





# An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?

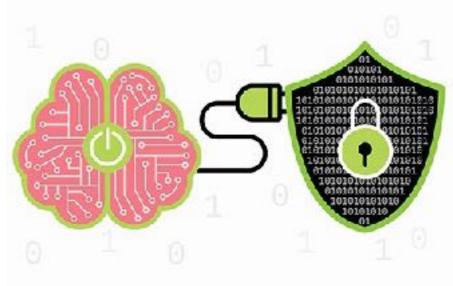
Tamara Broderick
Associate Professor,
MIT

With Ryan Giordano, Rachael Meager





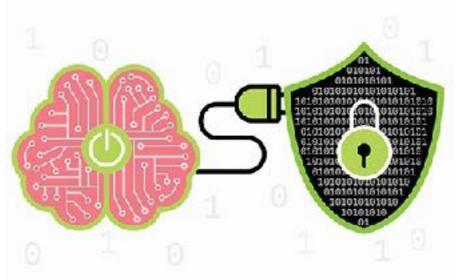






 More data & better computation → data analyses increasingly drive life-changing decisions

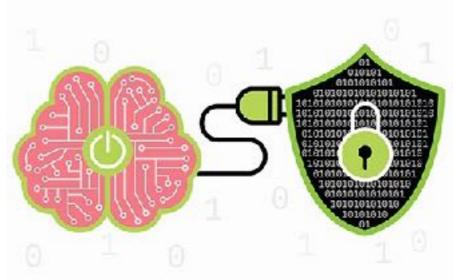






 One question: Would you be concerned if dropping a small fraction of data changed substantive conclusions?

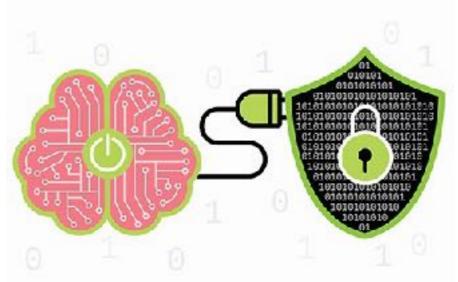






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- Challenge: Too expensive to check every data subset

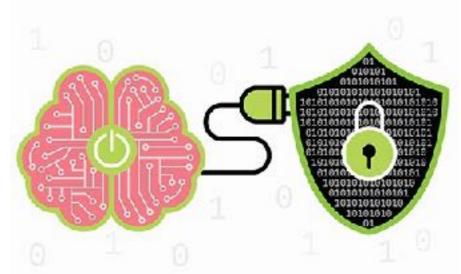






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- E.g. in a study of microcredit with ~16,500 data points, we find a single data point that drives the sign of the effect

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- Even if doesn't bother you, should be up front about it

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  - If analysis takes 1 second, check takes >33 years

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- How should we drop data subsets?
- Why is dropping data subsets computationally expensive?
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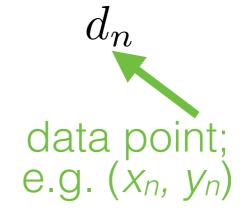
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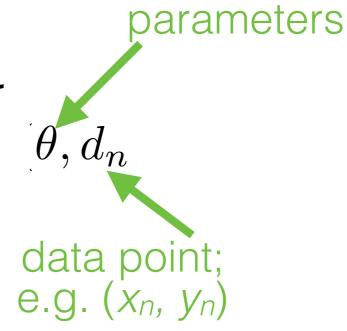
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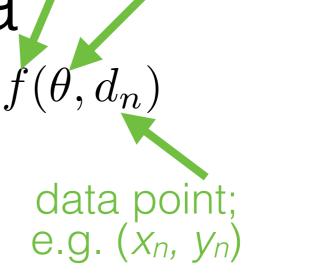
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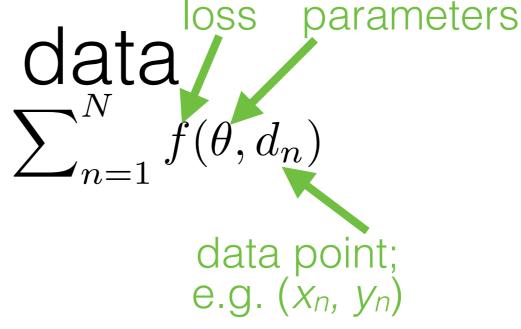
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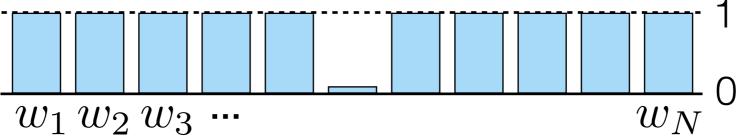
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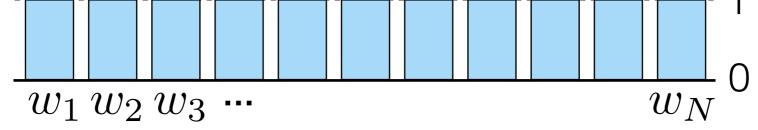


