Variational Bayes and beyond:

Foundations of scalable Bayesian inference (Part II)

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http://www.tamarabroderick.com/tutorials.html

Approximate Bayesian inference

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Use q^* to approximate $p(\cdot|y)$

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- Stochastic variational inference (SVI) [Hoffman et al 2013]

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- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Air pollution: Particulate matter

[Krongut 2020]

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- Sensor readings of log PM2.5 $y = (y_1, \ldots, y_N)$

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[Krongut 2020]

[MacKay 2003; Bishop 2006]

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$$p(\mu, \tau|y) \neq f_1(\mu, y) f_2(\tau, y)$$

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"variational parameters"

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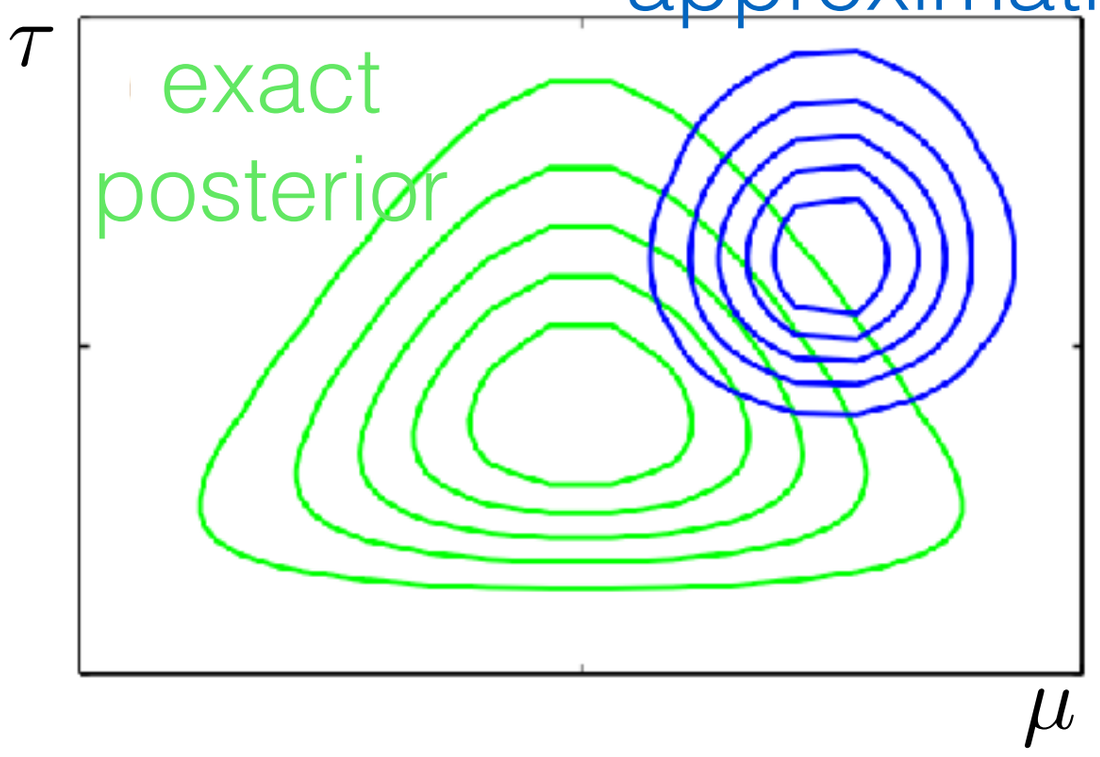

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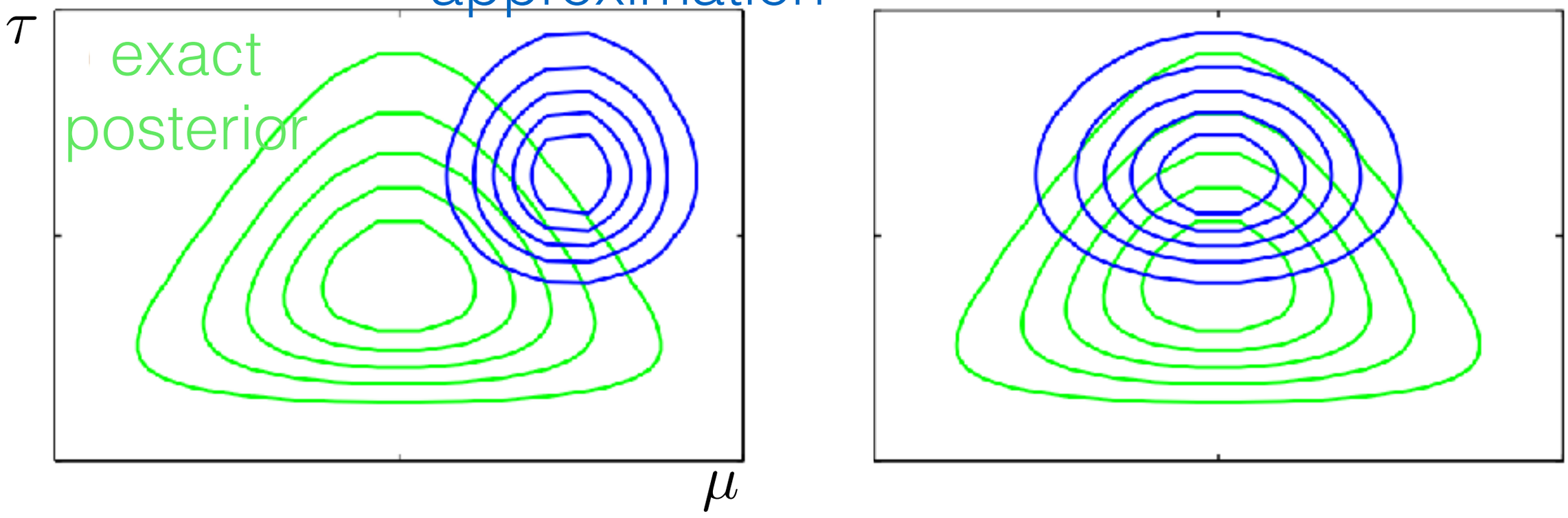

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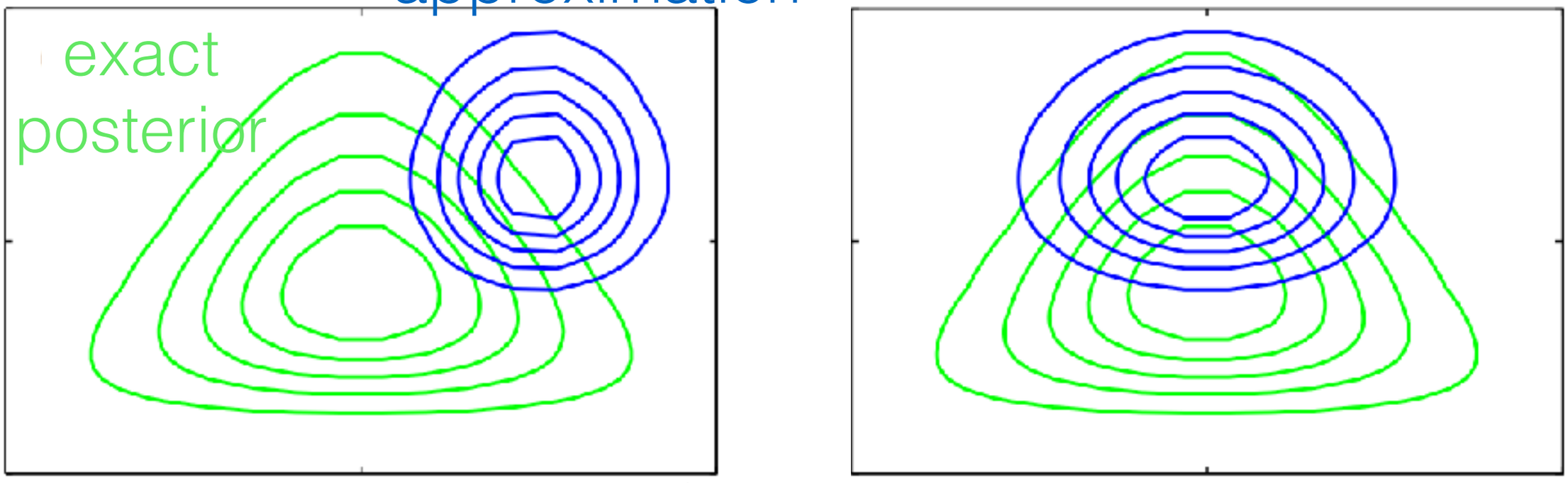
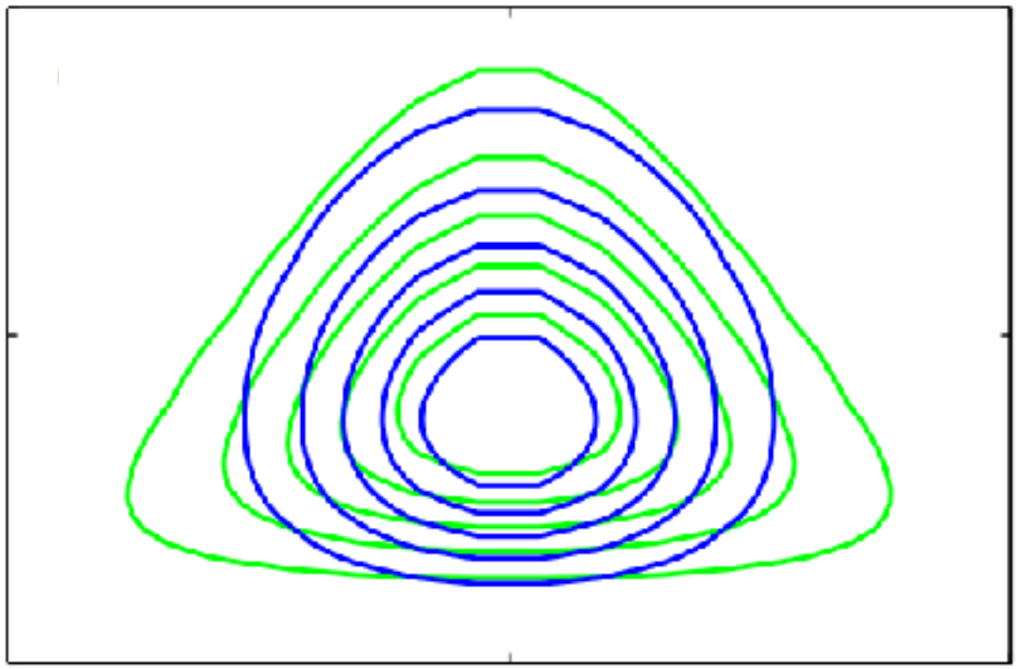
approximation

τ

exact posterior



μ



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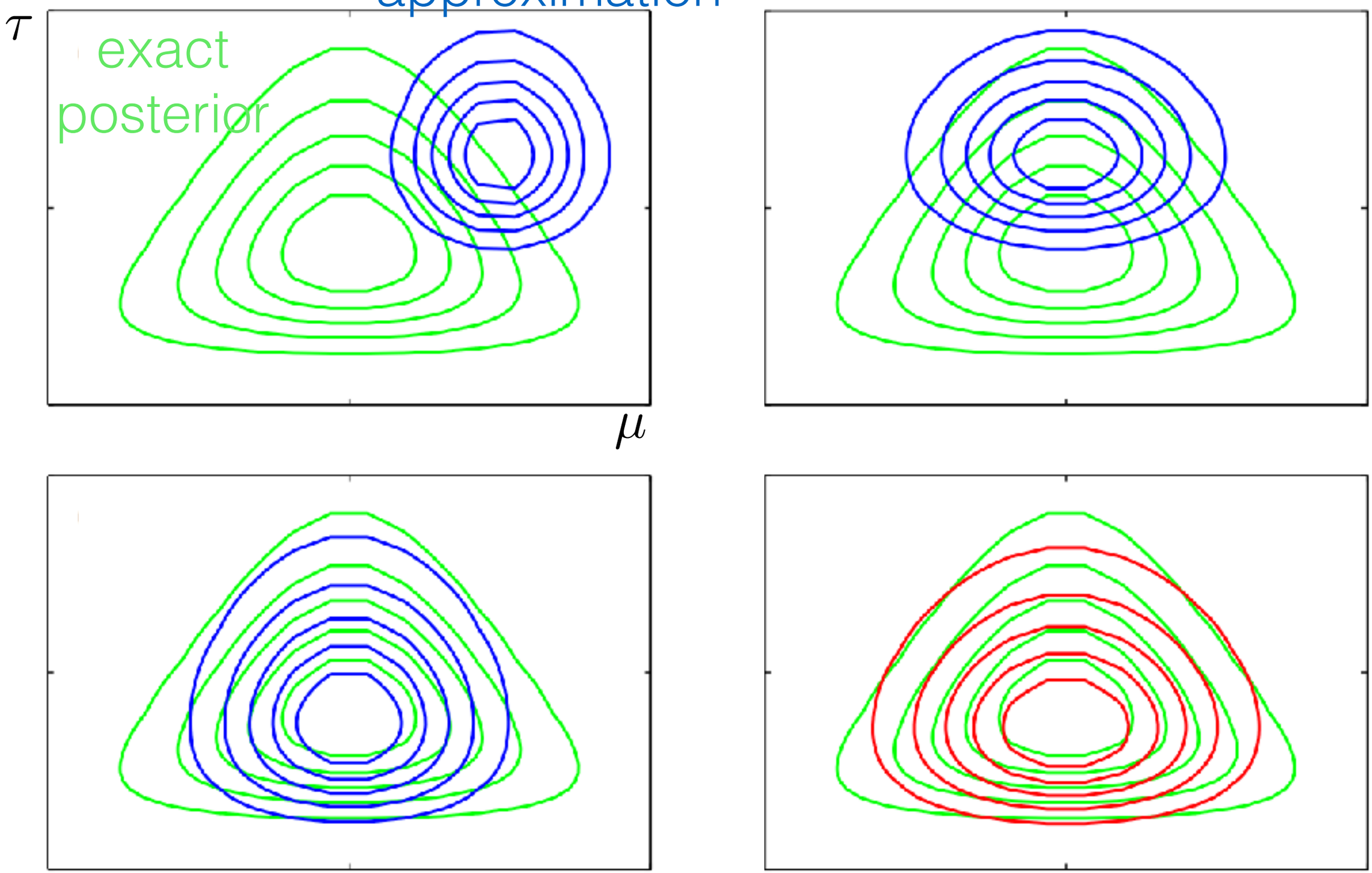

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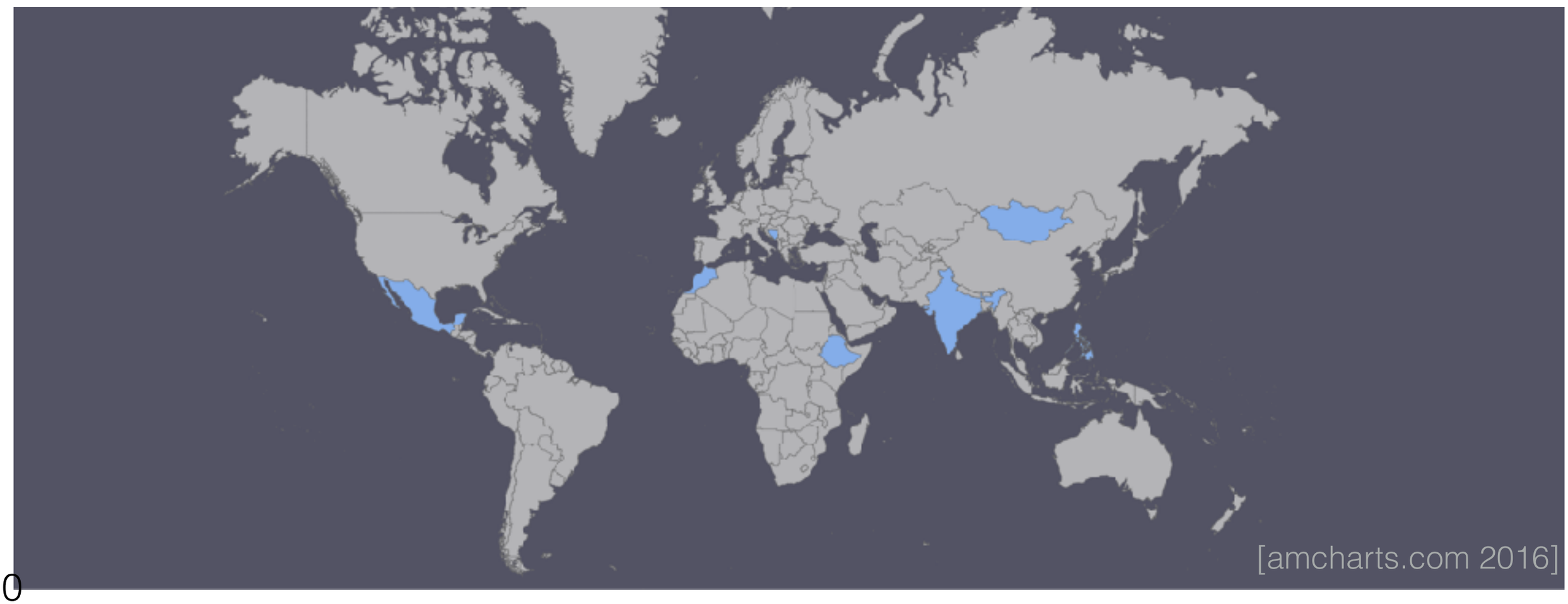
Microcredit Experiment



