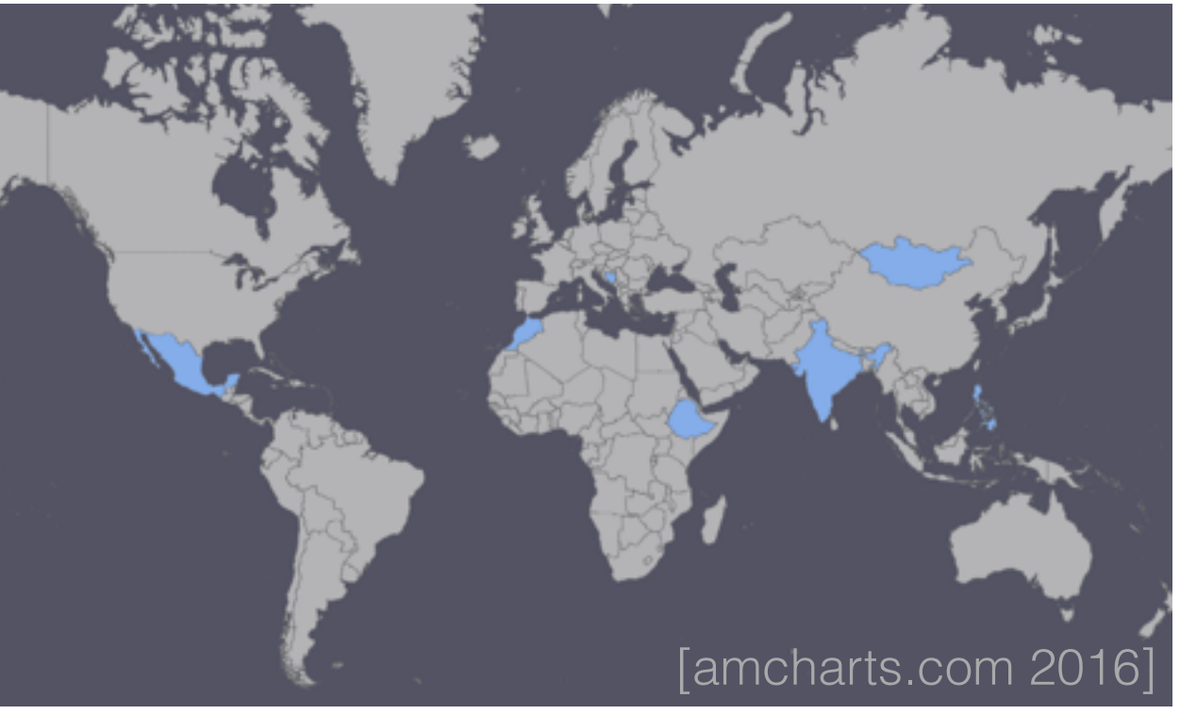
Coresets for automated, scalable Bayesian inference

Tamara Broderick
ITT Career Development
Assistant Professor,
MIT

With: Trevor Campbell, Jonathan H. Huggins

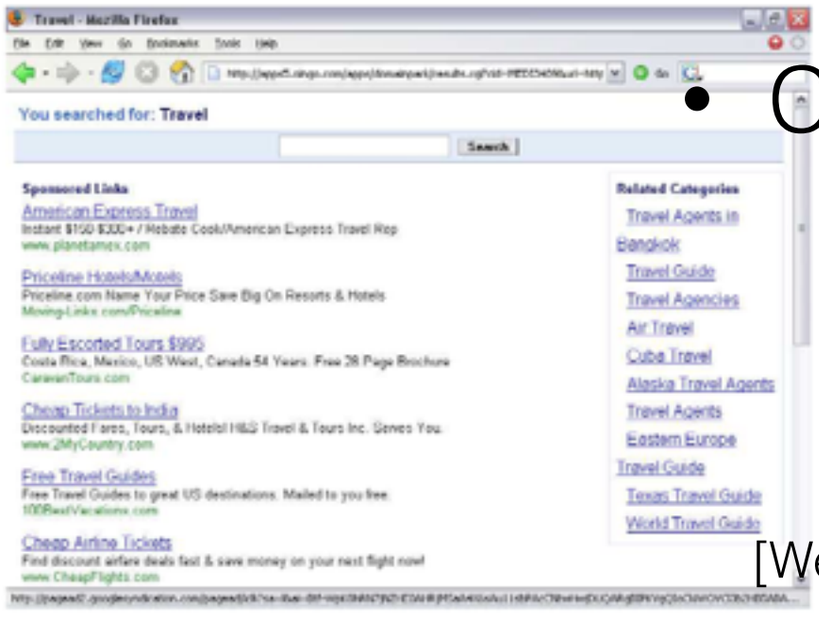
Bayesian inference

- Microcredit



- Fuel consumption

- Cybersecurity



[Webb, Caverlee, Pu 2006]

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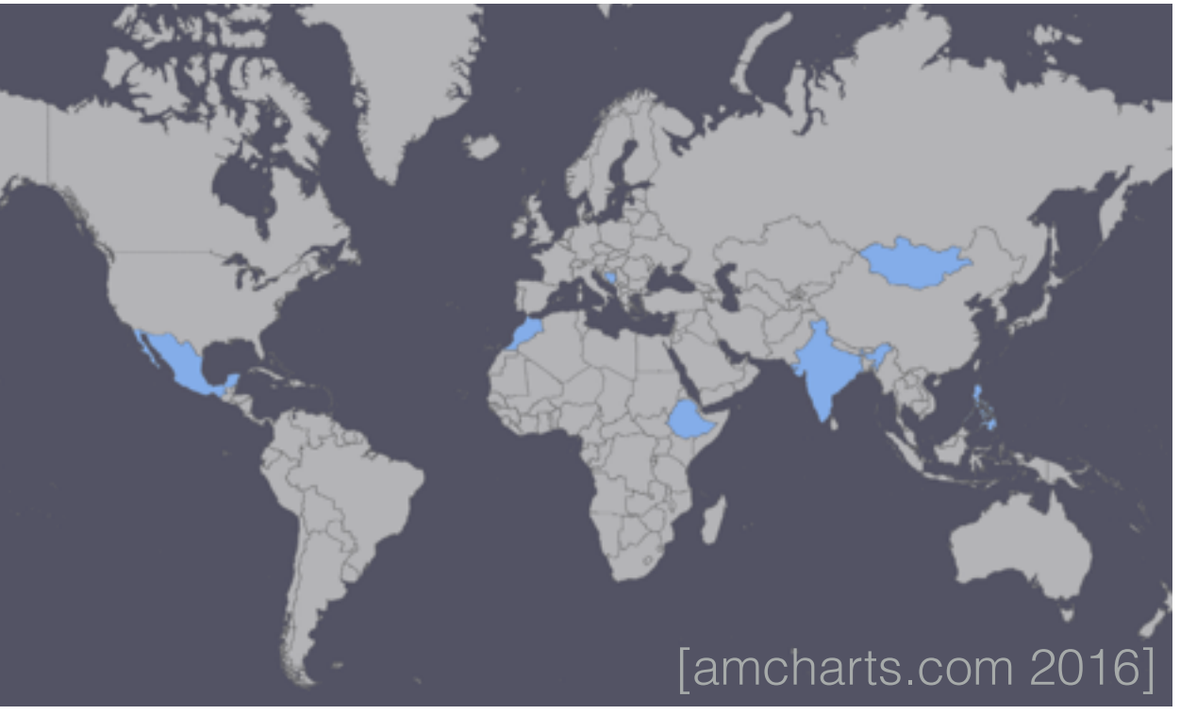

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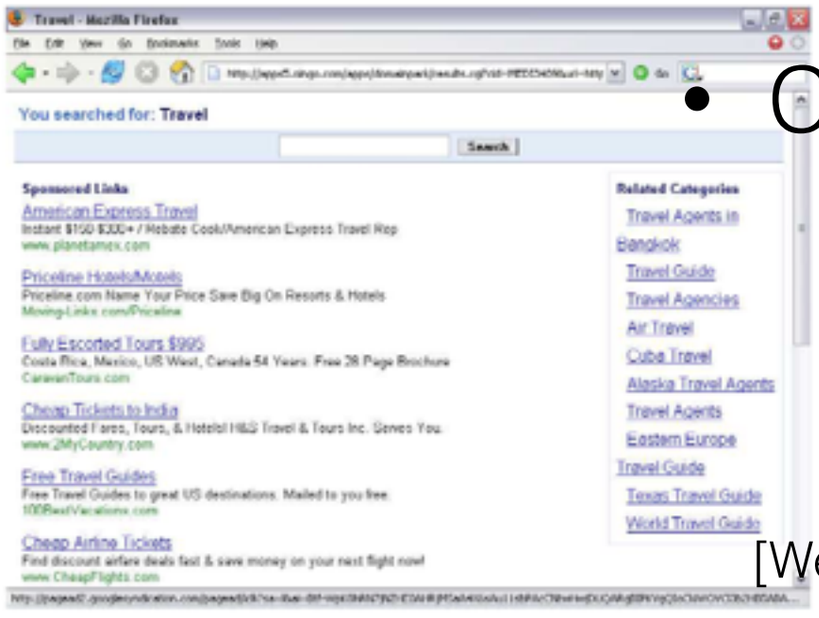

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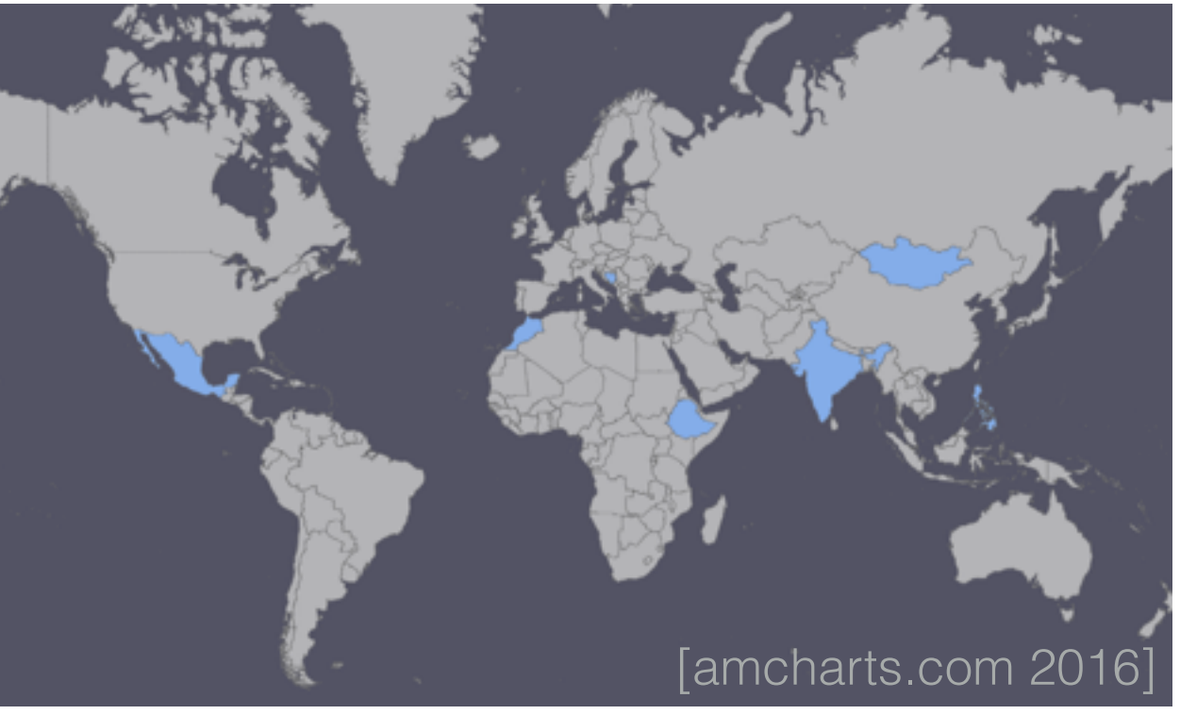

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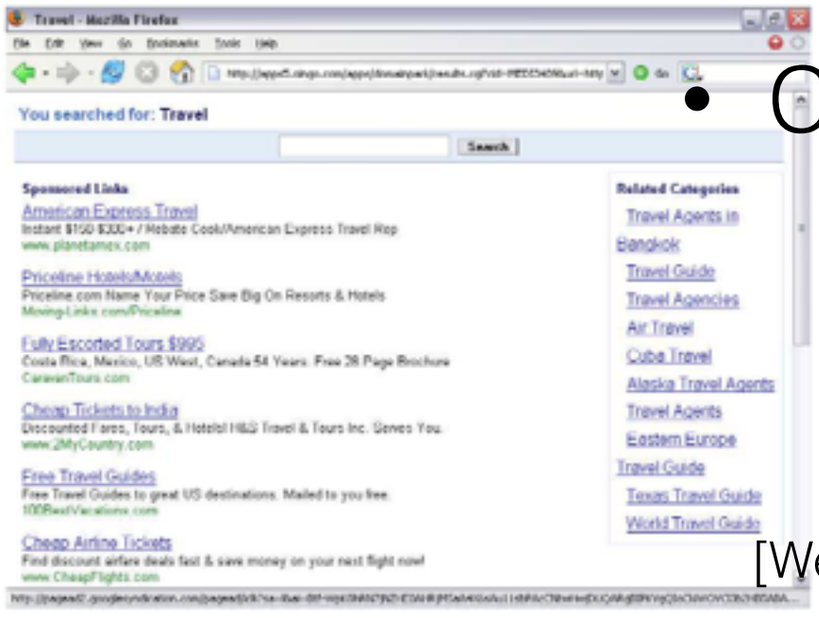

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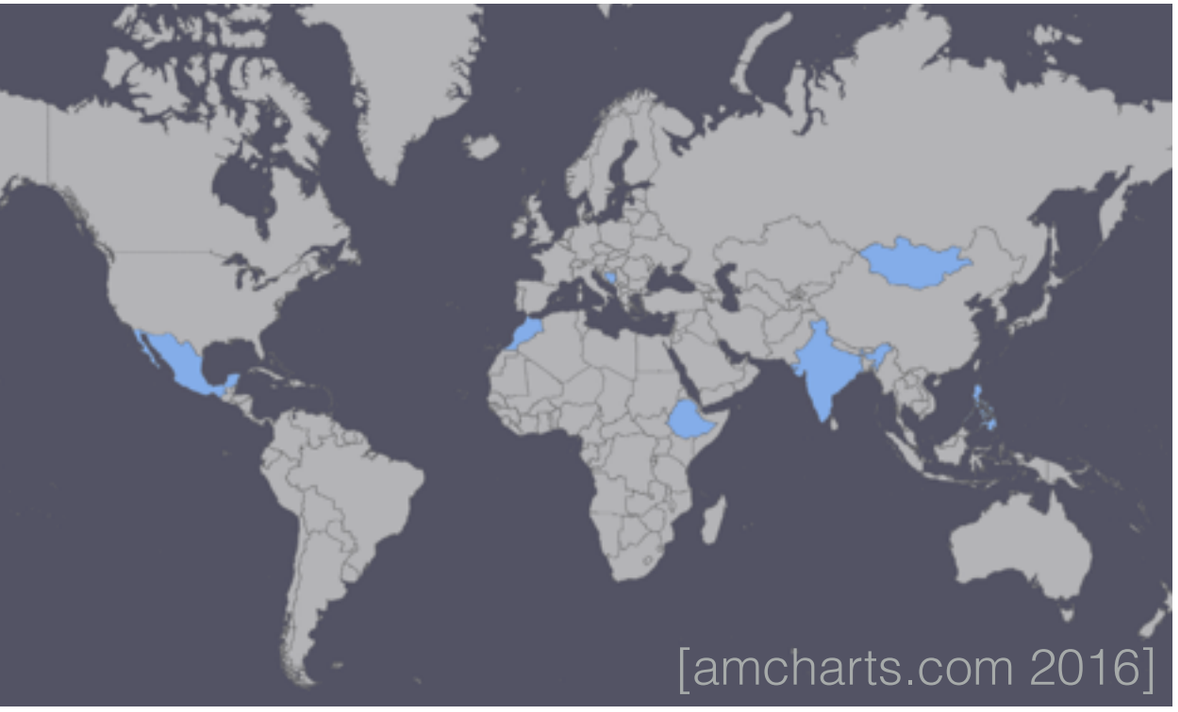

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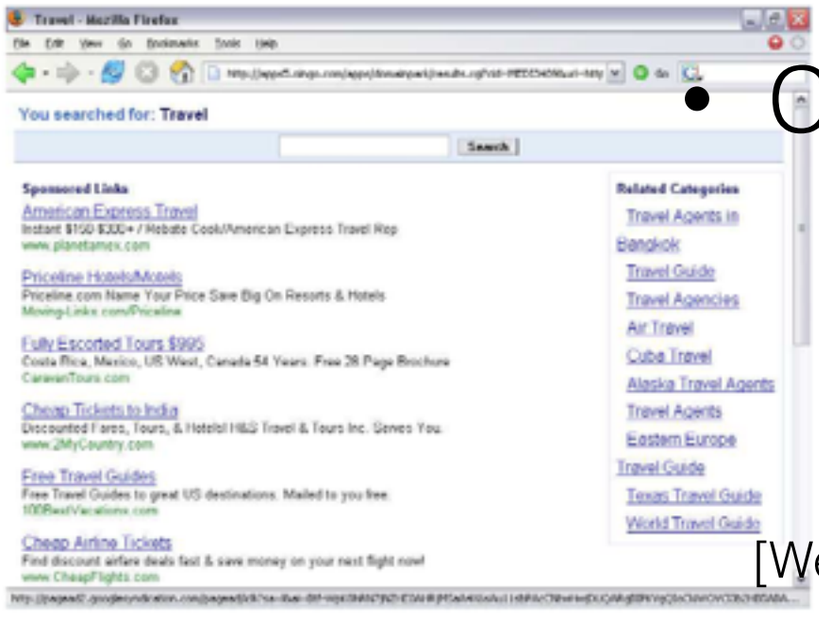

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- Challenge: existing methods can be slow (and/or tedious, unreliable)
- Our proposal: use *efficient summarization* of data
- *Coresets* for **scalable**, **automated** approximate Bayes algorithms with **error bounds for finite data**

Roadmap

- Approximate Bayes review
- The “core” of the data set
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$p(\theta)$

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$p(y|\theta)p(\theta)$

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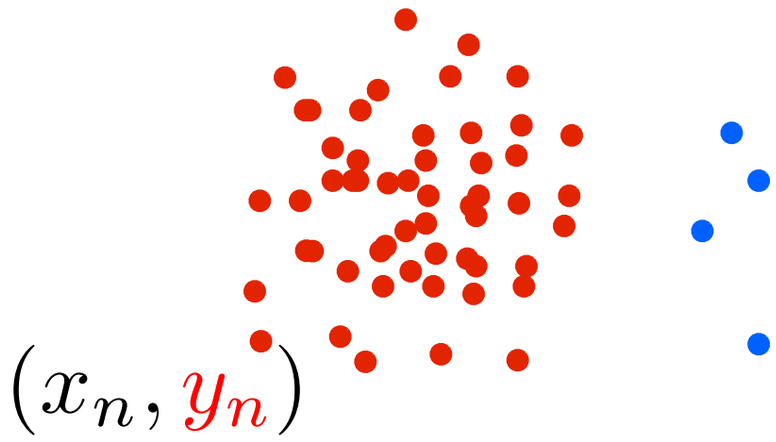

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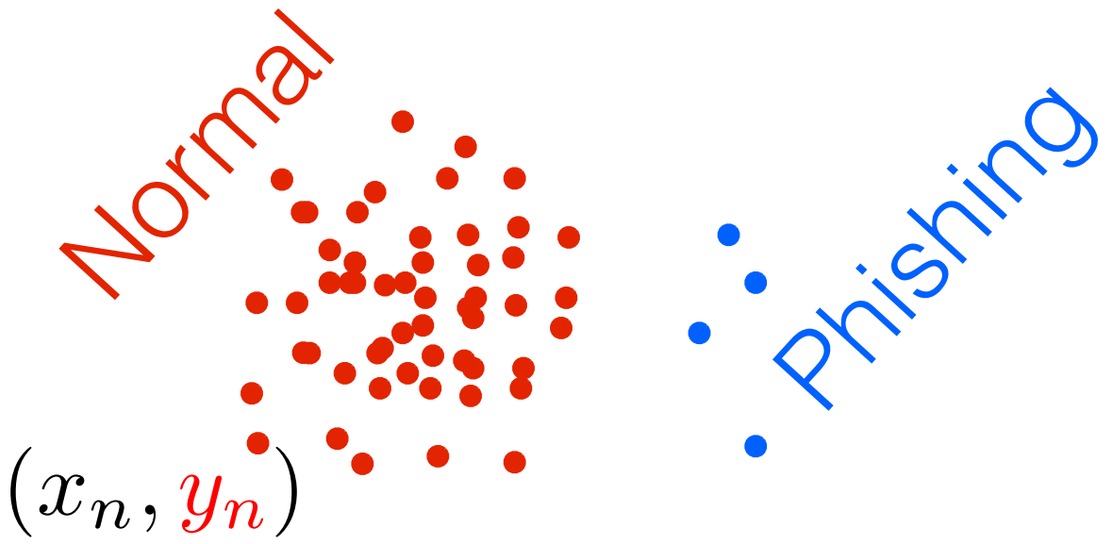

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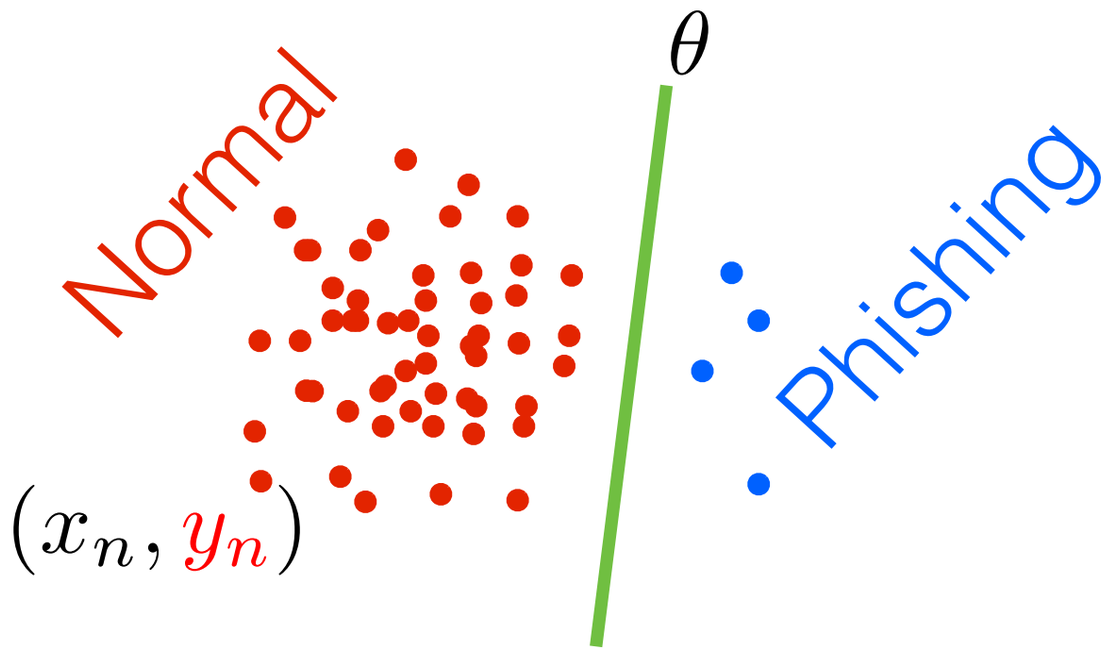

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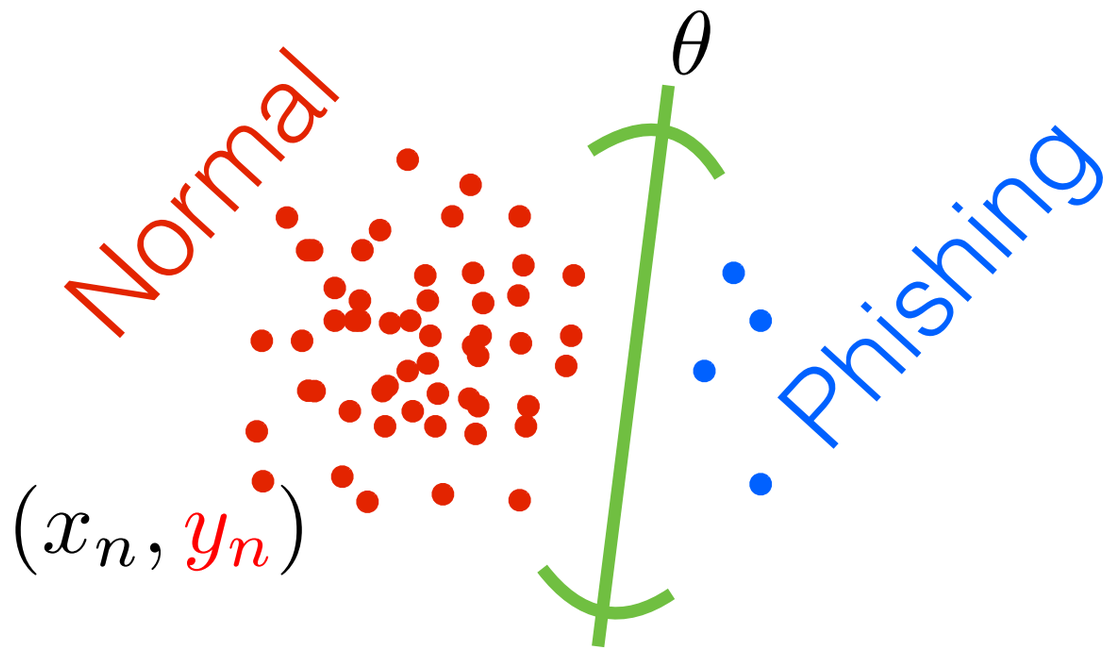

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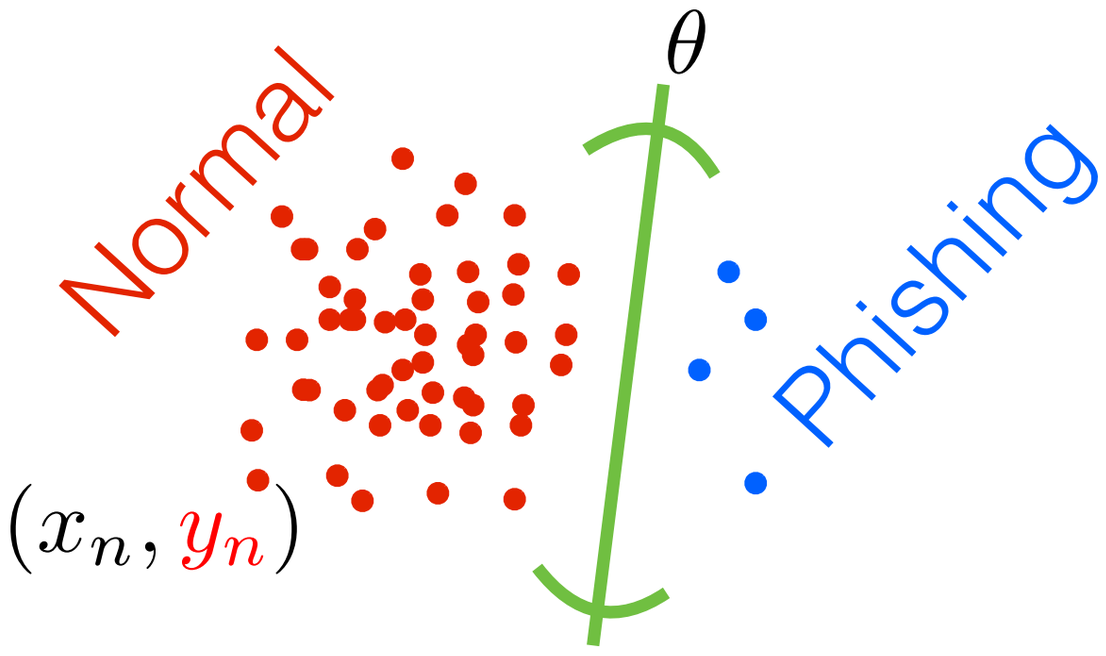

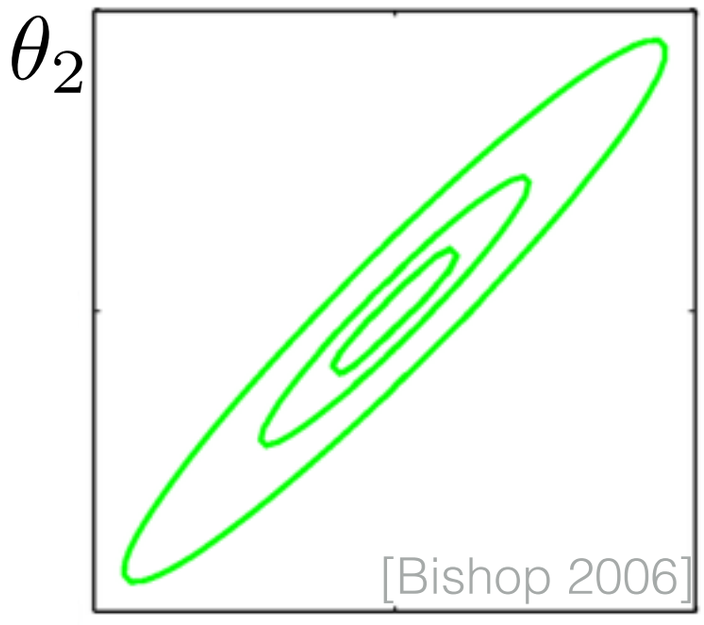

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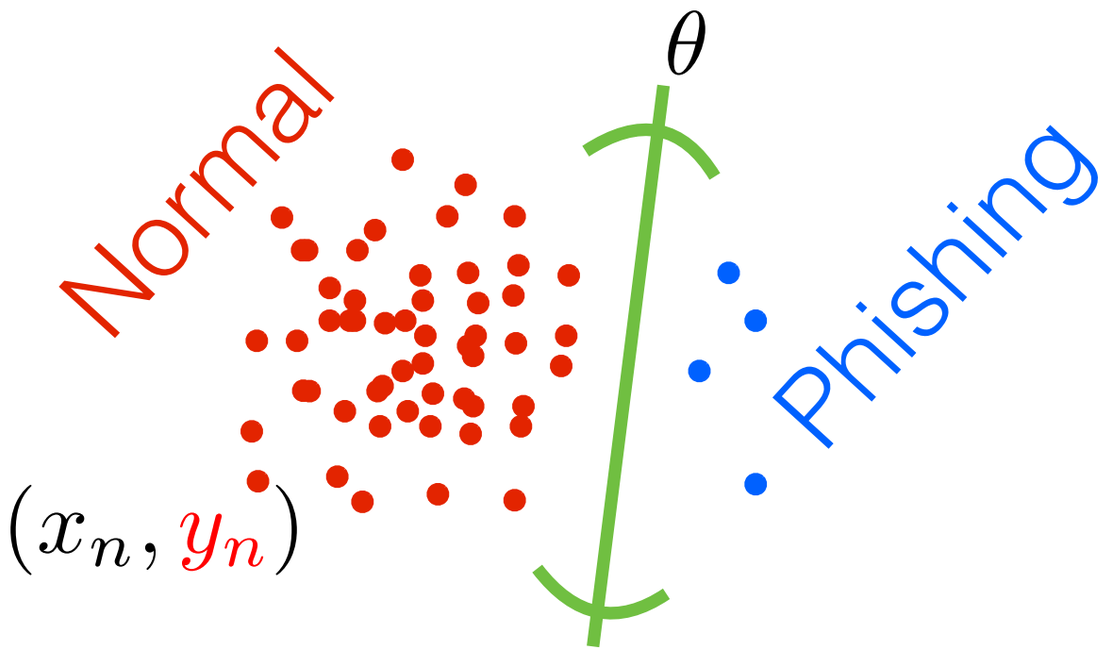