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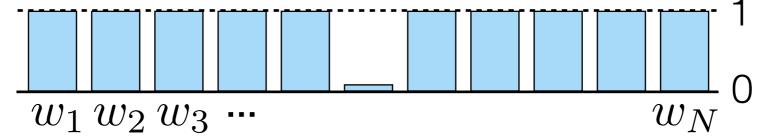
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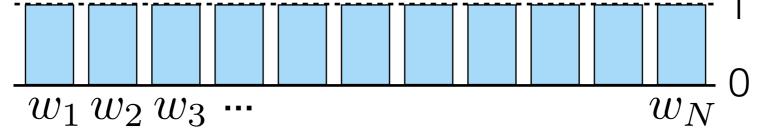


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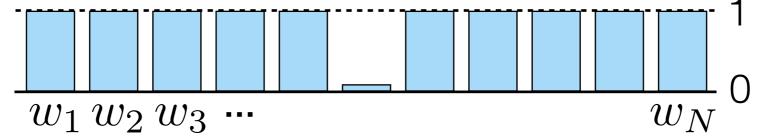
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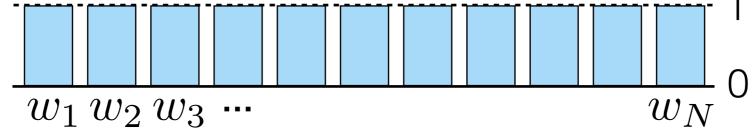


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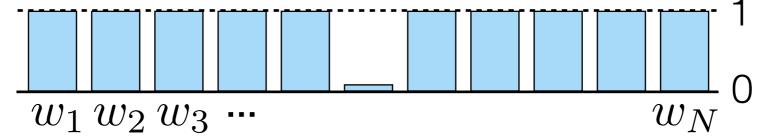
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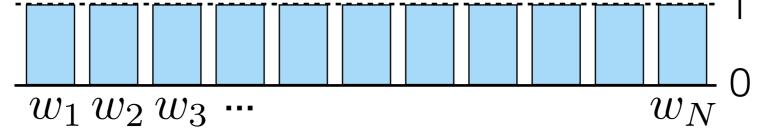


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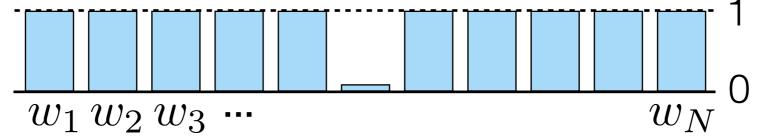
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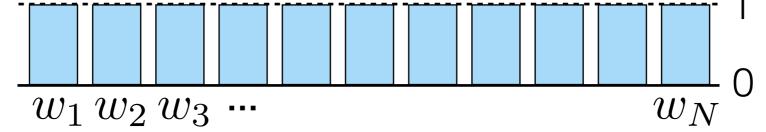


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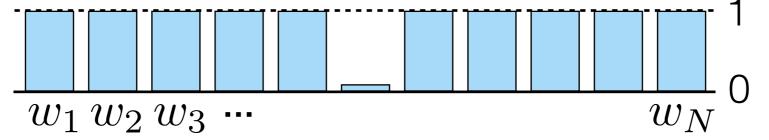
data point; e.g.  $(x_n, y_n)$ 

parameters

- A quantity of interest  $\phi$ 
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  - E.g.  $\phi = \hat{\theta}_p 1.96\sigma_p$
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• Dropping a data point:  $w = (1, \dots, 1, 0, 1, \dots, 1)$ 

