Nonparametric Bayesian Methods: Models, Algorithms, and Applications (Part III)

Tamara Broderick
Associate Professor
MIT

http://www.tamarabroderick.com/tutorials.html

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

- Big questions
  - Why NPBayes?
  - What does an infinite/growing number of parameters really mean (in NPBayes)?
  - Why is NPBayes challenging but practical?

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

- Big questions
  - Why NPBayes?
  - What does an infinite/growing number of parameters really mean (in NPBayes)?
  - Why is NPBayes challenging but practical?

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

- Big questions
  - Why NPBayes?
  - What does an infinite/growing number of parameters really mean (in NPBayes)?
  - Why is NPBayes challenging but practical?

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

- Big questions
  - Why NPBayes? Learn more as acquire more data
  - What does an infinite/growing number of parameters really mean (in NPBayes)?
  - Why is NPBayes challenging but practical?

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

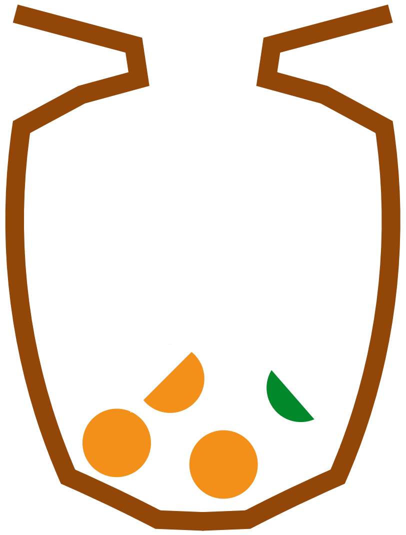
- Big questions
  - Why NPBayes? Learn more as acquire more data
  - What does an infinite/growing number of parameters really mean (in NPBayes)? Components vs. clusters; latent vs. realized
  - Why is NPBayes challenging but practical?

Roadmap

- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

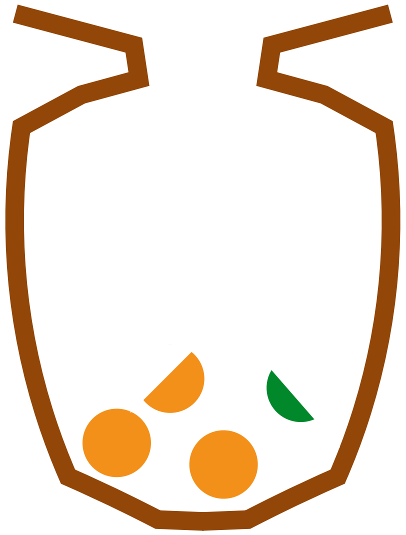
- Big questions
  - Why NPBayes? Learn more as acquire more data
  - What does an infinite/growing number of parameters really mean (in NPBayes)? Components vs. clusters; latent vs. realized
  - Why is NPBayes challenging but practical? Infinite dimensional parameter, but finitely many parameters realized

Marginal cluster assignments



Marginal cluster assignments

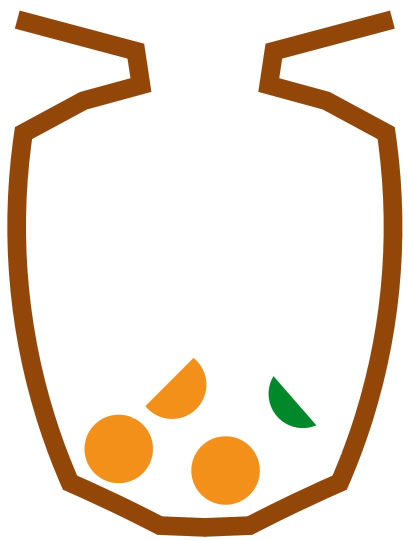
- Integrate out the frequencies



Marginal cluster assignments

- Integrate out the frequencies

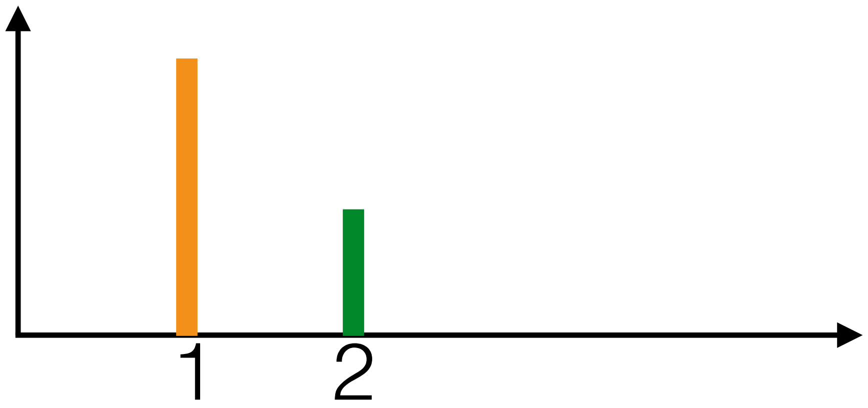
$$\rho_1 \sim \mathrm{Beta}(a_1, a_2), z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$$

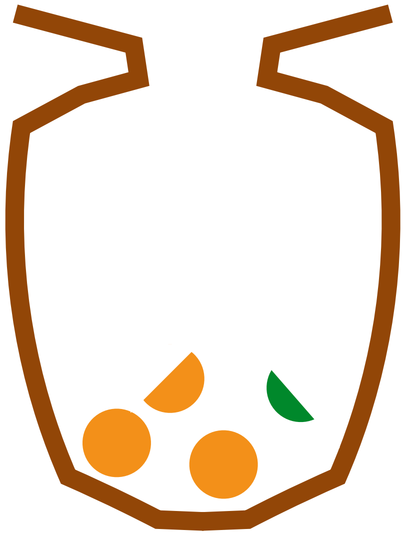


Marginal cluster assignments

- Integrate out the frequencies

$\rho_1 \sim \mathrm{Beta}(a_1, a_2), z_n \stackrel{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$

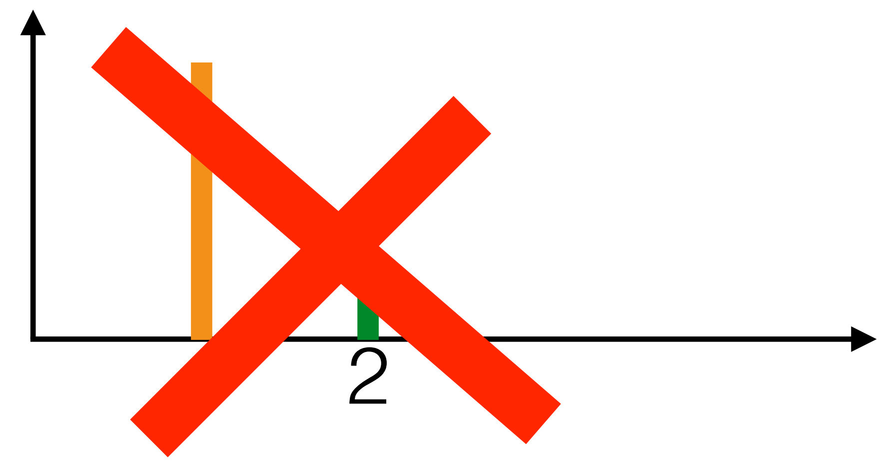


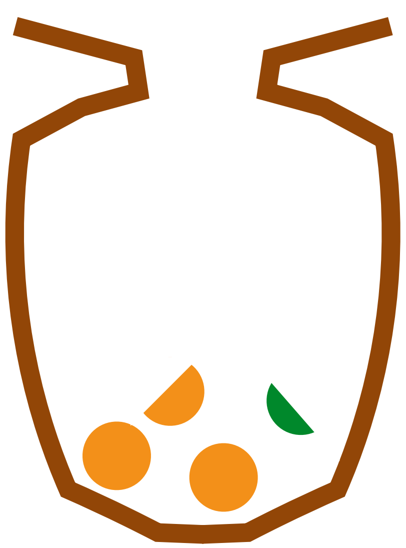


Marginal cluster assignments

- Integrate out the frequencies

$\rho_1 \sim \mathrm{Beta}(a_1, a_2), z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$



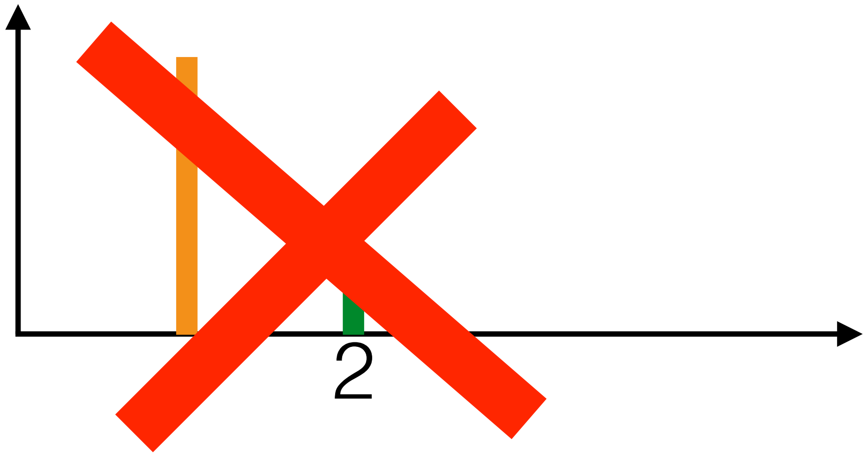


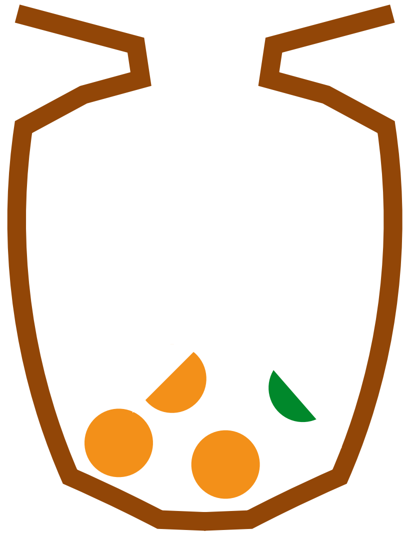
Marginal cluster assignments

- Integrate out the frequencies

$$\rho_1 \sim \mathrm{Beta}(a_1, a_2),\, z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$$

$$p(z_n = 1 | z_1, \ldots, z_{n-1}) = \frac{a_{1,n}}{a_{1,n} + a_{2,n}}$$



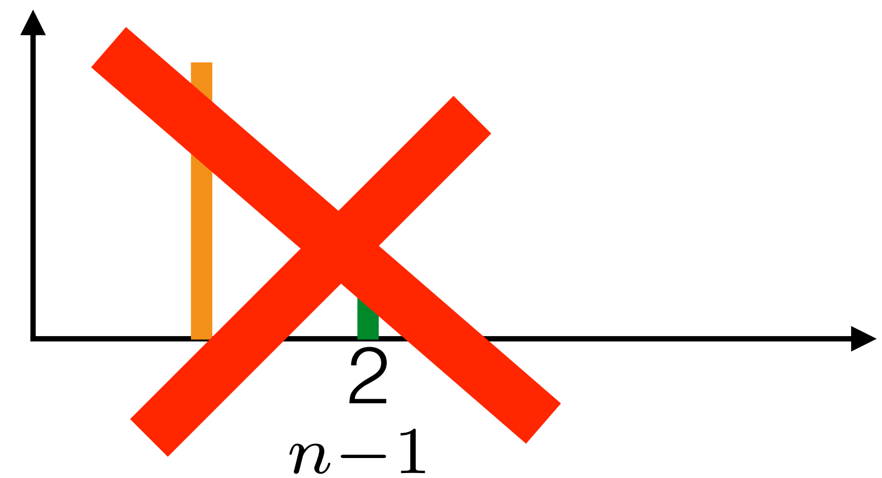


Marginal cluster assignments

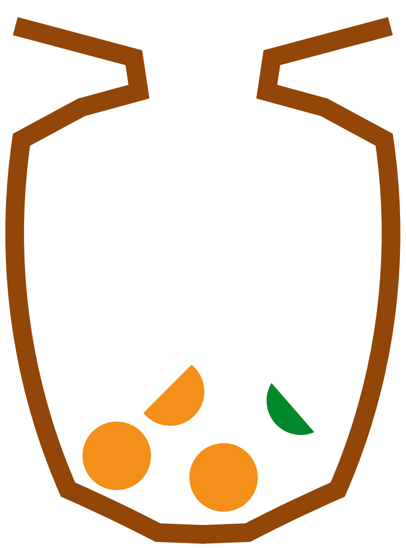
- Integrate out the frequencies

$$\rho_1 \sim \mathrm{Beta}(a_1, a_2),\, z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$$

$$p(z_n = 1 | z_1, \ldots, z_{n-1}) = \frac{a_{1,n}}{a_{1,n} + a_{2,n}}$$



$$a_{1,n} := a_1 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 1\},\, a_{2,n} = a_2 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 2\}$$

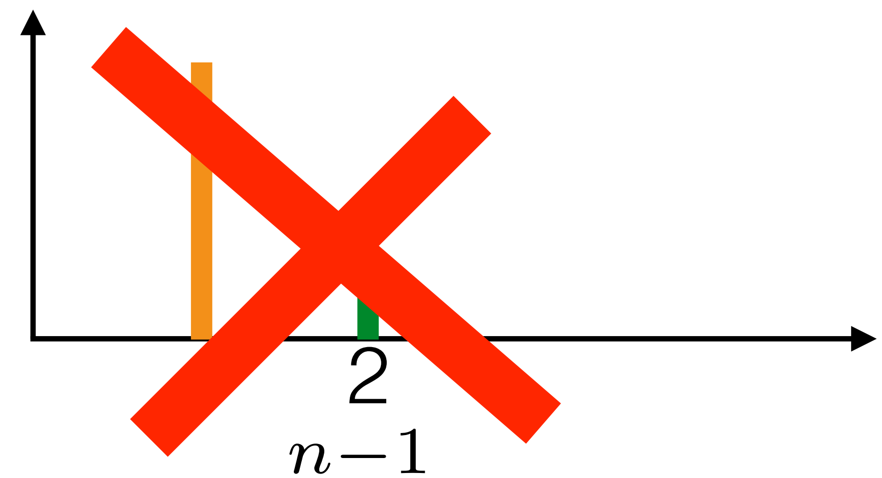


Marginal cluster assignments

- Integrate out the frequencies

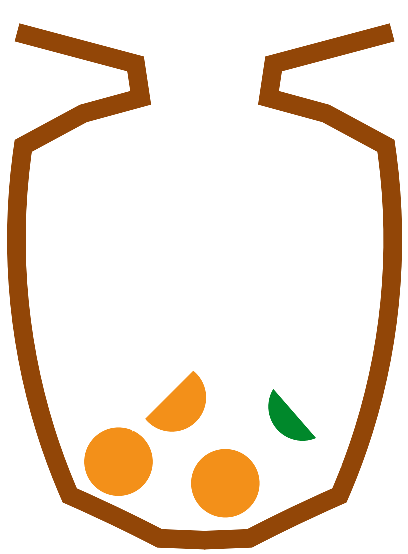
$$\rho_1 \sim \text{Beta}(a_1, a_2),\, z_n \overset{iid}{\sim} \text{Cat}(\rho_1, \rho_2)$$

$$p(z_n = 1 | z_1, \ldots, z_{n-1}) = \frac{a_{1,n}}{a_{1,n} + a_{2,n}}$$



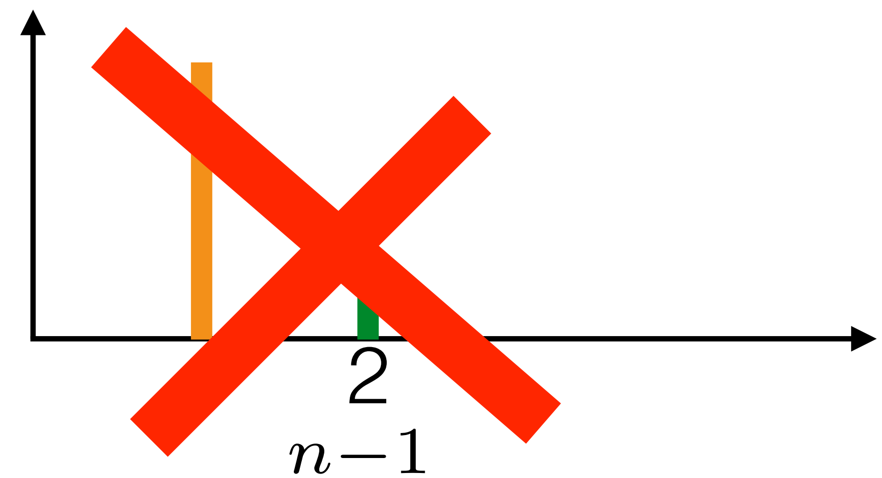
$$a_{1,n} := a_1 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 1\},\, a_{2,n} = a_2 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 2\}$$

- Pólya urn
  - Choose any ball with prob proportional to its mass
  - Replace and add ball of same color



Marginal cluster assignments

- Integrate out the frequencies

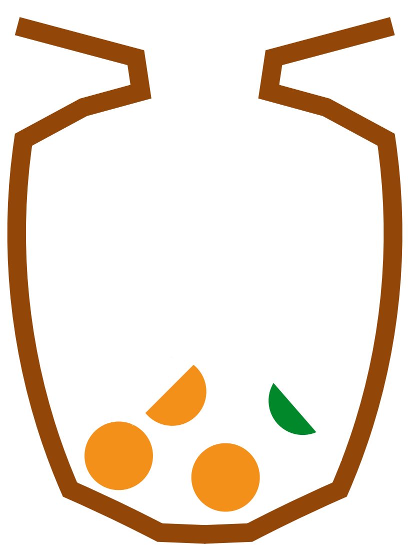


$$\rho_1 \sim \mathrm{Beta}(a_1, a_2),\ z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$$

$$p(z_n = 1 | z_1, \ldots, z_{n-1}) = \frac{a_{1,n}}{a_{1,n} + a_{2,n}}$$

$$a_{1,n} := a_1 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 1\},\ a_{2,n} = a_2 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 2\}$$

- Pólya urn
  - Choose any ball with prob proportional to its mass
  - Replace and add ball of same color



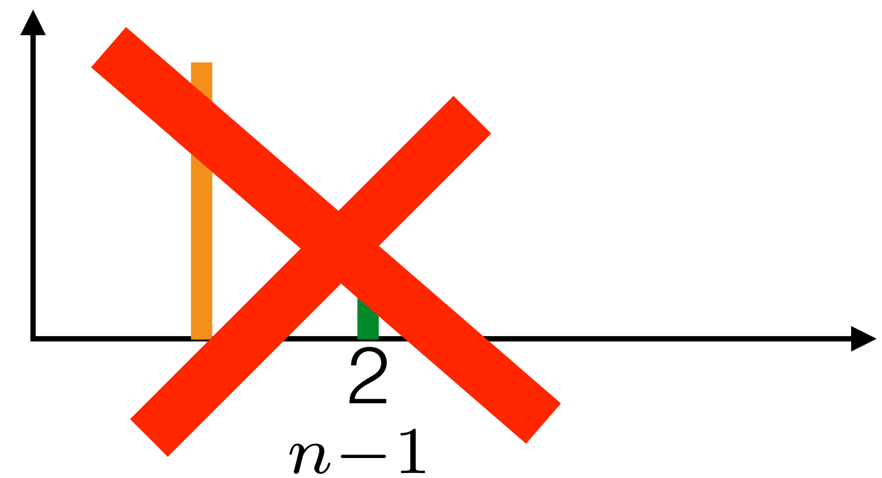
$$\lim_{n \to \infty} \frac{\#\ \text{orange}}{\#\ \text{total}} = \rho_{\text{orange}} \overset{d}{=} \mathrm{Beta}(a_{\text{orange}}, a_{\text{green}})$$

Marginal cluster assignments

- Integrate out the frequencies

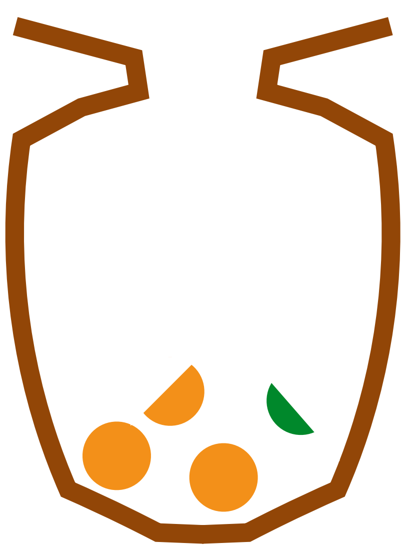
$$\rho_1 \sim \mathrm{Beta}(a_1, a_2),\, z_n \overset{iid}{\sim} \mathrm{Cat}(\rho_1, \rho_2)$$

$$p(z_n = 1 | z_1, \ldots, z_{n-1}) = \frac{a_{1,n}}{a_{1,n} + a_{2,n}}$$



$$a_{1,n} := a_1 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 1\}, a_{2,n} = a_2 + \sum_{m=1}^{n-1} \mathbf{1}\{z_m = 2\}$$

- Pólya urn
  - Choose any ball with prob proportional to its mass
  - Replace and add ball of same color



$$\lim_{n \to \infty} \frac{\#\ \text{orange}}{\#\ \text{total}} = \rho_{\text{orange}} \overset{d}{=} \mathrm{Beta}(a_{\text{orange}}, a_{\text{green}})$$

$$\mathrm{PolyaUrn}(a_{\text{orange}}, a_{\text{green}})$$

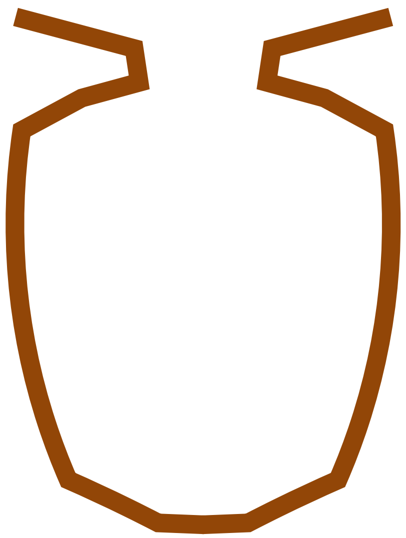
Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn

[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

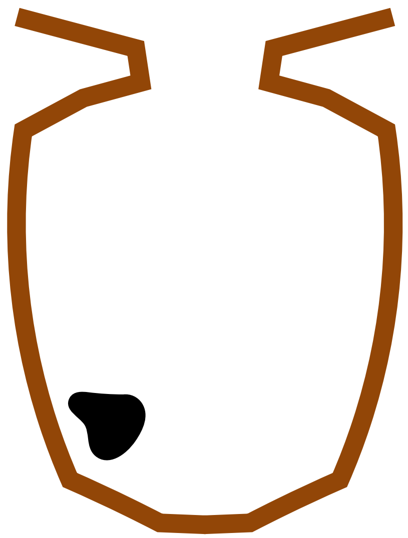
- Hoppe urn / Blackwell-MacQueen urn



[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

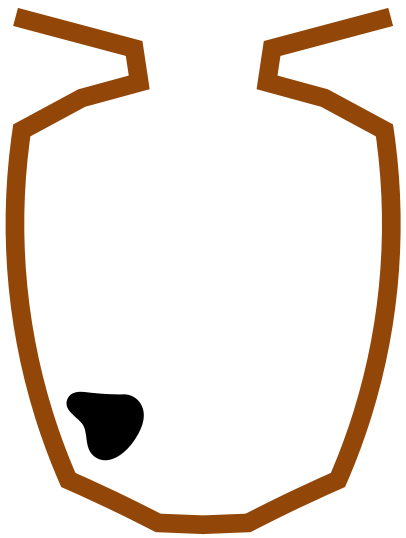
- Hoppe urn / Blackwell-MacQueen urn



[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn

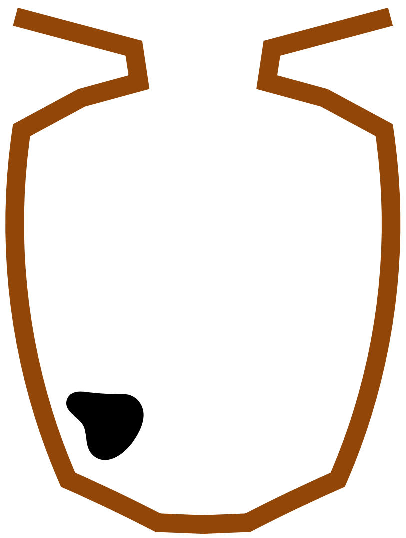


- Choose ball with prob proportional to its mass

[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn

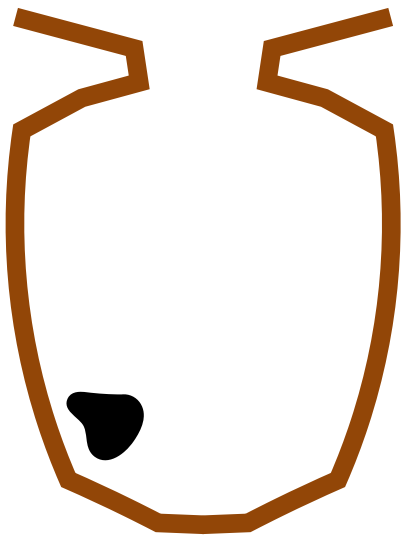


- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color

[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn

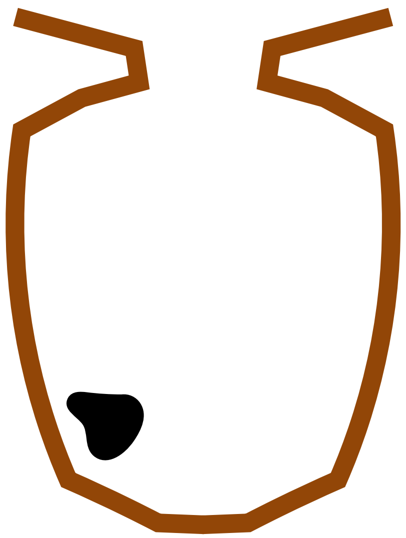


  - Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color

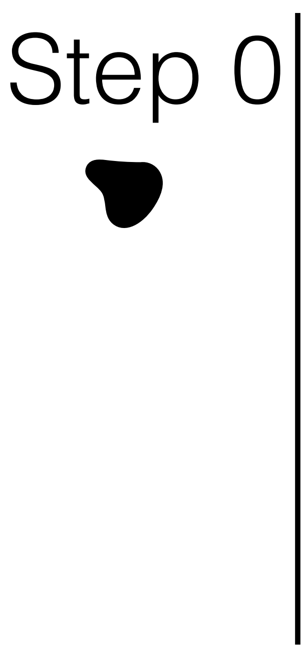
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



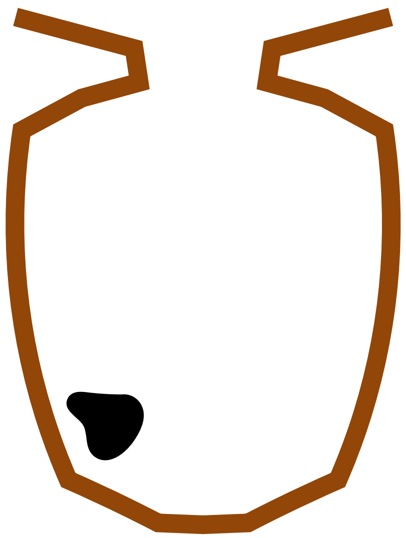
- Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color



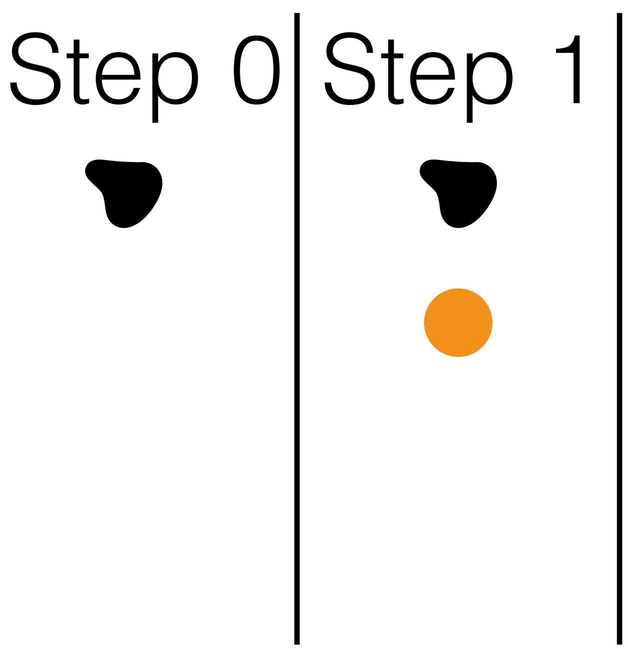
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



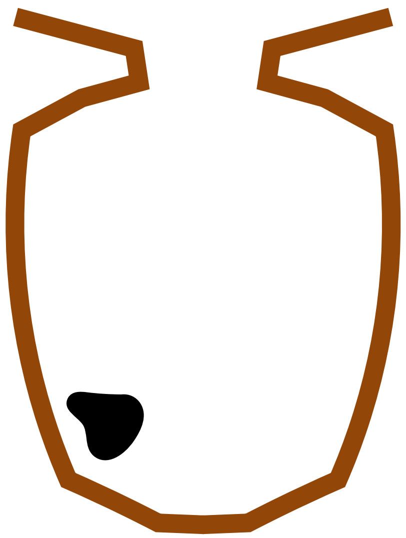
  - Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color



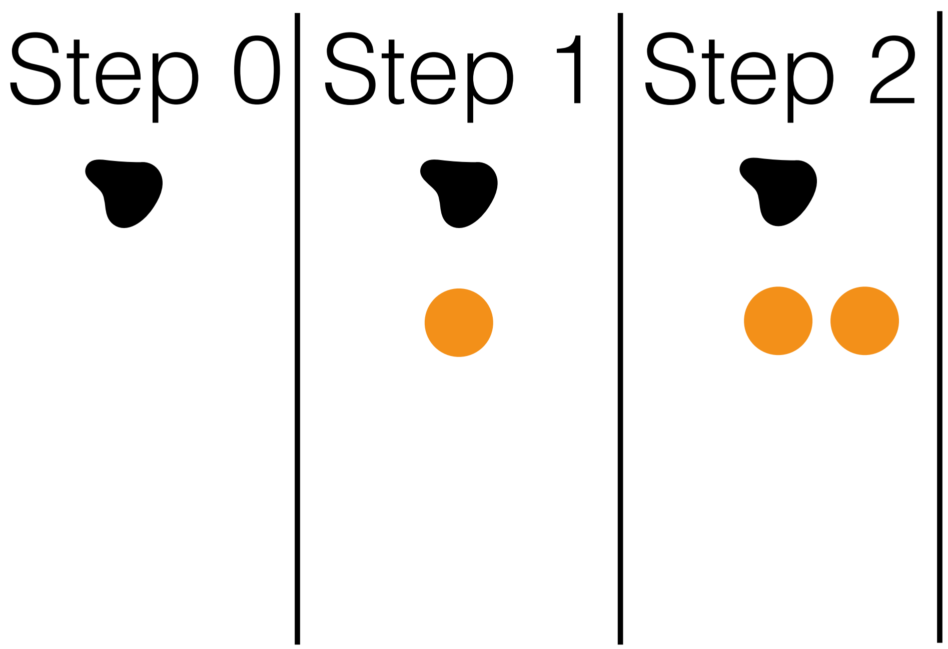
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



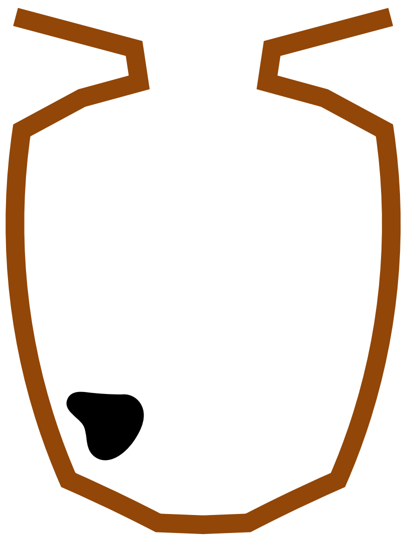
- Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color



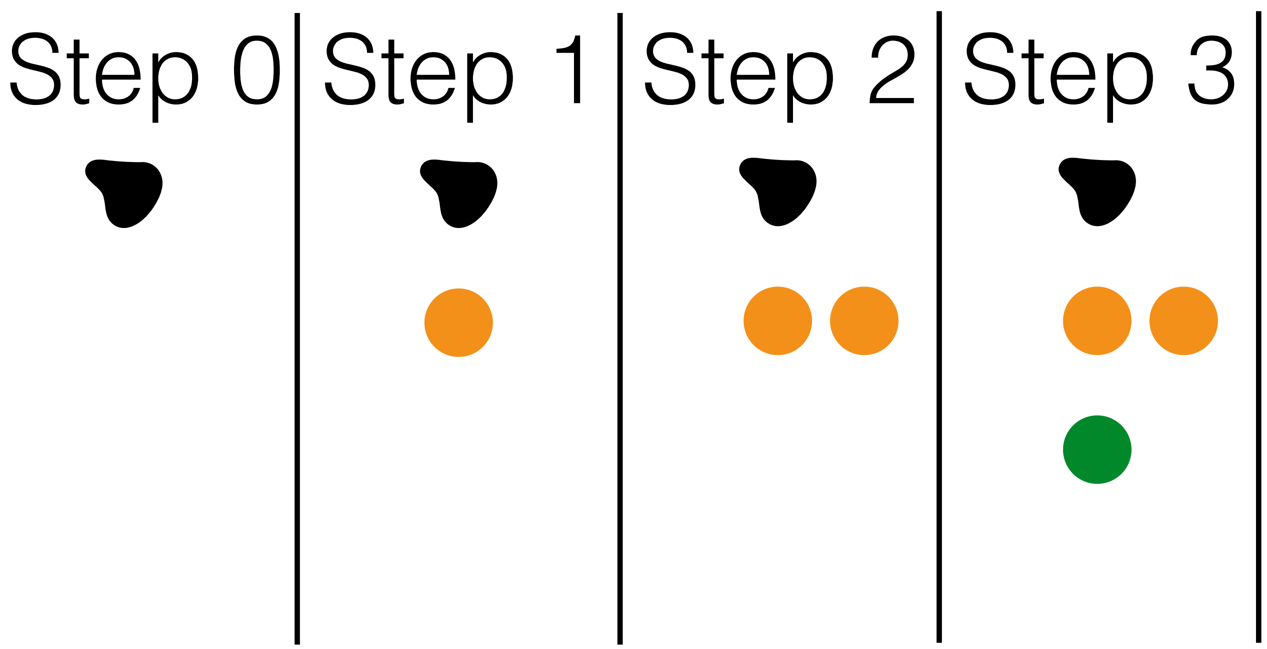
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



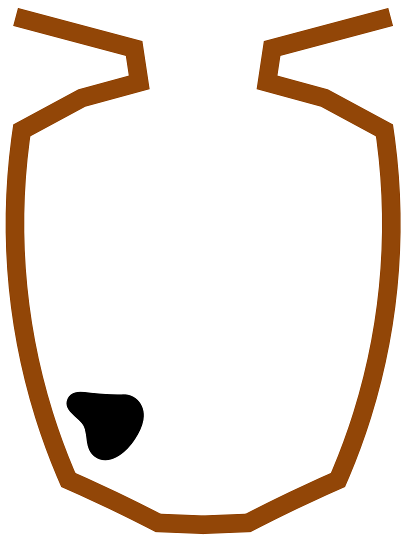
- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



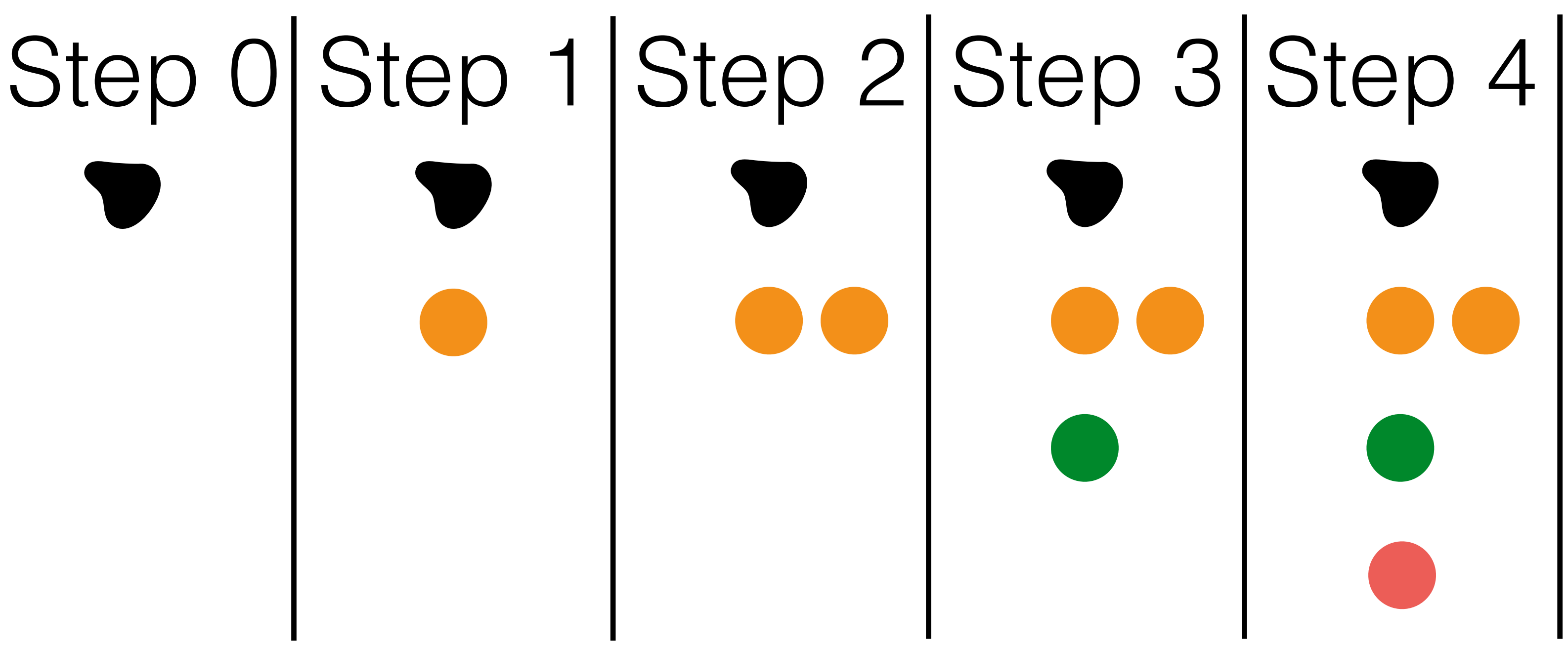
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



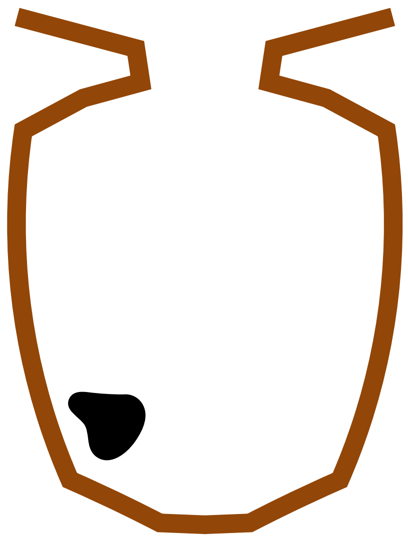
- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



