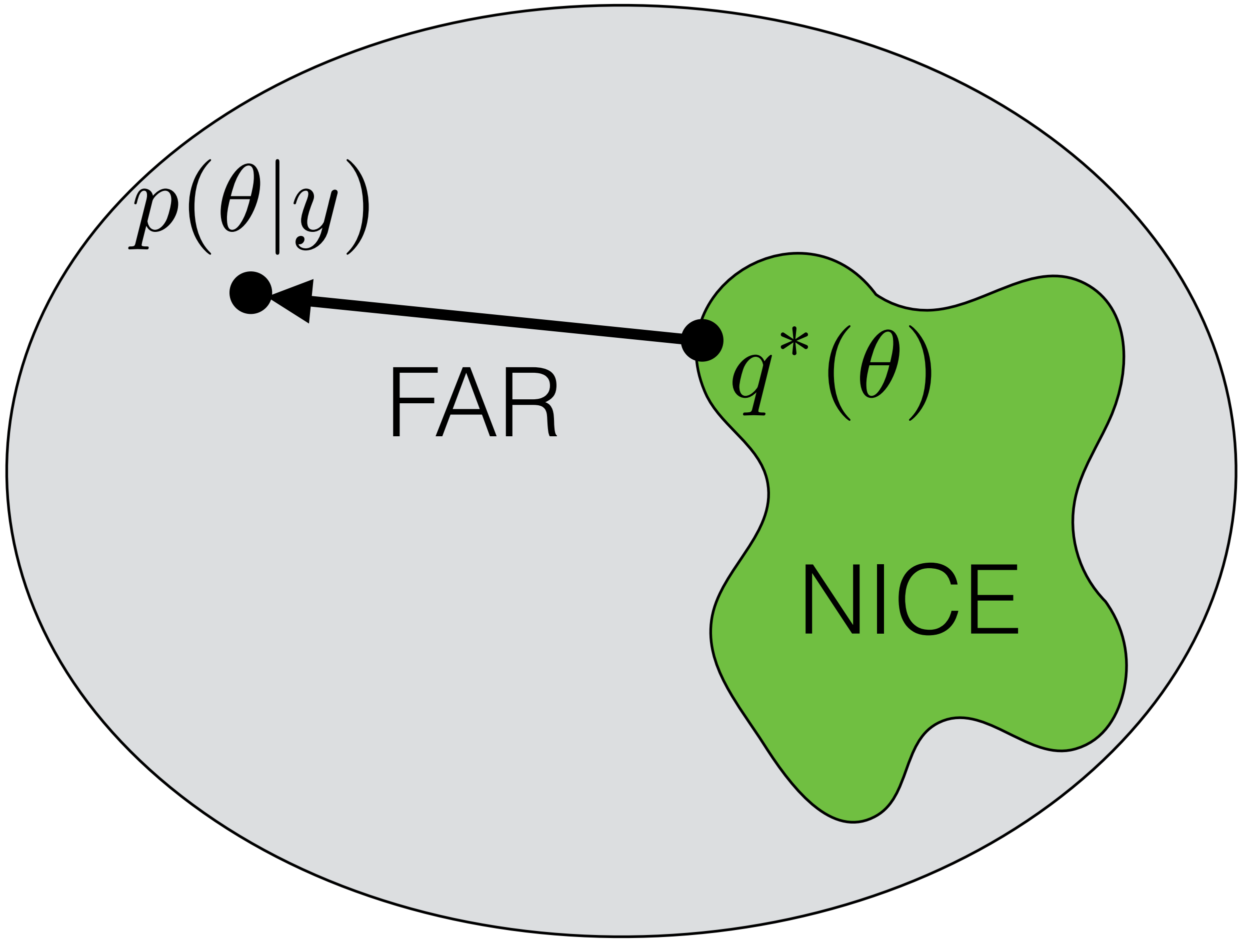

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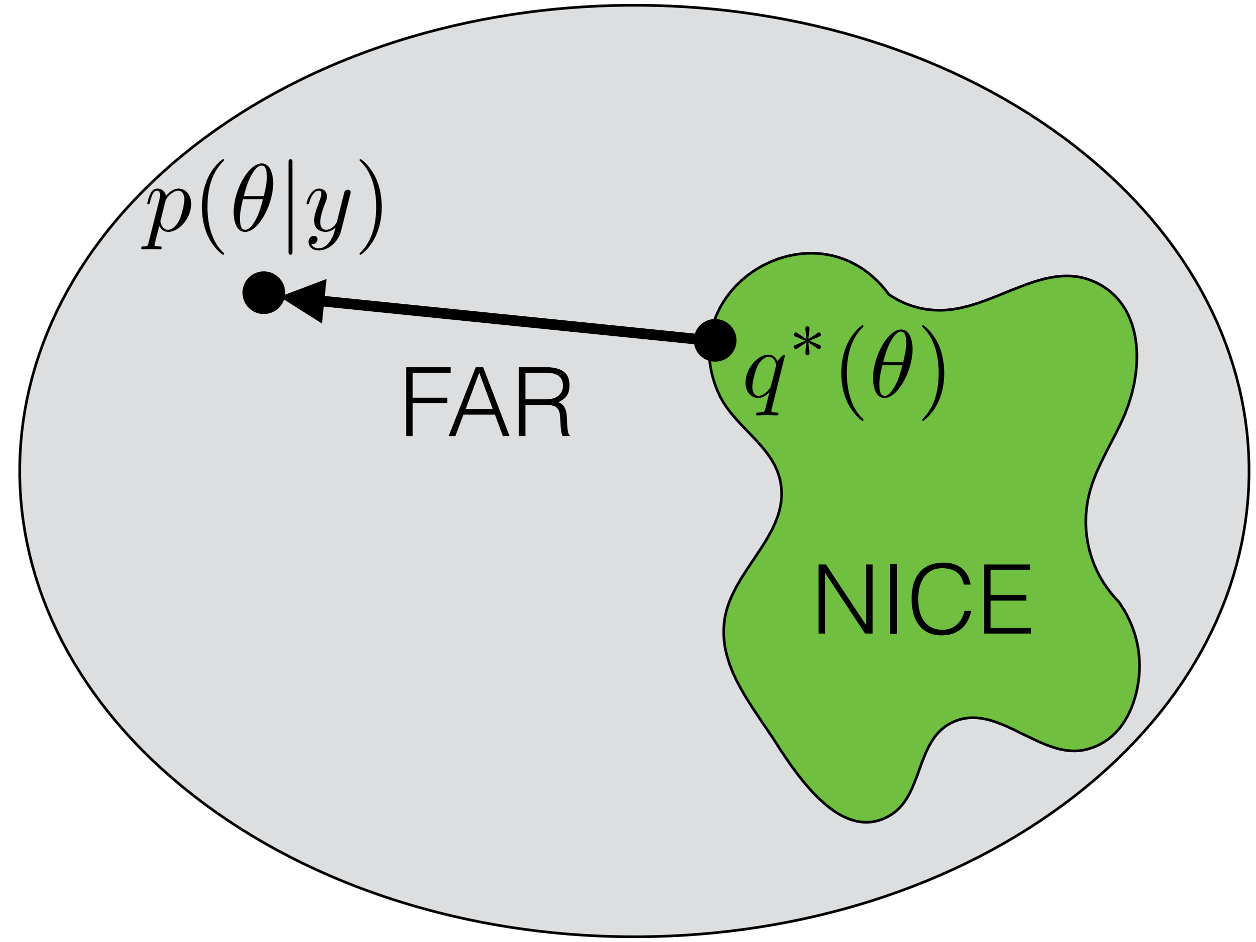

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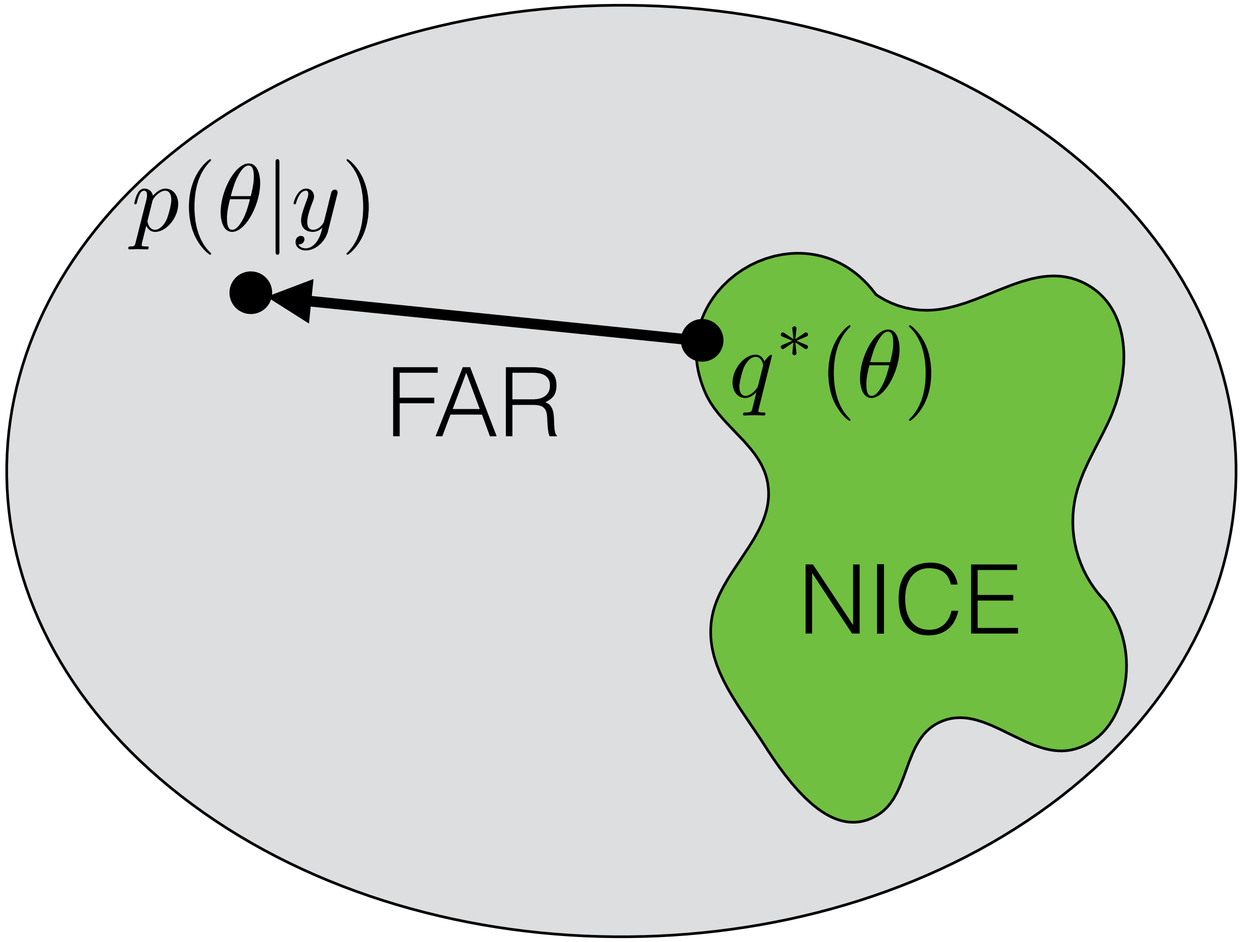

