

# 6.036/6.862: Introduction to Machine Learning

**Lecture:** starts Tuesdays 9:35am (Boston time zone)

**Course website:** [introml.odl.mit.edu](http://introml.odl.mit.edu)

**Who's talking?** Prof. Tamara Broderick

**Questions?** [discourse.odl.mit.edu](https://discourse.odl.mit.edu) (“Lecture 6” category)

**Materials:** Will all be available at course website

## Last Time(s)

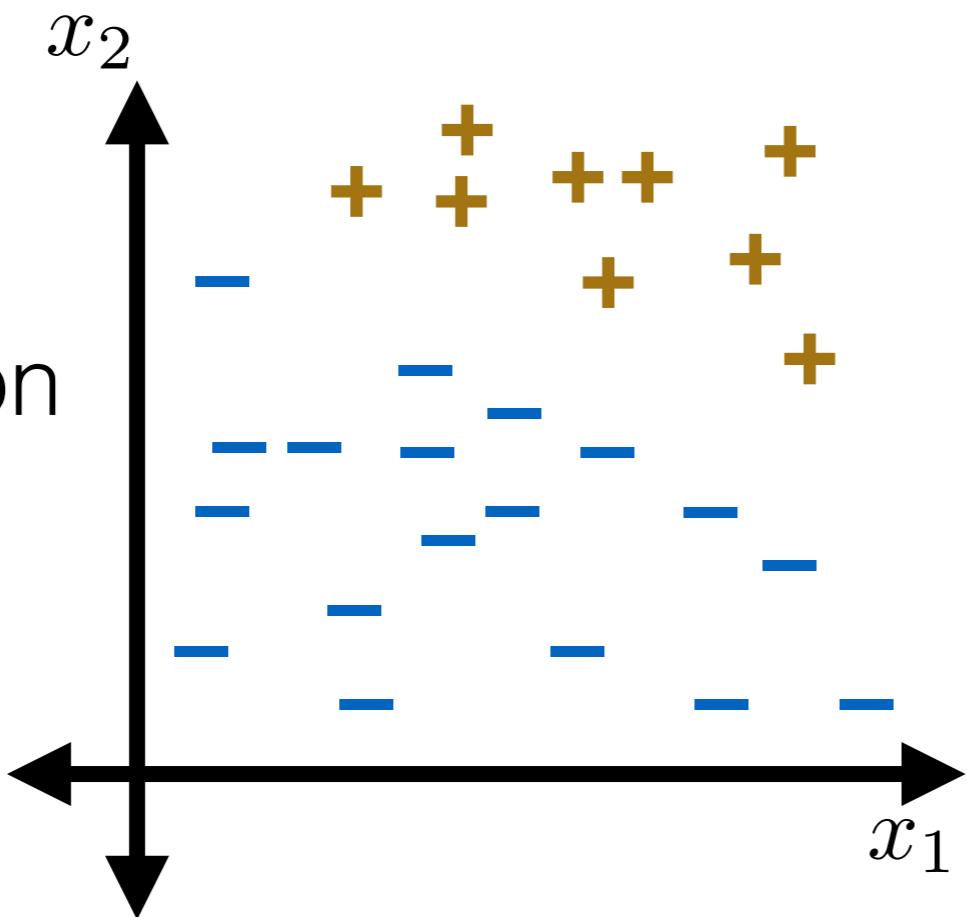
- I. Linear classification
- II. Linear regression
- III. Choosing features

## Today's Plan

- I. Step-function features
- II. Neural nets: hypothesis class
- III. Neural nets: learning

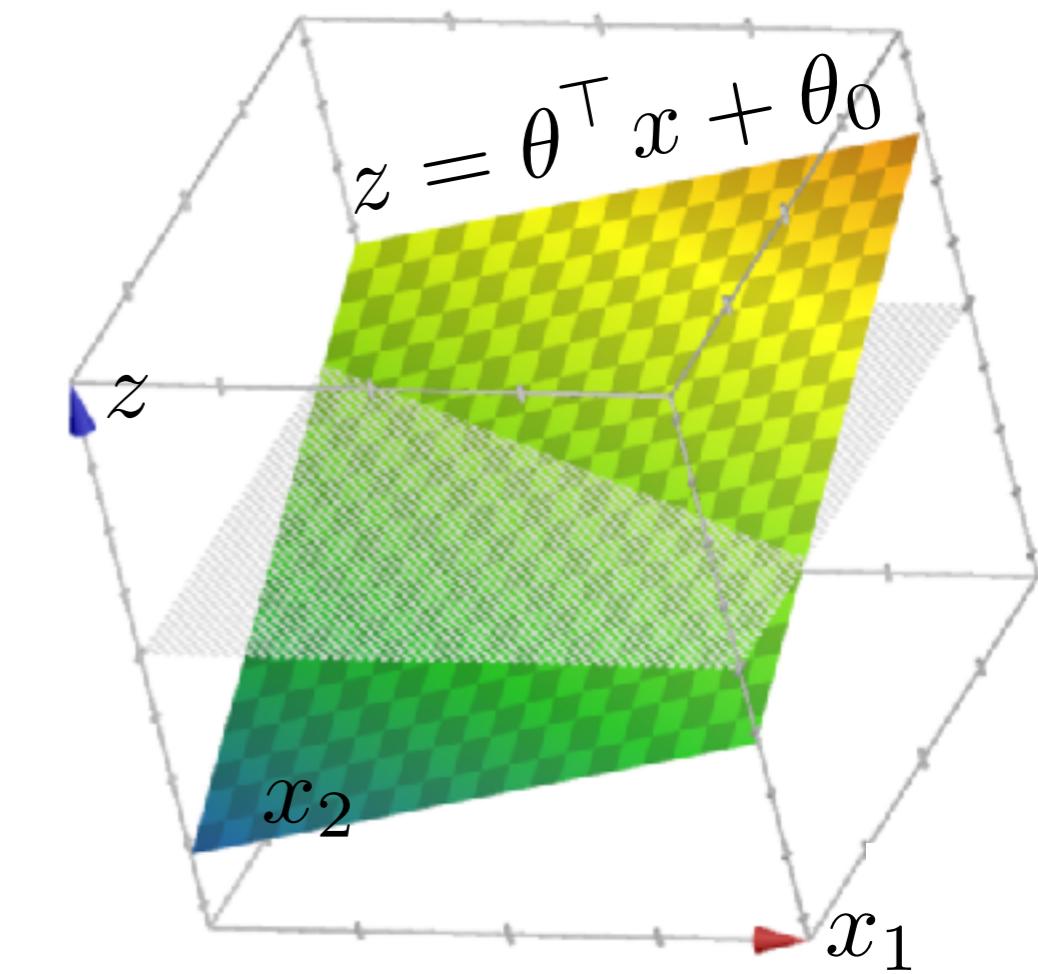
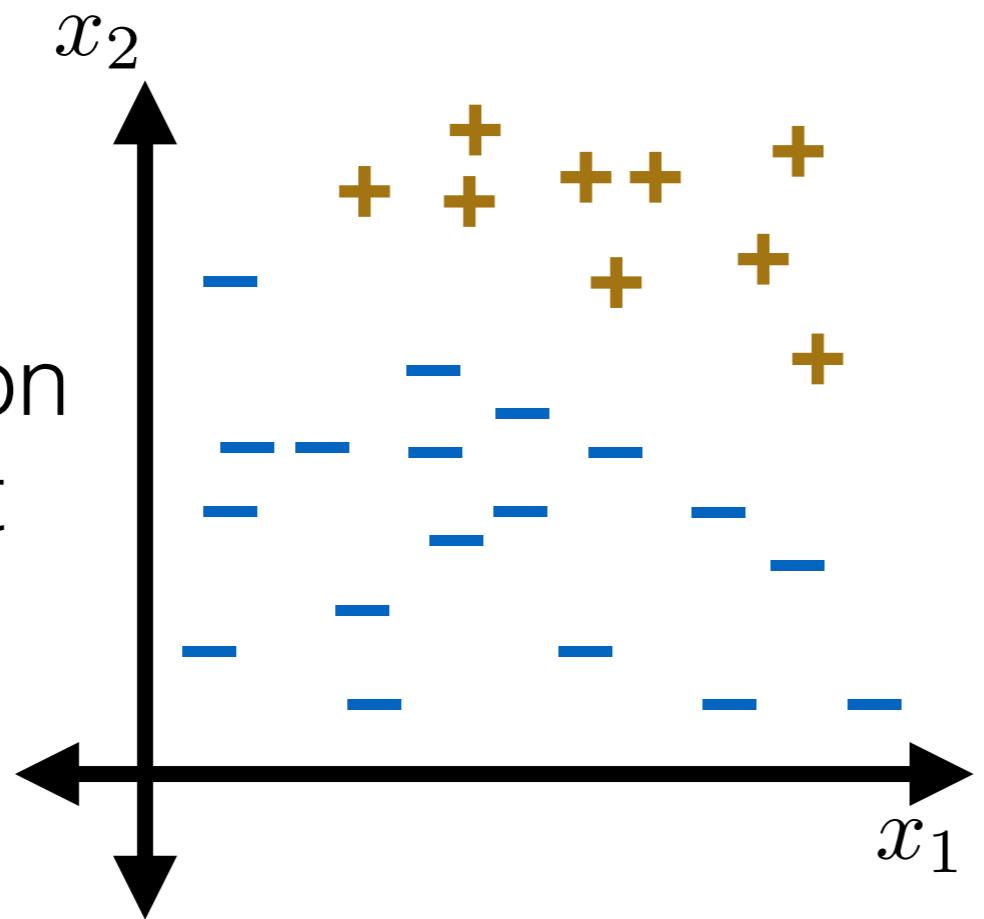
# Recall

- Linear classification with default features:



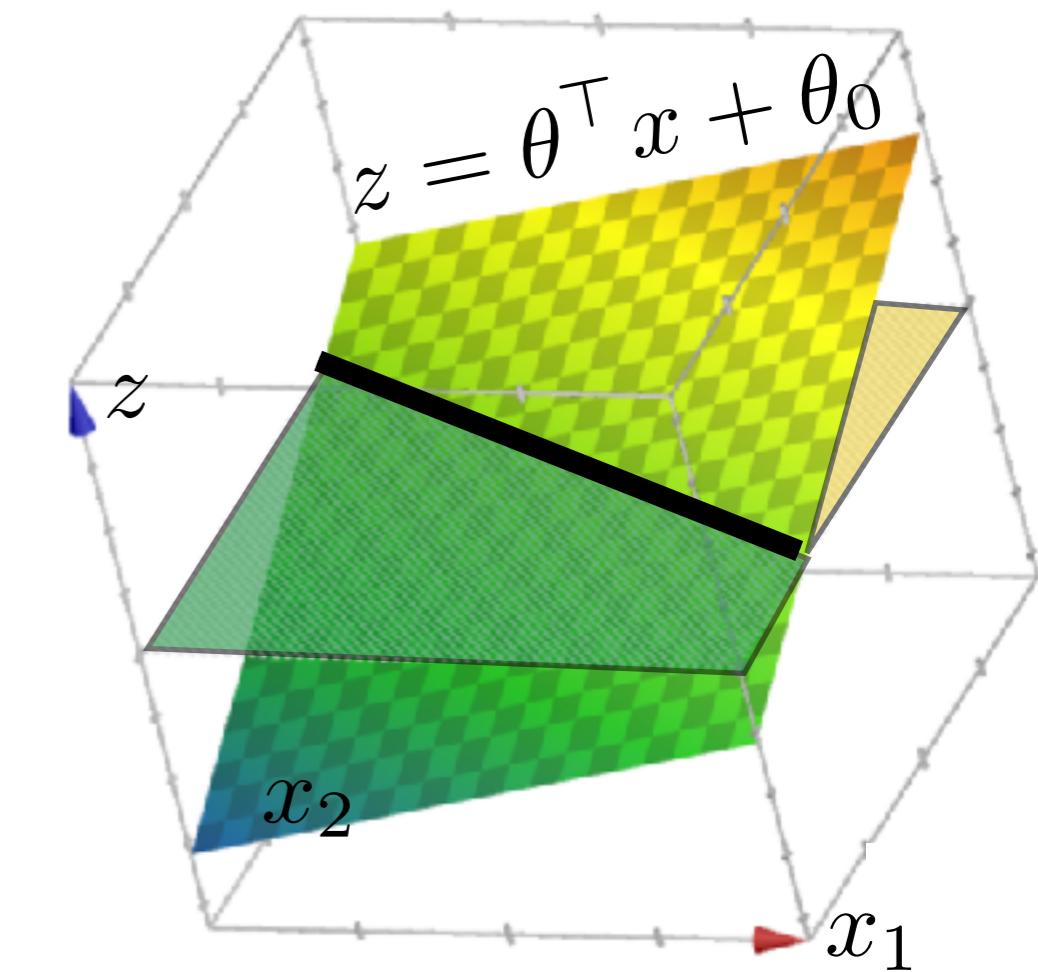
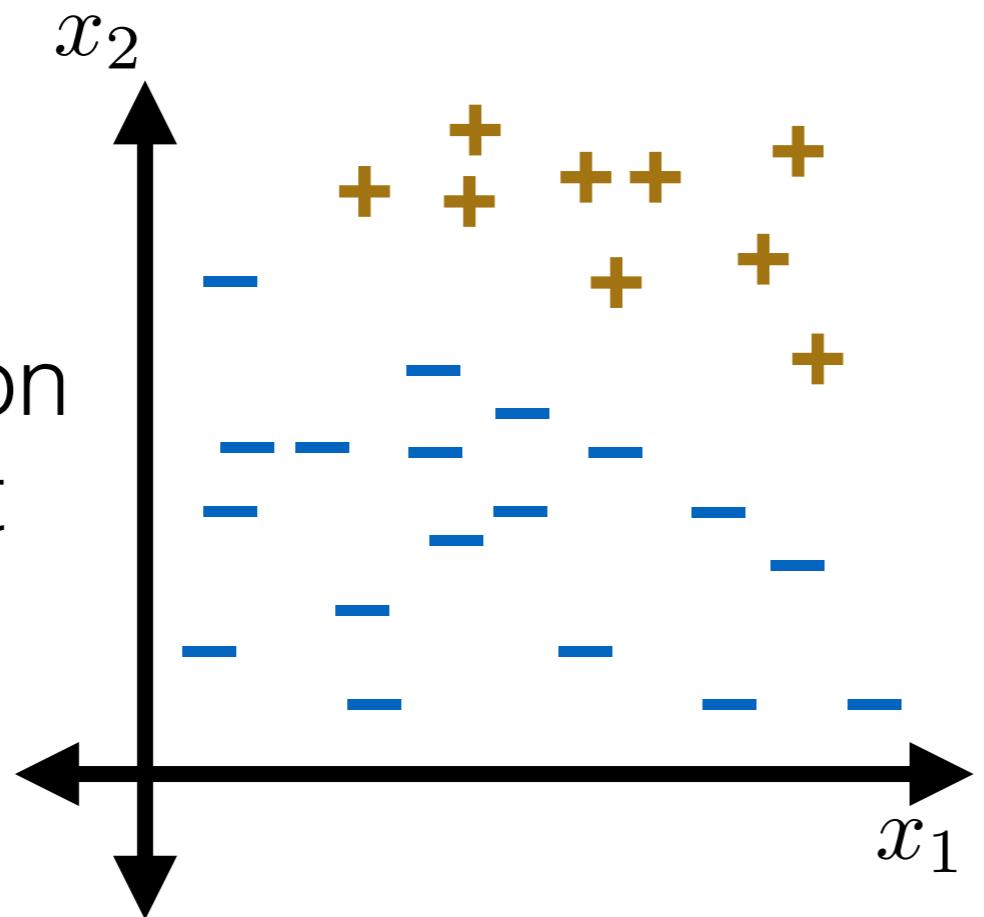
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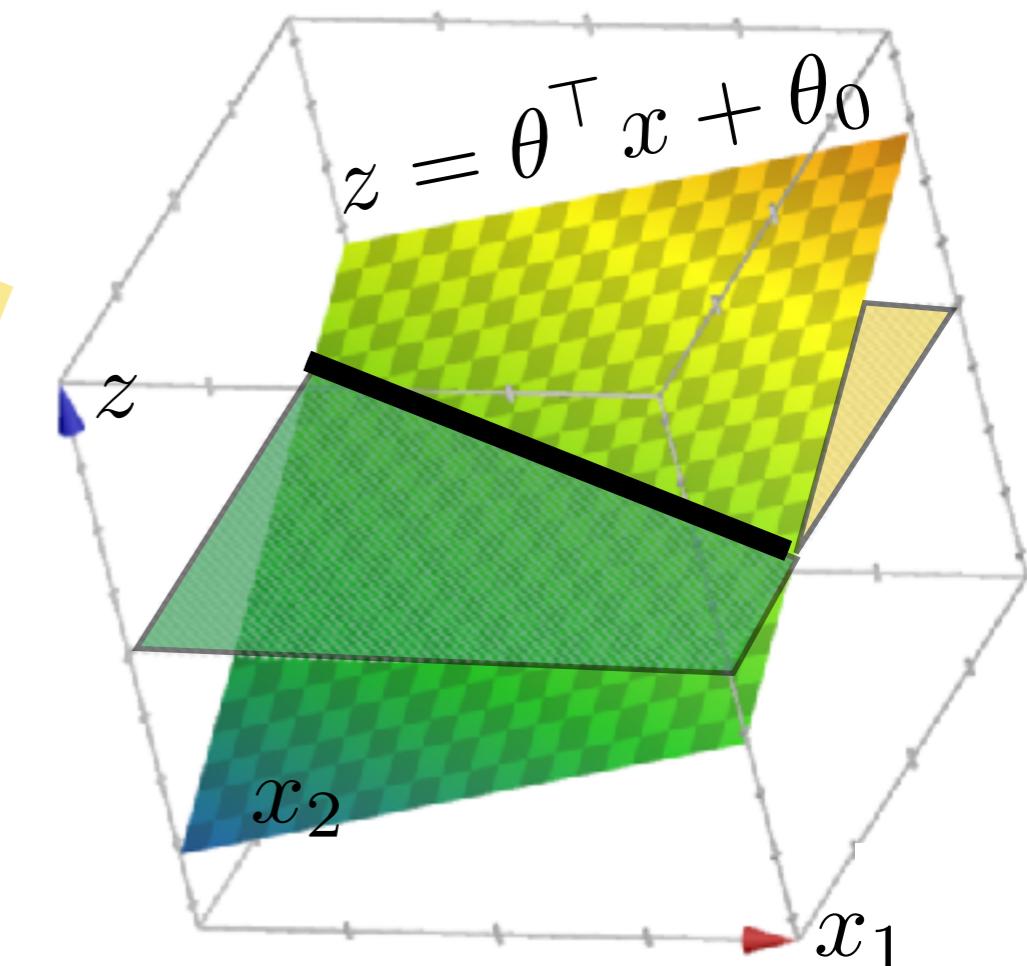
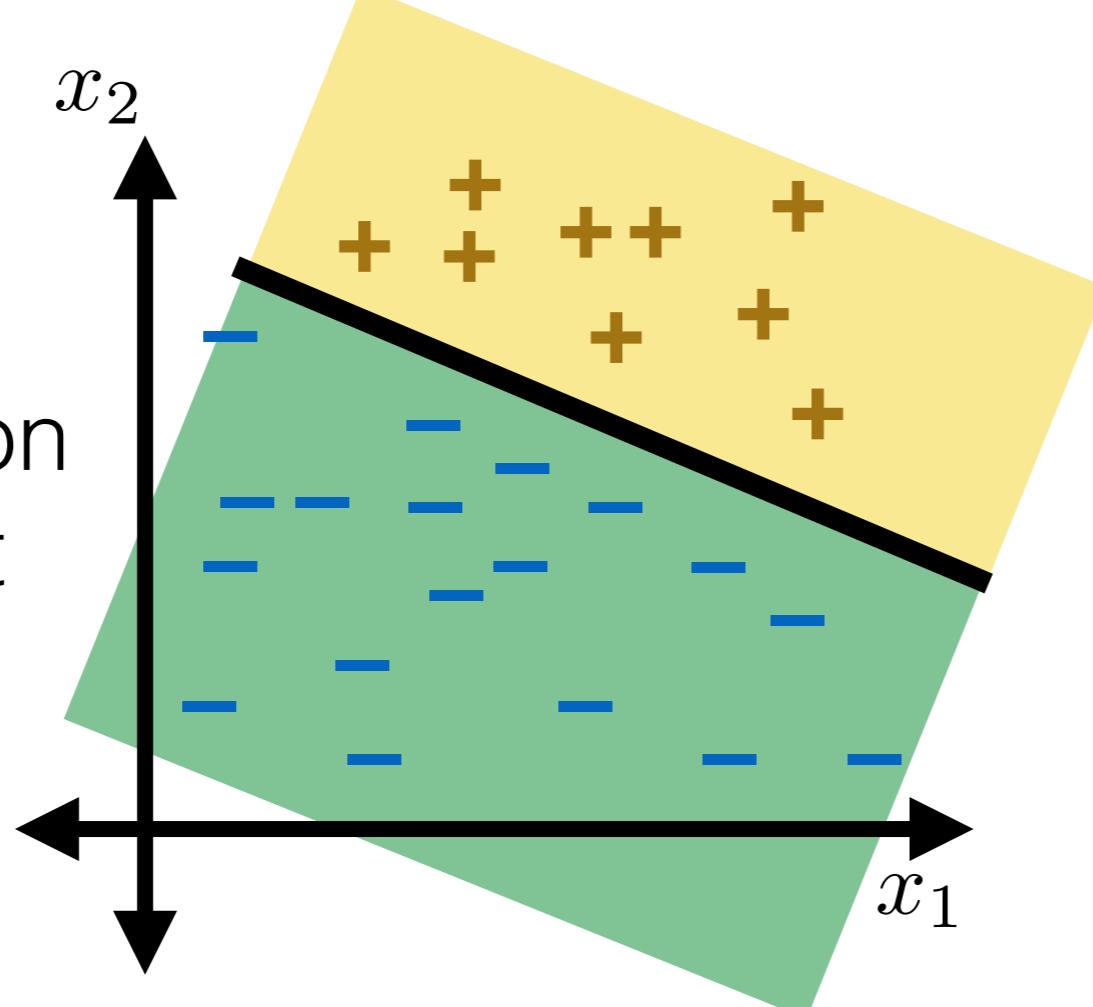
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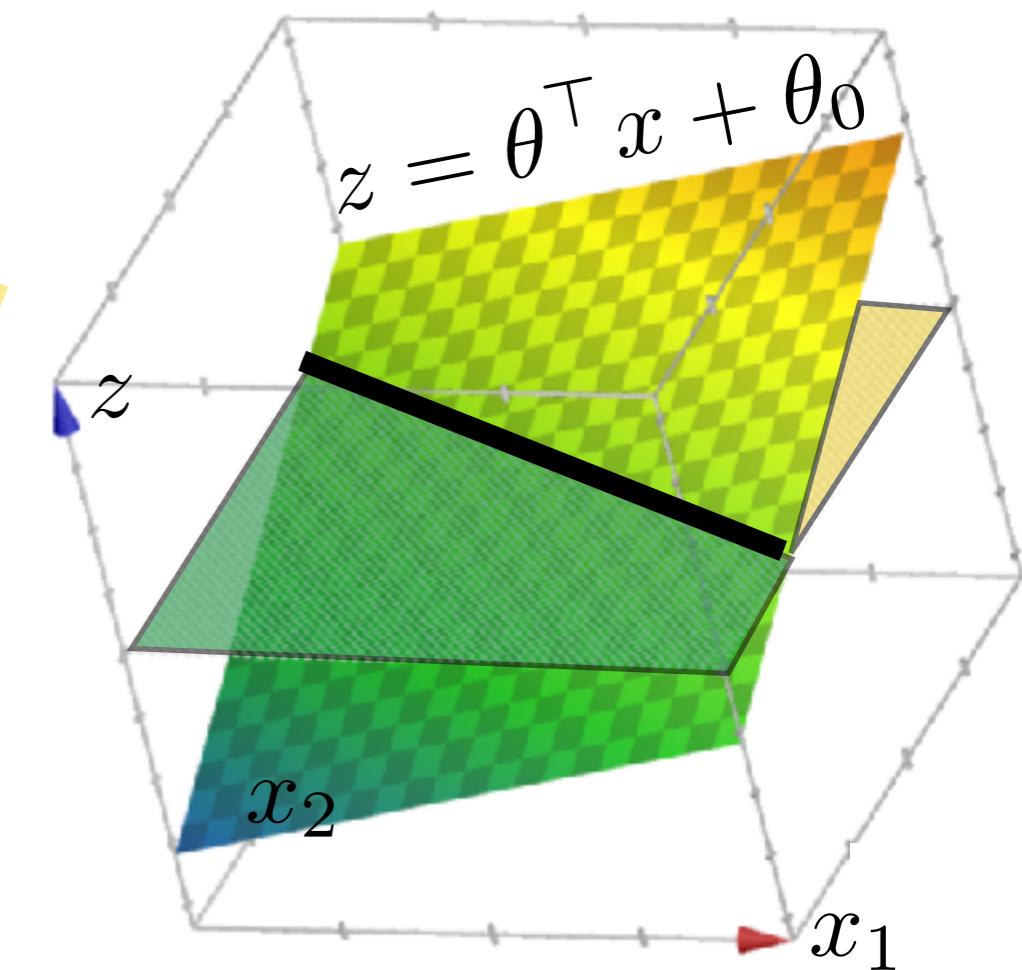
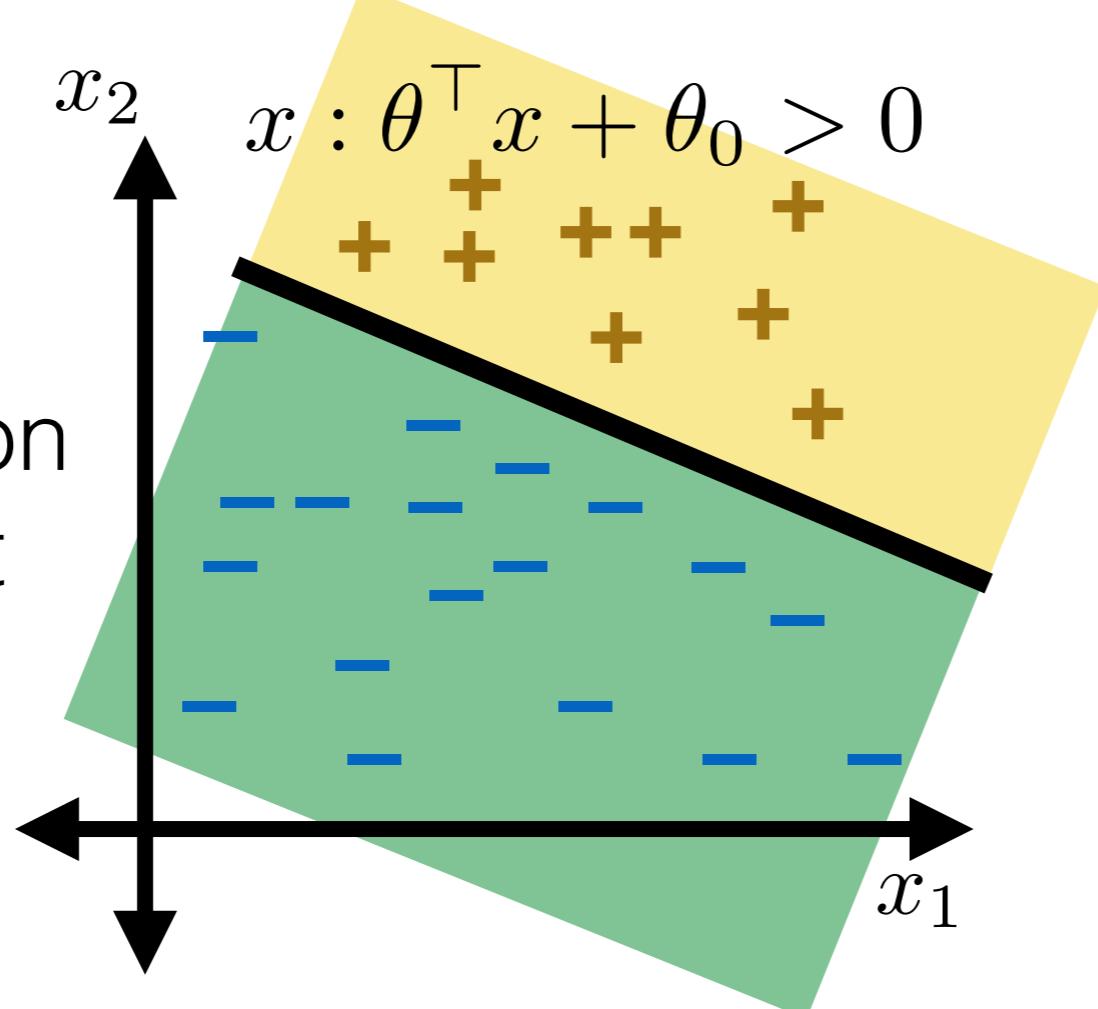
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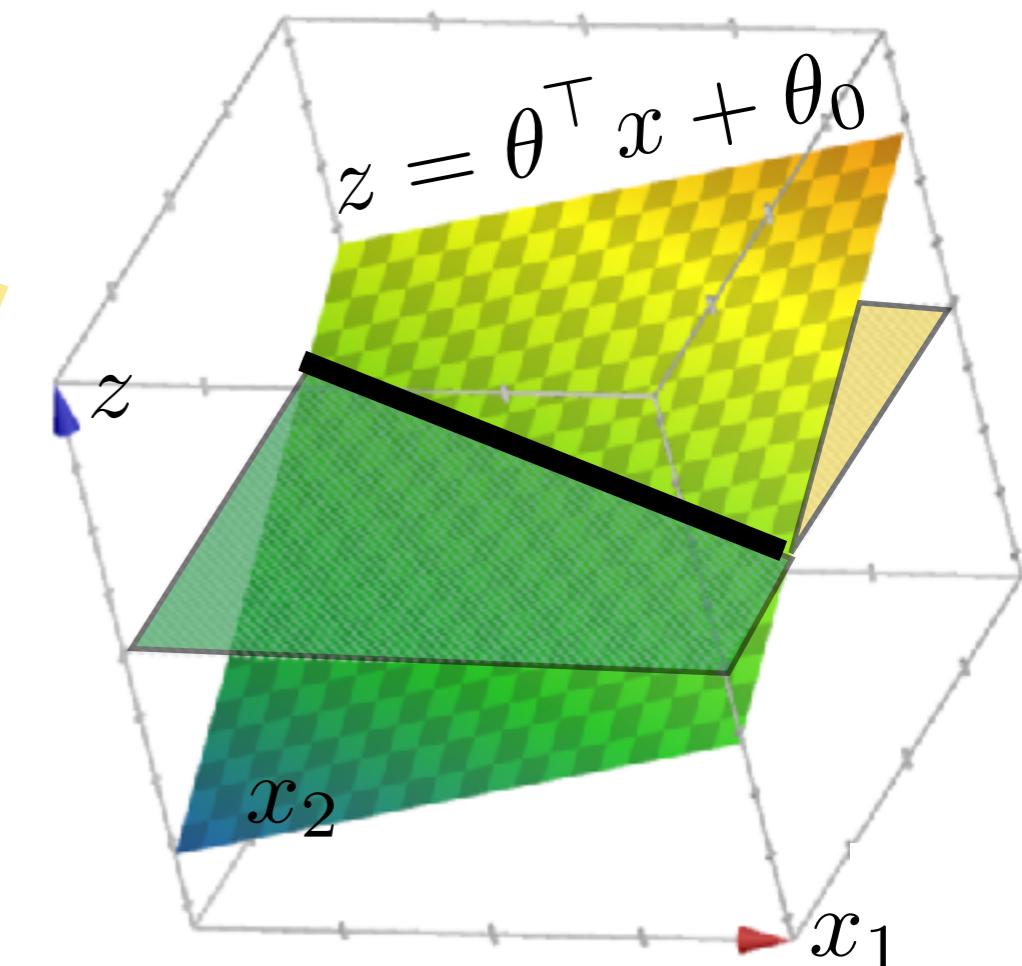
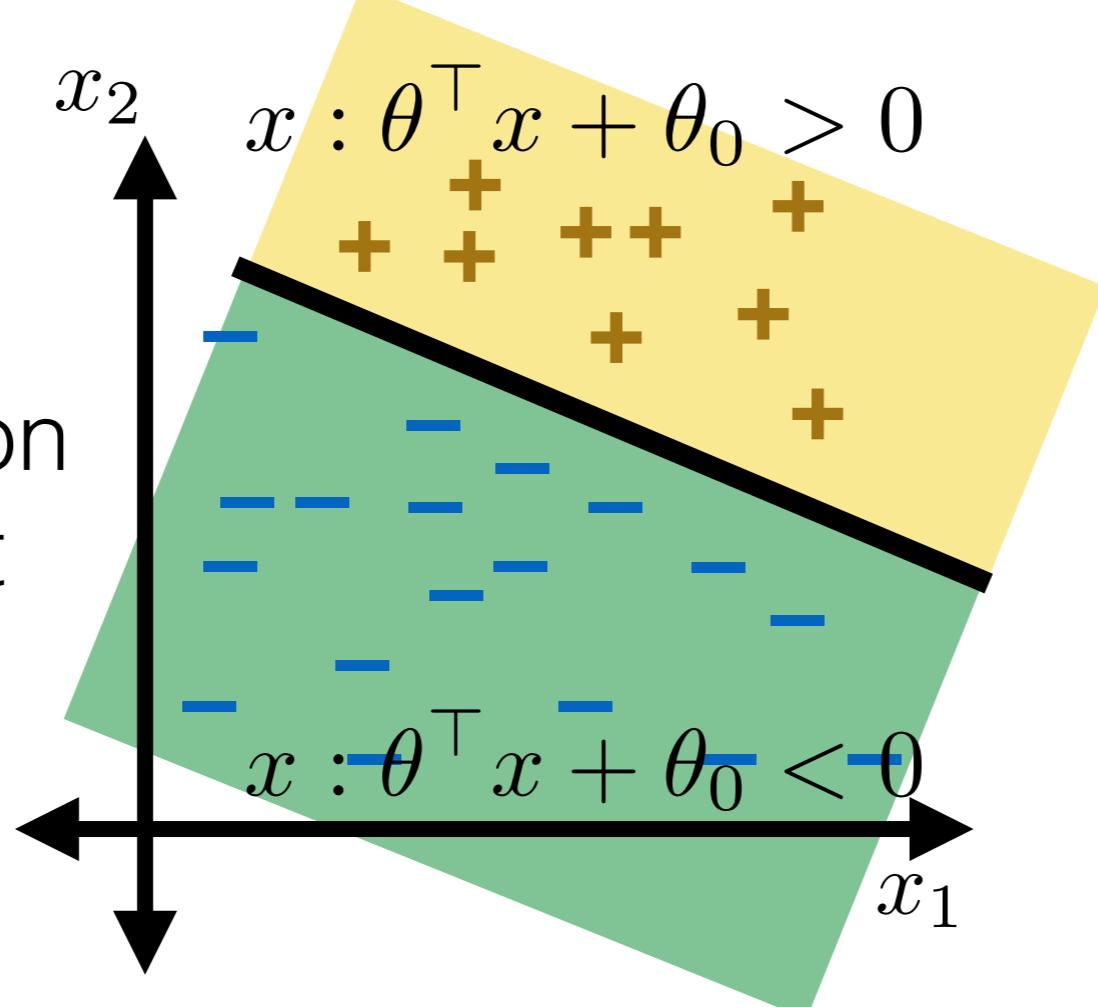
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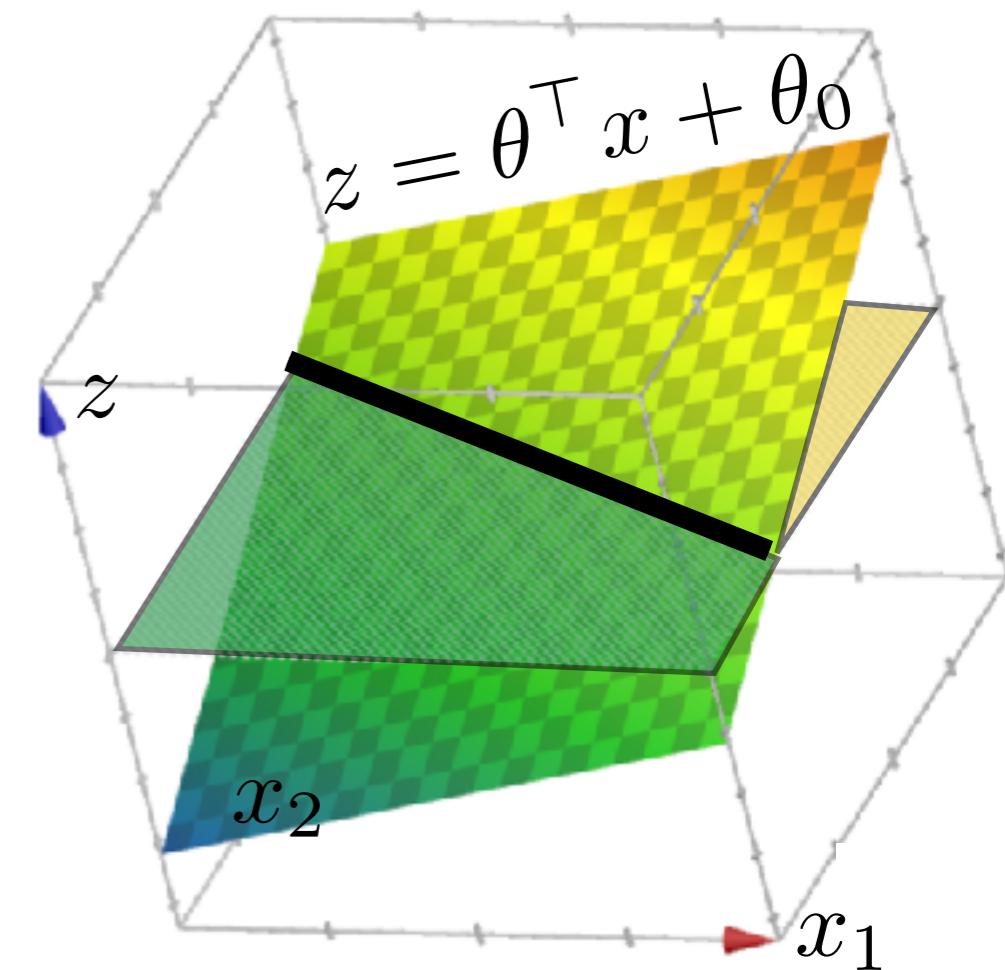
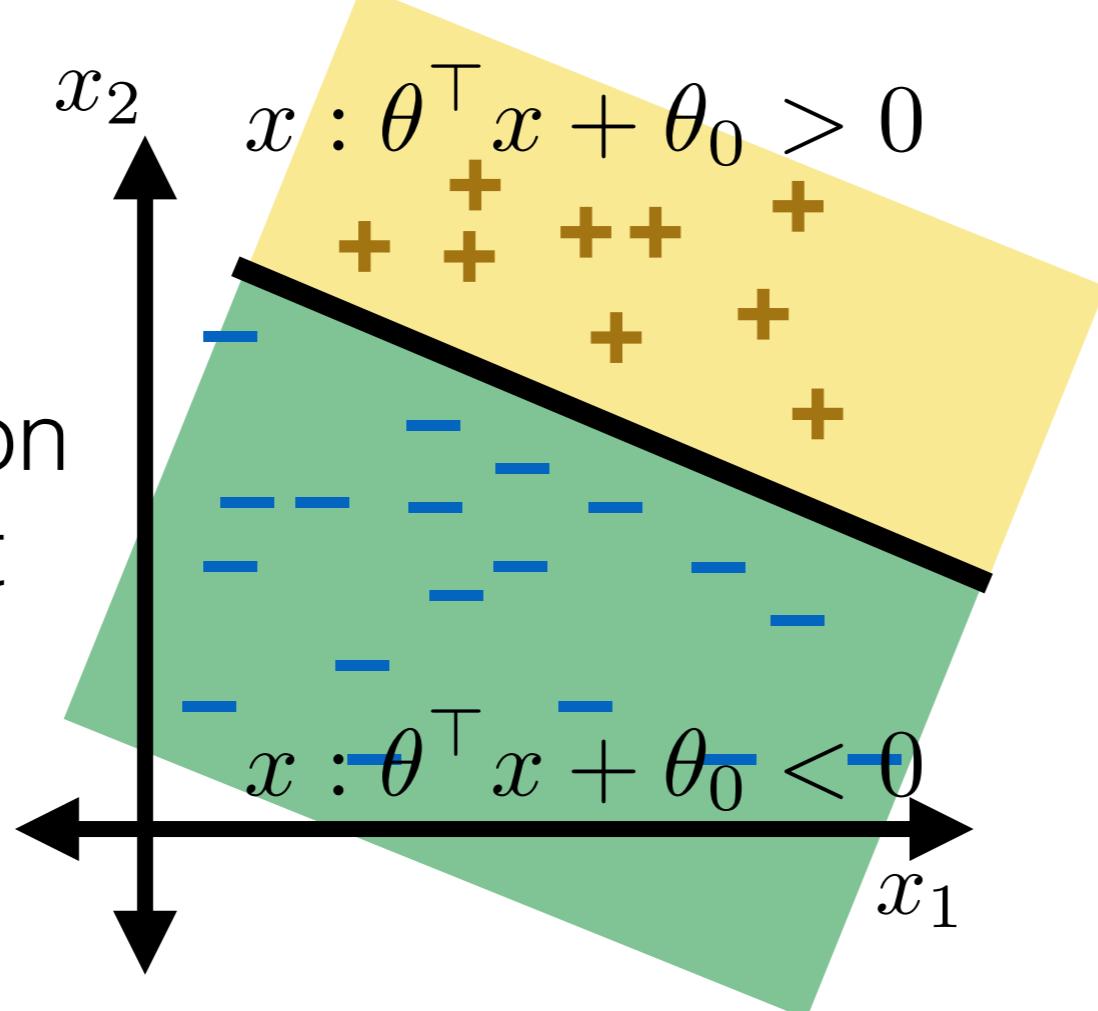
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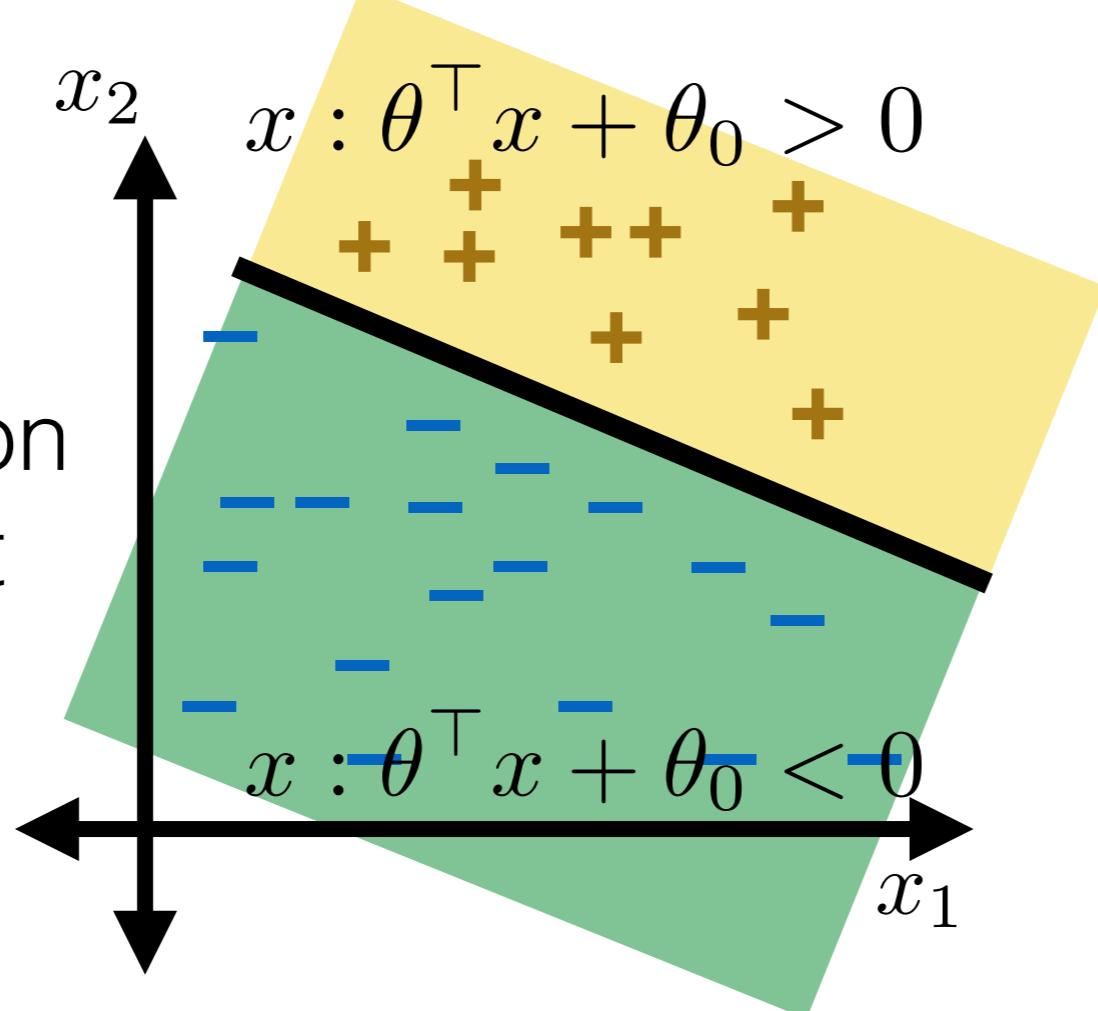
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- Linear classification with default features:
- Linear classification with polynomial features:



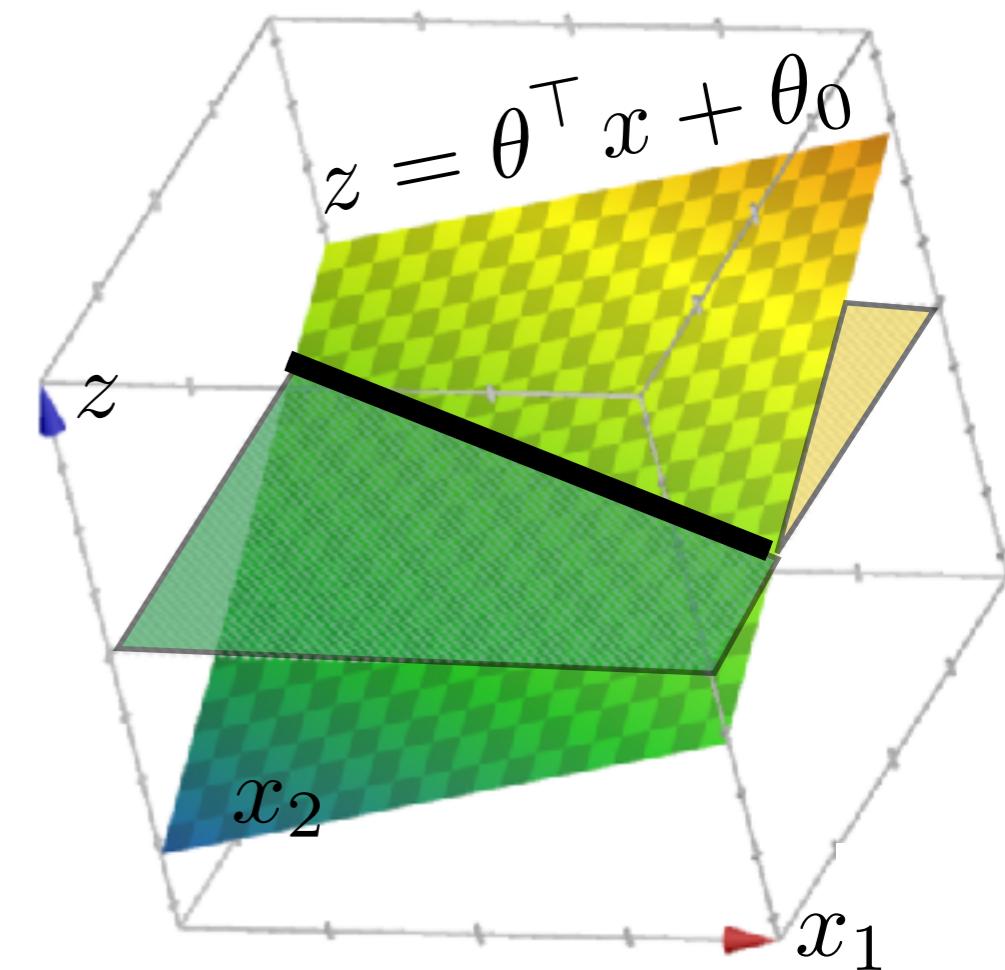
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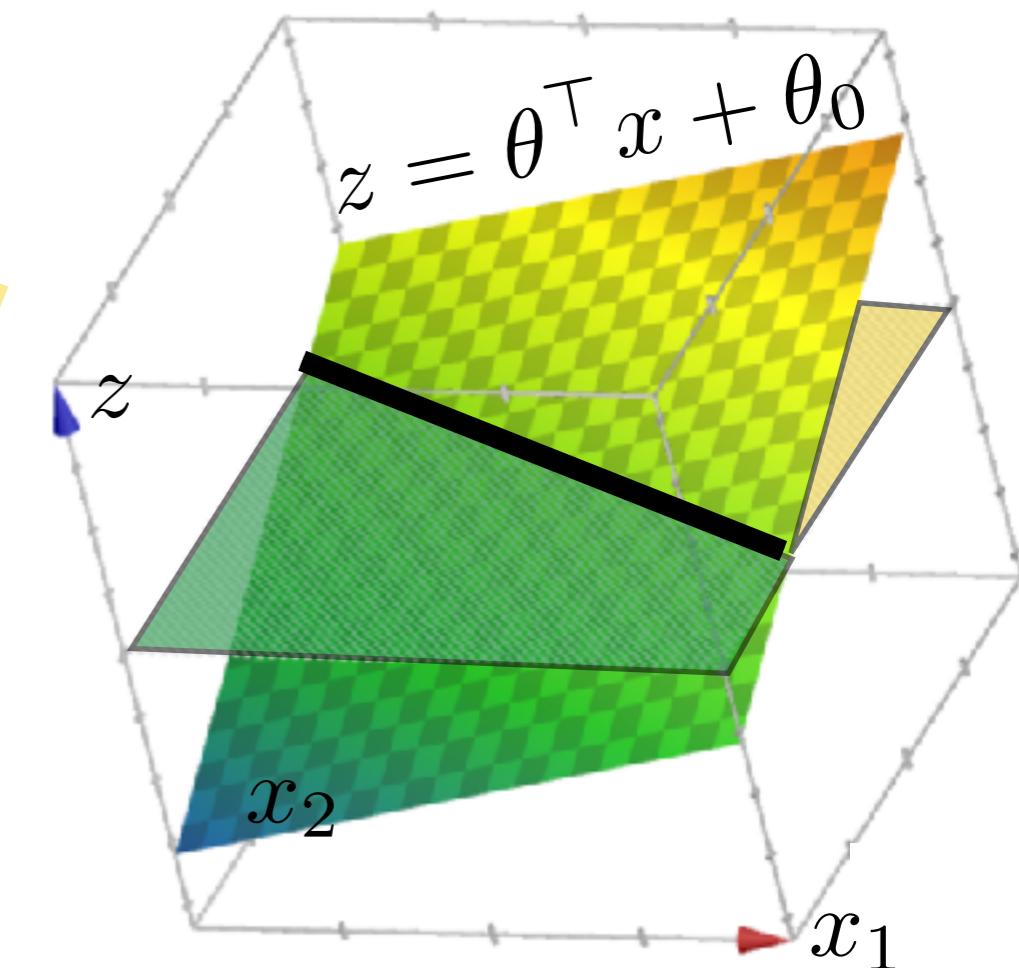
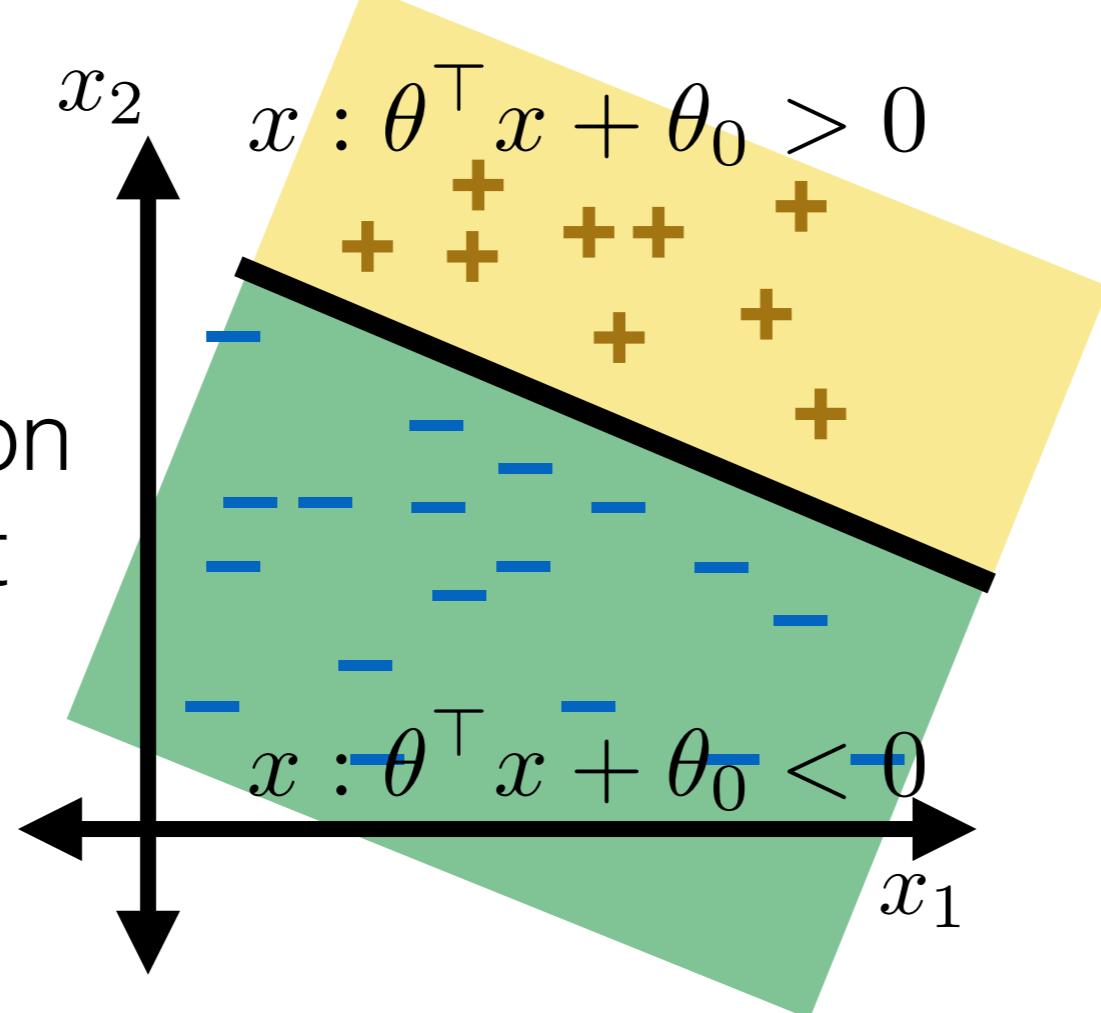
- Linear classification with polynomial features:

$$\phi(x) = [x_1, x_2, x_1^2, x_1 x_2, x_2^2]^\top$$



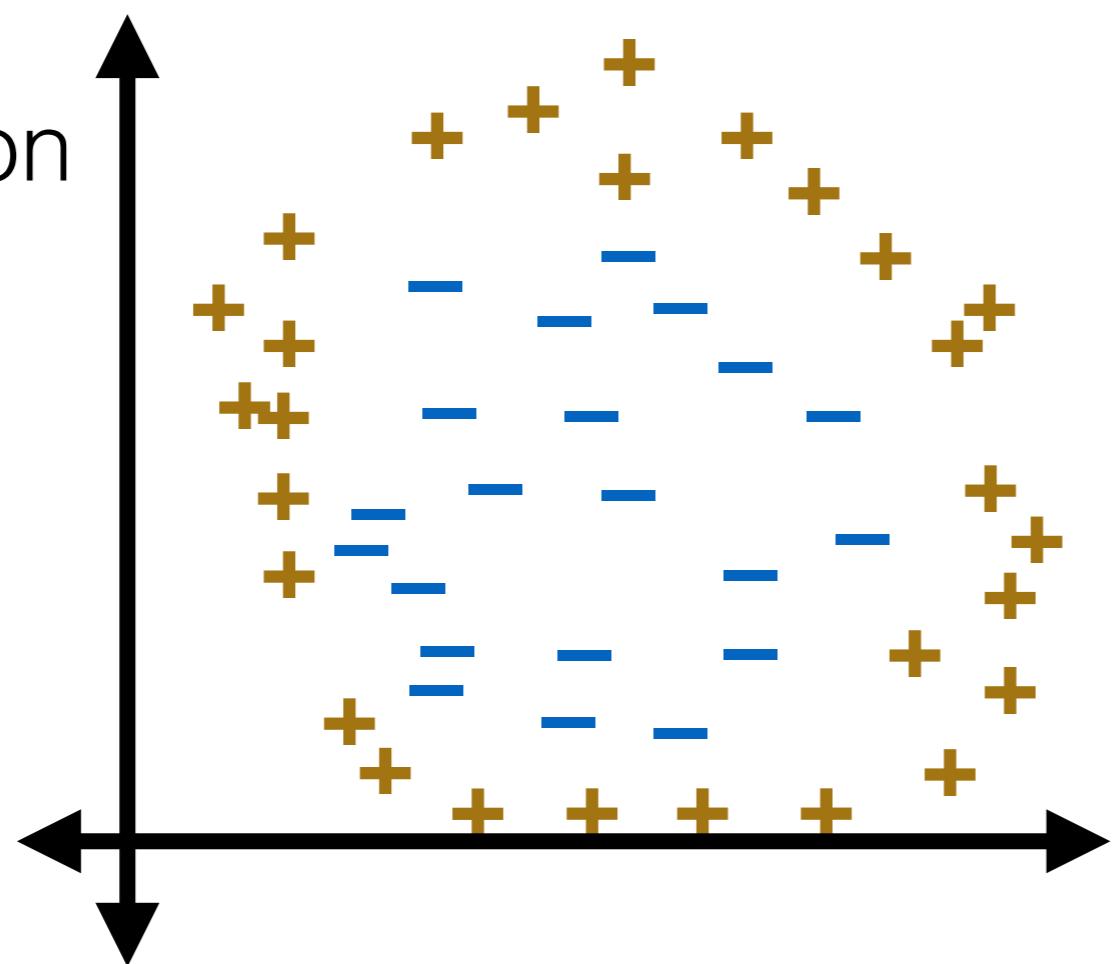
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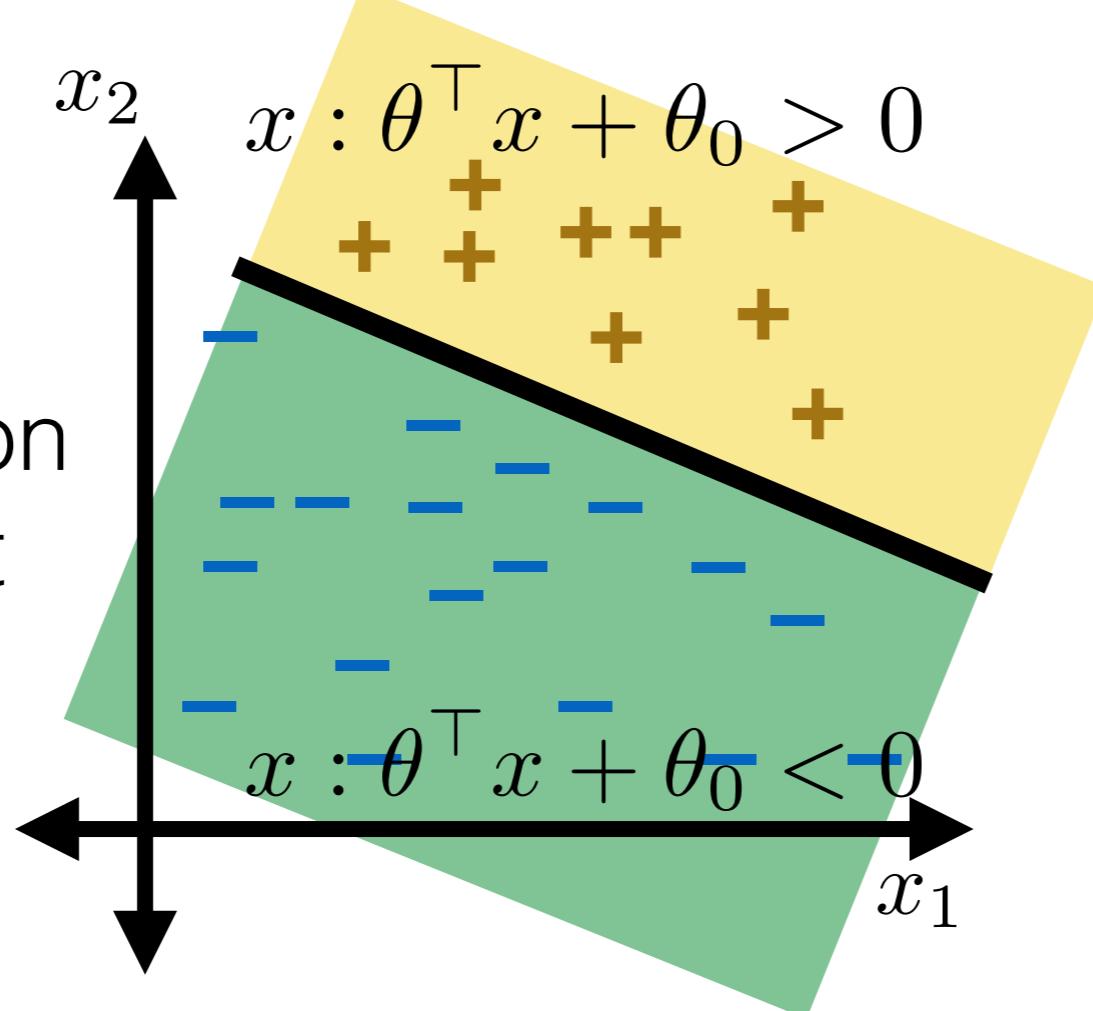
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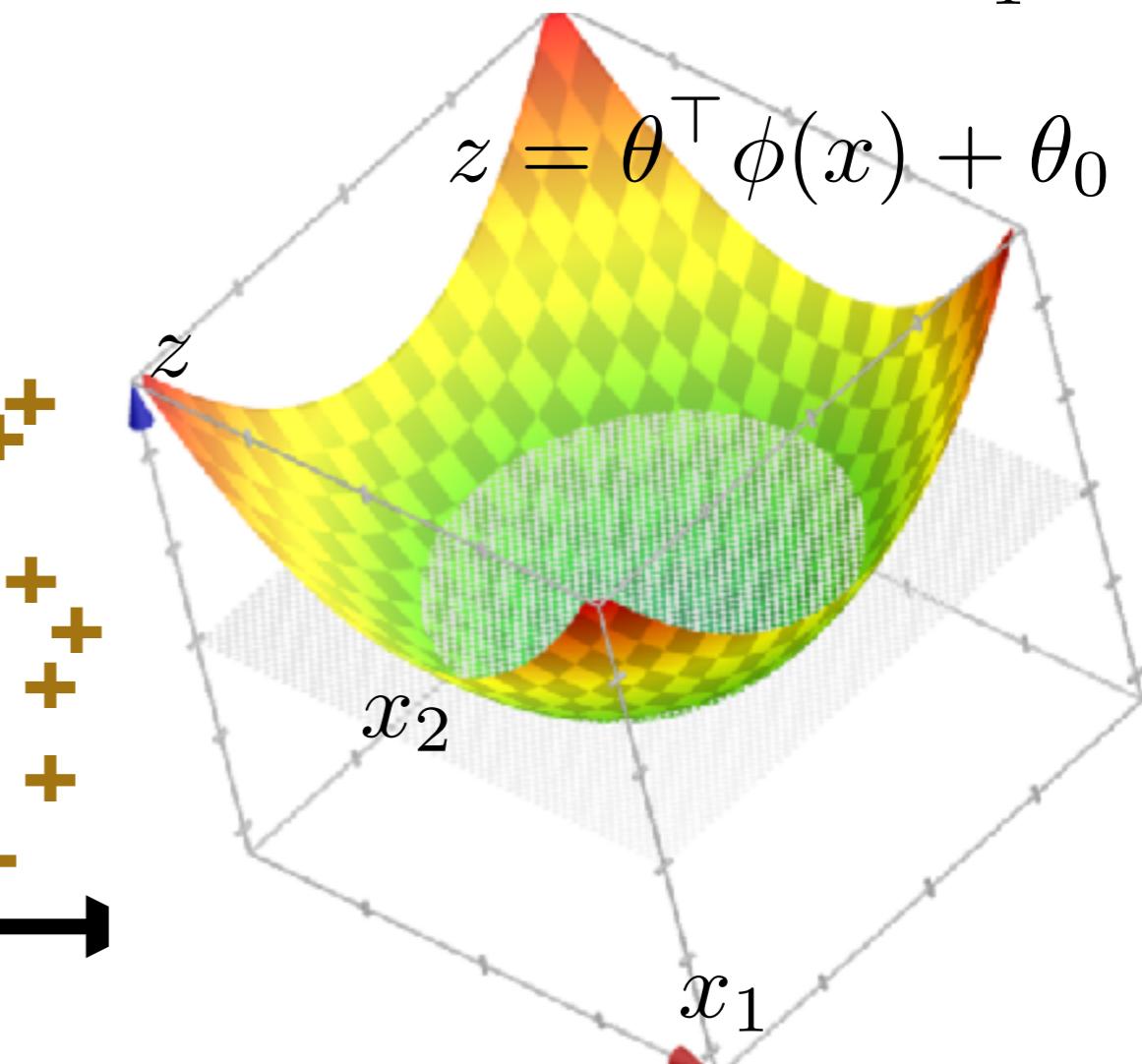
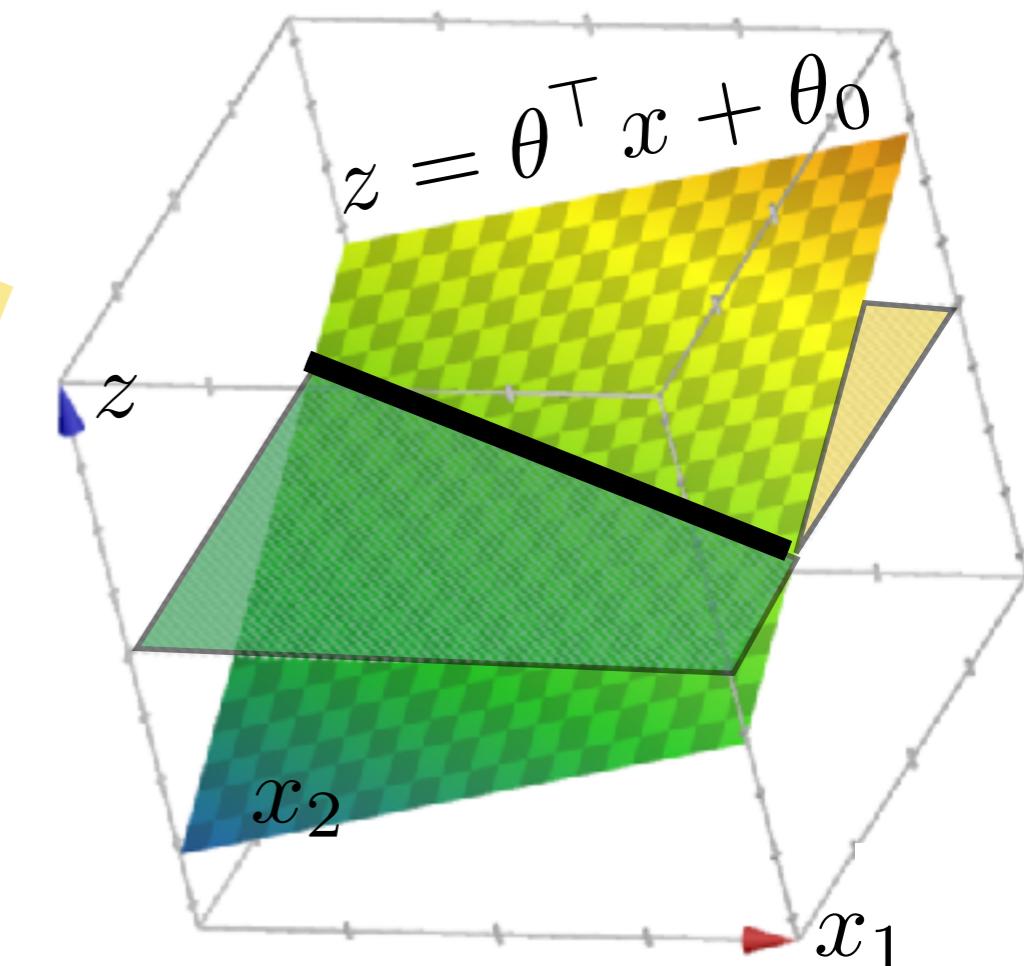
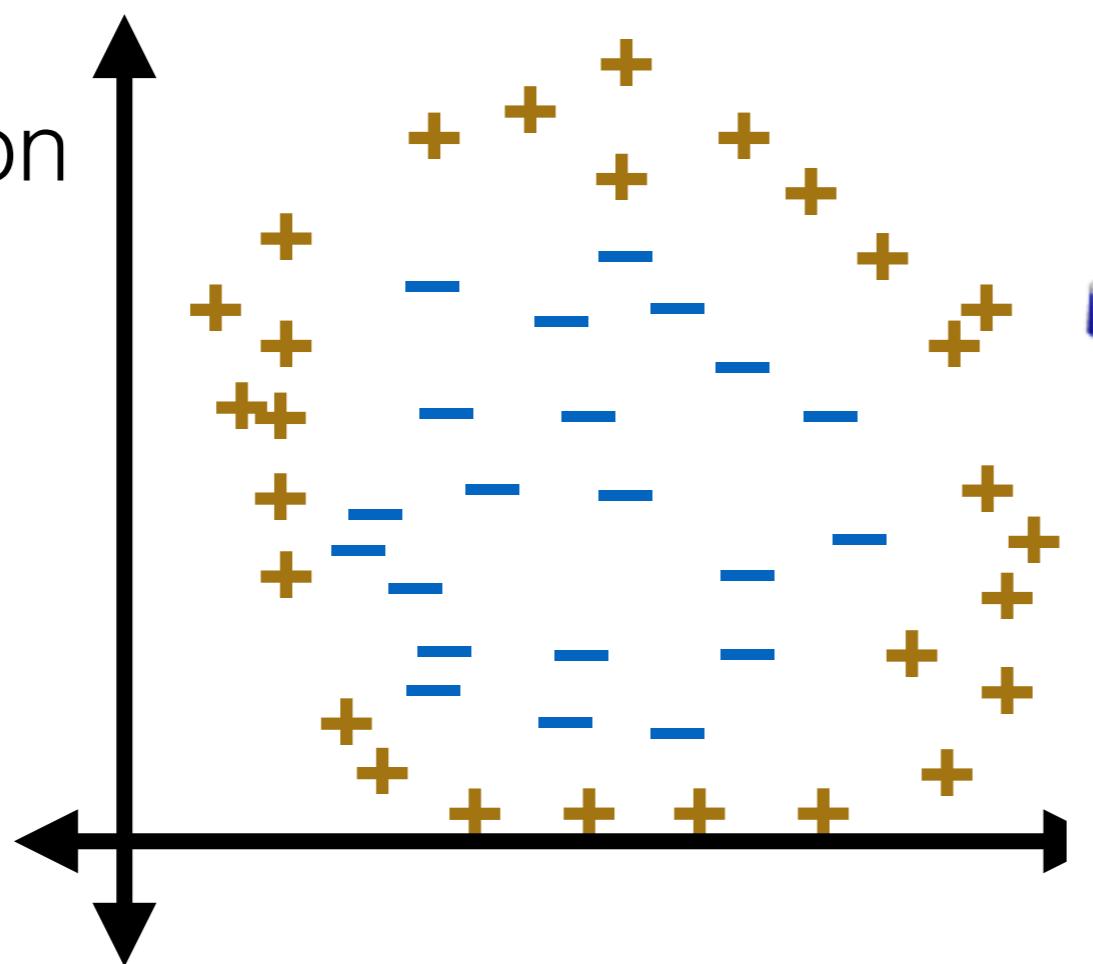
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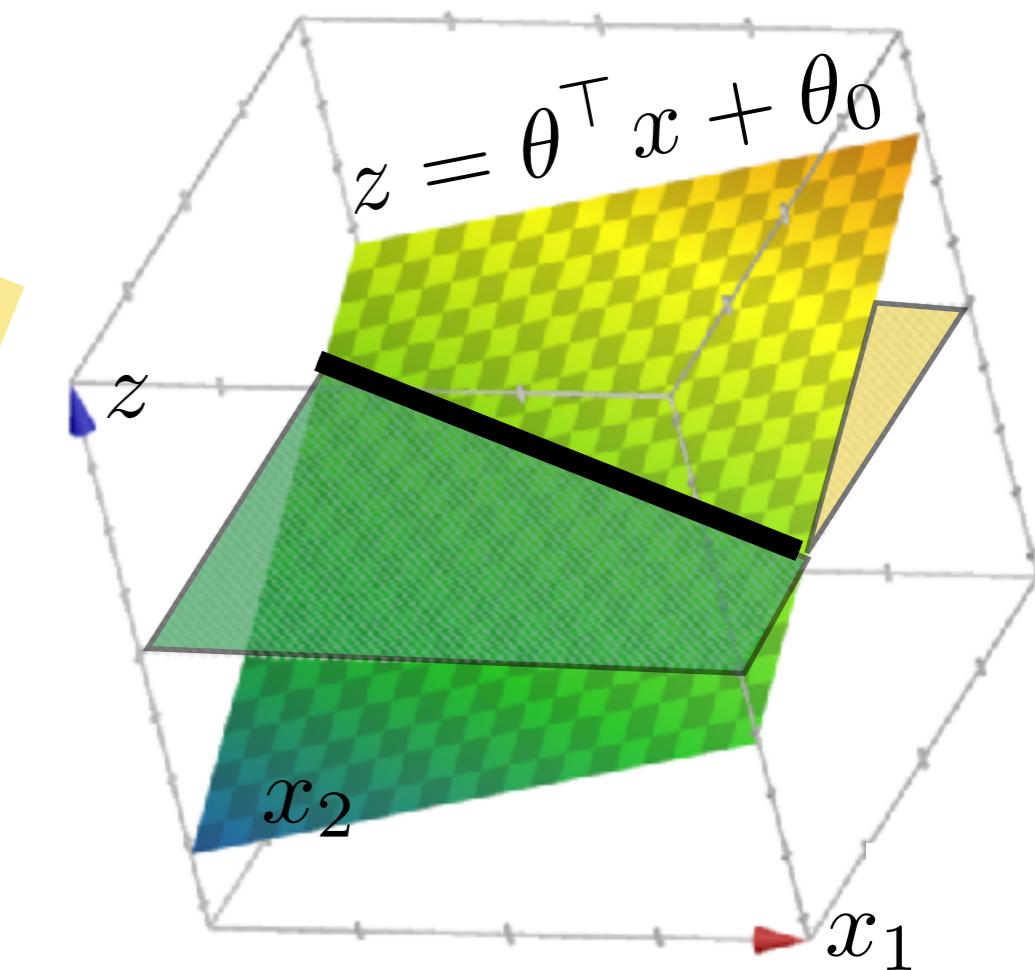
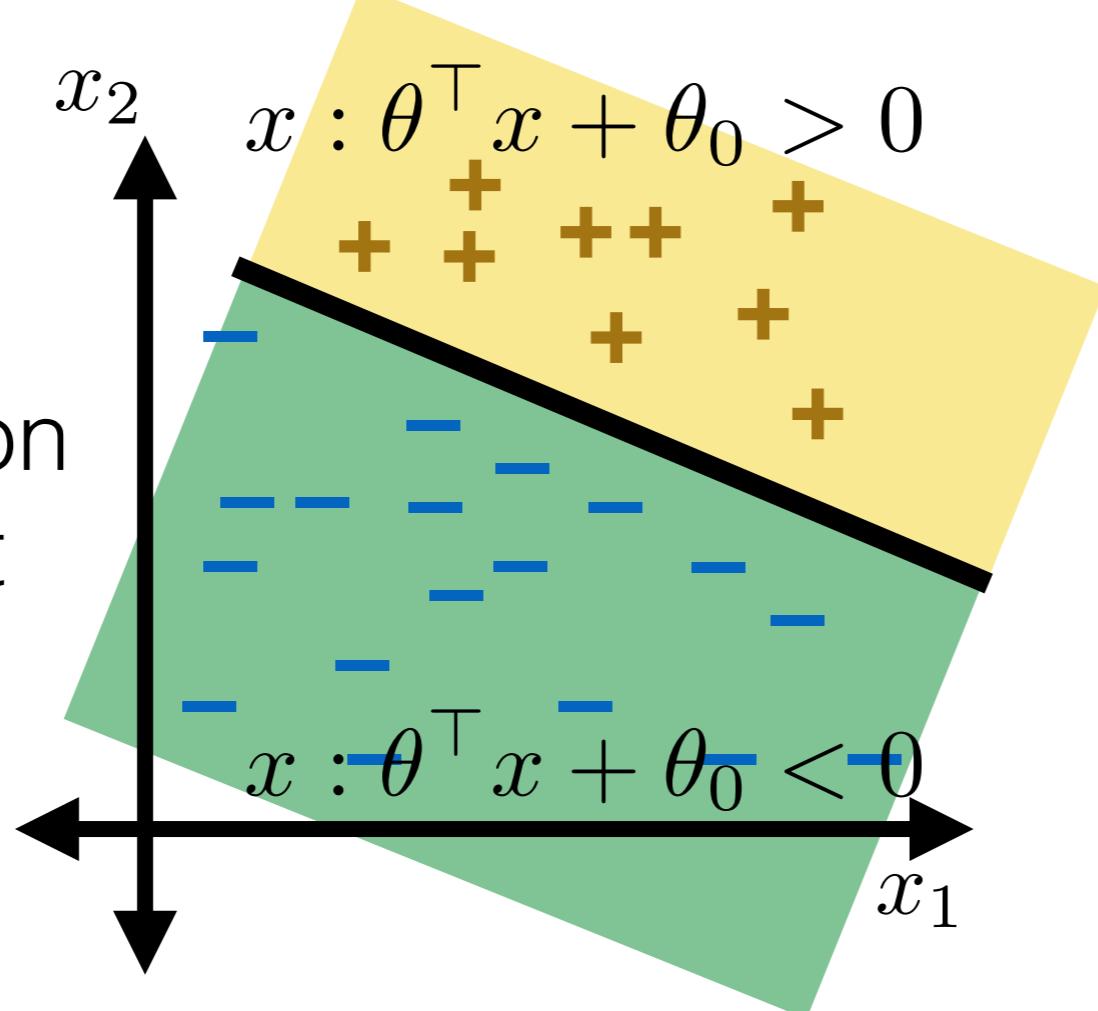
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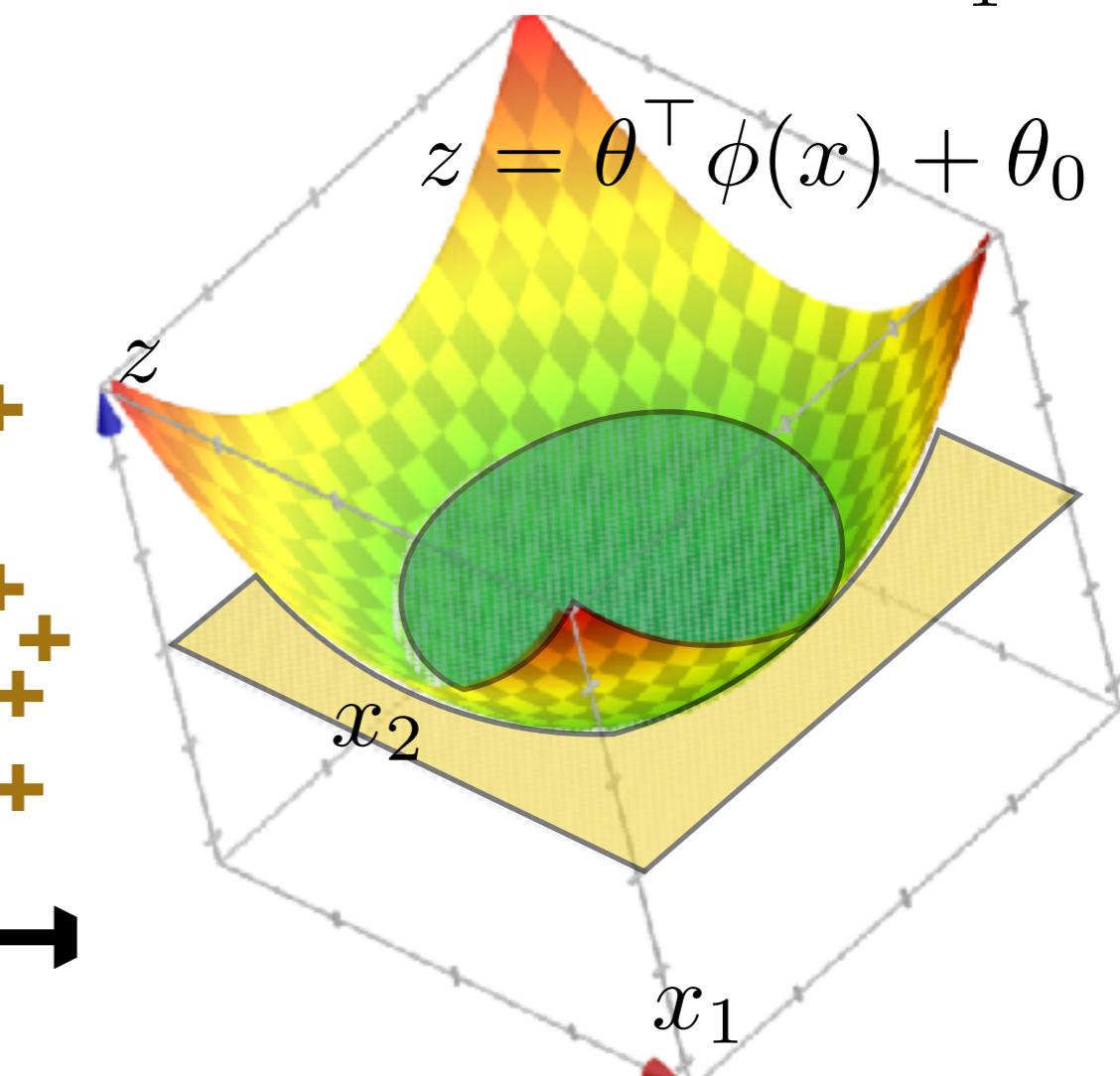
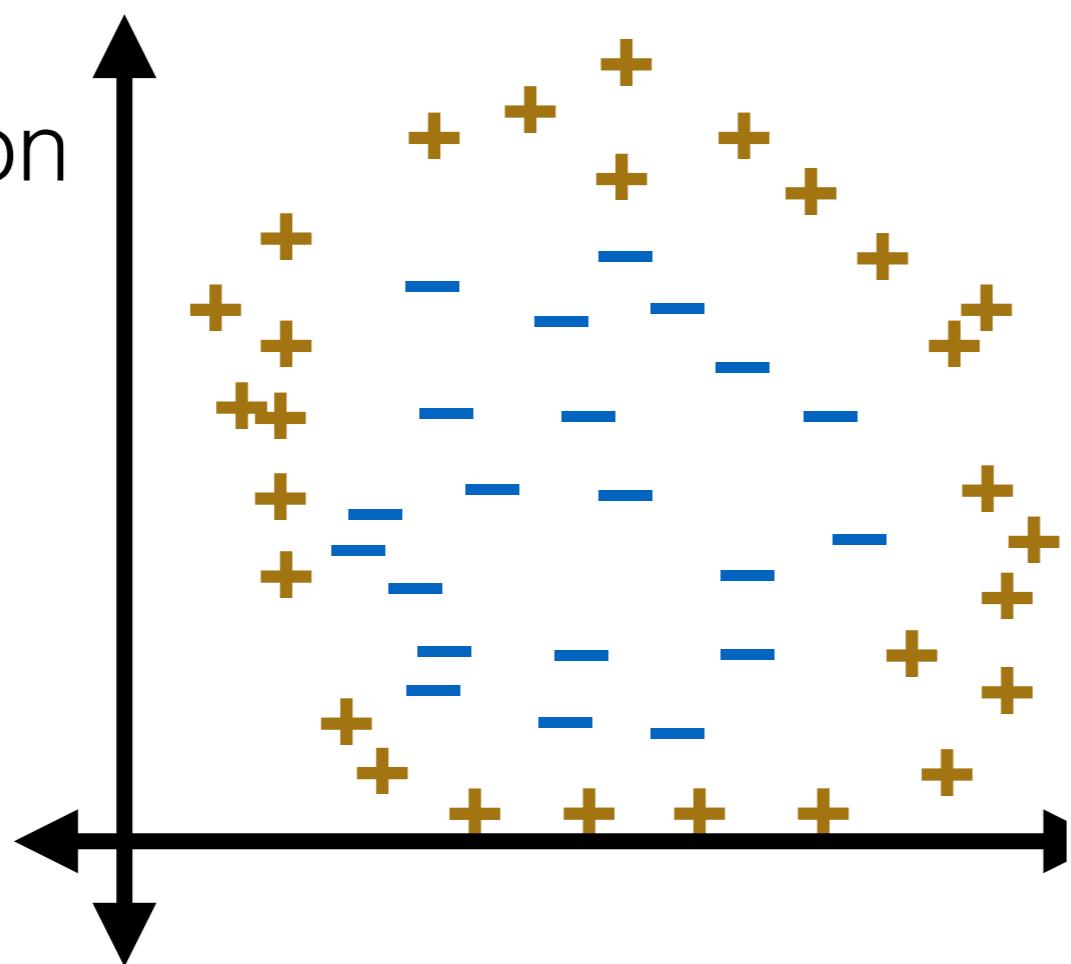
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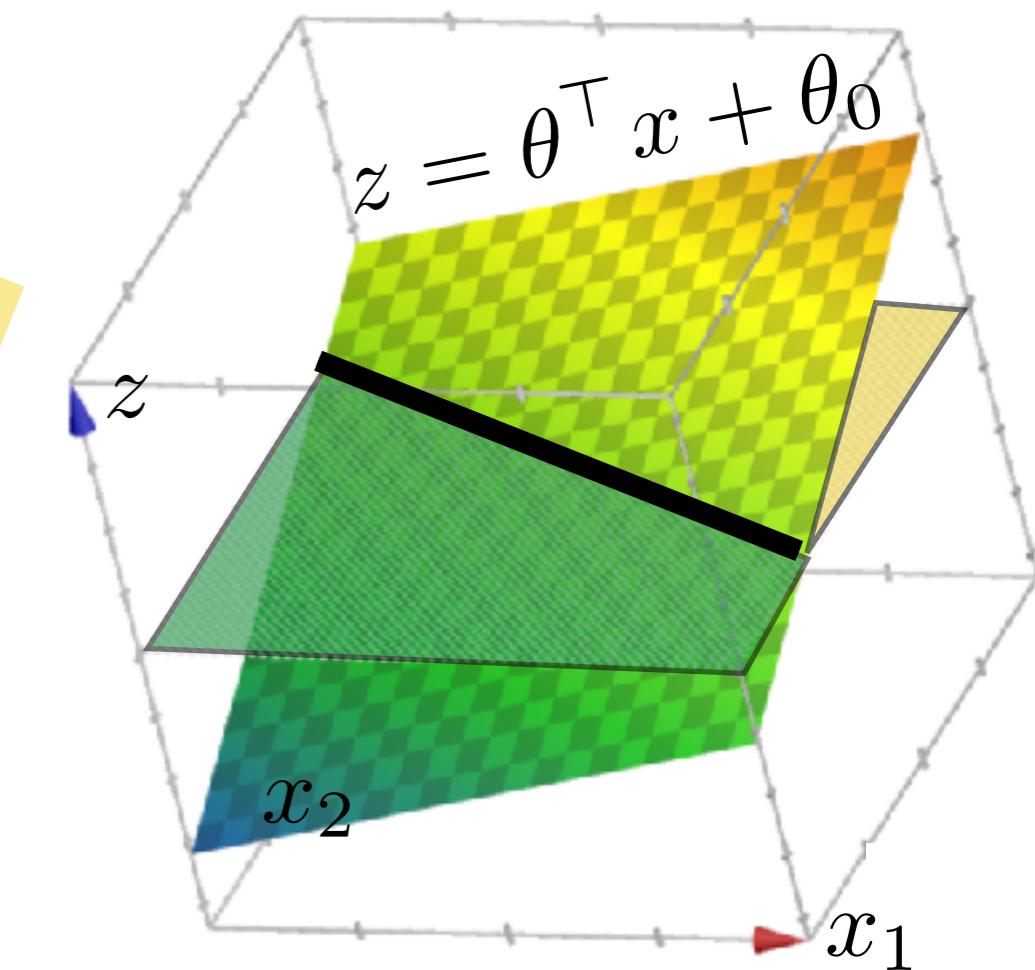
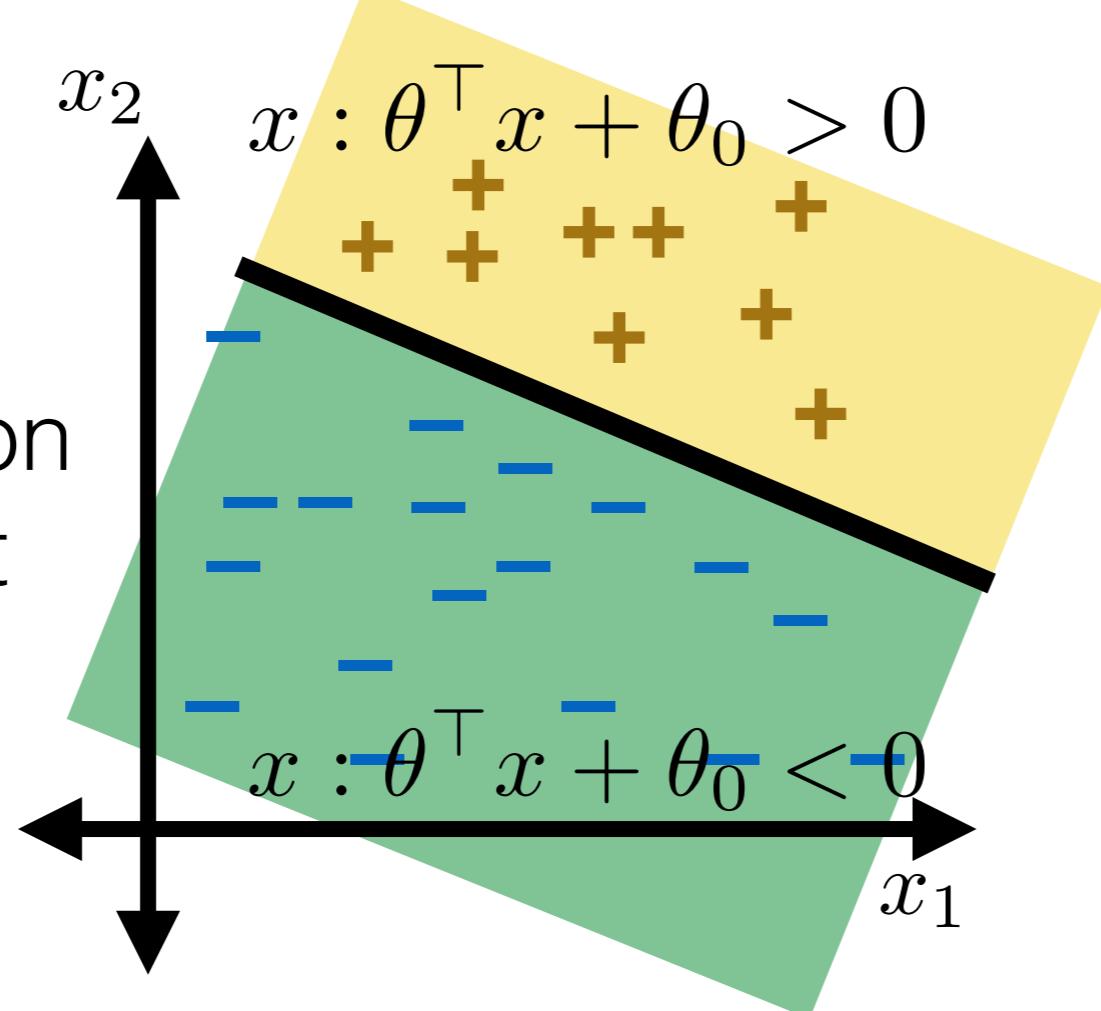
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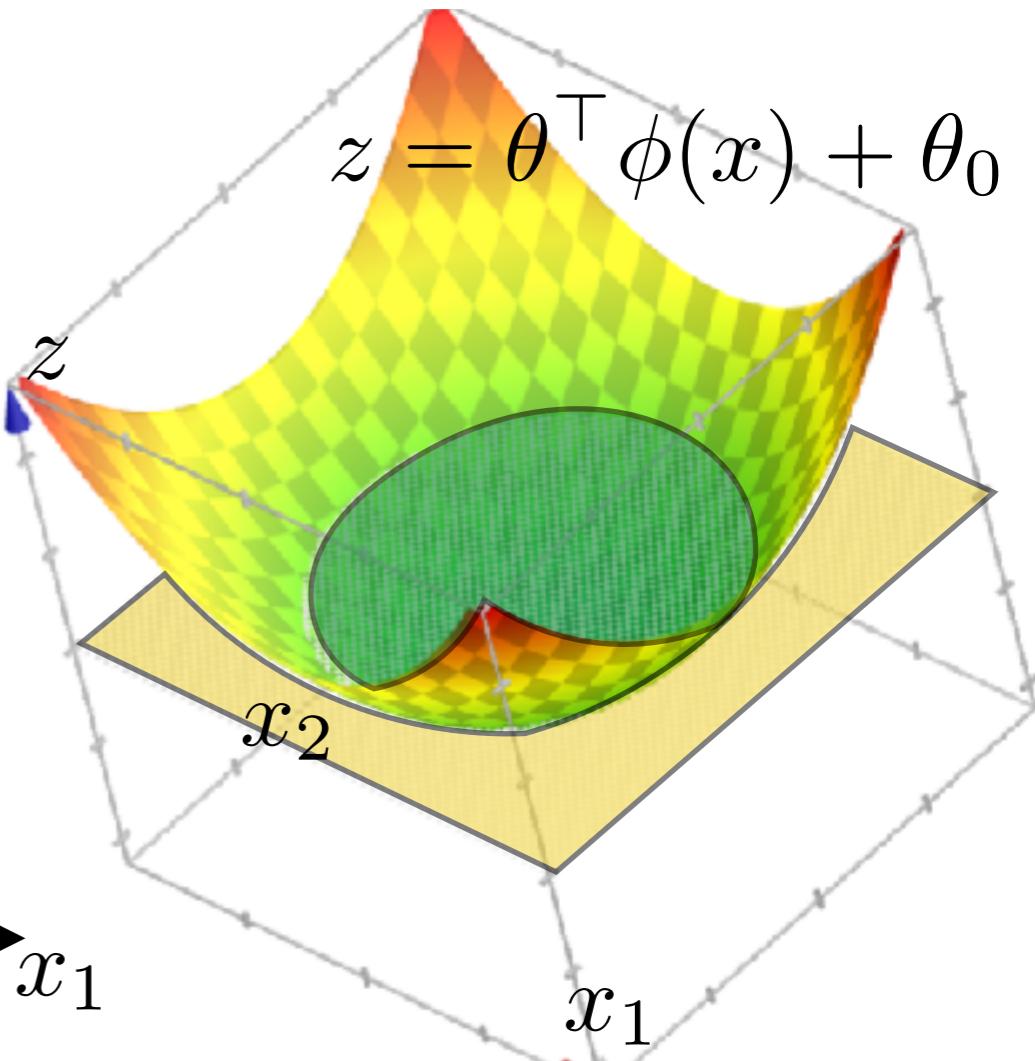
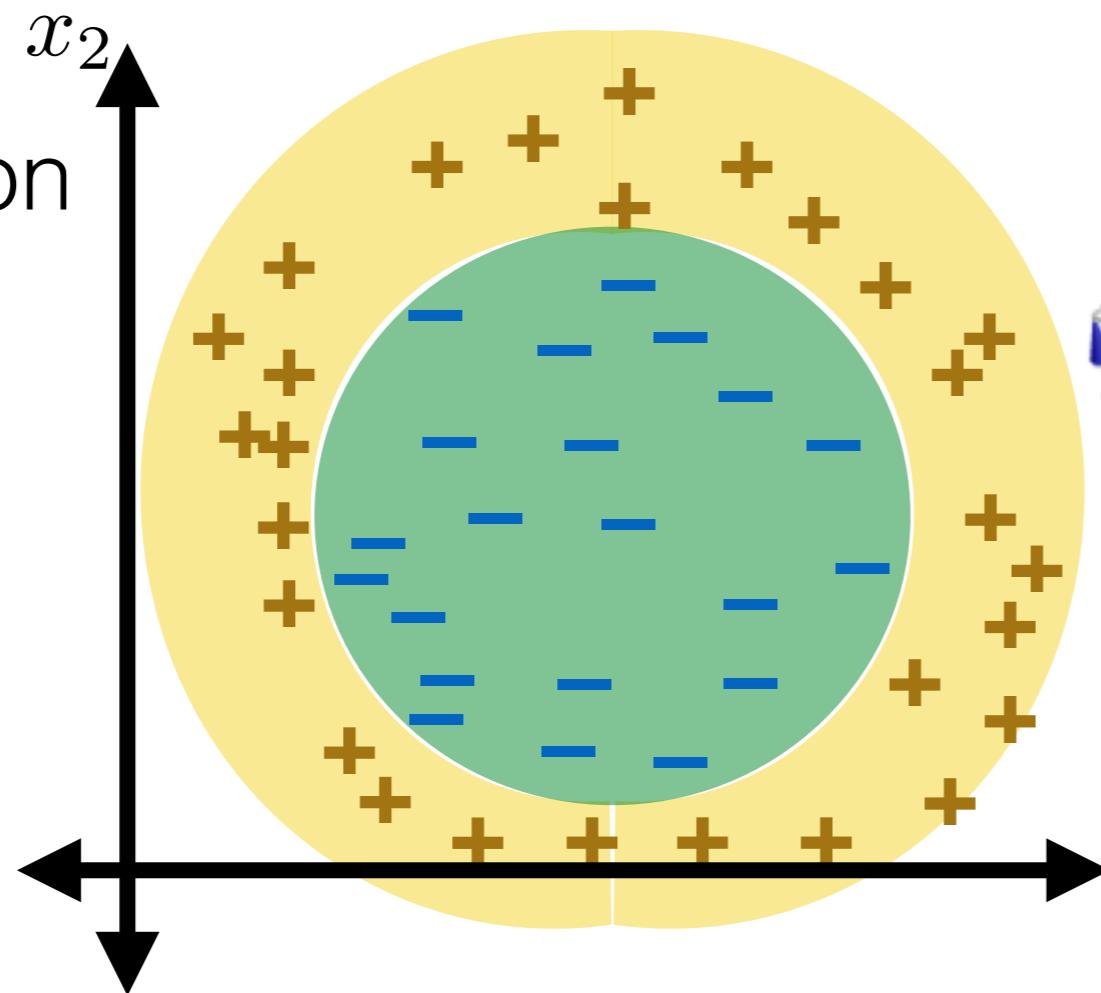
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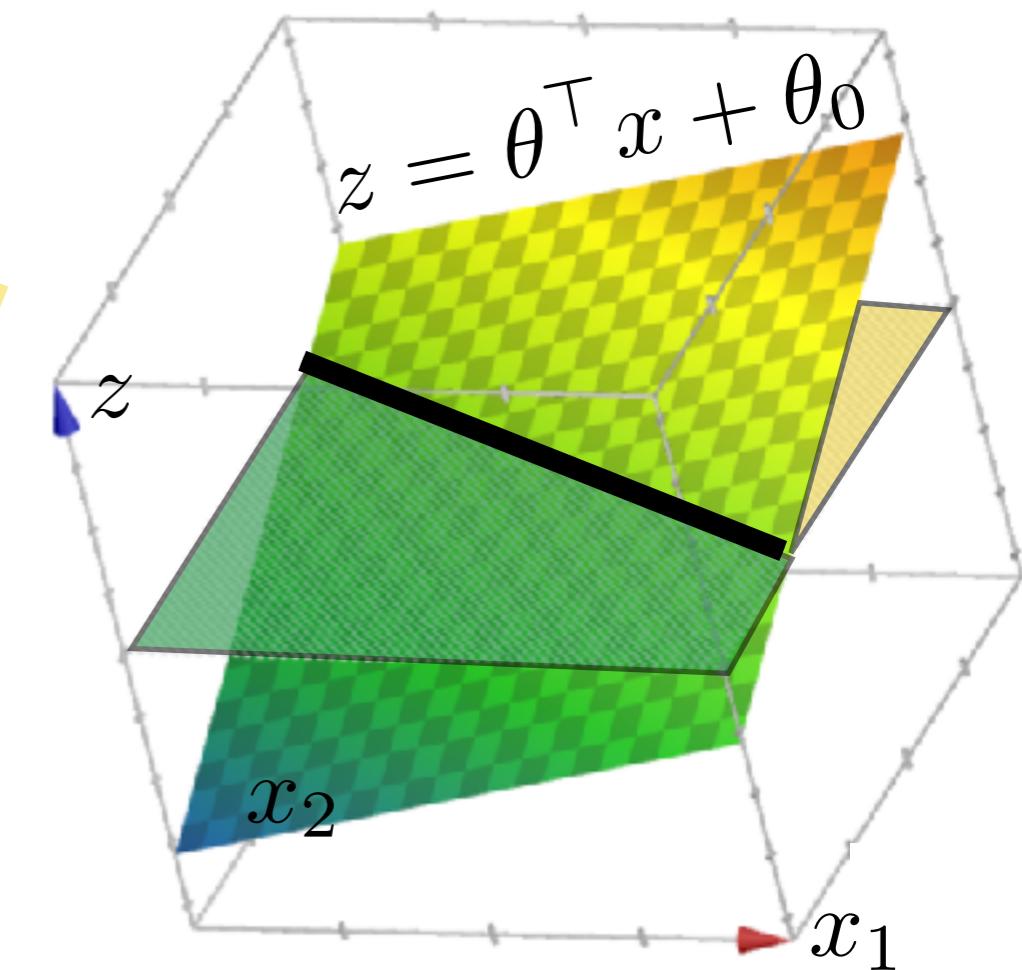
