





An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?

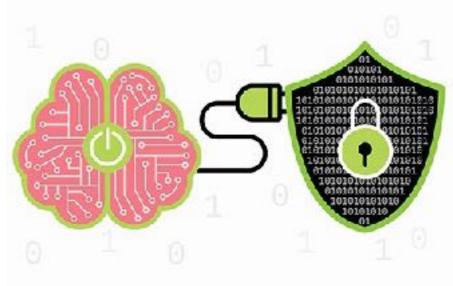
Tamara Broderick
Associate Professor,
MIT

With Ryan Giordano, Rachael Meager





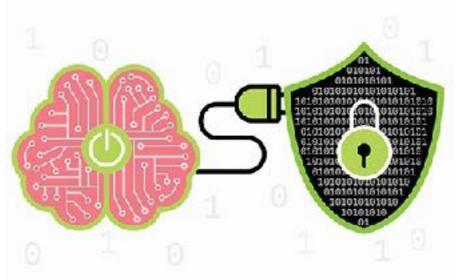






 More data & better computation → data analyses increasingly drive life-changing decisions

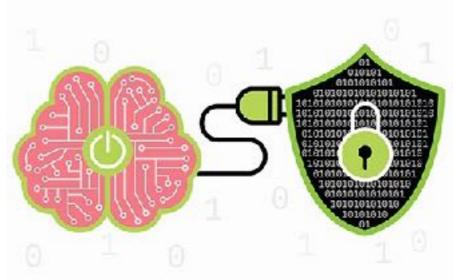






 One question: Would you be concerned if dropping a small fraction of data changed substantive conclusions?

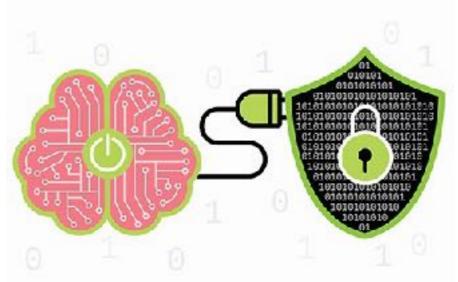






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- Challenge: Too expensive to check every data subset

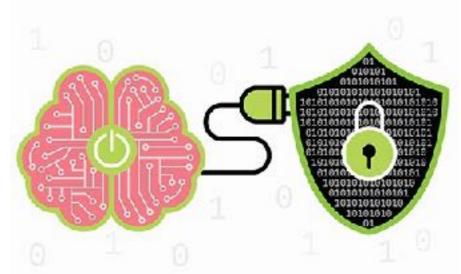






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- E.g. in a study of microcredit with ~16,500 data points, we find a single data point that drives the sign of the effect

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- Even if doesn't bother you, should be up front about it

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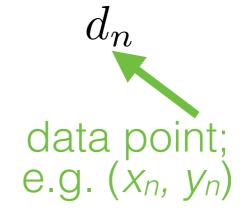
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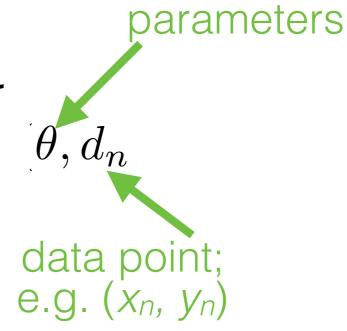
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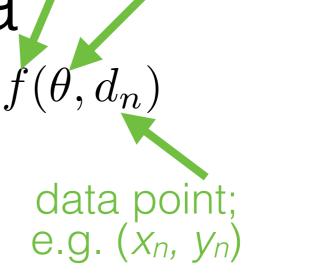
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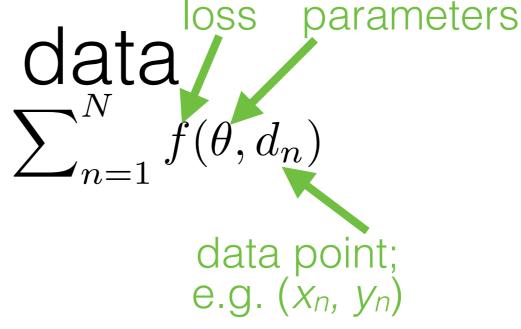
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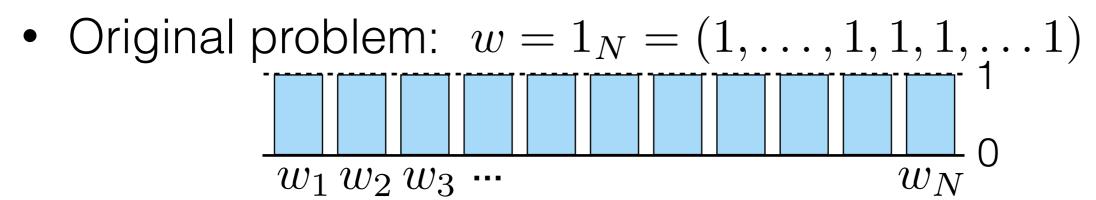
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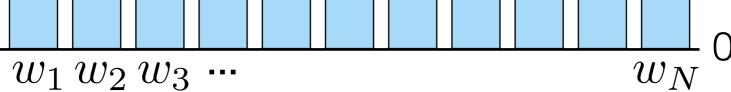
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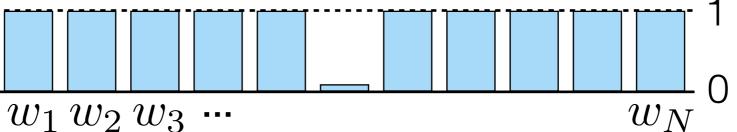
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parameters