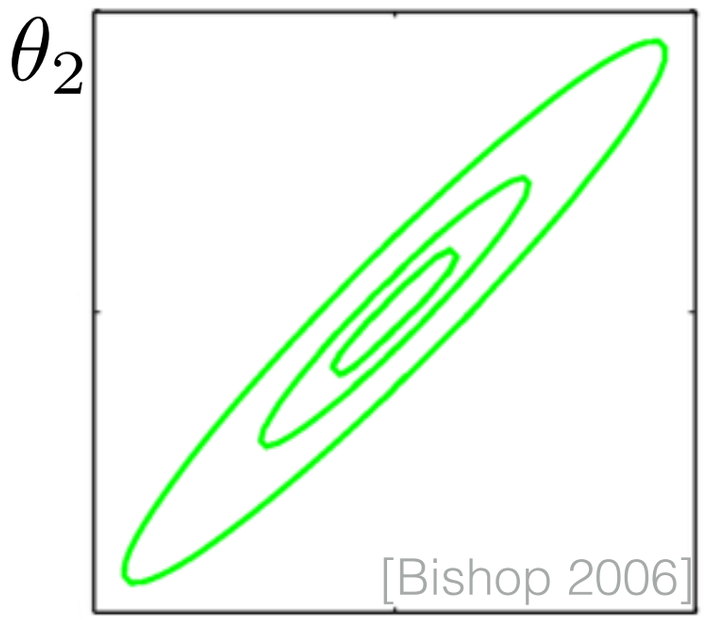

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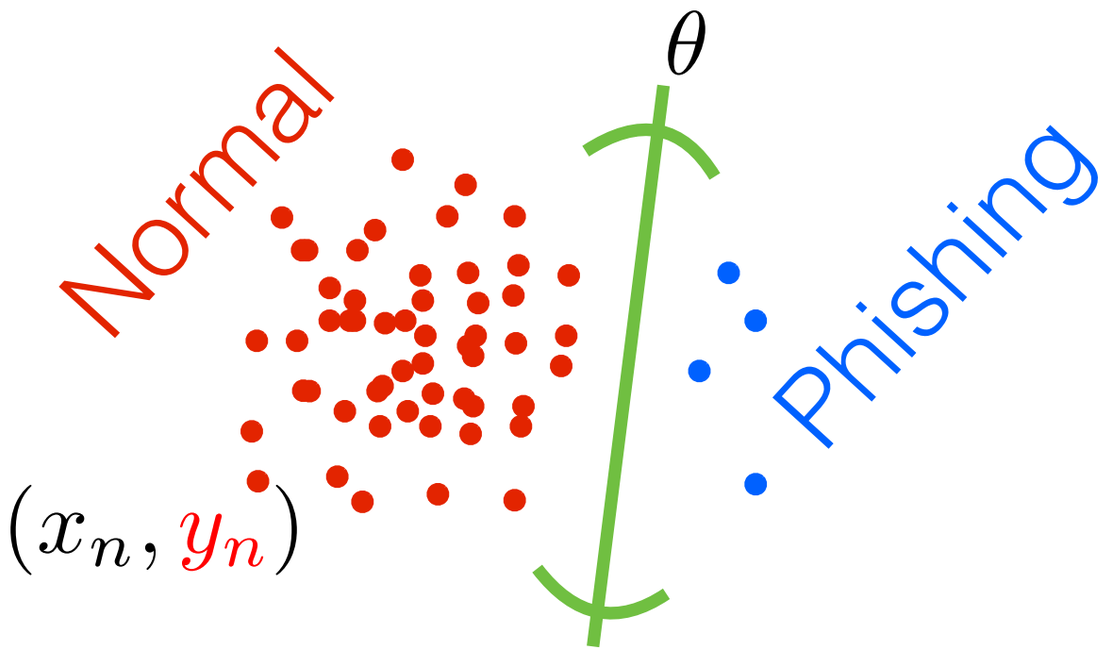

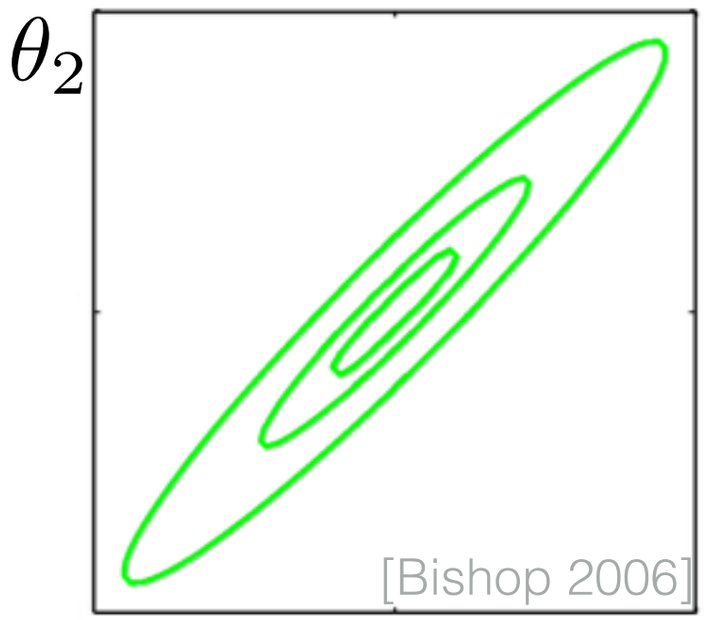

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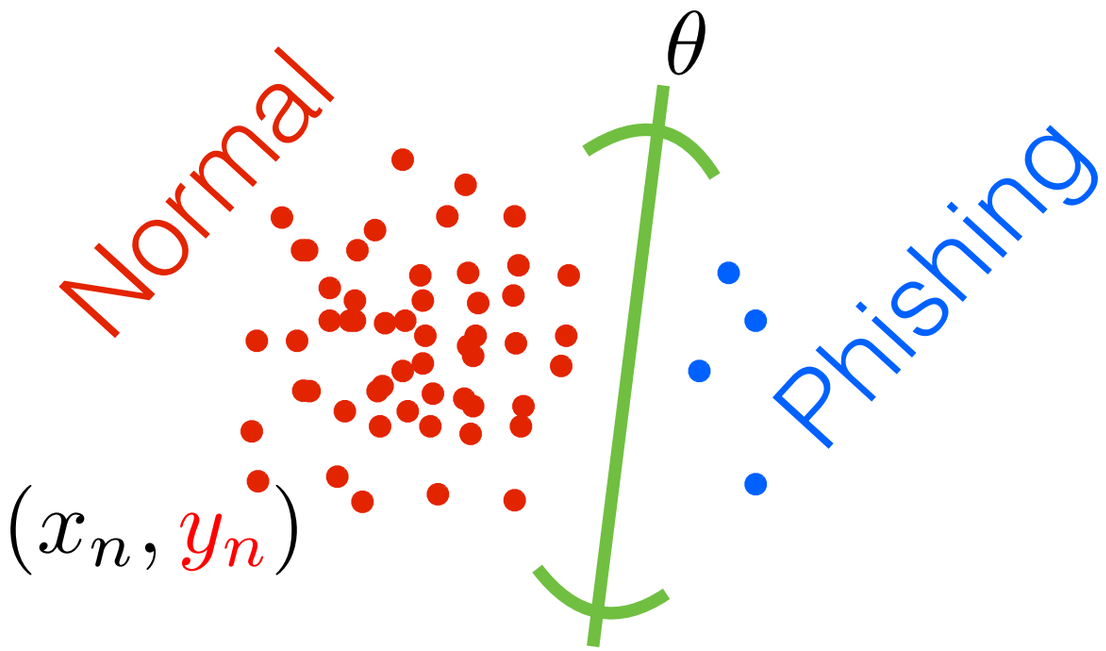

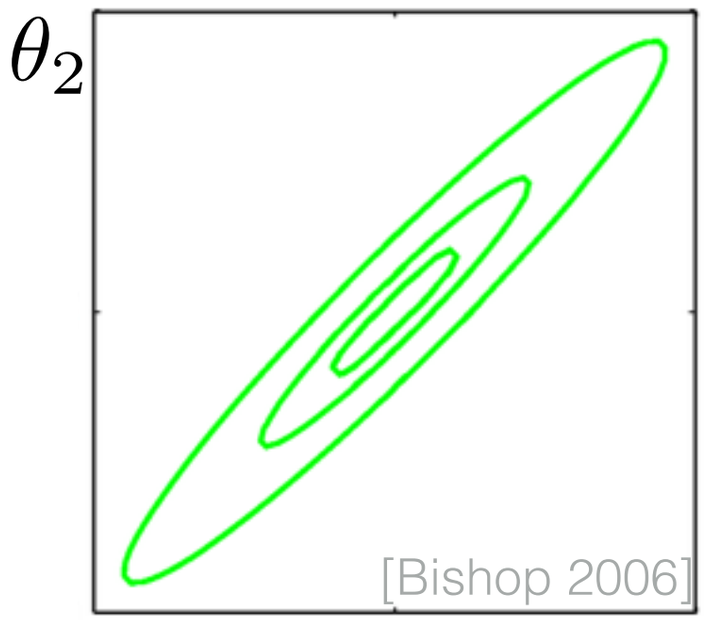

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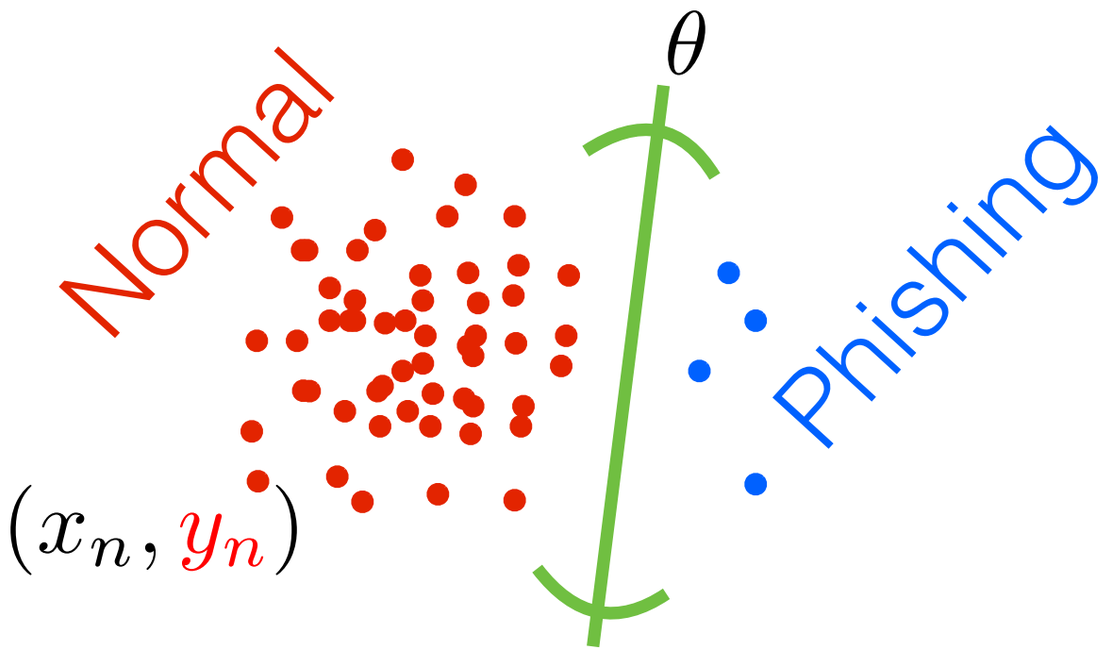

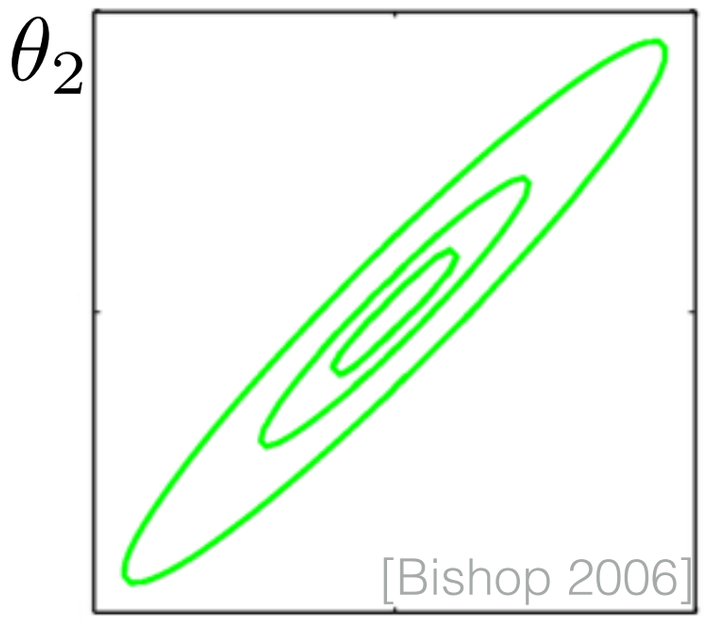

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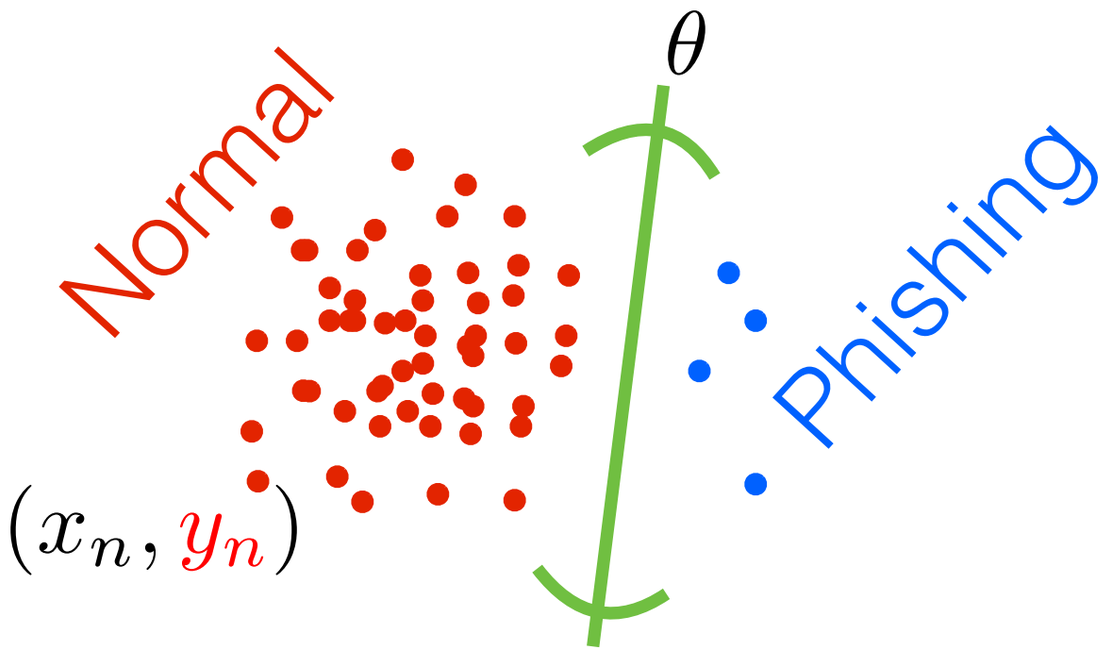

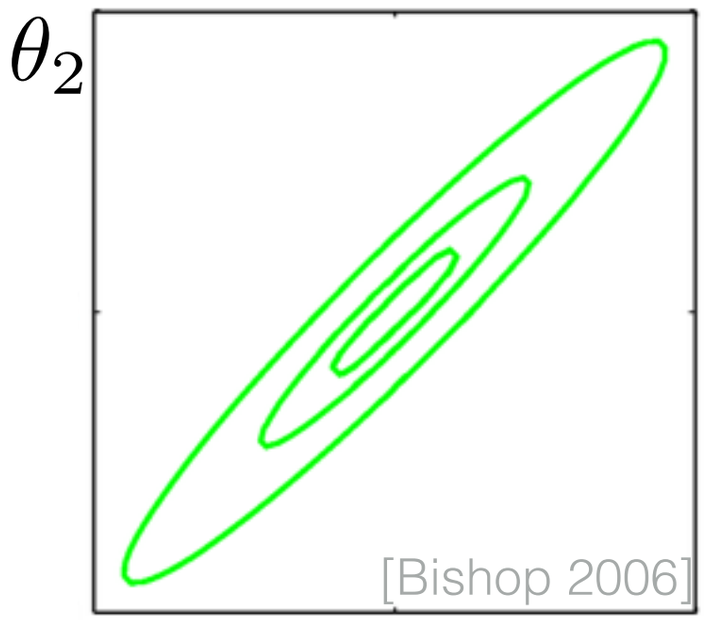

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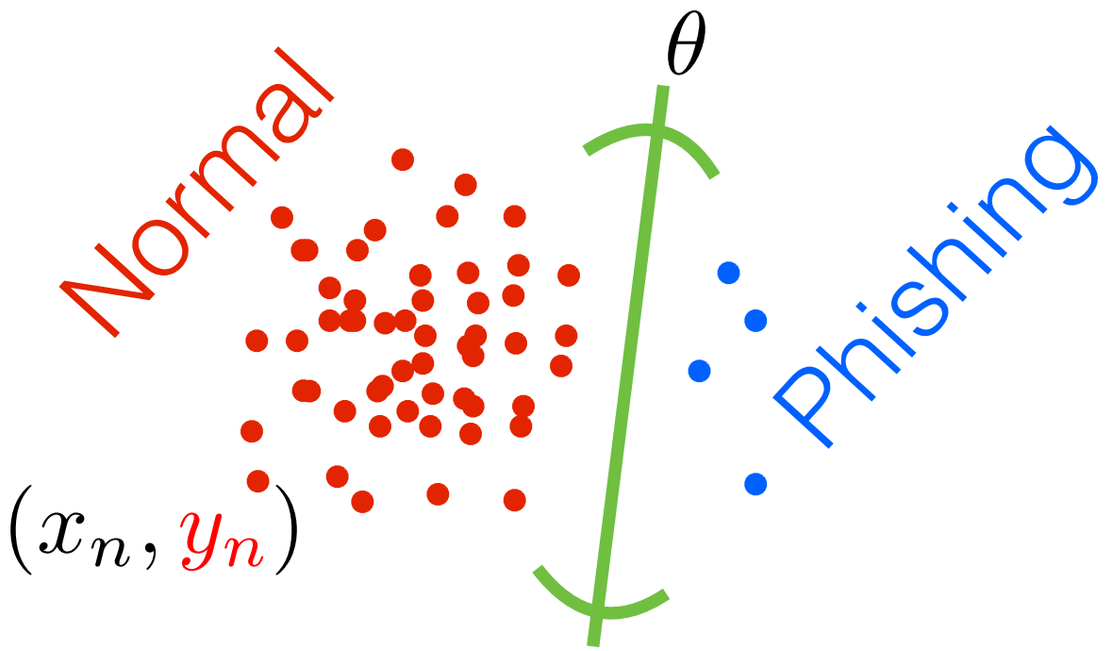

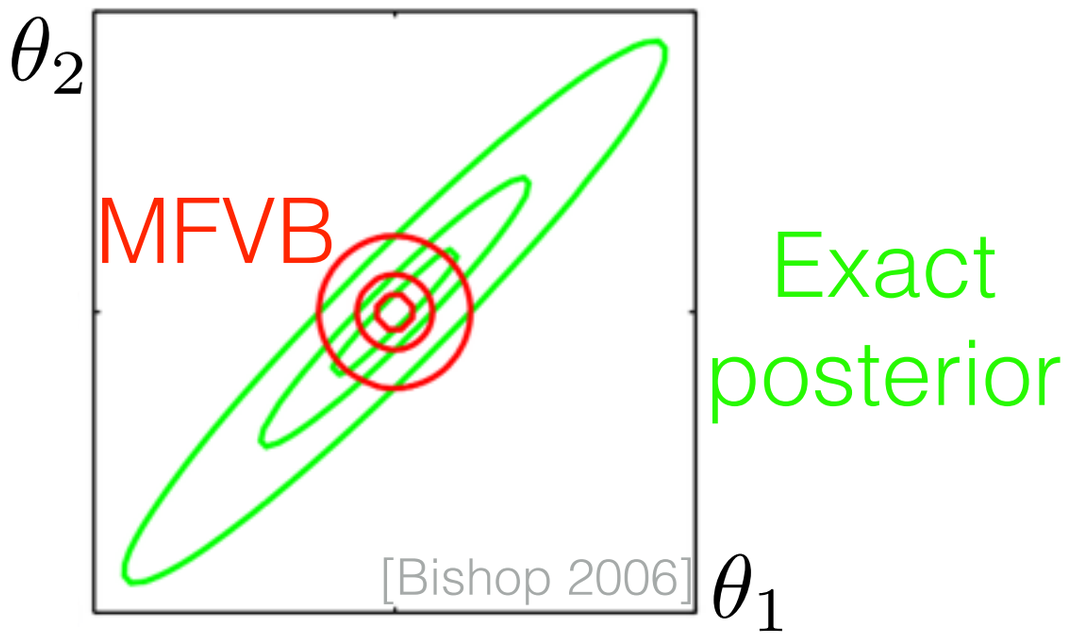

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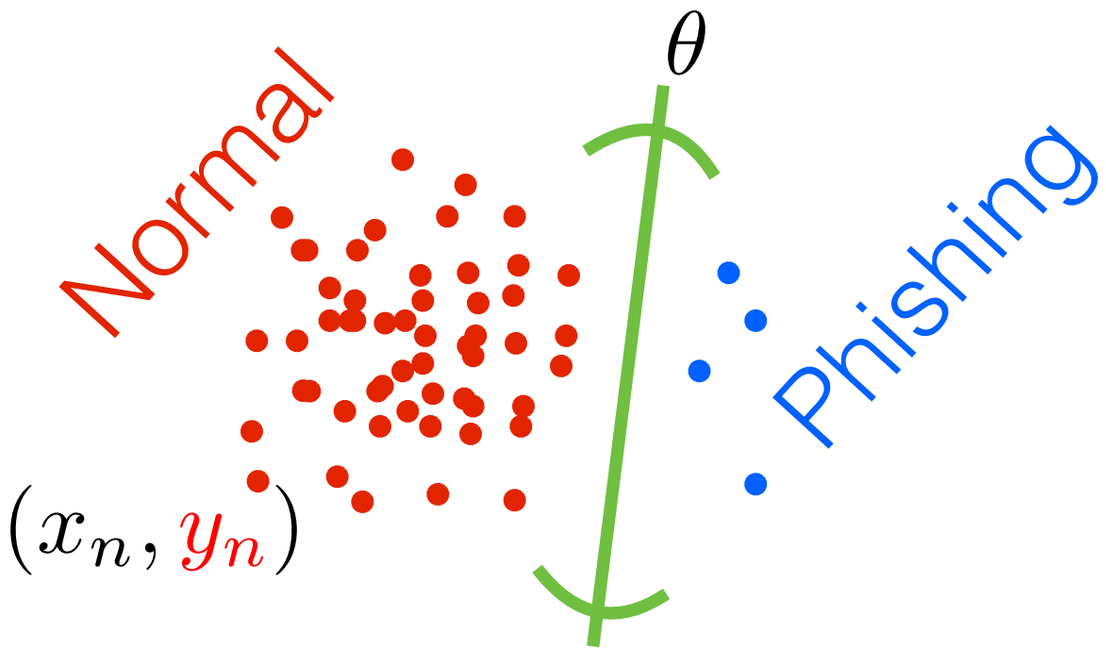

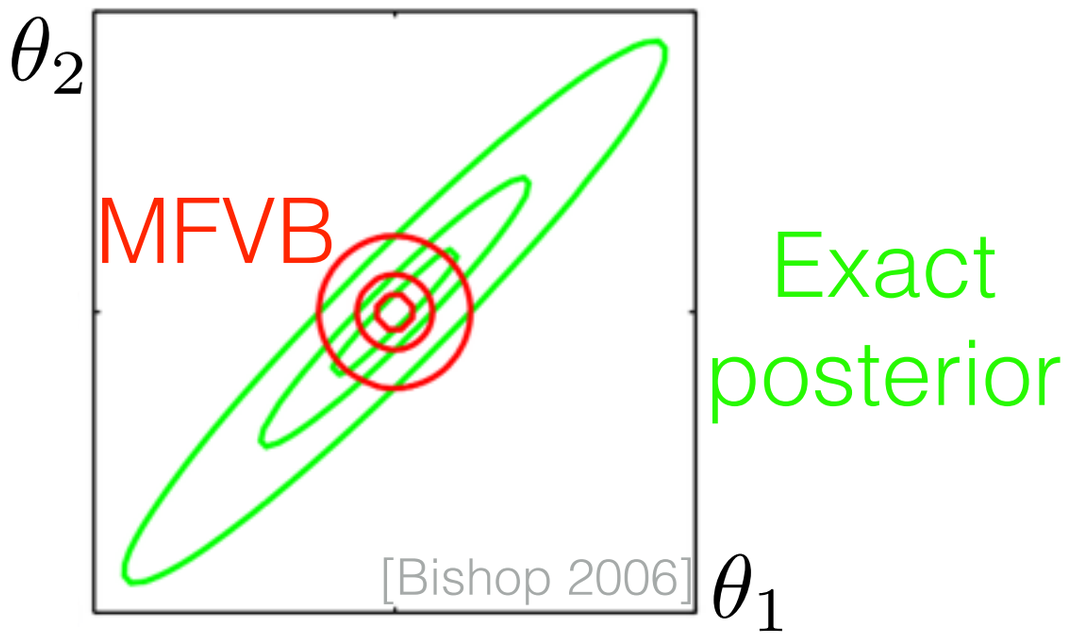

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- Automation: e.g. Stan, NUTS, ADVI

[http://mc-stan.org/ ; Hoffman, Gelman 2014; Kucukelbir, Tran, Ranganath, Gelman, Blei 2017]

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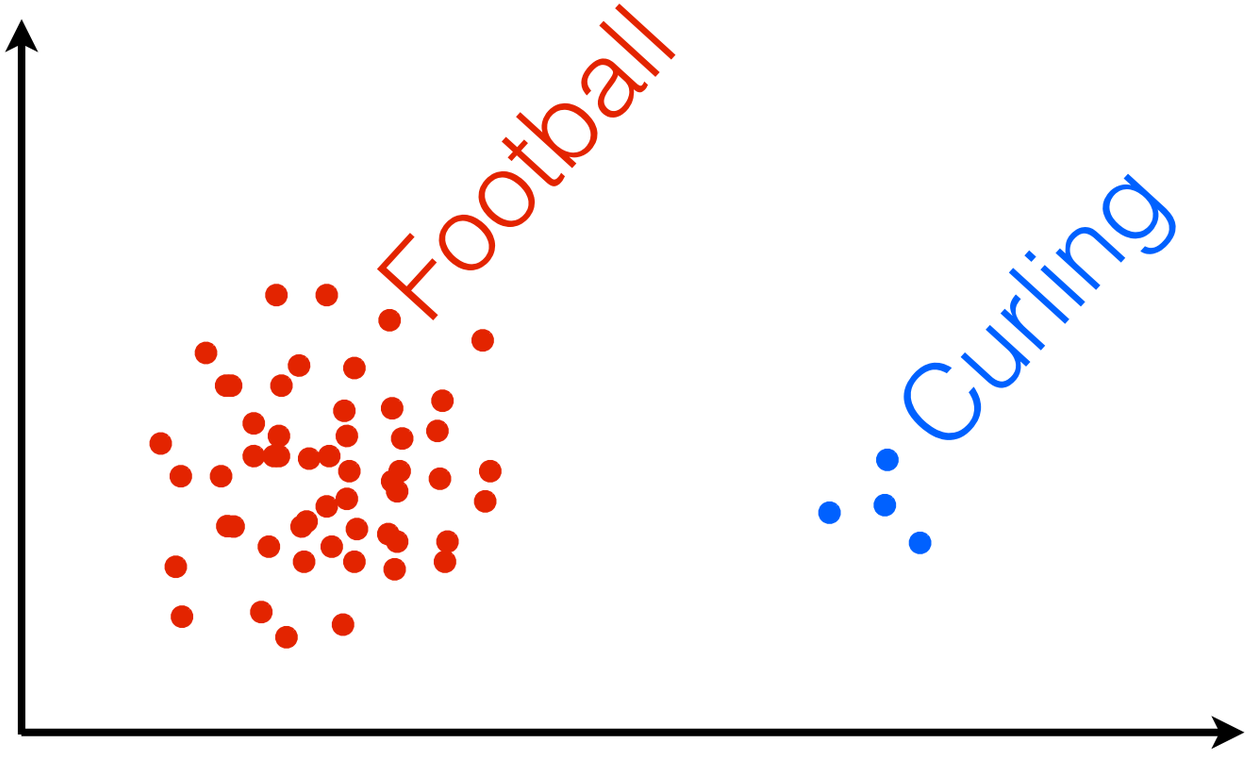

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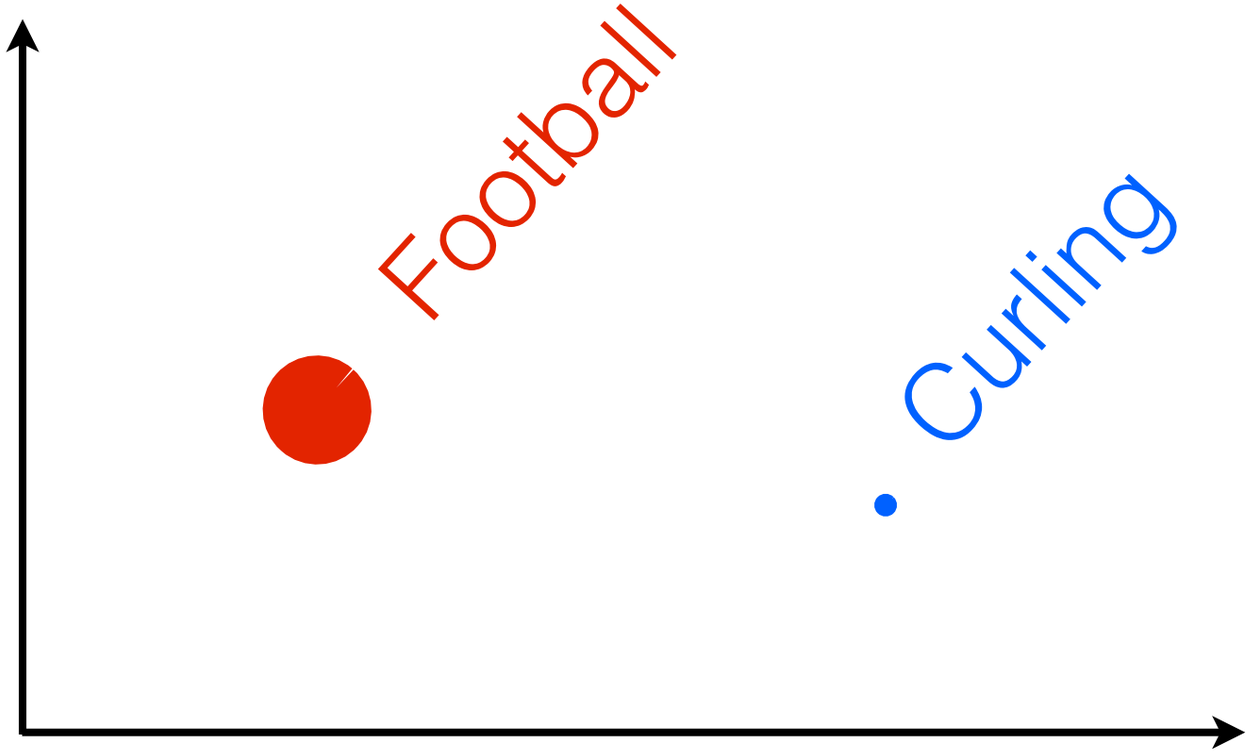

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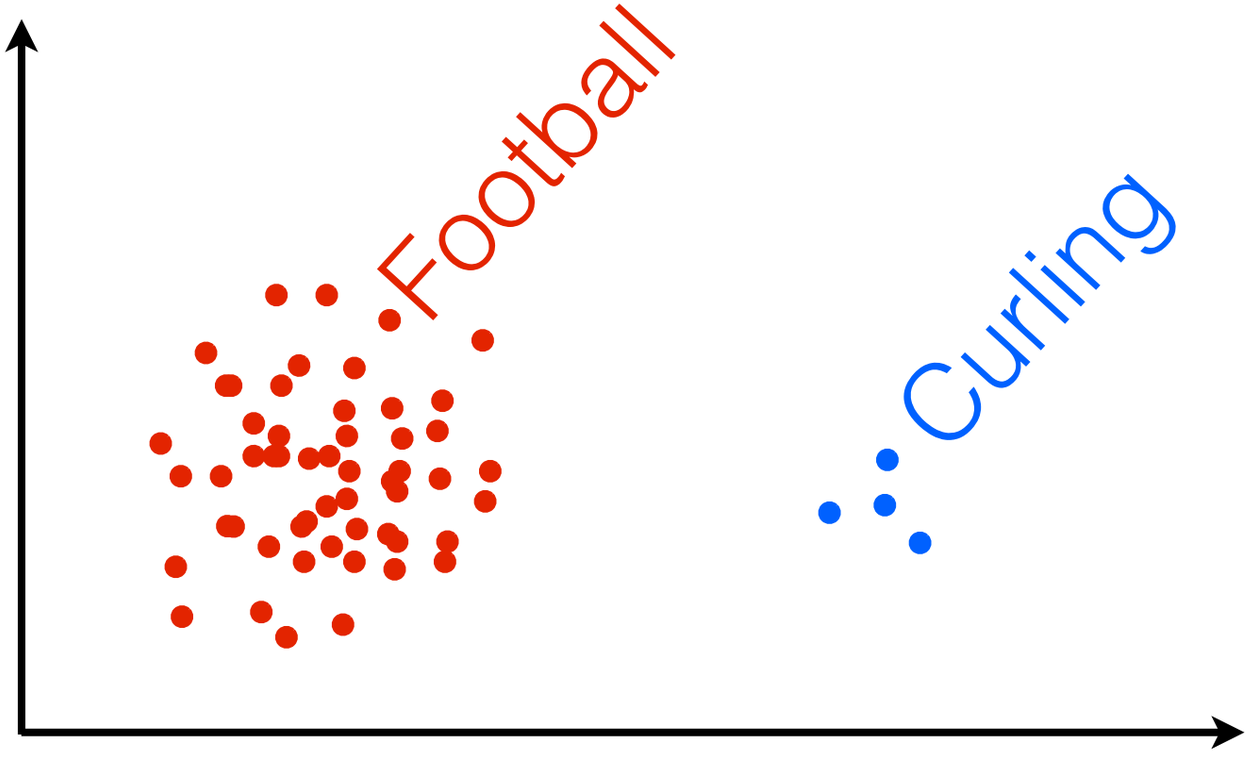