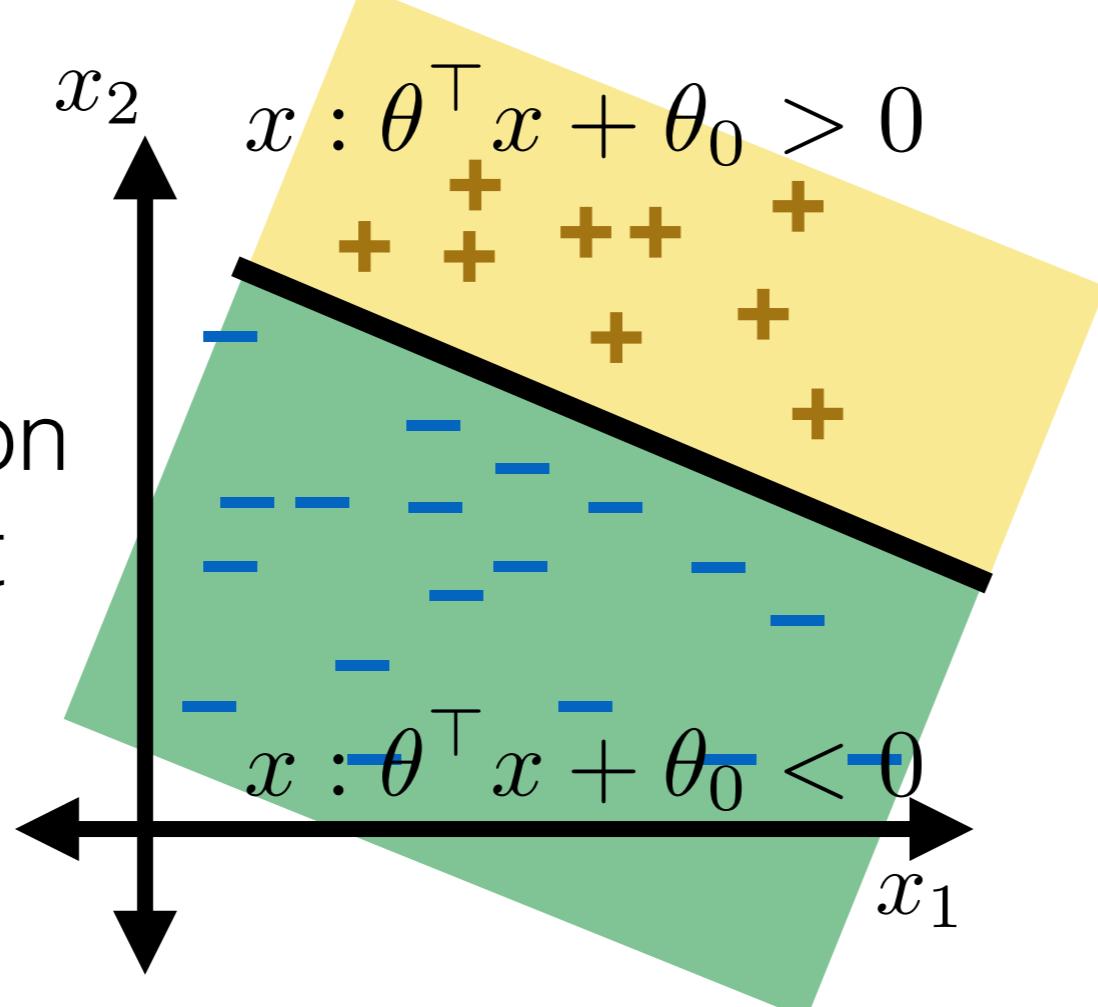
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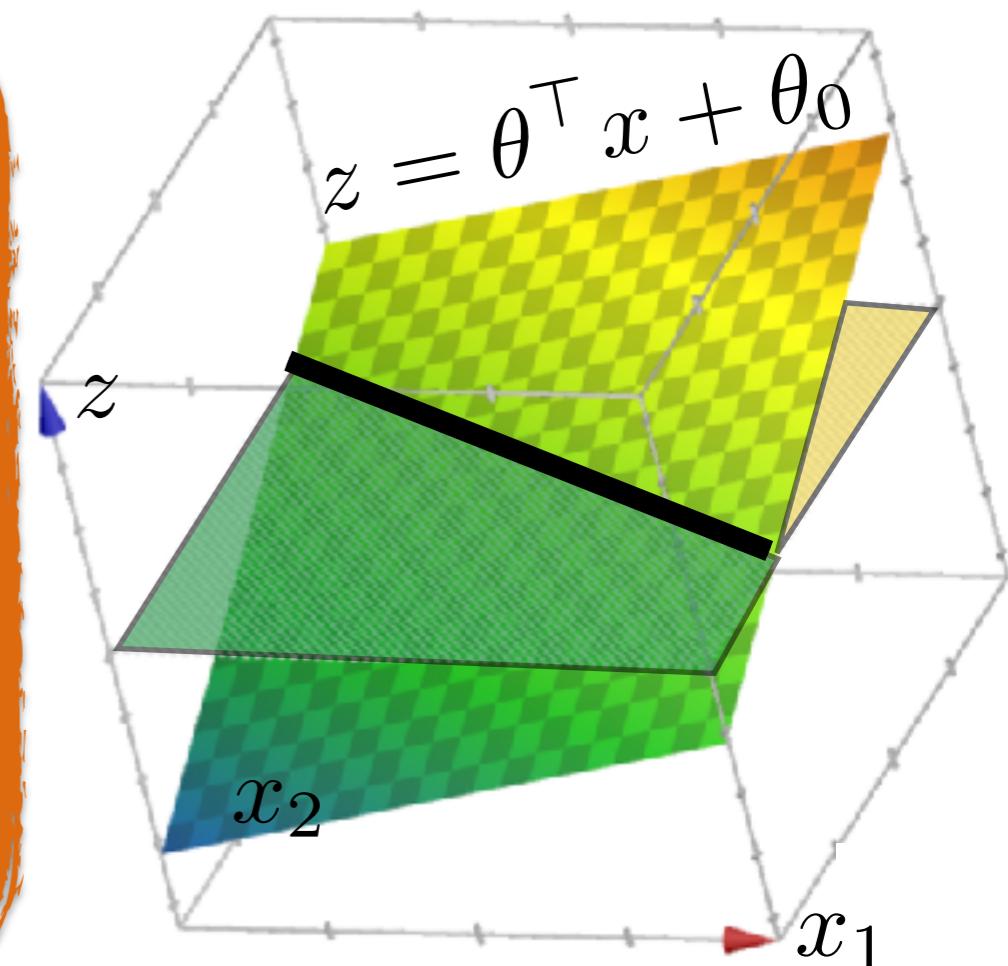
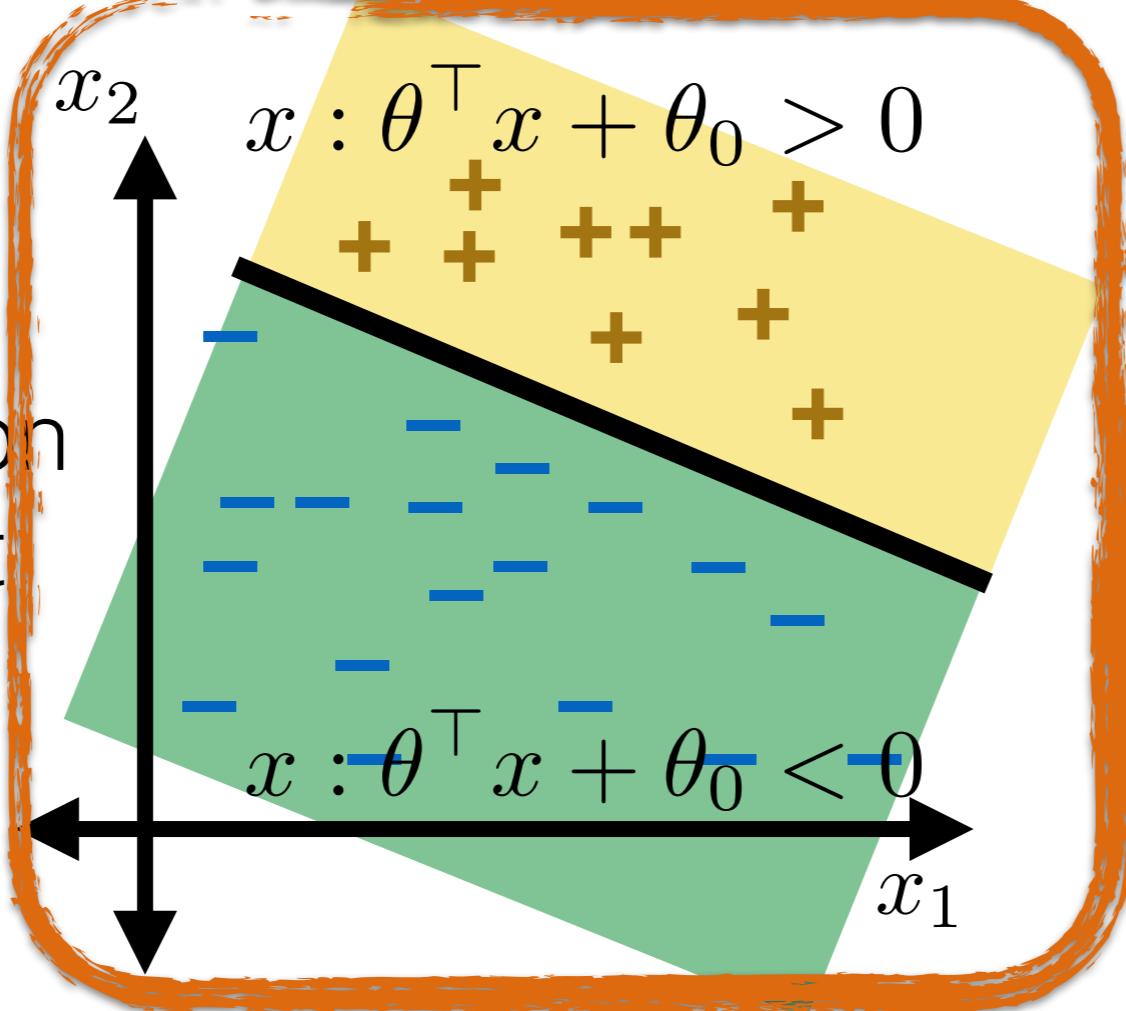
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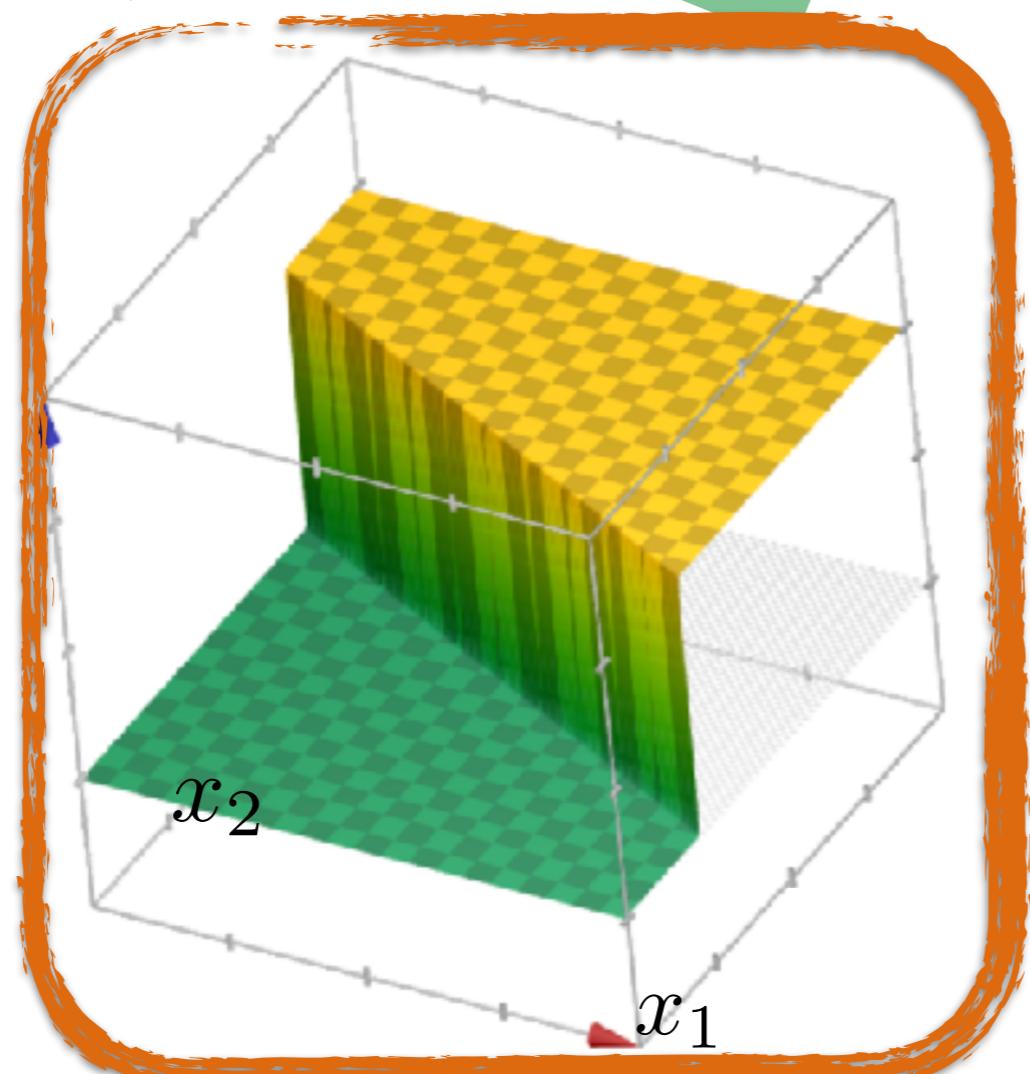
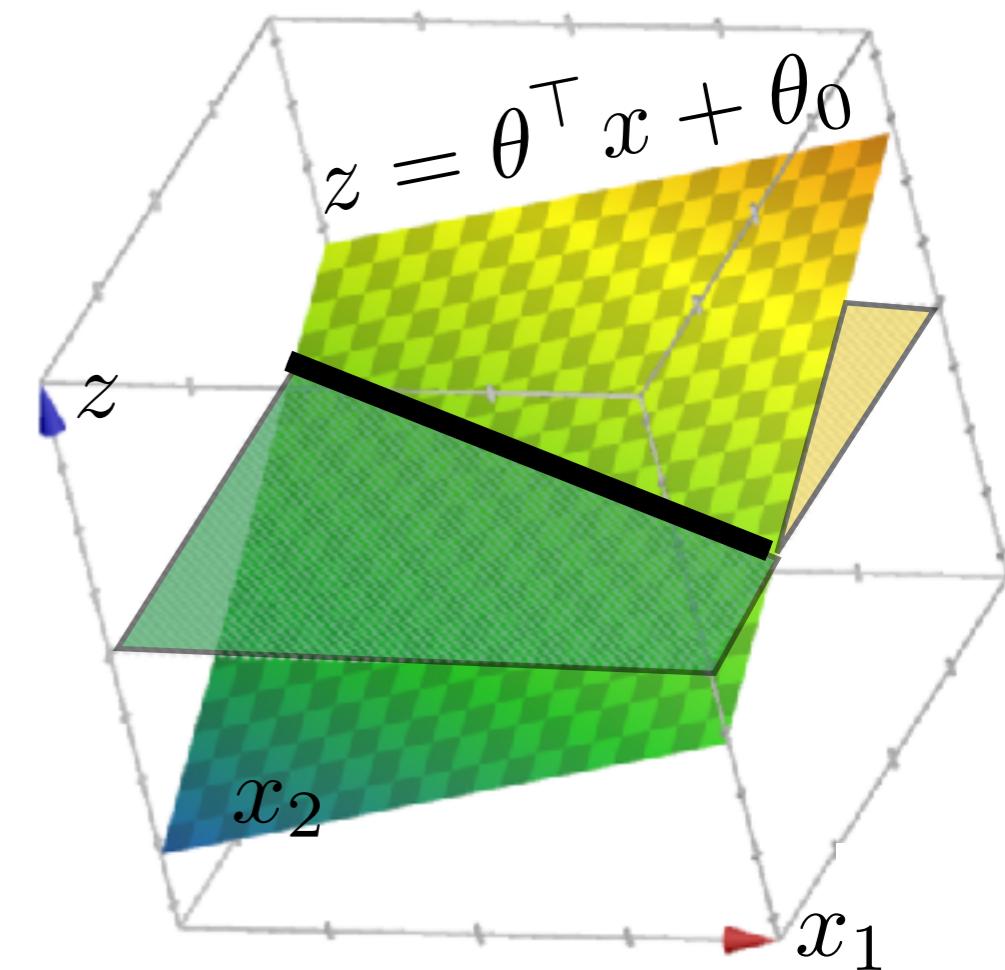
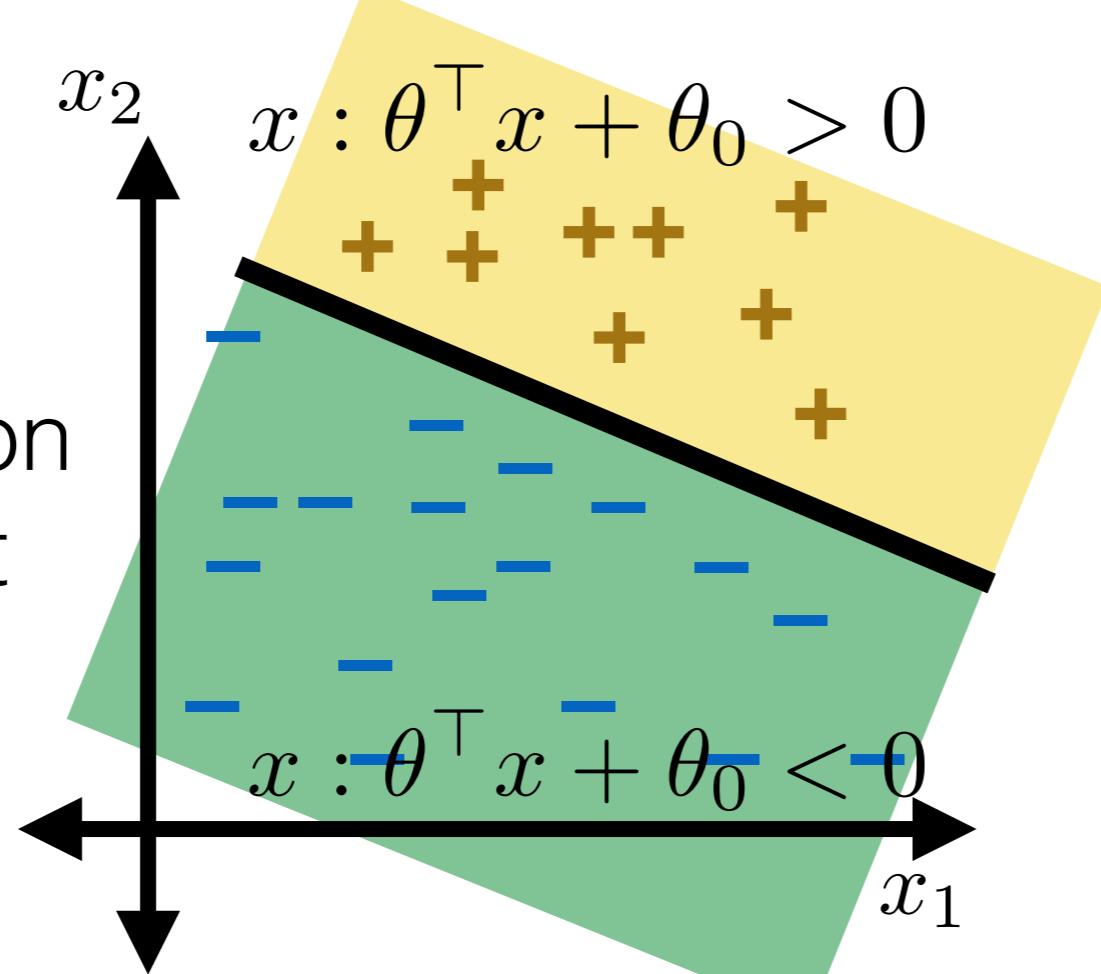
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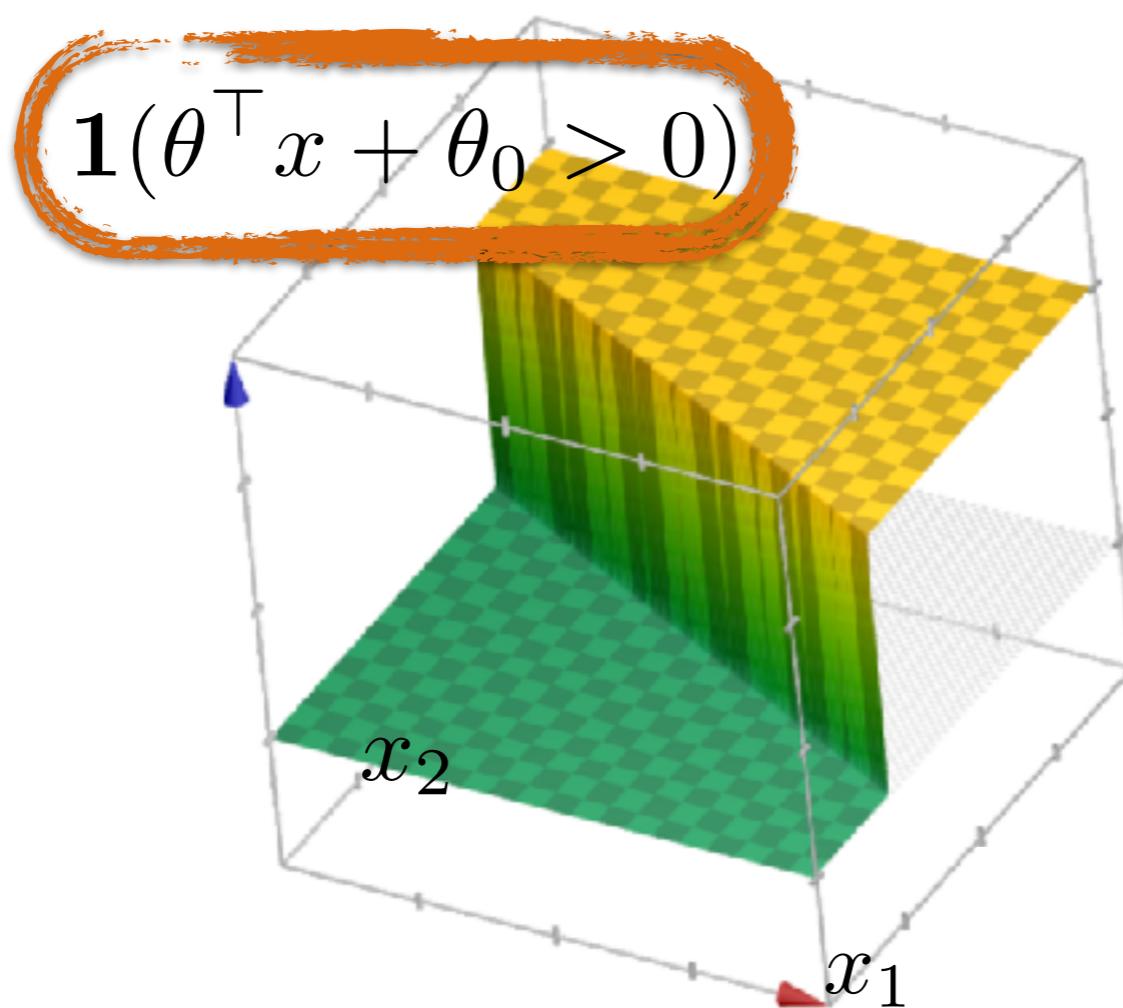
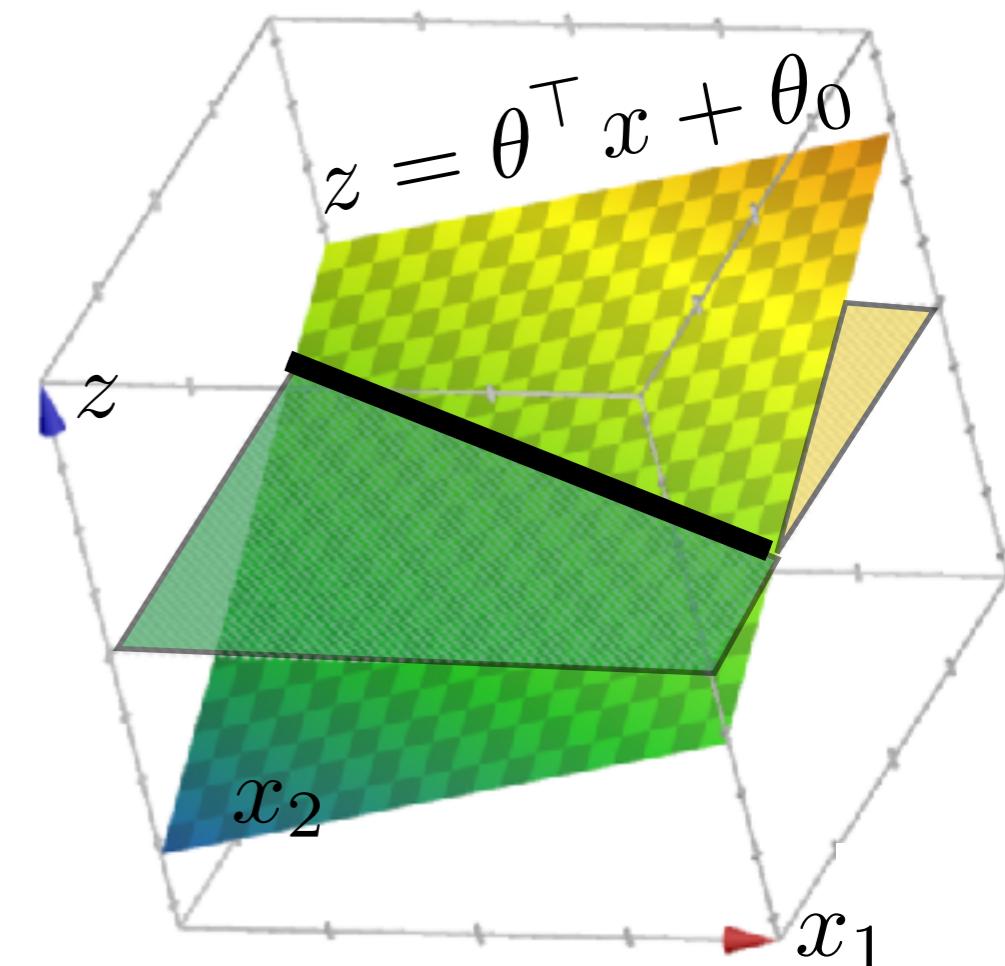
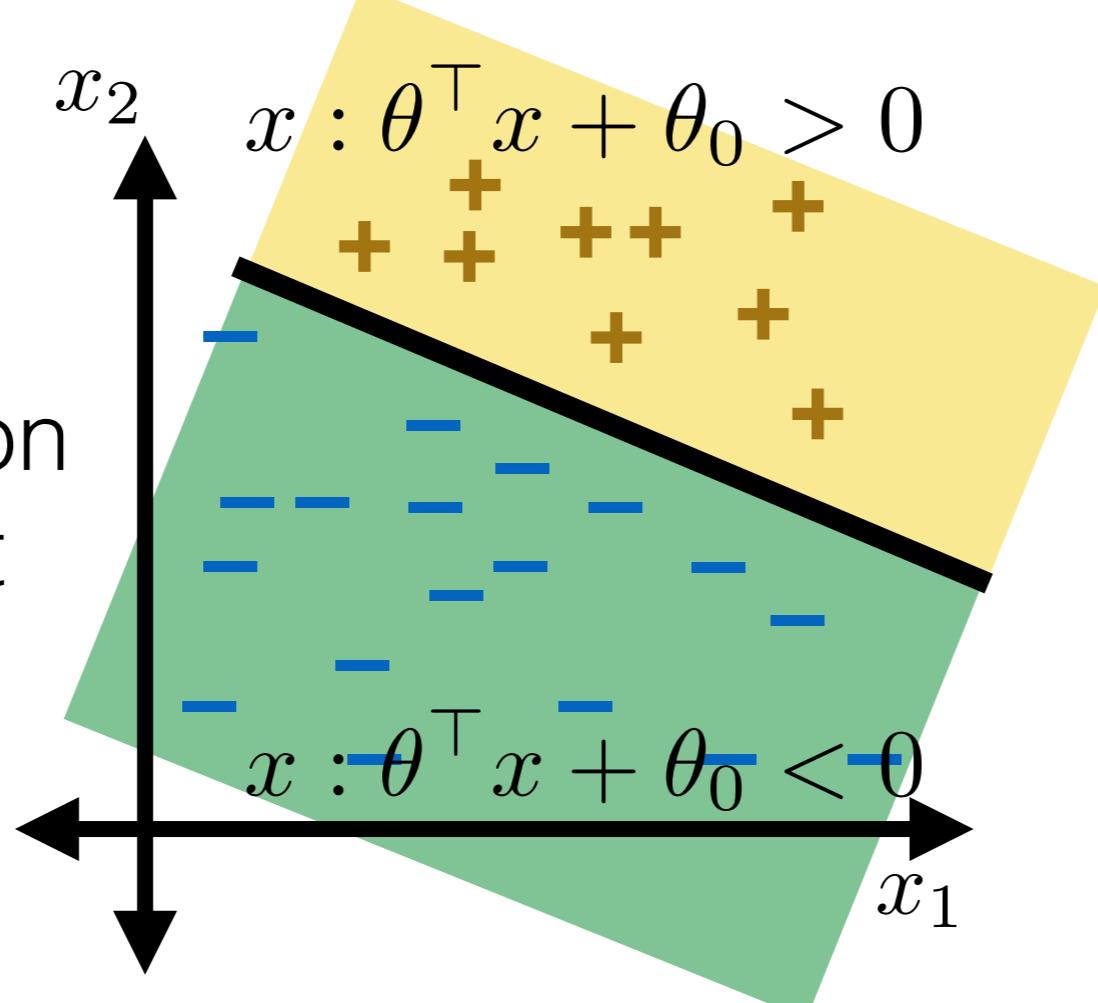
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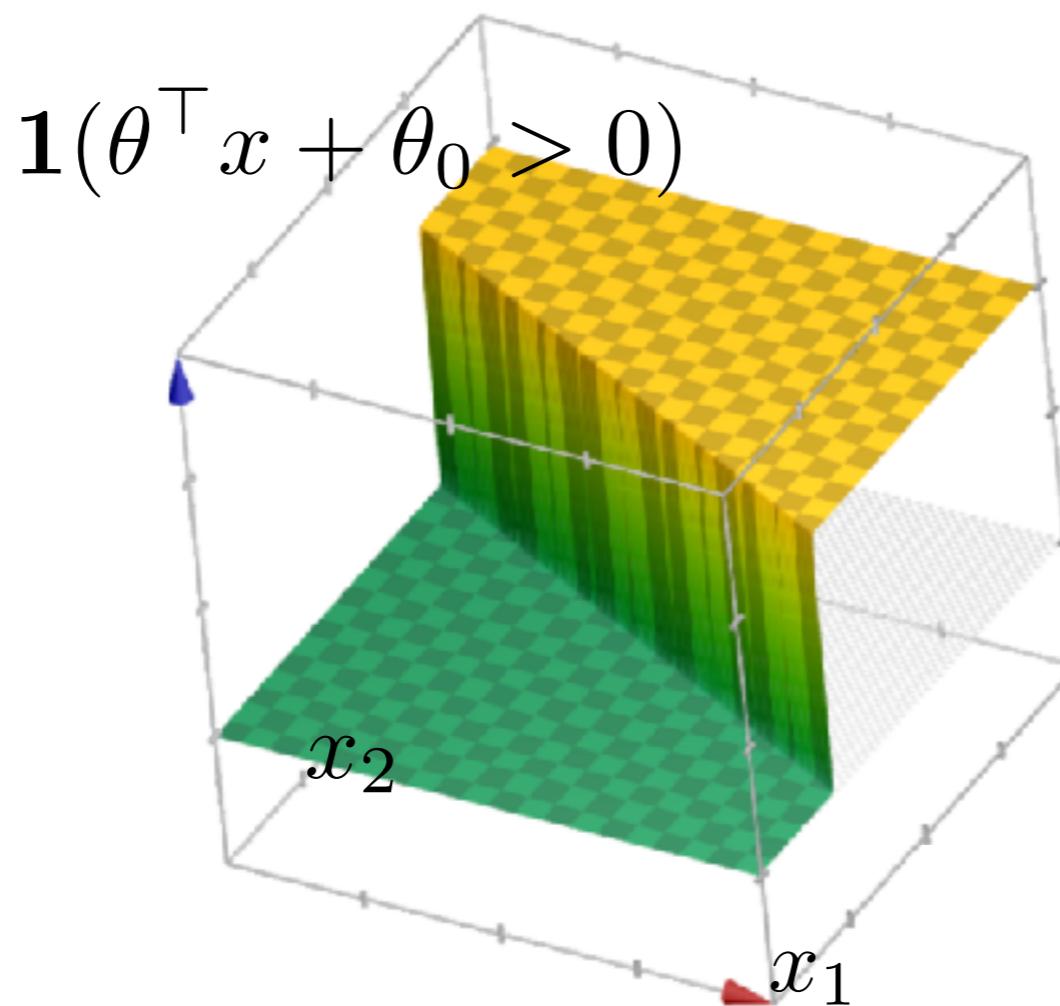
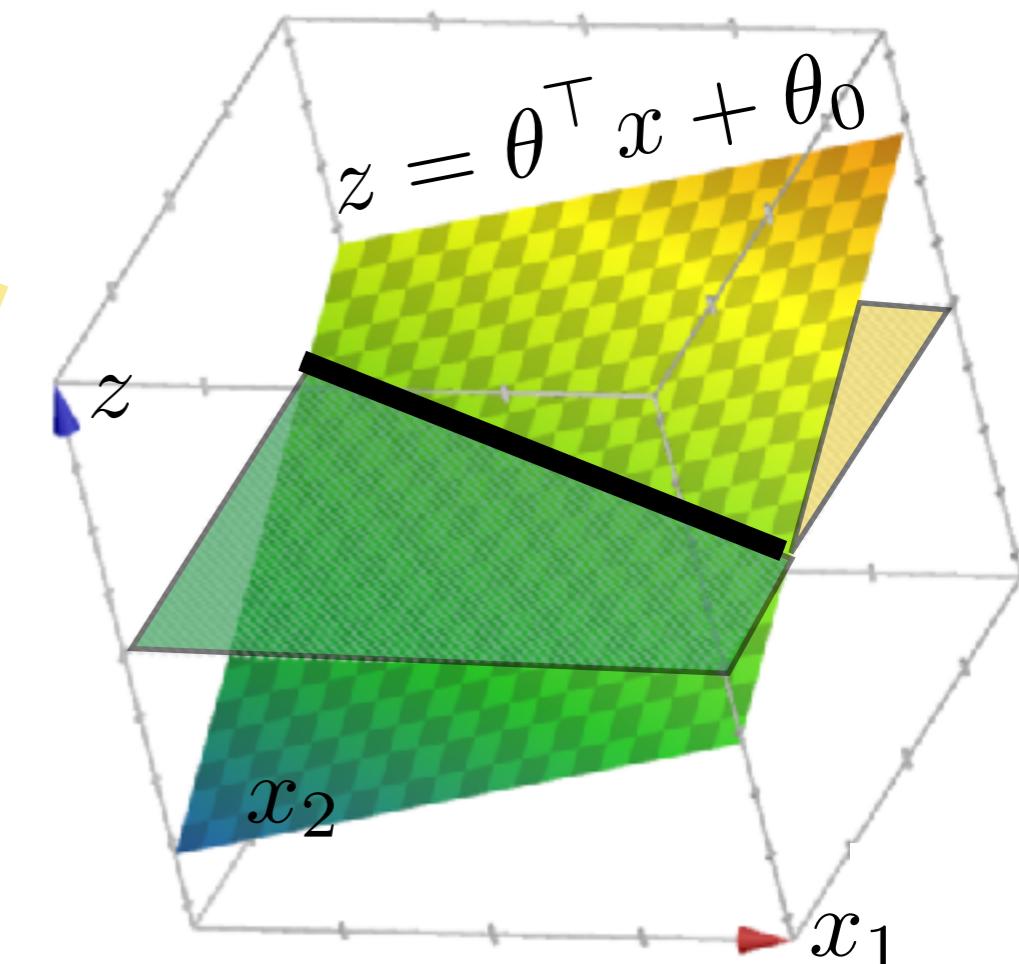
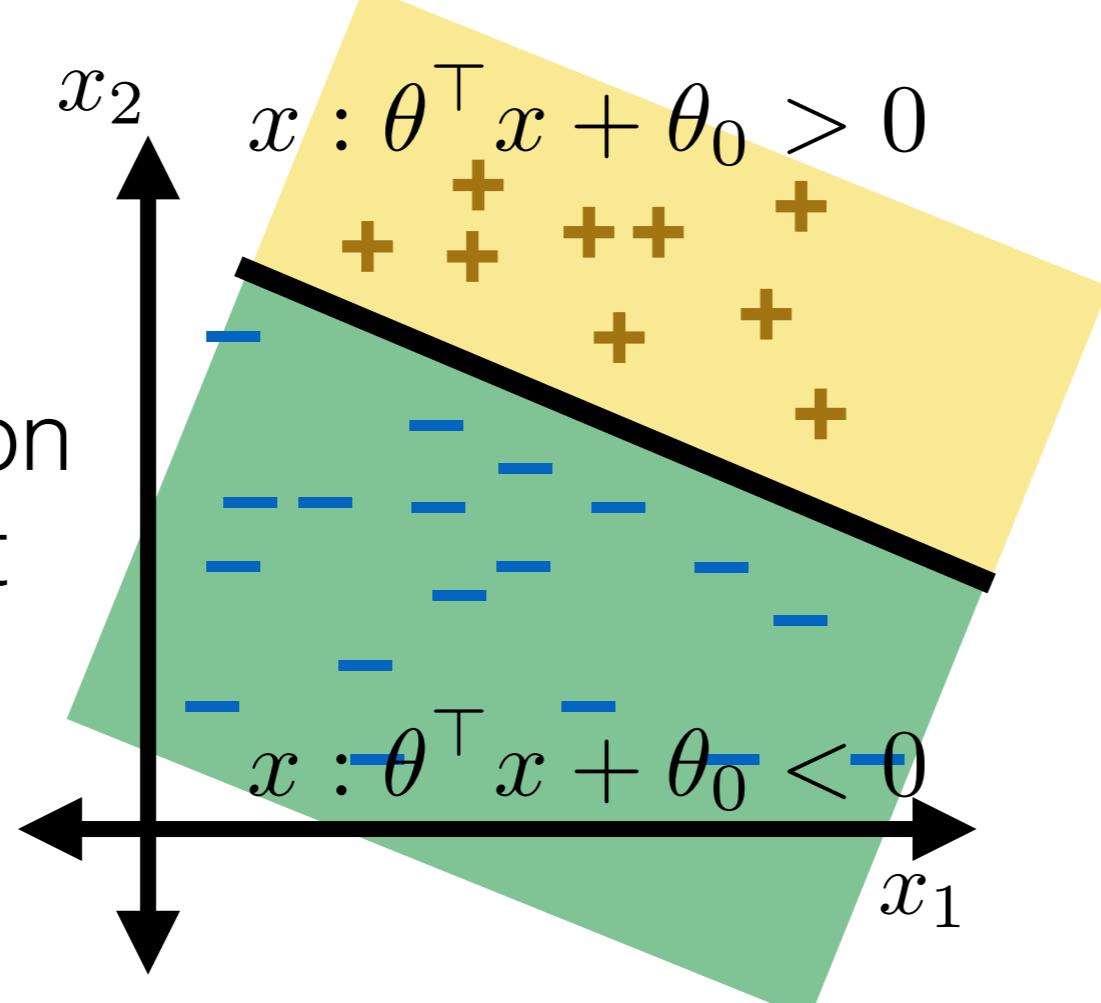
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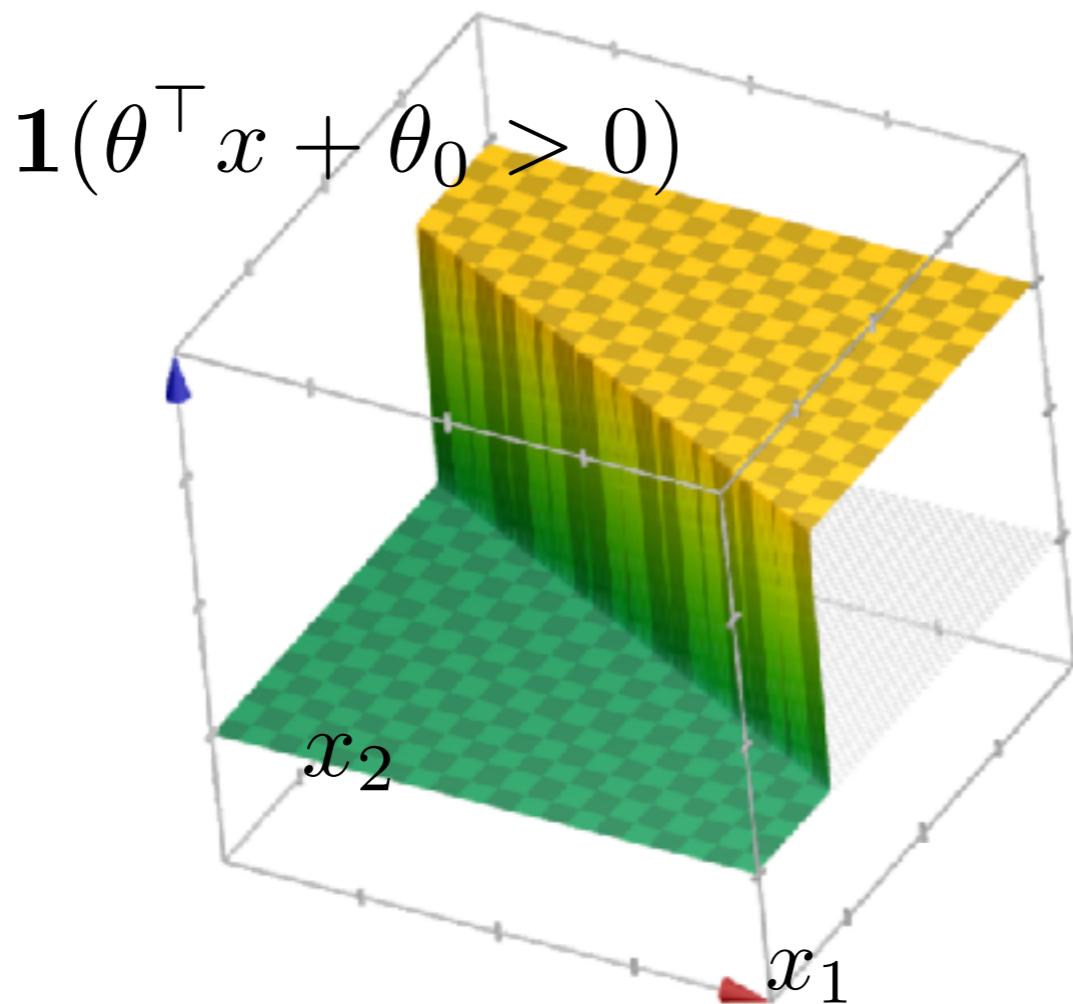
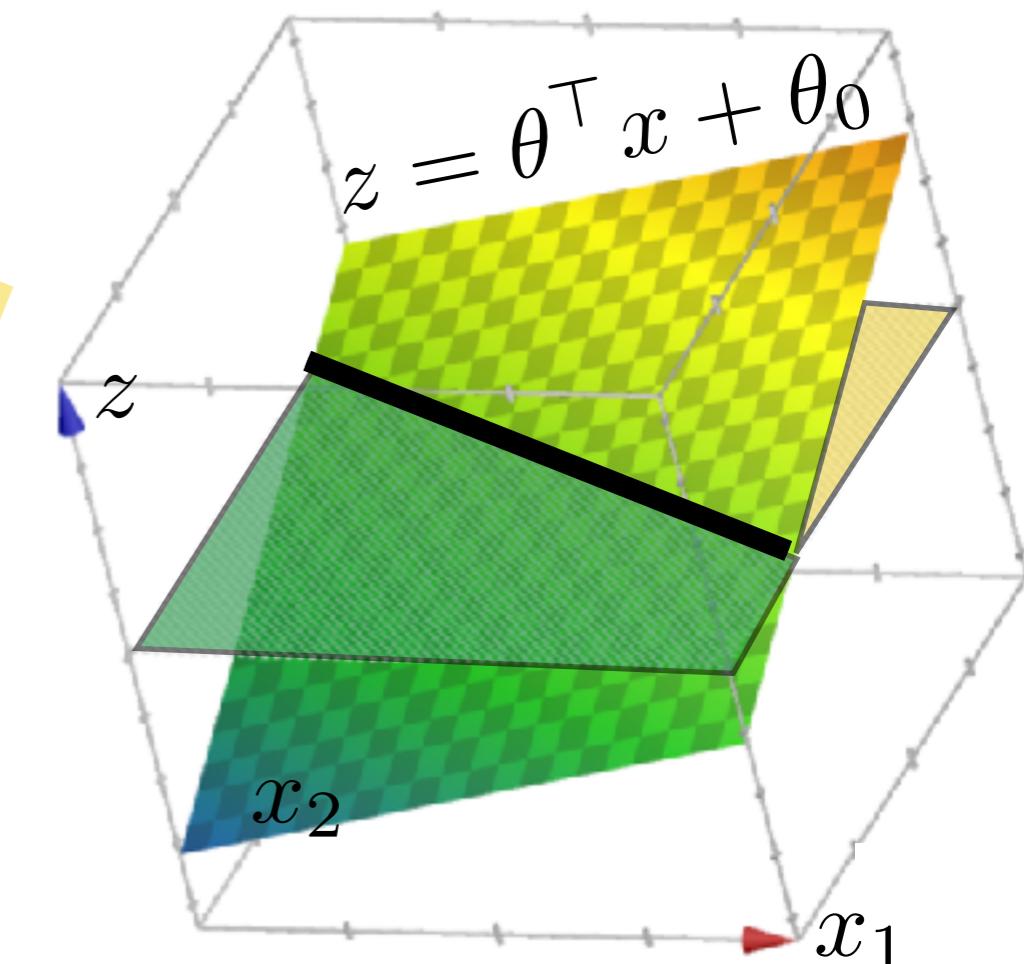
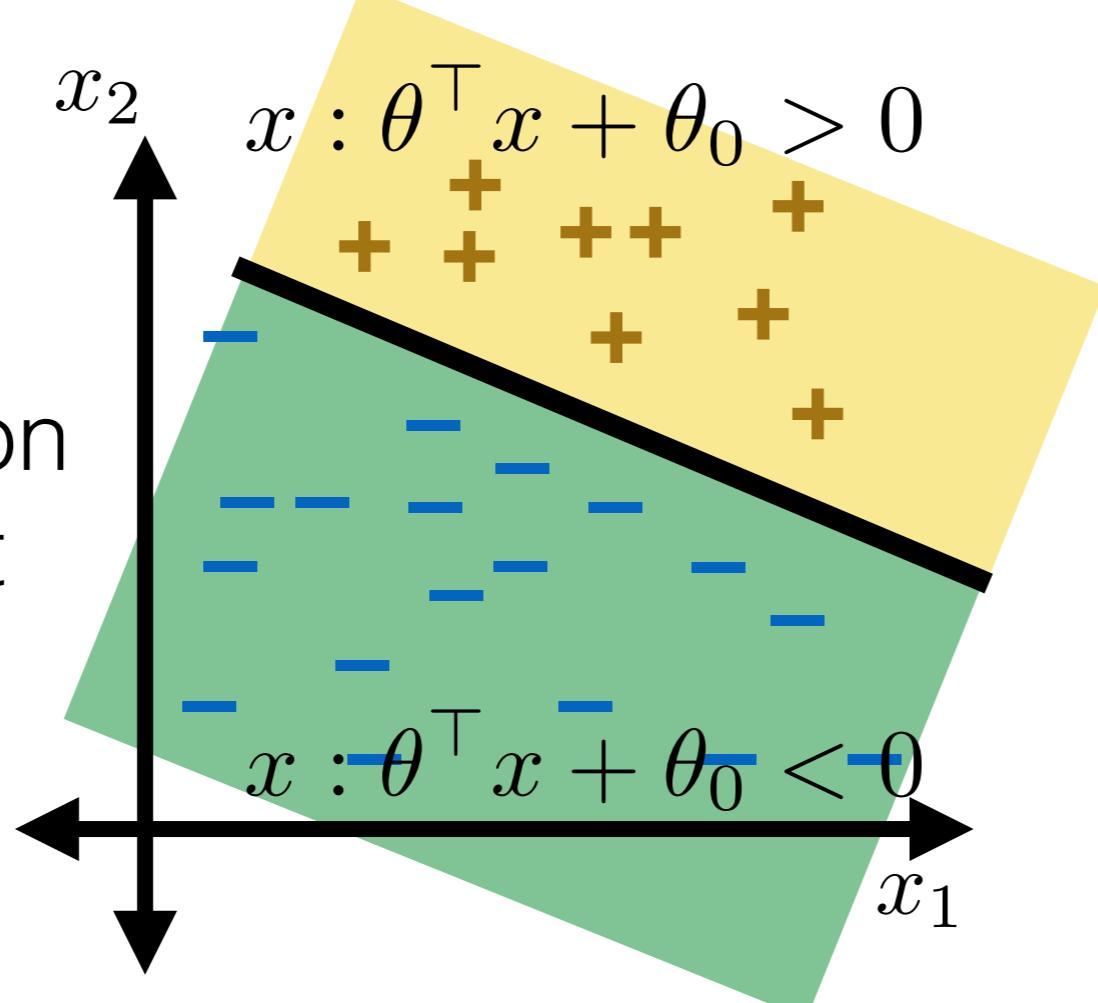
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- We're used to using step functions to classify