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• Dropping a data point: $w = (1, \dots, 1, 0, 1, \dots, 1)$



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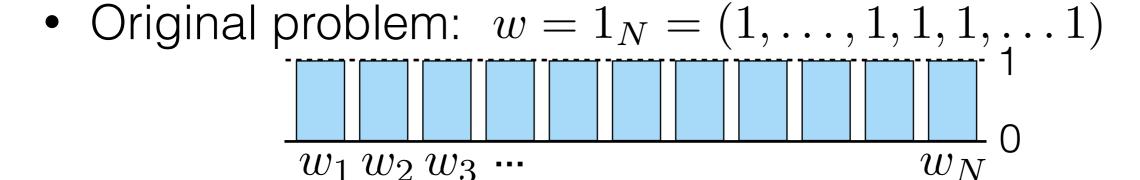
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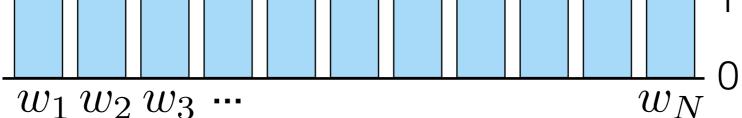
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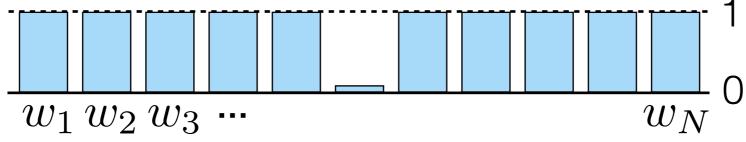
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Simulations from linear model with Gaussian noise