Microcredit Experiment

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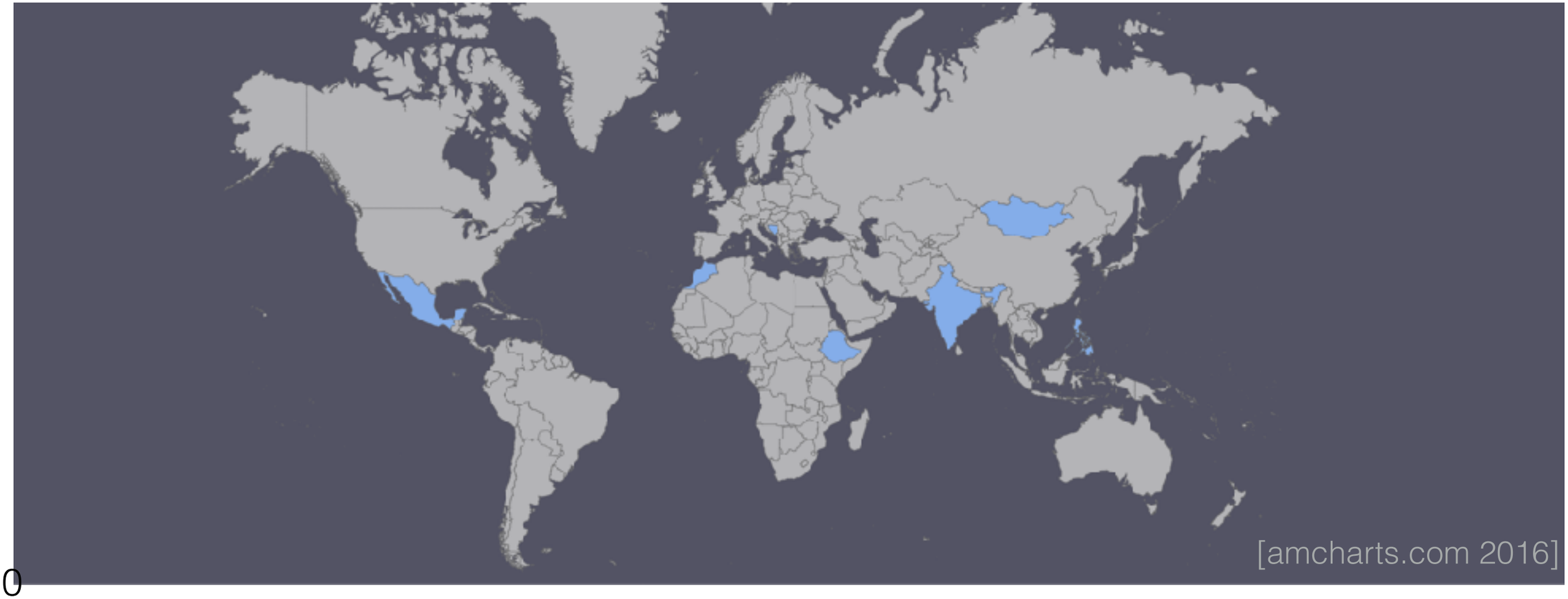

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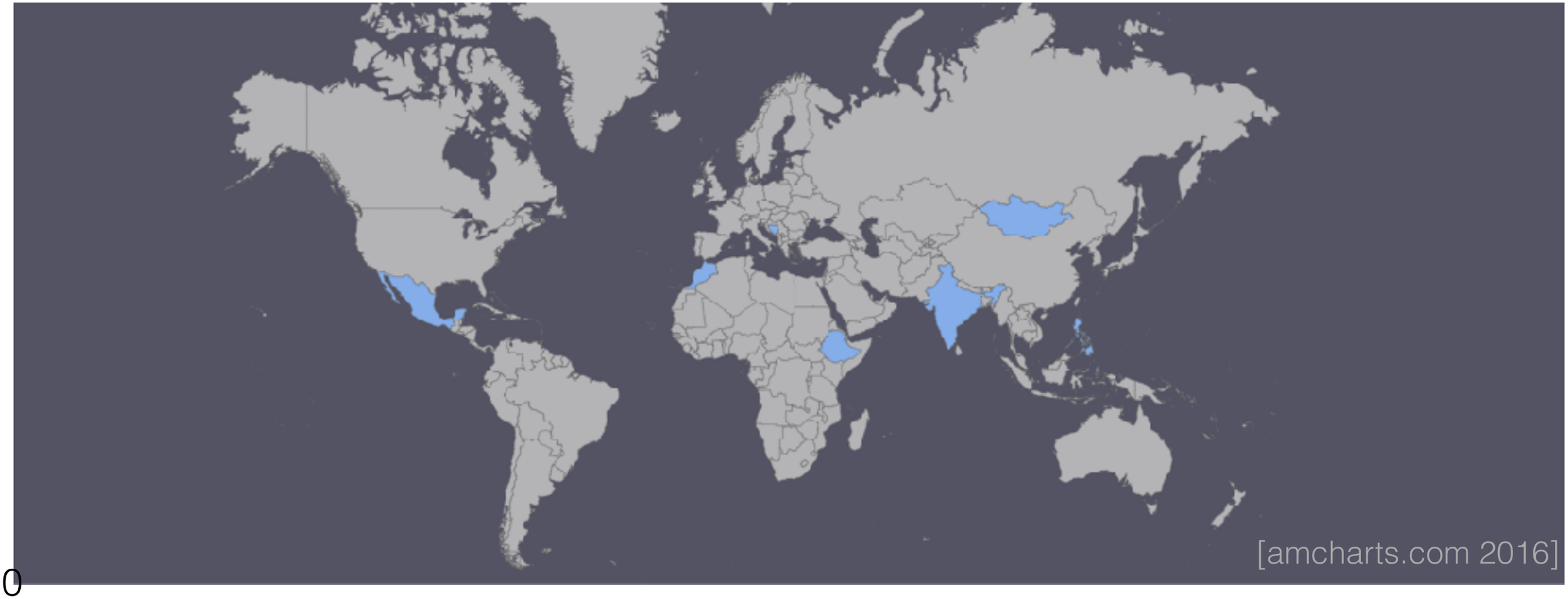

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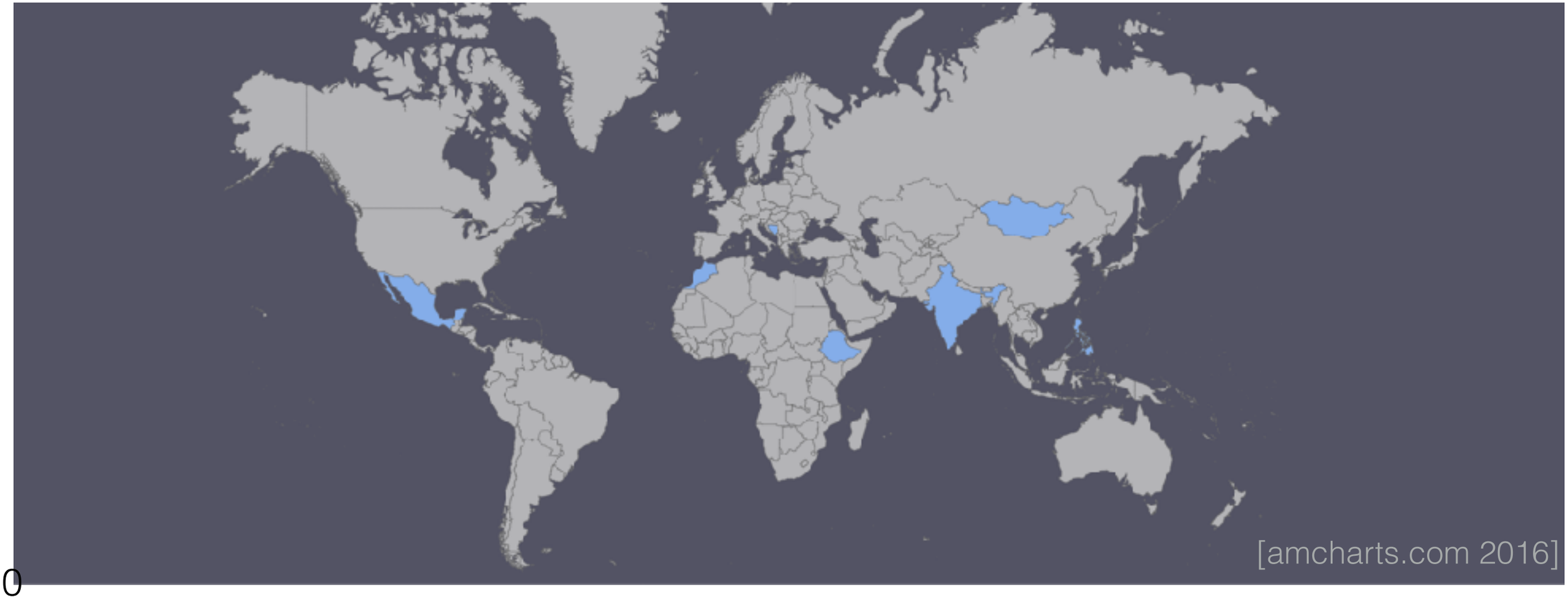

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$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \text{Sep\&LKJ}(\eta, c, d)$$

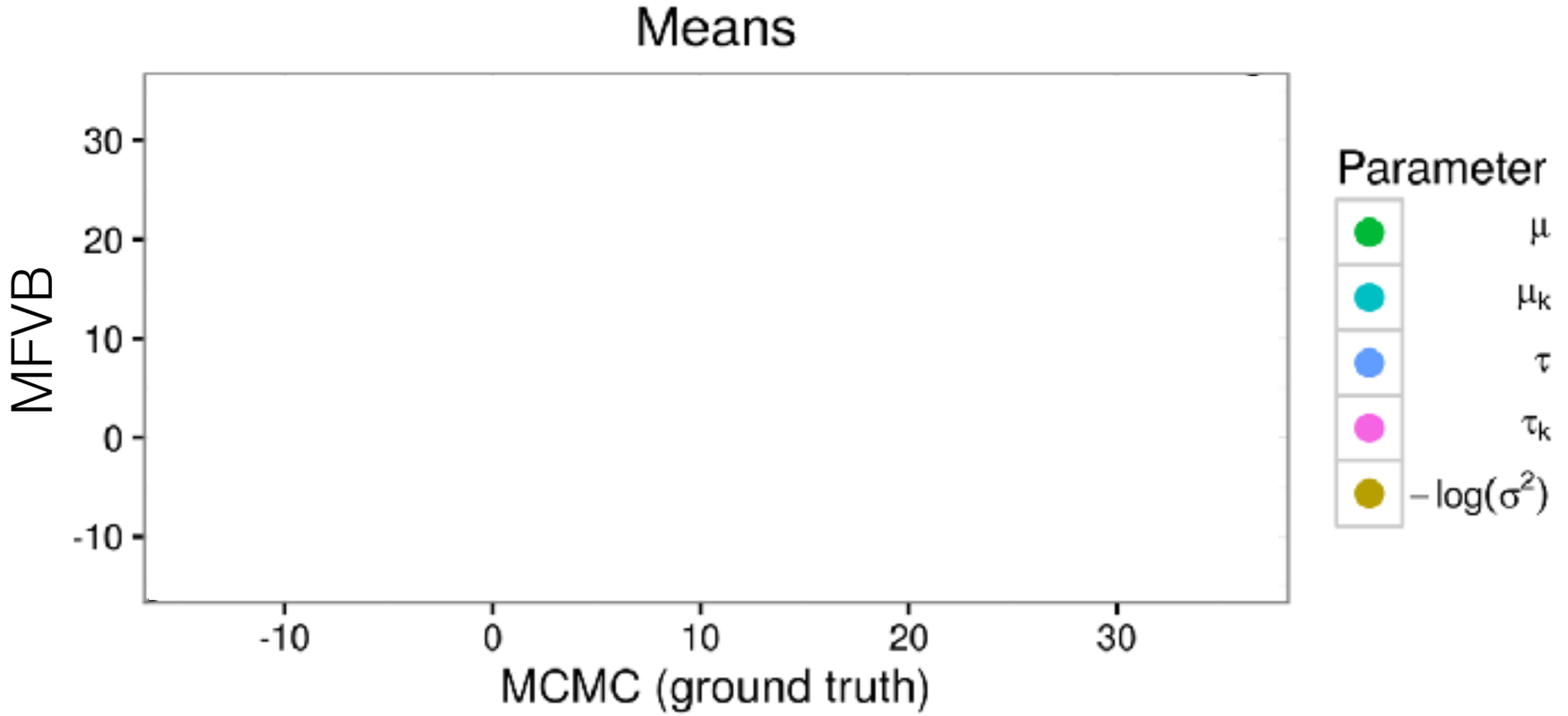
Microcredit

MFVB: Do we need to check the output?

Microcredit

MFVB: How will we know if it's working?

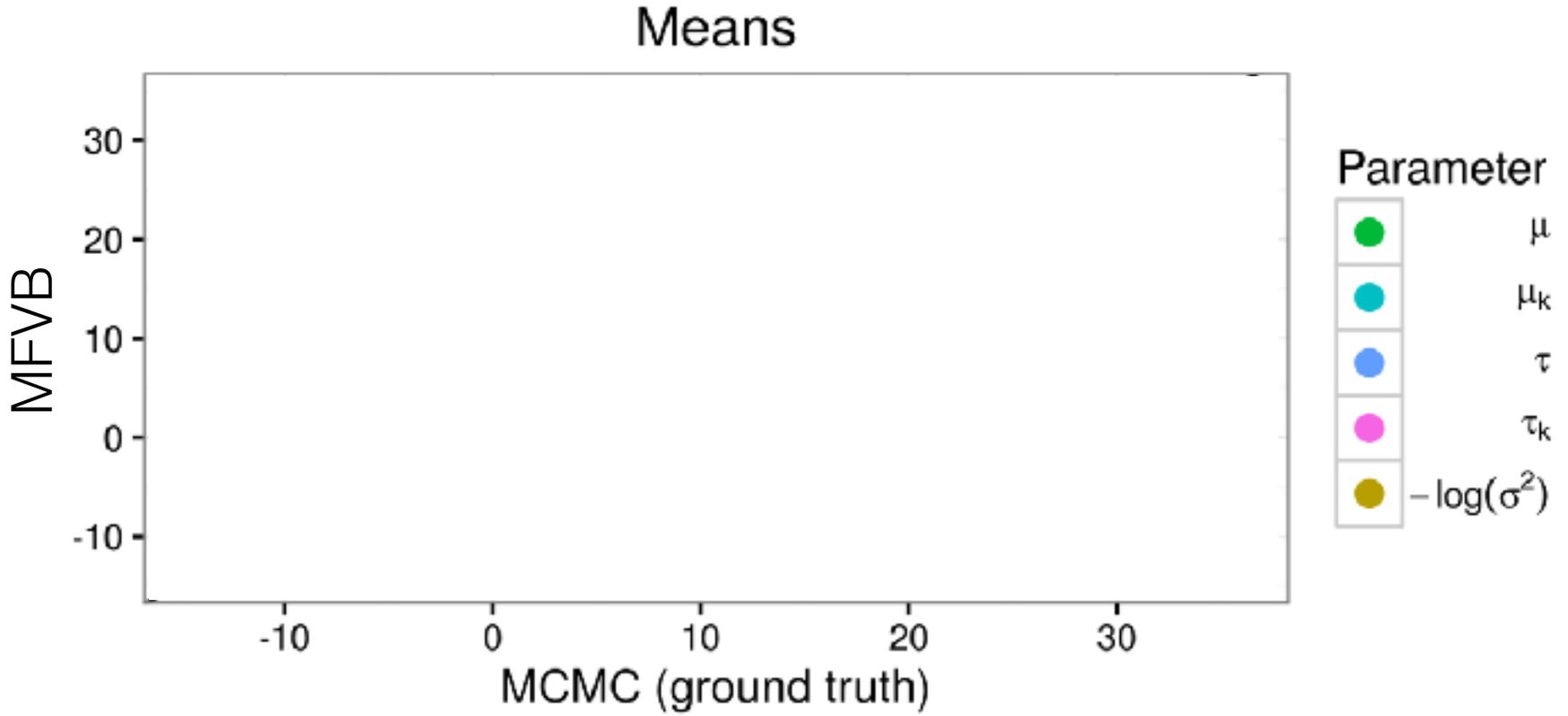
Microcredit



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

Microcredit

- *One set* of 2500 MCMC draws: **45 minutes**



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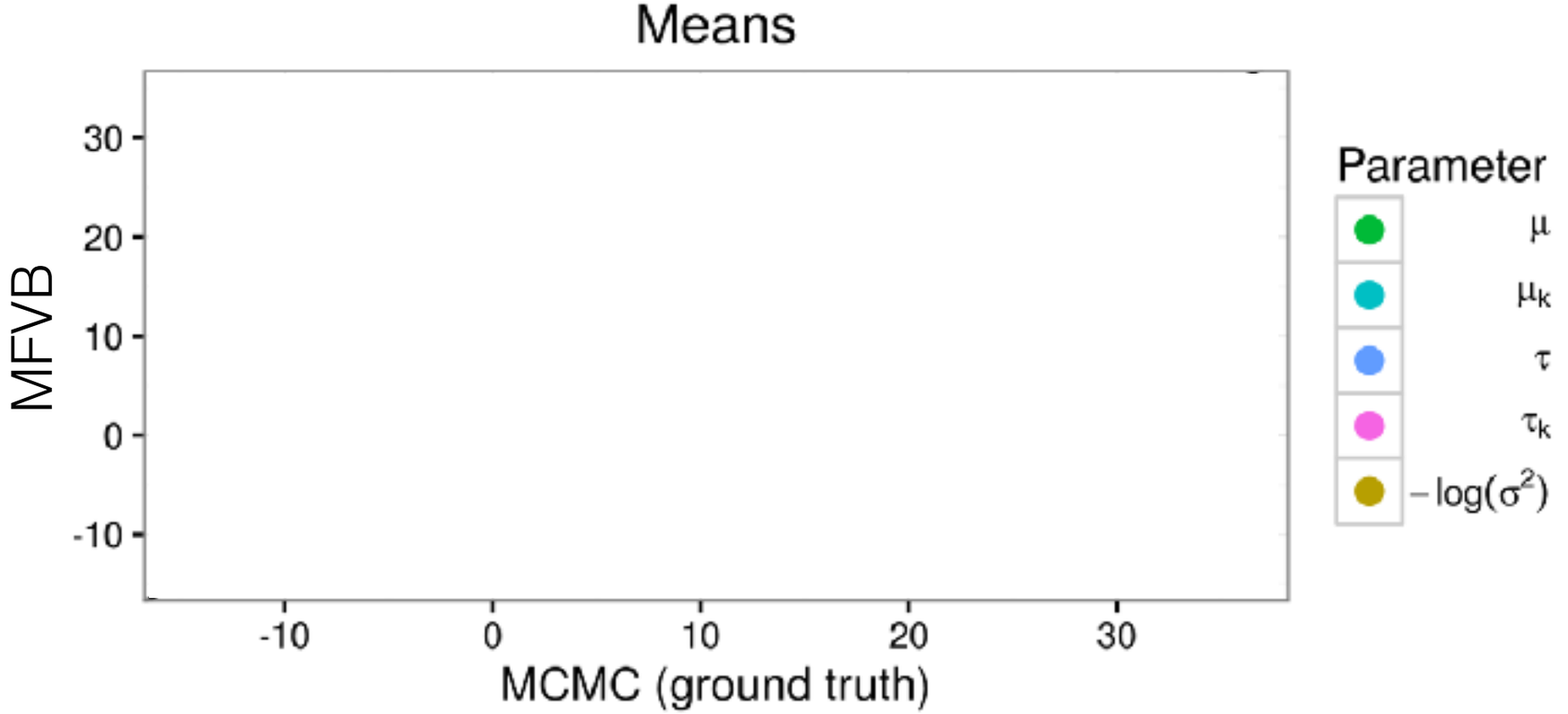
- MFVB optimization: **<1 min**