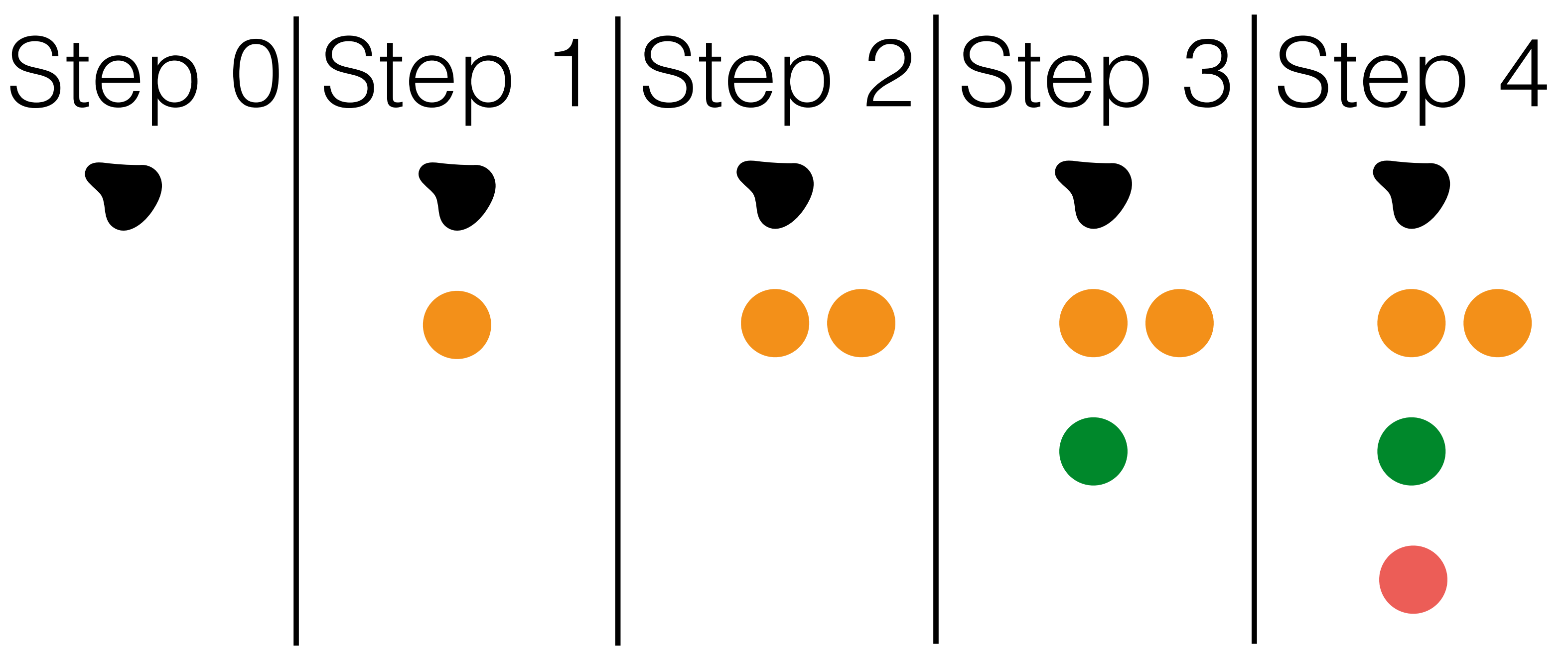
[Blackwell, MacQueen 1973; Hoppe 1984]

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



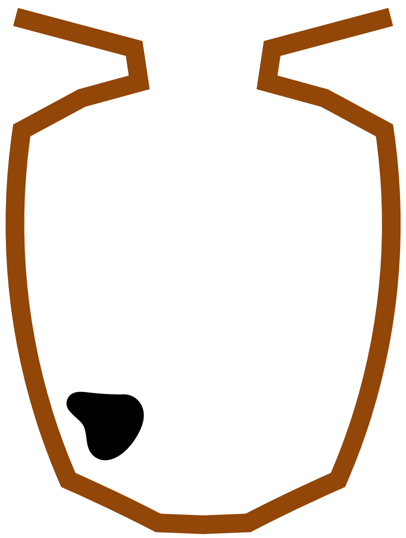
- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



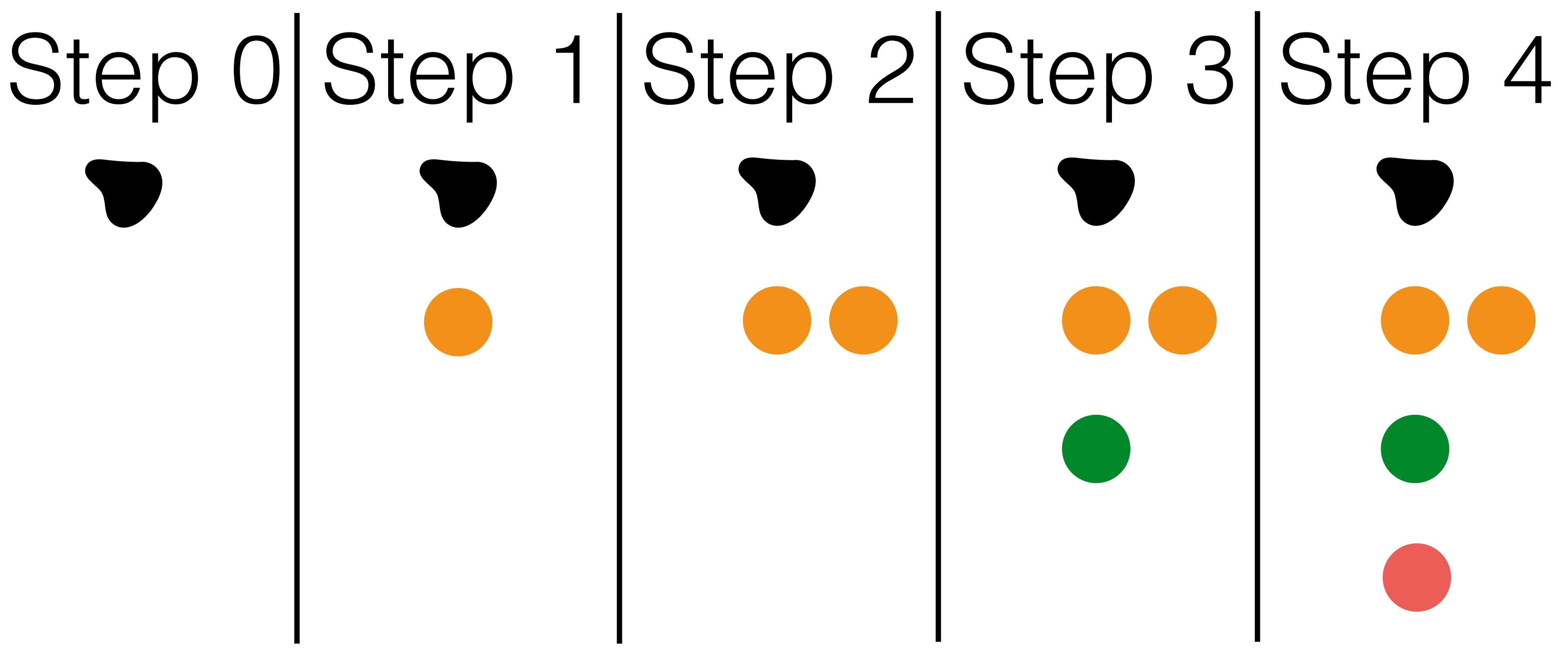
$(\#\text{orange}, \#\text{other})$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



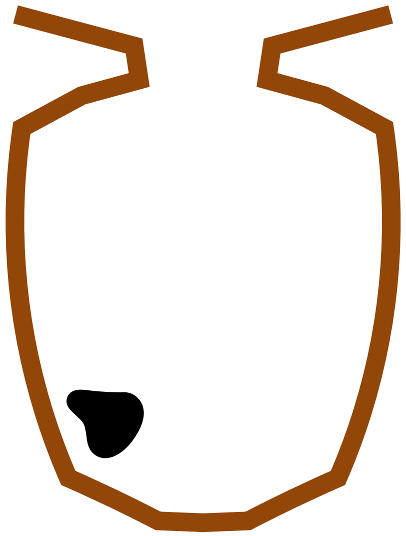
  - Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color



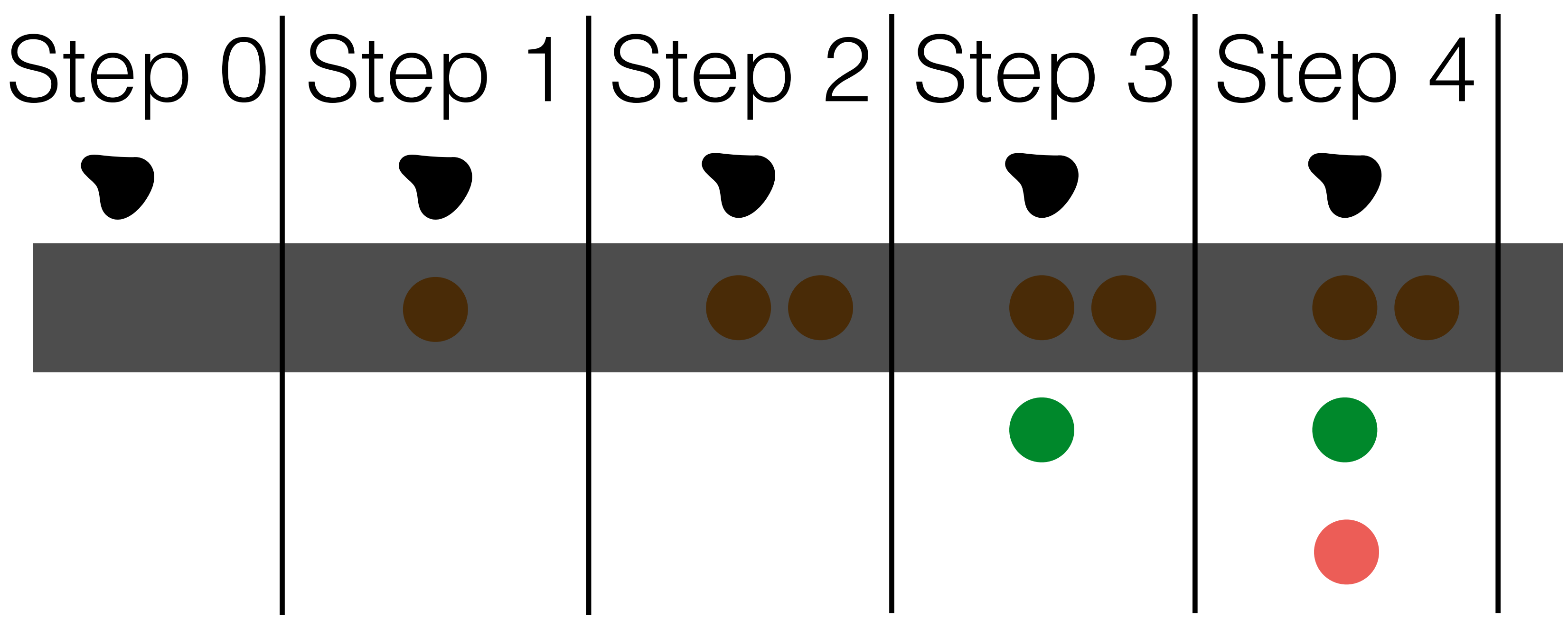
$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



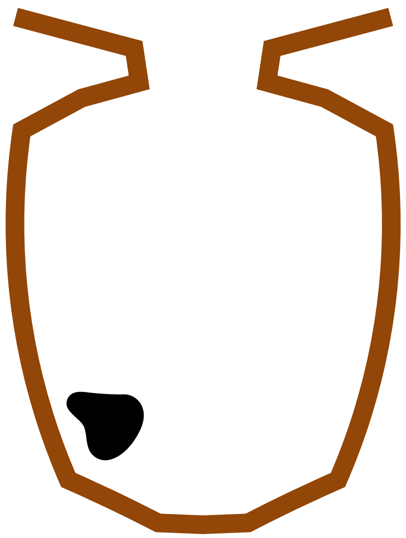
- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



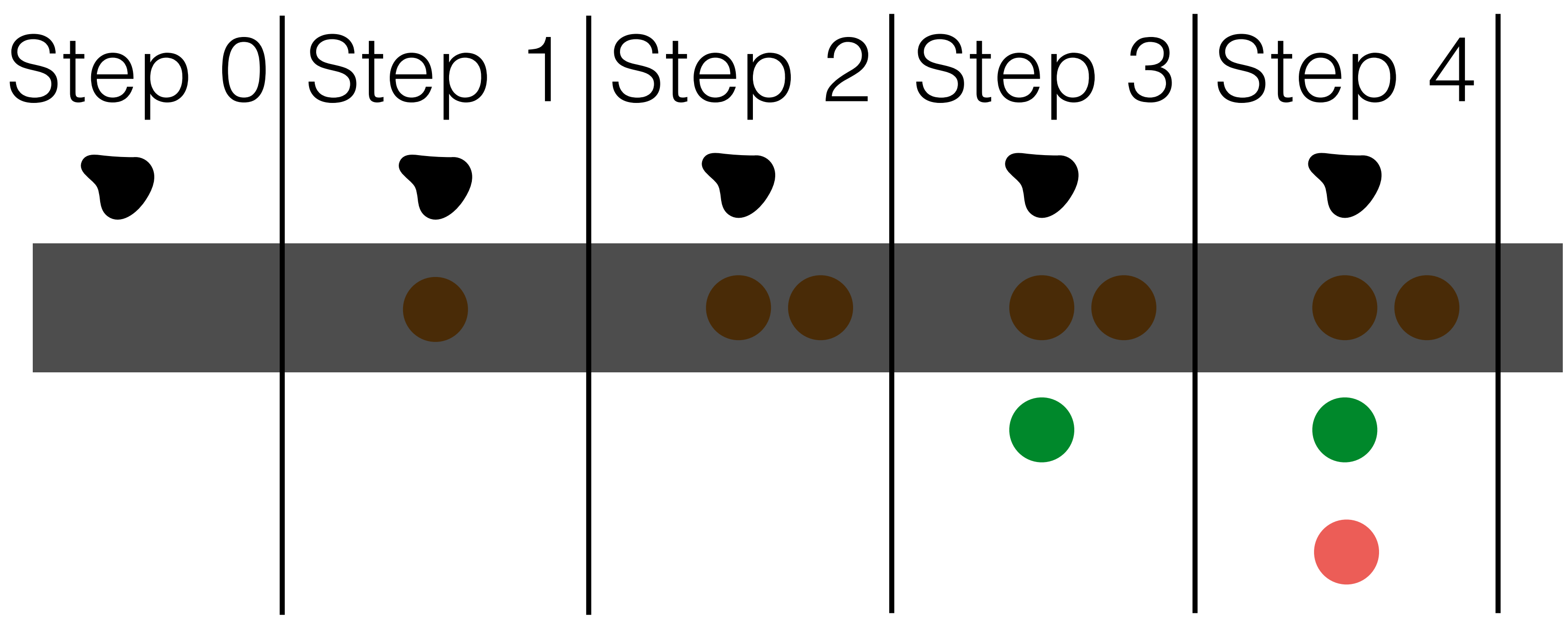
$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color

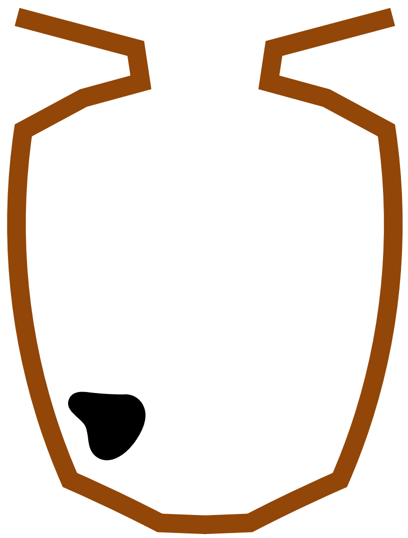


$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

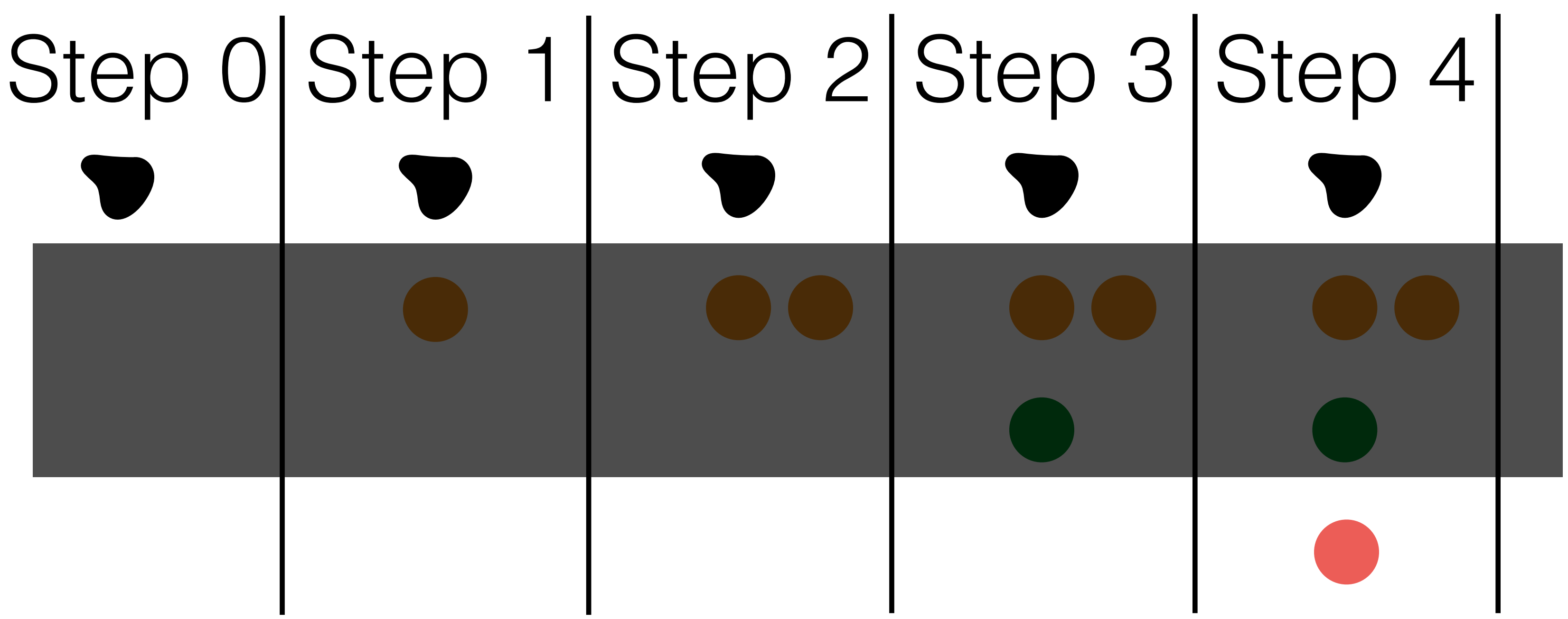
- not orange: $(\#\text{green}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color

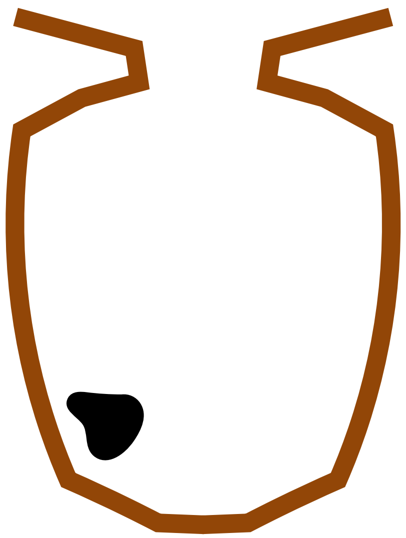


$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

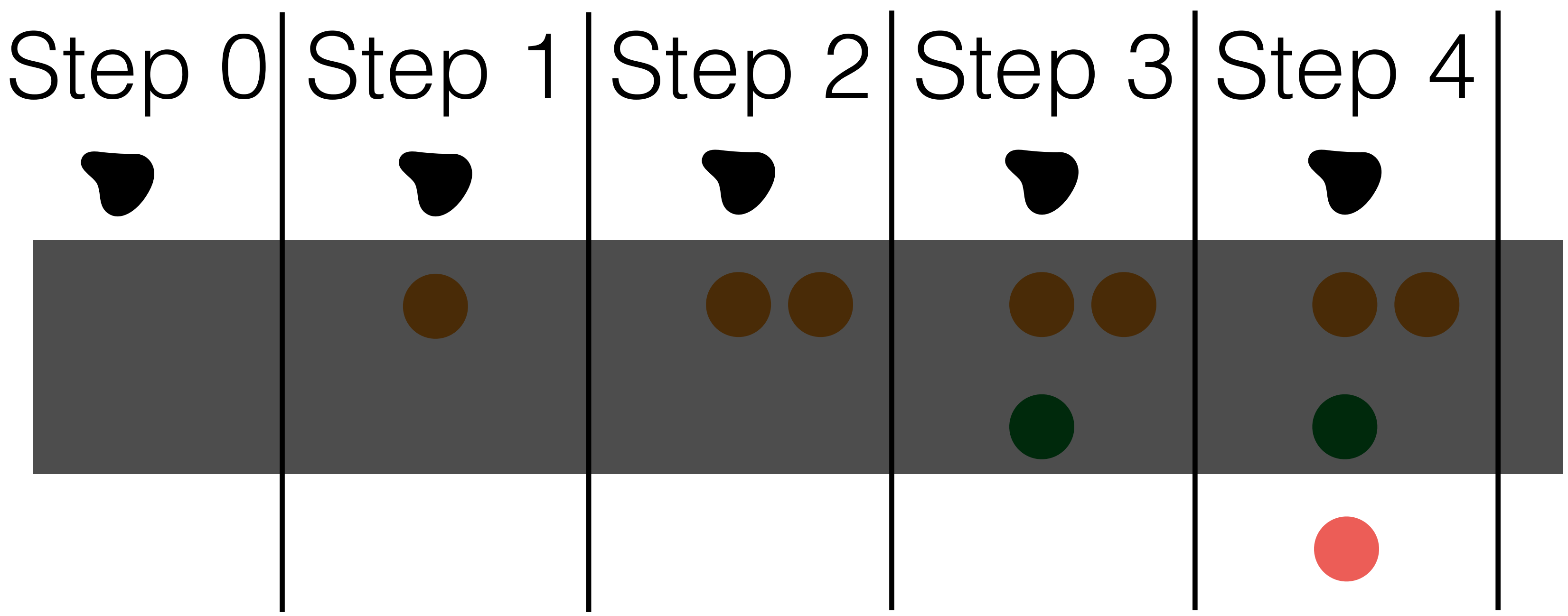
- not orange: $(\#\text{green}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color

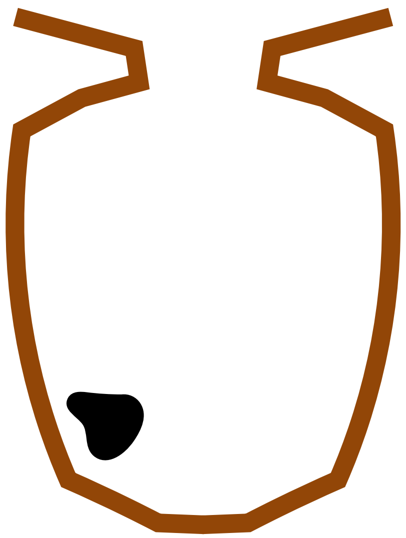


$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

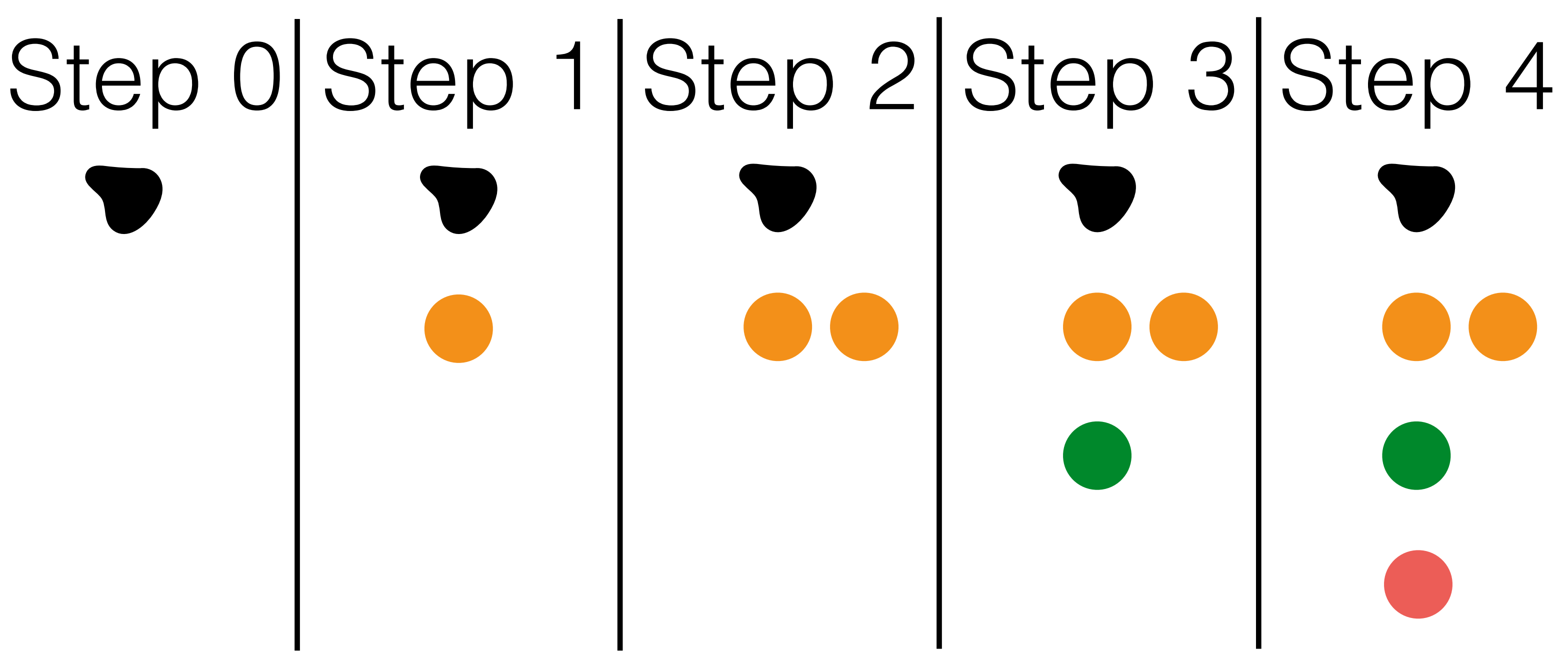
- not orange: $(\#\text{green}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\text{red}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color

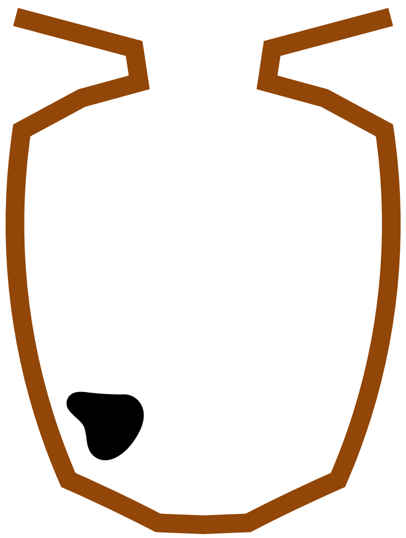


$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

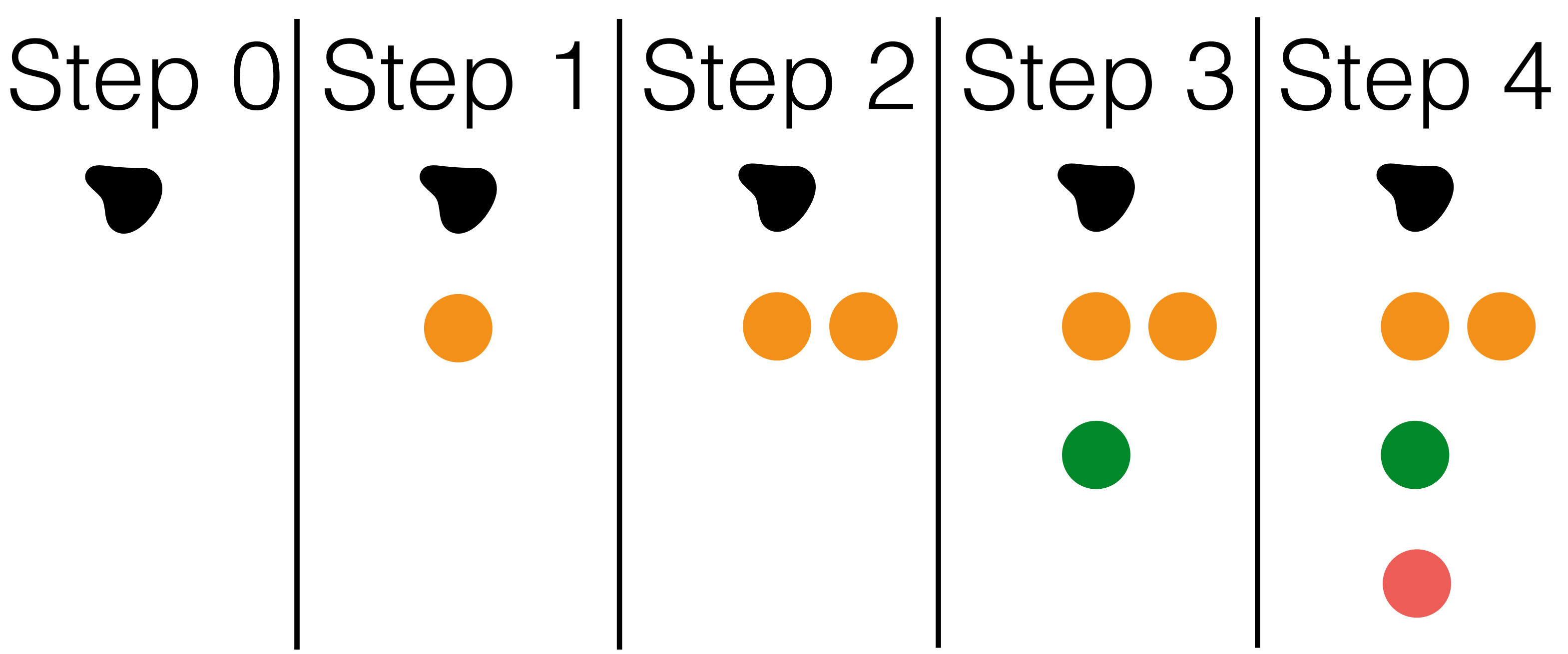
- not orange: $(\#\text{green}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\text{red}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



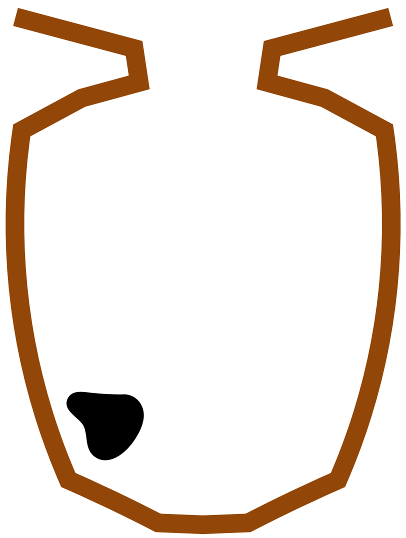
$$V_k \stackrel{iid}{\sim} \mathrm{Beta}(1, \alpha)$$

$$(\#\text{orange}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$$

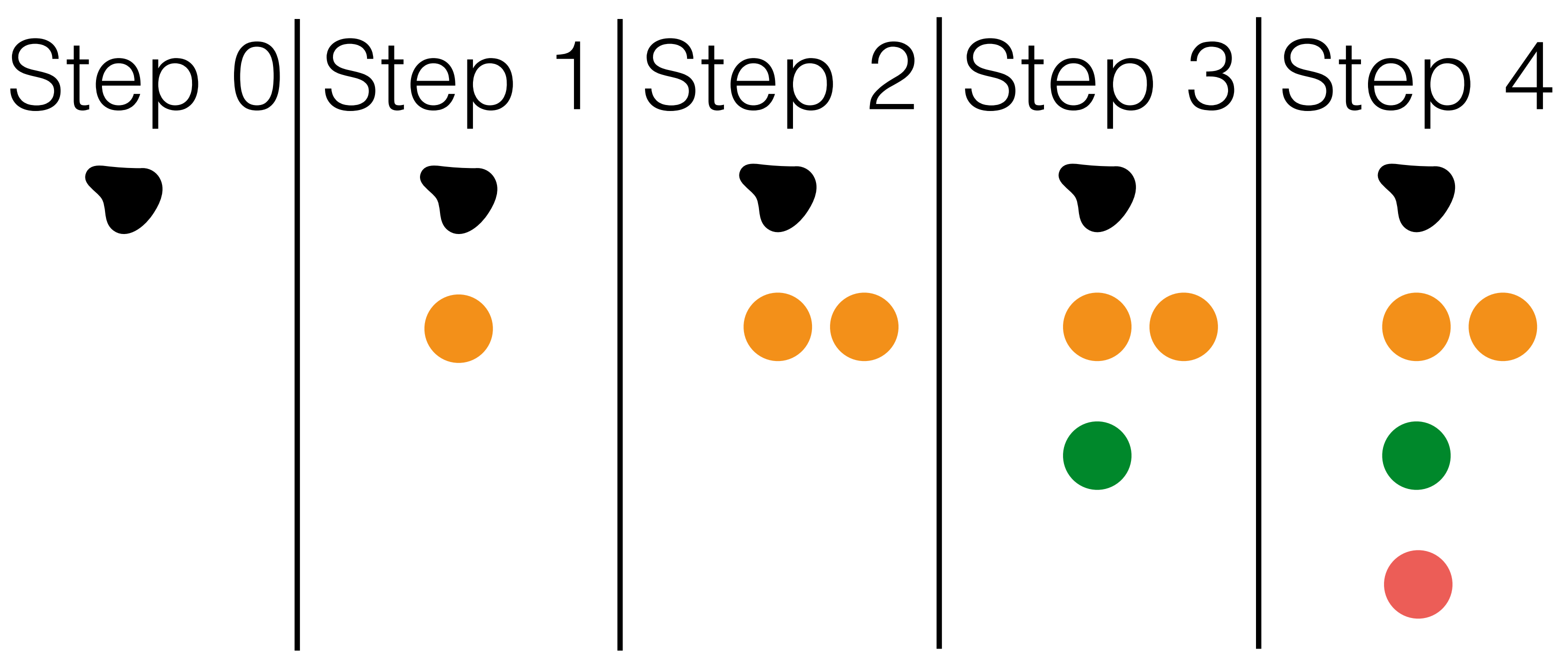
- not orange: $(\#\text{green}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\text{red}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



$$V_k \overset{iid}{\sim} \mathrm{Beta}(1, \alpha)$$

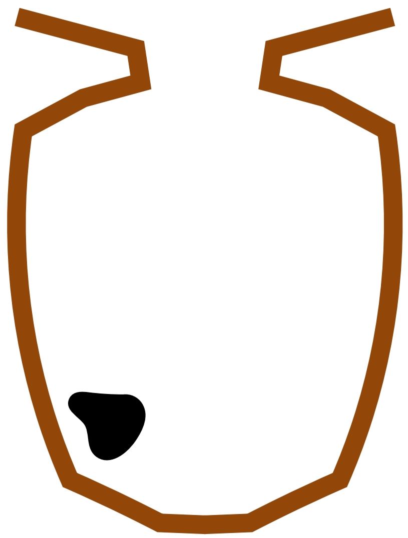
$$\rho_1 = V_1$$

$$(\#\mathrm{orange}, \#\mathrm{other}) = \mathrm{PolyaUrn}(1, \alpha)$$

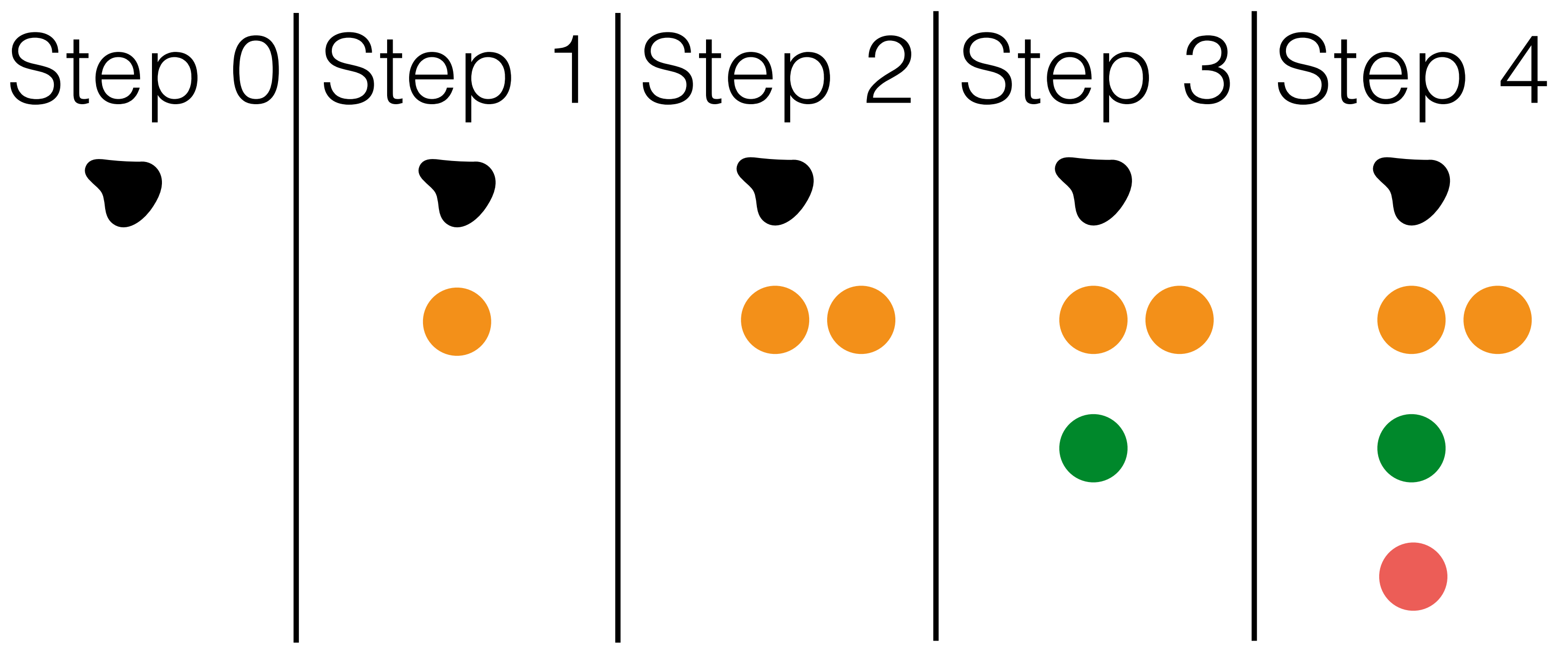
- not orange: $(\#\mathrm{green}, \#\mathrm{other}) = \mathrm{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\mathrm{red}, \#\mathrm{other}) = \mathrm{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
  - If black, replace and add ball of new color
  - Else, replace and add ball of same color



$$V_k \overset{iid}{\sim} \text{Beta}(1, \alpha)$$

$$\rho_1 = V_1$$

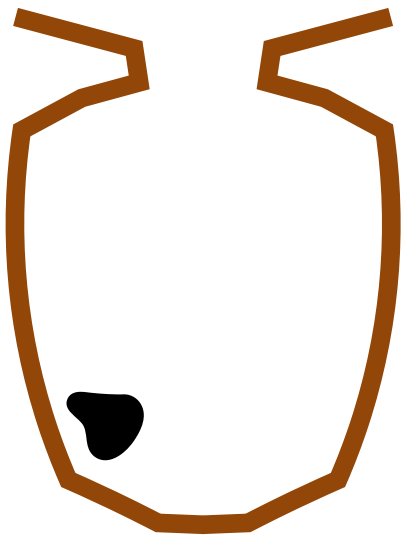
$$\rho_2 = (1 - V_1)V_2$$

$$(\#\text{orange}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$$

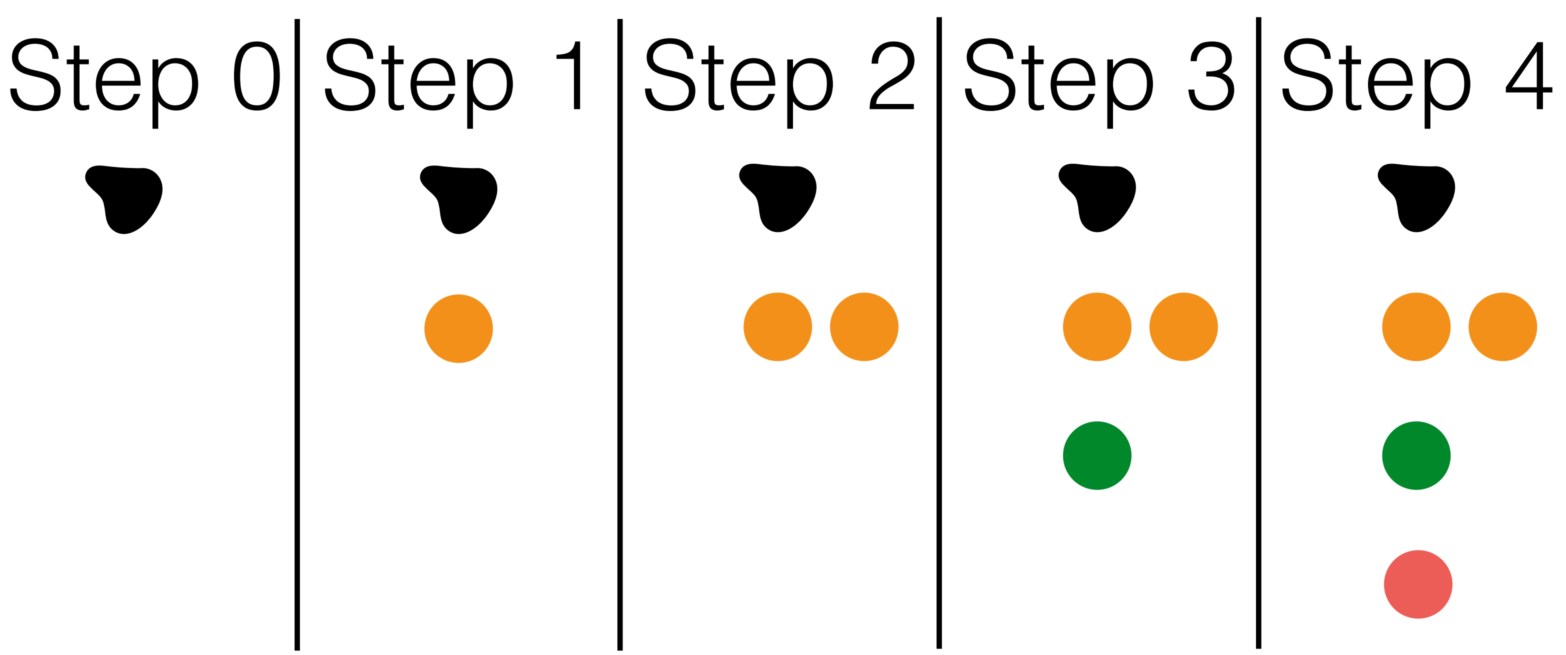
- not orange: $(\#\text{green}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\text{red}, \#\text{other}) = \text{PolyaUrn}(1, \alpha)$