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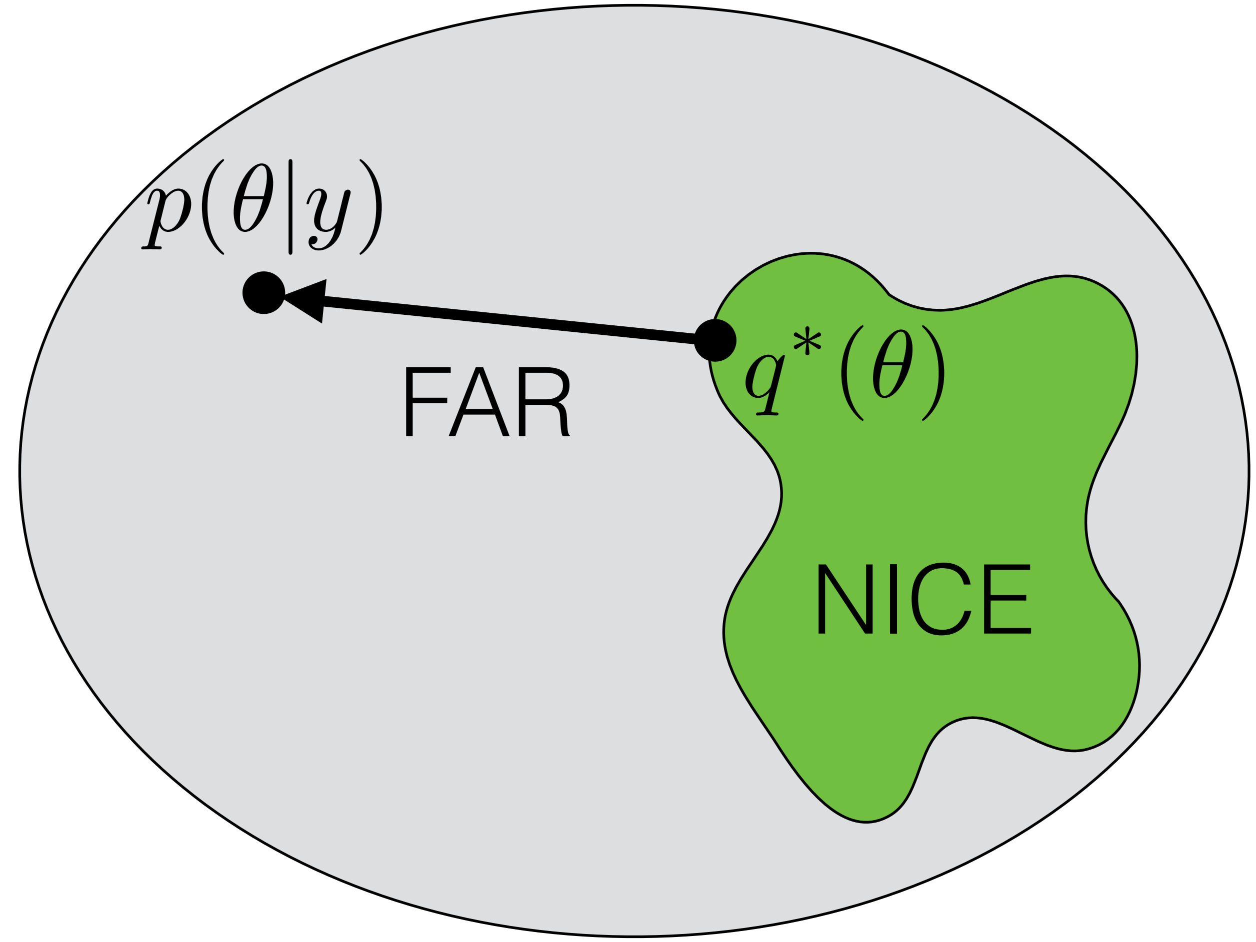

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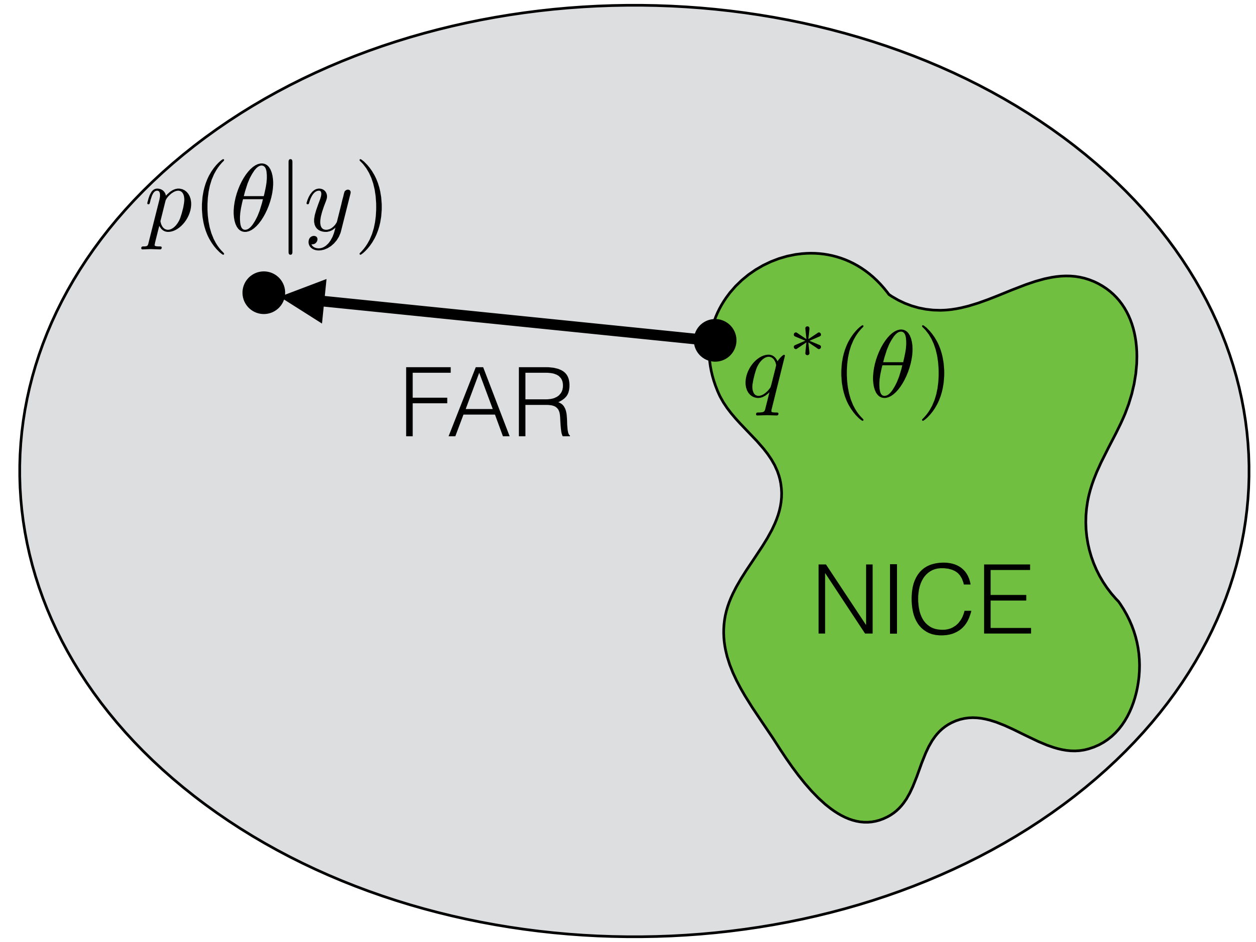

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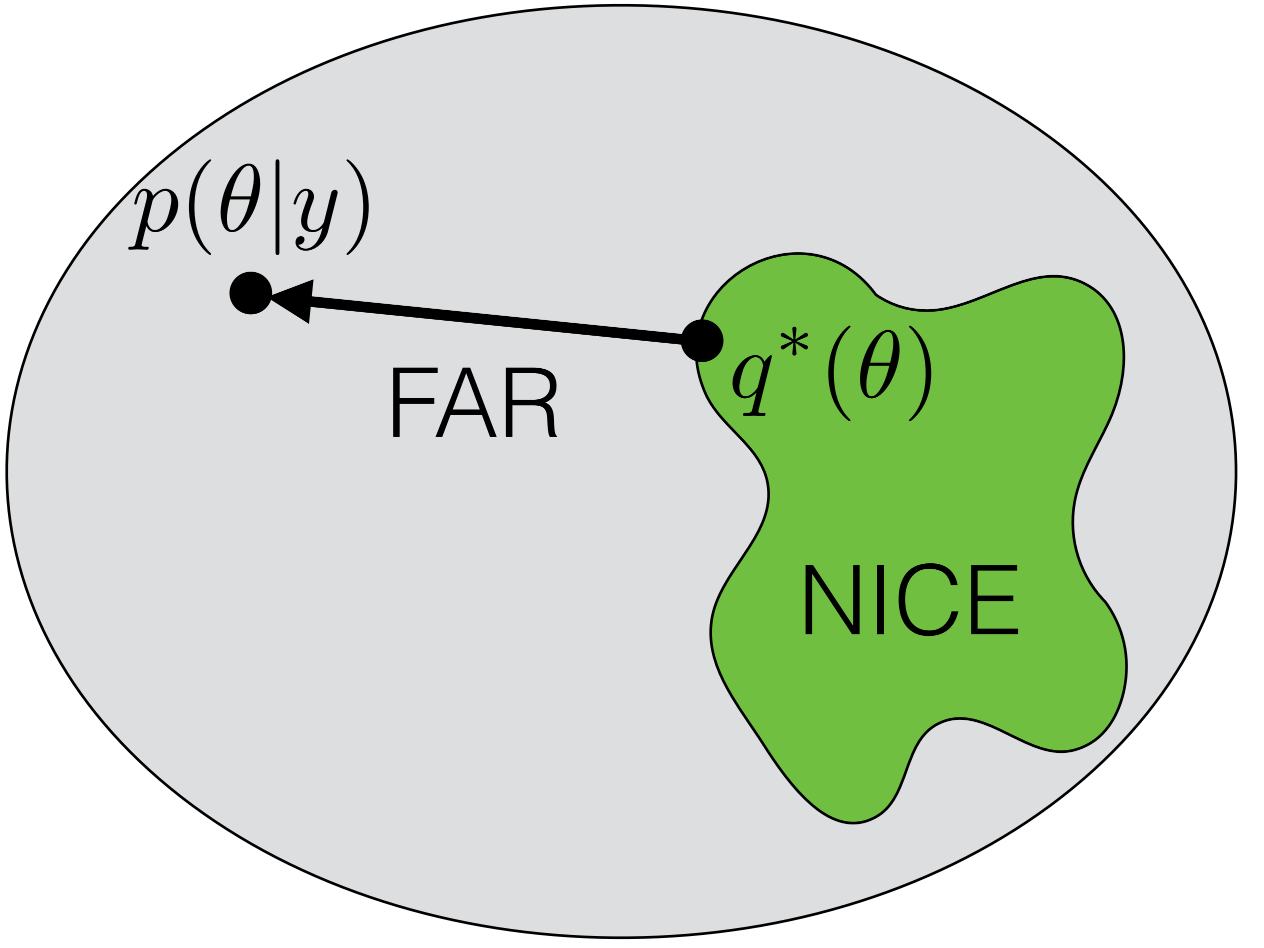

