Nonparametric Bayesian Statistics

Tamara Broderick
ITT Career Development Assistant Professor
Electrical Engineering & Computer Science
MIT

Nonparametric Bayes

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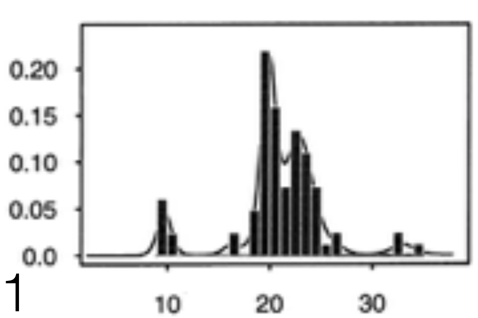

[Escobar, West 1995; Ghosal et al 1999]

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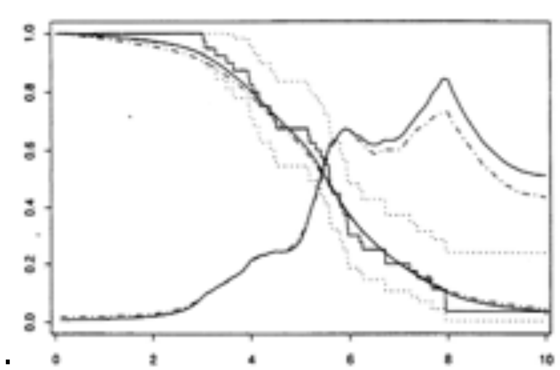
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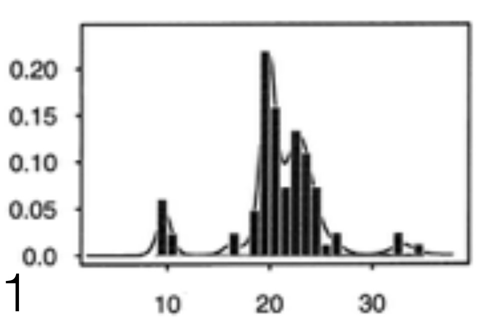

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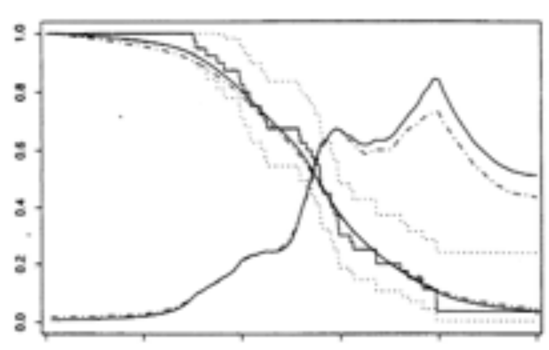

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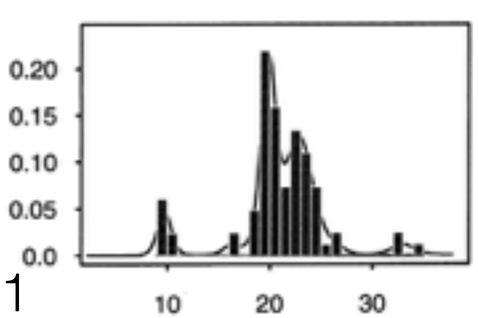

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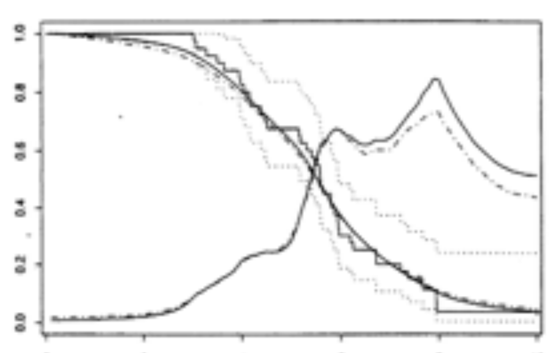

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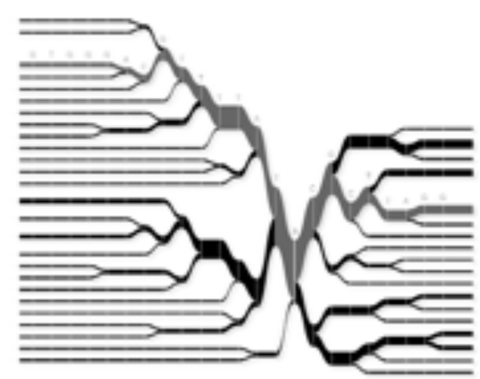

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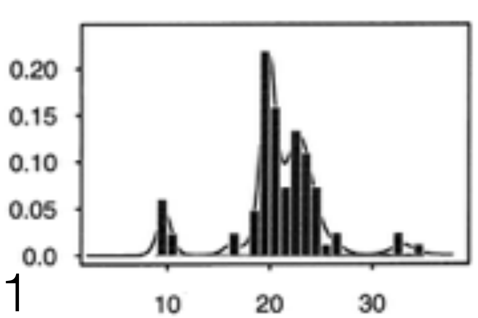

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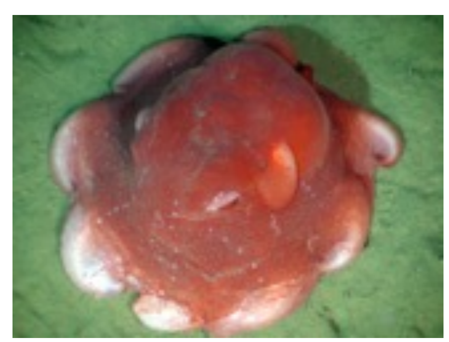

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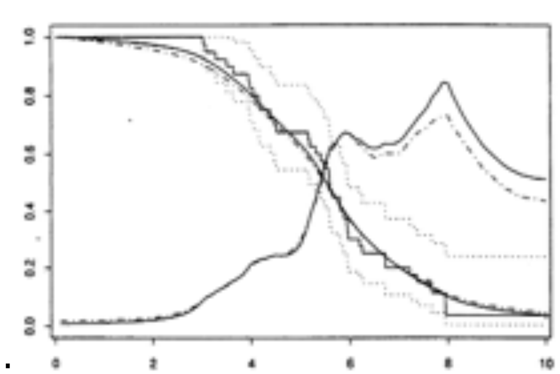

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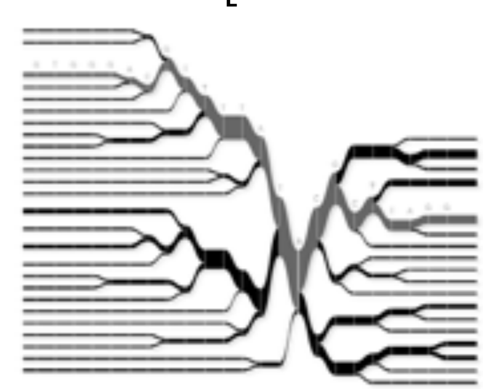

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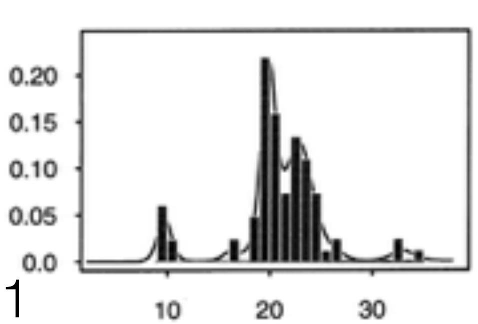

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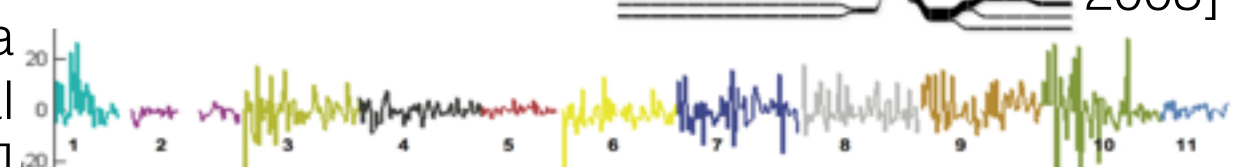

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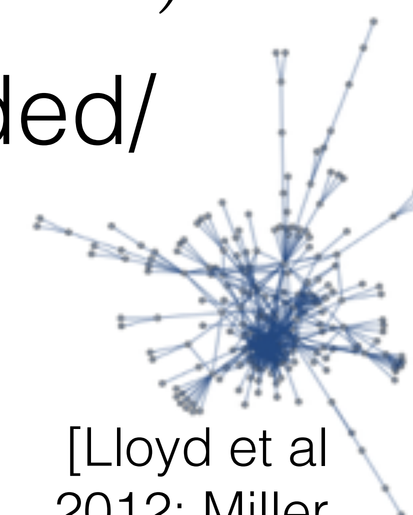

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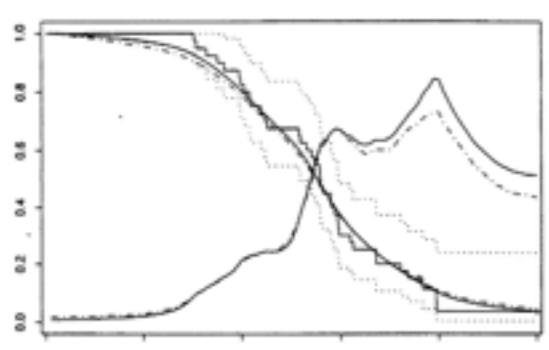

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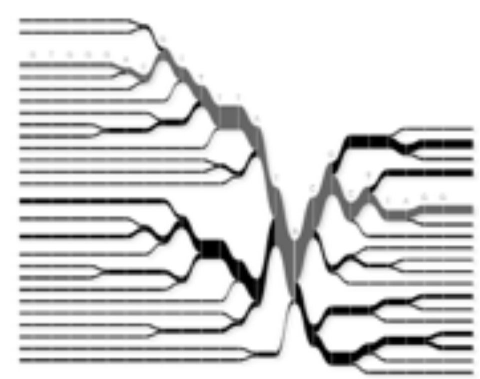

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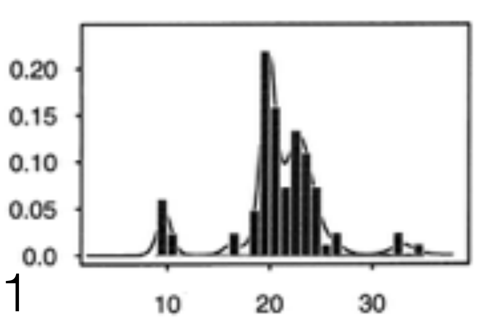

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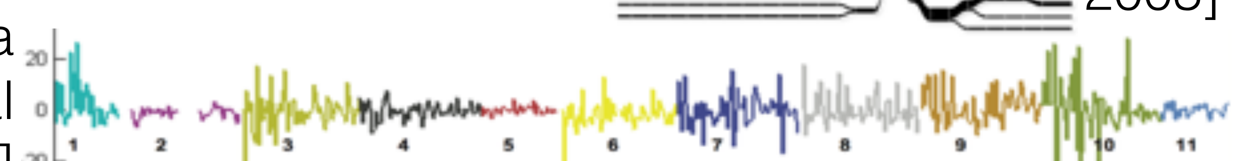

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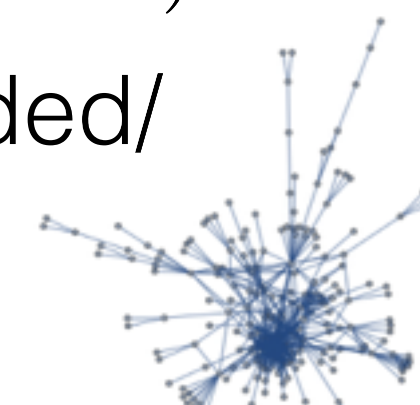

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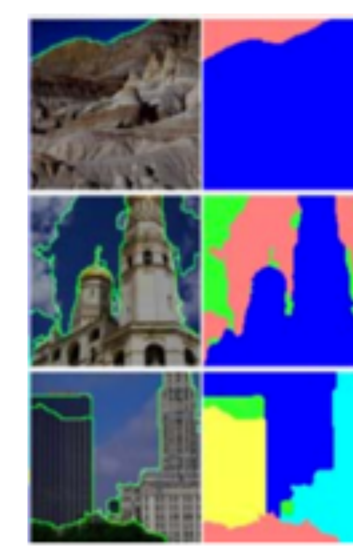
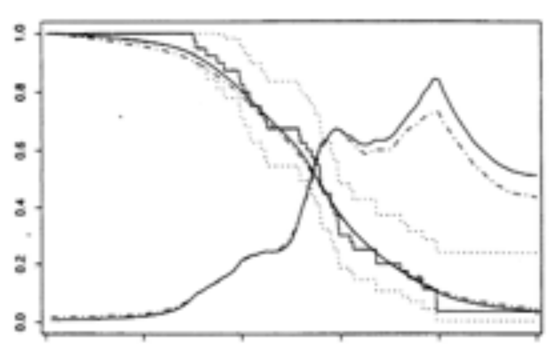


[Sudderth, Jordan 2009]



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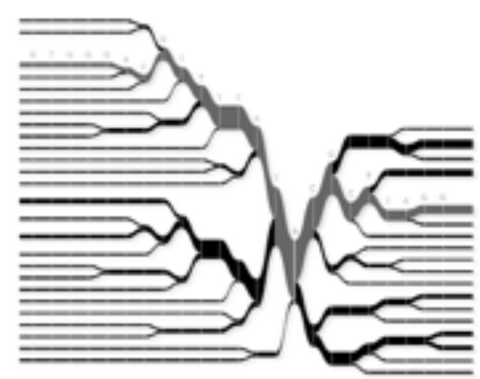

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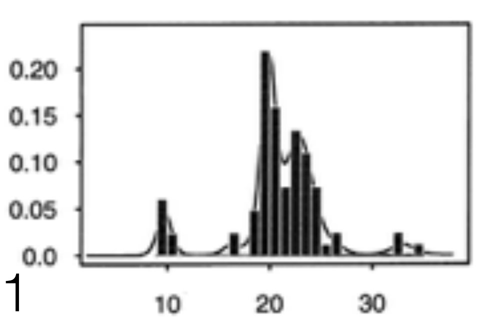

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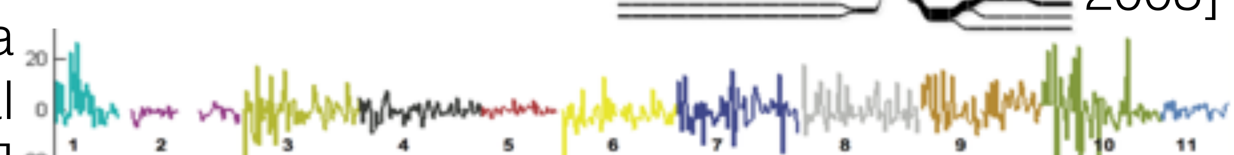

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- Dirichlet process

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    - Background for intuition

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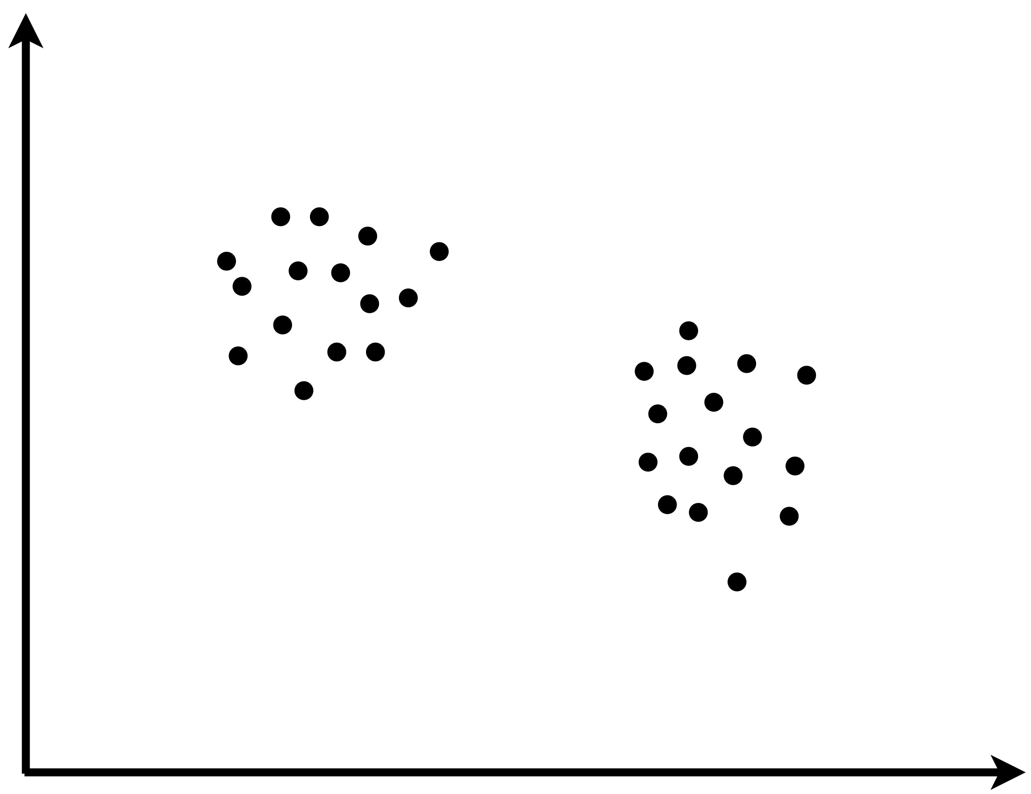
- Venture further into the wild world of Nonparametric Bayesian statistics

Generative model



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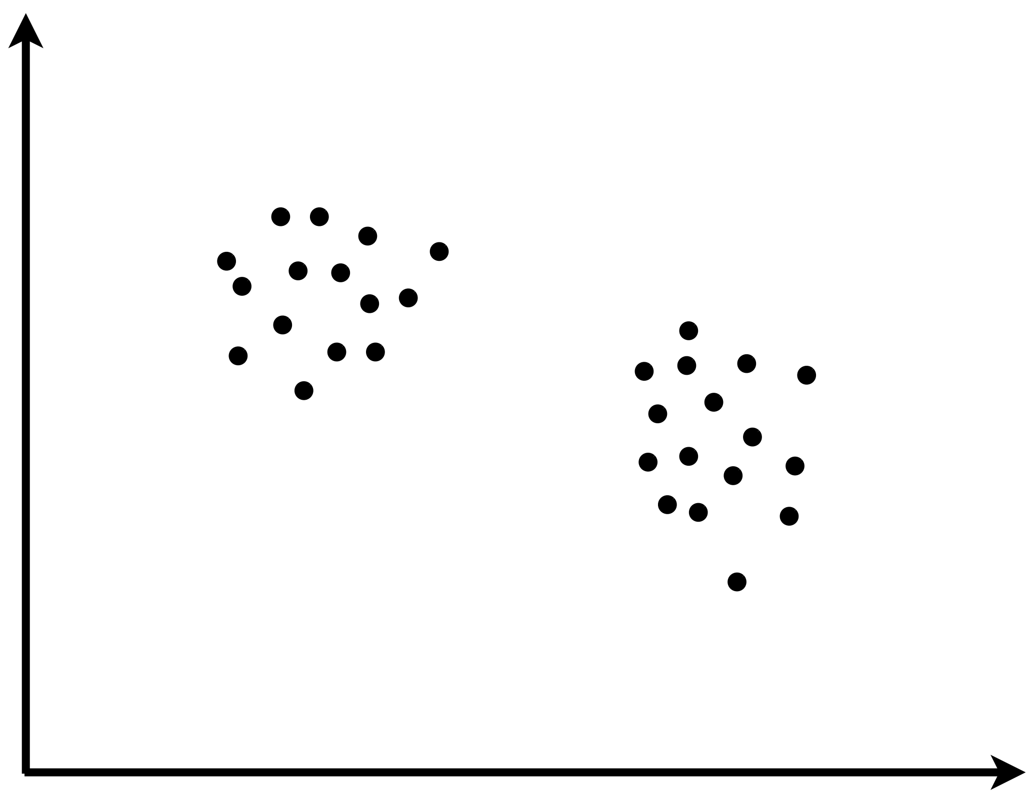

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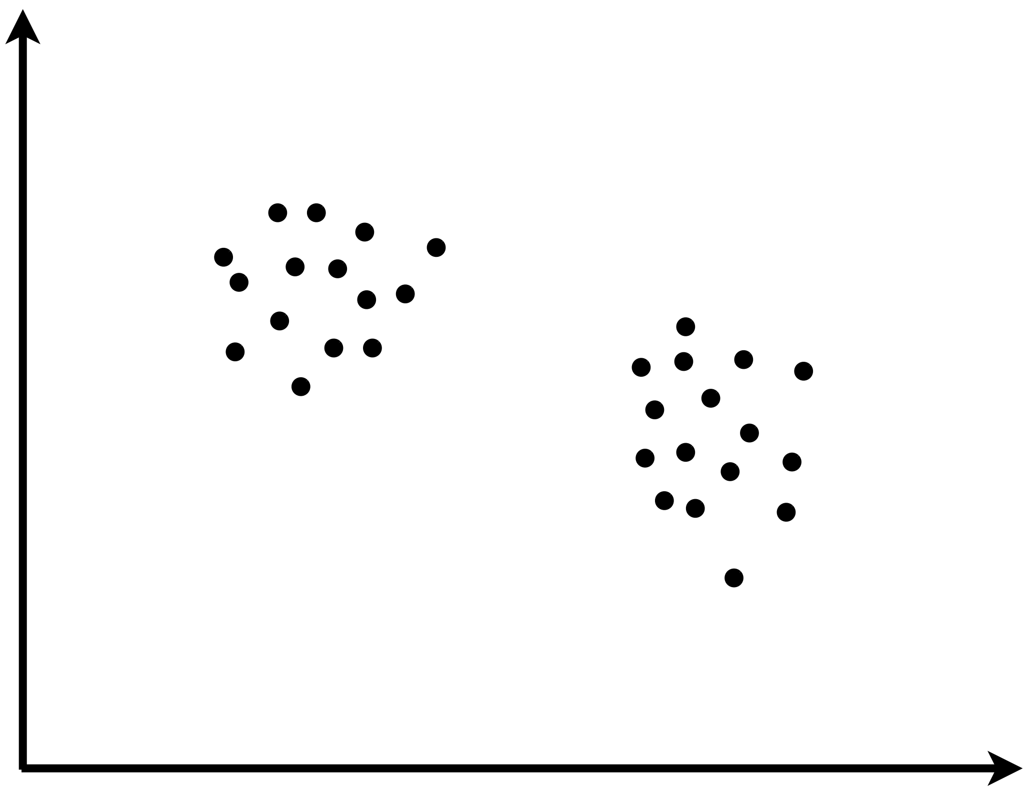

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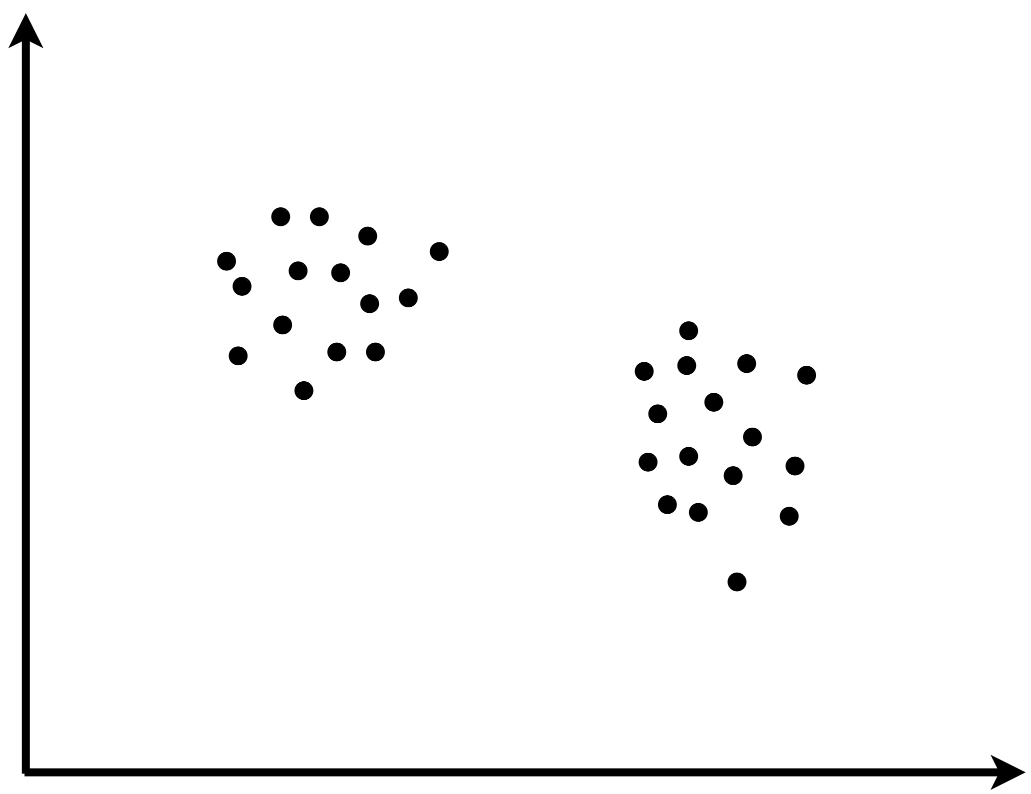

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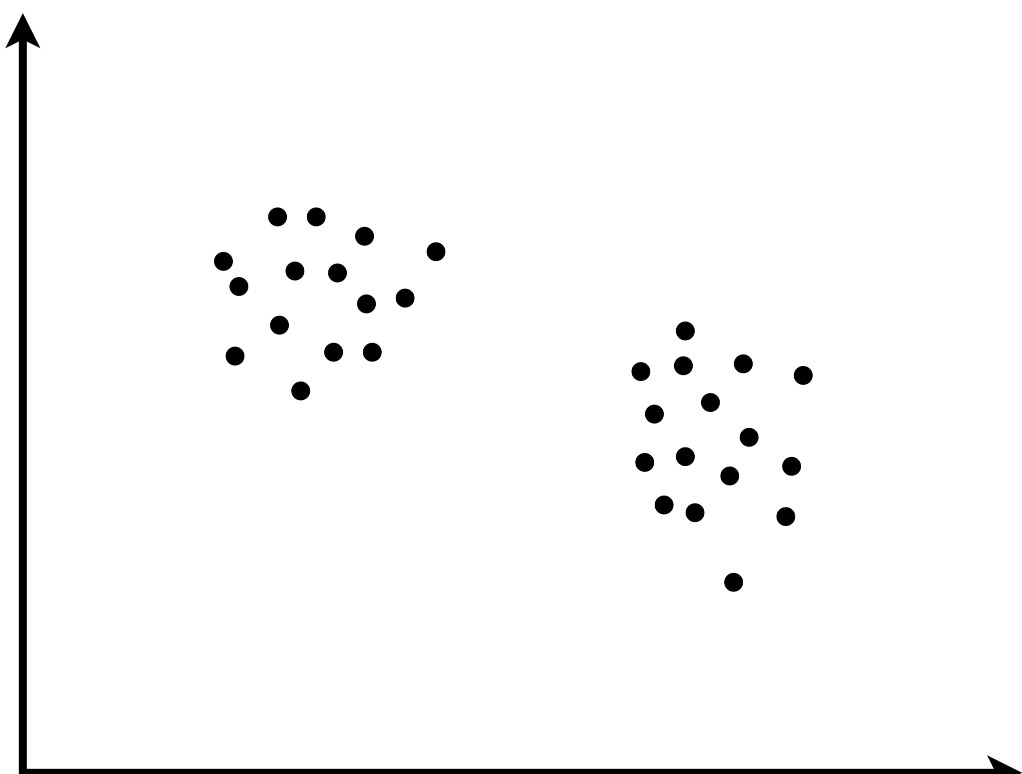

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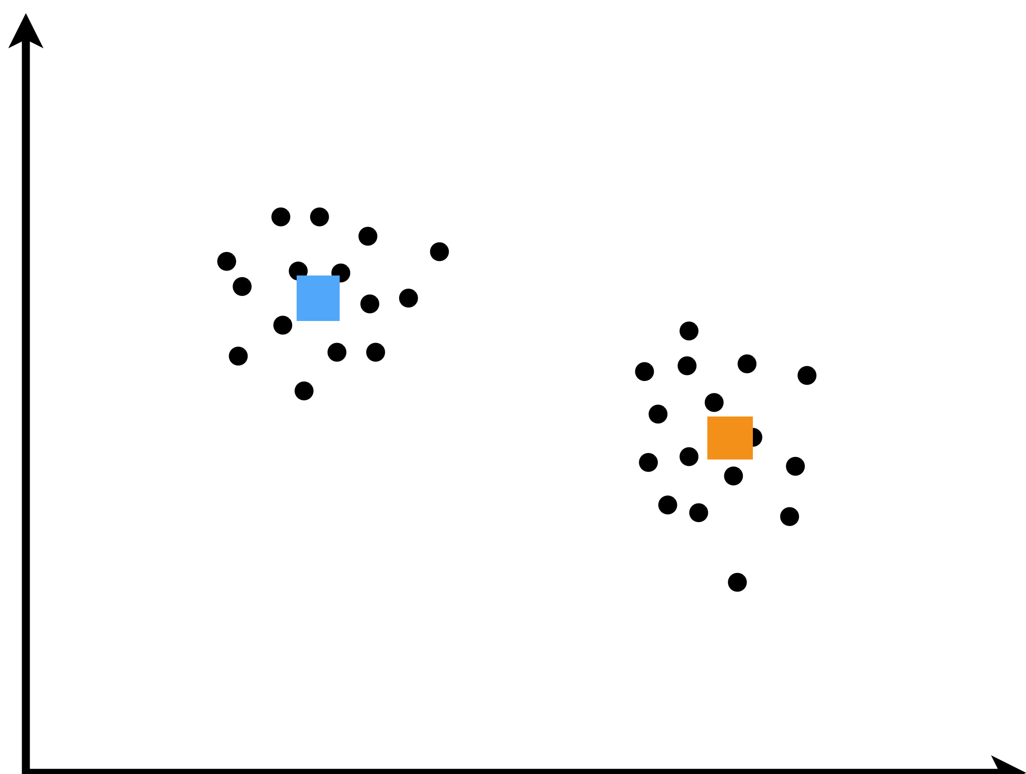

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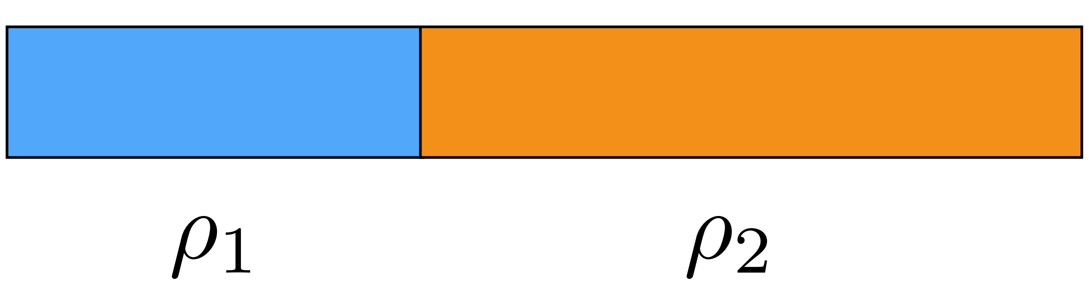

