MIT EECS ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

MIT STATISTICS + DATA SCIENCE CENTER

CSAIL

Variational Bayes and beyond: Bayesian inference for big data

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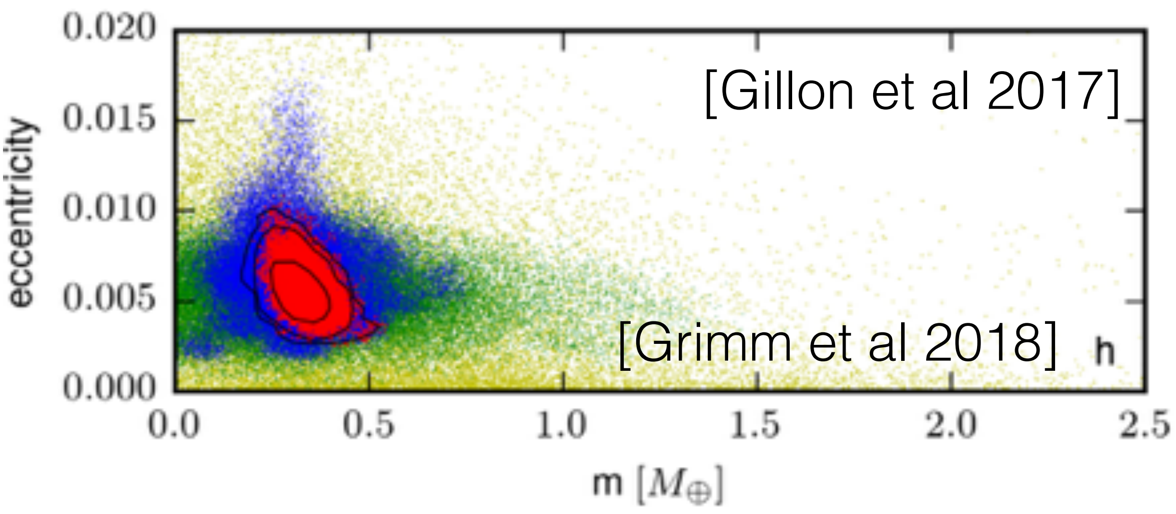
ITT Career Development
Assistant Professor,
MIT

http://www.tamarabroderick.com/tutorials.html

Bayesian inference

Bayesian inference



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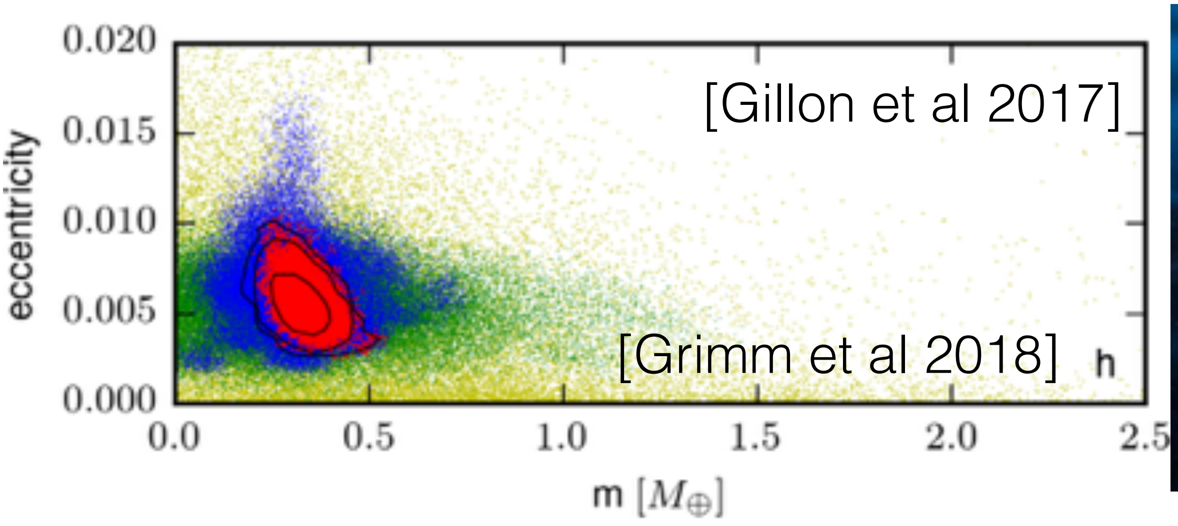

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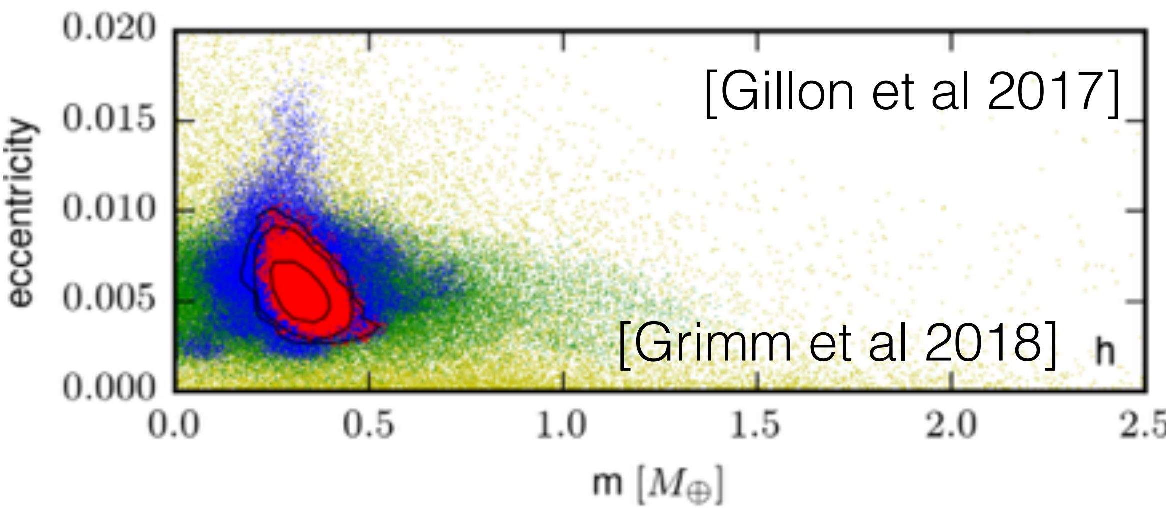

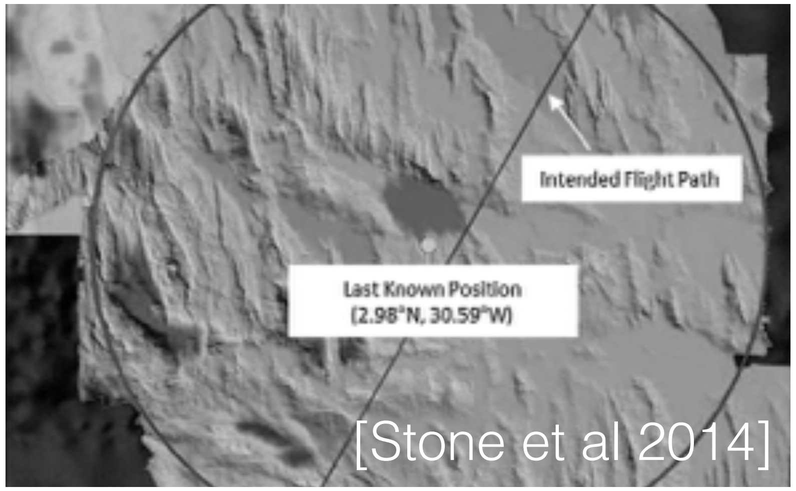

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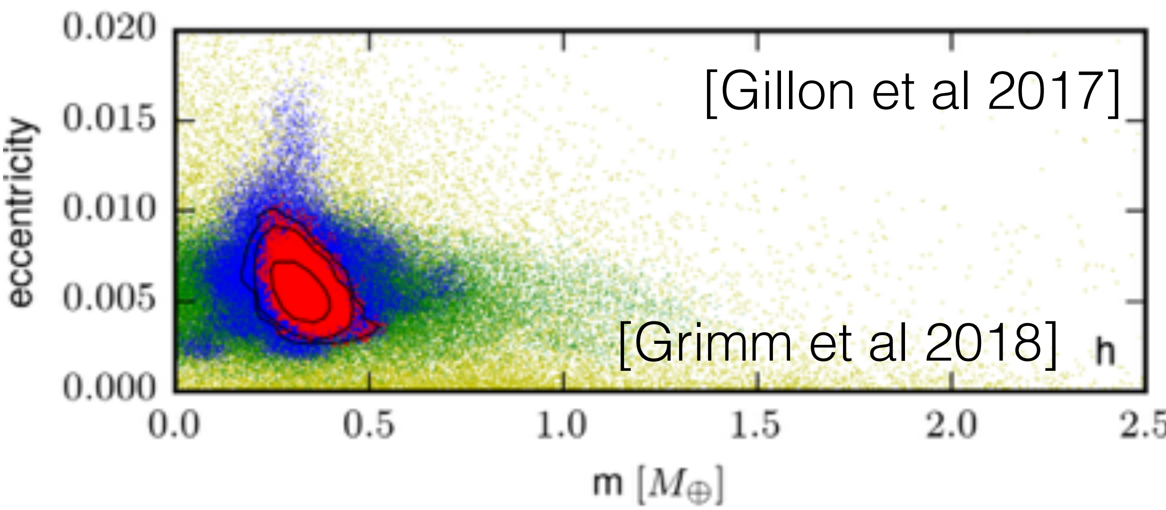

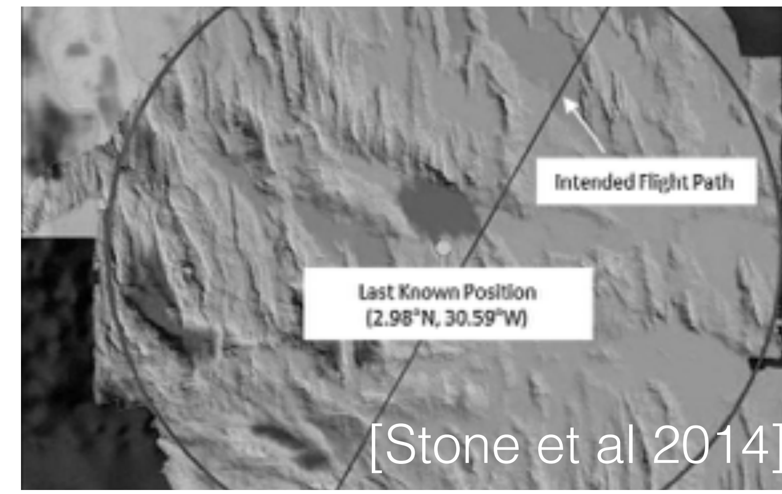

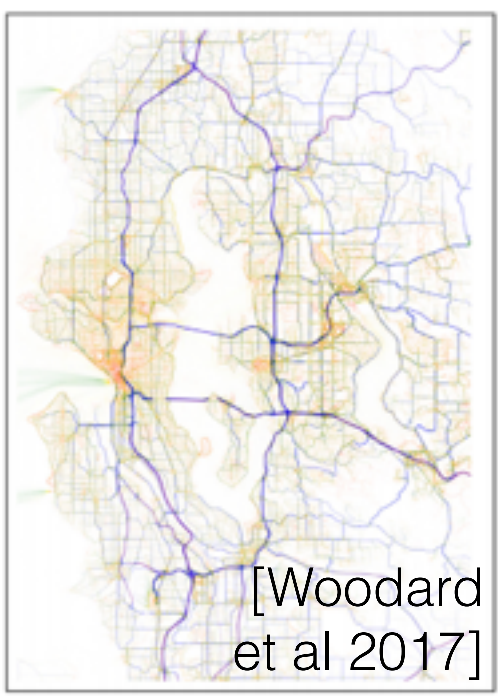

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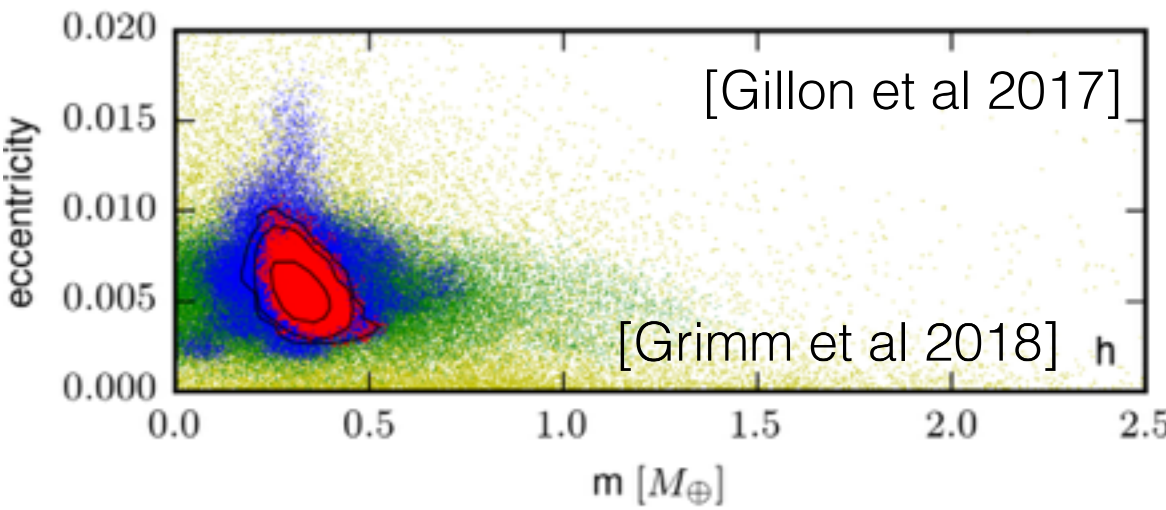

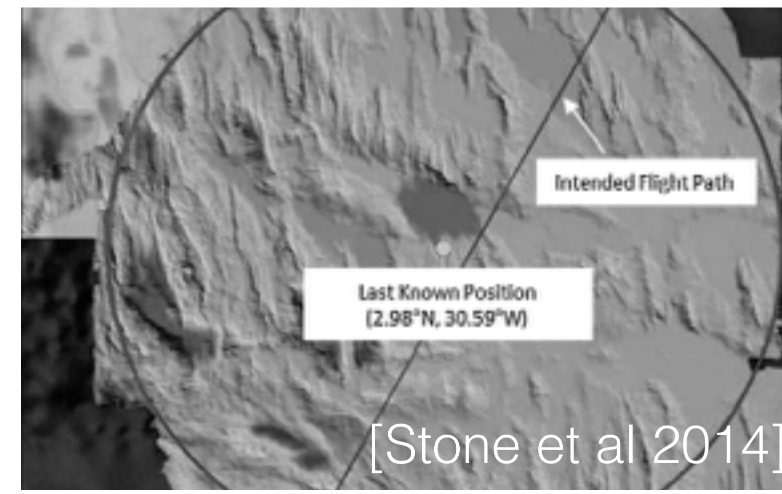

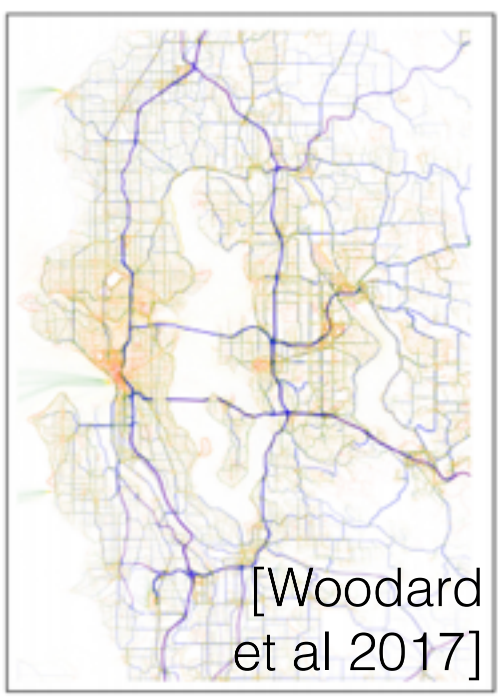

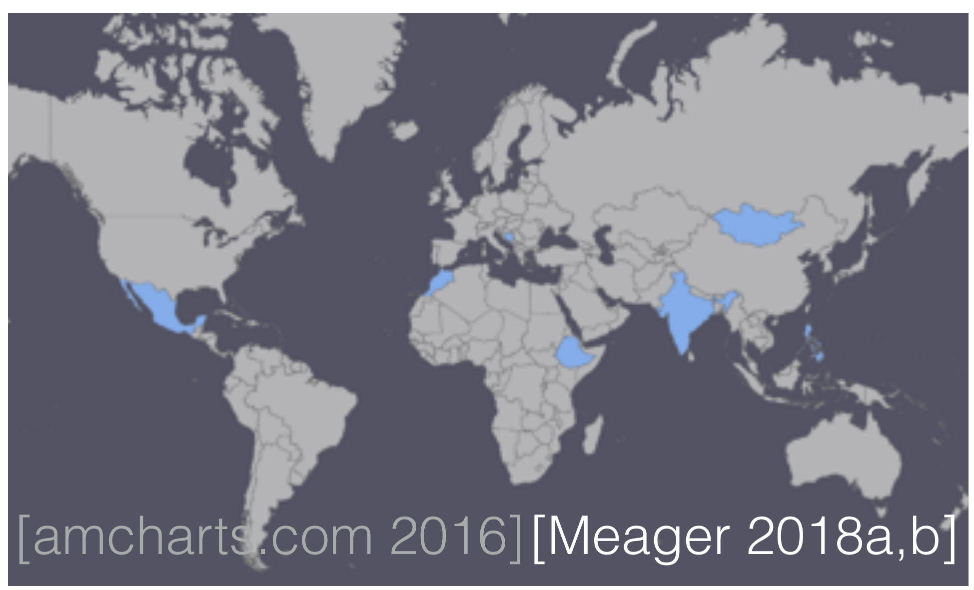

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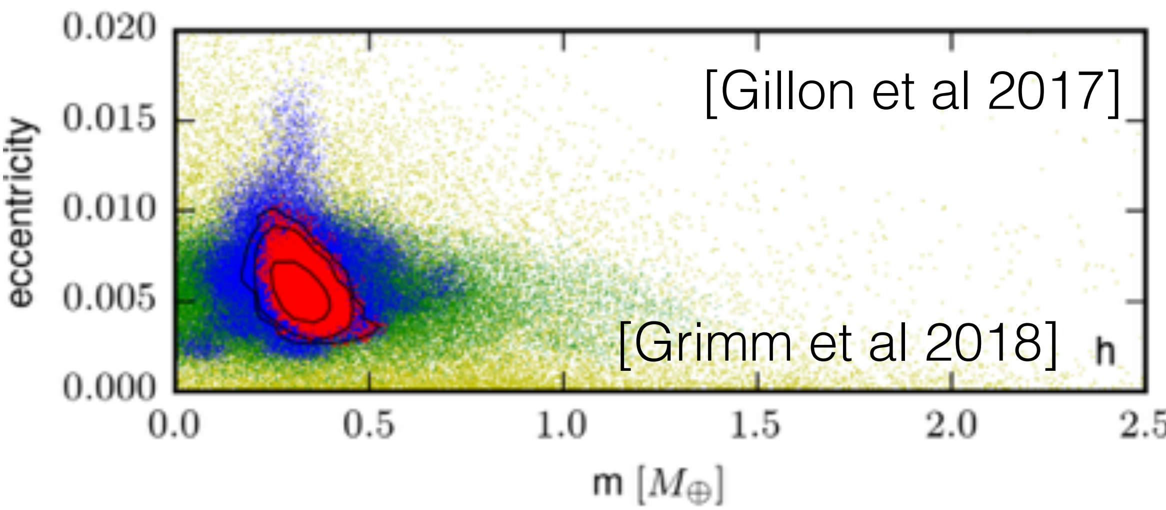
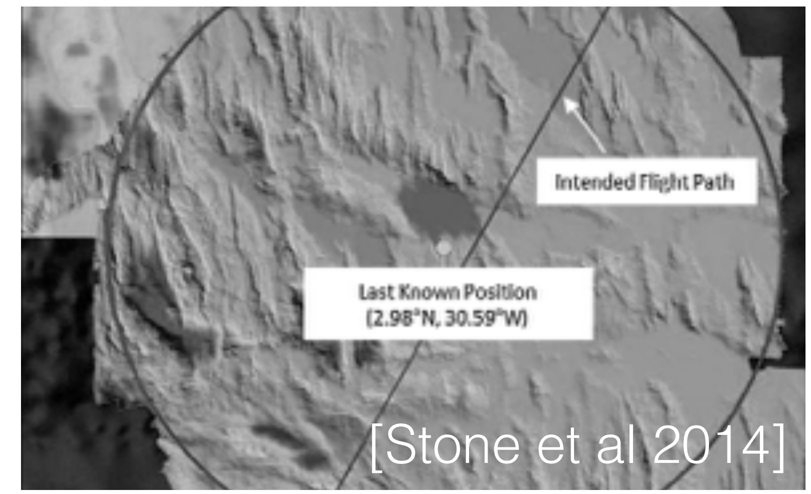
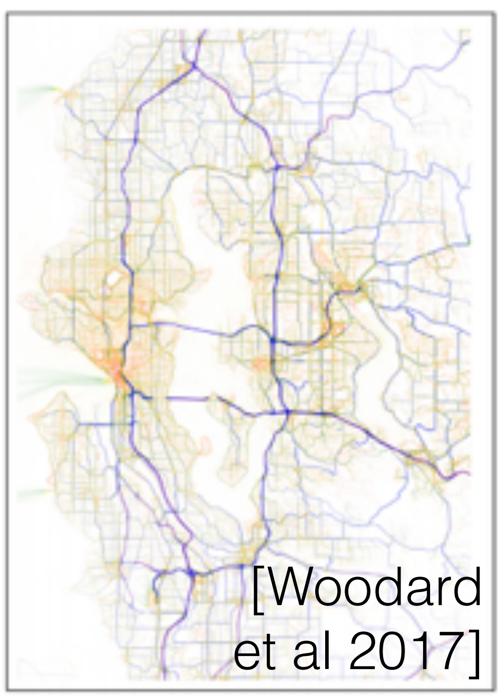


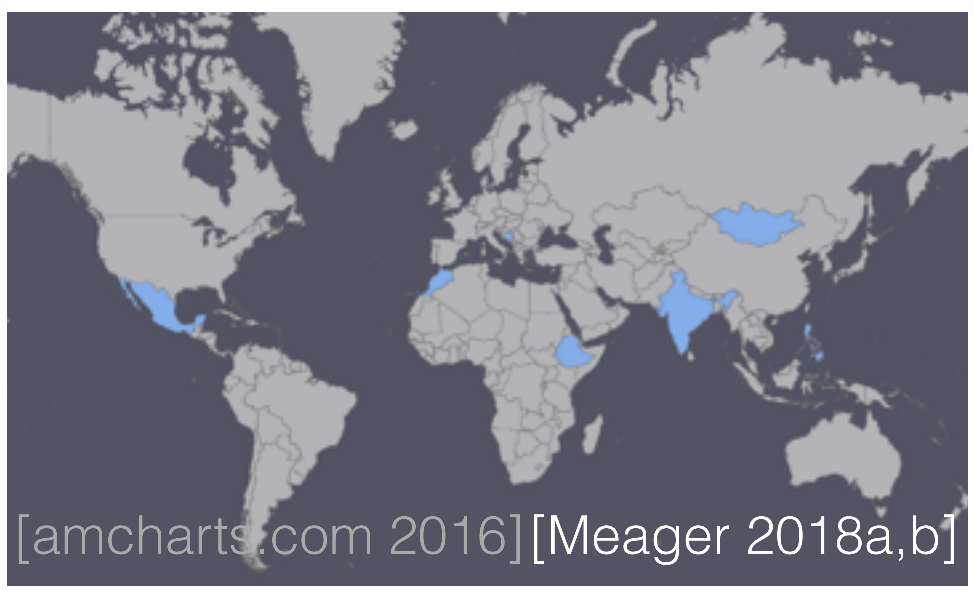
[Gillon et al 2017]
[Grimm et al 2018]


[ESO/
L. Calçada/
M. Kornmesser
2017]
[Abbott et al 2016a,b]



[Stone et al 2014]


[Woodard
et al 2017]


[amcharts.com 2016]
[Meager 2018a,b]


[Chati, Balakrishnan
2017]
[Julian Hertzog 2016]

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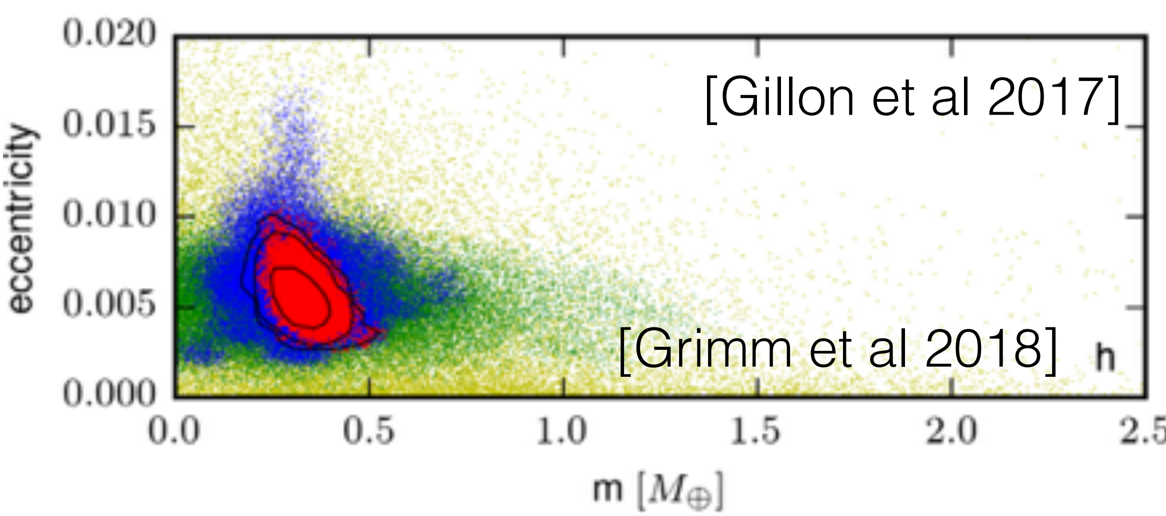
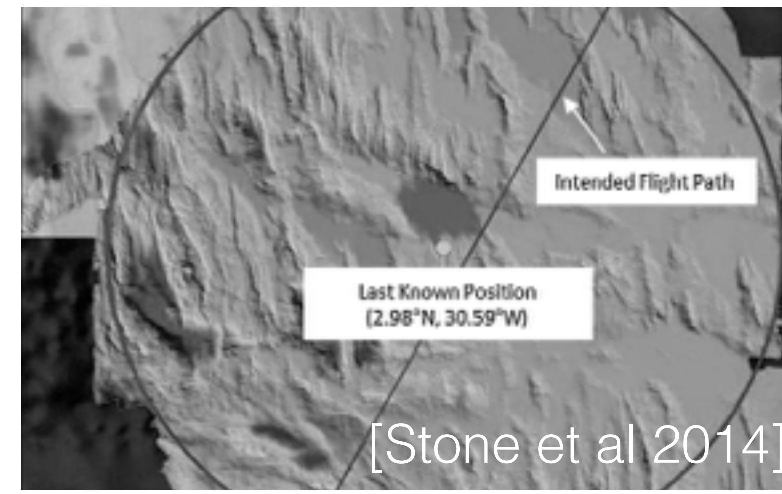
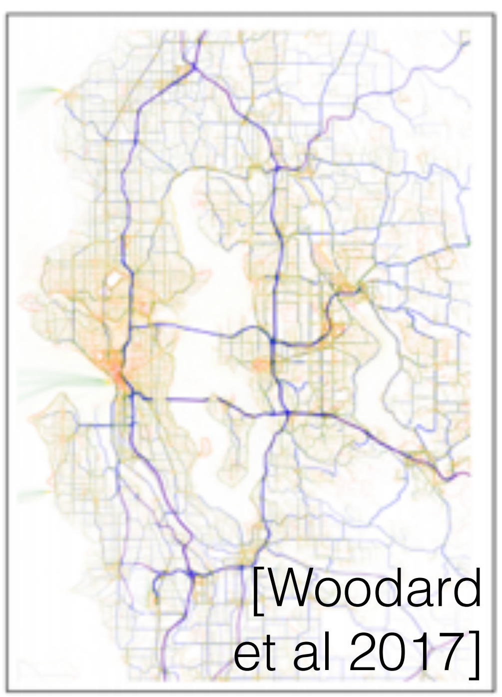
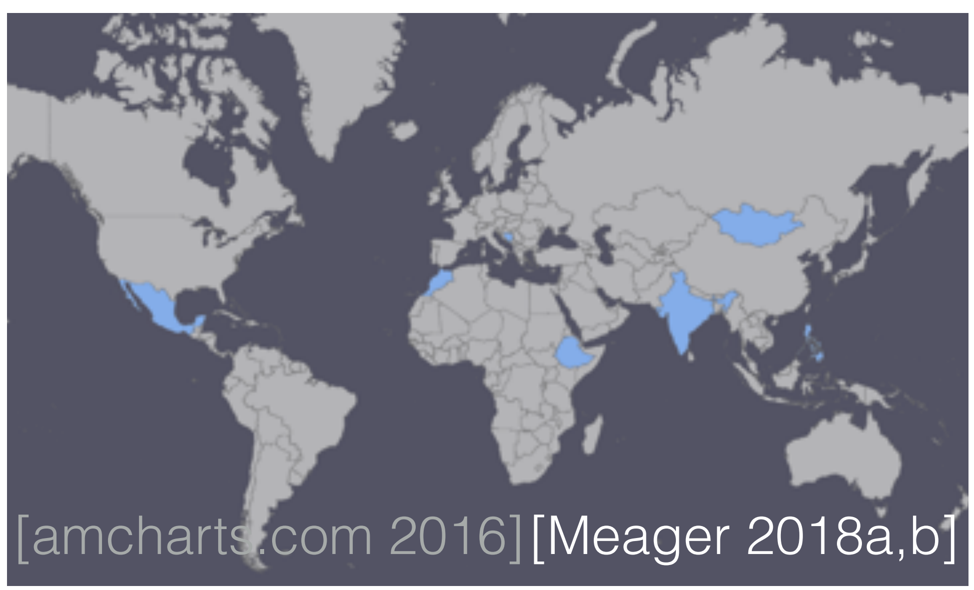


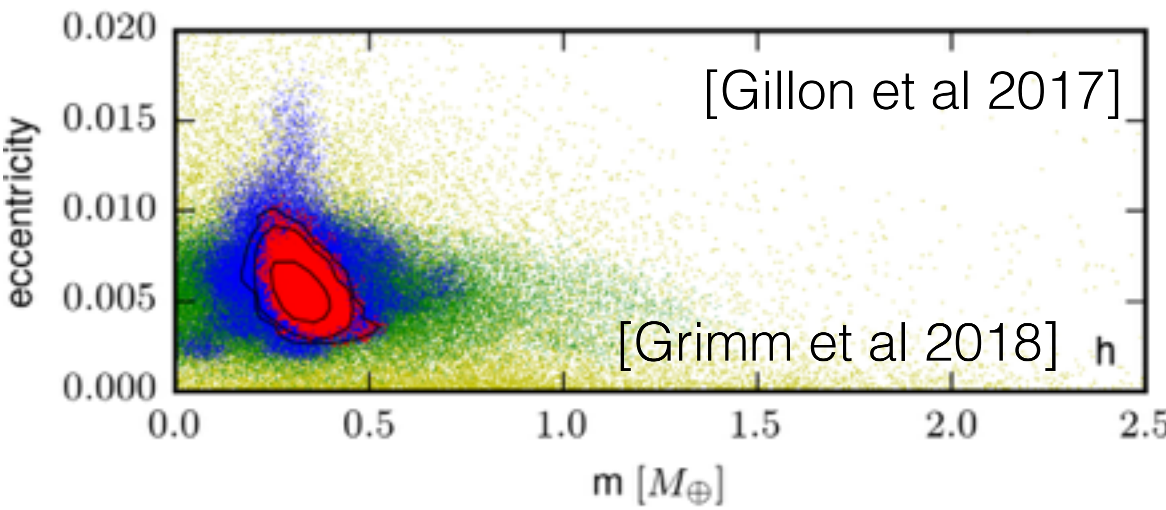


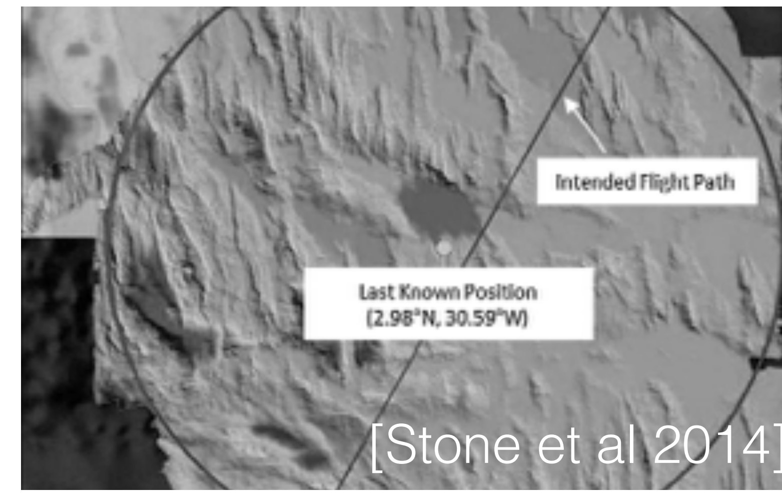


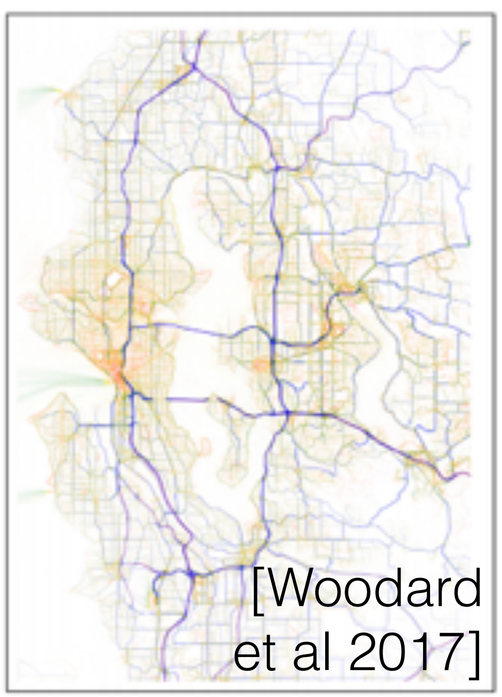


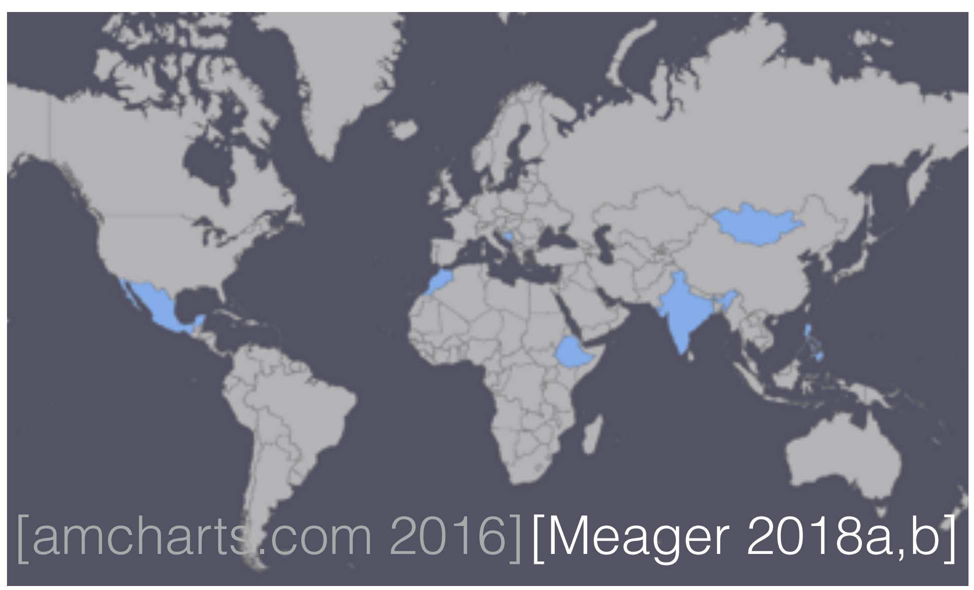
Bayesian inference

- Analysis goals: Point estimates, coherent uncertainties
  - Interpretable, complex, modular; expert information









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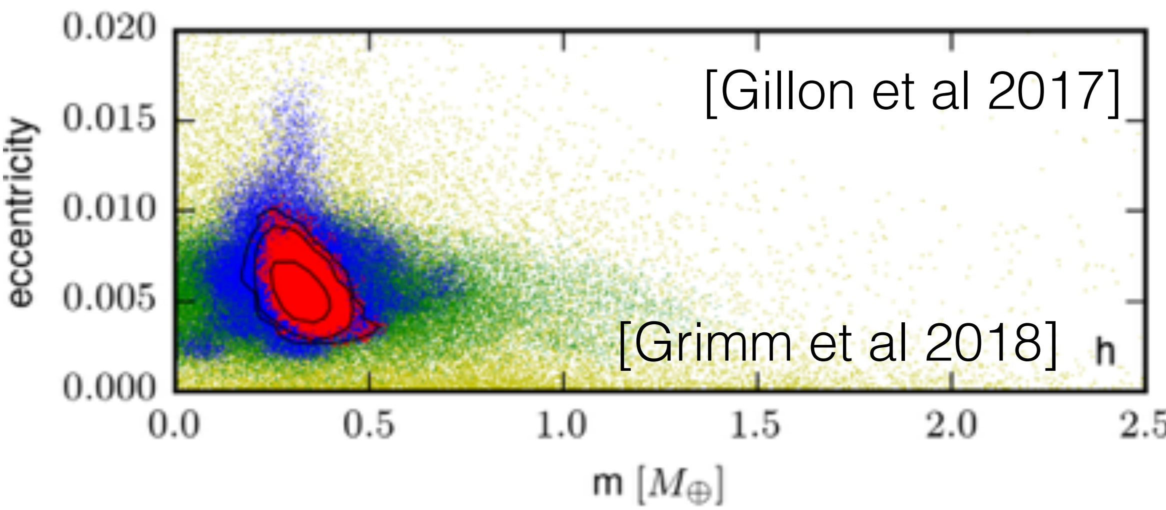

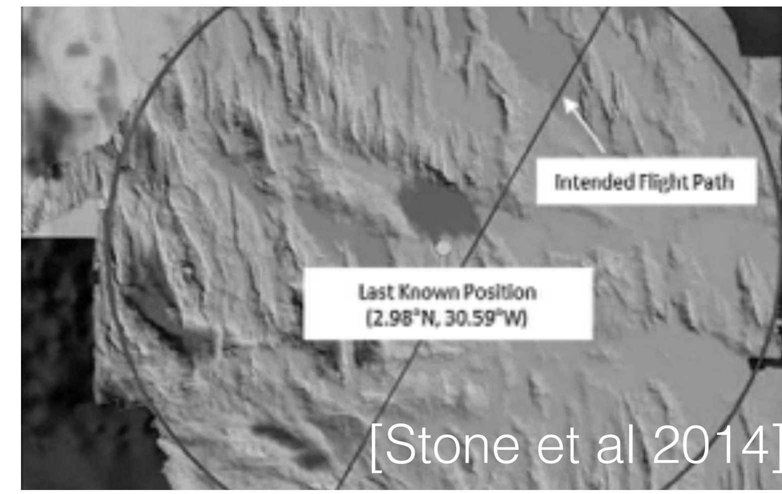

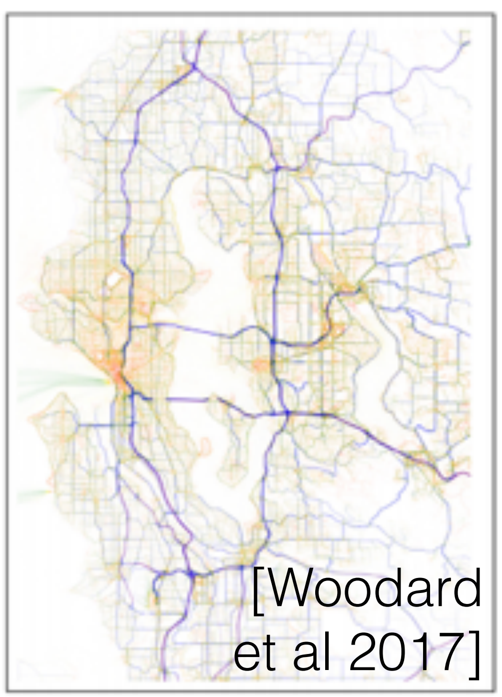

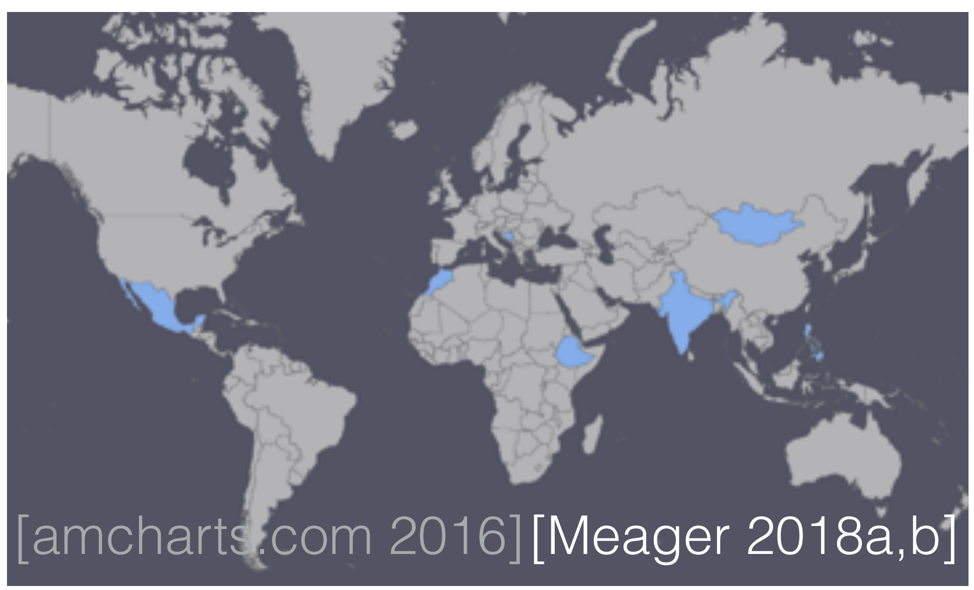

[mc-stan.org]

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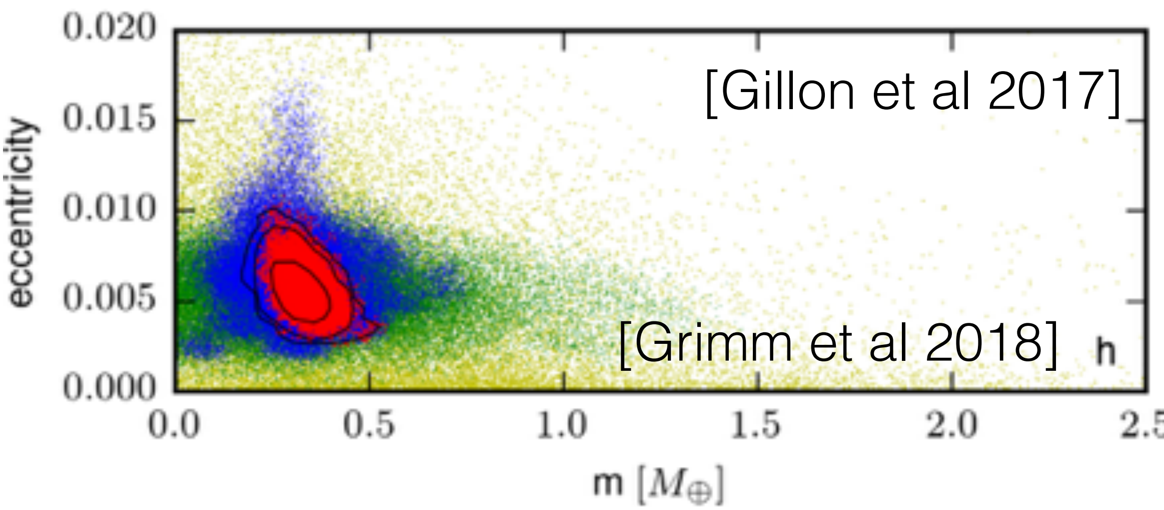

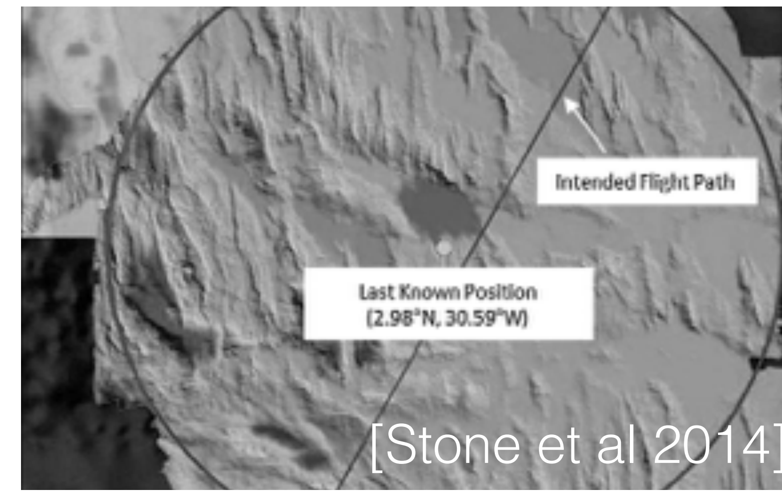

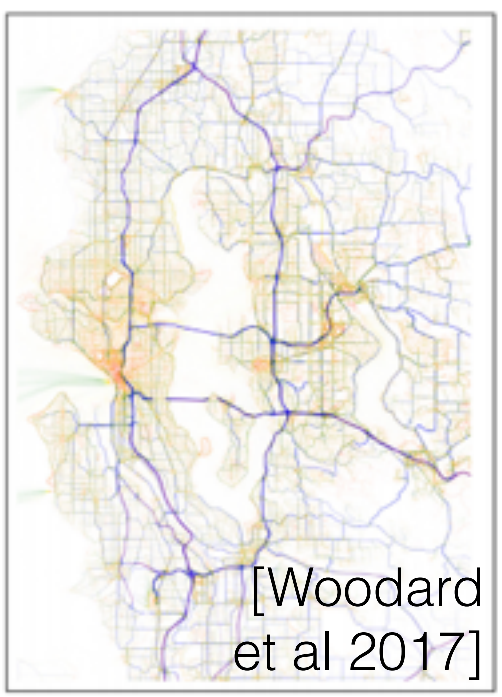

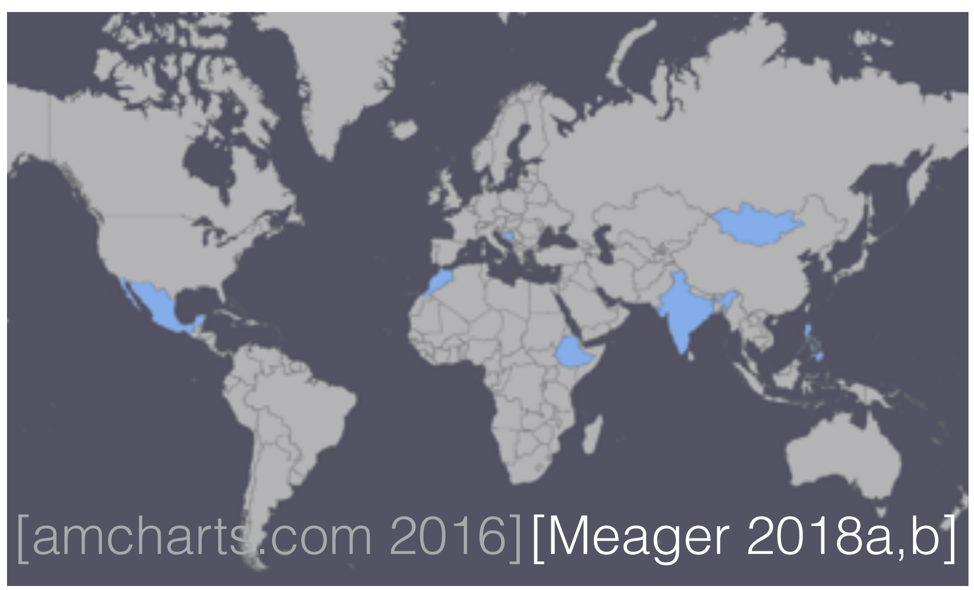

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- Challenge: fast (compute, user), reliable inference

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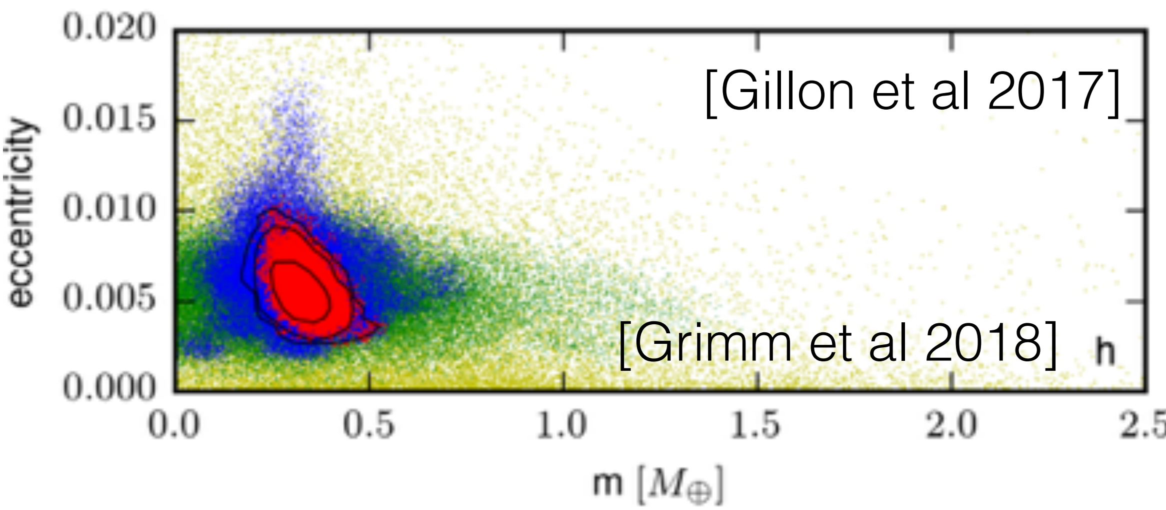

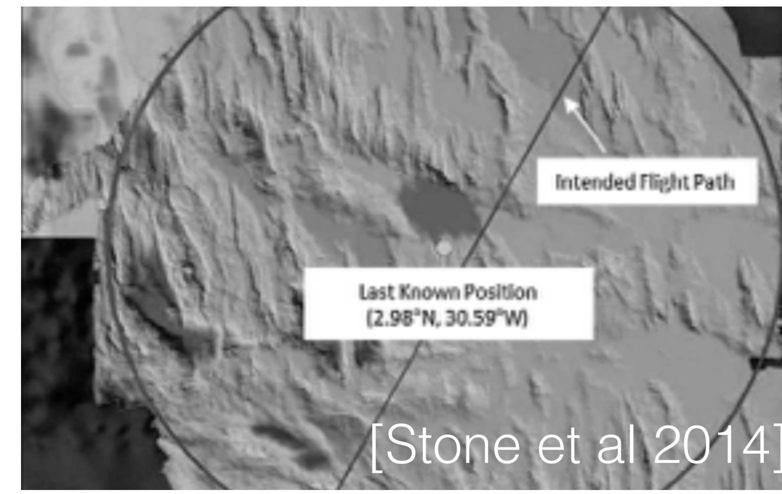

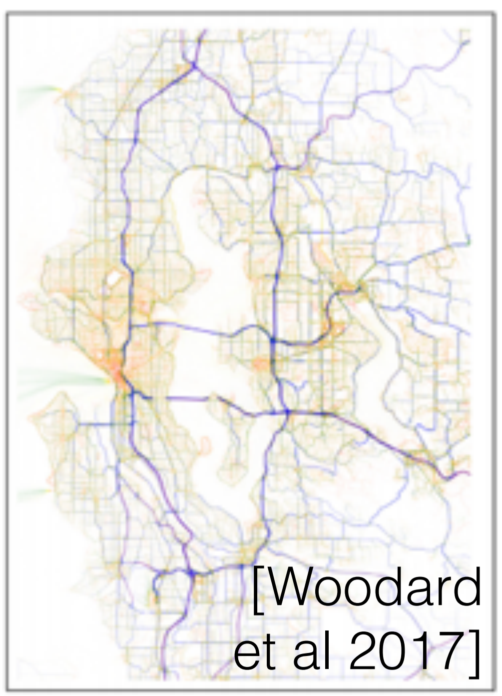

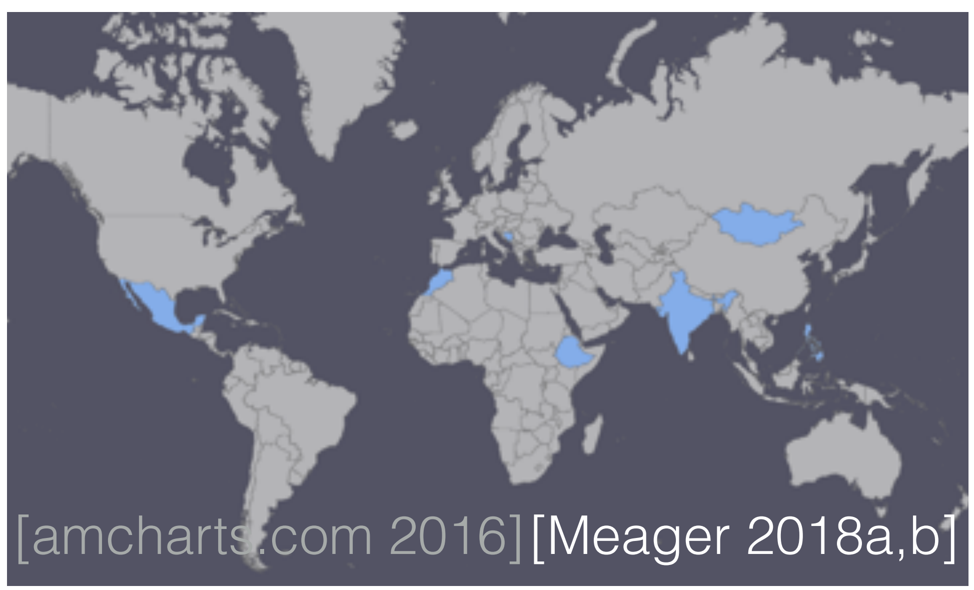

[mc-stan.org]

- Challenge: fast (compute, user), reliable inference
- Uncertainty doesn't have to disappear in large data sets

Variational Bayes

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- Modern problems: often large data, large dimensions

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[Blei et al 2003]

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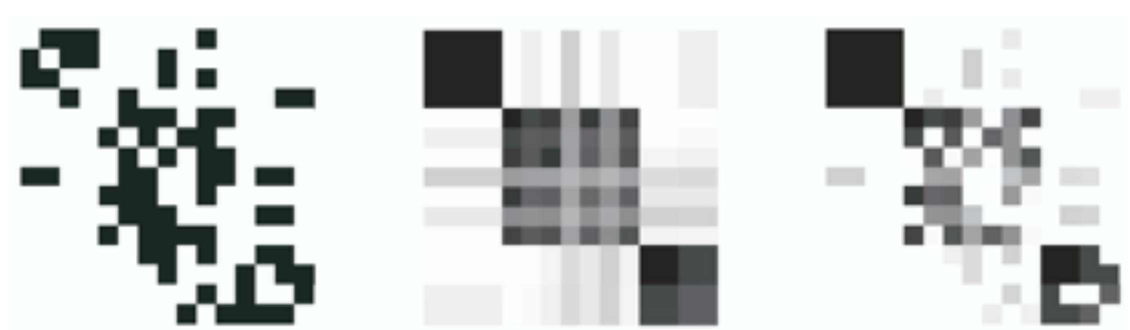

[Airoldi et al 2008]

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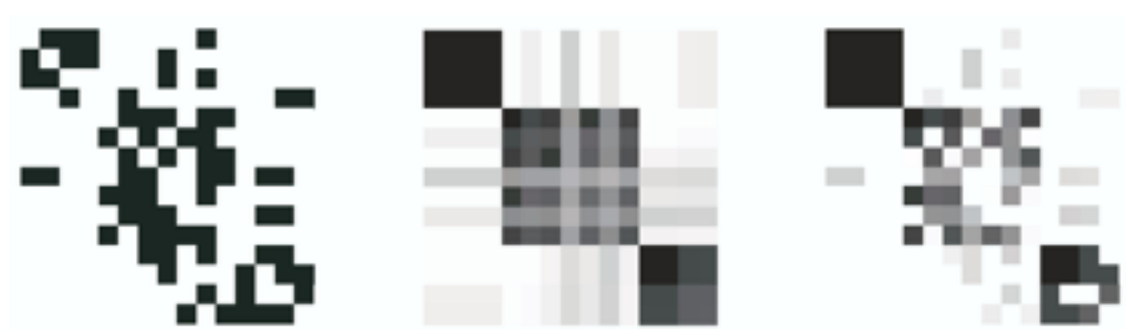

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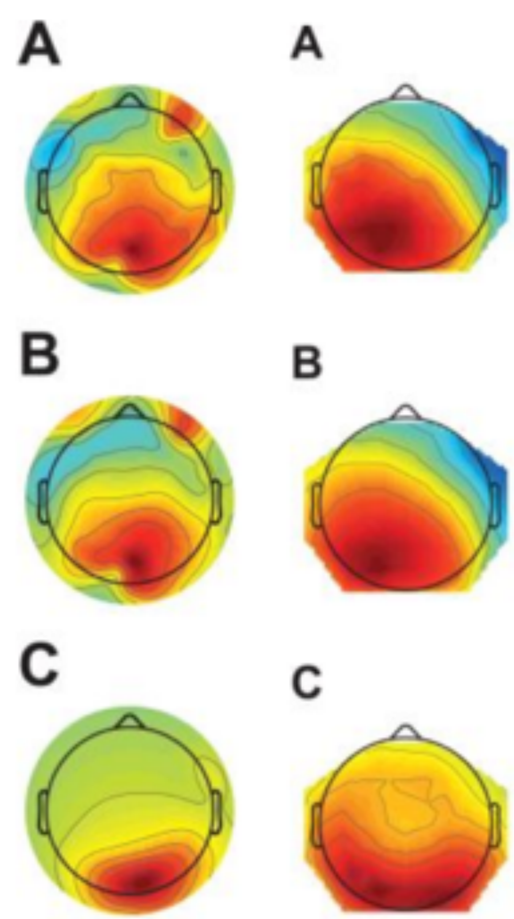

[Gershman et al 2014]

[Blei et al 2018]