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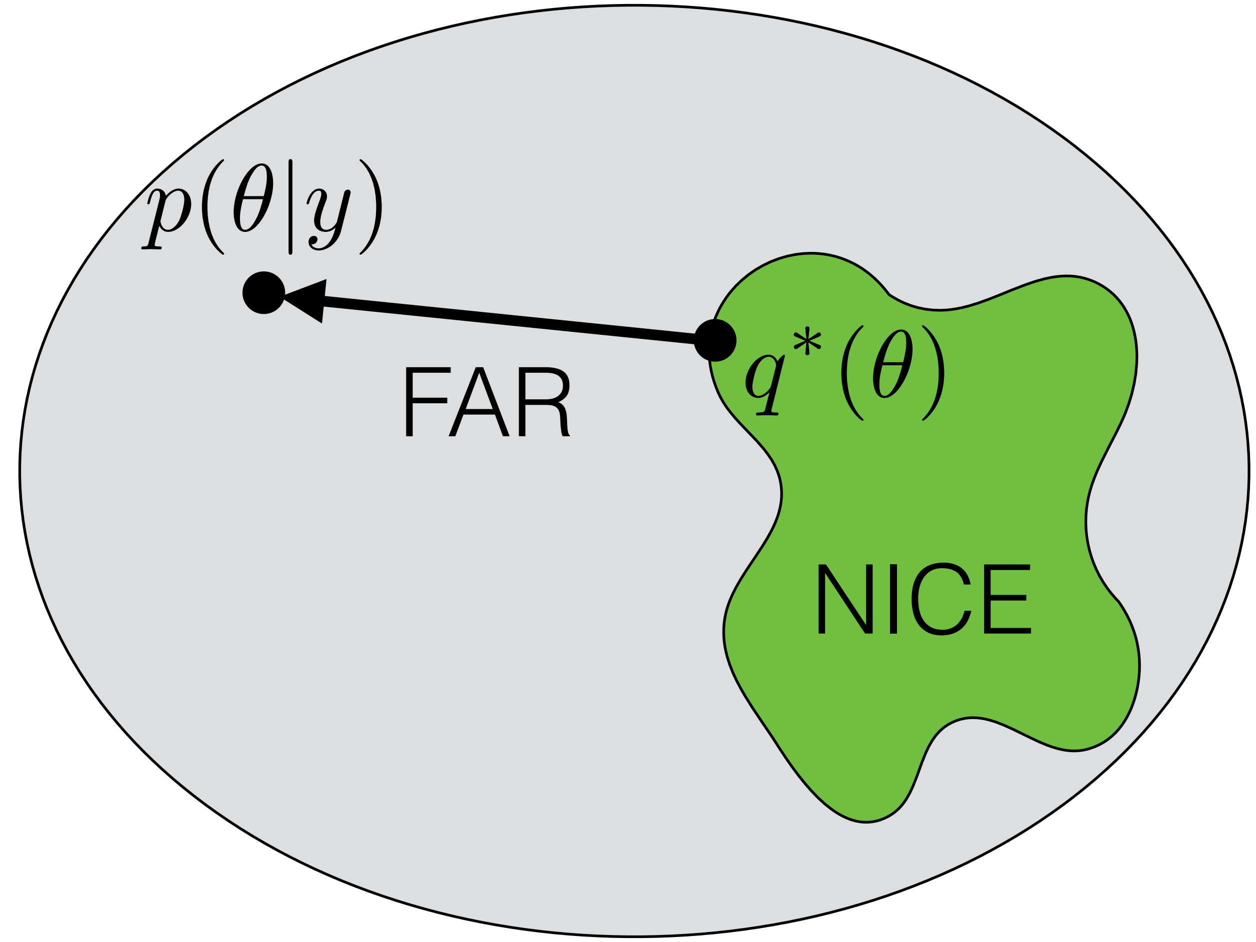

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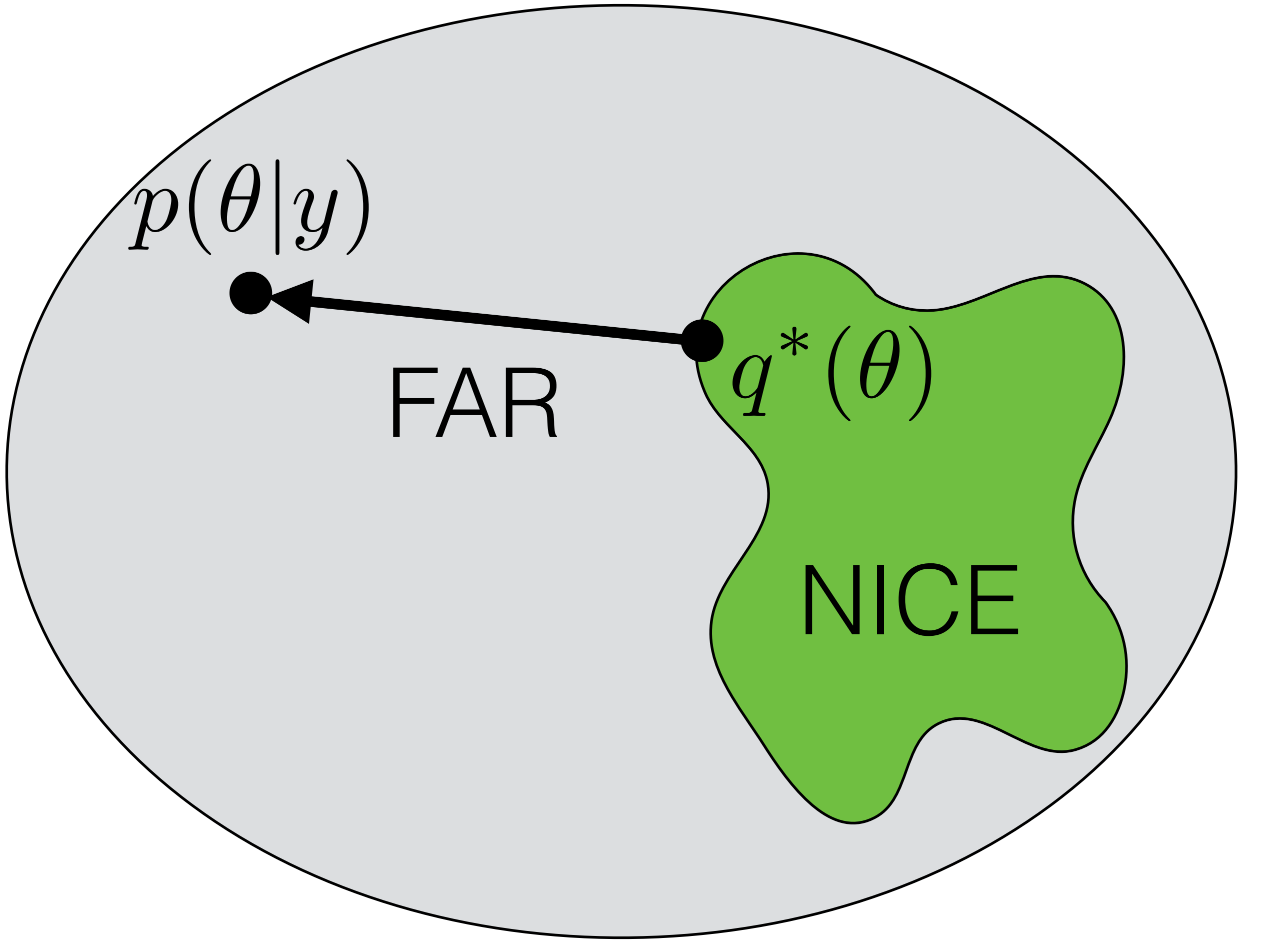

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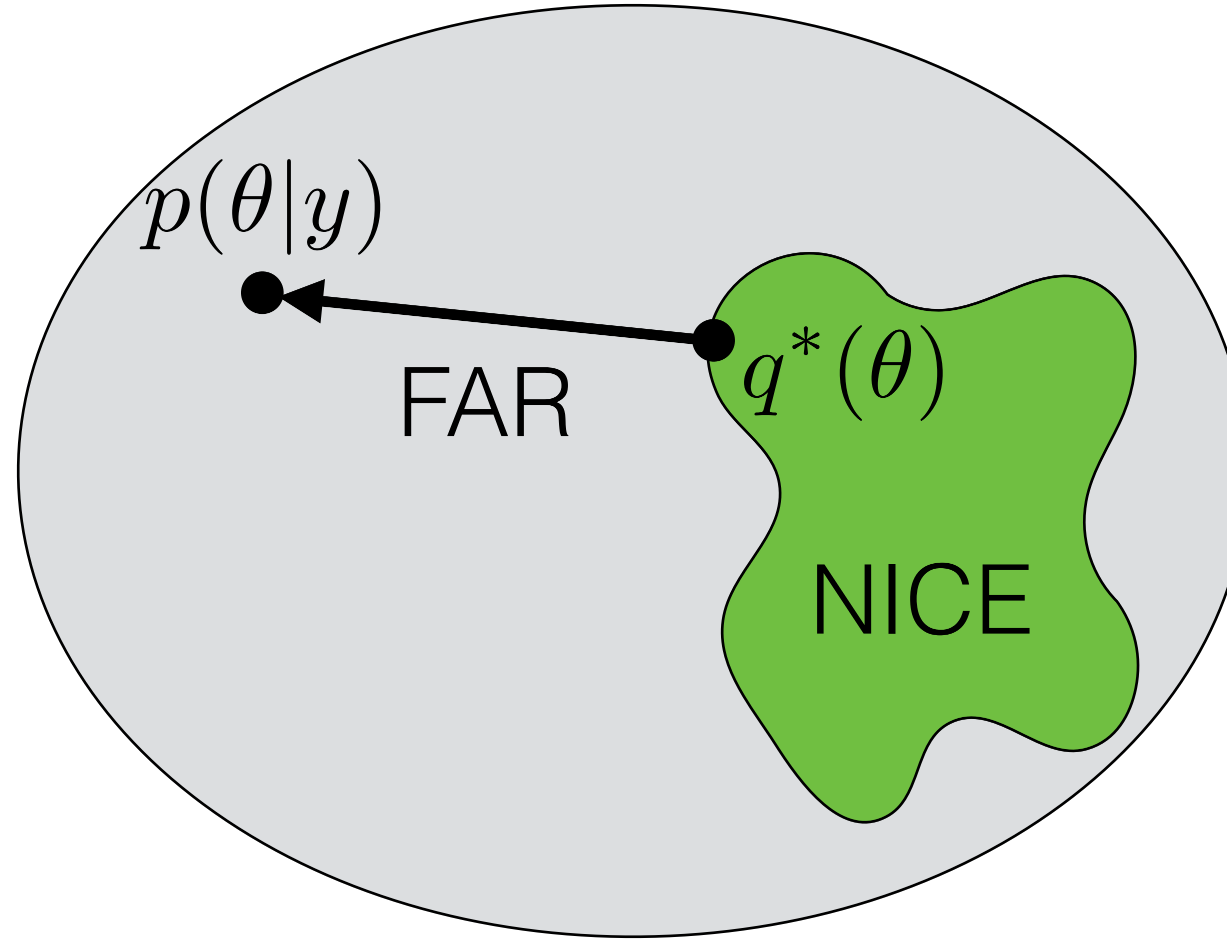