Marginal cluster assignments

- Hoppe urn / Blackwell-MacQueen urn



- Choose ball with prob proportional to its mass
    - If black, replace and add ball of new color
    - Else, replace and add ball of same color



$$V_k \overset{iid}{\sim} \mathrm{Beta}(1, \alpha)$$

$$\rho_1 = V_1$$

$$\rho_2 = (1 - V_1)V_2$$

$$\rho_3 = [\prod_{k=1}^{2}(1 - V_k)]V_3$$

$$(\#\text{orange}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$$

- not orange: $(\#\text{green}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$
- not orange, green: $(\#\text{red}, \#\text{other}) = \mathrm{PolyaUrn}(1, \alpha)$

Chinese restaurant process

Chinese restaurant process

- Same thing we just did

Chinese restaurant process

- Same thing we just did
- Each customer walks into the restaurant

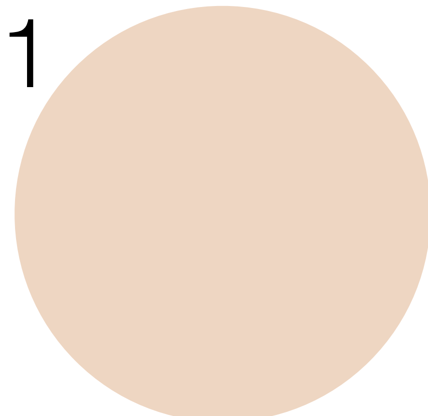
Chinese restaurant process

- Same thing we just did
- Each customer walks into the restaurant
    - Sits at existing table with prob proportional to # people there

Chinese restaurant process

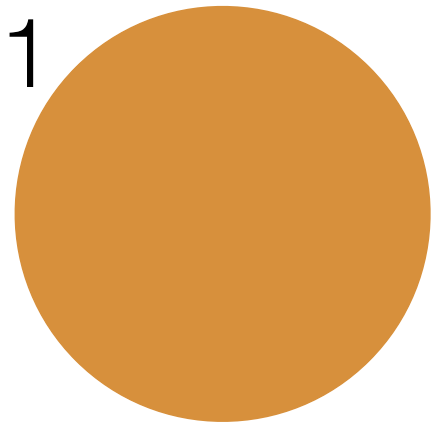
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



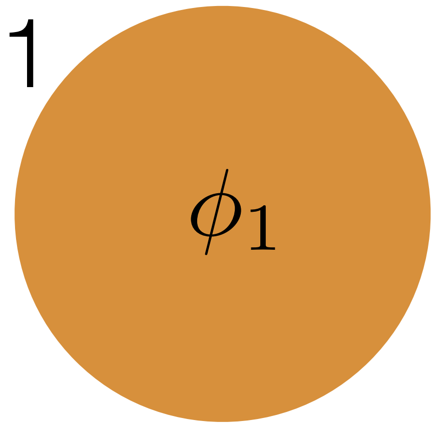
- Same thing we just did
- Each customer walks into the restaurant
    - Sits at existing table with prob proportional to # people there
    - Forms new table with prob proportional to α

Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process

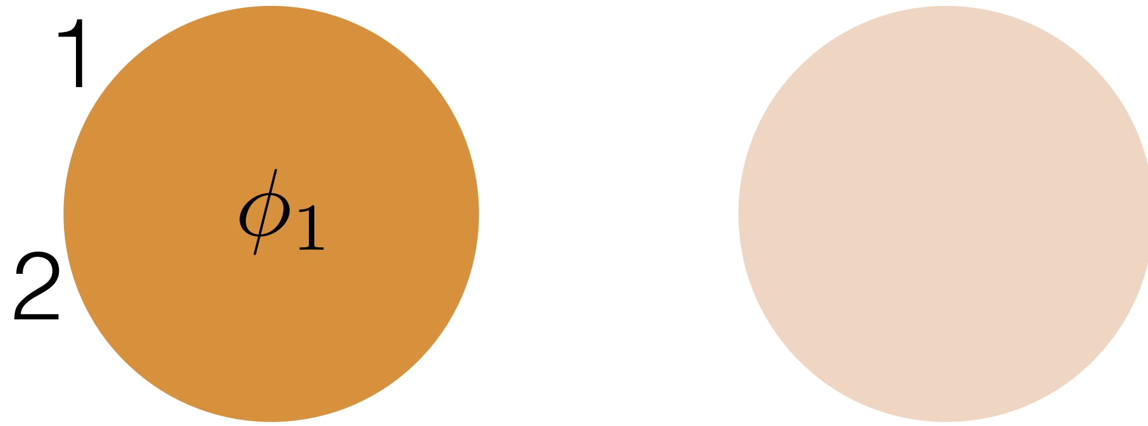


- Same thing we just did
- Each customer walks into the restaurant
    - Sits at existing table with prob proportional to # people there
    - Forms new table with prob proportional to α

Chinese restaurant process

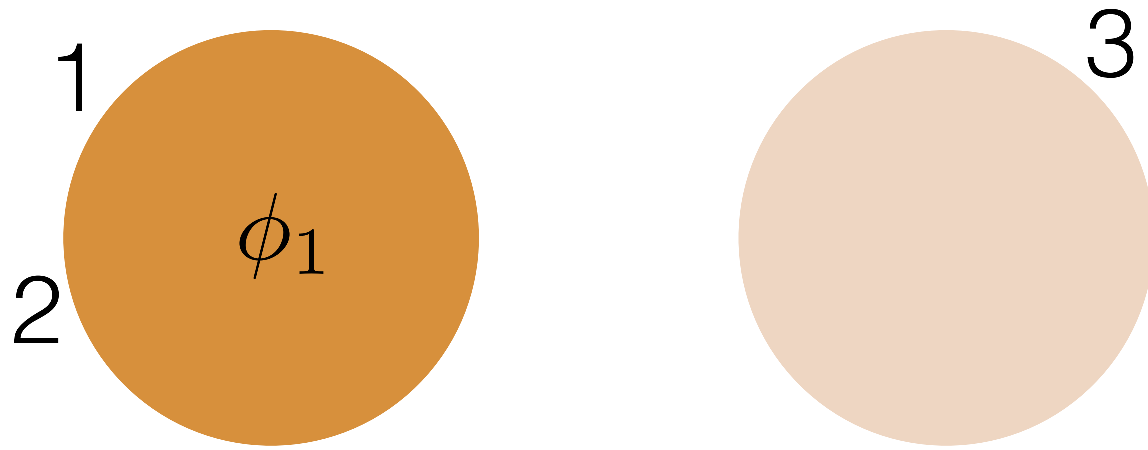
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
    - Sits at existing table with prob proportional to # people there
    - Forms new table with prob proportional to α

Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

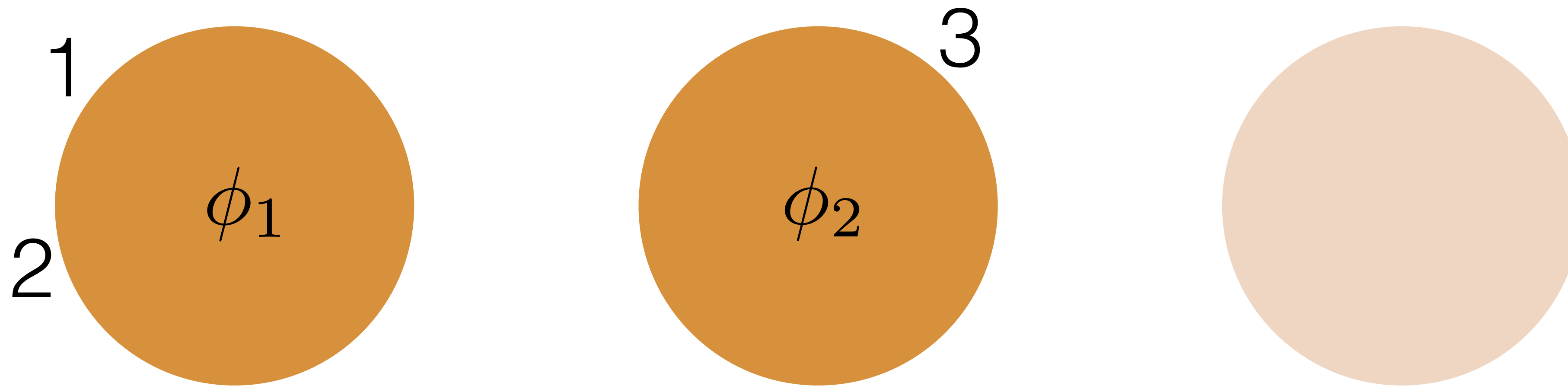
Chinese restaurant process

- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process

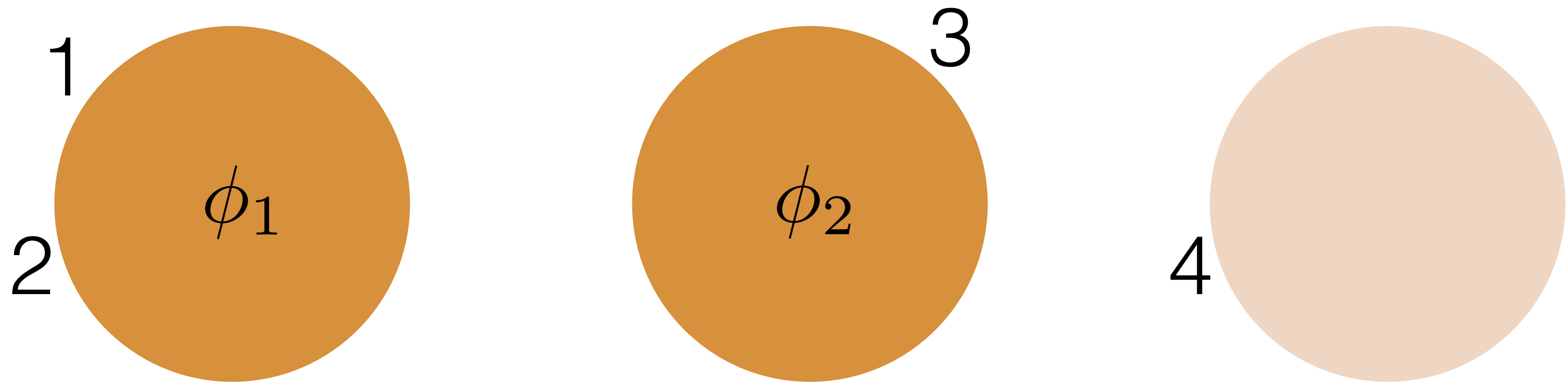
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



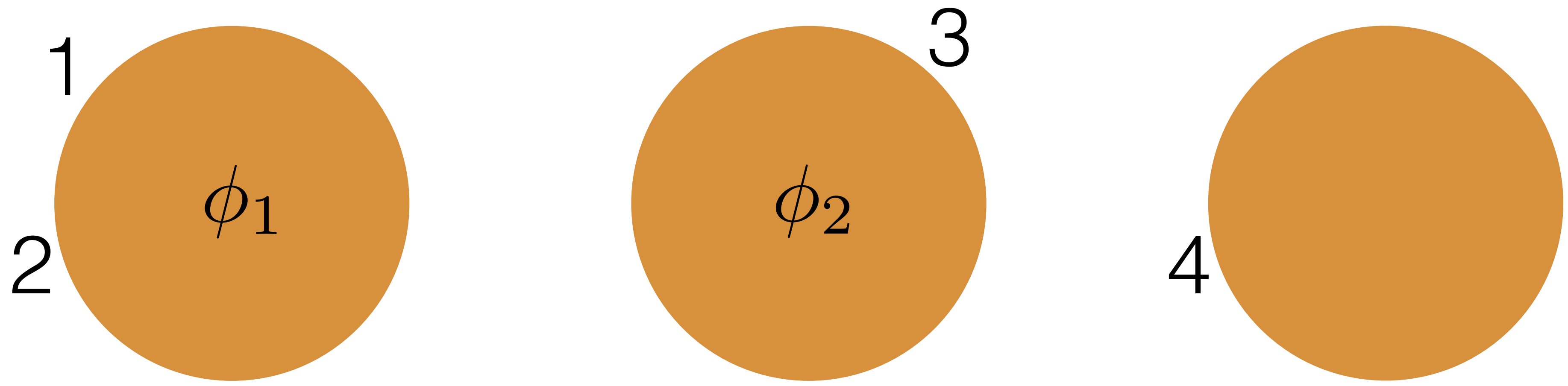
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



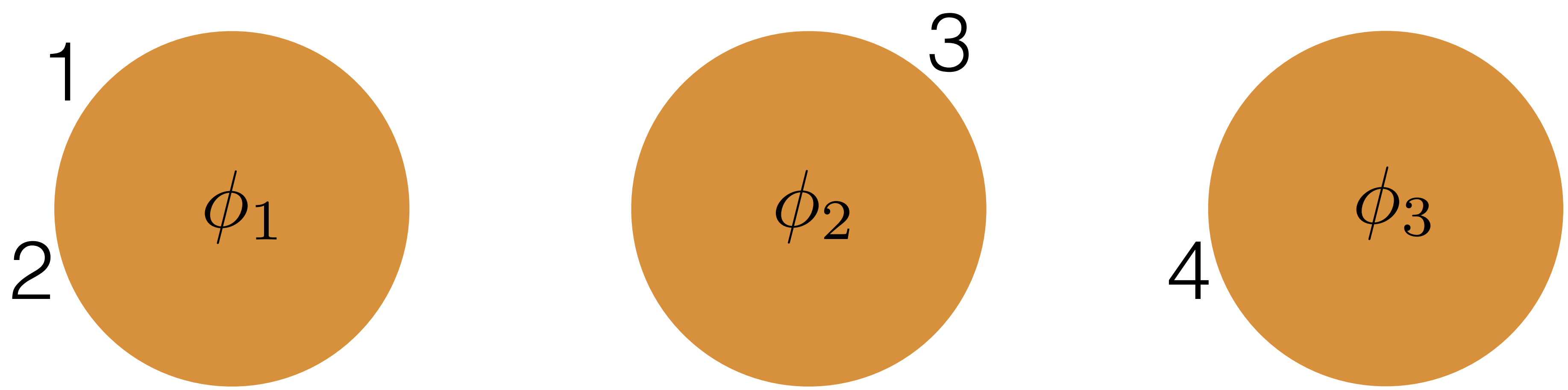
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



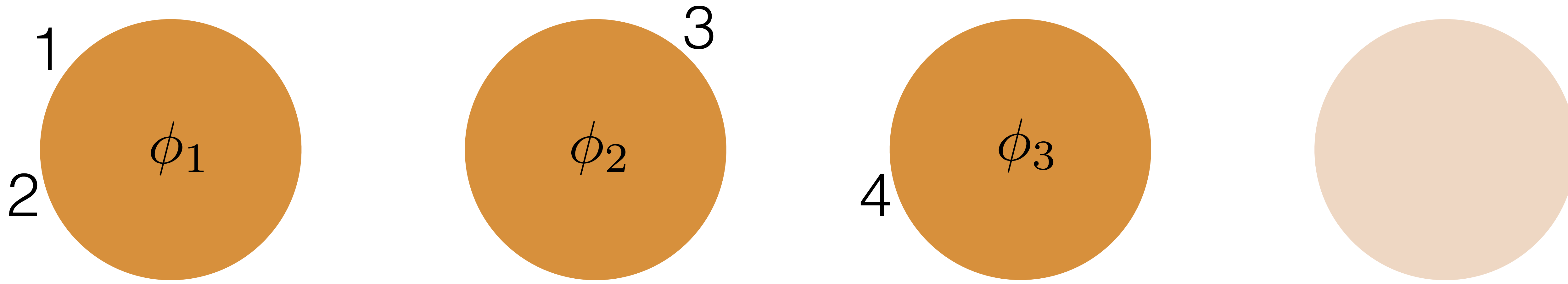
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



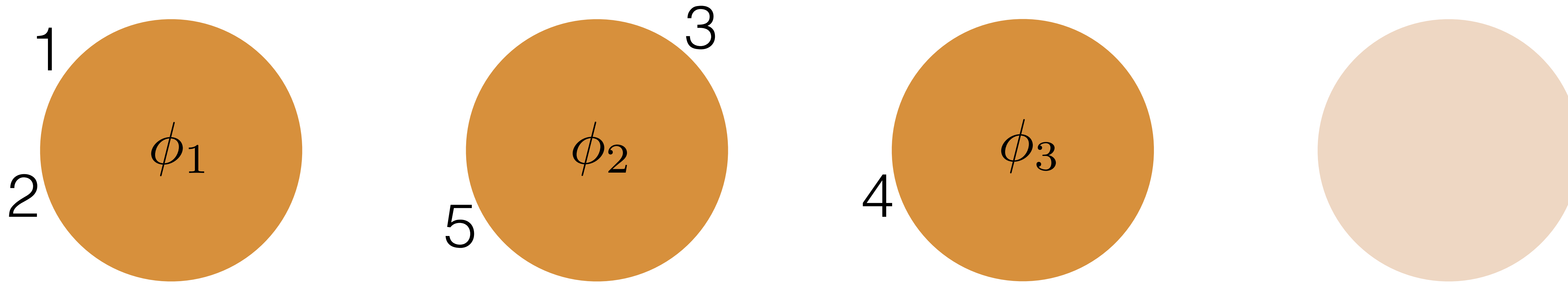
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



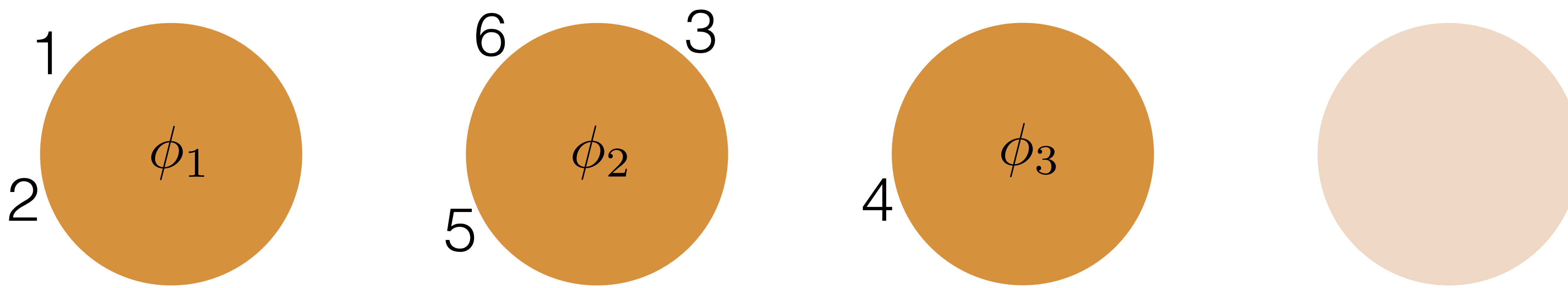
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



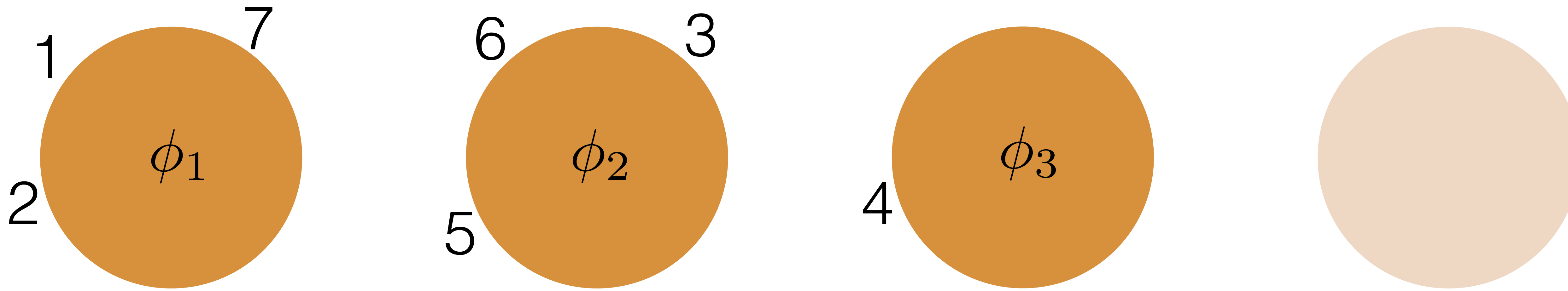
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



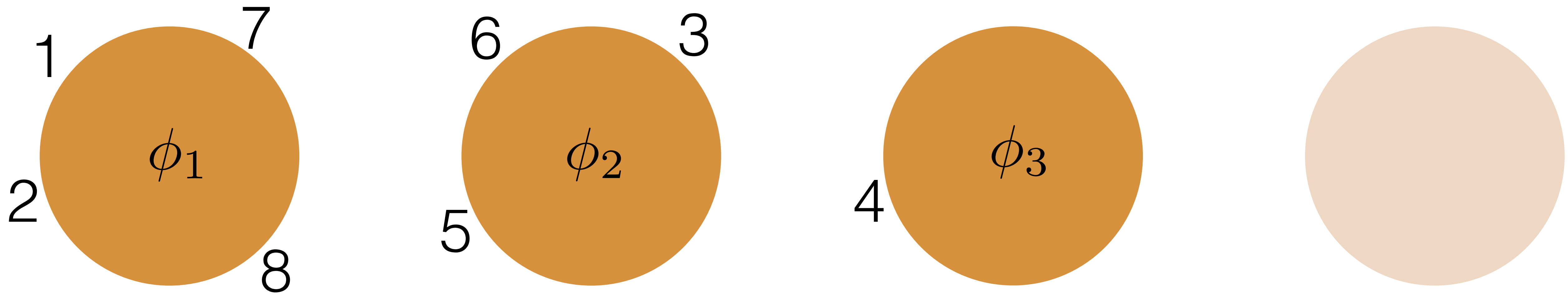
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



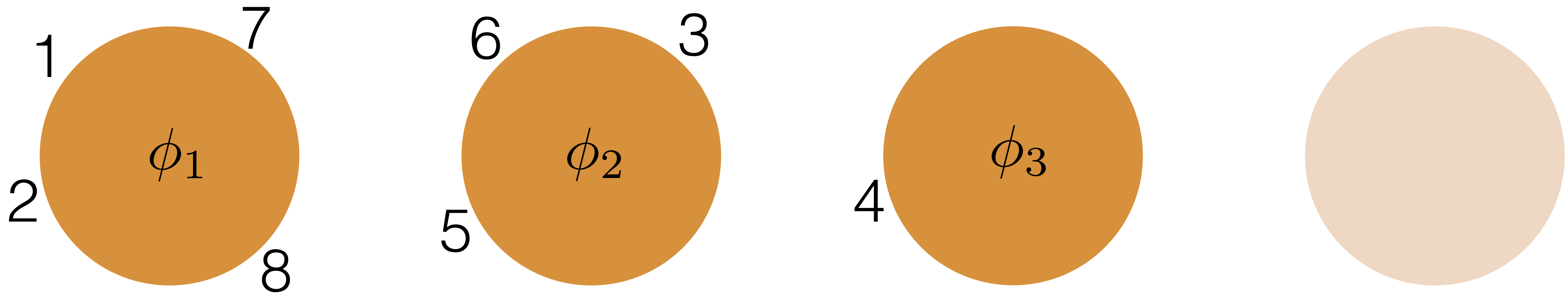
- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

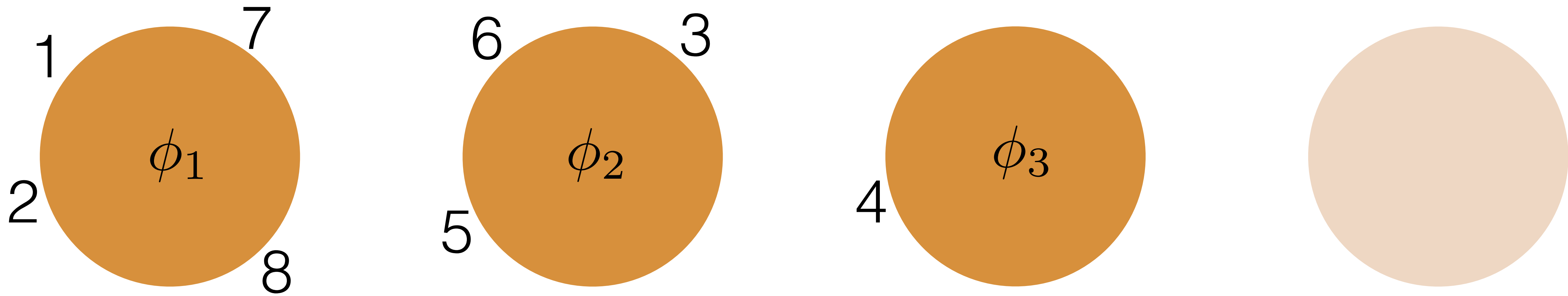
Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α

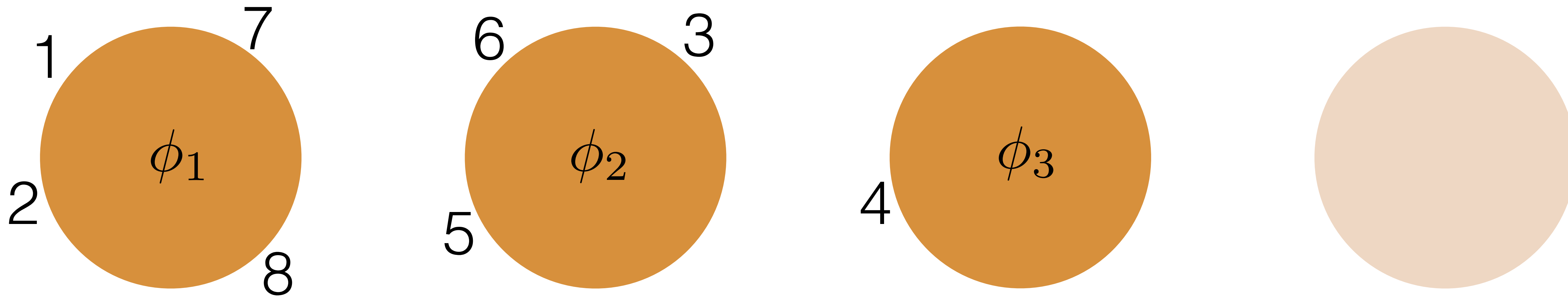
[Aldous 1983]

Chinese restaurant process



- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

Chinese restaurant process

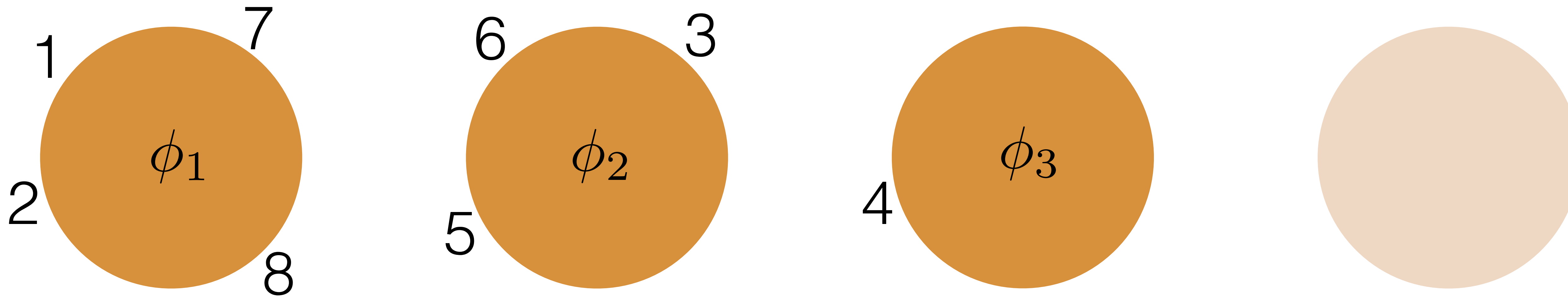


- Same thing we just did
- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

So far: Dirichlet process, Chinese restaurant process

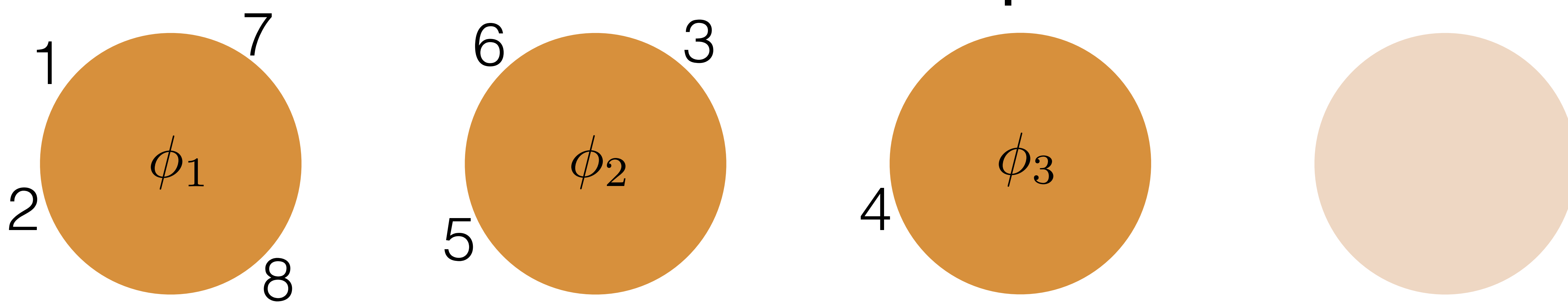
- Infinity of parameters, growing number of parameters

Chinese restaurant process



- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

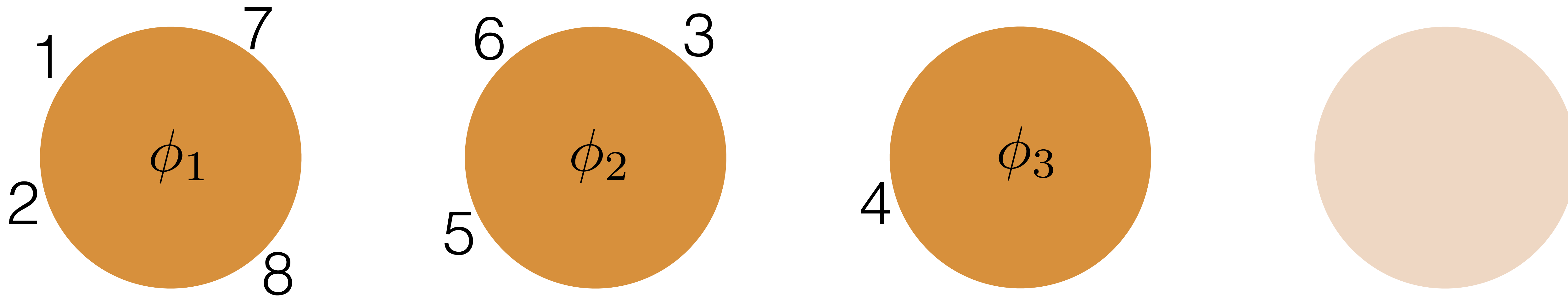
Chinese restaurant process



- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$

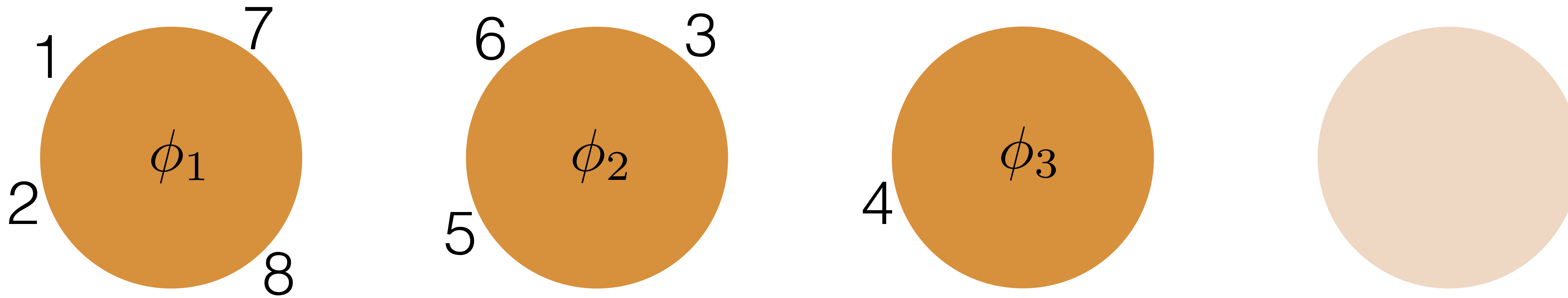
Chinese restaurant process



- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$
$$\Rightarrow \Pi_8 = \{\{1, 2, 7, 8\}, \{3, 5, 6\}, \{4\}\}$$

Chinese restaurant process

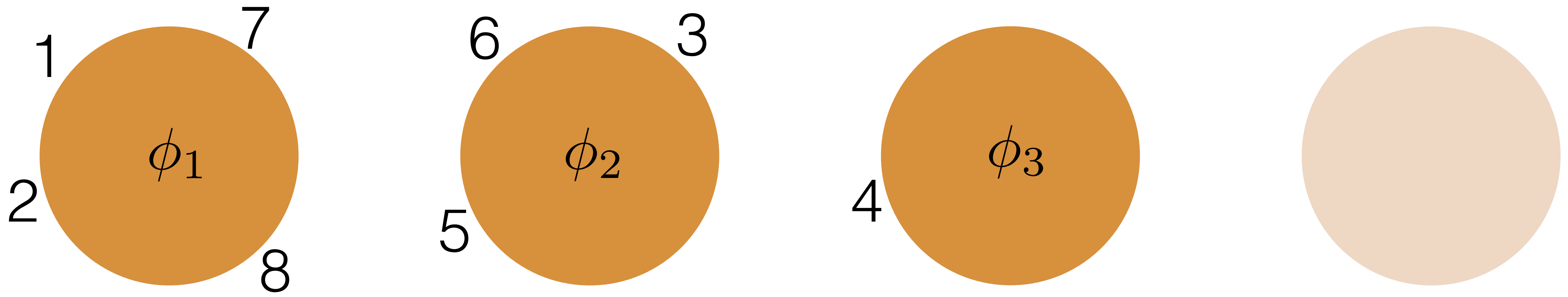


- Each customer walks into the restaurant
  - Sits at existing table with prob proportional to # people there
  - Forms new table with prob proportional to α
- Marginal for the Categorical likelihood with GEM prior

$$z_1 = z_2 = z_7 = z_8 = 1, z_3 = z_5 = z_6 = 2, z_4 = 3$$
$$\Rightarrow \Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}$$