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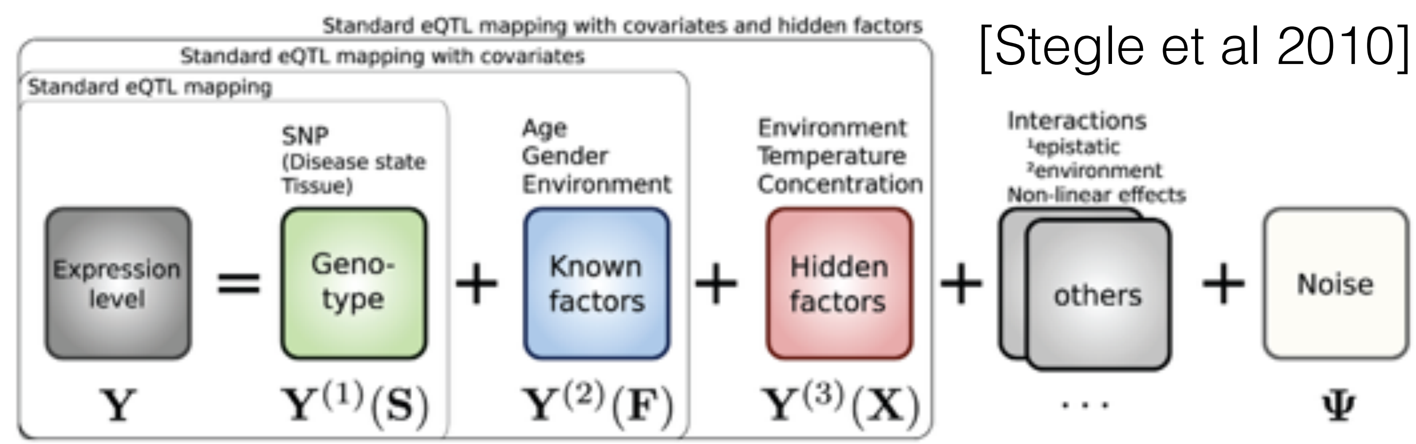
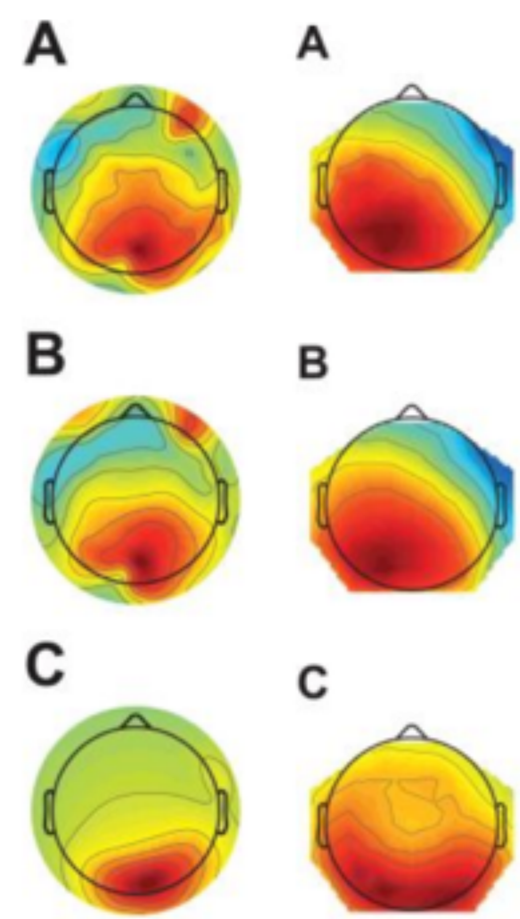
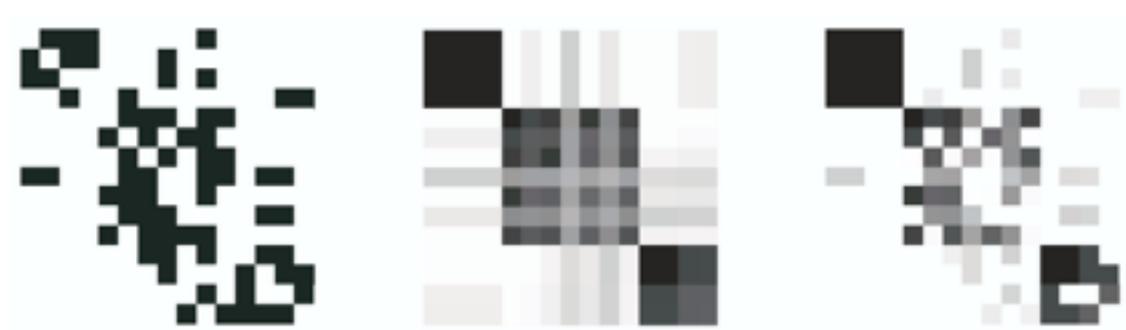


[Stegle et al 2010]





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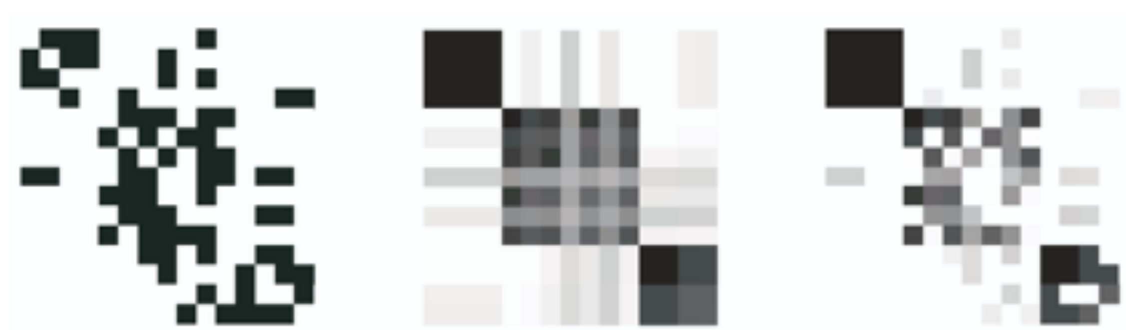

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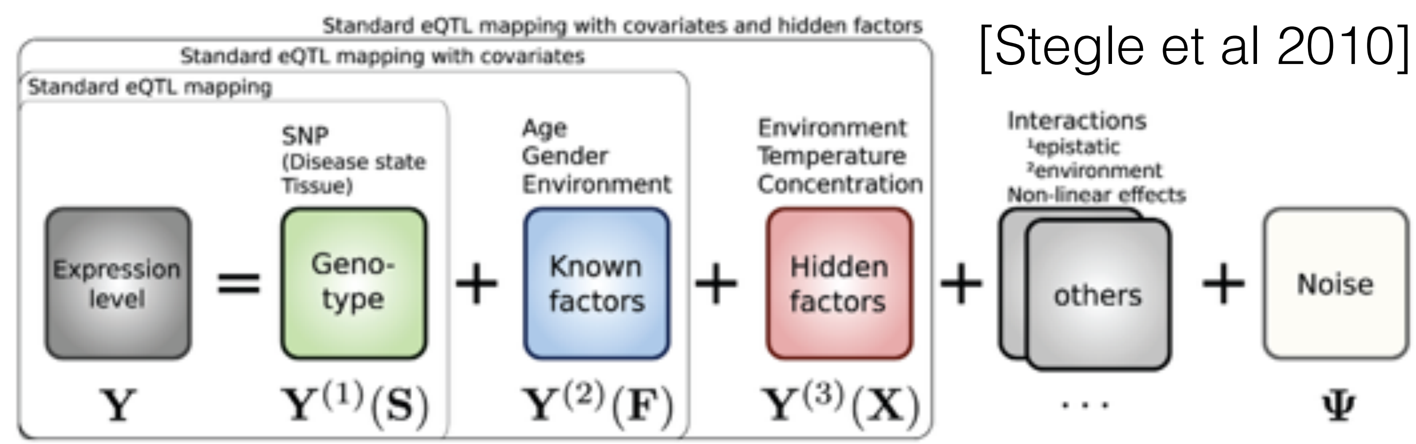

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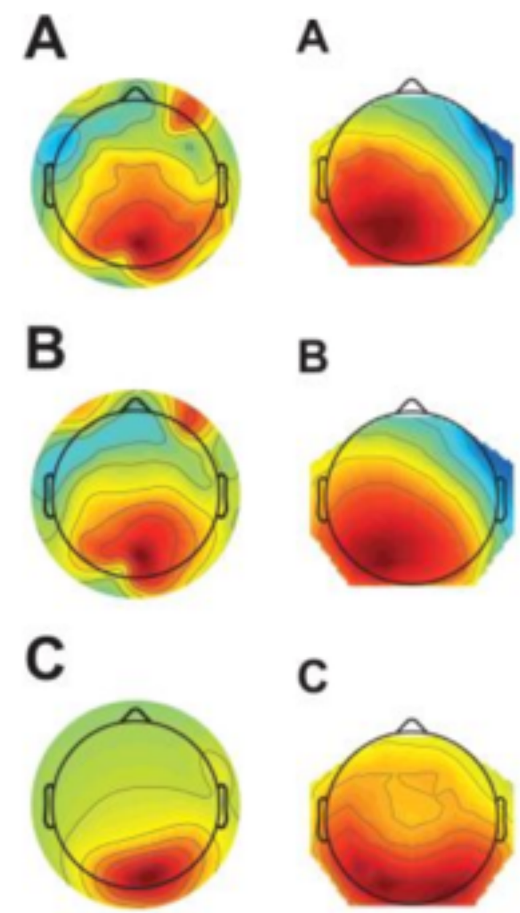

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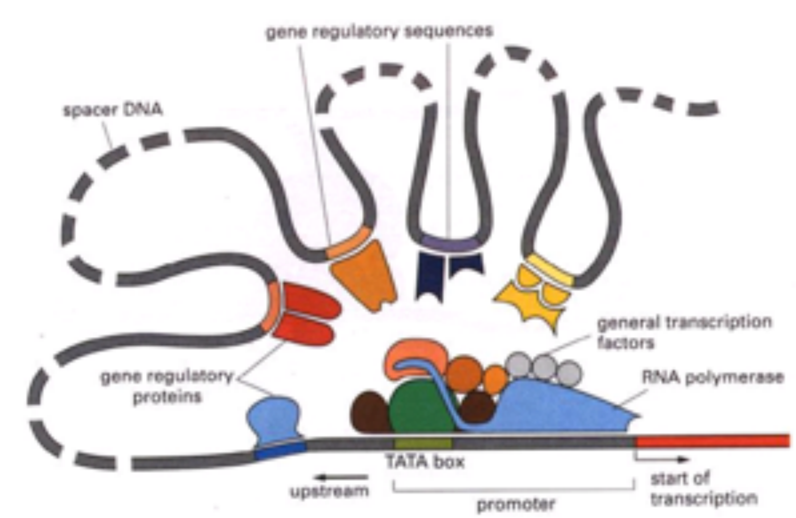

[Xing et al 2004]
[Xing 2003]

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Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
  - Mean-field variational Bayes (MFVB)
- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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parameters

$$p(\theta)$$

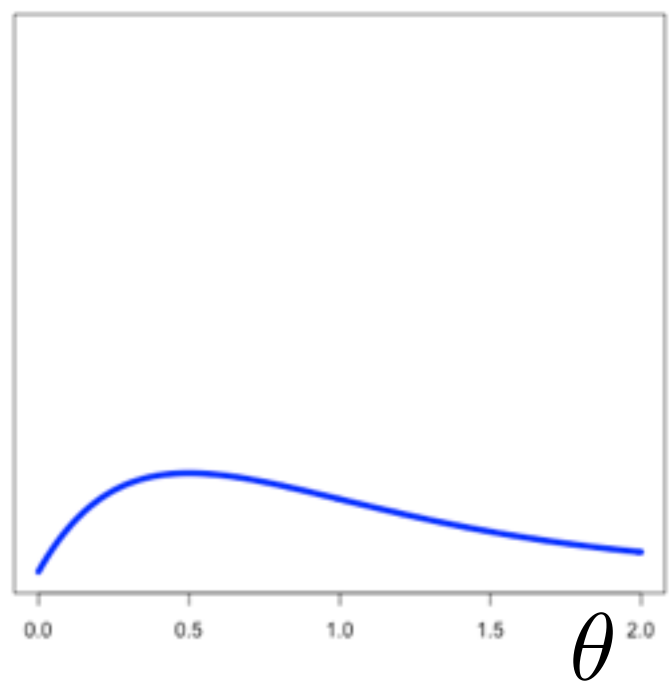
prior

Bayesian inference

parameters

$p(\theta)$

prior

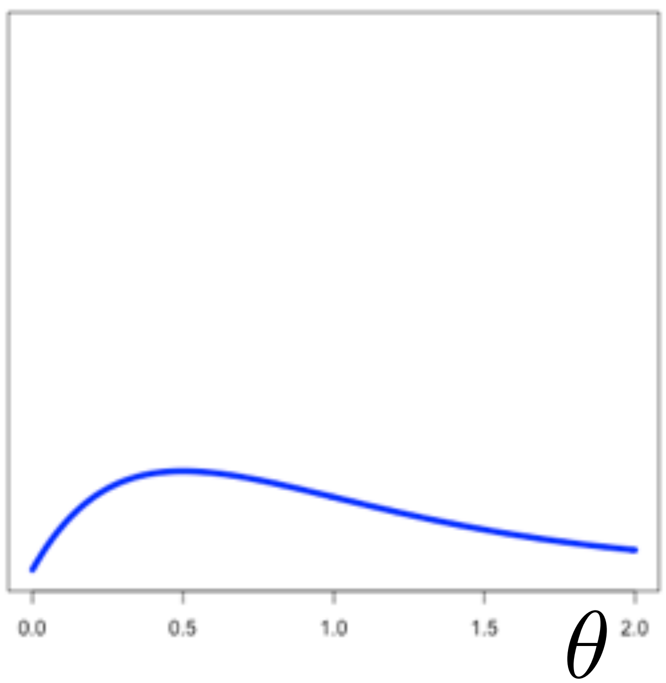


Bayesian inference

parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior

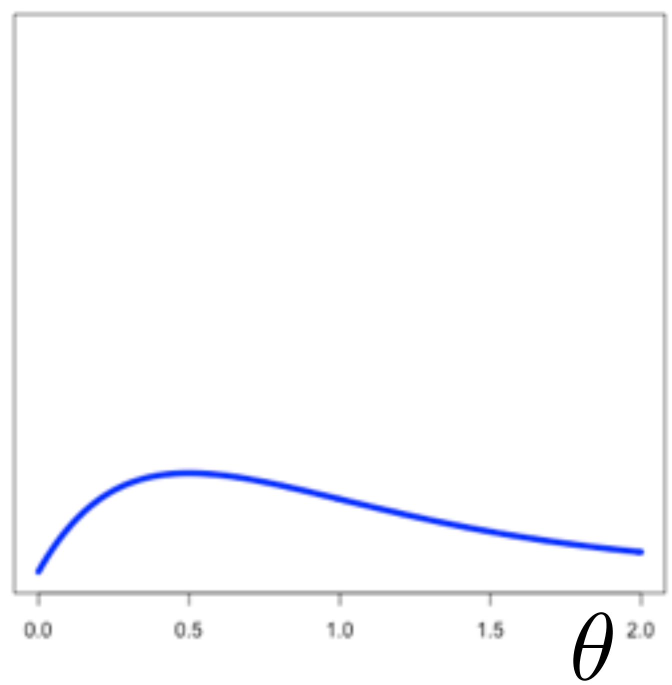


Bayesian inference

data parameters

$$p(y_{1:N}|\theta)p(\theta)$$

likelihood prior

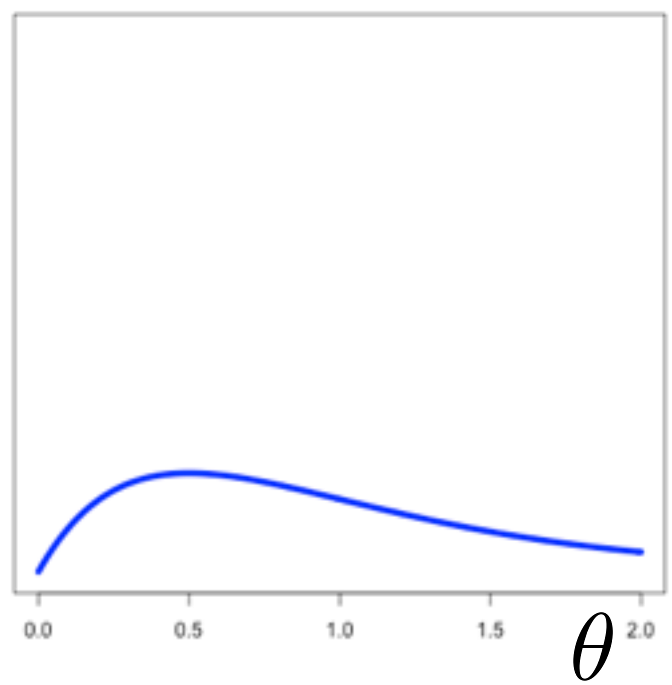


Bayesian inference

data parameters

$$p(\theta|y_{1:N}) \propto_\theta p(y_{1:N}|\theta)p(\theta)$$

posterior likelihood prior



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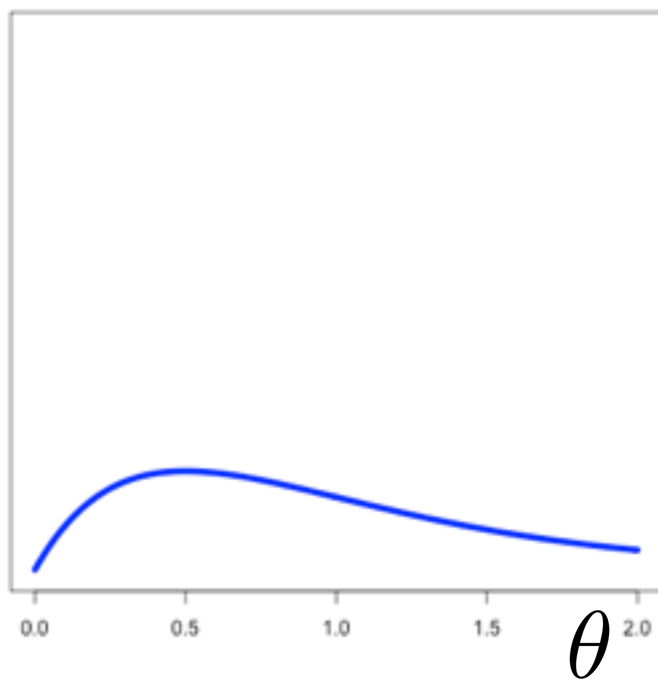

Bayes Theorem

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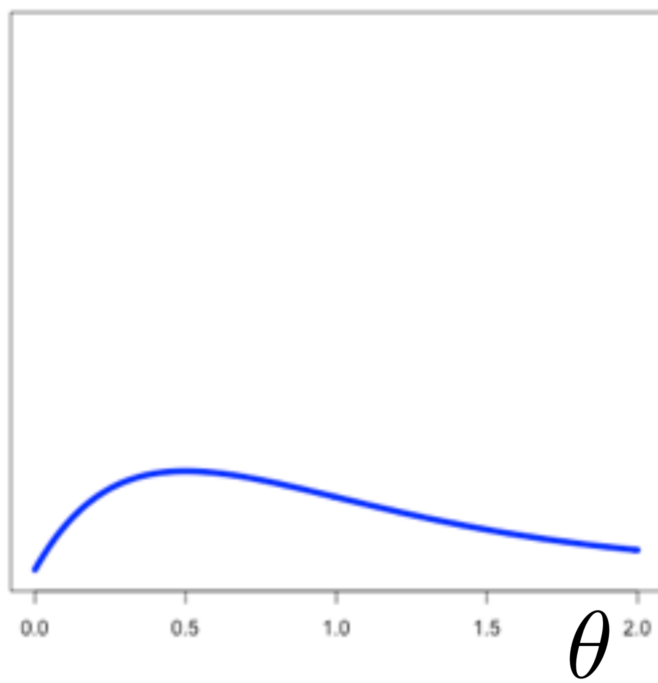

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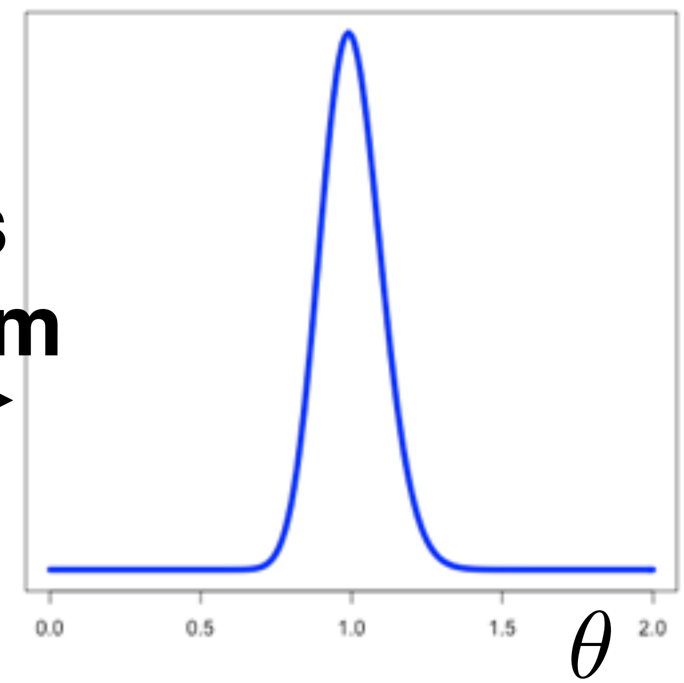

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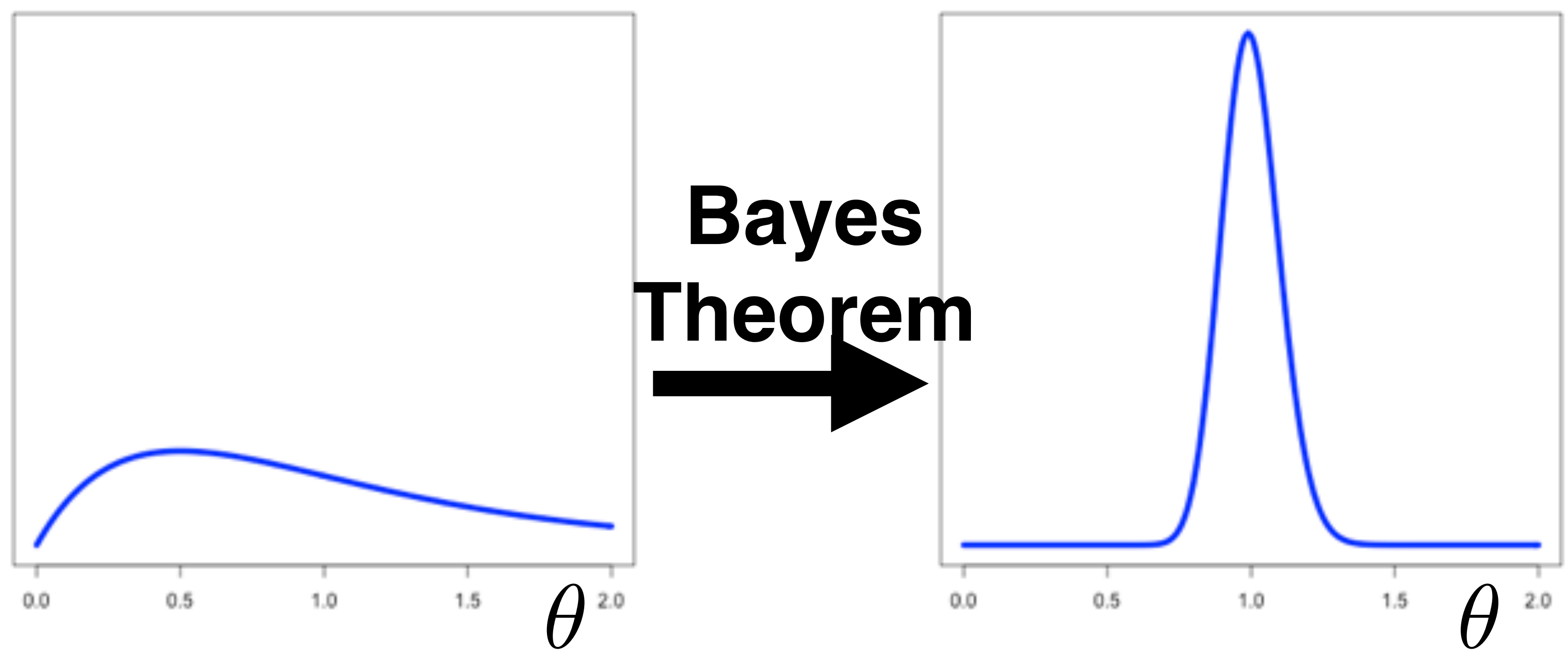

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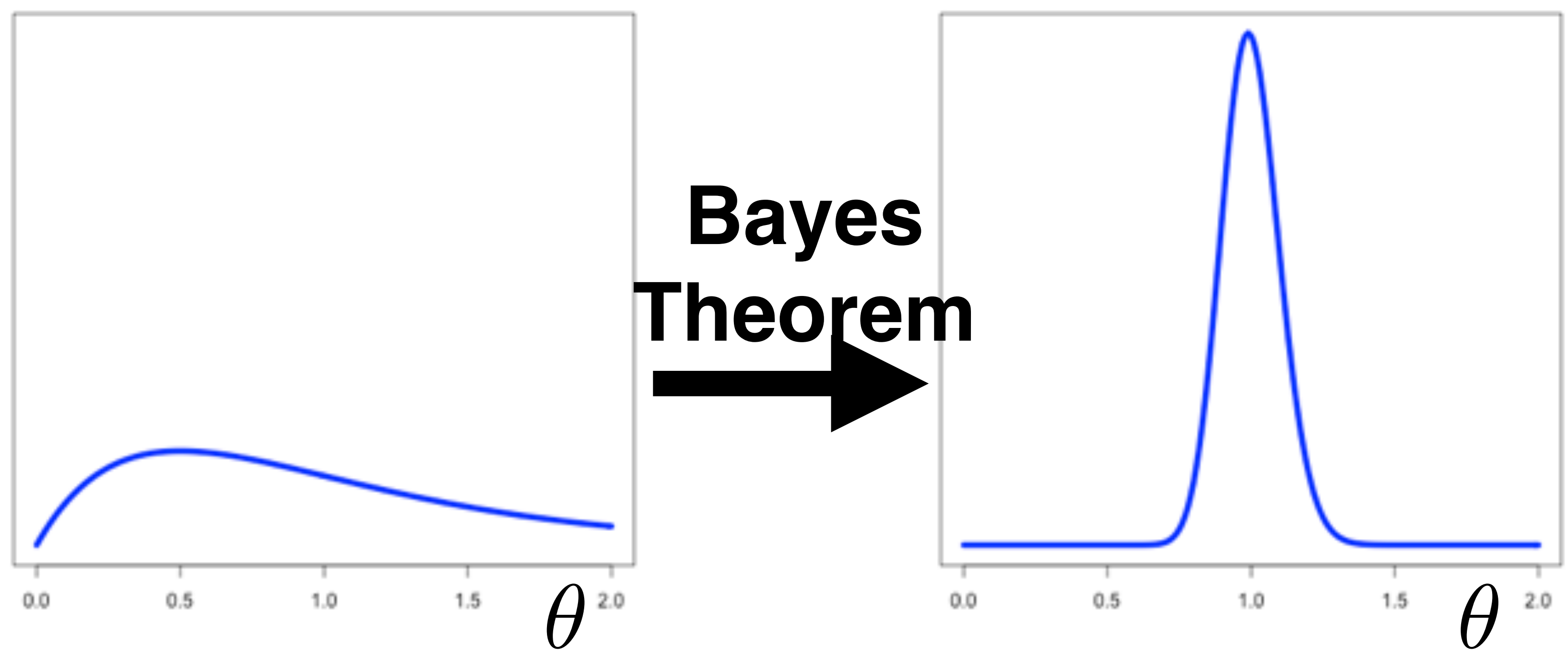

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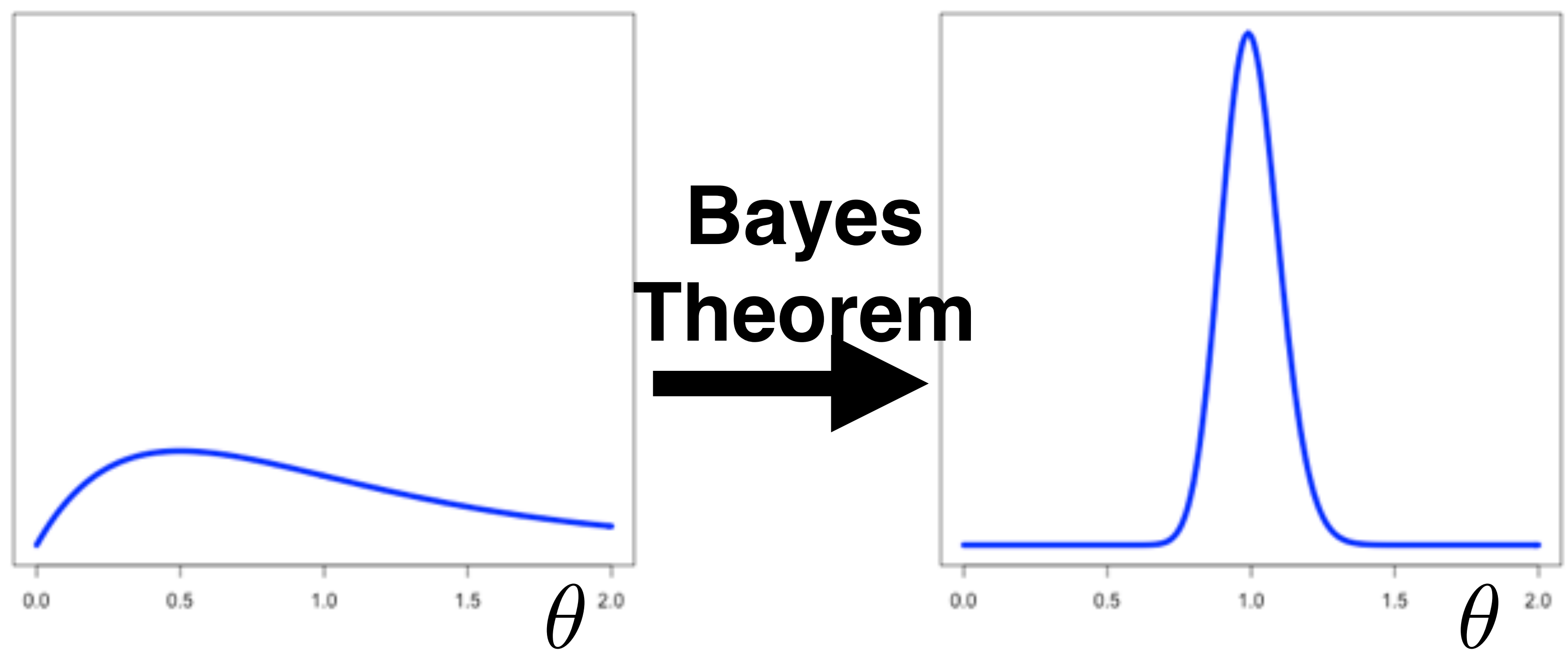

1. Build a model: choose prior & choose likelihood
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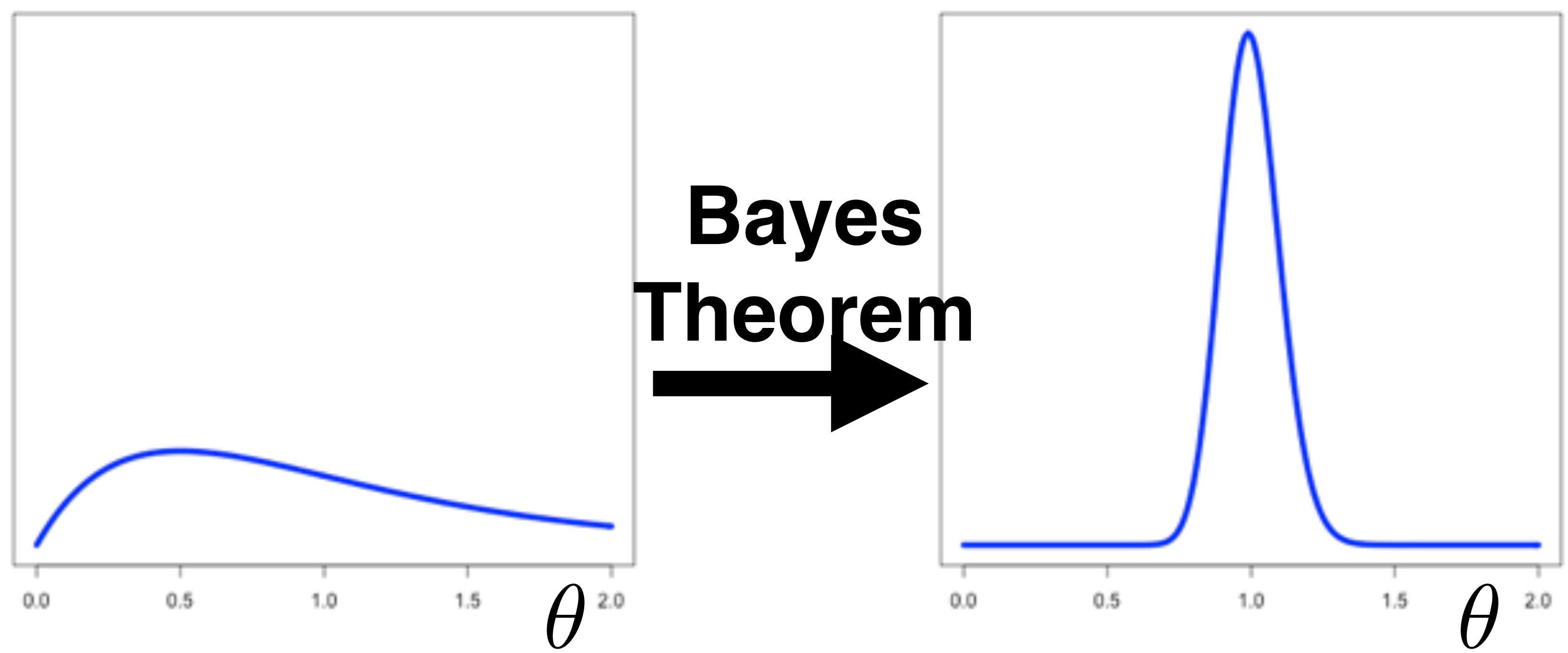

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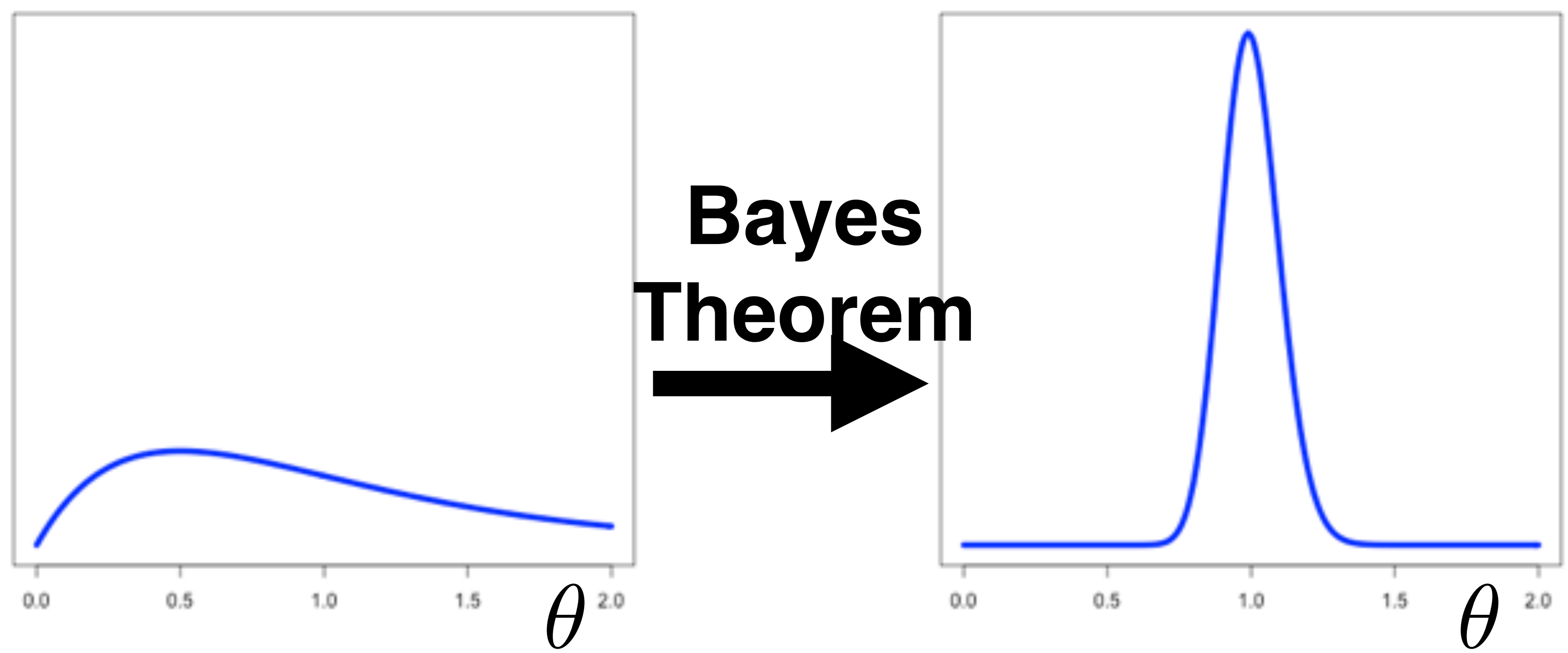

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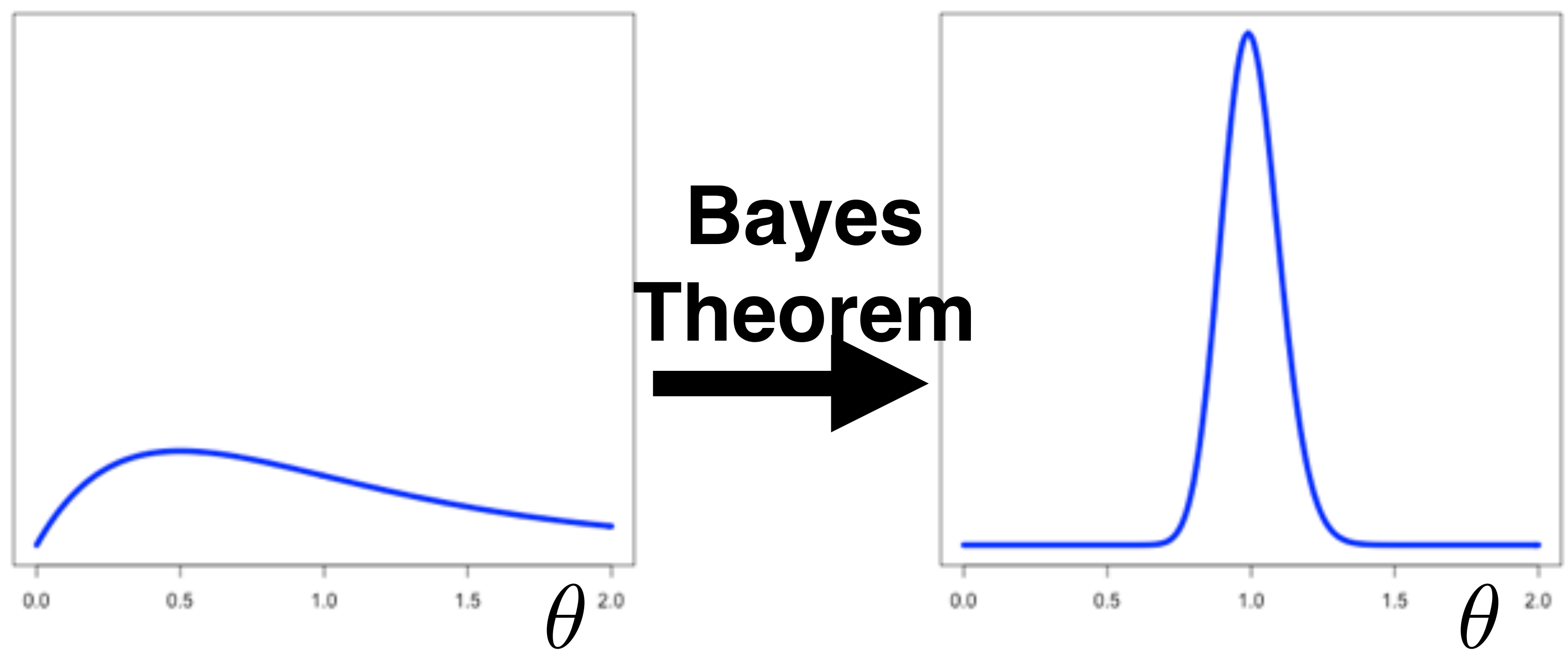

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Bayesian inference

data parameters

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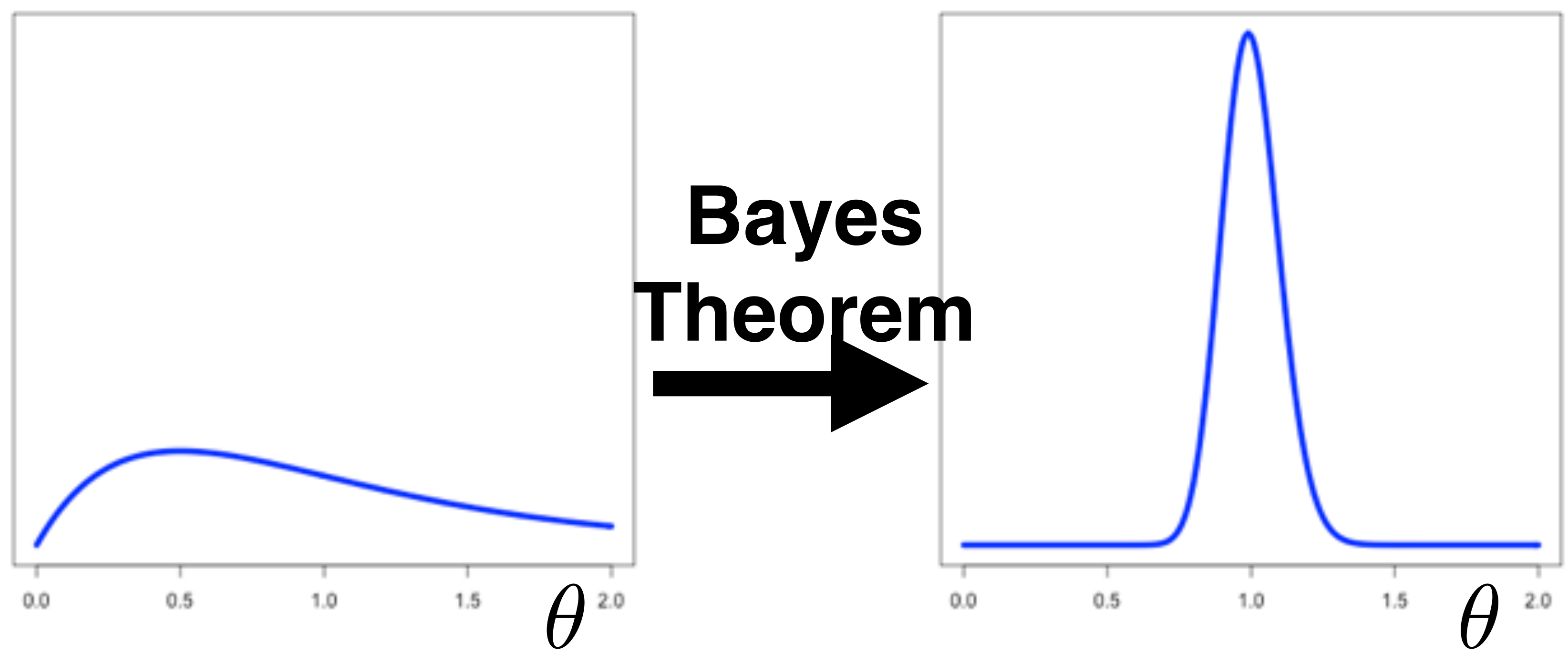

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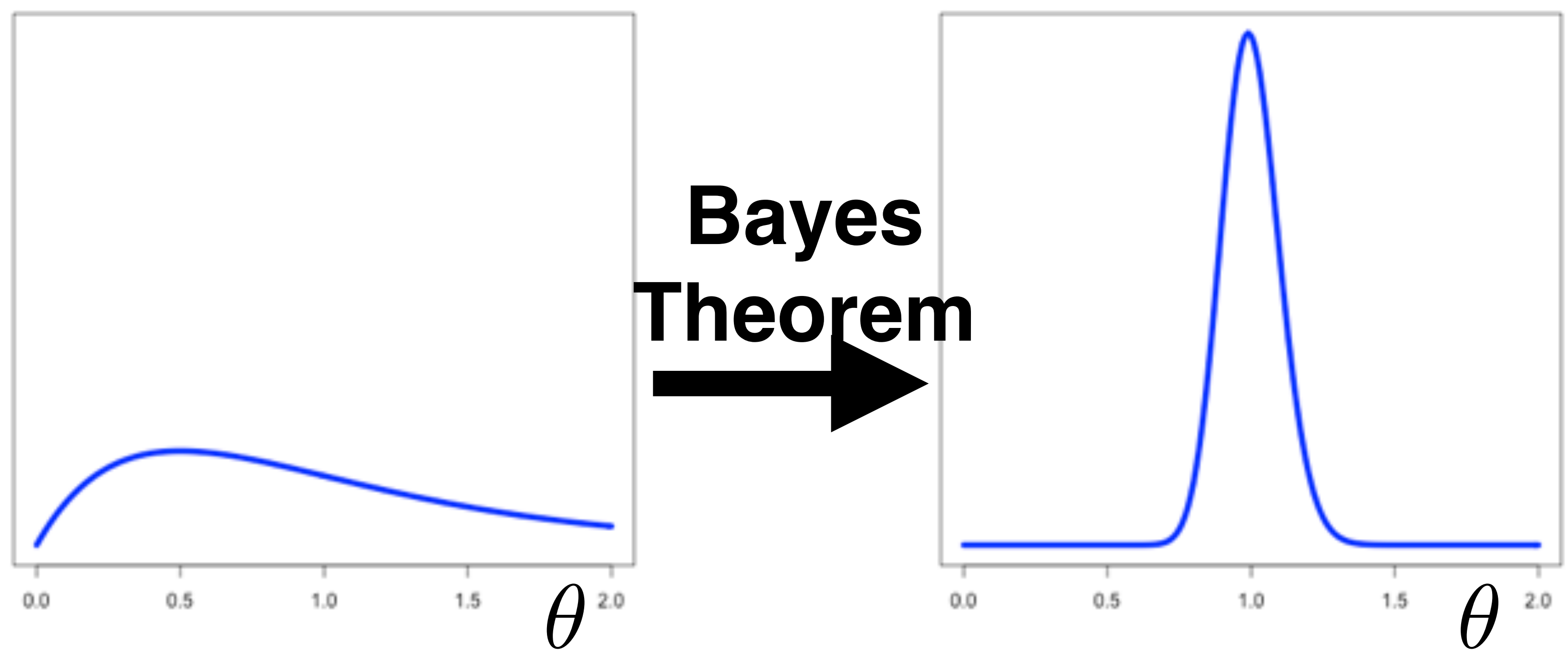

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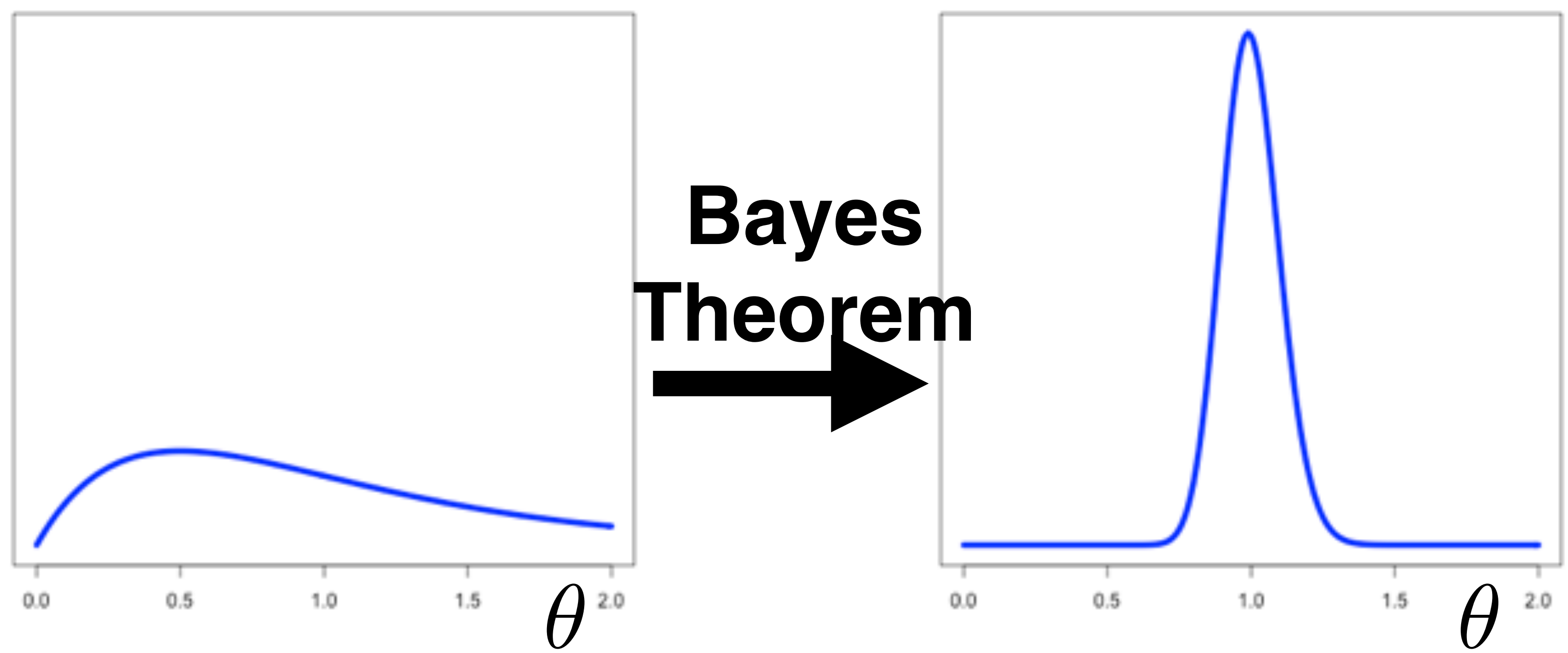

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- Gold standard: Markov Chain Monte Carlo (MCMC)

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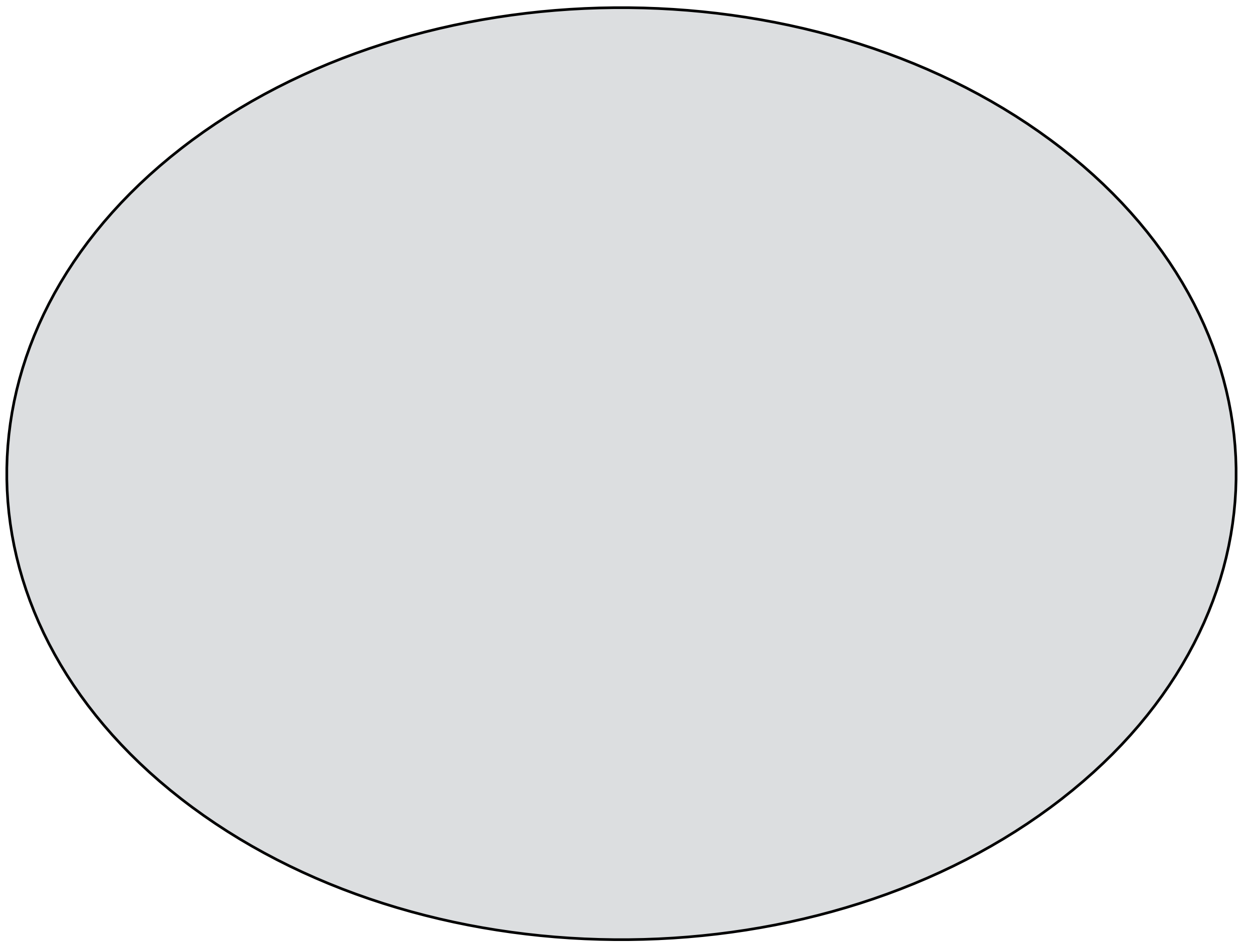

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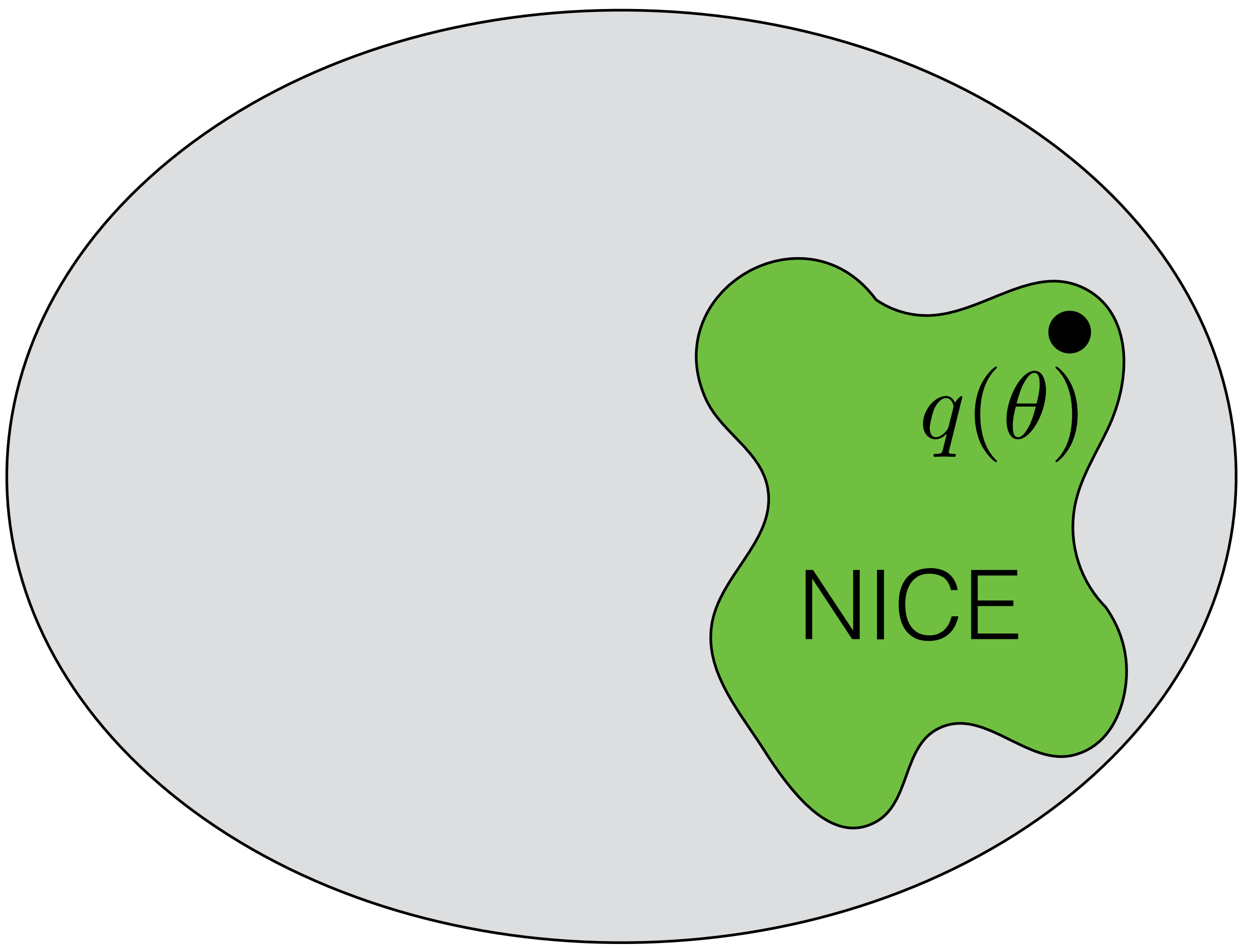

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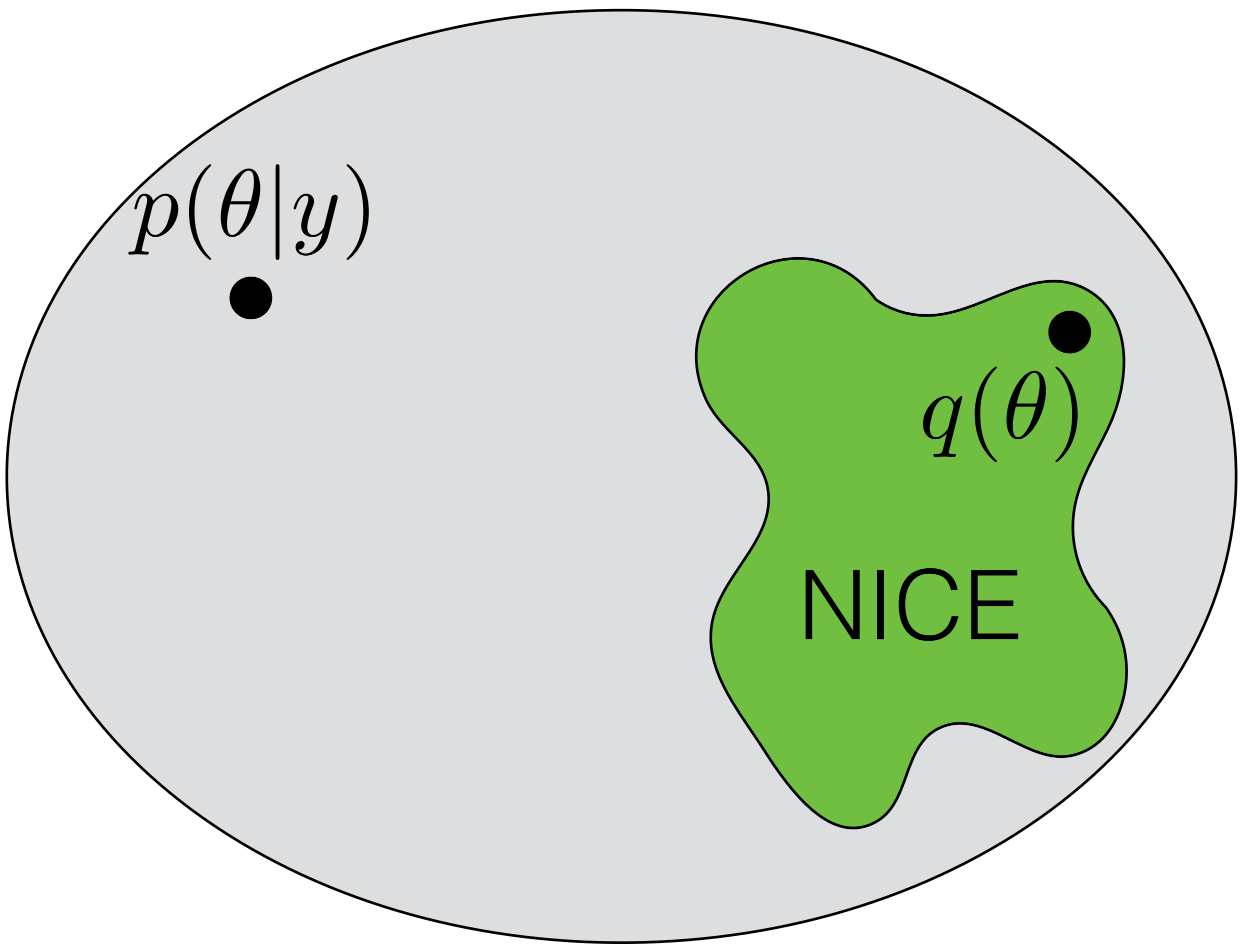