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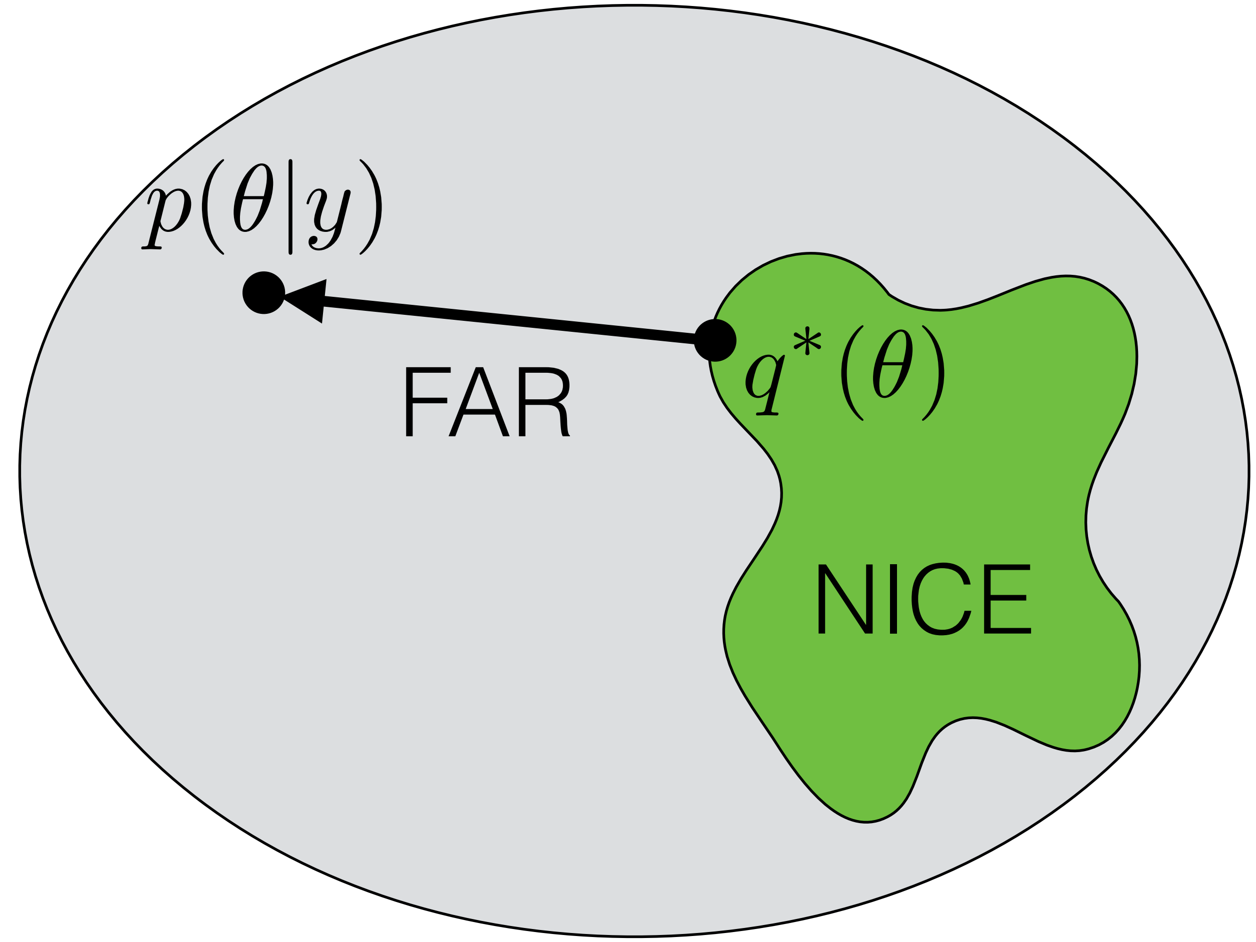

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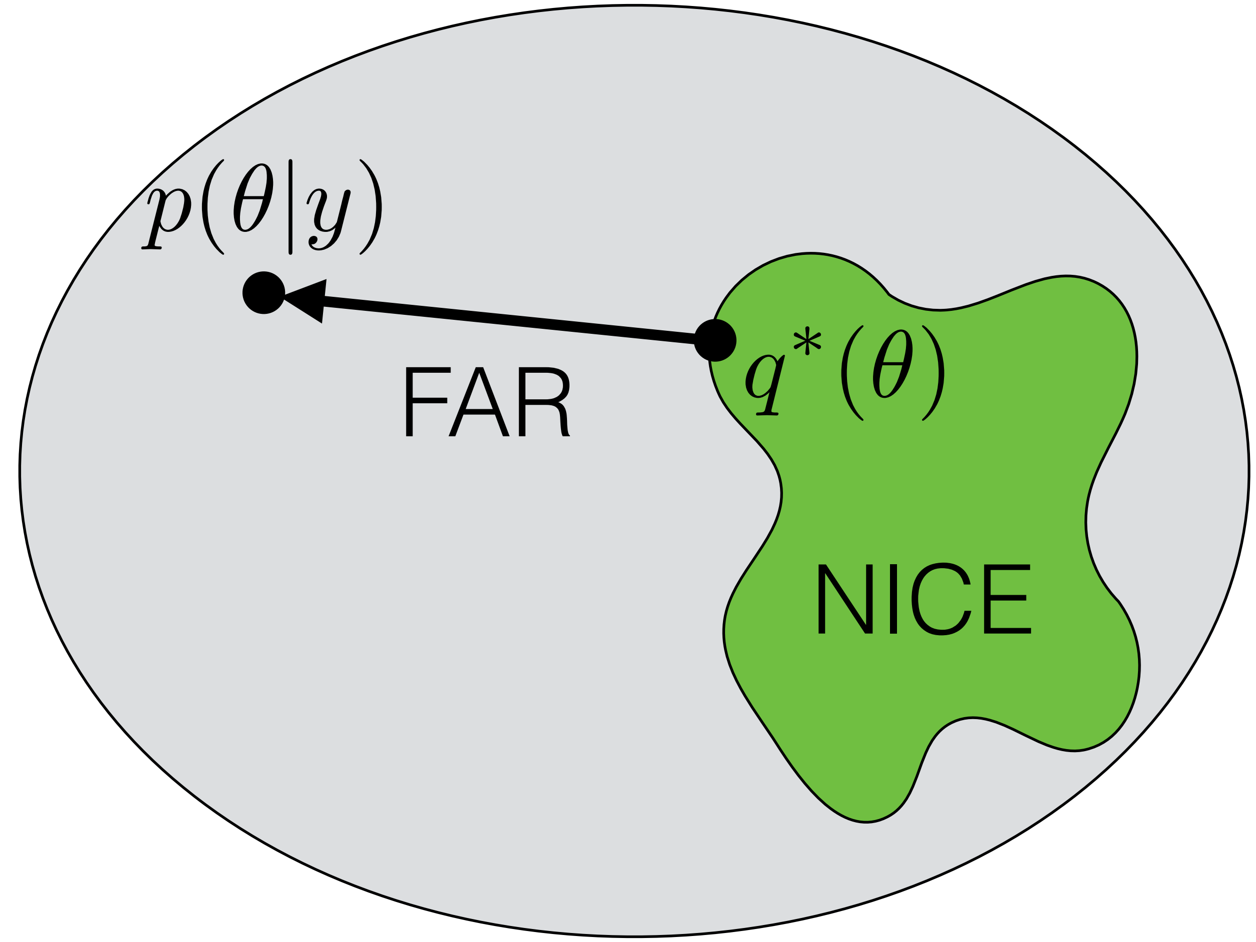

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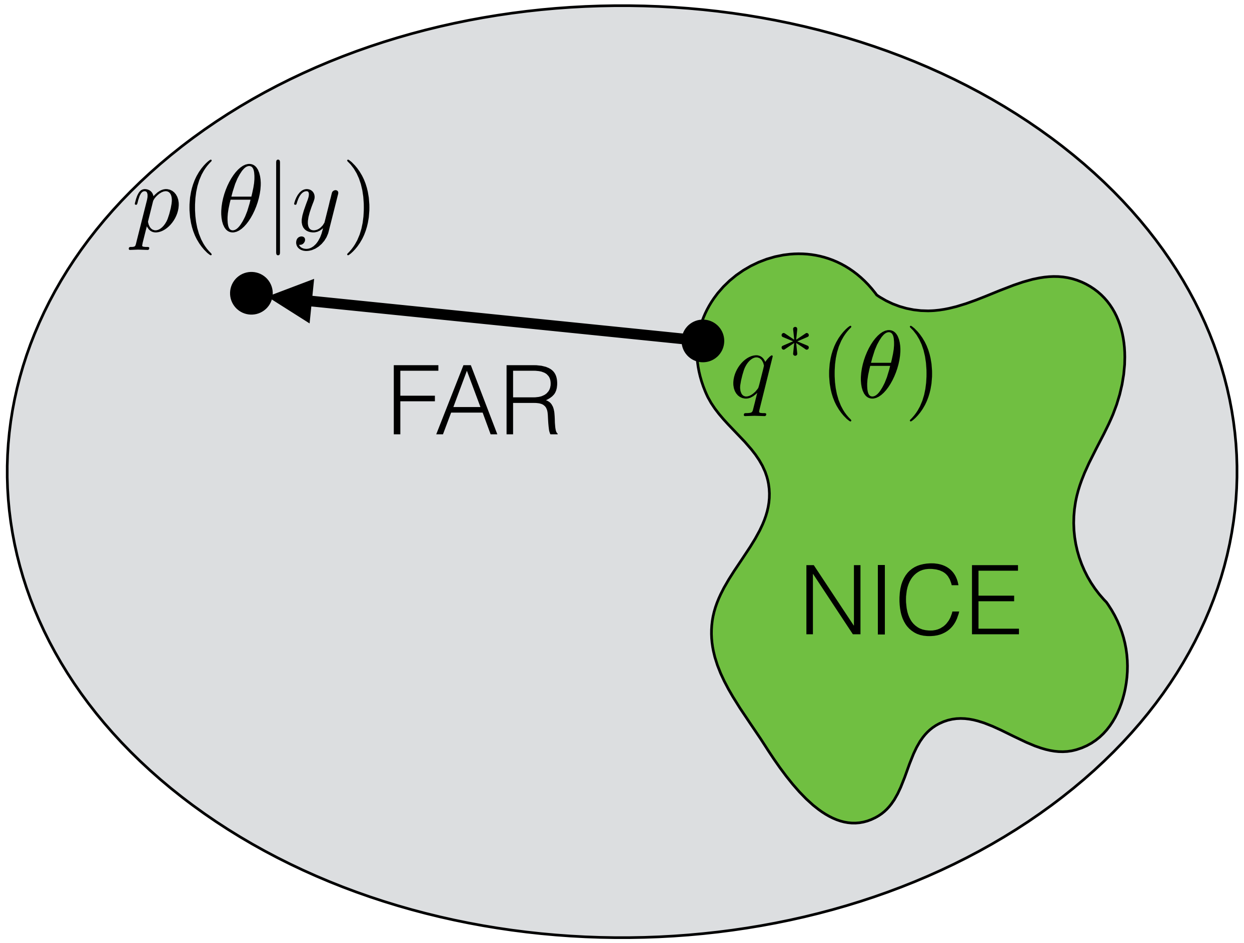