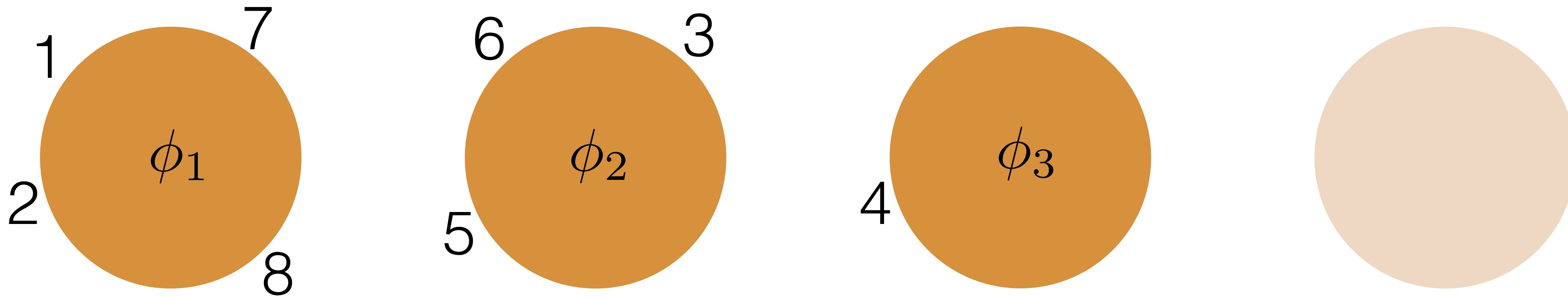
- *Partition of [8]*: set of mutually exclusive & exhaustive sets of $[8] := \{1, \ldots, 8\}$

Chinese restaurant process



- Probability of this seating:

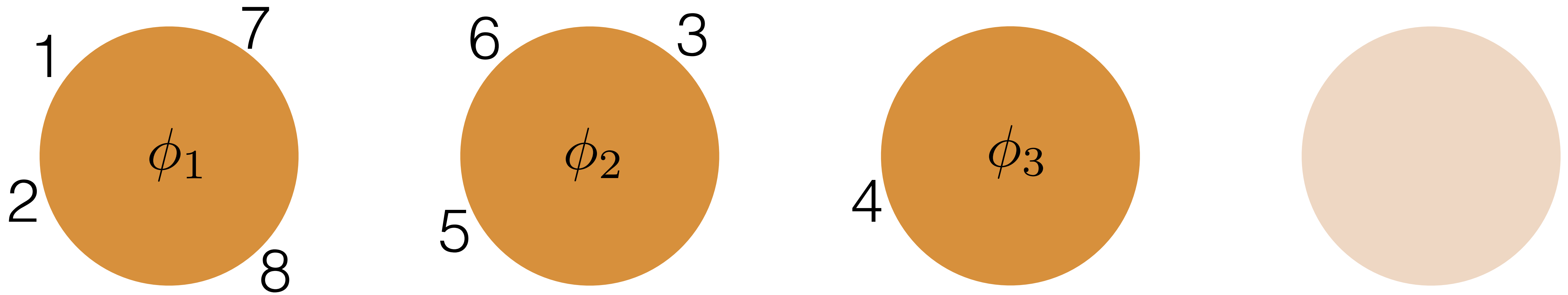
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha}$$

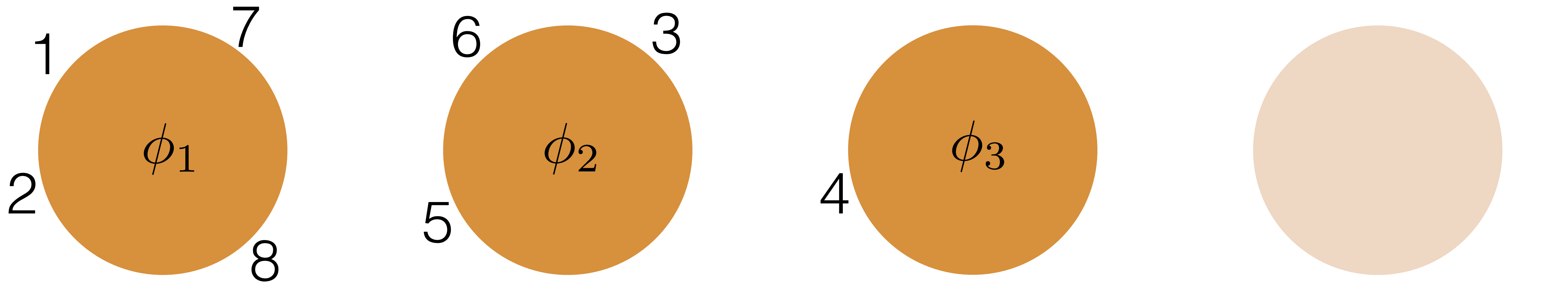
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha + 1}$$

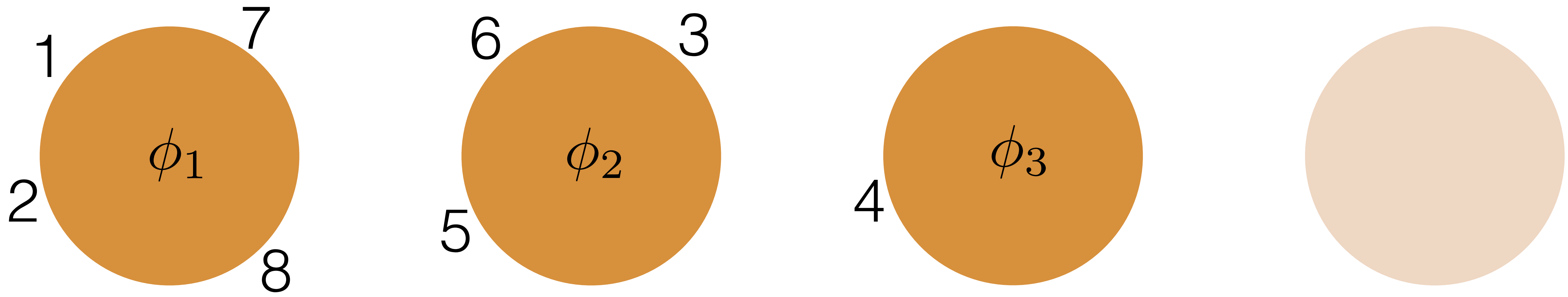
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha + 1} \cdot \frac{\alpha}{\alpha + 2}$$

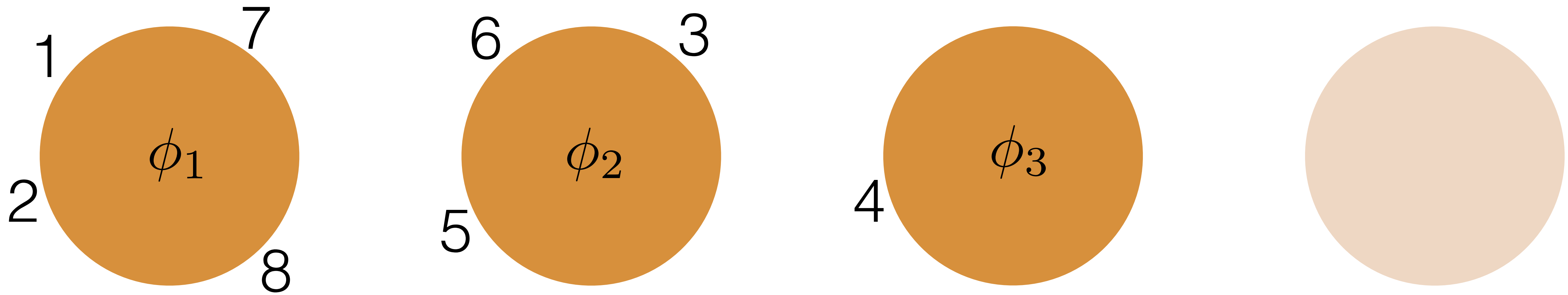
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha + 1} \cdot \frac{\alpha}{\alpha + 2} \cdot \frac{\alpha}{\alpha + 3}$$

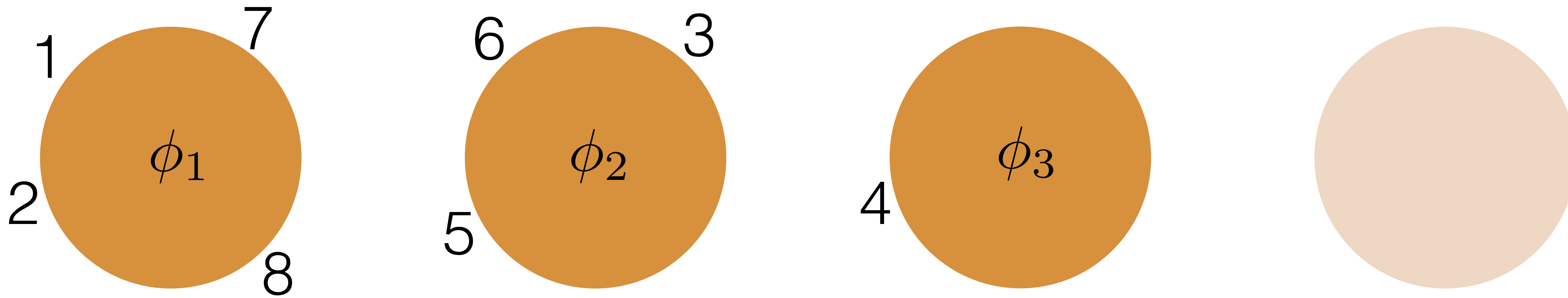
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4}$$

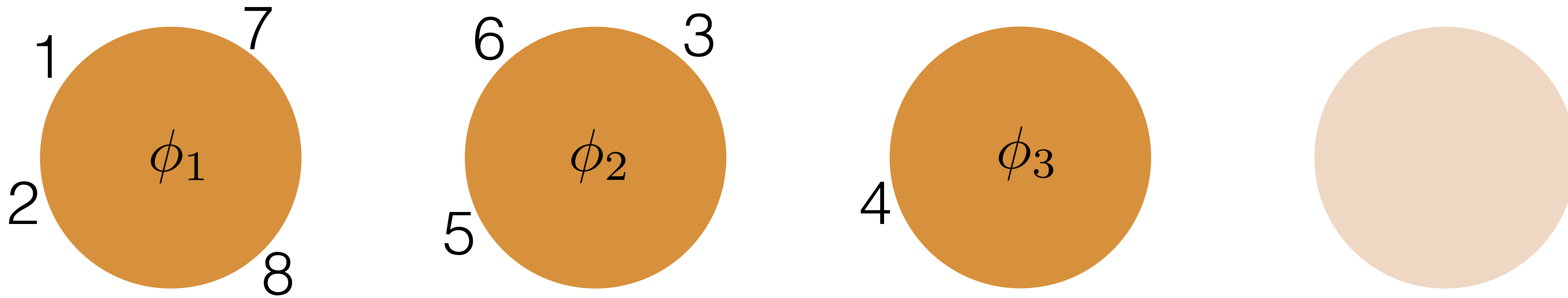
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5}$$

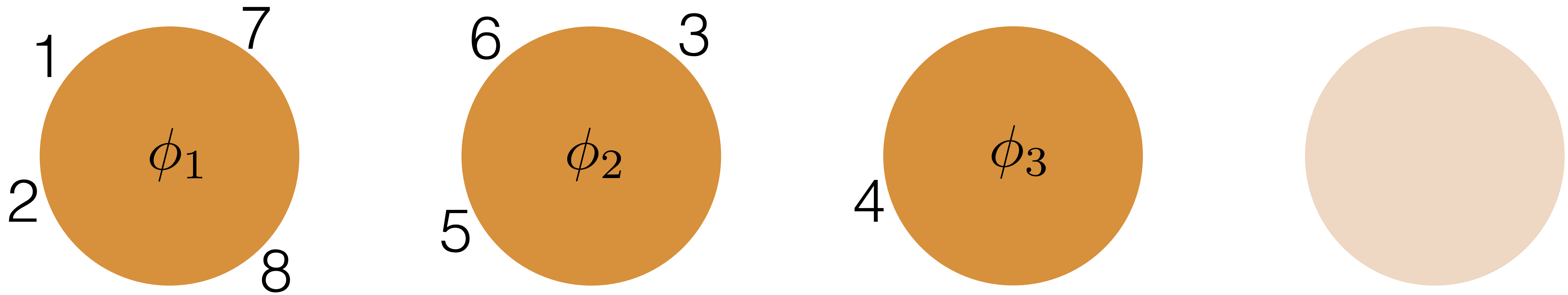
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6}$$

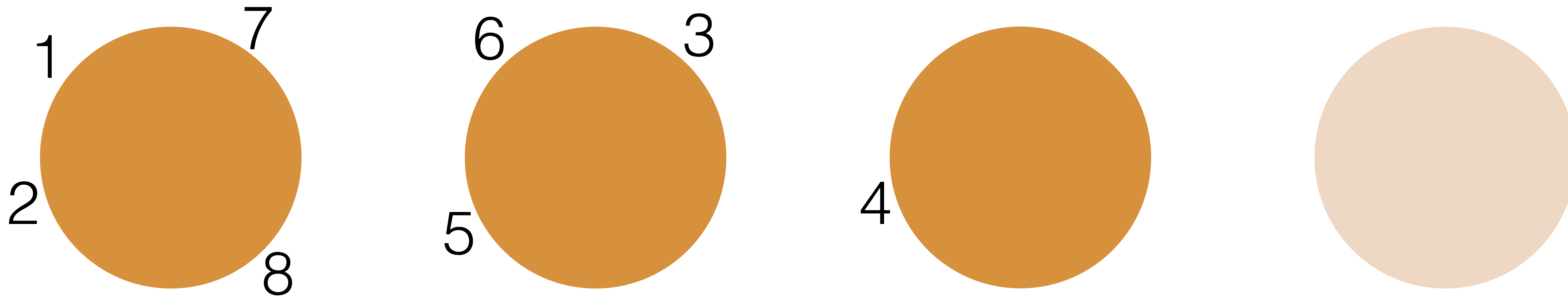
Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

Chinese restaurant process

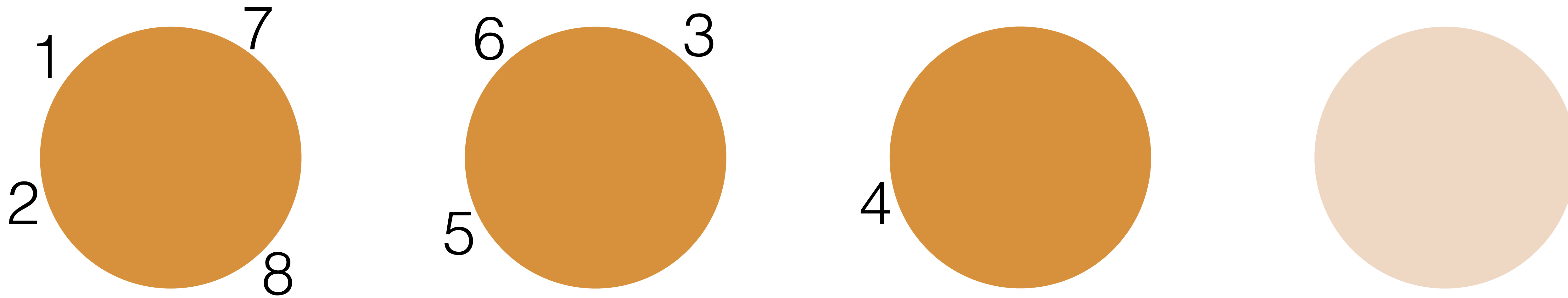


- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

Chinese restaurant process

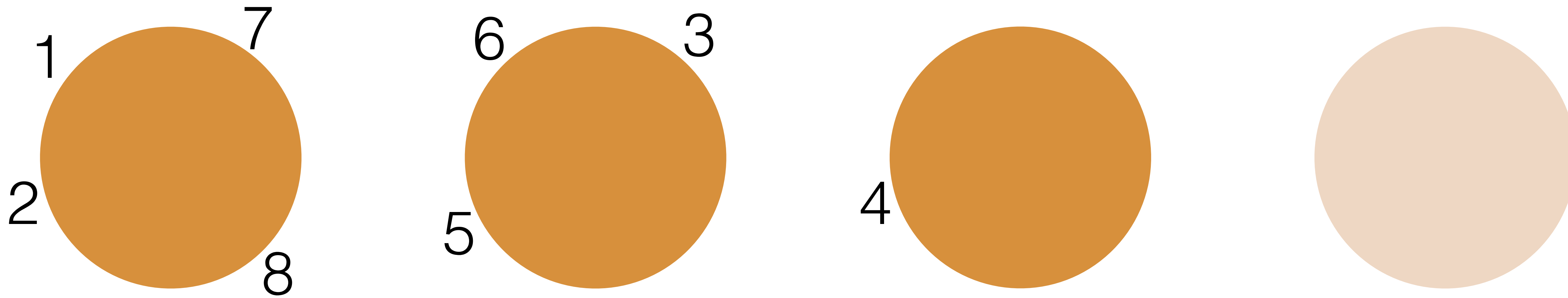


- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

Chinese restaurant process



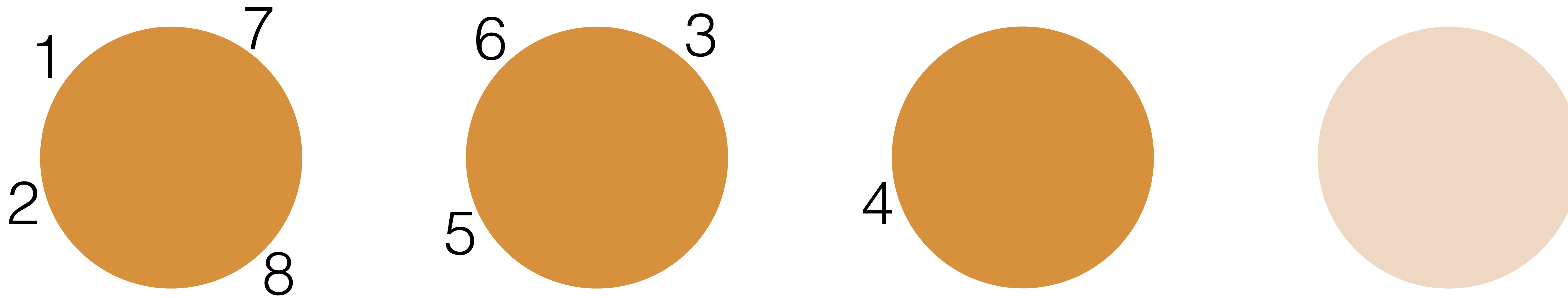
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



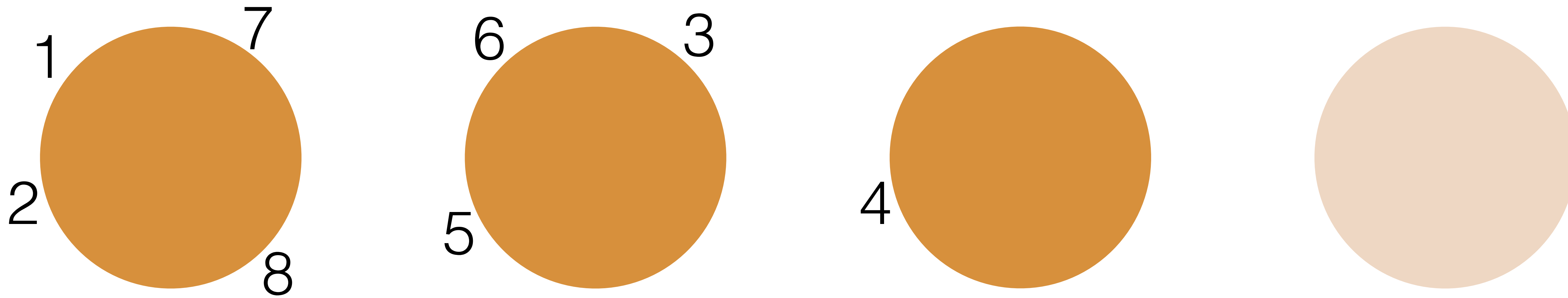
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



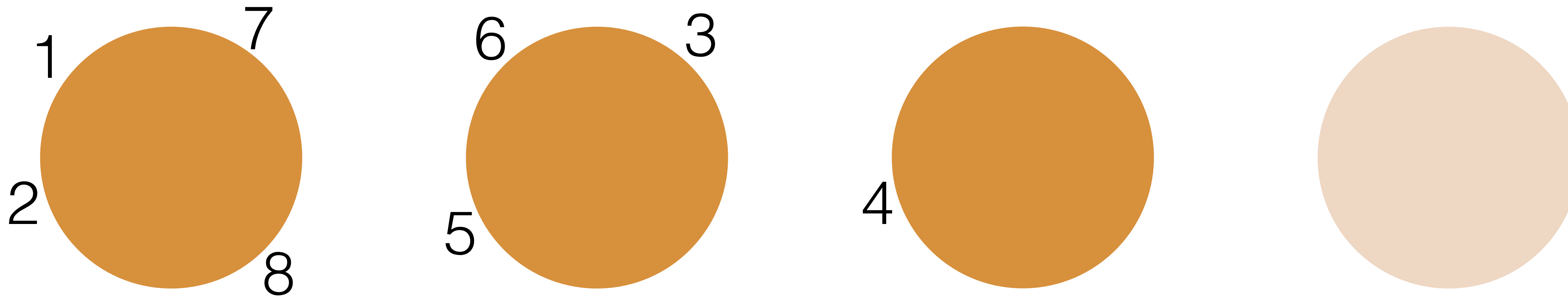
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



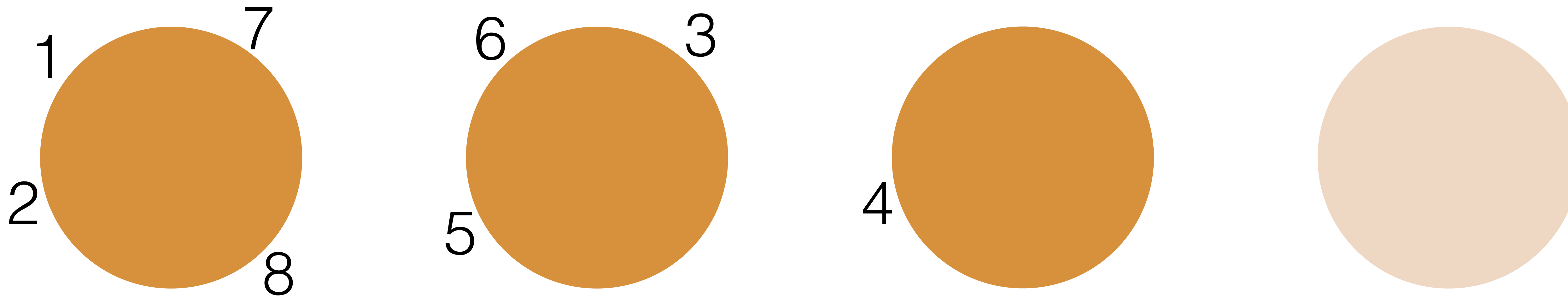
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



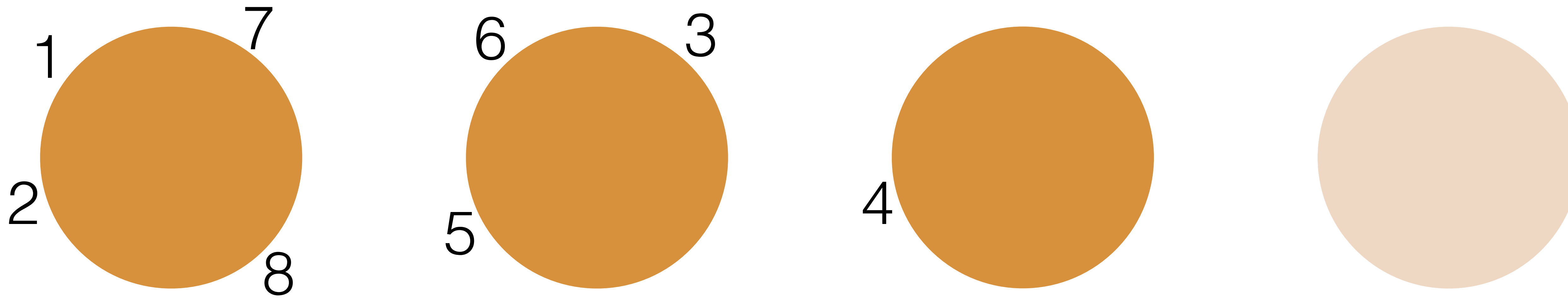
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha+N-1)}$$

Chinese restaurant process



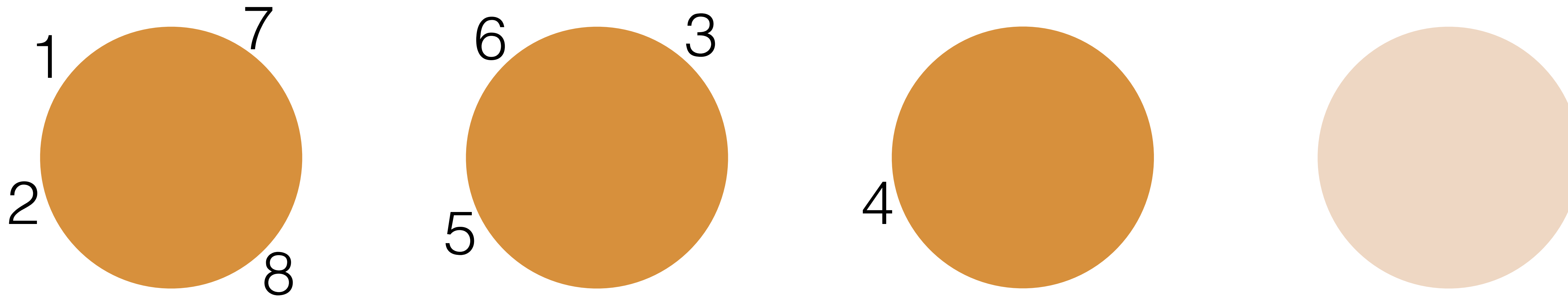
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, n_k at table *k*):

$$\frac{\alpha^{K_N}}{\alpha \cdots (\alpha+N-1)}$$

Chinese restaurant process



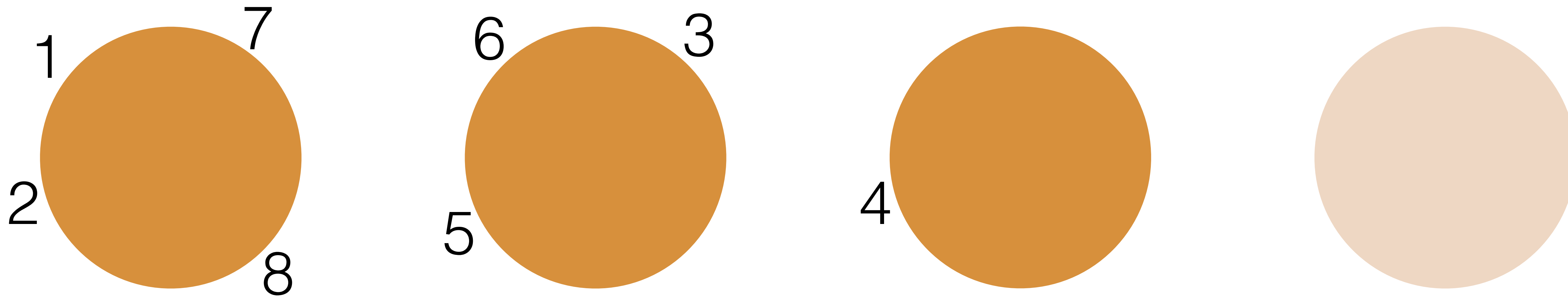
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of N customers (K_N tables, n_k at table k):

$$\frac{\alpha^{K_N} \prod_{k=1}^{K_N} (n_k - 1)!}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



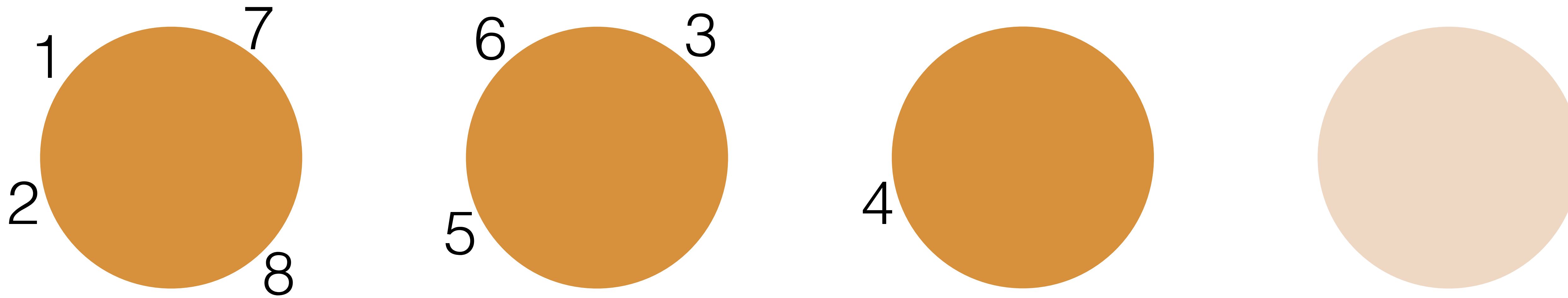
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, #*C* at table *C*):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)}$$

Chinese restaurant process



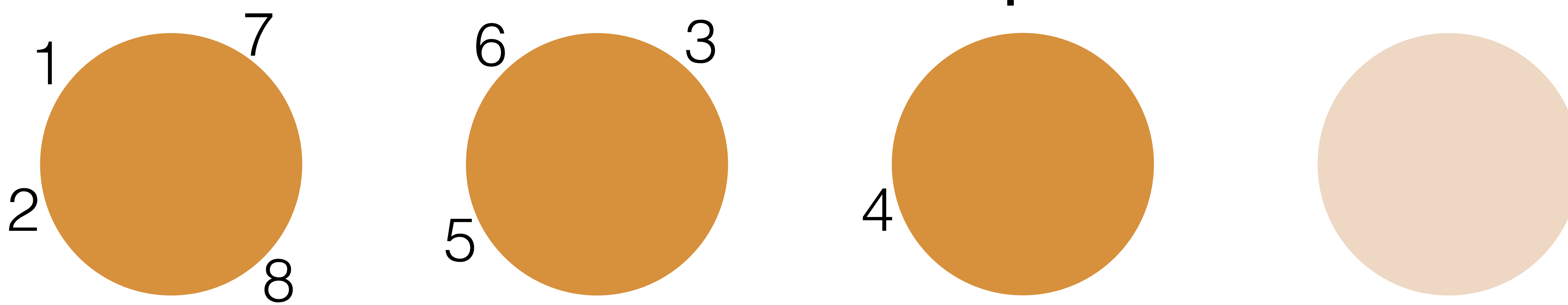
- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

Chinese restaurant process



- Probability of this seating:

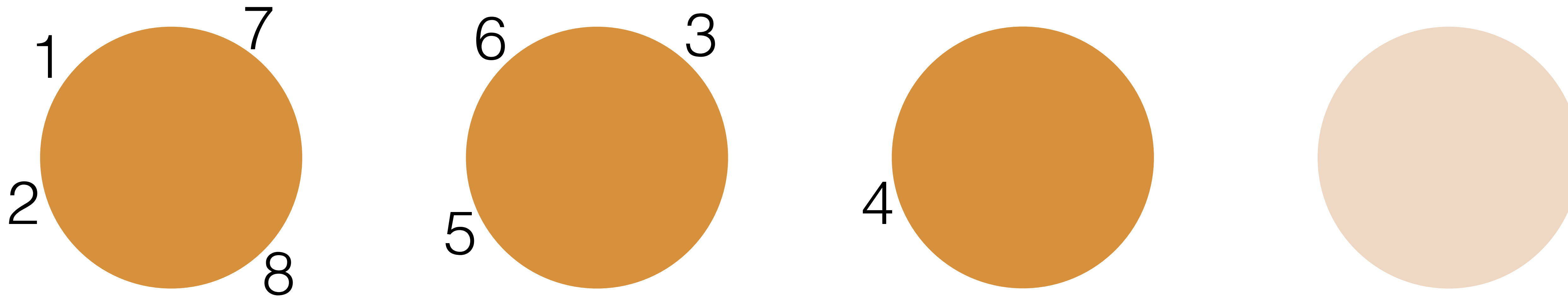
$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of *N* customers (K_N tables, #*C* at table *C*):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- Prob doesn't depend on customer order: *exchangeable*

Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

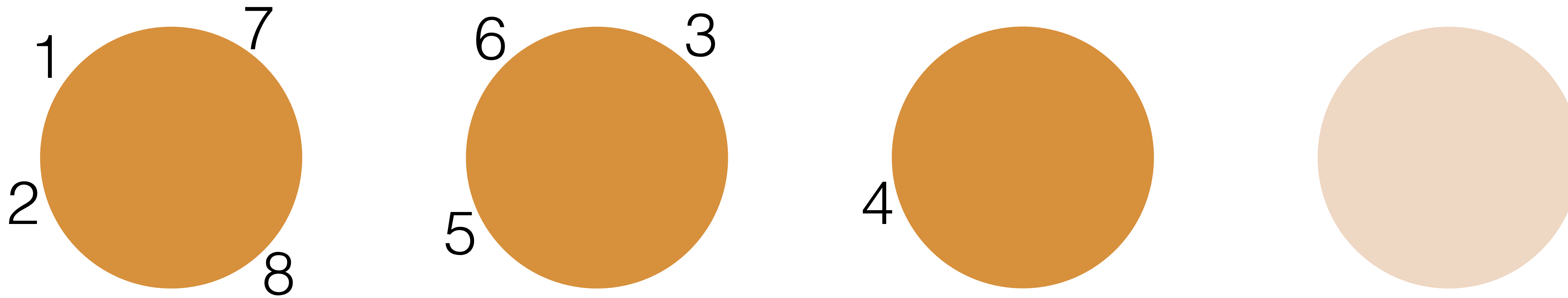
- Probability of *N* customers (K_N tables, #*C* at table *C*):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of N customers (K_N tables, #C at table C):

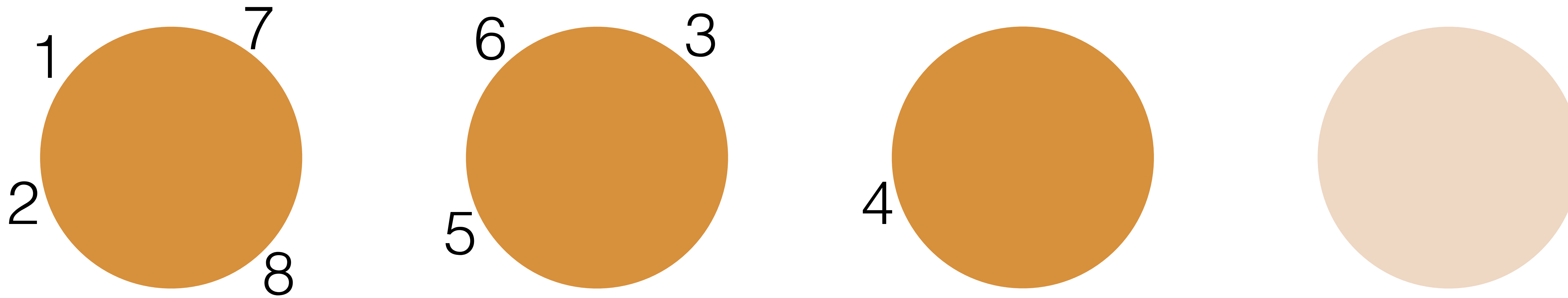
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

- Can always pretend n is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$

Chinese restaurant process



- Probability of this seating:

$$\frac{\alpha}{\alpha} \cdot \frac{1}{\alpha+1} \cdot \frac{\alpha}{\alpha+2} \cdot \frac{\alpha}{\alpha+3} \cdot \frac{1}{\alpha+4} \cdot \frac{2}{\alpha+5} \cdot \frac{2}{\alpha+6} \cdot \frac{3}{\alpha+7}$$

- Probability of N customers (K_N tables, $\#C$ at table C):

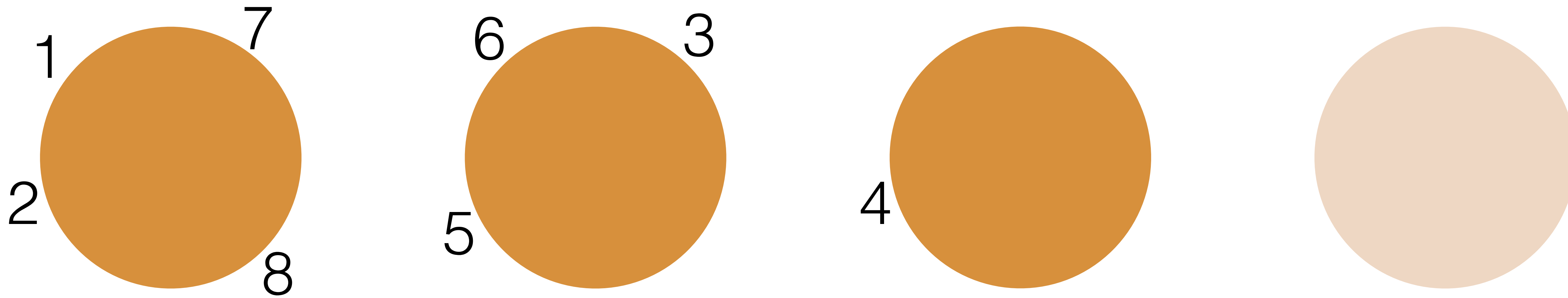
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- Prob doesn't depend on customer order: *exchangeable*

$$\mathbb{P}(\Pi_8 = \{\{1,2,7,8\},\{3,5,6\},\{4\}\}) = \mathbb{P}(\Pi_8 = \{\{2,3,8,1\},\{4,6,7\},\{5\}\})$$

- Can always pretend n is the last customer and calculate $p(\Pi_N | \Pi_{N,\text{-}n})$
    - e.g. $\Pi_{8,\text{-}5} = \{\{1,2,7,8\},\{3,6\},\{4\}\}$

Chinese restaurant process



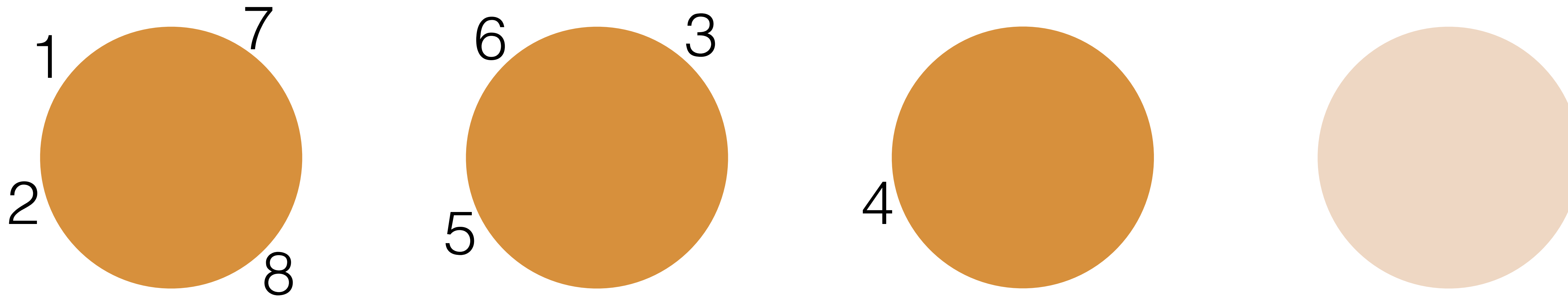
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) =$$