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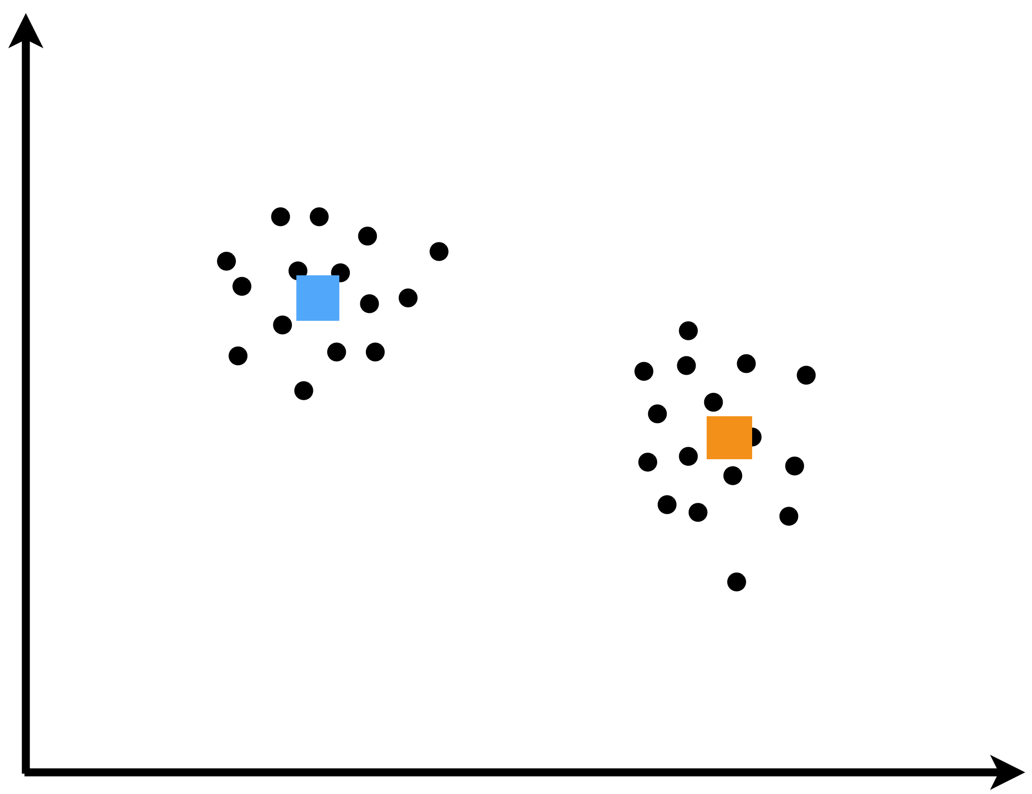

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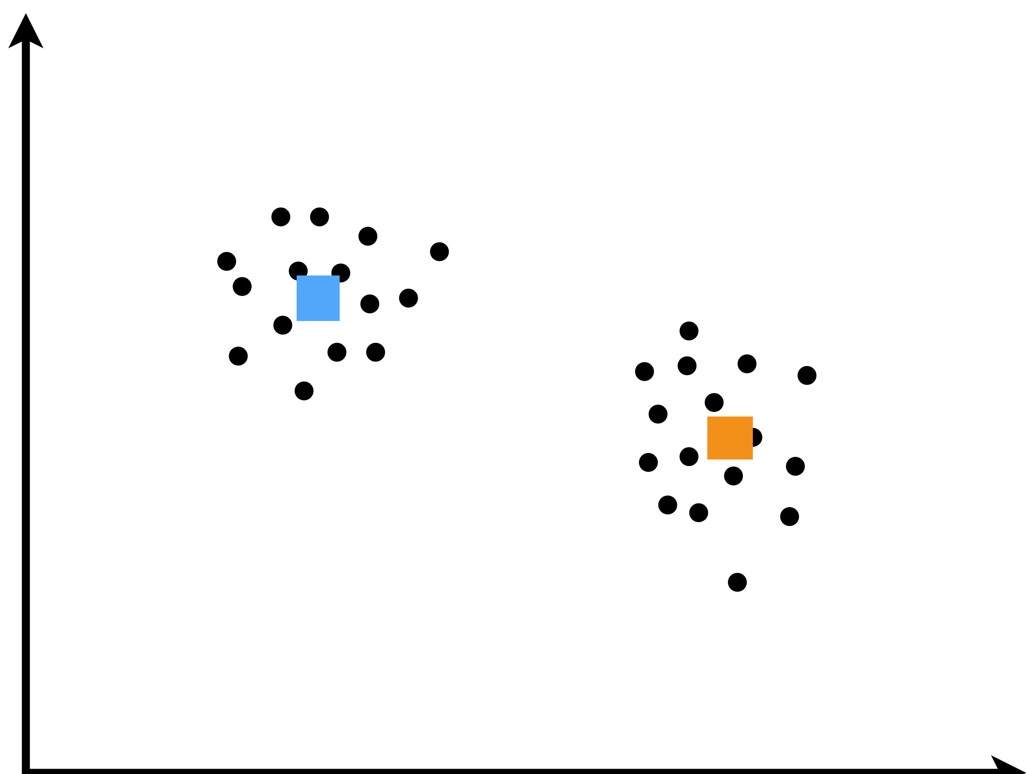

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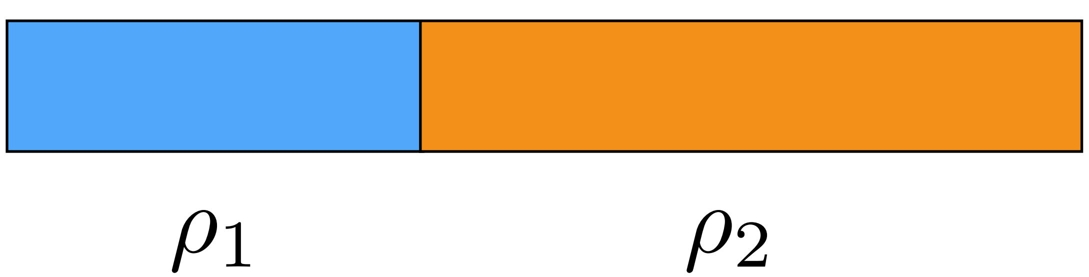

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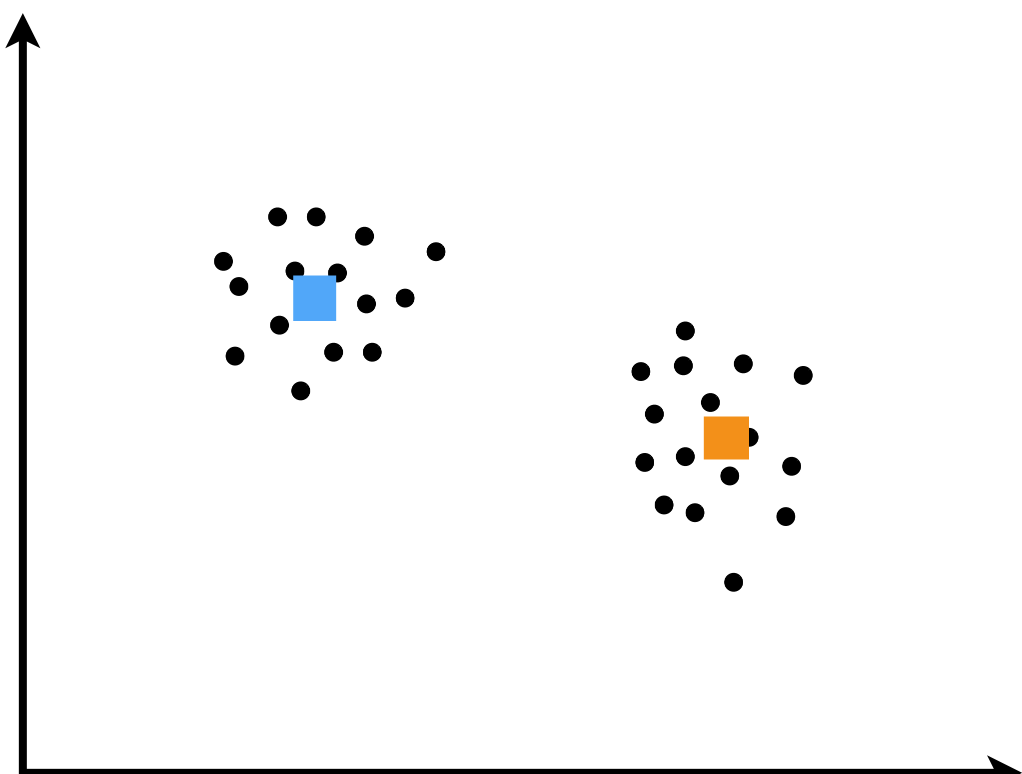

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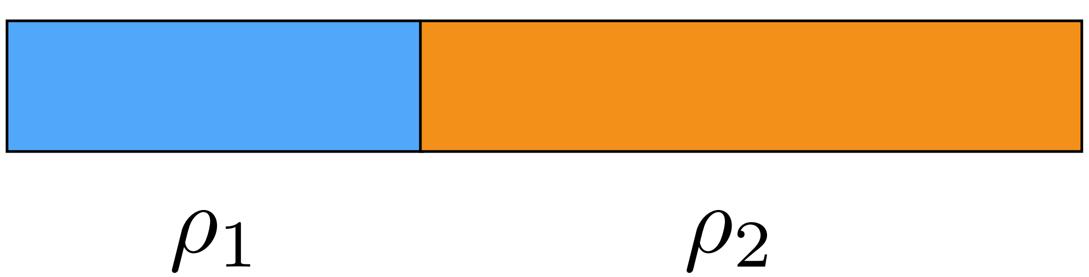

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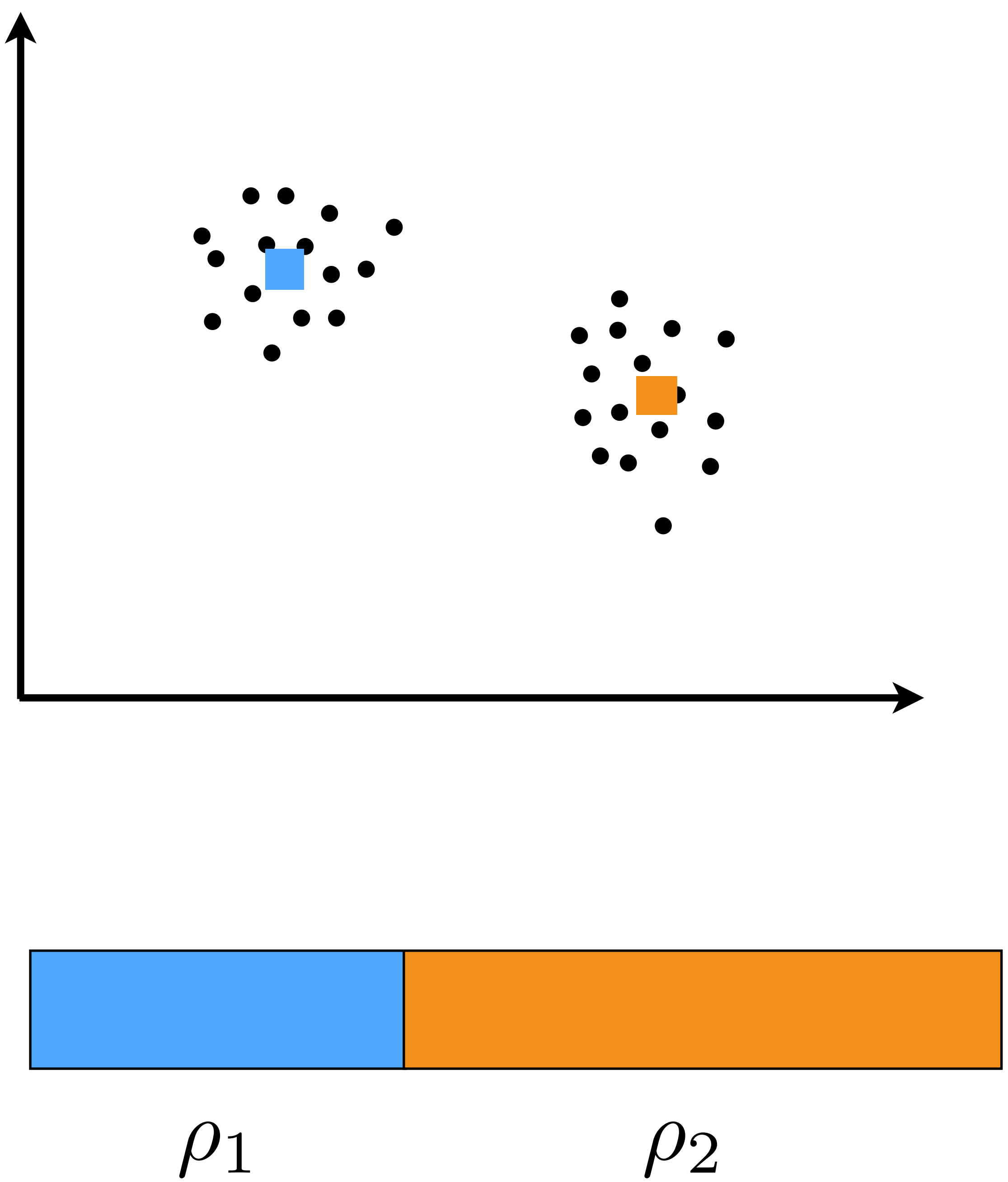

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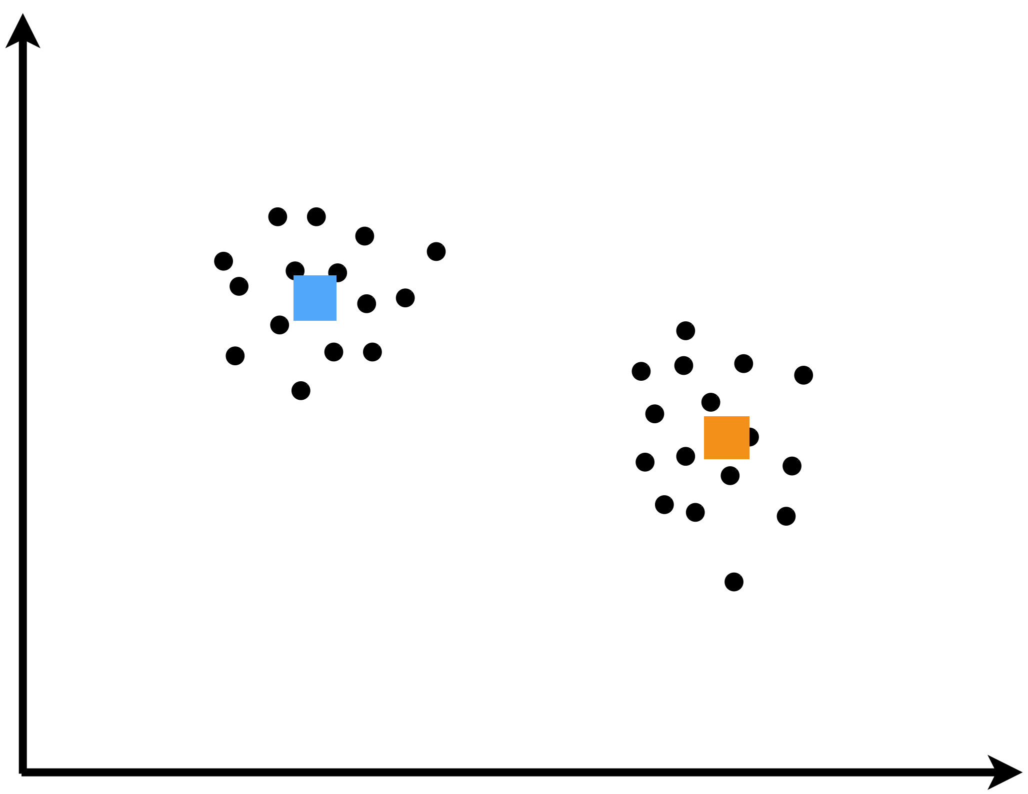

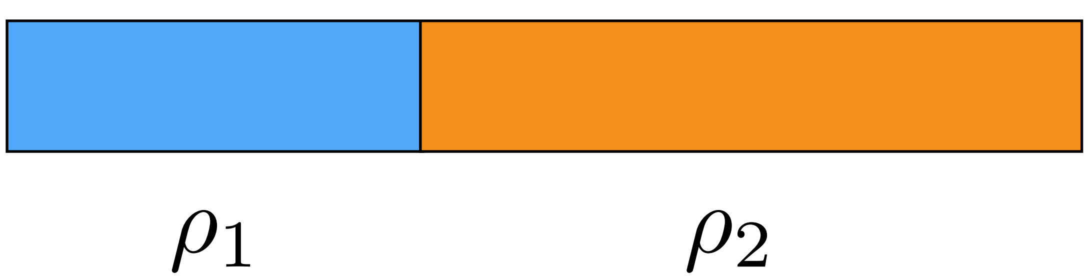

- Finite Gaussian mixture model (*K*=2 clusters)

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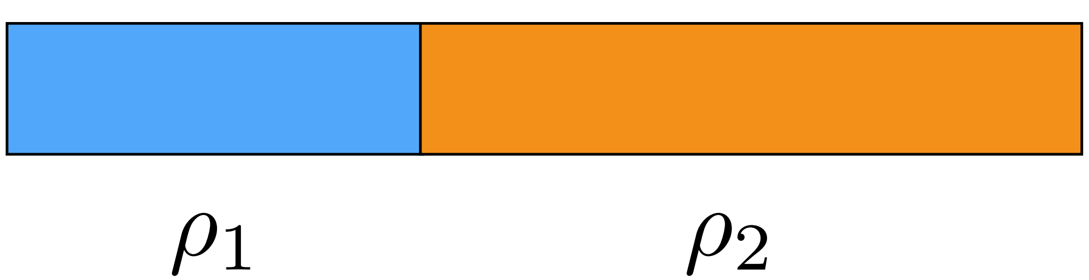

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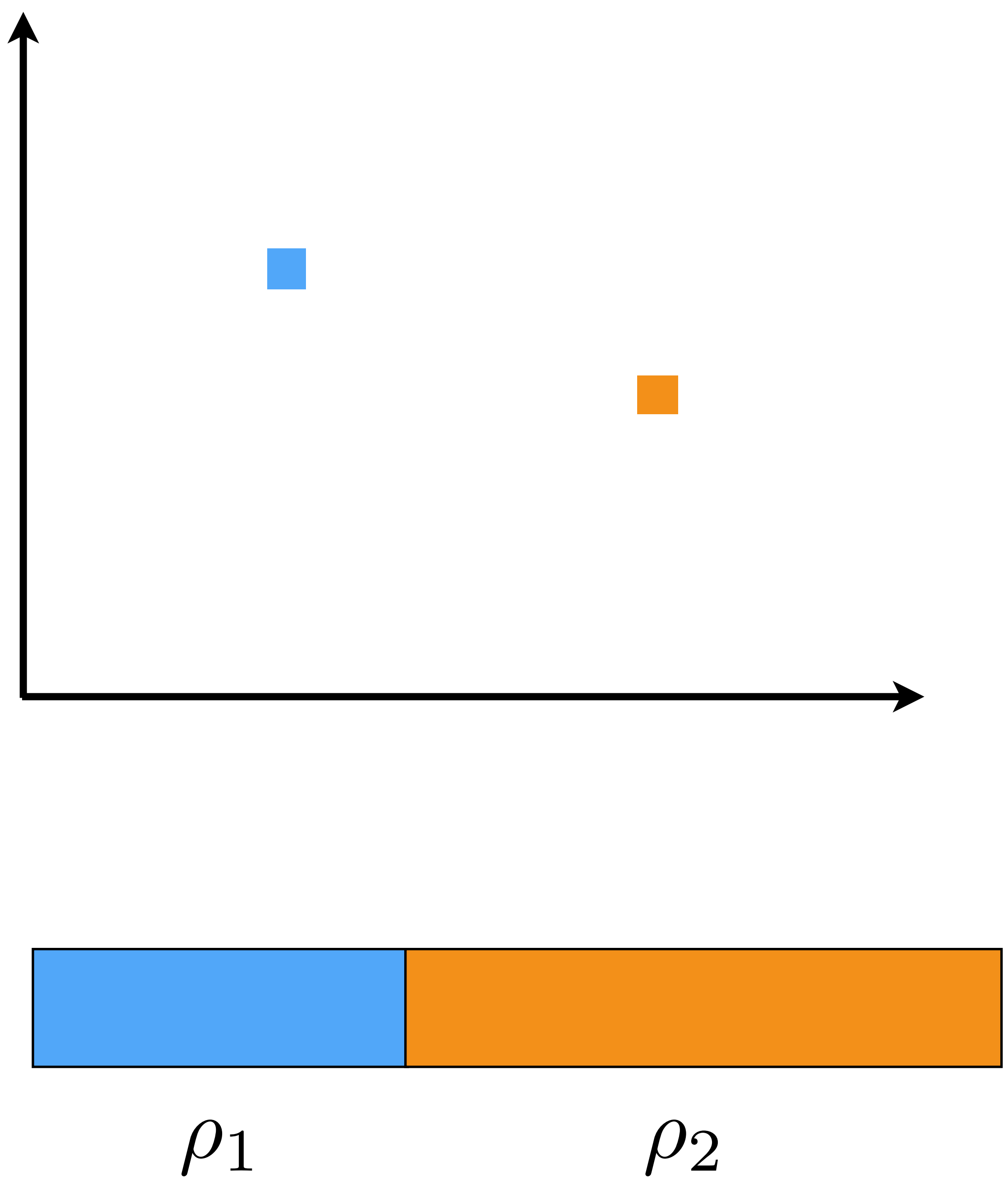

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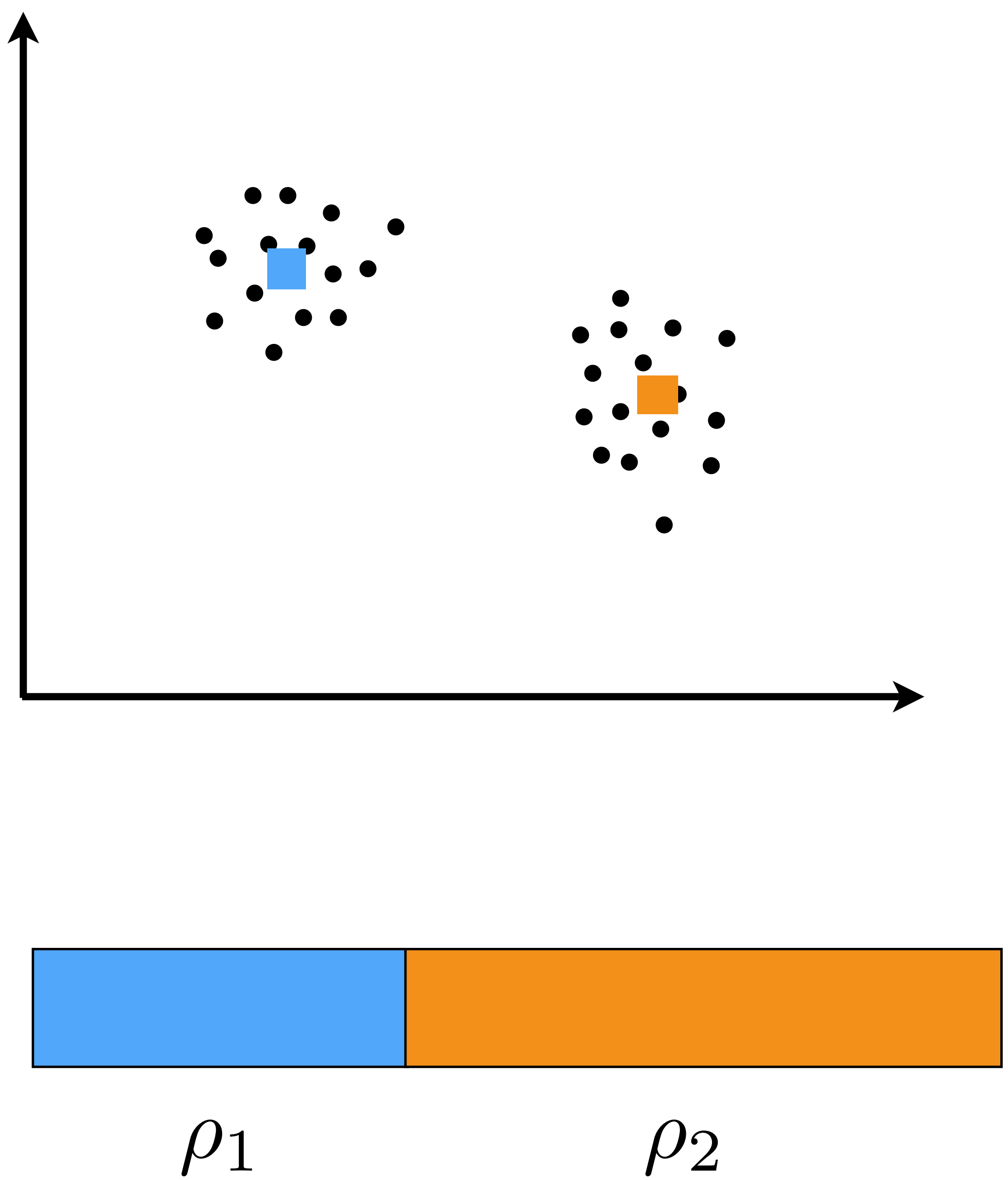

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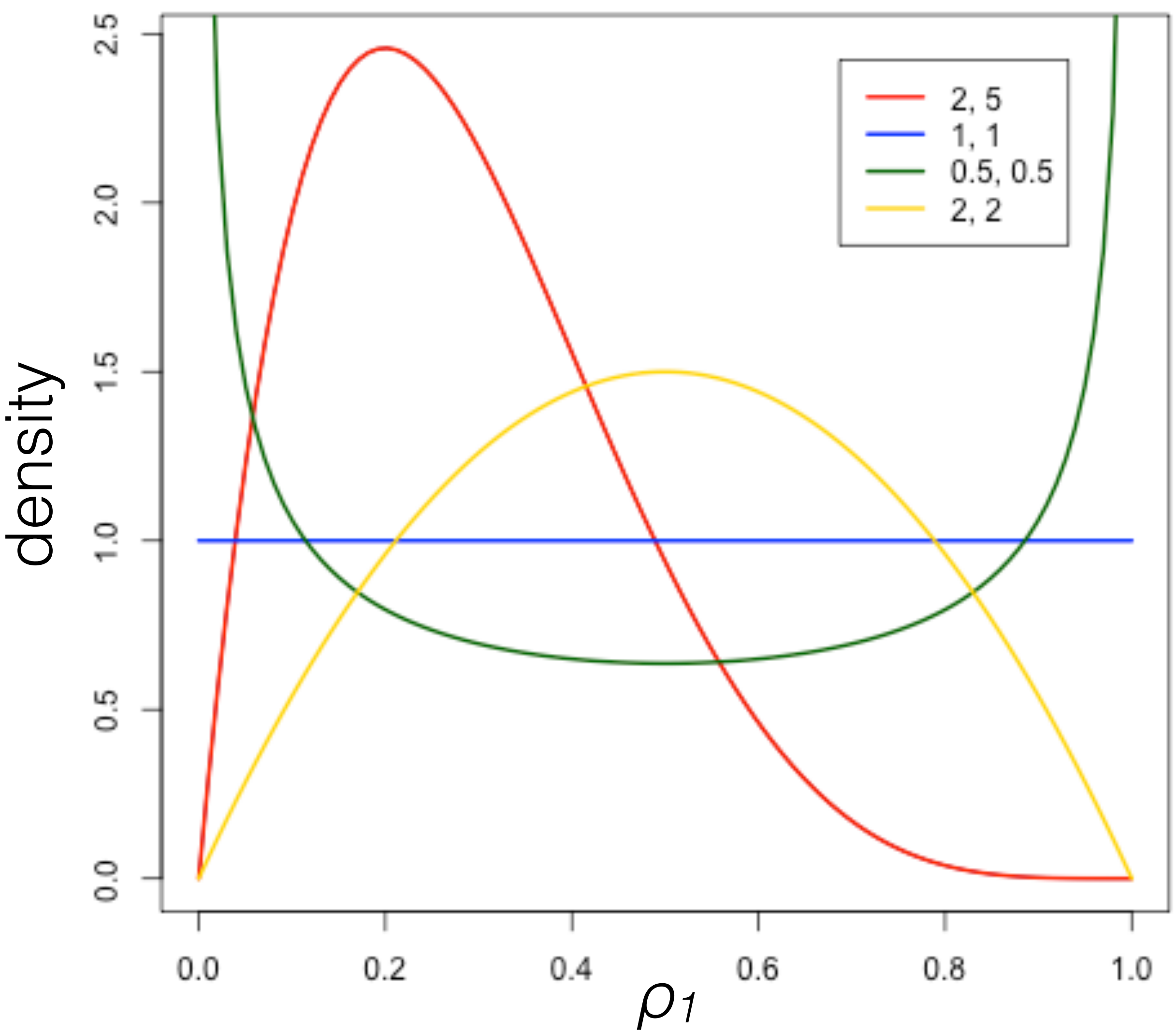

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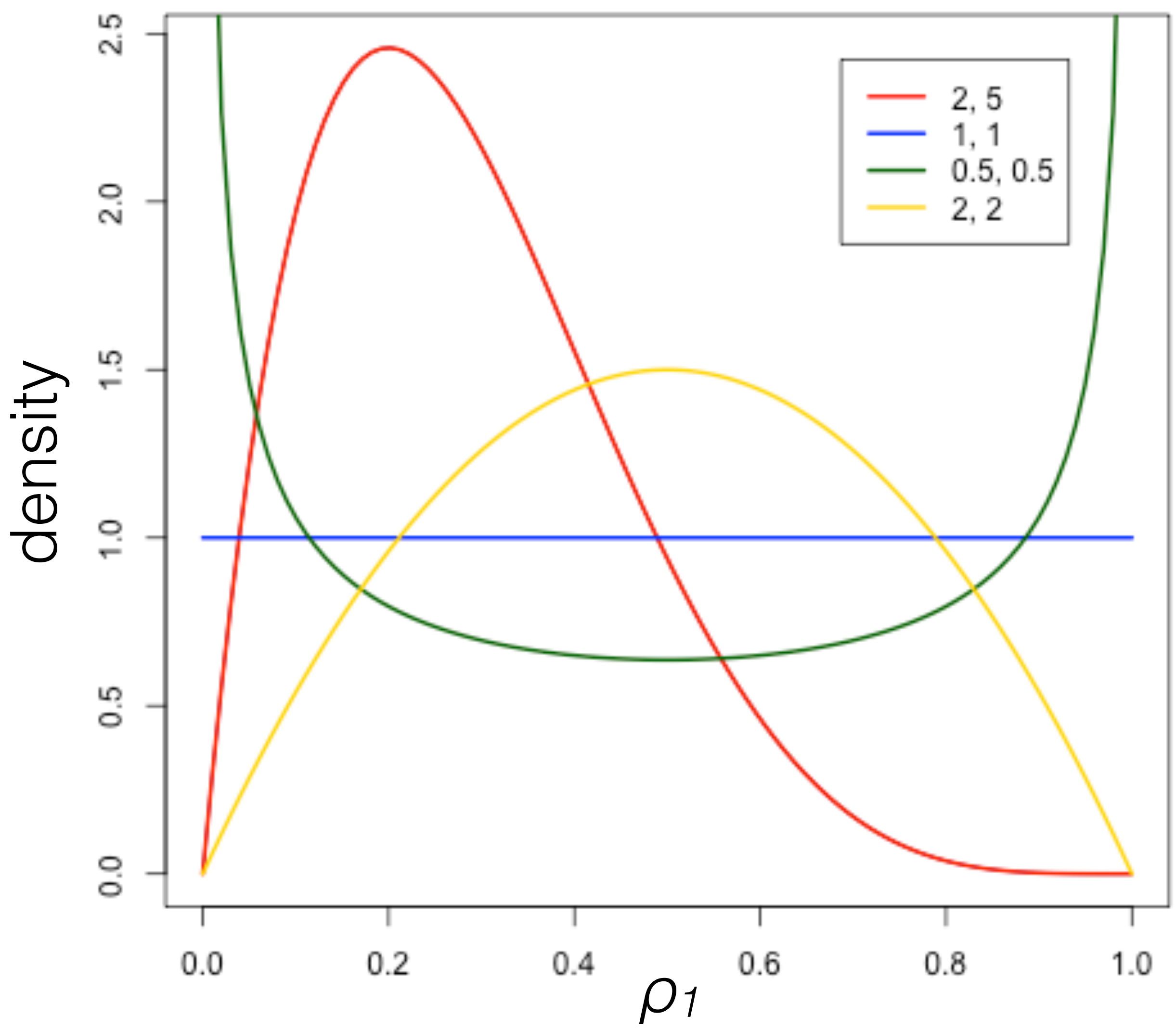