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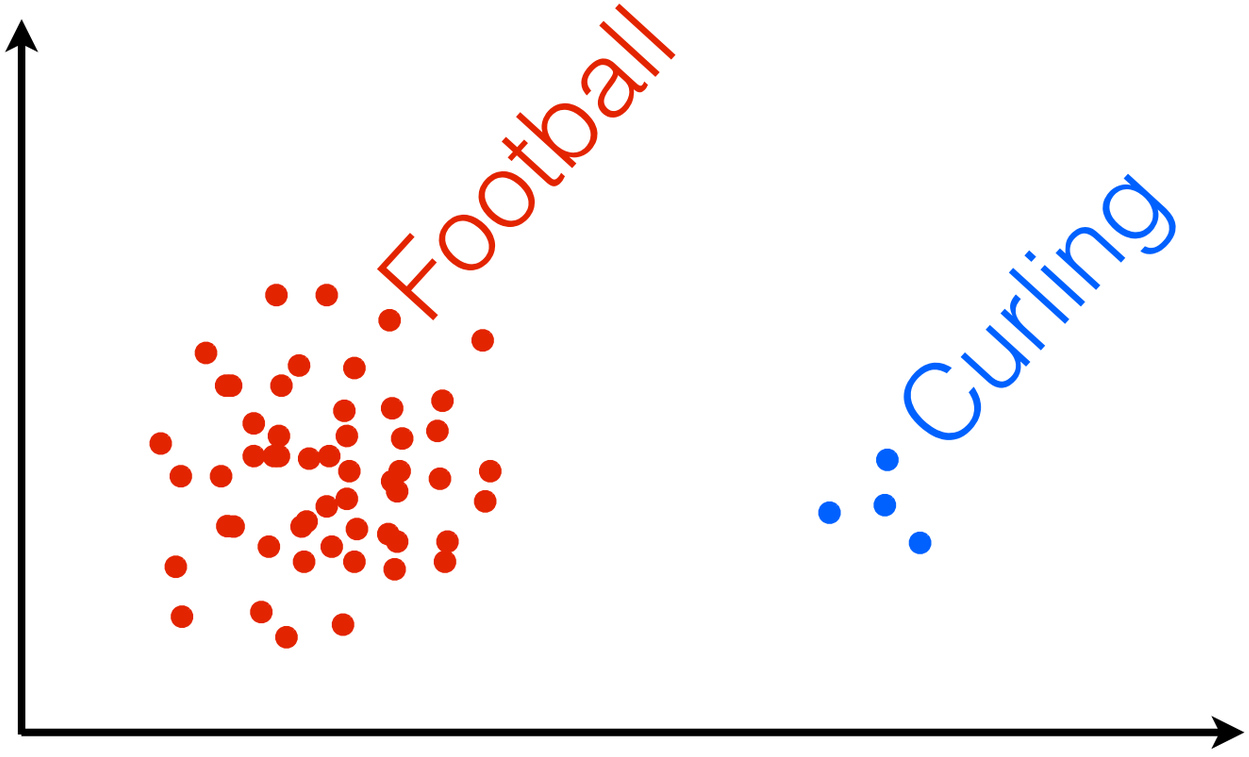

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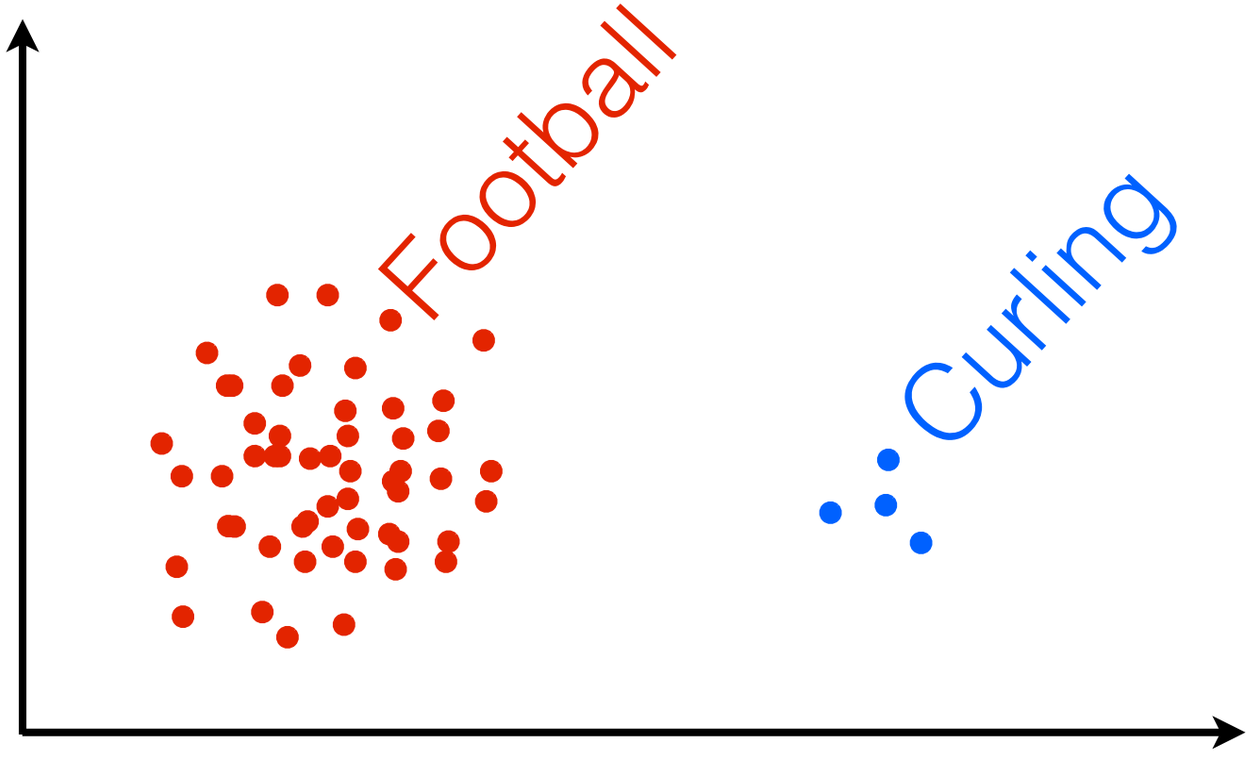

- Theoretical guarantees on quality

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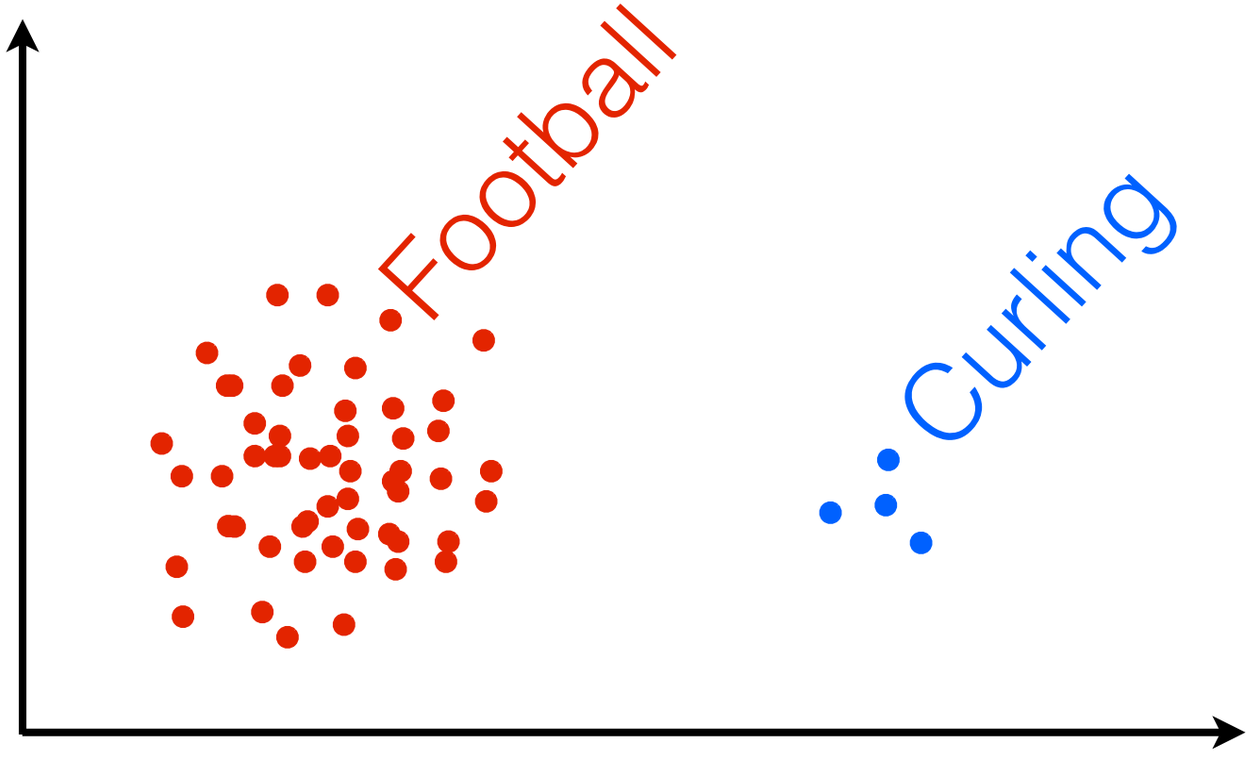

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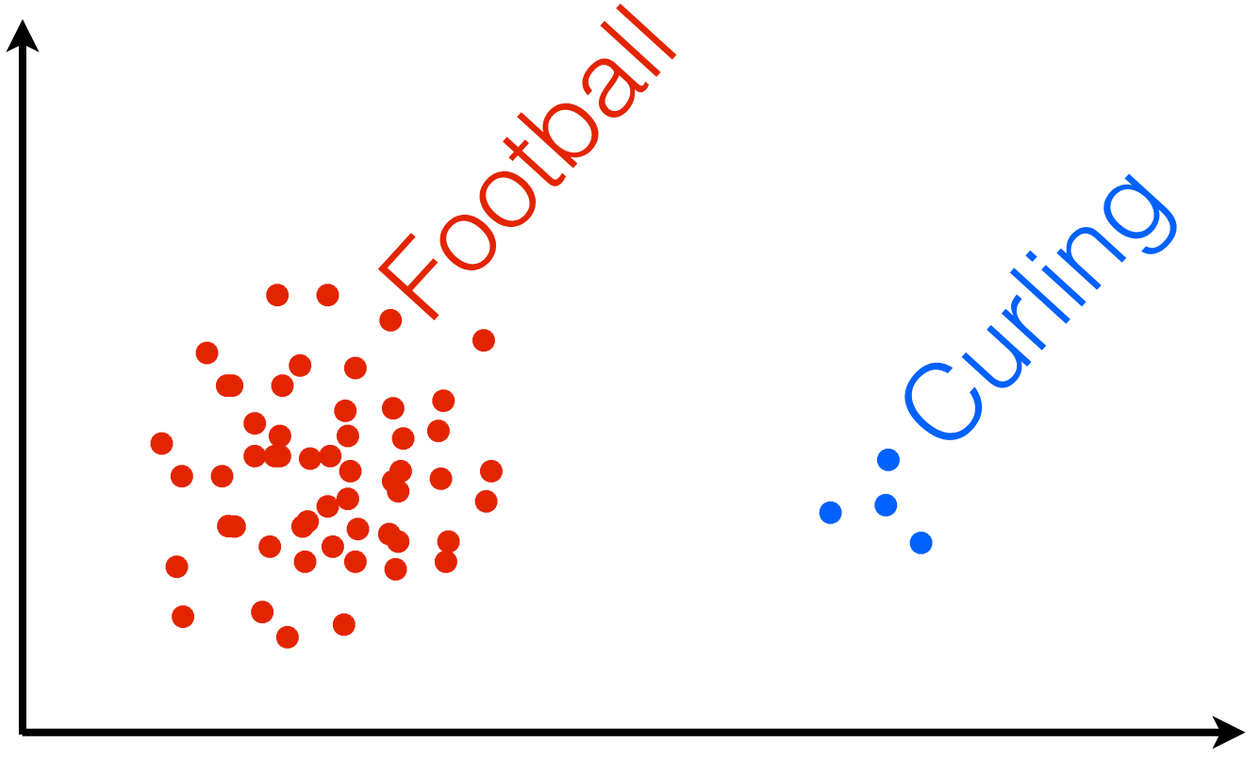

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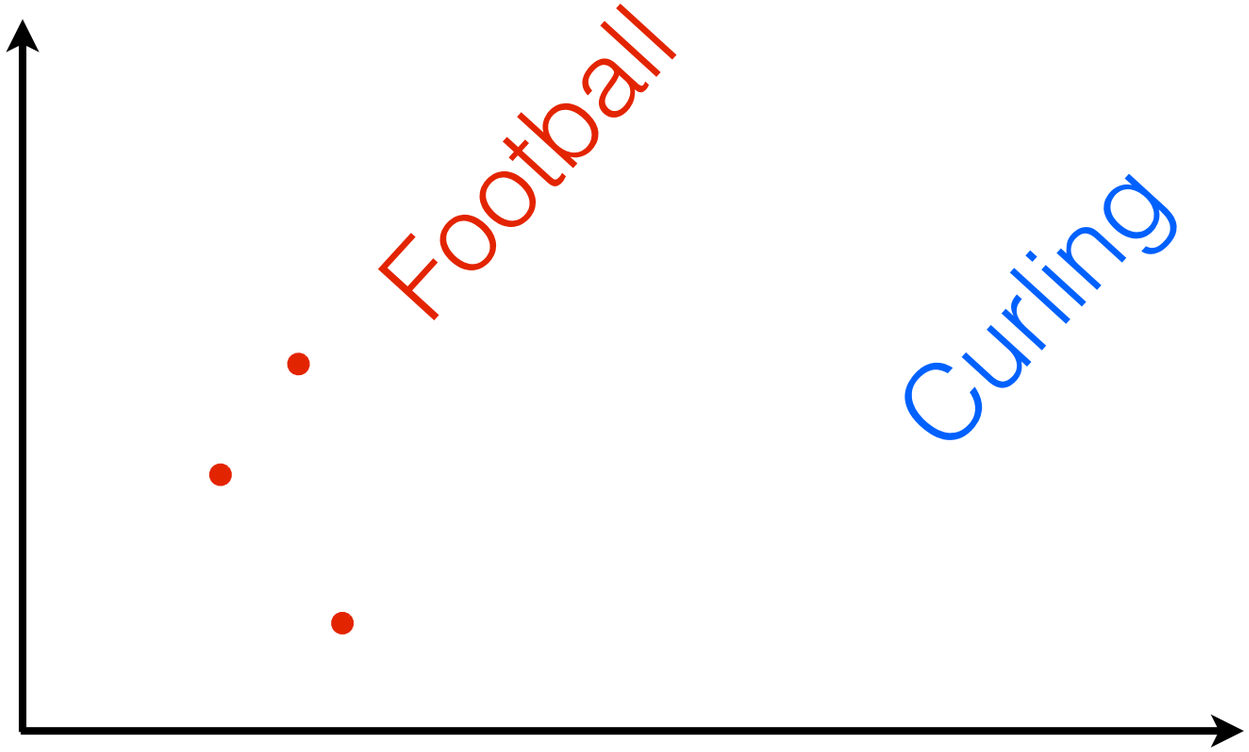

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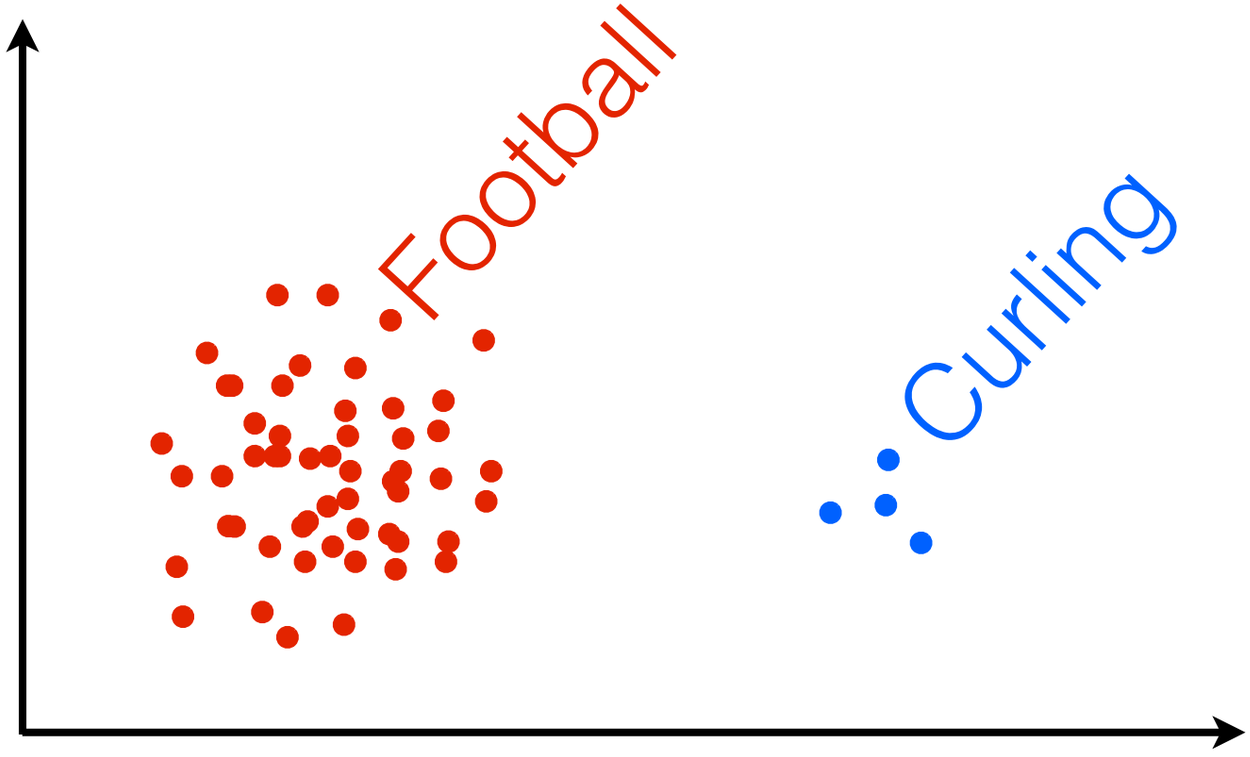

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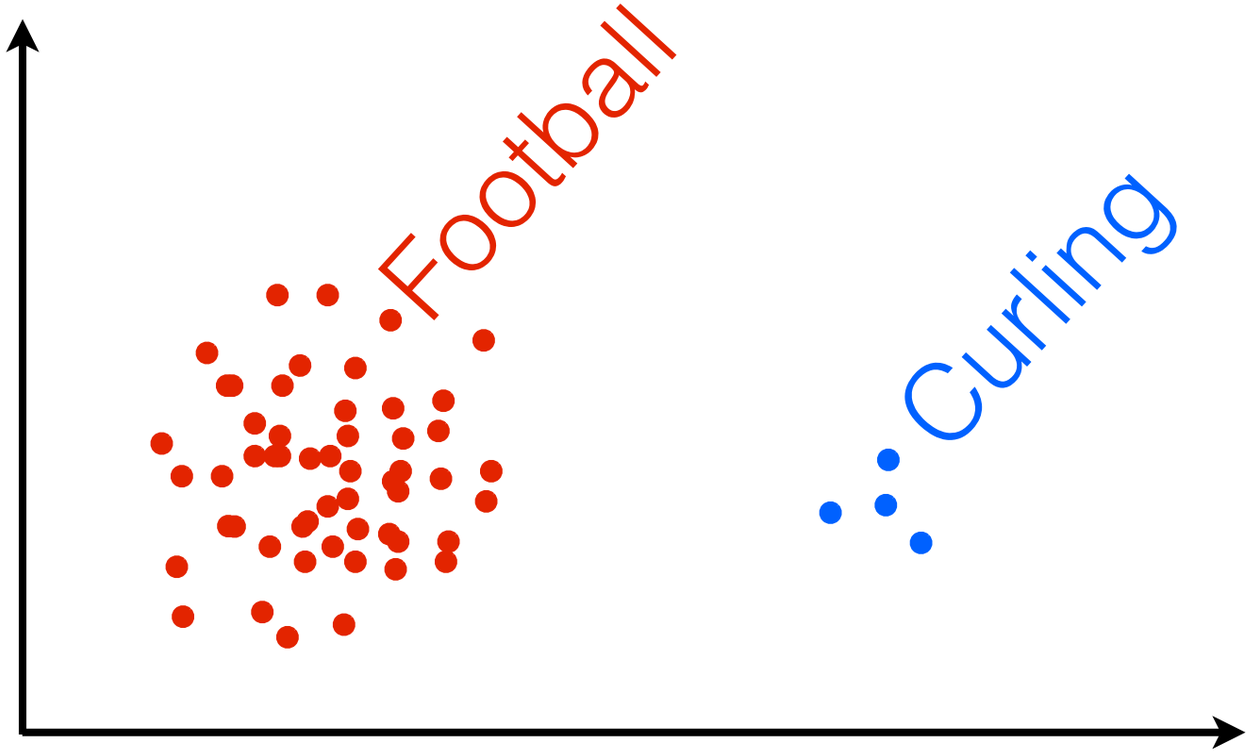

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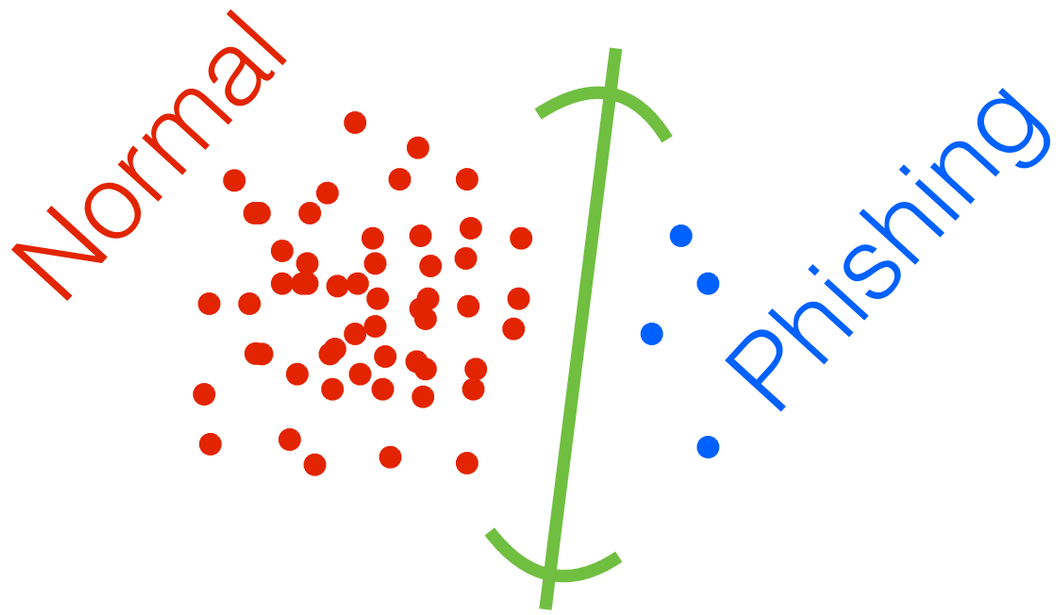
- How to develop coresets for Bayes?

[Agarwal et al 2005; Feldman & Langberg 2011; DuMouchel et al 1999; Madigan et al 1999; Huggins, Campbell, Broderick 2016; Campbell, Broderick 2017; Campbell, Broderick 2018]

Bayesian coresets

- Posterior $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$



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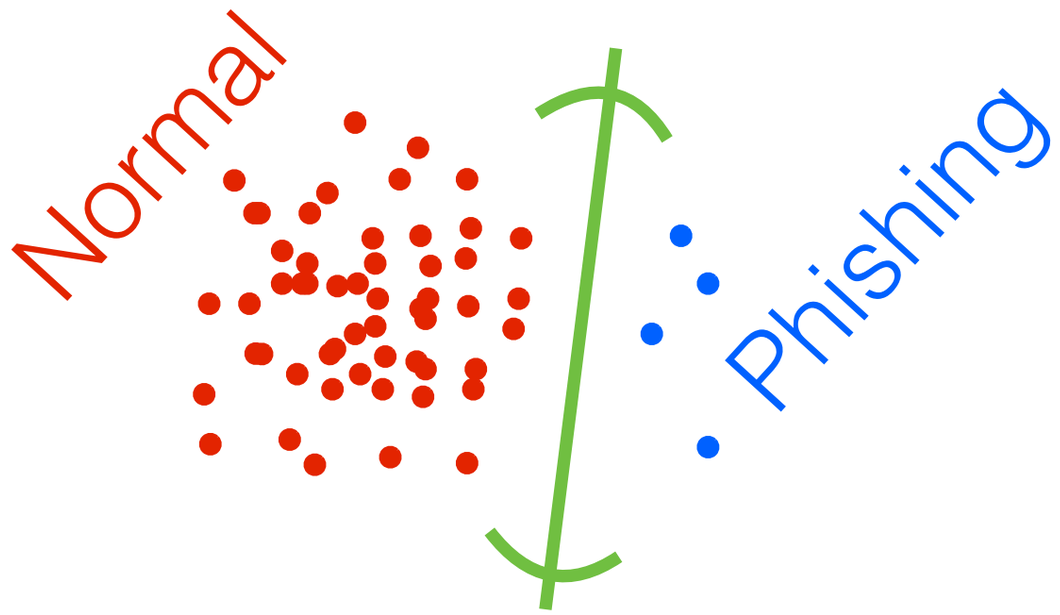

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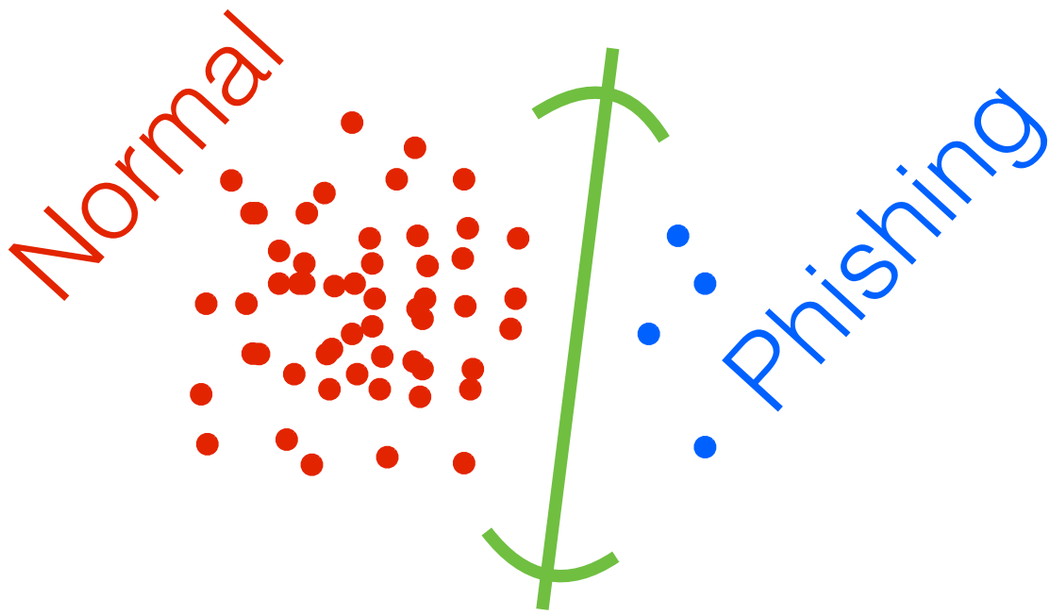

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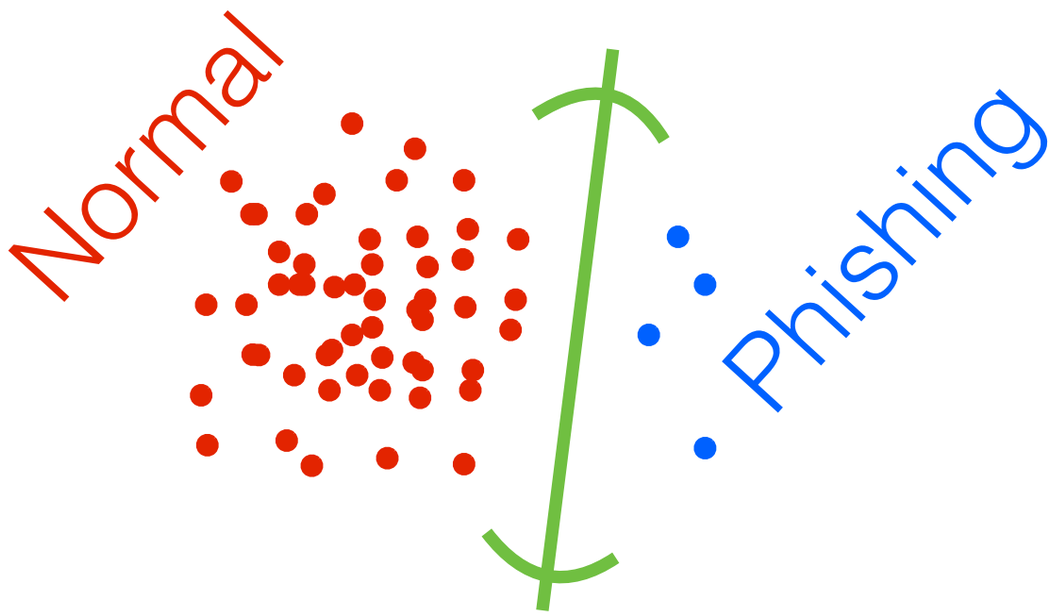
$$\|w\|_0 \ll N$$



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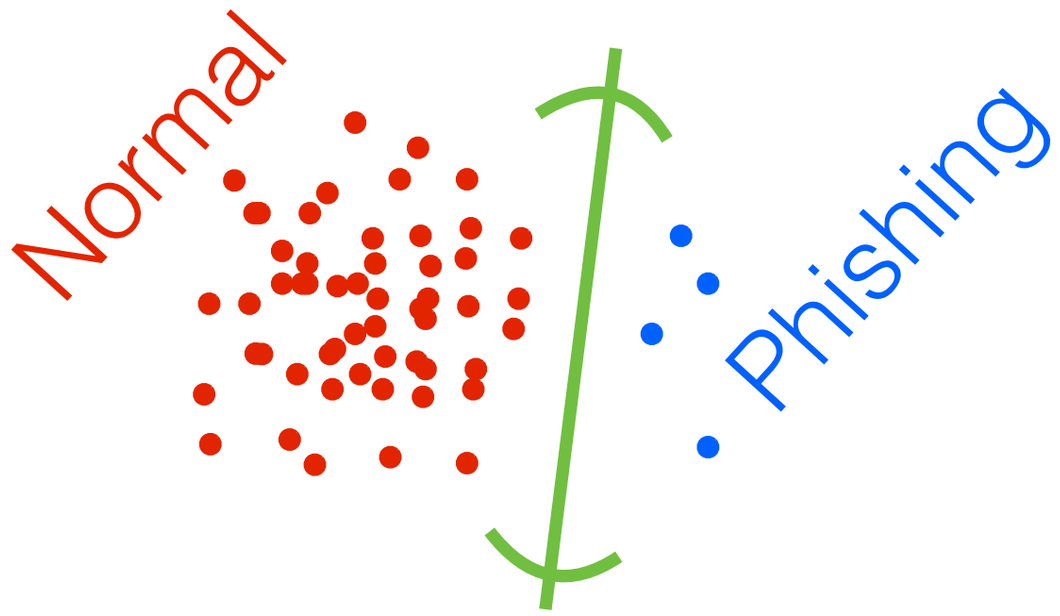

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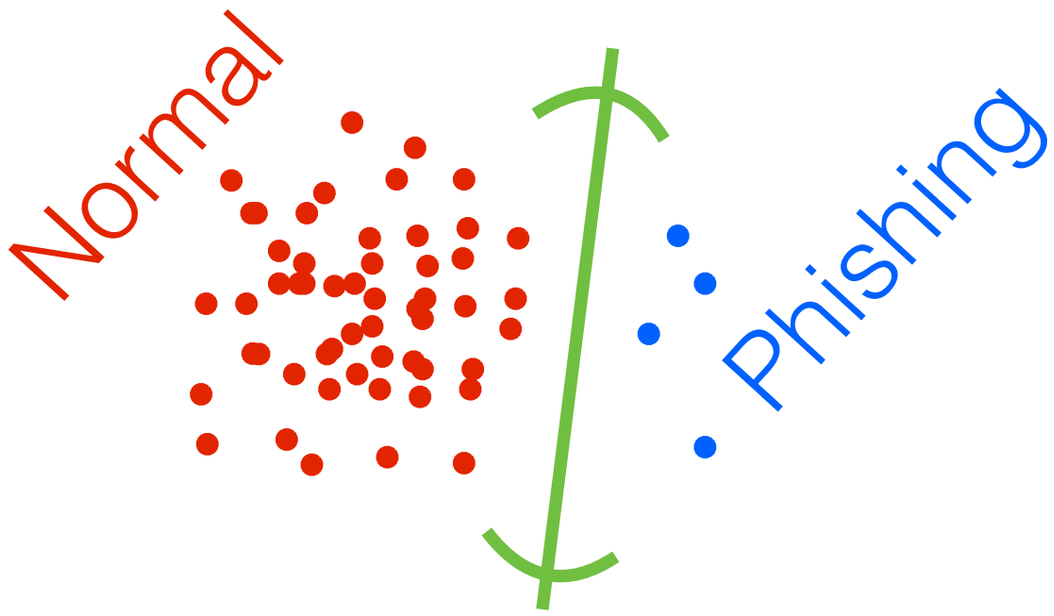