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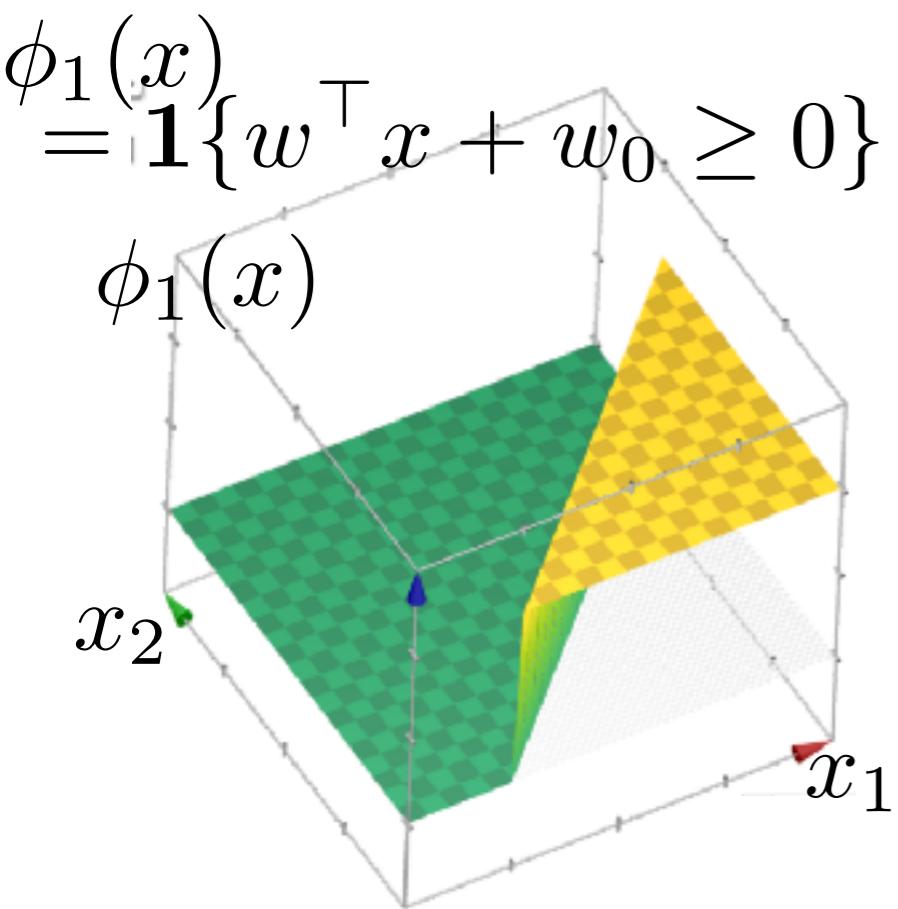
- We're used to using step functions to classify
- New idea today: we'll use step functions as *features*, with their own parameters