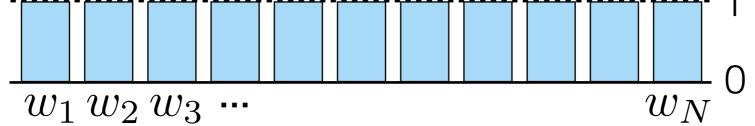


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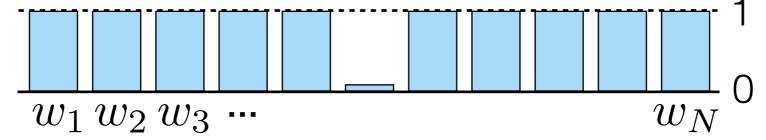
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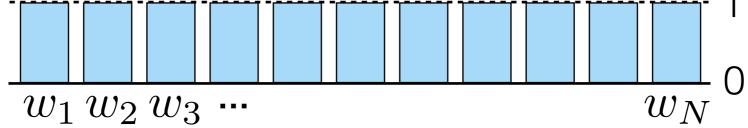


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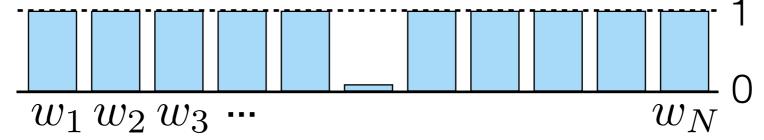
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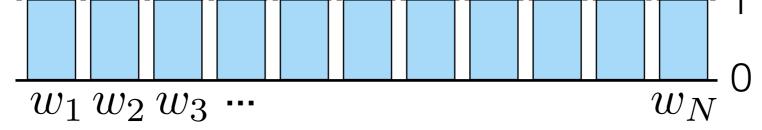


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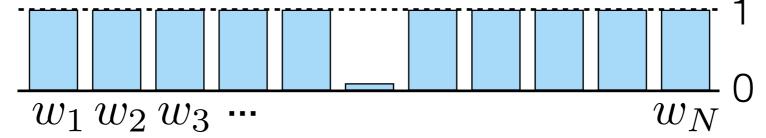
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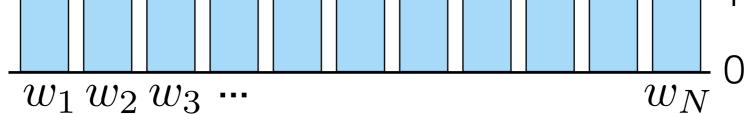


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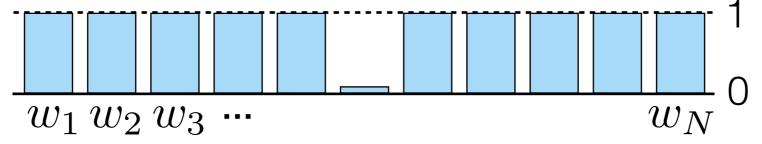
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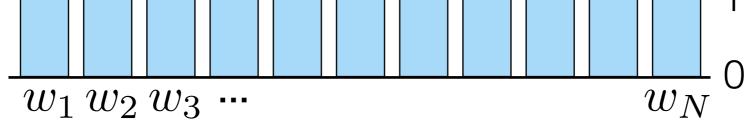


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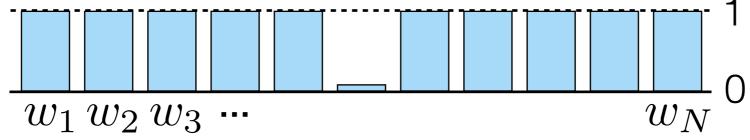
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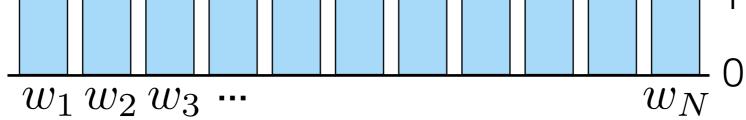