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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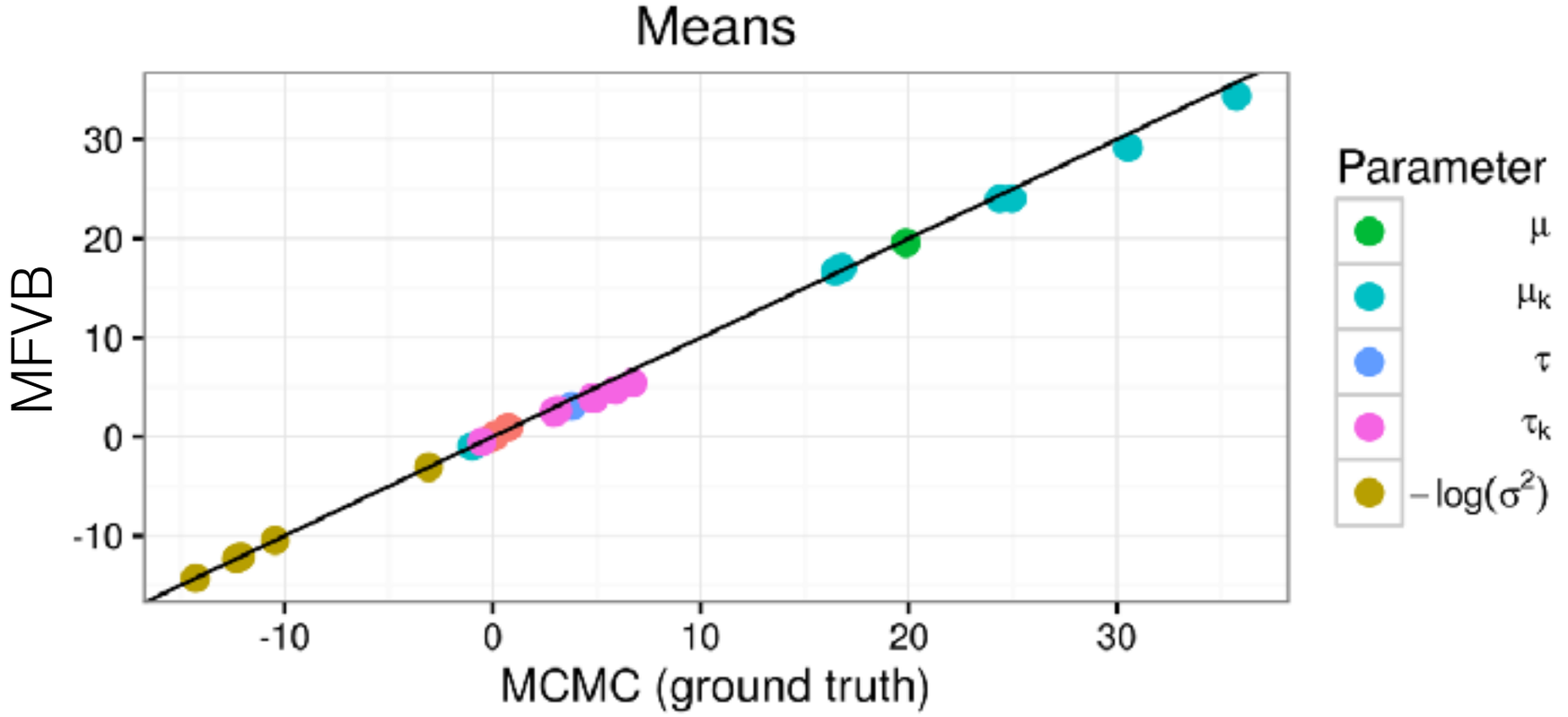

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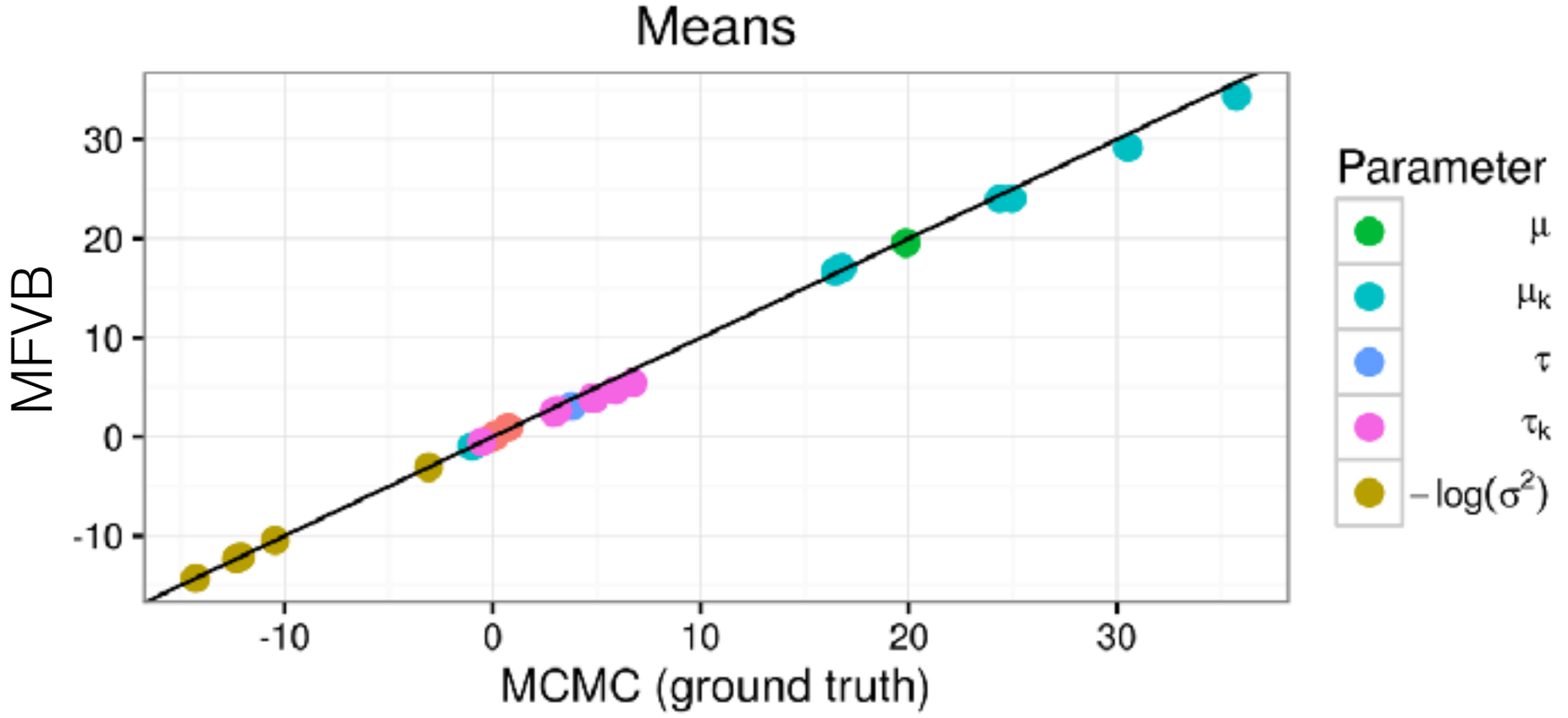

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?

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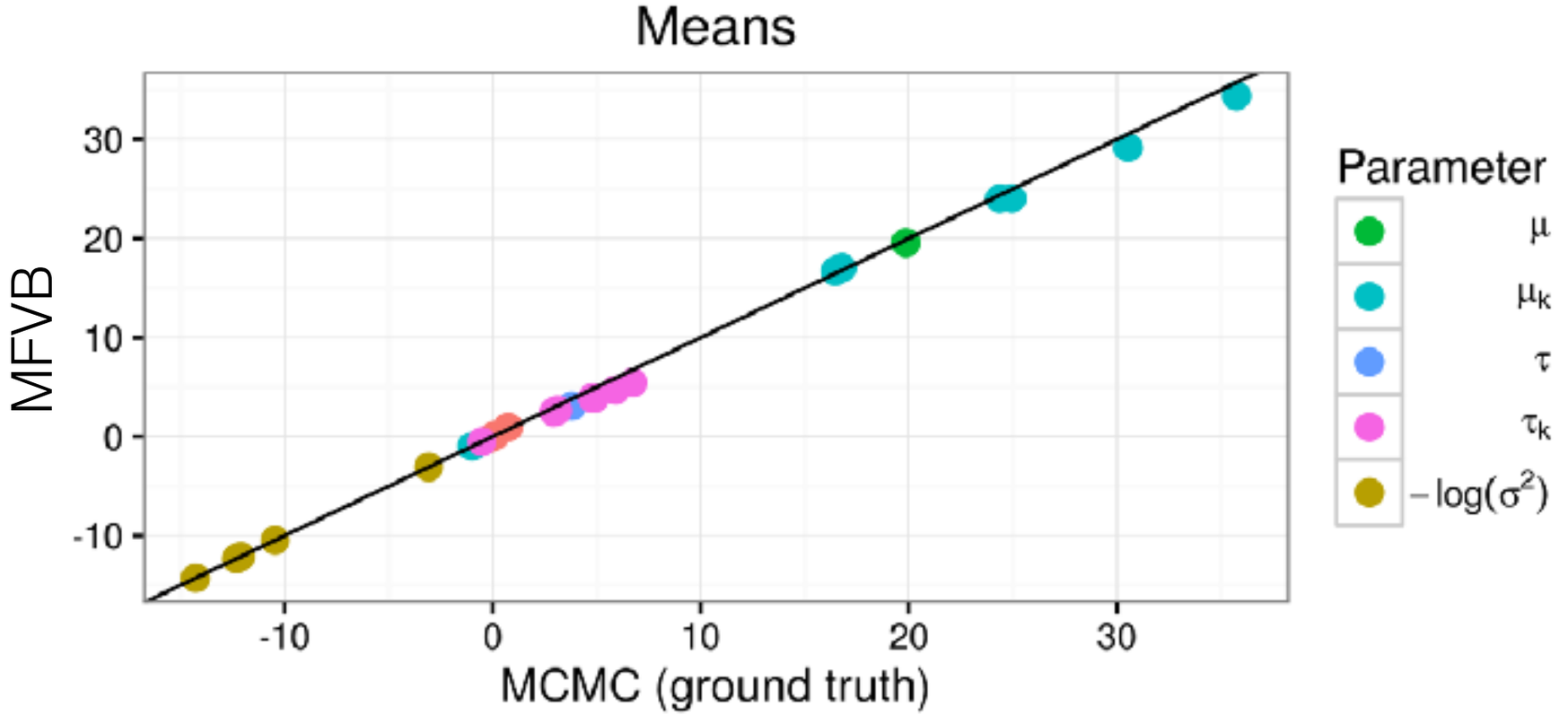

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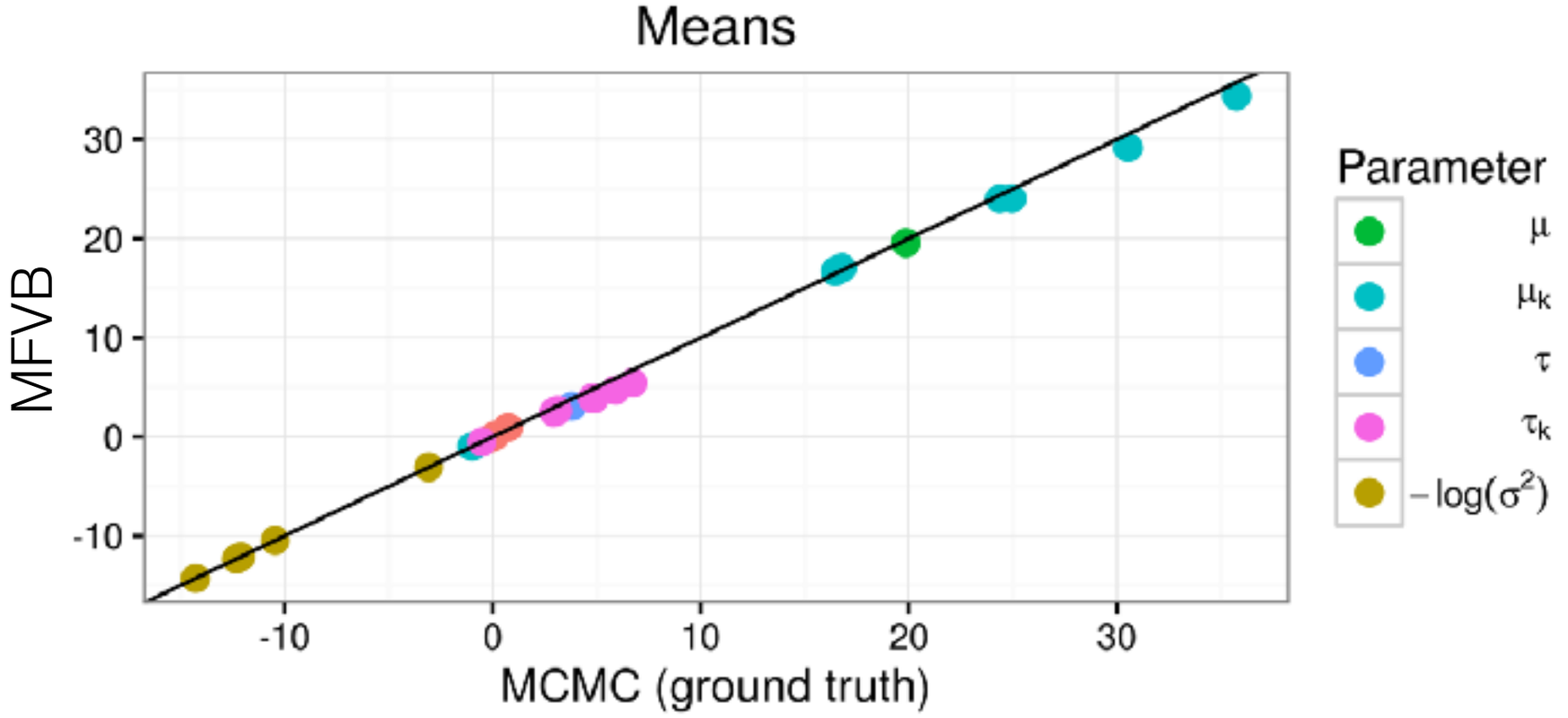