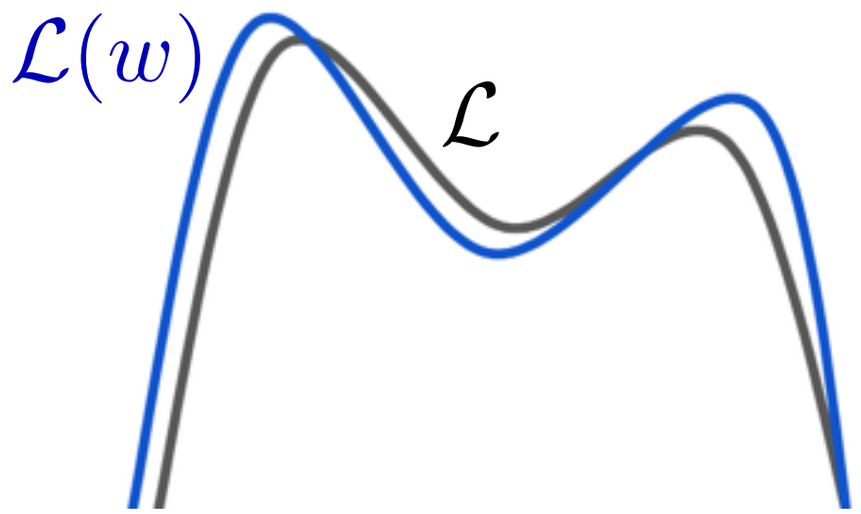

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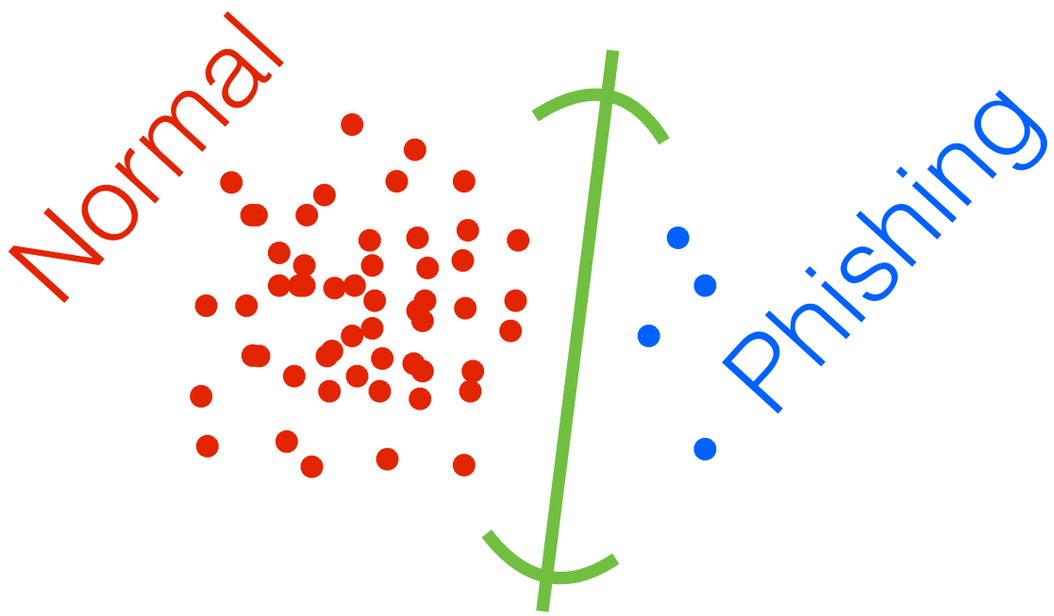

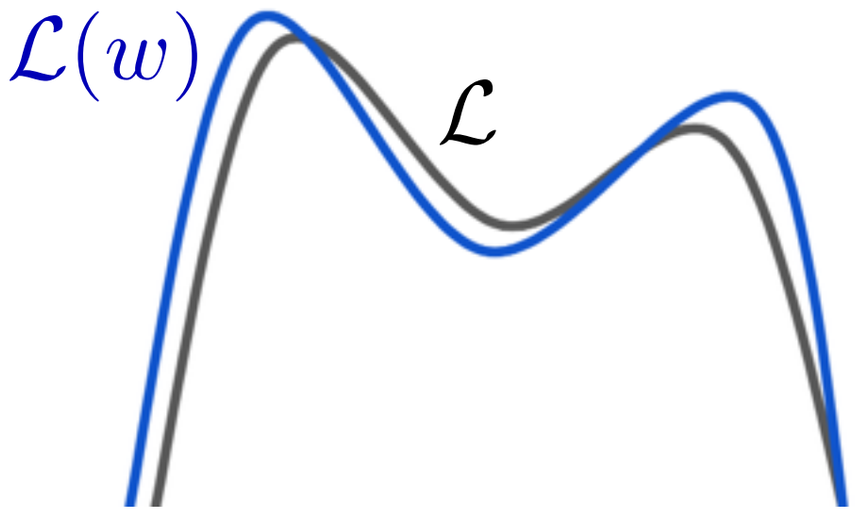

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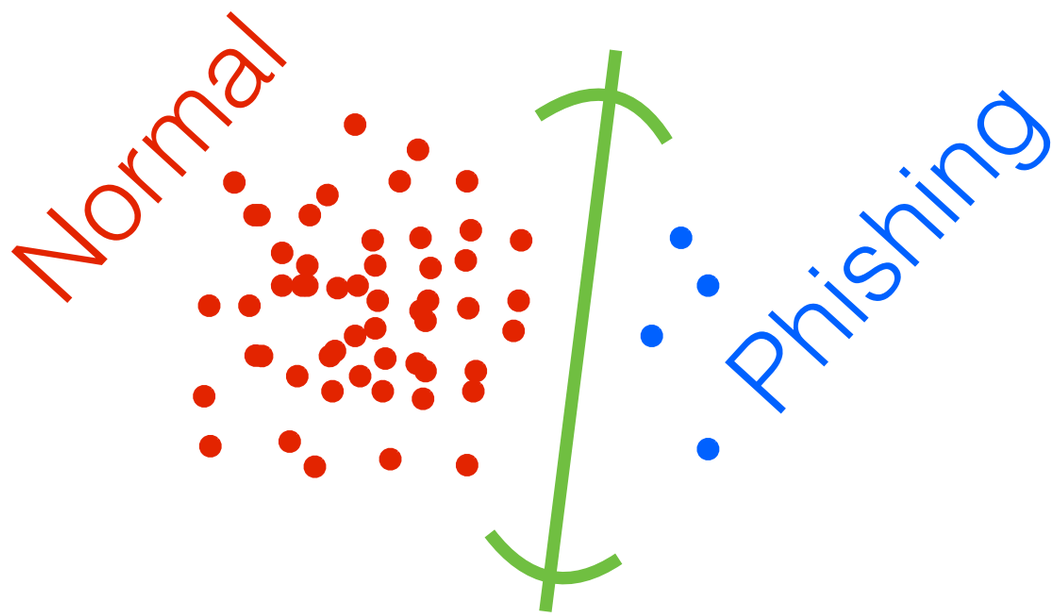
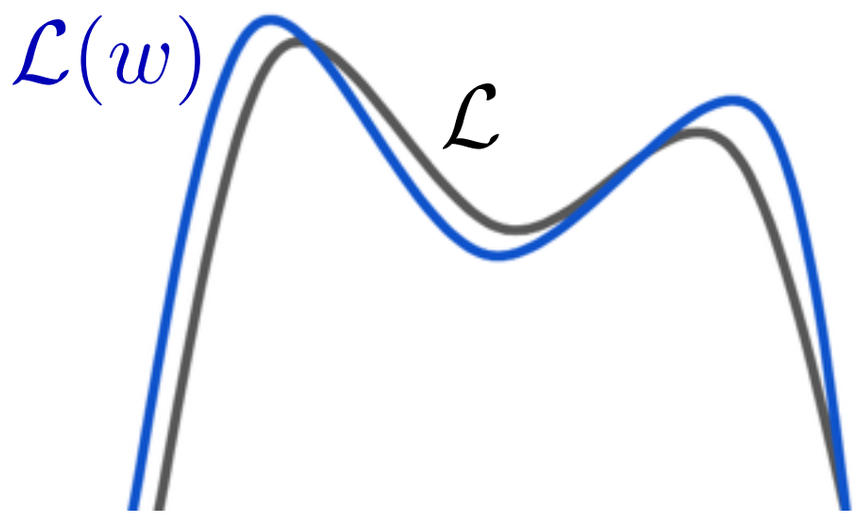
    - Approximate posterior close in Wasserstein distance





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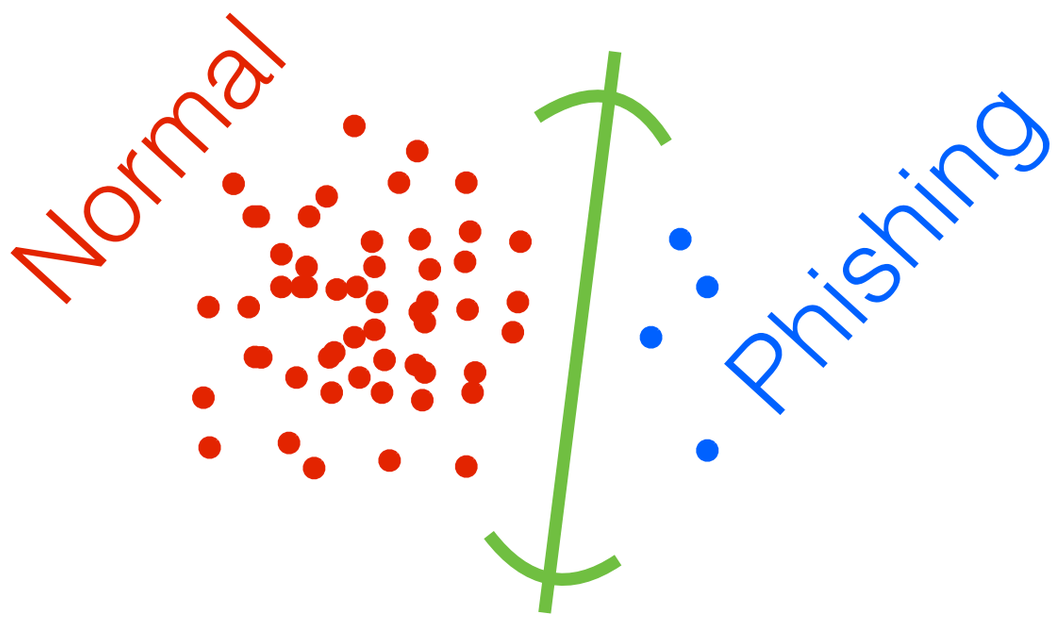

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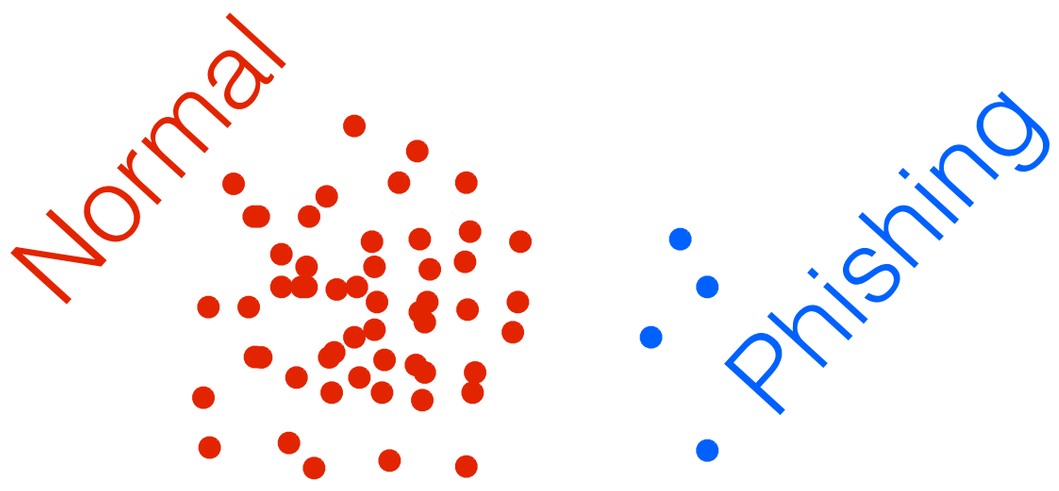

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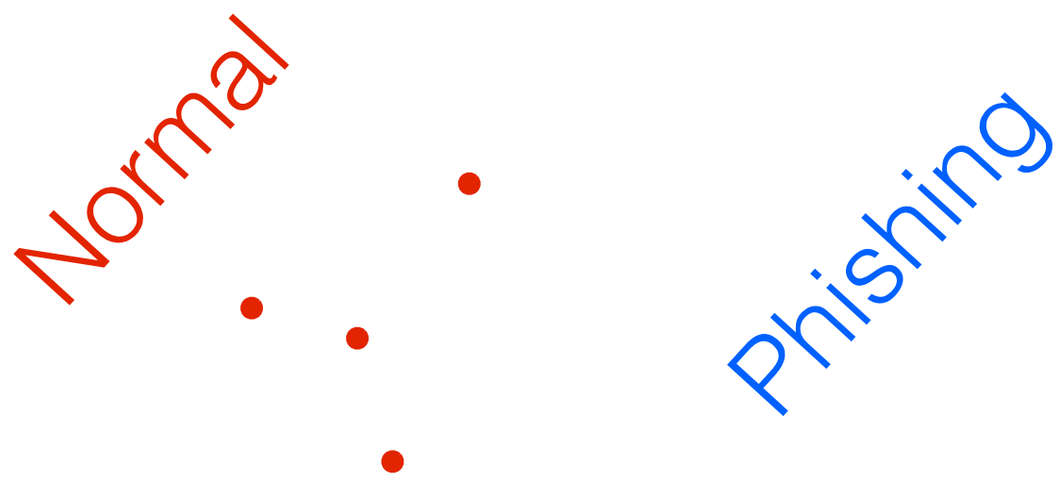

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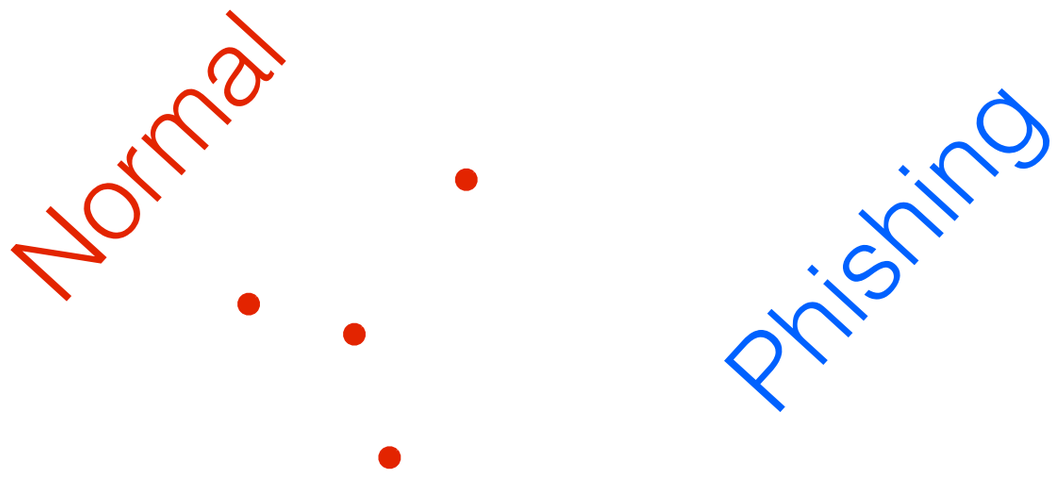

- Might miss important data

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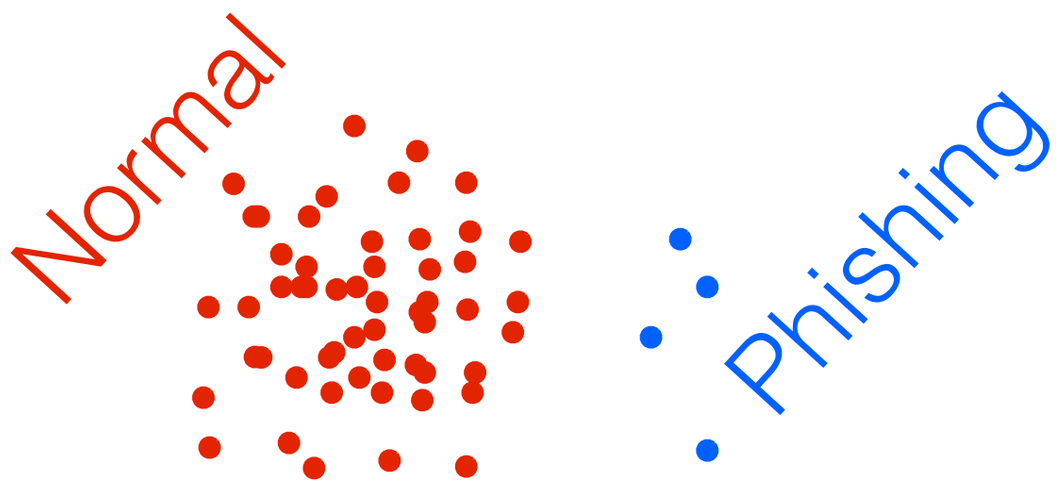

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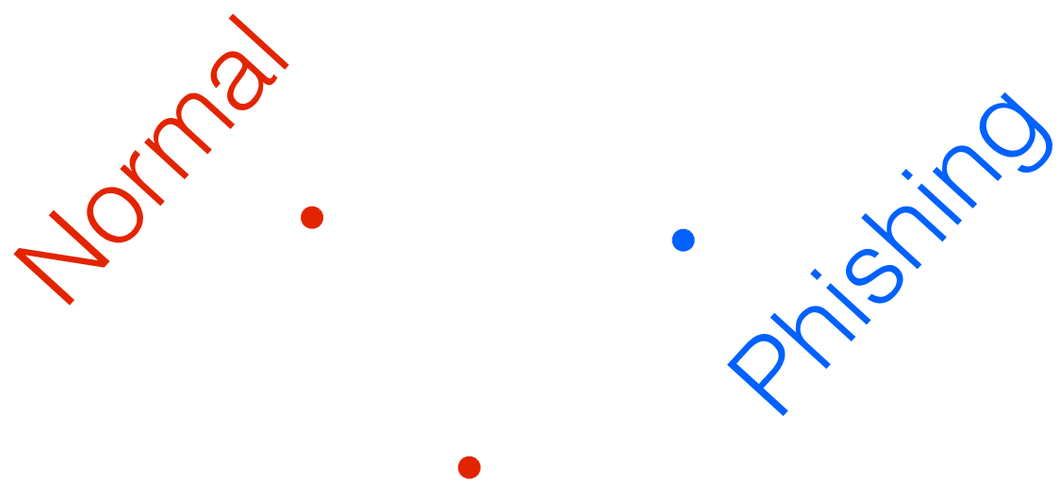

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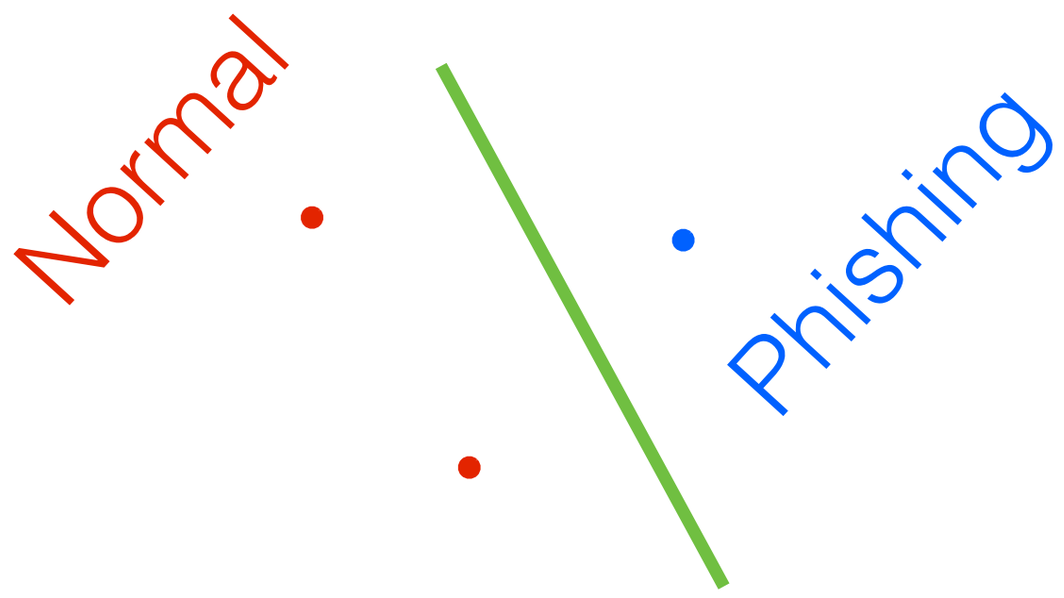

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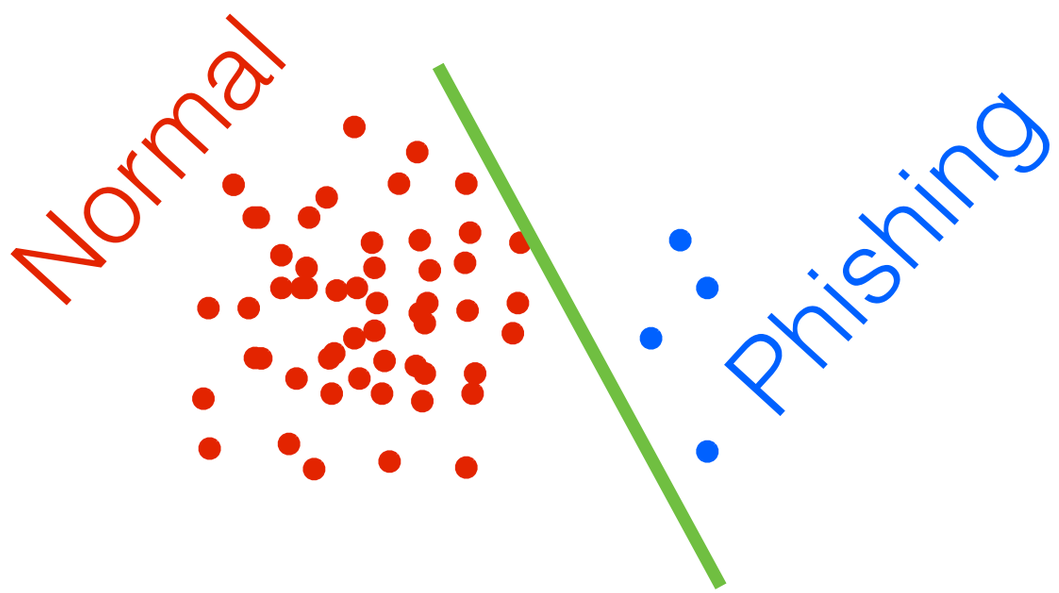

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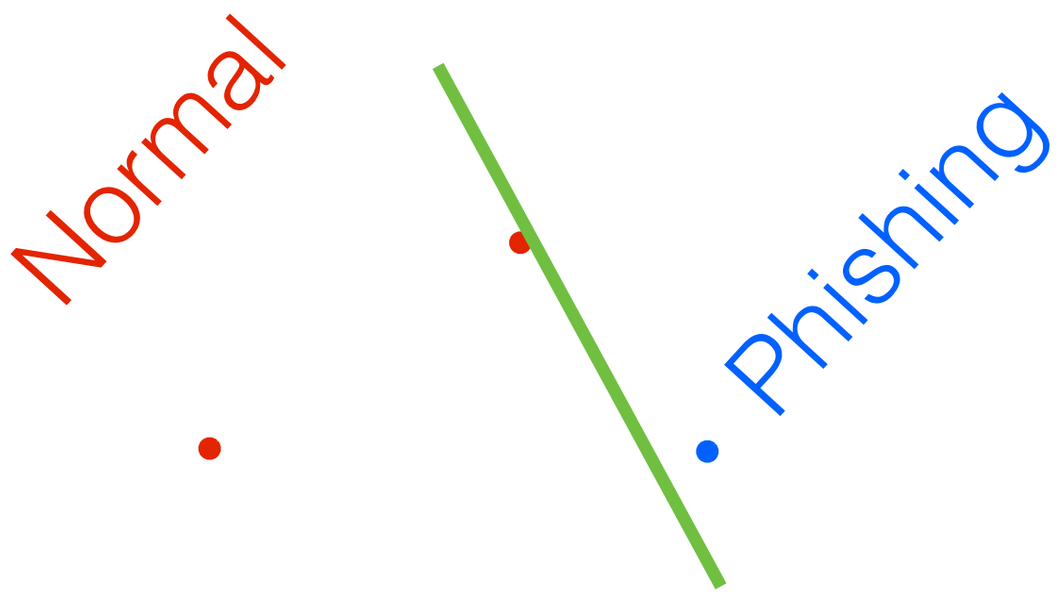

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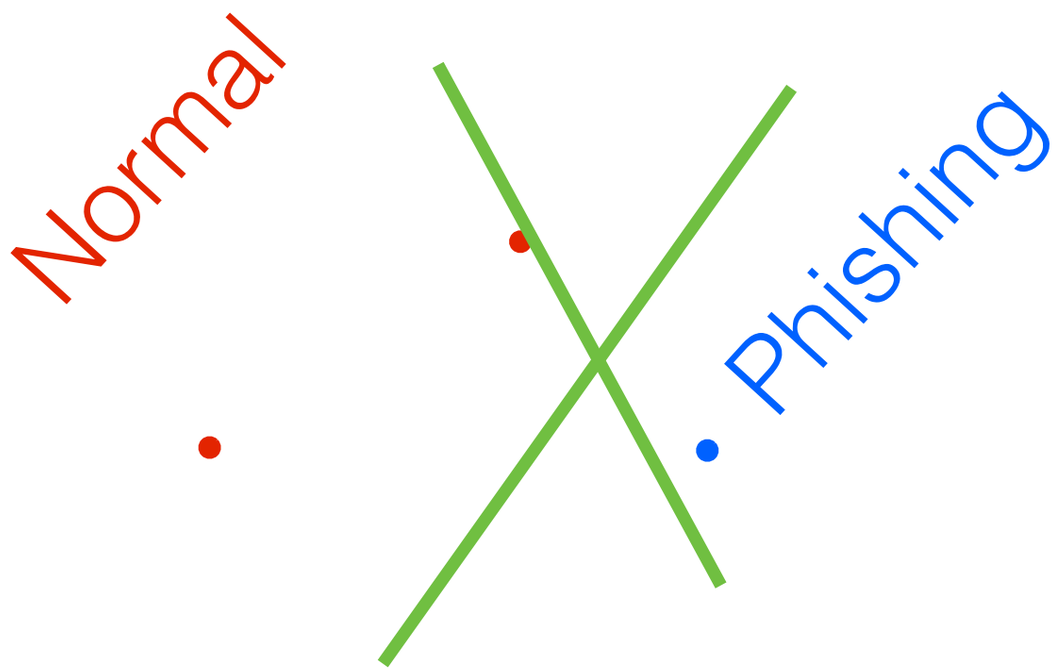

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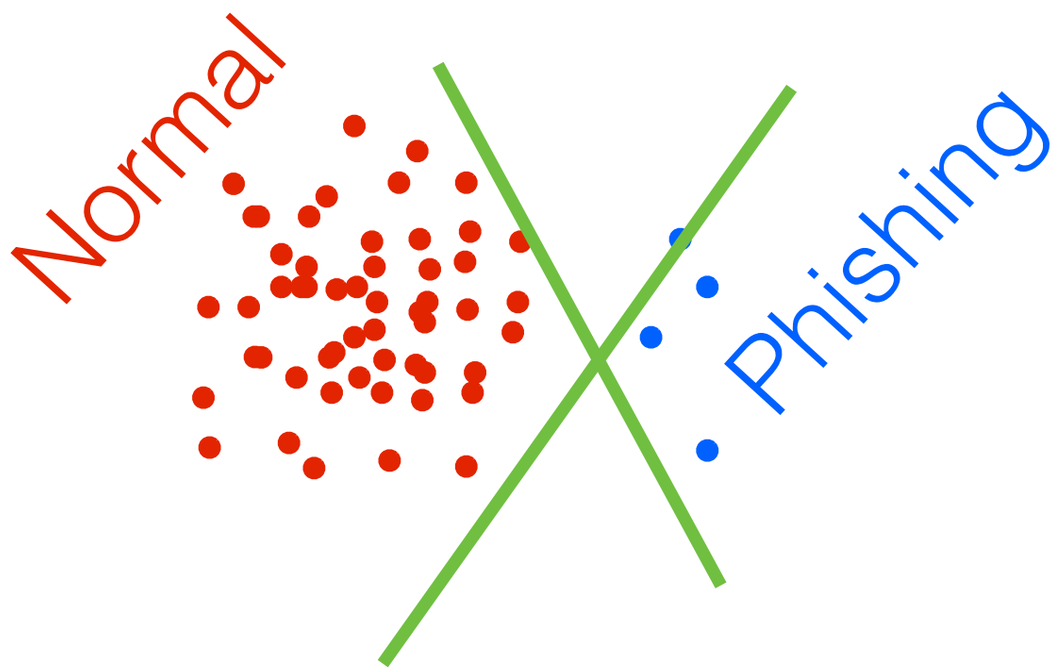

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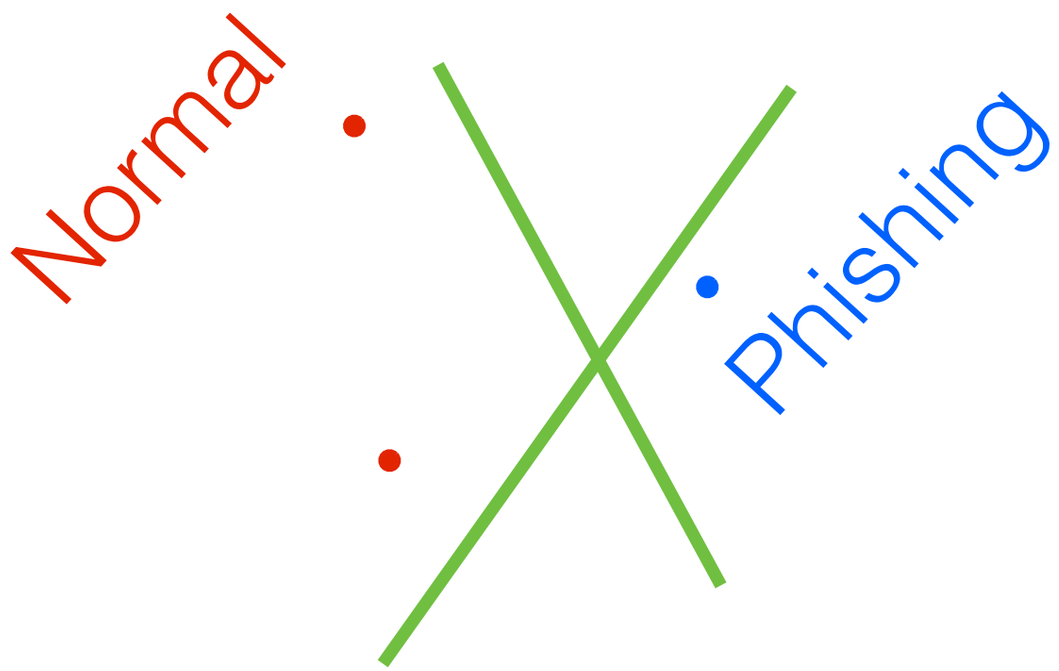

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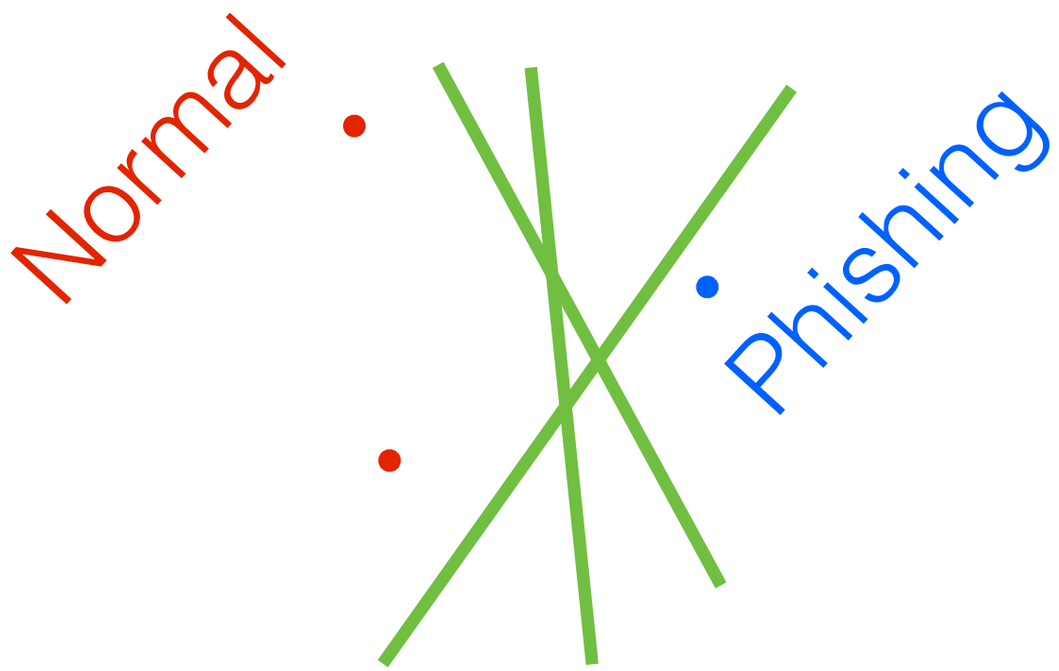