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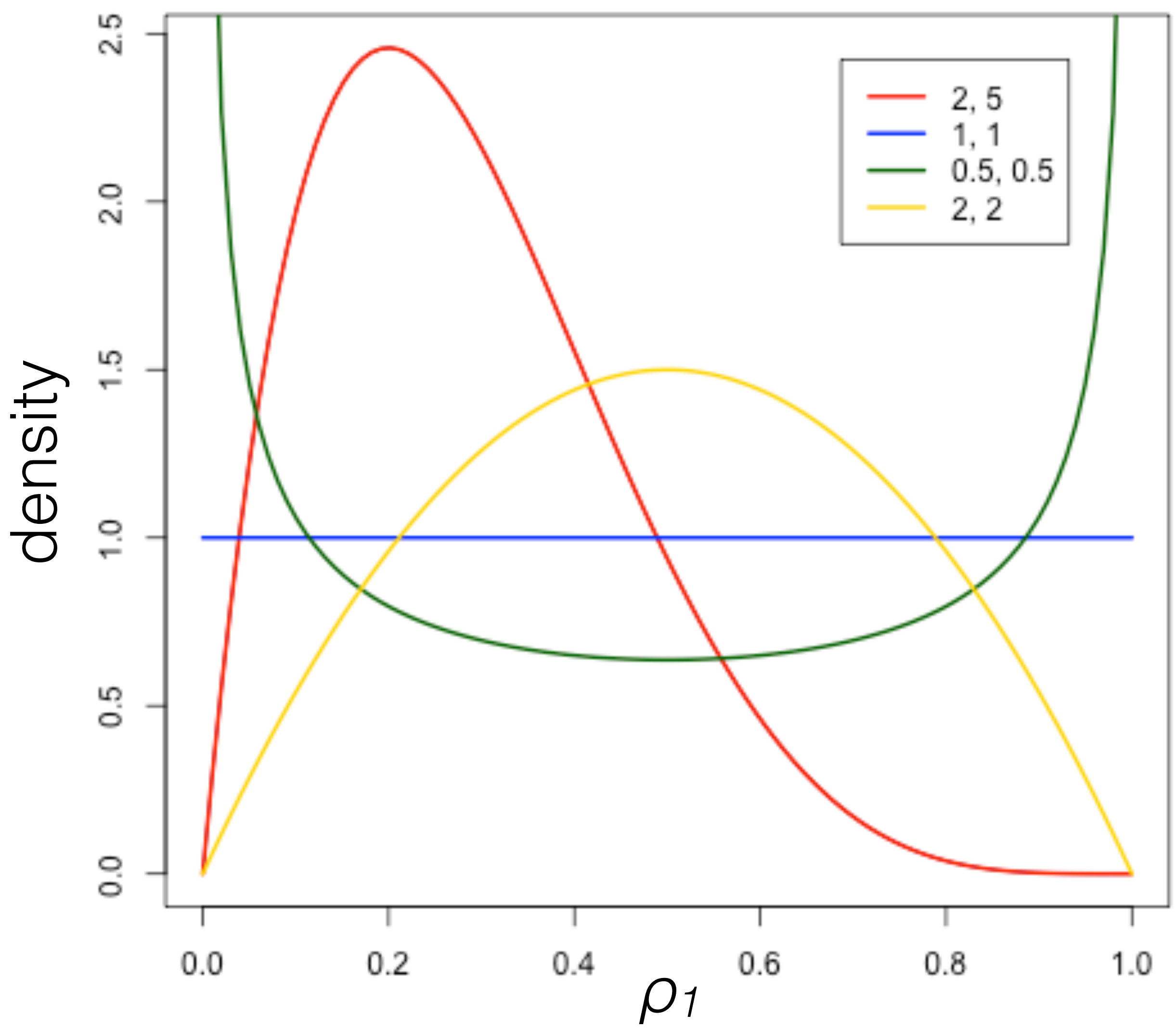

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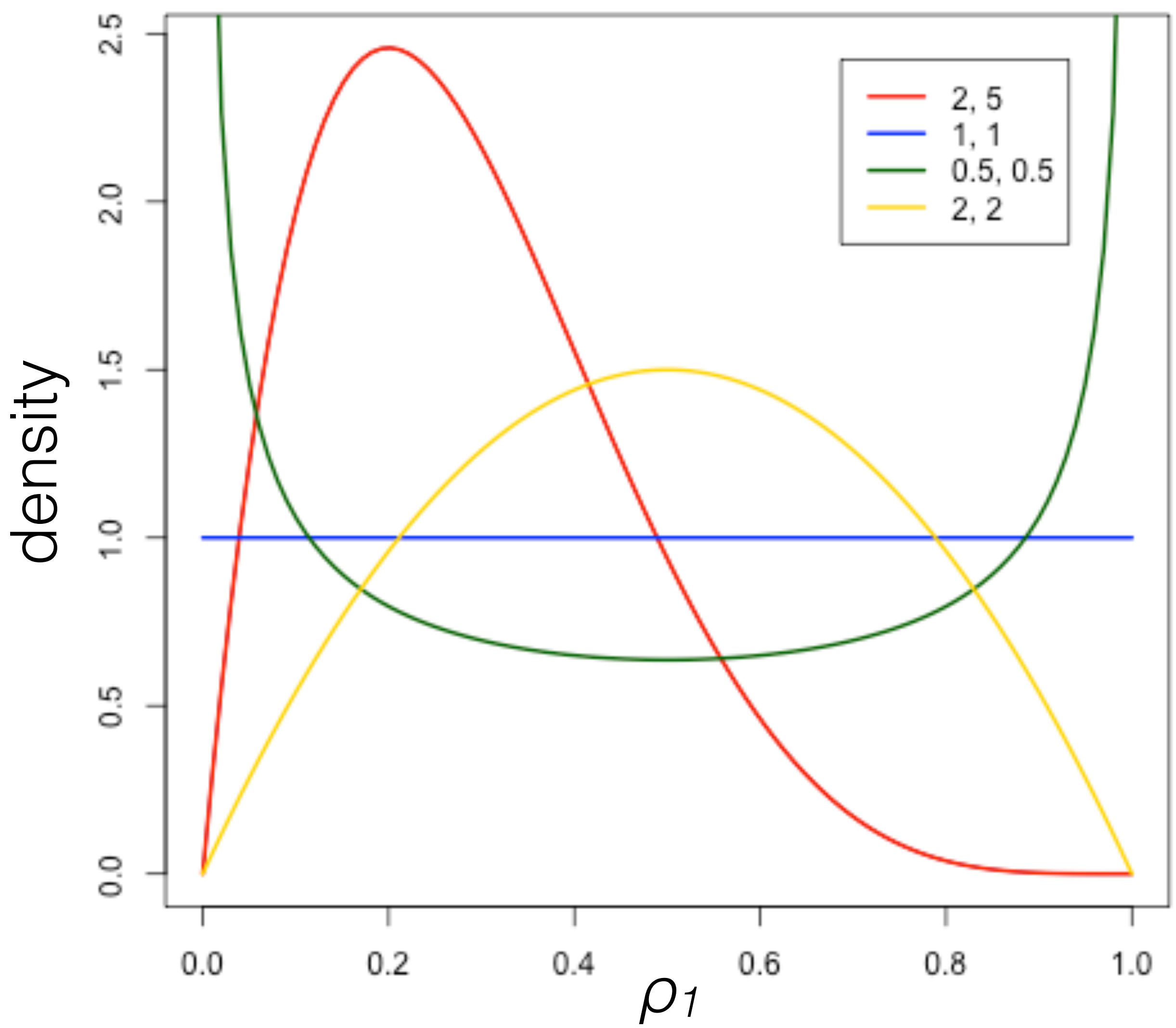

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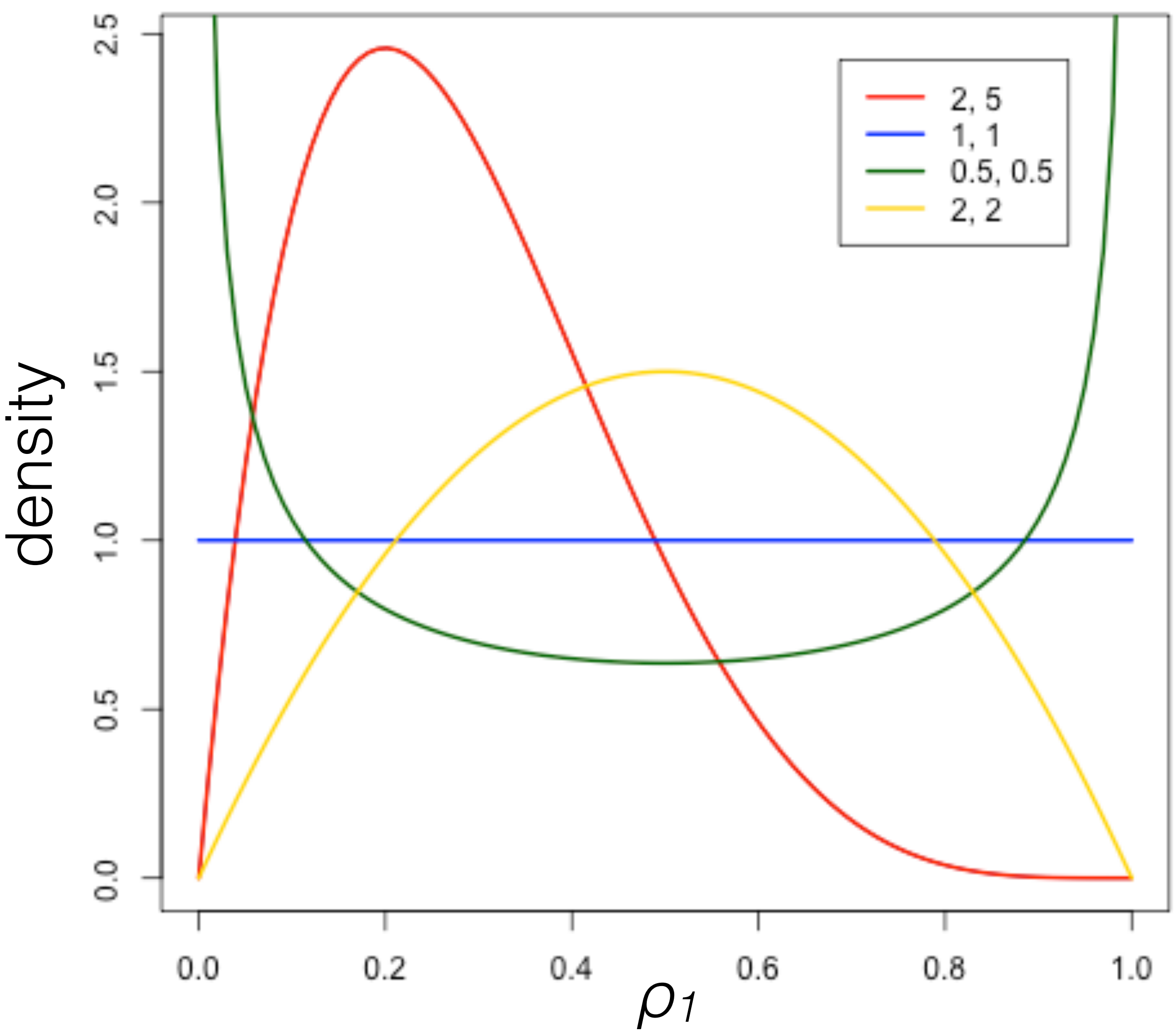

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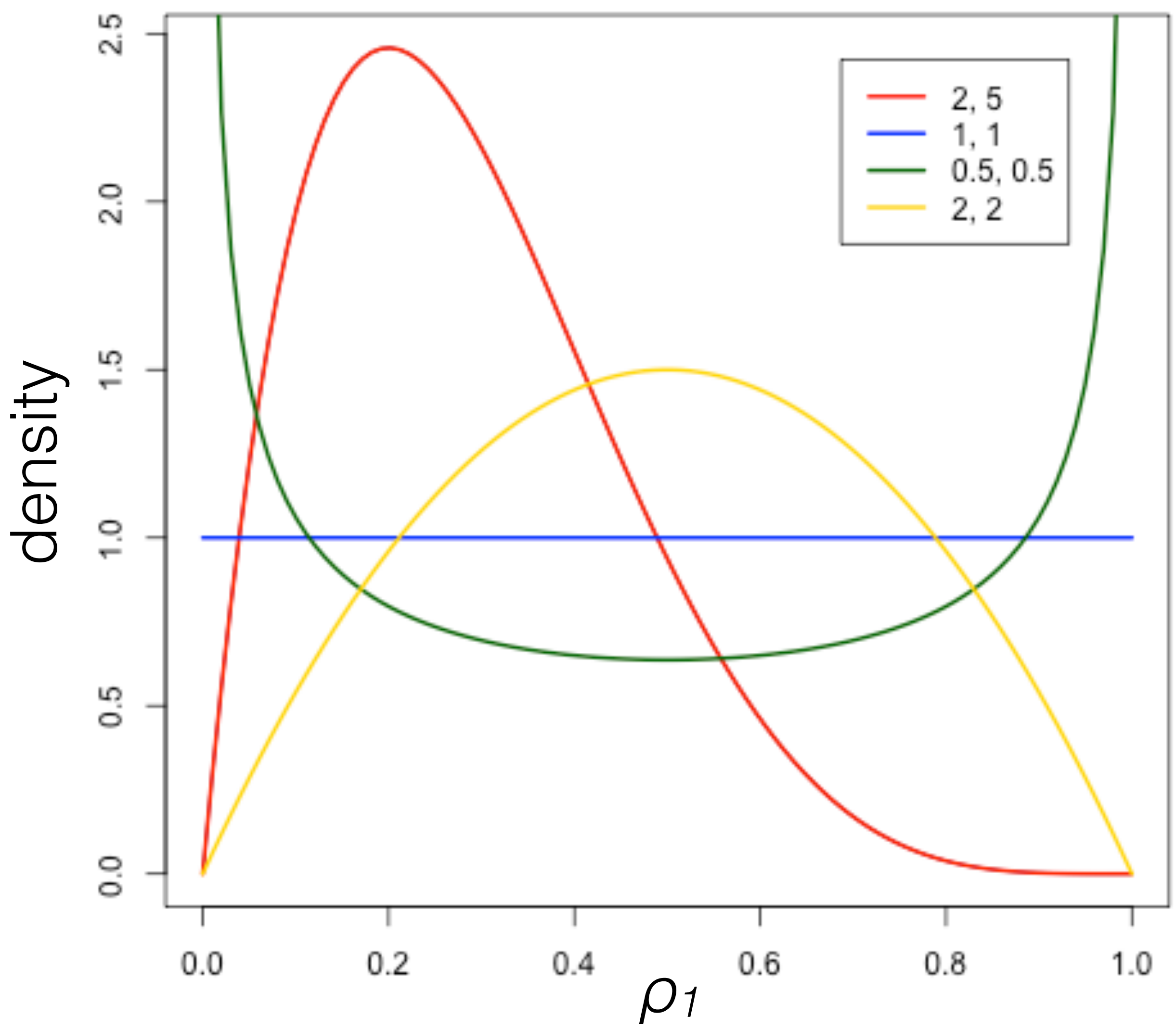

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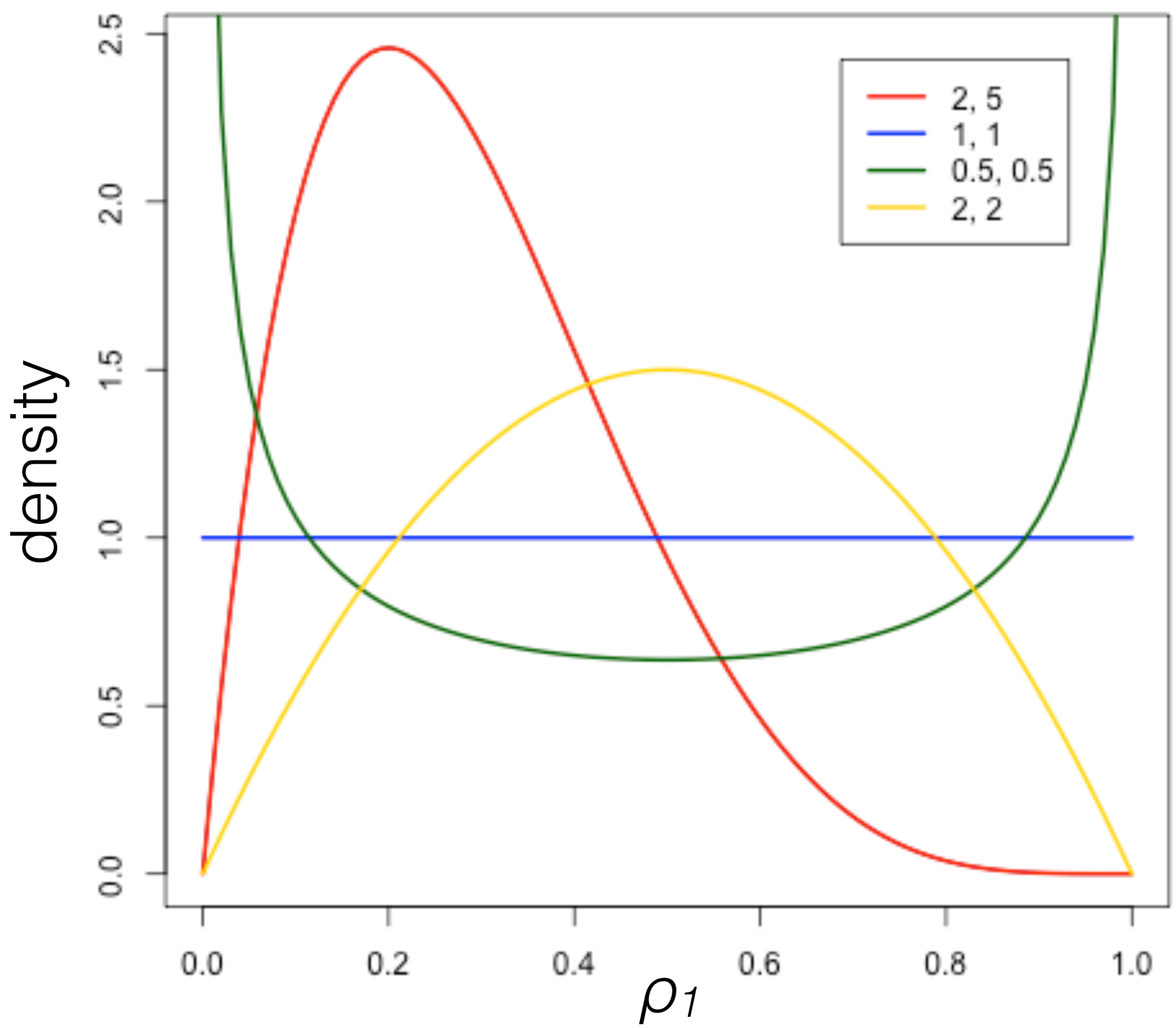

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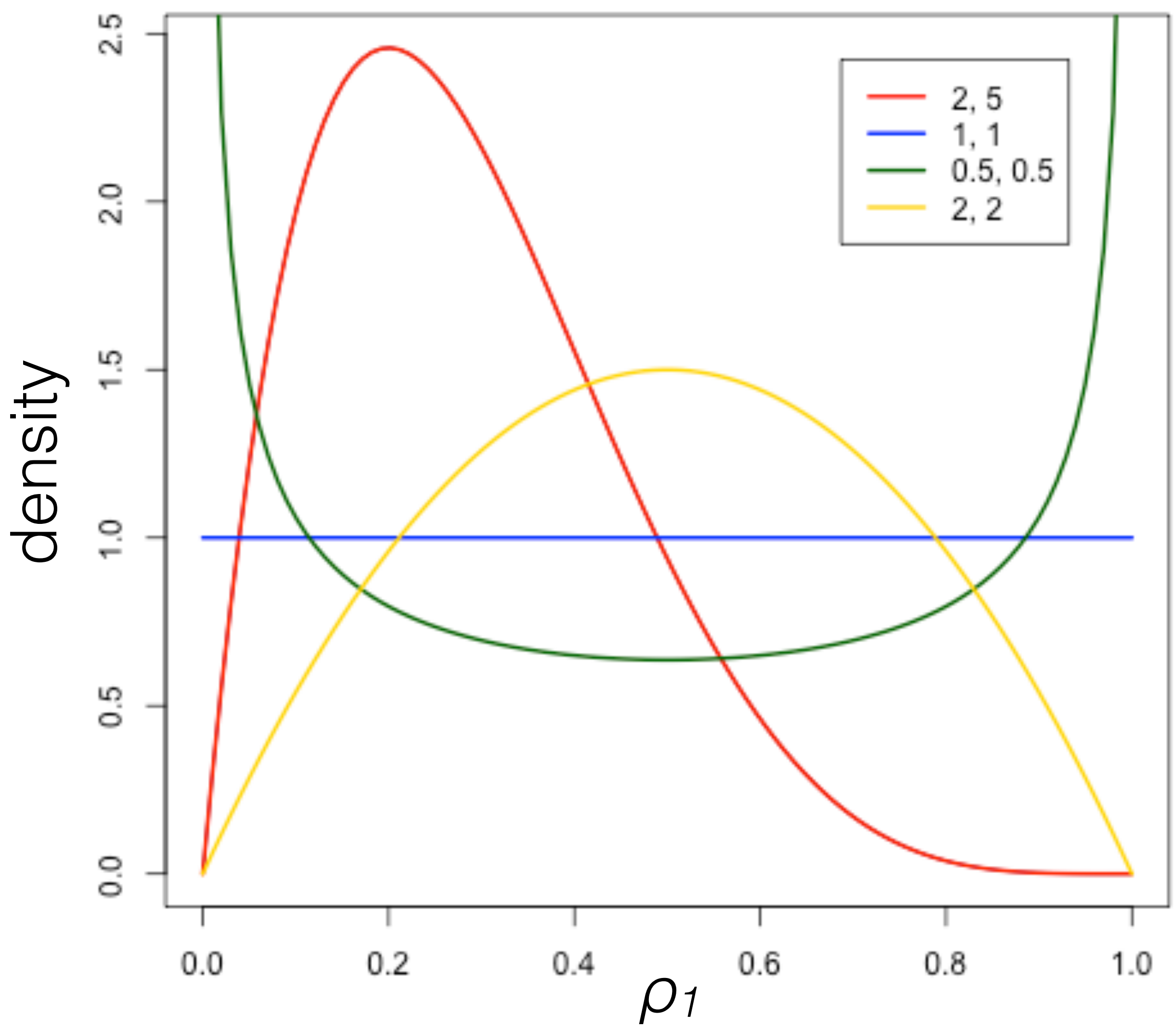

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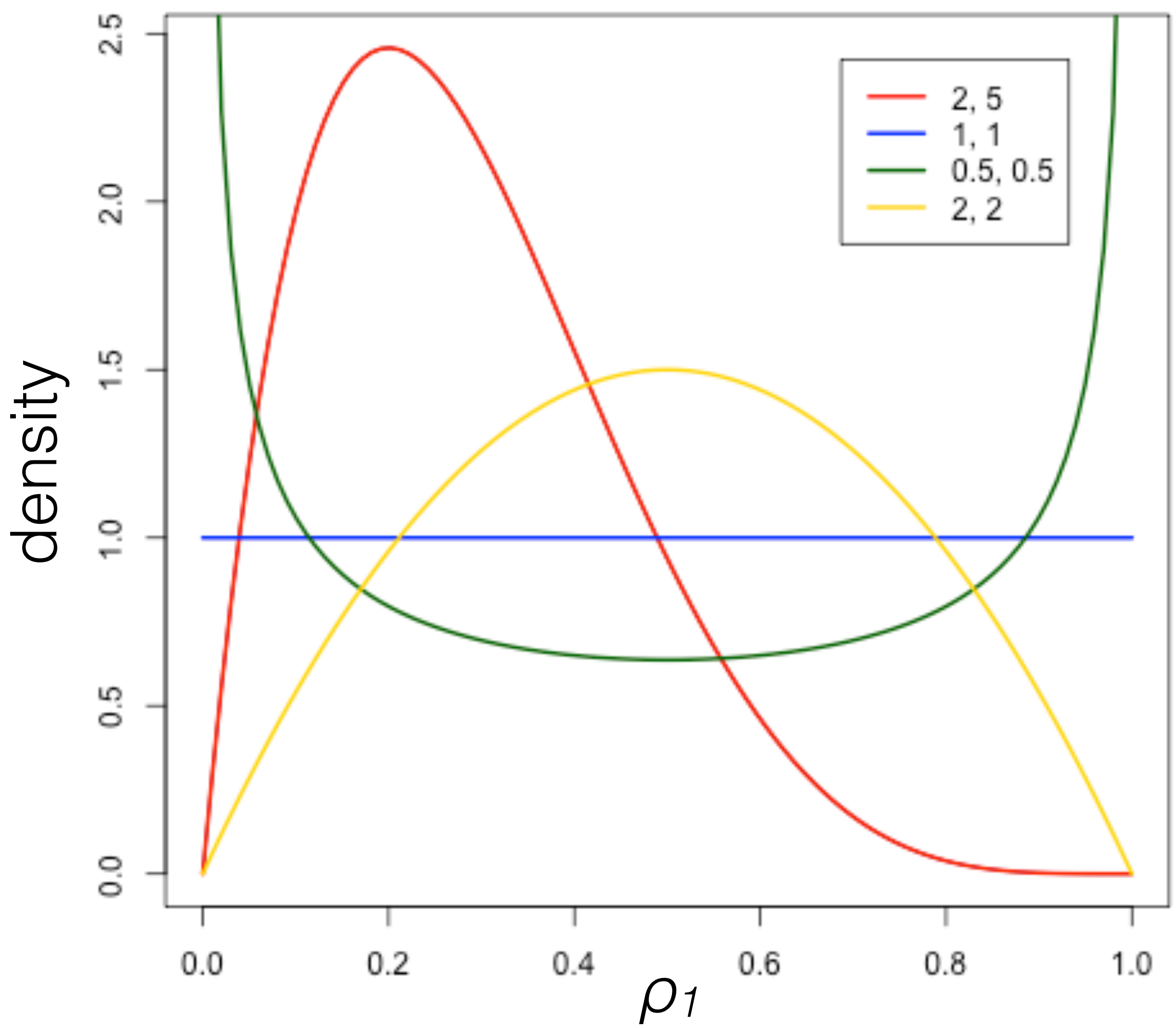

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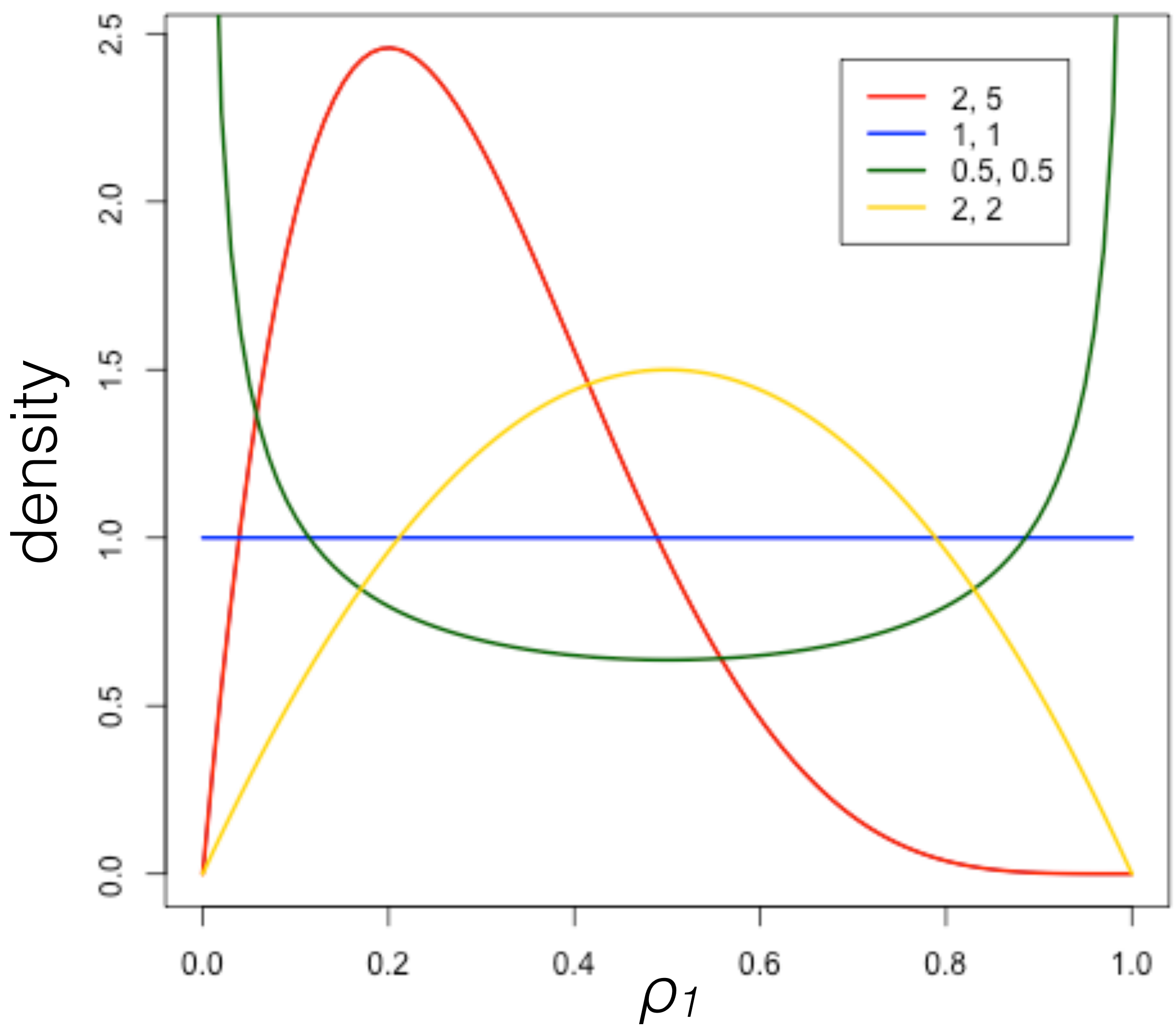