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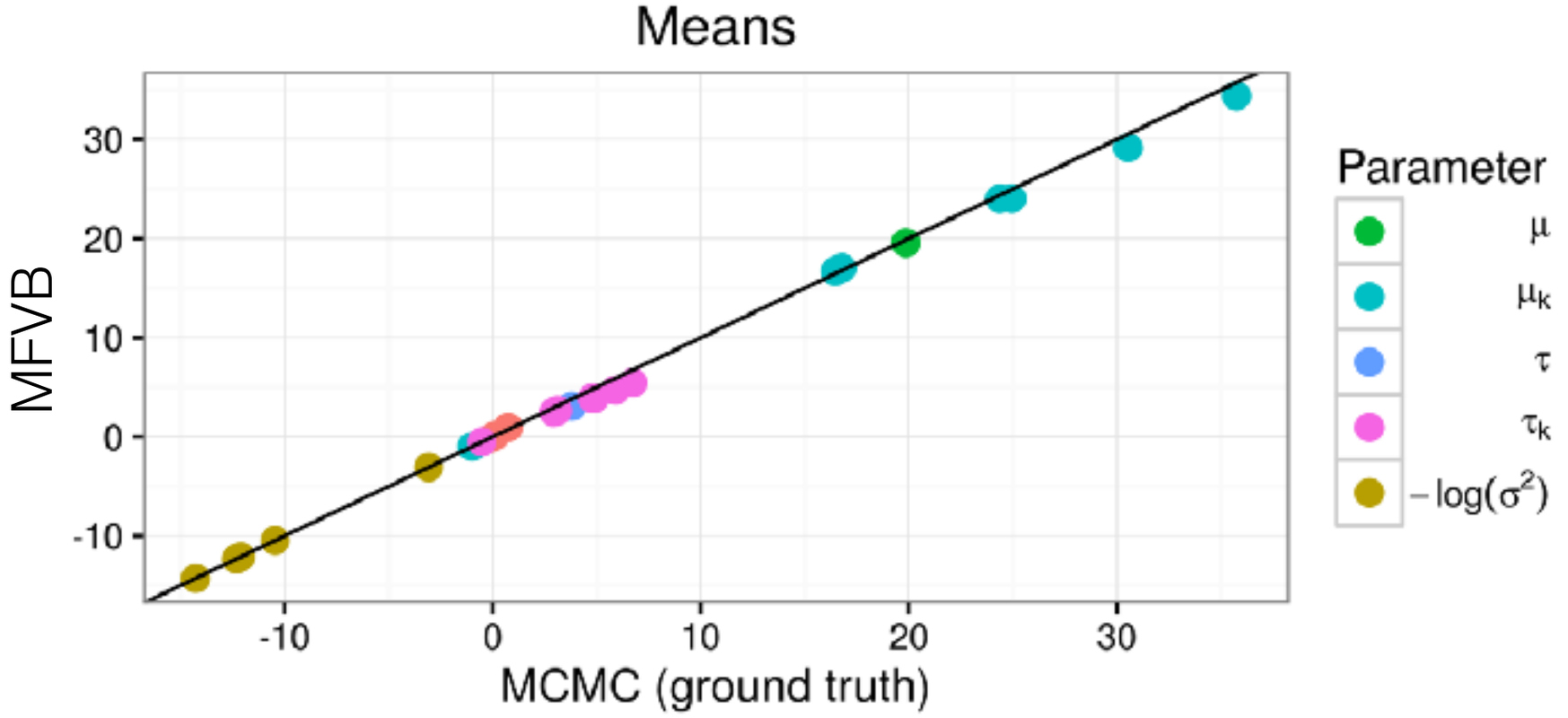

Criteo Online Ads Experiment

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- Q: How predictive of conversion are different features?
- Logistic GLMM; N = 61,895 subset to compare to MCMC

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

[Giordano, Broderick, Jordan 2018]

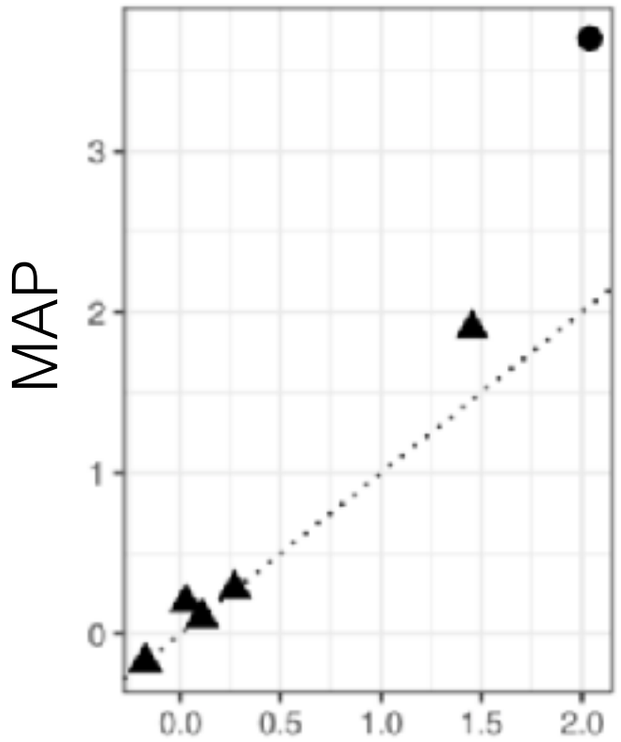
Criteo Online Ads Experiment

- MAP: **12 s**

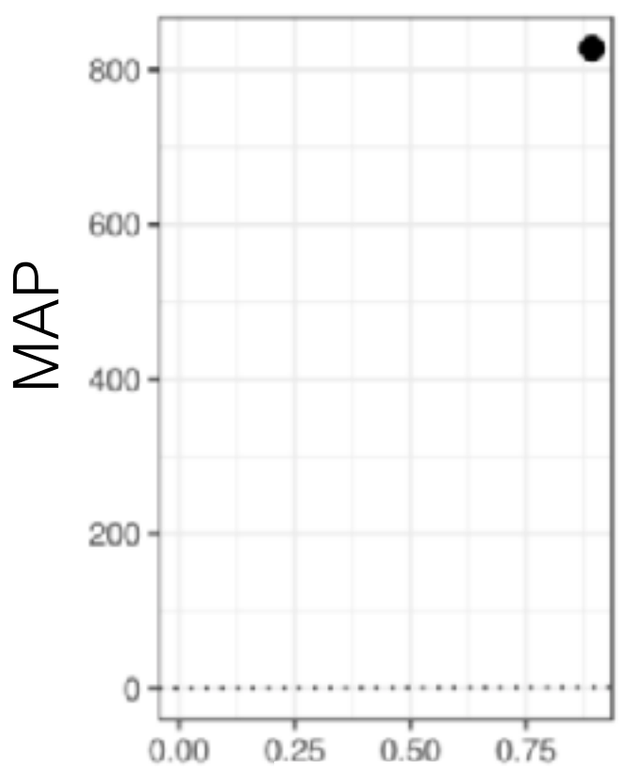
[Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

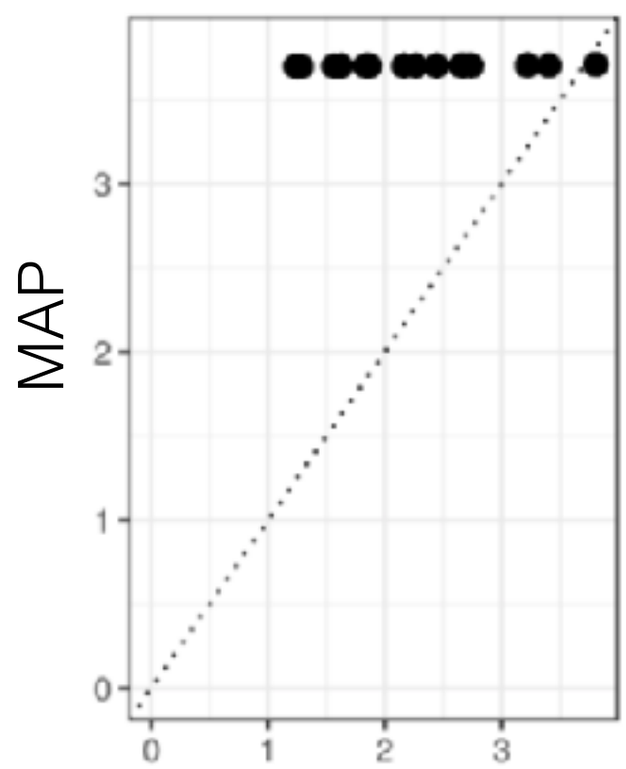
Global parameters (-τ)



Global parameter τ



Local parameters



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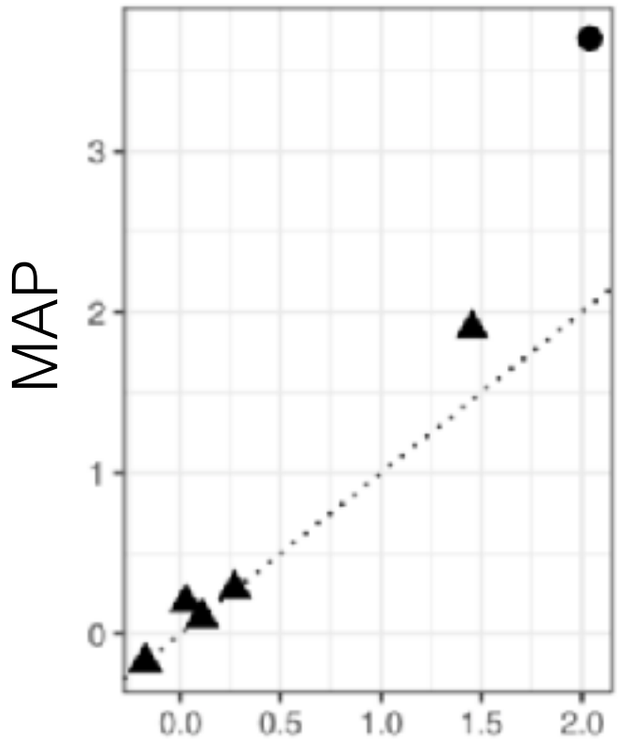

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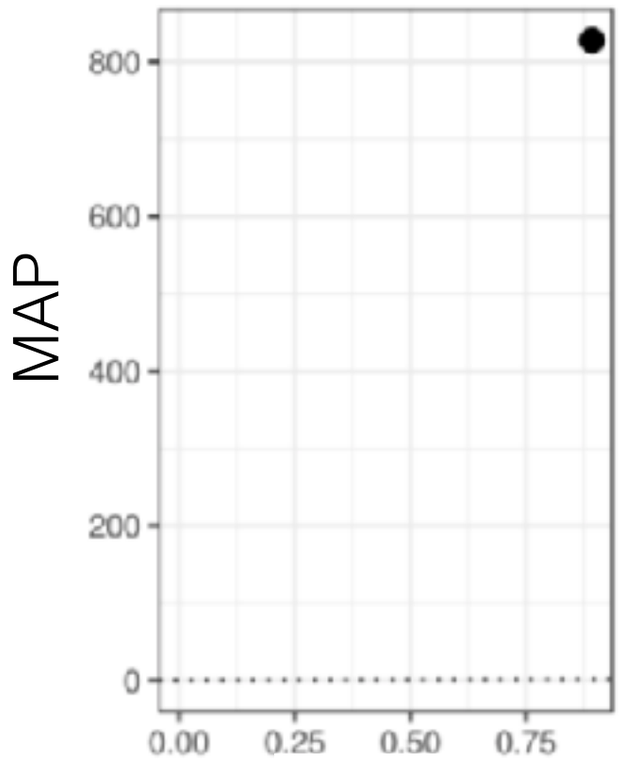

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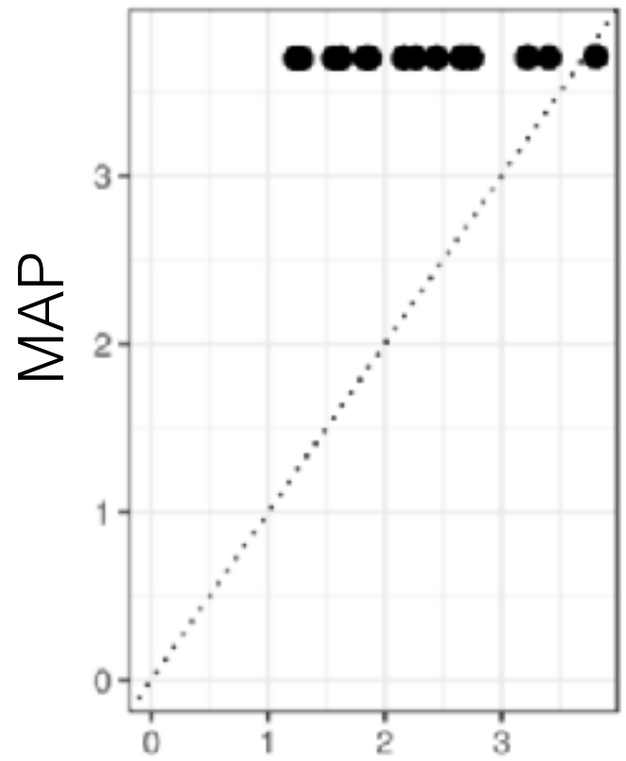

- MAP: **12 s**
- MFVB: **57 s**

[Giordano, Broderick, Jordan 2018]

Criteo Online Ads Experiment

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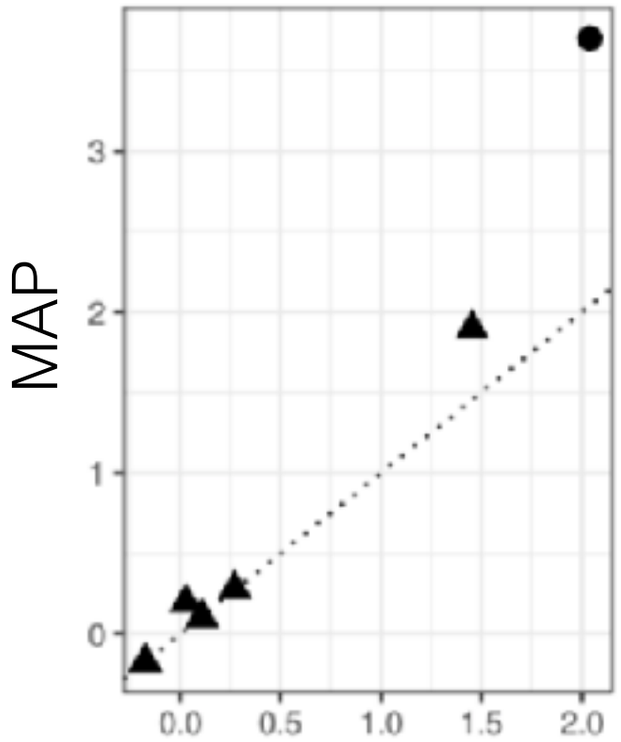

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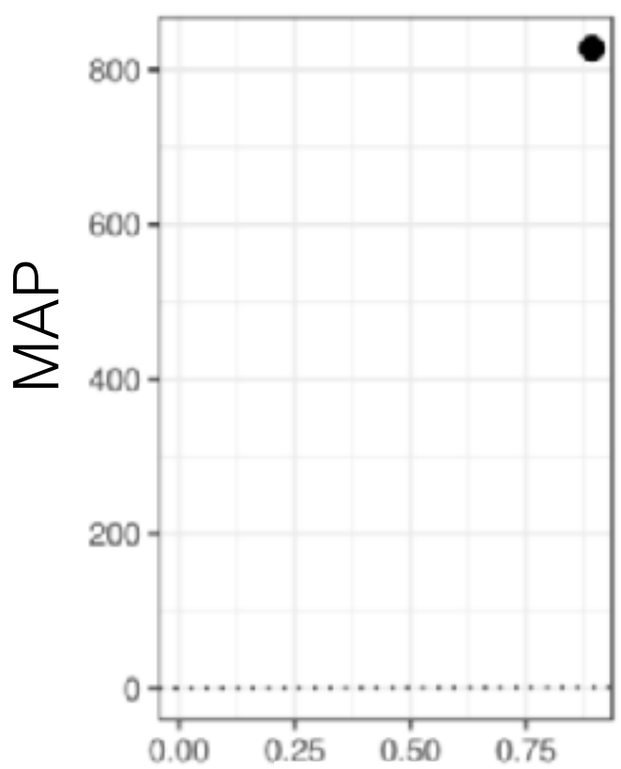

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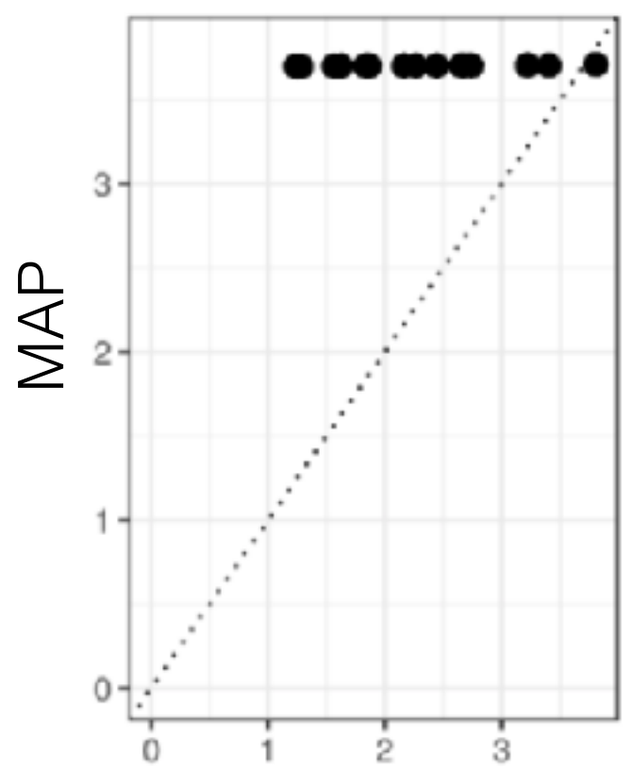