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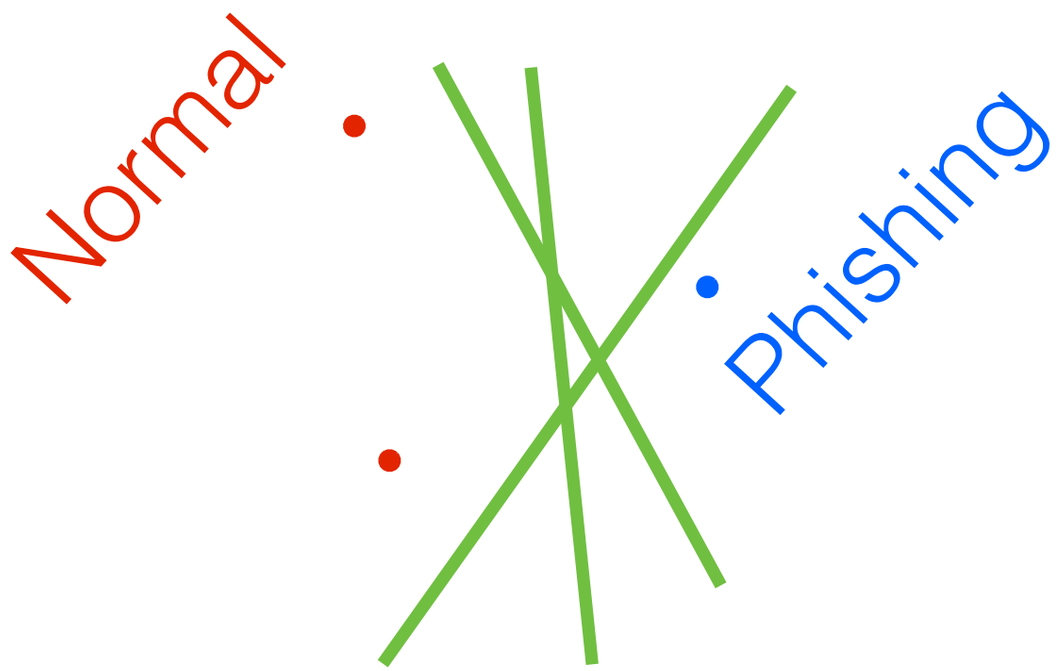

- Might miss important data
- Noisy estimates

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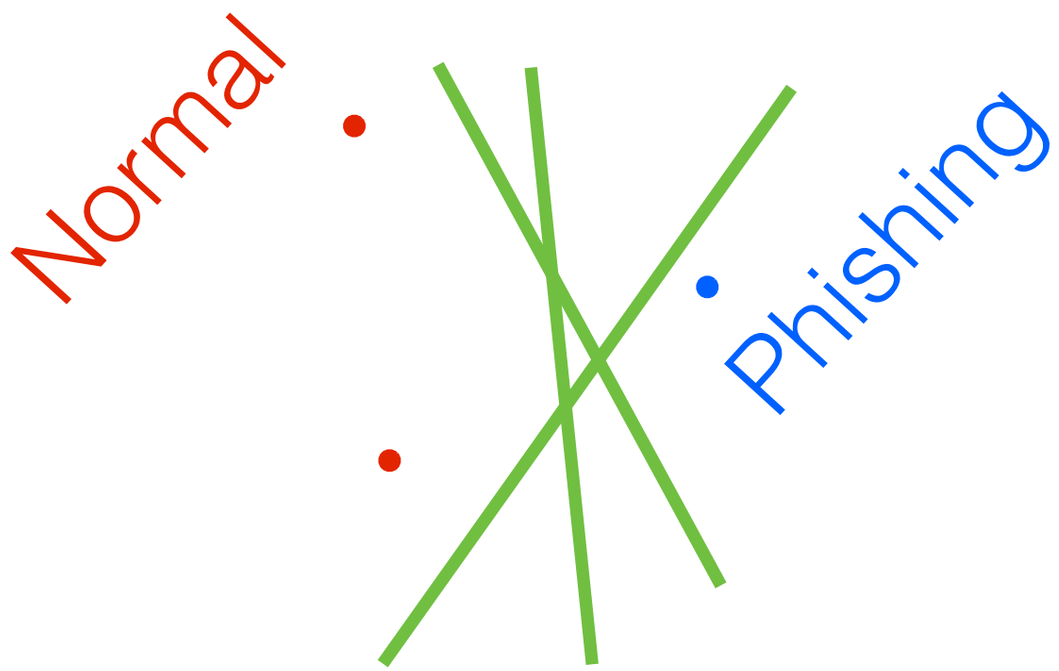

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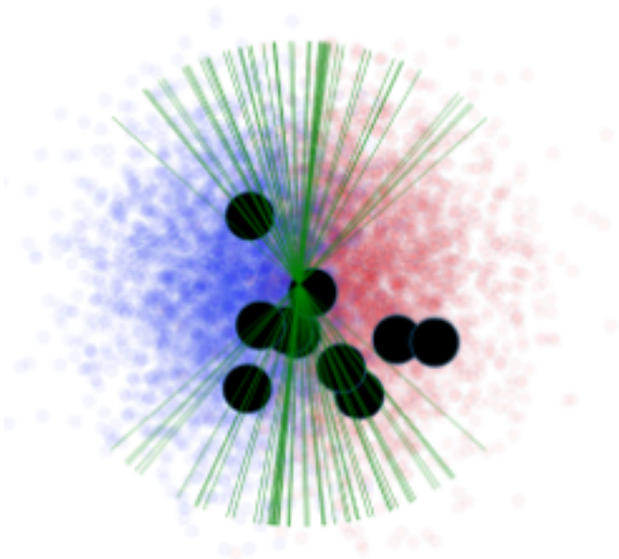
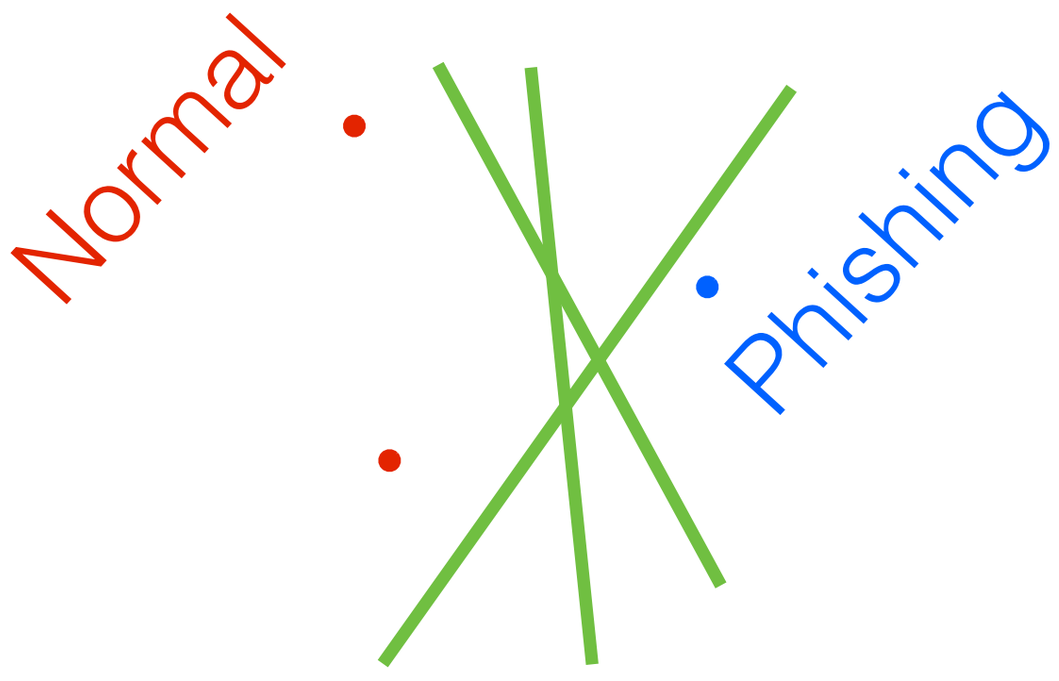


$M = 10$

Uniform subsampling revisited



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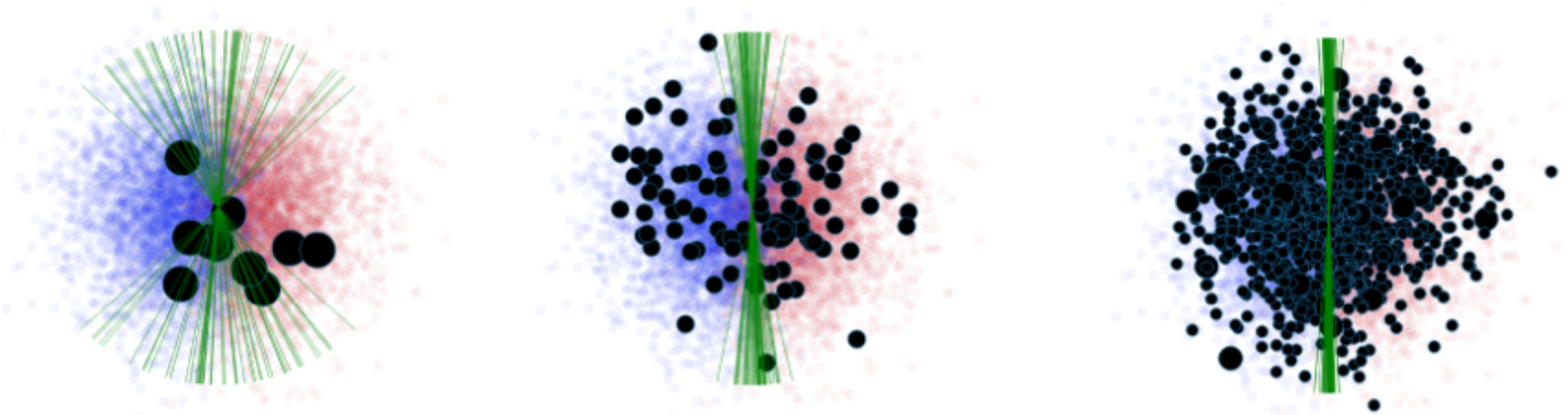

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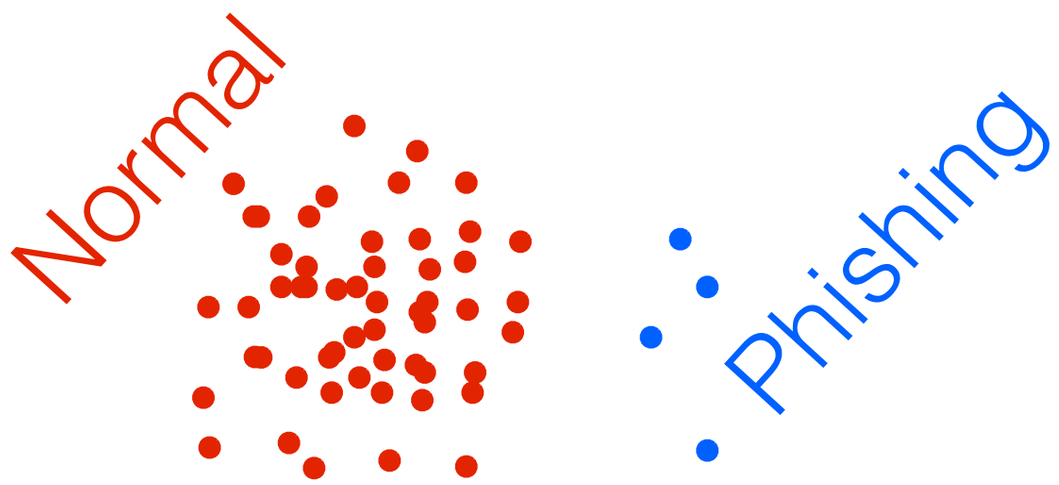

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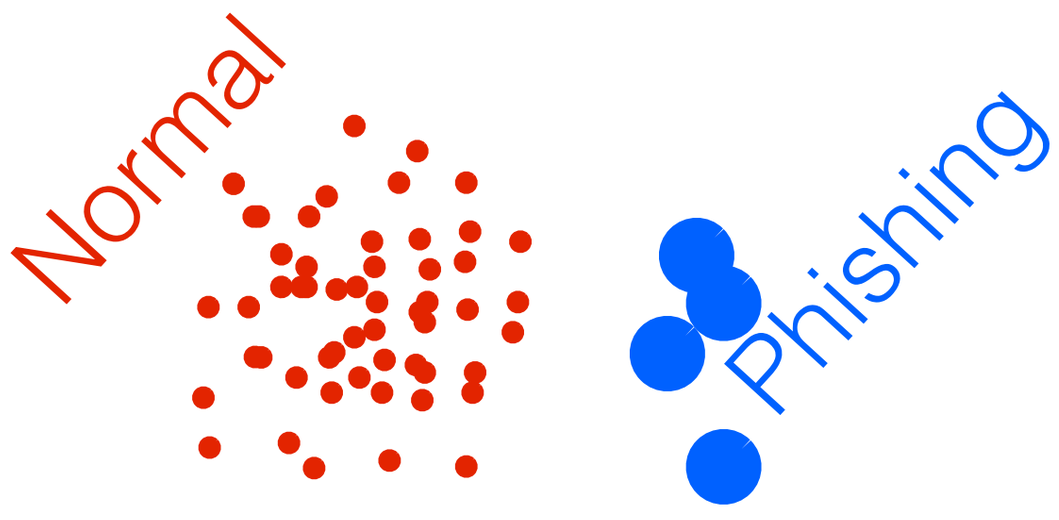

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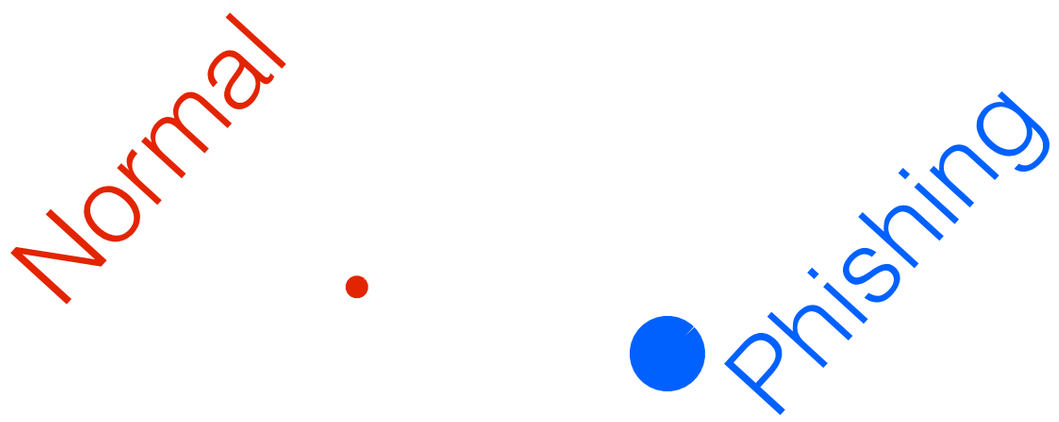

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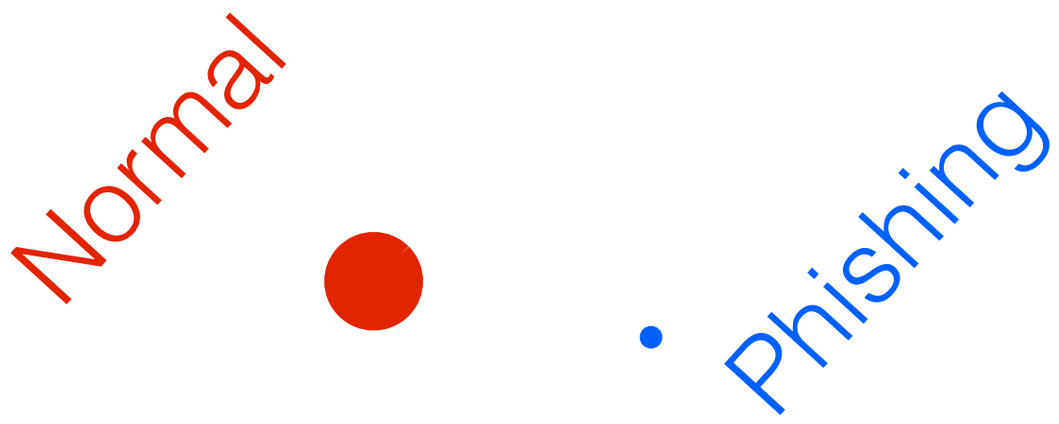

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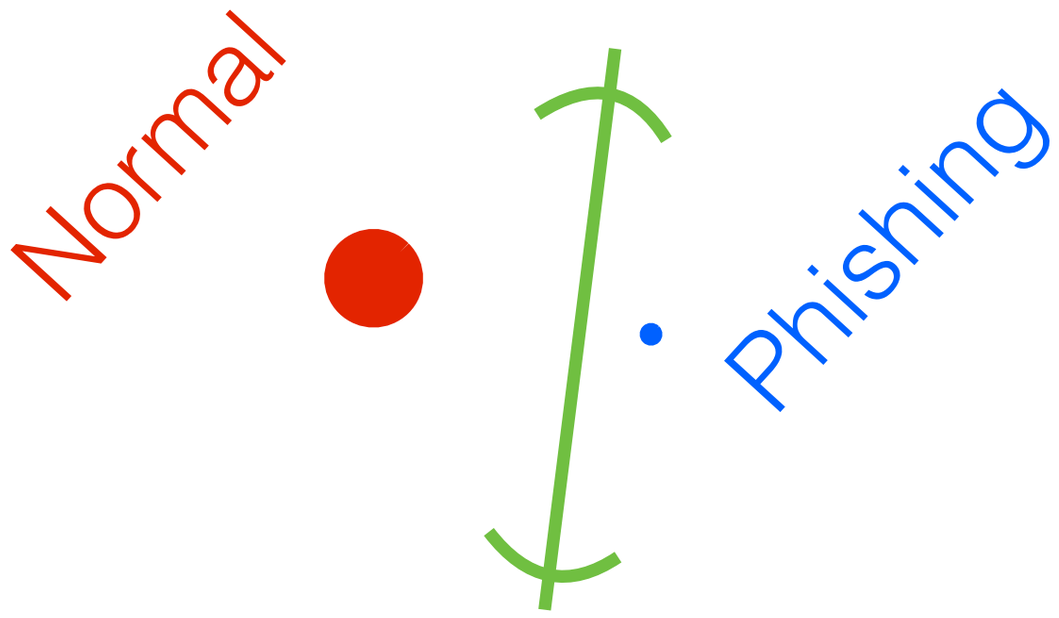

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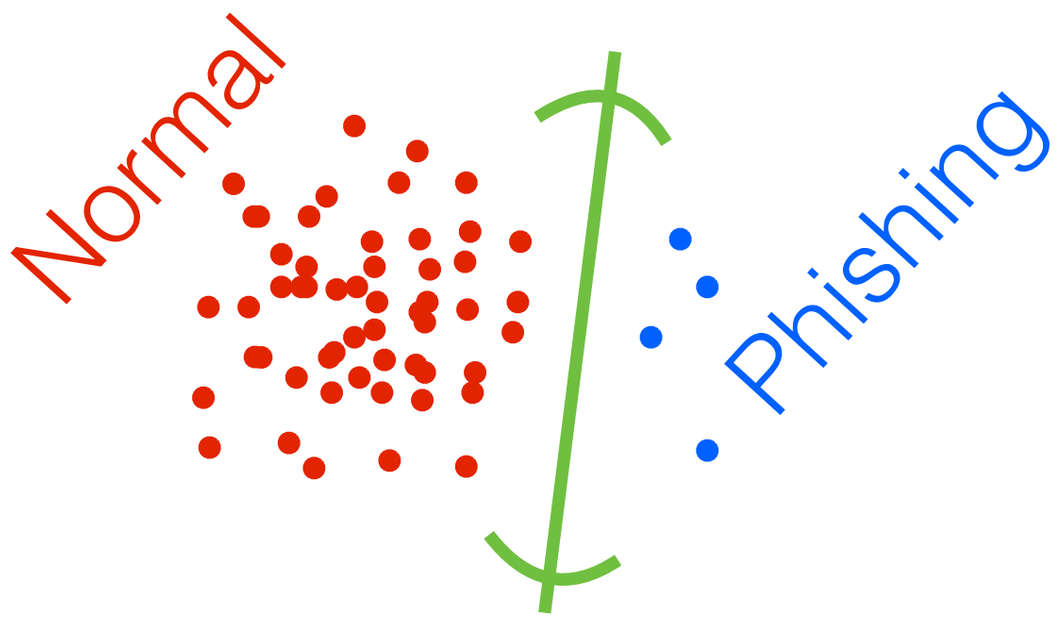

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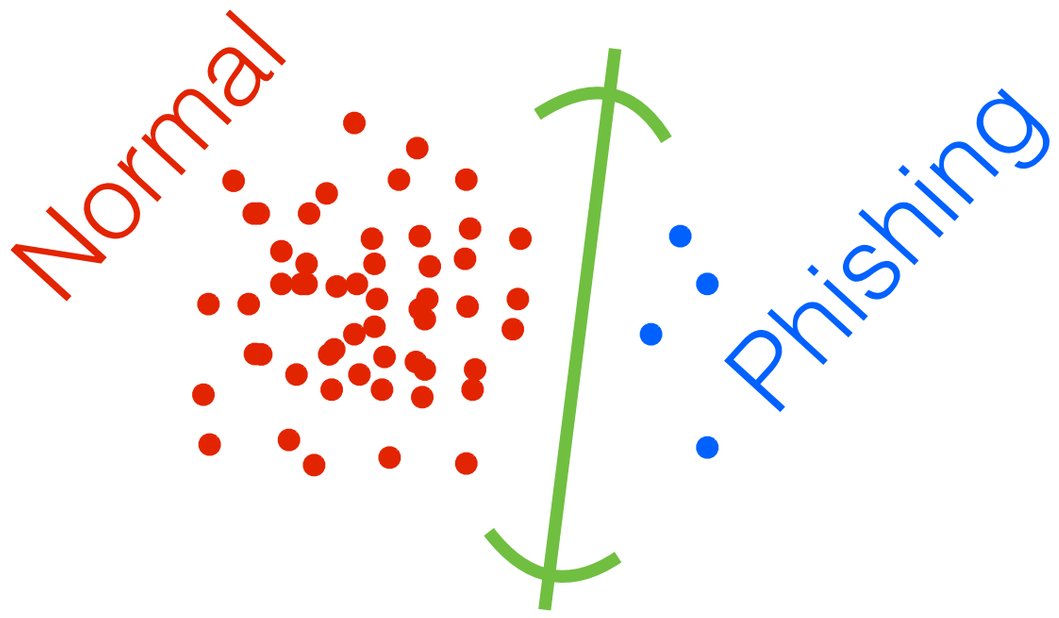
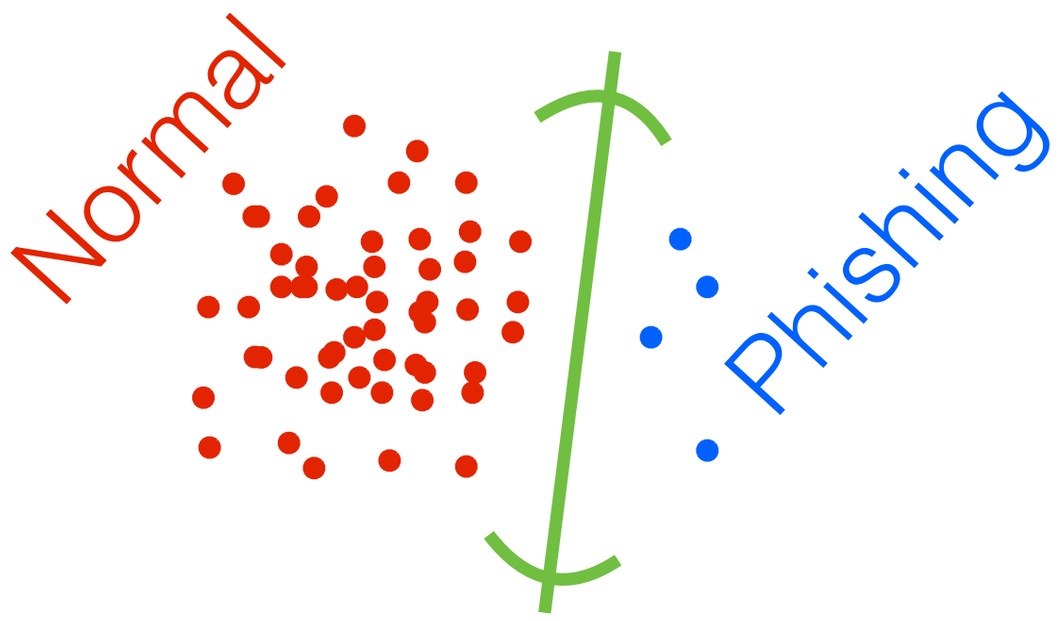


$$\sigma_n \propto \|\mathcal{L}_n\|$$

Importance sampling



$$\sigma := \sum_{n=1}^{N} \|\mathcal{L}_n\|$$

$$\sigma_n := \|\mathcal{L}_n\| / \sigma$$

Importance sampling

Thm sketch (CB). $\delta \in (0,1)$. W.p. $\geq 1 - \delta$, after M iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma\bar{\eta}}{\sqrt{M}}\left(1 + \sqrt{2\log\frac{1}{\delta}}\right)$$

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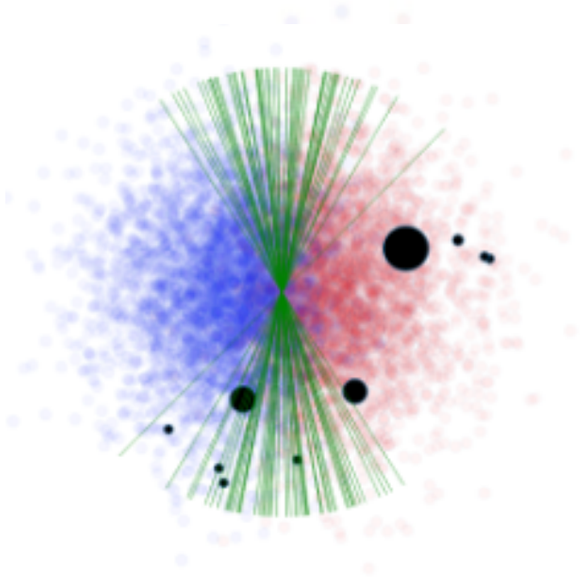

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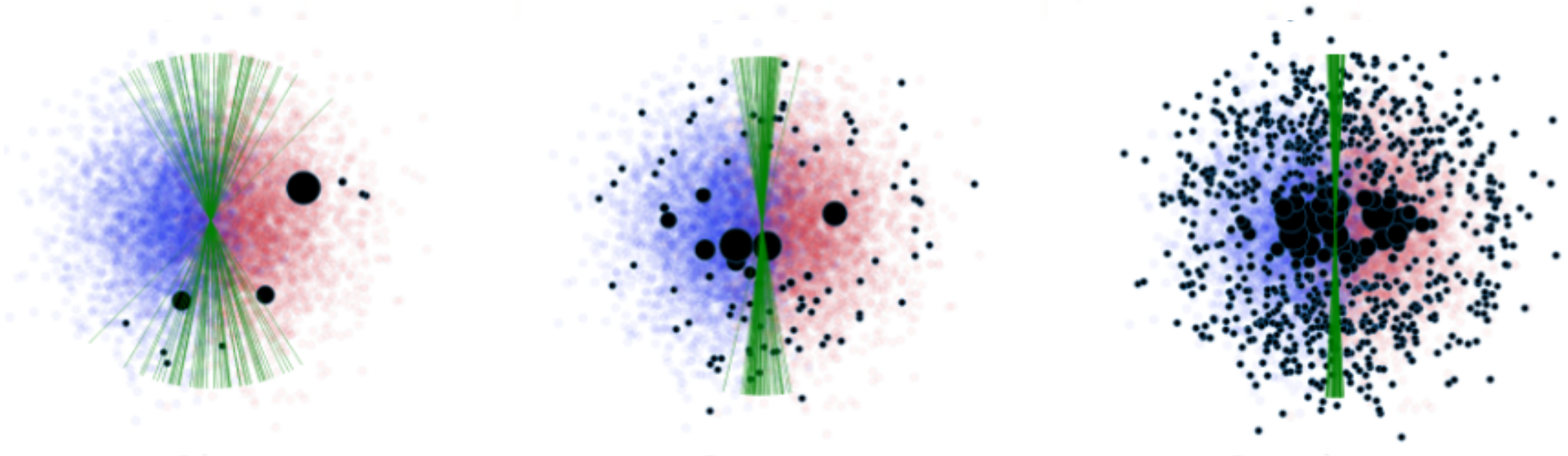

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Hilbert coresets

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$$\text{s.t. } w \geq 0, \|w\|_0 \leq M$$

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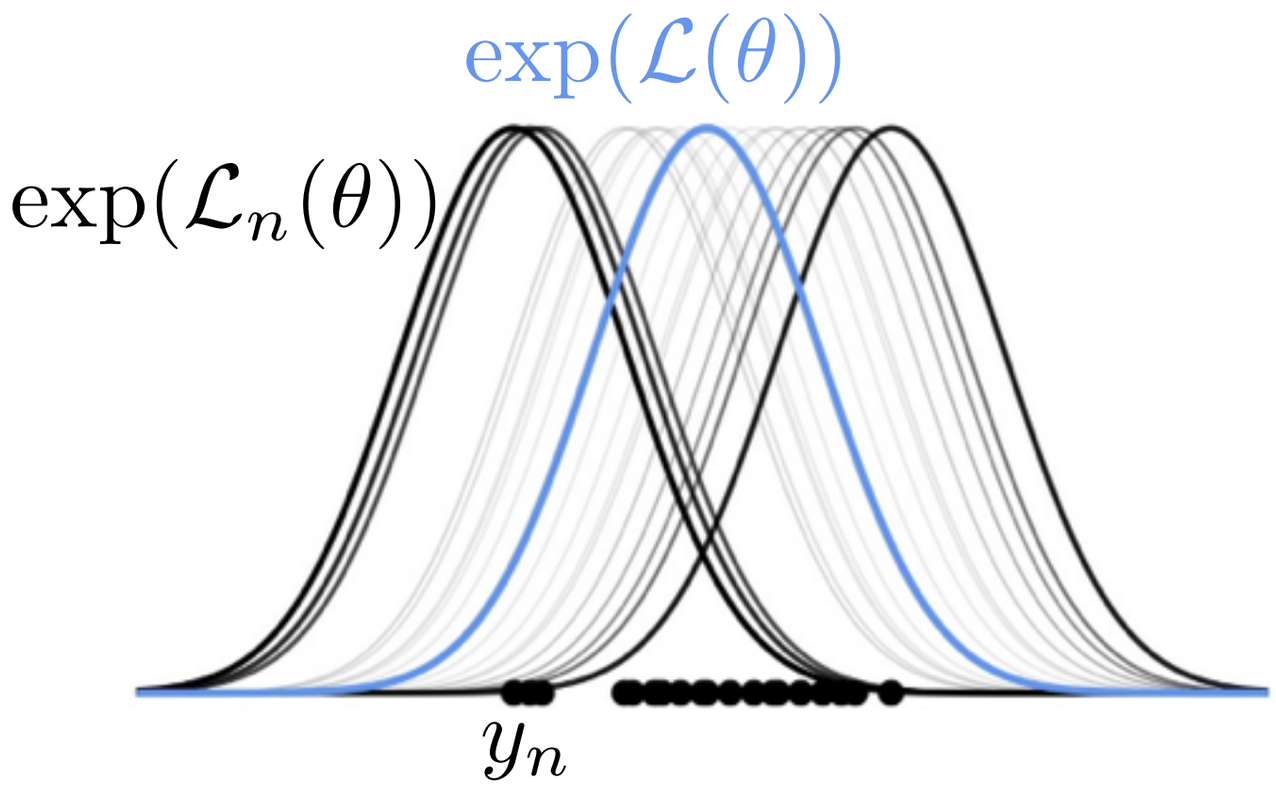