Chinese restaurant process



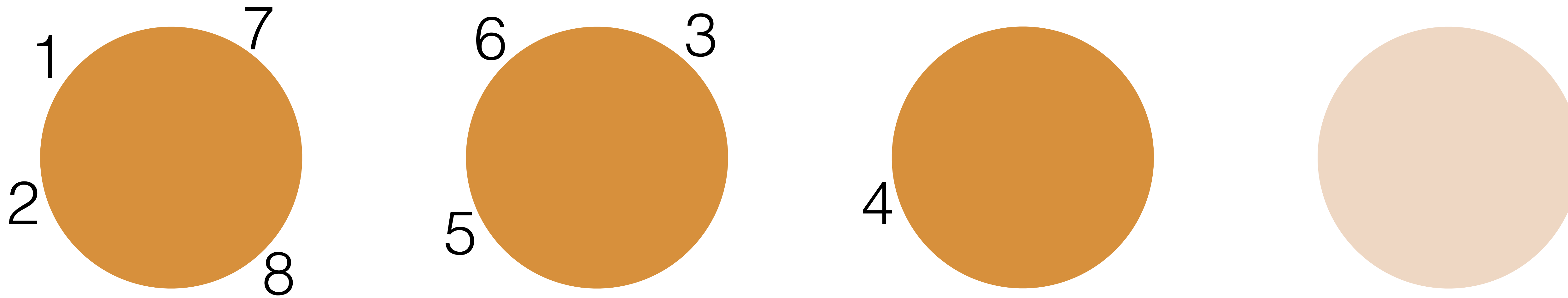
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \Bigg\{$$

Chinese restaurant process



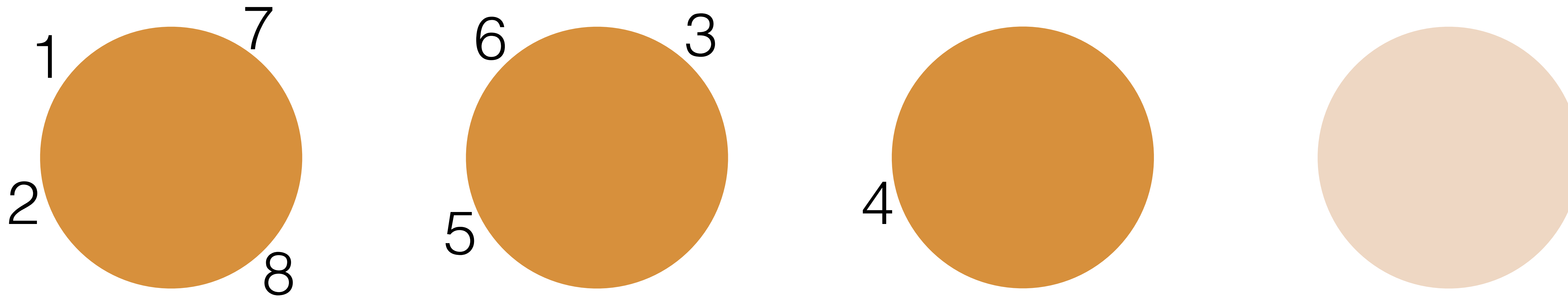
- Probability of N customers (K_N tables, #C at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

Chinese restaurant process



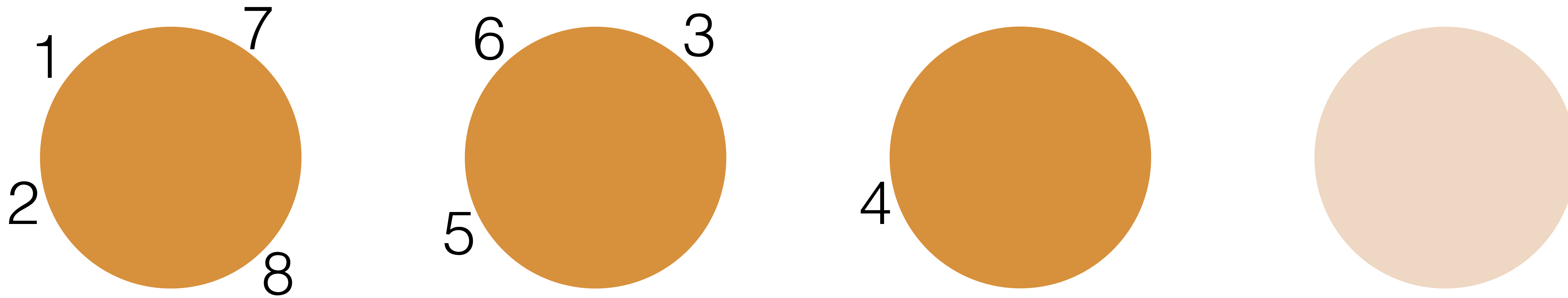
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

Chinese restaurant process



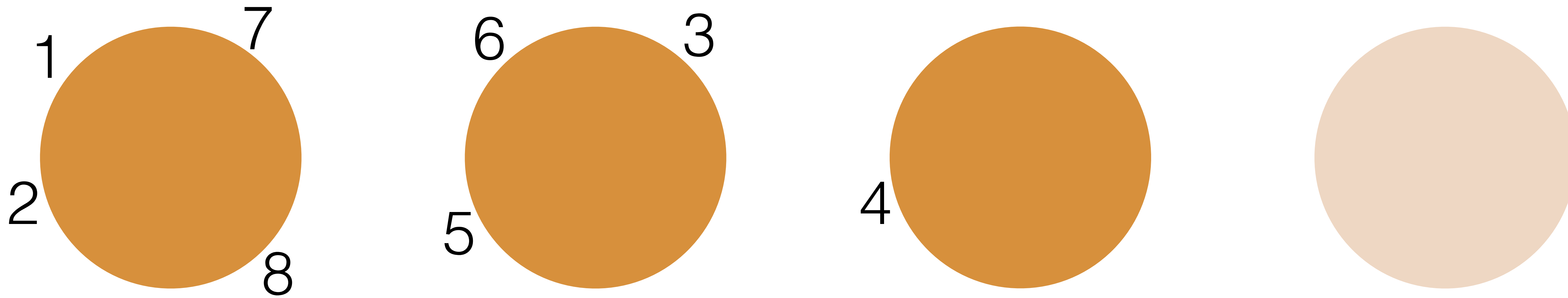
- Probability of N customers (K_N tables, $\#C$ at table C):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha+N-1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):

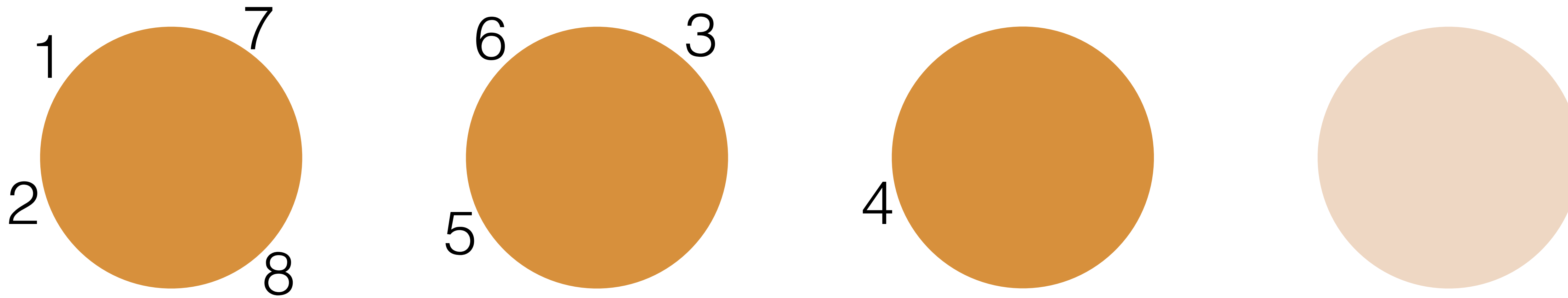
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

- Gibbs sampling review:

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):

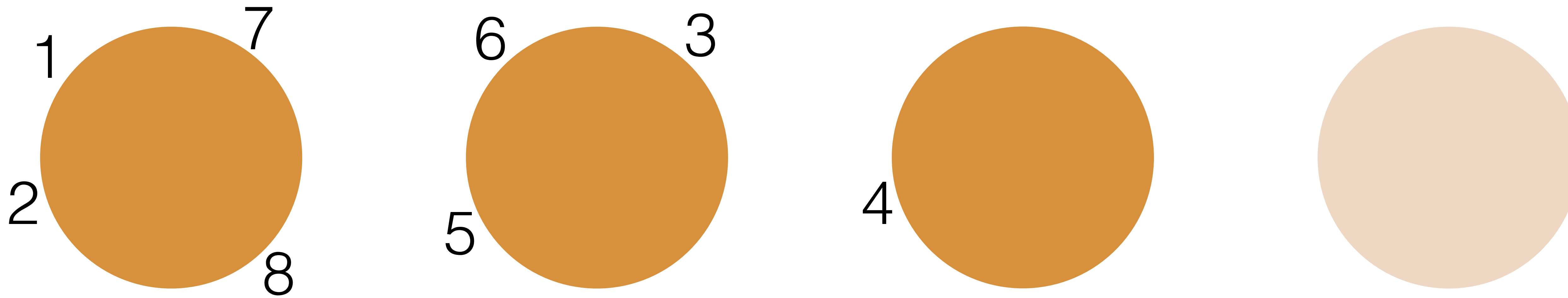
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):

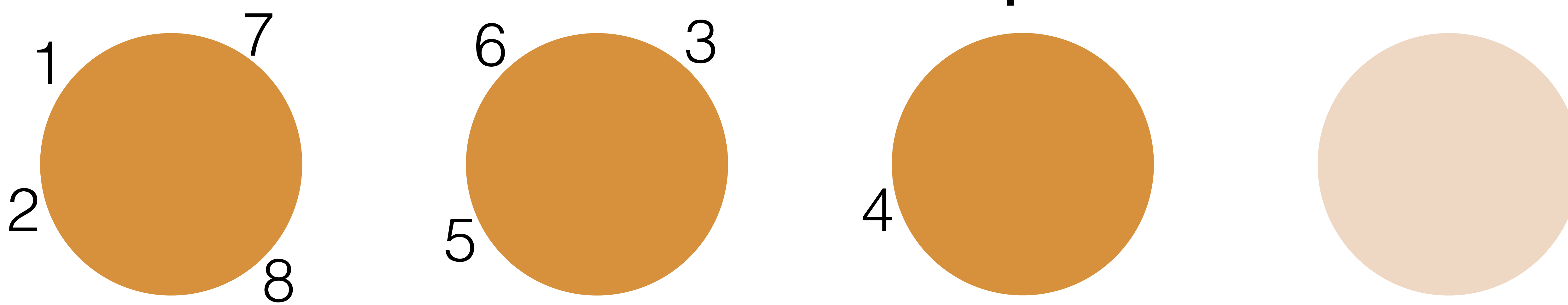
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

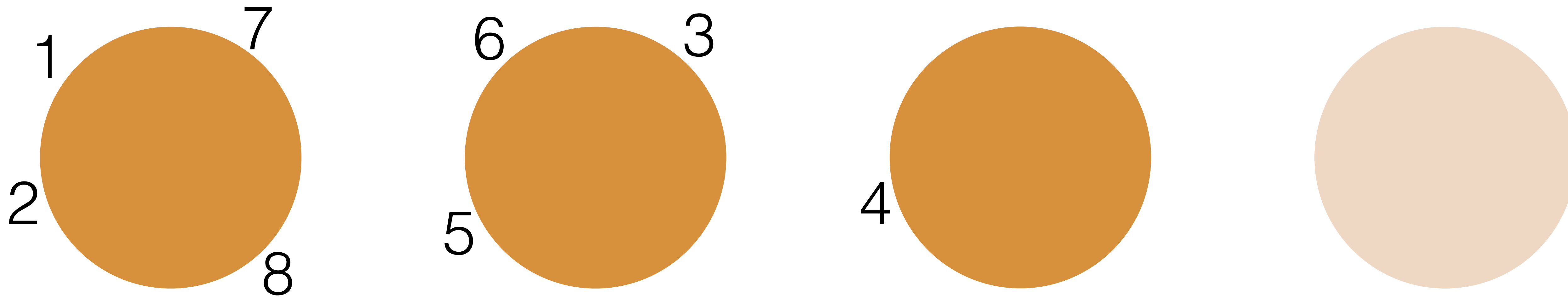
- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$
- So:
$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha+N-1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} & \text{if } n \text{ starts a new cluster} \end{cases}$$
- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$
  - t th step: $v_1^{(t)} \sim p(v_1 | v_2^{(t-1)}, v_3^{(t-1)})$

Chinese restaurant process



- Probability of N customers (K_N tables, $\#C$ at table C):

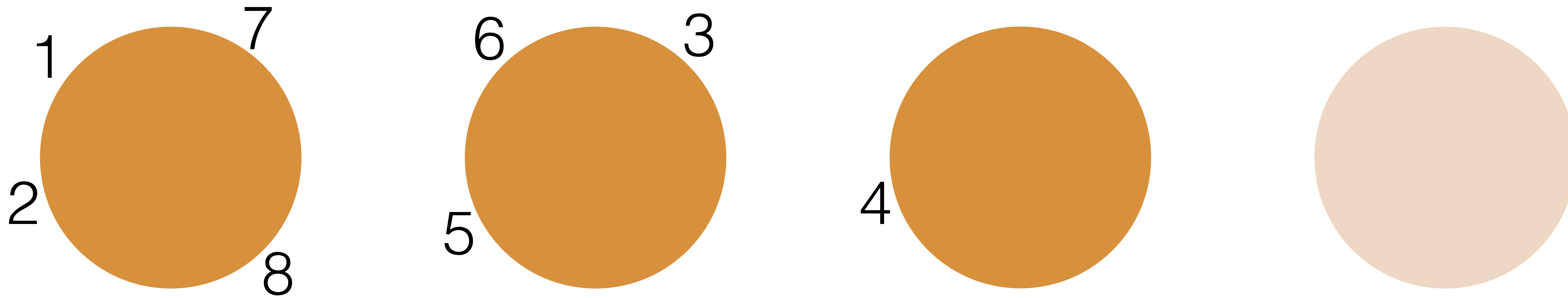
$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So: $$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha+N-1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$
  - t th step: $v_1^{(t)} \sim p(v_1 | v_2^{(t-1)}, v_3^{(t-1)})$

$$v_2^{(t)} \sim p(v_2 | v_1^{(t)}, v_3^{(t-1)})$$

Chinese restaurant process



- Probability of *N* customers (K_N tables, #*C* at table *C*):

$$\frac{\alpha^{K_N} \prod_{C \in \Pi_N} (\#C - 1)!}{\alpha \cdots (\alpha + N - 1)} = \mathbb{P}(\Pi_N = \pi_N)$$

- So:

$$p(\Pi_N | \Pi_{N,-n}) = \begin{cases} \frac{\#C}{\alpha + N - 1} & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} & \text{if } n \text{ starts a new cluster} \end{cases}$$

- Gibbs sampling review: target distribution $p(v_1, v_2, v_3)$
  - Start: $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$
  - t th step: $v_1^{(t)} \sim p(v_1 | v_2^{(t-1)}, v_3^{(t-1)})$

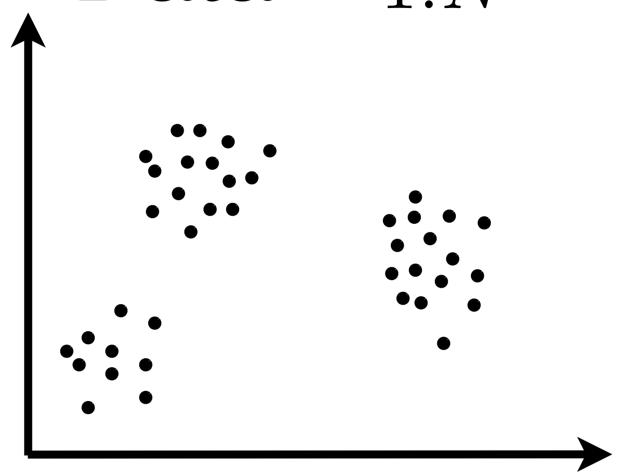
    $v_2^{(t)} \sim p(v_2 | v_1^{(t)}, v_3^{(t-1)})$

    $v_3^{(t)} \sim p(v_3 | v_1^{(t)}, v_2^{(t)})$

CRP mixture model: inference

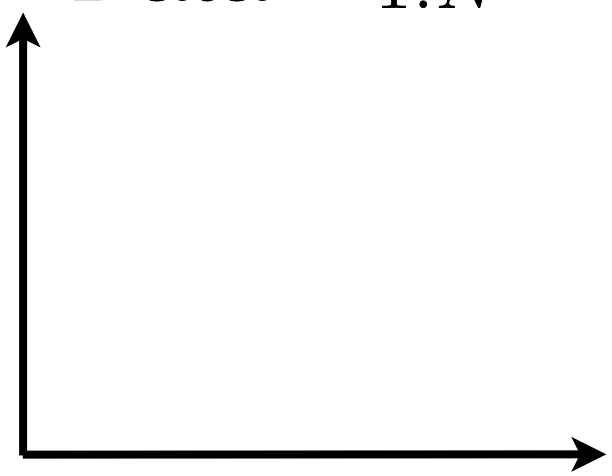
CRP mixture model: inference

- Data $x_{1:N}$



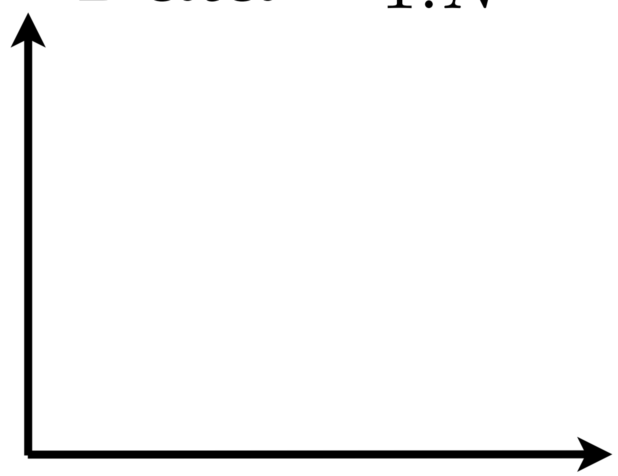
CRP mixture model: inference

- Data $x_{1:N}$



CRP mixture model: inference

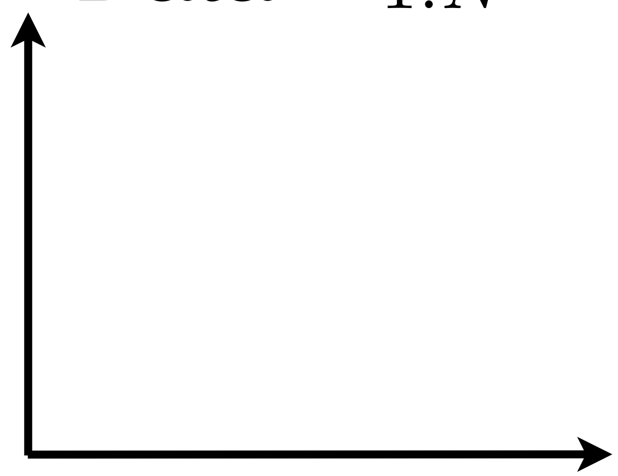
- Data $x_{1:N}$



- Generative model

CRP mixture model: inference

- Data $x_{1:N}$

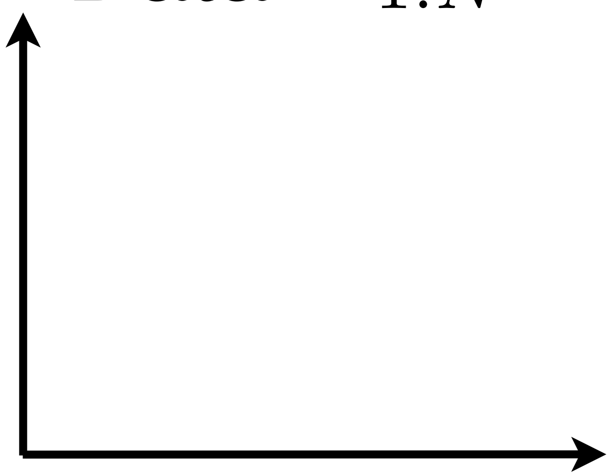


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

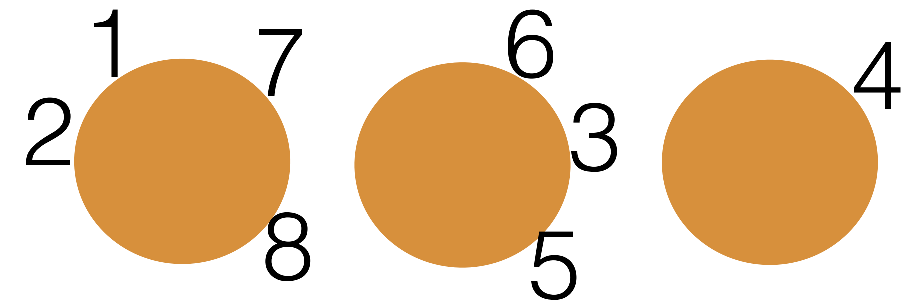
CRP mixture model: inference

- Data $x_{1:N}$



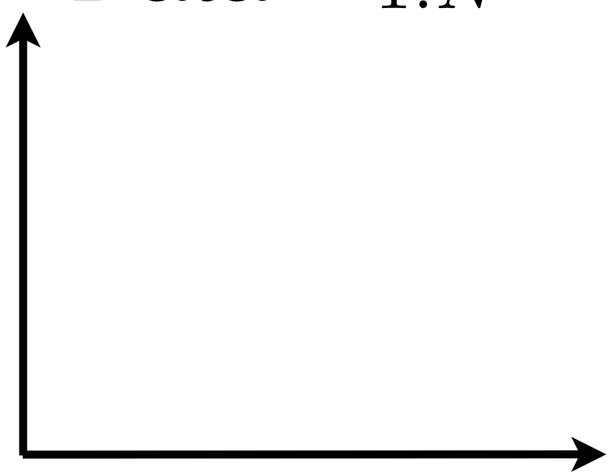
- Generative model

  $\Pi_N \sim \mathrm{CRP}(N, \alpha)$



CRP mixture model: inference

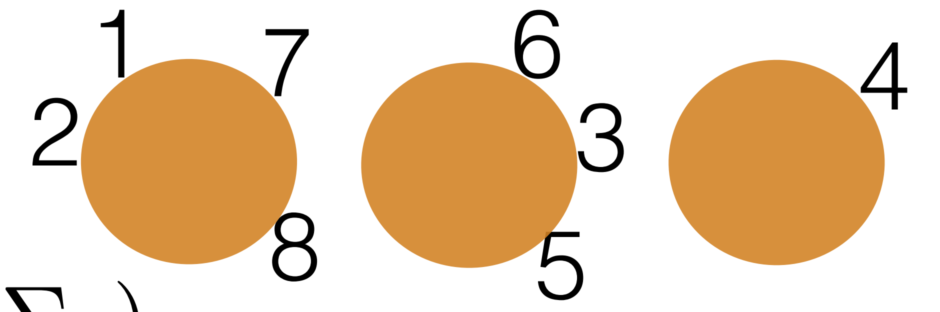
- Data $x_{1:N}$



- Generative model

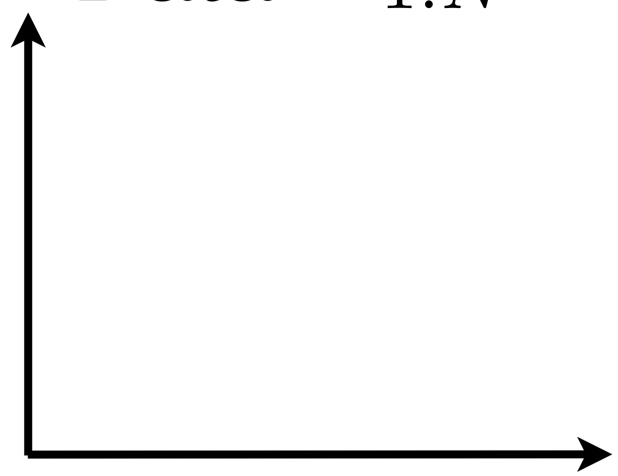
$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$



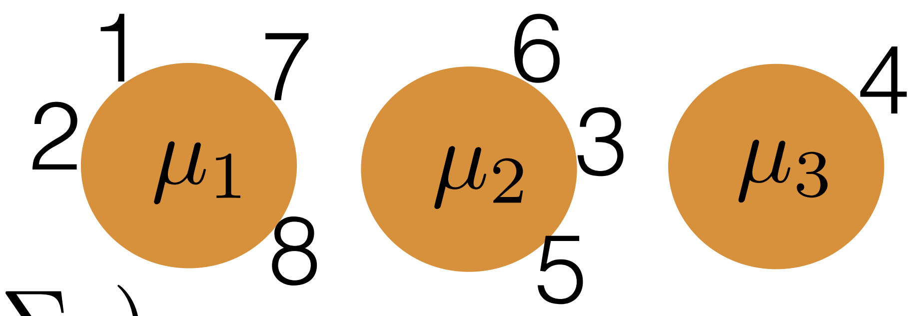
CRP mixture model: inference

- Data $x_{1:N}$



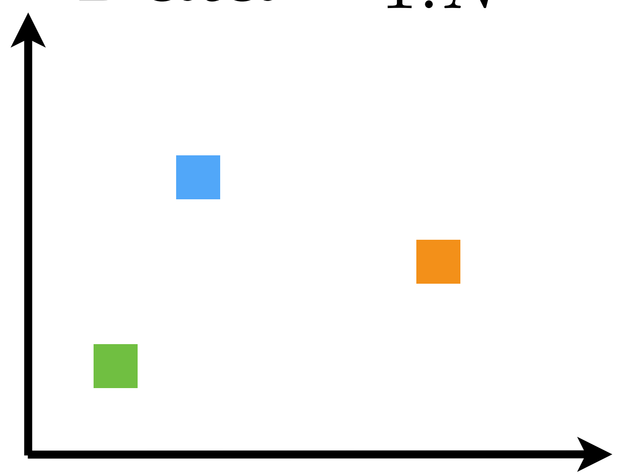
- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$



CRP mixture model: inference

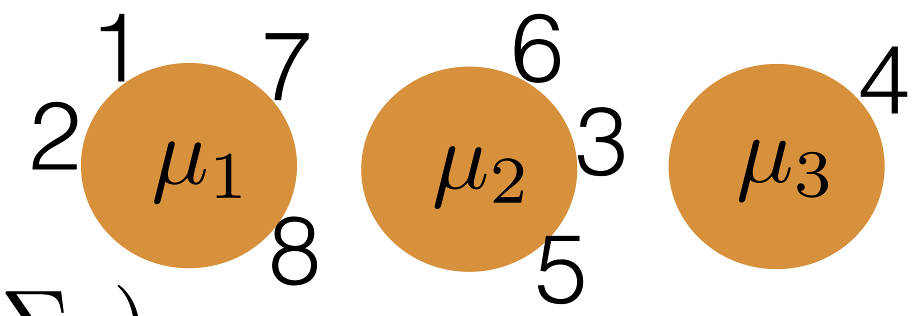
- Data $x_{1:N}$



- Generative model

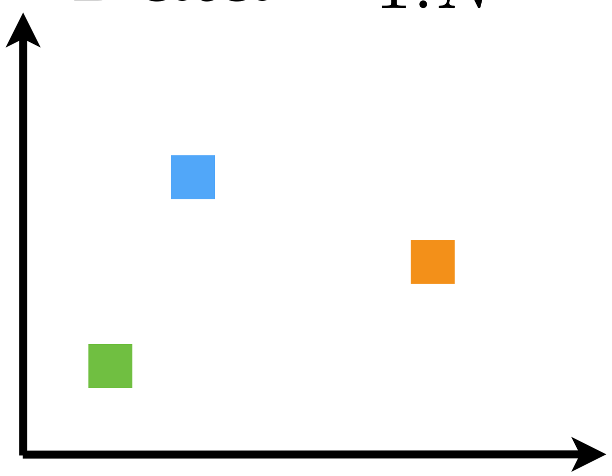
$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \mu_C \stackrel{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$



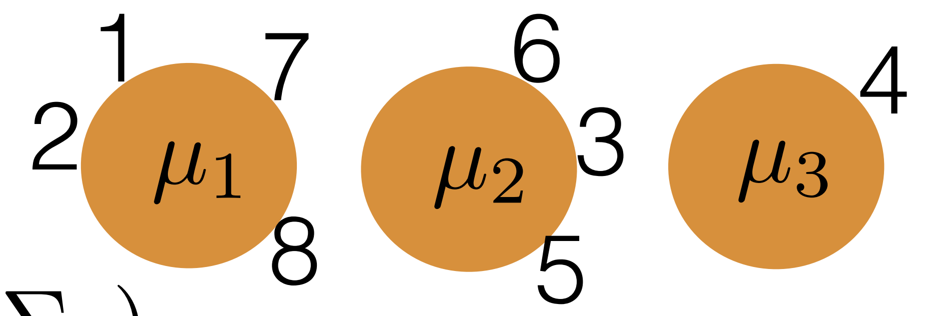
CRP mixture model: inference

- Data $x_{1:N}$



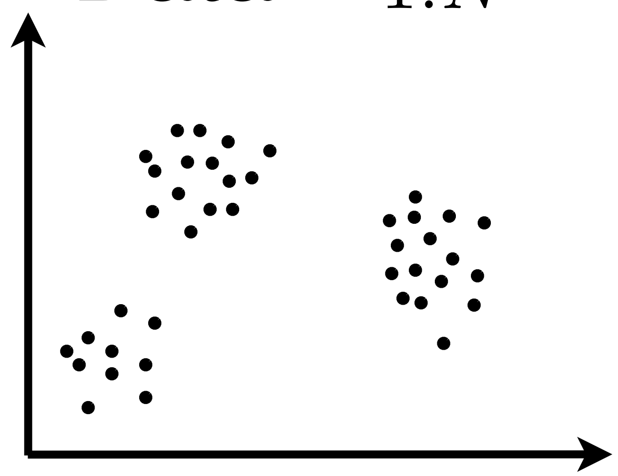
- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



CRP mixture model: inference

- Data $x_{1:N}$

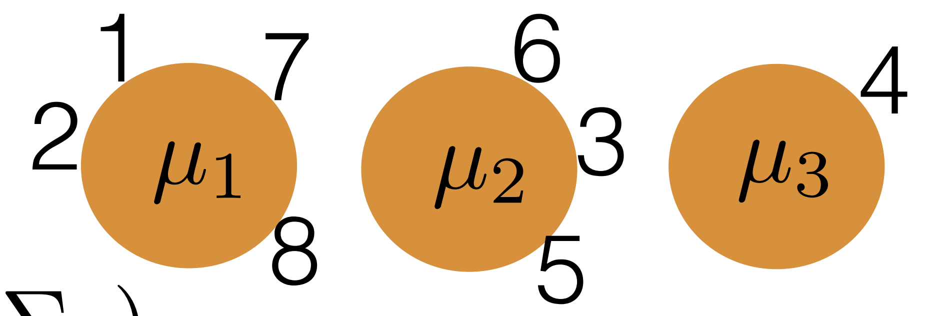


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

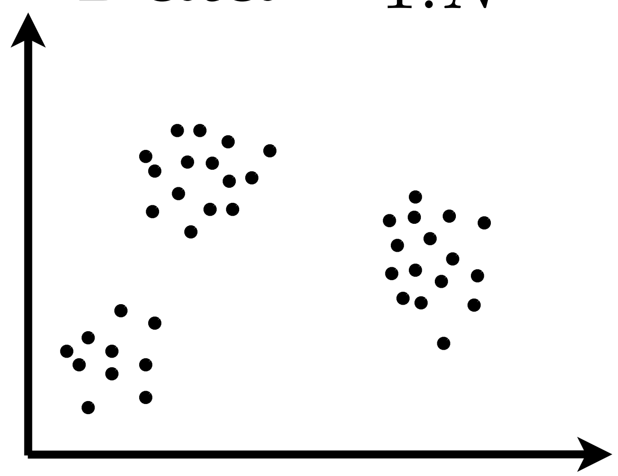
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$

$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



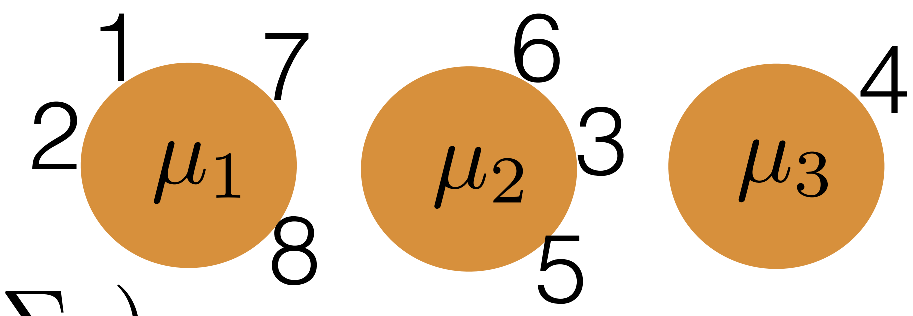
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

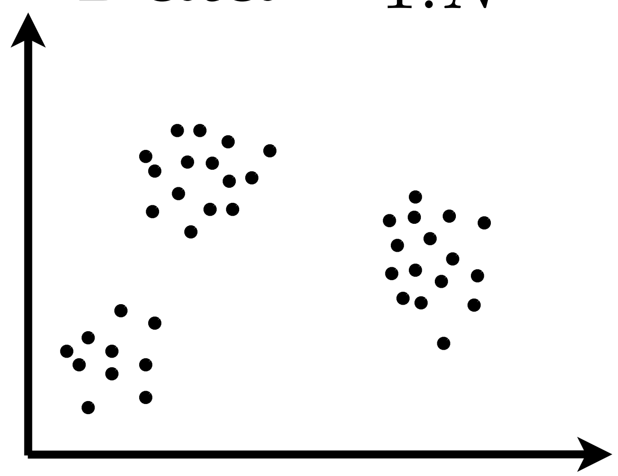
$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior

CRP mixture model: inference

- Data $x_{1:N}$

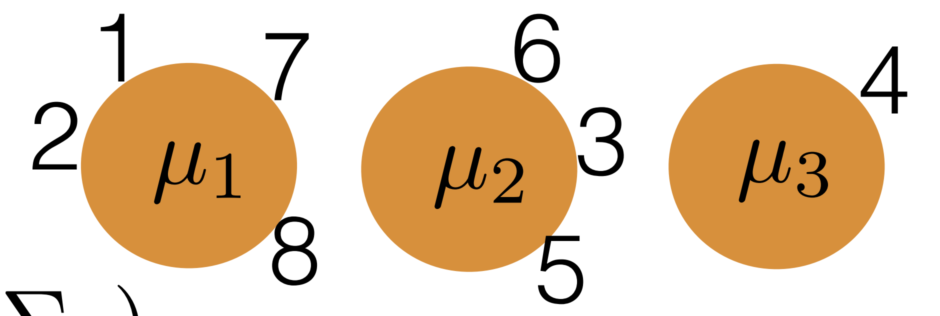


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$

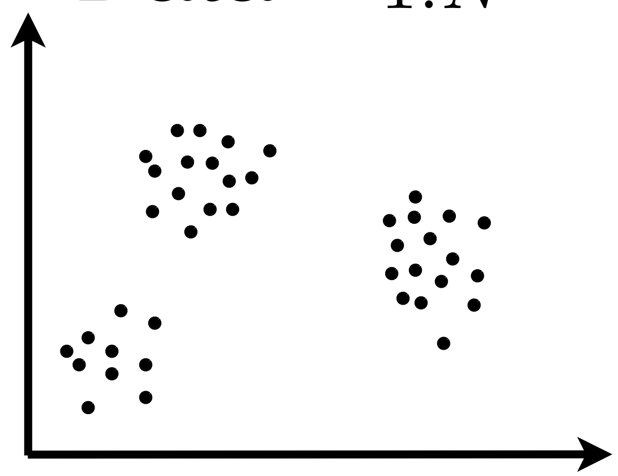
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



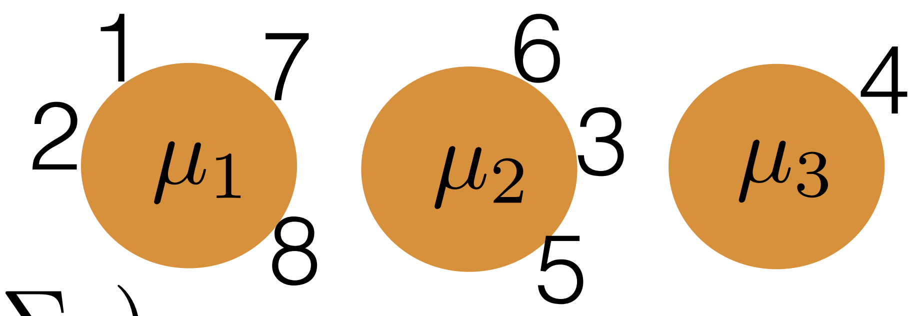
- Want: posterior $p(\Pi_N | x_{1:N})$

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

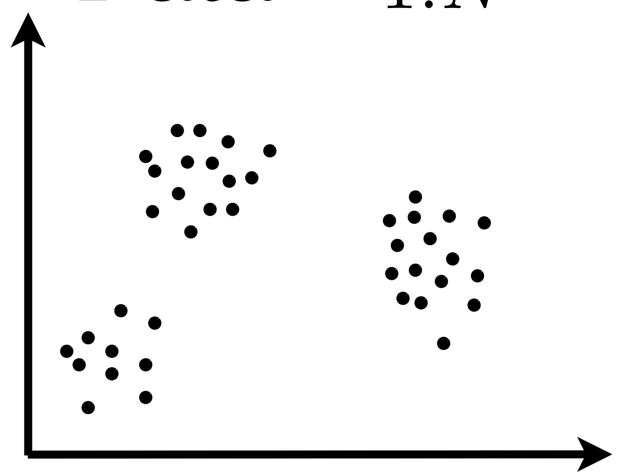


$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$

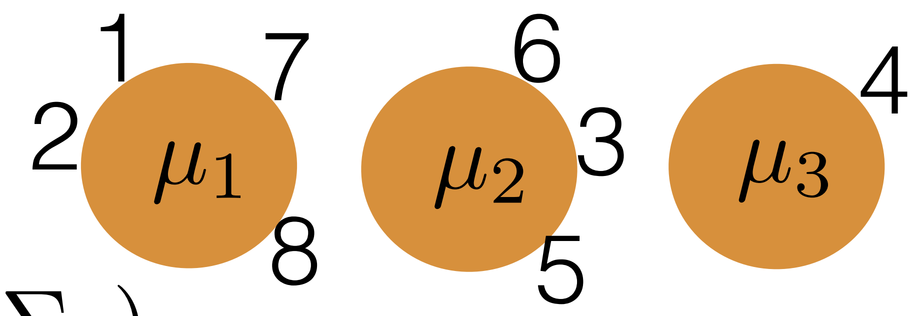
- Want: posterior $p(\Pi_N | x_{1:N})$
- Gibbs sampler:

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model



$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$

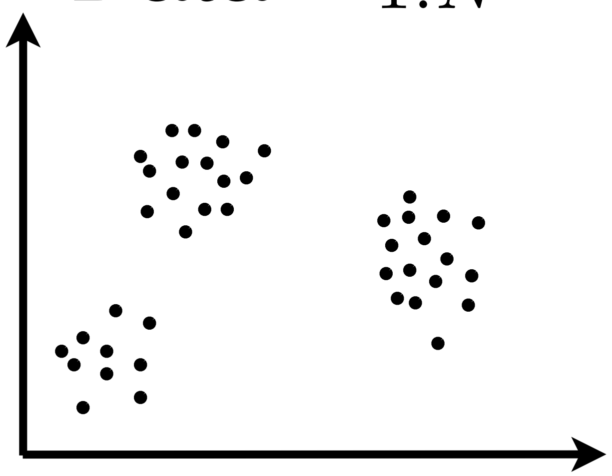
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

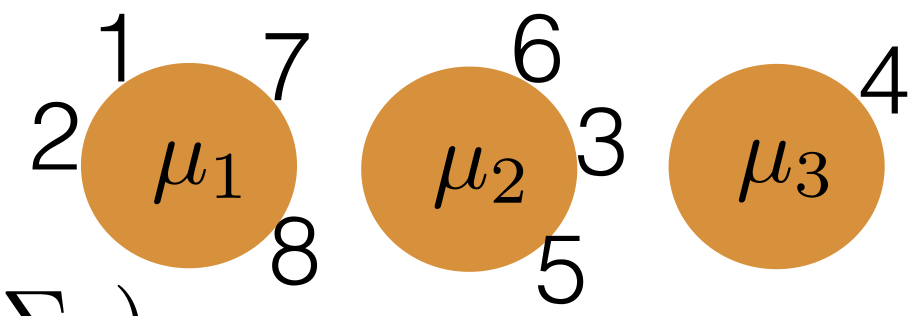
$$p(\Pi_N | \Pi_{N,\text{-}n}, x)$$

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model
  $$\Pi_N \sim \text{CRP}(N, \alpha)$$
  $$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
  $$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



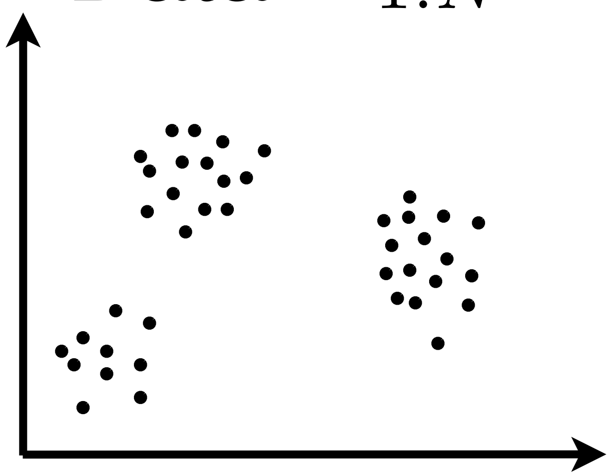
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

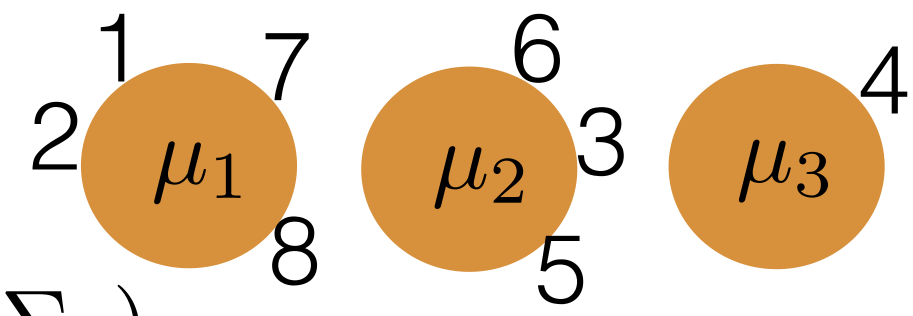
$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \left\{ \right.$$

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model



$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$

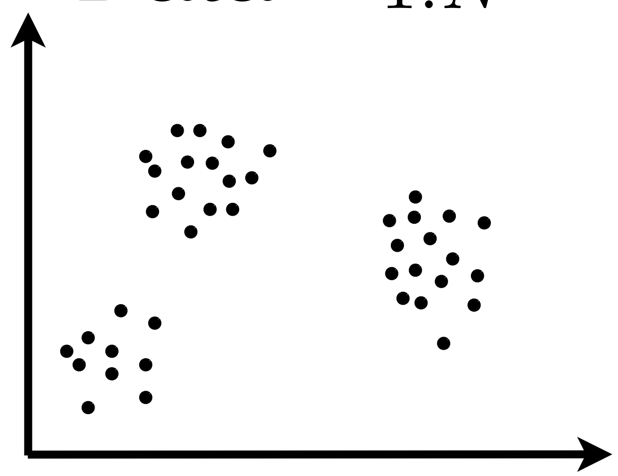
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} & \text{if } n \text{ joins cluster } C \\ & \end{cases}$$

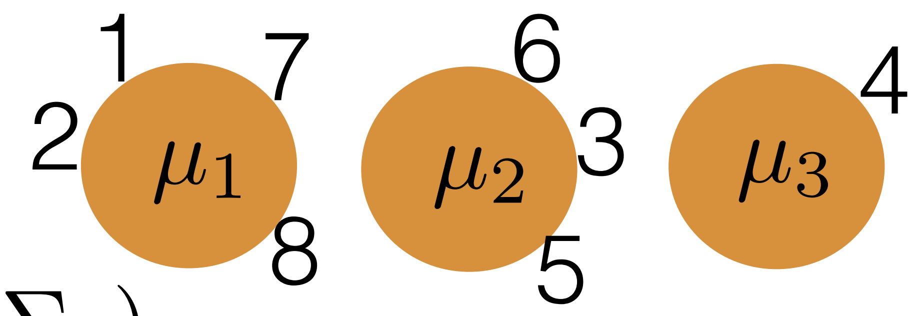
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



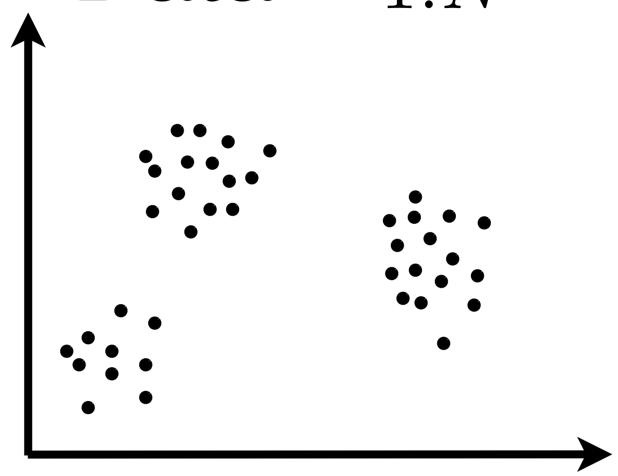
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

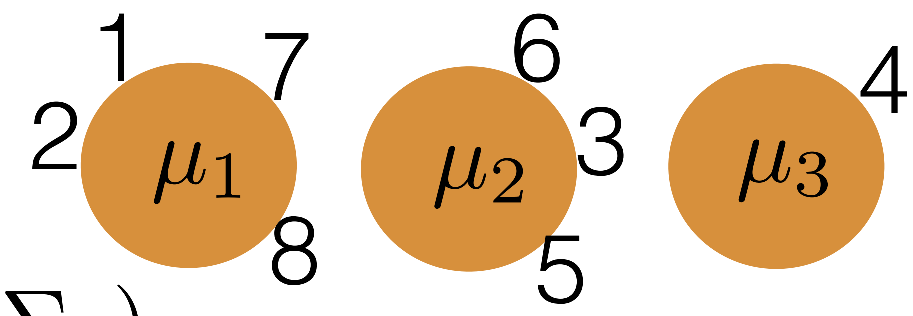
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



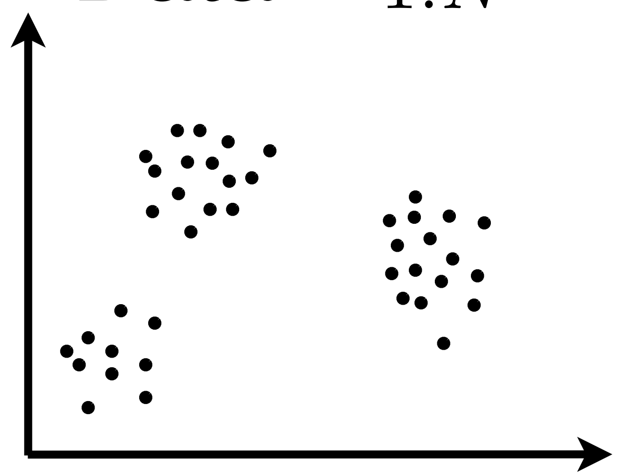
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

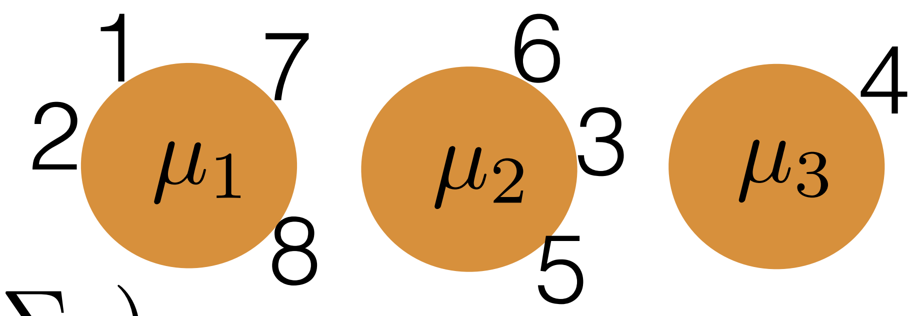
$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C \cup \{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ & \text{if } n \text{ starts a new cluster} \end{cases}$$

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model



$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$

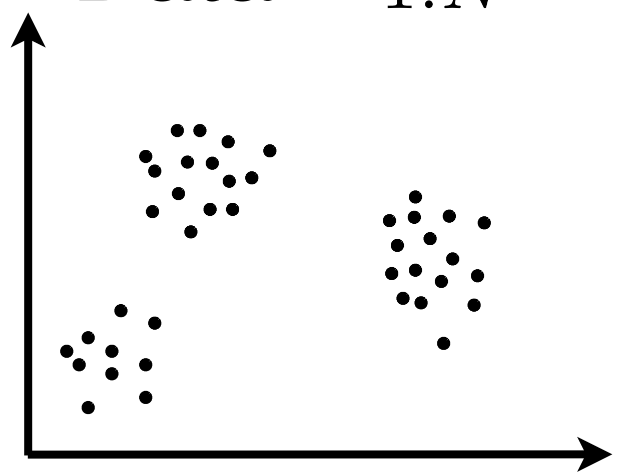
- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

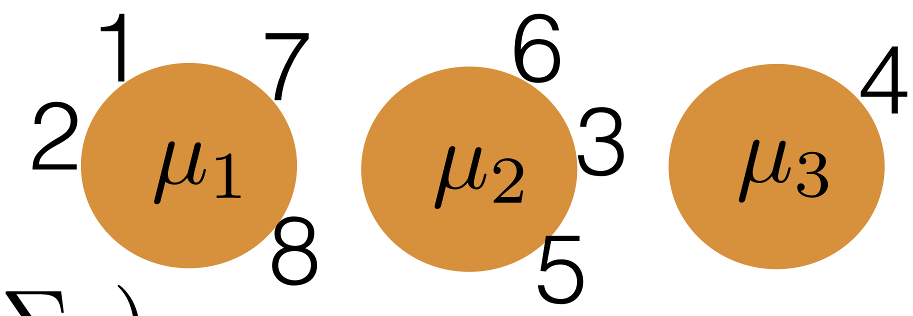
$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

CRP mixture model: inference

- Data $x_{1:N}$



- Generative model



$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$

- Want: posterior $p(\Pi_N | x_{1:N})$

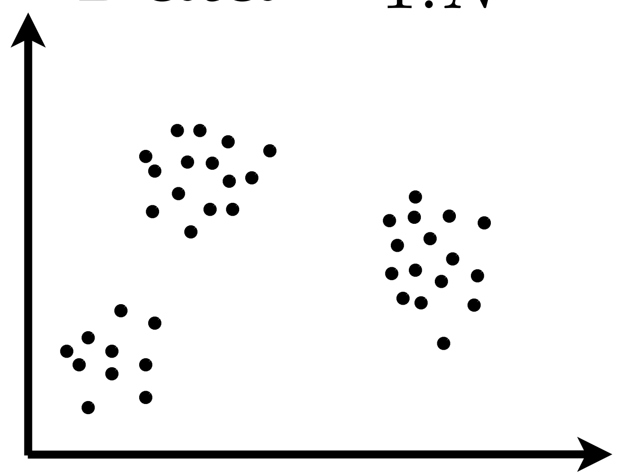
- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C\cup\{n\}} | x_C) =$

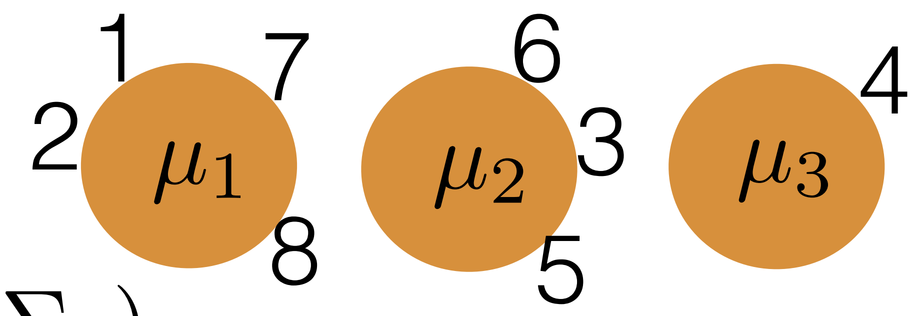
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$

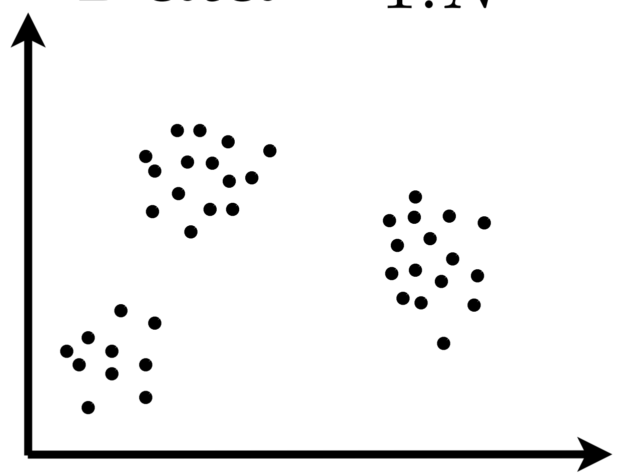
- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C\cup\{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

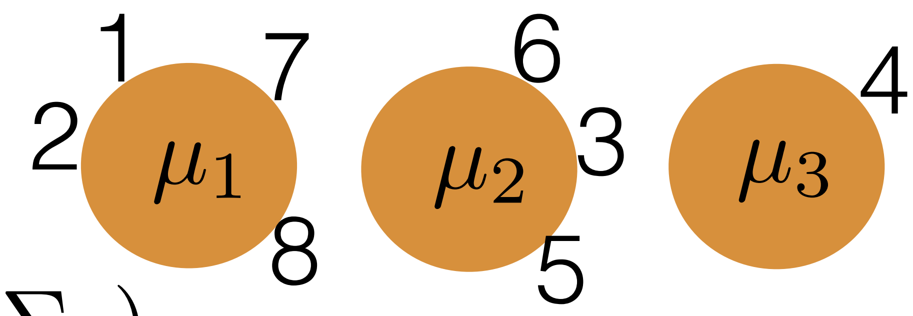
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$
- Gibbs sampler:

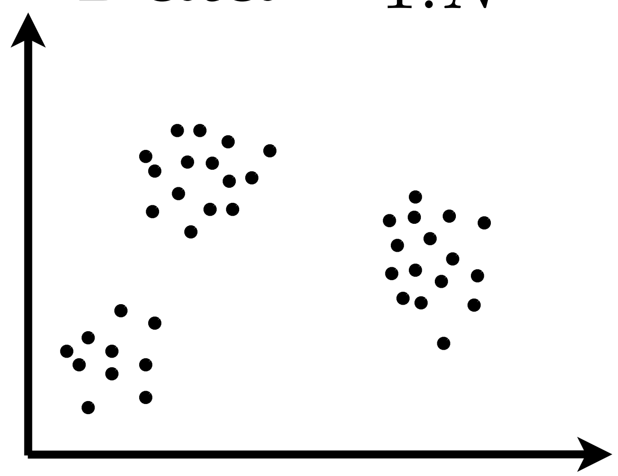
$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C\cup\{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$
$$\tilde{m} := \tilde{\Sigma}\left(\Sigma^{-1} \sum_{m\in C} x_m + \Sigma_0^{-1}\mu_0\right)$$

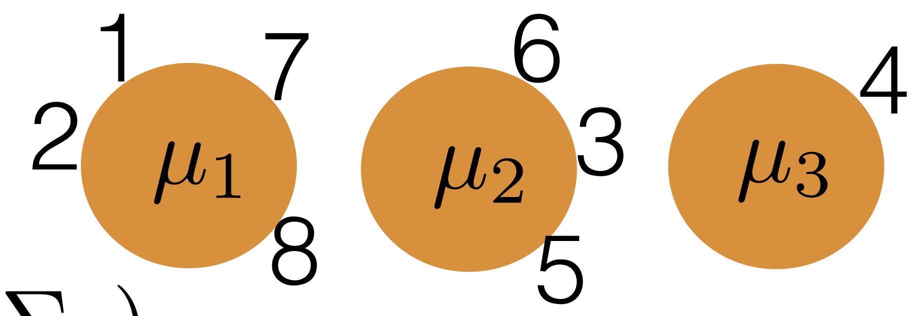
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

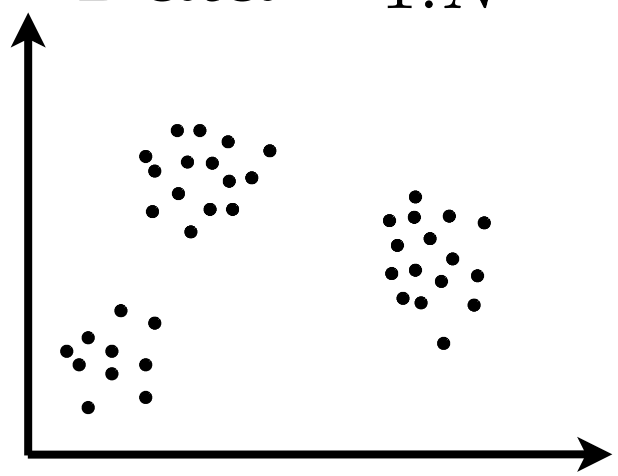
- For completeness: $p(x_{C\cup\{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$
$$\tilde{m} := \tilde{\Sigma}\left(\Sigma^{-1} \sum_{m \in C} x_m + \Sigma_0^{-1}\mu_0\right)$$

[MacEachern 1994; Neal 1992; Neal 2000]

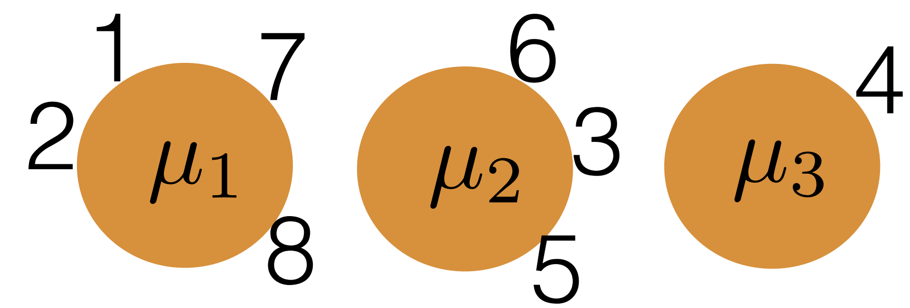
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \phi_C \overset{iid}{\sim} G_0$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C \cup \{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

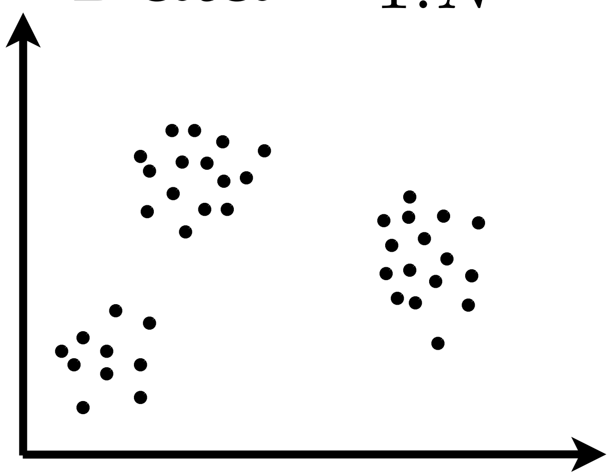
- For completeness: $p(x_{C \cup \{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$
$$\tilde{m} := \tilde{\Sigma}\left(\Sigma^{-1} \sum_{m \in C} x_m + \Sigma_0^{-1}\mu_0\right)$$

[MacEachern 1994; Neal 1992; Neal 2000]

CRP mixture model: inference

- Data $x_{1:N}$

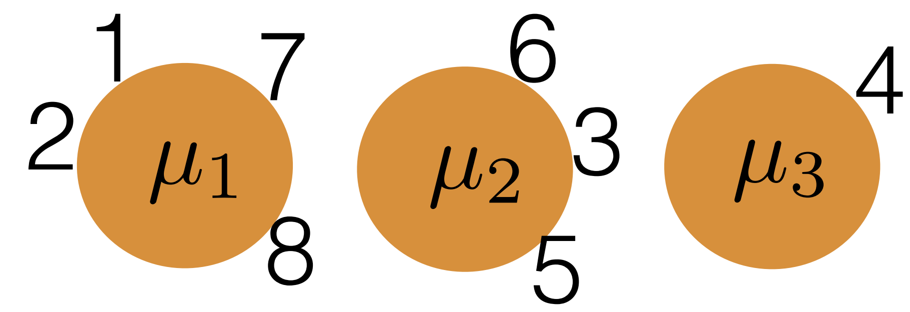


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \boxed{\phi_C \overset{iid}{\sim} G_0}$$

$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \boxed{F(\phi_C)}$$



- Want: posterior $p(\Pi_N | x_{1:N})$
- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha + N - 1} p(x_{C \cup \{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha + N - 1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C \cup \{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

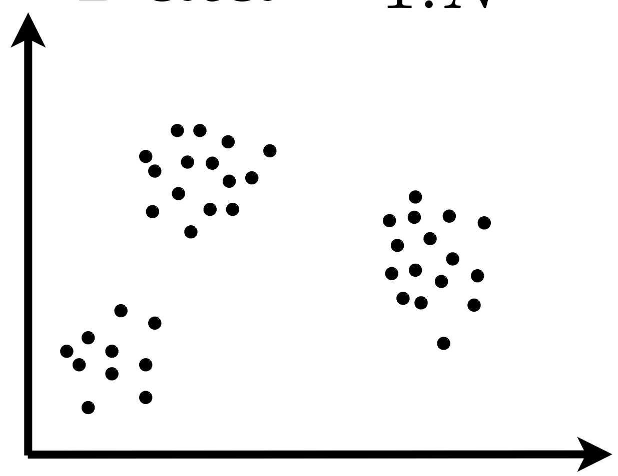
$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$

$$\tilde{m} := \tilde{\Sigma} \left(\Sigma^{-1} \sum_{m \in C} x_m + \Sigma_0^{-1} \mu_0 \right)$$

[MacEachern 1994; Neal 1992; Neal 2000]

CRP mixture model: inference

- Data $x_{1:N}$

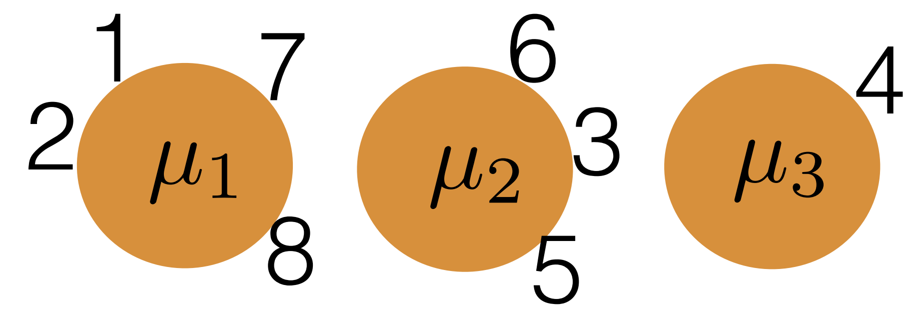


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \boxed{\phi_C \overset{iid}{\sim} G_0}$$

$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \boxed{F(\phi_C)}$$



- Want: posterior $p(\Pi_N | x_{1:N})$

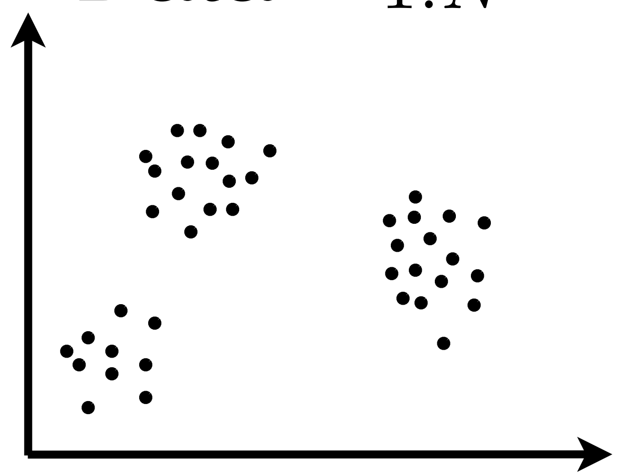
- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C\cup\{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

[MacEachern 1994; Neal 1992; Neal 2000]

CRP mixture model: inference

- Data $x_{1:N}$

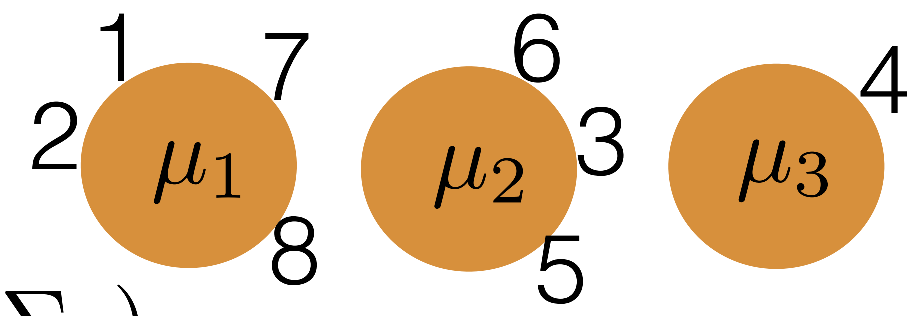


- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$

$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$

$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C \cup \{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C \cup \{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

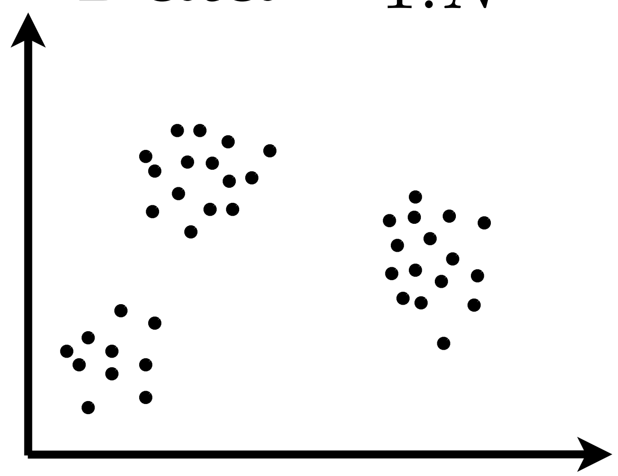
$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$

$$\tilde{m} := \tilde{\Sigma} \left(\Sigma^{-1} \sum_{m \in C} x_m + \Sigma_0^{-1} \mu_0 \right)$$

[MacEachern 1994; Neal 1992; Neal 2000]

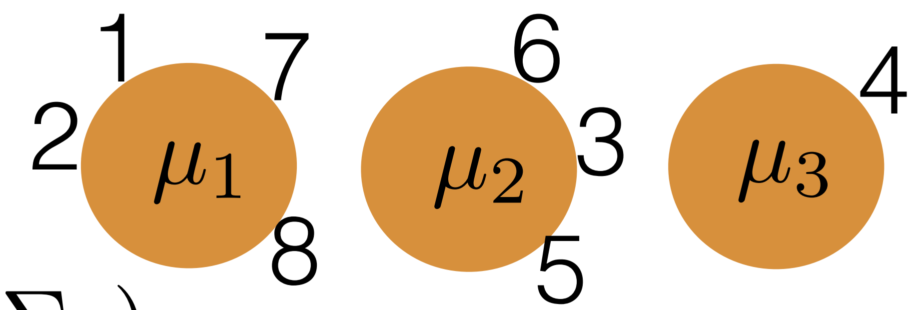
CRP mixture model: inference

- Data $x_{1:N}$



- Generative model

$$\Pi_N \sim \mathrm{CRP}(N, \alpha)$$
$$\forall C \in \Pi_N, \mu_C \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0)$$
$$\forall C \in \Pi_N, \forall n \in C, x_n \overset{indep}{\sim} \mathcal{N}(\mu_C, \Sigma)$$



- Want: posterior $p(\Pi_N | x_{1:N})$

- Gibbs sampler:

$$p(\Pi_N | \Pi_{N,\text{-}n}, x) \propto \begin{cases} \frac{\#C}{\alpha+N-1} p(x_{C \cup \{n\}} | x_C) & \text{if } n \text{ joins cluster } C \\ \frac{\alpha}{\alpha+N-1} p(x_{\{n\}}) & \text{if } n \text{ starts a new cluster} \end{cases}$$

- For completeness: $p(x_{C \cup \{n\}} | x_C) = \mathcal{N}(\tilde{m}, \tilde{\Sigma} + \Sigma)$

$$\tilde{\Sigma}^{-1} := \Sigma_0^{-1} + (\#C)\Sigma^{-1}$$
$$\tilde{m} := \tilde{\Sigma}\left(\Sigma^{-1} \sum_{m \in C} x_m + \Sigma_0^{-1}\mu_0\right)$$

[demo]

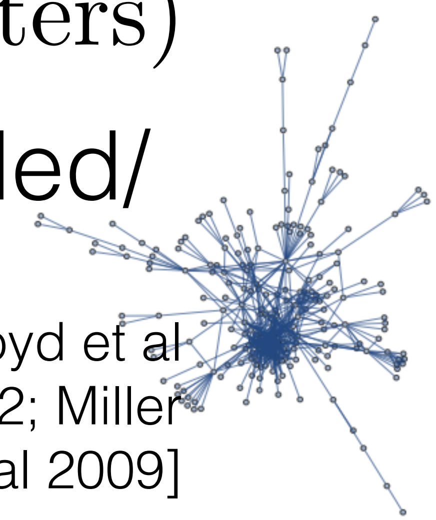
[MacEachern 1994; Neal 1992; Neal 2000]

Nonparametric Bayes

- Bayesian methods that are not parametric
- Bayesian

$$\mathbb{P}(\text{parameters}|\text{data}) \propto \mathbb{P}(\text{data}|\text{parameters})\mathbb{P}(\text{parameters})$$

- Not parametric (i.e. not finite parameter, unbounded/growing/infinite number of parameters)



[Lloyd et al 2012; Miller et al 2009]

[Eaton 2020]

[Fox et al 2014]

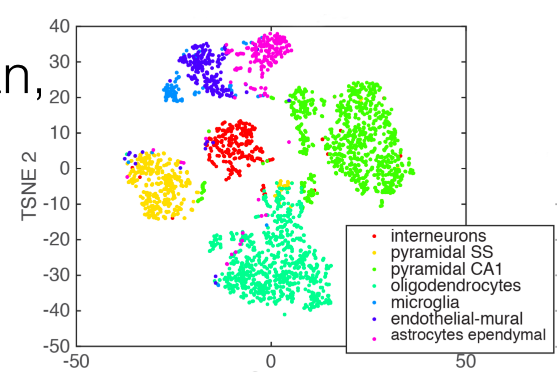
[MIT xPRO]

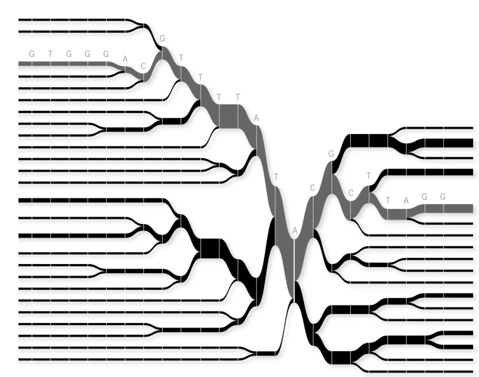
[Lan et al 2015]

[ESO/ L. Calçada/ M. Kornmesser 2017]

[Del Pozzo et al 2017, 2018]

[Prabhakaran, Azizi, Carr, Pe'er 2016]



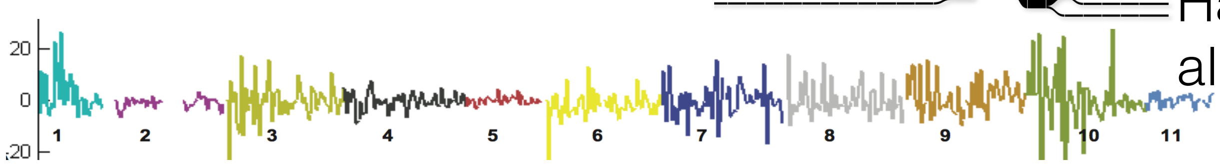


[Ewens 1972; Hartl, Clark 2003; Harris et al 2017]

[Xu et al 2015]

[Cassidy et al 2015]

[Saria et al 2010]



Roadmap

[slides, code:
www.tamarabroderick.com/tutorials.html]

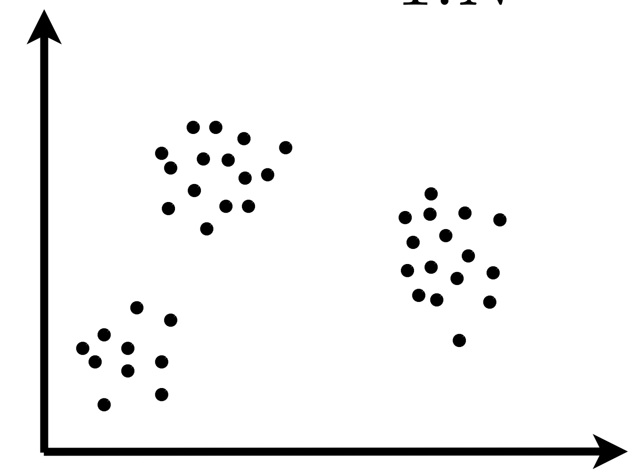
- Example problem: clustering
- Example NPBayes model: Dirichlet process
- Chinese restaurant process
- Inference
- Venture further into the wild world of Nonparametric Bayes

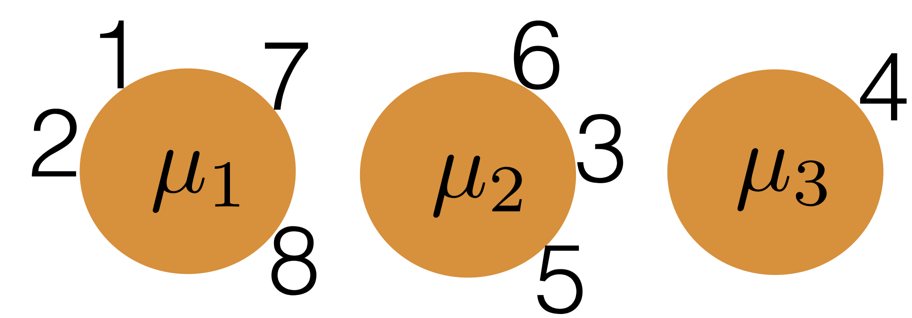
- Big questions
  - Why NPBayes? Learn more as acquire more data
  - What does an infinite/growing number of parameters really mean (in NPBayes)? Components vs. clusters; latent vs. realized
  - Why is NPBayes challenging but practical? Infinite dimensional parameter, but finitely many parameters realized

Exercises

- Can you find a formula for the expected # clusters from a Hoppe-urn(α) after N data points? What happens as $N \to \infty$
- Code a CRP mixture model simulator
- Derive the CRP mixture model Gibbs sampler in the slides
- Extend the CRP mixture model Gibbs sampler in the slides to sample the cluster-specific parameters as well