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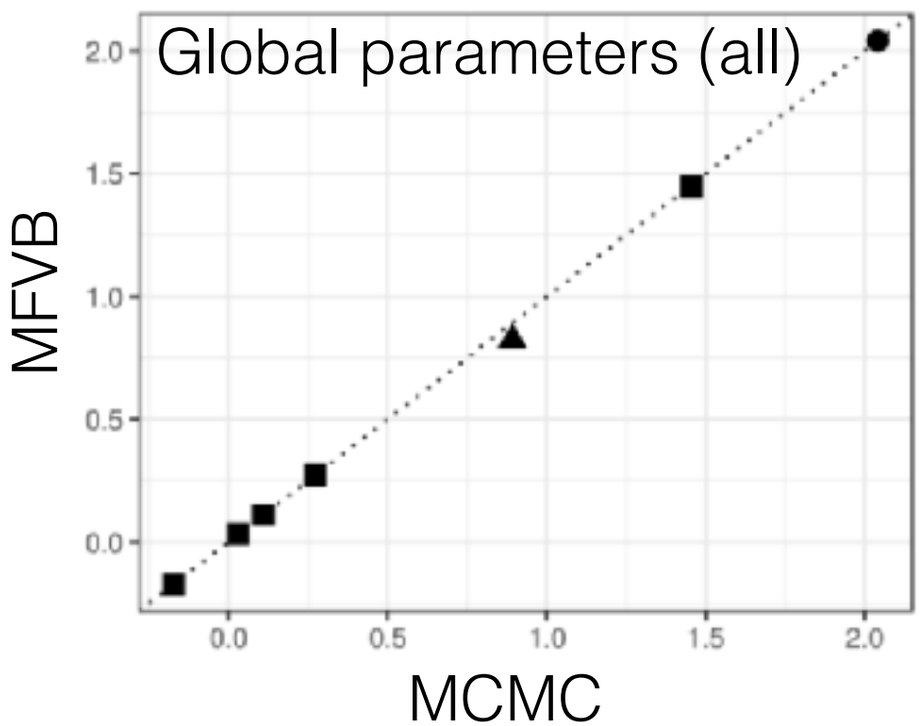

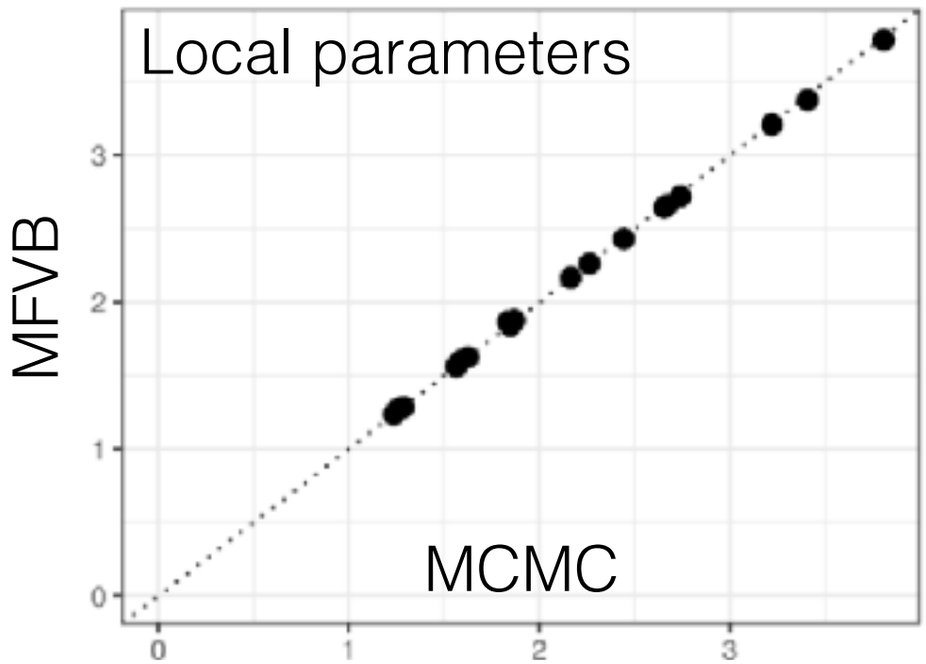

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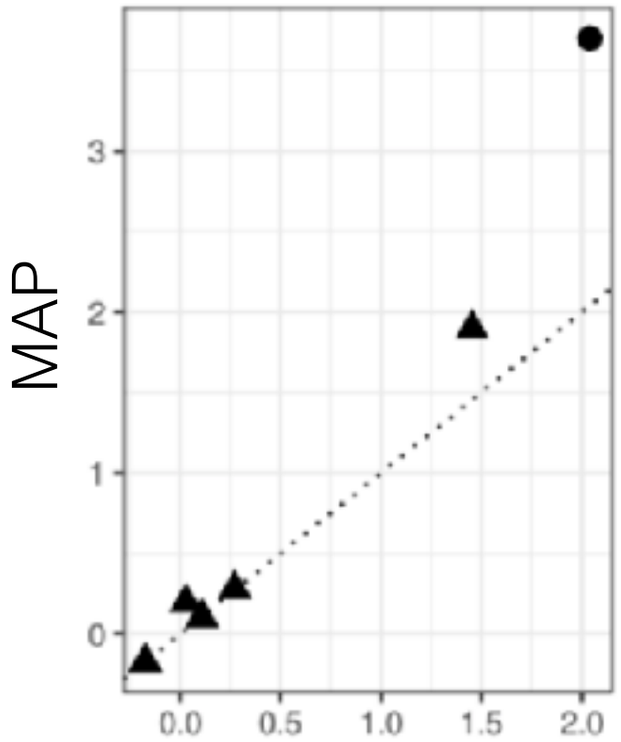

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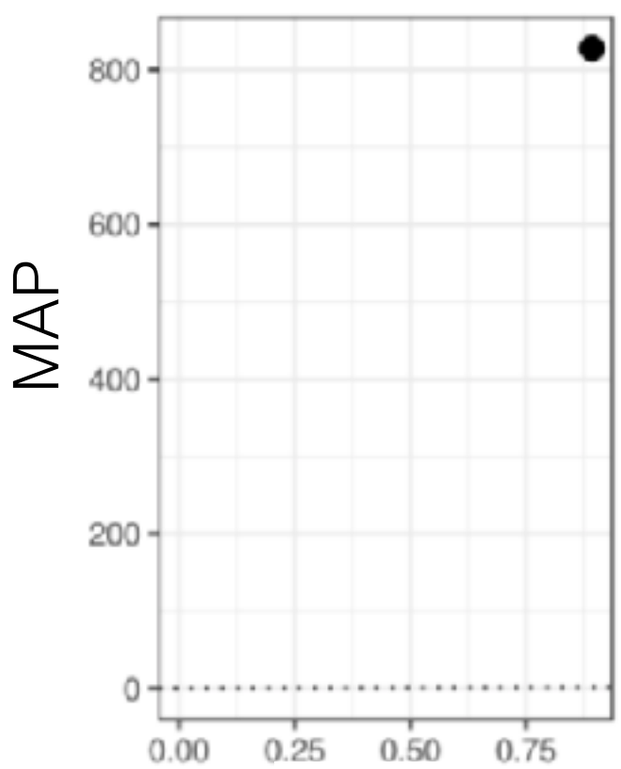

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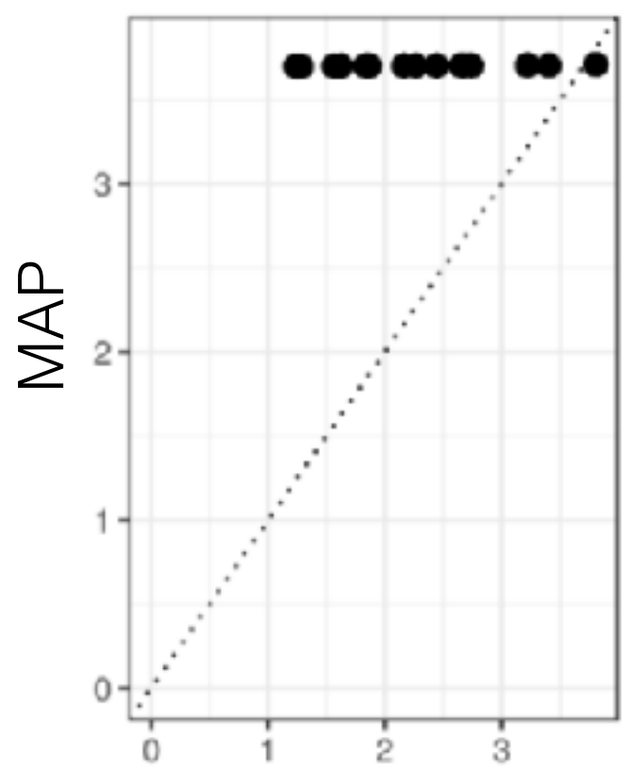

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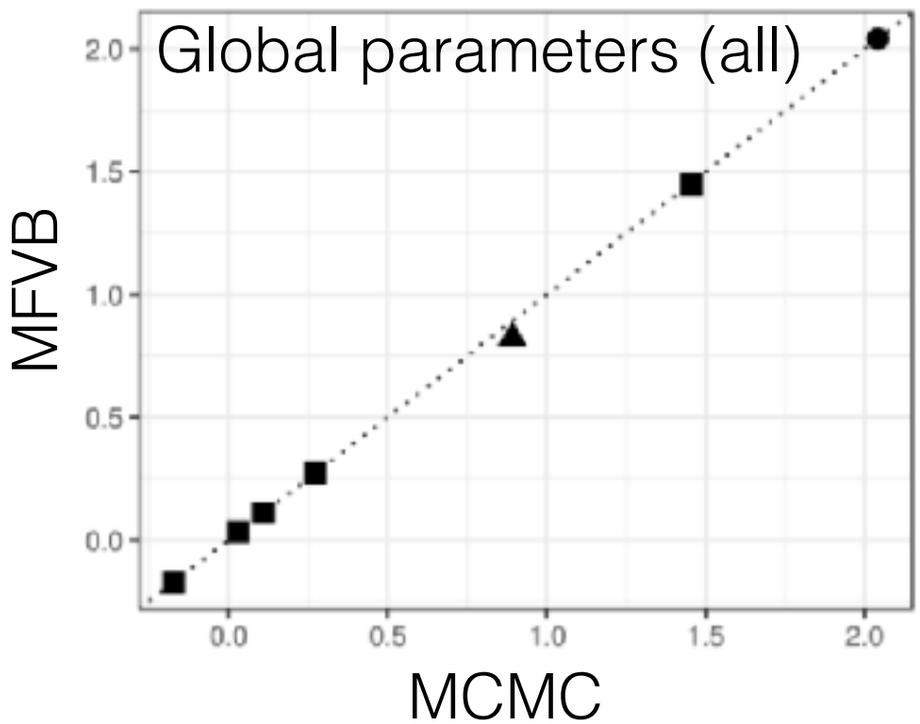
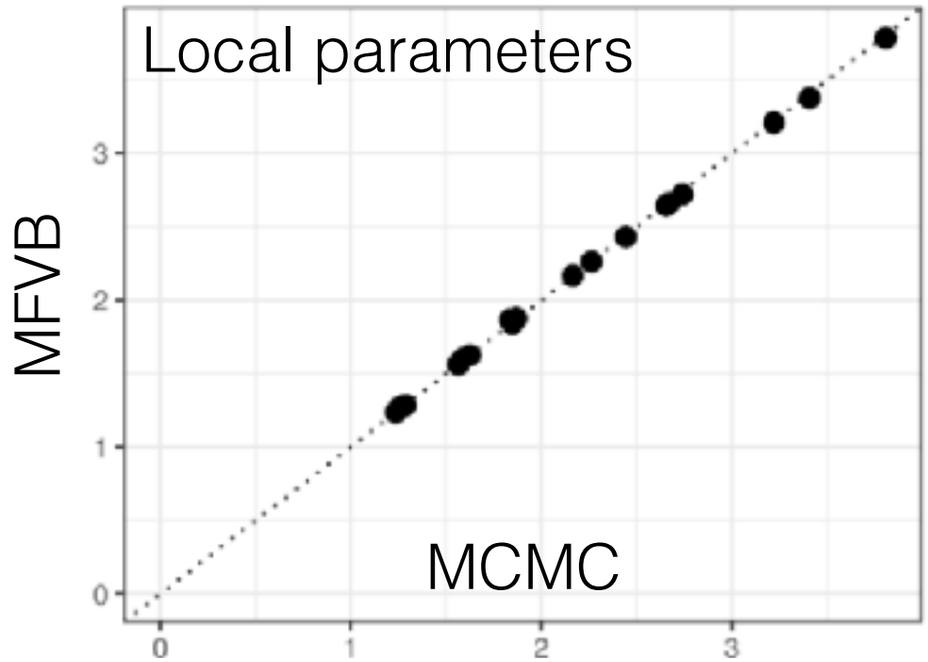
- MCMC (5K samples): 21,066 s (**5.85 h**)





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Why use MFVB?

- Topic discovery

"Arts"	"Budgets"	"Children"	"Education"
NEW	MILLION	CHILDREN	SCHOOL
FILM	TAX	WOMEN	STUDENTS
SHOW	PROGRAM	PEOPLE	SCHOOLS
MUSIC	BUDGET	CHILD	EDUCATION
MOVIE	BILLION	YEARS	TEACHERS
PLAY	FEDERAL	FAMILIES	HIGH
MUSICAL	YEAR	WORK	PUBLIC
BEST	SPENDING	PARENTS	TEACHER
ACTOR	NEW	SAYS	BENNETT
FIRST	STATE	FAMILY	MANIGAT
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LOVE	CONGRESS	LIFE	HAITI

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[Blei et al 2003]

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Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use VB?
- When can we trust VB?
- Where do we go from here?

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What about uncertainty?

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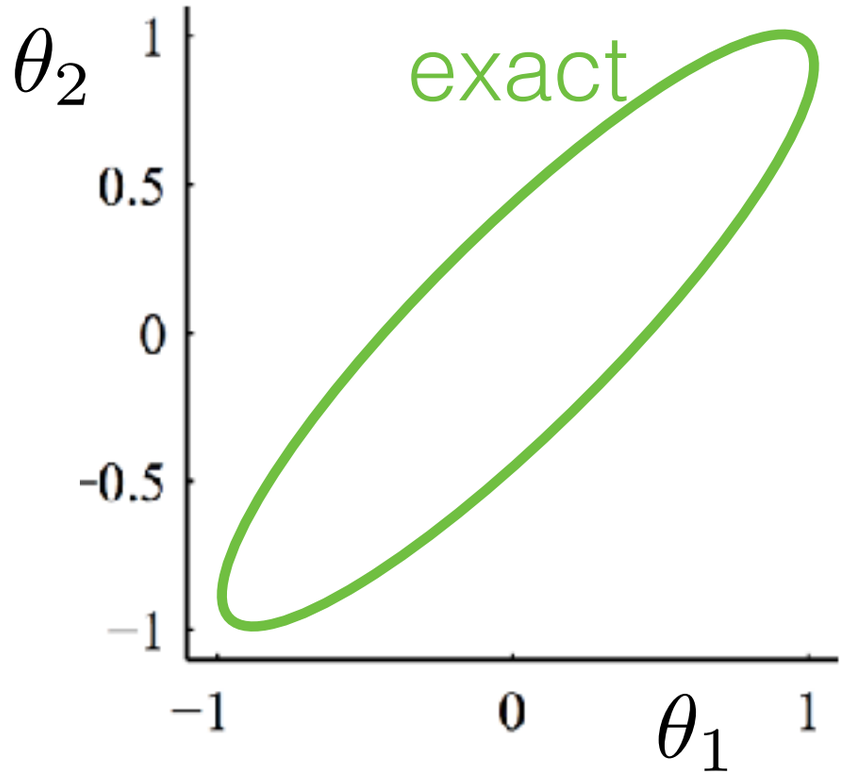

[Turner & Sahani 2011; MacKay 2003; Bishop 2006; Wang, Titterington 2004]

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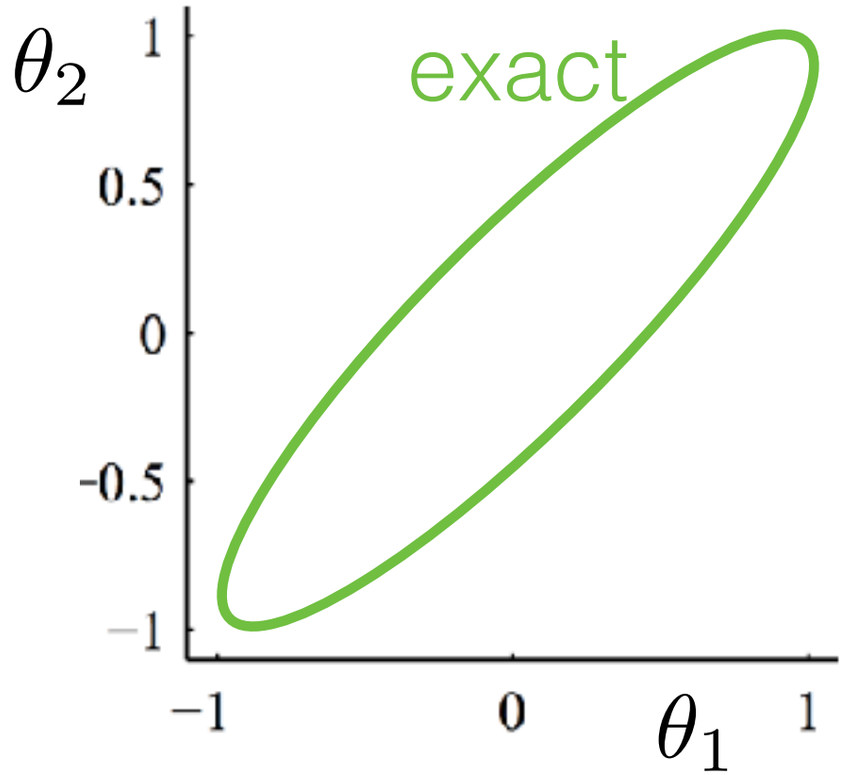