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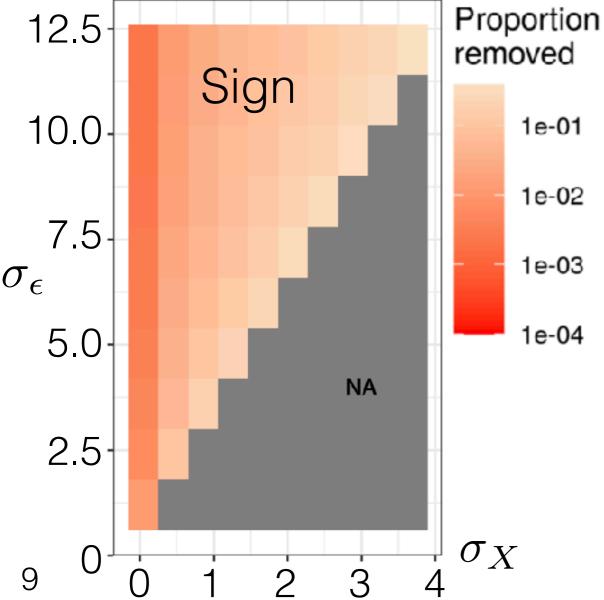
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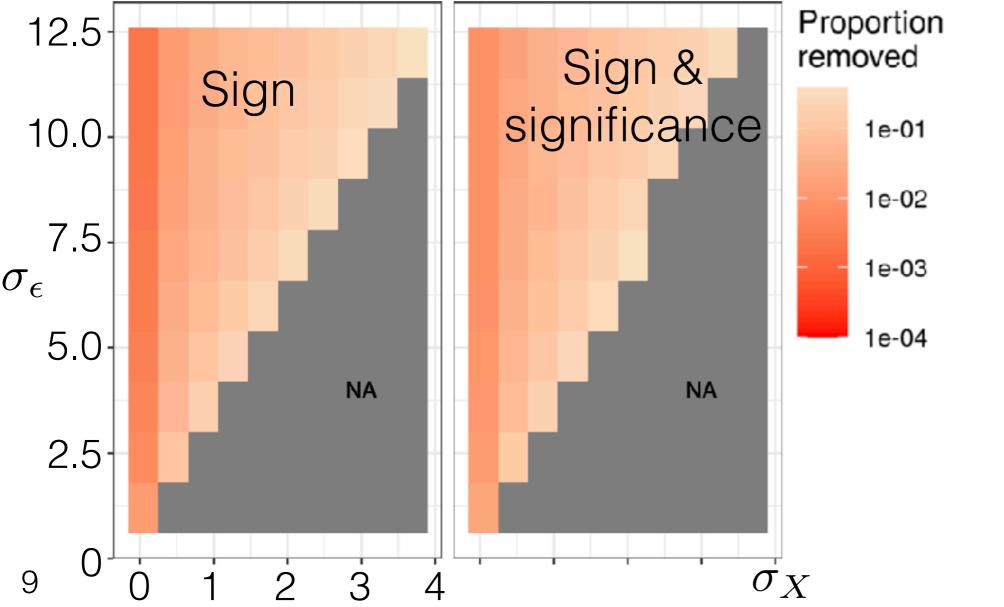
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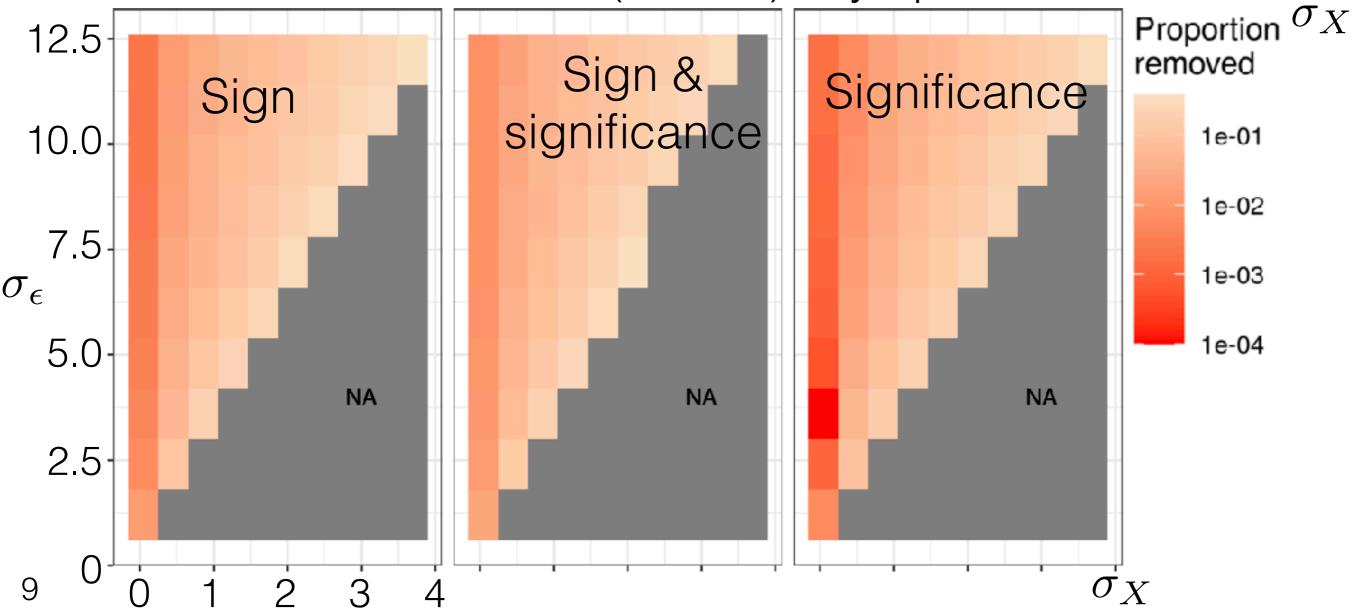
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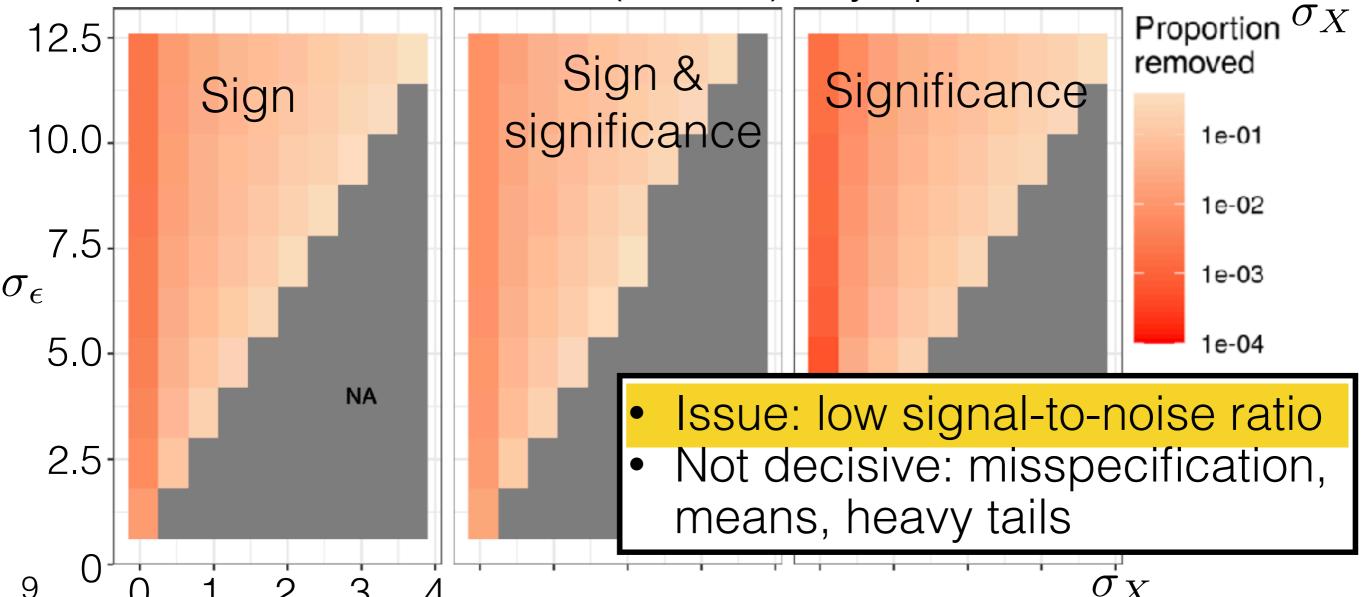
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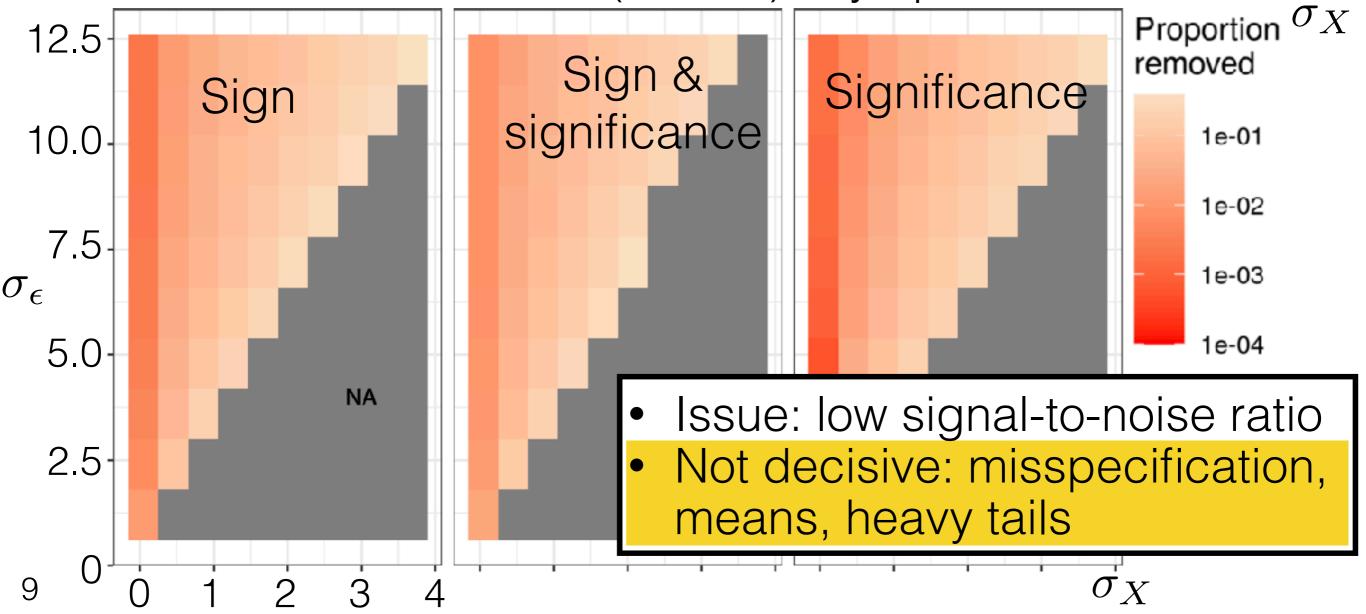
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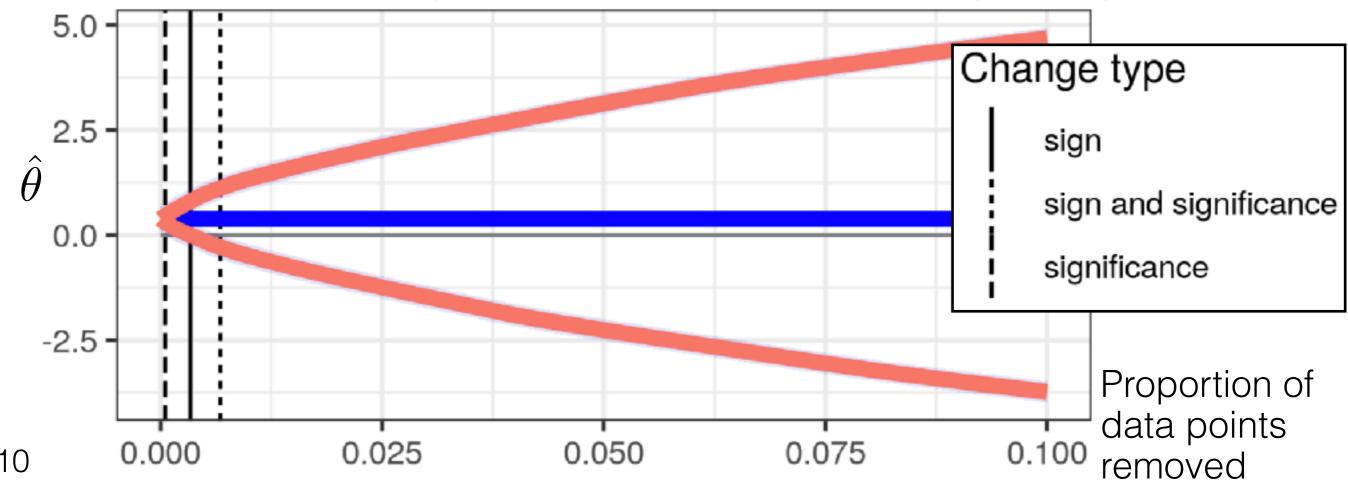
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 - Cf. the classical "infinitesimal jackknife" [Jaeckel 1972; Clarke 1983]

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Code, readme, and examples:

https://github.com/rgiordan/zaminfluence

- Try it out on your data analysis and email us!
 tbroderick@mit.edu, rgiordan@mit.edu,
 r.meager@lse.ac.uk
- See also: "Transparency and Reproducibility in Artificial Intelligence," Nature Matters Arising, 2020.
- Introduction to ML: tamarabroderick.com/ml.html

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See also:

- R Giordano, W Stephenson, R Liu, MI Jordan, T Broderick. A Swiss Army infinitesimal jackknife. AISTATS 2019.
- W Stephenson, T Broderick. Approximate Cross-Validation in High Dimensions with Guarantees. AISTATS 2020.
- WT Stephenson, M Udell, T Broderick. Approximate Cross-Validation with Low-Rank Data in High Dimensions. *NeurIPS* 2020.
- S Ghosh*, WT Stephenson*, TD Nguyen, SK Deshpande, T Broderick.
 Approximate Cross-Validation for Structured Models. *NeurIPS* 2020. (*equal)