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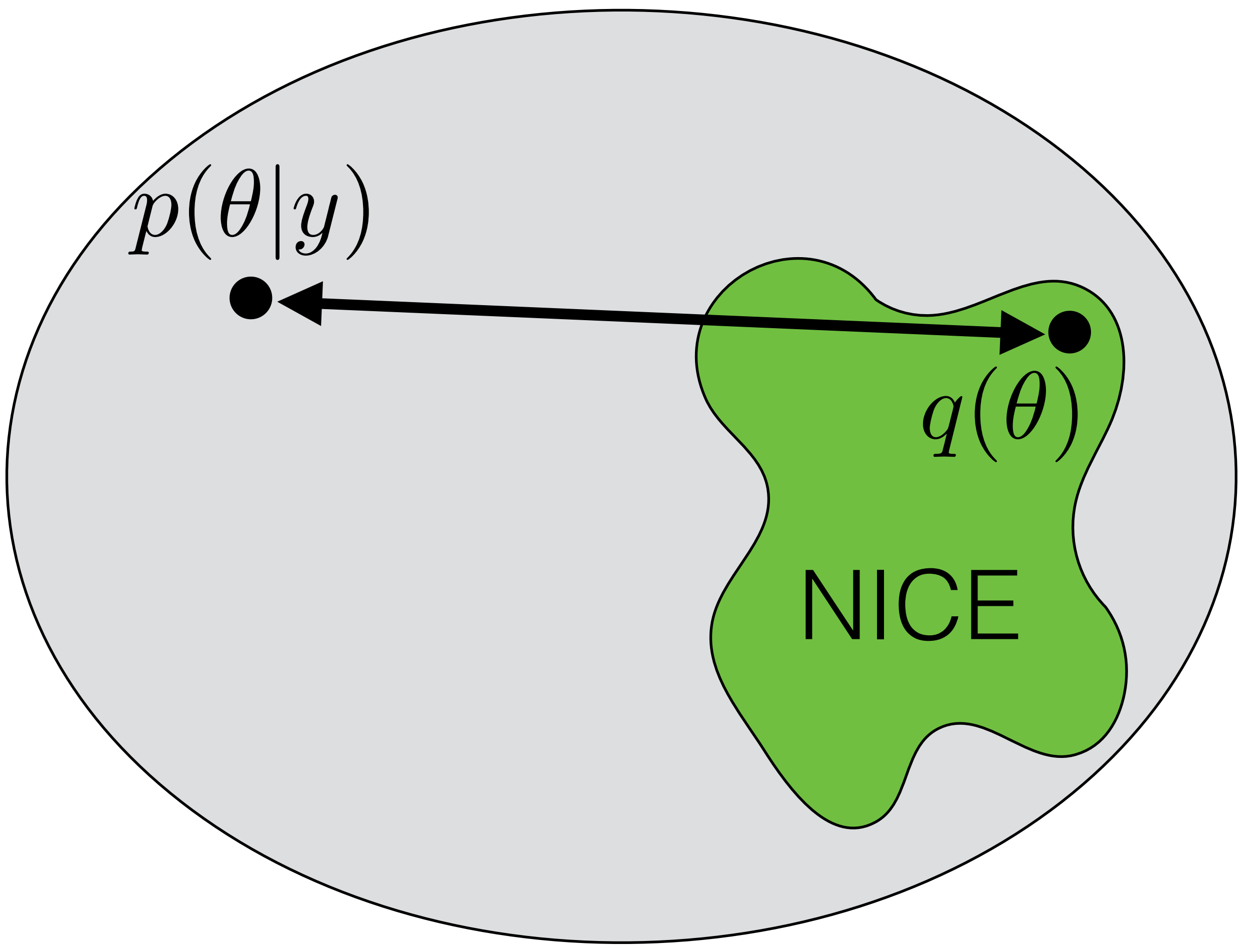

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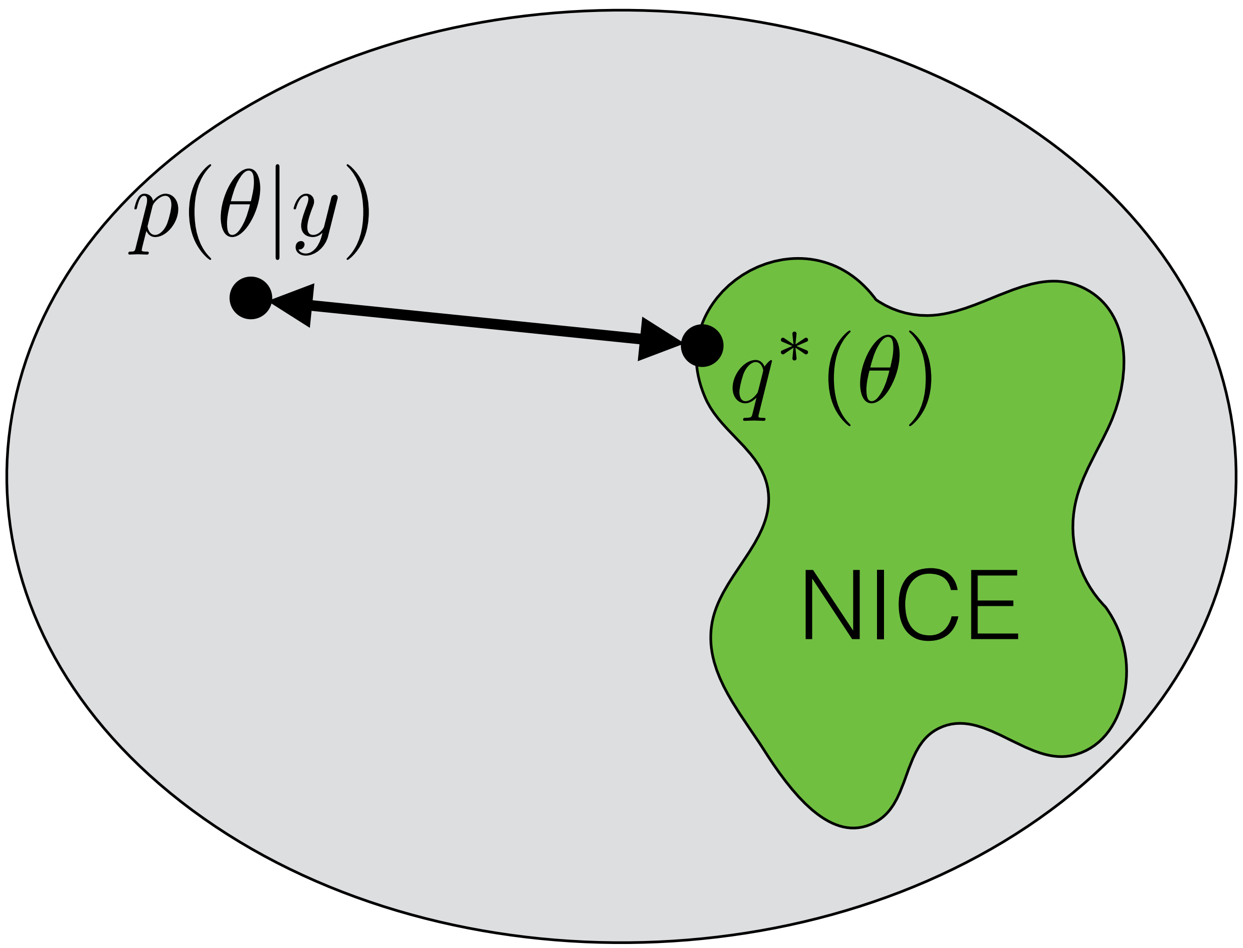

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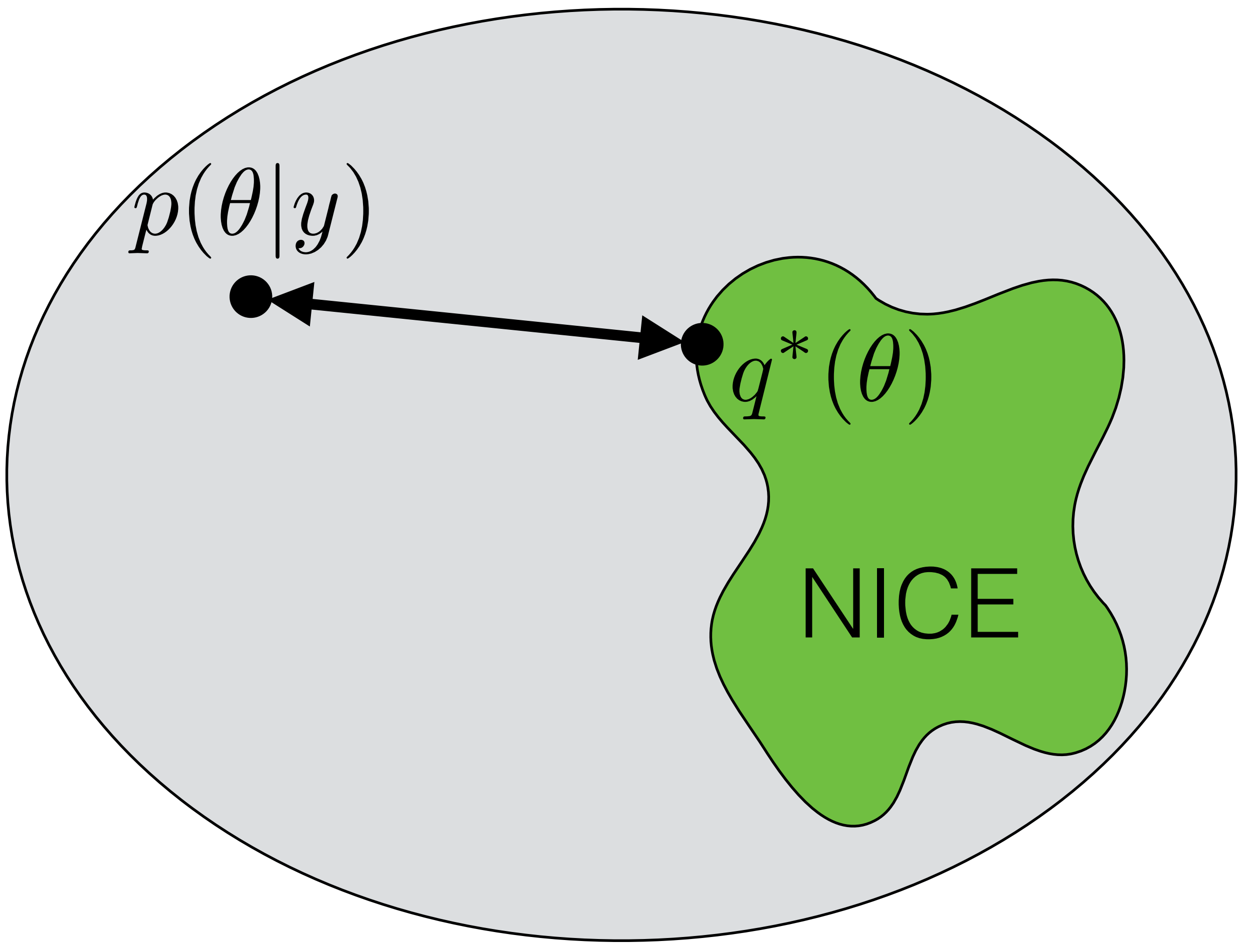

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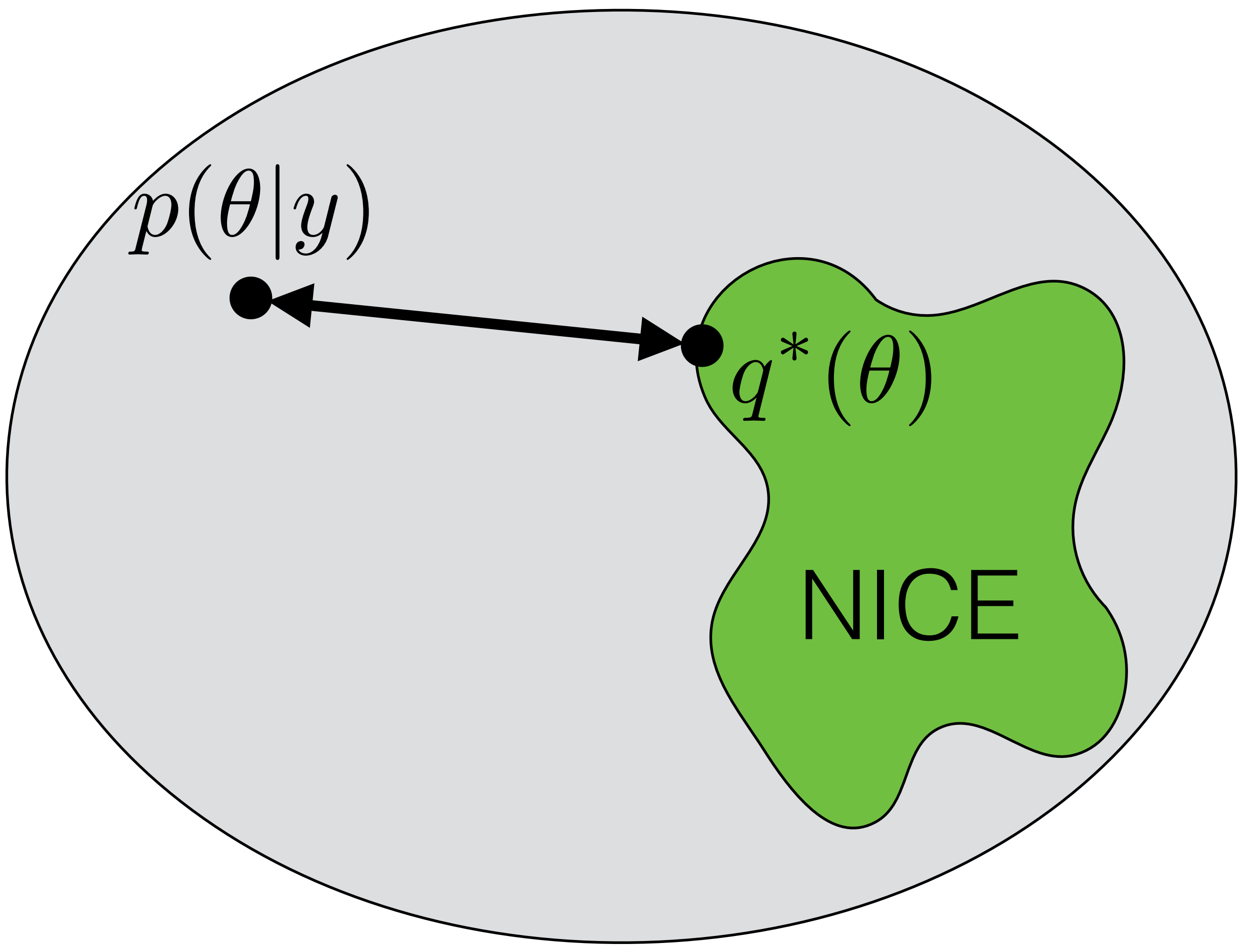

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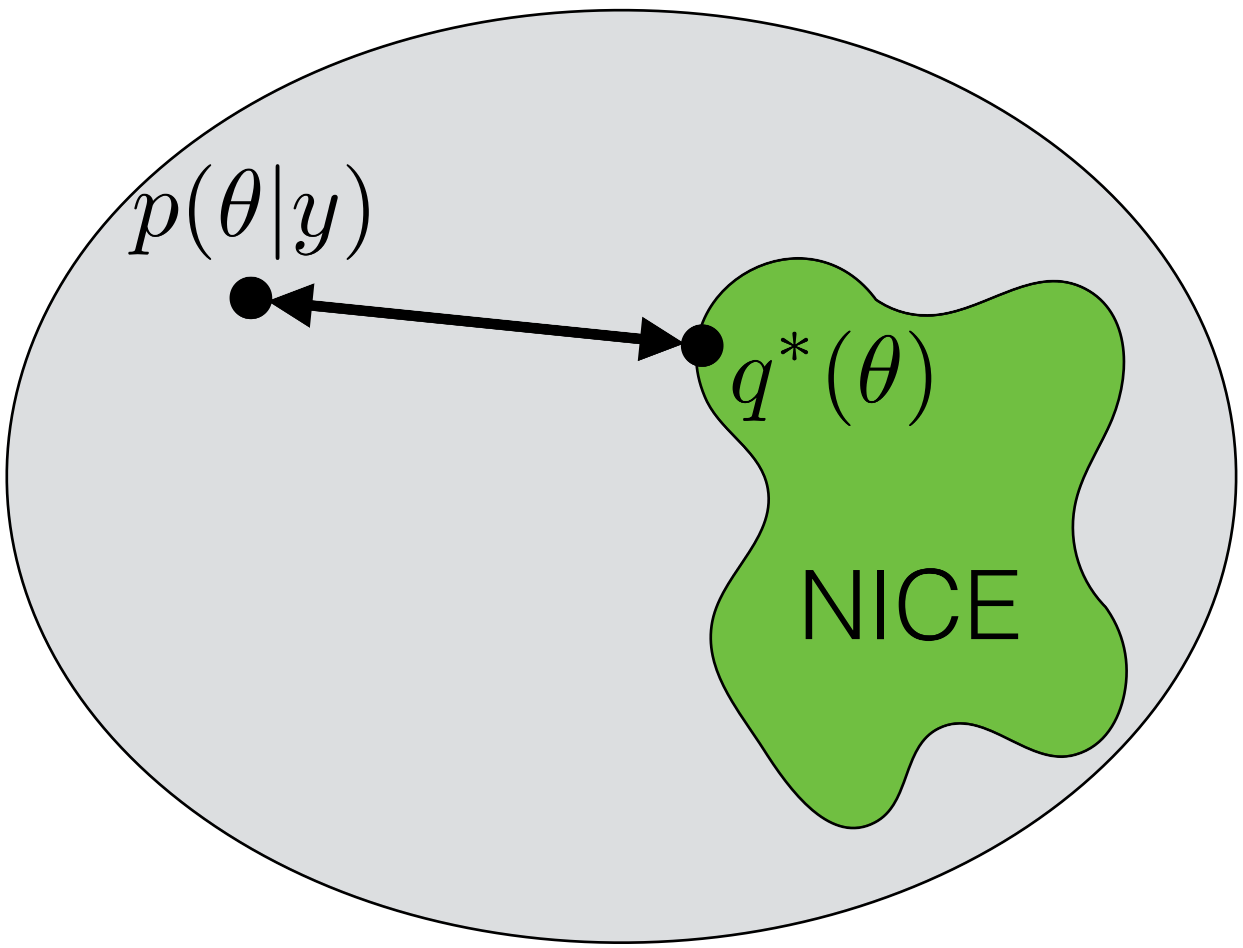

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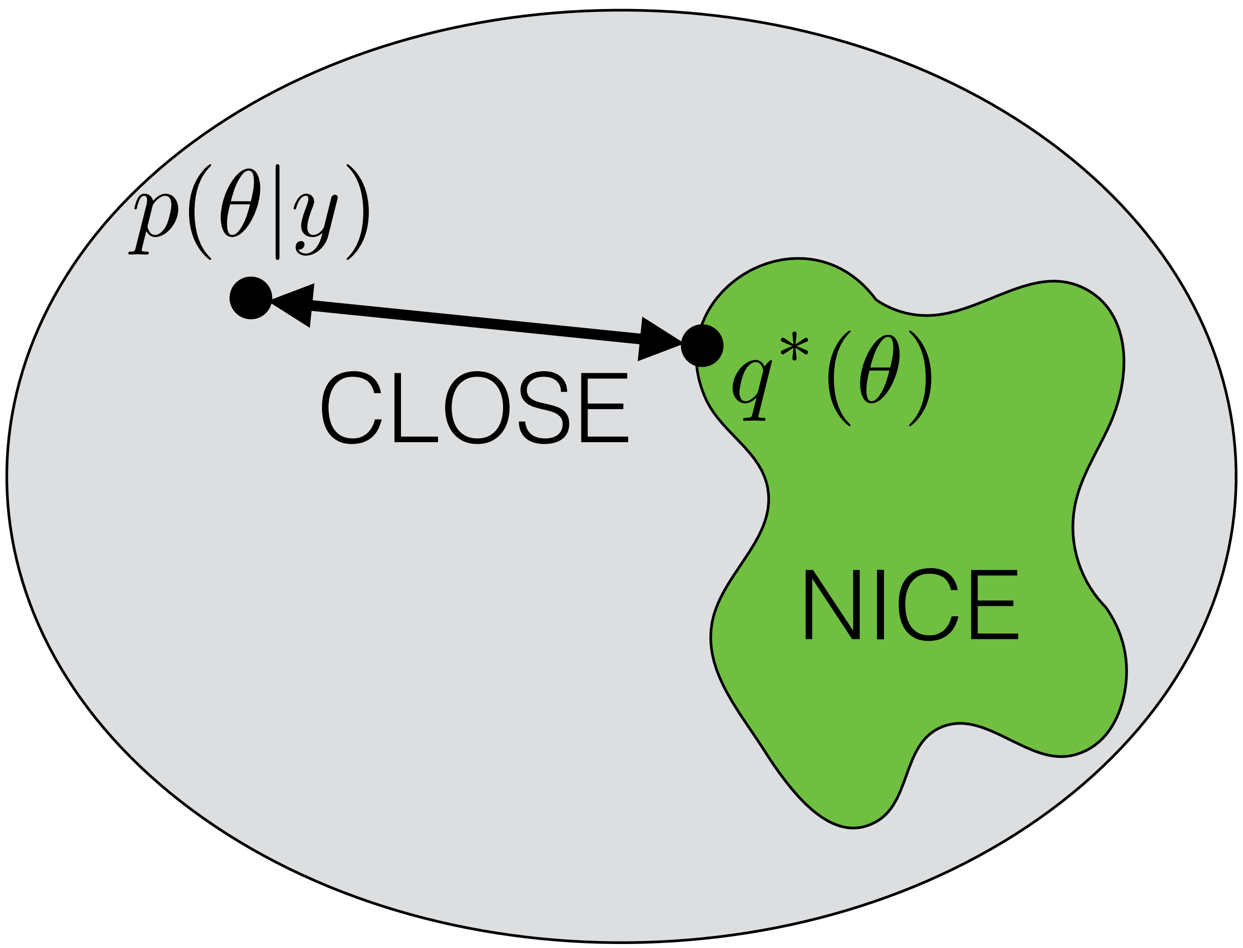

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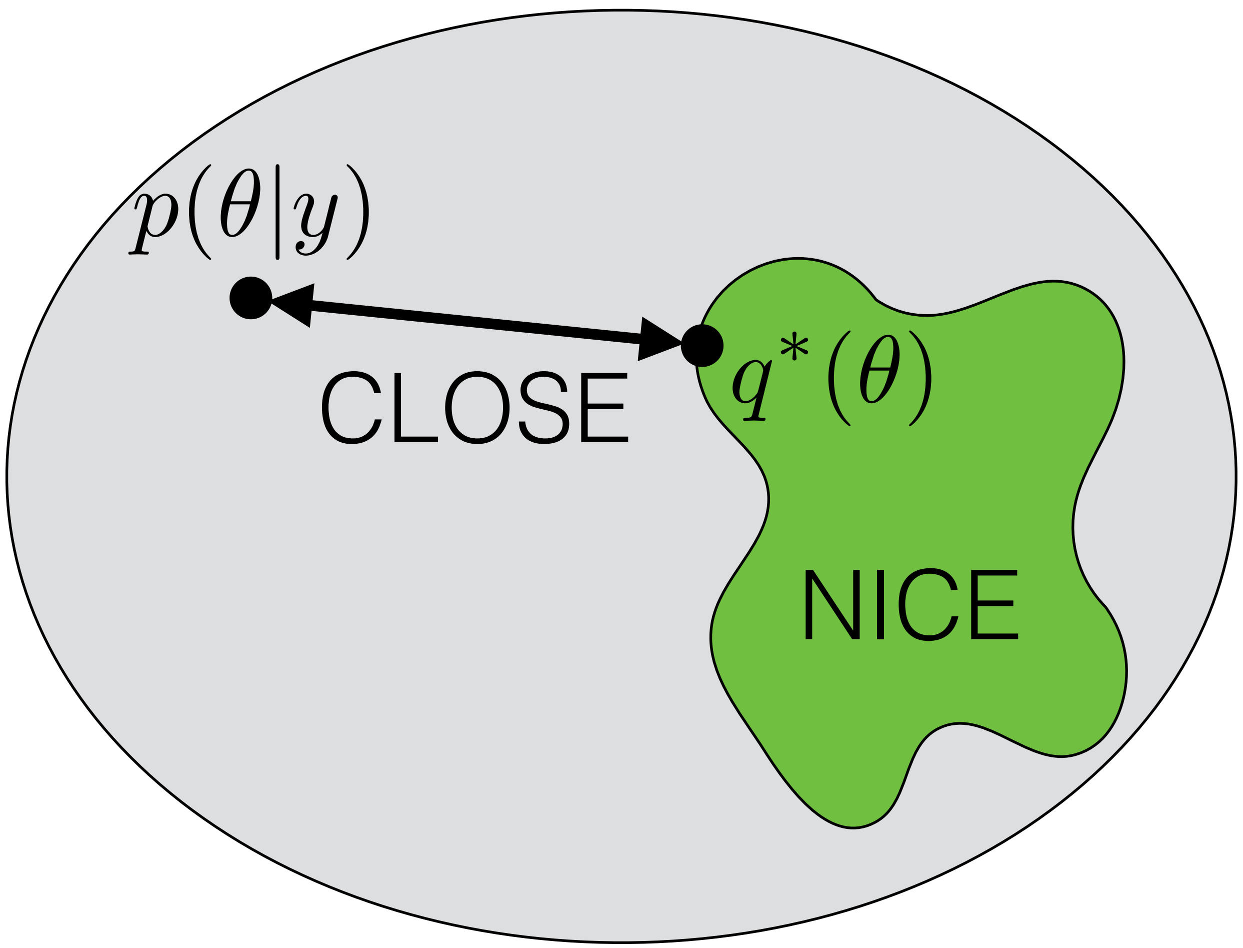

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[board]

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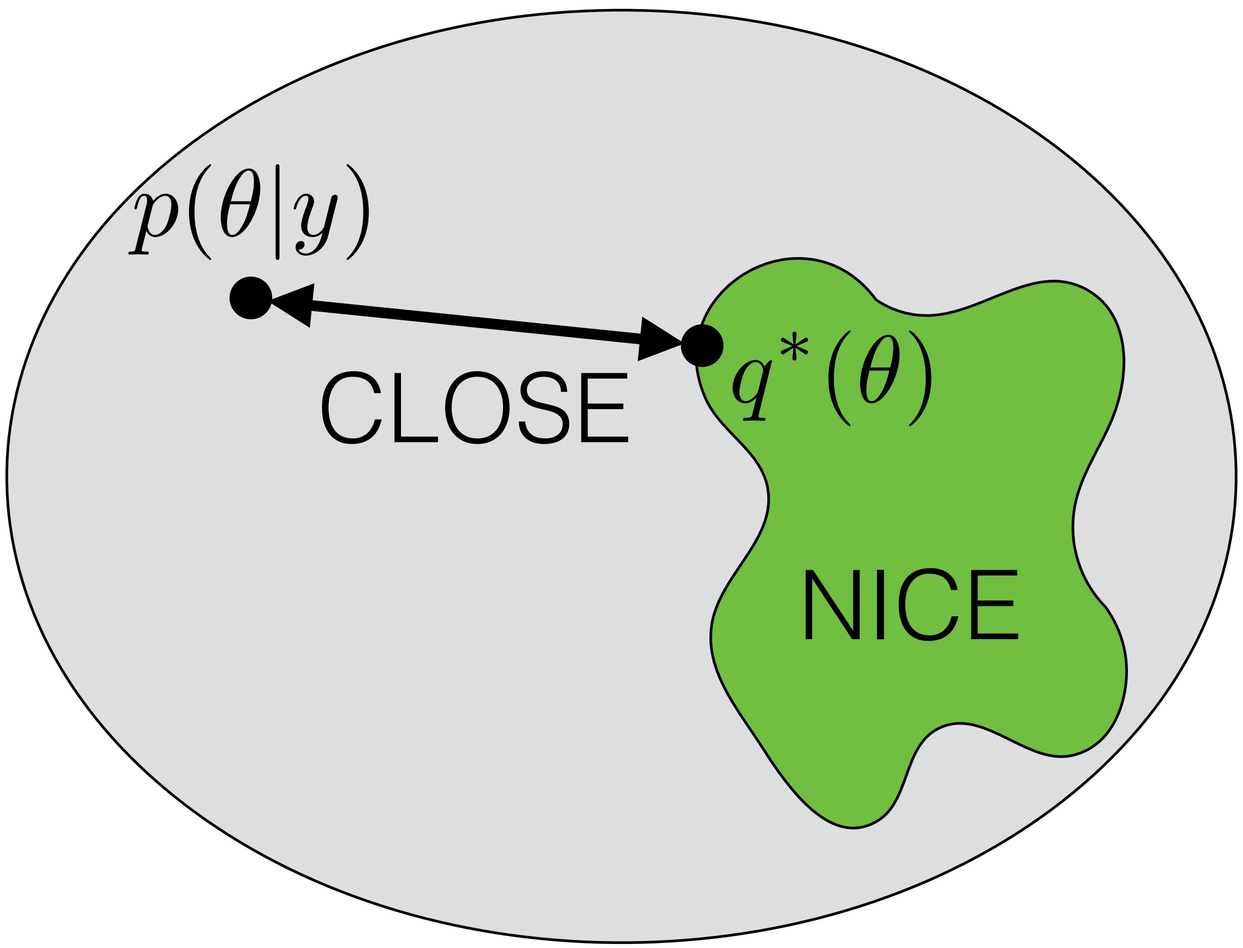

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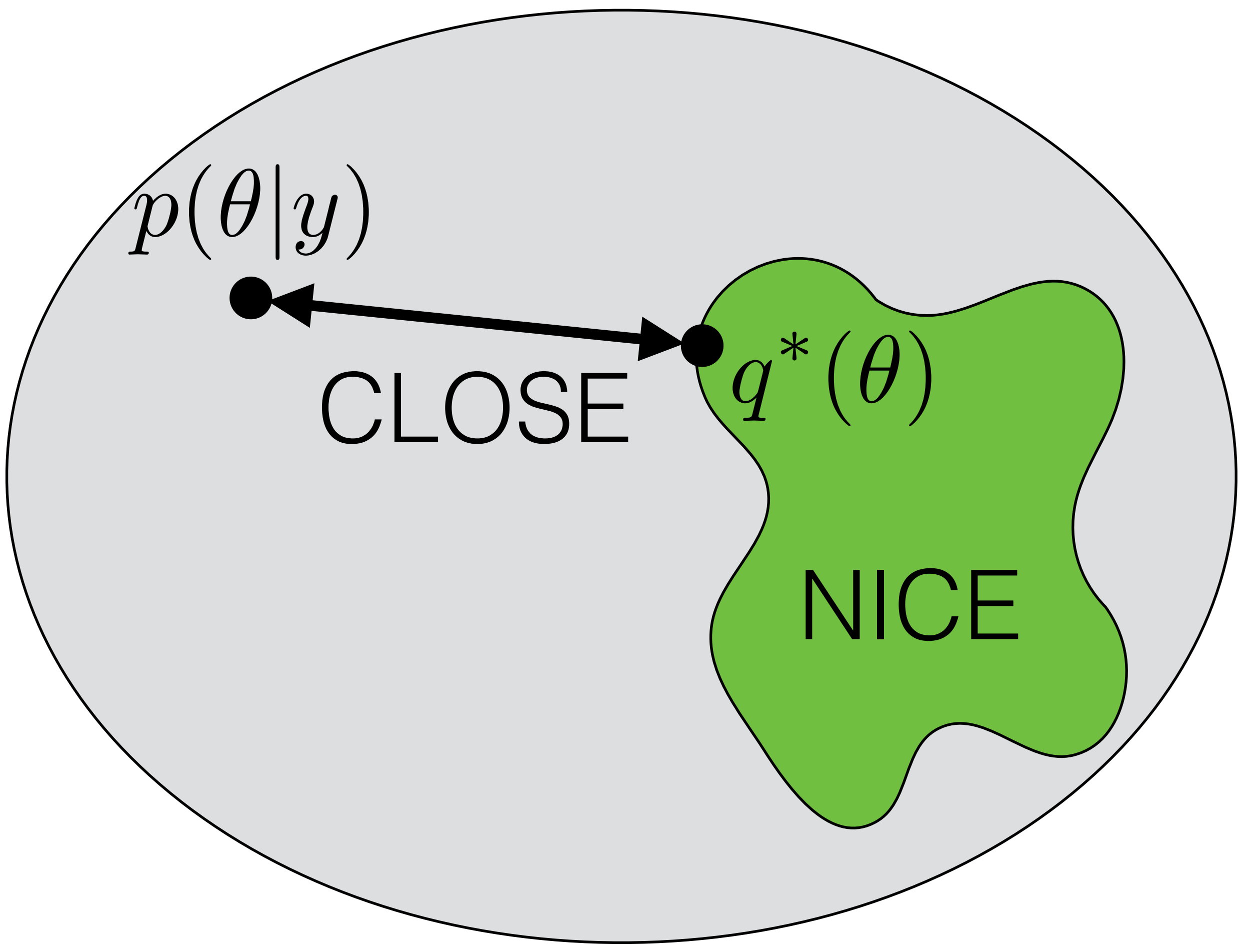

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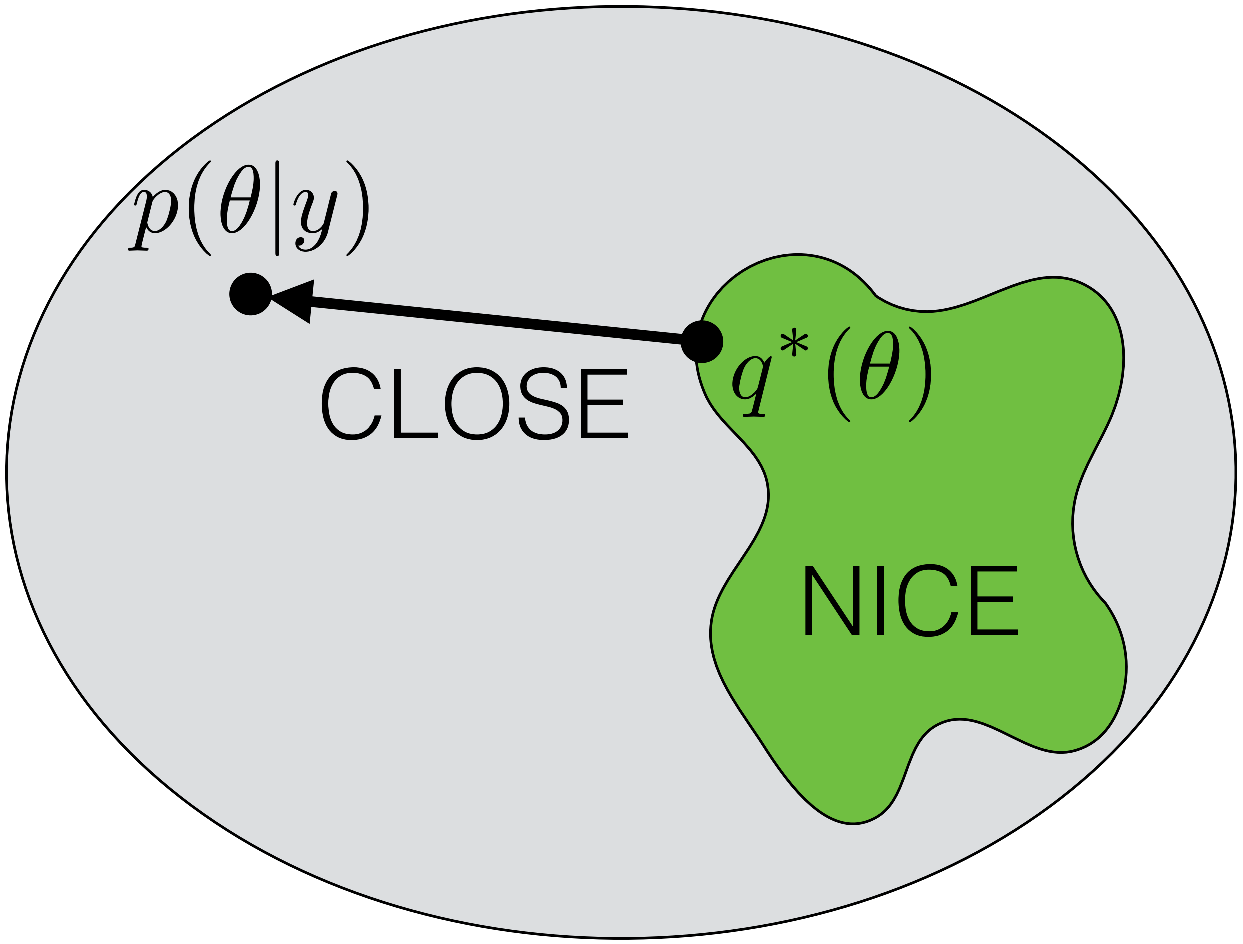

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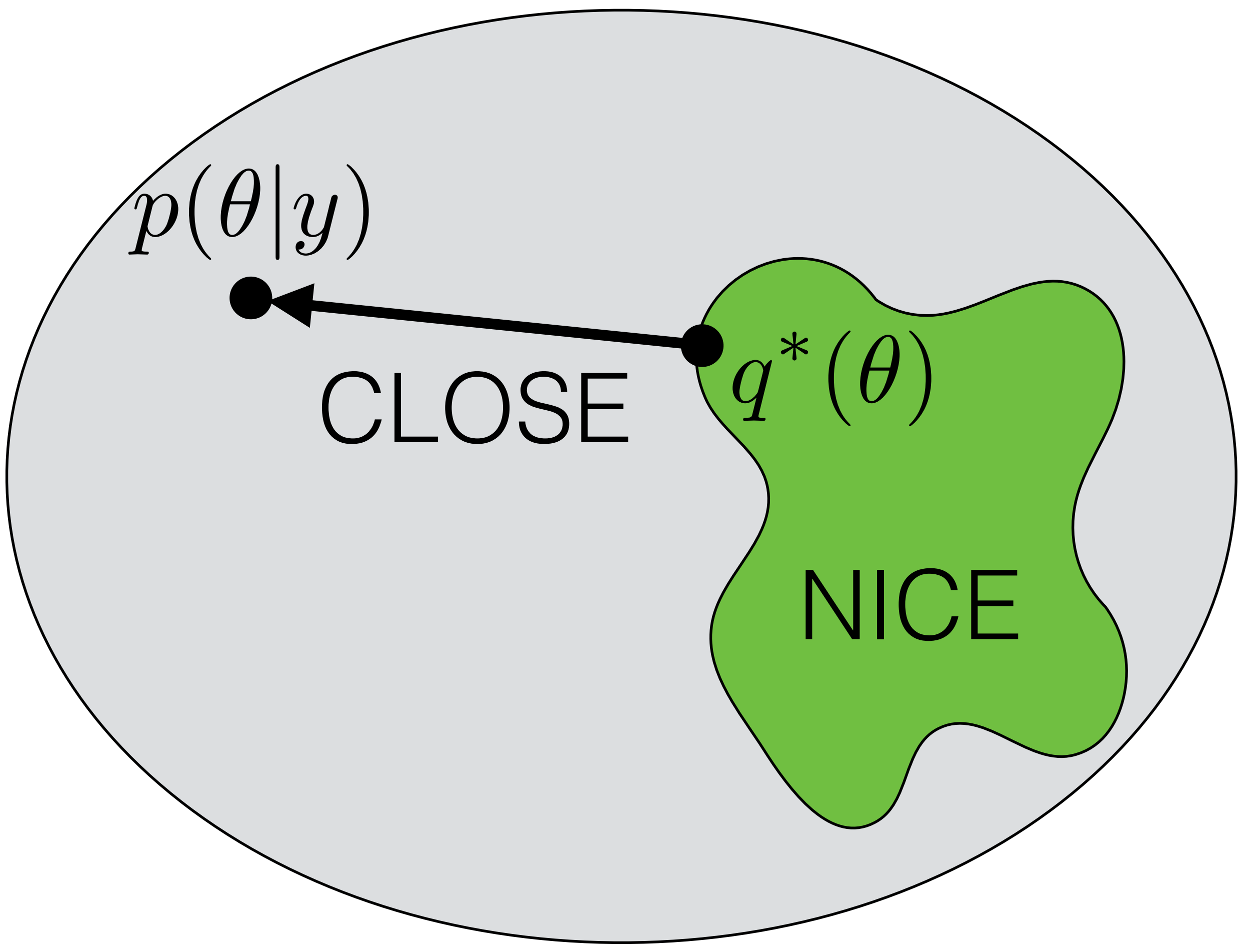

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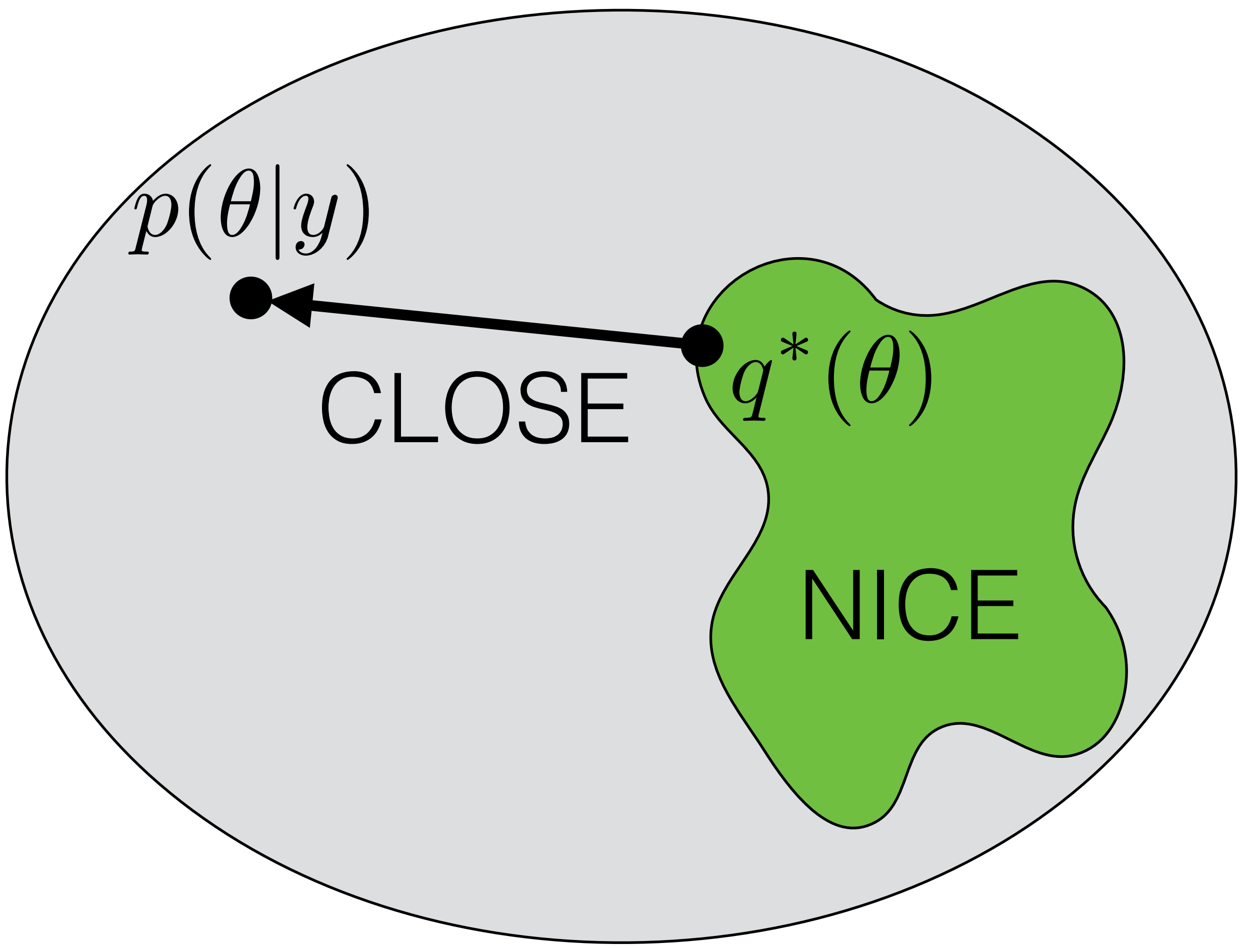

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Approximate Bayesian Inference

[Bardenet, Doucet, Holmes 2017]

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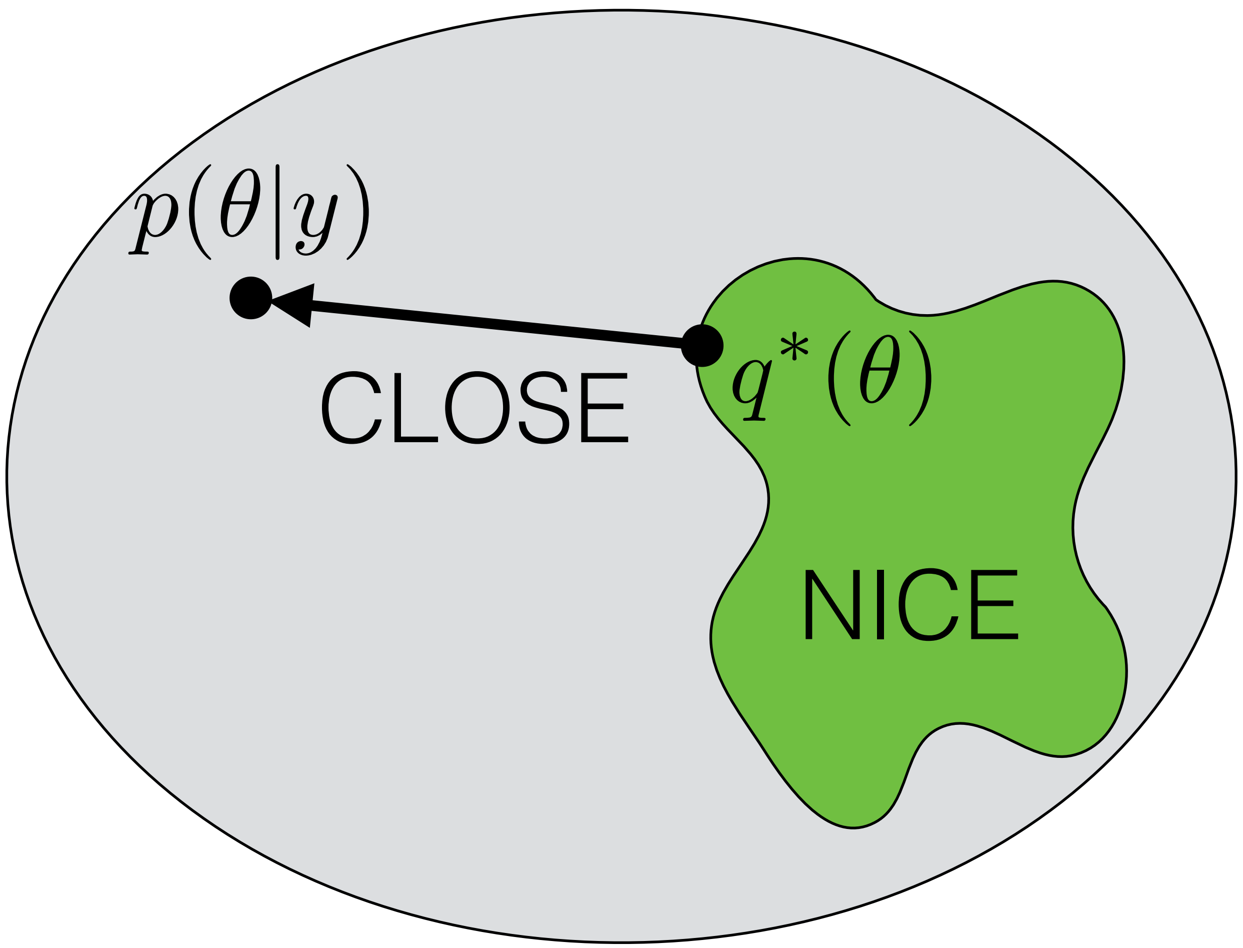

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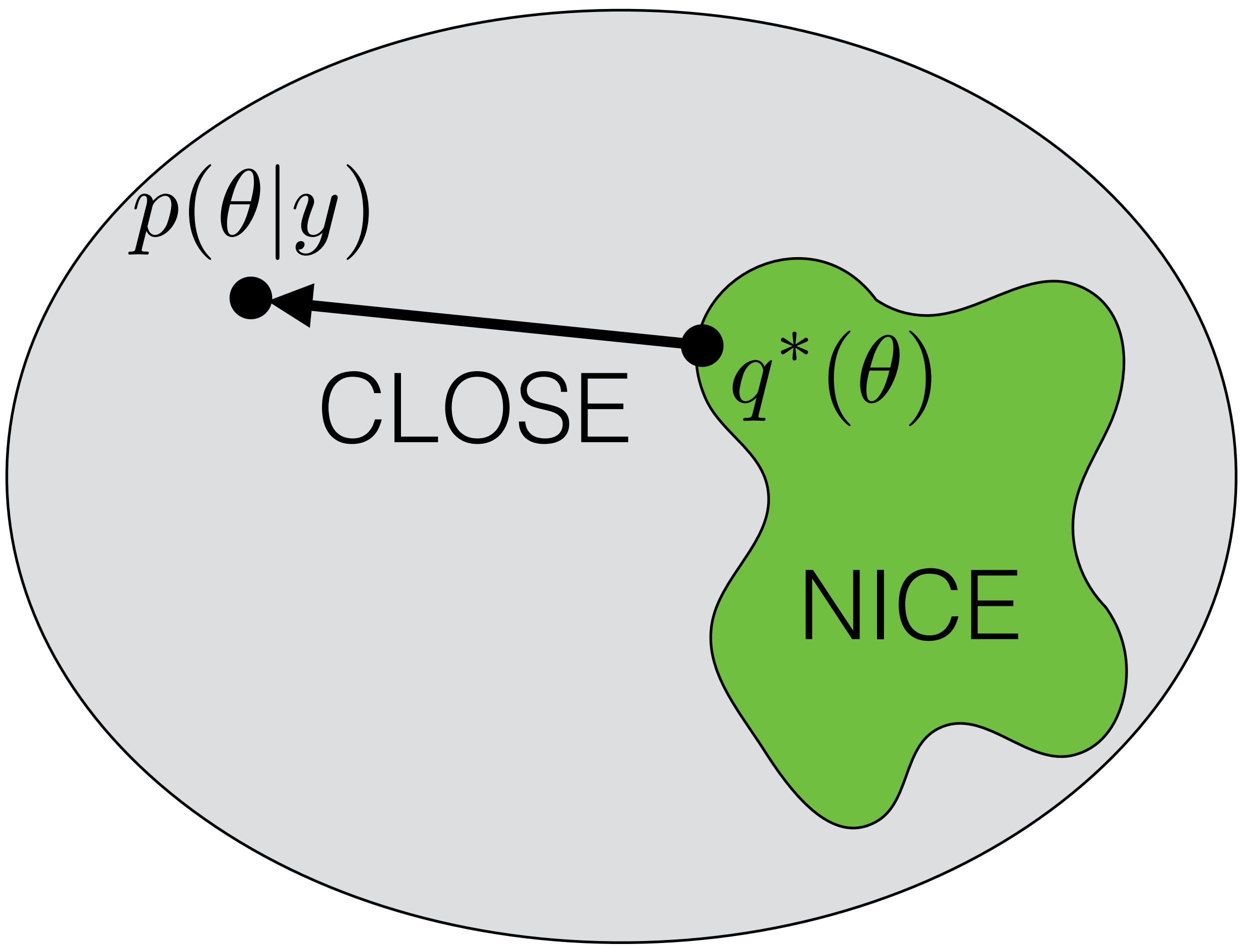

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$$KL(q(\cdot)||p(\cdot|y))$$

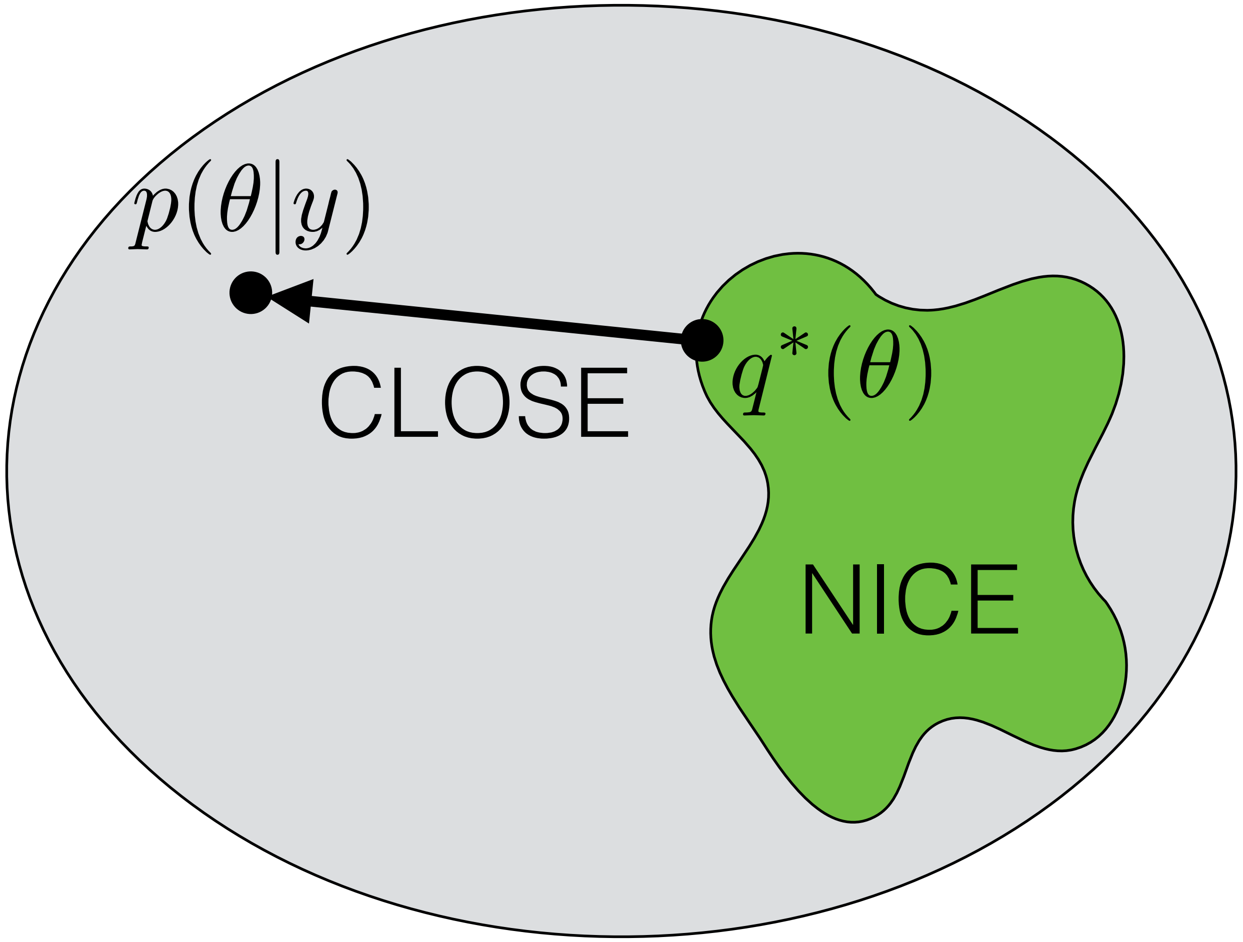
- VB practical success: point estimates and prediction, fast, streaming, distributed (3.6M Wikipedia, 350K Nature)

[Broderick, Boyd, Wibisono, Wilson, Jordan 2013]

Why KL?