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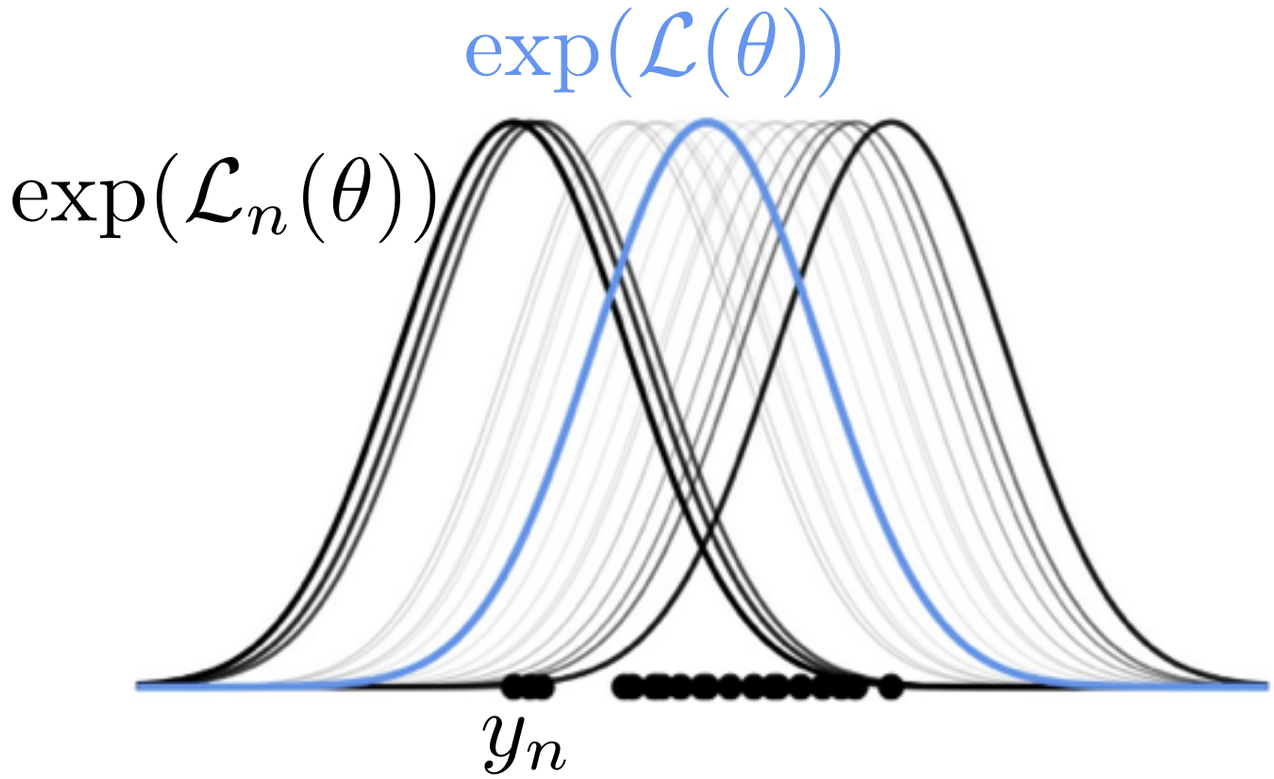

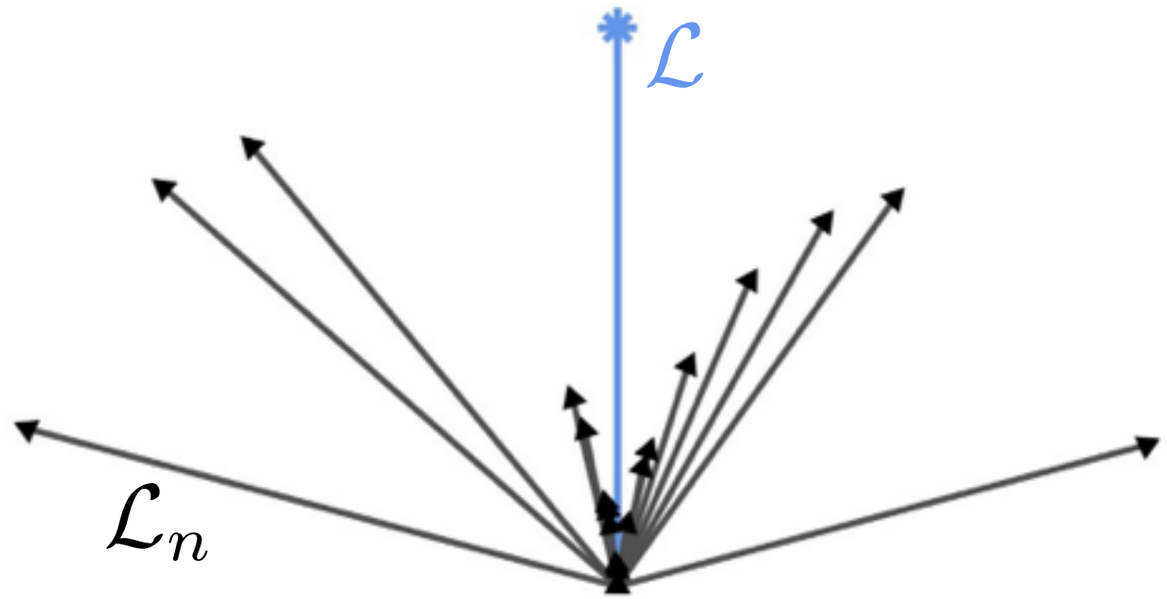

Hilbert coresets

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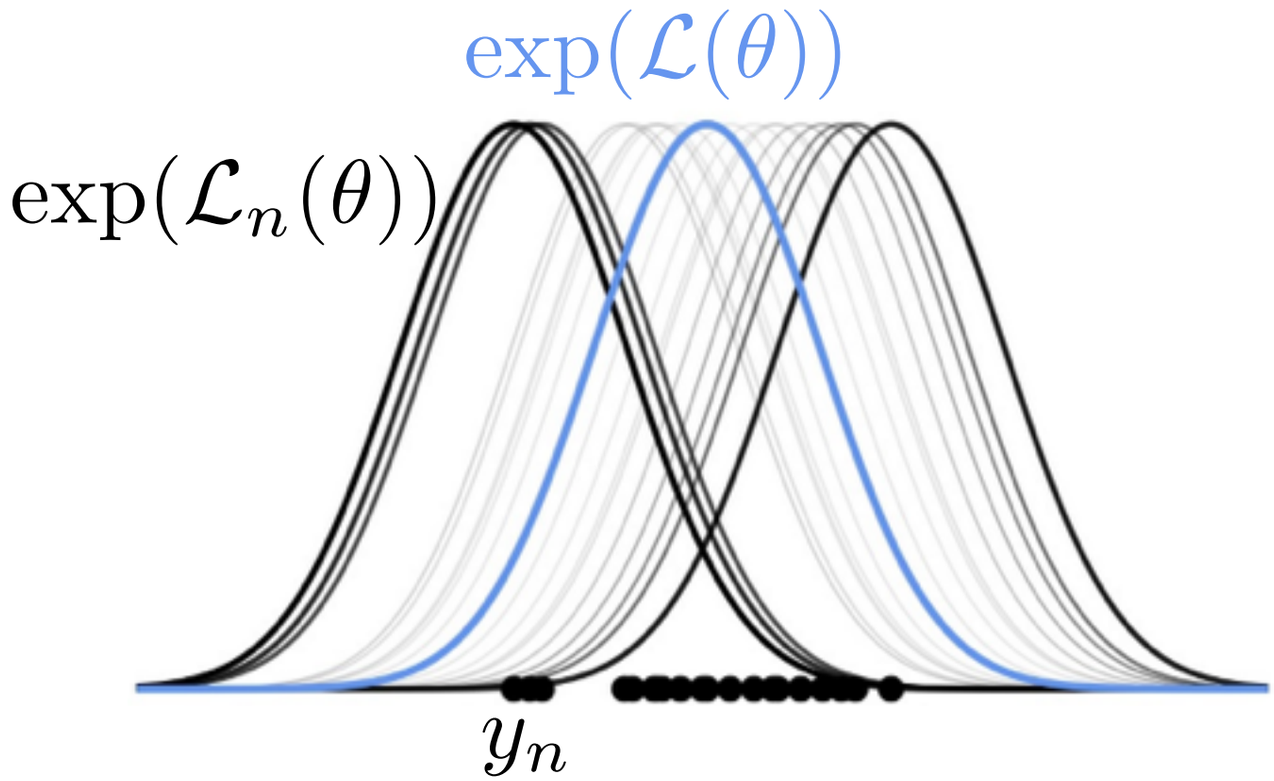

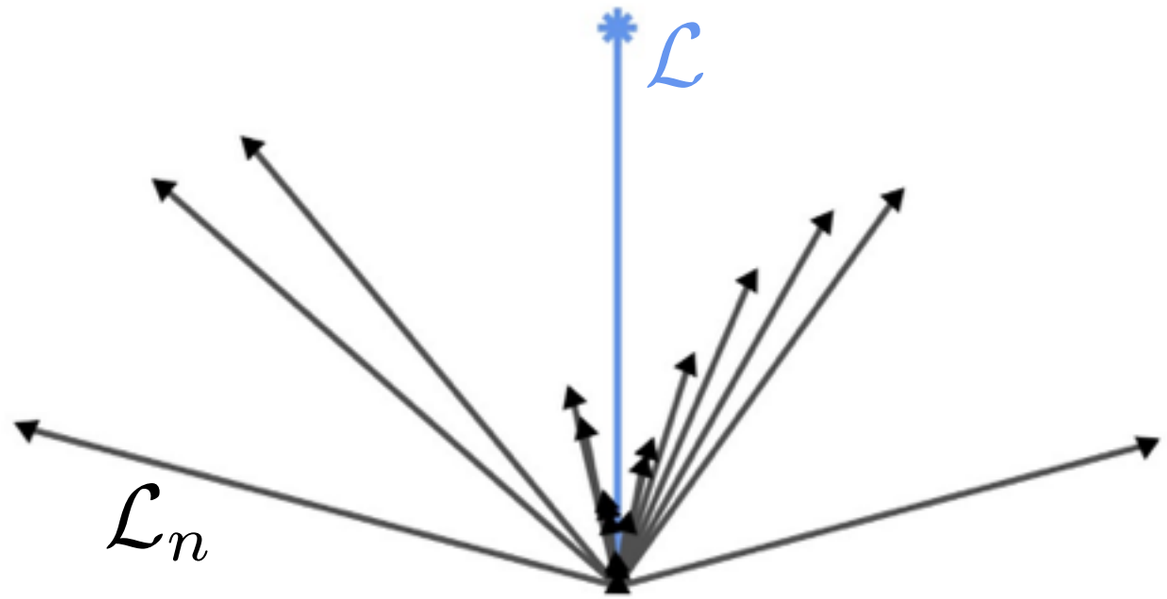

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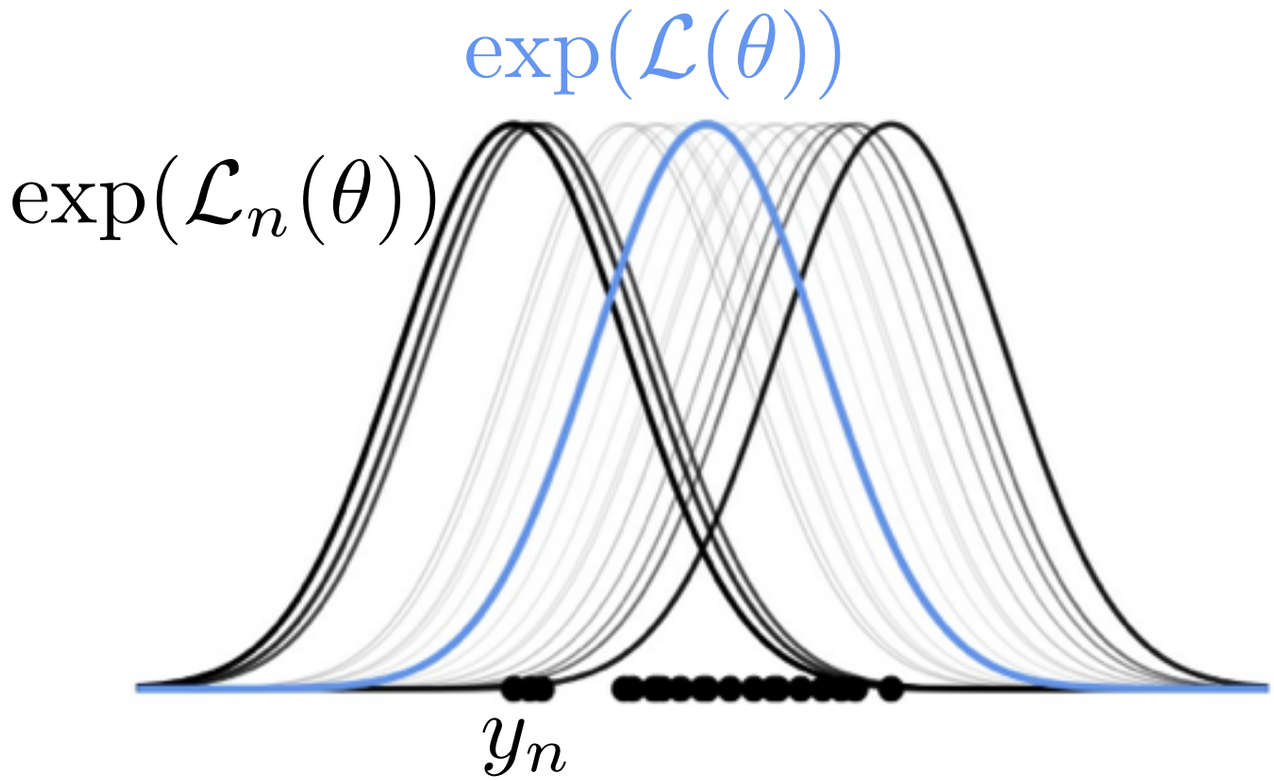

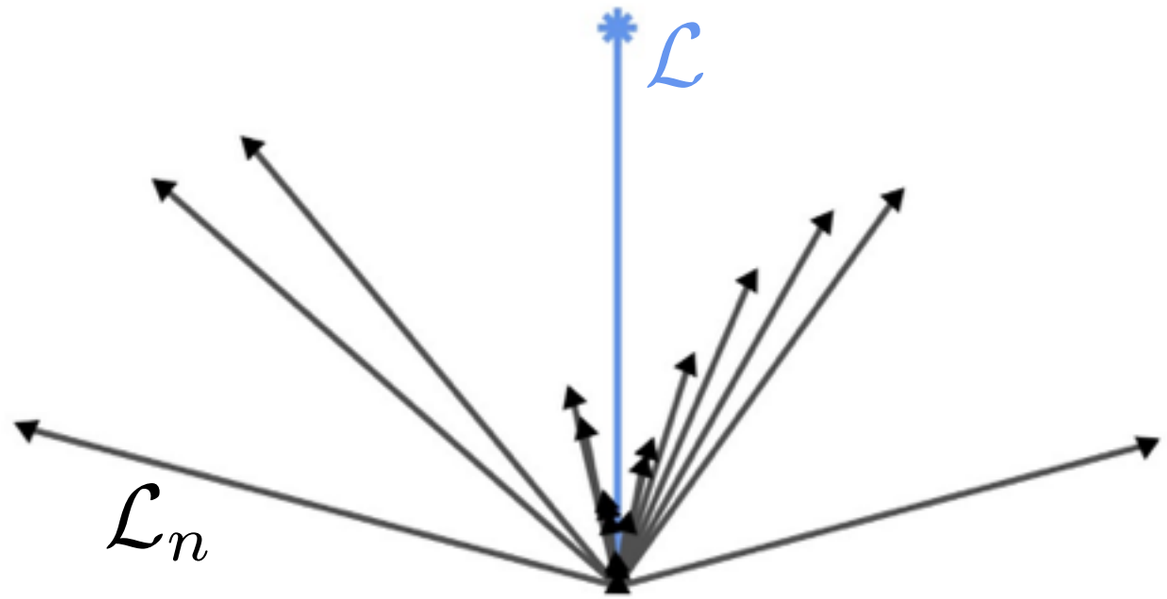

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Roadmap

- Approximate Bayes review
- The “core” of the data set
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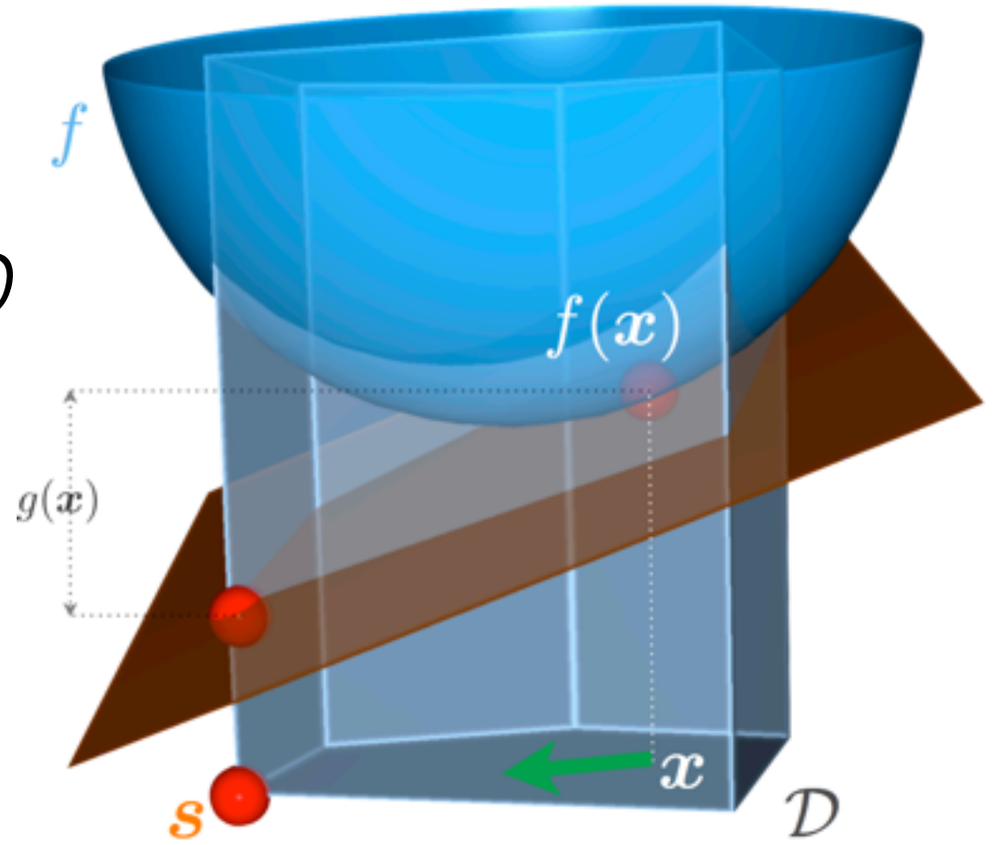
Convex optimization on a polytope *D*



[Jaggi 2013]

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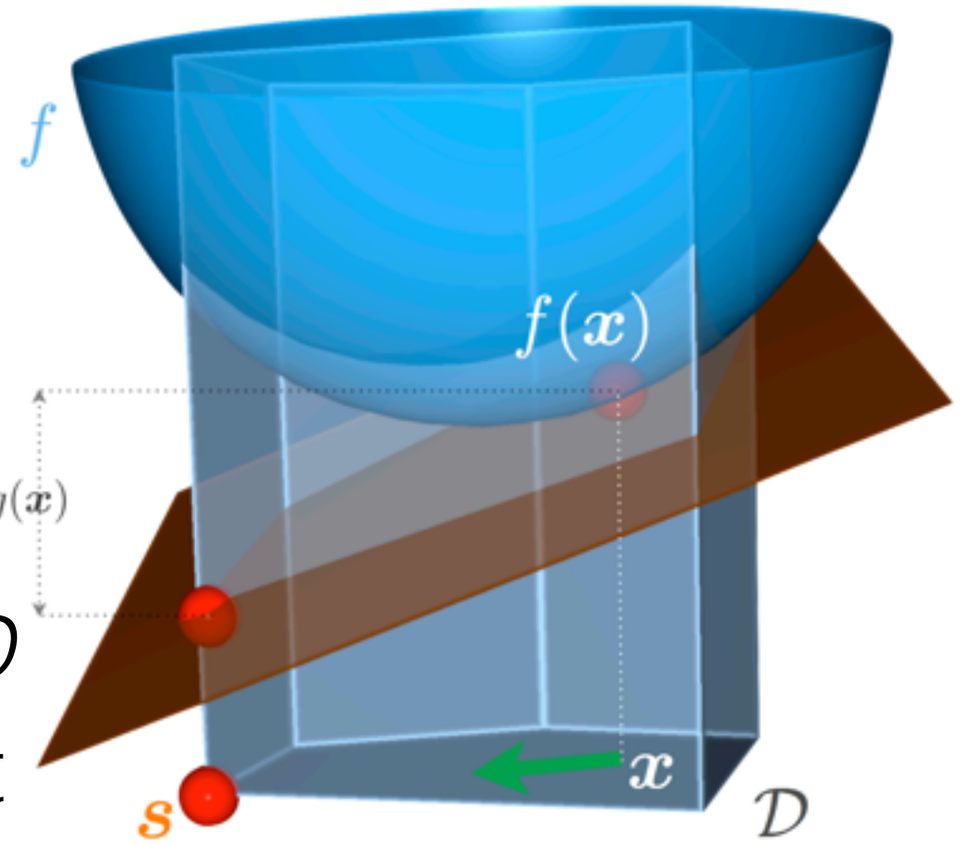

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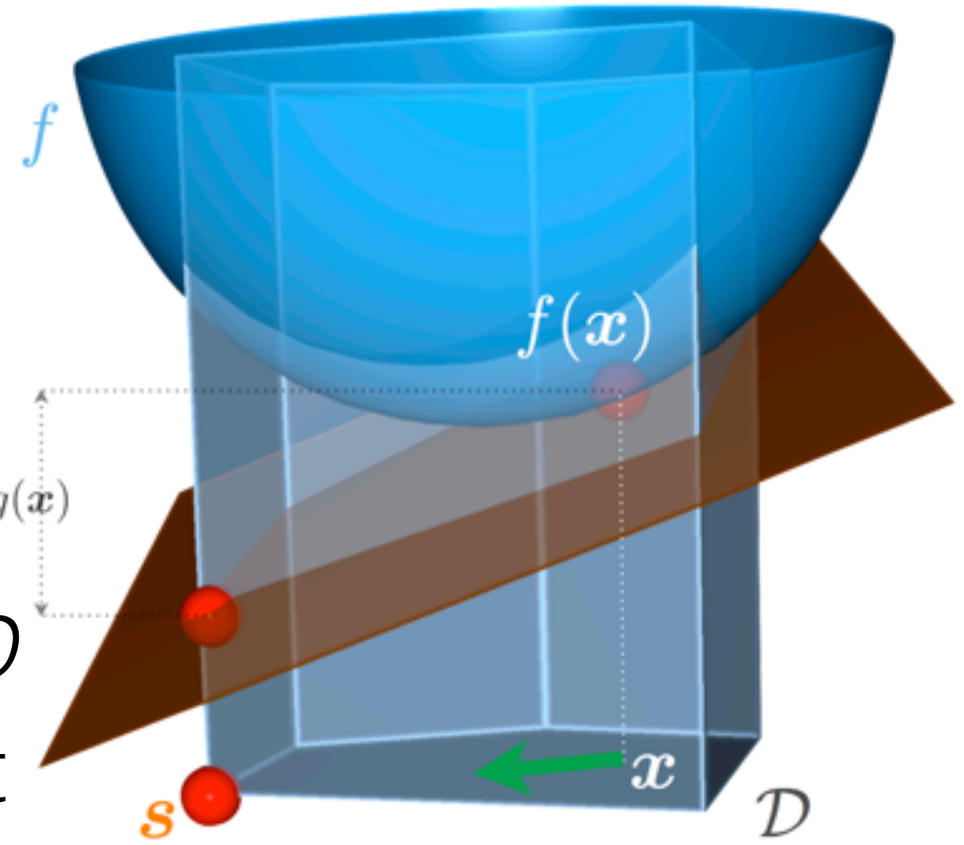

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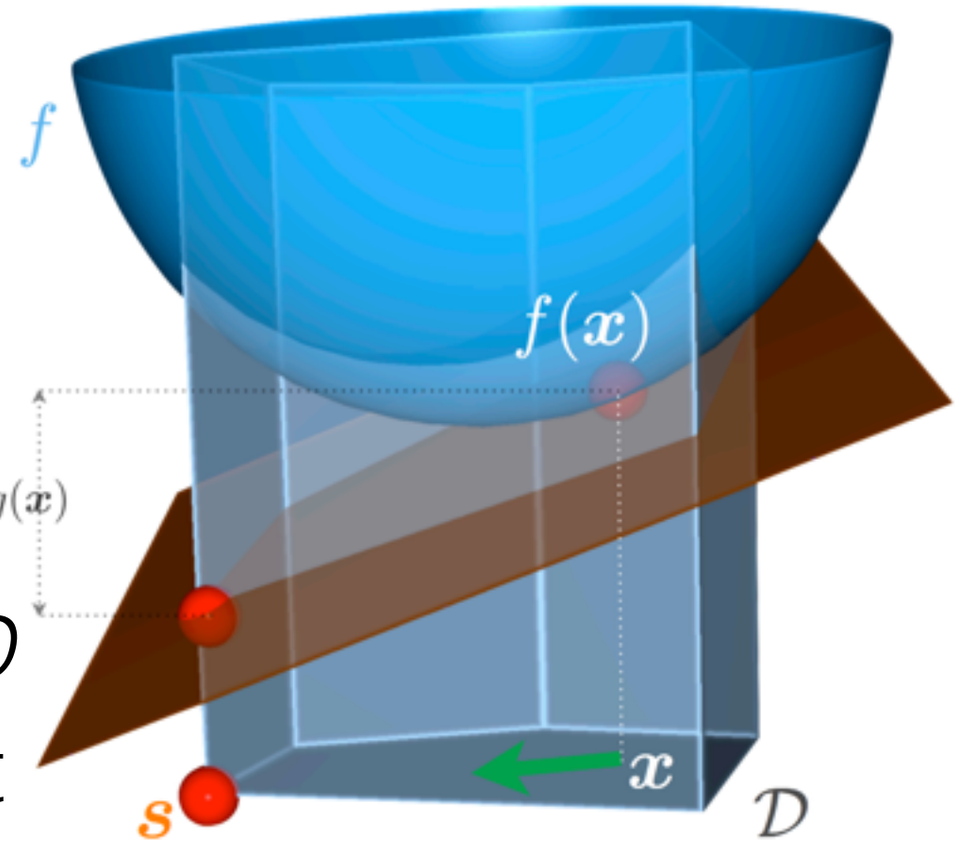

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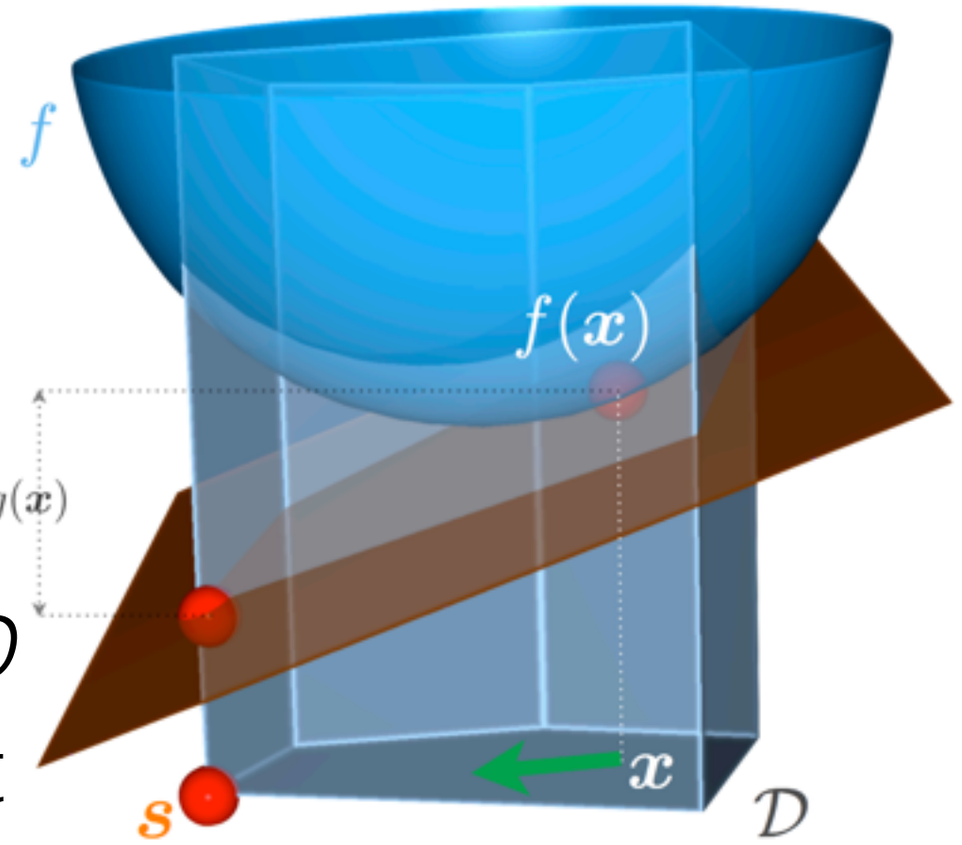

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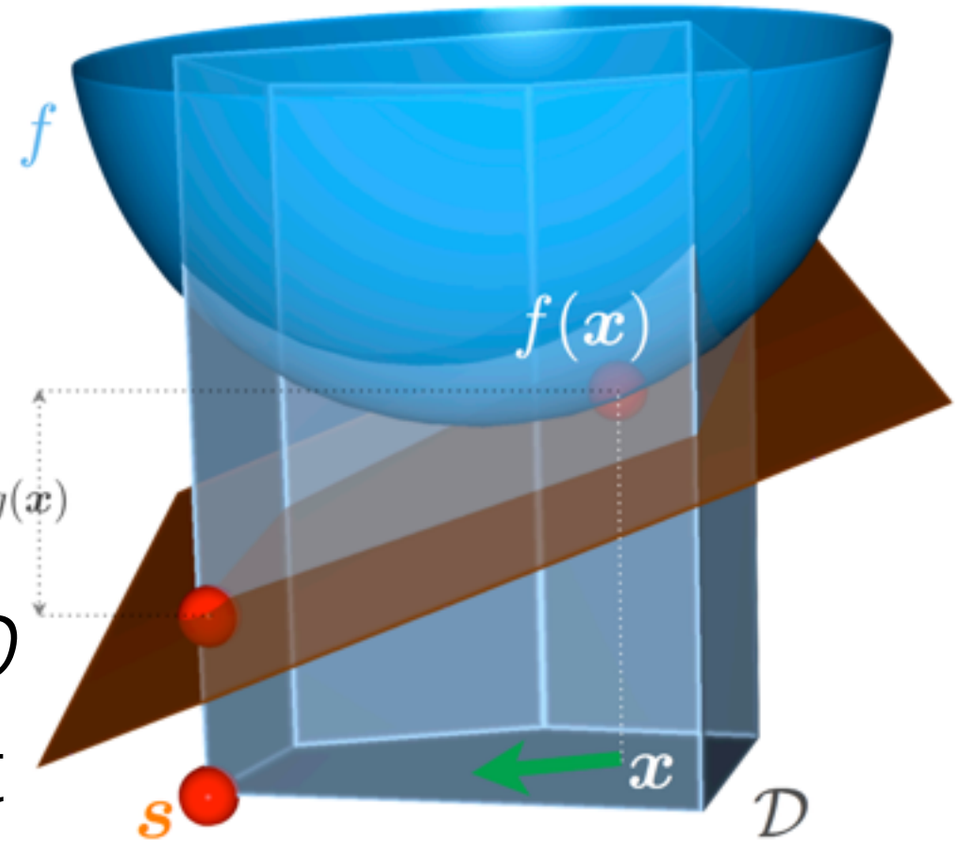

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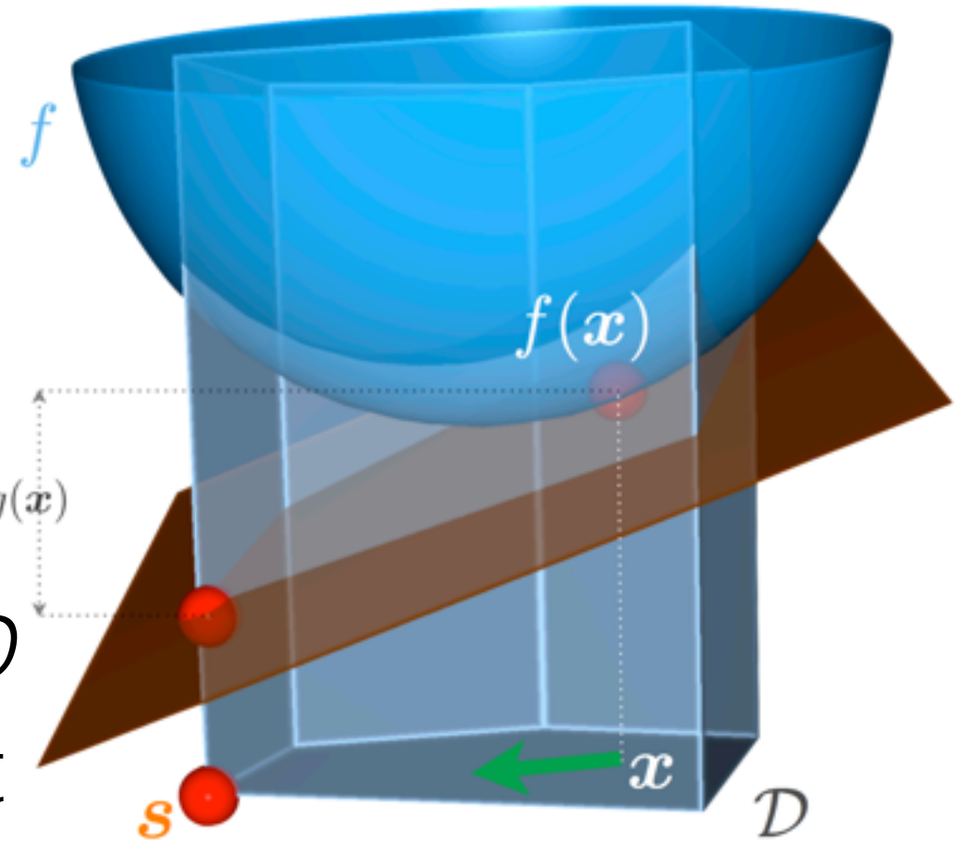