- Conjugate linear regression

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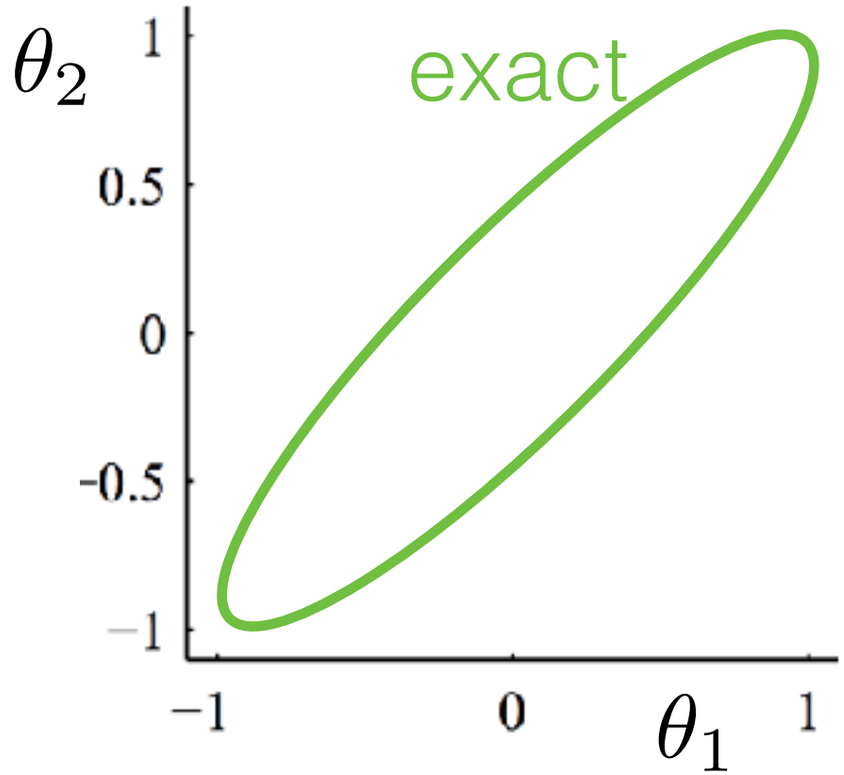

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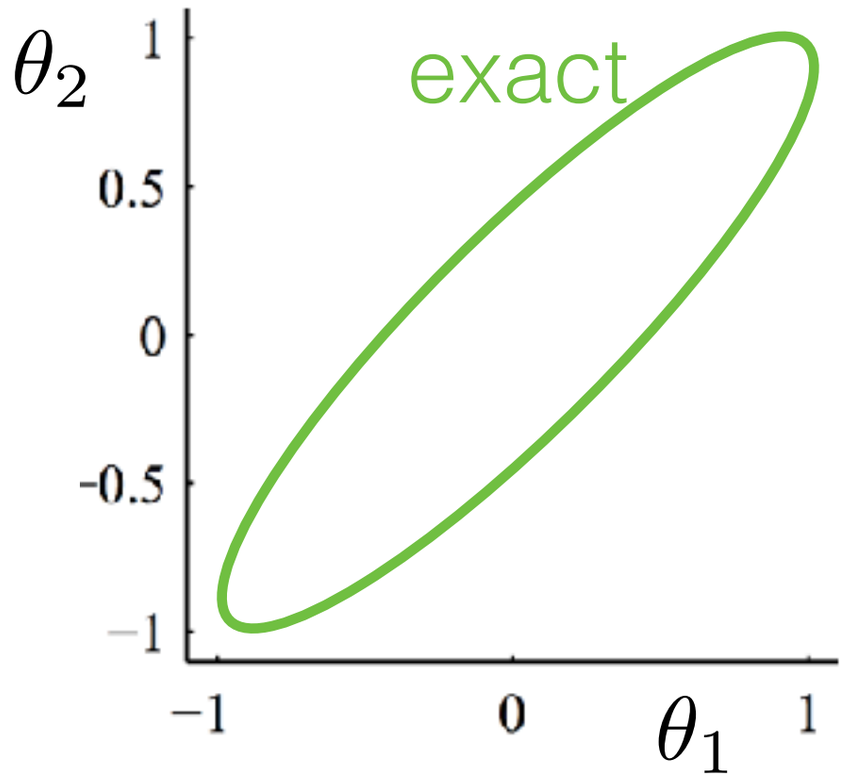

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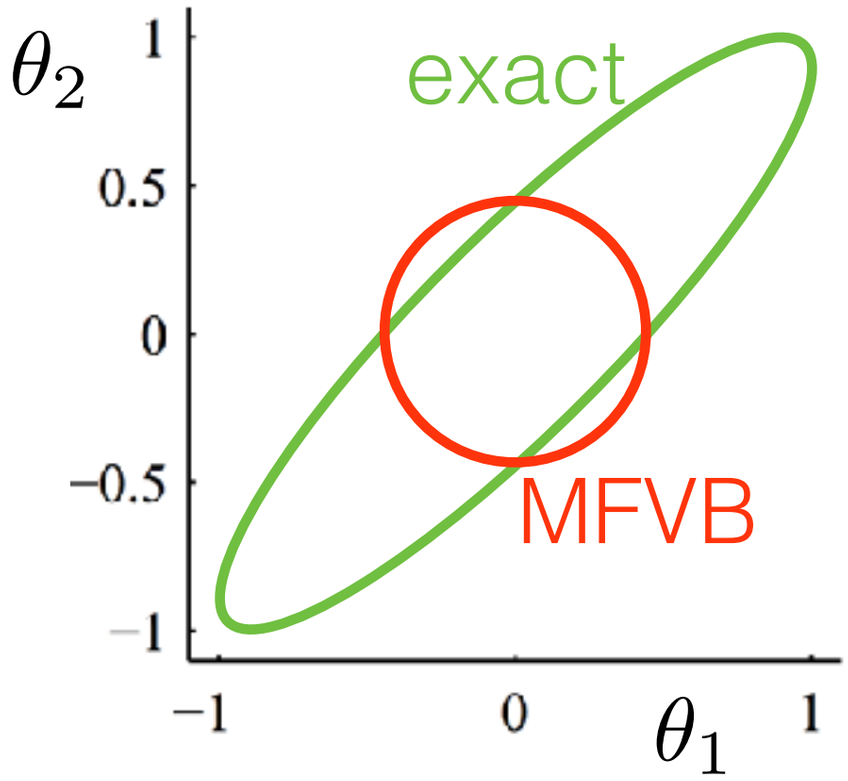

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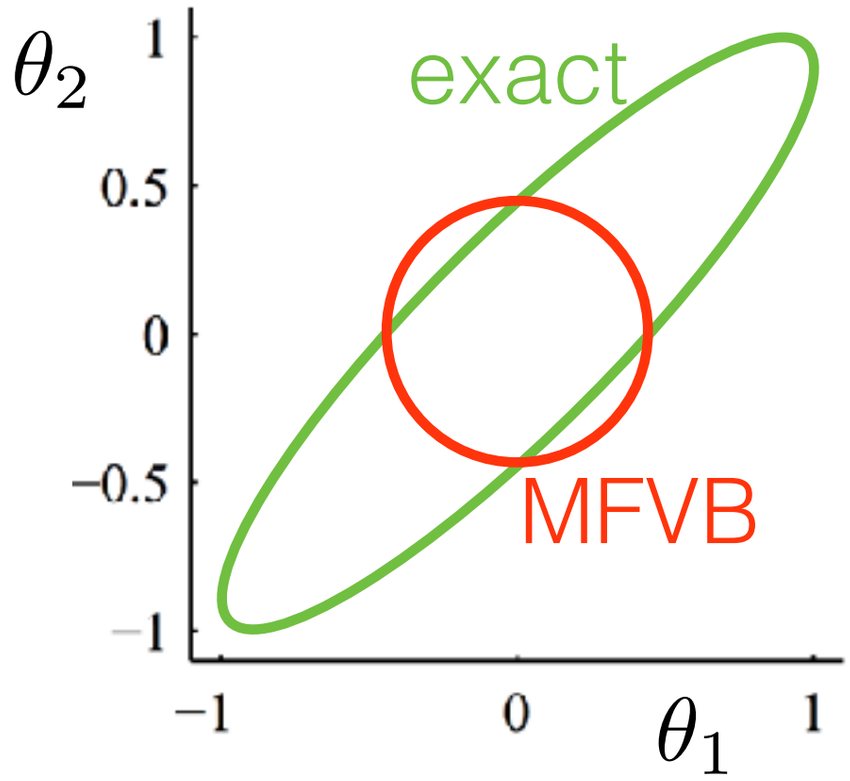

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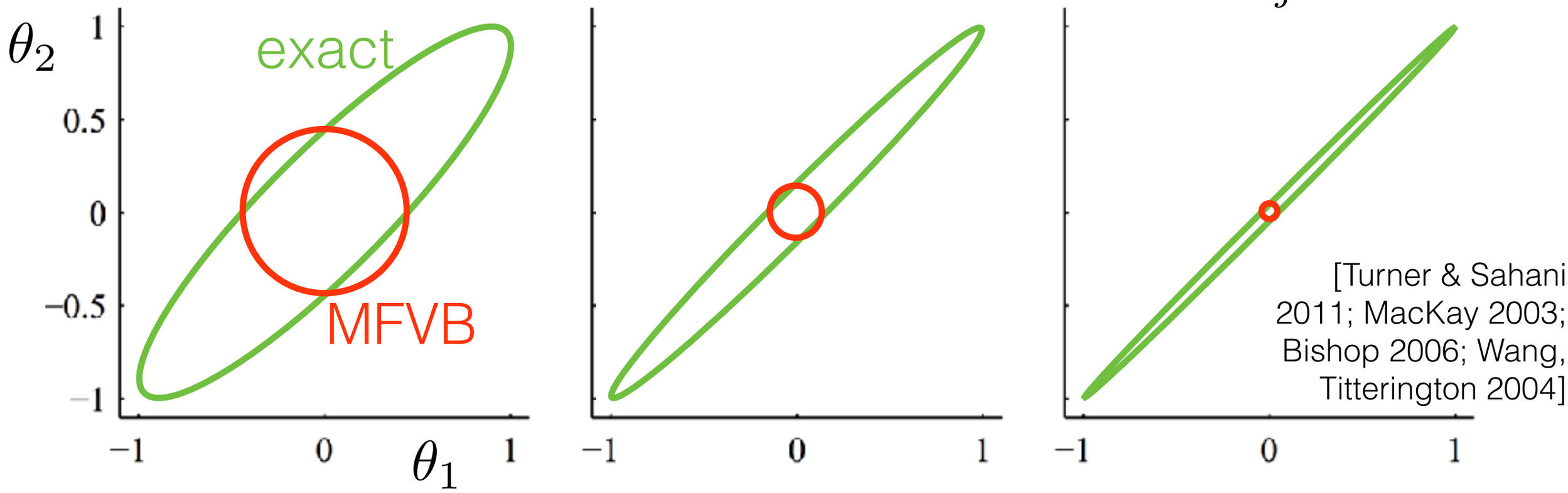

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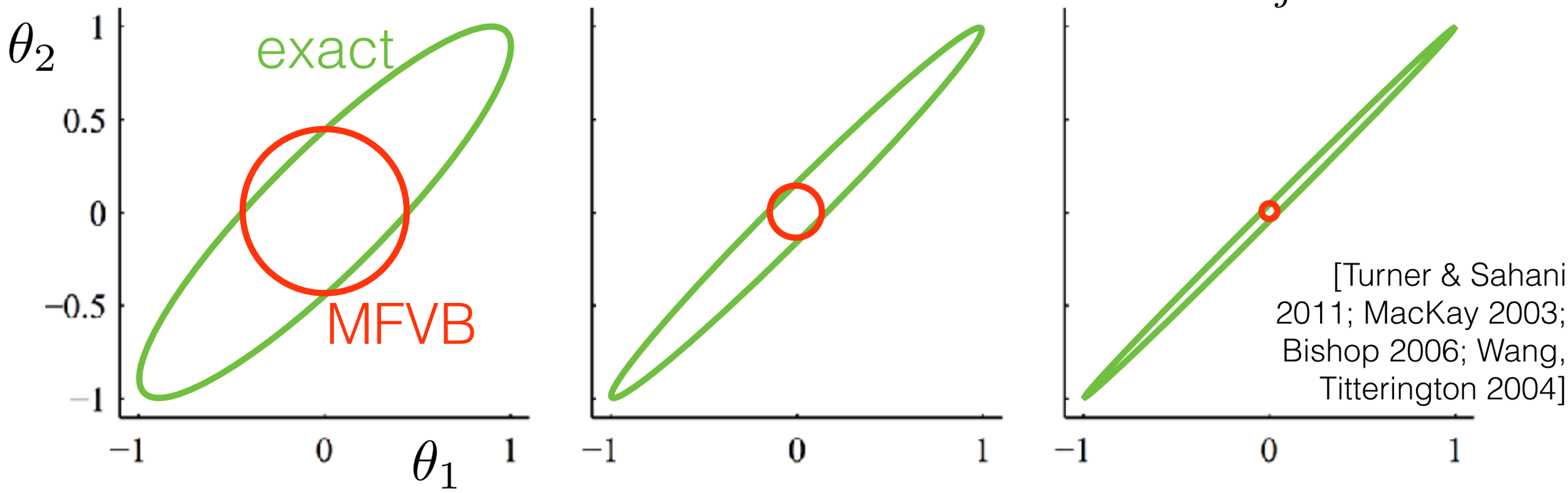

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- Underestimates variance (sometimes severely)
- No covariance estimates

What about uncertainty?

- Microcredit

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

What about uncertainty?

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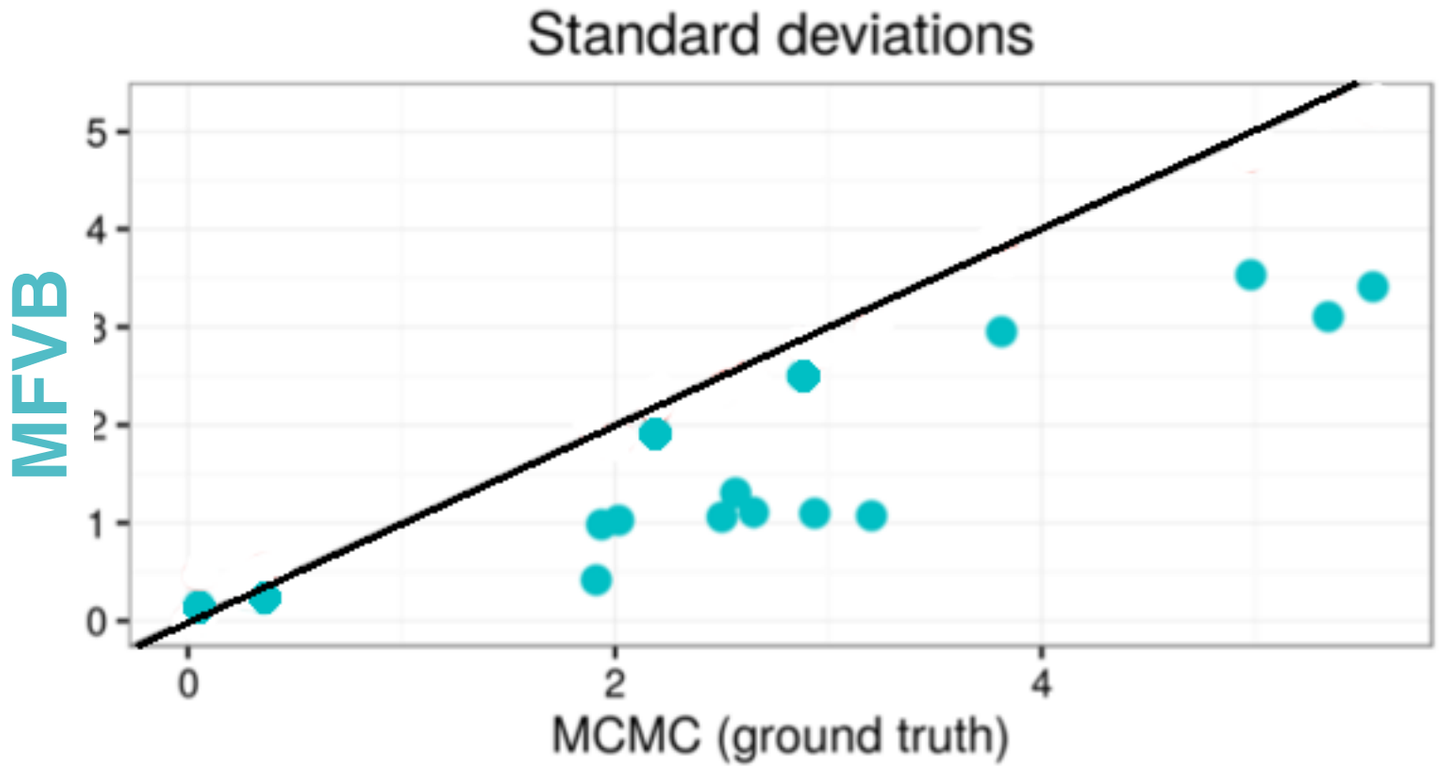