- Variational Bayes

$$q^* = \mathrm{argmin}_{q \in Q} \mathrm{KL}\left(q(\cdot) || p(\cdot|y)\right)$$



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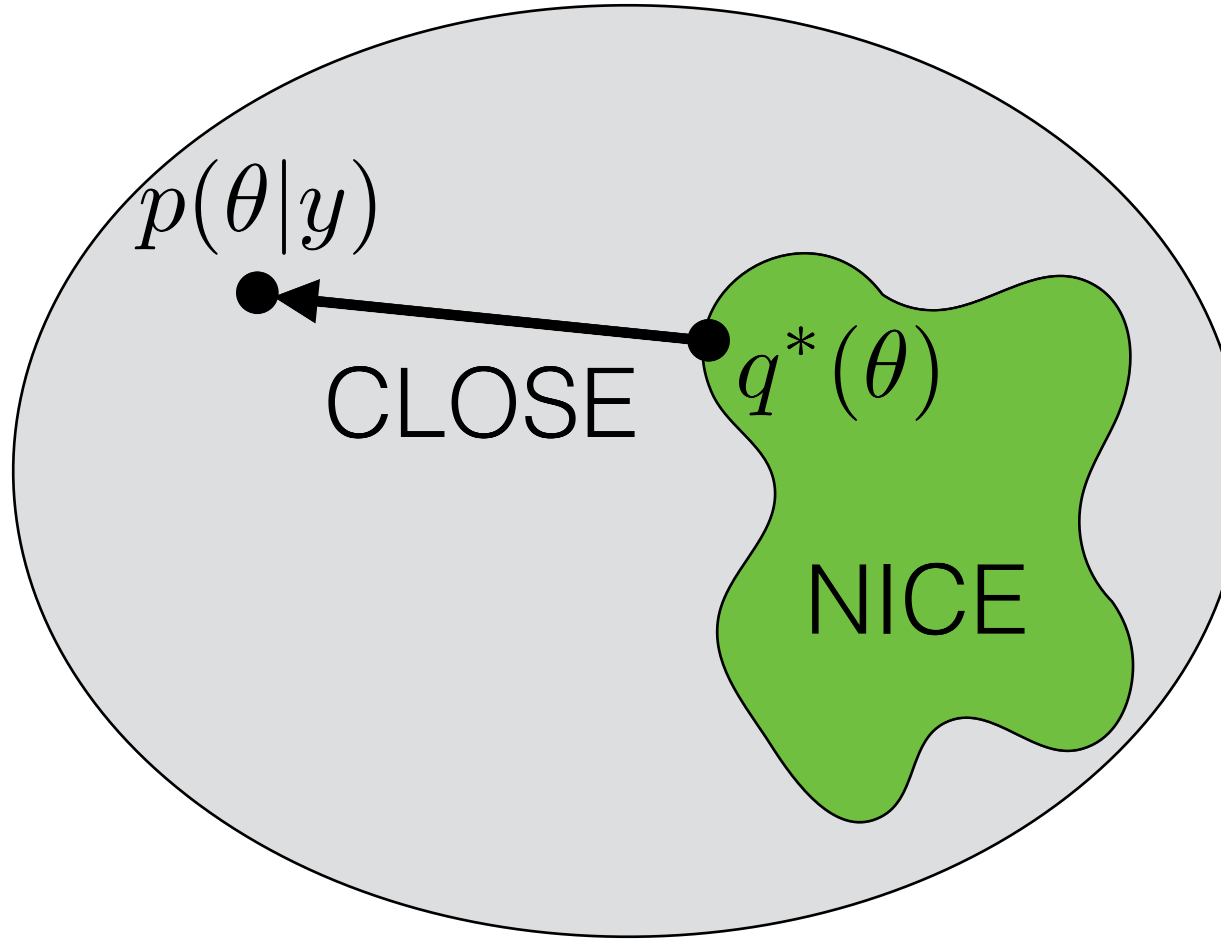

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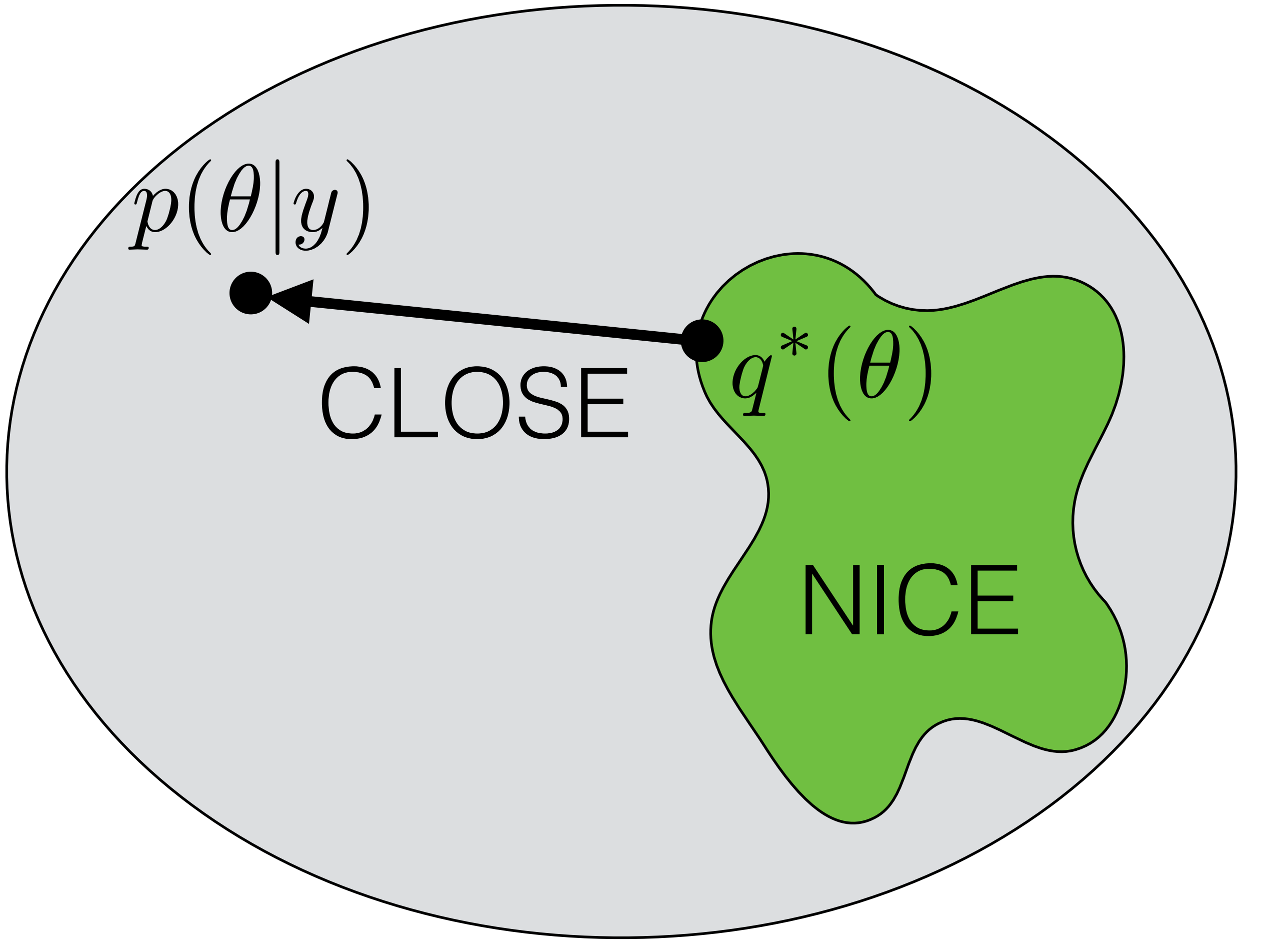

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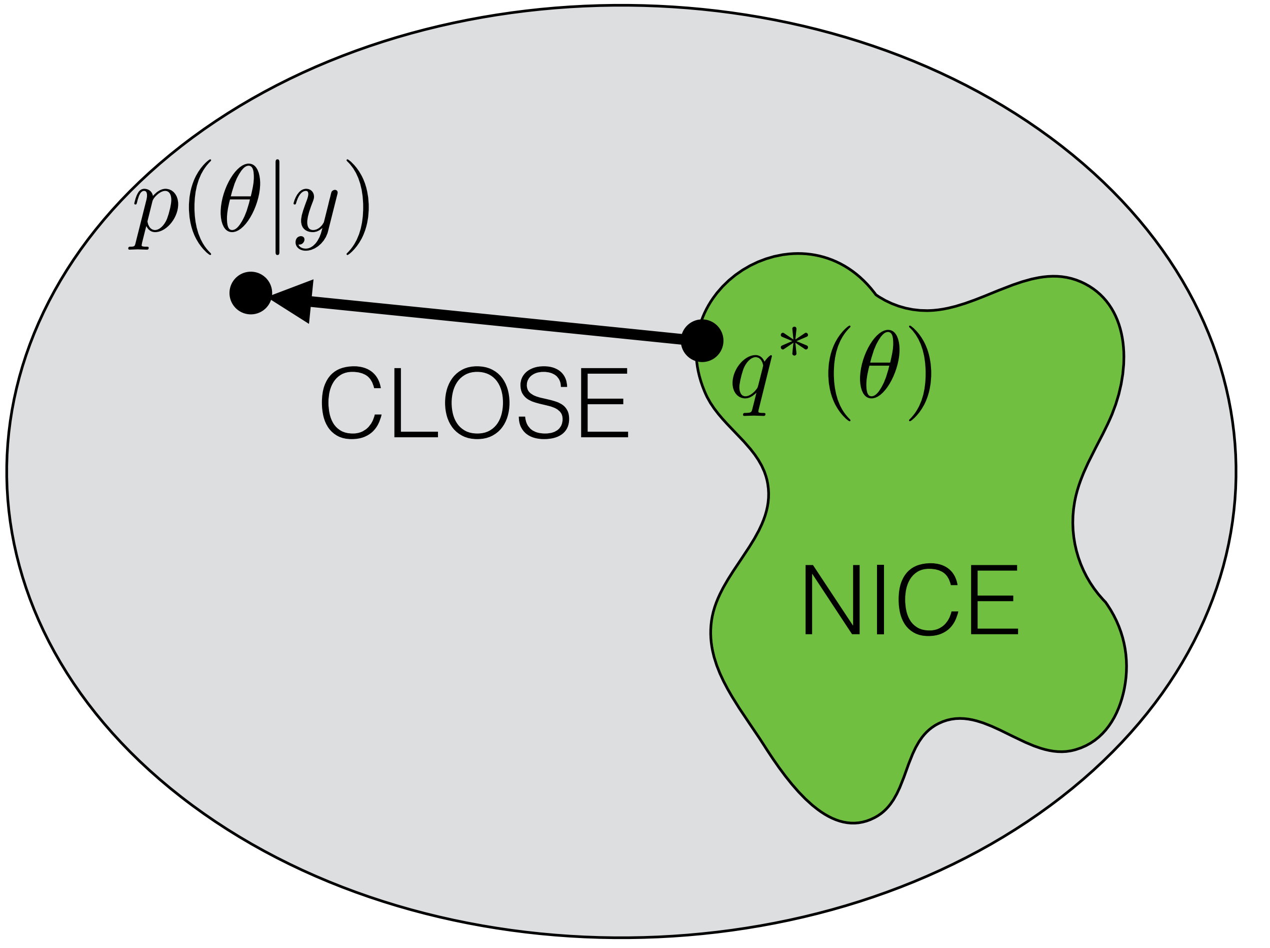

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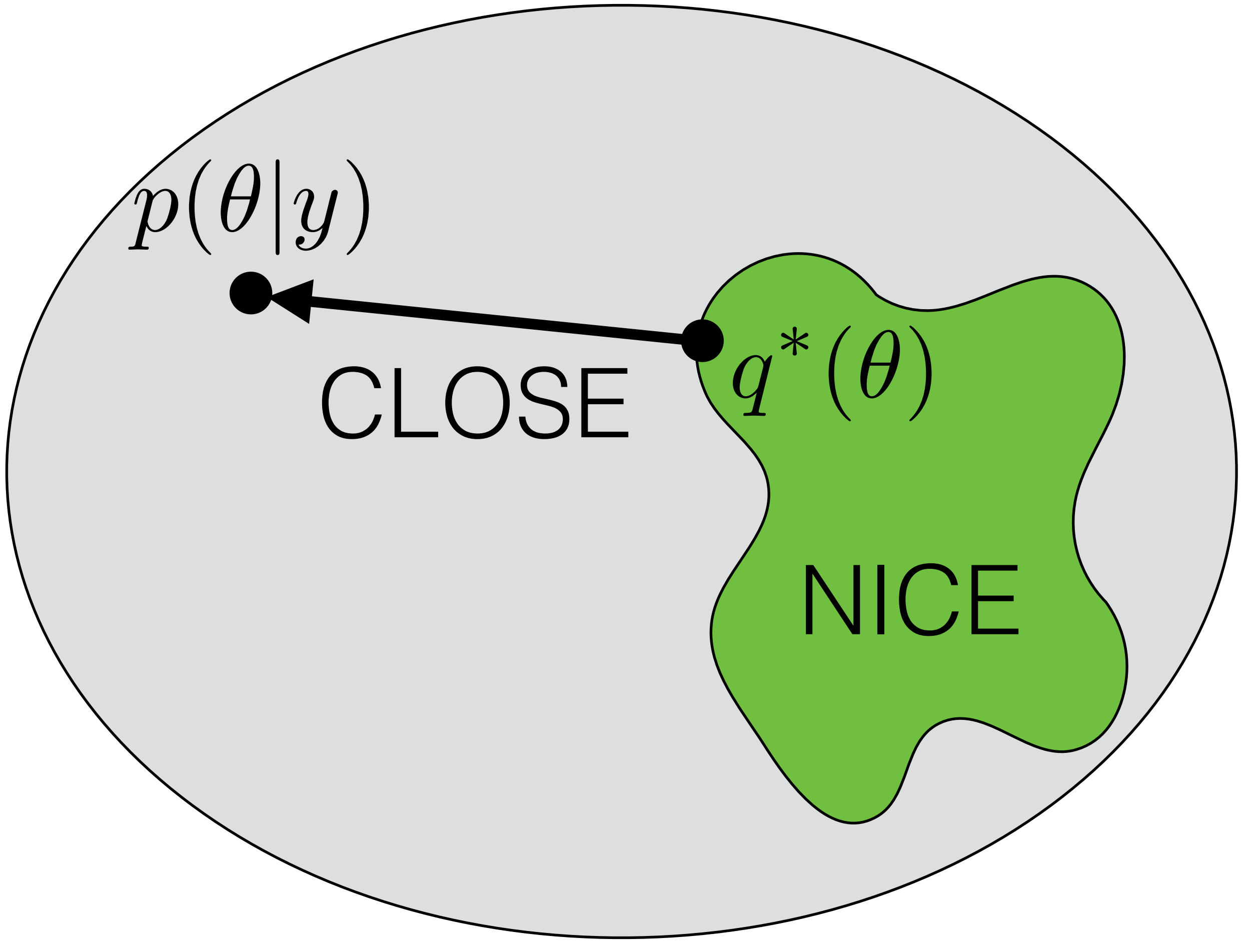

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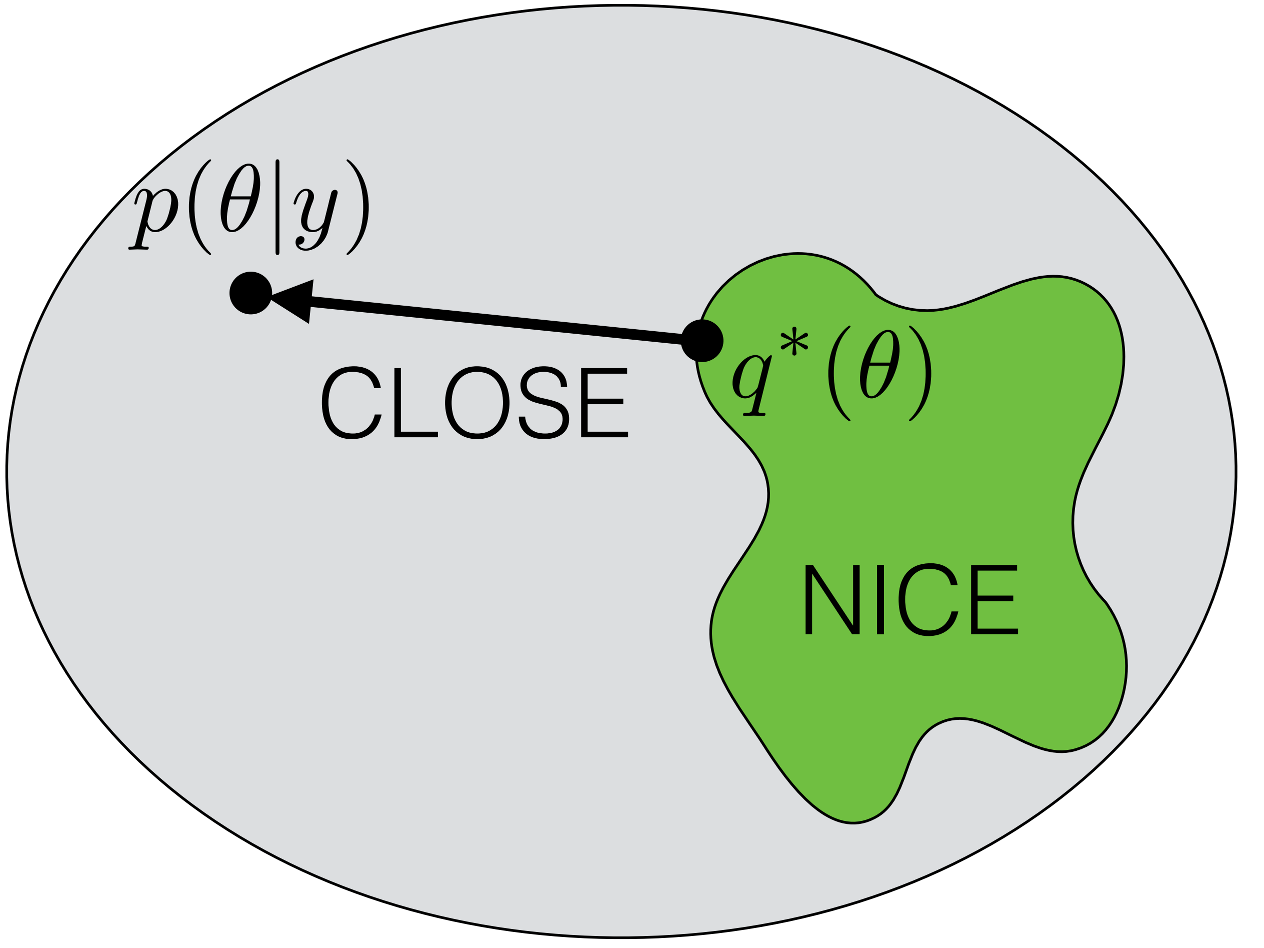

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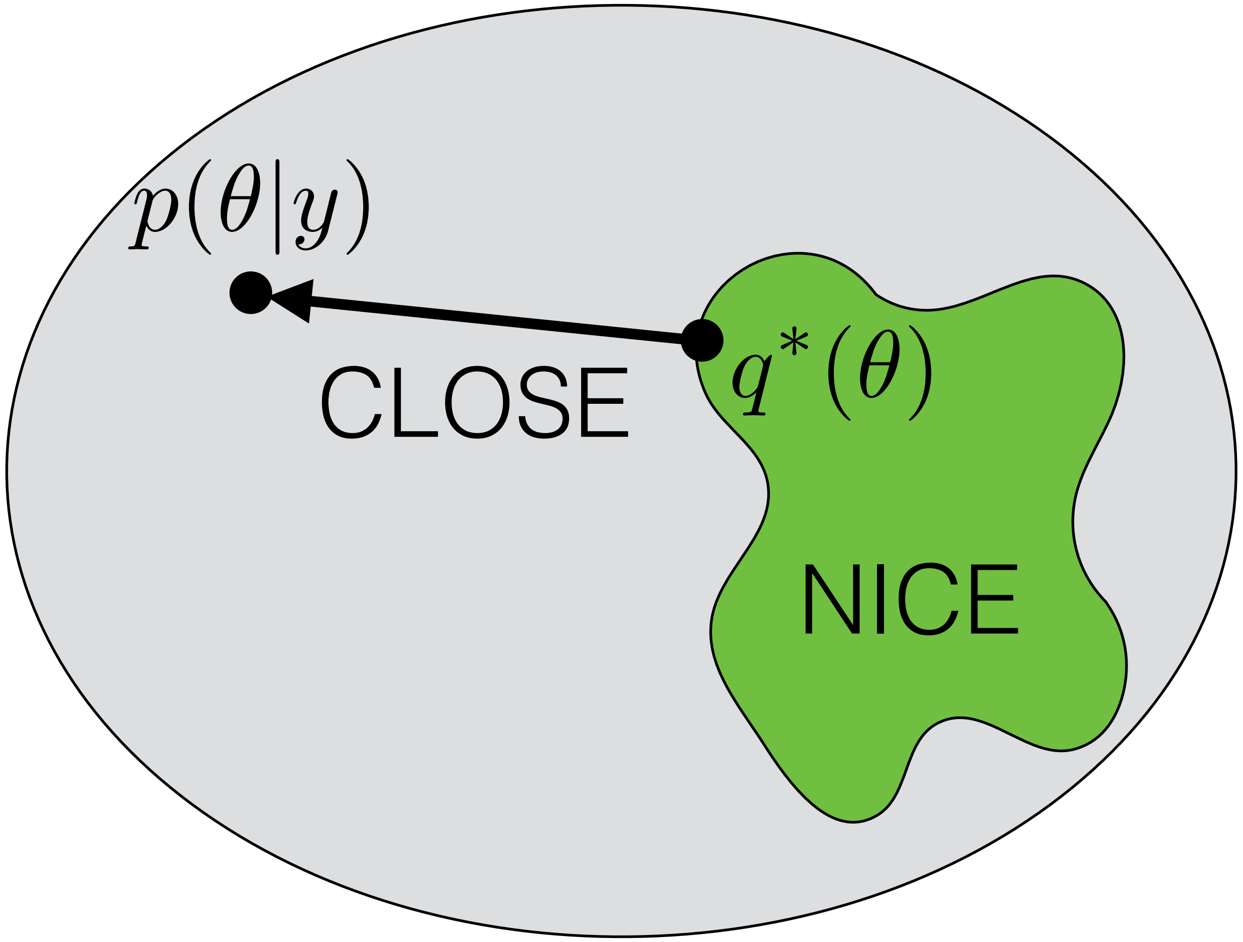

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"Evidence lower bound" (ELBO)

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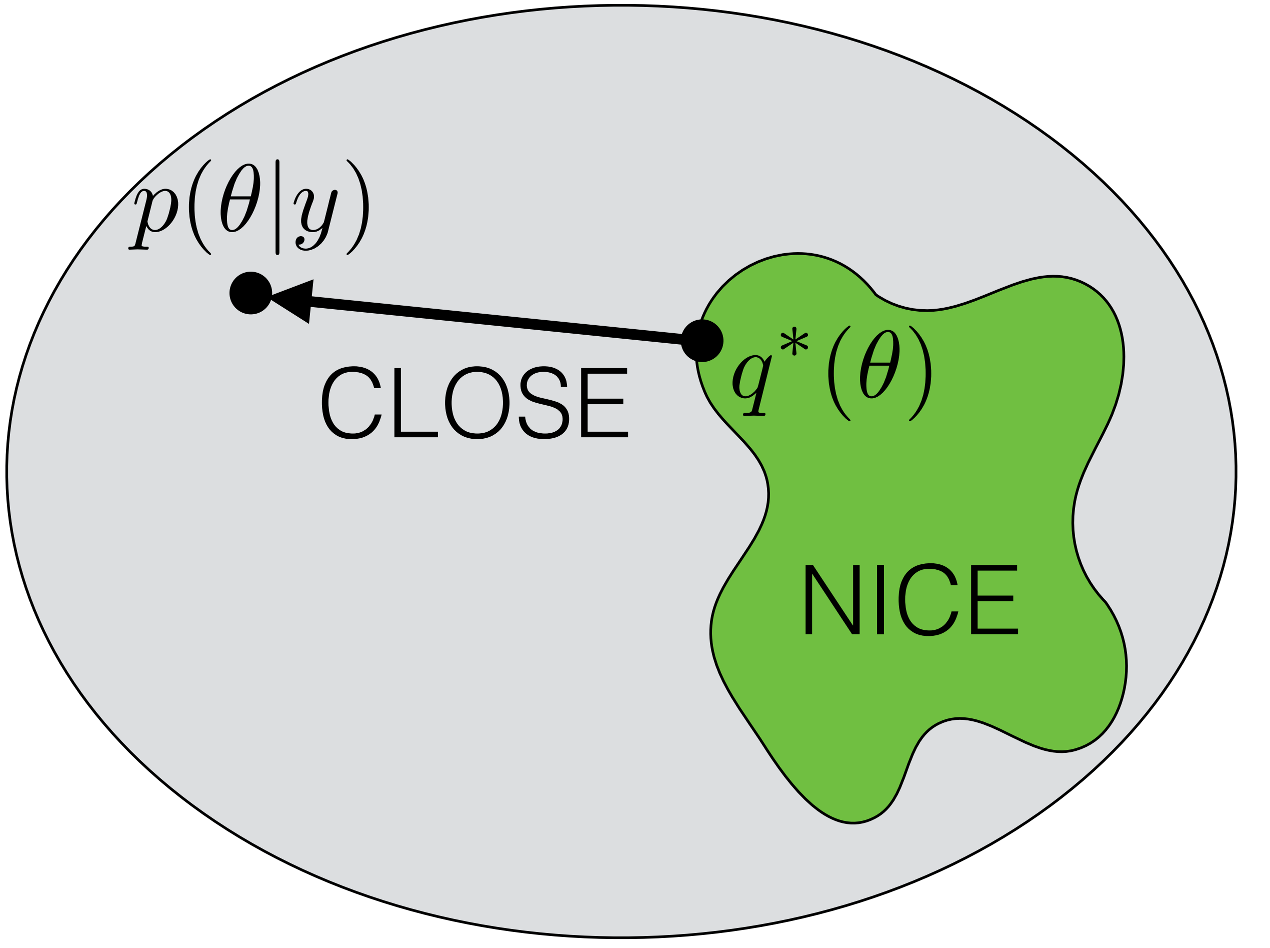

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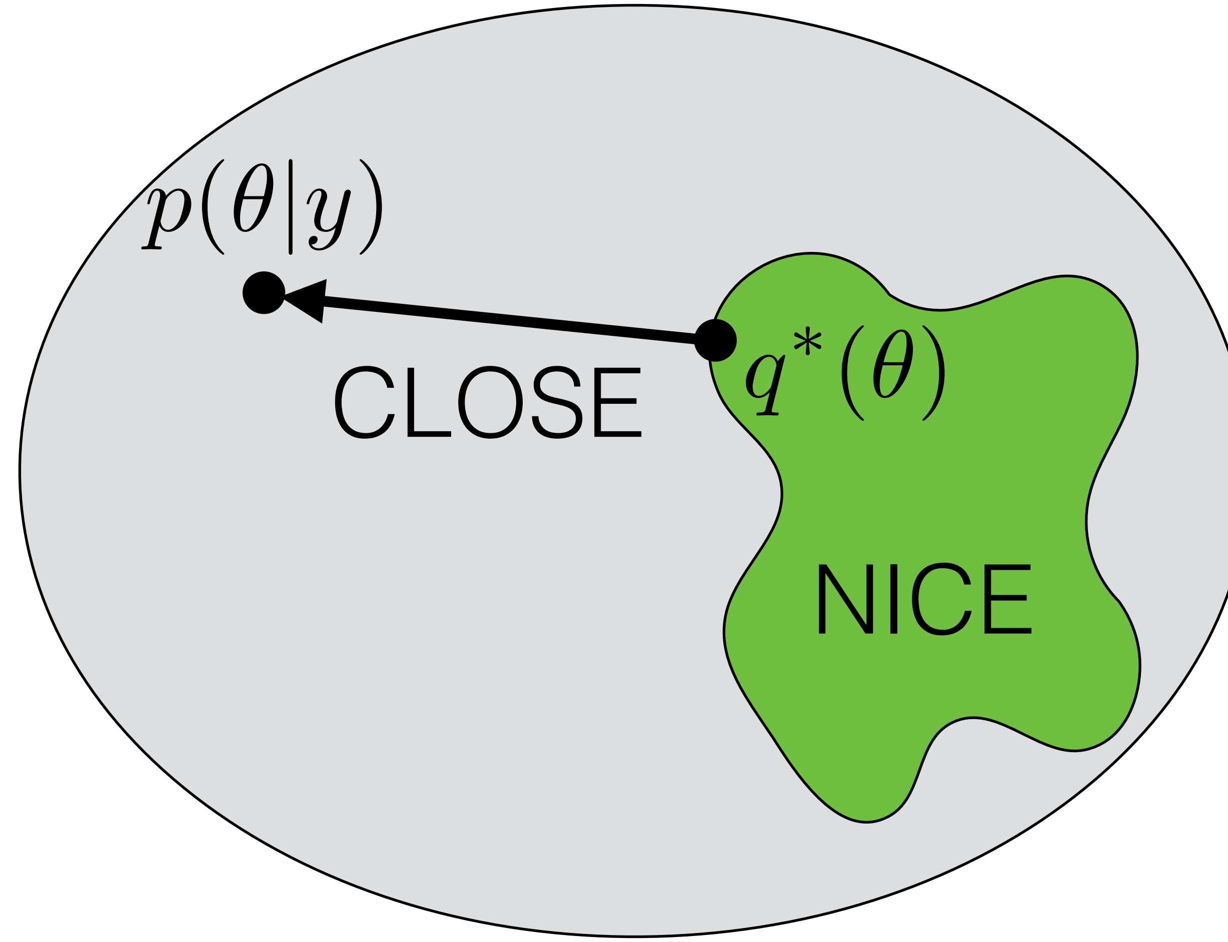

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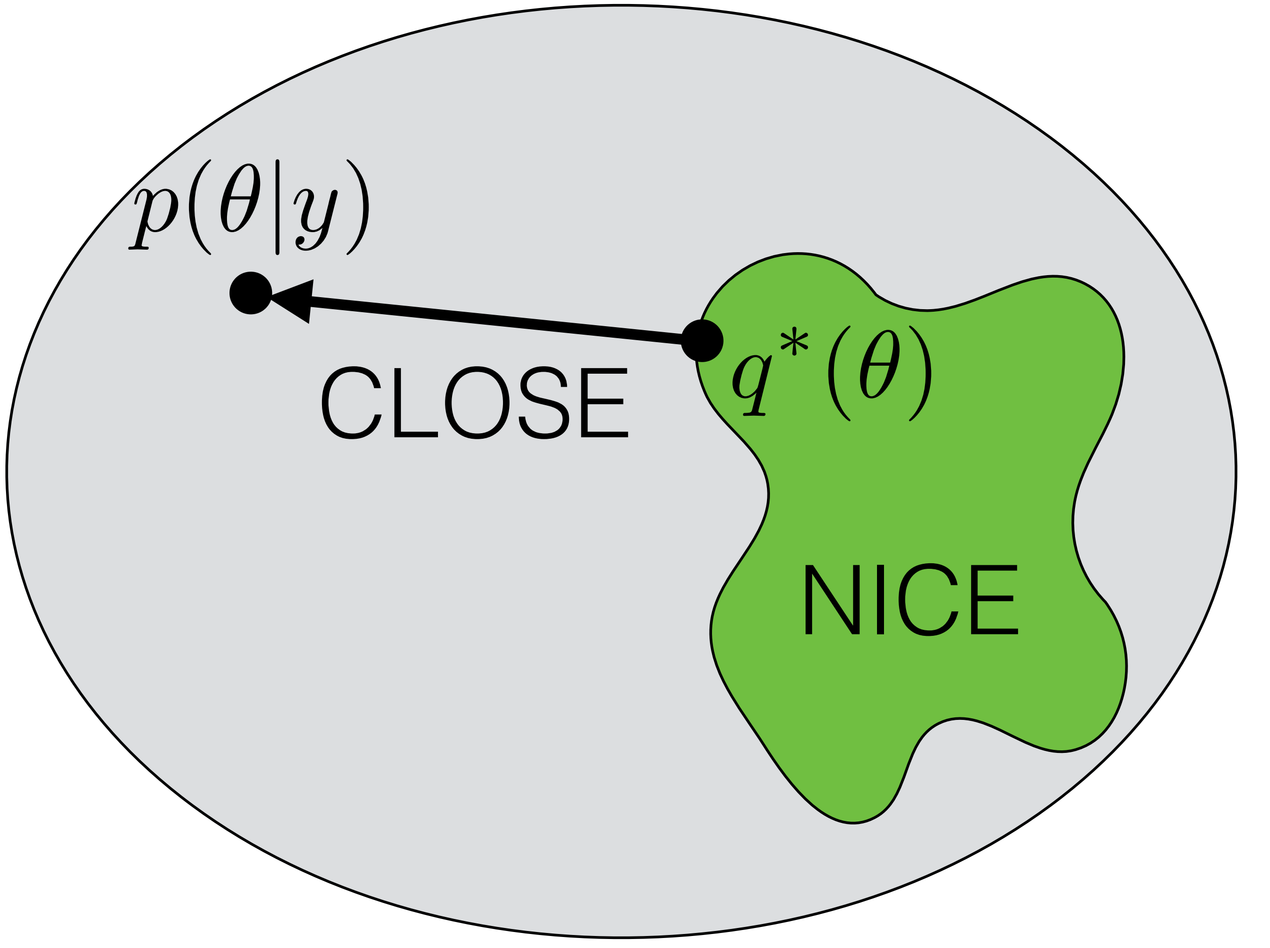

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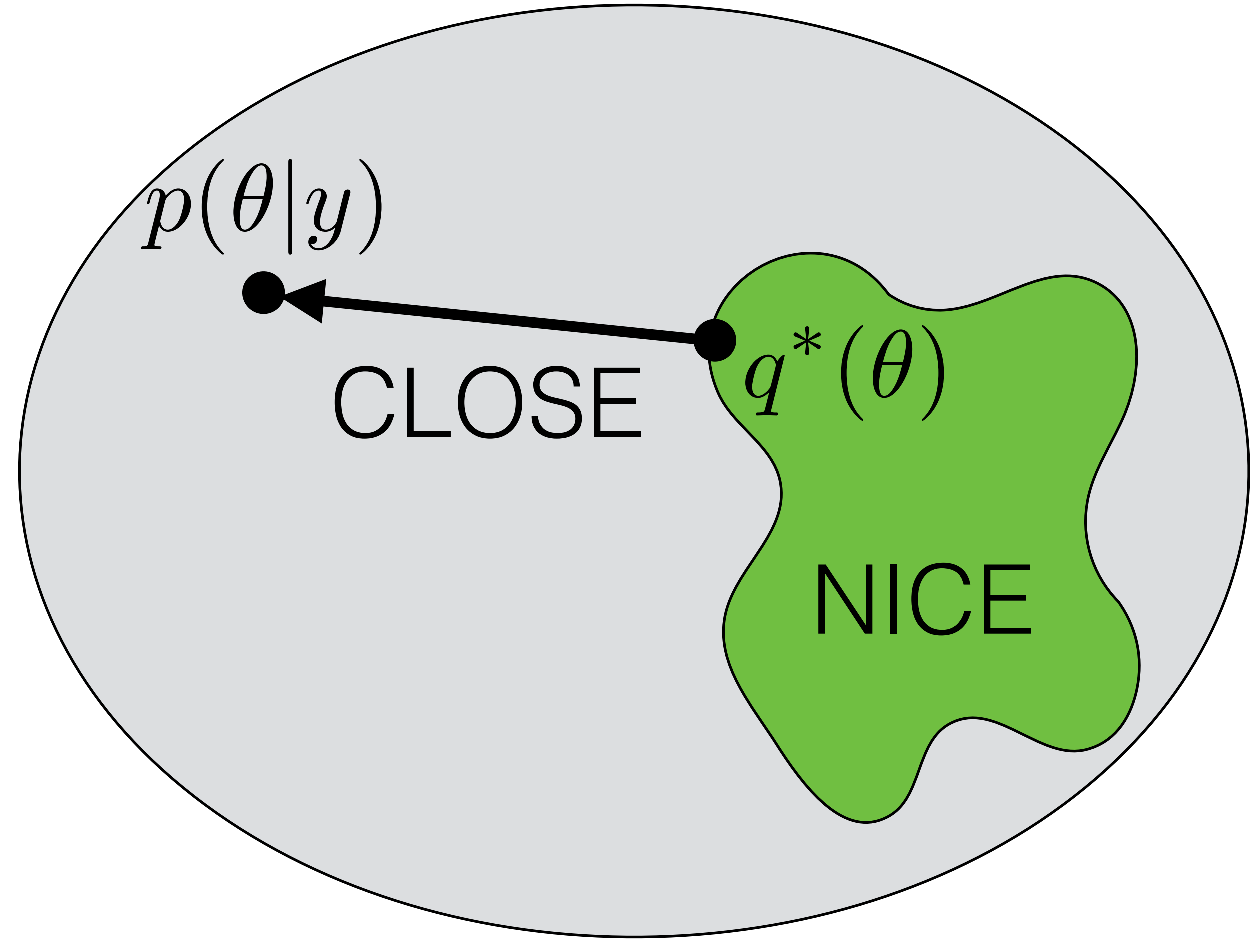

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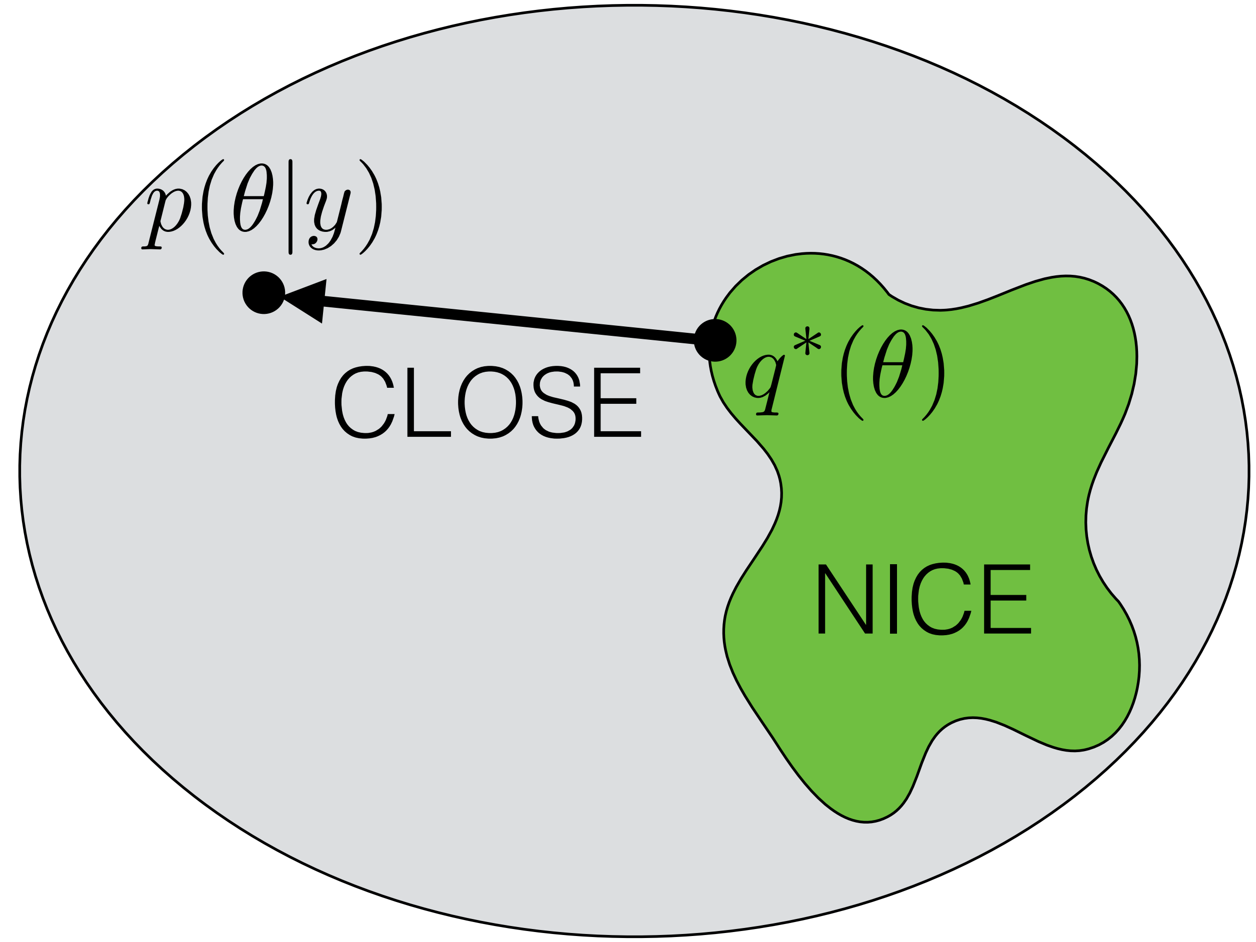

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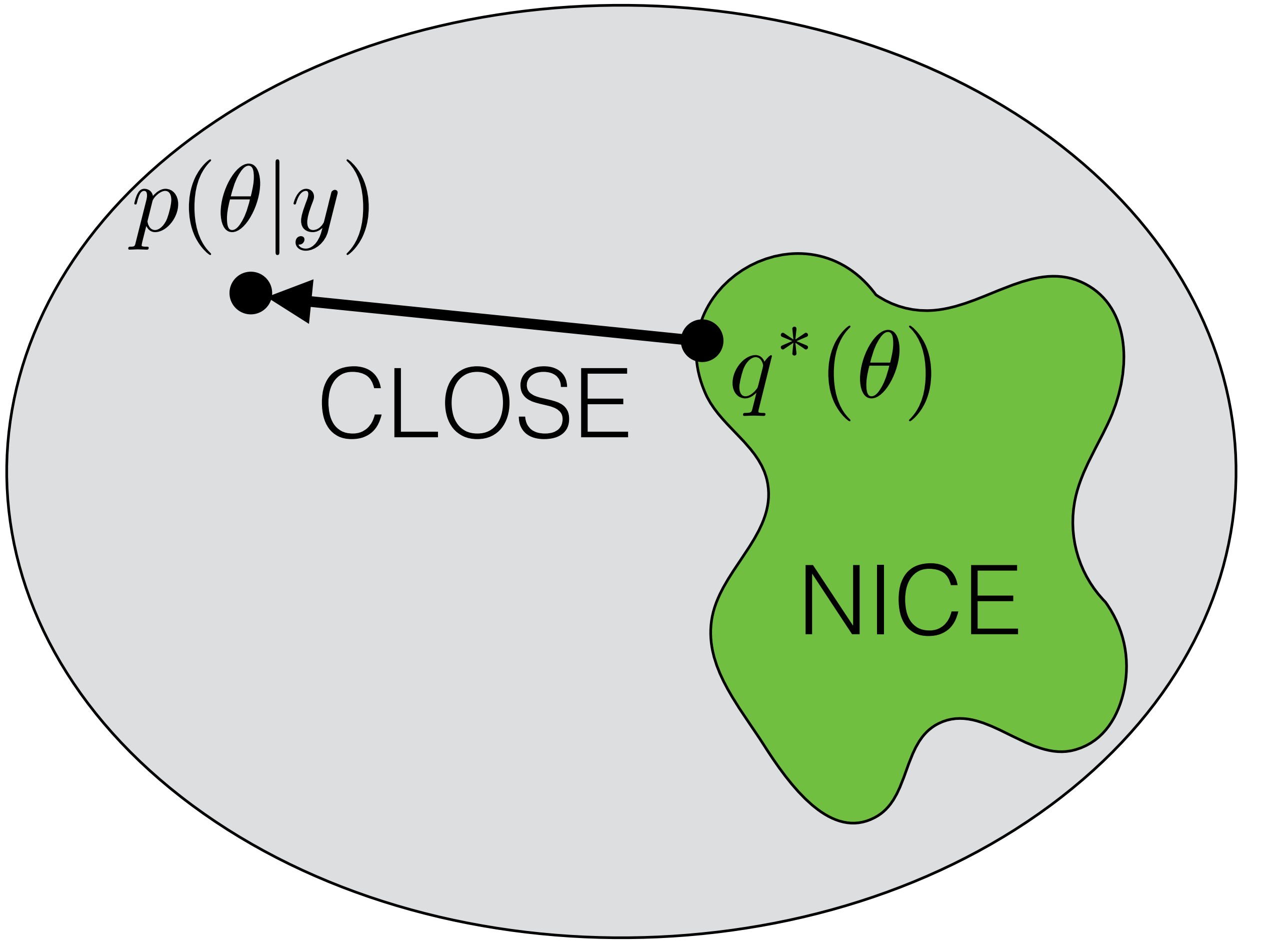

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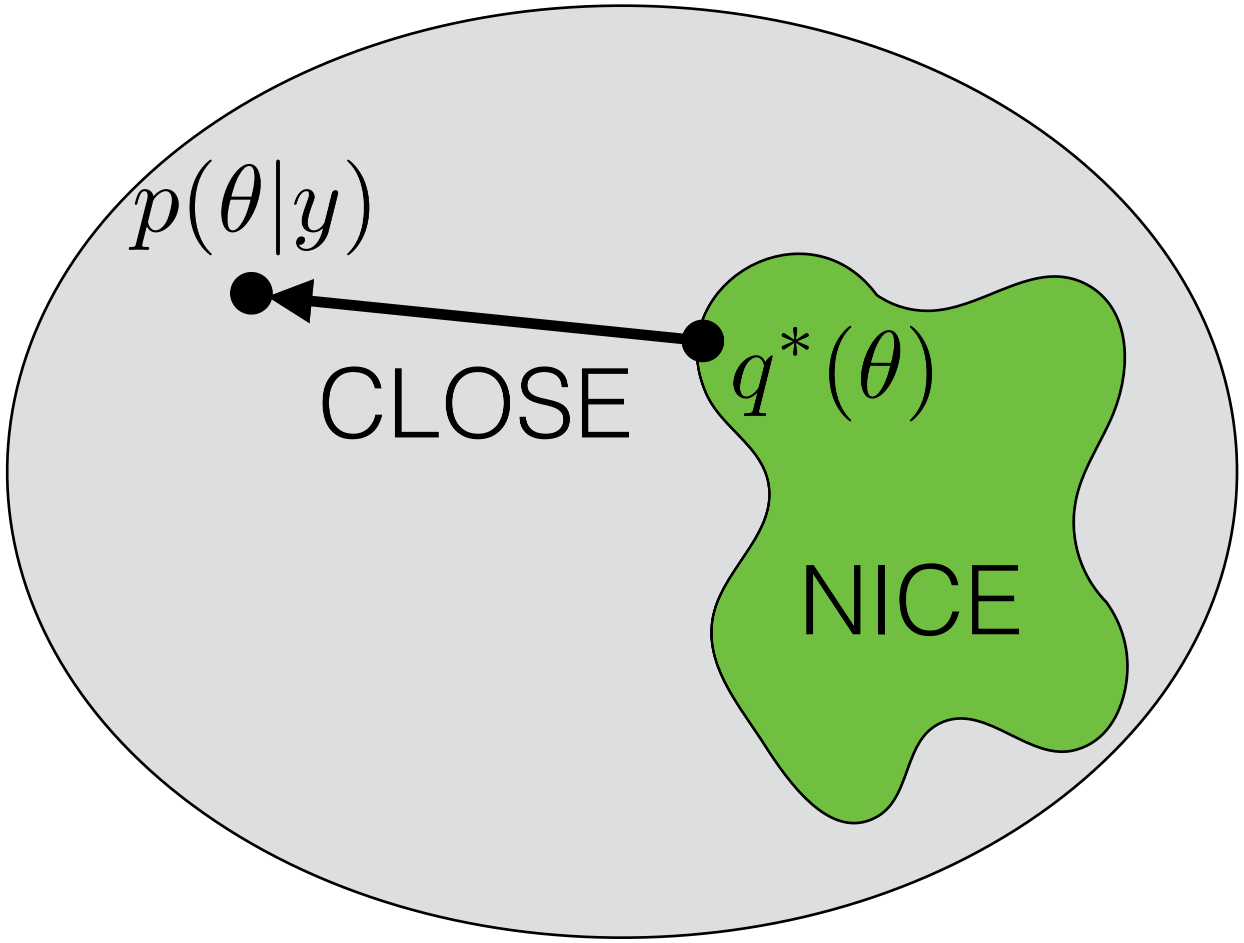

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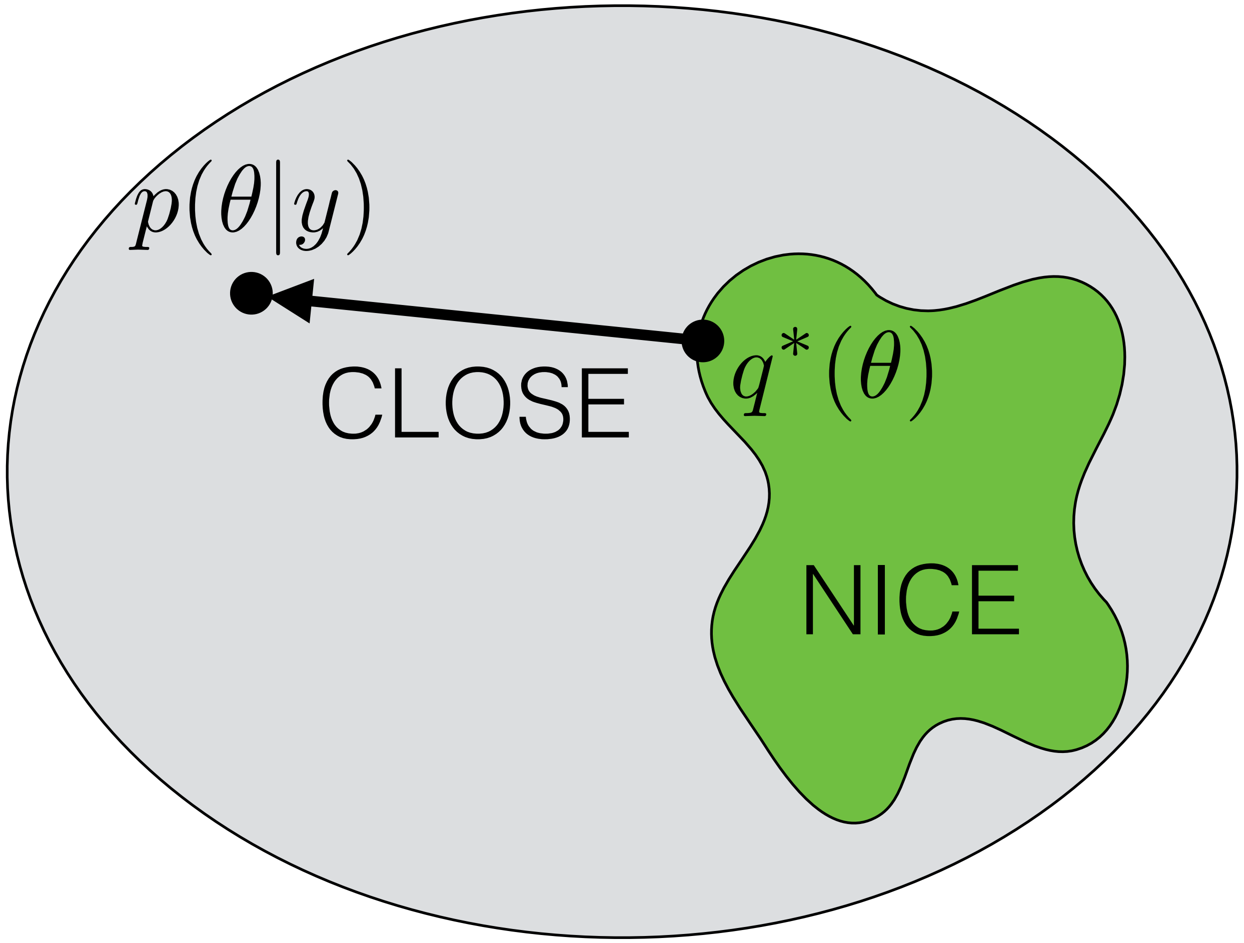

Choose "NICE" distributions

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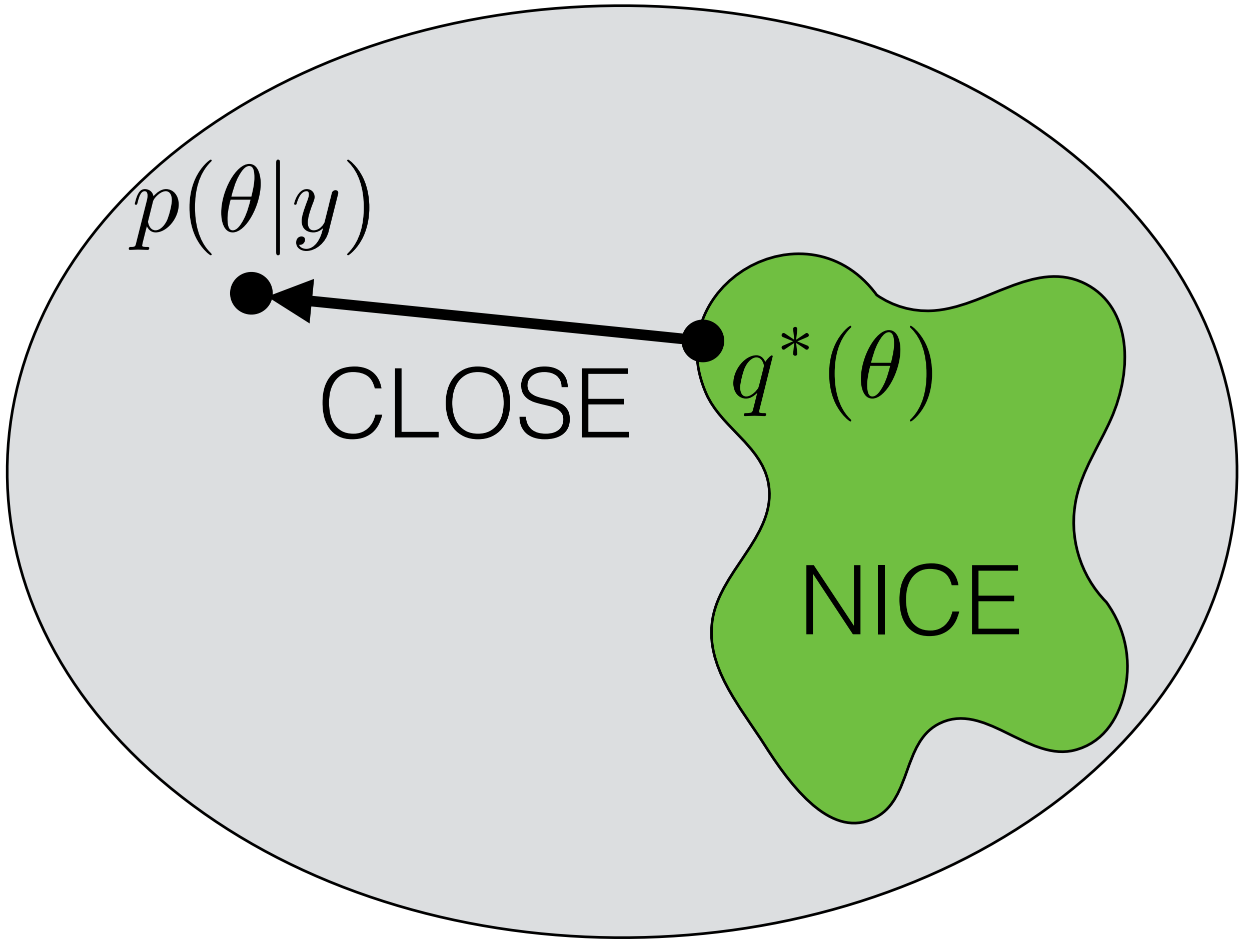

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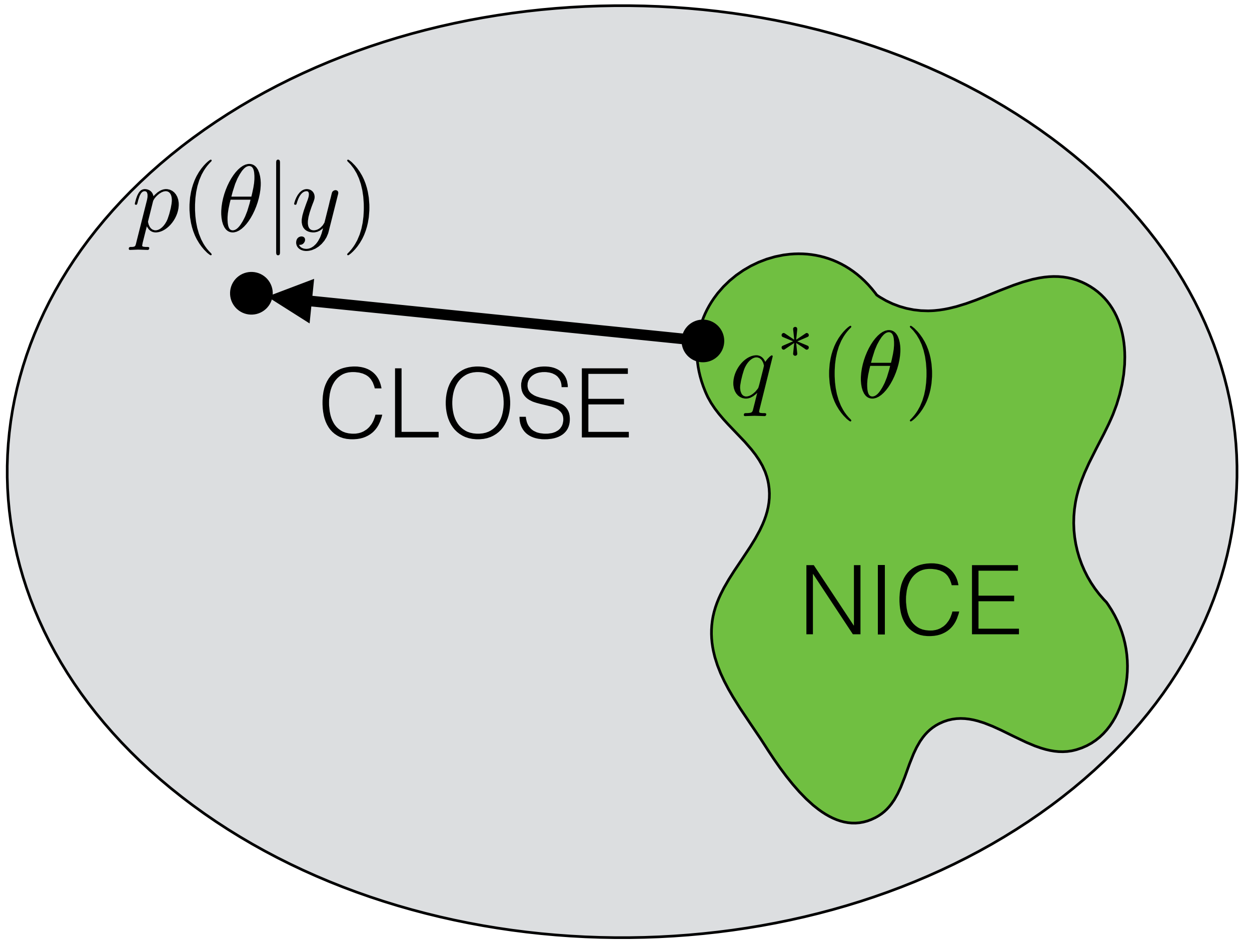

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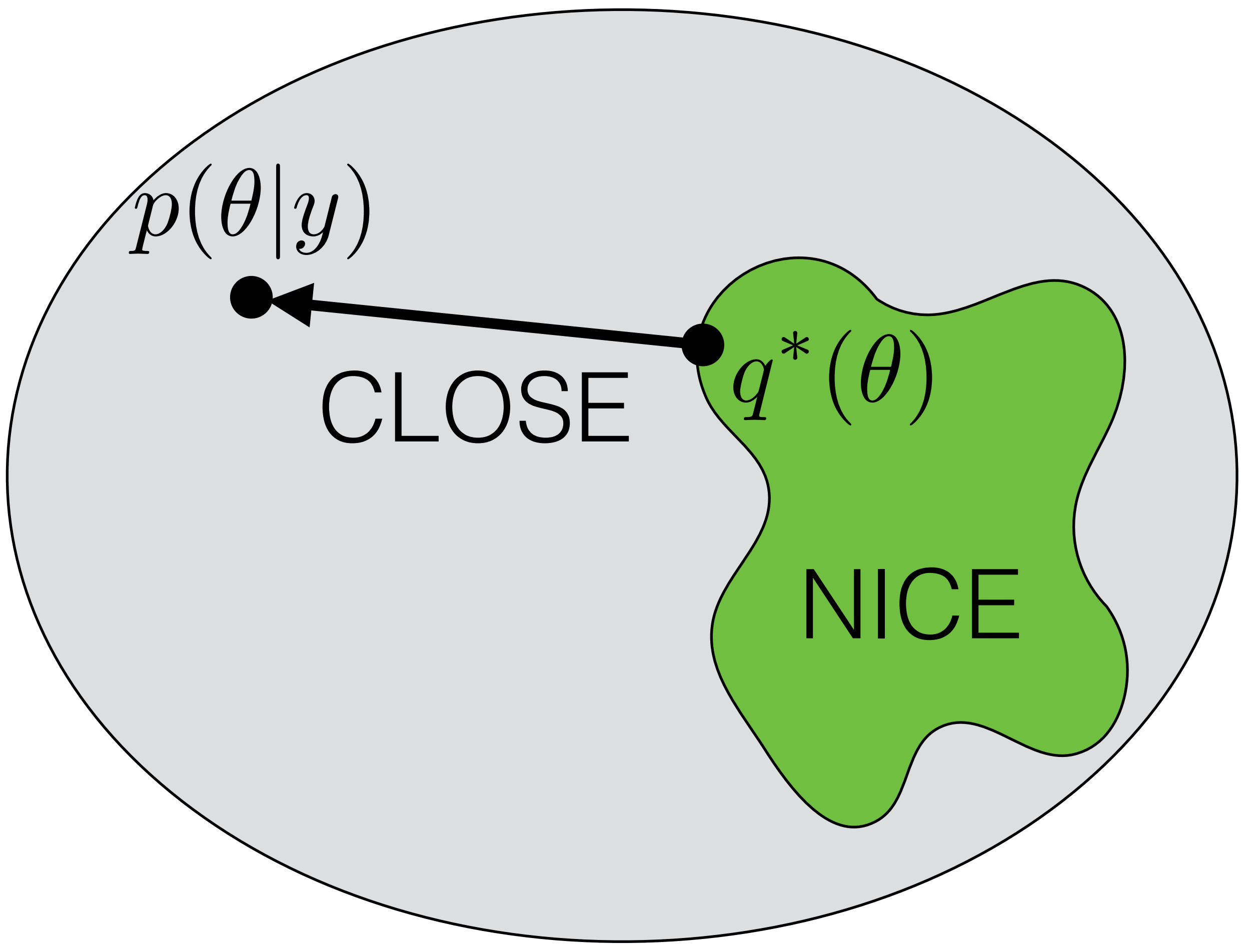

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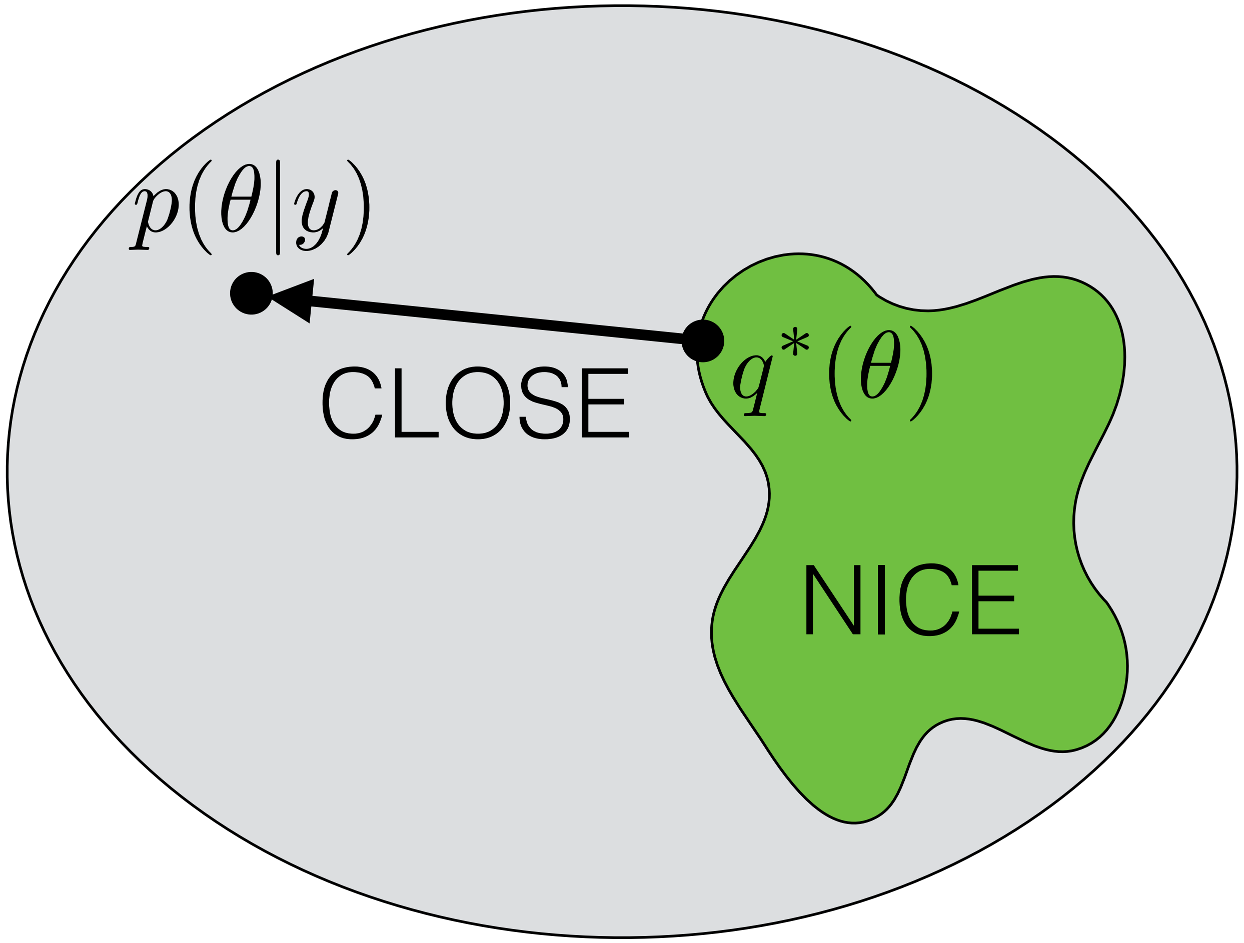