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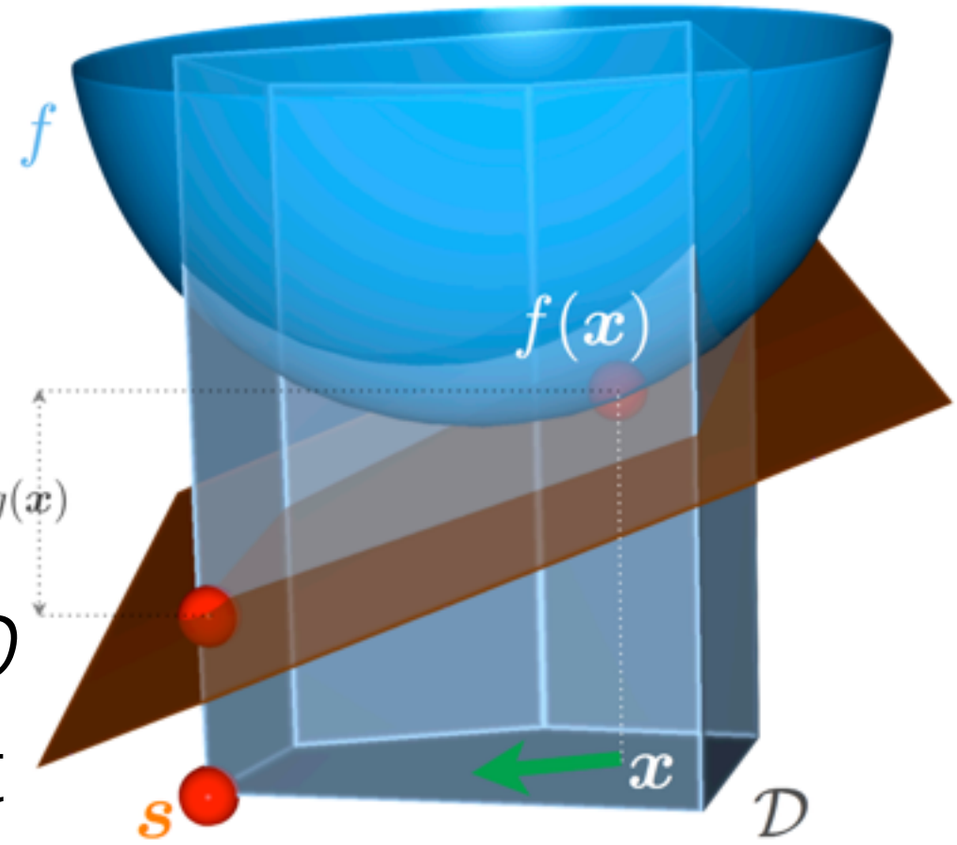

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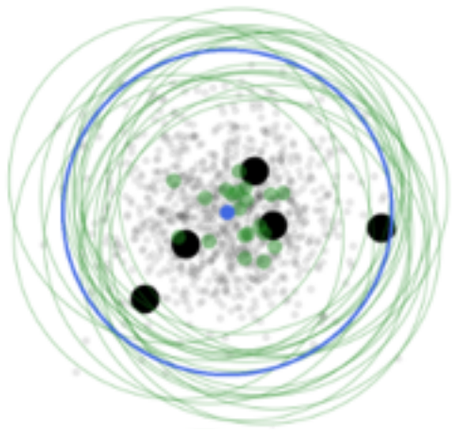
Thm sketch (CB). After *M* iterations,

$$\|\mathcal{L}(w) - \mathcal{L}\| \leq \frac{\sigma \bar{\eta}}{\sqrt{\alpha^{2M} + M}}$$

Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

Uniform subsampling



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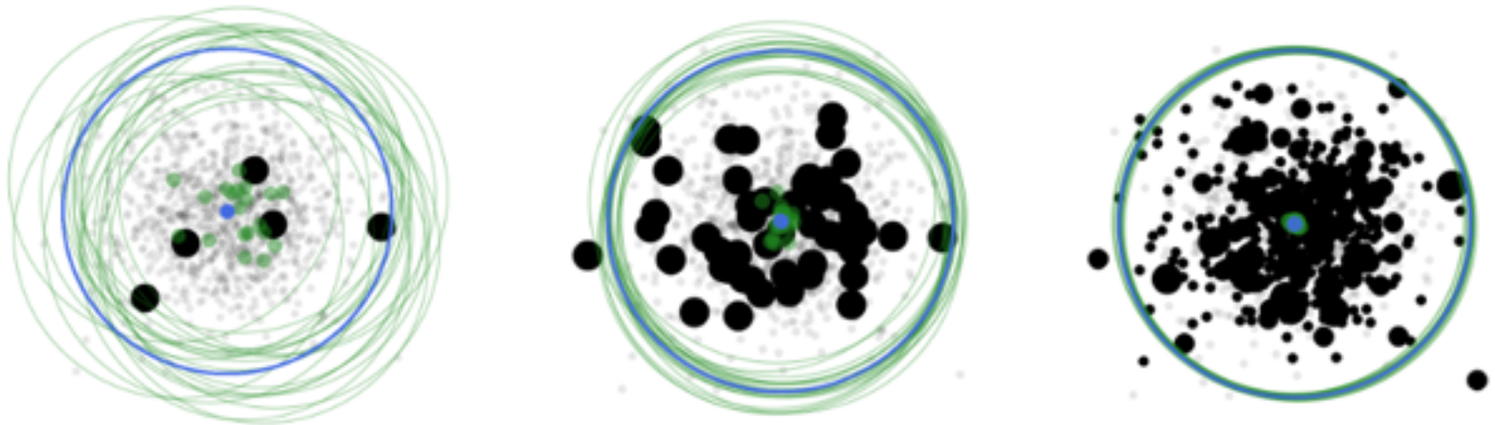

$M = 5$ $M = 50$ $M = 500$

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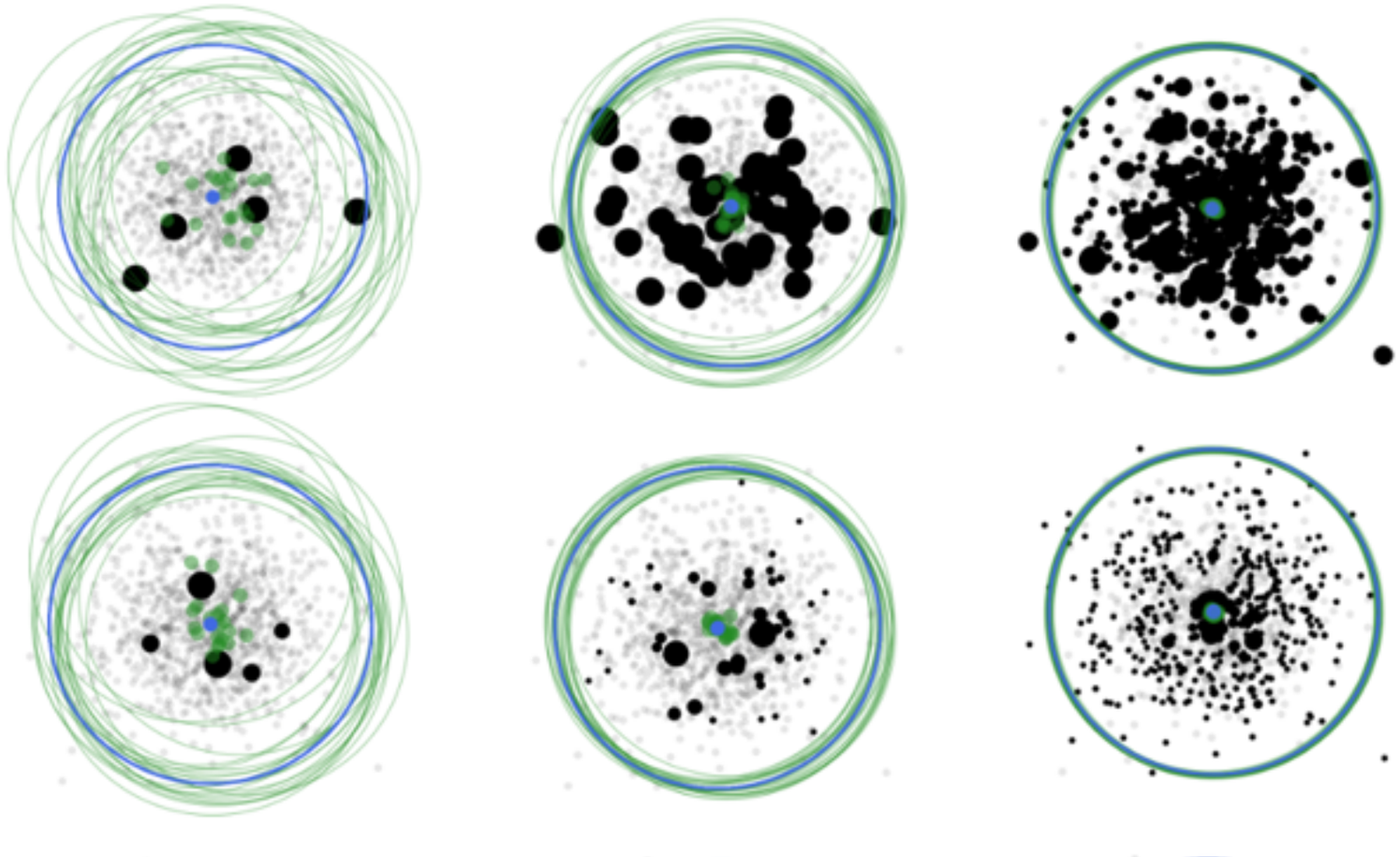

Importance sampling

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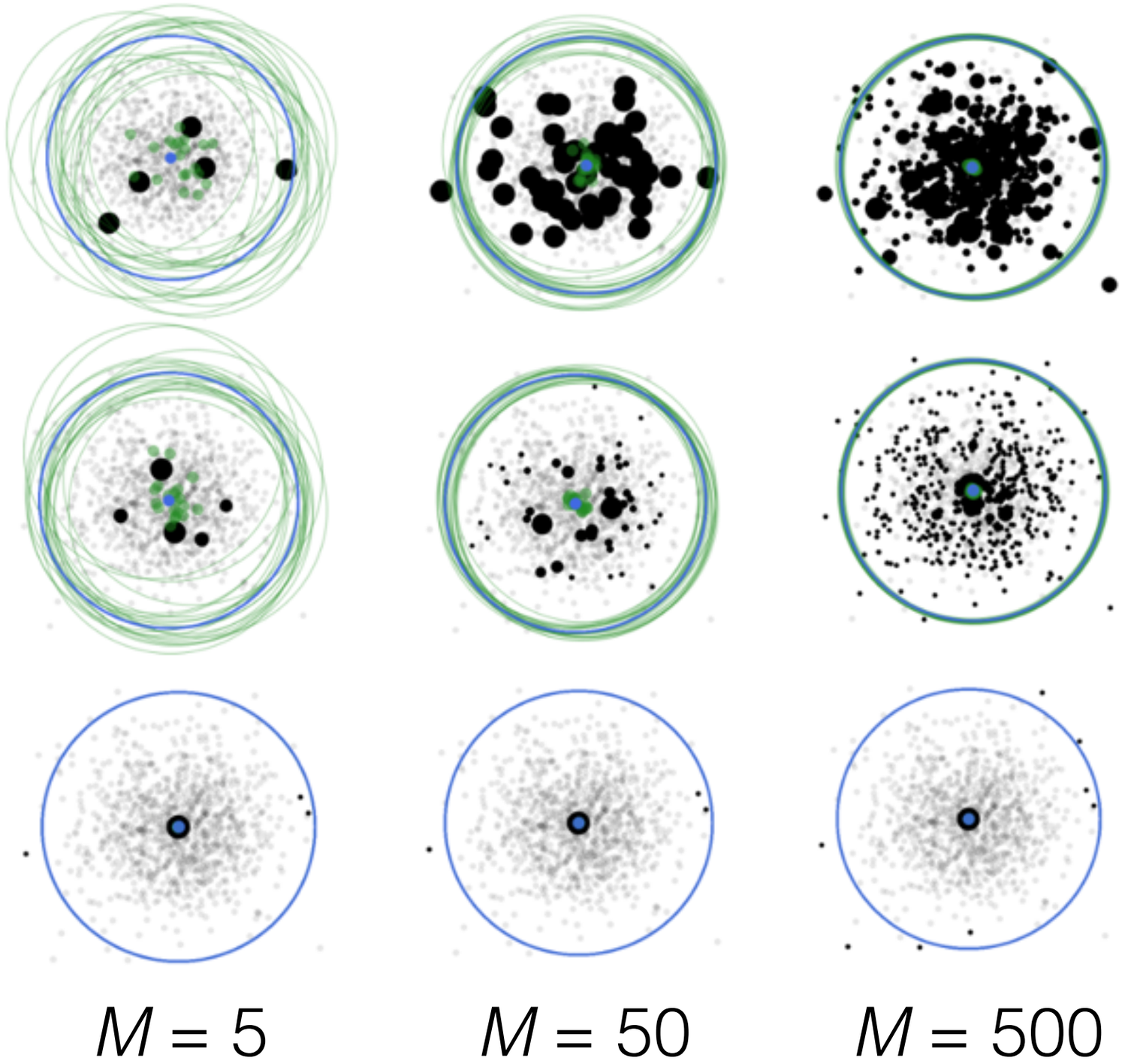
Frank-Wolfe



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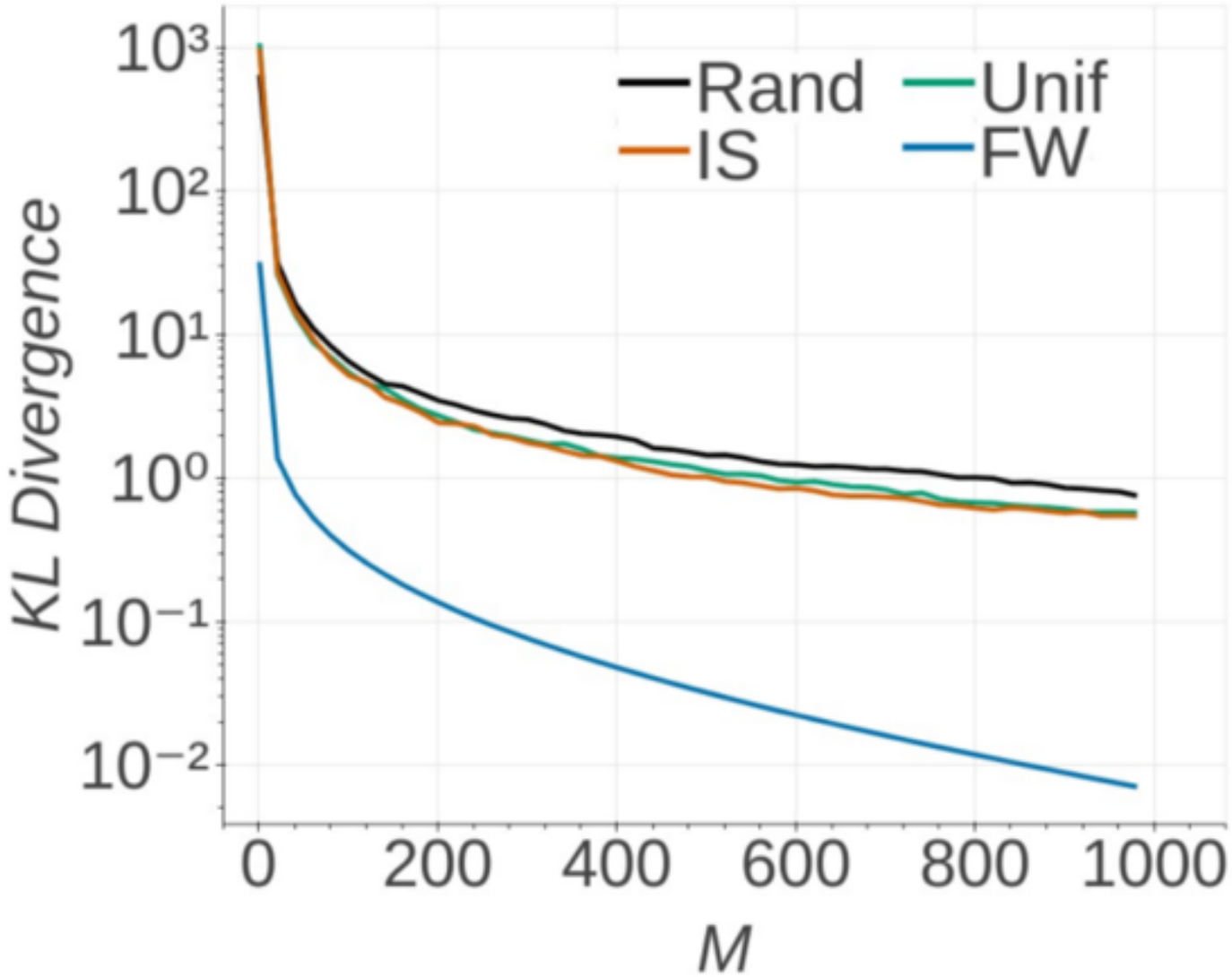
lower error



Logistic regression (simulated)

- 10K data points

Uniform subsampling

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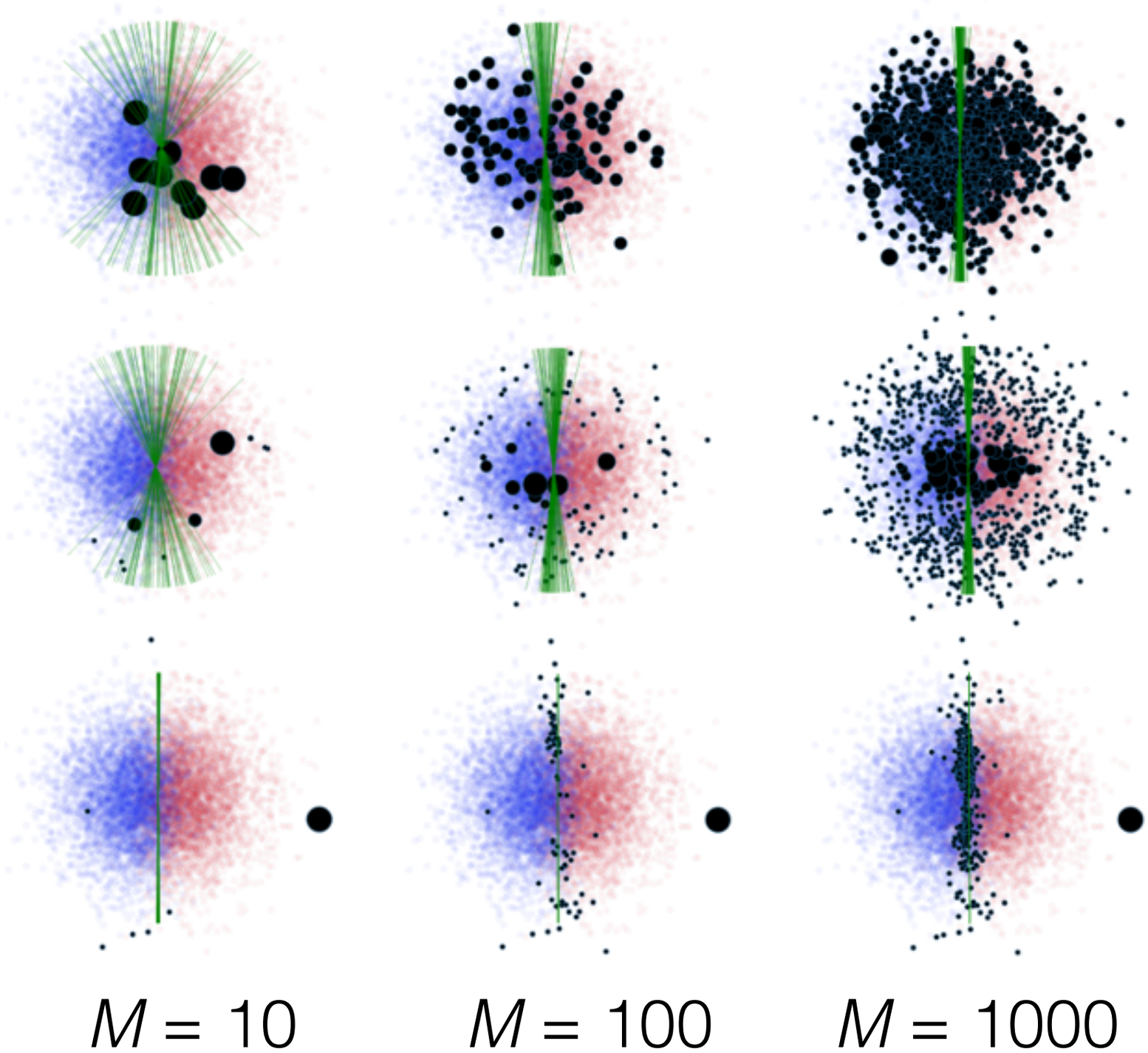
Frank-Wolfe



$M = 10$ $M = 100$ $M = 1000$

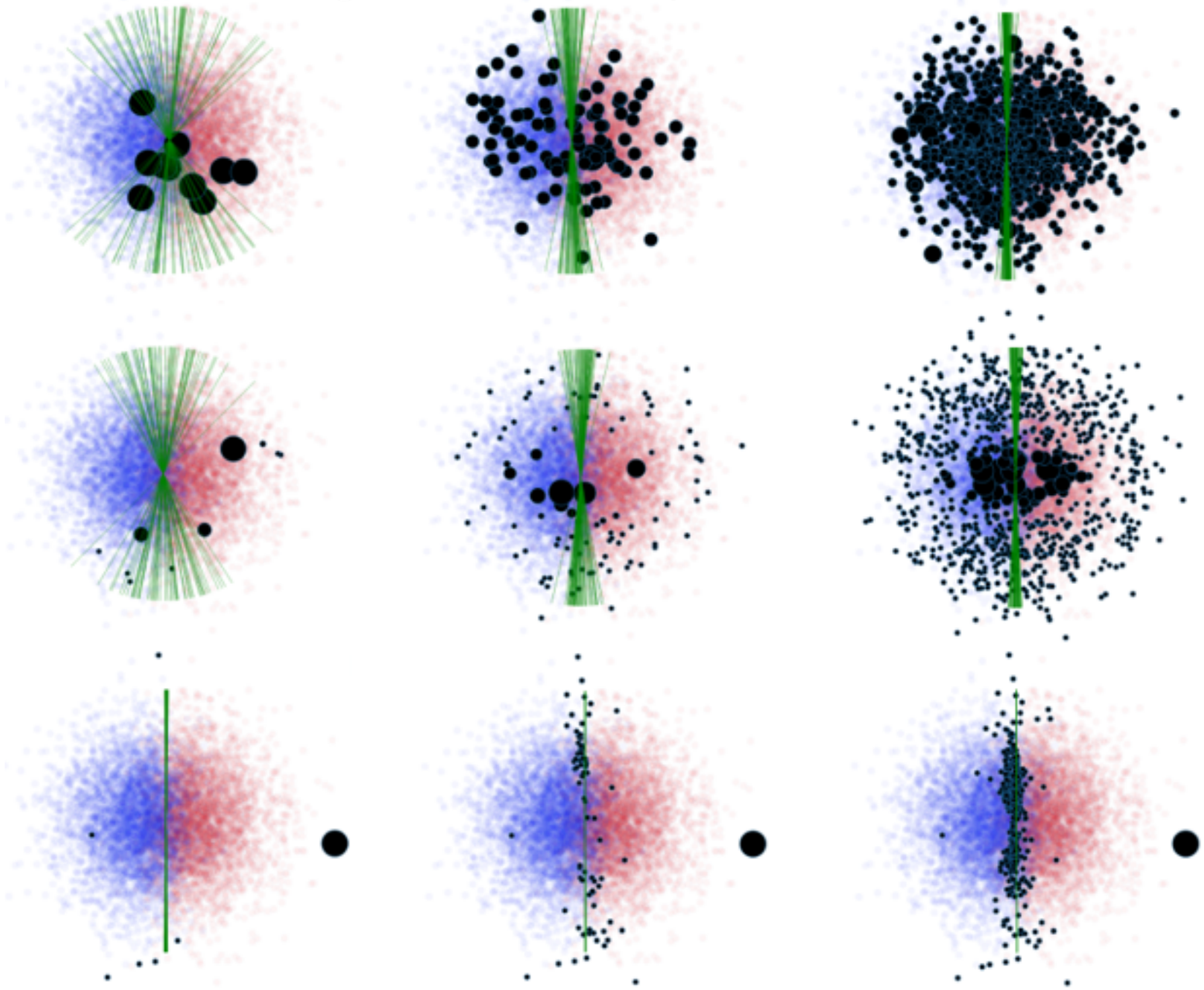
Logistic regression (simulated)

- 10K data points
- similar for Poisson regression, spherical clustering

Uniform subsampling

Importance sampling

Frank-Wolfe



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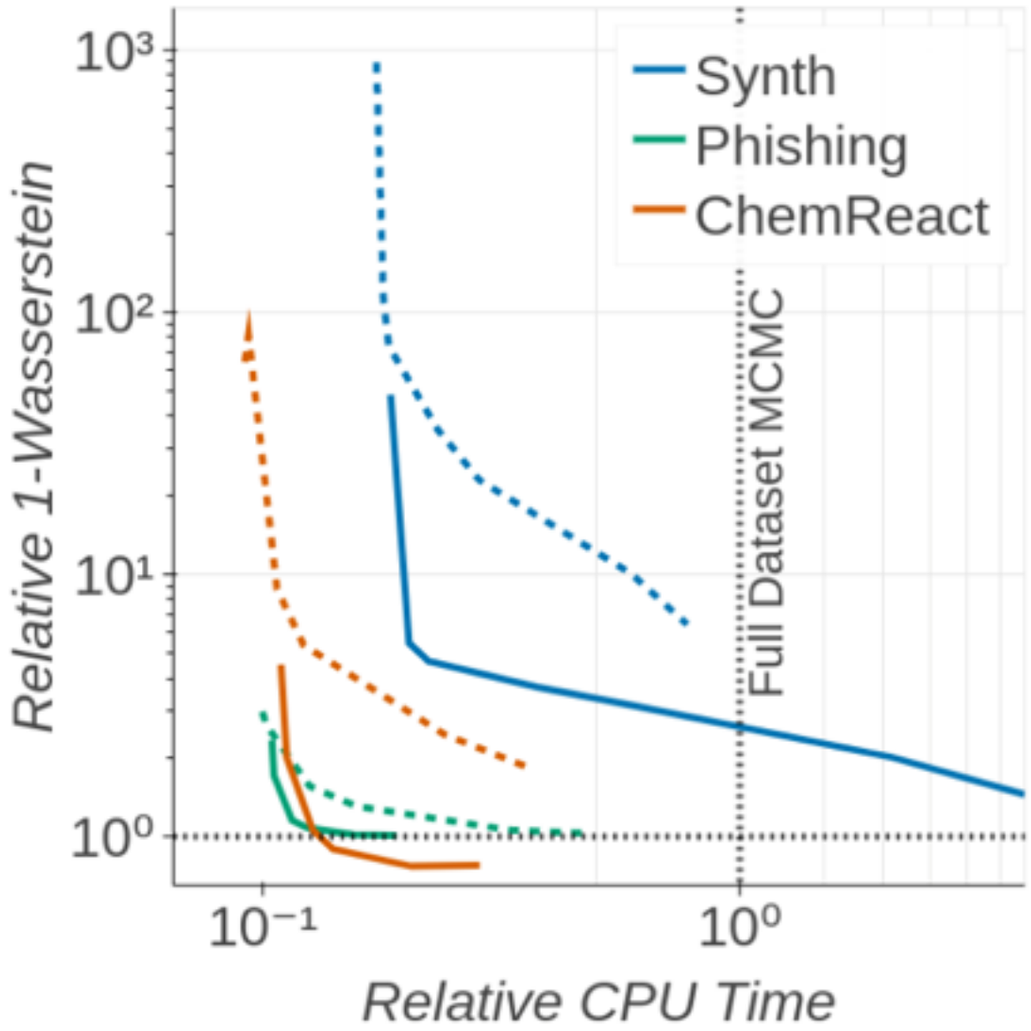
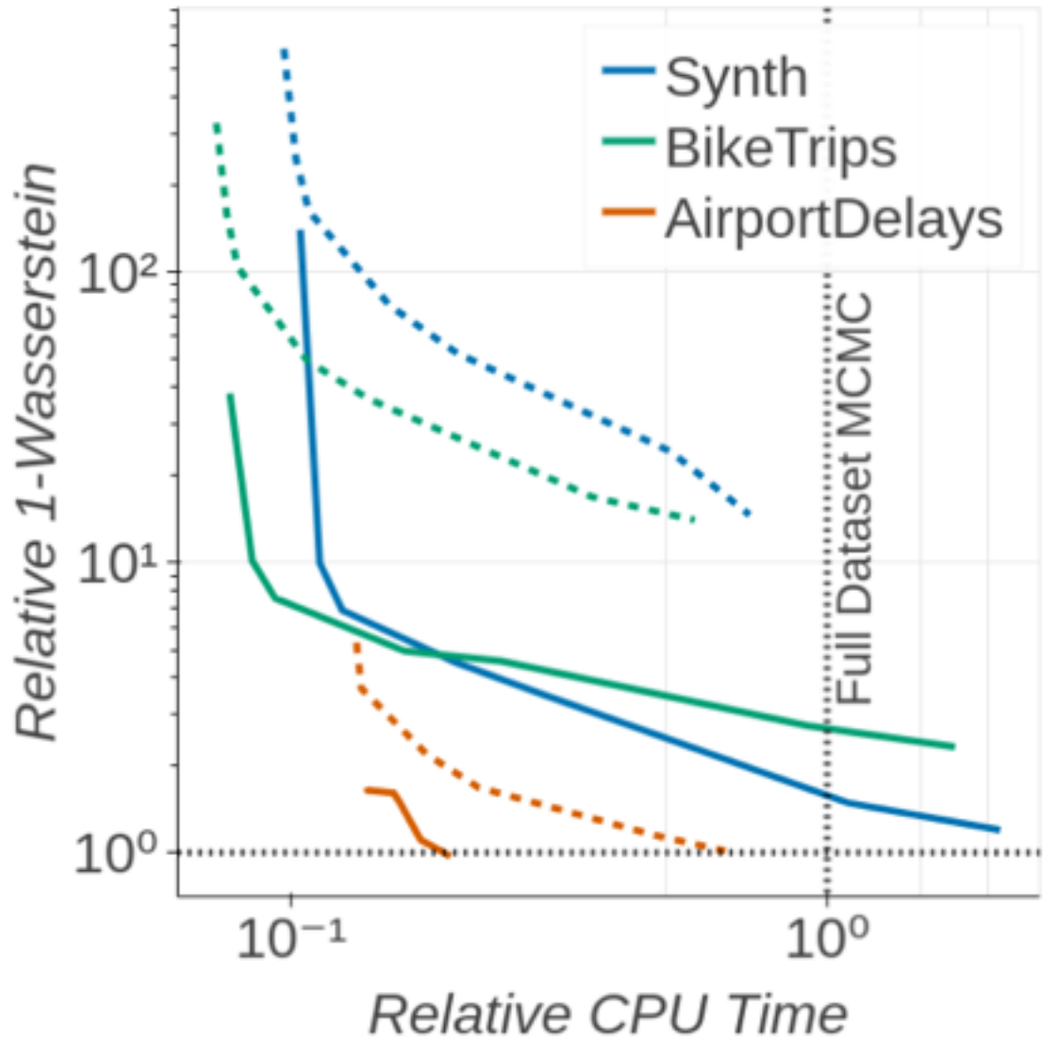
Real data experiments

lower error

Logistic regression



Poisson regression



uniform subsampling

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Conclusions

- *Coresets* for **scalable**, **automated** approx. Bayes algorithms with **error bounds on quality for finite data**

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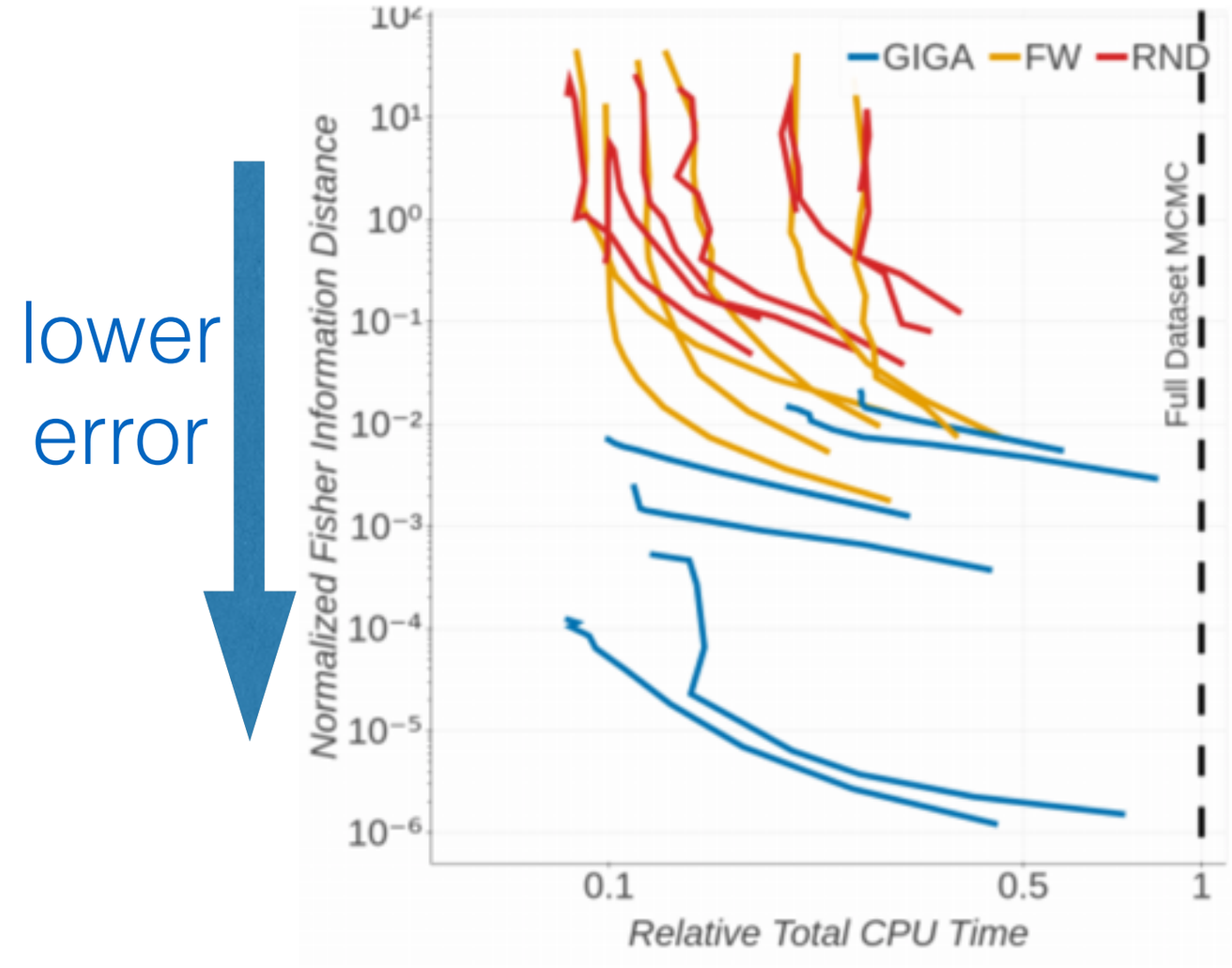


[Campbell, Broderick 2018]

References

T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. ArXiv:1710.05053.