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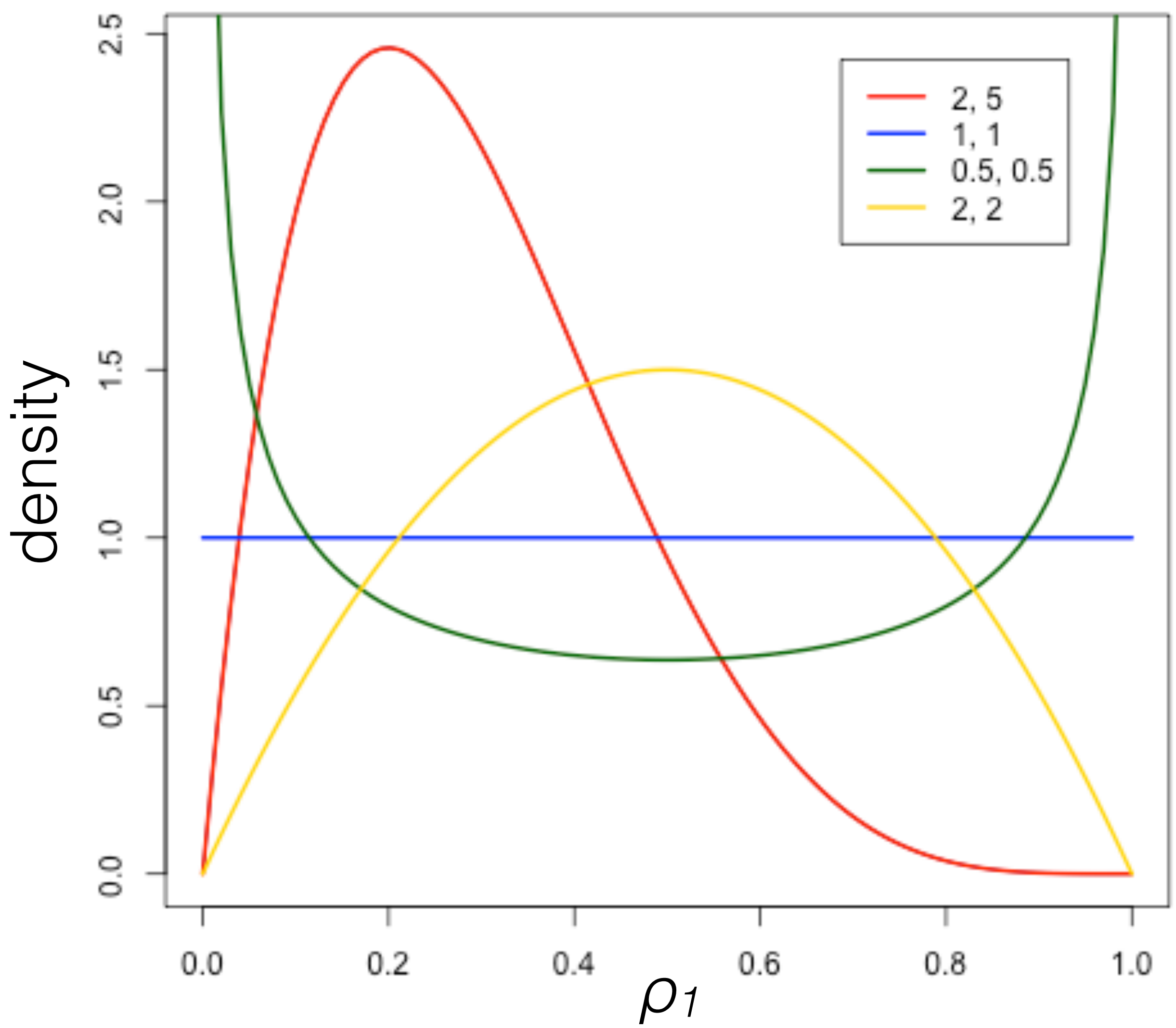

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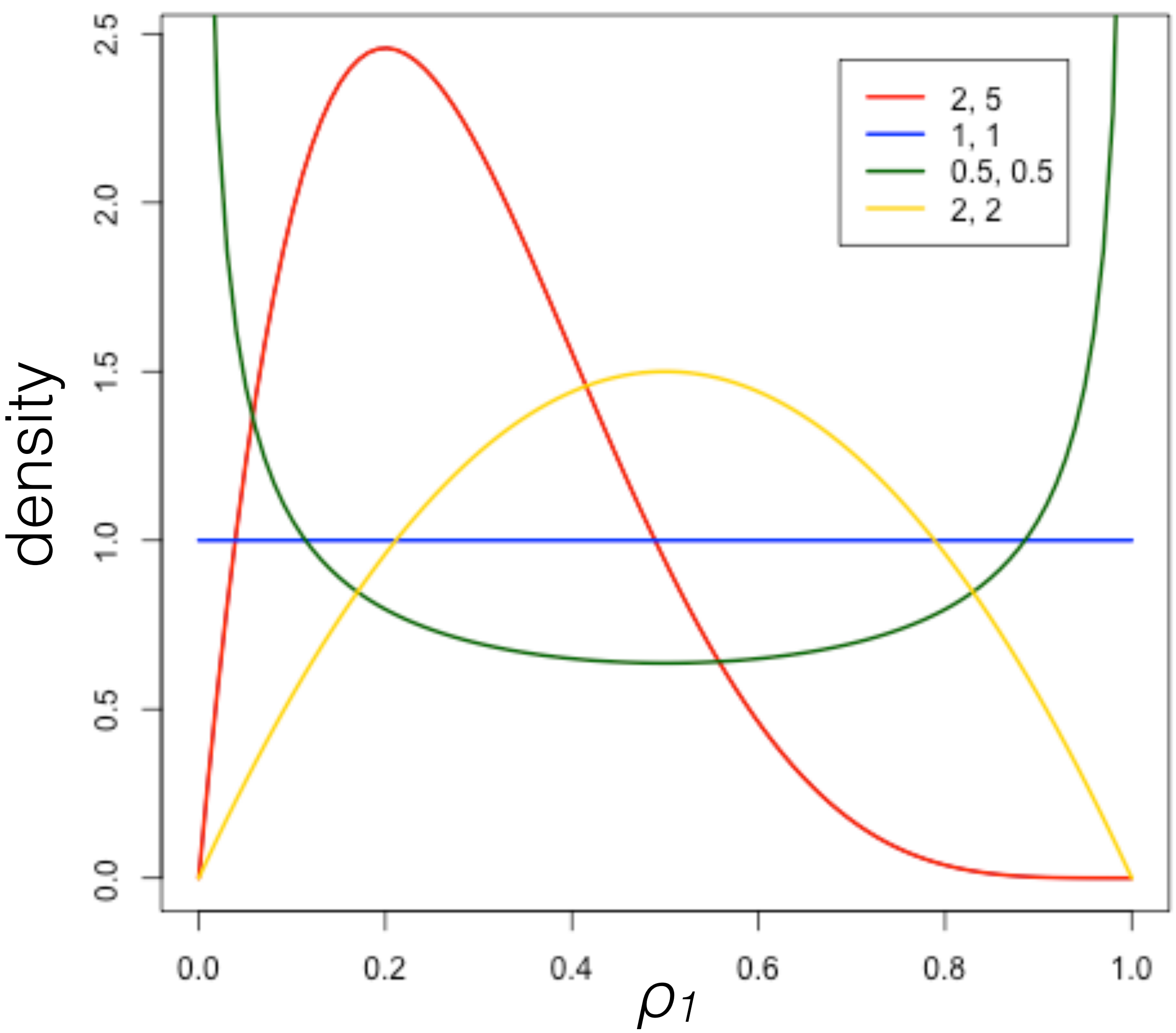

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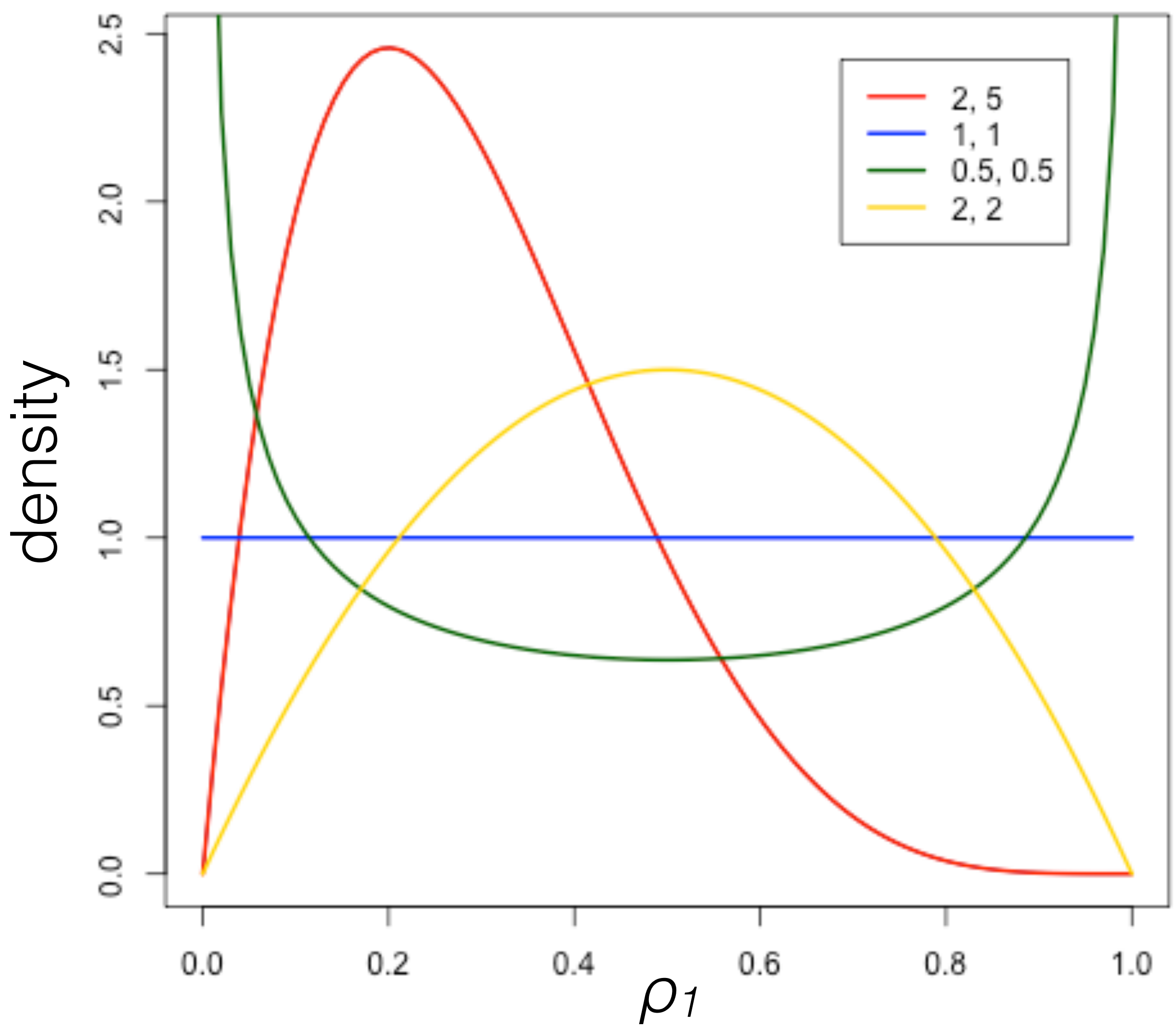

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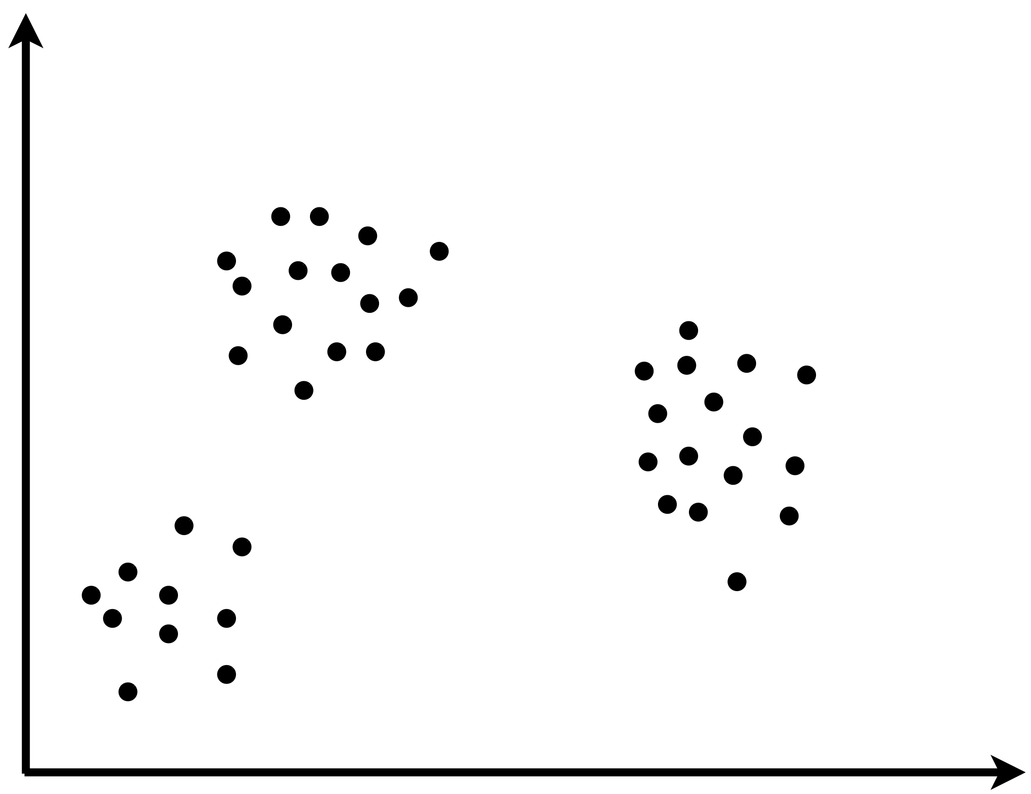

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ρ_1 ρ_2 ρ_3

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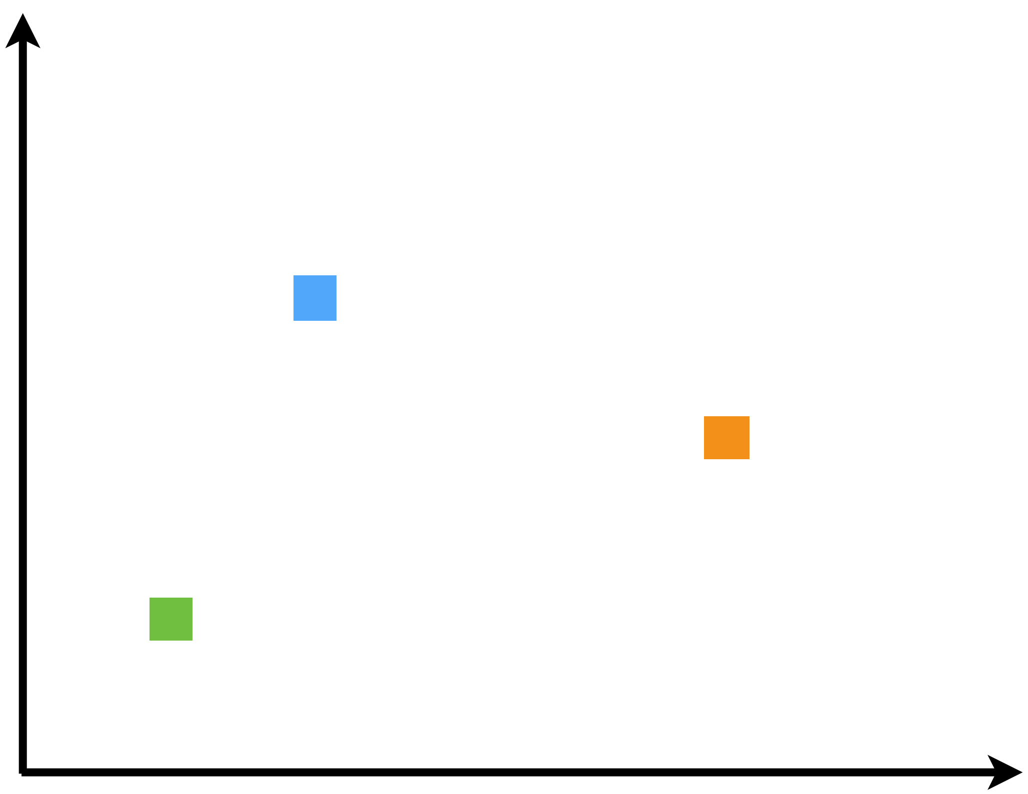

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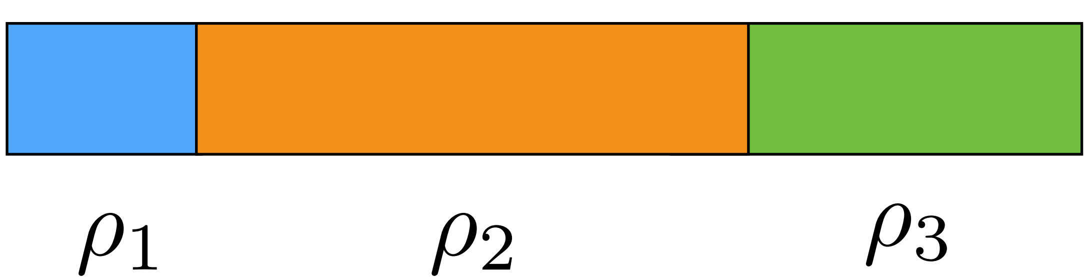

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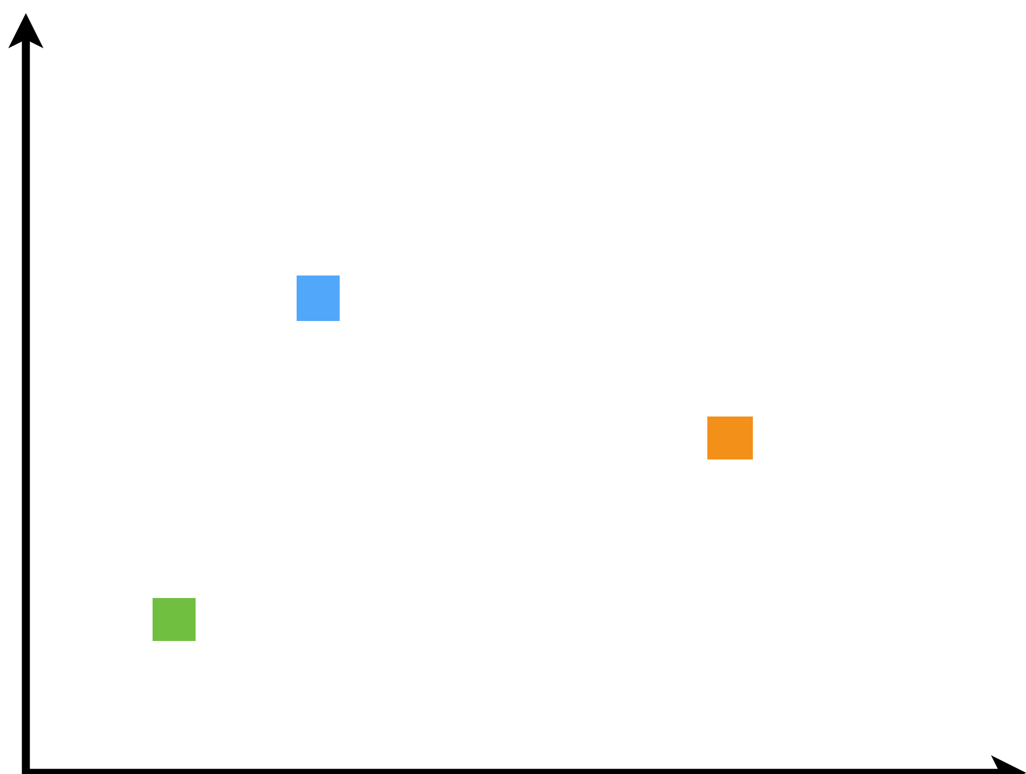

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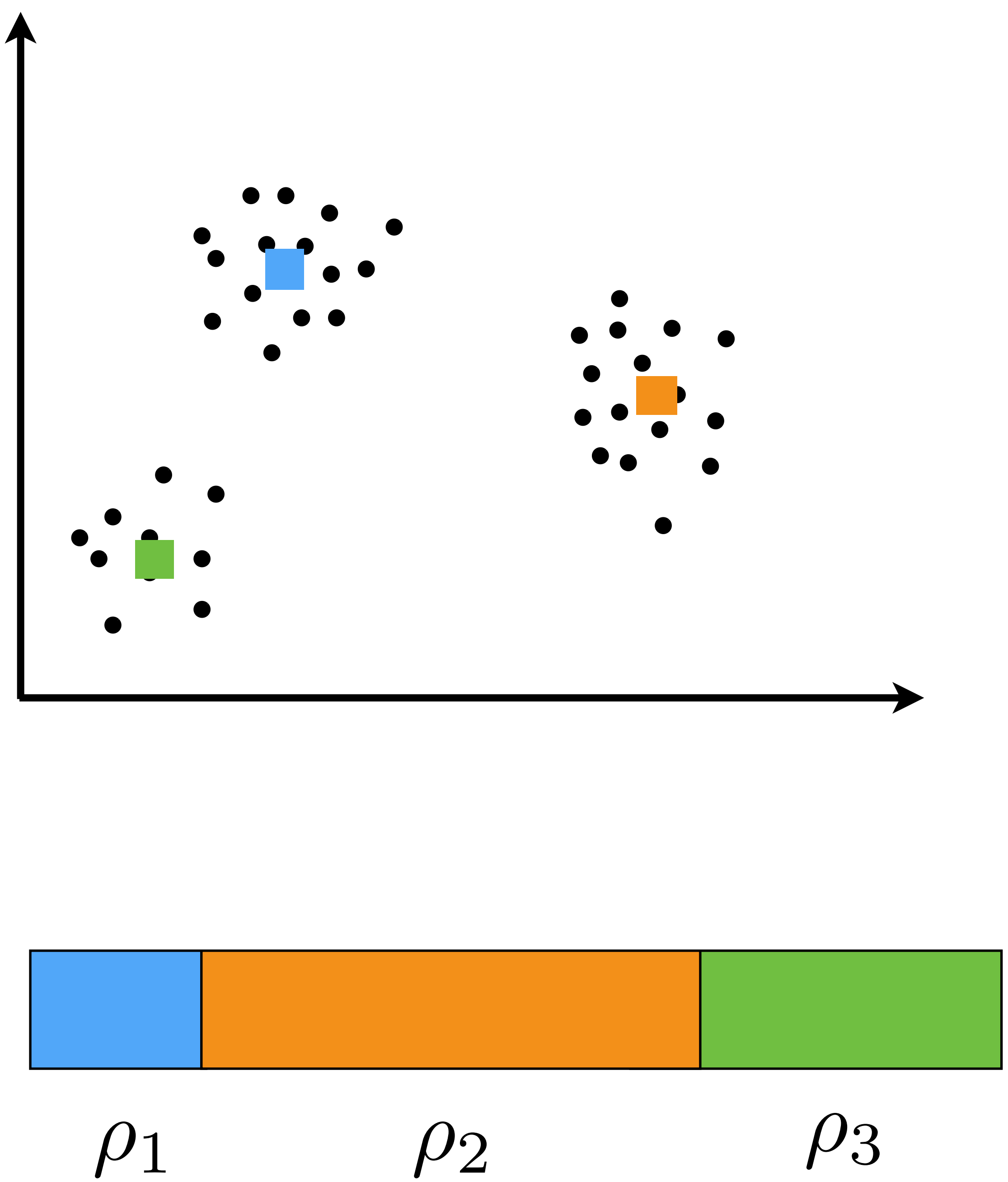

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$$z_n \overset{iid}{\sim} \text{Categorical}(\rho_{1:K})$$

$$x_n \overset{indep}{\sim} \mathcal{N}(\mu_{z_n}, \Sigma)$$

Dirichlet distribution review

$$\mathrm{Dirichlet}(\rho_{1:K}|a_{1:K}) = \frac{\Gamma(\sum_{k=1}^{K} a_k)}{\prod_{k=1}^{K} \Gamma(a_k)} \prod_{k=1}^{K} \rho_k^{a_k - 1} \qquad a_k > 0$$

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$$a_k > 0$$

$$\rho_k \in (0,1)$$

$$\sum_k \rho_k = 1$$

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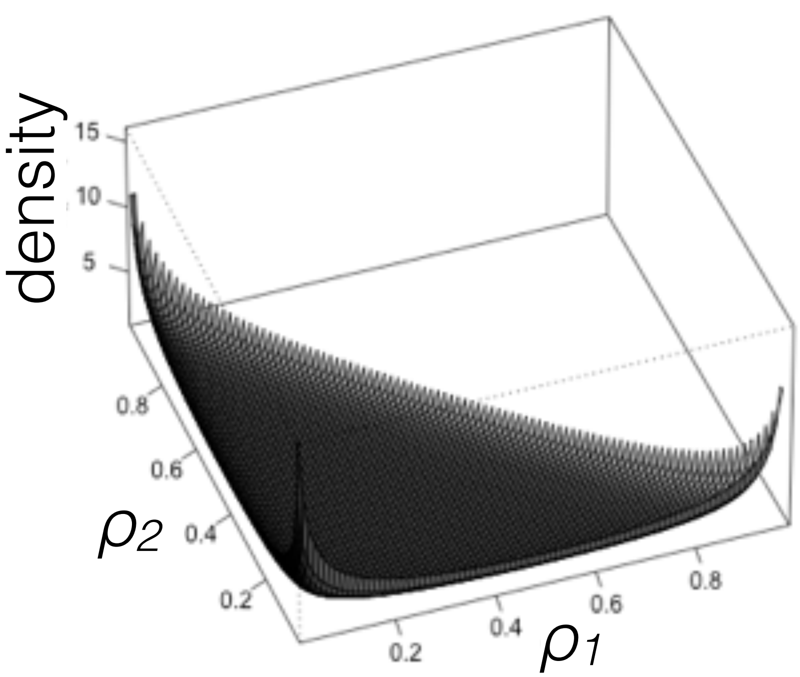
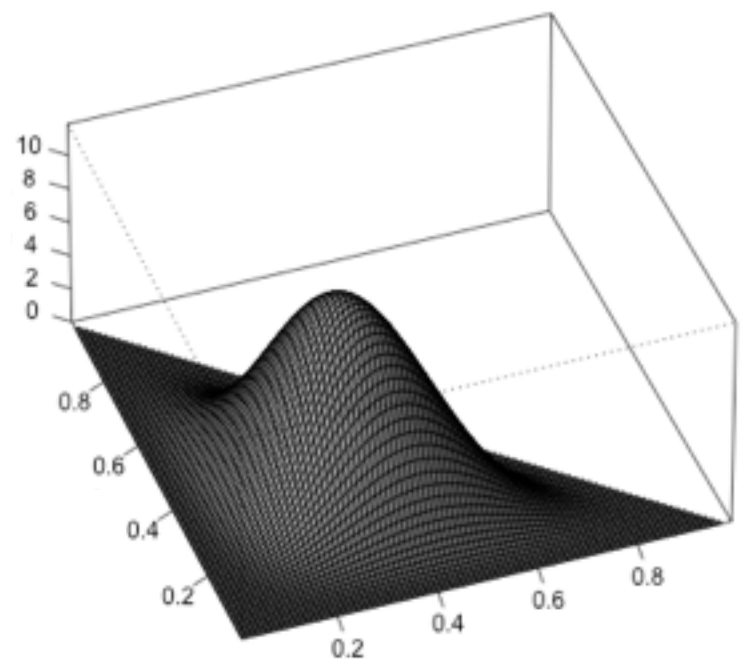
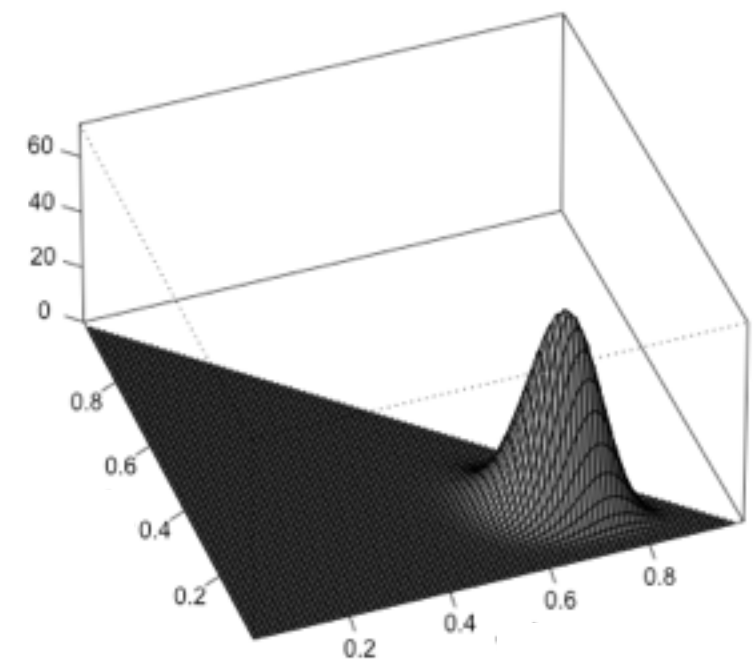
a = (0.5,0.5,0.5)



a = (5,5,5)



a = (40,10,10)



- What happens?