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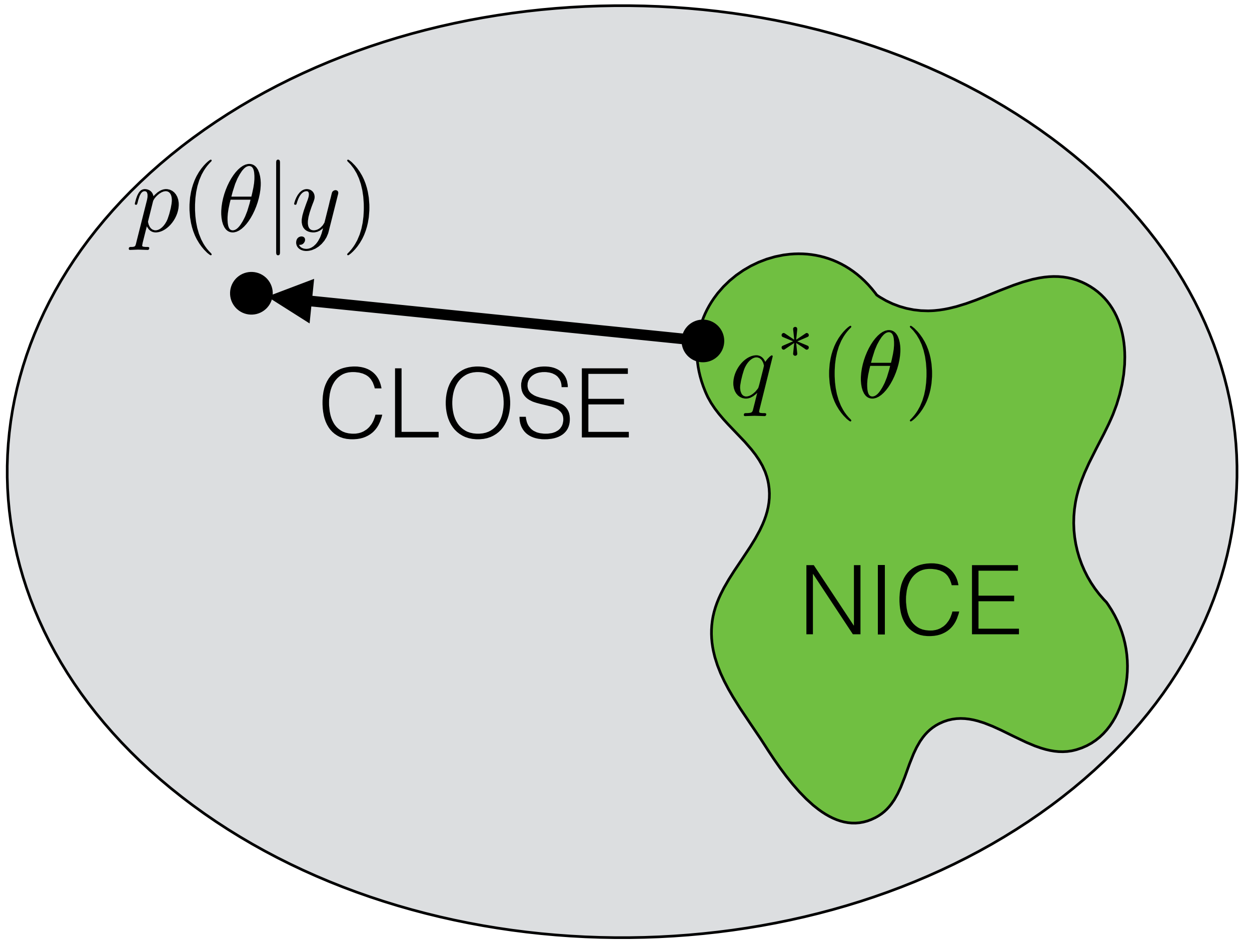

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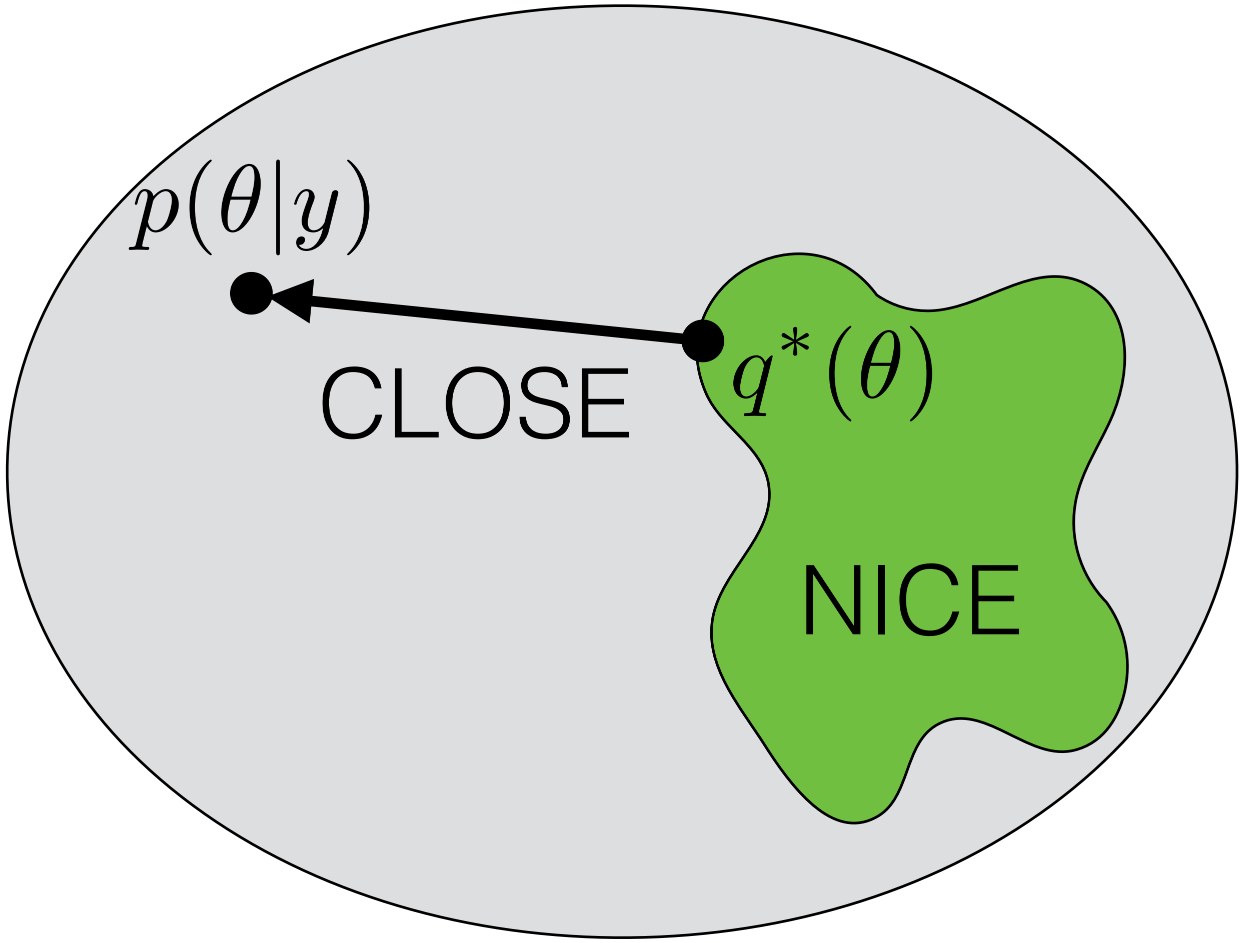

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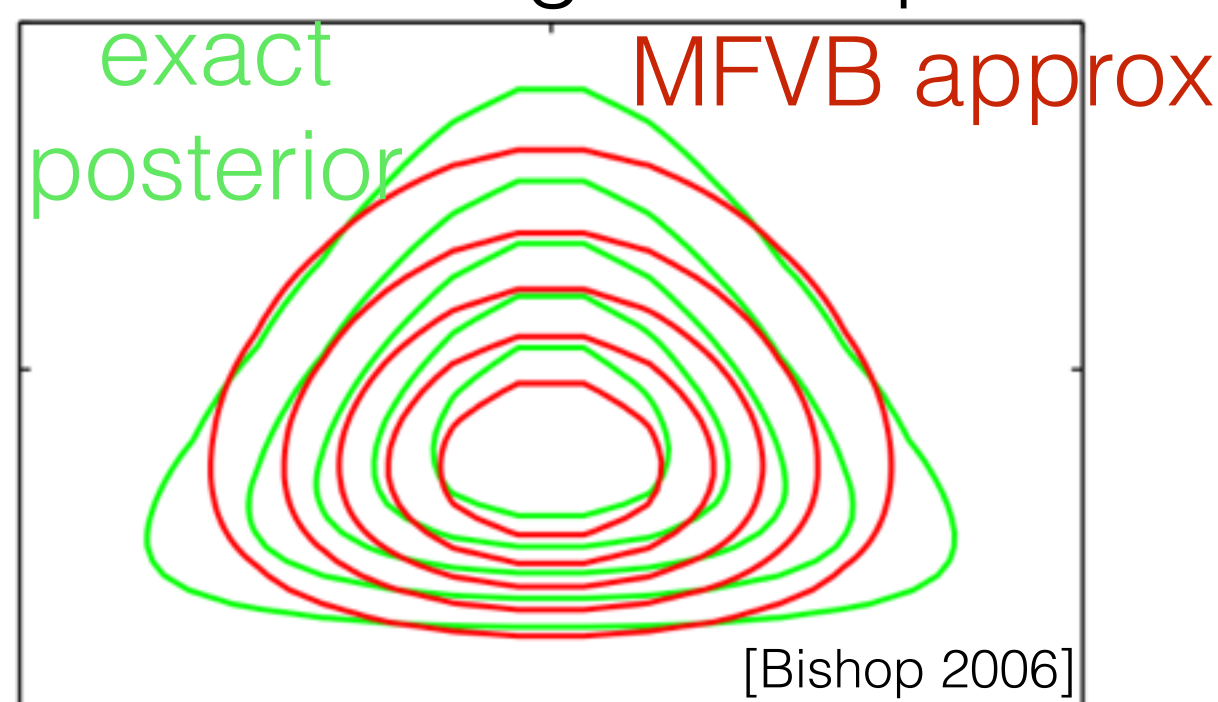

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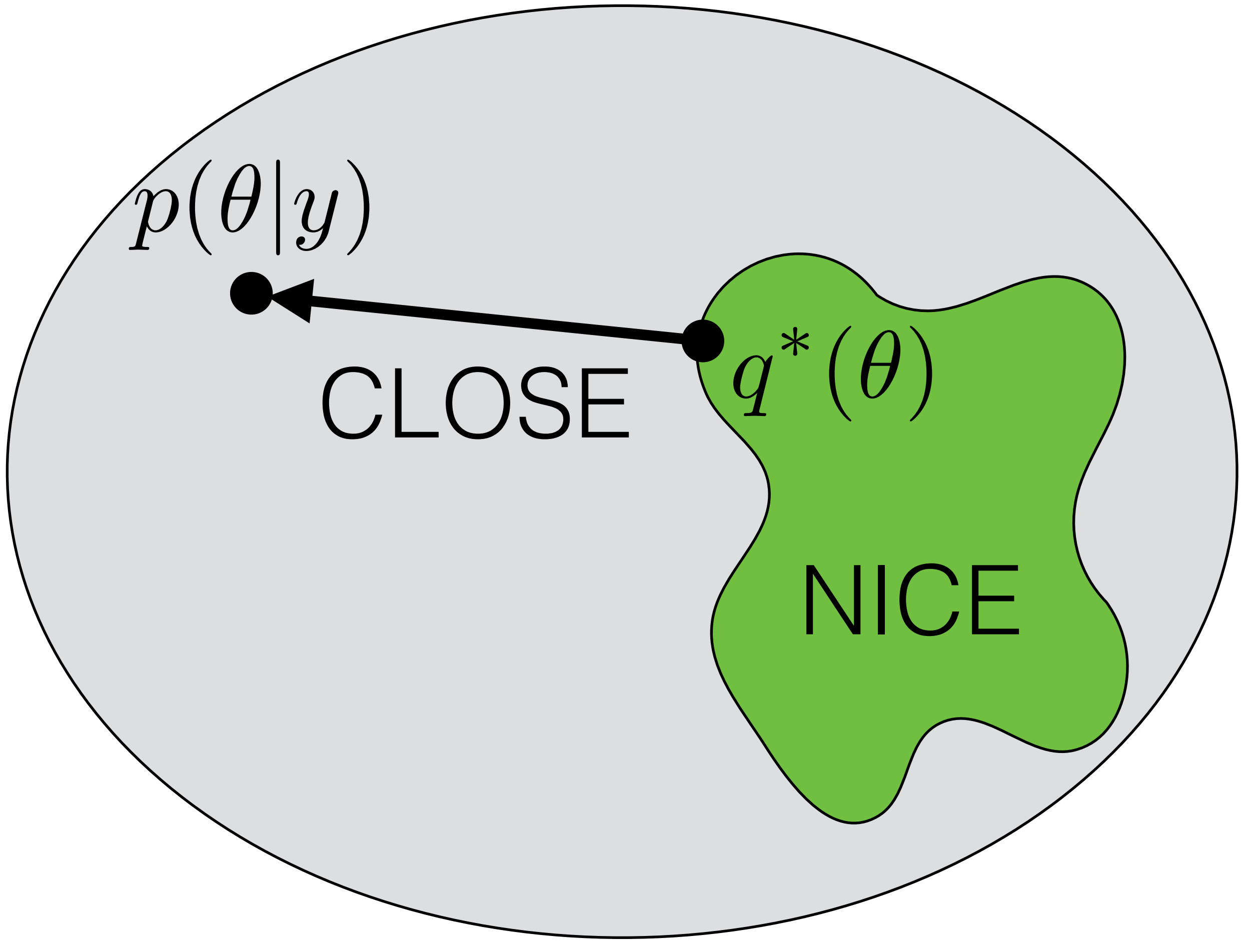

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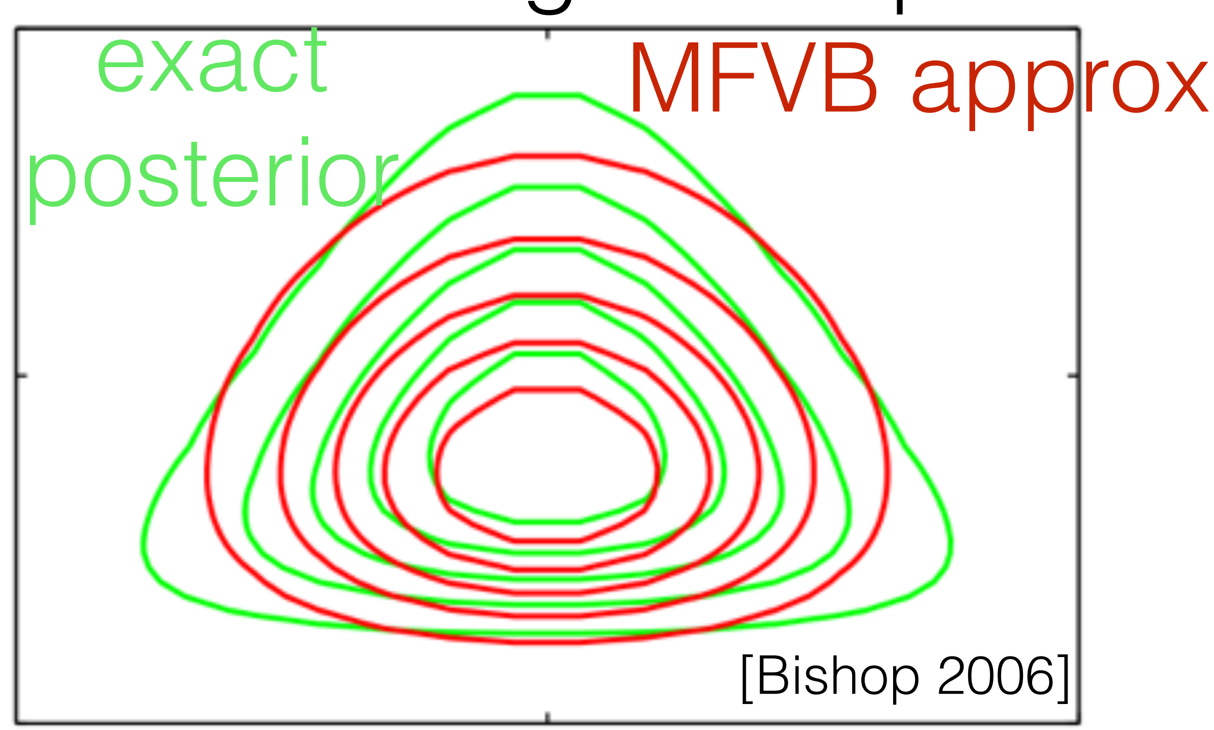

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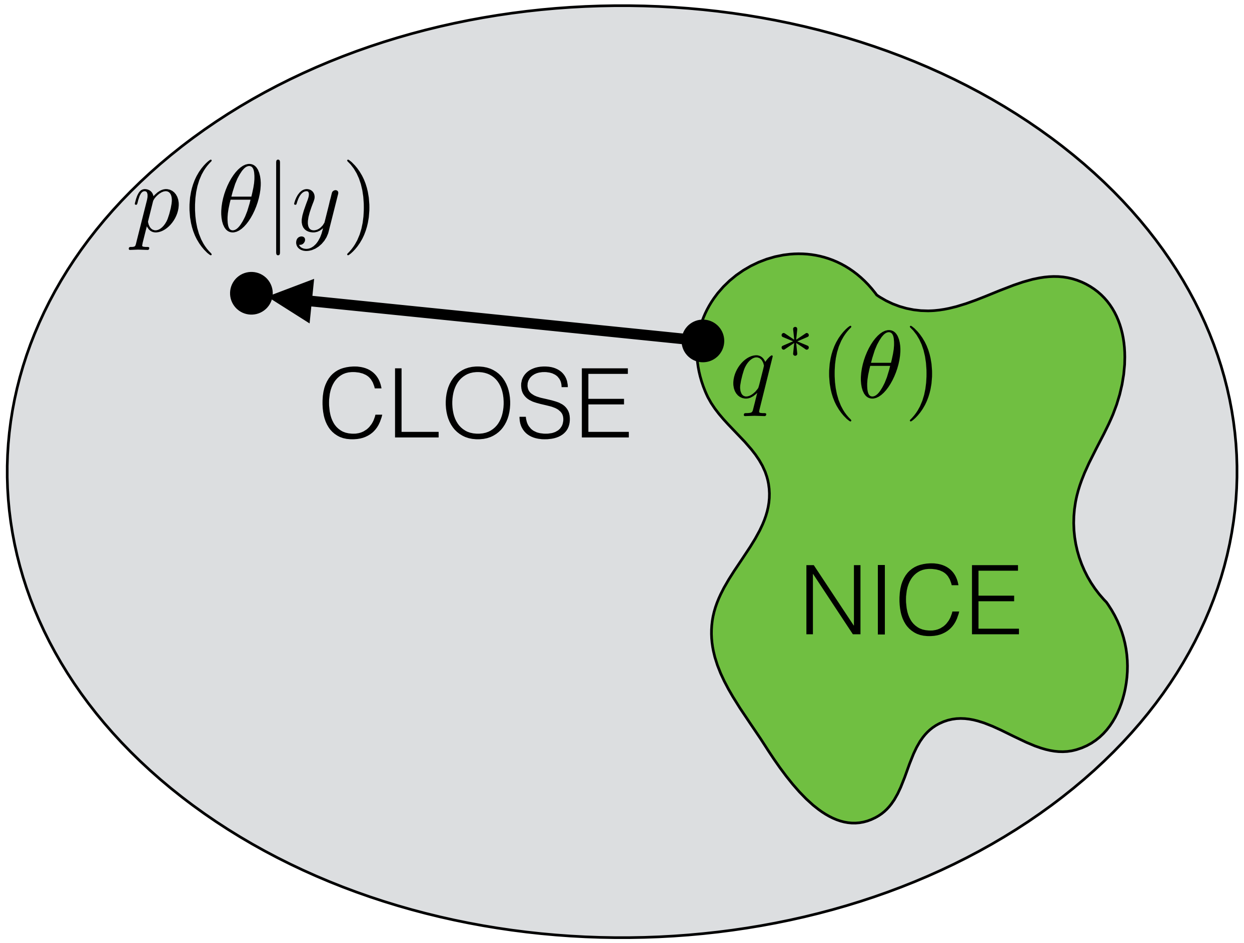

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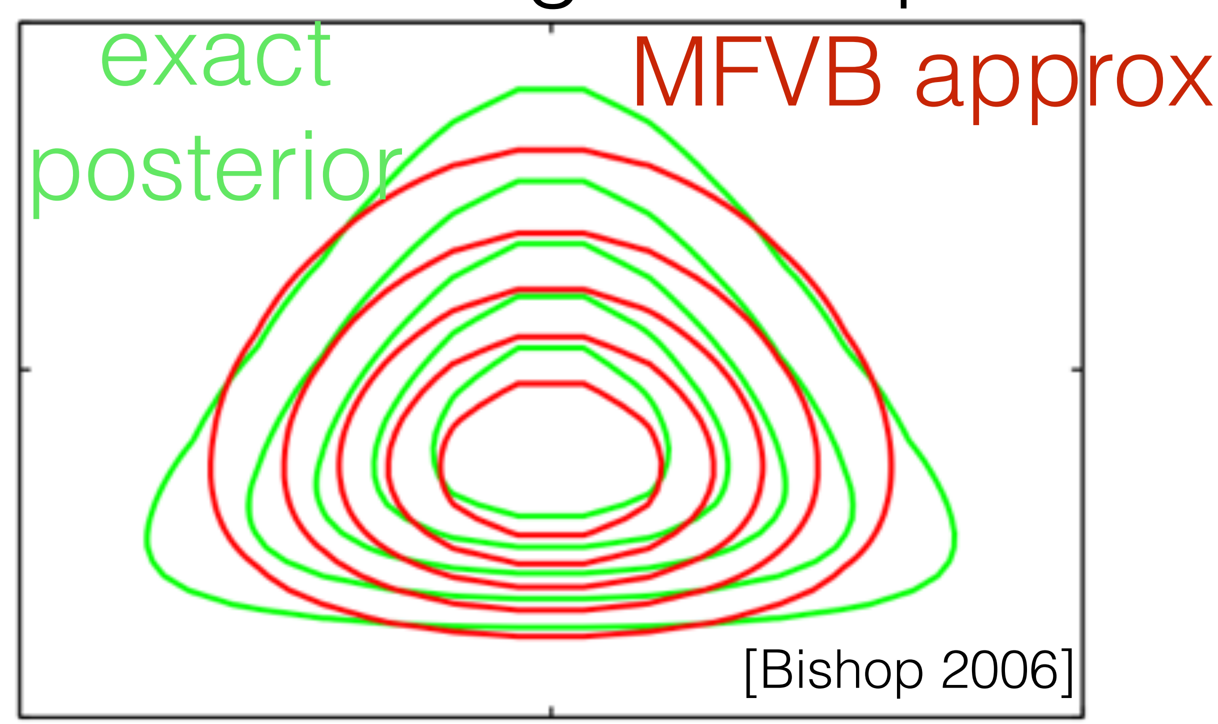

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- One option: Coordinate descent in $q_1, \ldots, q_J$

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Use q^* to approximate $p(\cdot|y)$

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- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Roadmap

- Bayes & Approximate Bayes review
- What is:
  - Variational Bayes (VB)
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- Why use MFVB?
- When can we trust MFVB?
- Where do we go from here?

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- Catalogued midge wing lengths (mm) $y = (y_1, \dots, y_N)$

[CSIRO 2004]

[Hoff 2009; Grogan, Wirth 1981; MacKay 2003; Bishop 2006]

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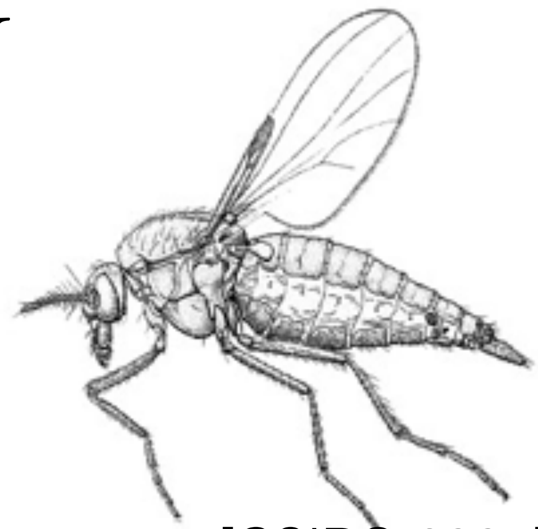

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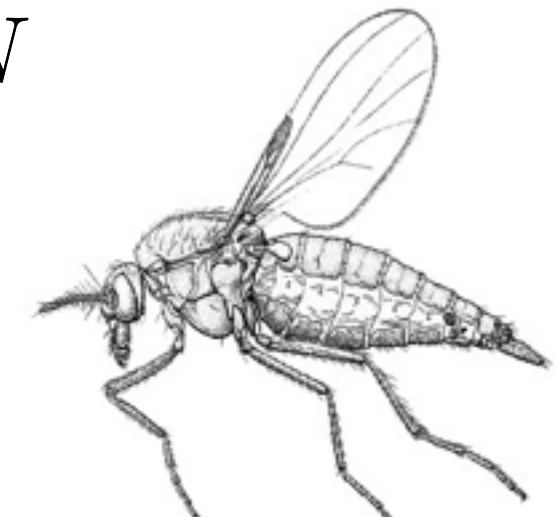

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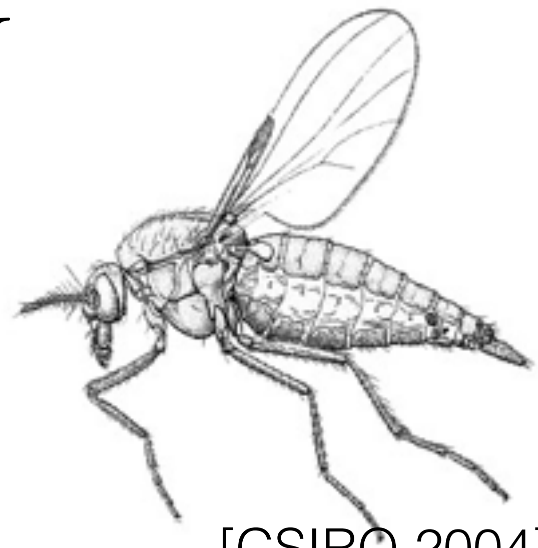

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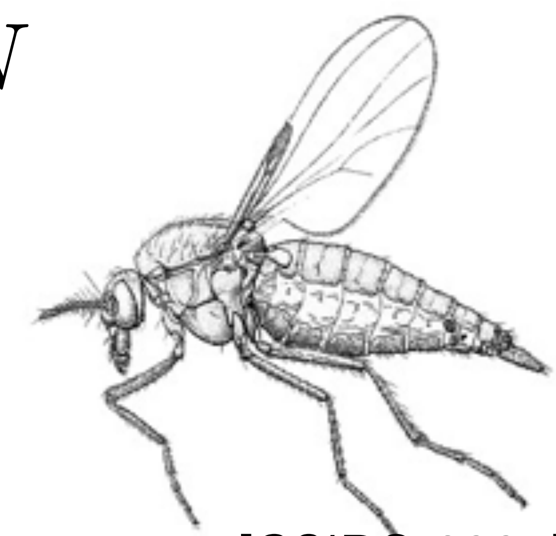

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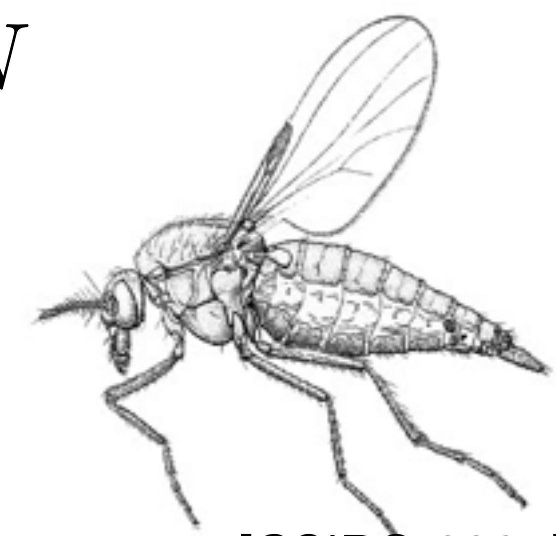

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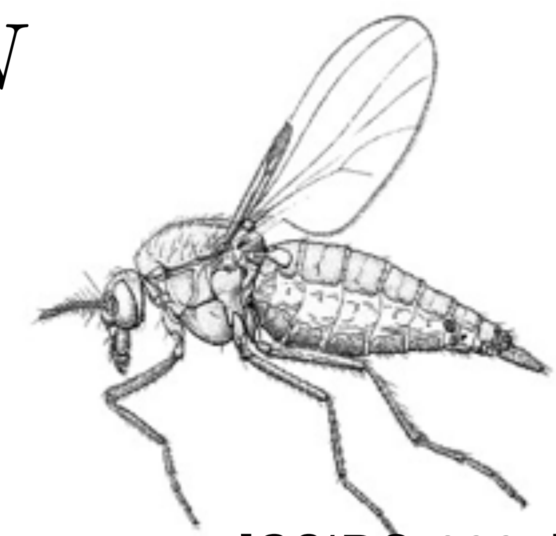

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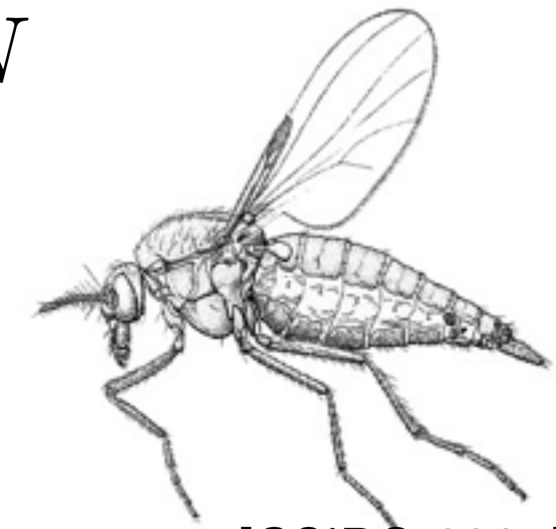

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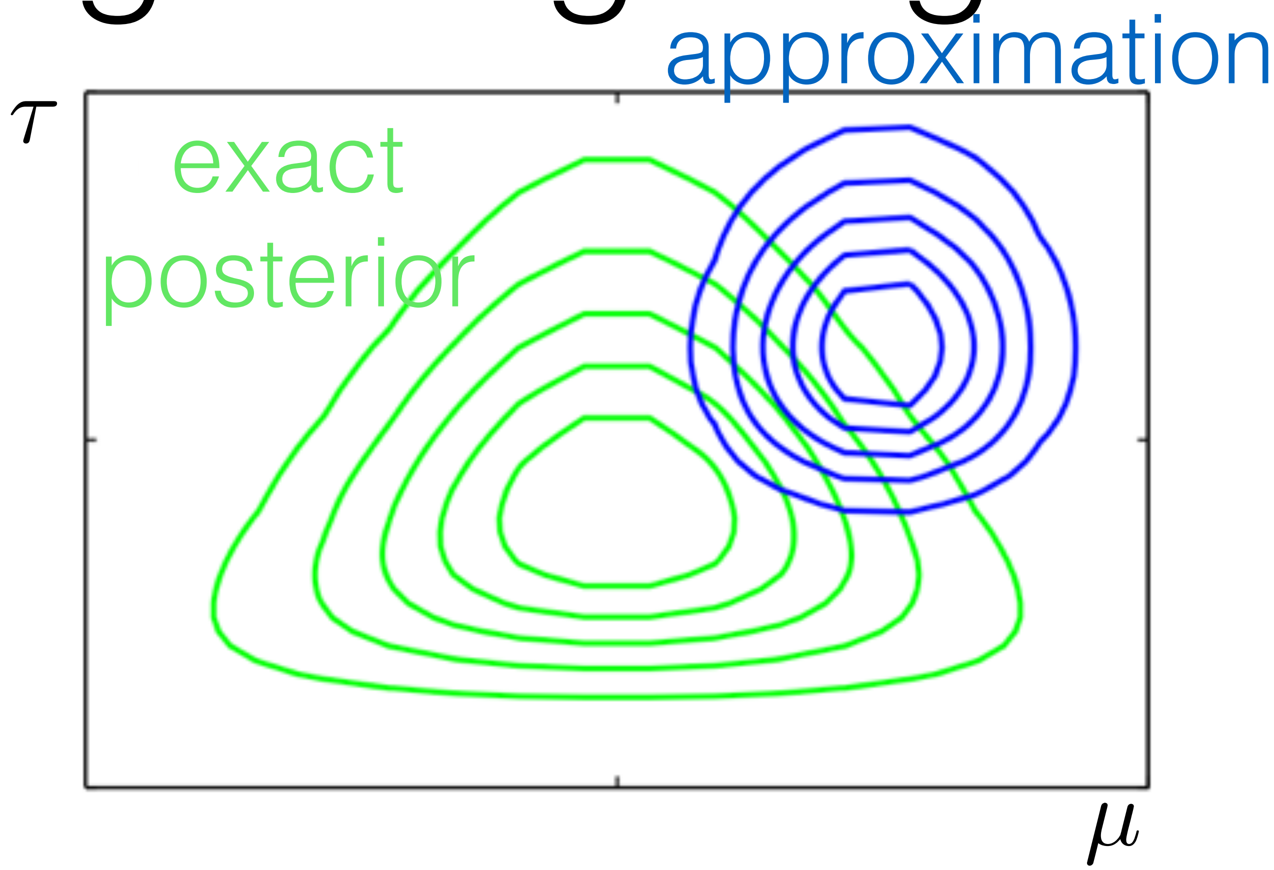

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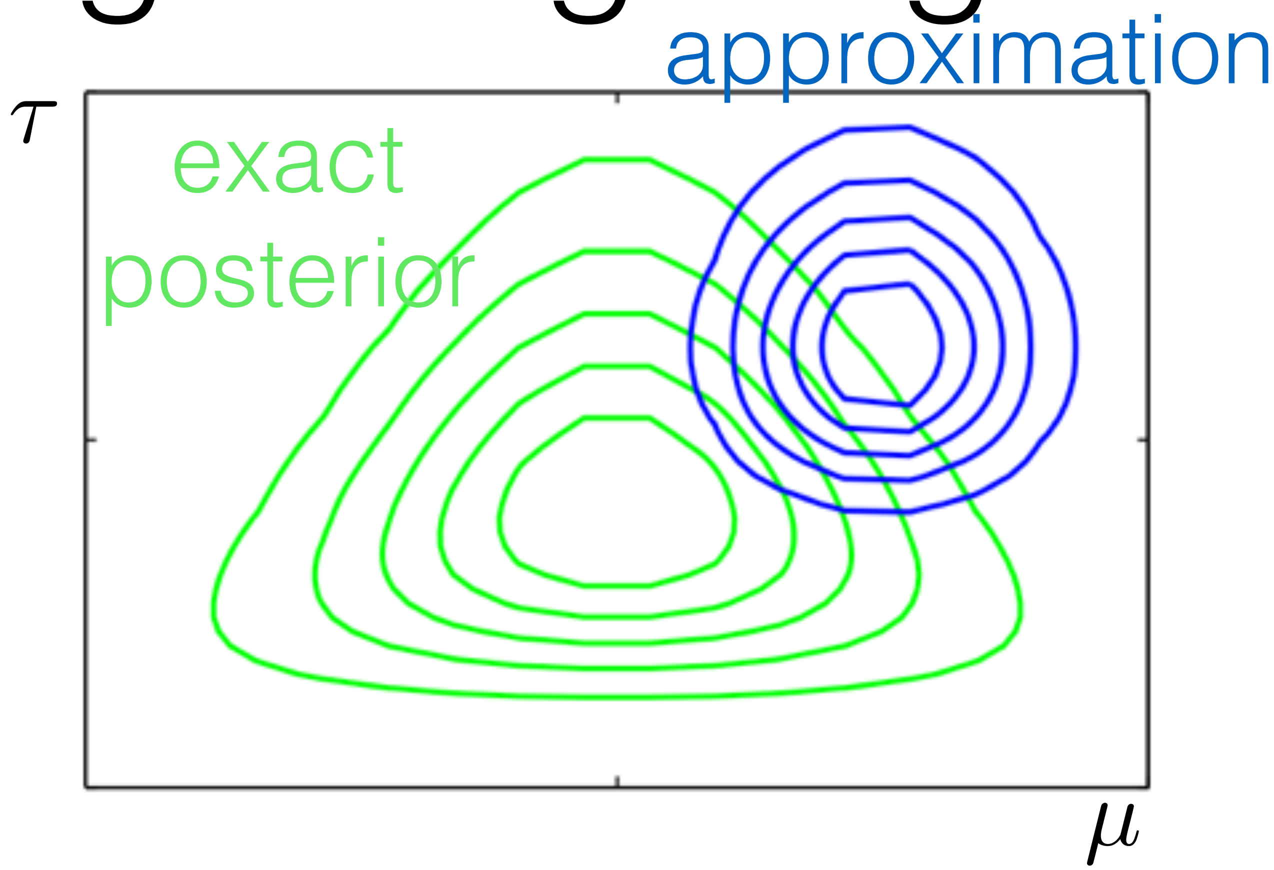

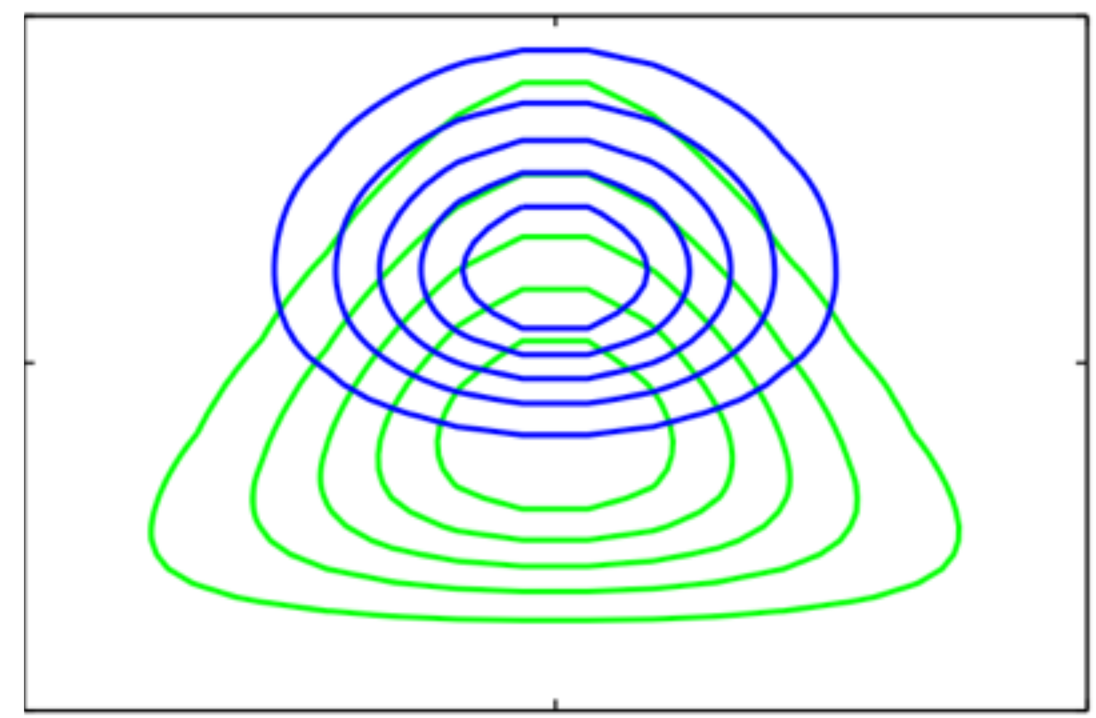

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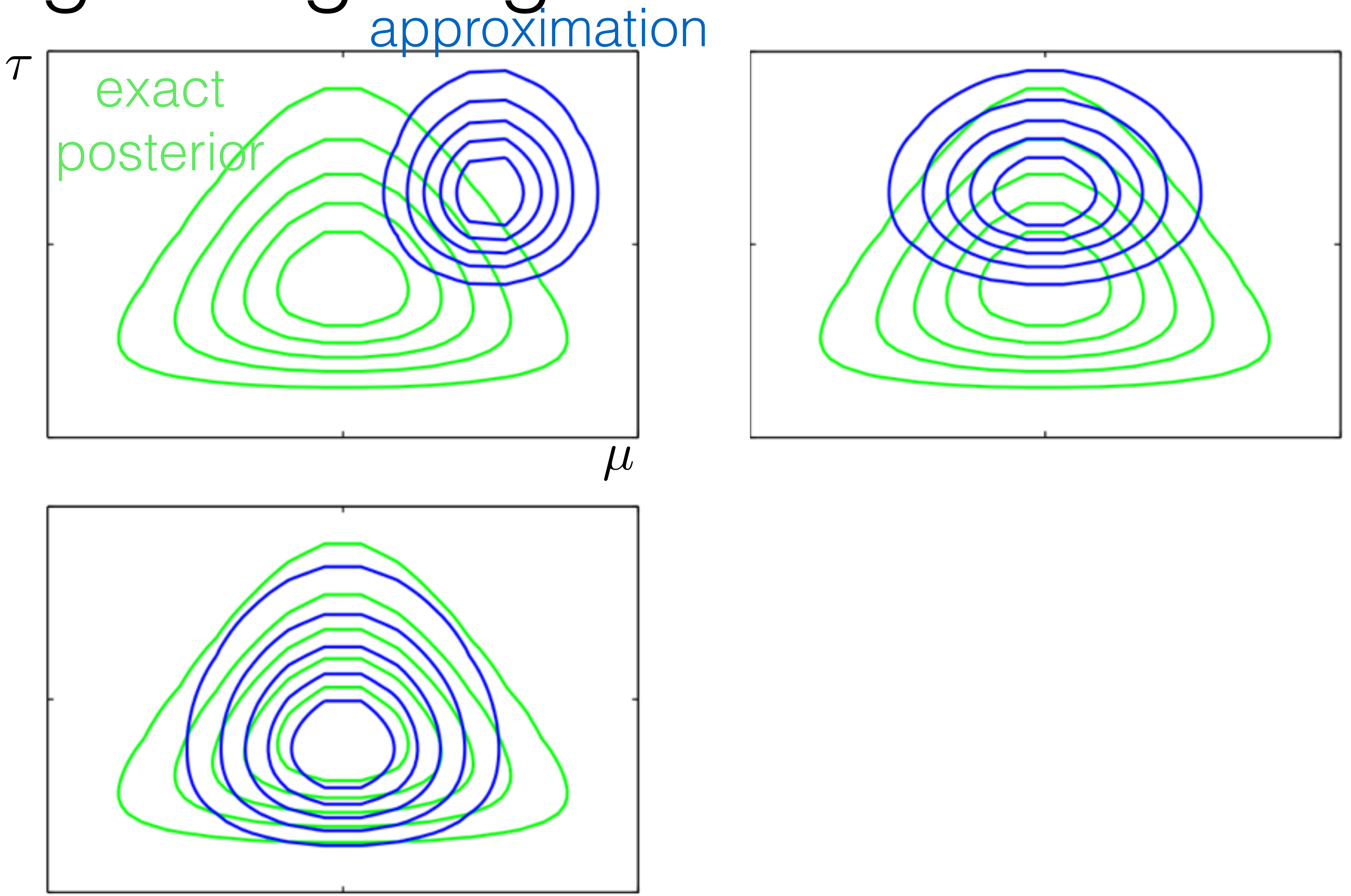

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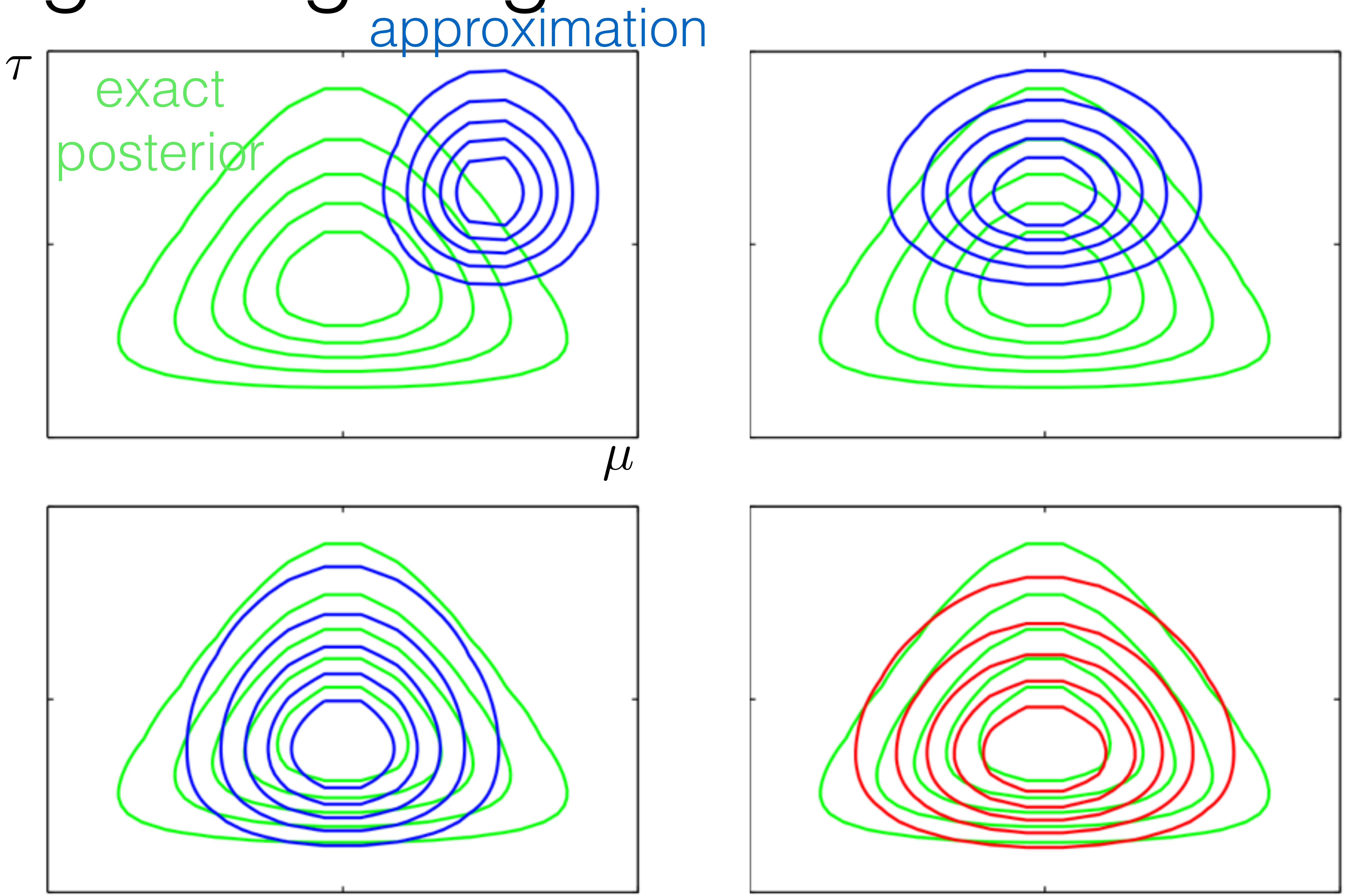

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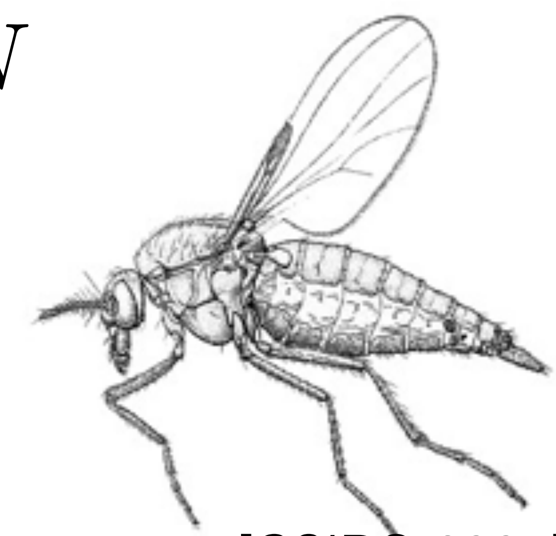

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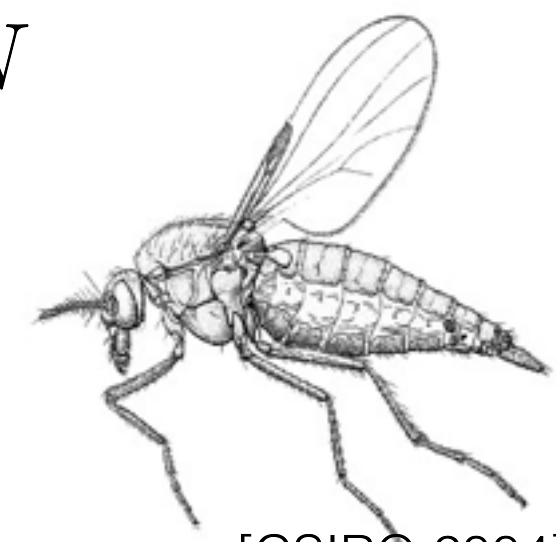

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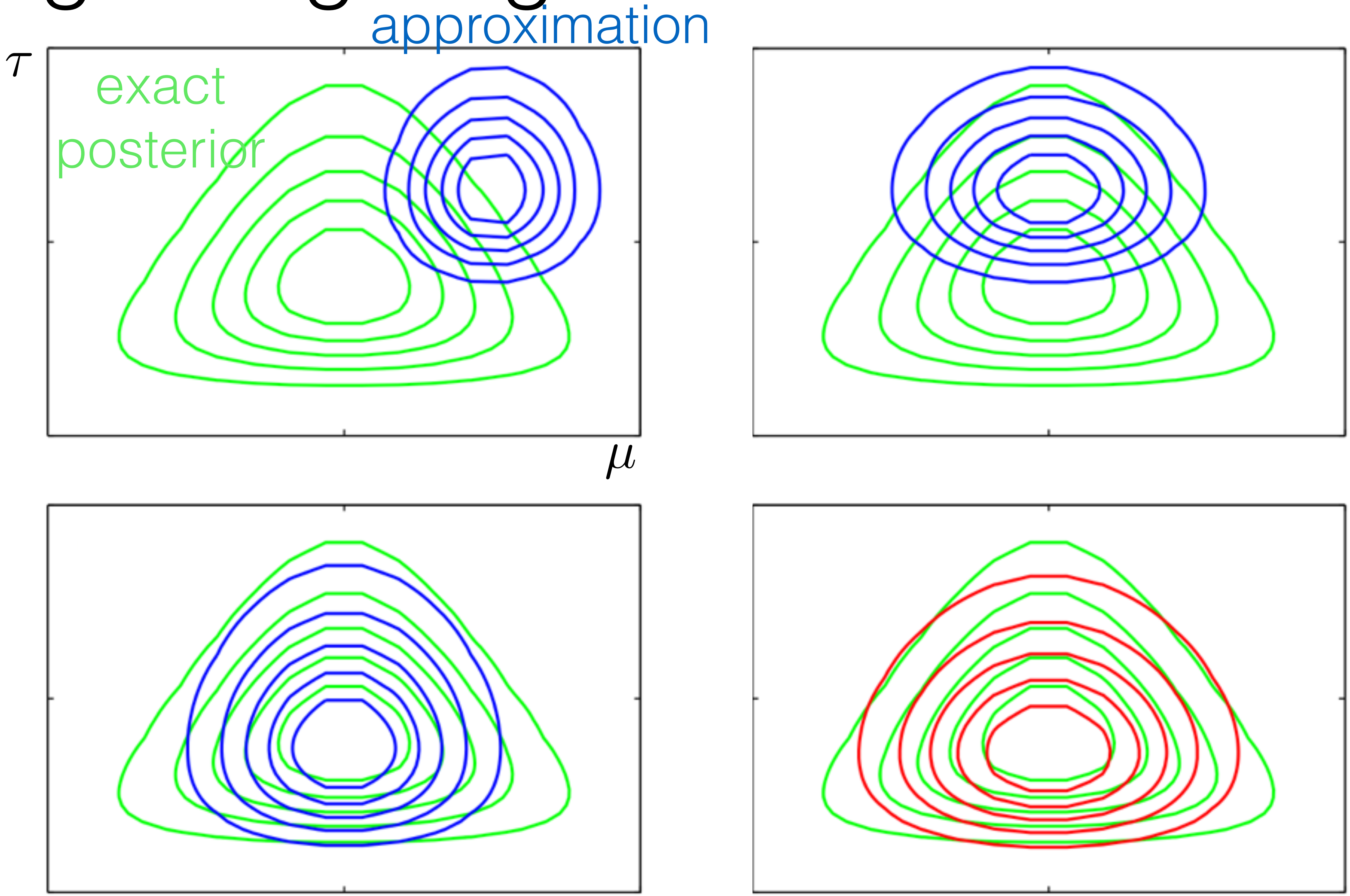

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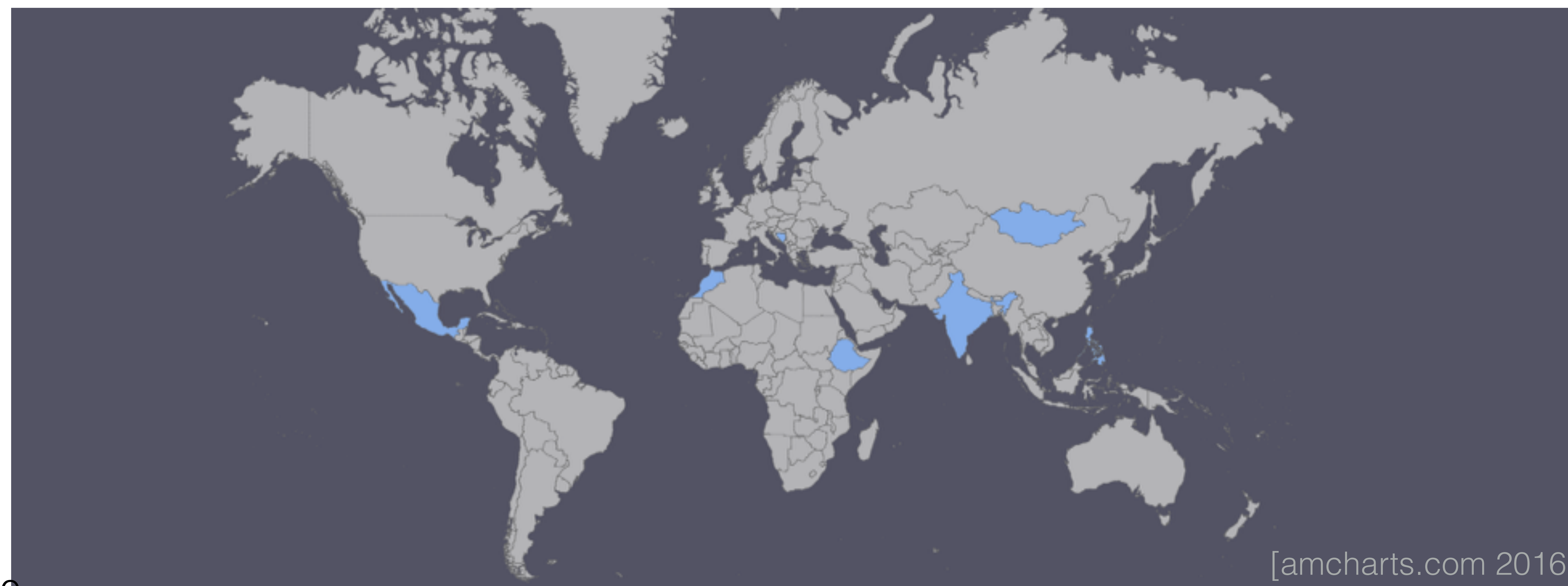
Microcredit Experiment



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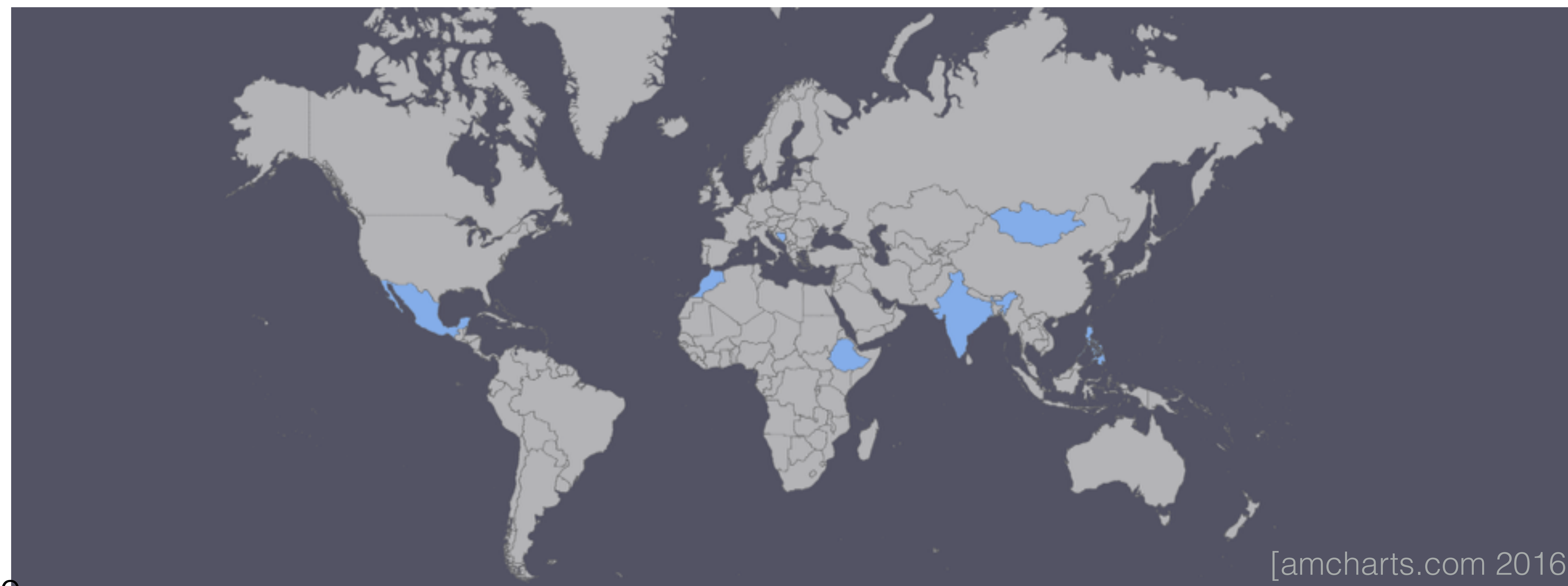

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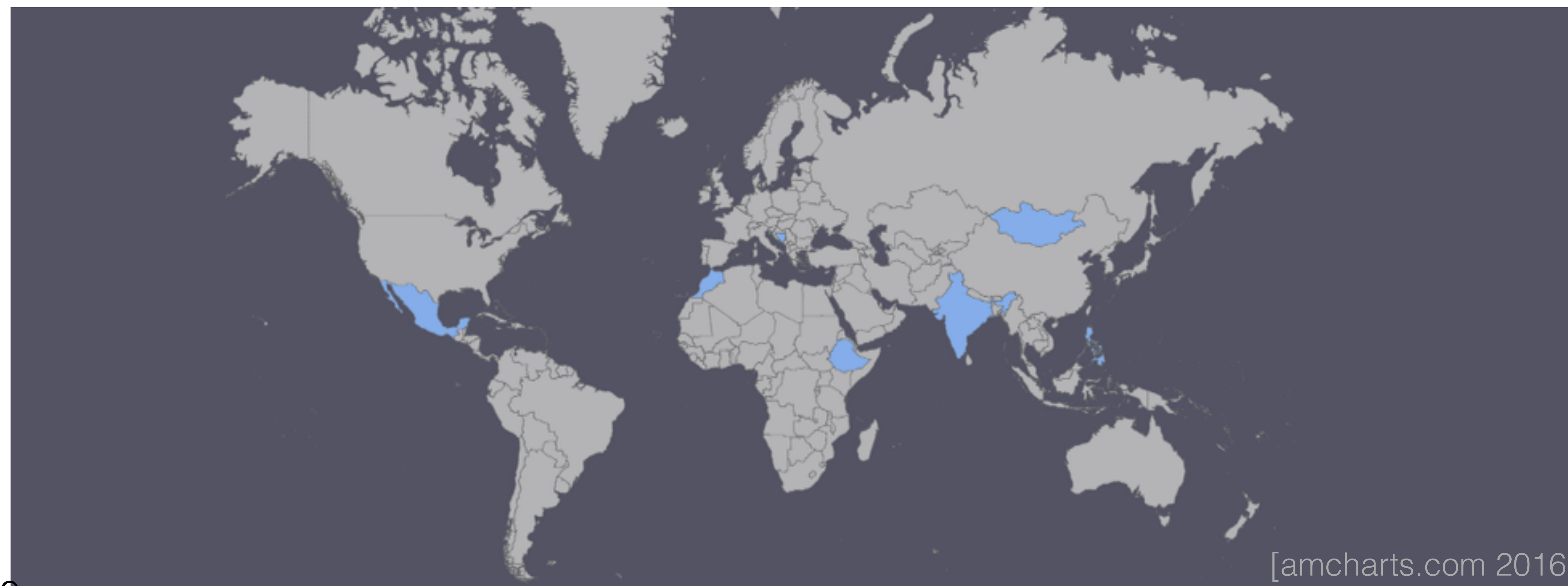

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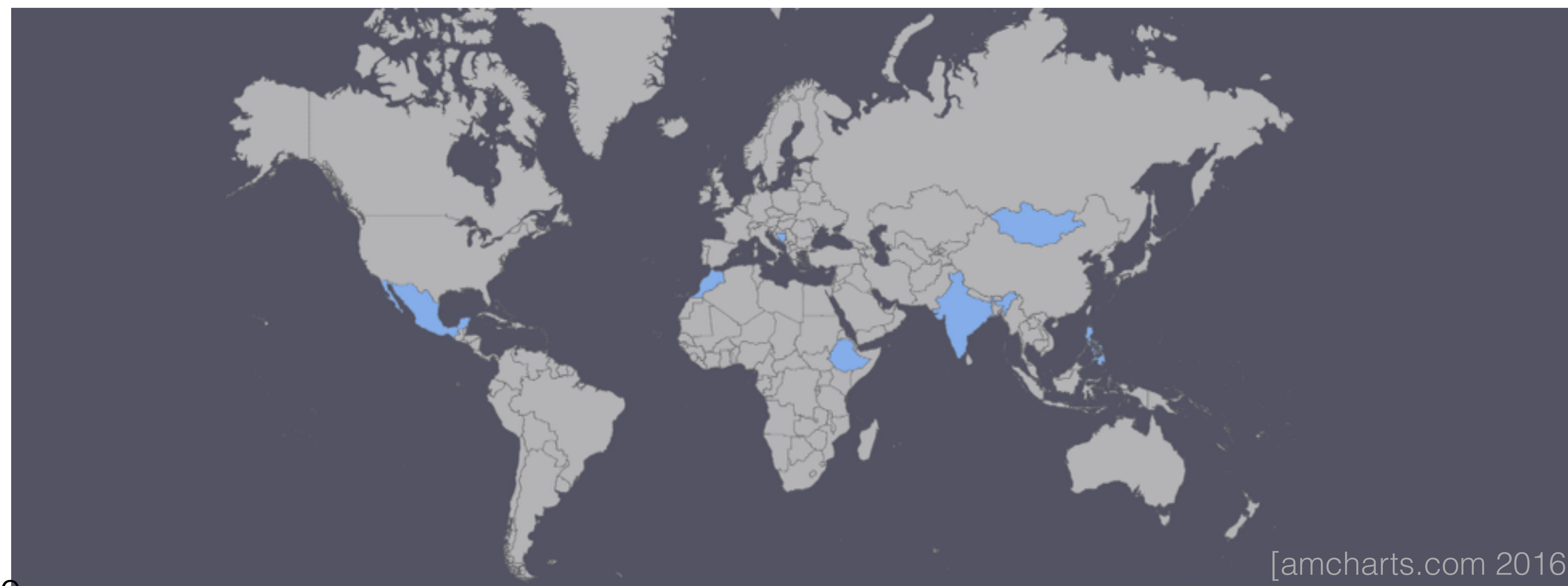

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$$y_{kn}$$

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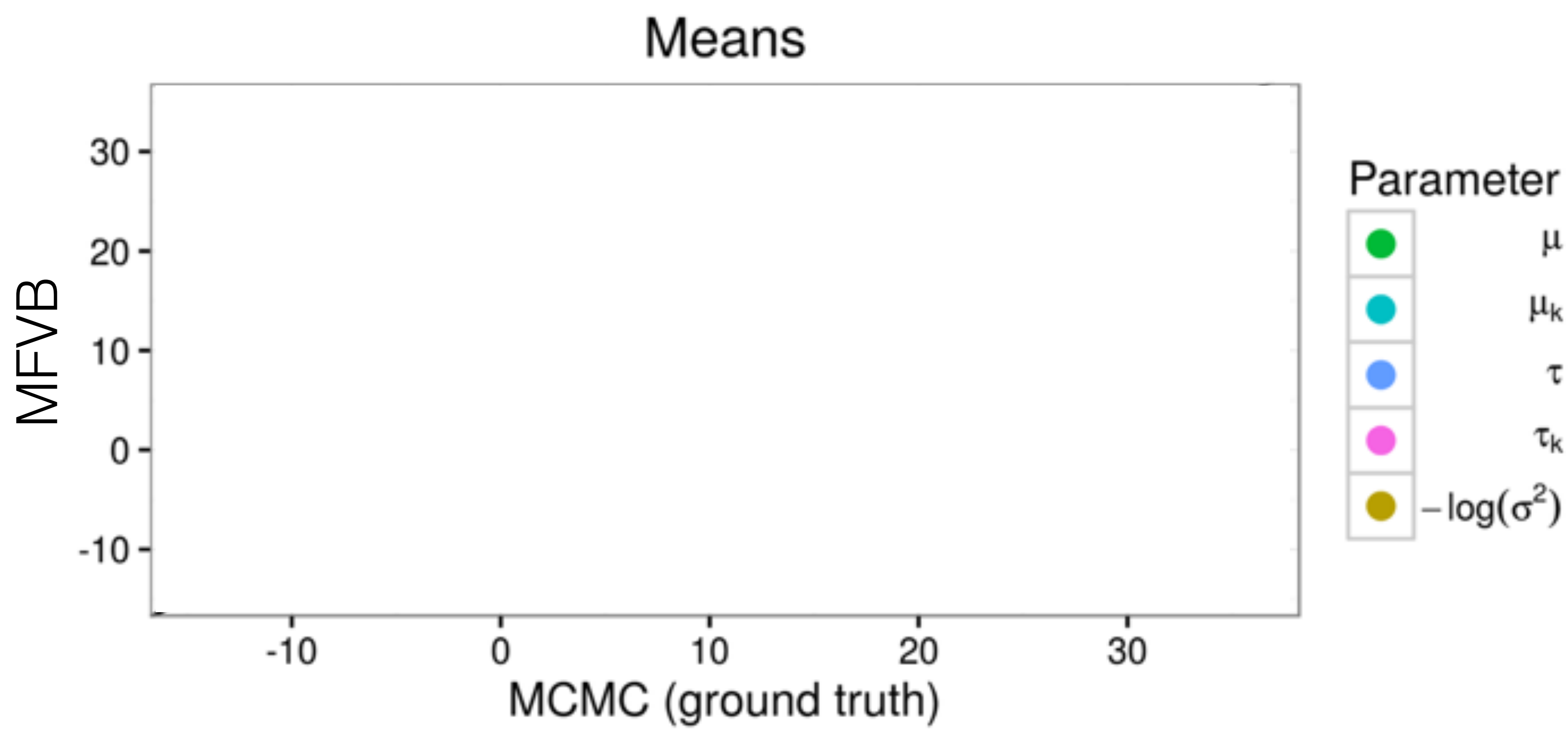
$$\sigma_k^{-2} \overset{iid}{\sim} \Gamma(a, b) \qquad C \sim \mathrm{Sep\&LKJ}(\eta, c, d)$$

Microcredit

MFVB: How will we know if it’s working?