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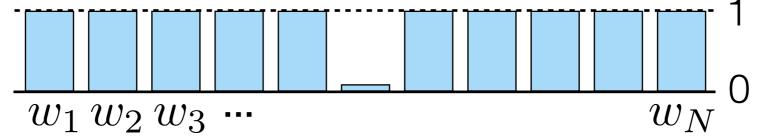
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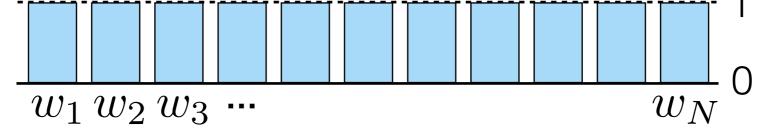


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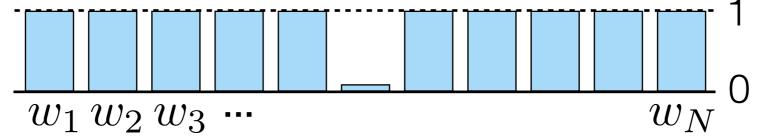
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Simulations from linear model with Gaussian noise

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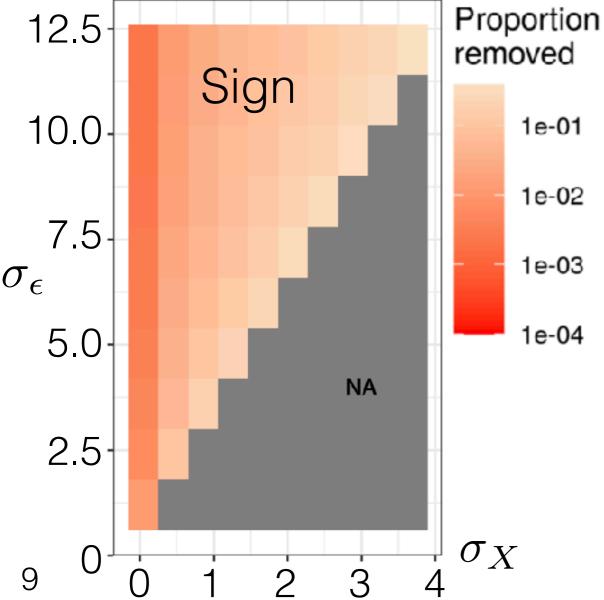
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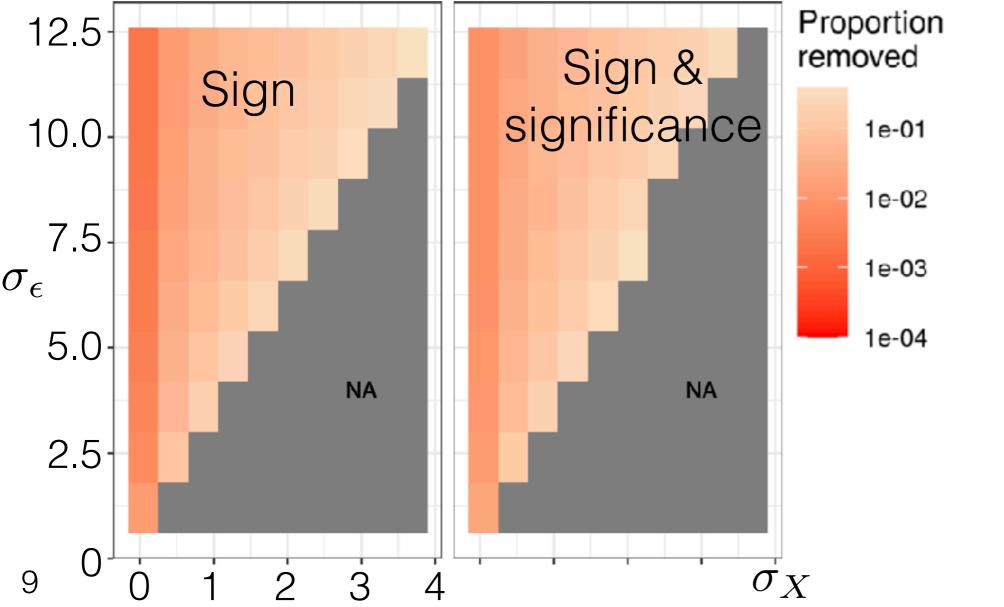