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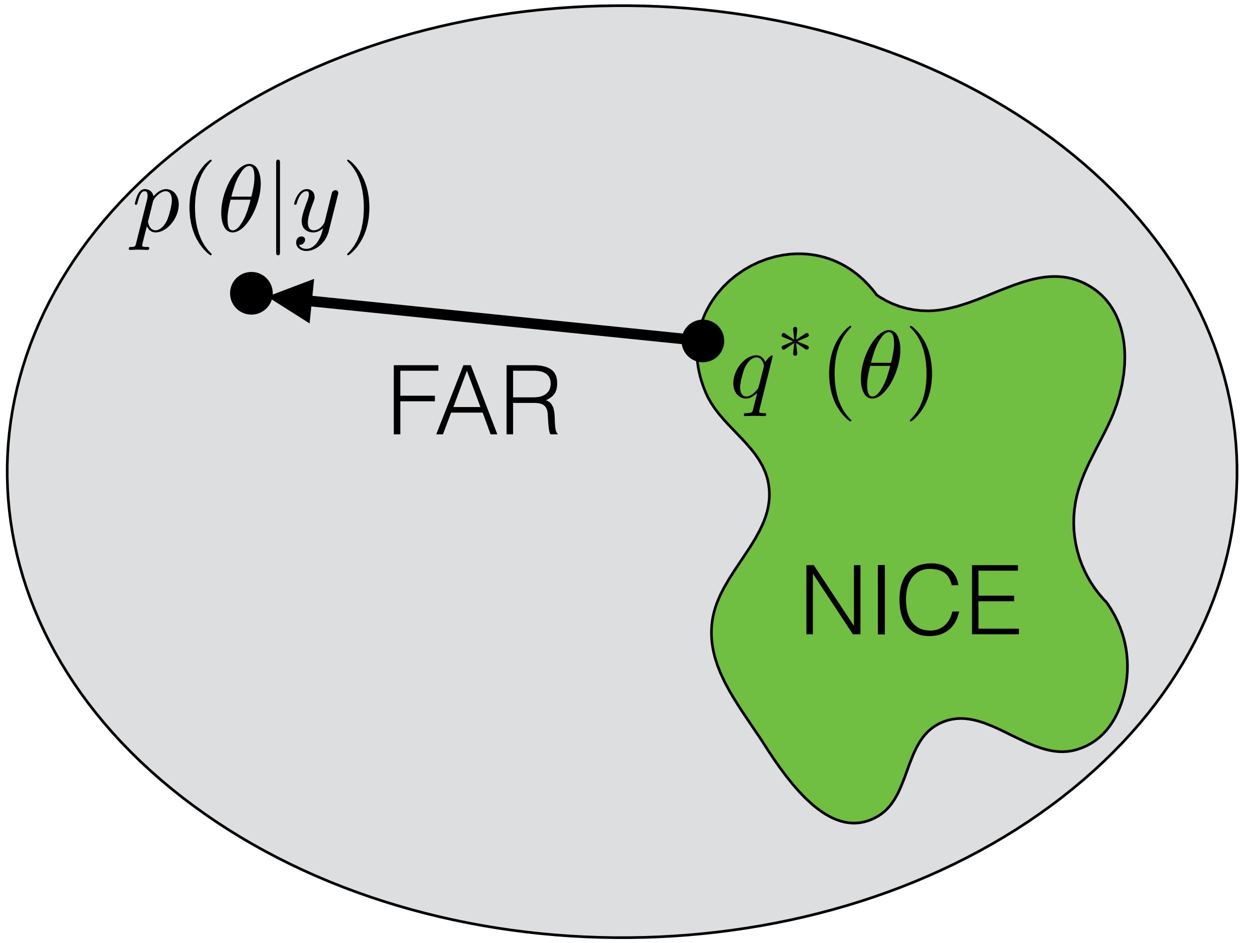

Choose "NICE" distributions

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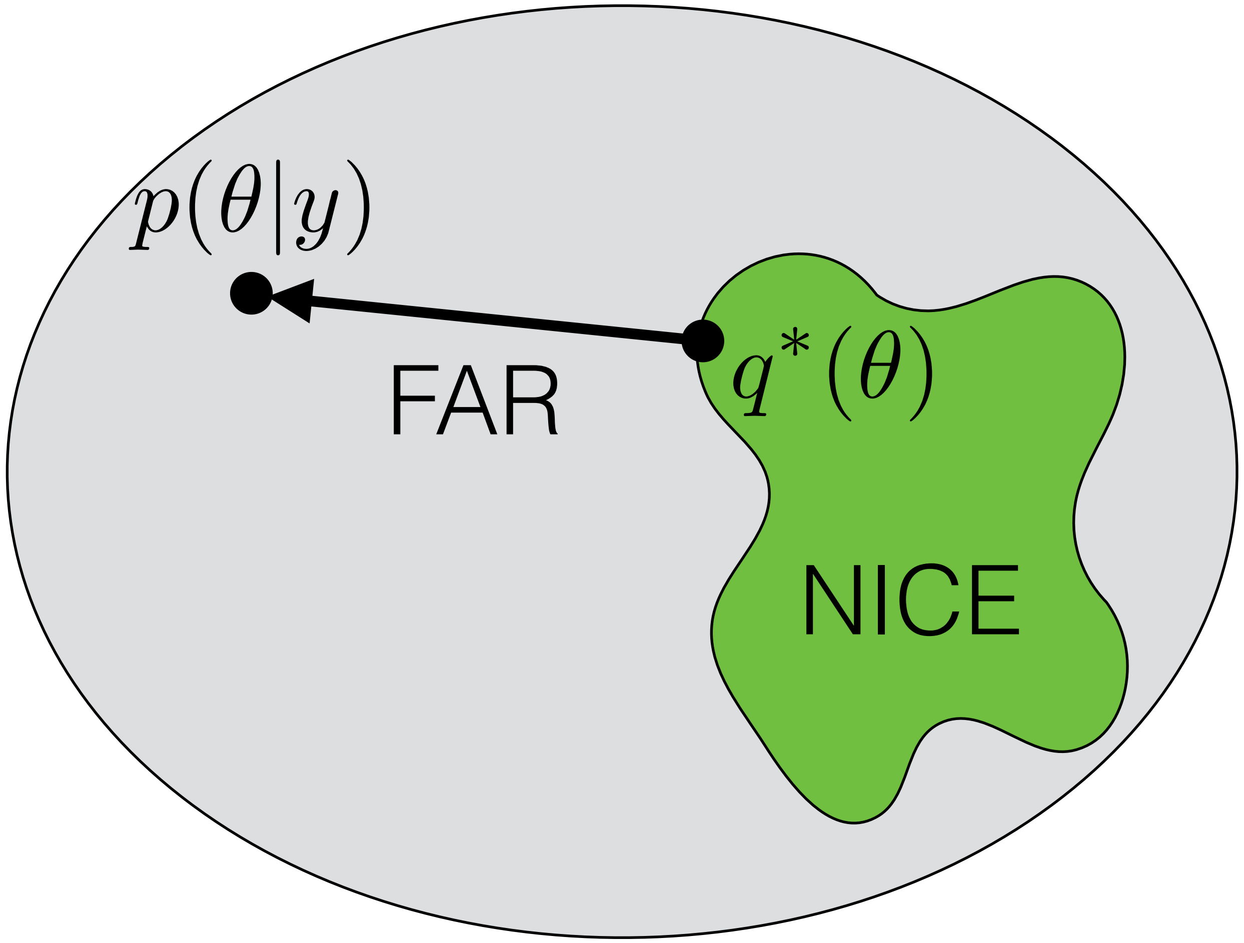

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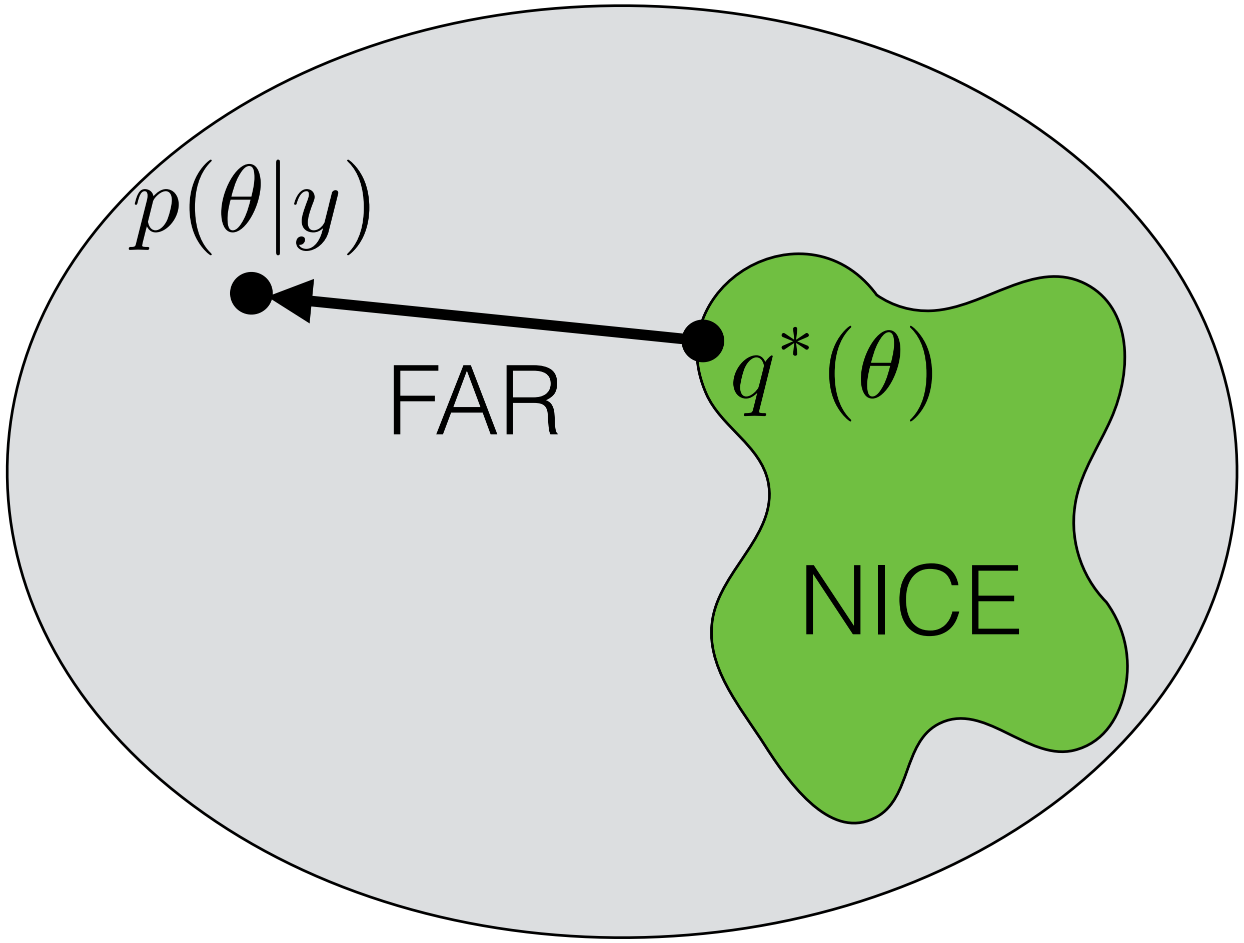