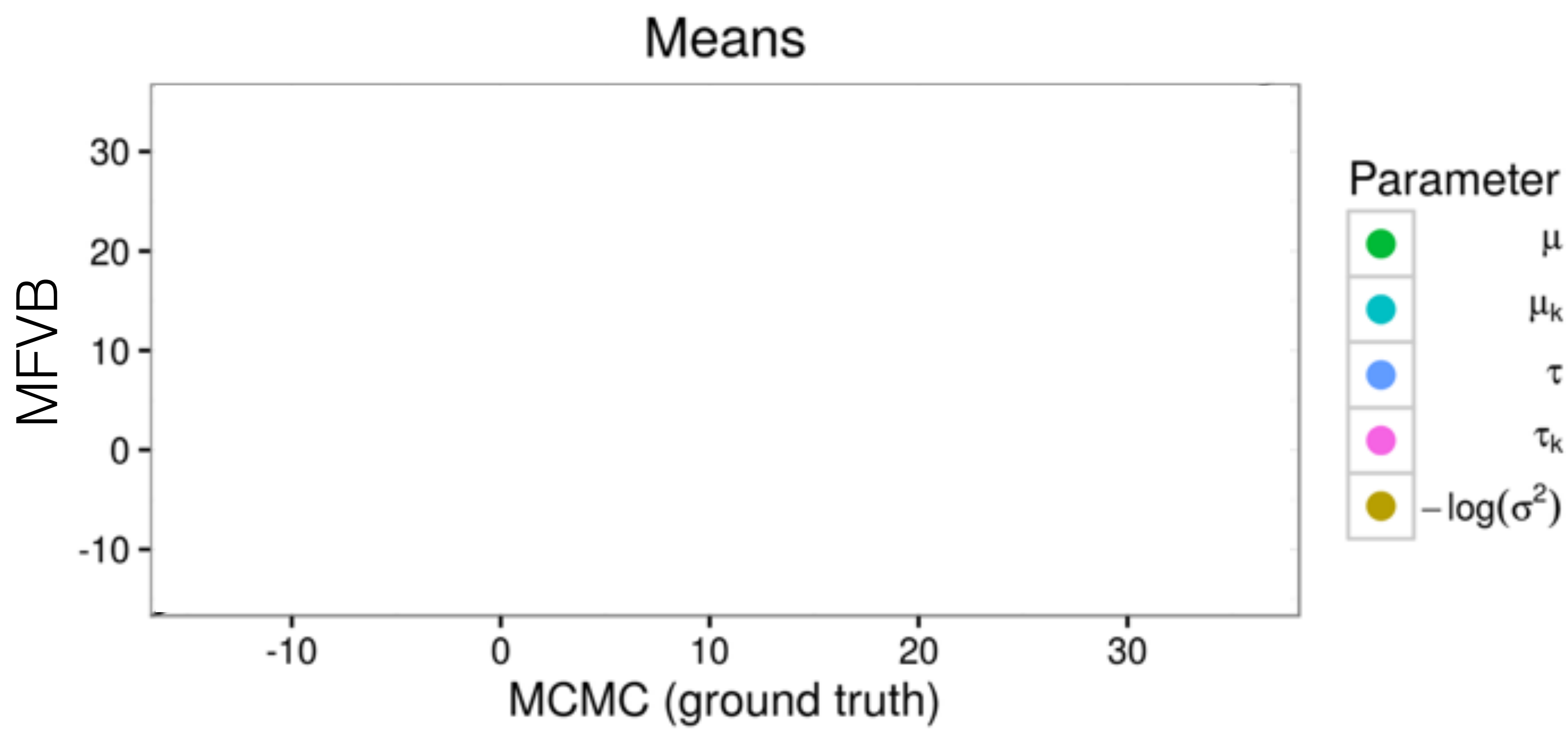
Microcredit



[Giordano, Broderick, Meager, Huggins, Jordan 2016]

Microcredit

- *One set* of 2500 MCMC draws: **45 minutes**



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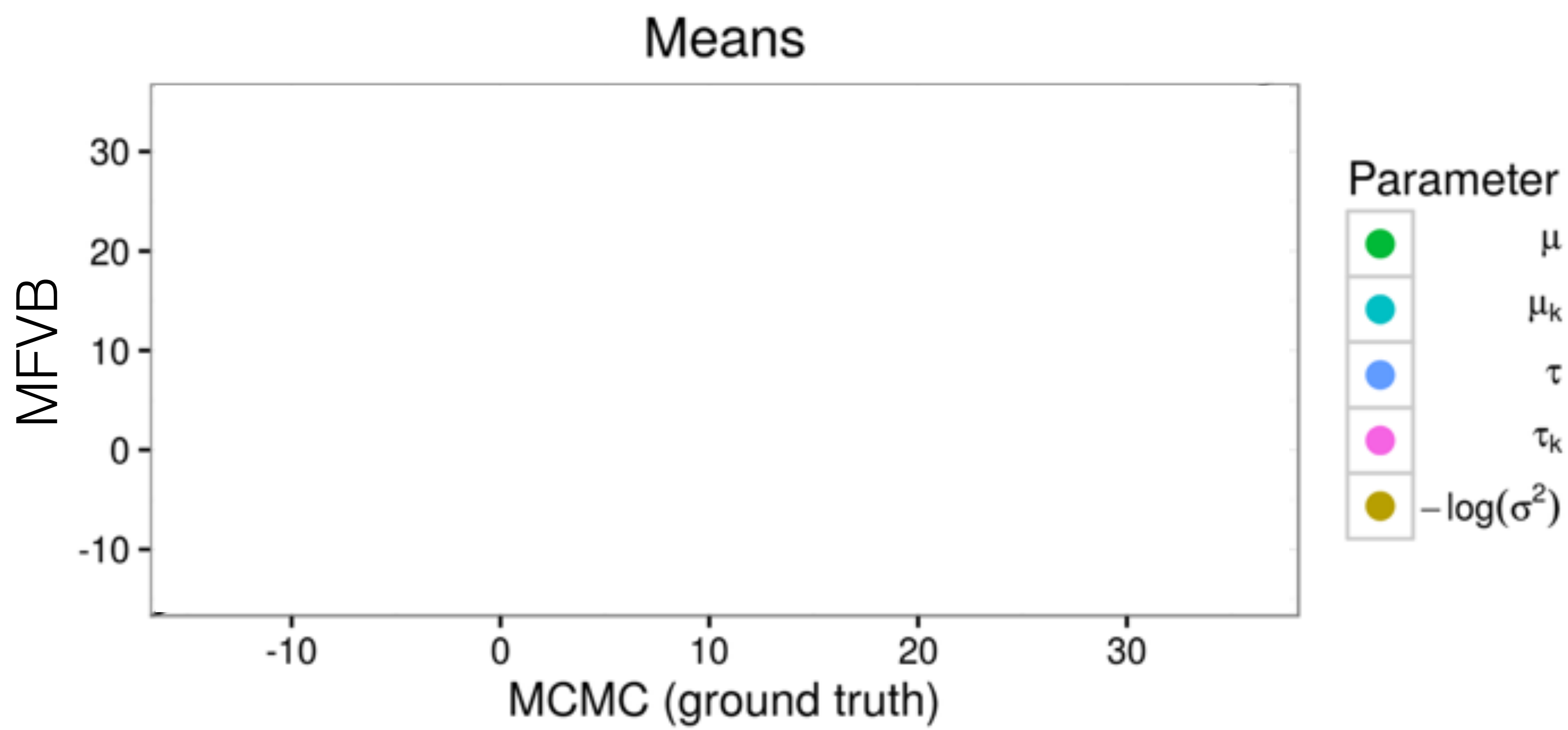

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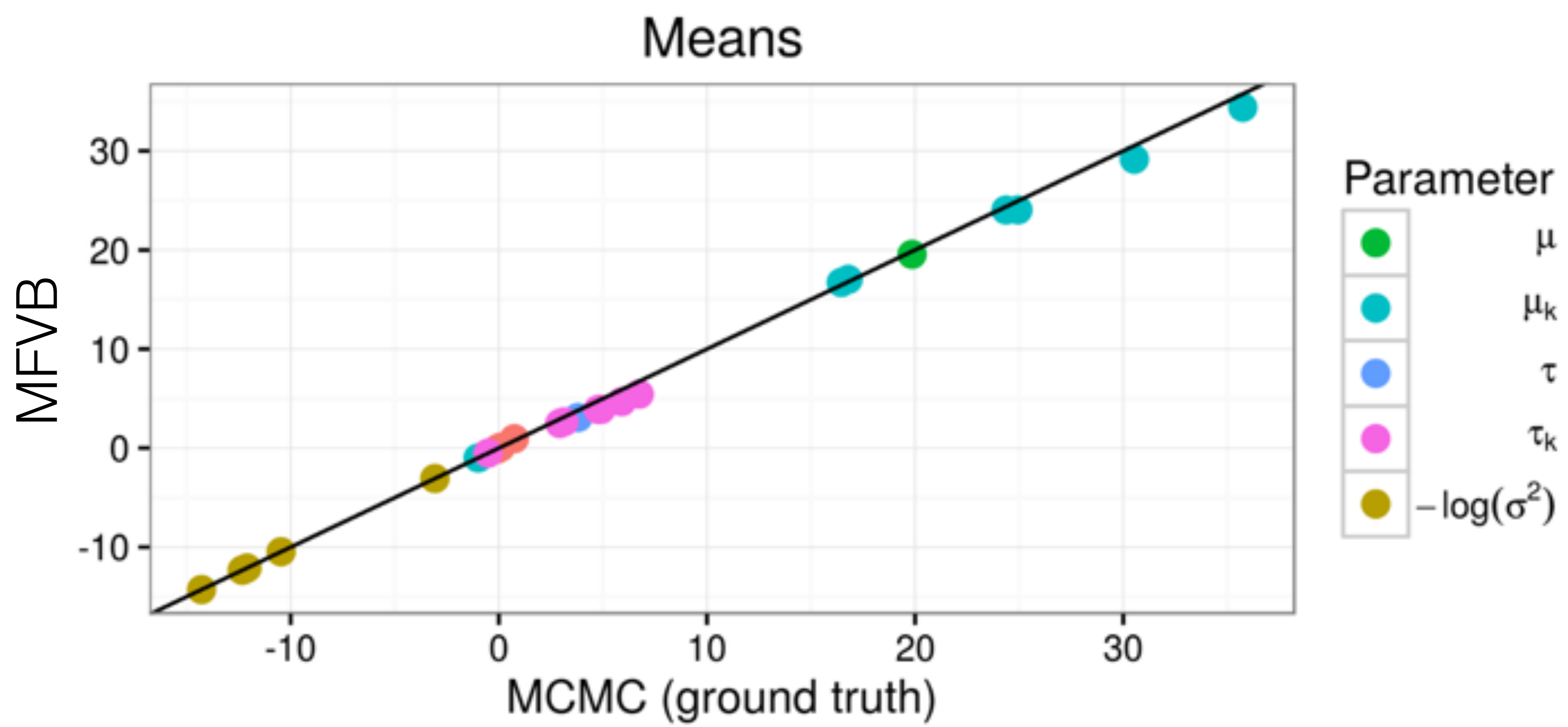

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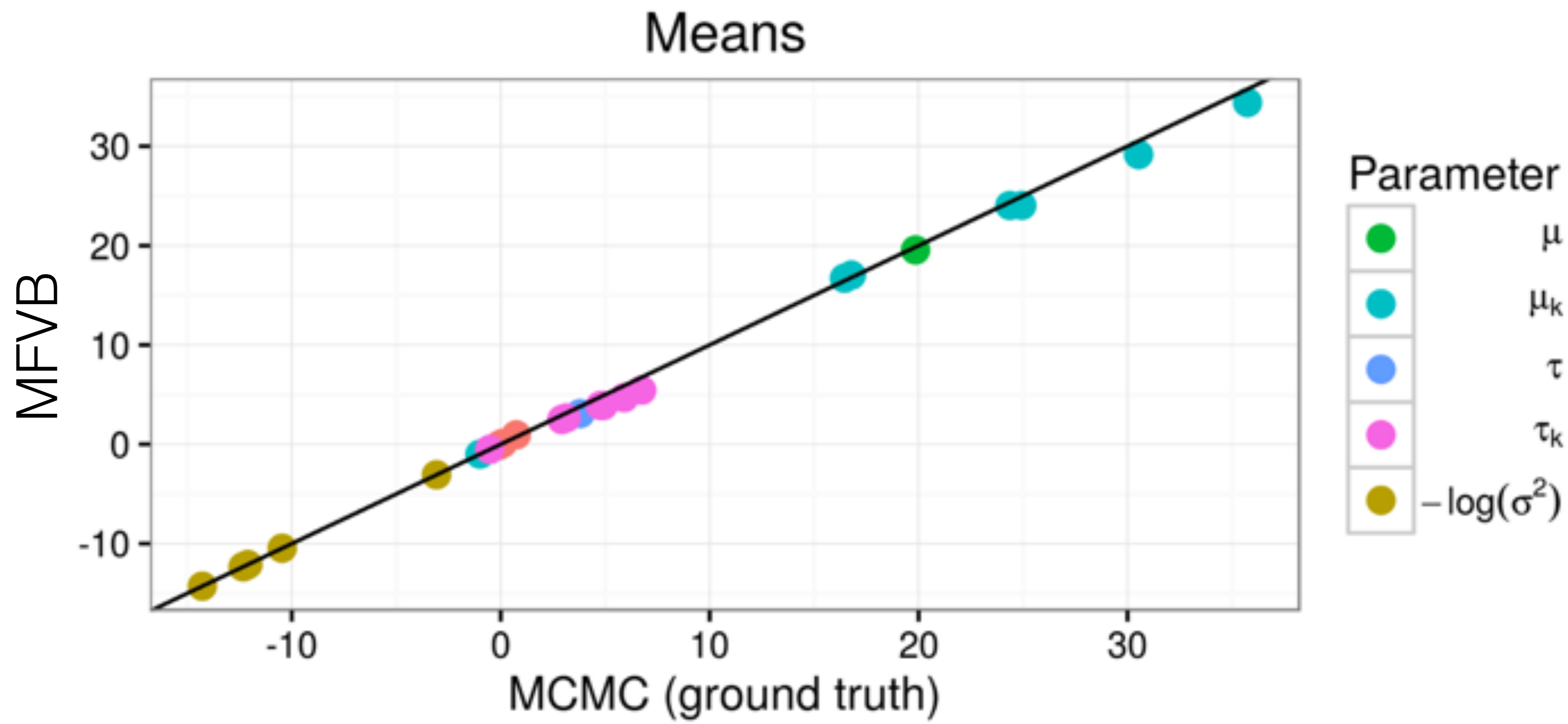

Criteo Online Ads Experiment

- Click-through conversion prediction
- Q: Will a customer (e.g.) buy a product after clicking?

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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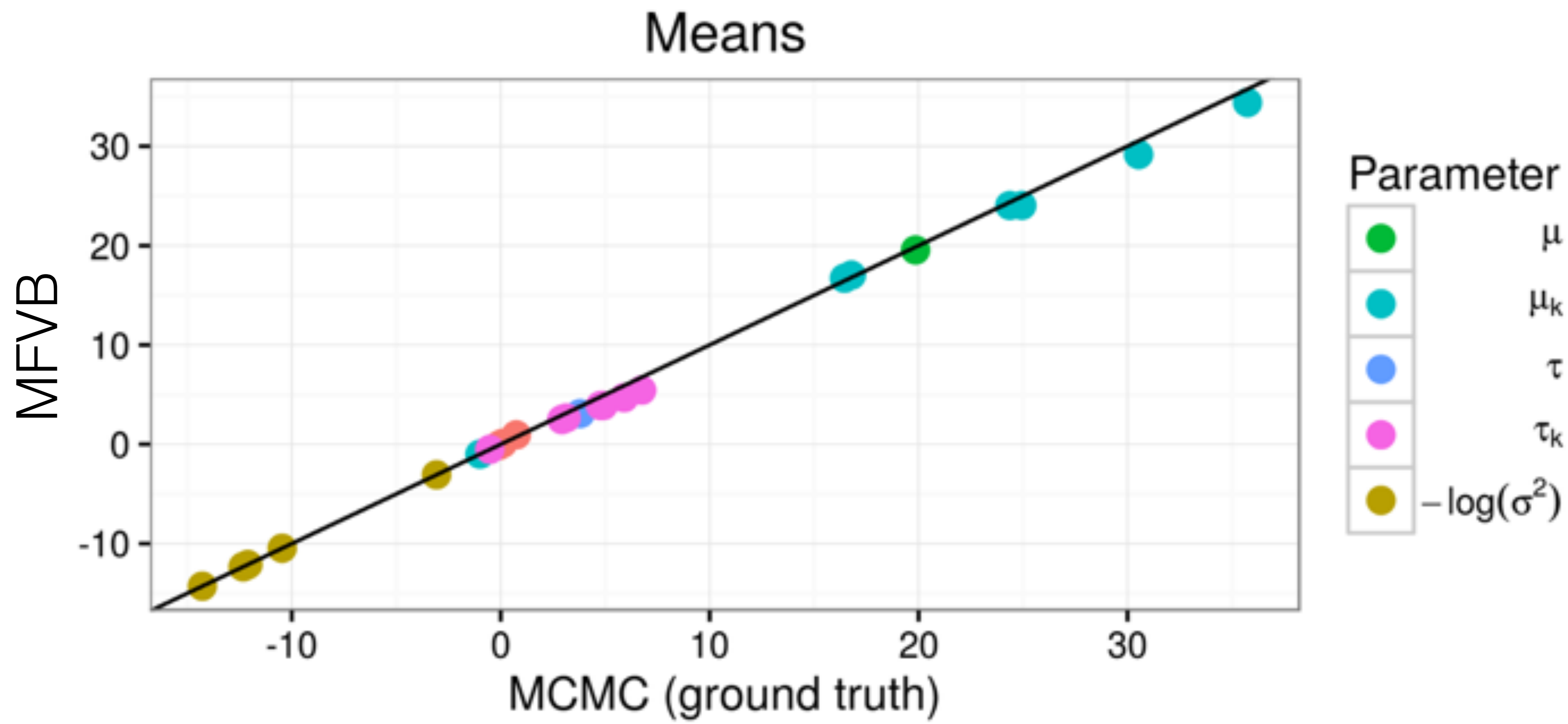

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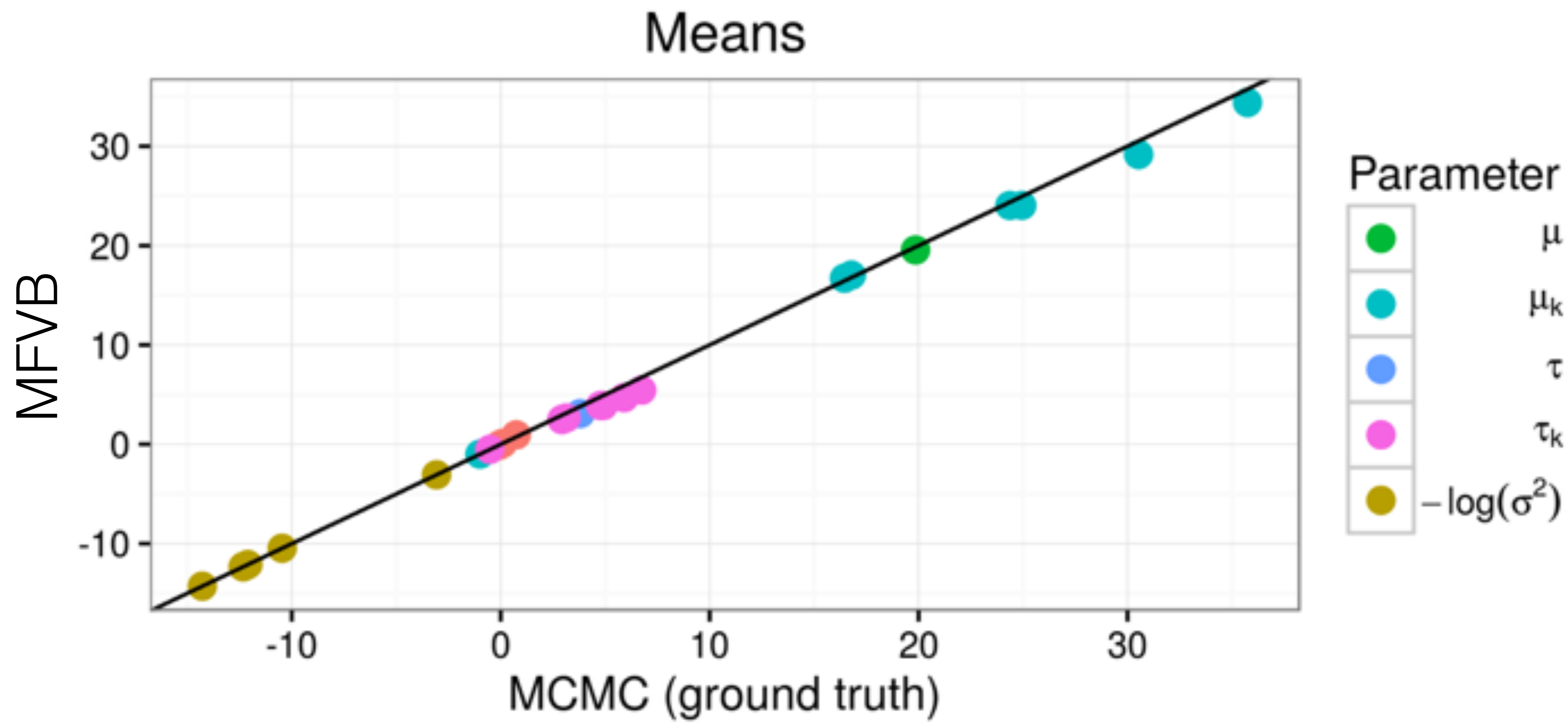

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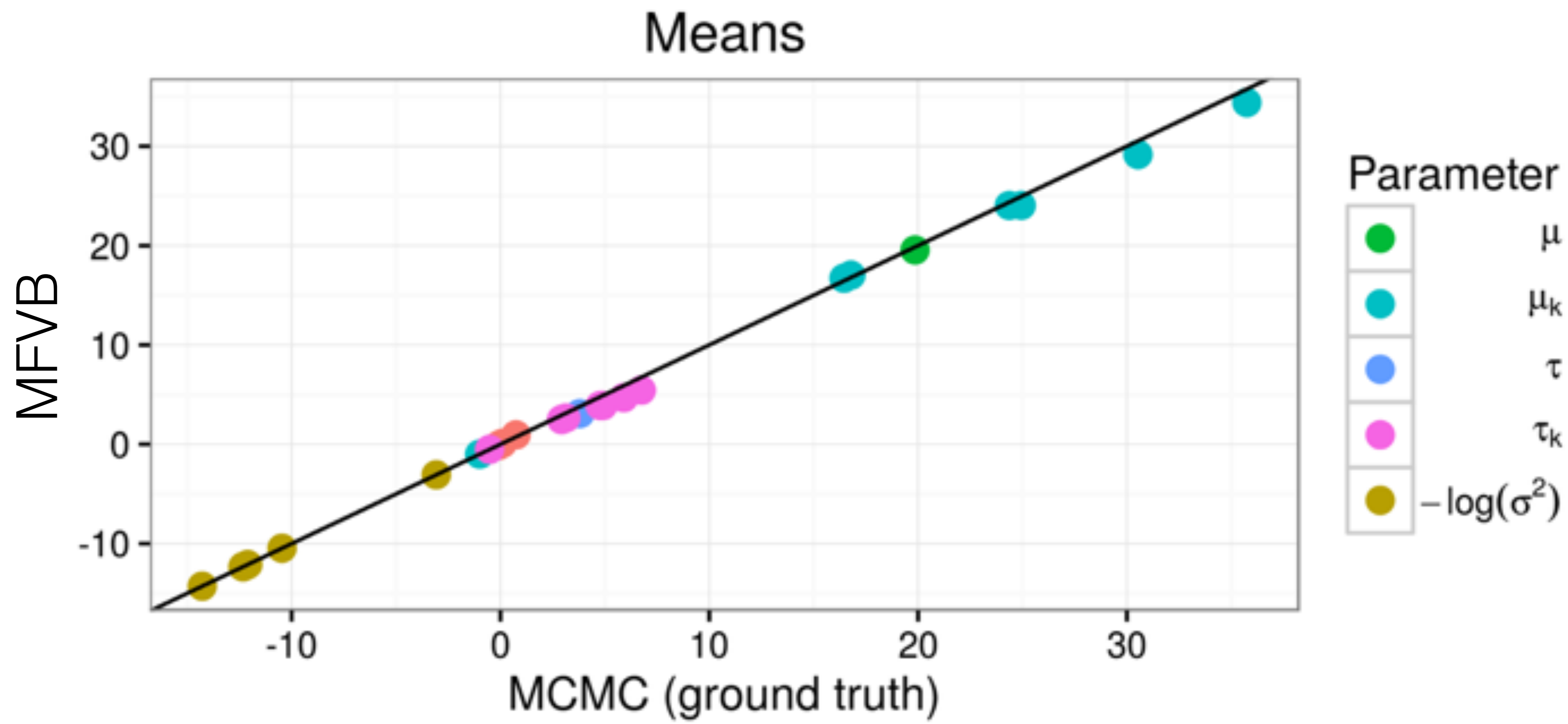

Criteo Online Ads Experiment

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- Q: How predictive of conversion are different features?
- Logistic GLMM; N = 61,895 subset to compare to MCMC

[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

- MAP: **12 s**

[Giordano, Broderick, Jordan 2017]

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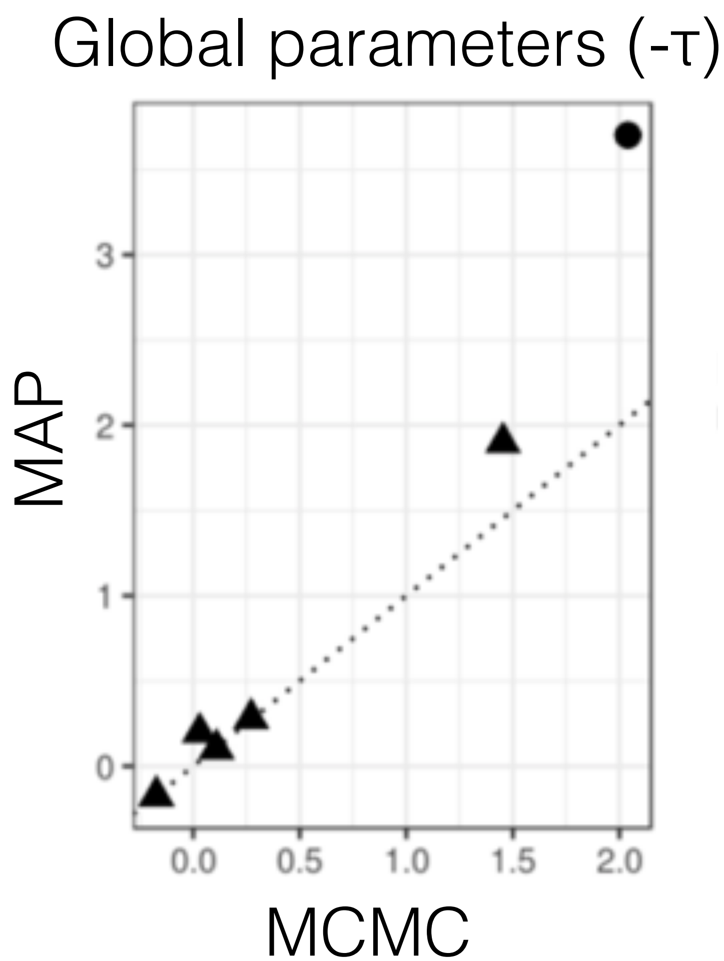

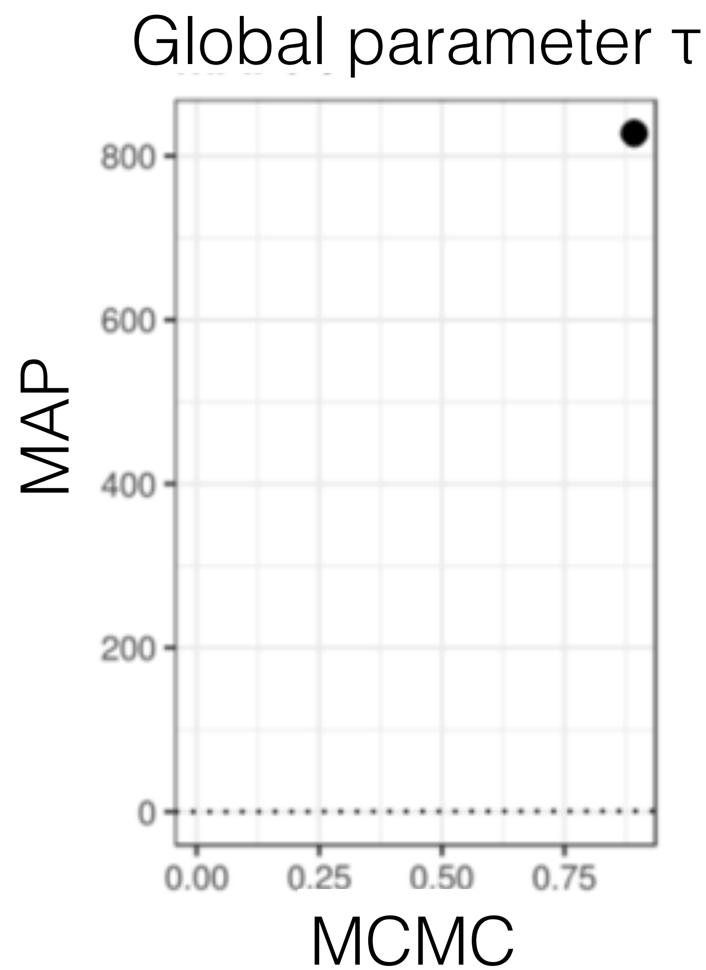

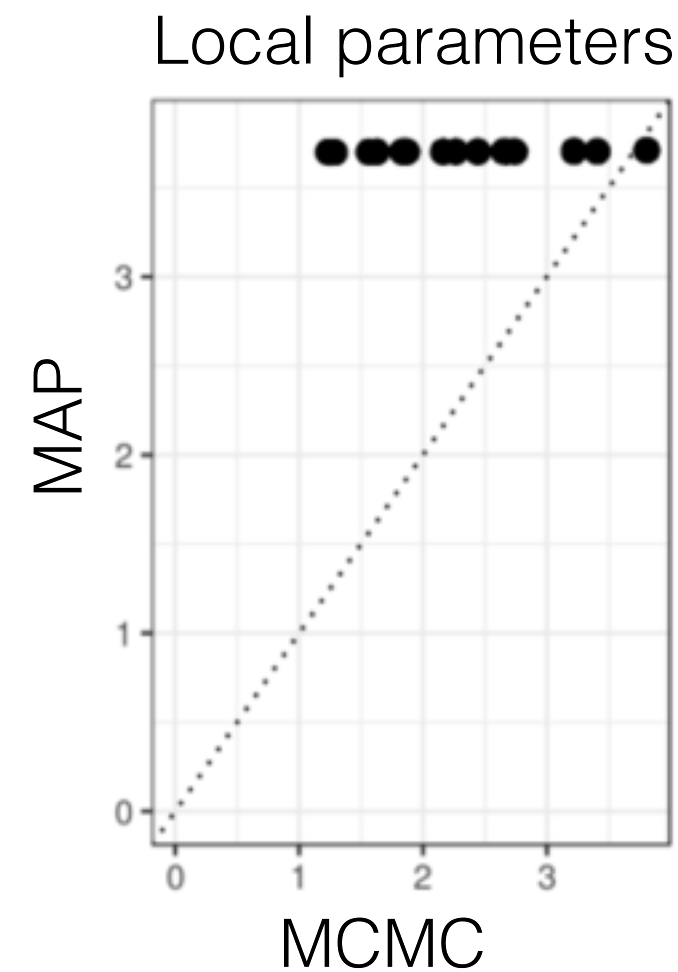

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[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

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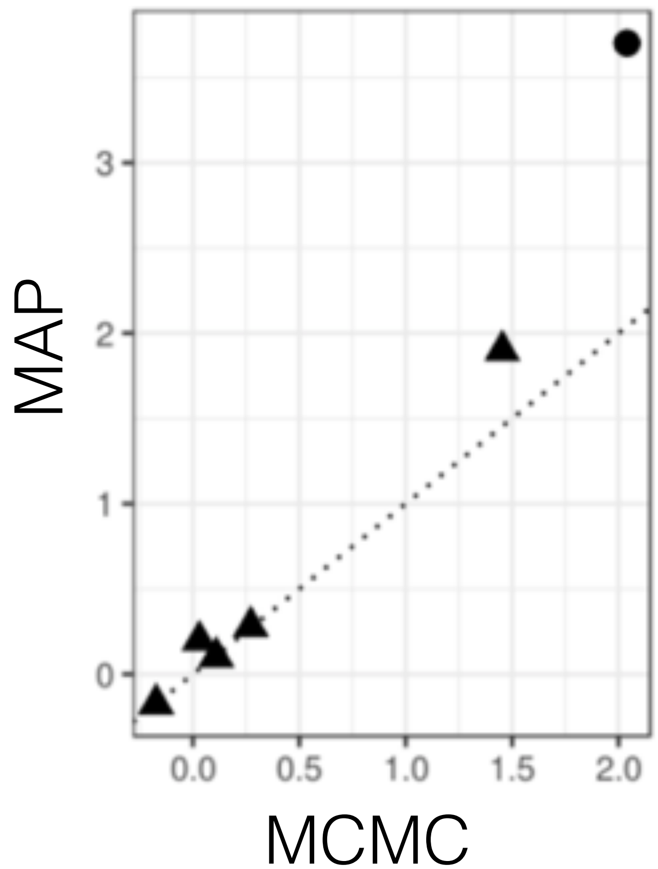
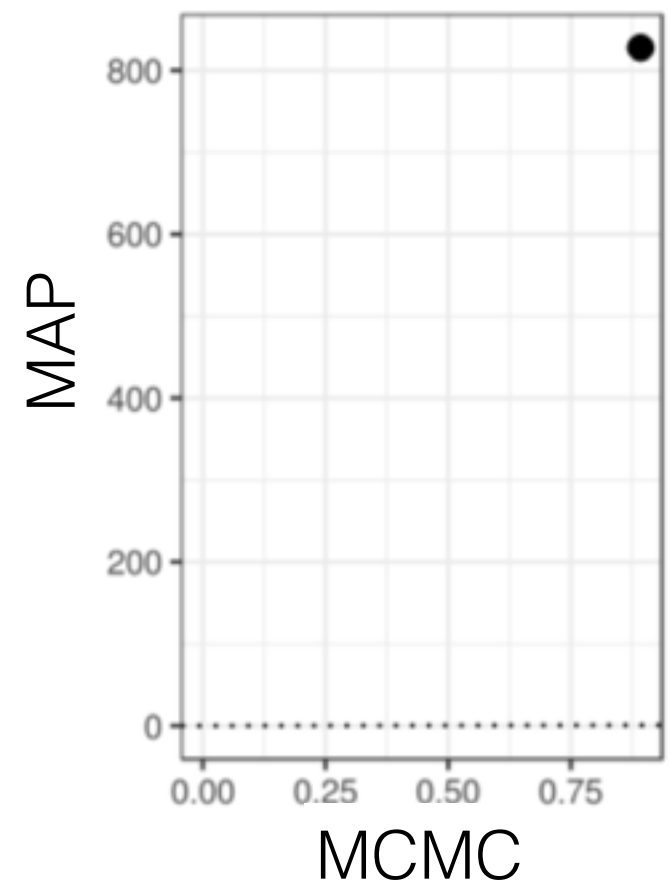
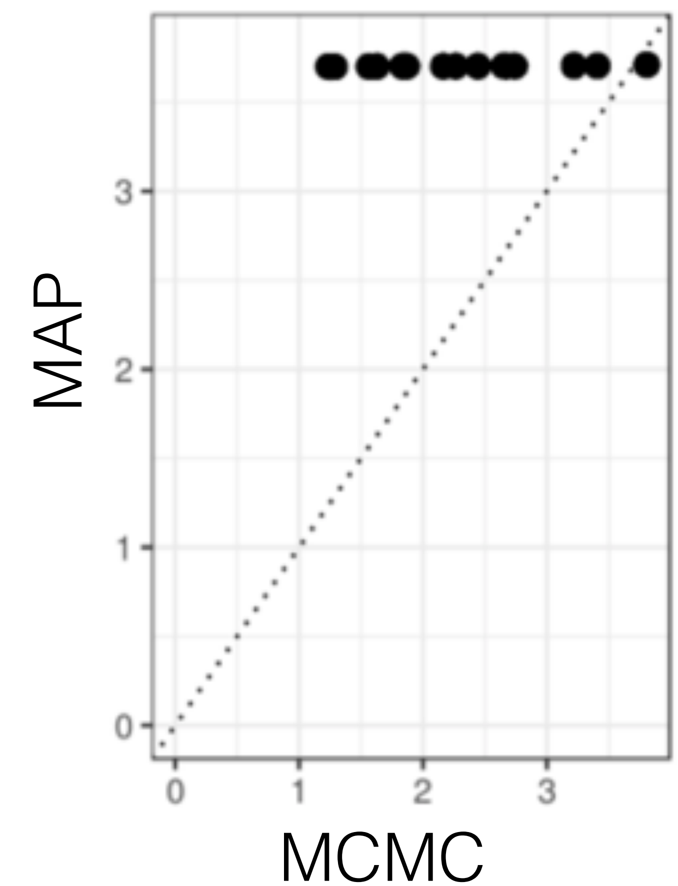


Global parameter τ



Local parameters

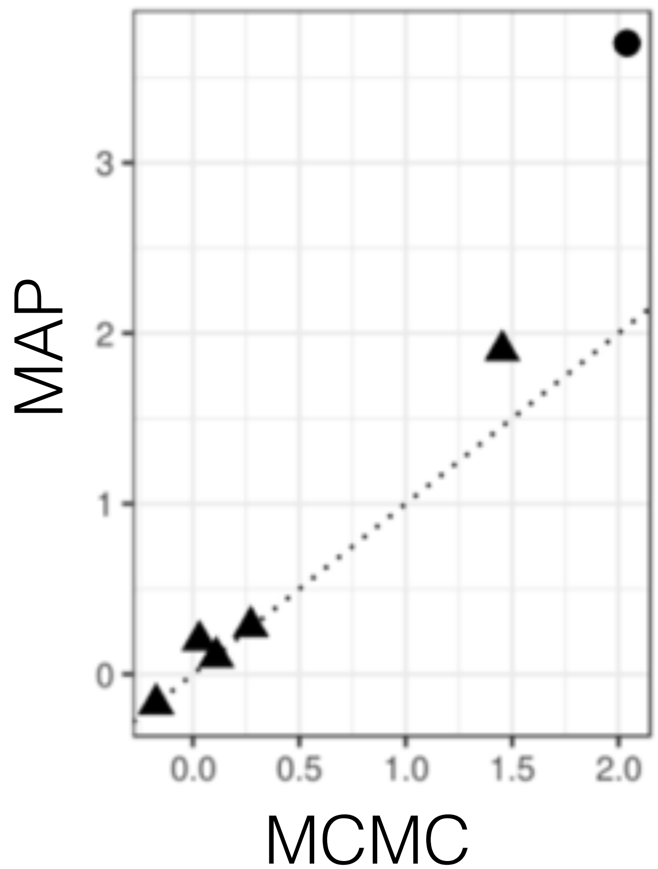


- MAP: **12 s**
- MFVB: **57 s**

[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

Global parameters (-τ)



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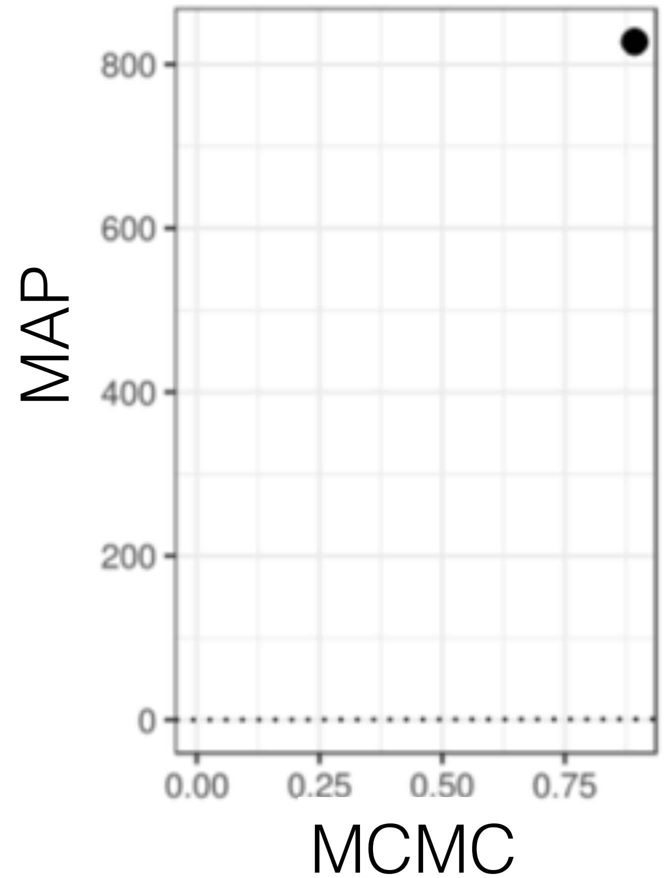
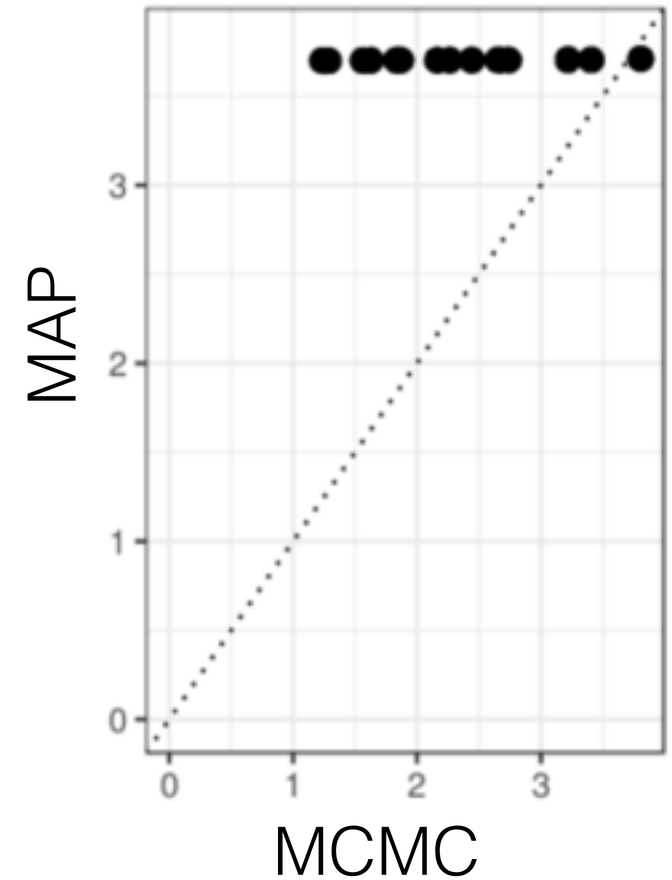


Local parameters



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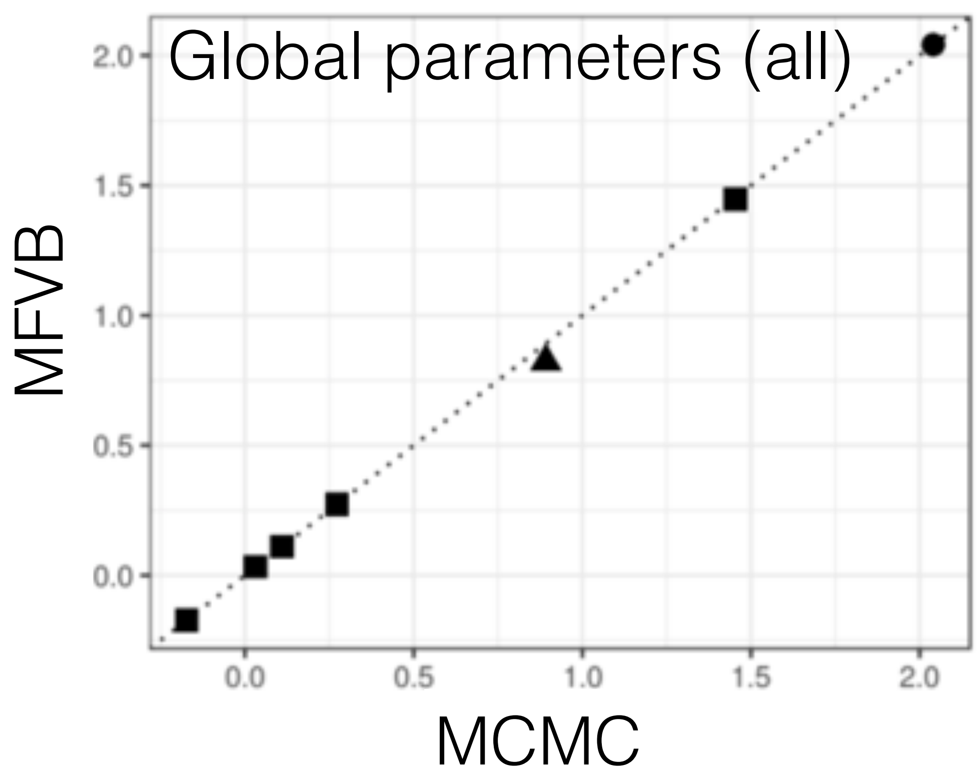
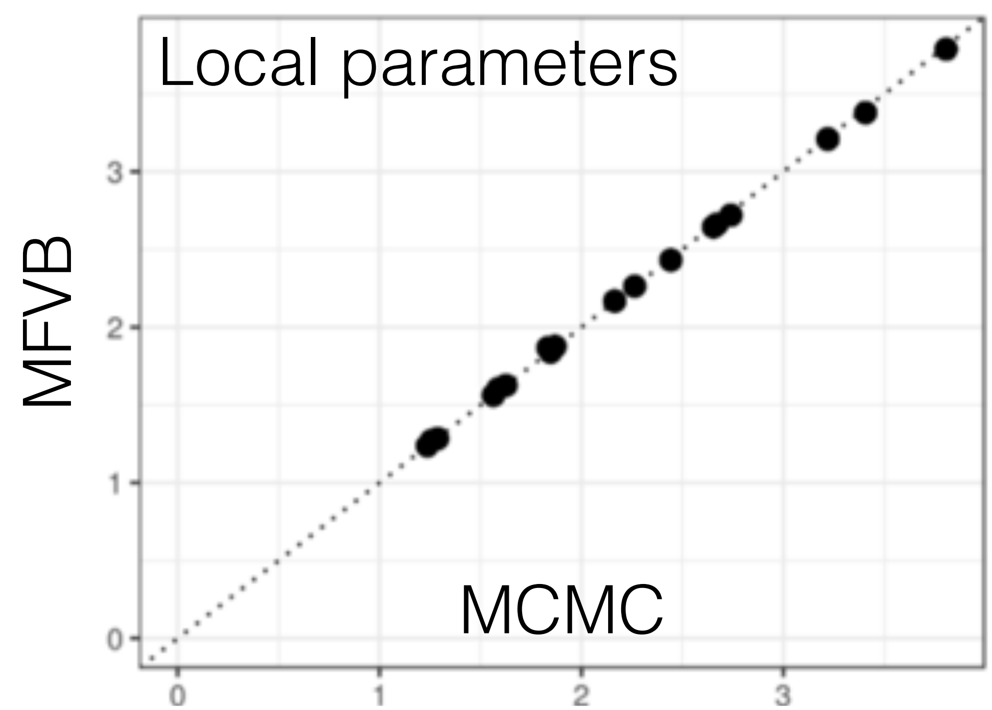
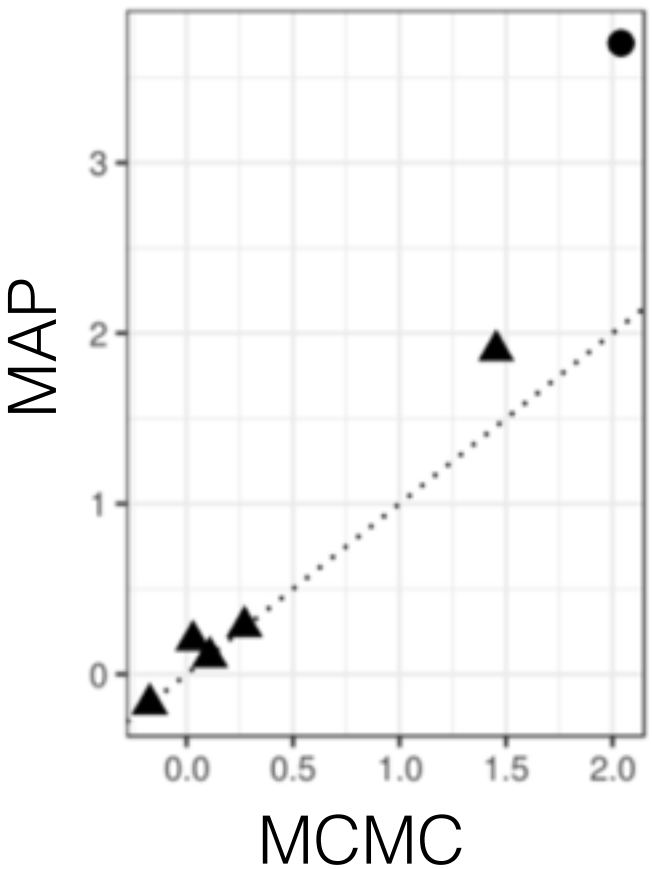




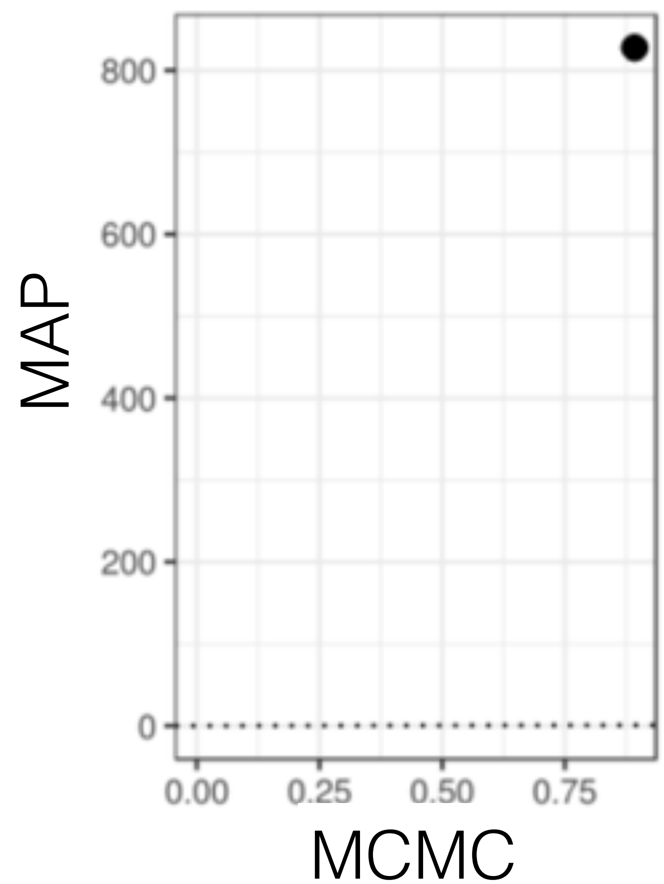
[Giordano, Broderick, Jordan 2017]

Criteo Online Ads Experiment

Global parameters (-τ)