- Data $x_{1:N}$

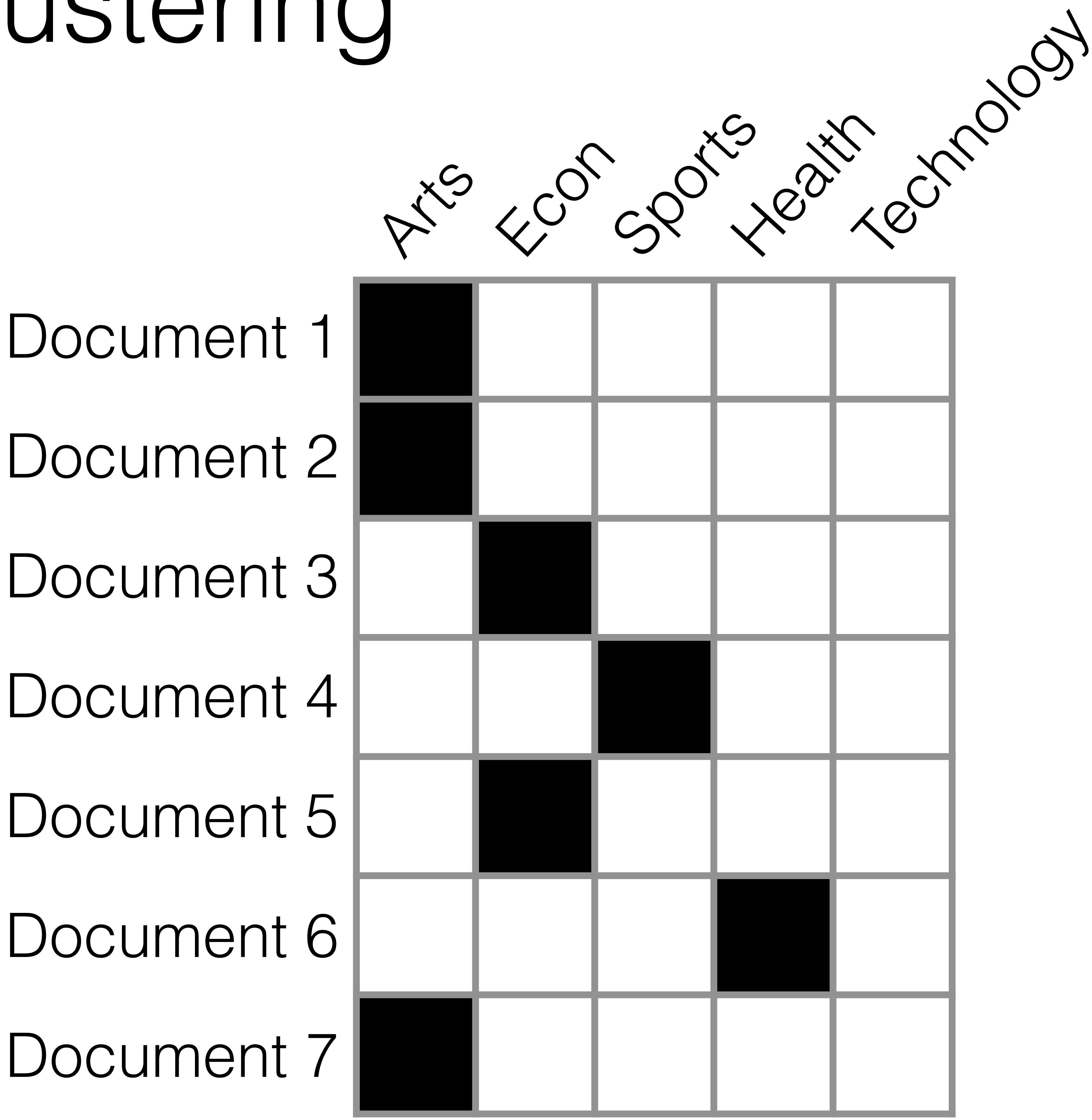




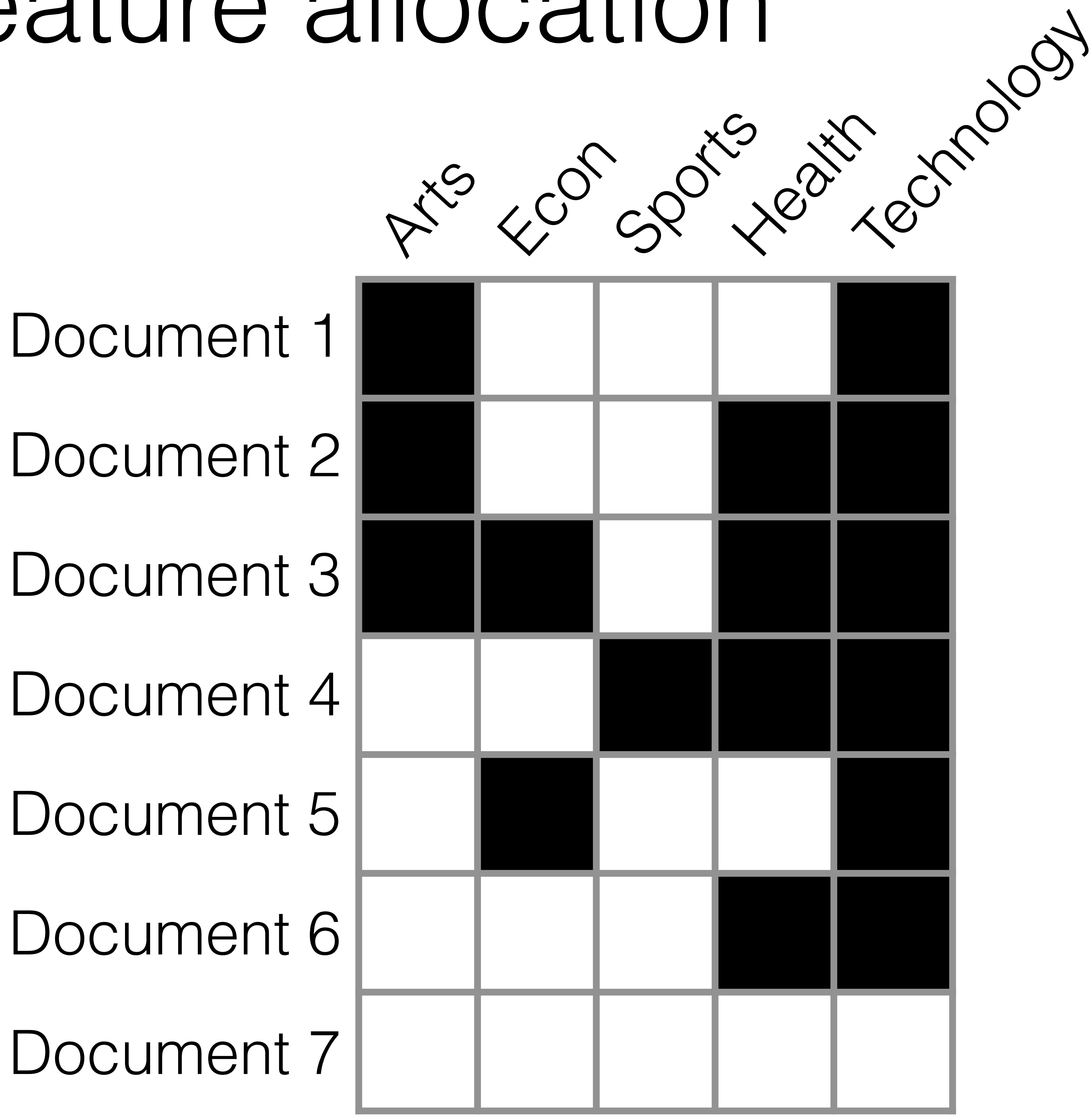
- Review slice sampling [Neal 2003], variational [Bishop 2006].
- Code a DPMM Gibbs sampler [Neal 2000]. Code a DPMM slice sampler [Walker 2007; Kalli, Griffin, Walker 2011].
- Read [Blei, Jordan 2006] & code DPMM variational inference

[Tiia Monto, https://commons.wikimedia.org/wiki/File:Trail_and_mountain.jpg]

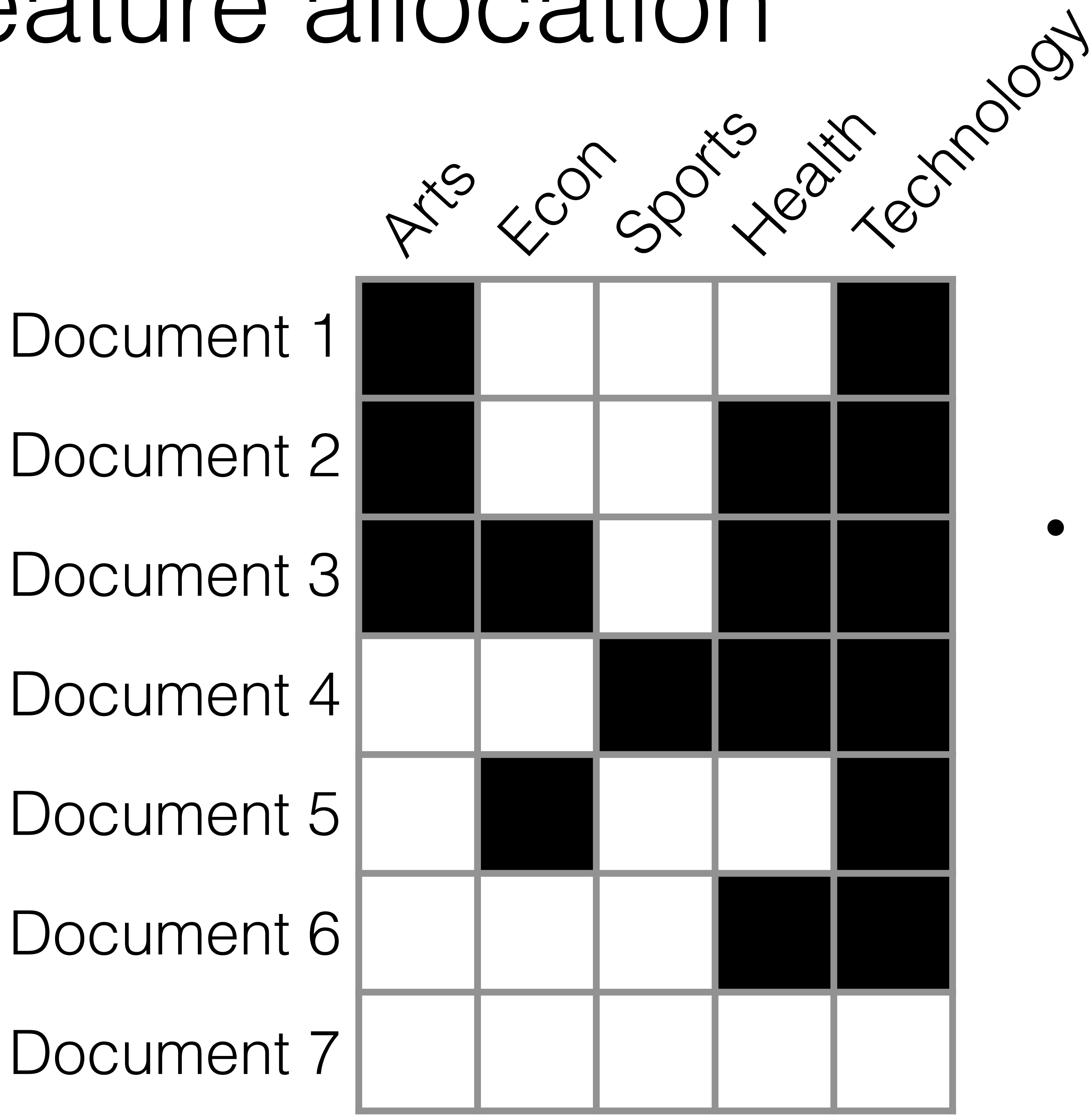
Clustering


Arts
Econ
Sports
Health
Technology
Document 1
Document 2
Document 3
Document 4
Document 5
Document 6
Document 7

Feature allocation

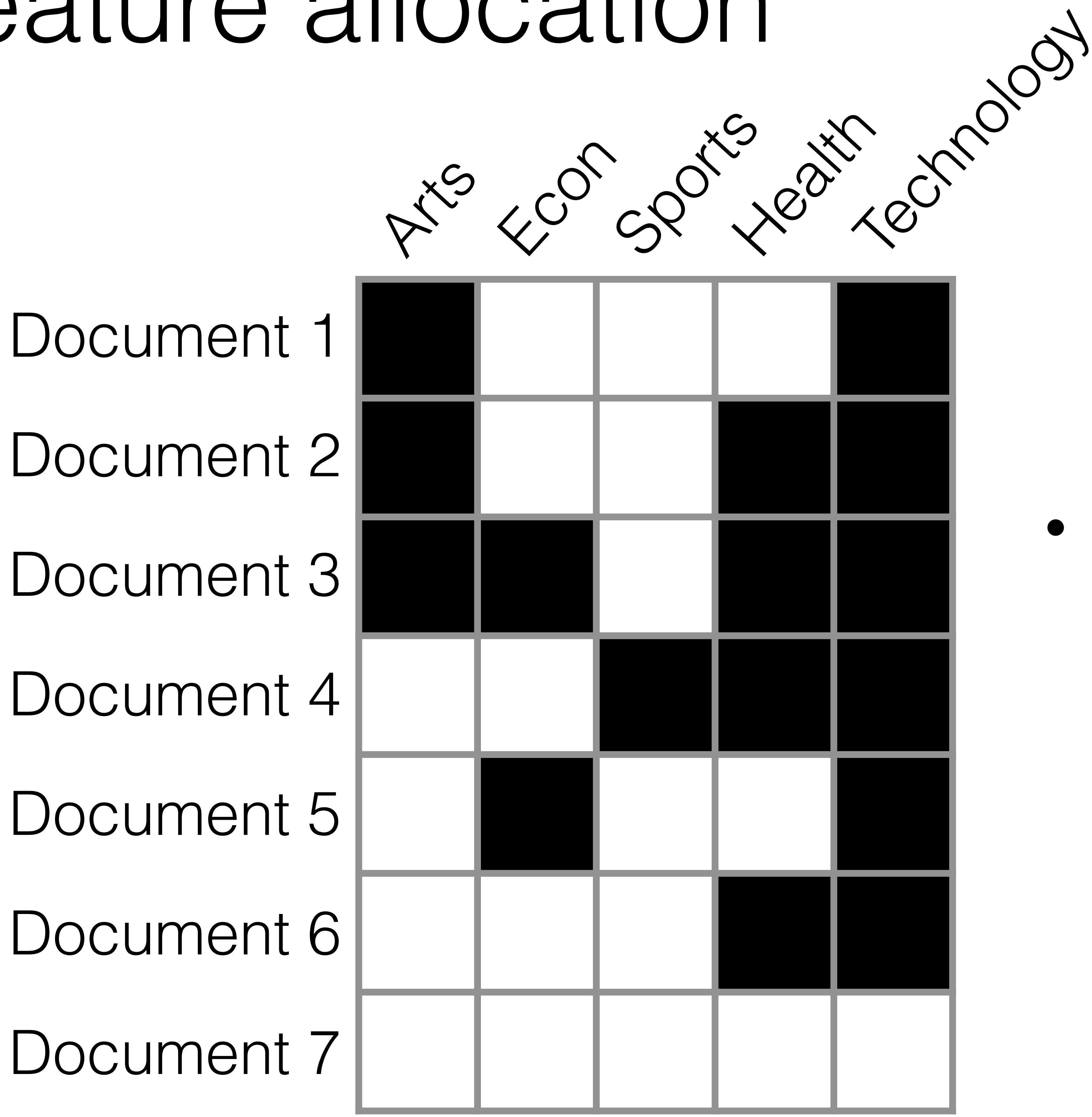


Feature allocation



- Indian buffet process

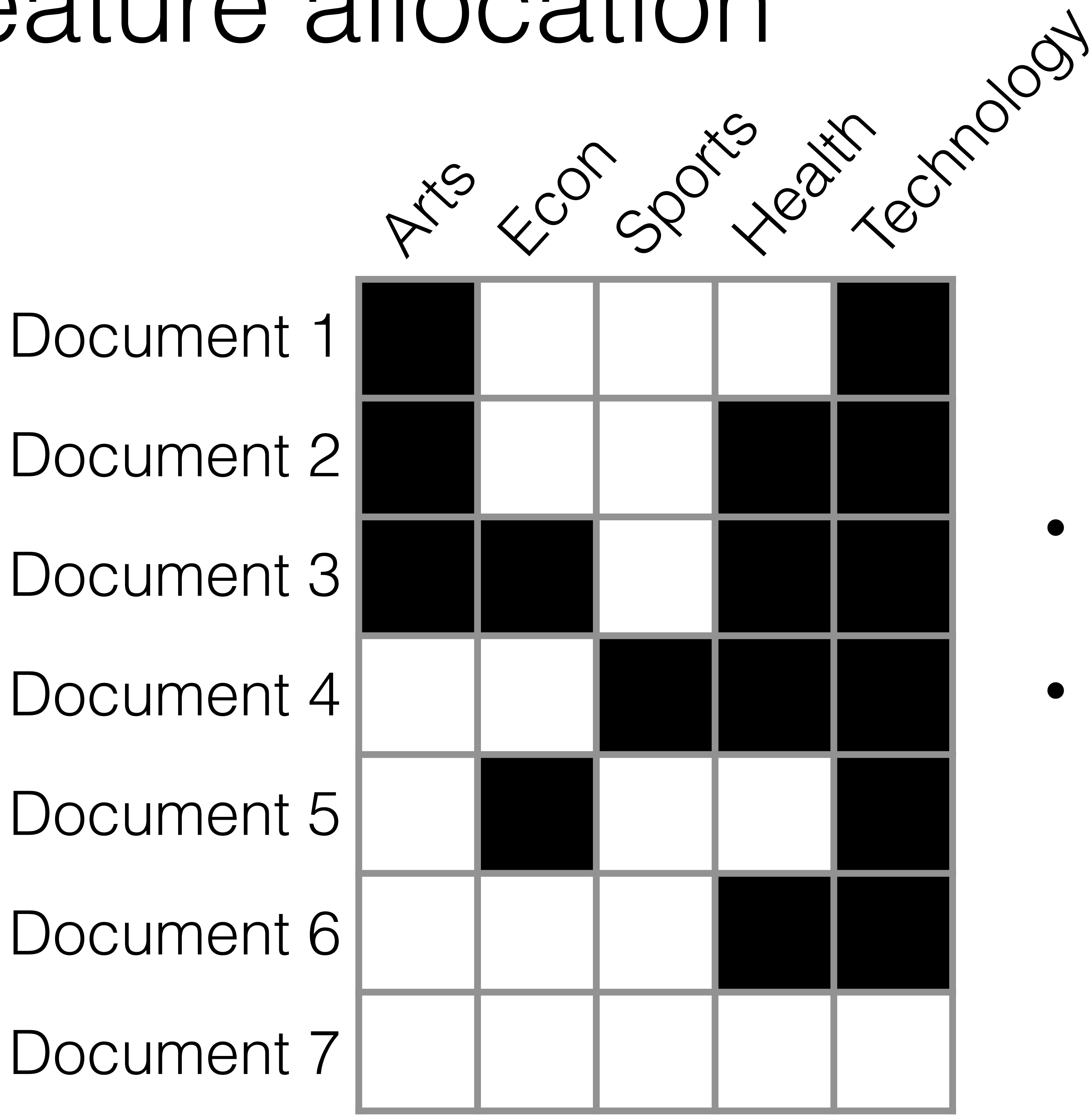
Feature allocation



- Indian buffet process

[Griffiths, Ghahramani 2005

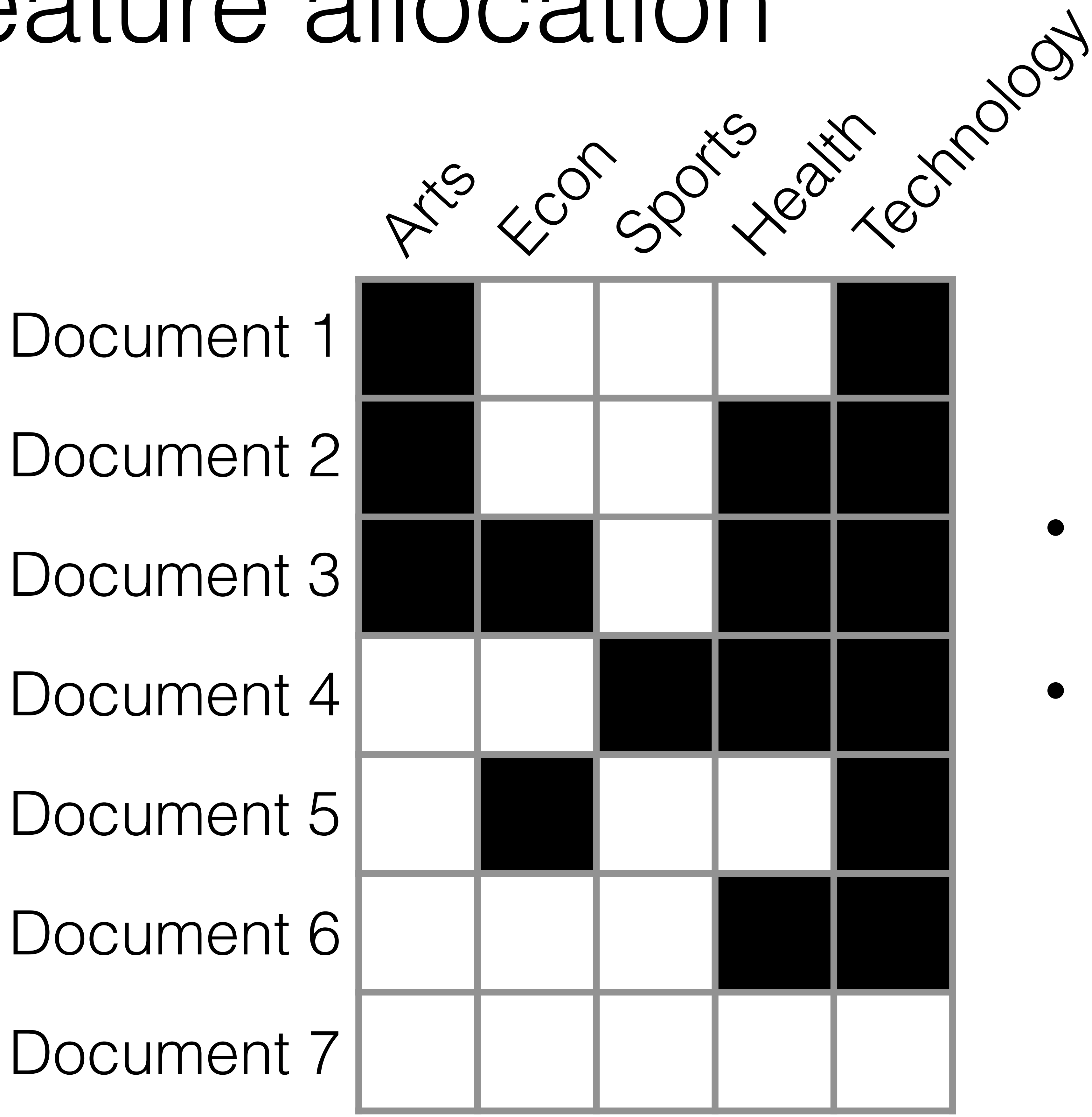
Feature allocation



- Indian buffet process
- Beta process

[Griffiths, Ghahramani 2005

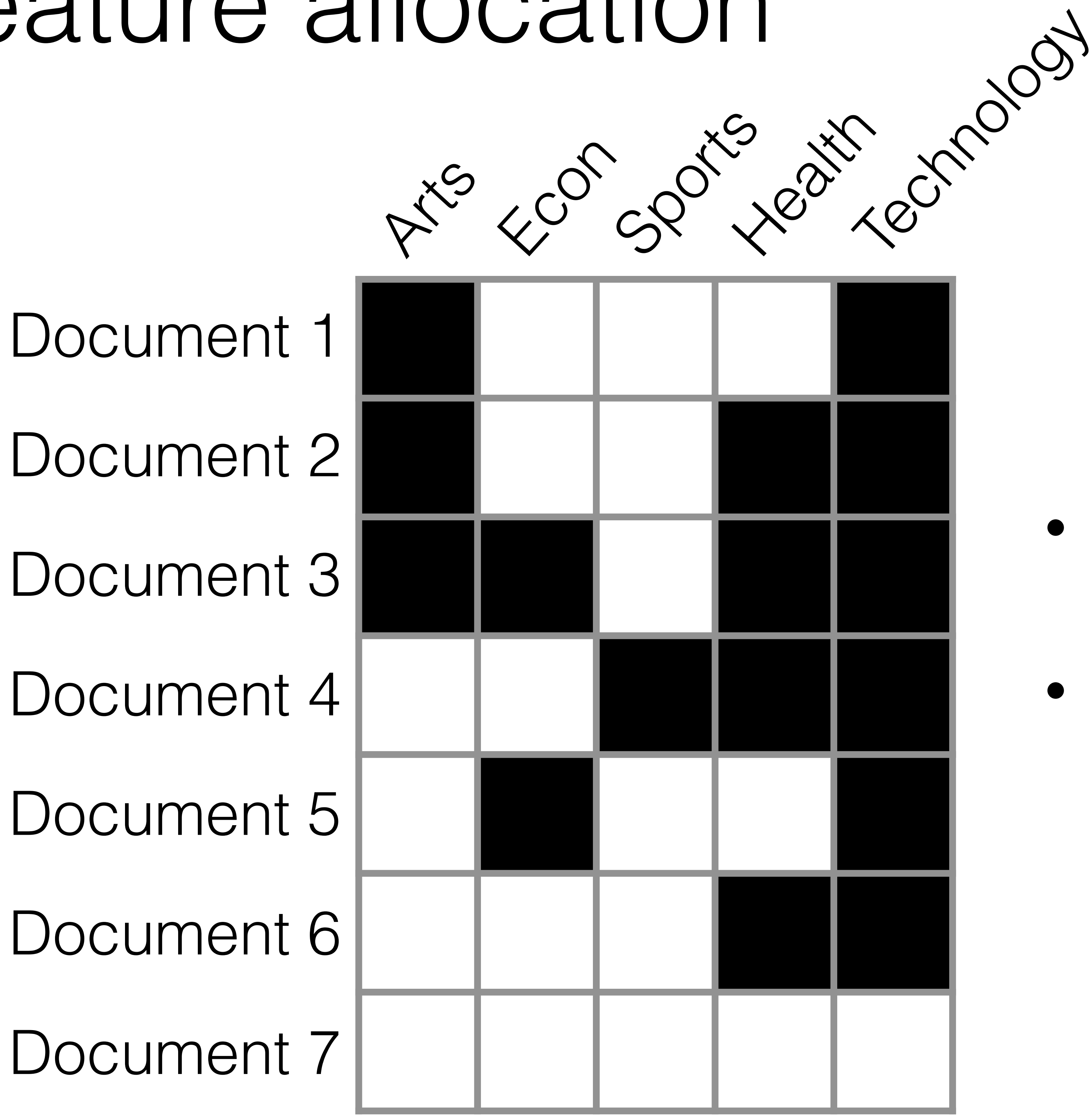
Feature allocation



- Indian buffet process
- Beta process

[Griffiths, Ghahramani 2005, Hjort 1990

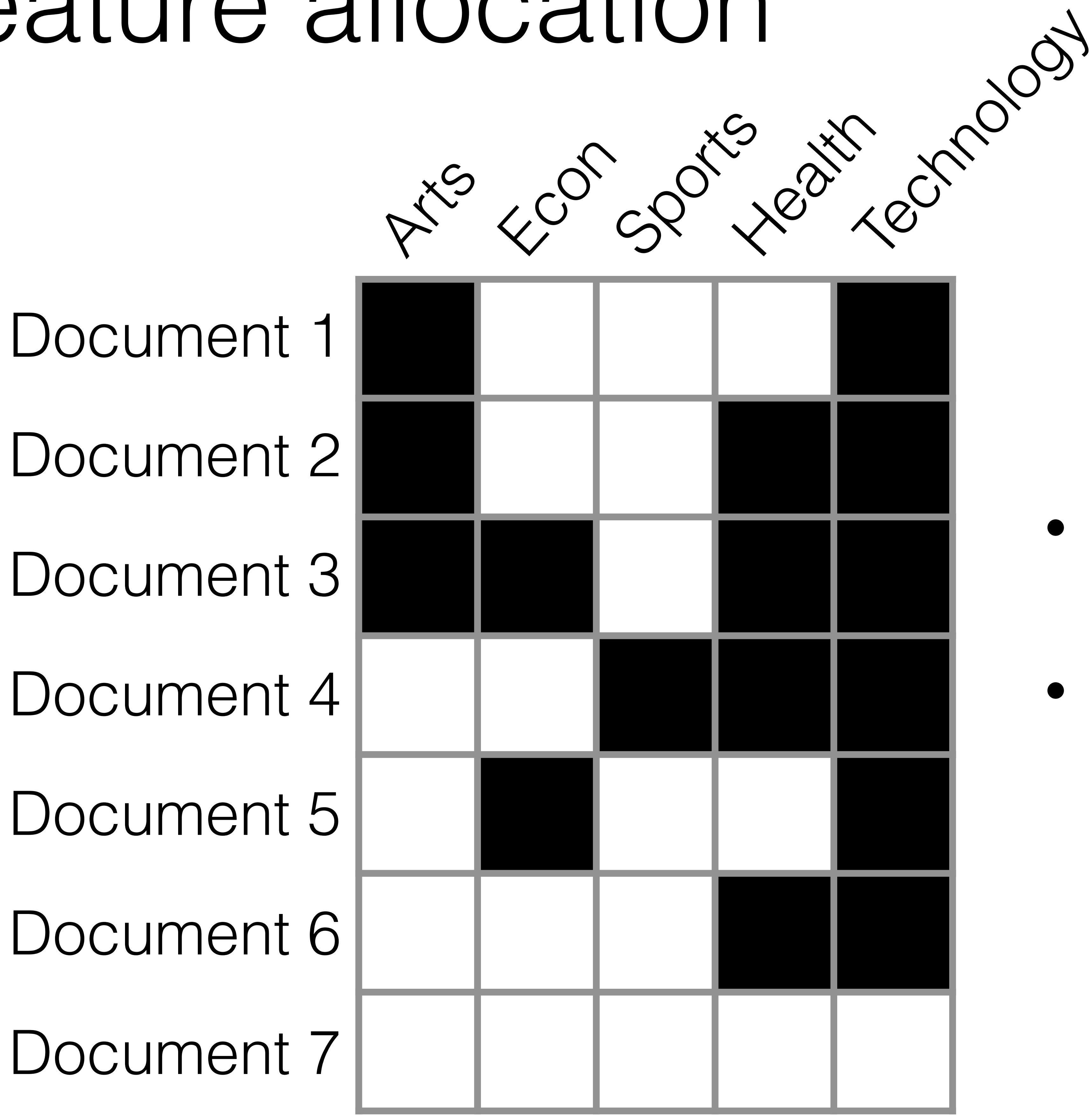
Feature allocation



- Indian buffet process
- Beta process

[Griffiths, Ghahramani 2005, Hjort 1990, Kim 1999, Thibaux, Jordan 2007, Broderick, Jordan, Pitman 2013]

Feature allocation



- Indian buffet process
- Beta process

[Griffiths, Ghahramani 2005, Hjort 1990, Kim 1999, Thibaux, Jordan 2007, Broderick, Jordan, Pitman 2013; Broderick, Pitman Jordan 2013]

Trait allocation

	Arts	Econ	Sports	Health	Technology
Document 1	30				15
Document 2	5			12	42
Document 3	3	9		19	28
Document 4			4	21	8
Document 5		14			26
Document 6				9	27
Document 7					

[Griffiths, Ghahramani 2005, Hjort 1990, Kim 1999, Thibaux, Jordan 2007, Broderick, Jordan, Pitman 2013; Broderick, Pitman Jordan 2013; Campbell, Cai, Broderick 2018]

Experimental design under a budget

[Masoero, Camerlenghi, Favaro, Broderick 2019]

Experimental design under a budget

[Masoero, Camerlenghi, Favaro, Broderick 2019]

Experimental design under a budget

- State-of-the-art performance under constant experimental conditions

[Masoero, Camerlenghi, Favaro, Broderick 2019]

Experimental design under a budget

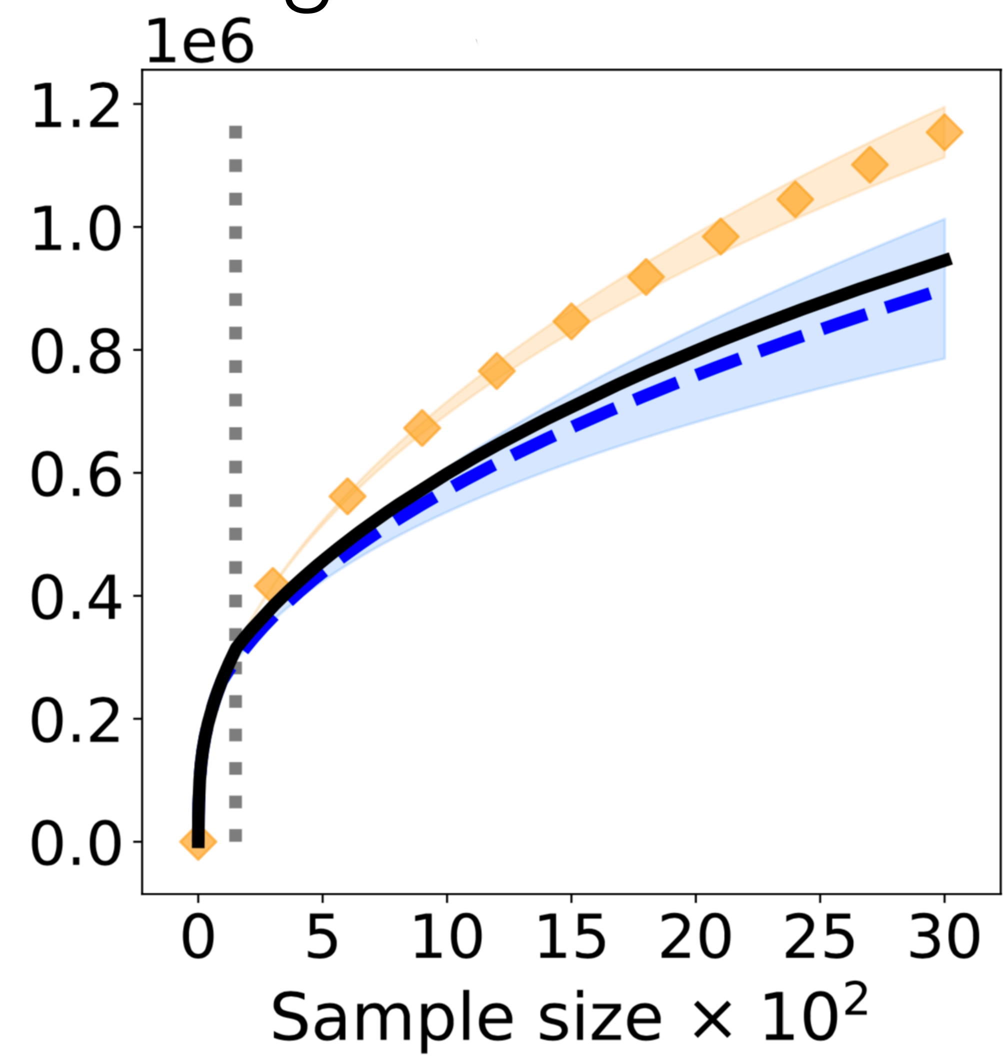
- State-of-the-art performance under constant experimental conditions
- Can estimate # new variants when experimental conditions change

[Masoero, Camerlenghi, Favaro, Broderick 2019]

Experimental design under a budget

- State-of-the-art performance under constant experimental conditions
- Can estimate # new variants when experimental conditions change

A subpopulation of the gnomAD data

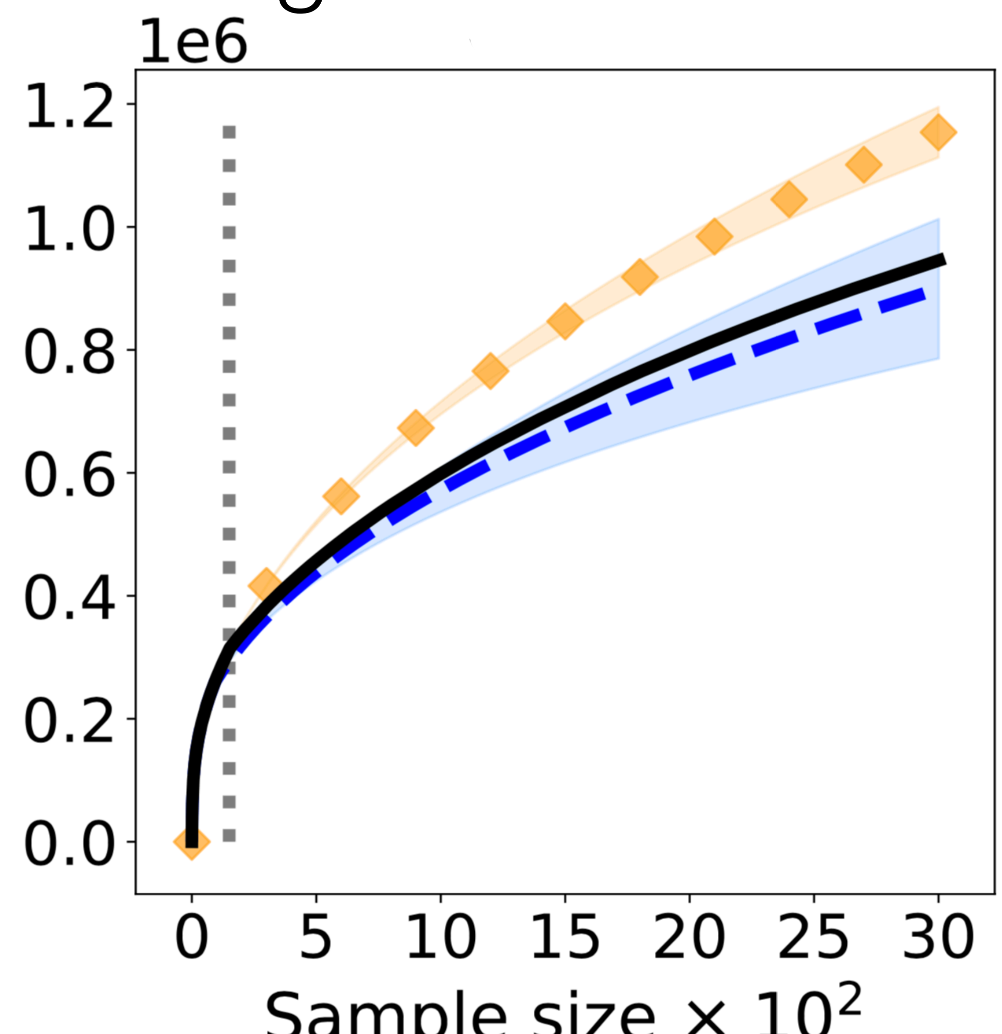


[Masoero, Camerlenghi, Favaro, Broderick 2019]

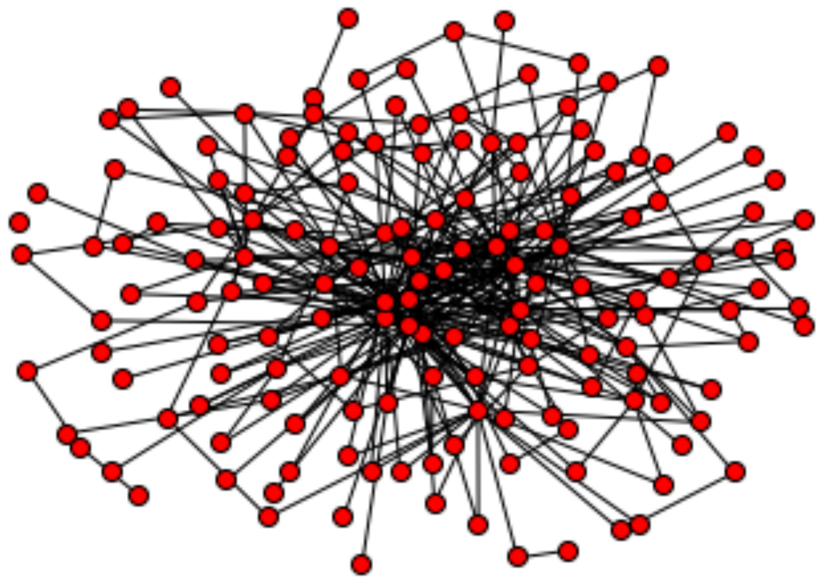
Experimental design under a budget

- State-of-the-art performance under constant experimental conditions
- Can estimate # new variants when experimental conditions change
- Can be used to optimize under a budget

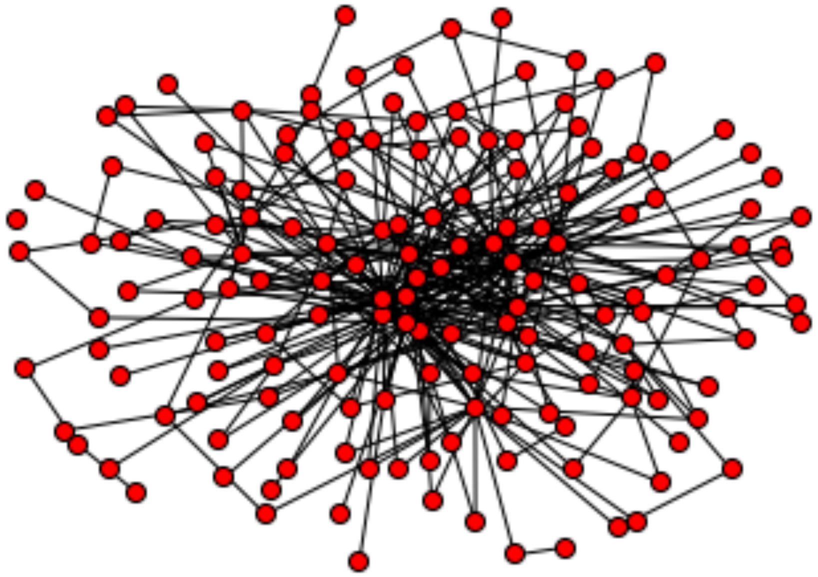
A subpopulation of the gnomAD data



[Masoero, Camerlenghi, Favaro, Broderick 2019]