# New features: step functions!

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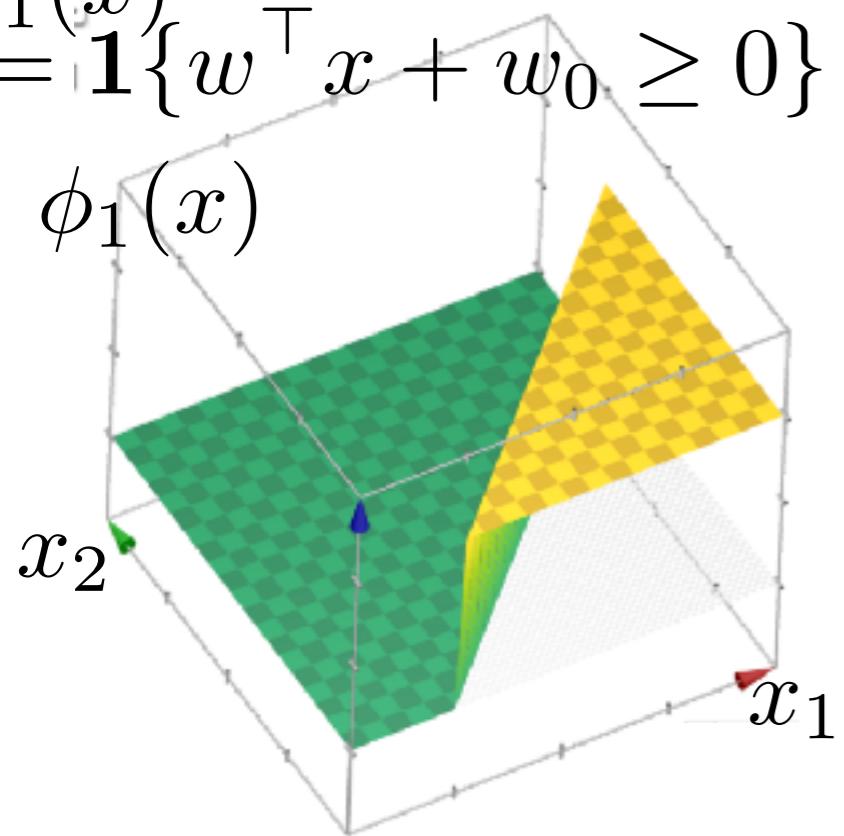
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$