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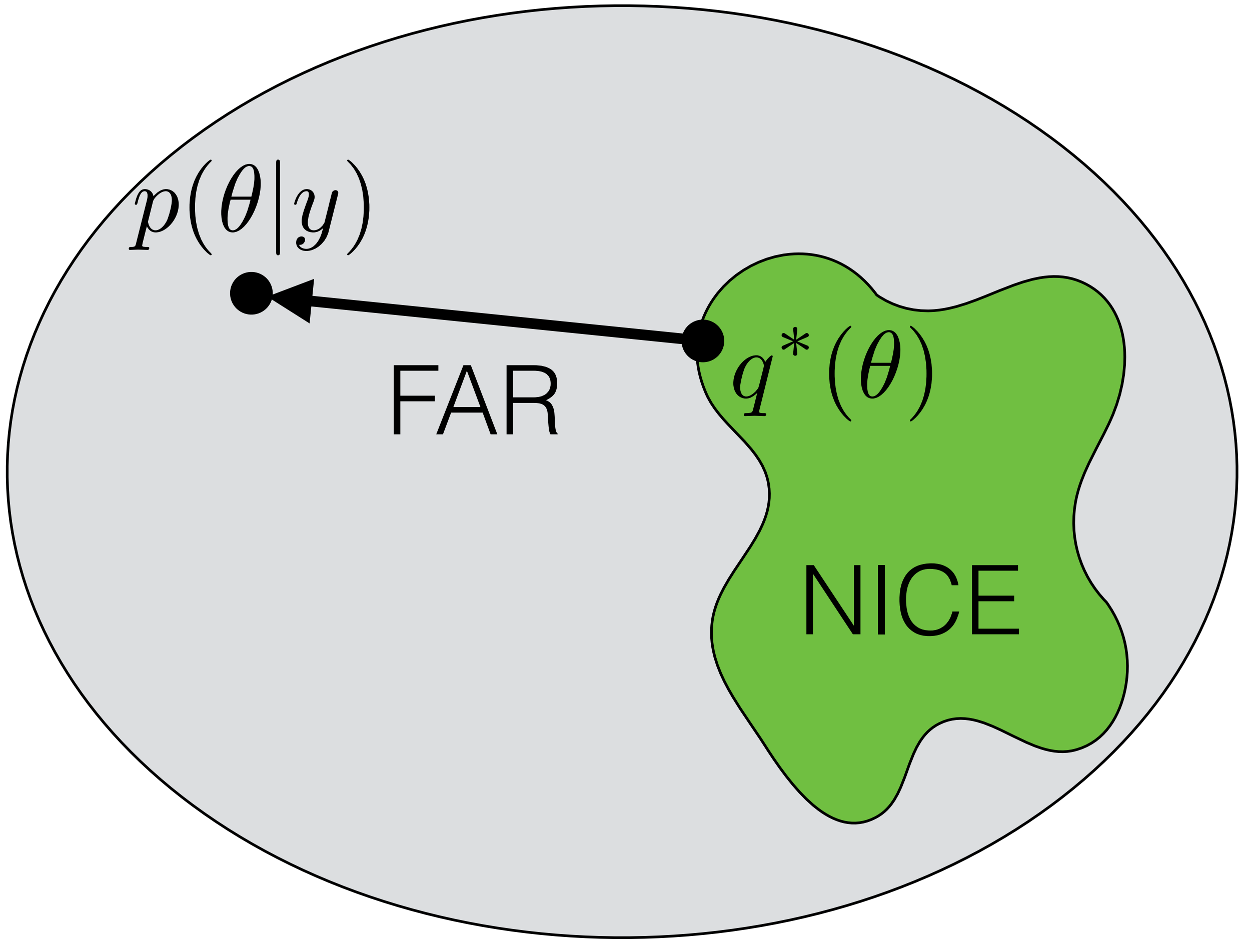

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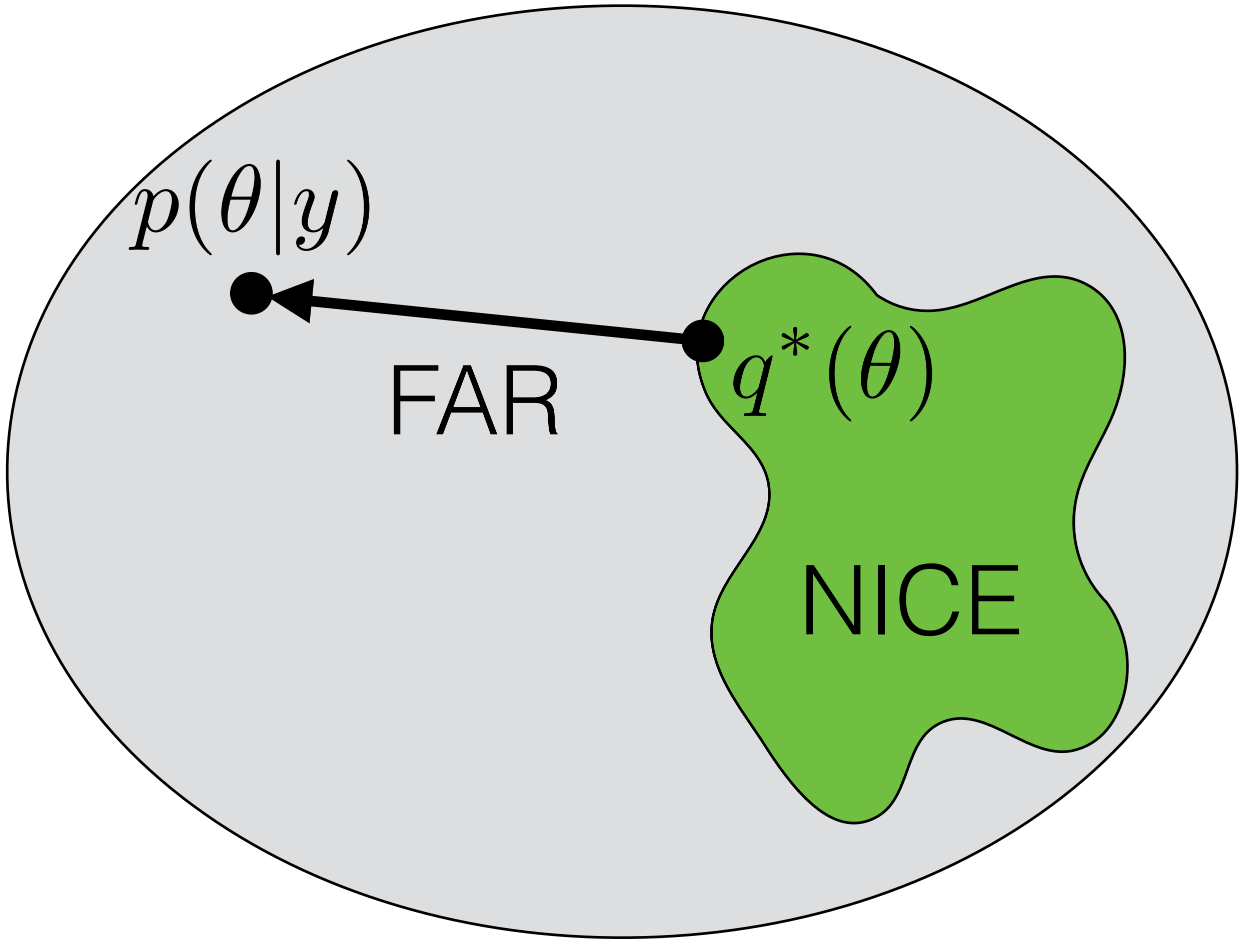

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- Often also exponential family