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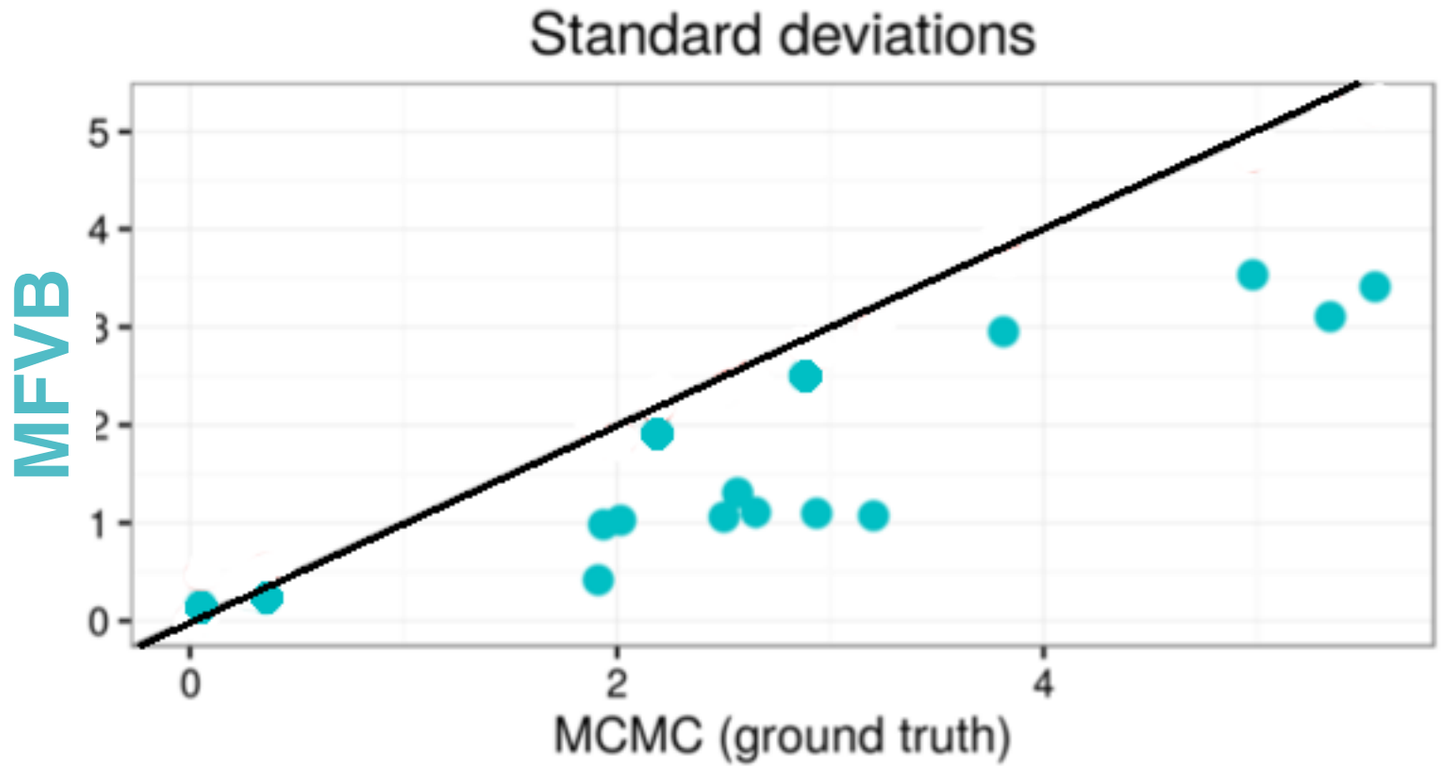
What about uncertainty?

- Microcredit effect
- τ mean:
  3.08 USD PPP



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

What about uncertainty?

- Microcredit effect
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- τ std dev:
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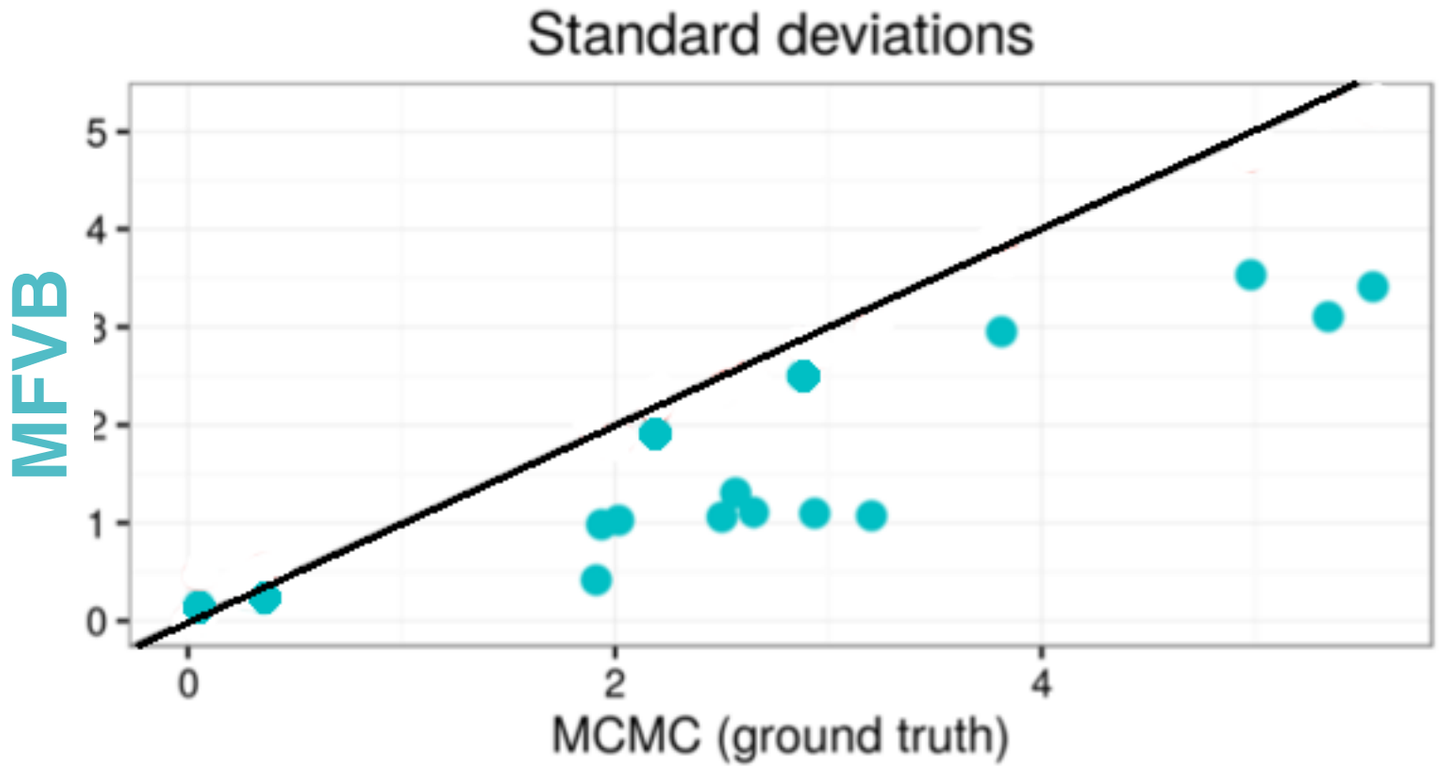

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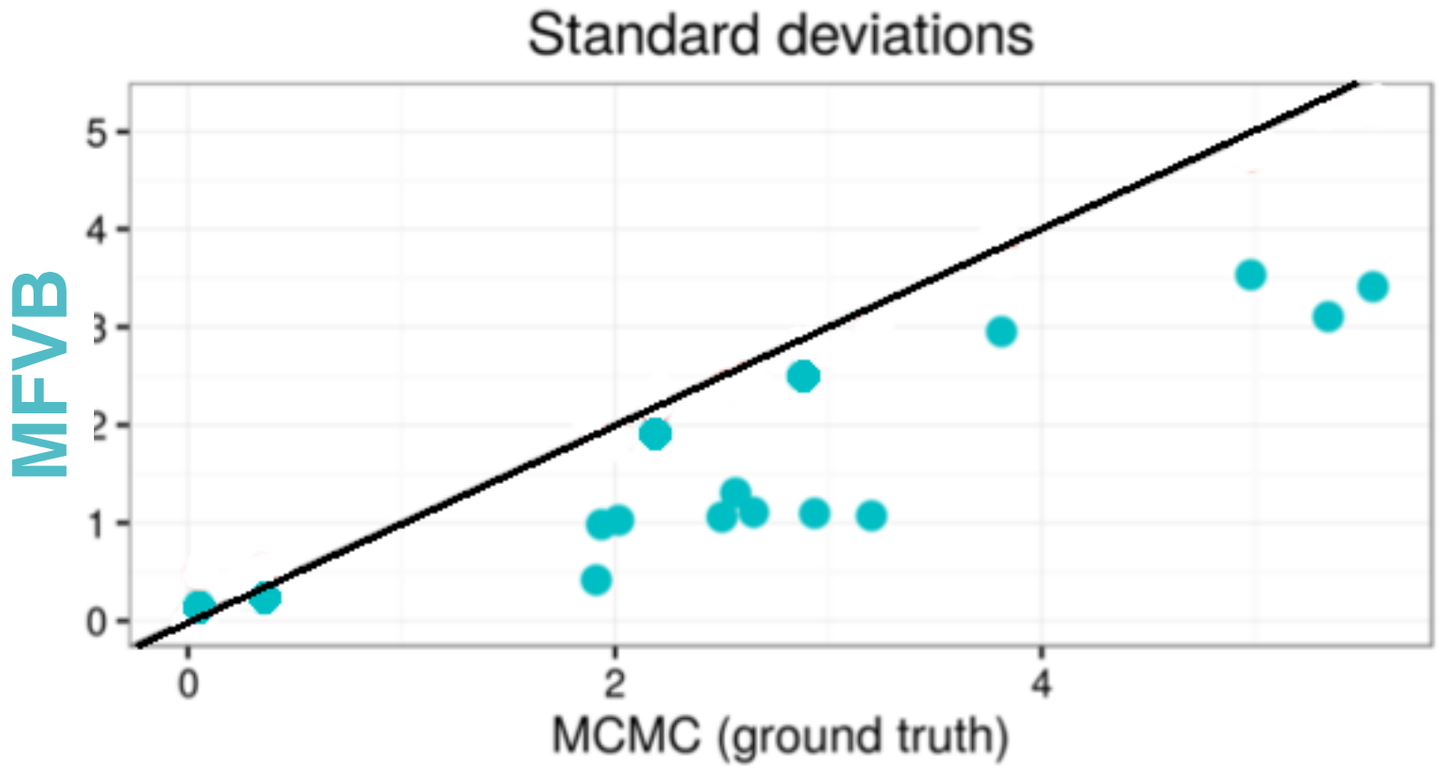

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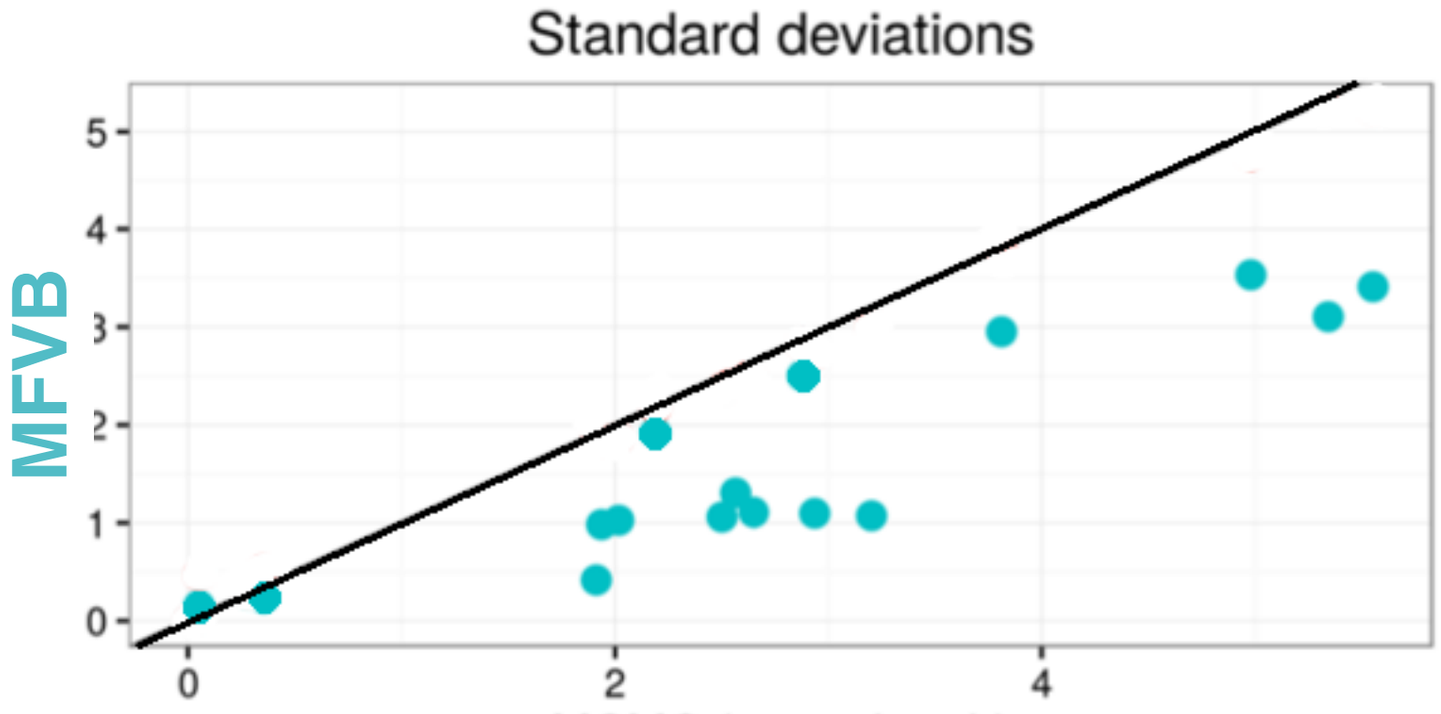
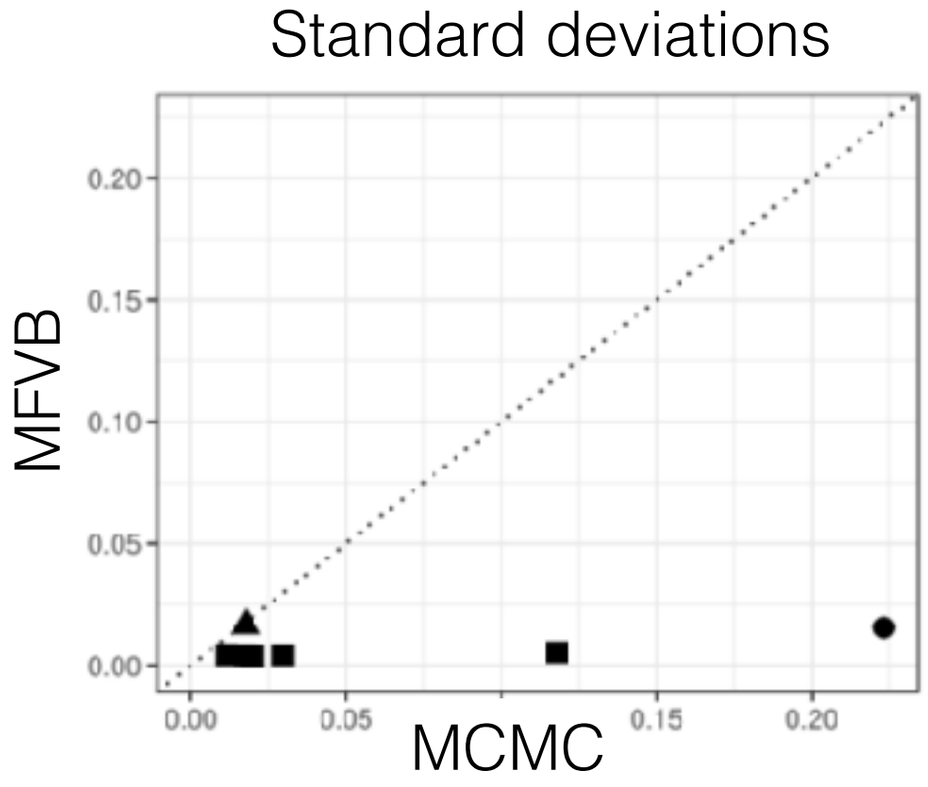


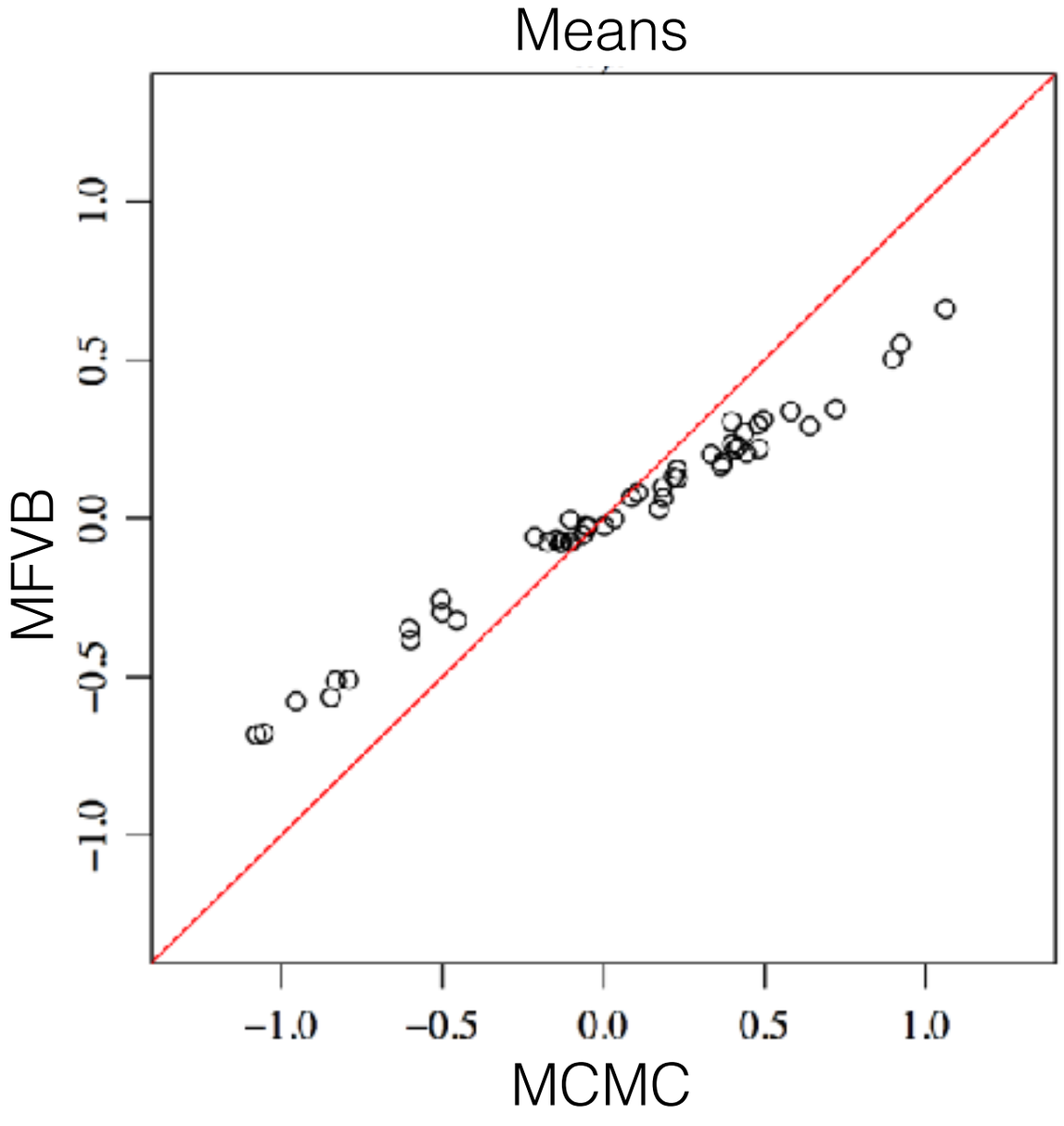
- Criteo online ads experiment



[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2018]

What about means?