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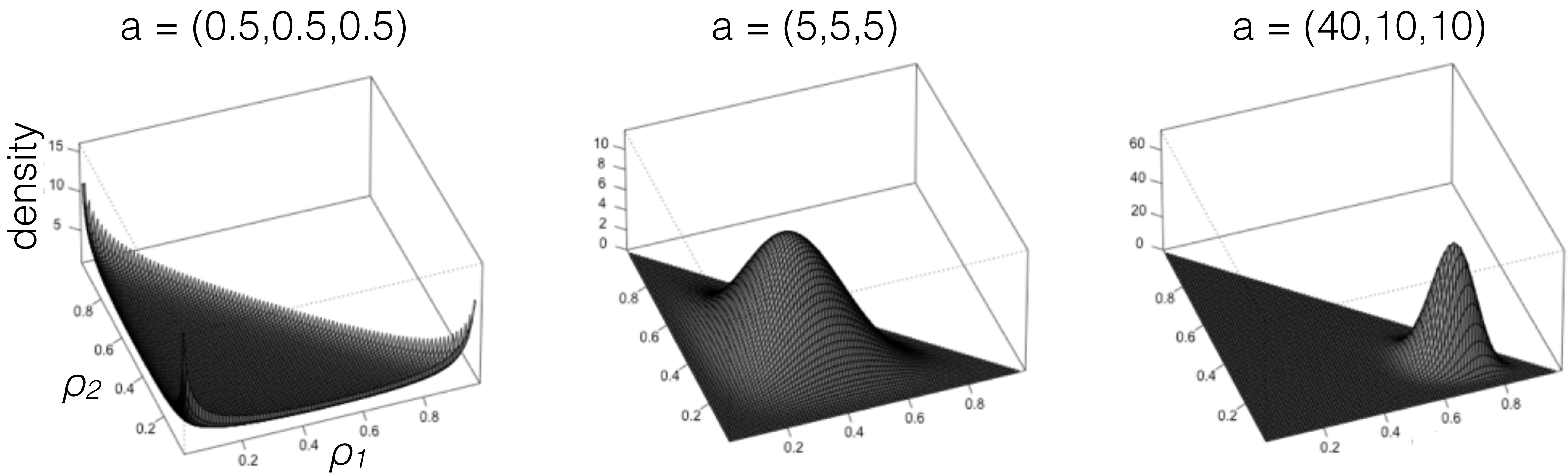

- What happens? $a = a_k = 1$

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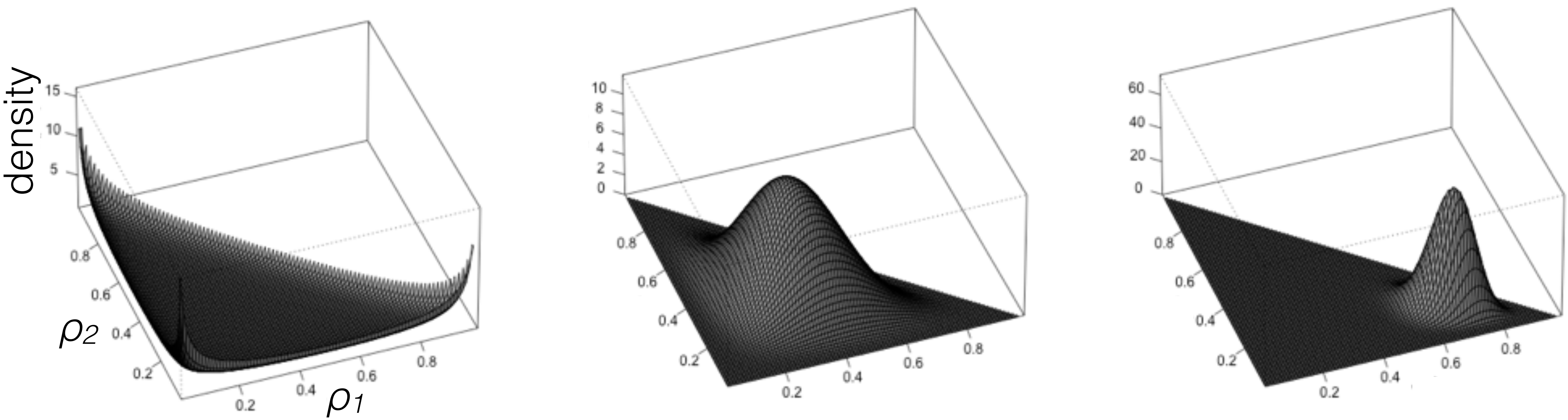

- What happens? $a = a_k = 1$ $\quad a = a_k \to 0$

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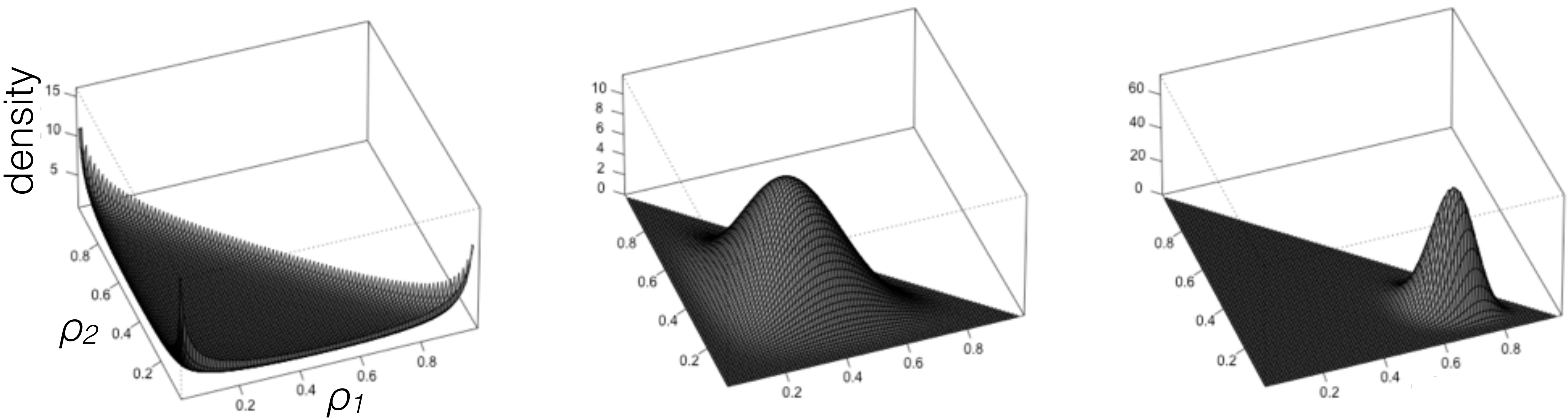

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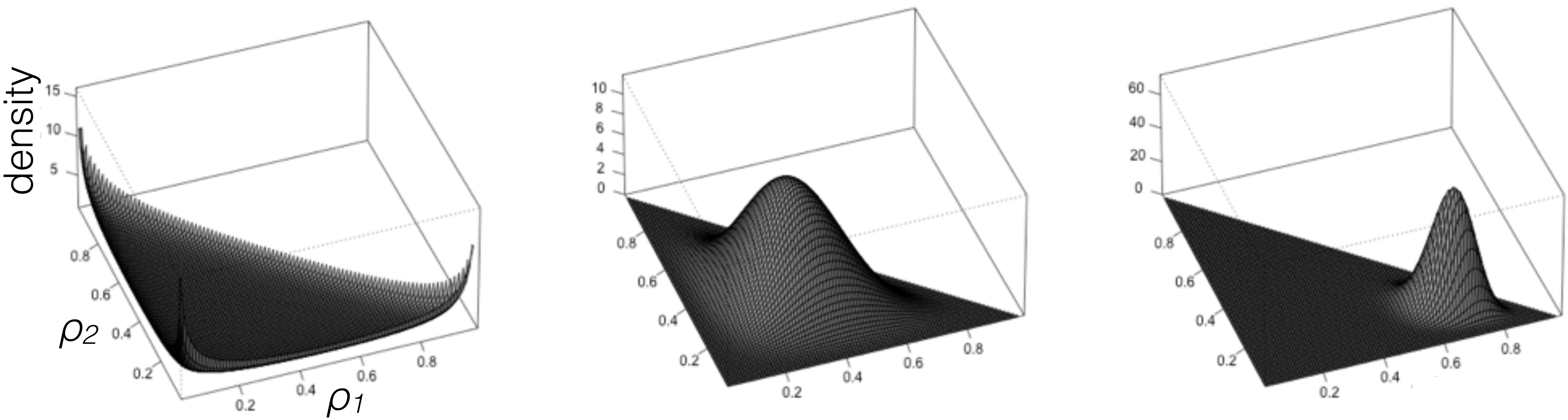

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[demo]

Dirichlet distribution review

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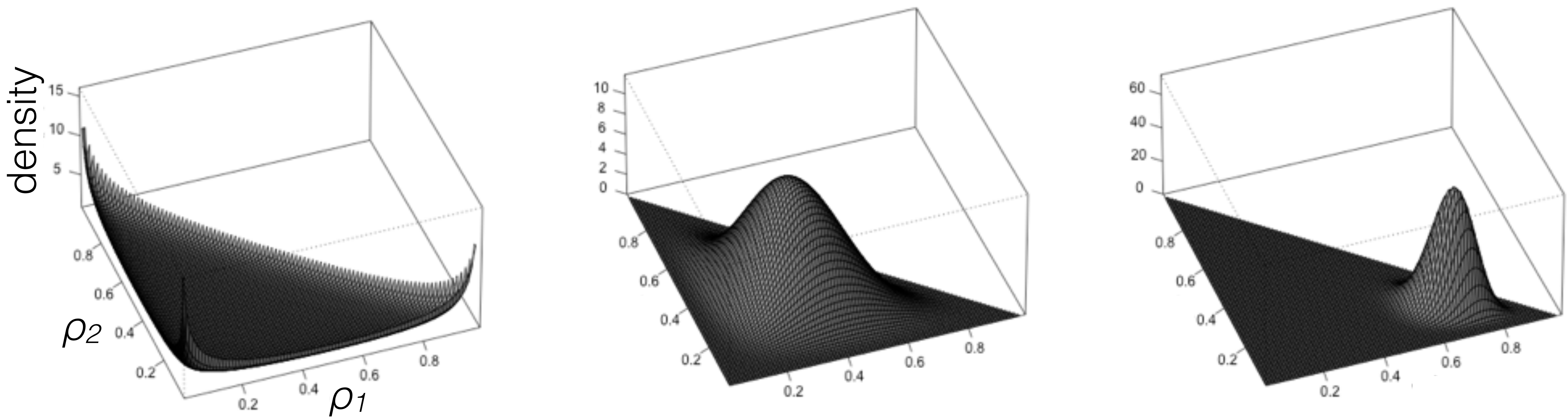

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- Dirichlet is conjugate to Categorical

[demo]

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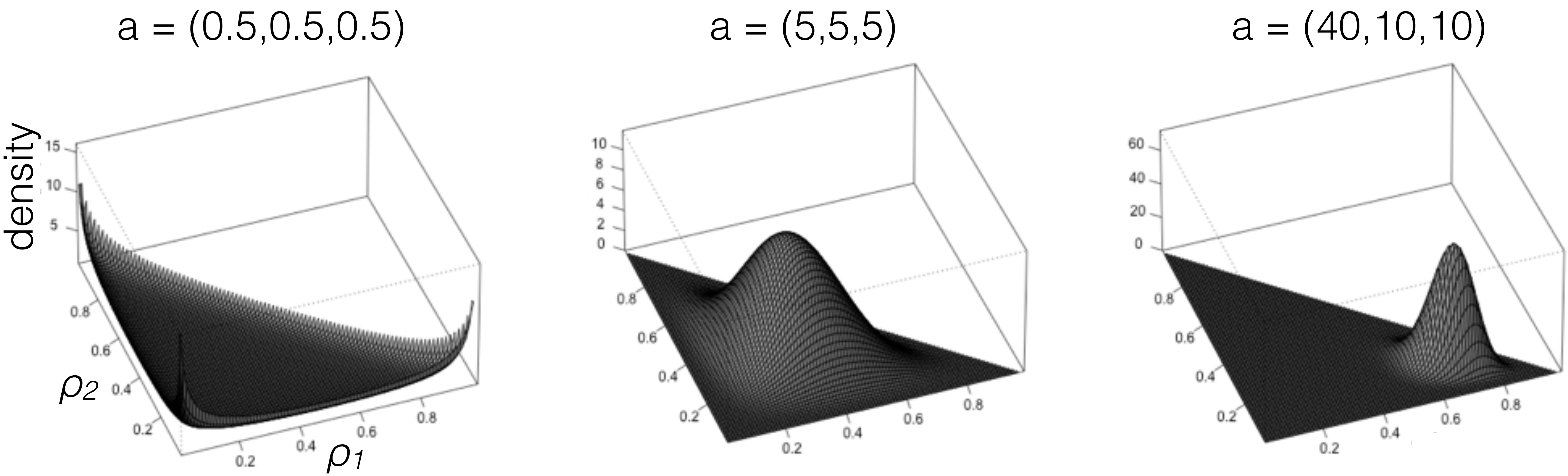

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[demo]

- Dirichlet is conjugate to Categorical

$\rho_{1:K} \sim \text{Dirichlet}(a_{1:K}), z \sim \text{Cat}(\rho_{1:K})$

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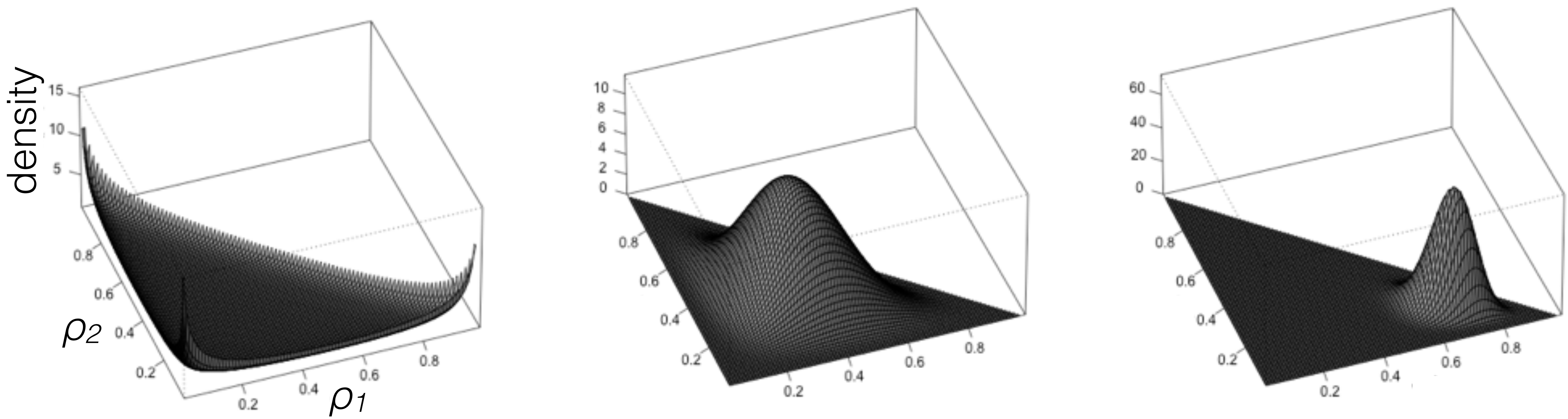

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[demo]

- Dirichlet is conjugate to Categorical

$$\rho_{1:K} \sim \mathrm{Dirichlet}(a_{1:K}), z \sim \mathrm{Cat}(\rho_{1:K})$$

$$\rho_{1:K}|z \stackrel{d}{=} \mathrm{Dirichlet}(a'_{1:K}), a'_k = a_k + \mathbf{1}\{z = k\}$$

What if $K > N$?

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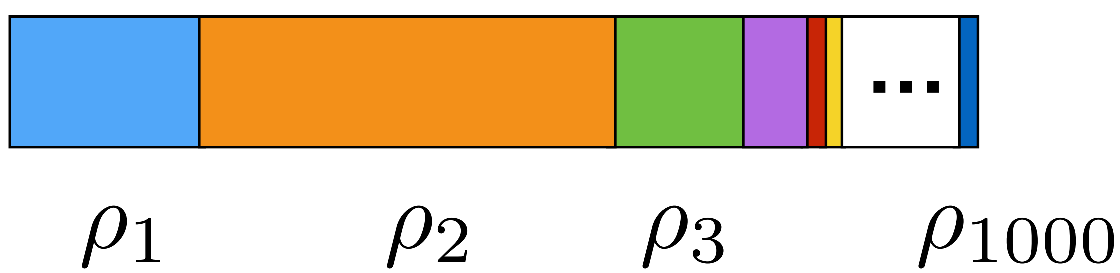
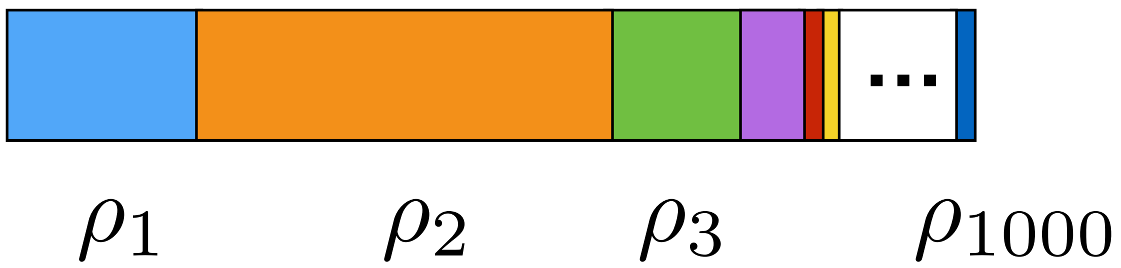


What if $K > N$?

- e.g. species sampling, topic modeling, groups on a social network, etc.



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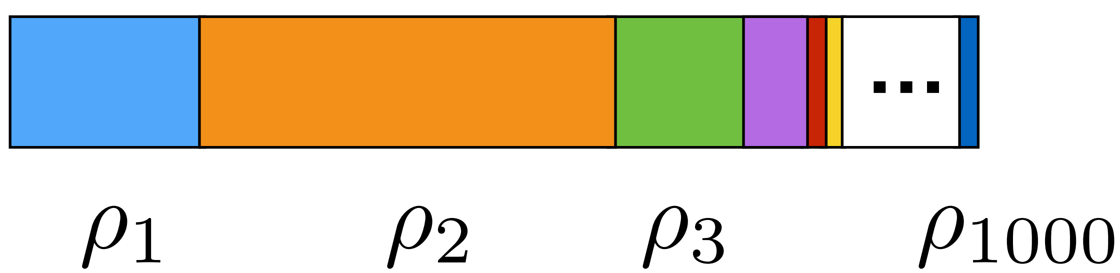

- Components: number of latent groups

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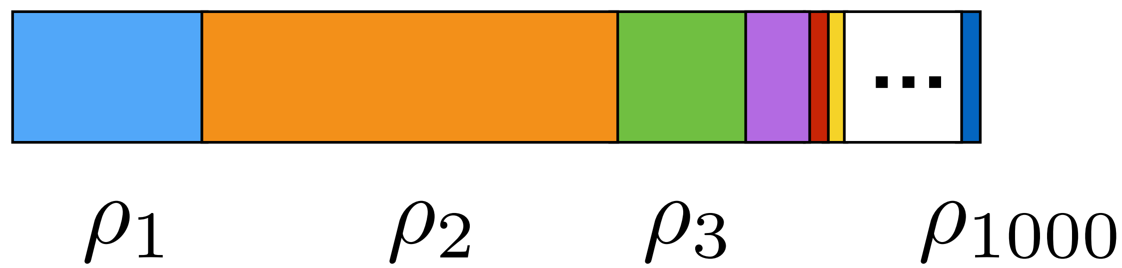

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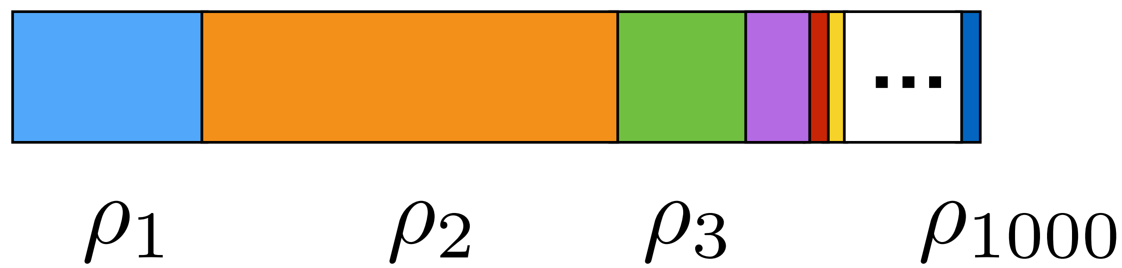

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- [demo 1, demo 2]

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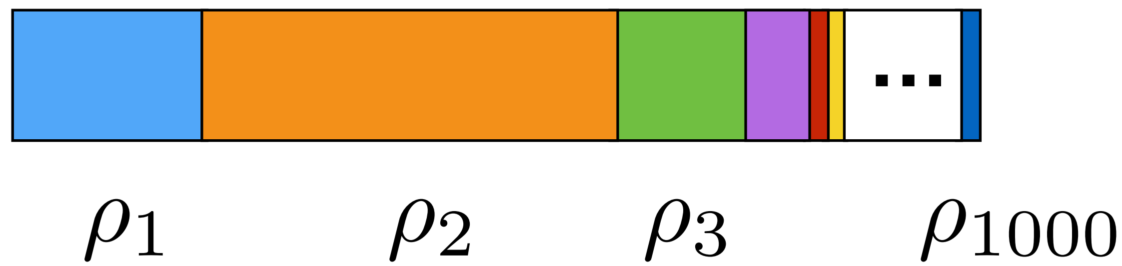

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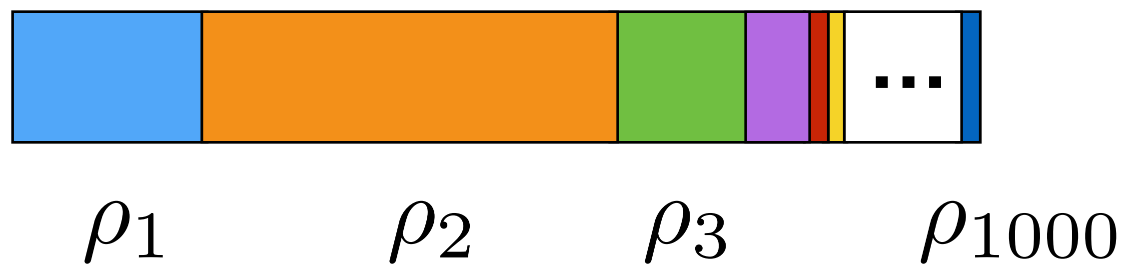

- Components: number of latent groups
- Clusters: number of components represented in the data
- [demo 1, demo 2]
- Number of clusters for N data points is $< K$ and random
- Number of clusters grows with N

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data

Choosing $K = \infty$

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- "Stick breaking"

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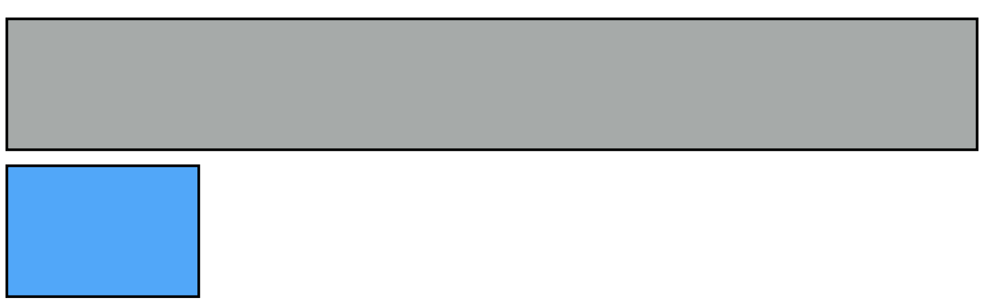

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$V_1 \sim \mathrm{Beta}(a_1, a_2 + a_3 + a_4)$

$\rho_1 = V_1$

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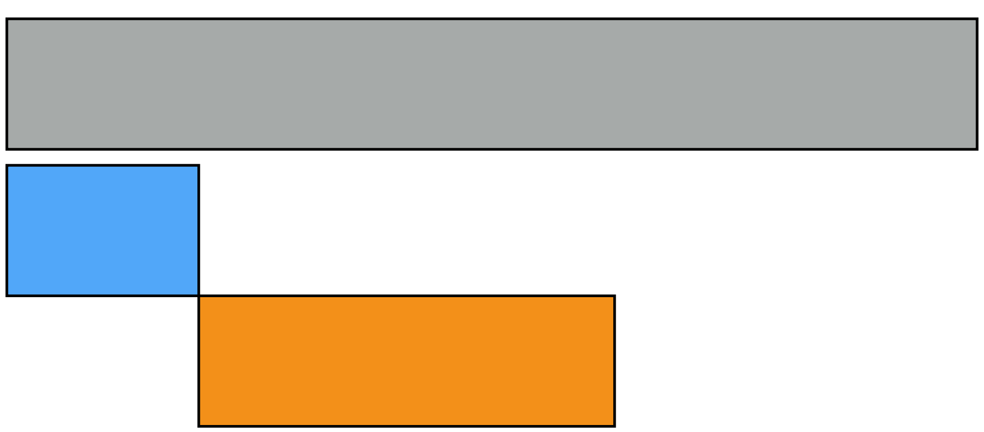

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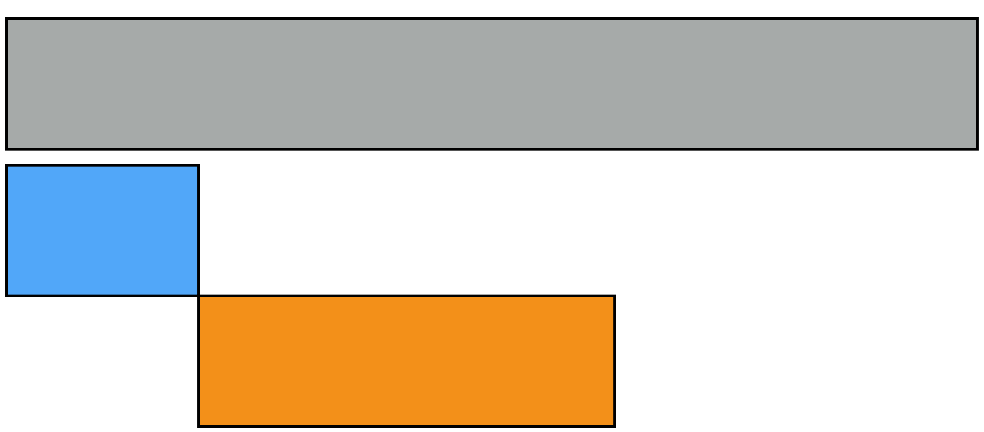

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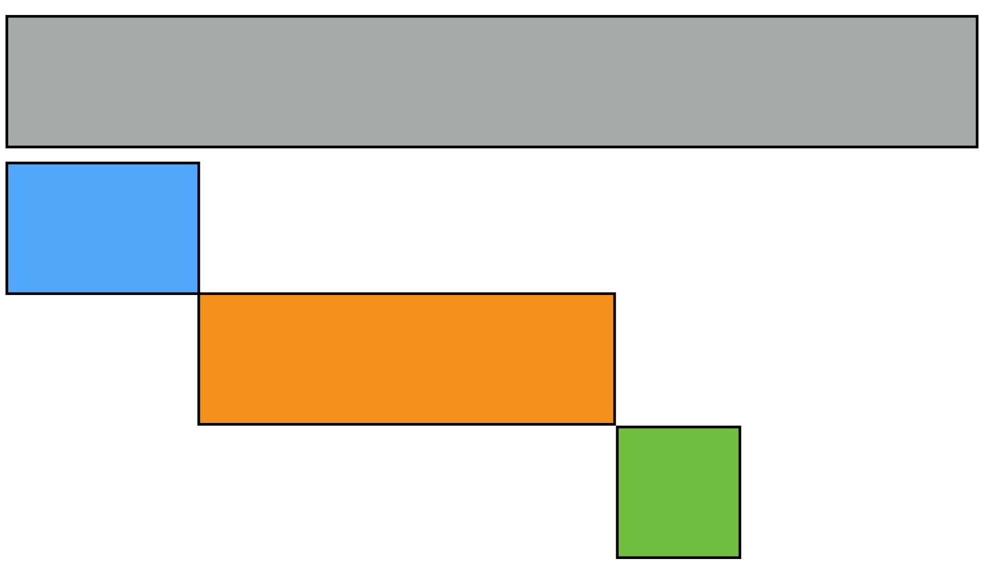

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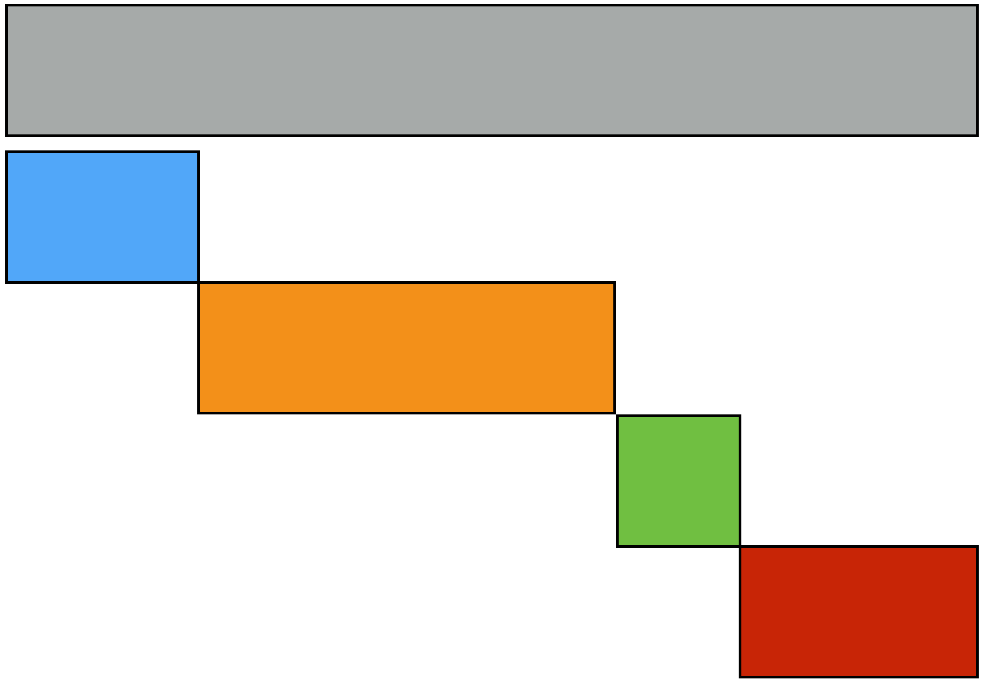

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$$\rho_4 = 1 - \sum_{k=1}^{3} \rho_k$$

Choosing $K = \infty$

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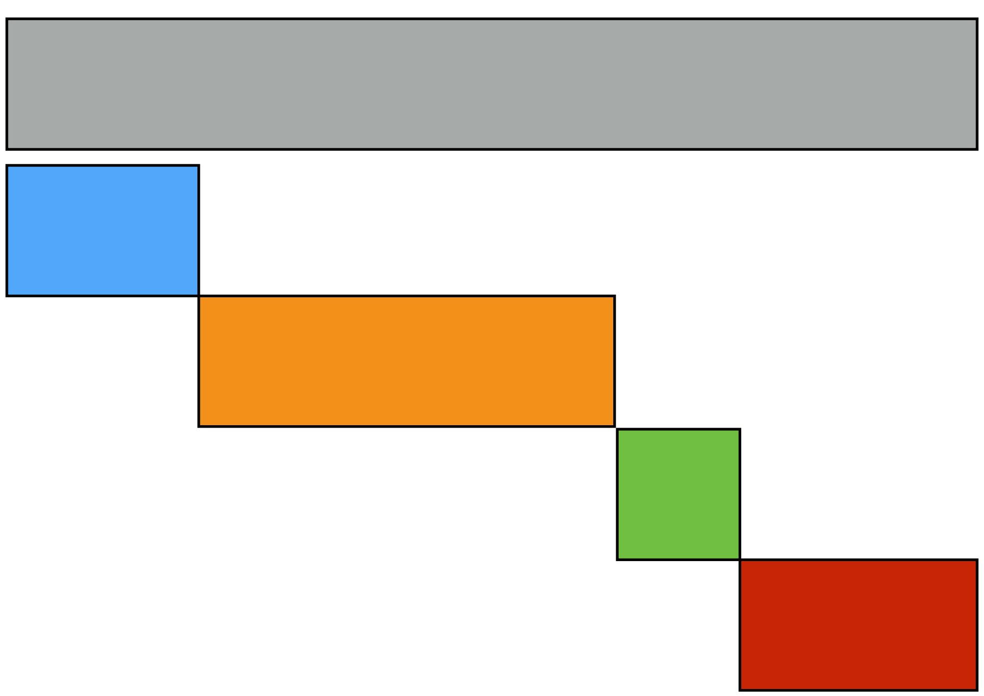

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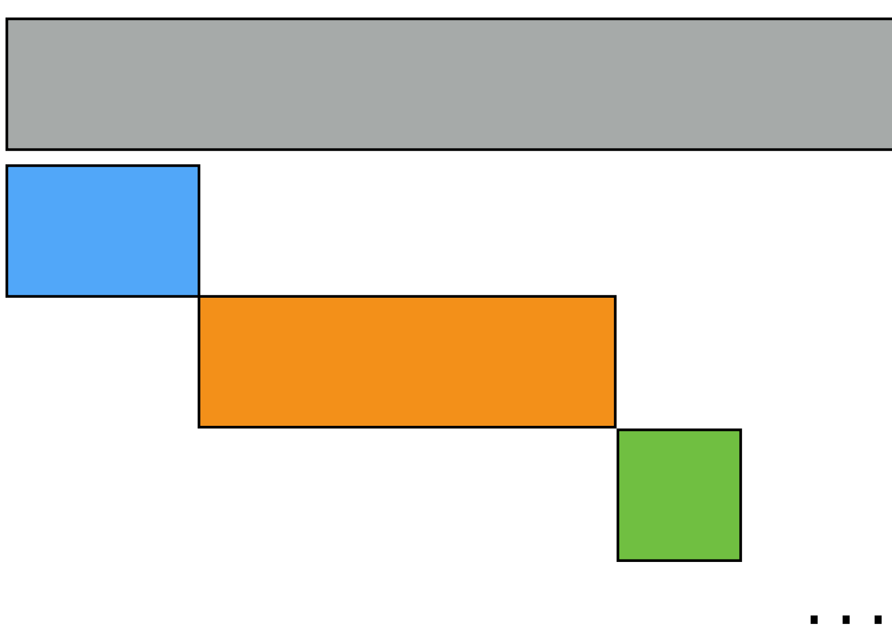

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Choosing $K = \infty$

- Here, difficult to choose finite K in advance (contrast with small K): don't know K, difficult to infer, streaming data
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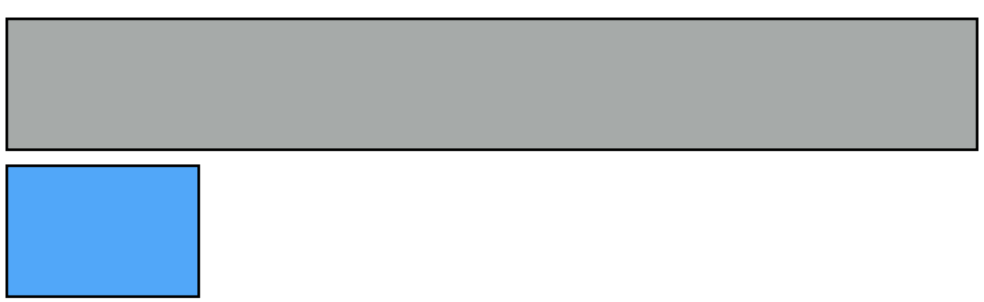