Global parameter τ



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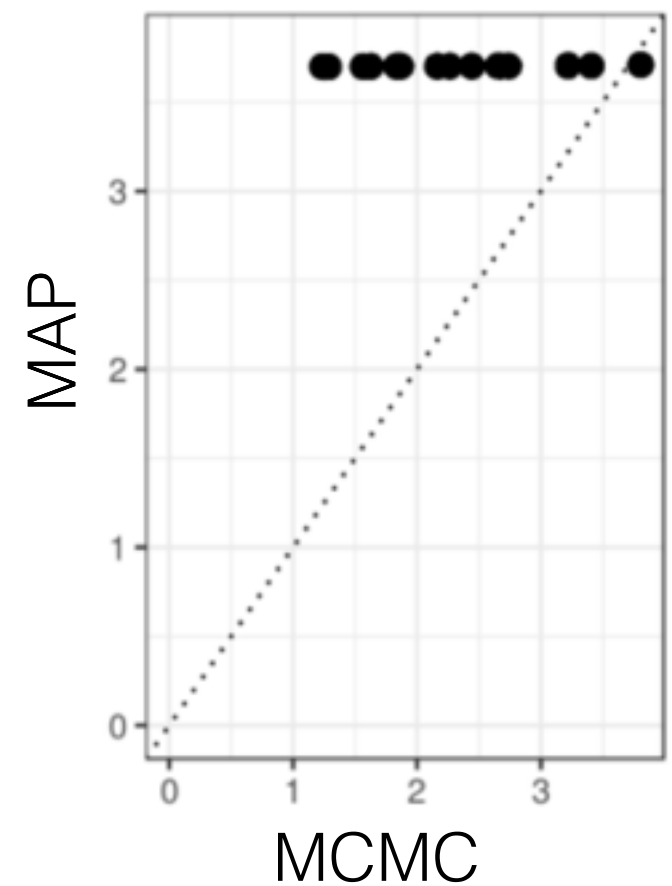

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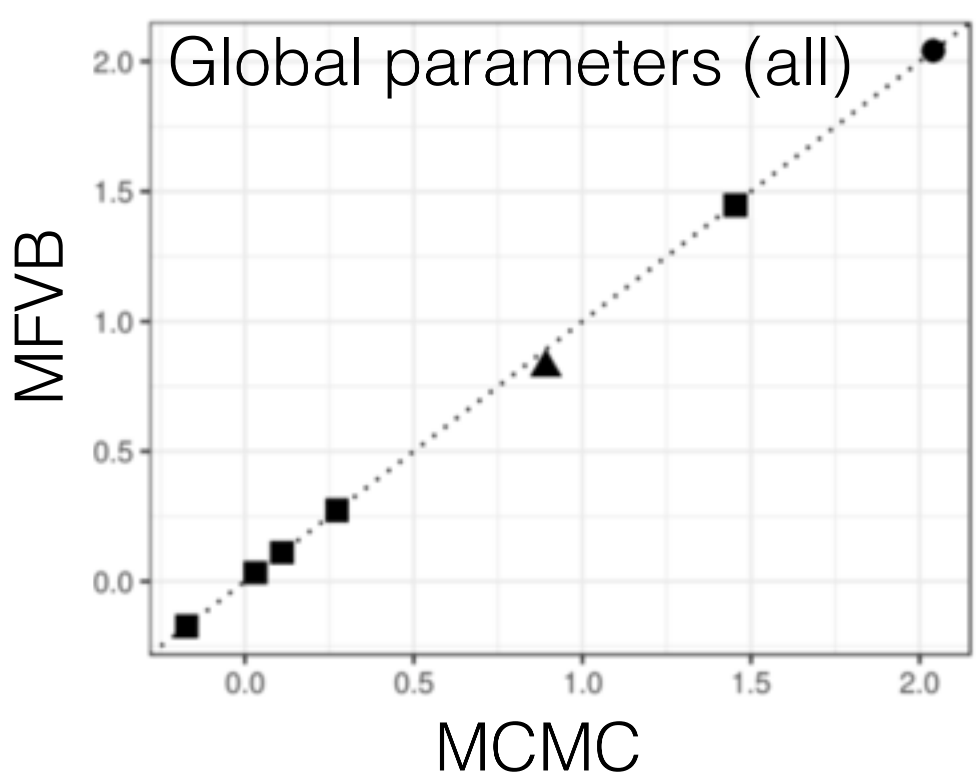
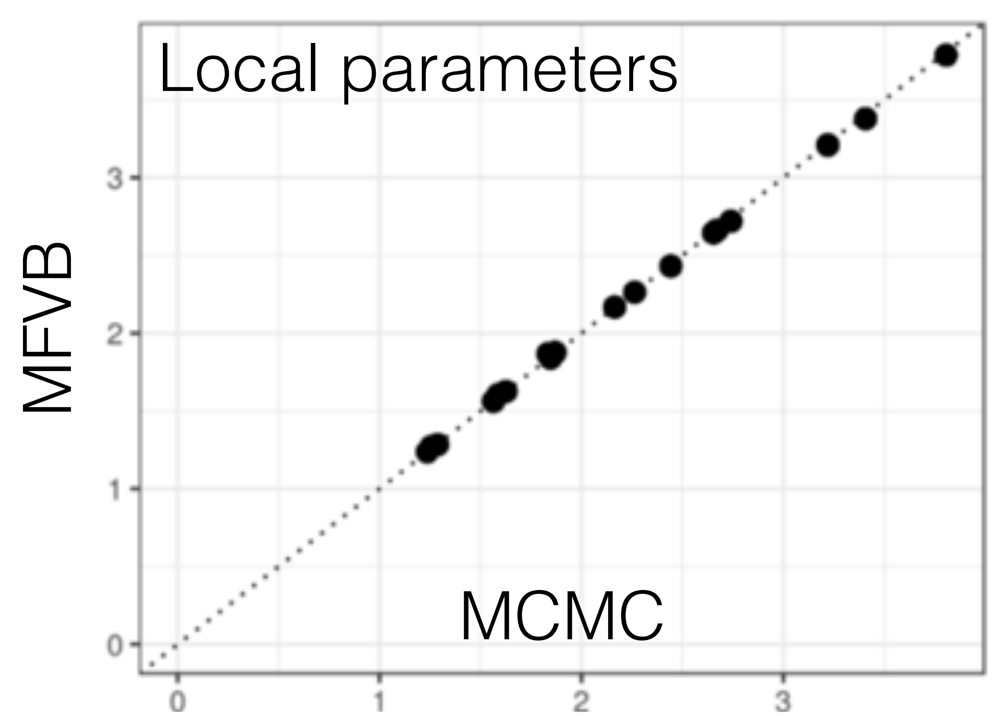
- MCMC (5K samples): 21,066 s (**5.85 h**)





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Roadmap

- Bayes & Approximate Bayes review
- What is:
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- Why use MFVB?
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Variational Bayes and beyond: Bayesian inference for big data

Tamara Broderick
ITT Career Development
Assistant Professor,
MIT

http://www.tamarabroderick.com/tutorials.html

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Approximate Bayesian inference

Use q^* to approximate $p(\cdot|y)$

Optimization

$$q^* = \text{argmin}_{q \in Q} f(q(\cdot), p(\cdot|y))$$

Variational Bayes

$$q^* = \text{argmin}_{q \in Q} KL(q(\cdot)||p(\cdot|y))$$

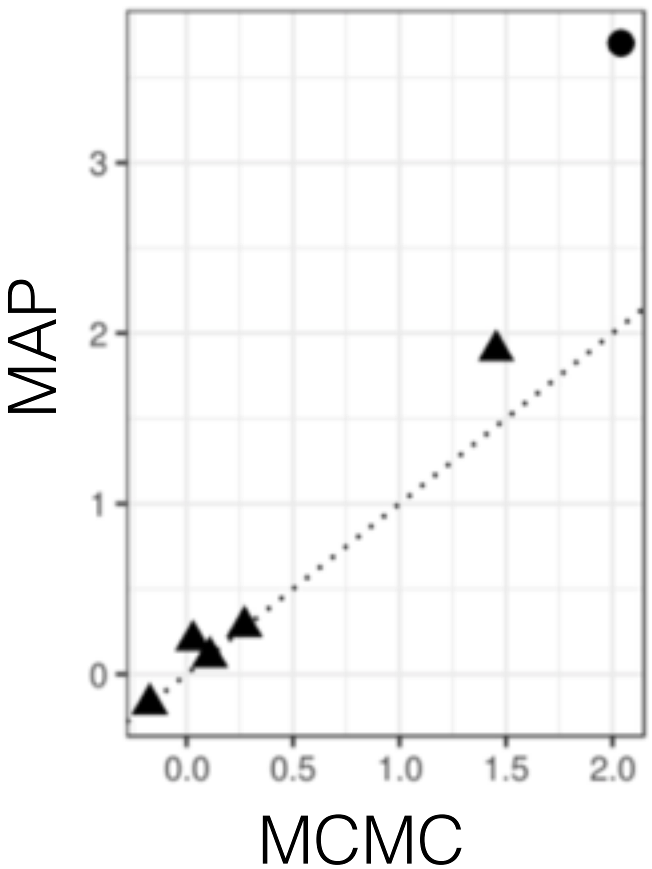
Mean-field variational Bayes

$$q^* = \text{argmin}_{q \in Q_{\text{MFVB}}} KL(q(\cdot)||p(\cdot|y))$$

- Coordinate descent
- Stochastic variational inference (SVI) [Hoffman et al 2013]
- Automatic differentiation variational inference (ADVI) [Kucukelbir et al 2015, 2017]

Criteo Online Ads Experiment

Global parameters (-τ)



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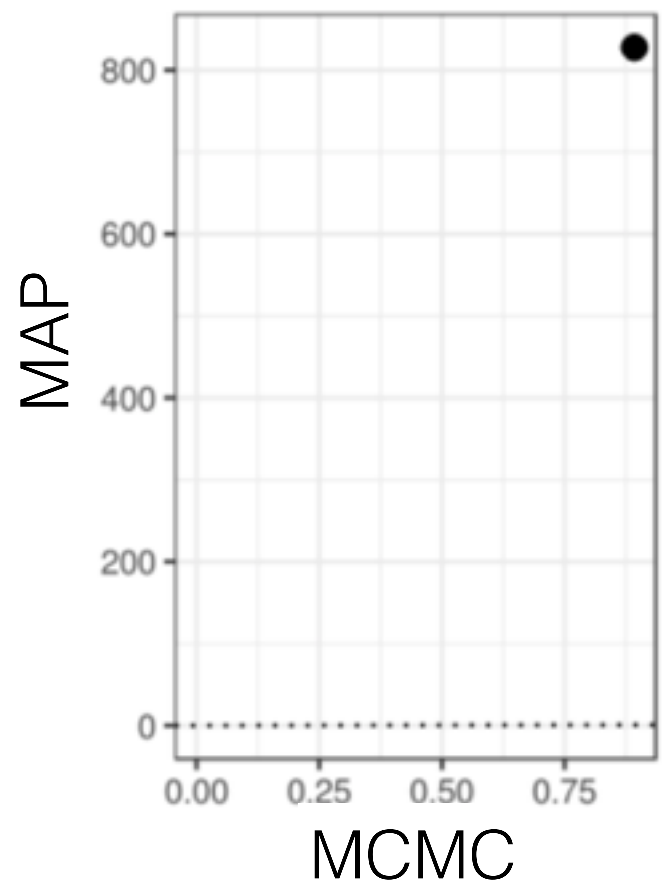

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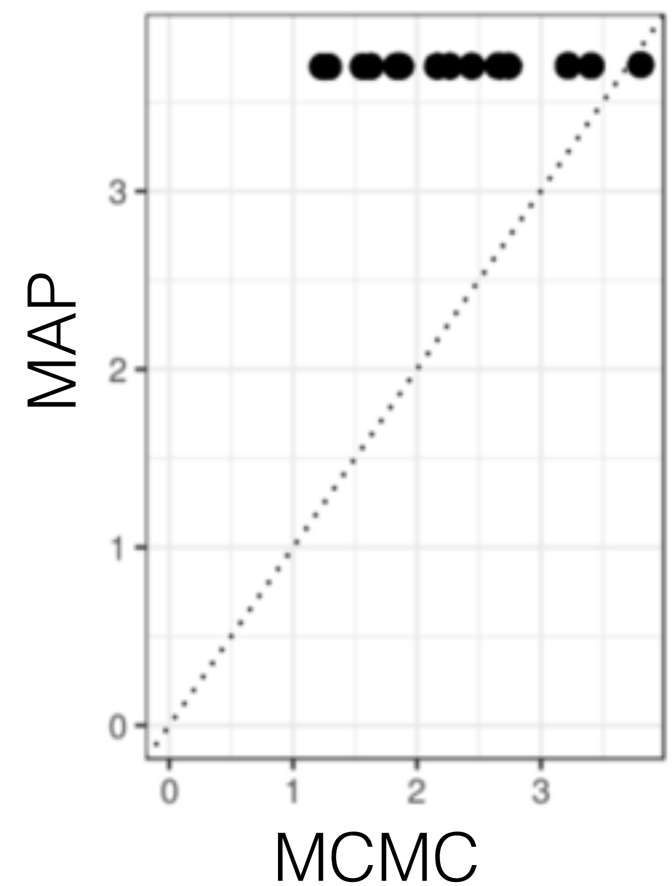

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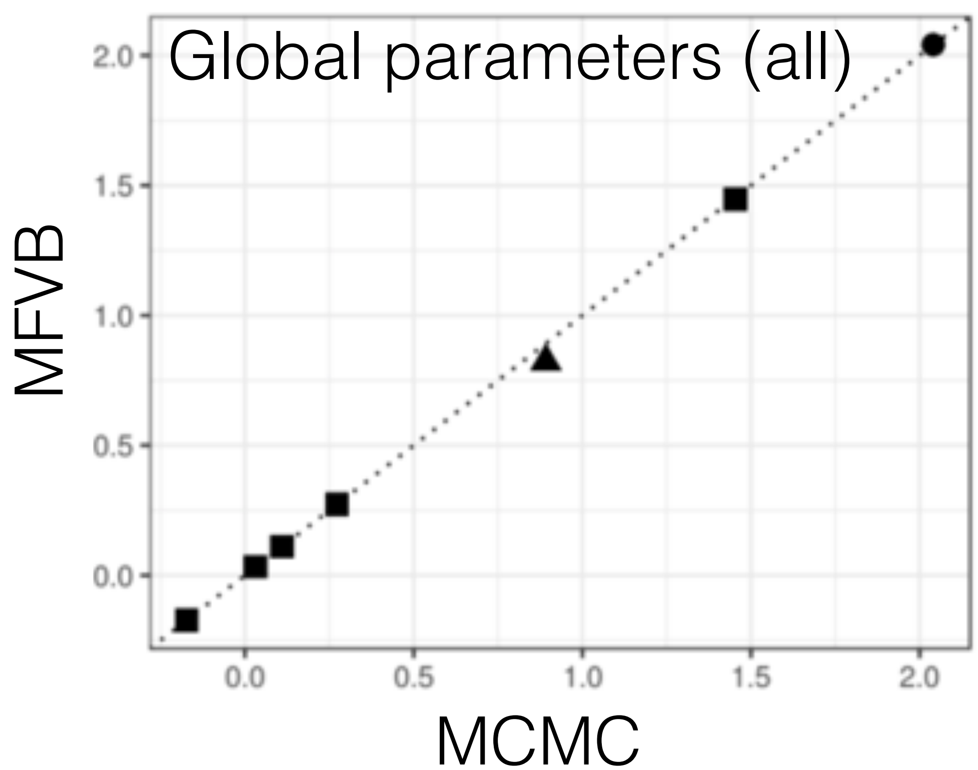

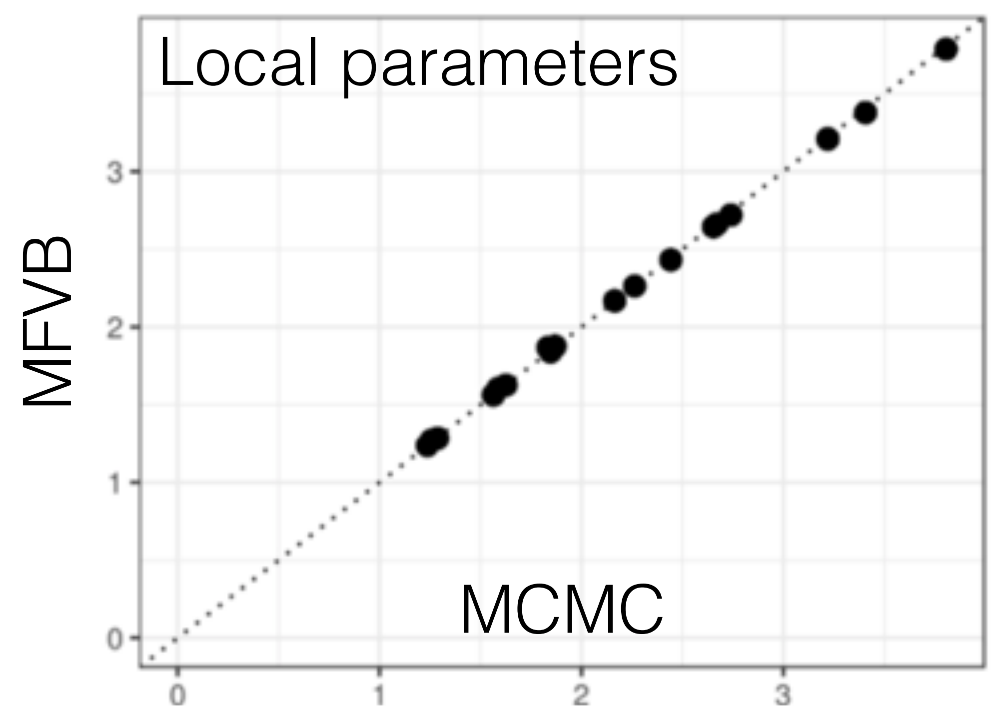

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$$KL(q||p(\cdot|y)) = \int_\theta q(\theta) \log \frac{q(\theta)}{p(\theta|y)} d\theta \qquad q(\theta) = \prod_{j=1}^{J} q_j(\theta_j)$$

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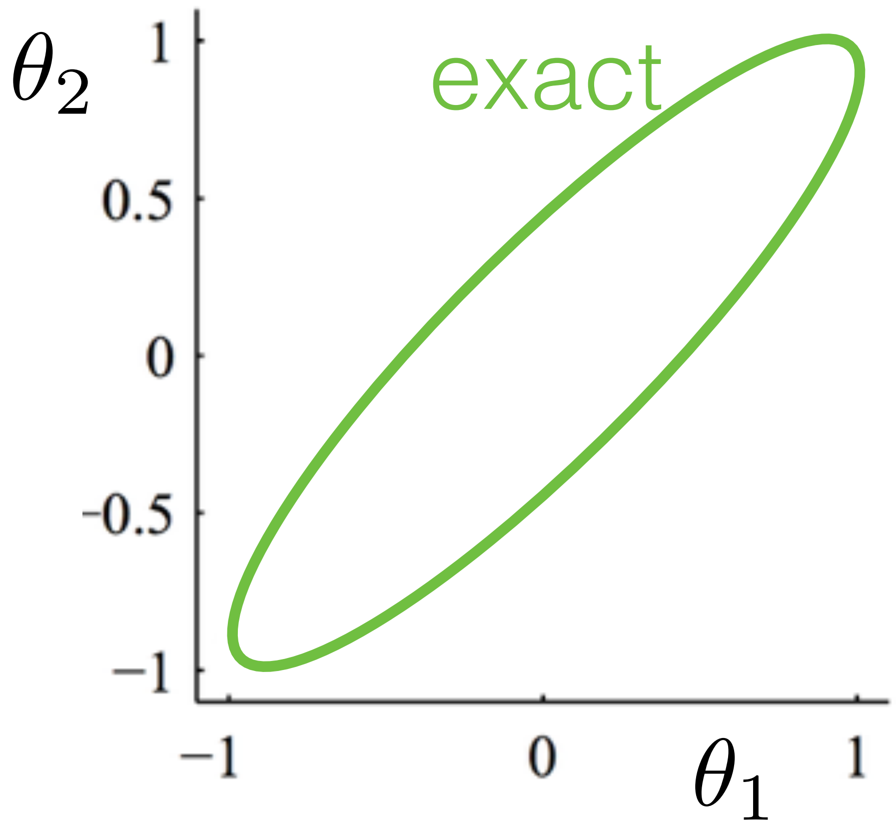

[Turner & Sahani 2011; MacKay 2003; Bishop 2006; Wang, Titterington 2004]

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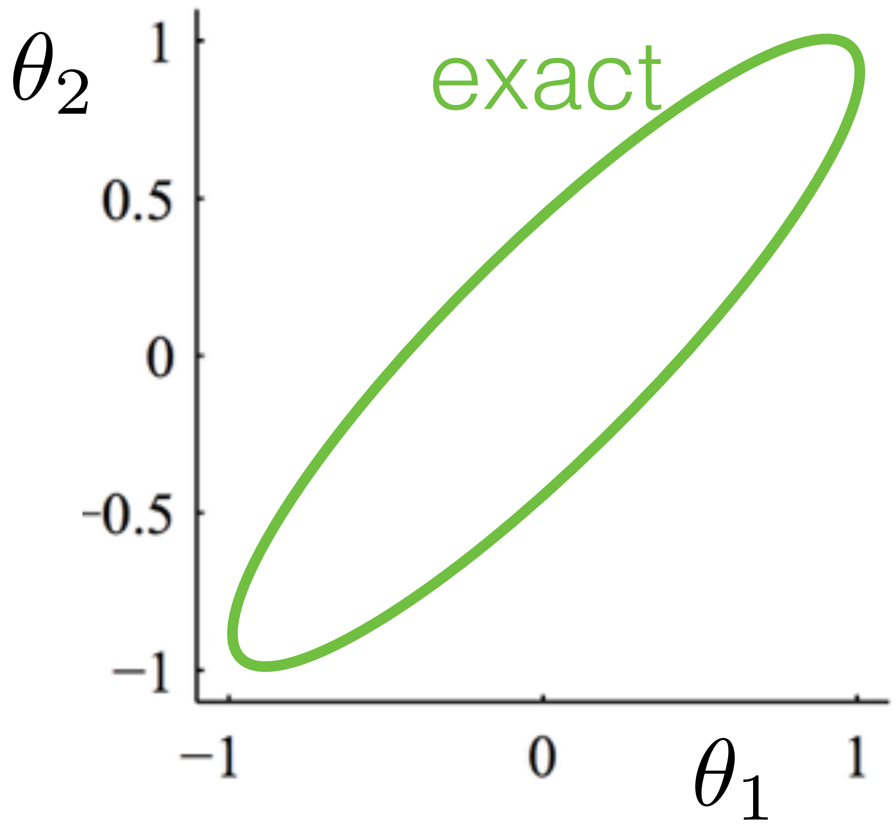

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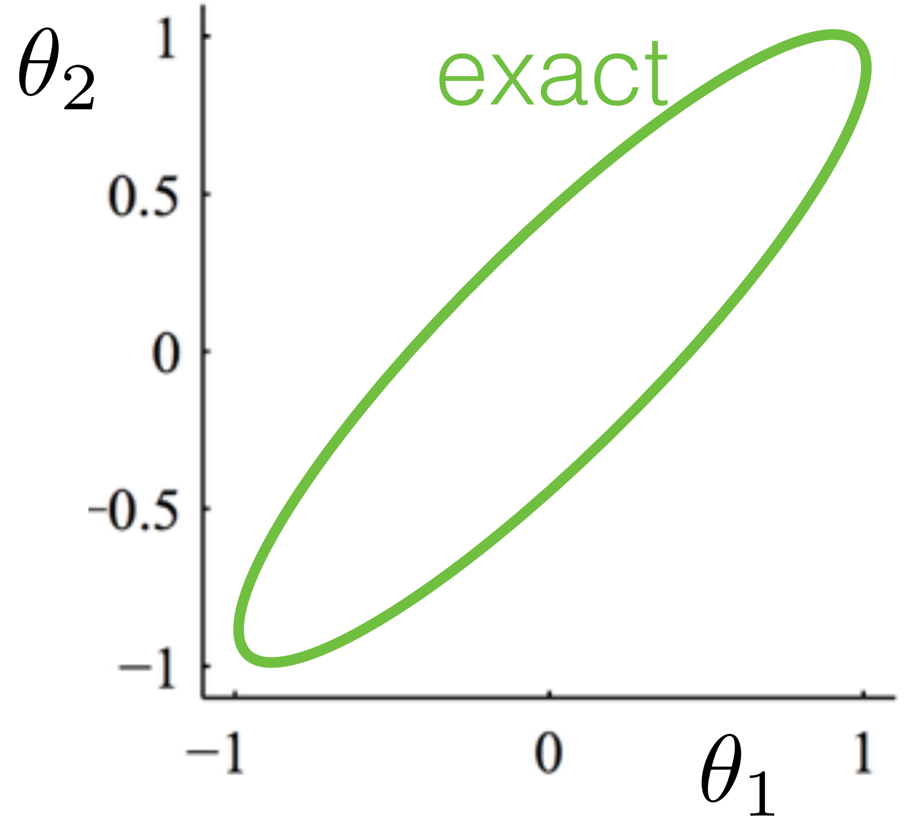

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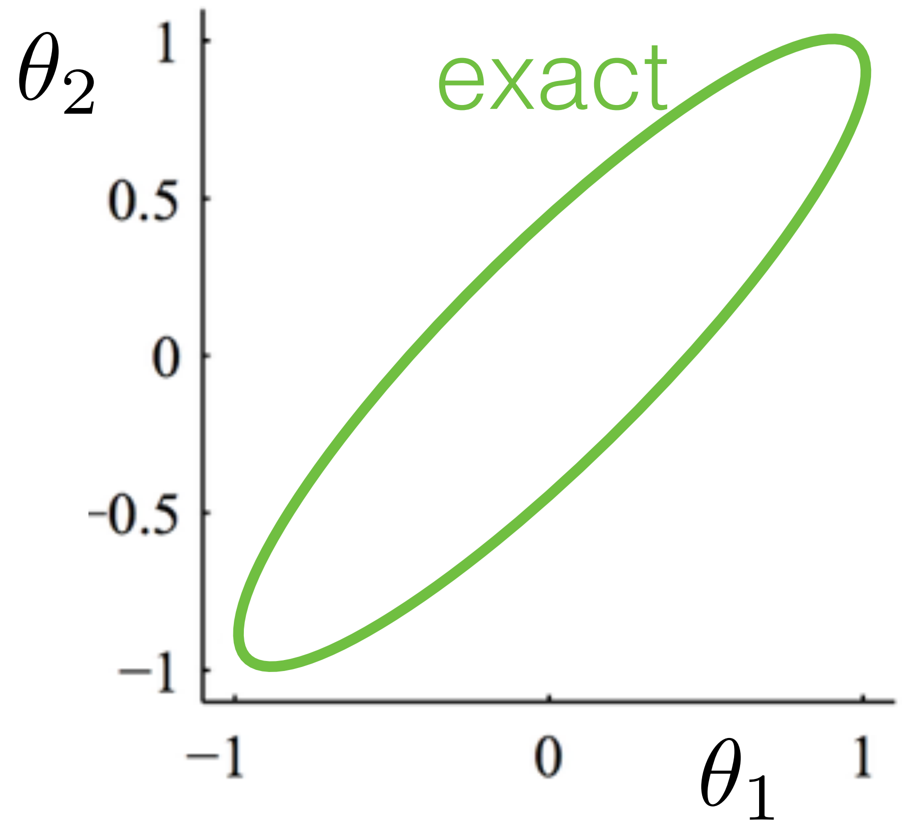

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[Exercise: derive the MFVB-CA steps. Hint: use precision matrix.]

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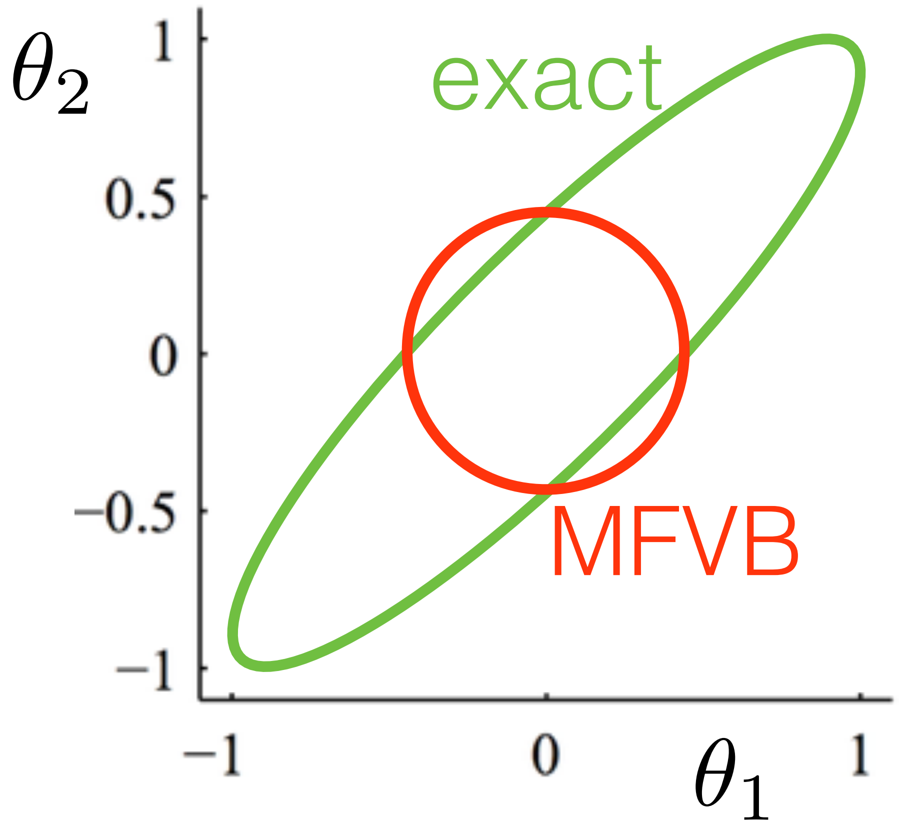

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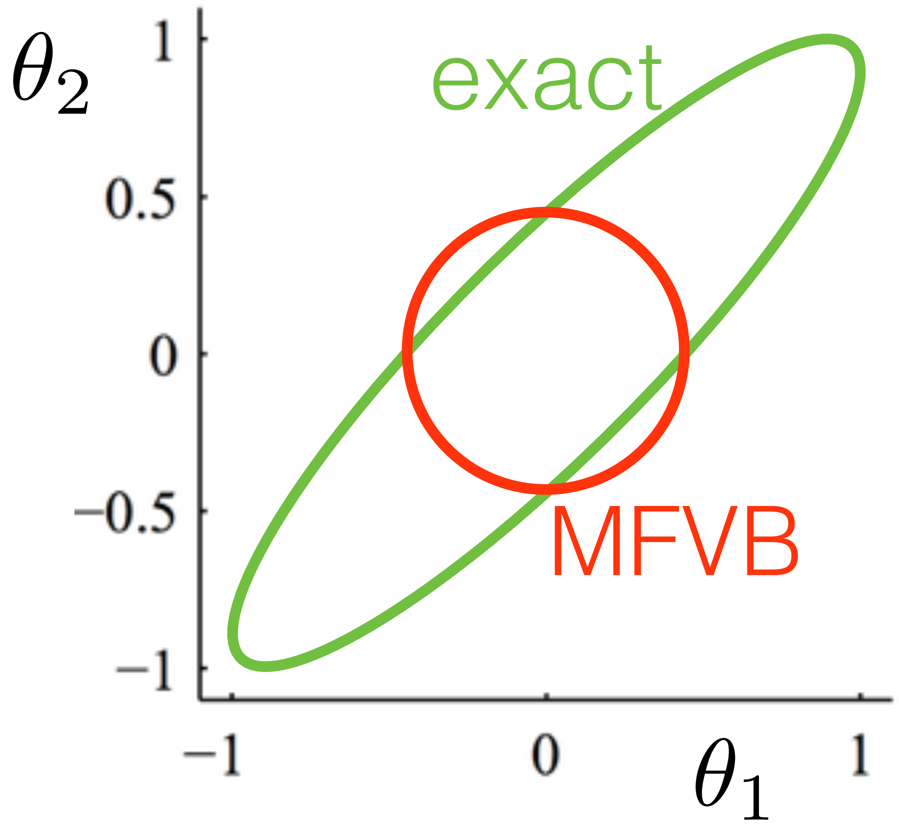

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- Underestimates variance (sometimes severely)

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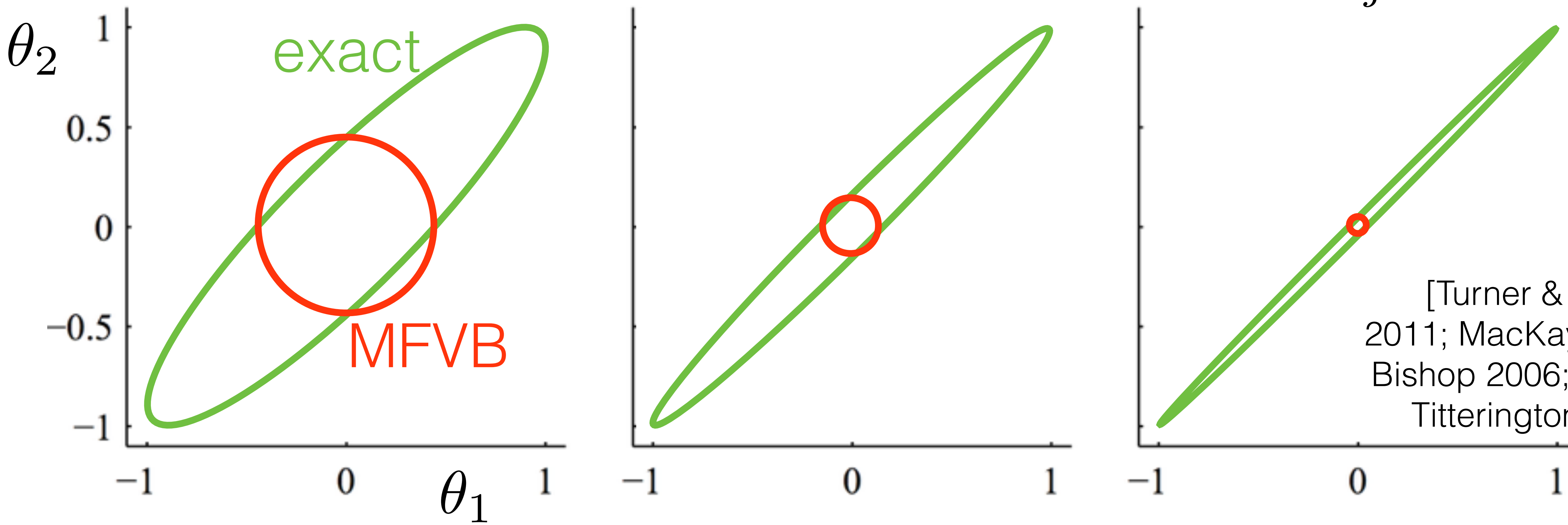

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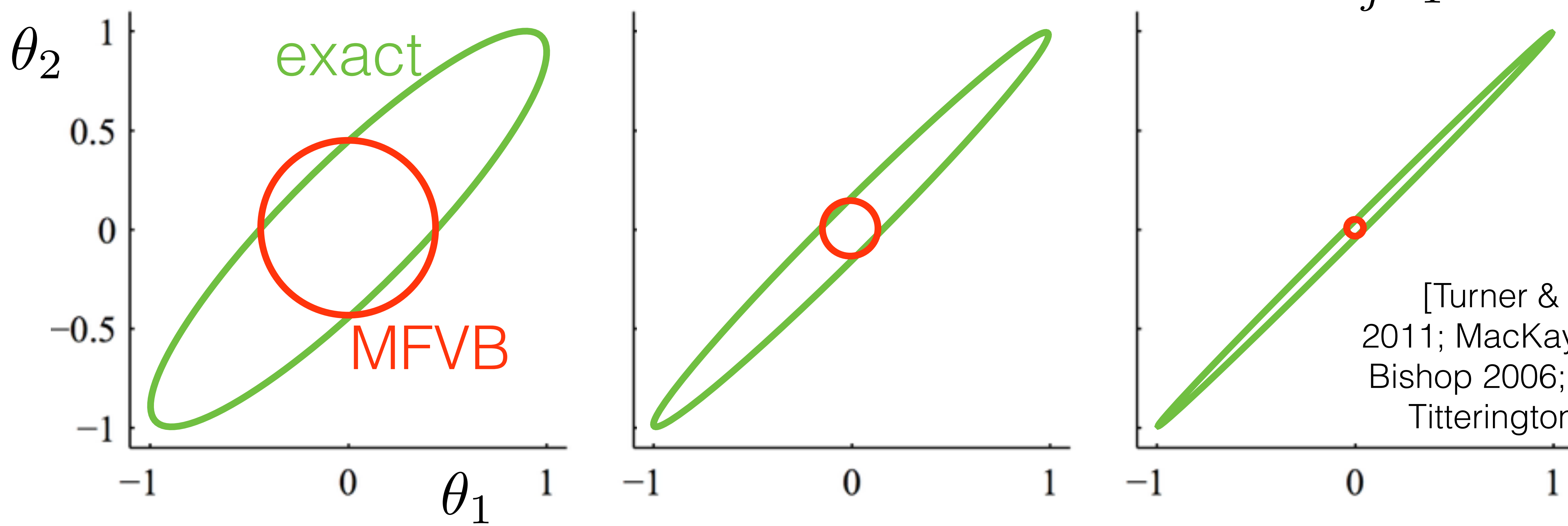

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- Underestimates variance (sometimes severely)
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What about uncertainty?

- Microcredit

[Giordano, Broderick, Meager, Huggins, Jordan 2016]

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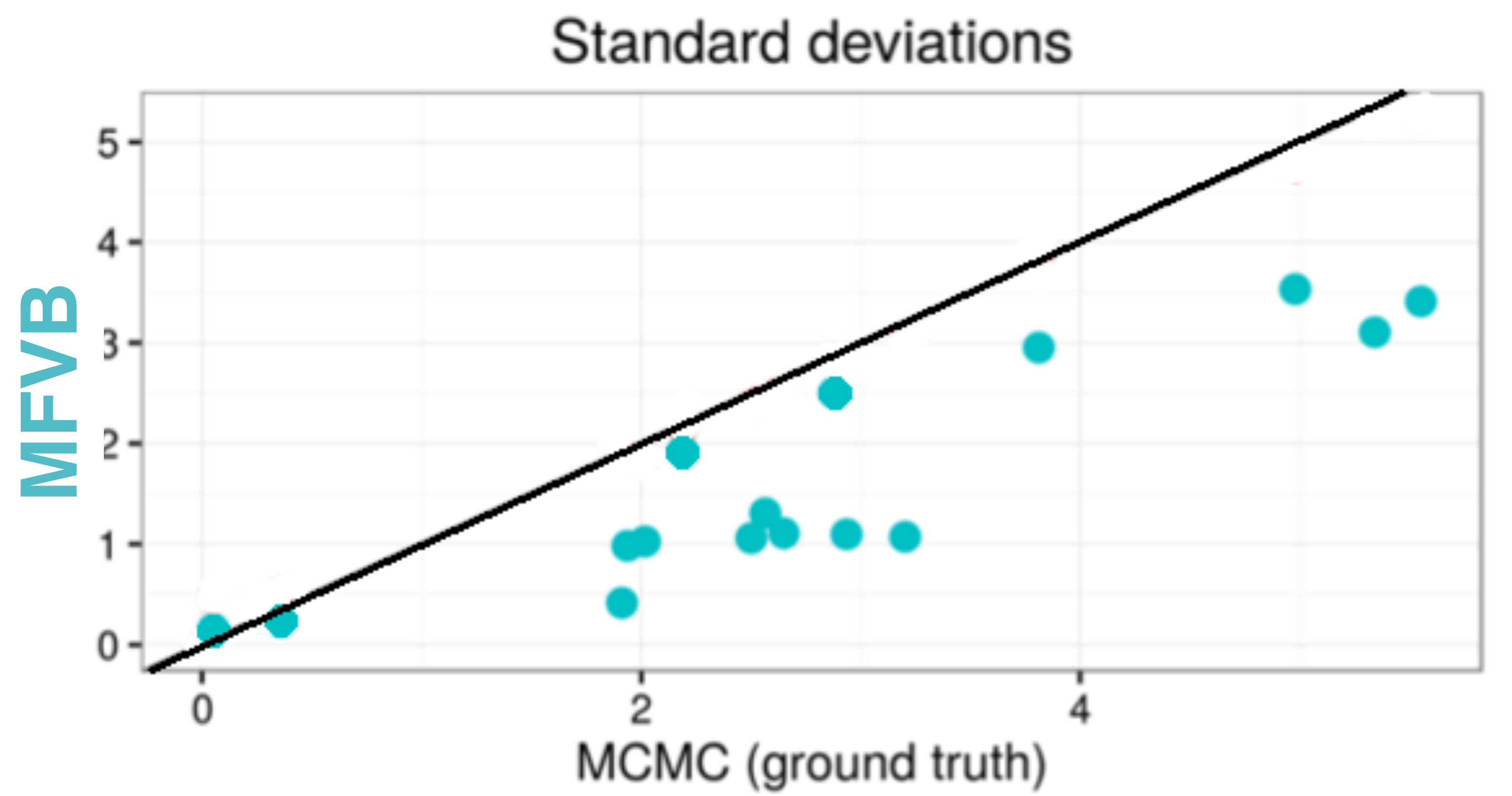

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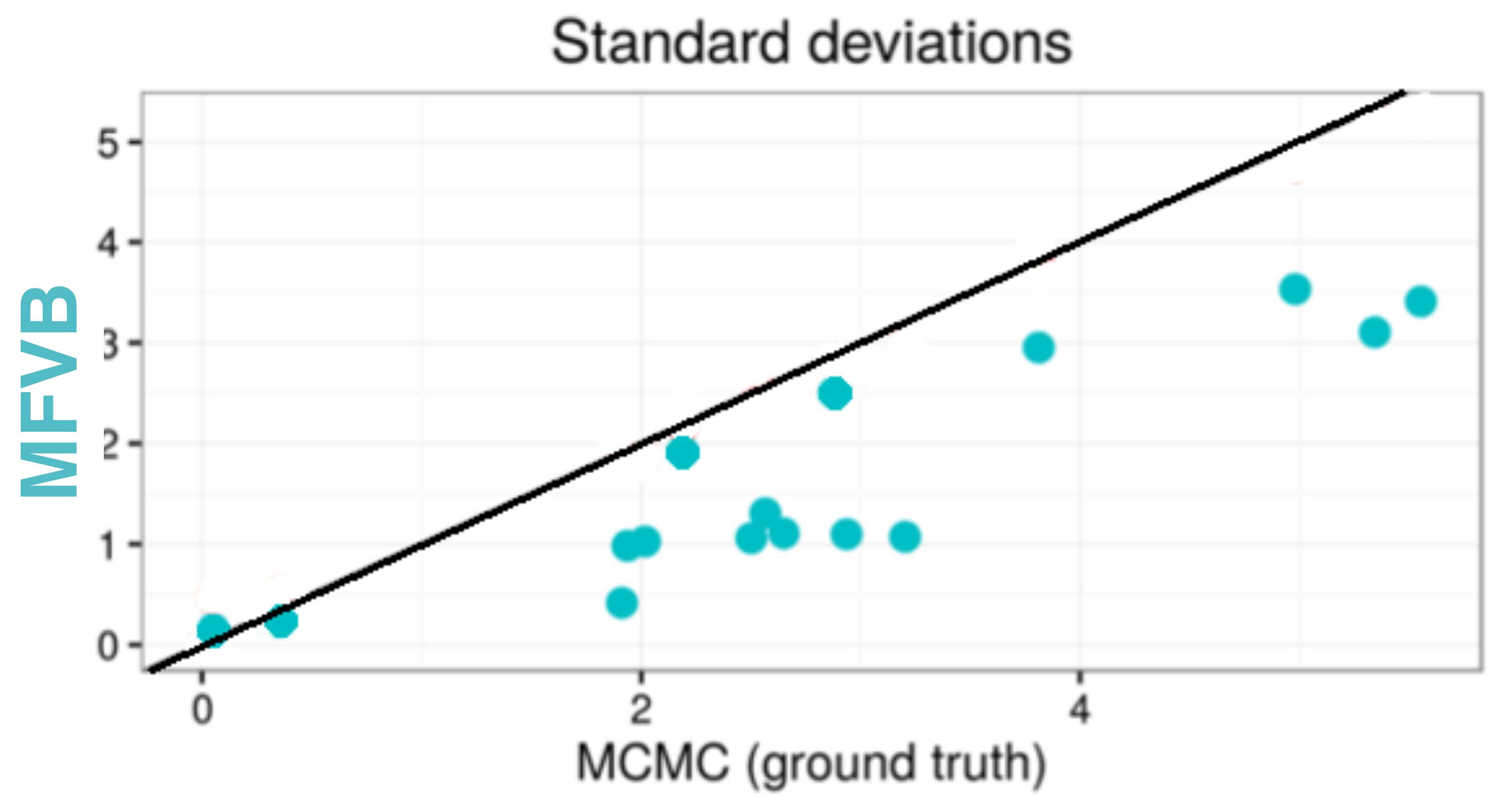

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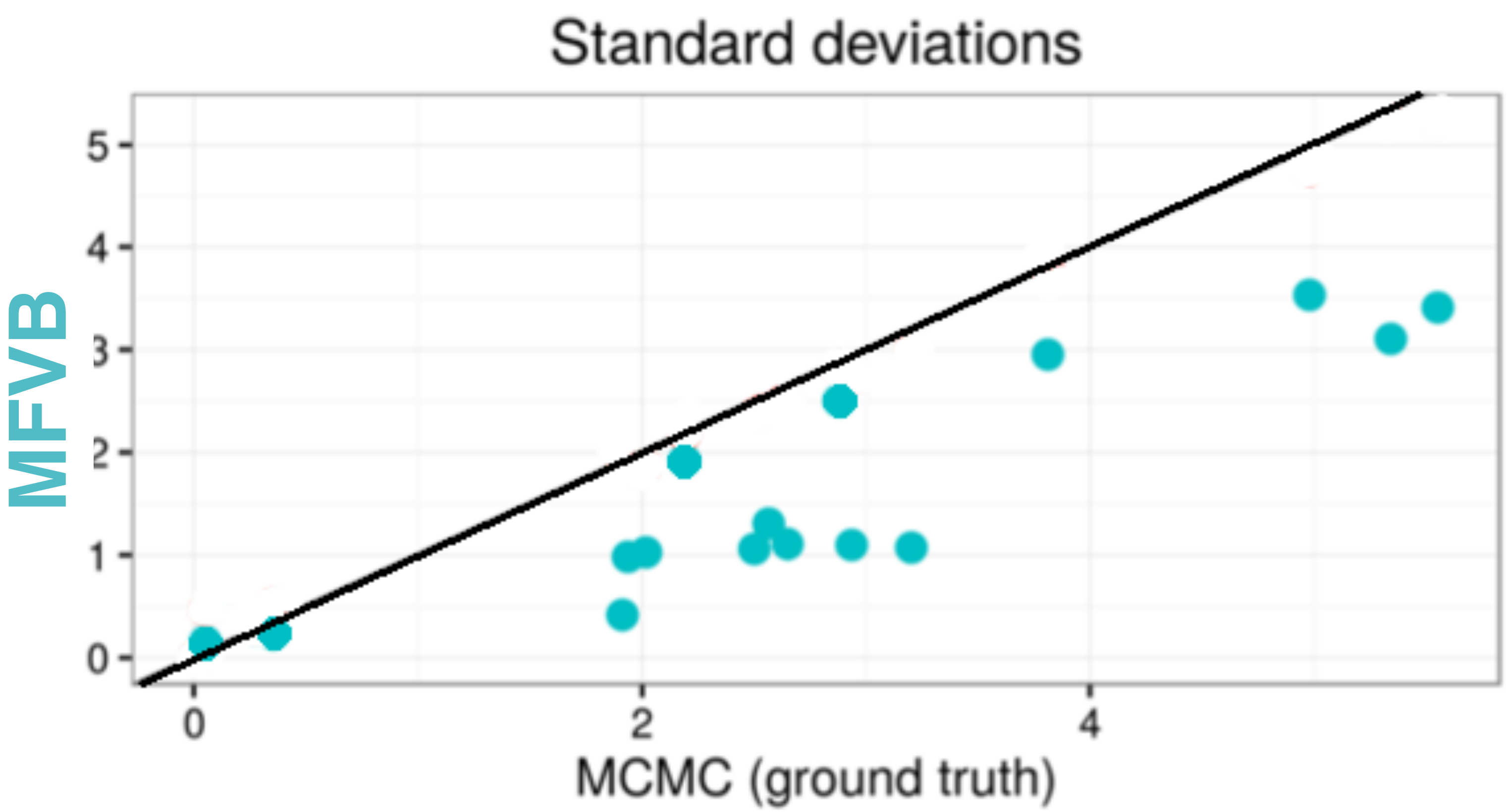

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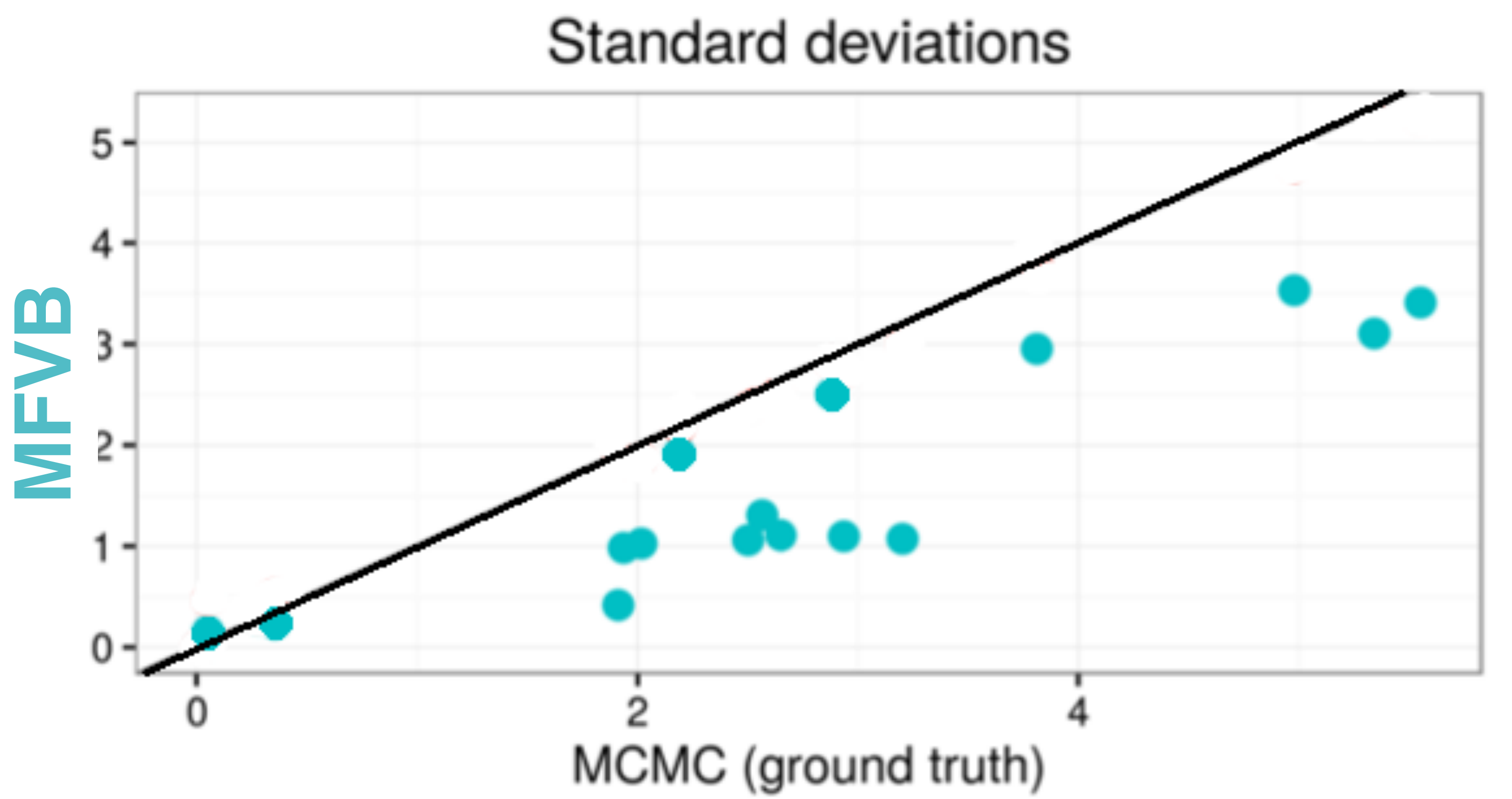

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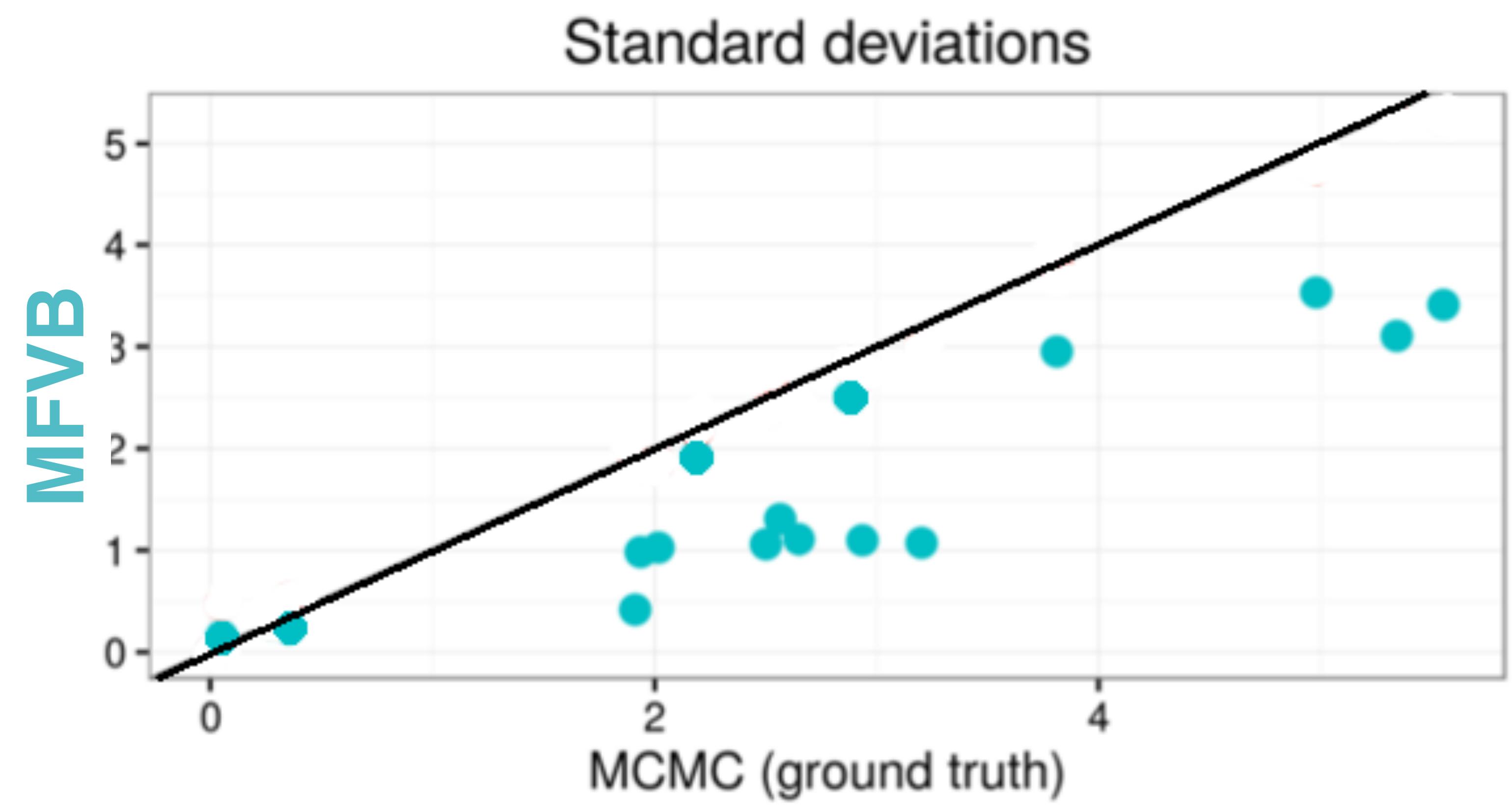
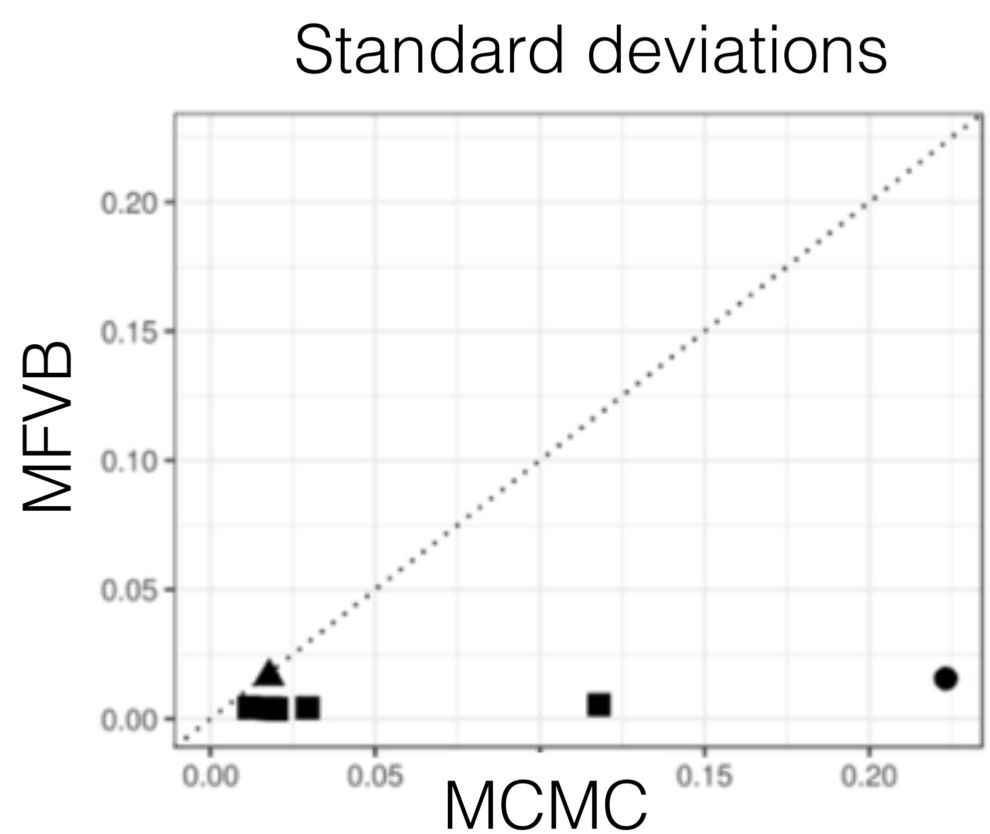


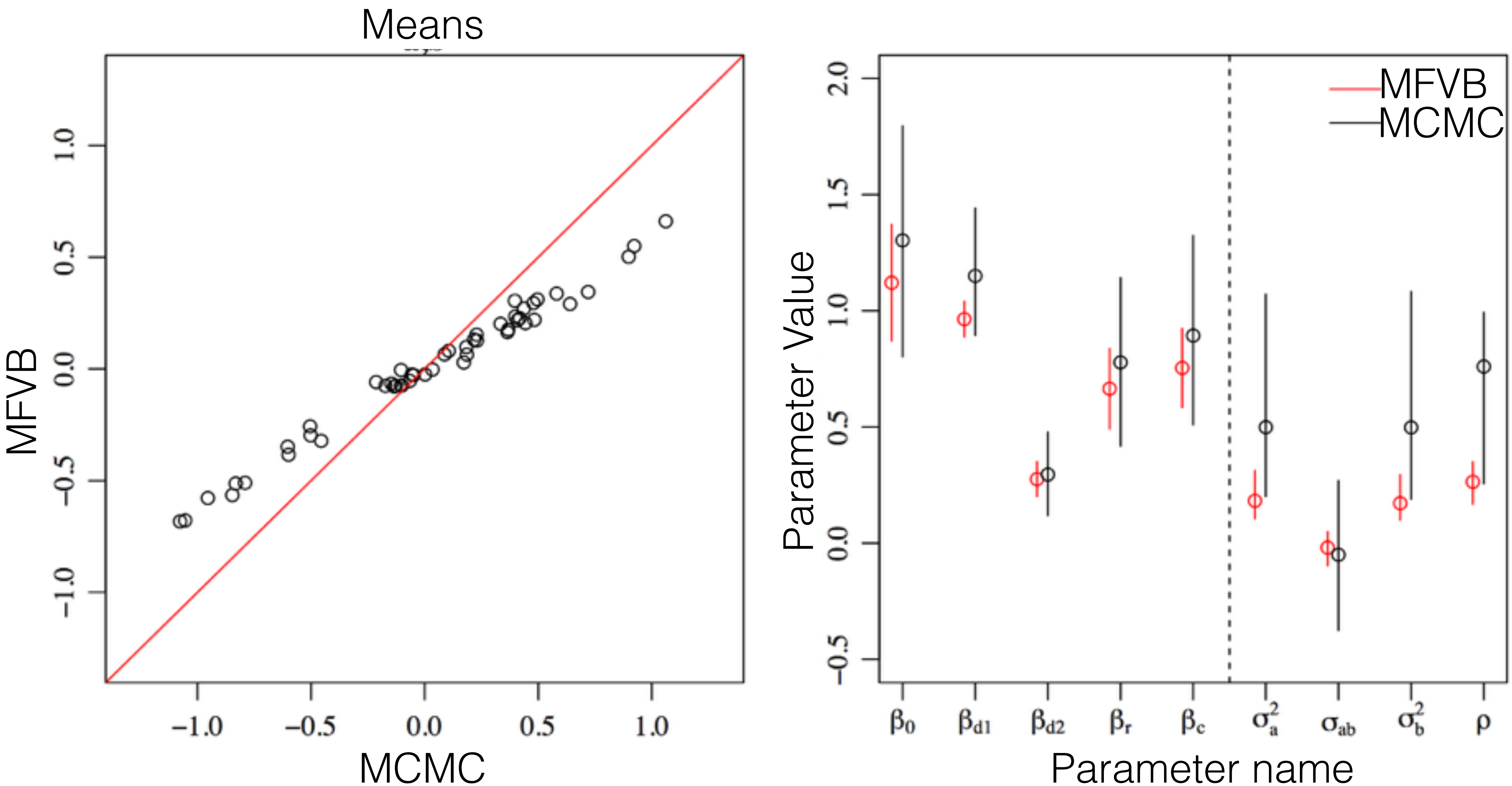
- Criteo online ads experiment



[Giordano, Broderick, Meager, Huggins, Jordan 2016; Giordano, Broderick, Jordan 2017]

What about means?