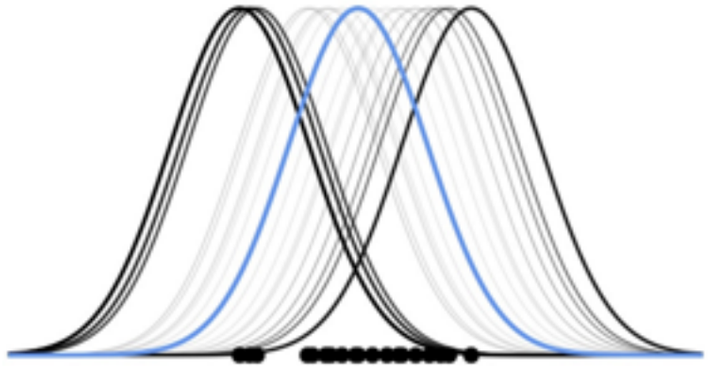
* Code: https://github.com/trevorcampbell/bayesian-coresets

T Campbell and T Broderick. Bayesian coreset construction via Greedy Iterative Geodesic Ascent. ArXiv:1802.01737.

JH Huggins, T Campbell, and T Broderick. Coresets for scalable Bayesian logistic regression. *NIPS* 2016.

JH Huggins, RP Adams, and T Broderick. PASS-GLM: Polynomial approximate sufficient statistics for scalable Bayesian GLM inference. *NIPS* 2017.

Practicalities



Practicalities

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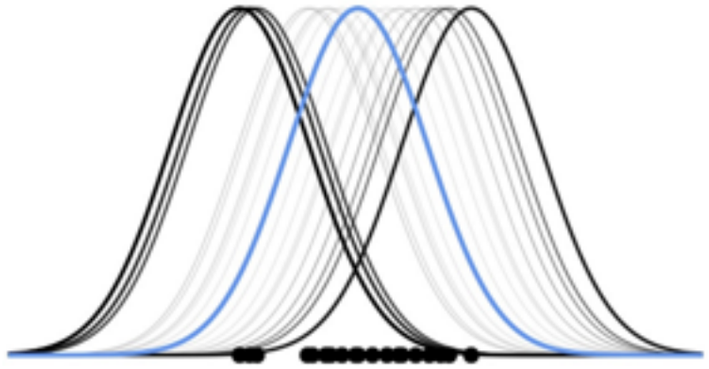

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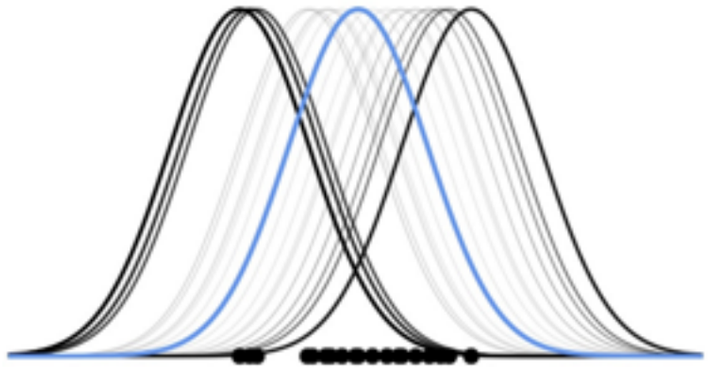

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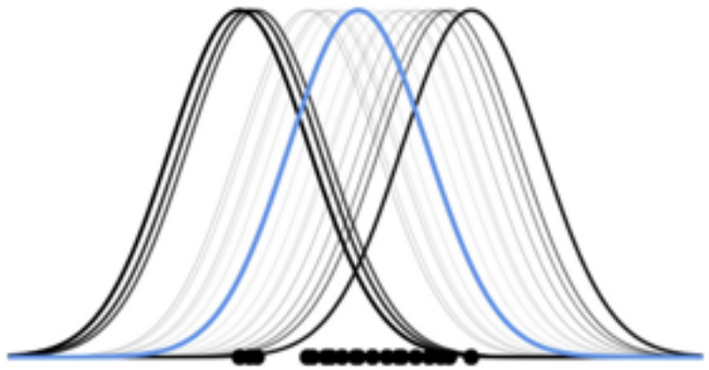

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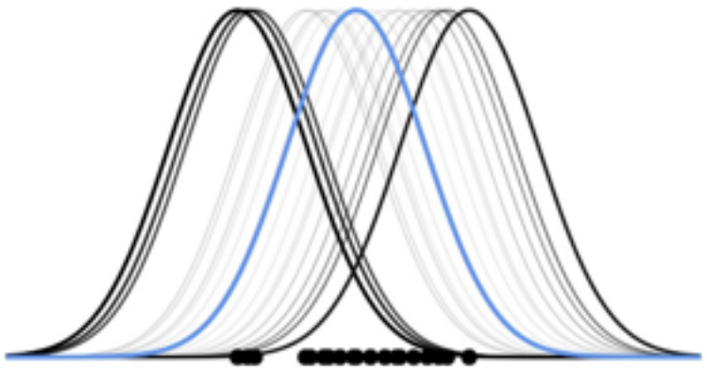

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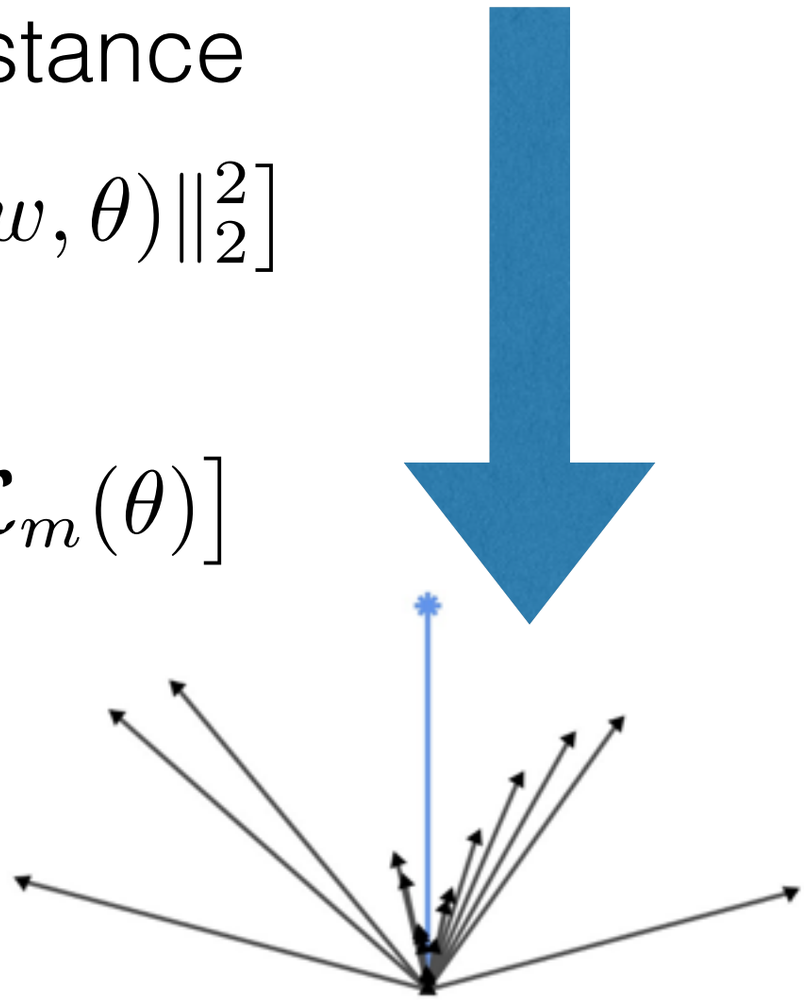

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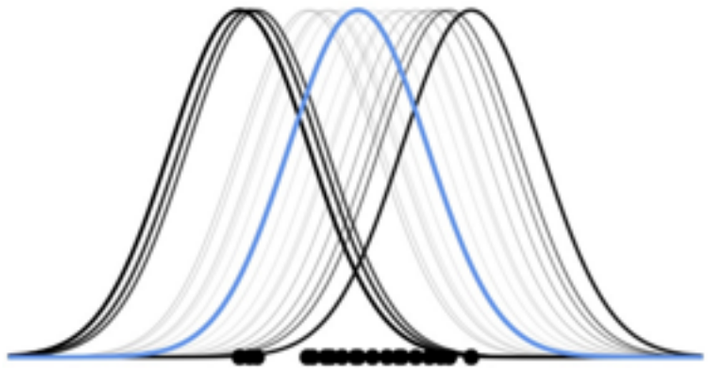

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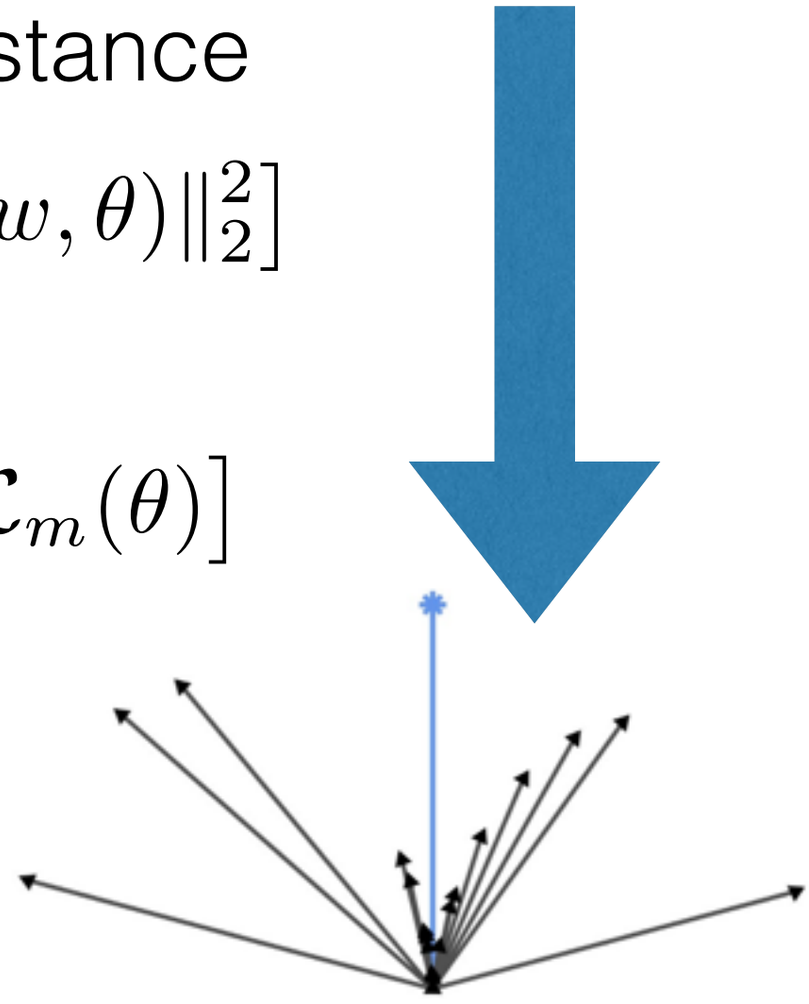

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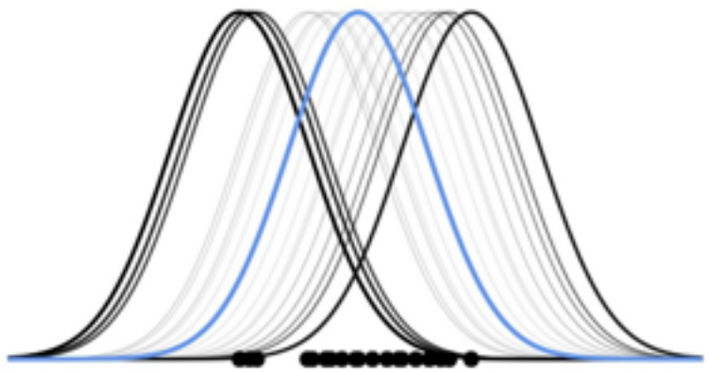

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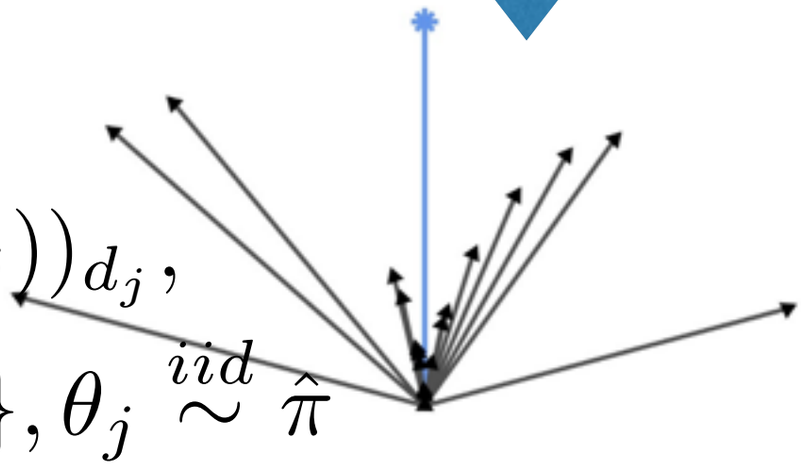
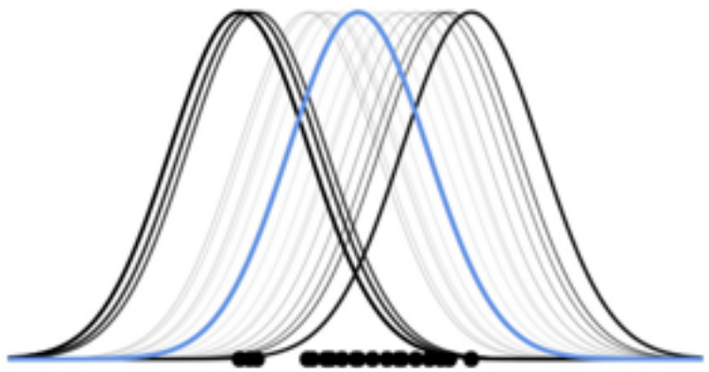


$$\langle \mathcal{L}_n, \mathcal{L}_m \rangle_{\hat{\pi},F} \approx \frac{D}{J}\sum_{j=1}^{J} (\nabla\mathcal{L}_n(\theta_j))_{d_j} (\nabla\mathcal{L}_m(\theta_j))_{d_j},$$

$$d_j \overset{iid}{\sim} \mathrm{Unif}\{1,\ldots,D\}, \theta_j \overset{iid}{\sim} \hat{\pi}$$

Practicalities



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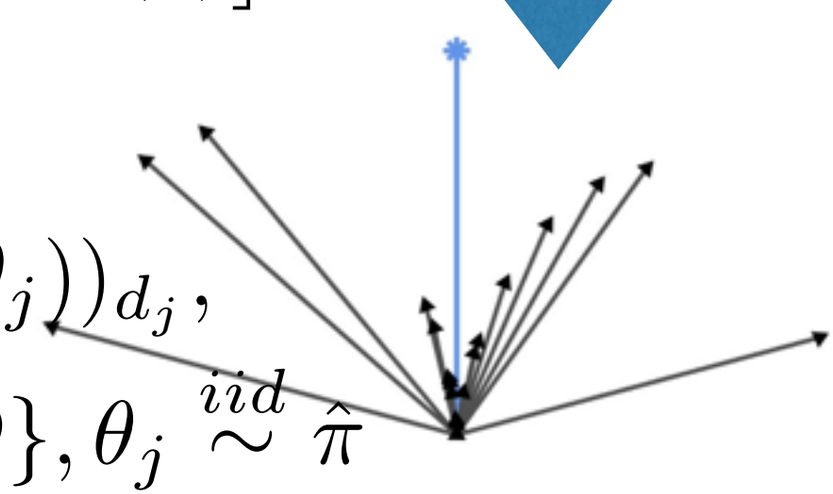
$$\|\mathcal{L}(w) - \mathcal{L}\|^2_{\hat{\pi},F} := \mathbb{E}_{\hat{\pi}}\left[\|\nabla\mathcal{L}(\theta) - \nabla\mathcal{L}(w,\theta)\|_2^2\right]$$

    - Associated inner product:

$$\langle \mathcal{L}_n, \mathcal{L}_m\rangle_{\hat{\pi},F} := \mathbb{E}_{\hat{\pi}}\left[\nabla\mathcal{L}_n(\theta)^T\nabla\mathcal{L}_m(\theta)\right]$$

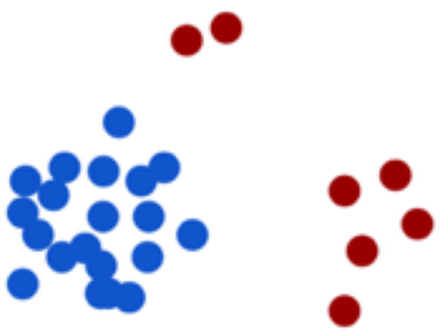
- Random feature projection

$$\langle \mathcal{L}_n, \mathcal{L}_m\rangle_{\hat{\pi},F} \approx \frac{D}{J}\sum_{j=1}^{J}(\nabla\mathcal{L}_n(\theta_j))_{d_j}(\nabla\mathcal{L}_m(\theta_j))_{d_j},$$
$$d_j \overset{iid}{\sim} \mathrm{Unif}\{1,\ldots,D\}, \theta_j \overset{iid}{\sim} \hat{\pi}$$



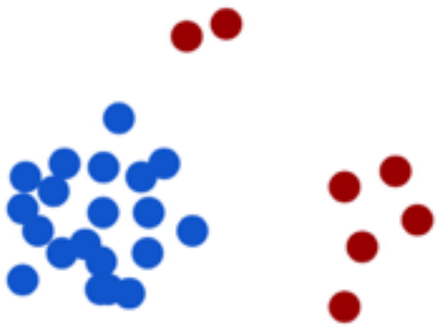
Thm sketch (CB). With high probability and large enough *J*, a good coreset after random feat. proj. is a good coreset for $(\mathcal{L}_n)_{n=1}^N$

Full pipeline



N
dataset size

Full pipeline

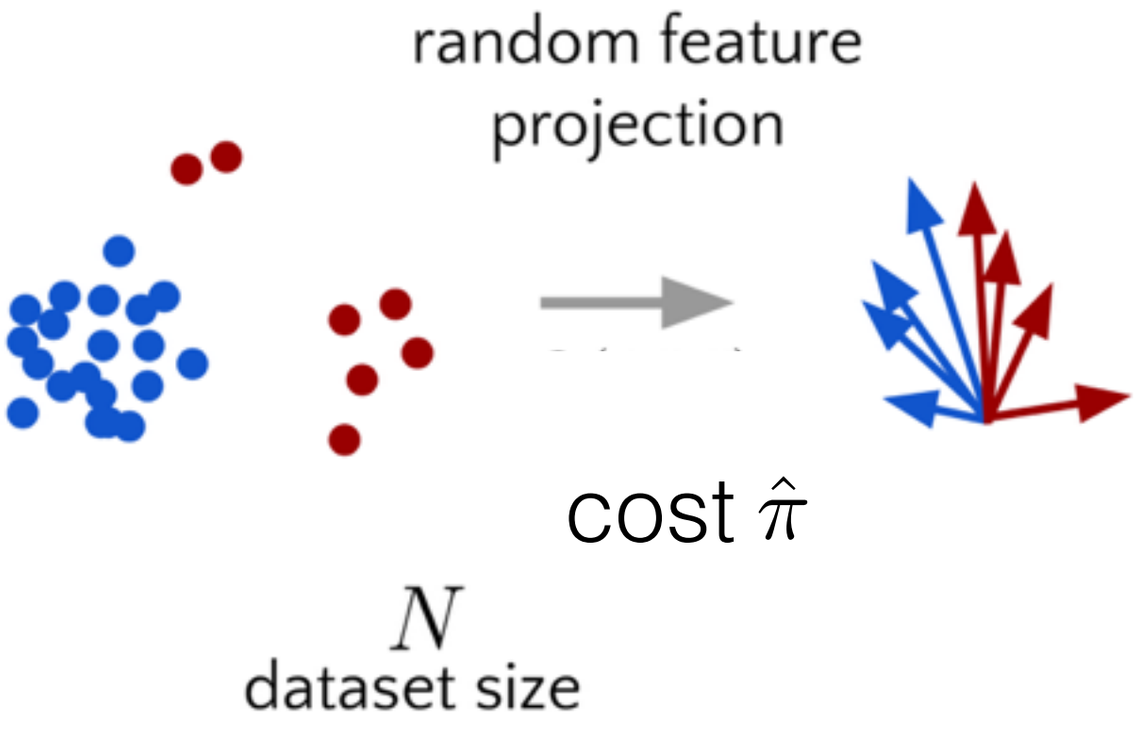


cost $\hat{\pi}$

N

dataset size

Full pipeline

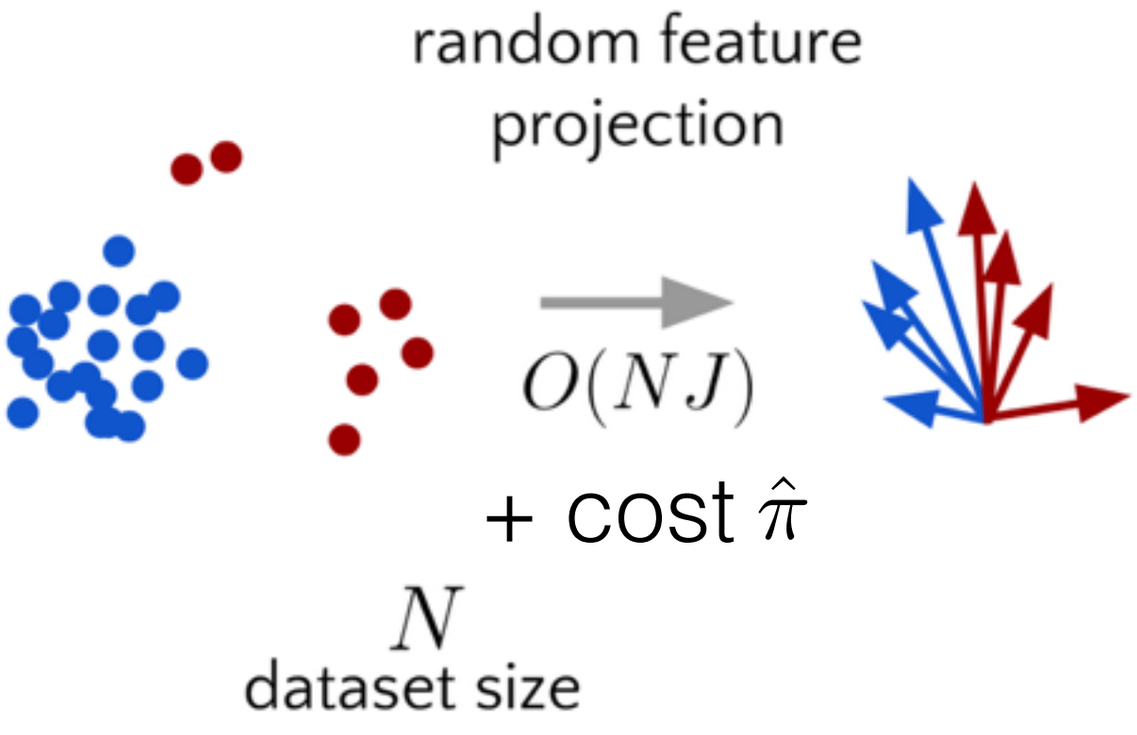


cost $\hat{\pi}$

N
dataset size

J
projection dim

Full pipeline

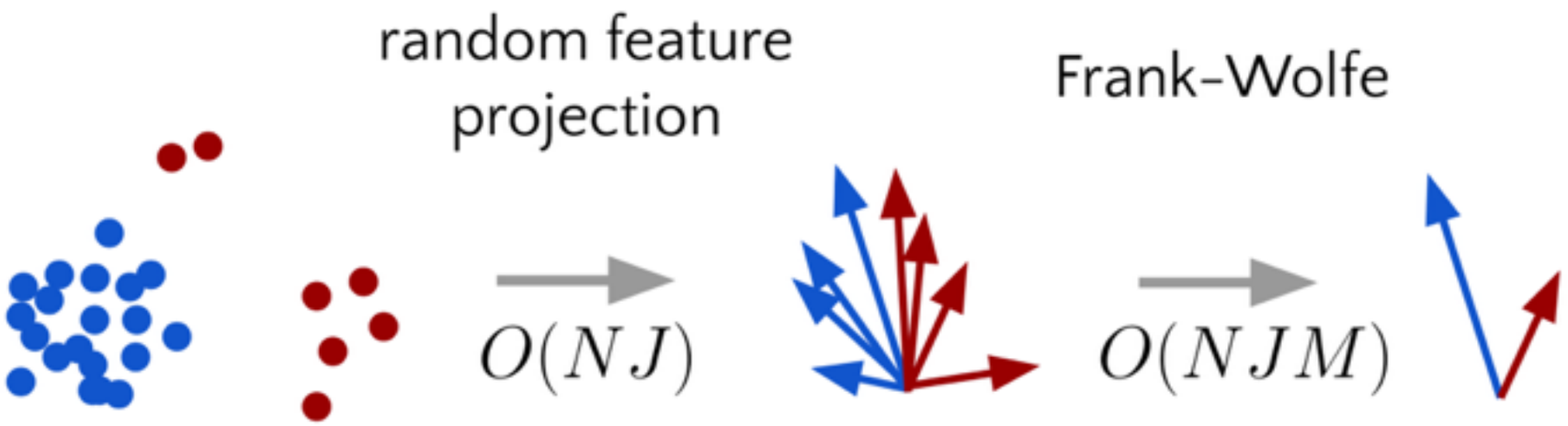


+ cost $\hat{\pi}$

N
dataset size

J
projection dim

Full pipeline

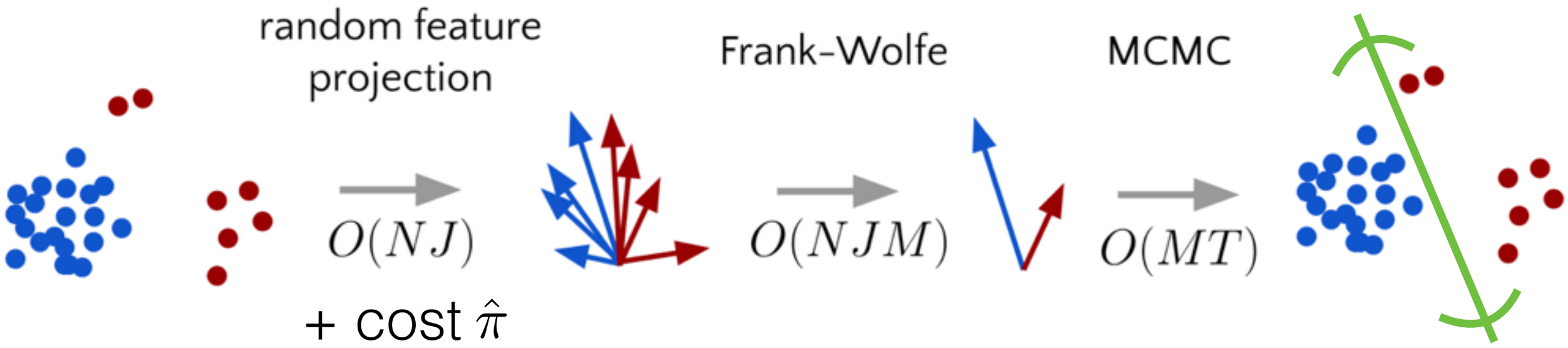


N
dataset size

M
coreset size

J
projection dim

Full pipeline



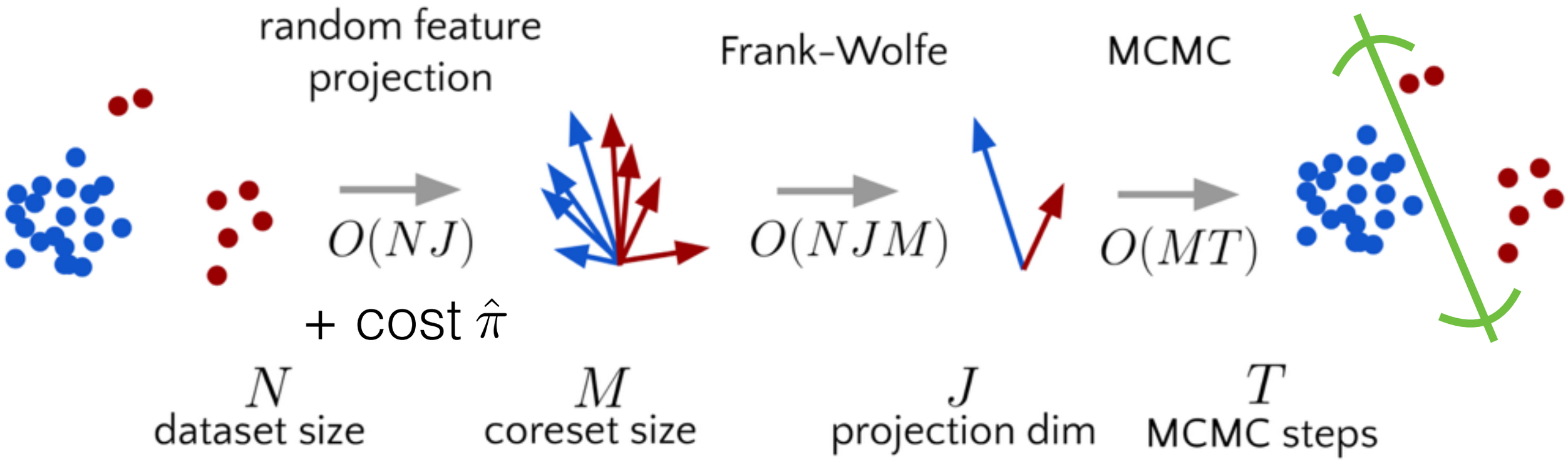
N dataset size

M coreset size

J projection dim

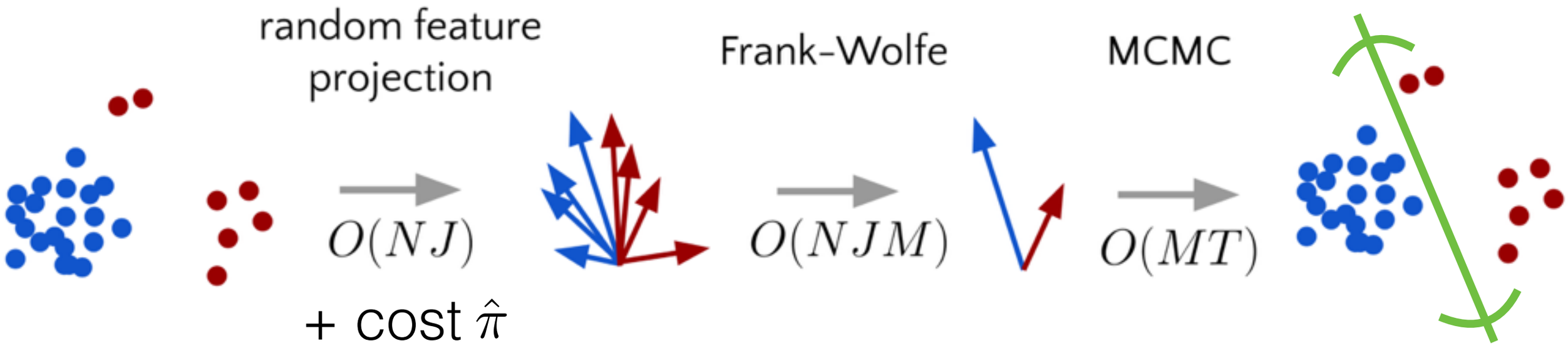
T MCMC steps

Full pipeline



- vs. $O(NT)$

Full pipeline



- vs. $O(NT)$
- Can make streaming, distributed