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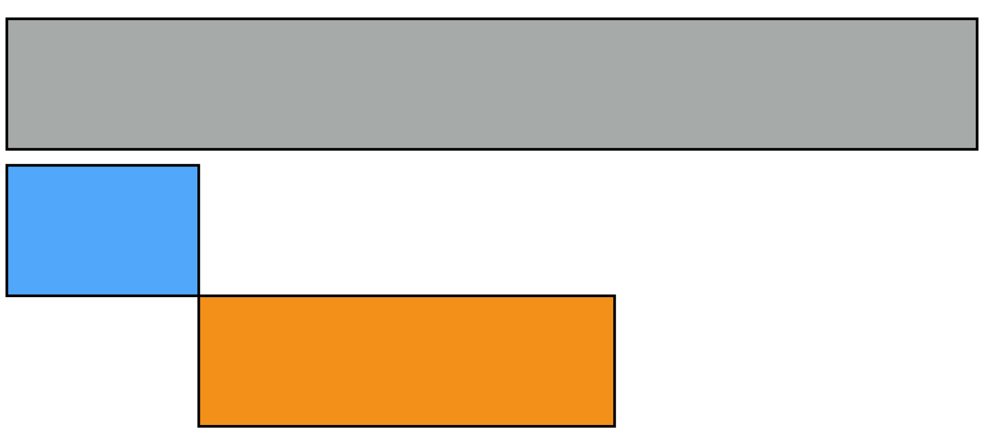

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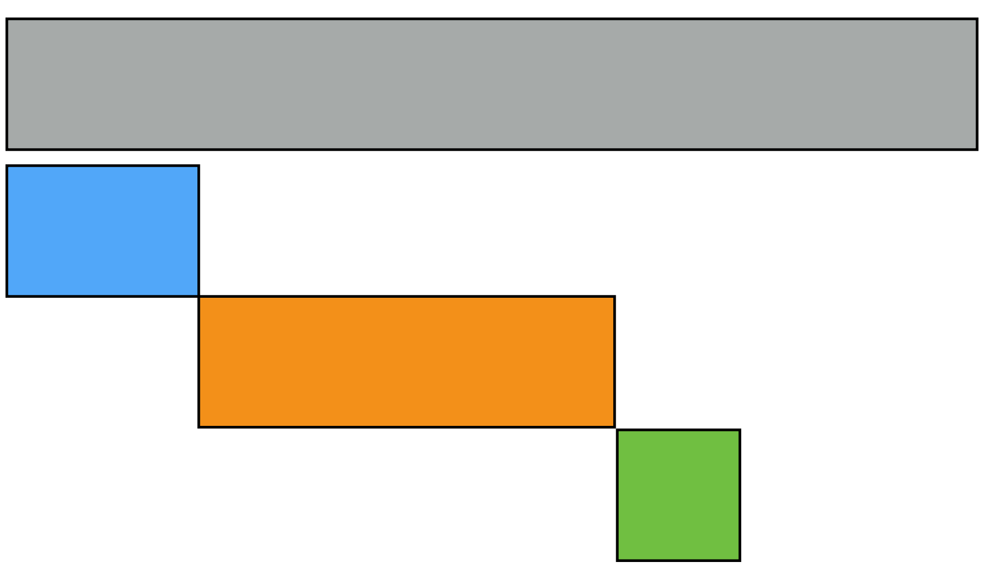

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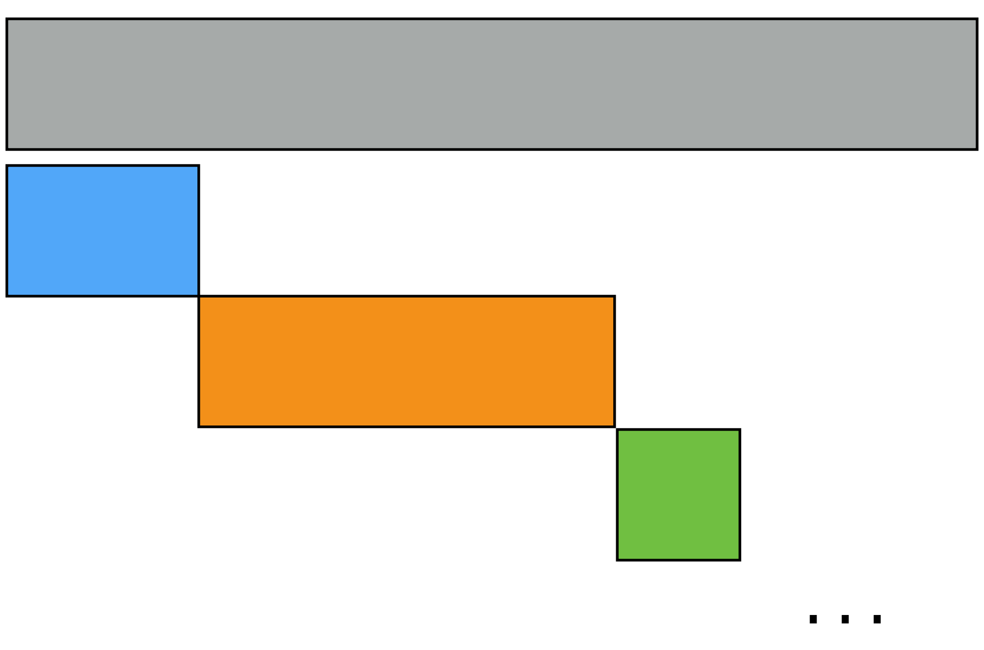

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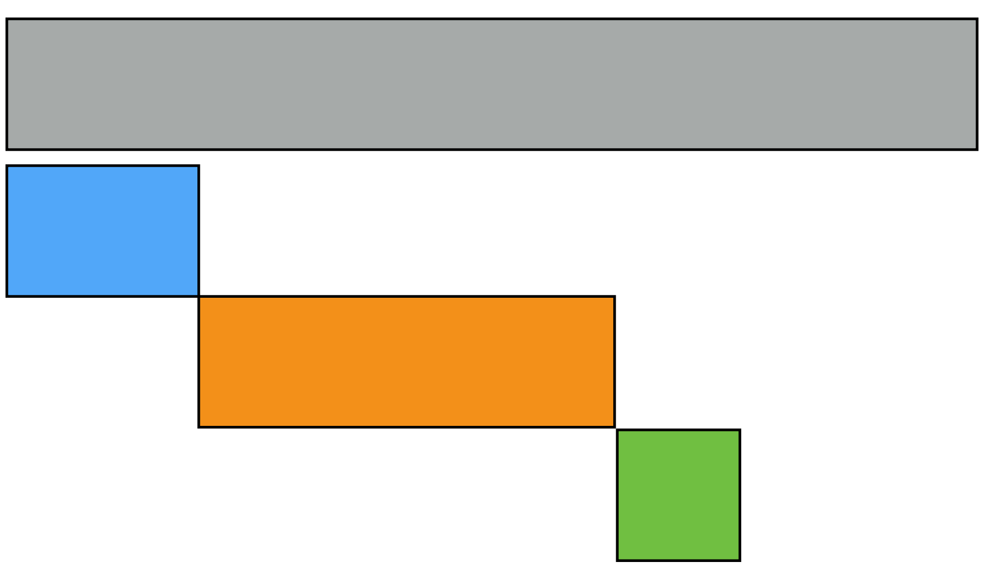

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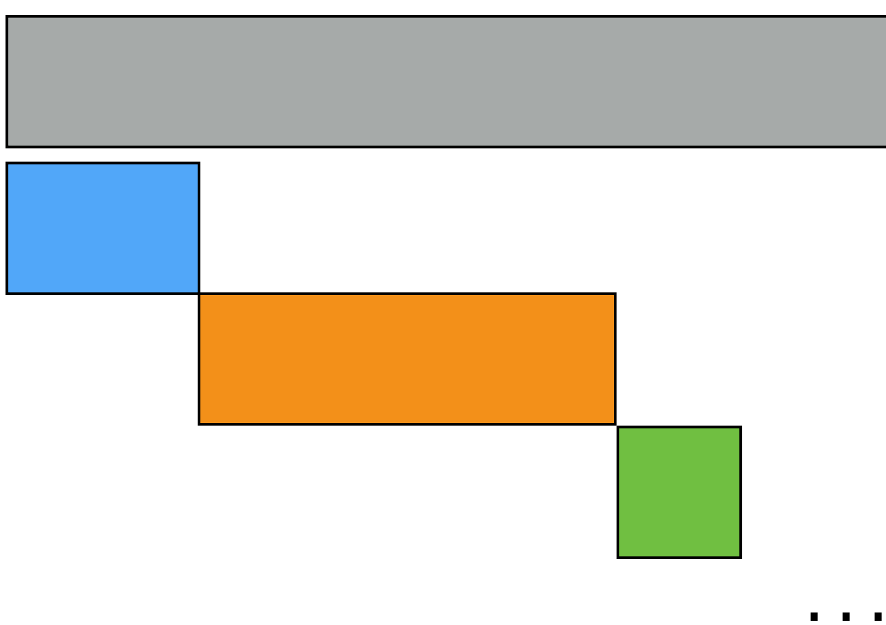

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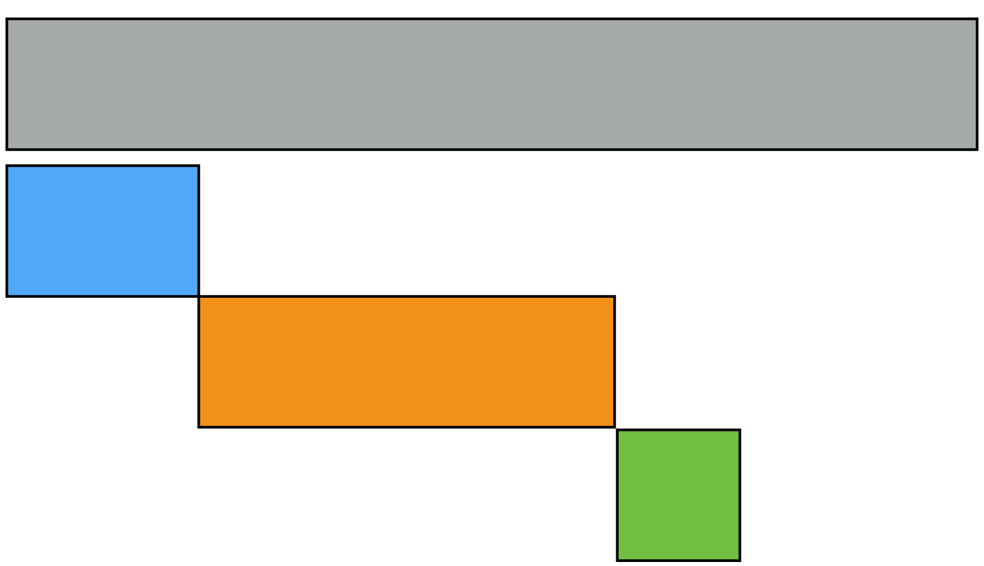

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[Ishwaran, James 2001]

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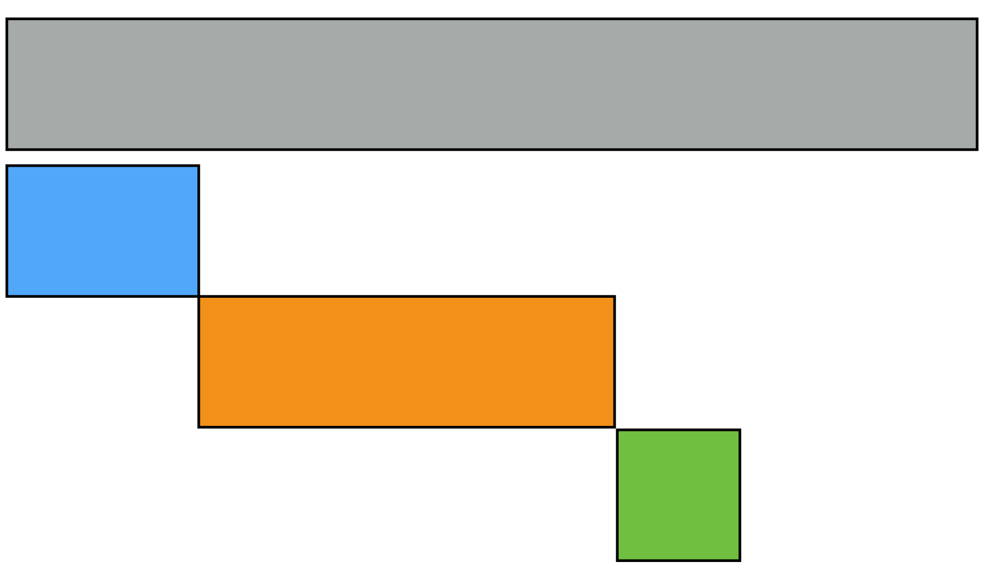

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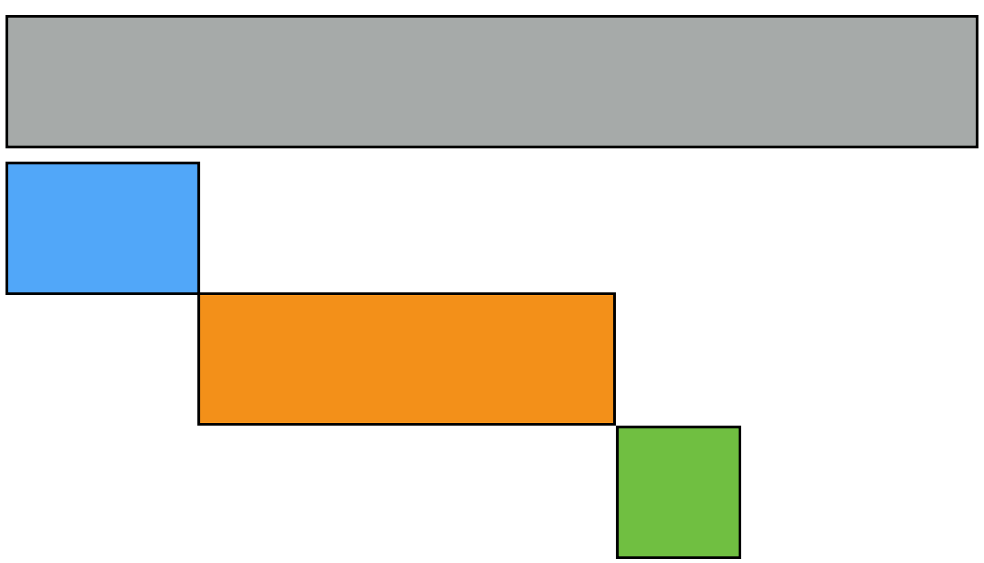

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[McCloskey 1965; Engen 1975; Patil and Taillie 1977; Ewens 1987; Sethuraman 1994; Ishwaran, James 2001]

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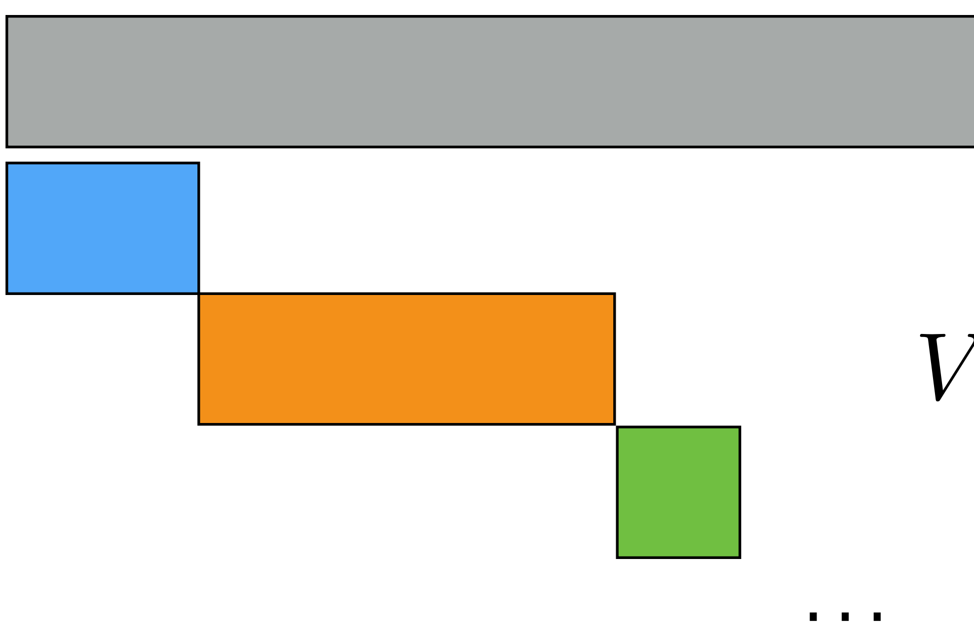

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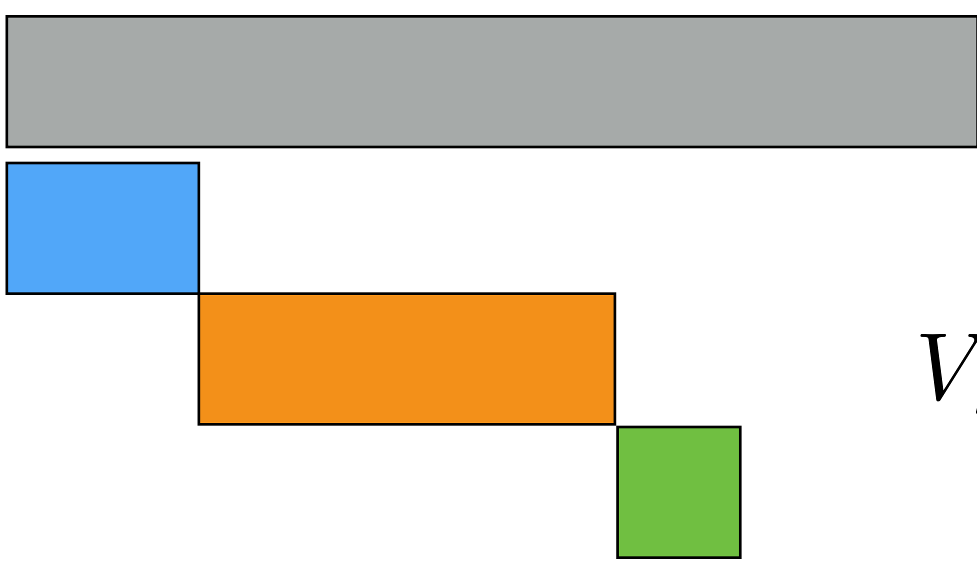

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[demo]

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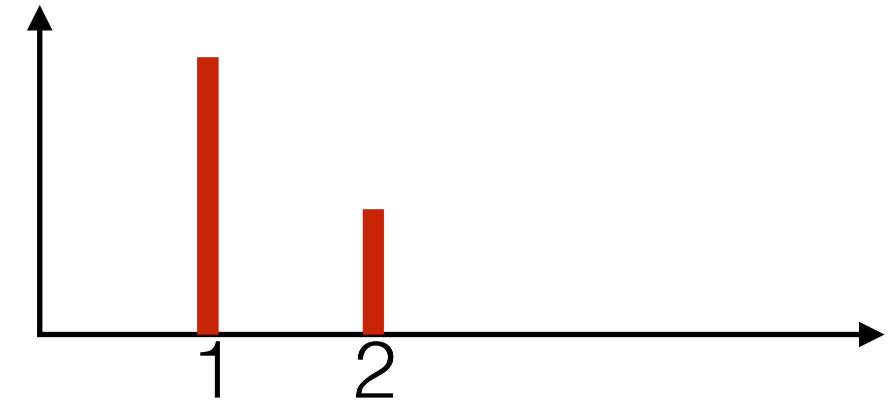
Distributions

Distributions

- Beta → random distribution over $1, 2$



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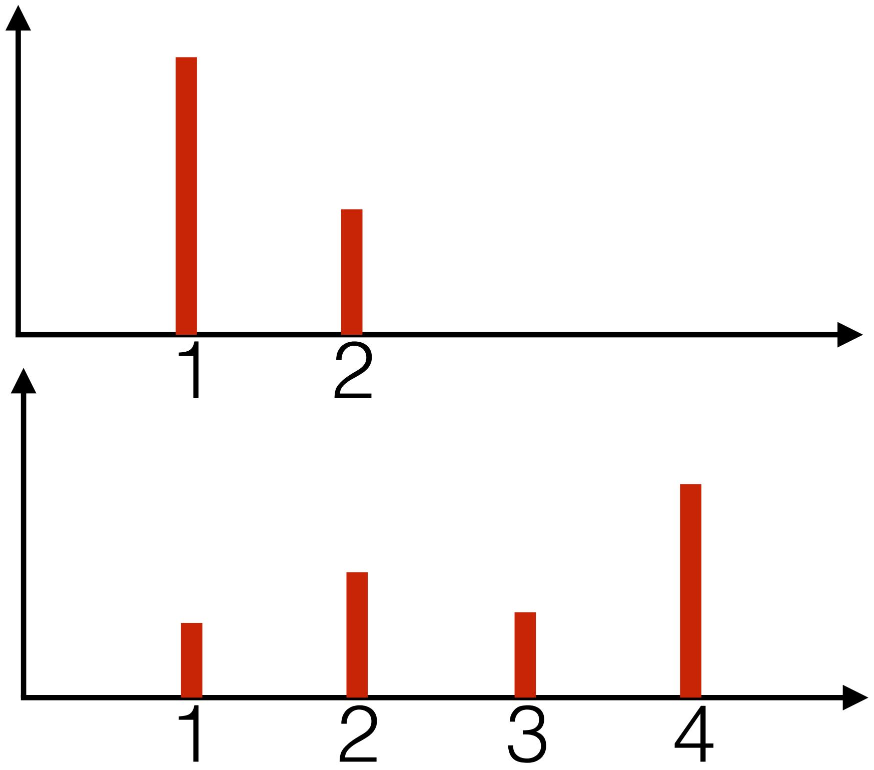

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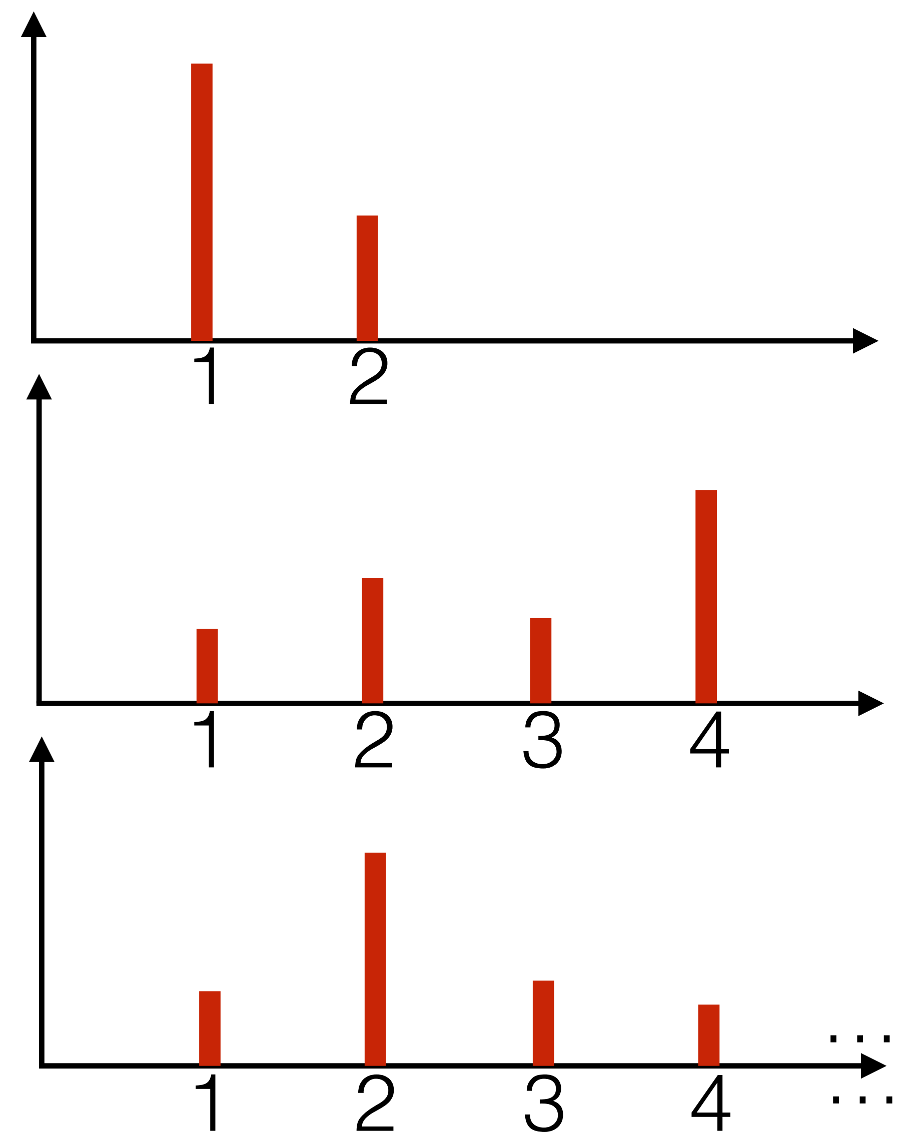

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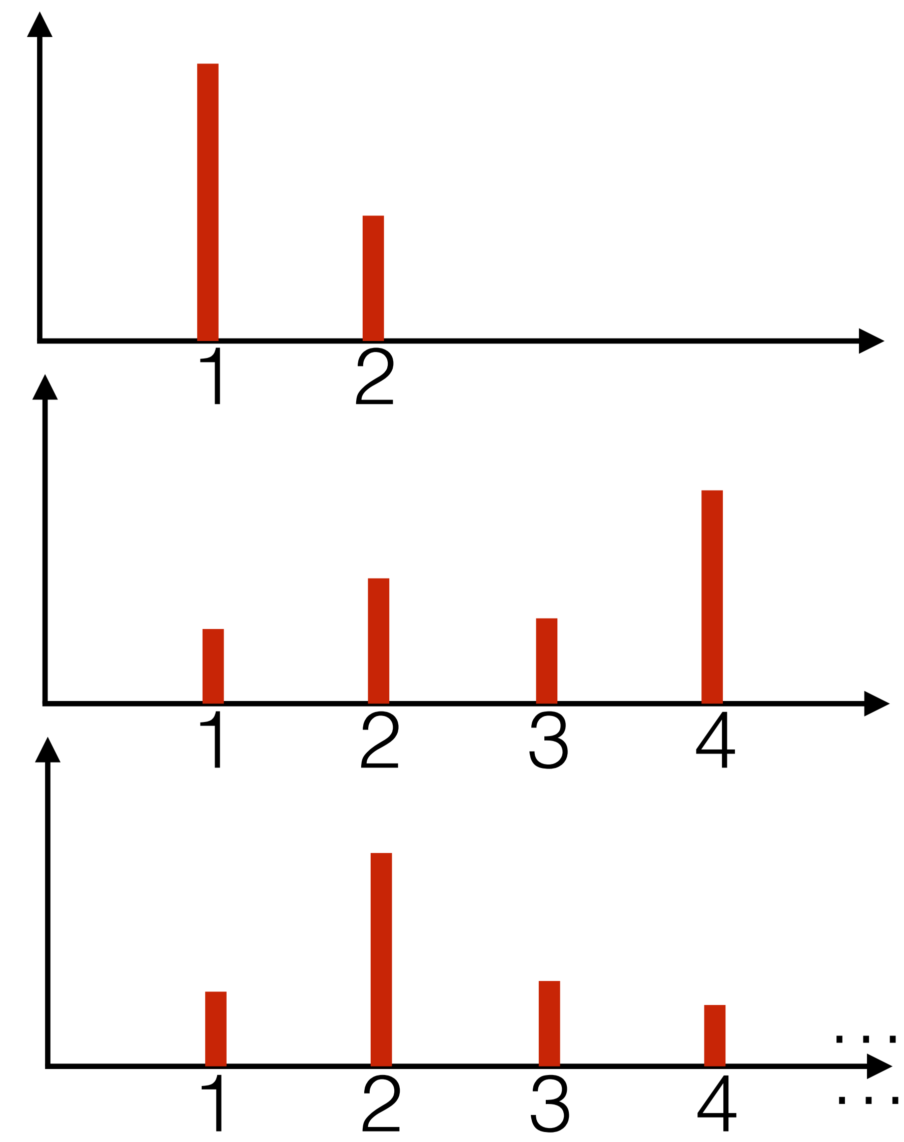

- Infinity of parameters: components
- Growing number of parameters: clusters

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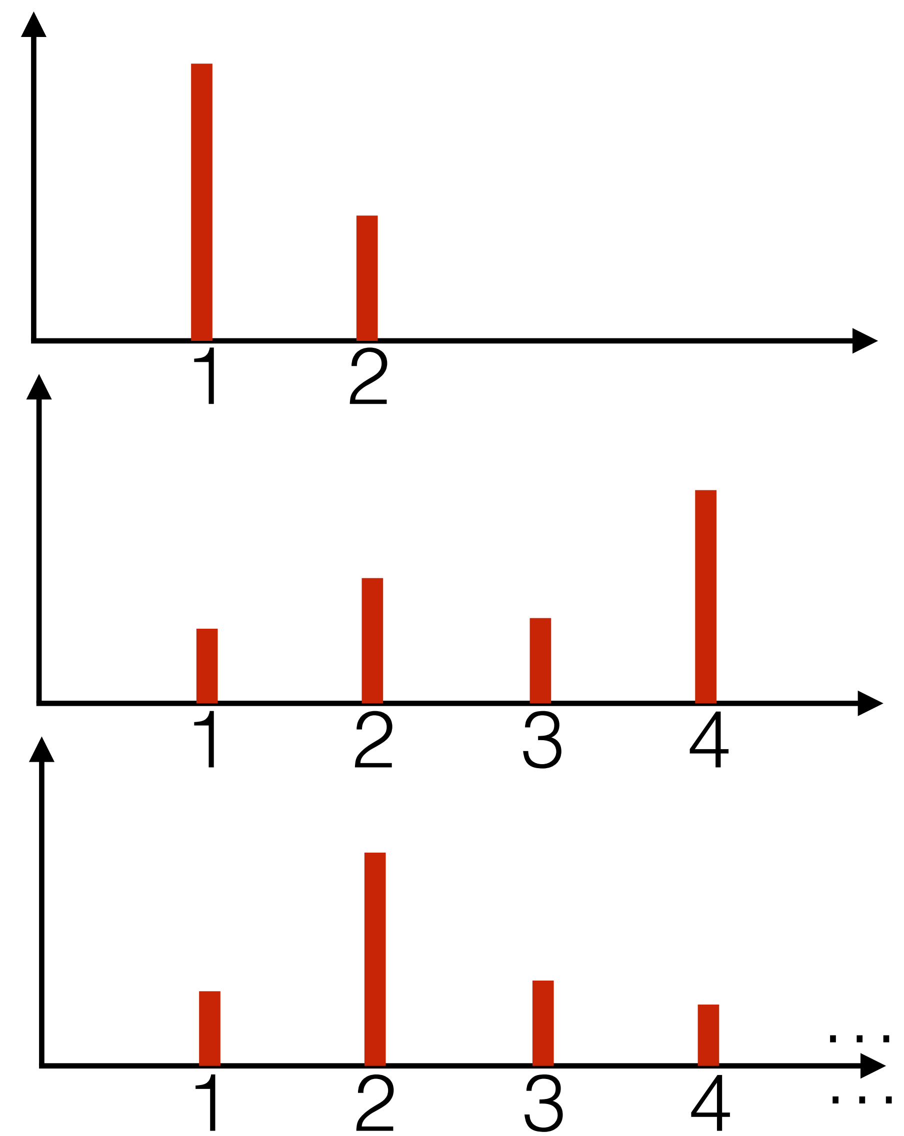