- Model for relational data with covariates
- When 1000+ nodes, MCMC > 1 day [Fosdick 2013, Ch 4]



[Fosdick 2013, Ch 4, Fig 4.3]

Posterior means: revisited

- Want to predict college GPA y_n

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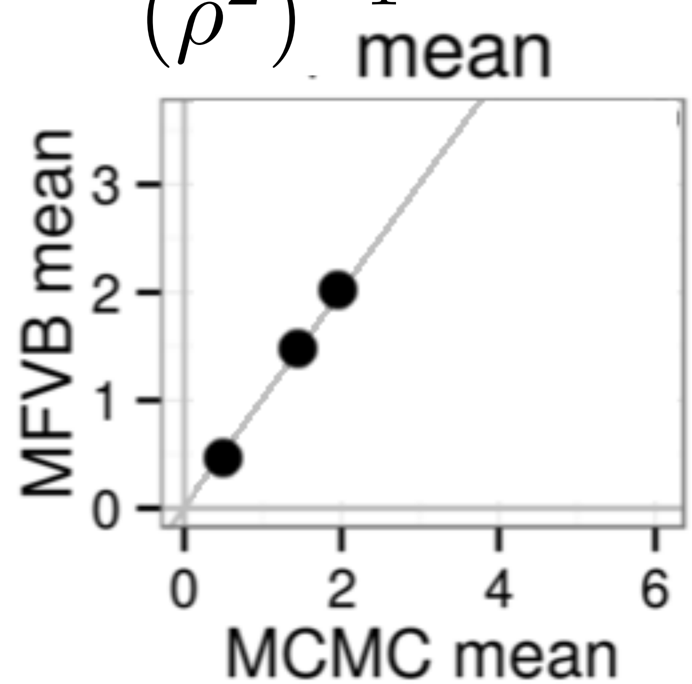
- Data simulated from model (3 data sets, 300 data points):

$(\rho^2)^{-1}$



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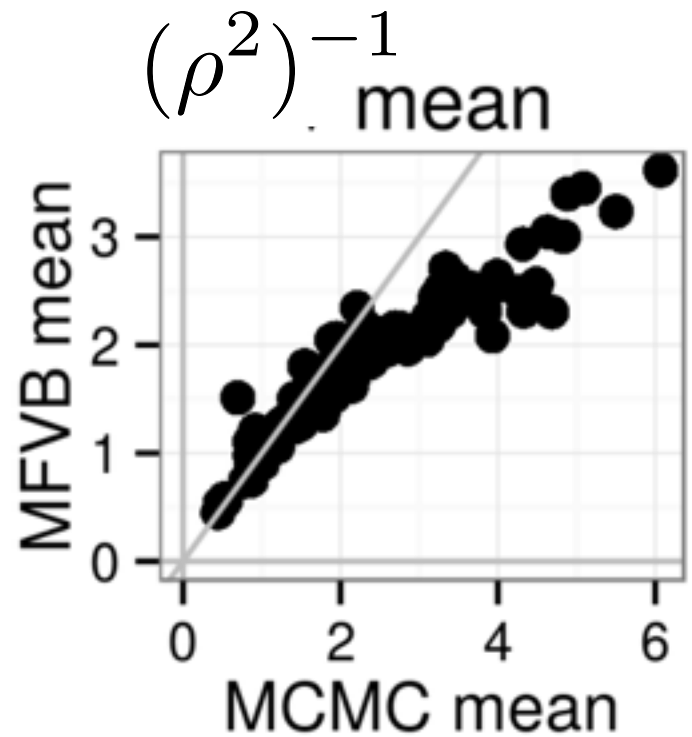

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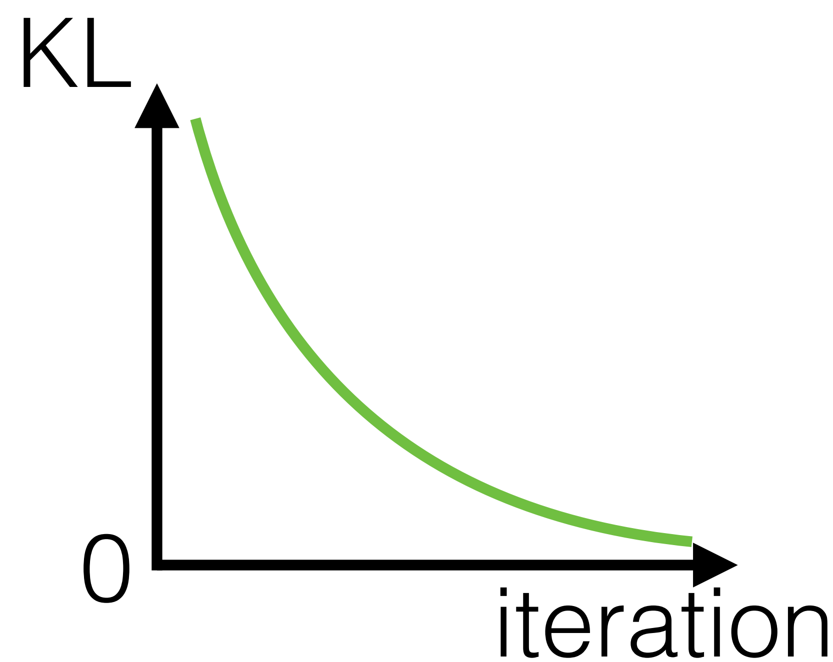

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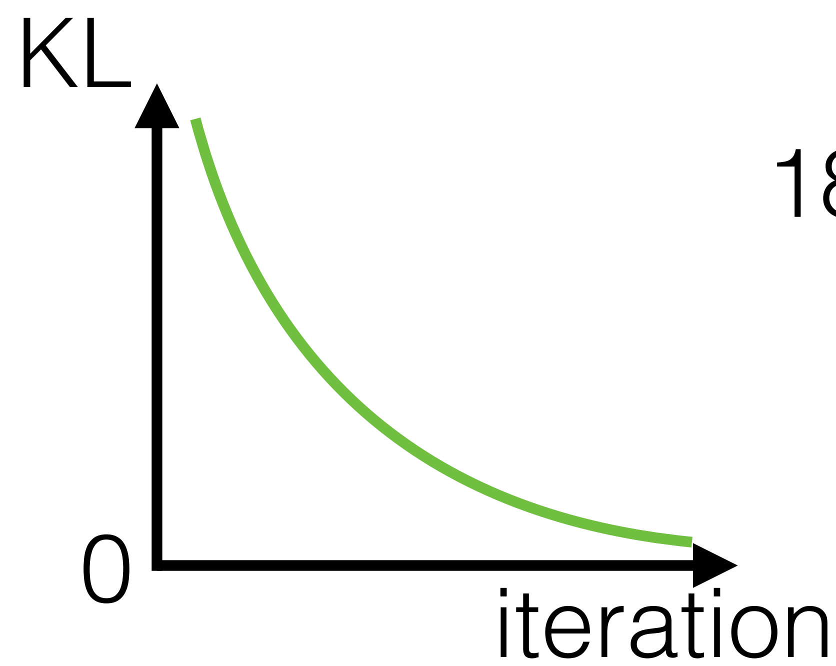

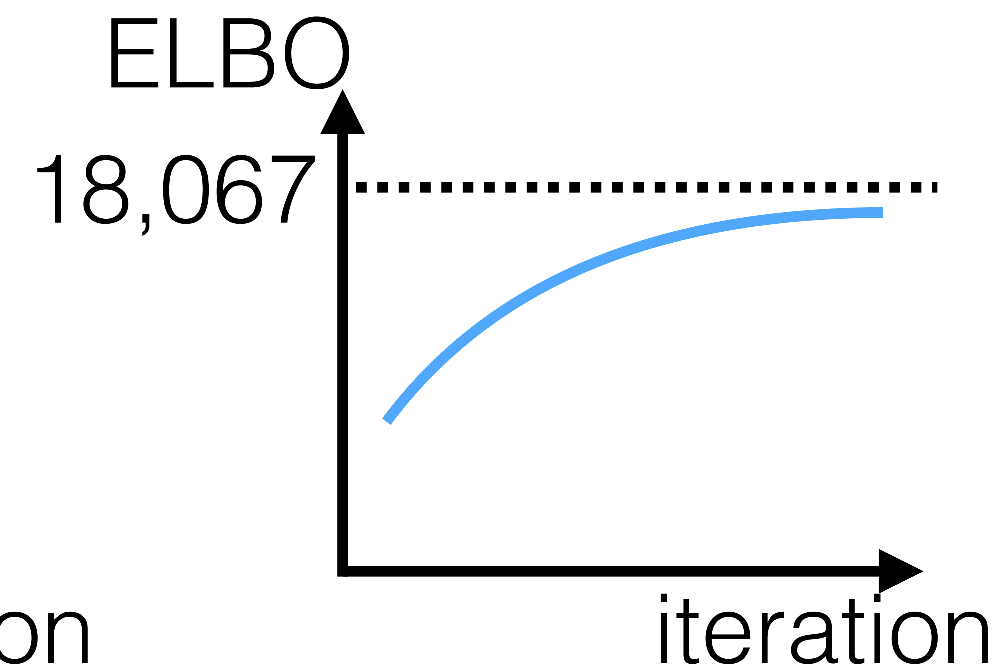

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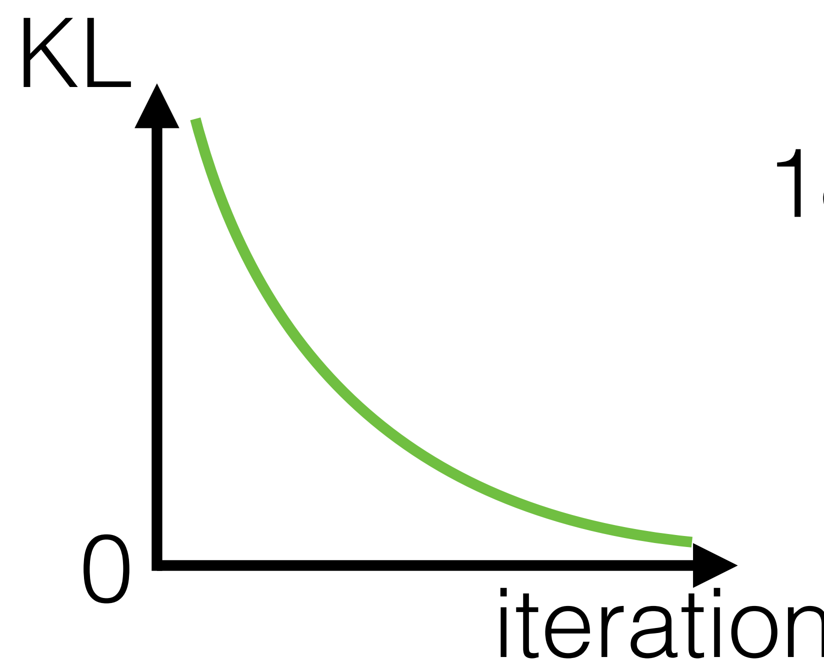

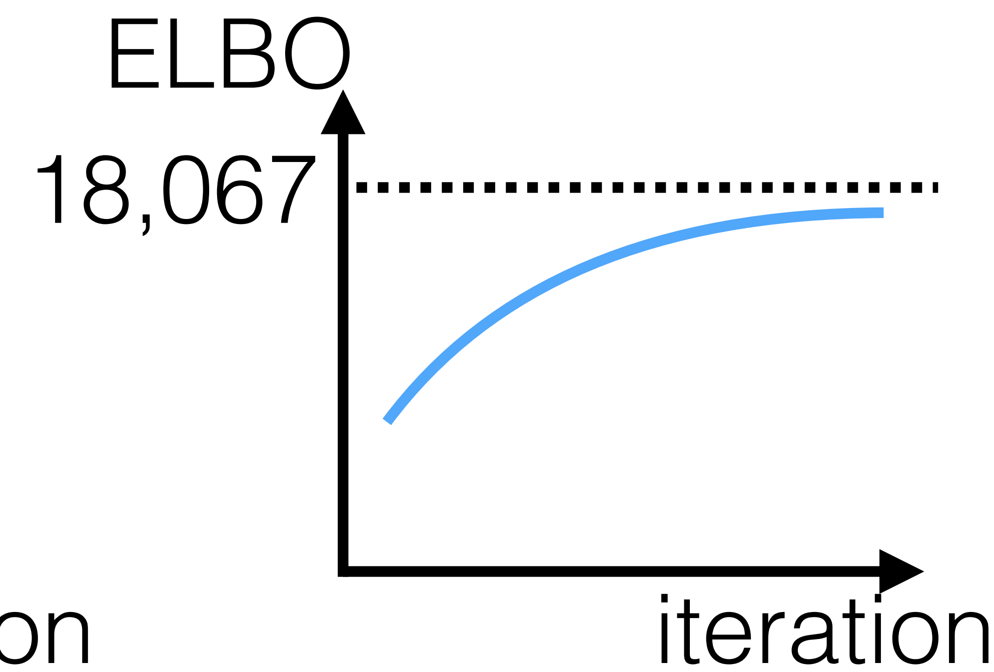

"Yes, but did it work? Evaluating variational inference" ICML 2018

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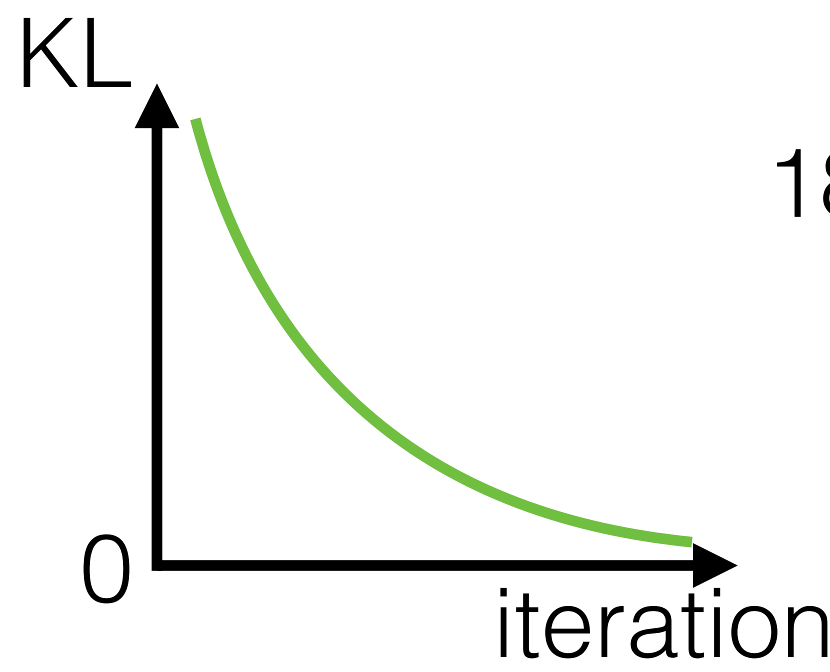

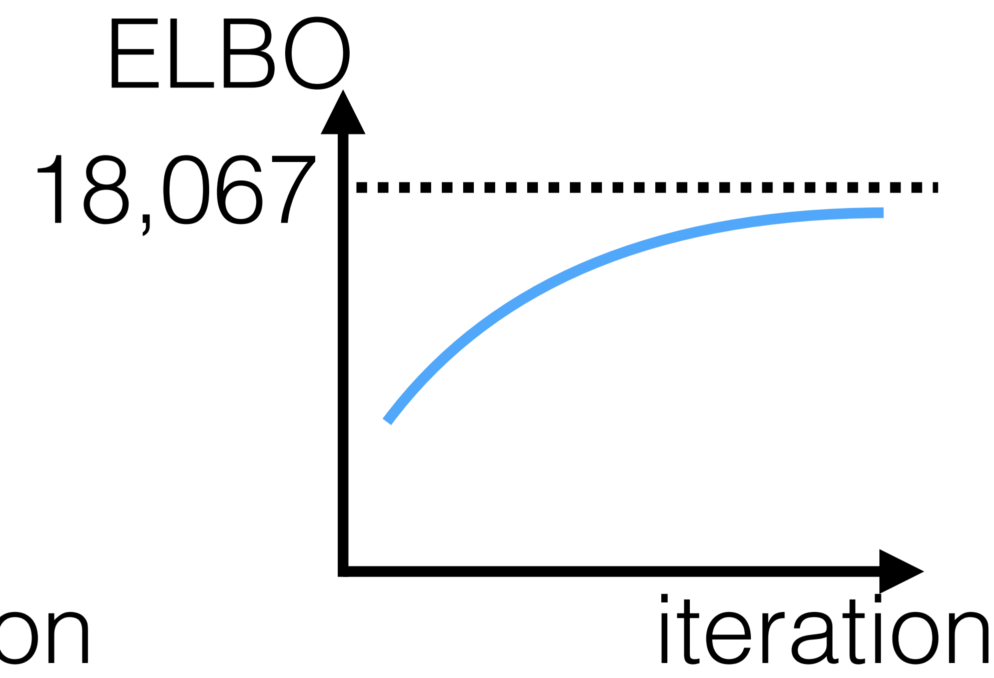

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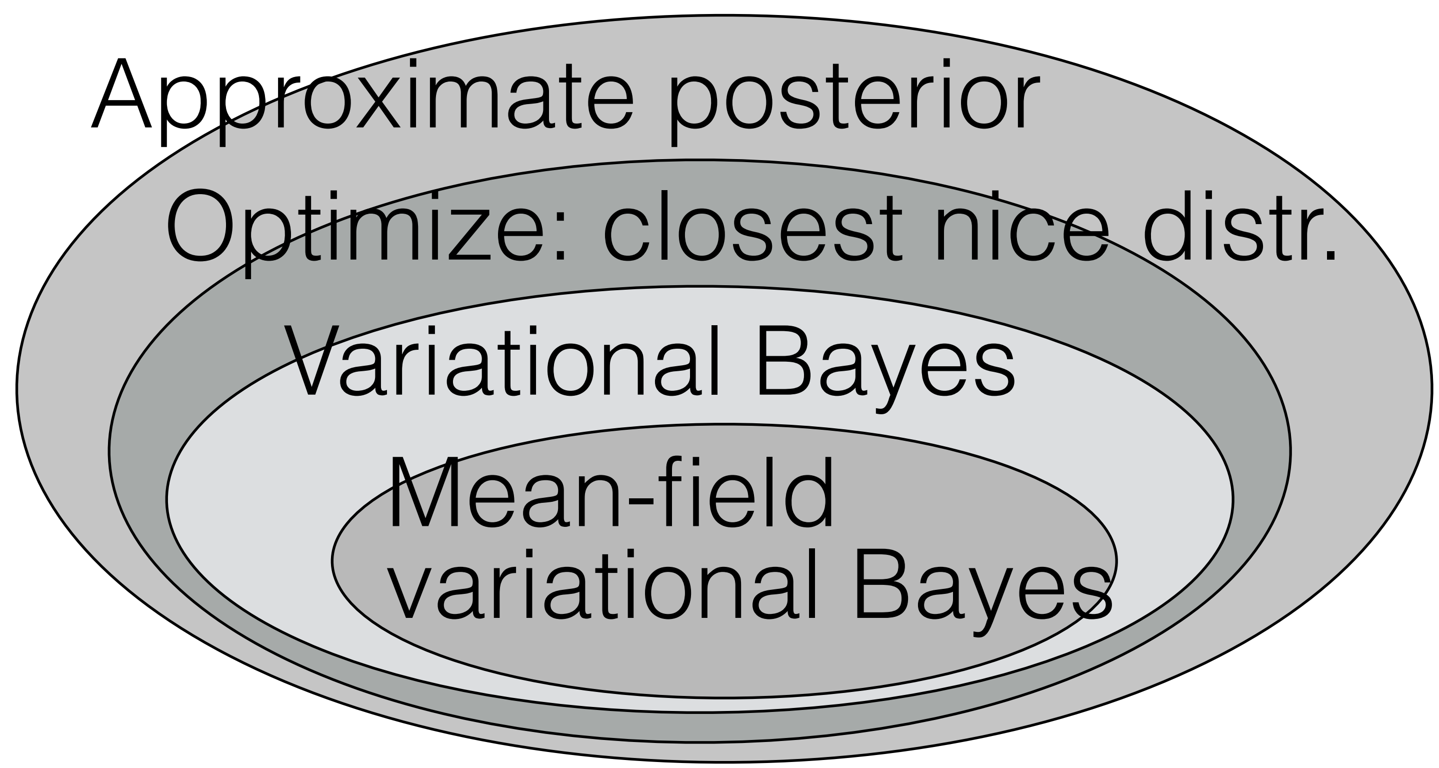

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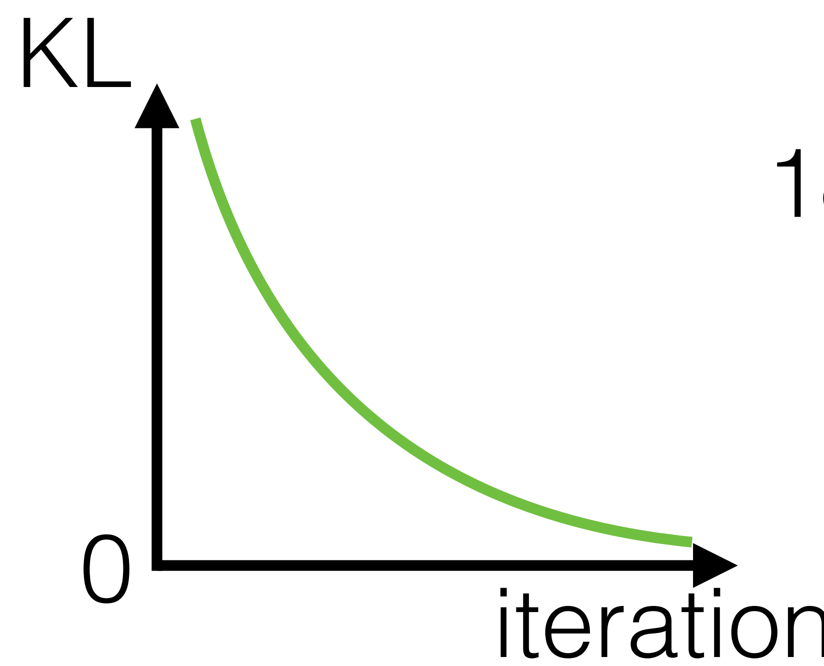
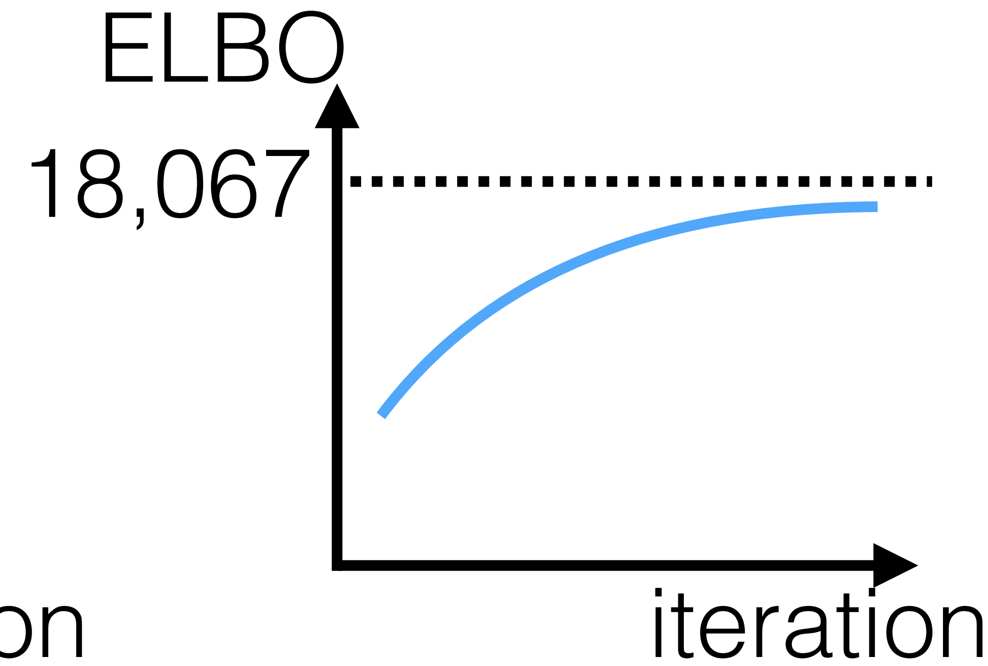
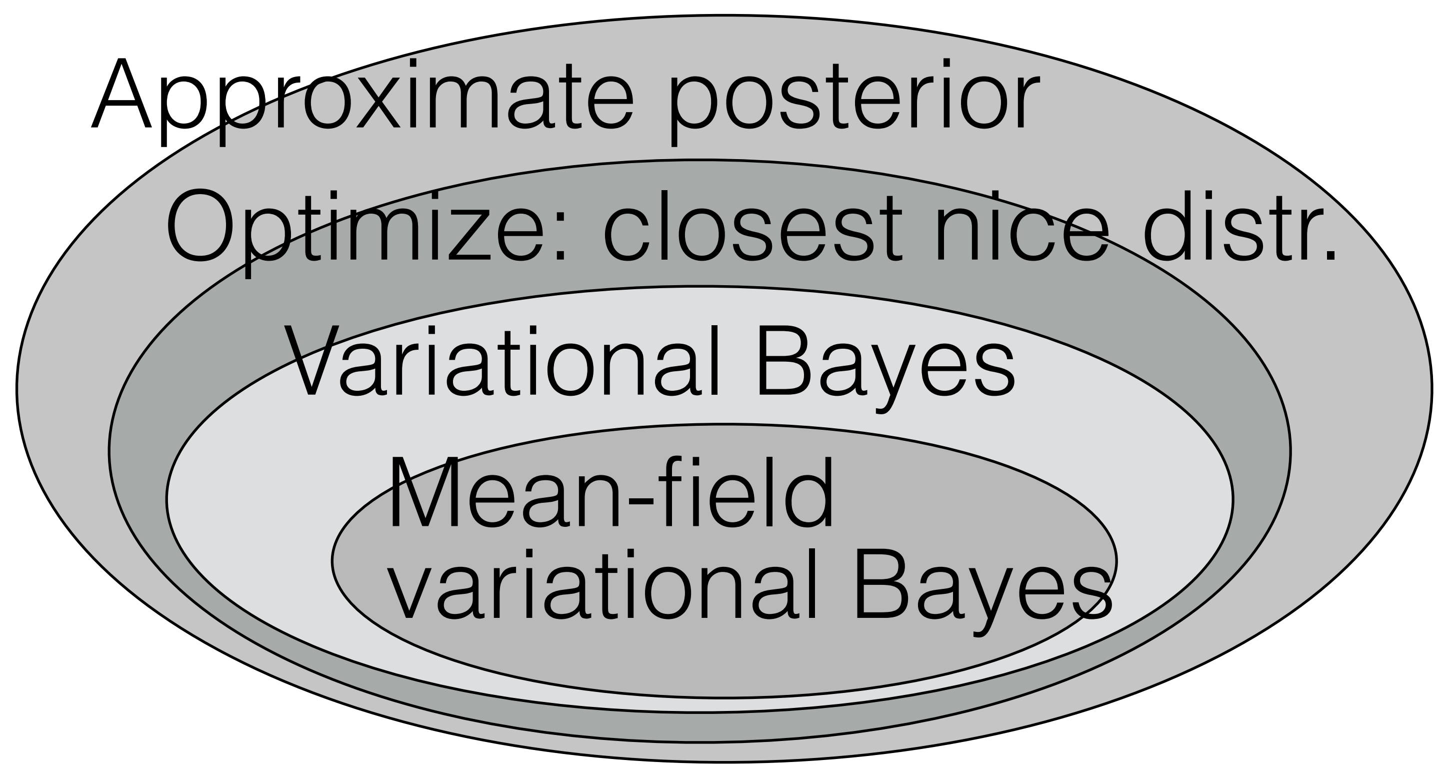




“Yes, but did it work? Evaluating variational inference” ICML 2018

- Richer “nice” set; alternative divergences



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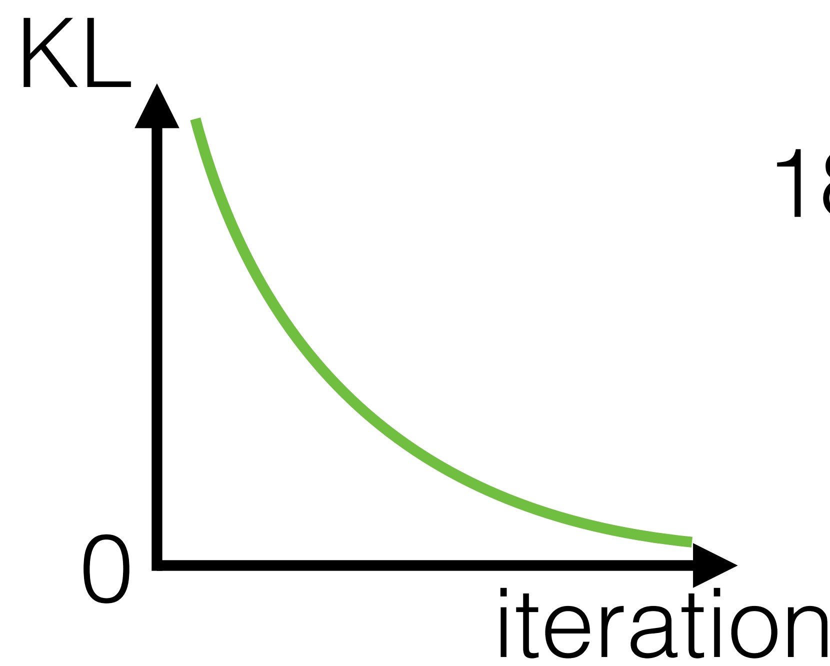
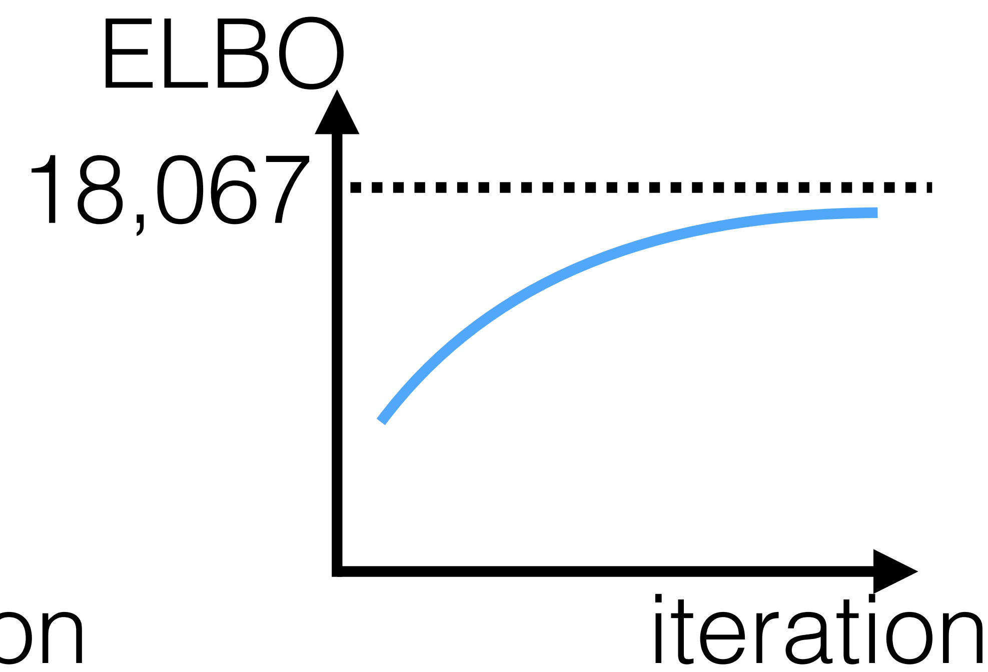
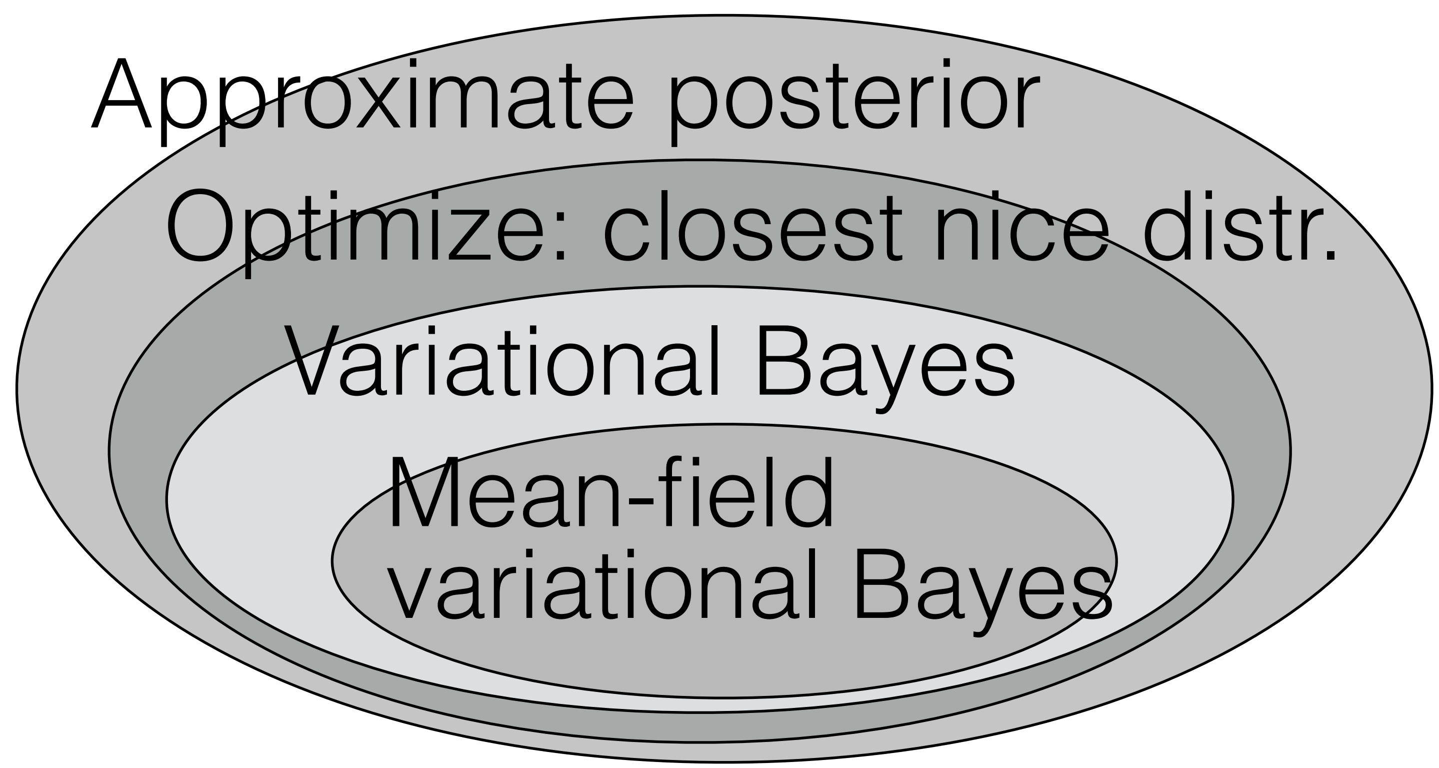




“Yes, but did it work? Evaluating variational inference” ICML 2018

- Richer “nice” set; alternative divergences

[Turner, Sahani 2011]



What can we do?

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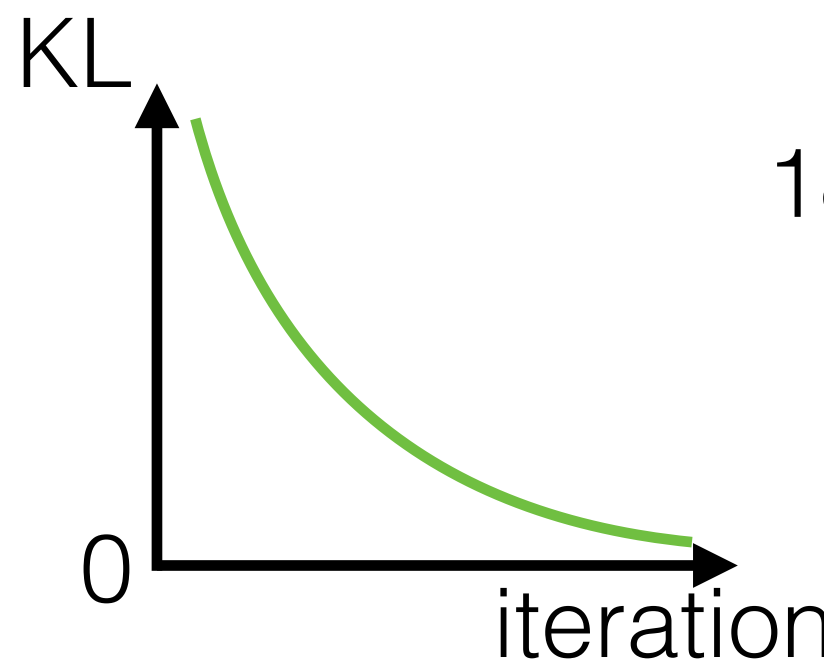
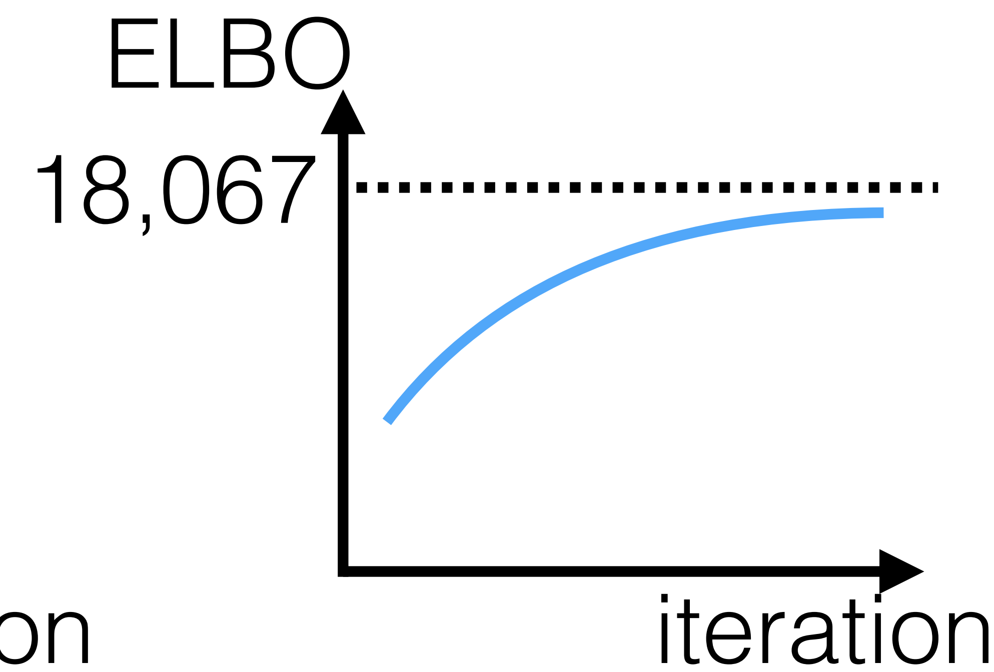
  - KL vs ELBO

[Gorham, Mackey 2015, 2017; Chwialkowski, Strathmann, Gretton 2016; Jitkrittum et al 2017; Talts et al 2018; Yao et al 2018, etc.]





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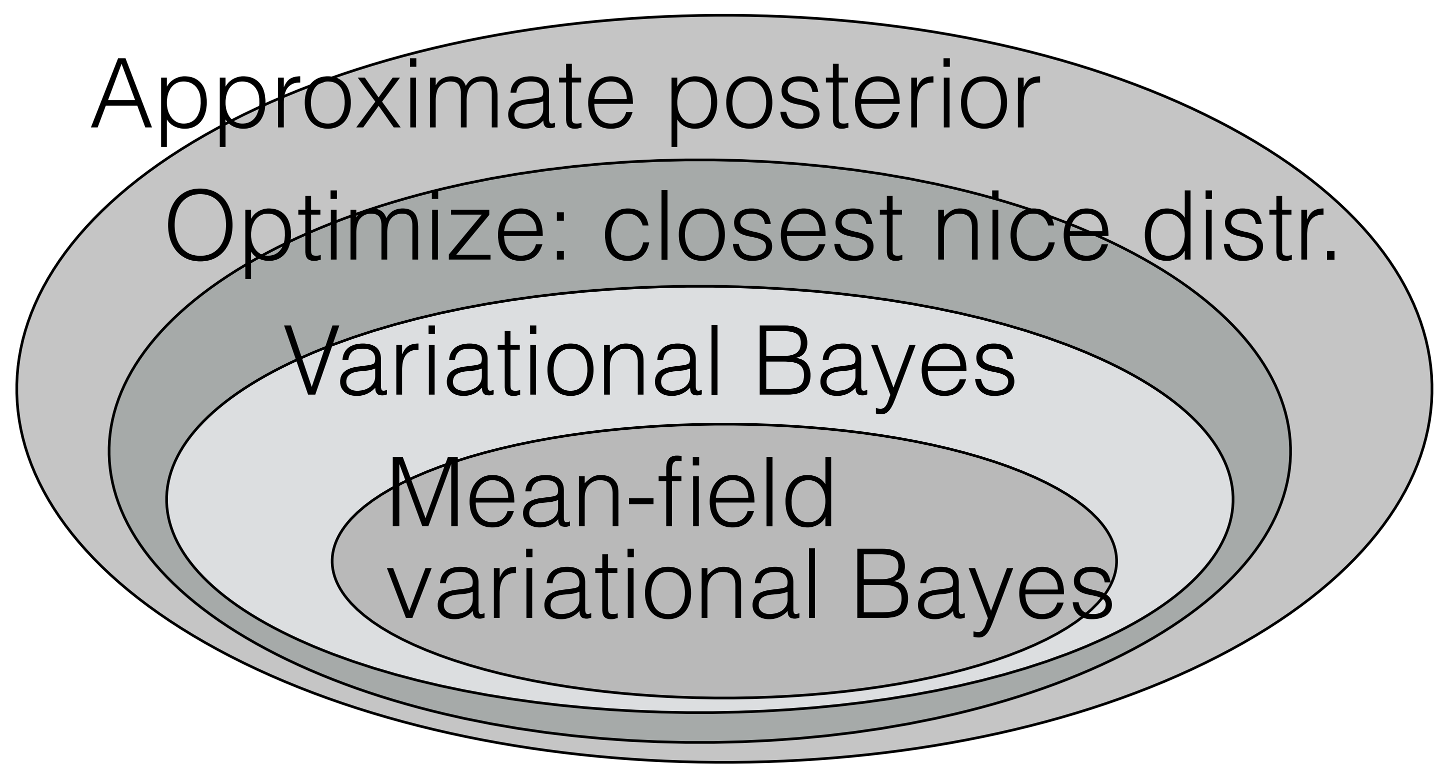
[Huggins, Kasprzak, Campbell, Broderick, 2018]



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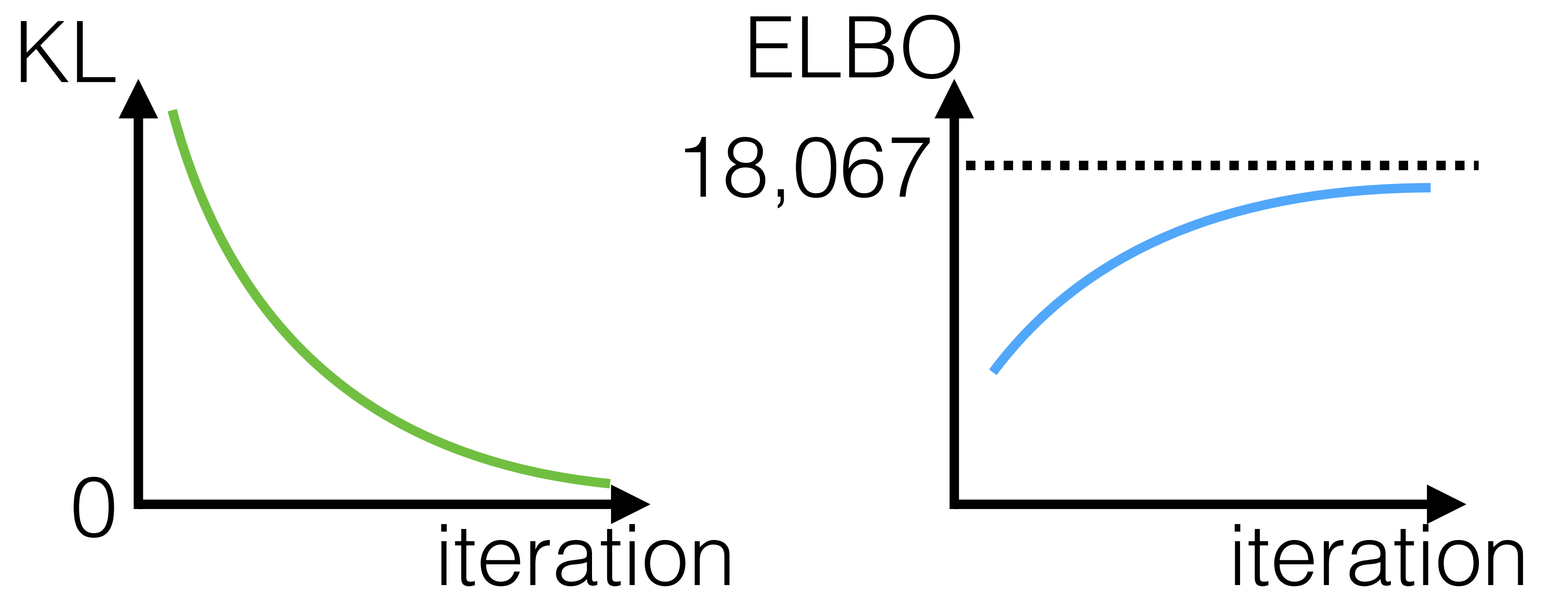

"Yes, but did it work? Evaluating variational inference" ICML 2018

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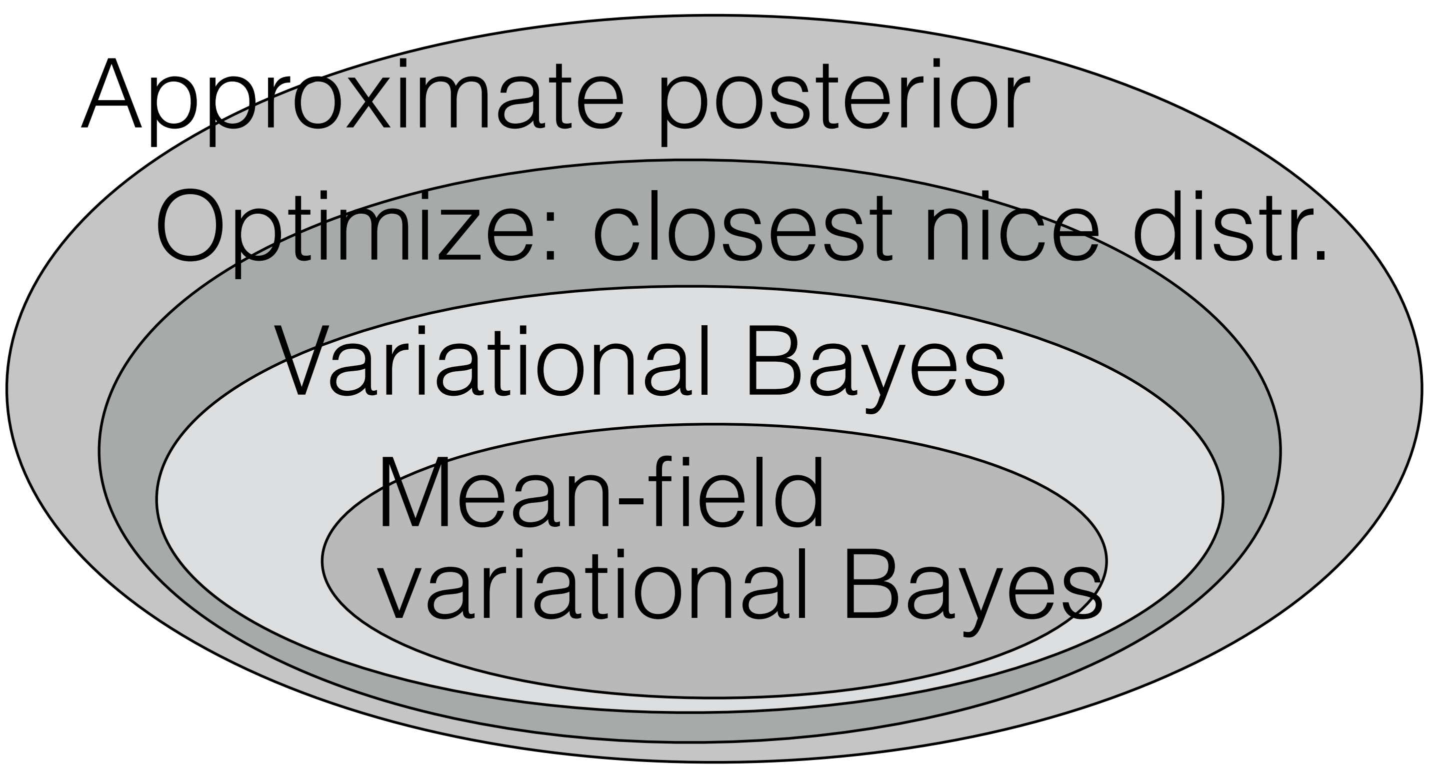
- Corrections [next]



What can we do?

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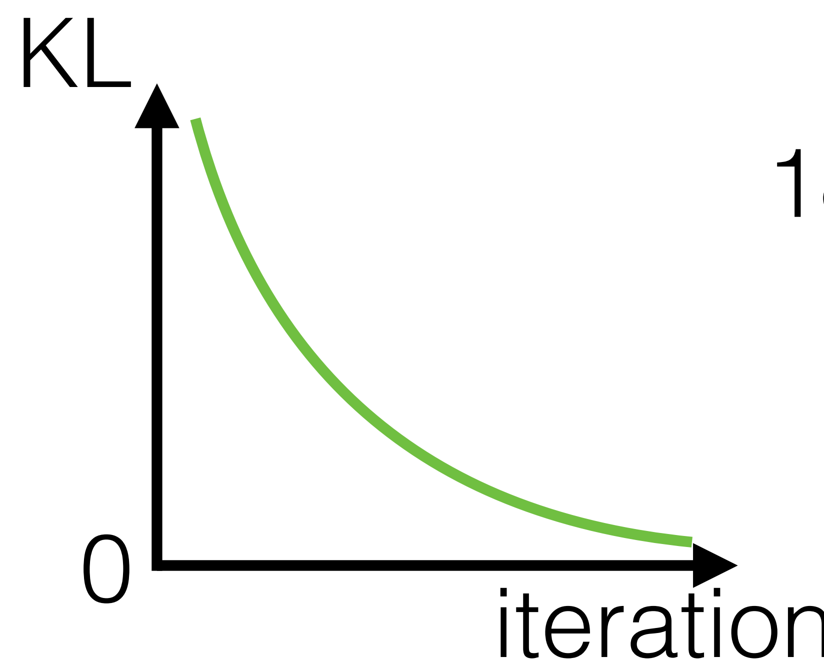

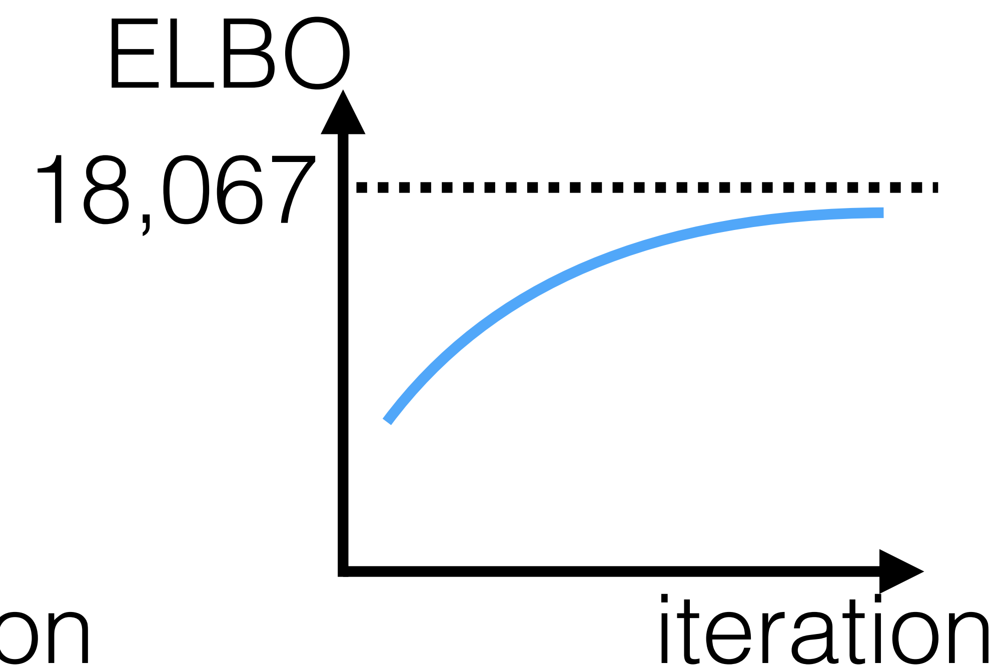

"Yes, but did it work? Evaluating variational inference" ICML 2018

- Richer "nice" set; alternative divergences

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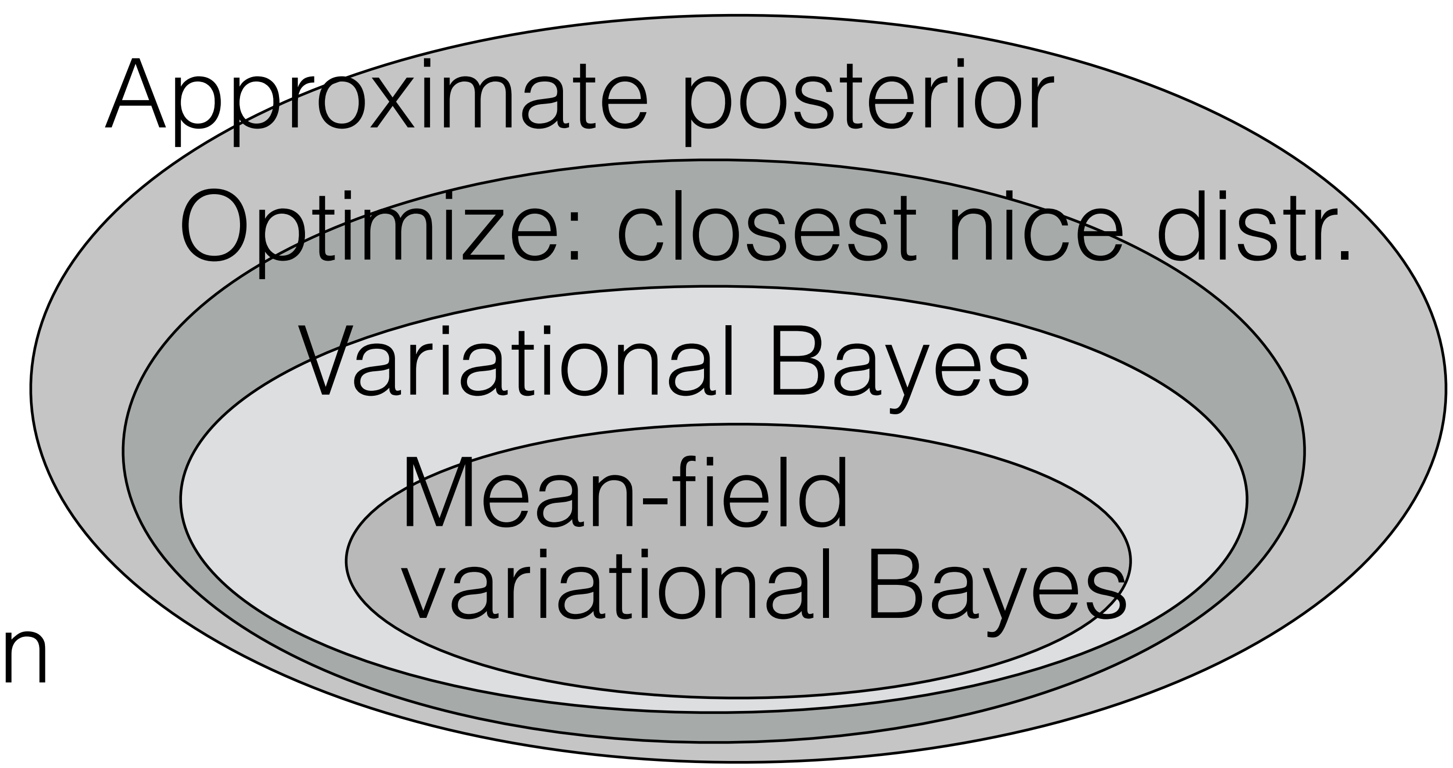
[Huggins, Kasprzak, Campbell, Broderick, 2018]

- Corrections [next]
- Theoretical guarantees on finite-data quality [next]



What to read next

Textbooks and Reviews

- Bishop. *Pattern Recognition and Machine Learning*, Ch 10. 2006.
- Blei, Kucukelbir, McAuliffe. Variational inference: A review for statisticians, *JASA* 2016.
- MacKay. *Information Theory, Inference, and Learning Algorithms*, Ch 33. 2003.
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More Experiments

- RJ Giordano, T Broderick, and MI Jordan. Linear response methods for accurate covariance estimates from mean field variational Bayes. *NIPS* 2015.
- RJ Giordano, T Broderick, R Meager, J Huggins, and MI Jordan. Fast robustness quantification with variational Bayes. *ICML Data4Good Workshop* 2016.
- RJ Giordano, T Broderick, and MI Jordan. Covariances, robustness, and variational Bayes. *Journal of Machine Learning Research*, to appear. ArXiv:1709.02536.
- J Huggins, T Campbell, M Kasprzak, T Broderick. Practical bounds on the error of Bayesian posterior approximations: A nonasymptotic approach, 2018. ArXiv: 1809.09505.

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JH Huggins, RP Adams, and T Broderick. PASS-GLM: Polynomial approximate sufficient statistics for scalable Bayesian GLM inference. *NIPS* 2017.

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