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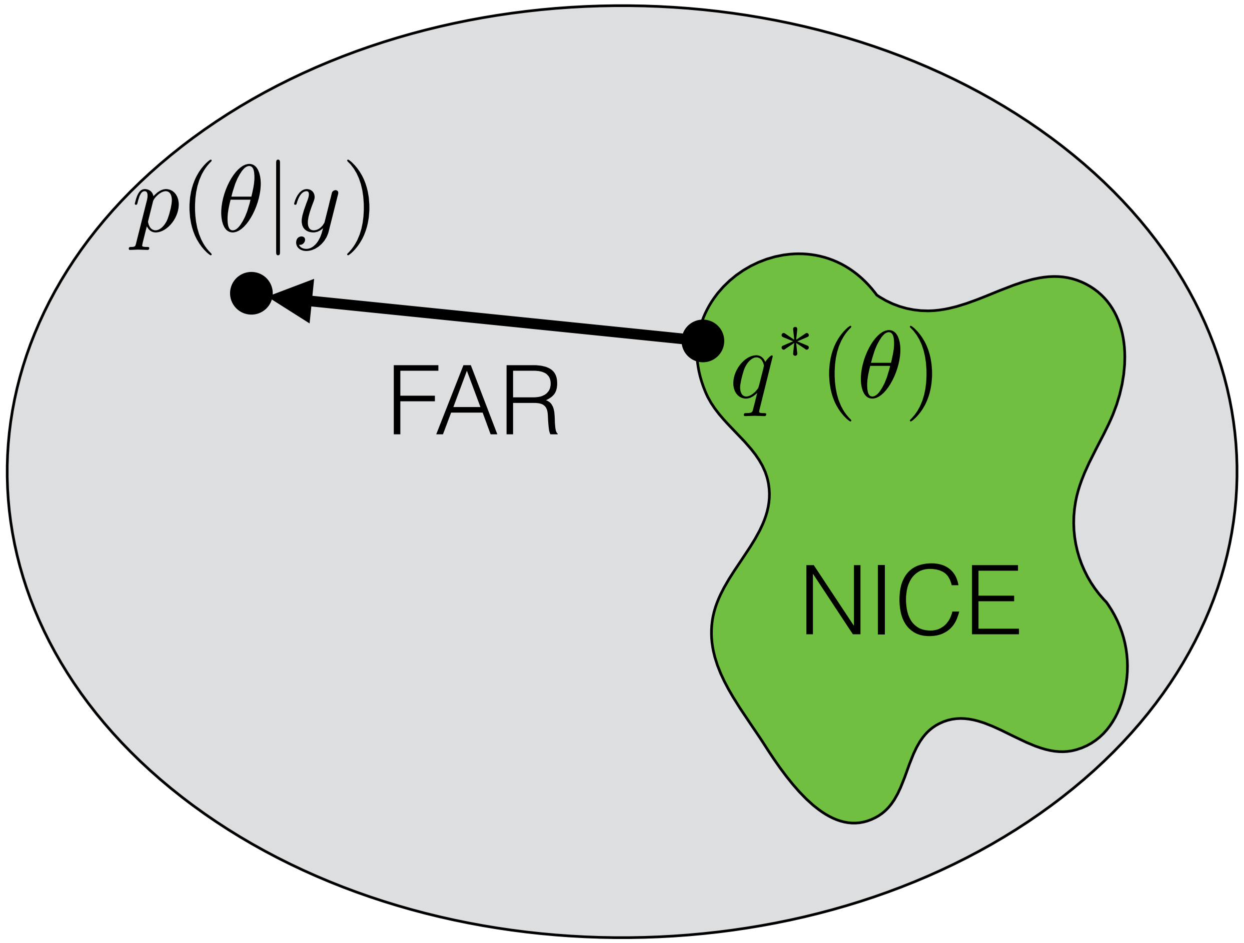

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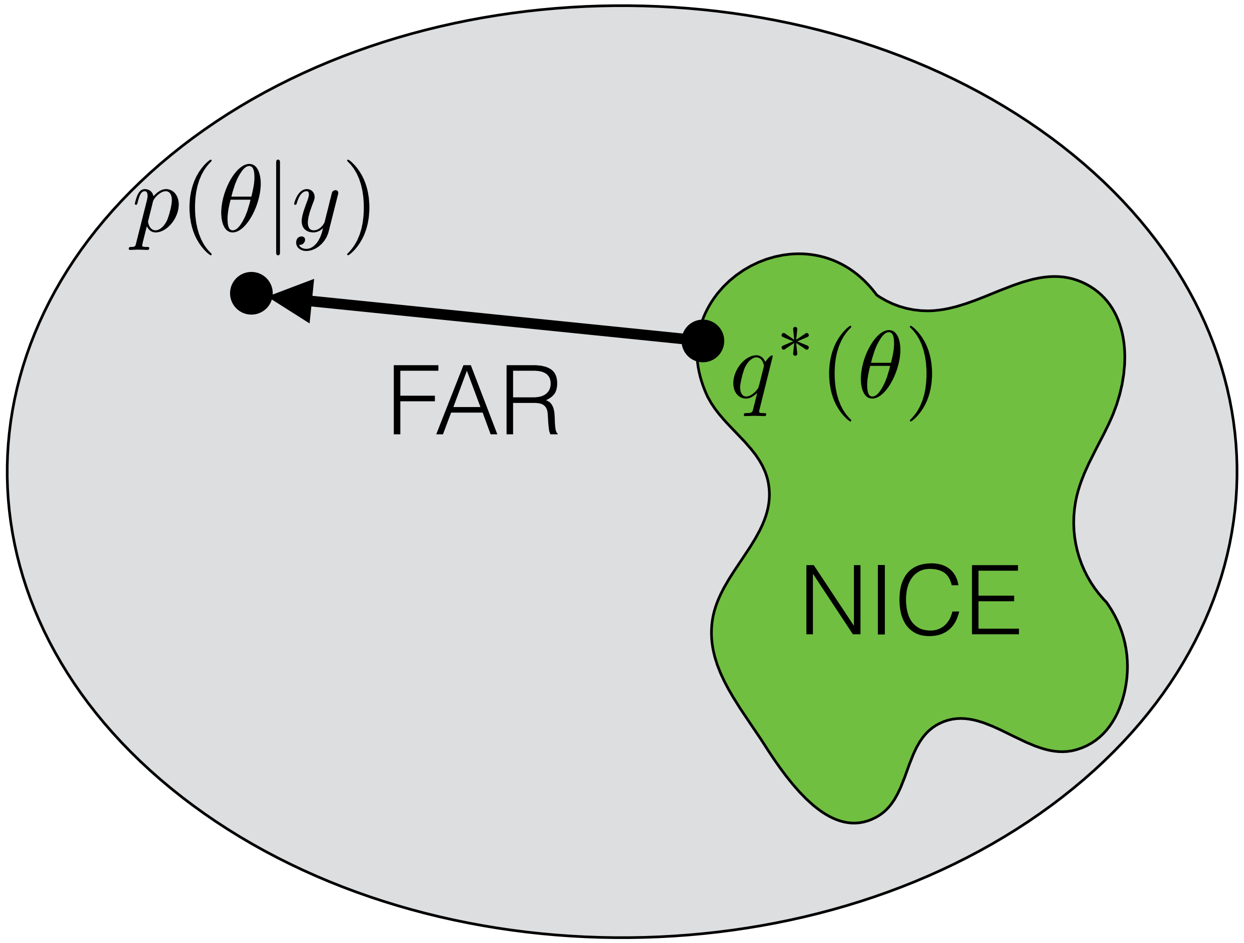

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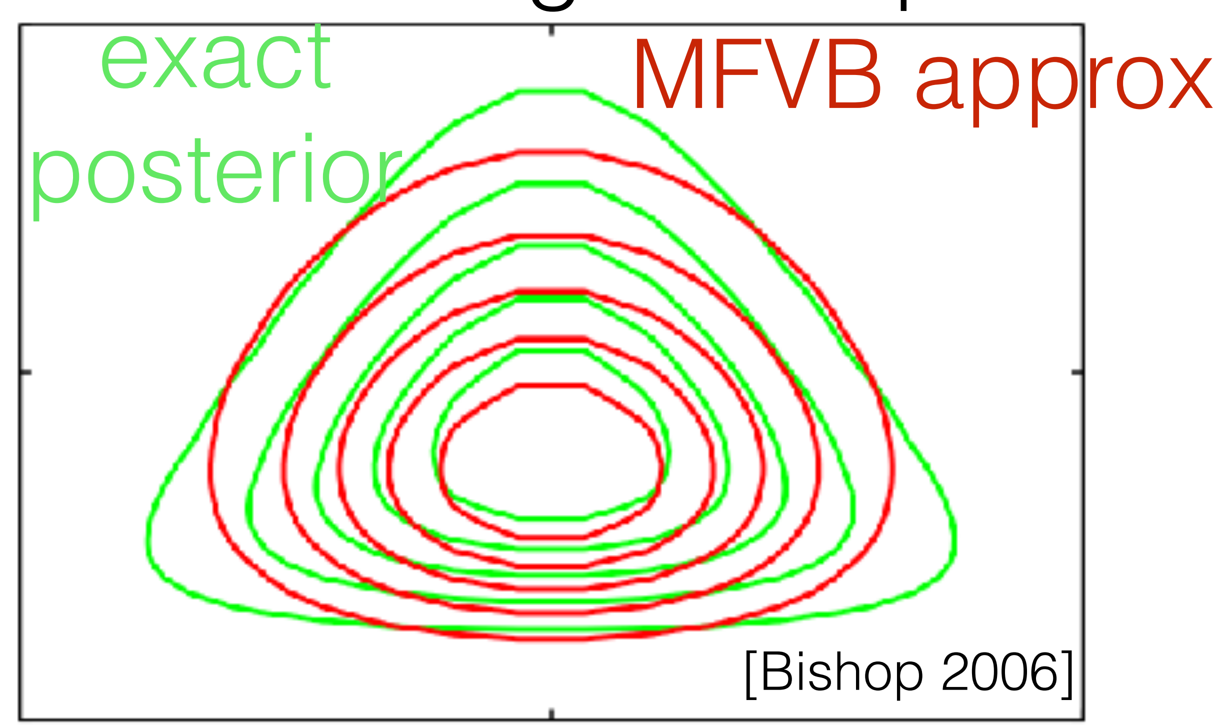

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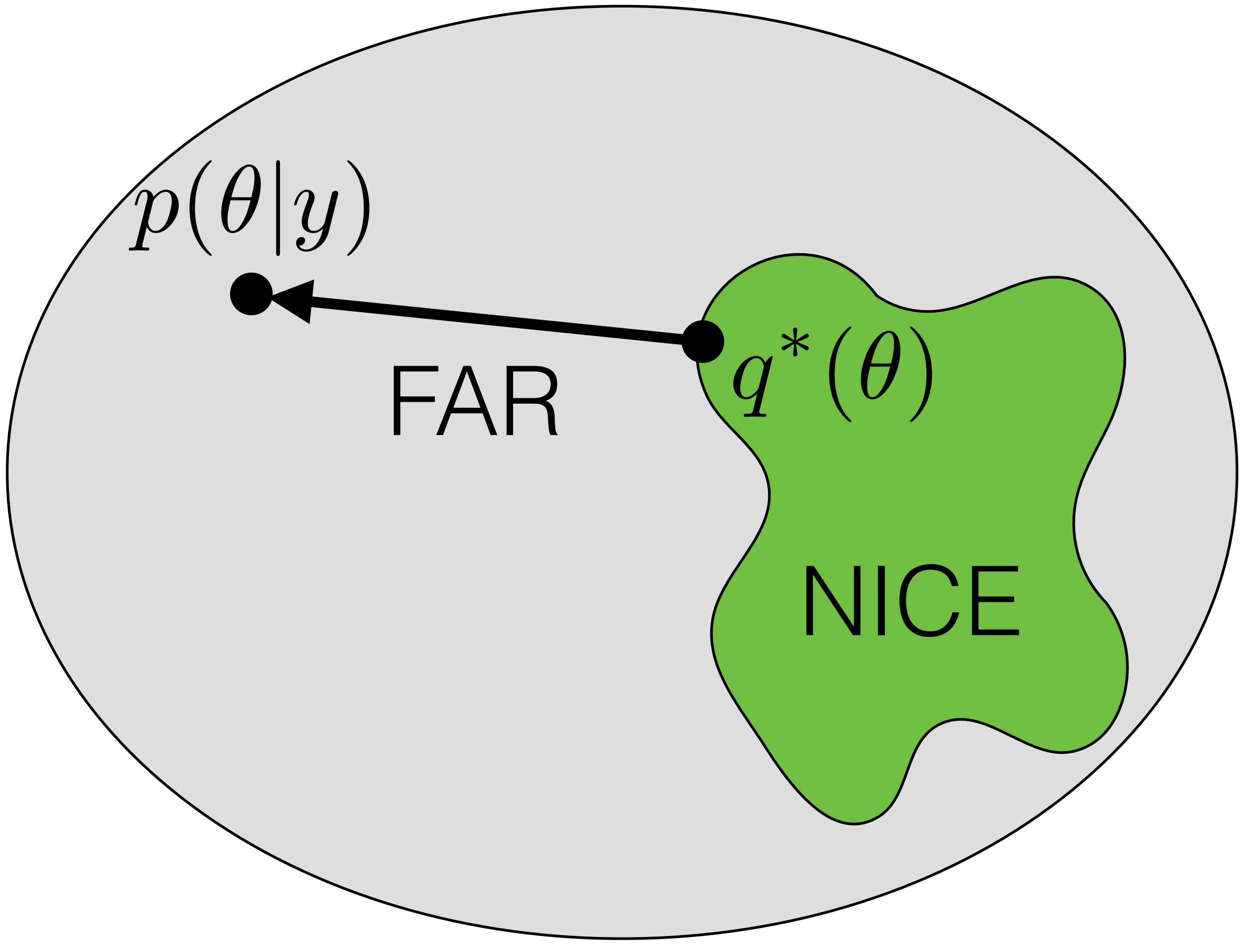