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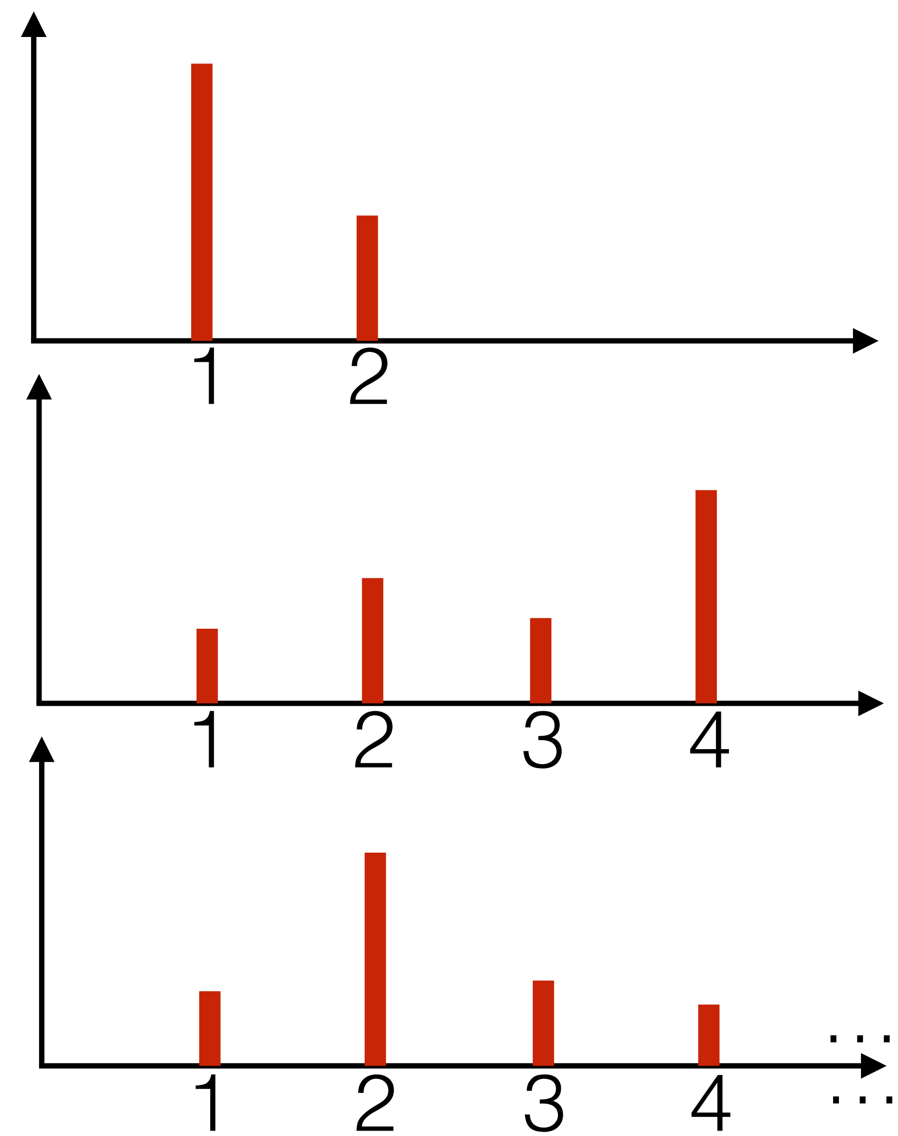

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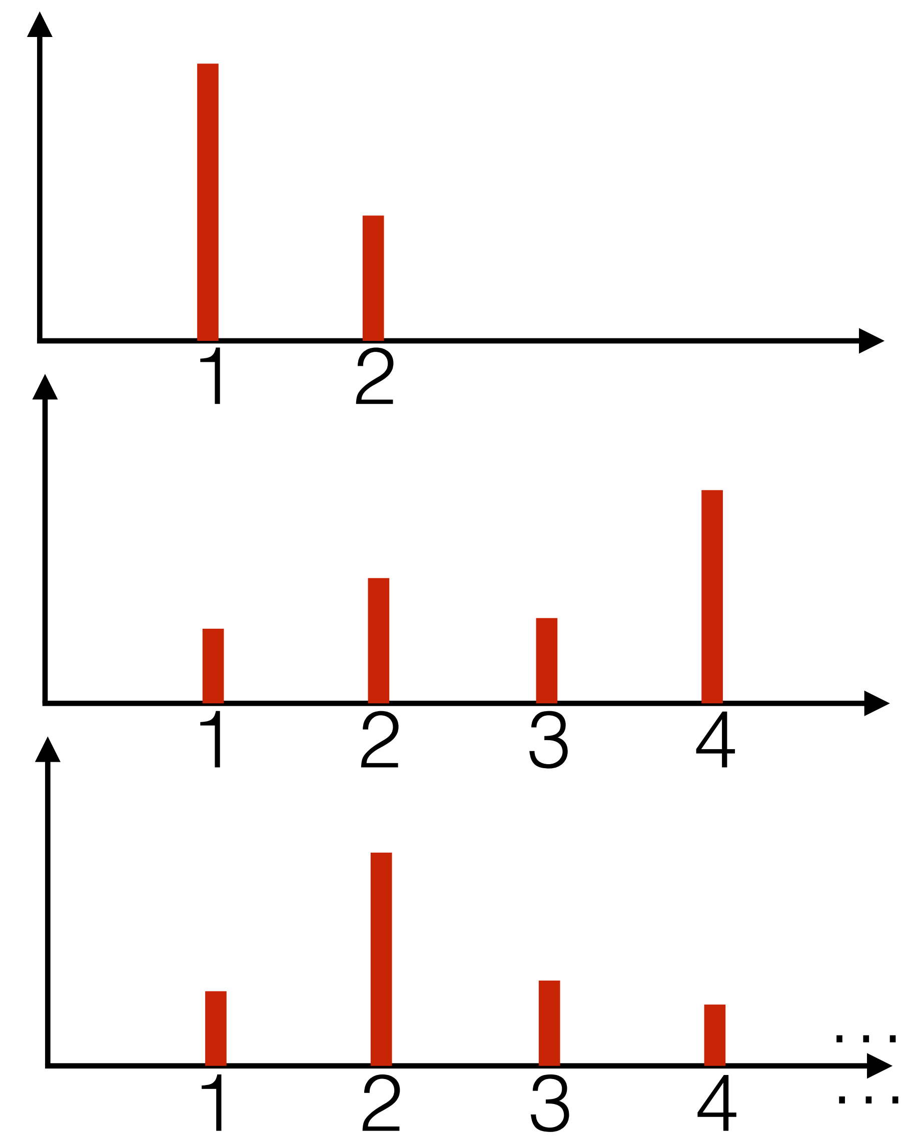

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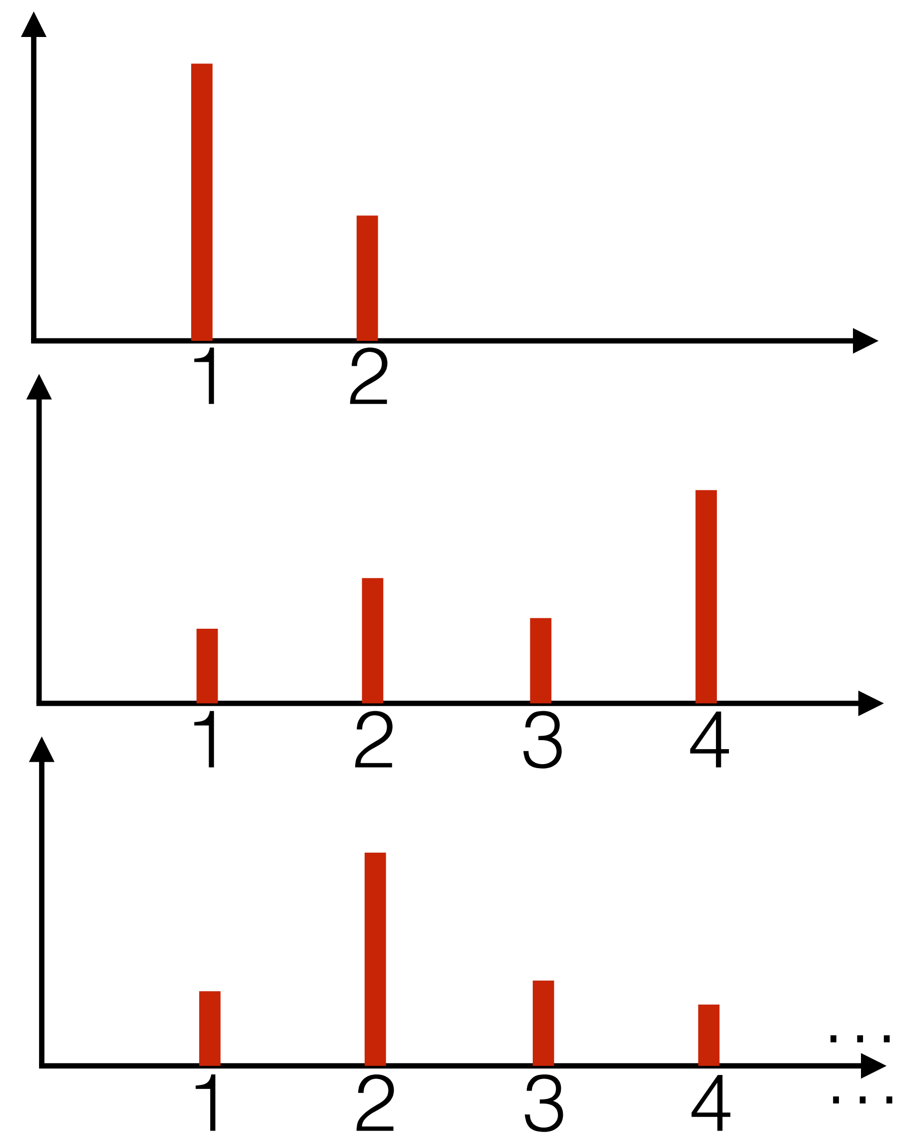

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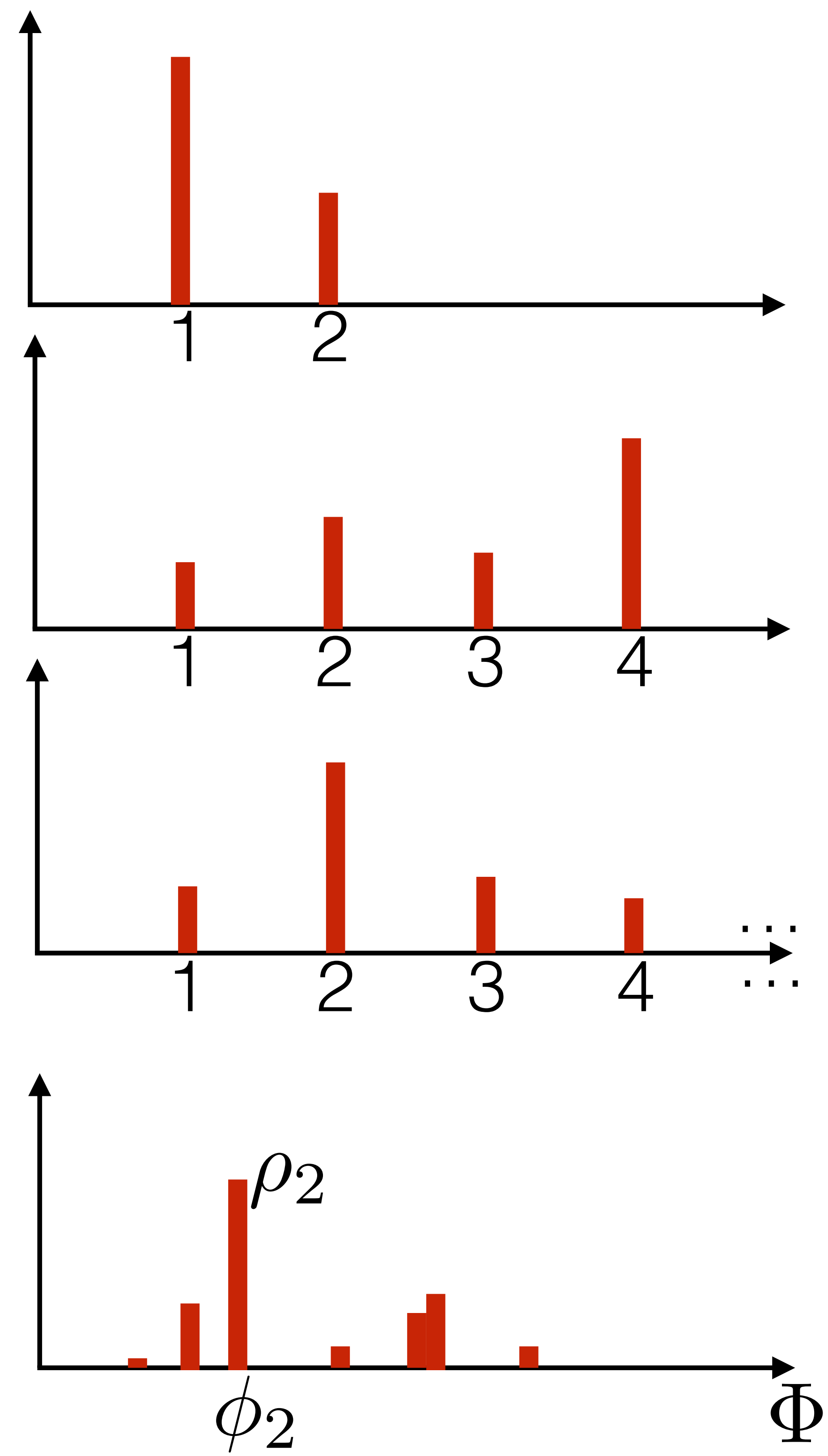

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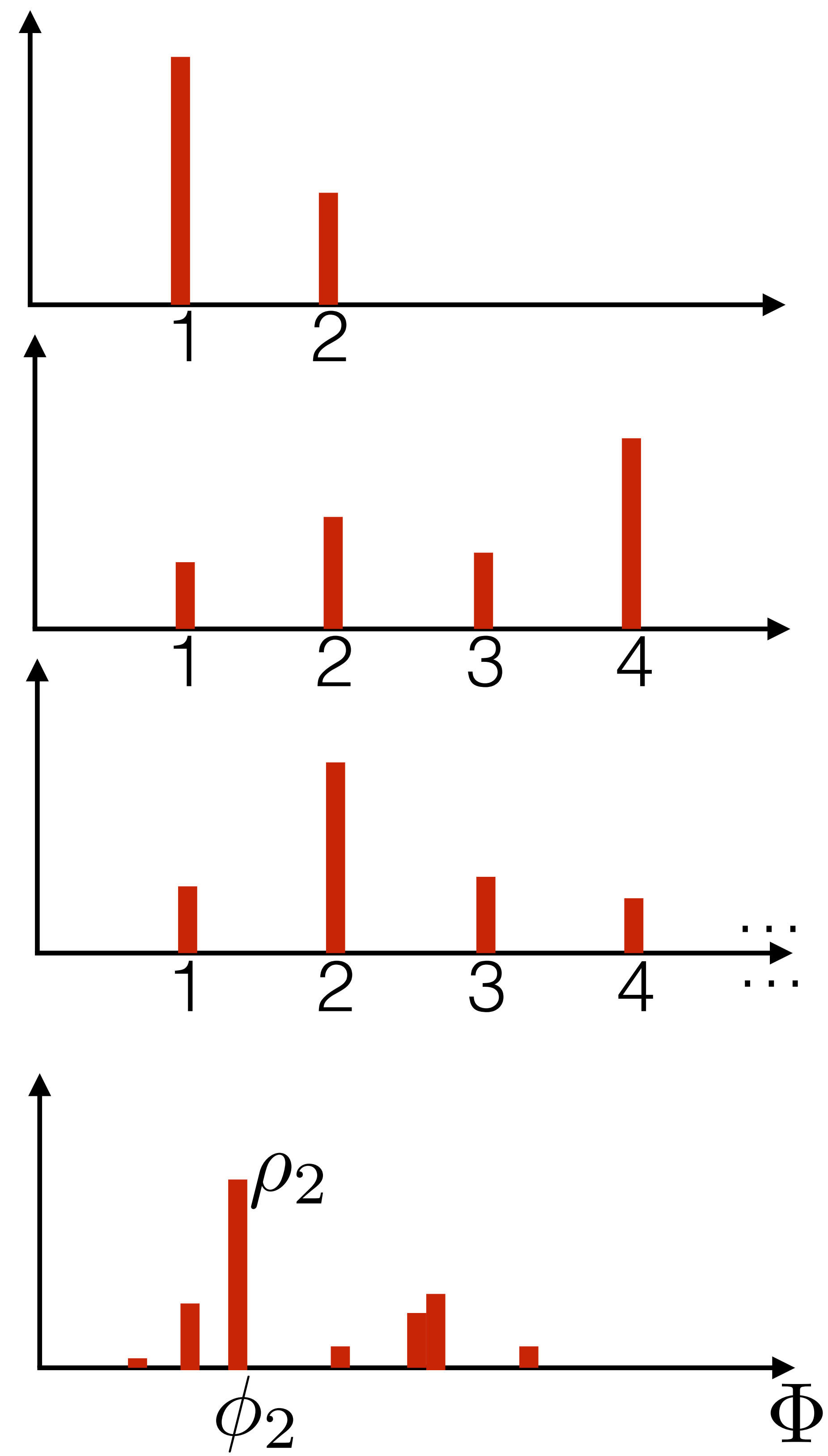

[Ferguson 1973]

Dirichlet process mixture model

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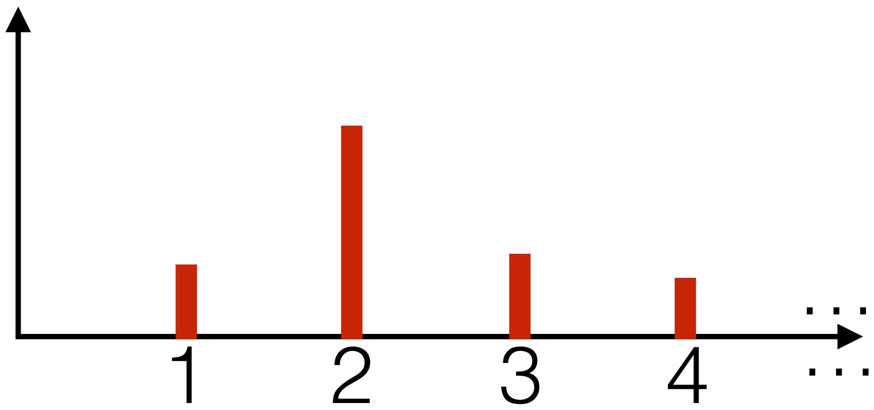

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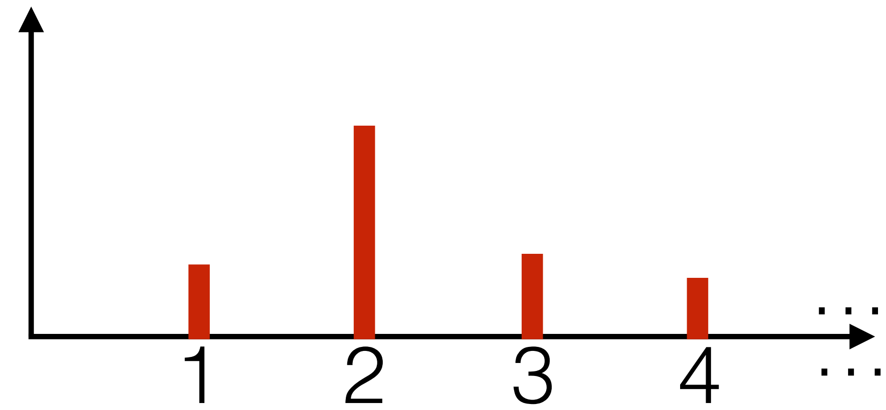

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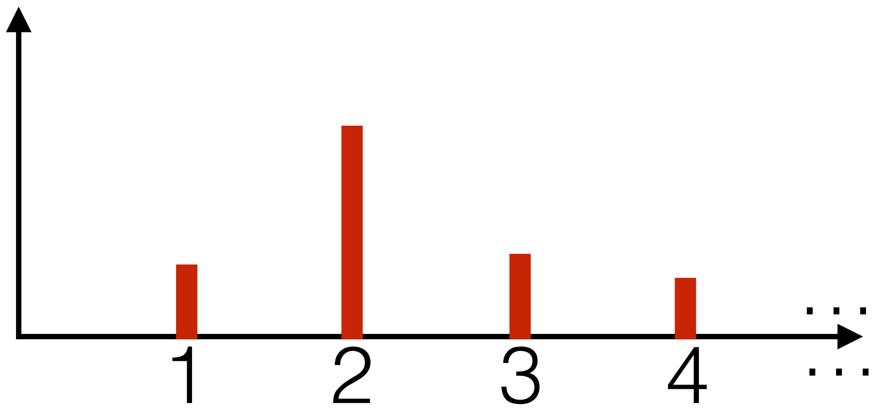

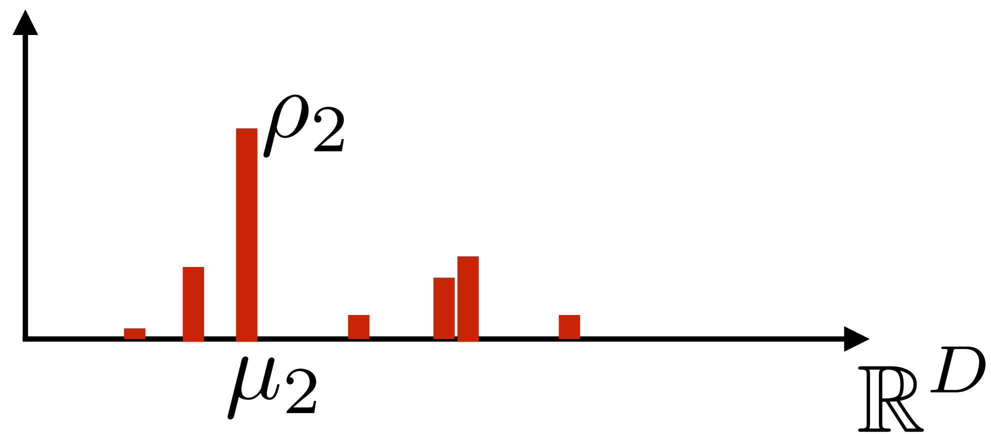

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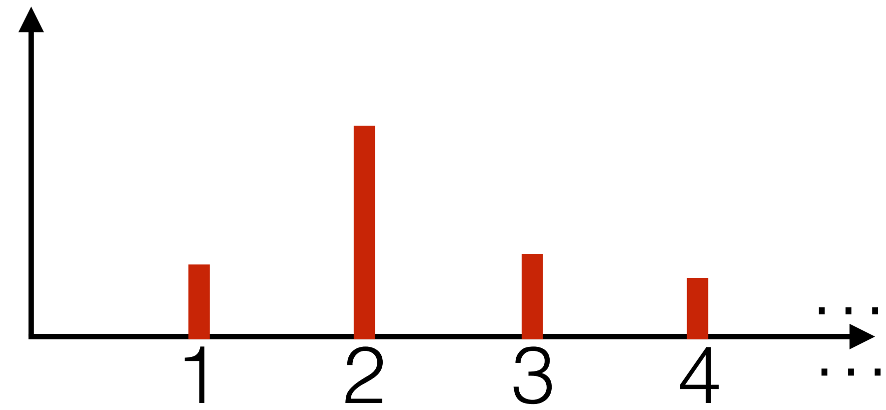

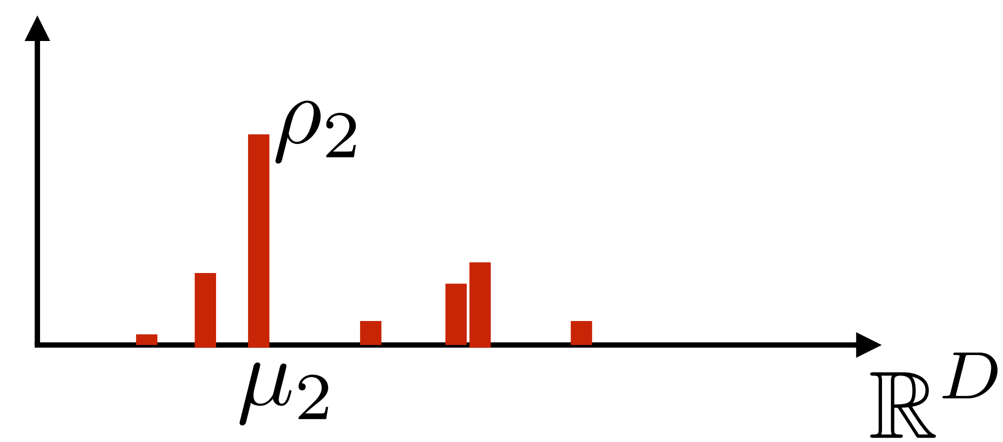

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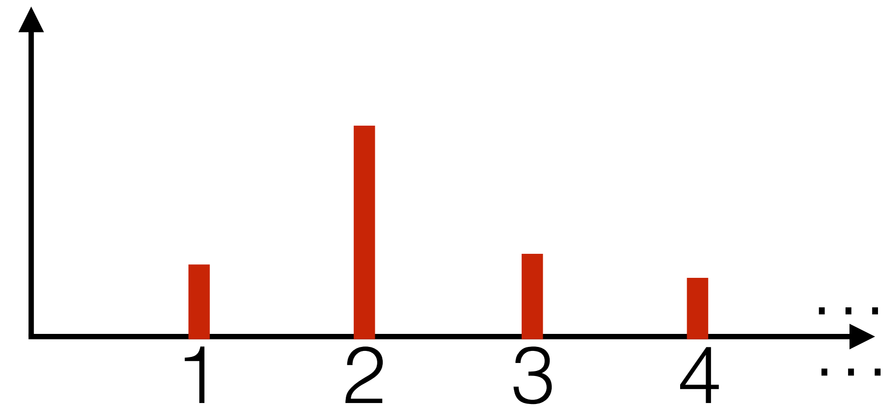

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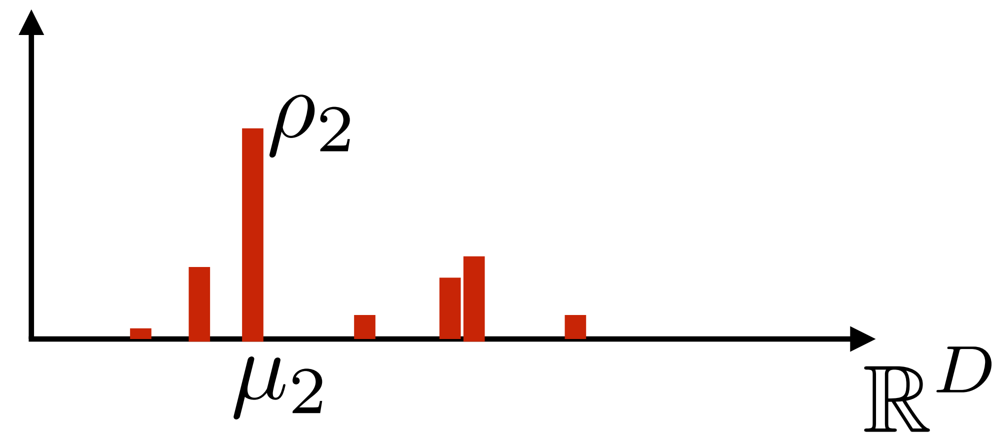

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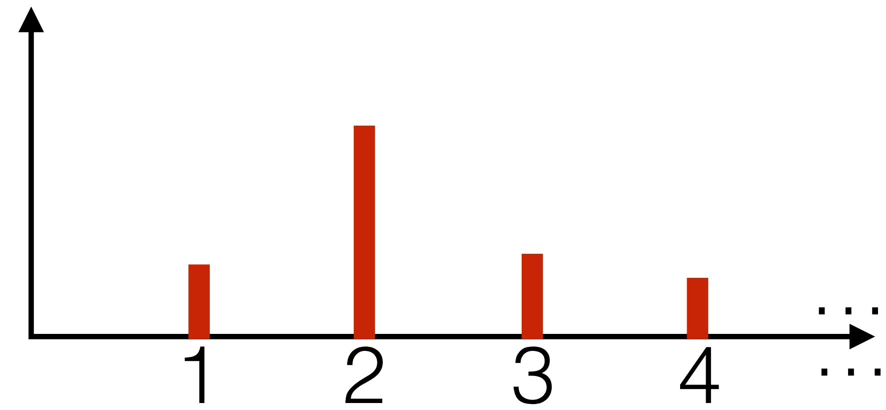
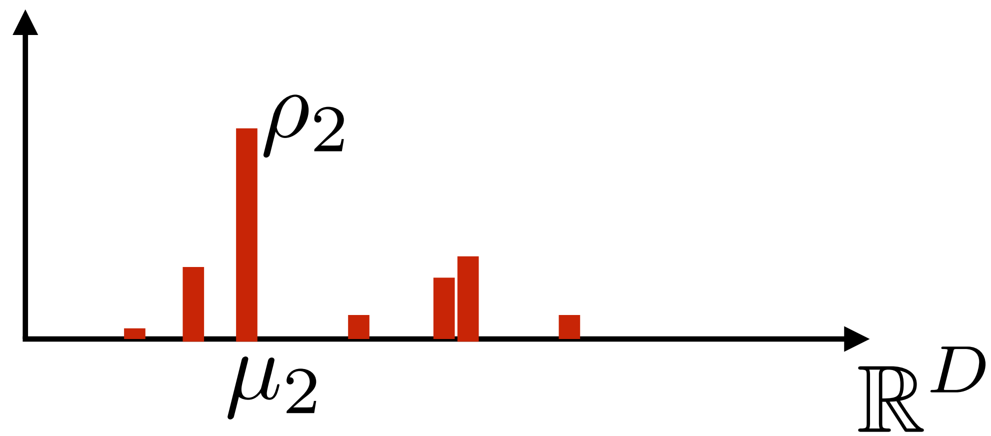
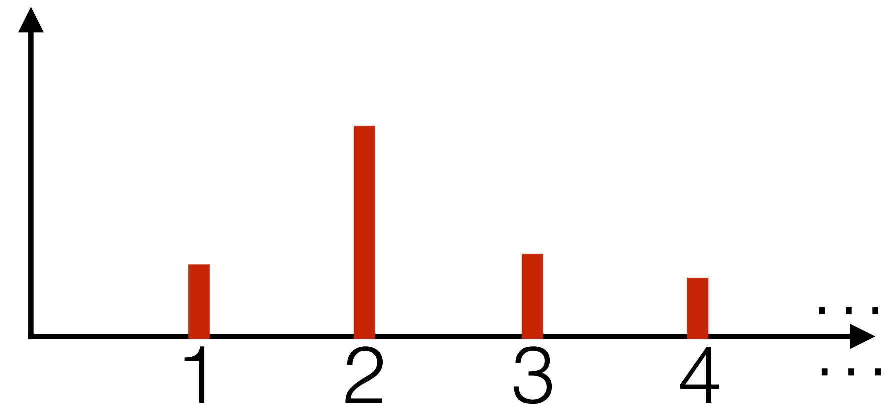




$$z_n \stackrel{iid}{\sim} \text{Categorical}(\rho)$$

Dirichlet process mixture model

- Gaussian mixture model



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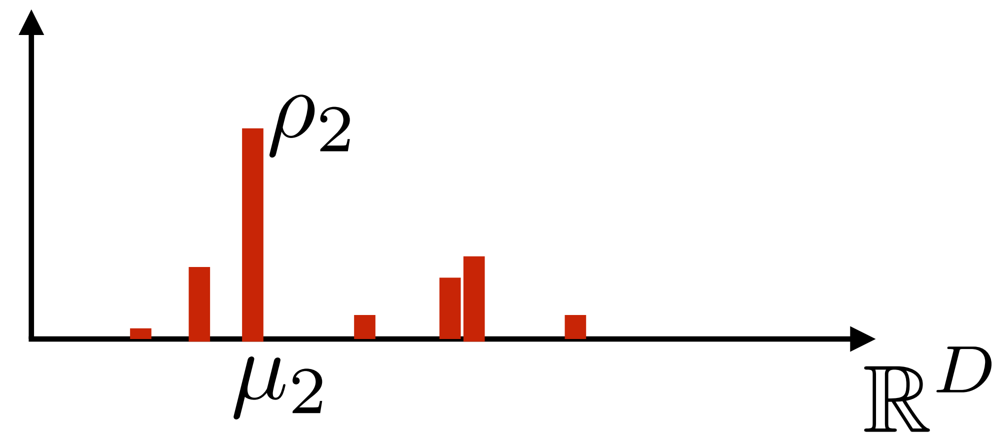