- Model for relational data with covariates
- When 1000+ nodes, MCMC > 1 day [Fosdick 2013, Ch 4]



[Fosdick 2013, Ch 4, Fig 4.3]

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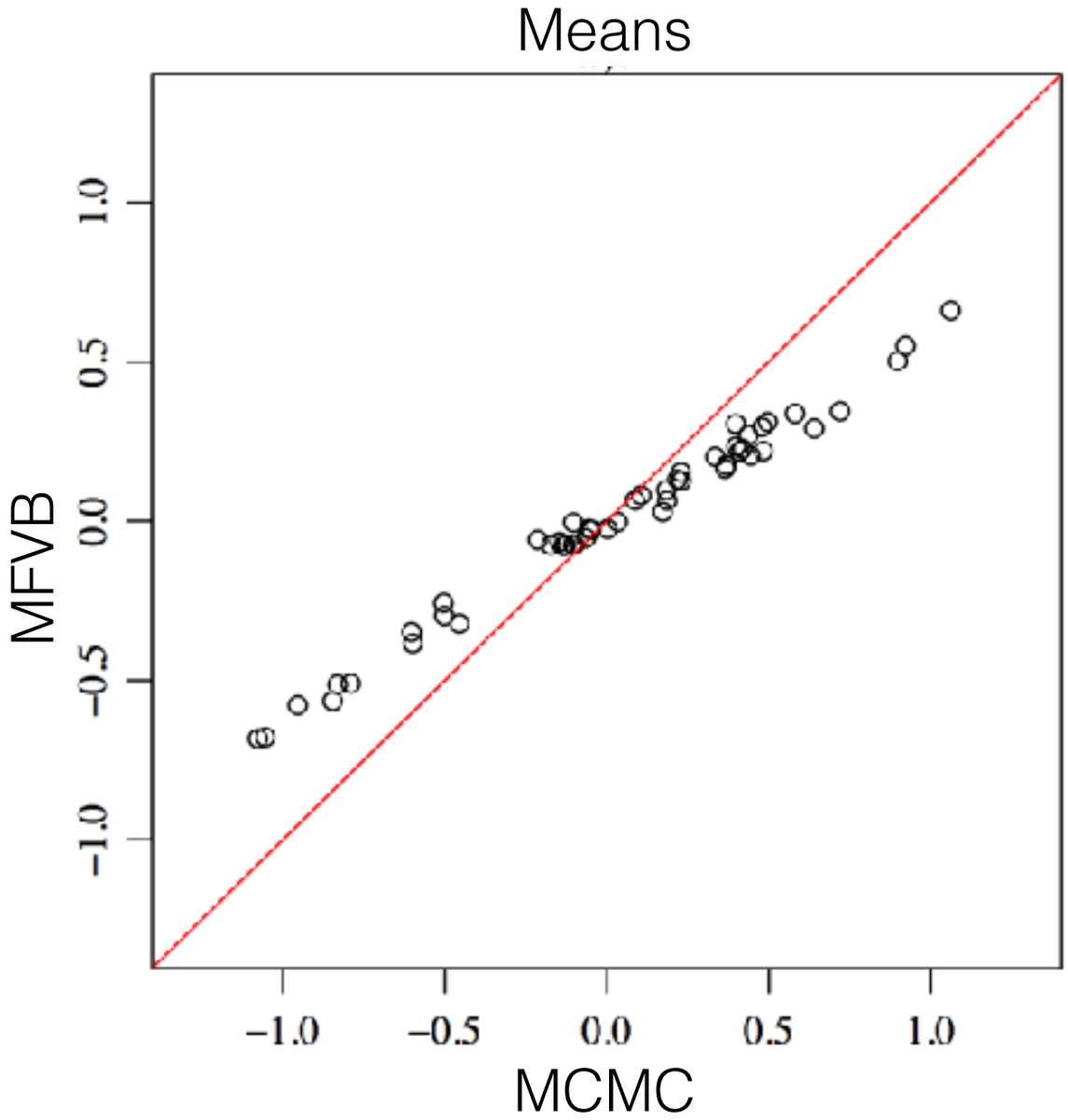

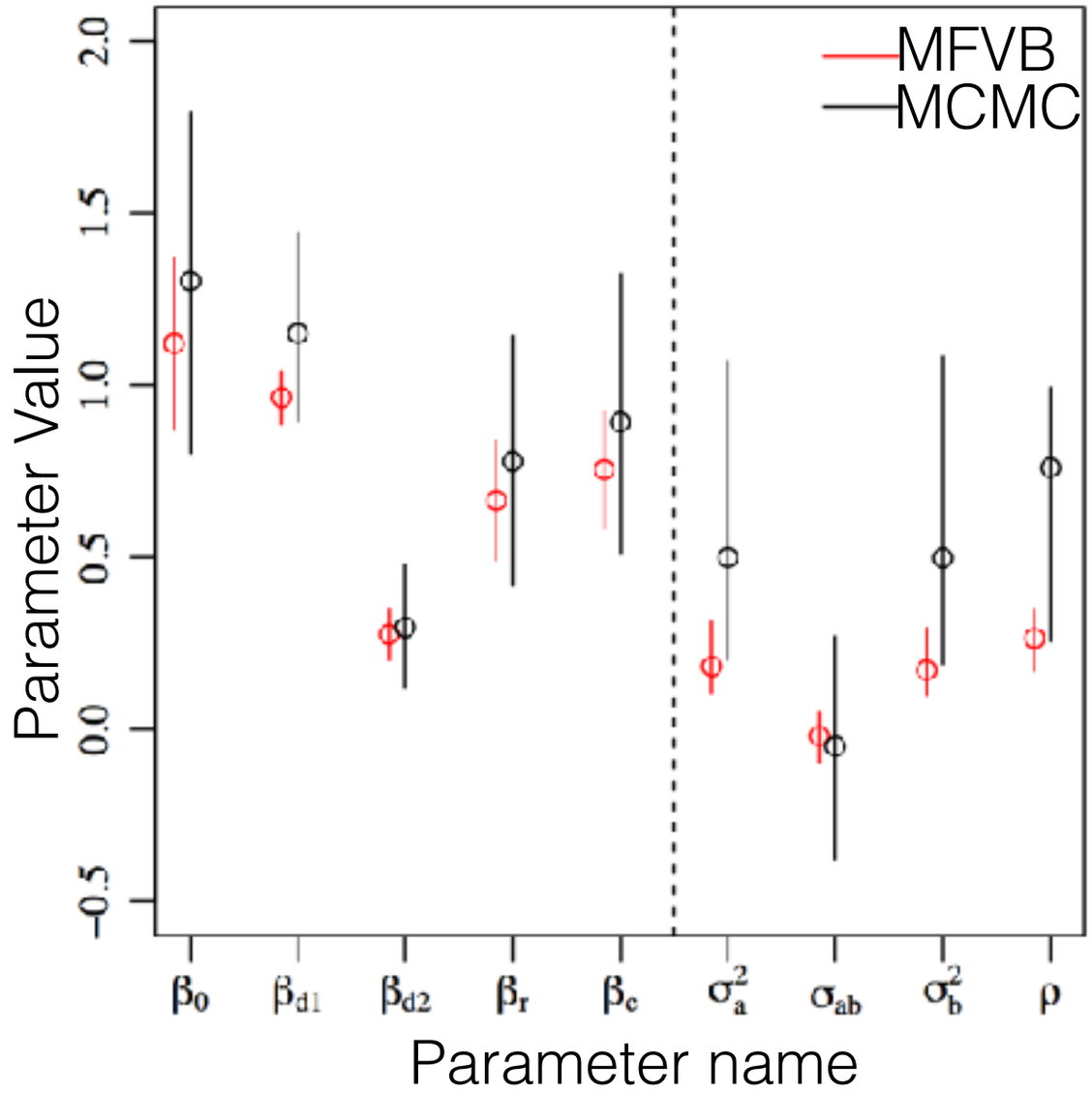

[Fosdick 2013, Ch 4, Fig 4.3]

Posterior means: revisited

- Want to predict college GPA y_n

[Giordano, Broderick, Jordan 2015]

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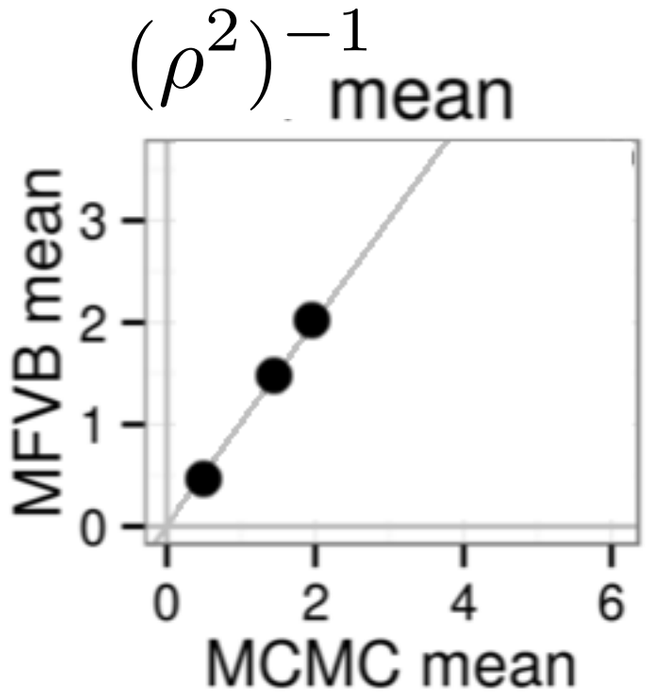

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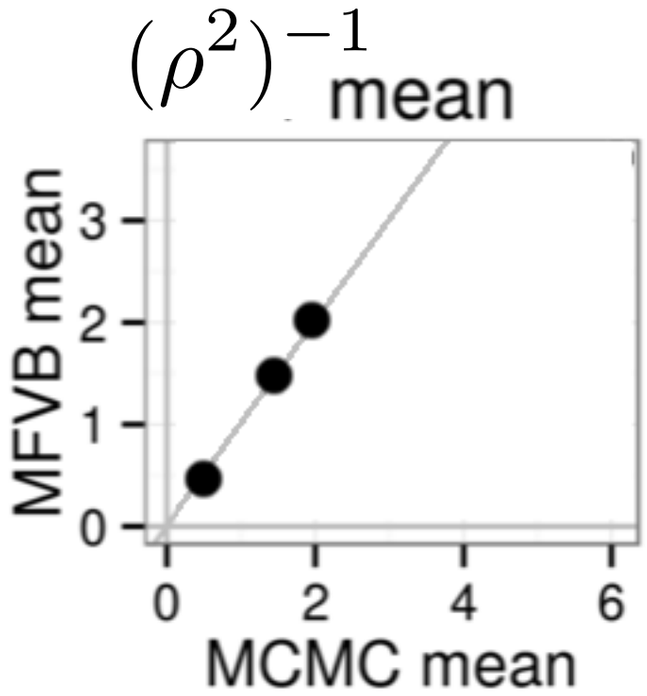

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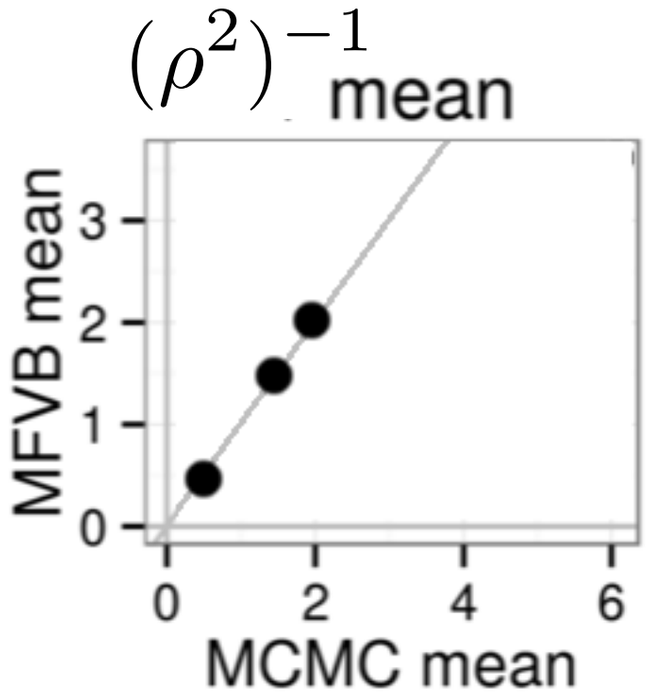

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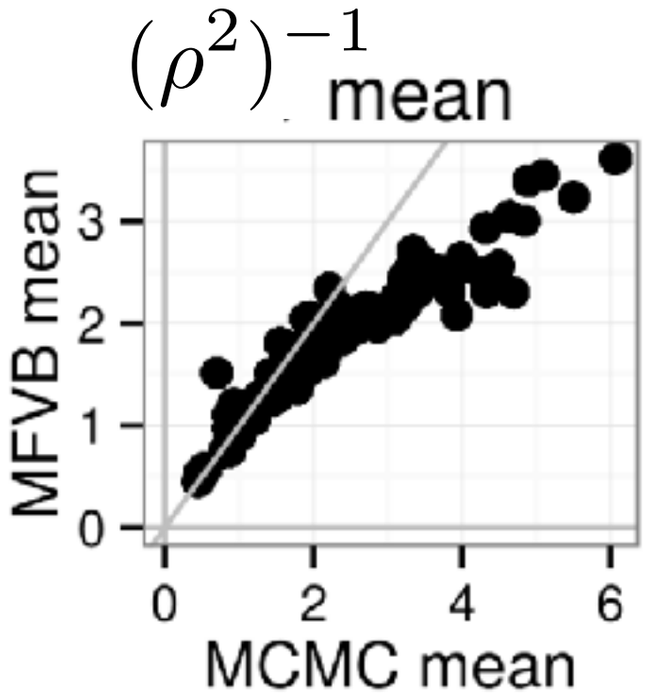

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Approximate Bayesian inference

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \text{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \text{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

Mean-field variational Bayes

$$q^* = \text{argmin}_{q \in Q_{\text{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

How deep is the issue?

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Algorithm

Implementation

How deep is the issue?

What to read next

Textbooks and Reviews

- Bishop. *Pattern Recognition and Machine Learning*, Ch 10. 2006.
- Blei, Kucukelbir, McAuliffe. Variational inference: A review for statisticians, *JASA* 2016.
- MacKay. *Information Theory, Inference, and Learning Algorithms*, Ch 33. 2003.
- Murphy. *Machine Learning: A Probabilistic Perspective*, Ch 21. 2012.
- Ormerod, Wand. Explaining variational approximations. *Amer Stat* 2010.
- Turner, Sahani. Two problems with variational expectation maximisation for time-series models. In *Bayesian Time Series Models*, 2011.
- Wainwright, Jordan. Graphical models, exponential families, and variational inference. *Foundations and Trends in Machine Learning*, 2008.

Our Experiments

- RJ Giordano, T Broderick, and MI Jordan. Linear response methods for accurate covariance estimates from mean field variational Bayes. *NeurIPS* 2015.
- RJ Giordano, T Broderick, R Meager, JH Huggins, and MI Jordan. Fast robustness quantification with variational Bayes. *ICML Data4Good Workshop* 2016.
- RJ Giordano, T Broderick, and MI Jordan. Covariances, robustness, and variational Bayes. *JMLR* 2018.
- J Huggins, M Kasprzak, T Campbell, T Broderick. Validated Variational Inference via Practical Posterior Error Bounds. ArXiv: 1910.04102. *AISTATS* 2020, to appear.
- T Campbell and T Broderick. Automated scalable Bayesian inference via Hilbert coresets. *JMLR* 2019.
- T Campbell and T Broderick. Bayesian Coreset Construction via Greedy Iterative Geodesic Ascent. *ICML* 2018.

References

Full references at end of final slides