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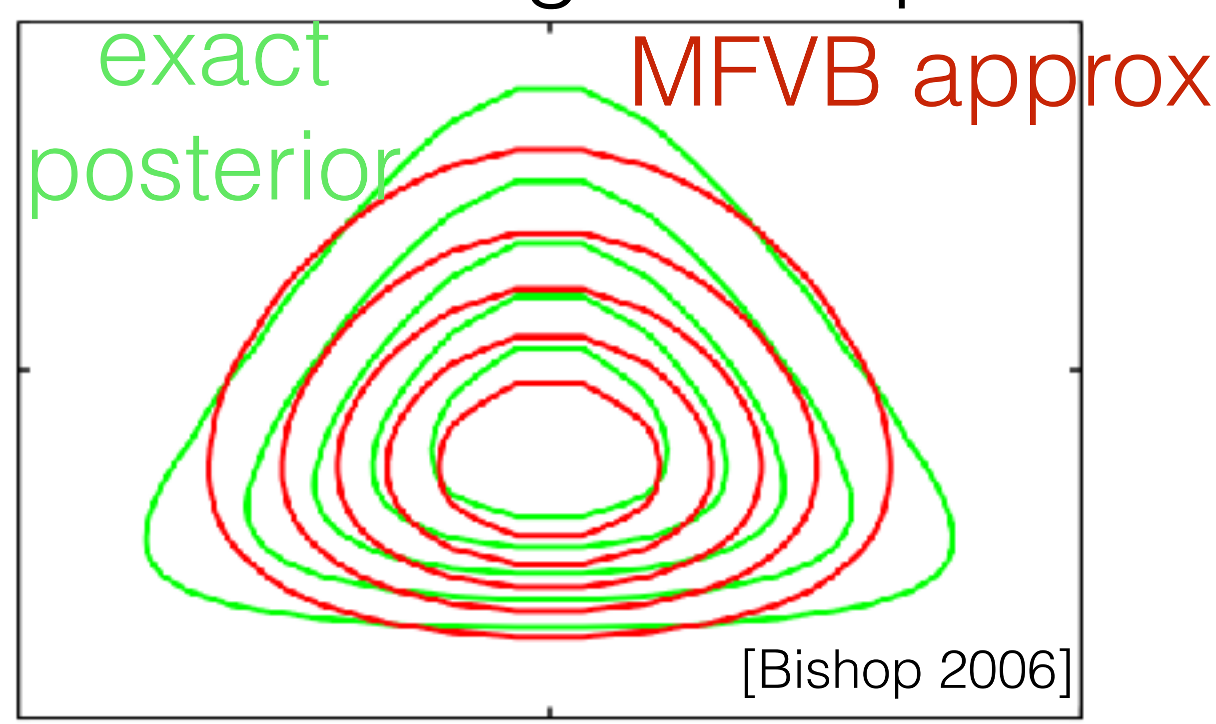

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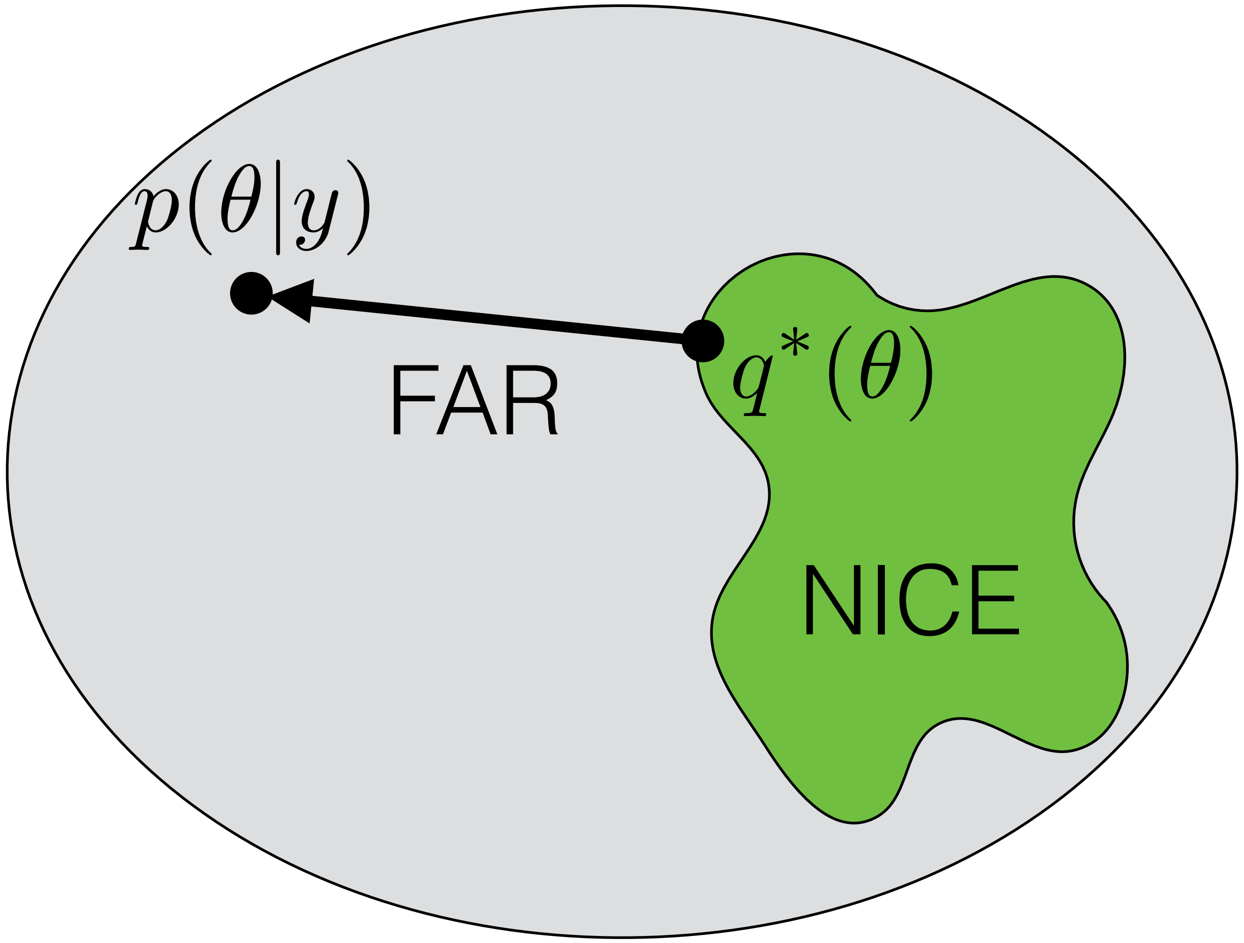

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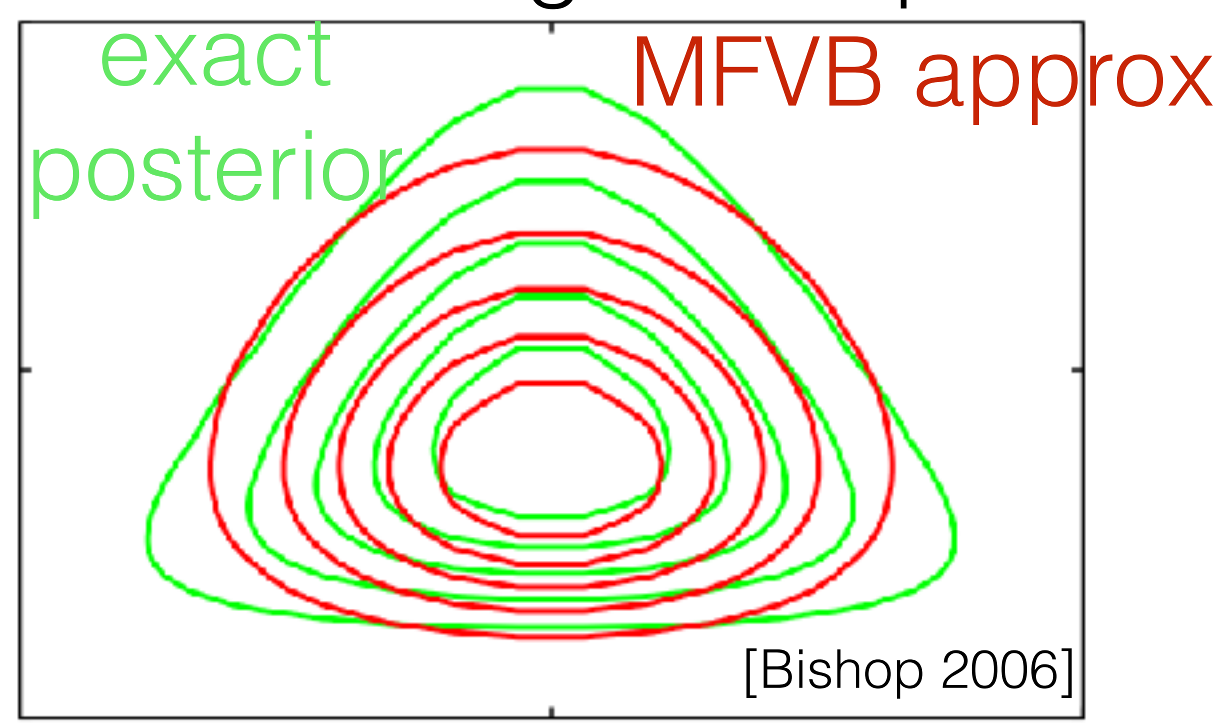

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- *One* option: Coordinate descent in $q_1, \ldots, q_J$

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References

Full references at end of final slides