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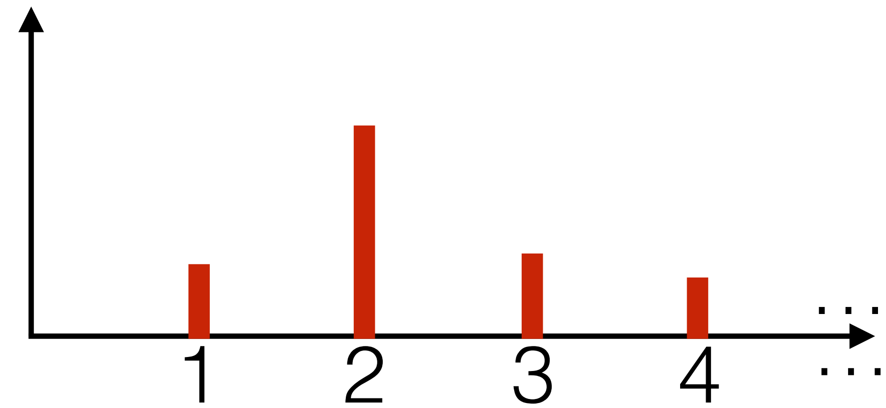

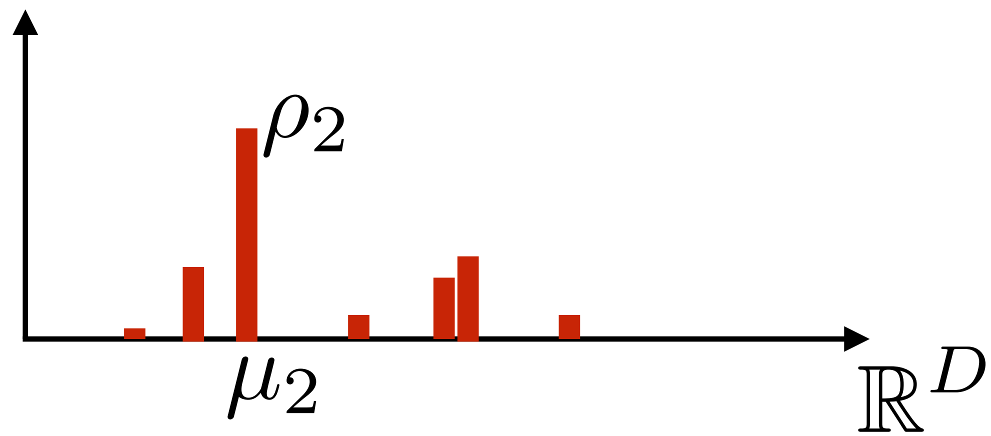

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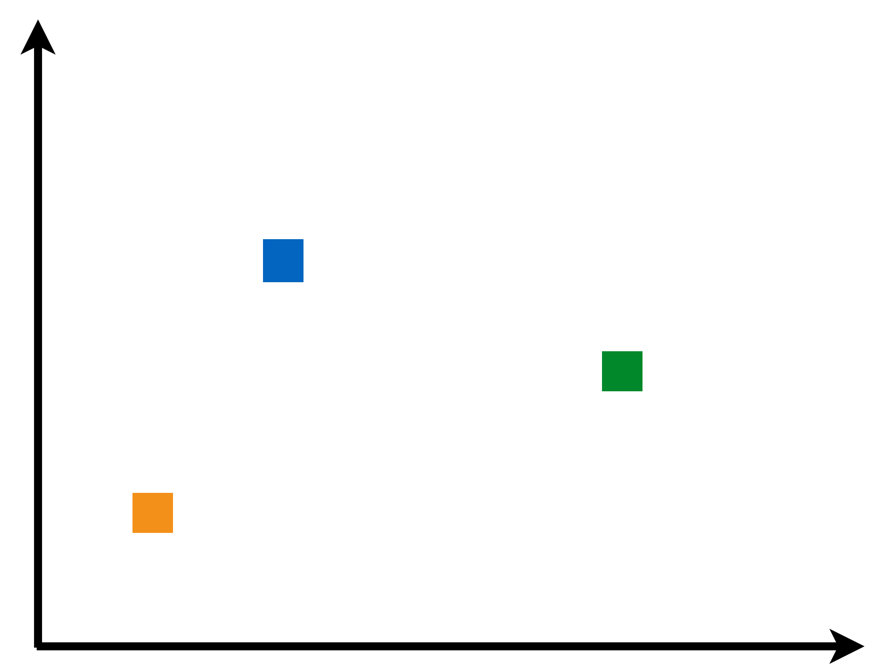

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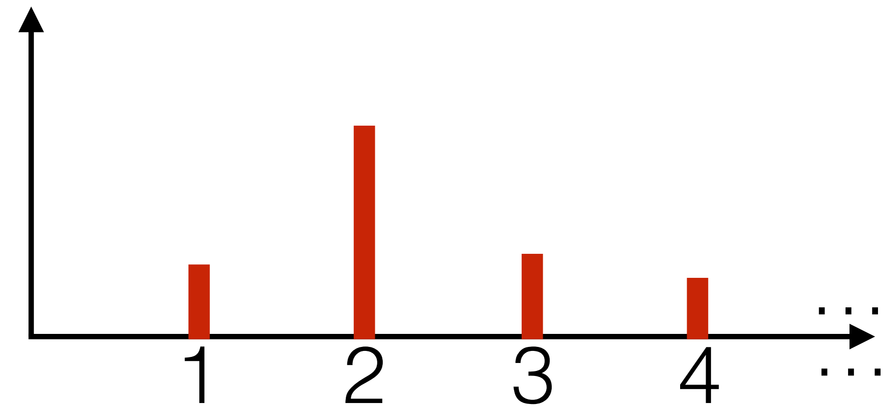

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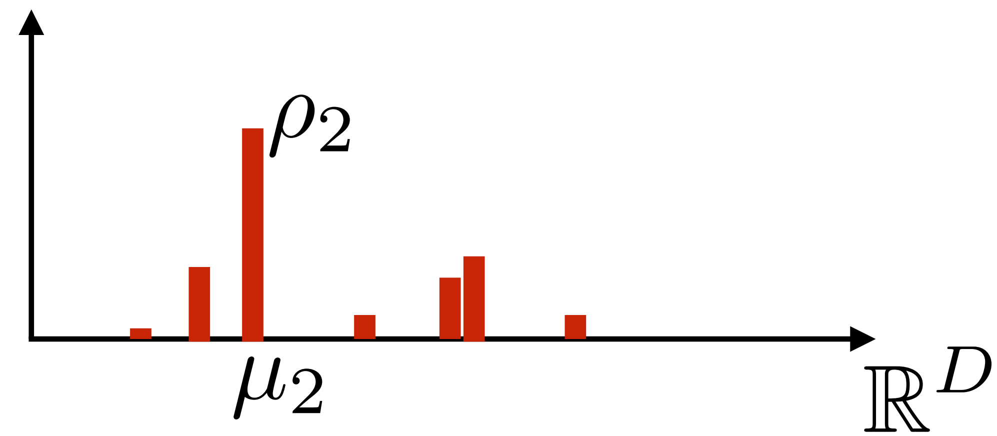

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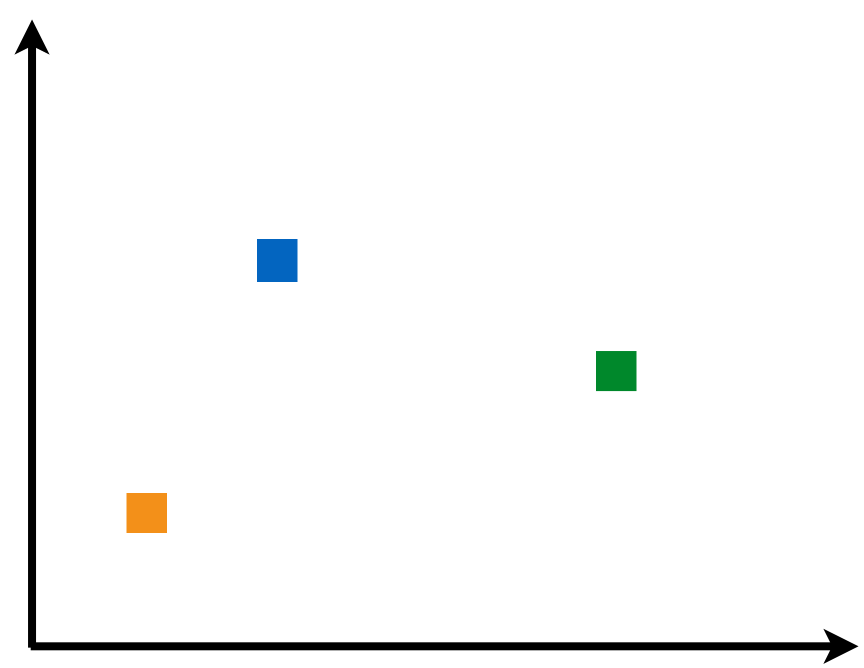
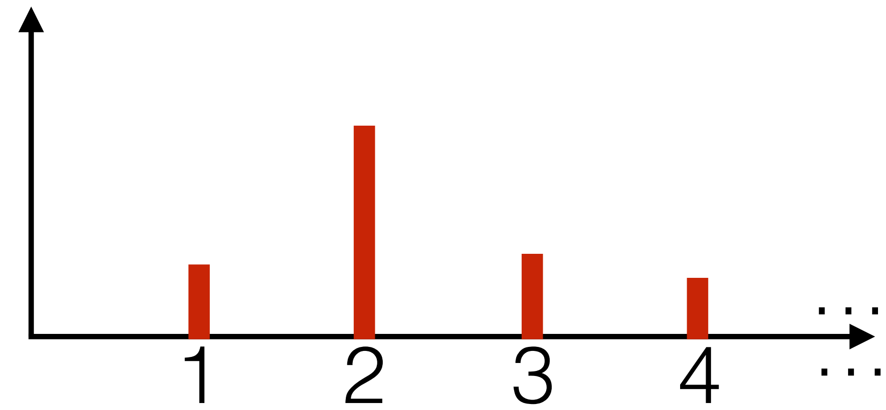
- i.e. $\mu_n^* \overset{iid}{\sim} G$



Dirichlet process mixture model

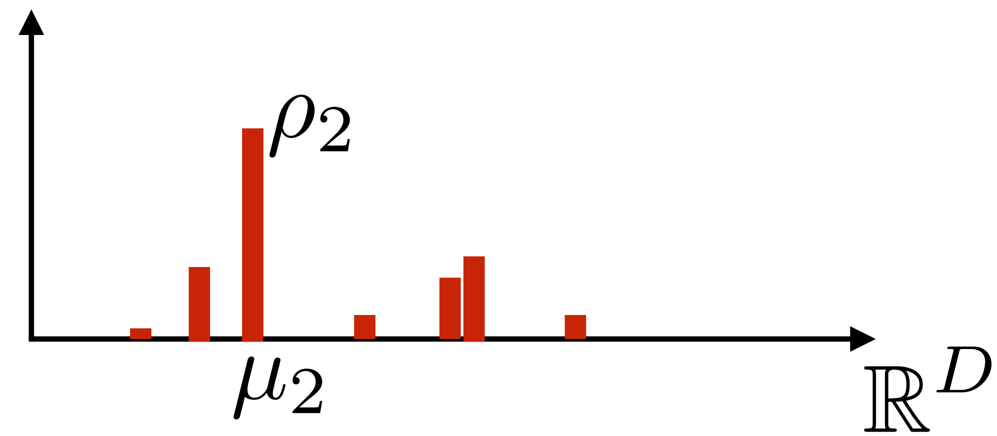
- Gaussian mixture model



$$\rho = (\rho_1, \rho_2, \ldots) \sim \mathrm{GEM}(\alpha)$$

$$\mu_k \overset{iid}{\sim} \mathcal{N}(\mu_0, \Sigma_0), k = 1, 2, \ldots$$

- i.e. $G = \sum_{k=1}^{\infty} \rho_k \delta_{\mu_k} \overset{d}{=} \mathrm{DP}(\alpha, \mathcal{N}(\mu_0, \Sigma_0))$



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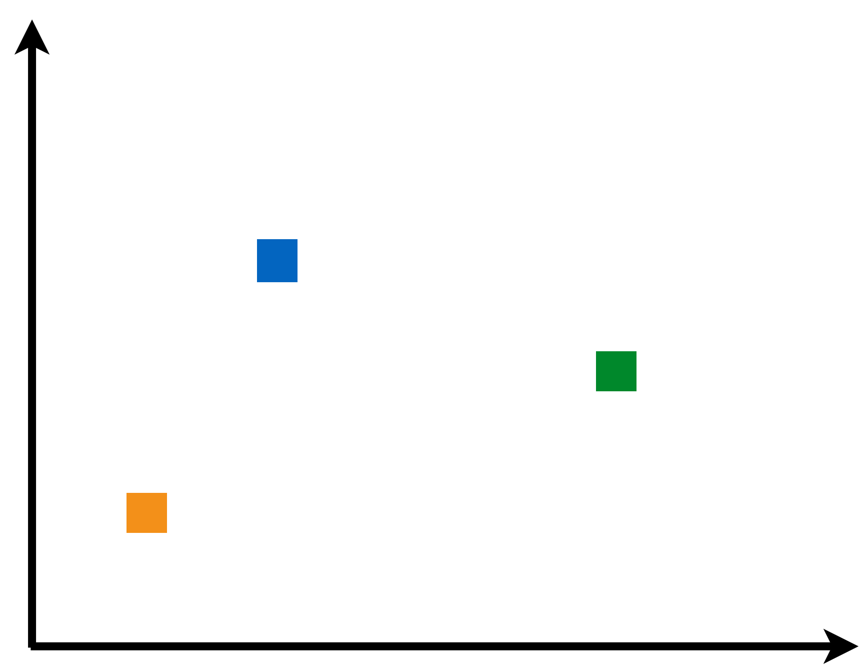

$$x_n \overset{indep}{\sim} \mathcal{N}(\mu_n^*, \Sigma)$$

Dirichlet process mixture model

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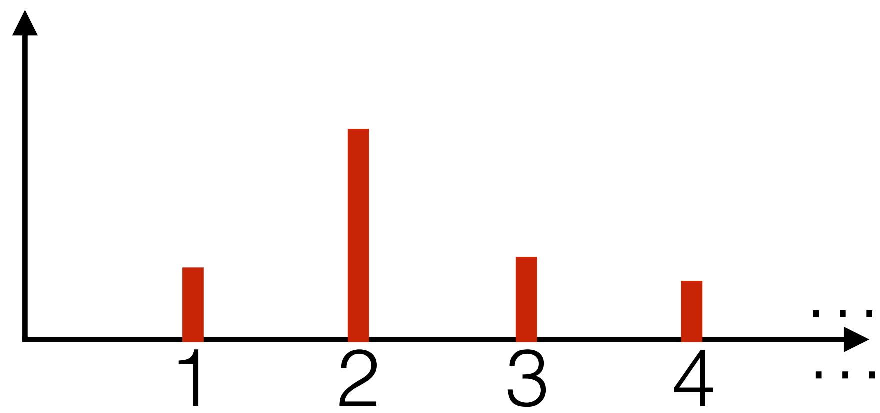

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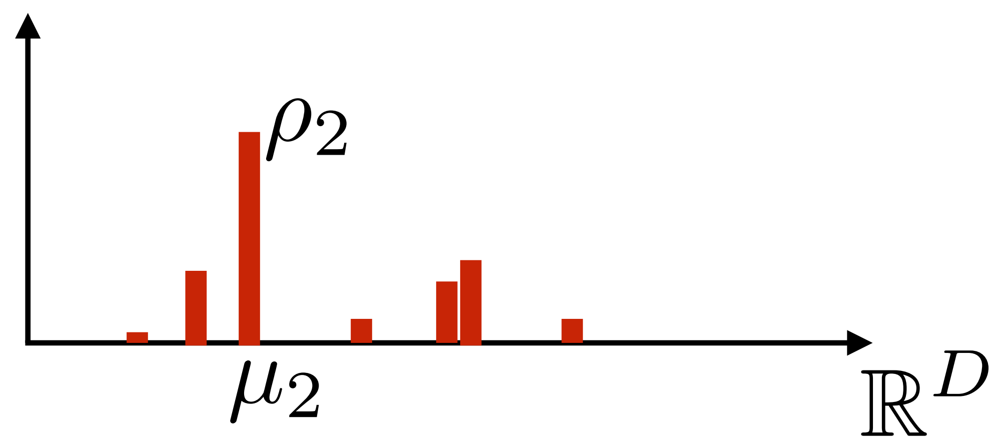

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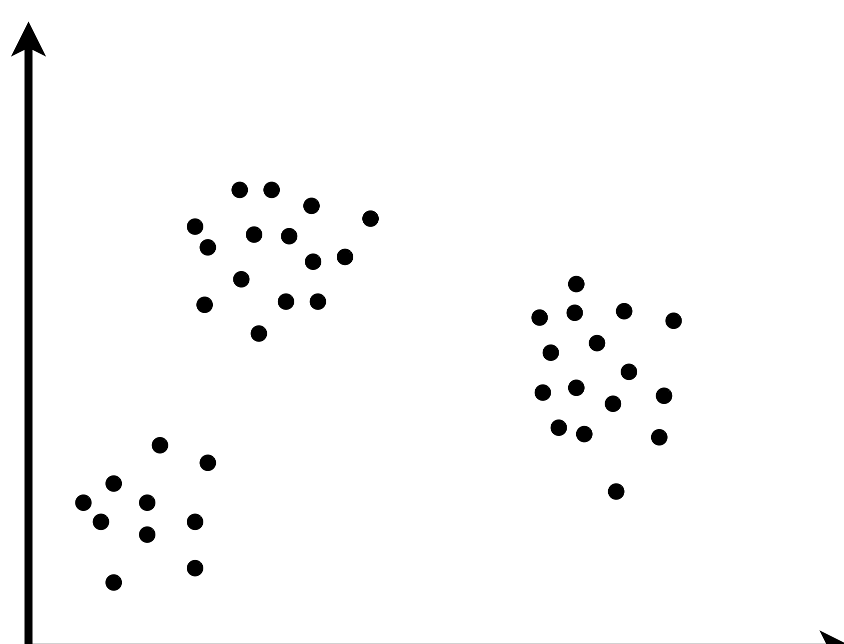

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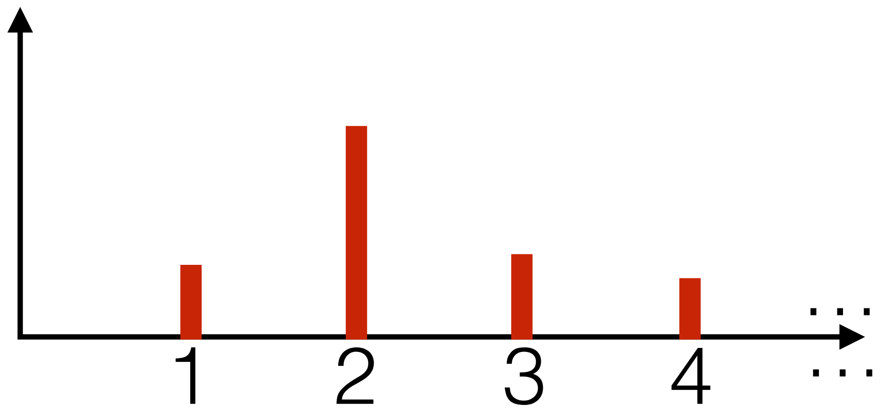
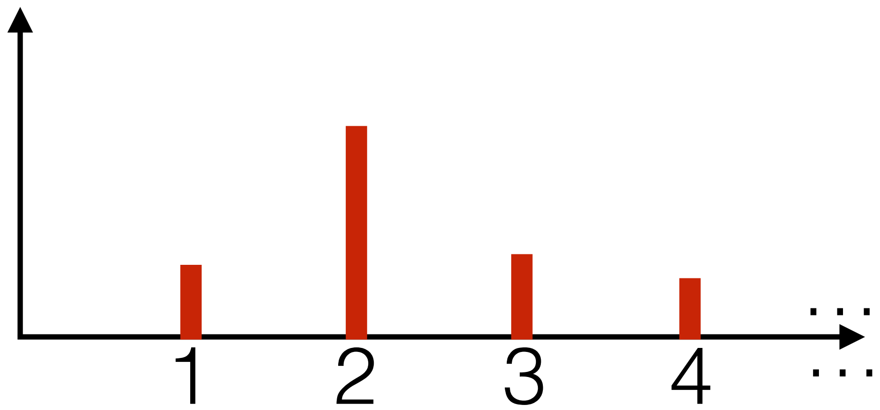
Exercises



Exercises

[slides, code: www.tamarabroderick.com/tutorials.html]



Exercises

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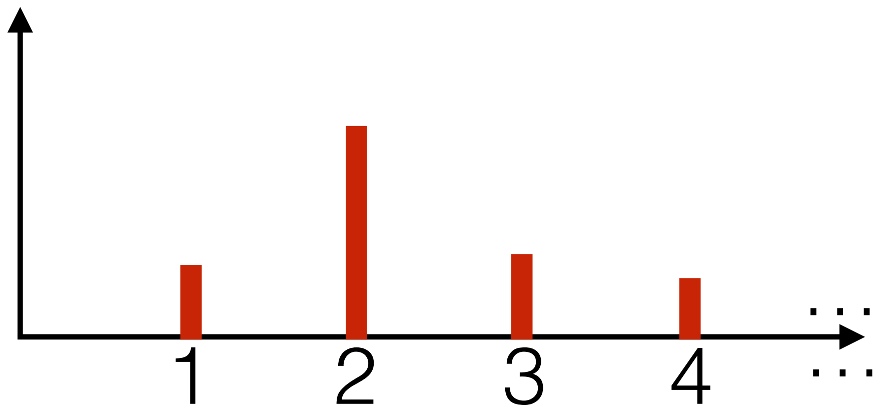
- Prove the beta (Dirichlet) is conjugate to the categorical



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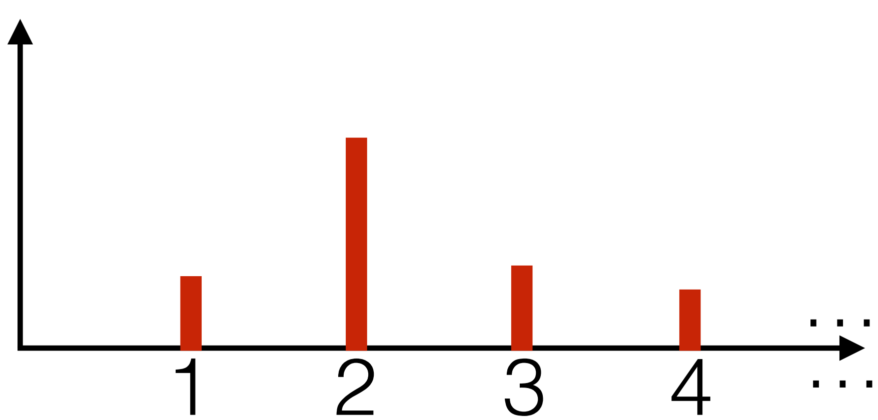
  - What is the posterior after N data points?



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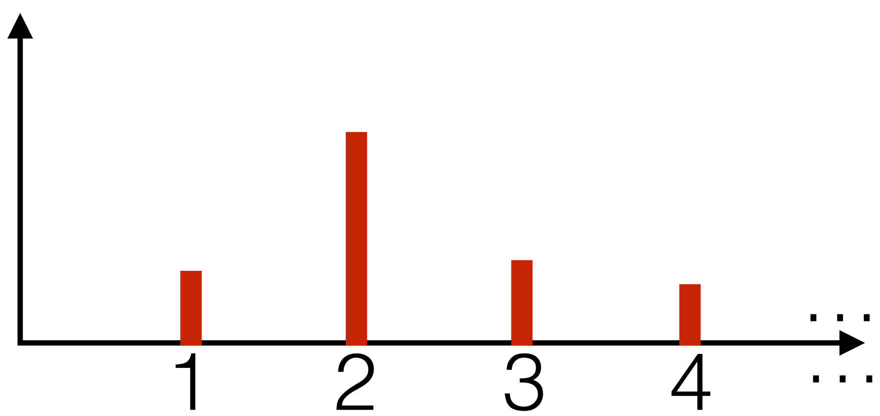

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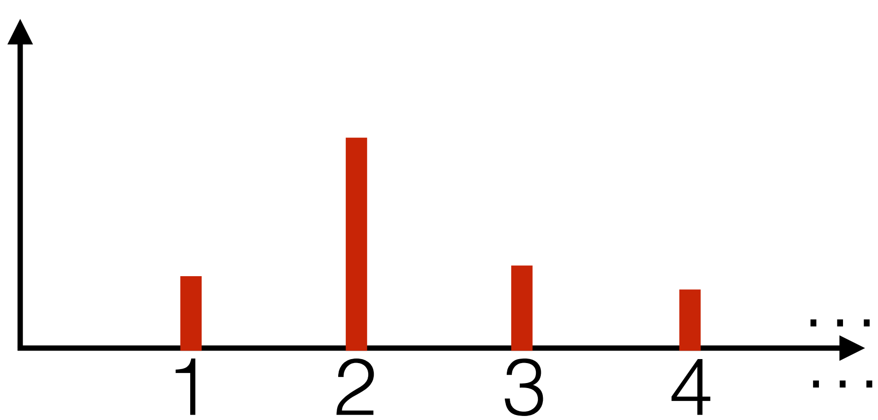

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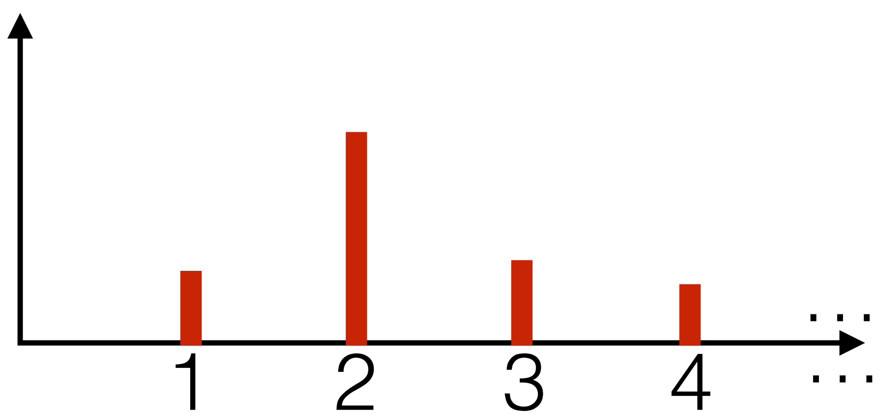

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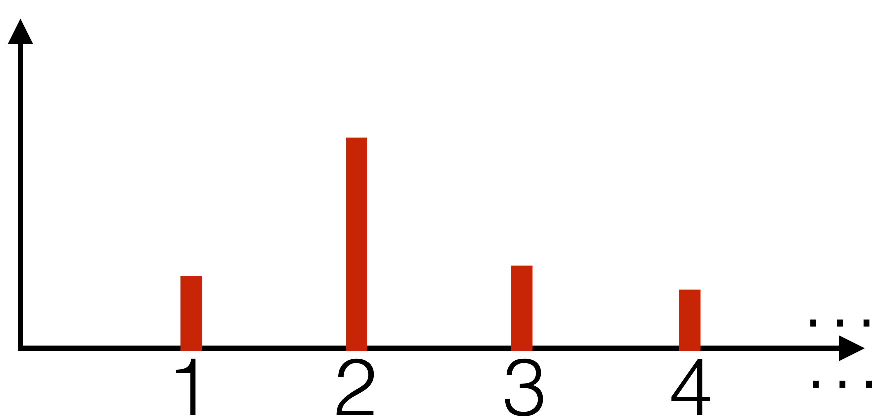

- Compare the number of clusters as *N* changes in the GEM case with the growth in the *K*=1000 case

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[slides, code: www.tamarabroderick.com/tutorials.html]

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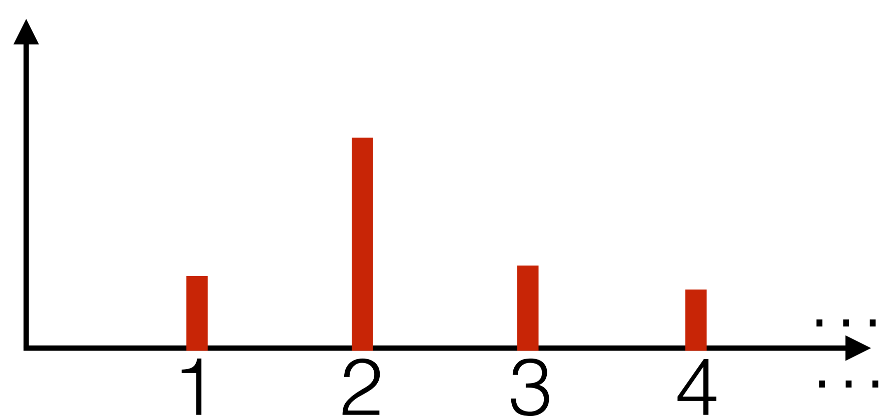

- Compare the number of clusters as *N* changes in the GEM case with the growth in the *K*=1000 case

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Exercises

[slides, code: www.tamarabroderick.com/tutorials.html]

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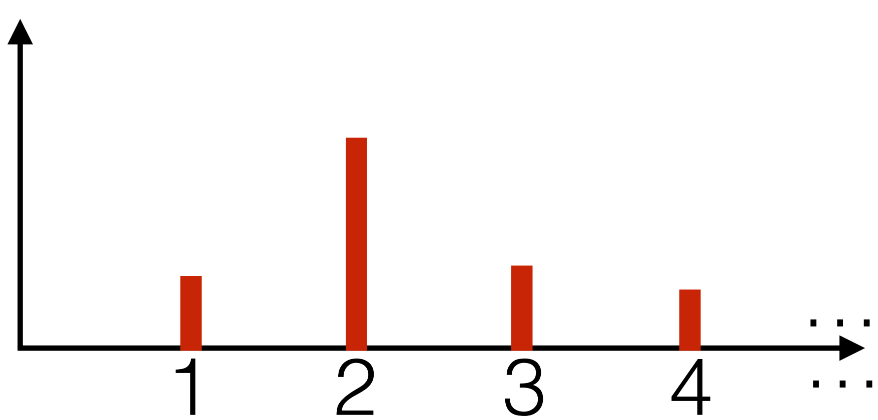

- Compare the number of clusters as *N* changes in the GEM case with the growth in the *K*=1000 case
- How does the growth in *N* change when you change α?
- How does the distribution of # clusters at *N* change with α?

References

A full reference list is provided at the end of the “Part III” slides.