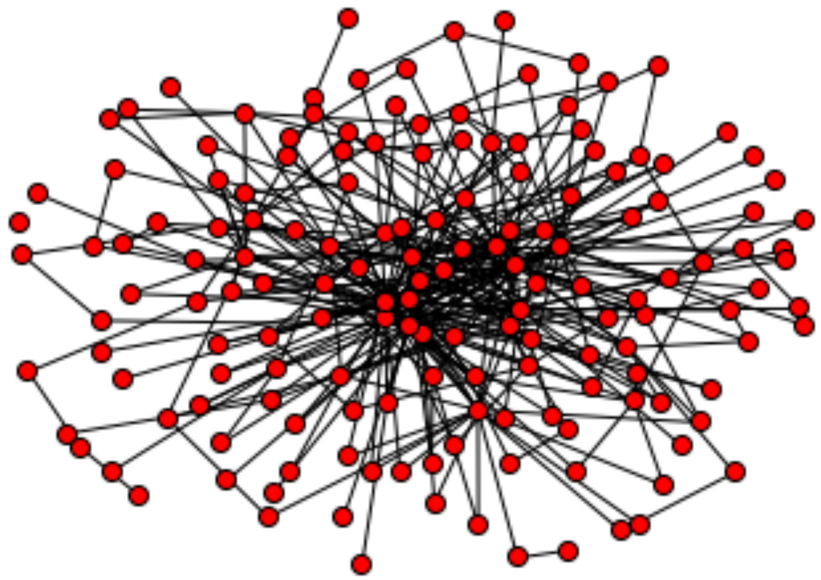
[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]



social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

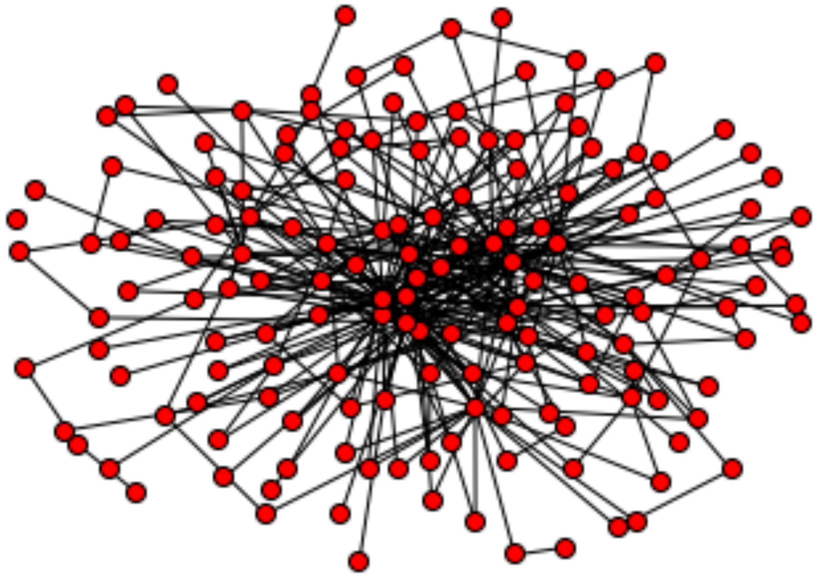
Probabilistic models for graphs

$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

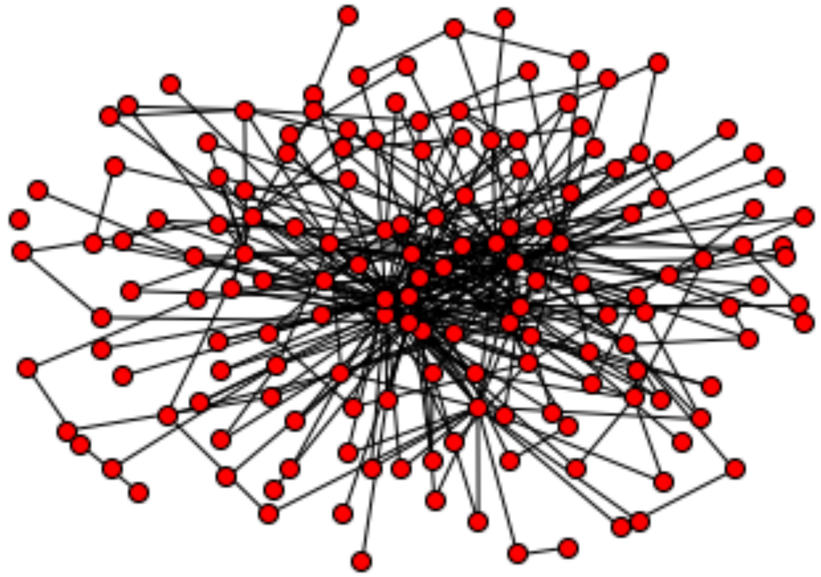
$$p\Big(\quad$$  $$\quad \Big)$$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

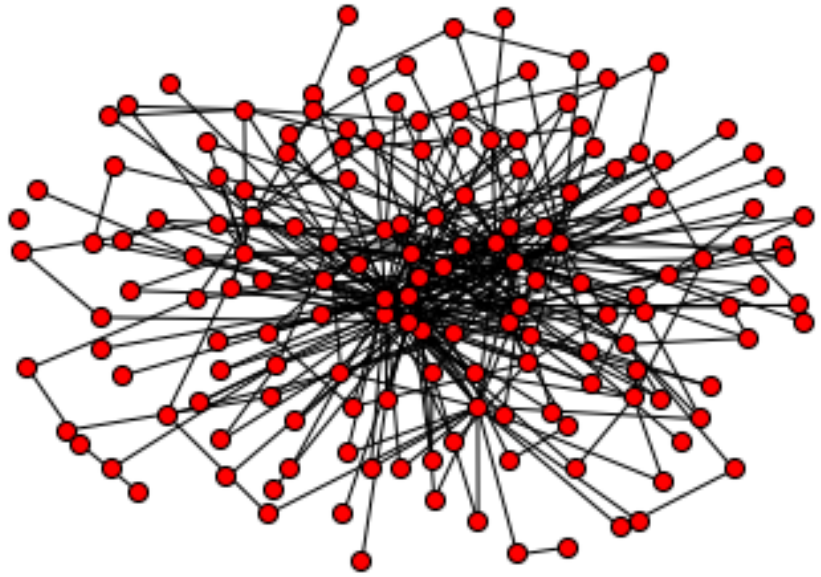
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models:

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008]

Probabilistic models for graphs

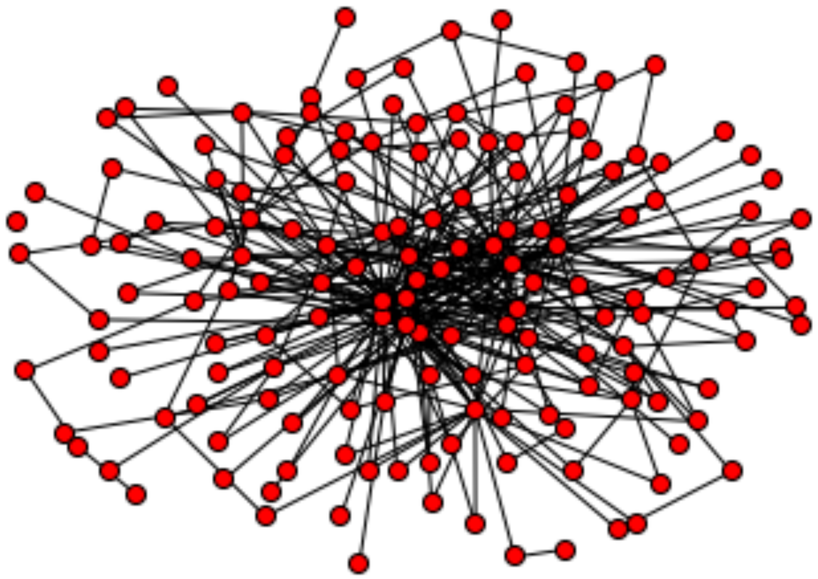
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models:
  - Stochastic block model

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008]

Probabilistic models for graphs

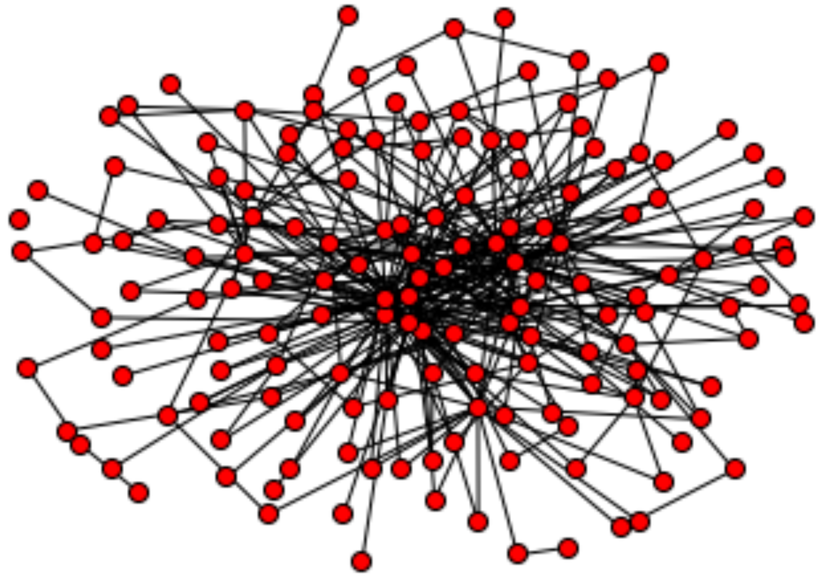
$p\Big($  $\Big)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models:
  - Stochastic block model
  - Mixed membership stochastic block model

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008]

Probabilistic models for graphs

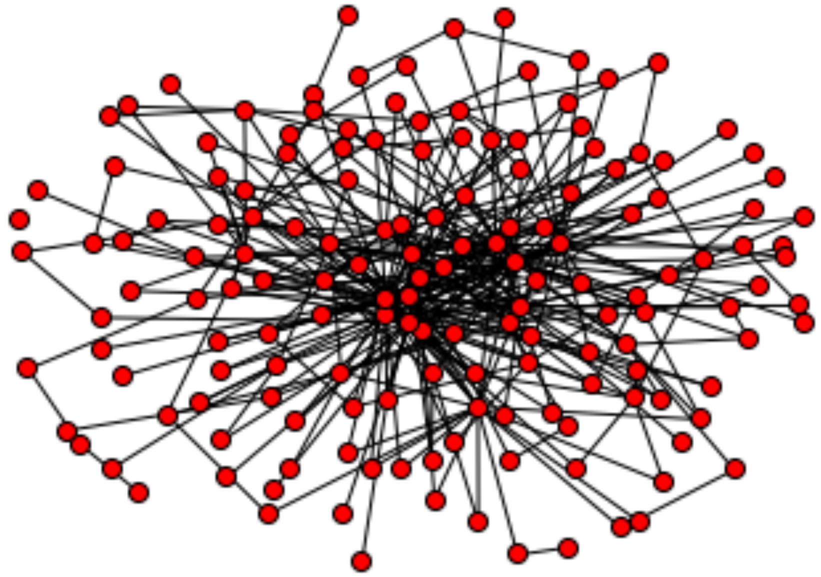
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models:
  - Stochastic block model
  - Mixed membership stochastic block model
  - Infinite relational model

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008]

Probabilistic models for graphs

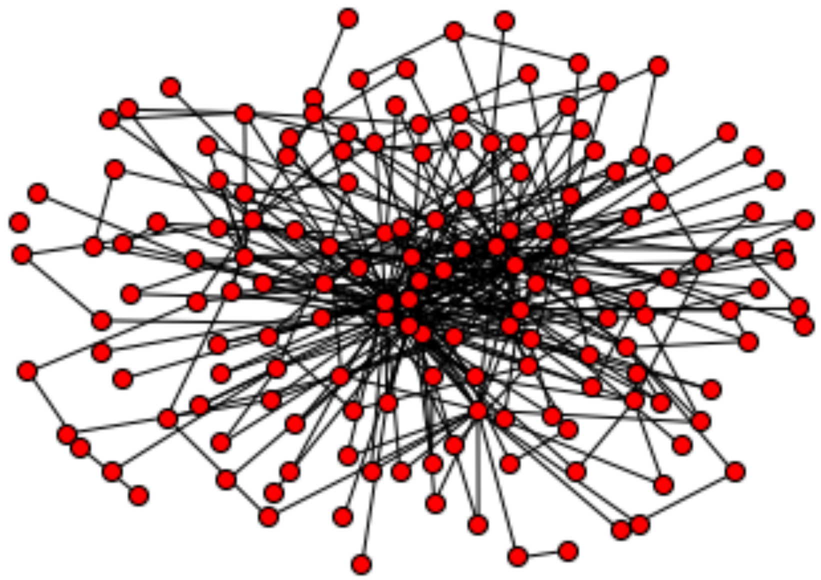
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models:
  - Stochastic block model
  - Mixed membership stochastic block model
  - Infinite relational model
  - Latent space model
  - Eigenmodel
  - Latent feature relational model
  - Infinite latent attribute model
  - Sparse matrix-variate Gaussian process block model
  - Random function model

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

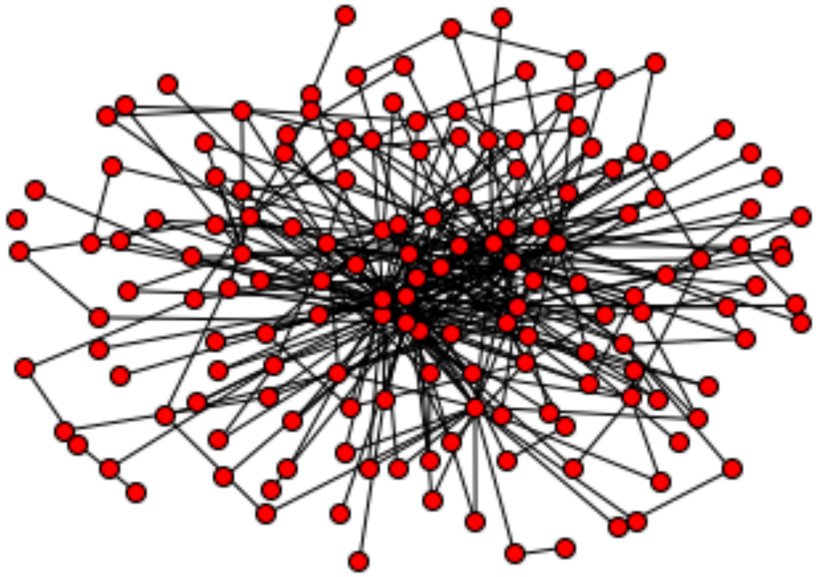
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models: Stochastic block, mixed membership stochastic block, infinite relational, and ***many*** more

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

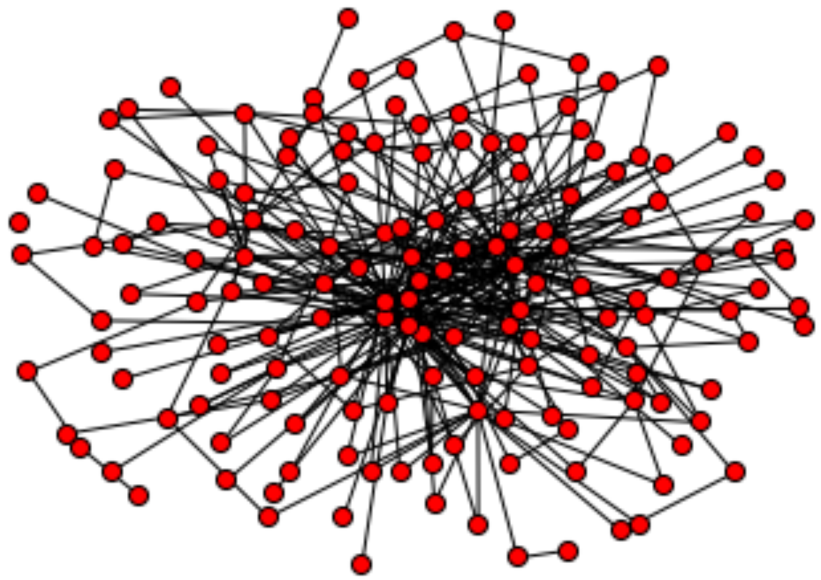
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models: Stochastic block, mixed membership stochastic block, infinite relational, and many more
- Assume: Adding more data doesn't change distribution of earlier data (*projectivity*)

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

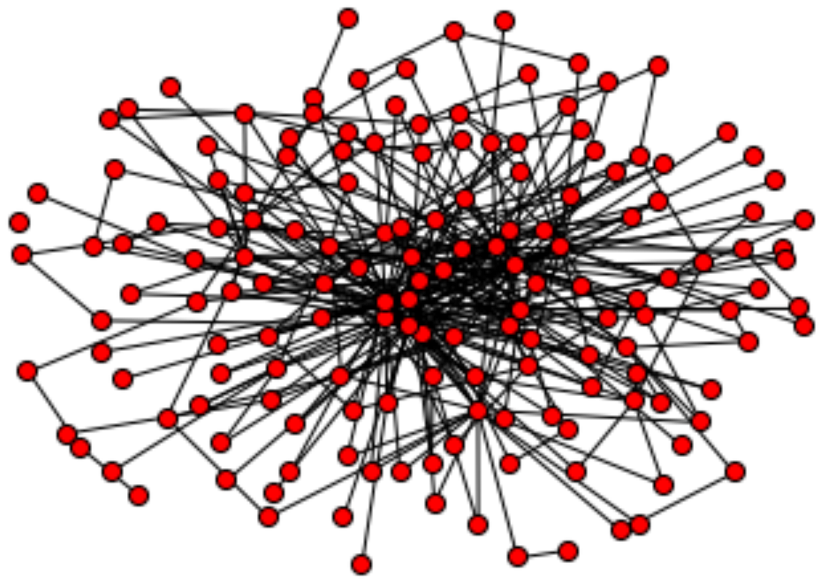
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models: Stochastic block, mixed membership stochastic block, infinite relational, and many more
- Assume: Adding more data doesn't change distribution of earlier data (*projectivity*)
- **Problem**: model misspecification, dense graphs

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

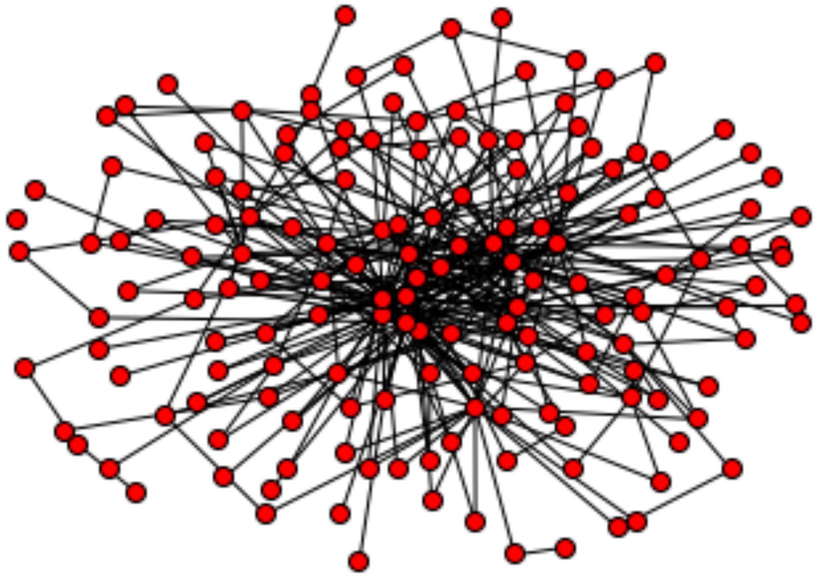
$p($  $)$

social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models: Stochastic block, mixed membership stochastic block, infinite relational, and many more
- Assume: Adding more data doesn't change distribution of earlier data (*projectivity*)
- **Problem**: model misspecification, dense graphs
- **Solution**: an alternative framework for sparse graphs

[Holland et al 1983; Kemp et al 2006; Xu et al 2007; Airoldi et al 2008; Lloyd et al 2012]

Probabilistic models for graphs

$p($  $)$

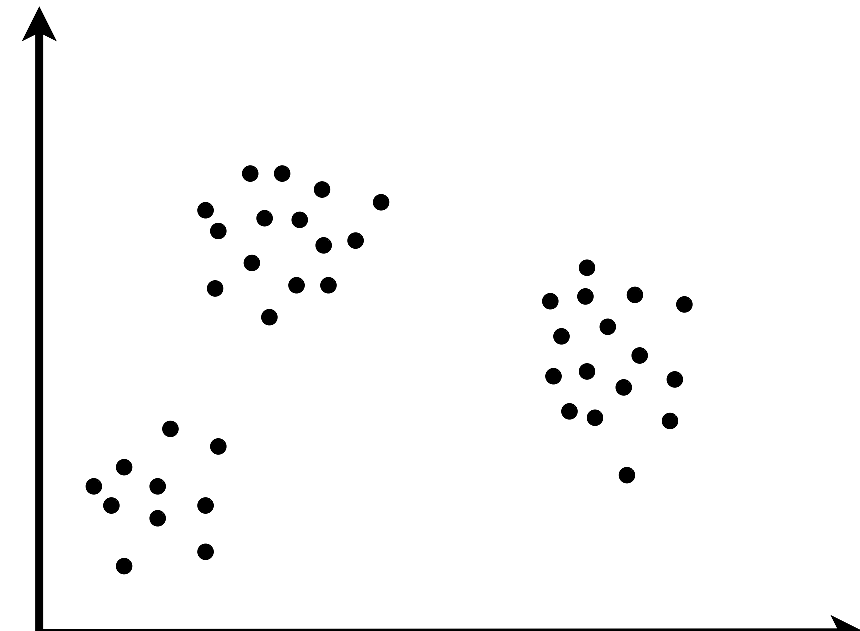
social: *Facebook, Twitter, email*
biological: *ecological, protein, gene*
transportation: *roads, railways*

- Interpretable, flexible, coherent uncertainties, prior info
- Example models: Stochastic block, mixed membership stochastic block, infinite relational, and many more
- Assume: Adding more data doesn't change distribution of earlier data (*projectivity*)
- **Problem**: model misspecification, dense graphs
- **Solution**: an alternative framework for sparse graphs
  - Don't miss independent graphs work by Crane & Dempsey

[Broderick, Cai 2015; Cai, Broderick 2015a,b; Crane, Dempsey 2015a,b,16a,b; Cai, Campbell, Broderick 2016; Campbell, Cai, Broderick 2016; Campbell, Cai, Broderick 2018; Williamson 2016]

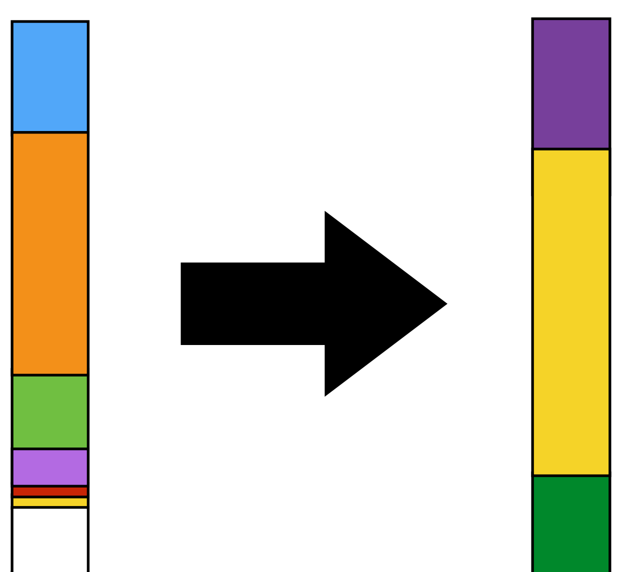
DP or not DP, that is the question

- GEM: 
- Compare to:
  - Finite (small K) mixture model



  - Finite (large K) mixture model

  - Time series

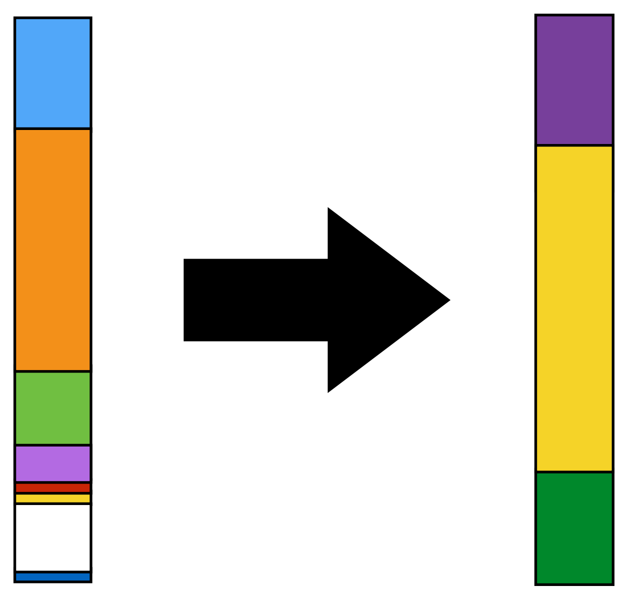


Local Exchangeability

Recall

- GEM: 

- vs time series



[Campbell, Syed, Yang, Jordan, Broderick 2019]

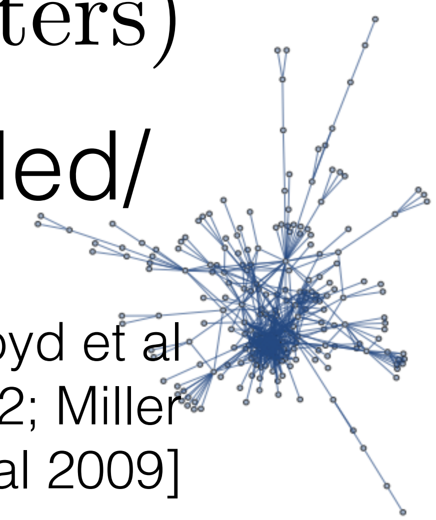
[Tiia Monto, https://commons.wikimedia.org/wiki/File:Trail_and_mountain.jpg]

Nonparametric Bayes

- Bayesian methods that are not parametric
- Bayesian

$$\mathbb{P}(\text{parameters}|\text{data}) \propto \mathbb{P}(\text{data}|\text{parameters})\mathbb{P}(\text{parameters})$$

- Not parametric (i.e. not finite parameter, unbounded/growing/infinite number of parameters)



[Lloyd et al 2012; Miller et al 2009]

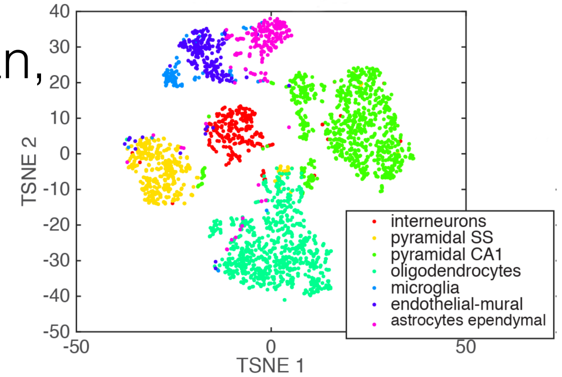
[Eaton 2020]

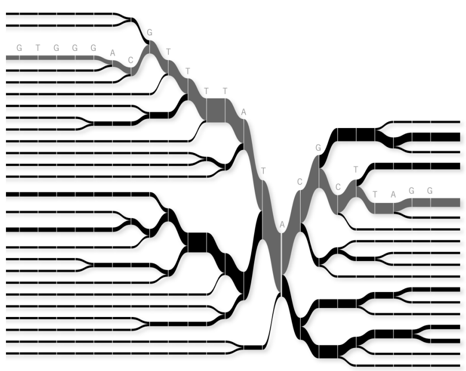
[Fox et al 2014]

[MIT xPRO]

[Lan et al 2015]

[Prabhakaran, Azizi, Carr, Pe'er 2016]





[Ewens 1972; Hartl, Clark 2003; Harris et al 2017]

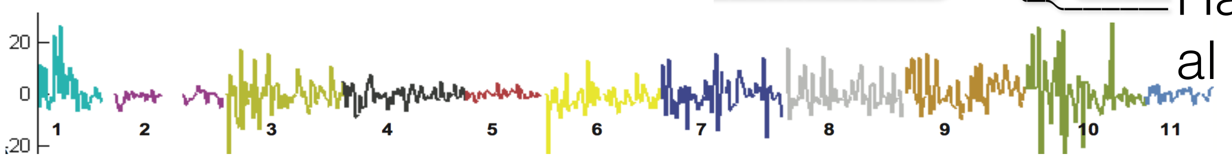
[Xu et al 2015]

[Cassidy et al 2015]

[ESO/ L. Calçada/ M. Kornmesser 2017]

[Del Pozzo et al 2017, 2018]

[Saria et al 2010]



References (1/7)

DJ Aldous. *Exchangeability and related topics*. Springer, 1983.

CE Antoniak. Mixtures of Dirichlet processes with applications to Bayesian nonparametric problems. *The Annals of Statistics*, 1974.

J Bertoin. *Random Fragmentation and Coagulation Processes*. Cambridge University Press, 2006.

D Blackwell and JB MacQueen. Ferguson distributions via Pólya urn schemes. *The Annals of Statistics*, 1973.

T Broderick, MI Jordan, and J Pitman. Beta processes, stick-breaking, and power laws. *Bayesian Analysis*, 2012.

T Broderick, MI Jordan, and J Pitman. Cluster and feature modeling from combinatorial stochastic processes. *Statistical Science*, 2013.

T Broderick, J Pitman, and MI Jordan. Feature allocations, probability functions, and paintboxes. *Bayesian Analysis*, 2013.

T Broderick, AC Wilson, and MI Jordan. Posteriors, conjugacy, and exponential families for completely random measures. *Bernoulli*, 2018.

D Cai, T Campbell, and T Broderick. Edge-exchangeable graphs and sparsity. *NeurIPS*, 2016.

- *NeurIPS 2015 Workshop on Networks in the Social & Information Sciences*.
- *NeurIPS 2015 Workshop, Bayesian Nonparametrics: The Next Generation.*

T Campbell, D Cai, and Broderick T. Exchangeable trait allocations. *Electronic Journal of Statistics*, 2018. https://arxiv.org/abs/1609.09147

- *NeurIPS 2016 Workshop on Adaptive & Scalable Nonparametric Methods in ML.*
- *NeurIPS 2016 Workshop on Practical Bayesian Nonparametrics.*

References (2/7)

T Campbell, JH Huggins, JP How, and T Broderick. Truncated random measures. *Bernoulli*, 2019. https://arxiv.org/abs/1603.00861

T Campbell, S Syed, CYang, MI Jordan, and T Broderick. Local exchangeability. https://arxiv.org/abs/1906.09507

H Crane and W Dempsey. Atypical scaling behavior persists in real world interaction networks. arXiv 1509.08184, 2015.

H Crane and W Dempsey. A framework for statistical network modeling. arXiv 1509.08185, 2015.

H Crane and W Dempsey. Edge exchangeable models for network data. arXiv 1603.04571, 2016.

H Crane and W Dempsey. Relational exchangeability. arXiv 1607.06762, 2016.

W Del Pozzo, TGF Li, and C Messenger. Cosmological inference using only gravitational wave observations of binary neutron stars. *Physical Review D*, 2017.

W Del Pozzo, CPL Berry, A Ghosh, TSF Haines, LP Singer, and A Vecchio. Dirichlet Process Gaussian-mixture model: An application to localizing coalescing binary neutron stars with gravitational-wave observations. *Monthly Notices of the Royal Astronomical Society,* 2018.

S Engen. A note on the geometric series as a species frequency model. *Biometrika*, 1975.

MD Escobar and M West. Bayesian density estimation and inference using mixtures. *Journal of the American Statistical Association*, 1995.

W Ewens. The sampling theory of selectively neutral alleles. *Theoretical Population Biology*, 1972.

W Ewens. Population genetics theory -- the past and the future. *Mathematical and Statistical Developments of Evolutionary Theory*, 1987.

TS Ferguson. A Bayesian analysis of some nonparametric problems. *The Annals of Statistics*, 1973.

References (3/7)

TS Ferguson. Bayesian density estimation by mixtures of normal distributions. *Recent Advances in Statistics*, 1983.

EB Fox, personal website. Retrieved in 2016 from: http://www.stat.washington.edu/~ebfox/research.html --- Associated paper: EB Fox, MC Hughes, EB Sudderth, and MI Jordan. *The Annals of Applied Statistics*, 2014.

S Ghosal, JK Ghosh, and RV Ramamoorthi. Posterior consistency of Dirichlet mixtures in density estimation. *The Annals of Statistics*, 1999.

S Goldwater, TL Griffiths, and M Johnson. Interpolating between types and tokens by estimating power-law generators. *NeurIPS*, 2005.

A Gnedin, B Hansen, and J Pitman. Notes on the occupancy problem with infinitely many boxes: general asymptotics and power laws. *Probability Surveys*, 2007.

TL Griffiths and Z Ghahramani. Infinite latent feature models and the Indian buffet process. *NeurIPS*, 2005.

K Harris, TL Parsons, UZ Ijaz, L Lahti, I Holmes, and C Quince. Linking statistical and ecological theory: Hubbell's unified neutral theory of biodiversity as a hierarchical Dirichlet process. *Proceedings of the IEEE*, 2017.

DL Hartl and AG Clark. *Principles of Population Genetics, Fourth Edition*. 2003.

E Hewitt and LJ Savage. Symmetric measures on Cartesian products. *Transactions of the American Mathematical Society*, 1955.

NL Hjort. Nonparametric Bayes estimators based on beta processes in models for life history data. *The Annals of Statistics*, 1990.

DN Hoover. Relations on probability spaces and arrays of random variables, *Preprint, Institute for Advanced Study*, 1979.

FM Hoppe. Pólya-like urns and the Ewens' sampling formula. *Journal of Mathematical Biology*, 1984.

References (4/7)

H Ishwaran and LF James. Gibbs sampling methods for stick-breaking priors. *Journal of the American Statistical Association*, 2001.

L James. Poisson latent feature calculus for generalized Indian buffet processes. arXiv preprint arXiv:1411.2936, 2014.

Y Kim. Nonparametric Bayesian estimators for counting processes. *The Annals of Statistics*, 1999.

JFC Kingman. The representation of partition structures. *Journal of the London Mathematical Society*, 1978.

JFC Kingman. On the genealogy of large populations. *Journal of Applied Probability*, 1982.

JFC Kingman. *Poisson processes*, 1992.

AS Lan, D Vats, AE Waters, and RG Baraniuk. Mathematical language processing: Automatic grading and feedback for open response mathematical questions. In *Proceedings of the Second (2015) ACM Conference on Learning@Scale*, 2015.

JR Lloyd, P Orbanz, Z Ghahramani, and DM Roy. Random function priors for exchangeable arrays with applications to graphs and relational data. *NeurIPS*, 2012.

SN MacEachern and P Müller. Estimating mixtures of Dirichlet process models. *Journal of Computational and Graphical Statistics*, 1998.

L Masoero, F Camerlenghi, S Favaro, and T Broderick. More for less: Predicting and maximizing genetic variant discovery via Bayesian nonparametrics. https://arxiv.org/abs/1912.05516

JW McCloskey. A model for the distribution of individuals by species in an environment. *Ph.D. thesis, Michigan State University*, 1965.

K Miller, MI Jordan, and TL Griffiths. Nonparametric latent feature models for link prediction. *NeurIPS*, 2009.

References (5/7)

RM Neal. Density modeling and clustering using Dirichlet diffusion trees. *Bayesian Statistics*, 2003.

P Orbanz. Construction of nonparametric Bayesian models from parametric Bayes equations. *NeurIPS*, 2009.

P Orbanz. Conjugate Projective Limits. arXiv preprint arXiv:1012.0363, 2010.

P Orbanz, DM Roy. Bayesian models of graphs, arrays and other exchangeable random structures. *IEEE TPAMI*, 2015.

GP Patil and C Taillie. Diversity as a concept and its implications for random communities. *Bulletin of the International Statistical Institute*, 1977.

J Pitman and M Yor. The two-parameter Poisson-Dirichlet distribution derived from a stable subordinator. *The Annals of Probability*, 1997.

S Prabhakaran, E Azizi, A Carr, and D Pe'er. Dirichlet process mixture model for correcting technical variation in single-cell gene expression data. *ICML*, 2016.

A Rodríguez, DB Dunson & AE Gelfand. The nested Dirichlet process. *Journal of the American Statistical Association*, 2008.

S Saria, D Koller, and A Penn. Learning individual and population traits from clinical temporal data. *NeurIPS*, 2010.

J Sethuraman. A constructive definition of Dirichlet priors. *Statistica Sinica*, 1994.

EB Sudderth and MI Jordan. Shared segmentation of natural scenes using dependent Pitman-Yor processes. *NeurIPS*, 2009.

YW Teh. A hierarchical Bayesian language model based on Pitman-Yor processes. *ACL*, 2006.

YW Teh, C Blundell, and L Elliott. Modelling genetic variations using fragmentation-coagulation processes. *NeurIPS*, 2011.

References (6/7)

YW Teh, MI Jordan, MJ Beal, and DM Blei. Hierarchical Dirichlet processes. *Journal of the American Statistical Association*, 2006.

R Thibaux and MI Jordan. Hierarchical beta processes and the Indian buffet process. *ICML*, 2007.

J Wakeley. *Coalescent Theory: An Introduction*, Chapter 3, 2008.

M West, P Müller, and MD Escobar. Hierarchical Priors and Mixture Models, With Application in Regression and Density Estimation. *Aspects of Uncertainty*, 1994.

Y Xu, P Müller, Y Yuan, K Gulukota, and Y Ji. MAD Bayes for tumor heterogeneity—feature allocation with exponential family sampling. *Journal of the American Statistical Association*, 2015.

Image References (7/7)

E Bowlby. NOAA/Olympic Coast NMS; NOAA/OAR/Office of Ocean Exploration - NOAA Photo Library. Retrieved in 2016 from: https://en.wikipedia.org/wiki/Opisthoteuthis_californiana#/media/File:Opisthoteuthis_californiana.jpg

JW Cassidy, C Caldas, and A Bruna. Maintaining tumor heterogeneity in patient-derived tumor xenografts. *Cancer research*, 2015.

ESO/L. Calçada/M. Kornmesser. 16 October 2017, 16:00:00. Obtained in 2018 from: https://commons.wikimedia.org/wiki/File:Artist%E2%80%99s_impression_of_merging_neutron_stars.jpg || Source: https://www.eso.org/public/images/eso1733a/ (Creative Commons Attribution 4.0 International License)

MIT xPro, 2017. Obtained in 2018 from: https://mitxpro.mit.edu/courses/course-v1:MITProfessionalX+DSx+2017_T2/about

J Eaton/Birdtour Liverpool. Via New Scientist. “Scientists have discovered five new species of songbird in Indonesia.” Obtained in Jan 2020 from: https://www.newscientist.com/article/2229579-scientists-have-discovered-five-new-species-of-songbird-in-indonesia/