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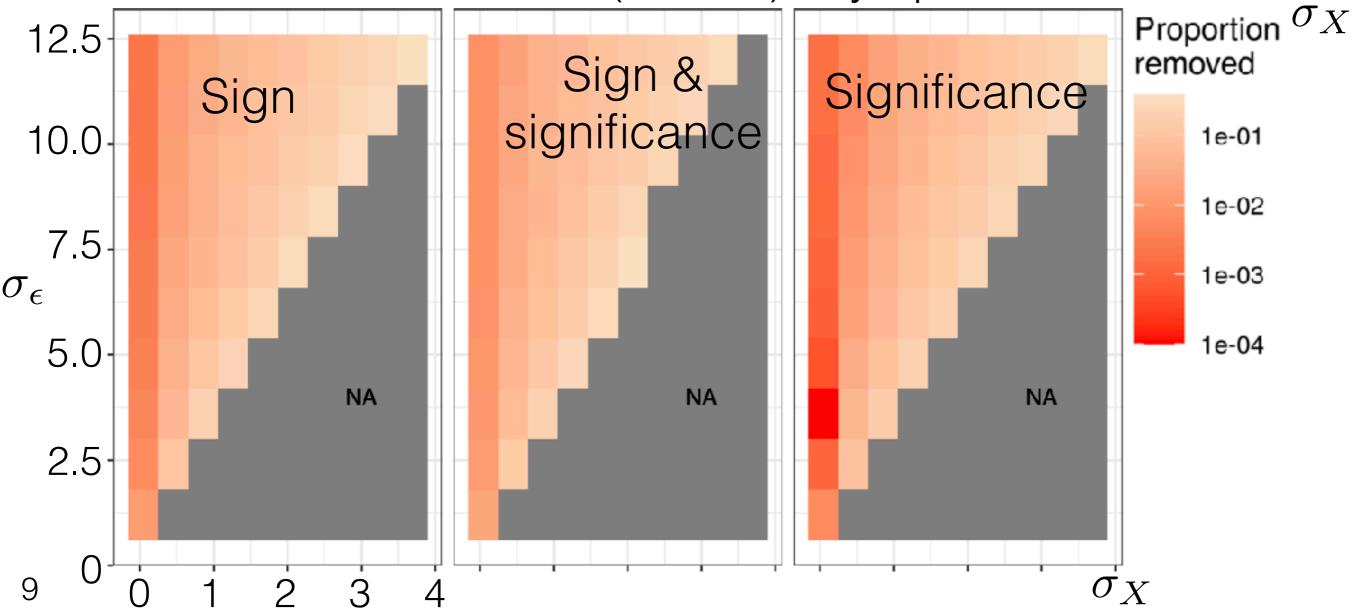
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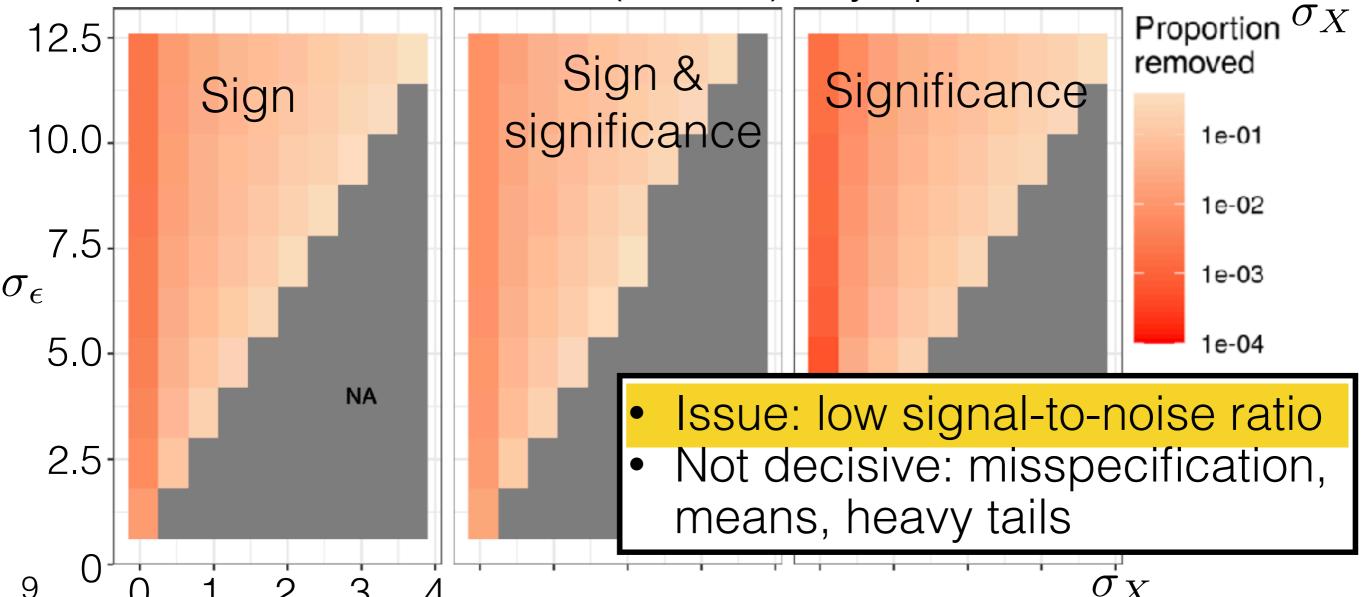
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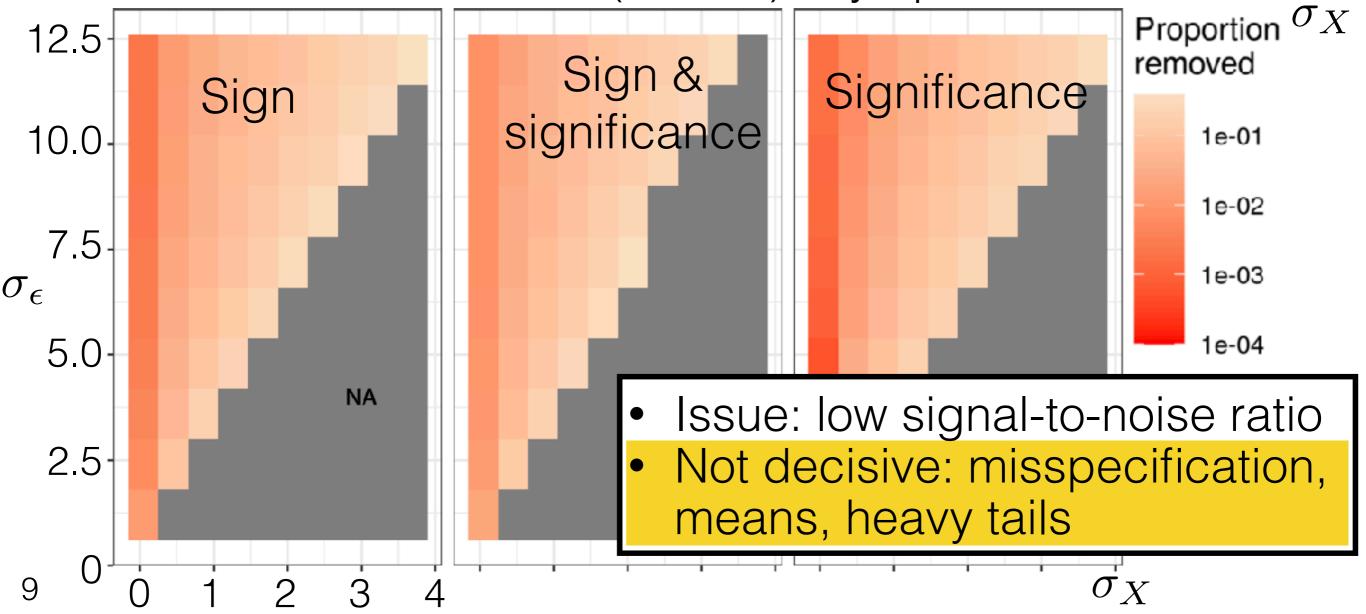
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# Oregon Medicaid Study

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  - We show: in linear regression, influence score = residual times leverage

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  - Cf. the classical "infinitesimal jackknife" [Jaeckel 1972; Clarke 1983]

# Try it out!

- We present a metric to check if there is a small fraction of data you can drop to change conclusions
- Paper: T Broderick, R Giordano, R Meager "An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?" 2020

https://arxiv.org/abs/2011.14999

Code, readme, and examples:

https://github.com/rgiordan/zaminfluence

Try it out on your data analysis and email us!

tbroderick@mit.edu, rgiordan@mit.edu, r.meager@lse.ac.uk

 Aside: "Transparency and Reproducibility in Artificial Intelligence," Nature Matters Arising, 2020.