# New features: step functions!

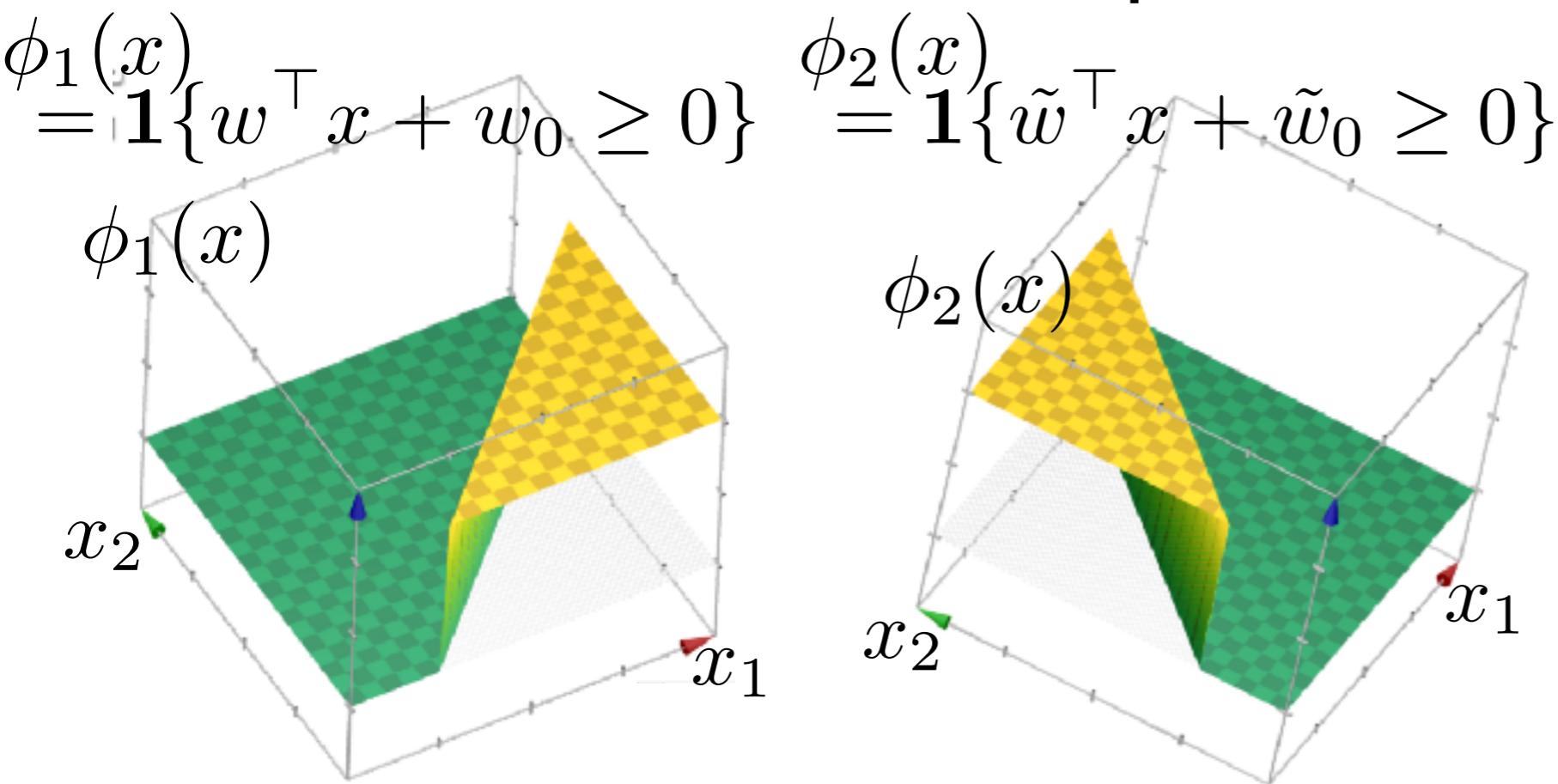


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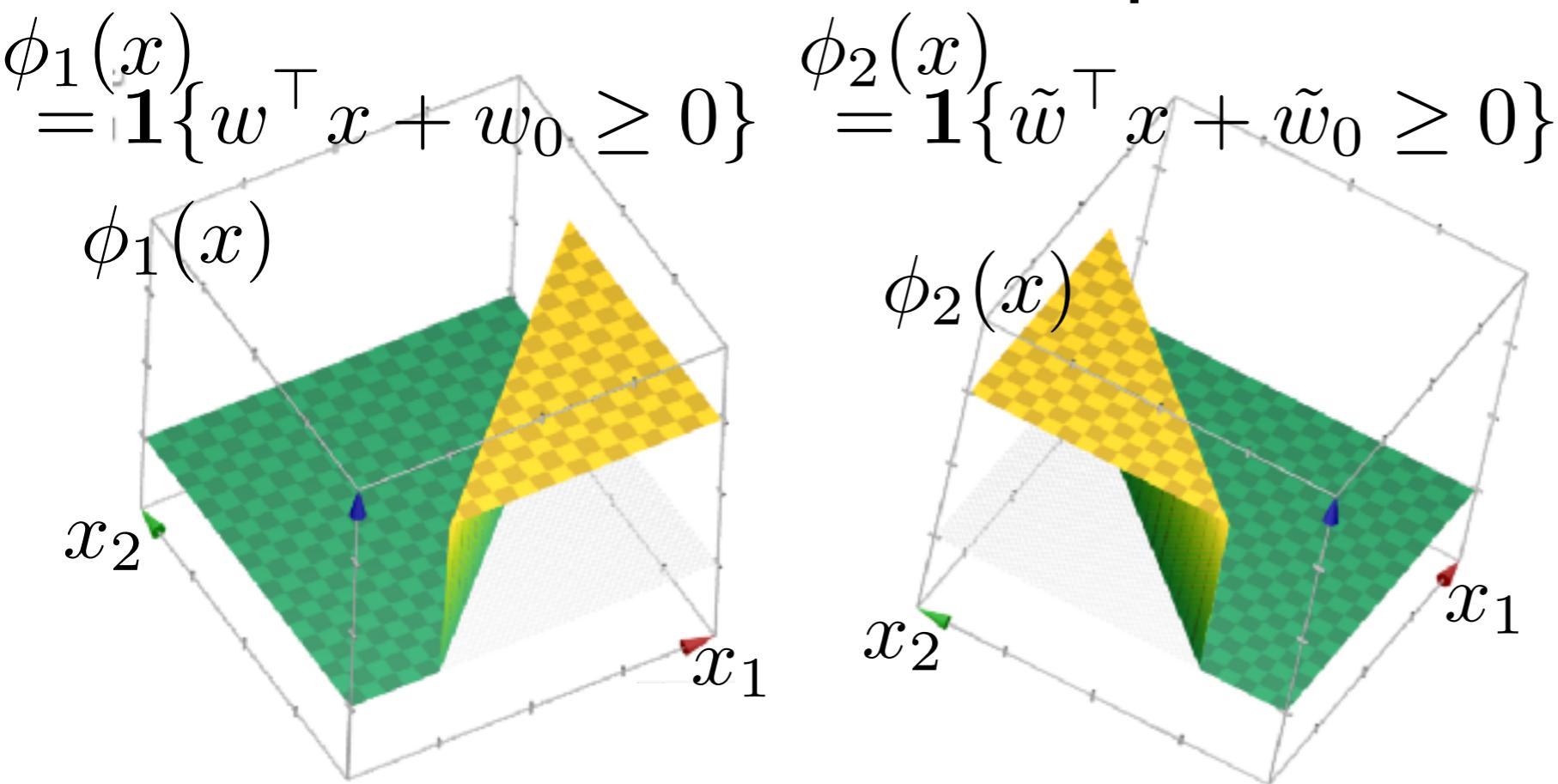
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\} \quad \phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



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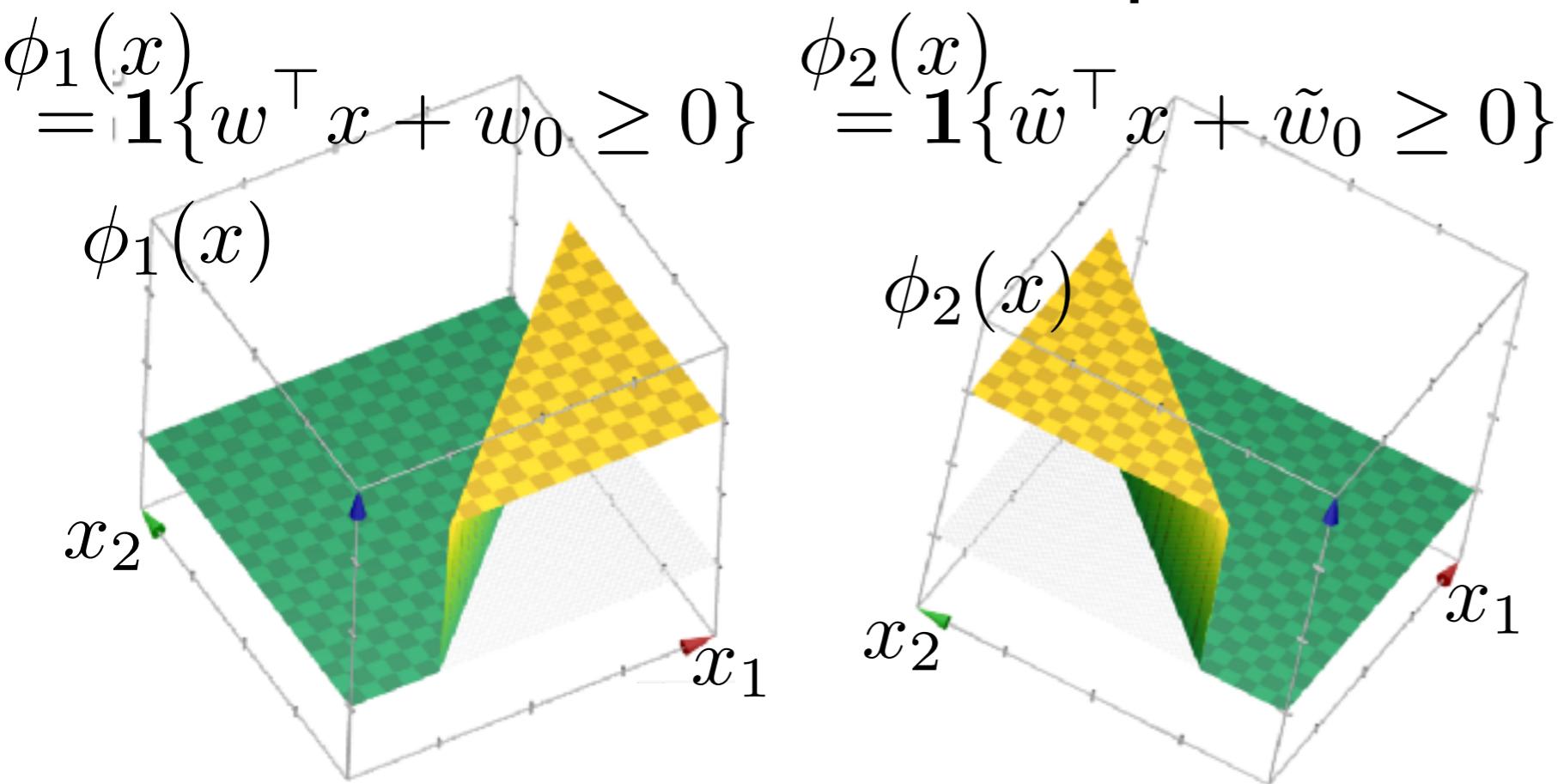


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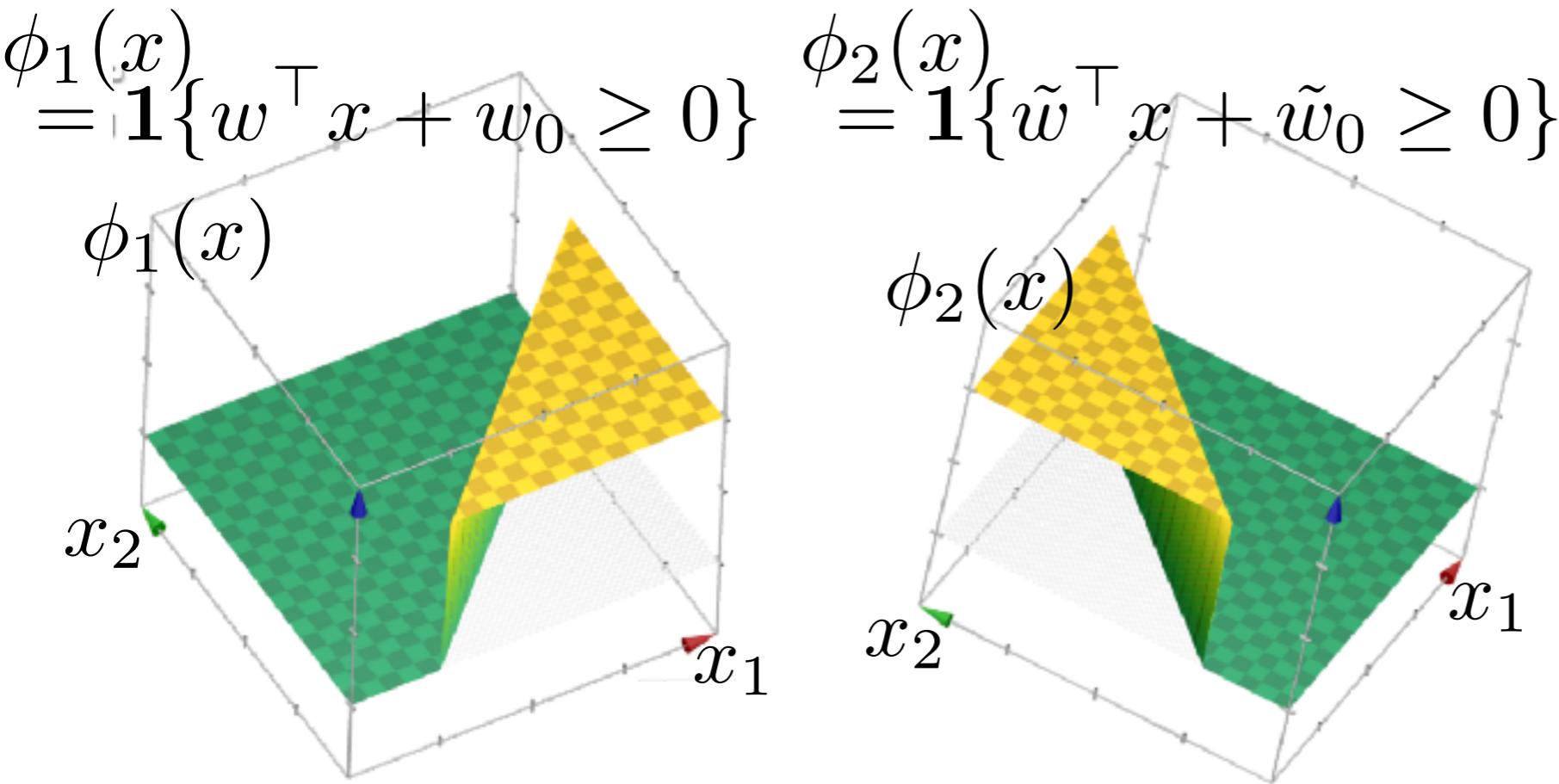
$$z = \theta^\top \phi(x) + \theta_0$$

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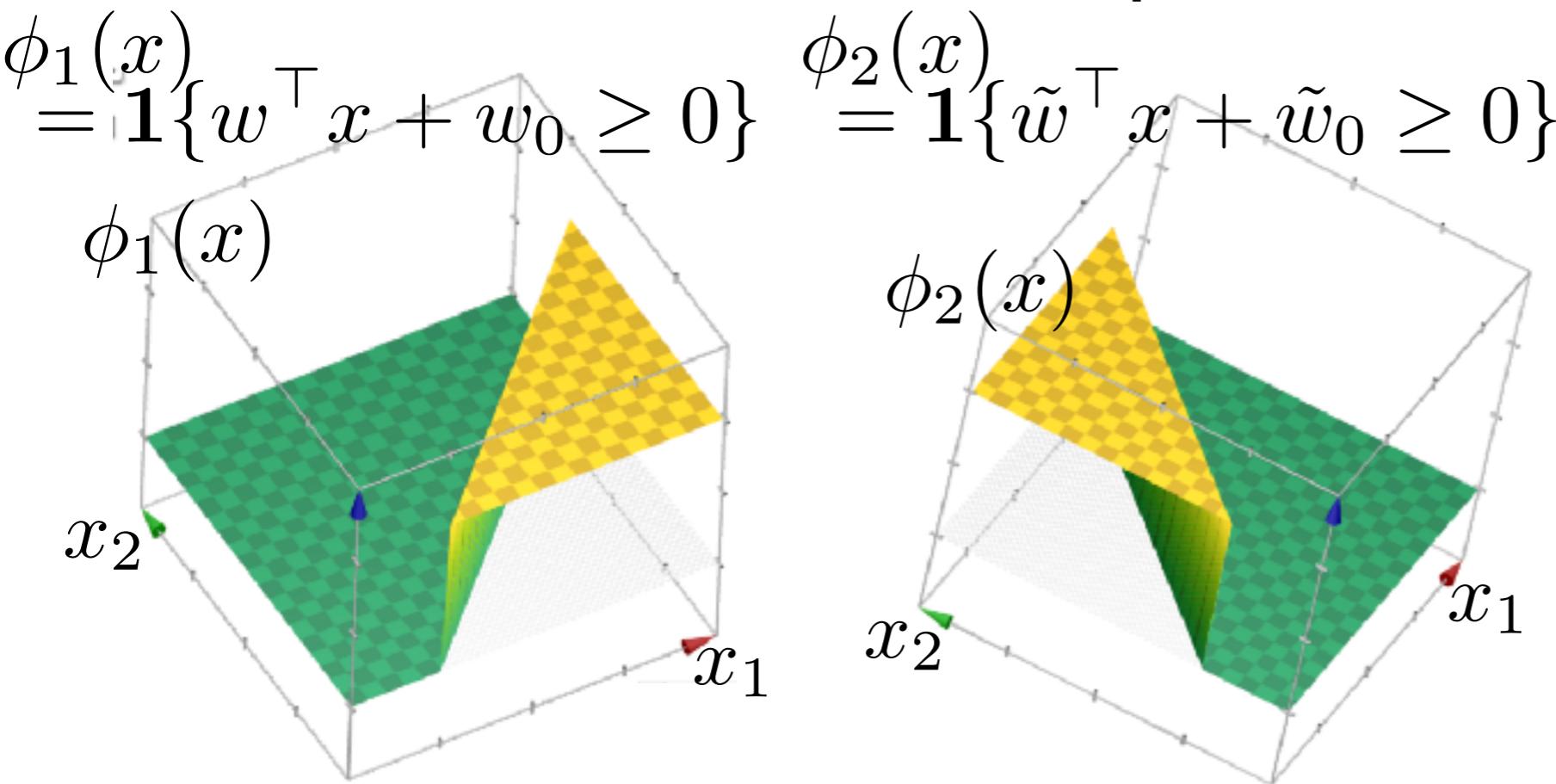
$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \end{aligned}$$

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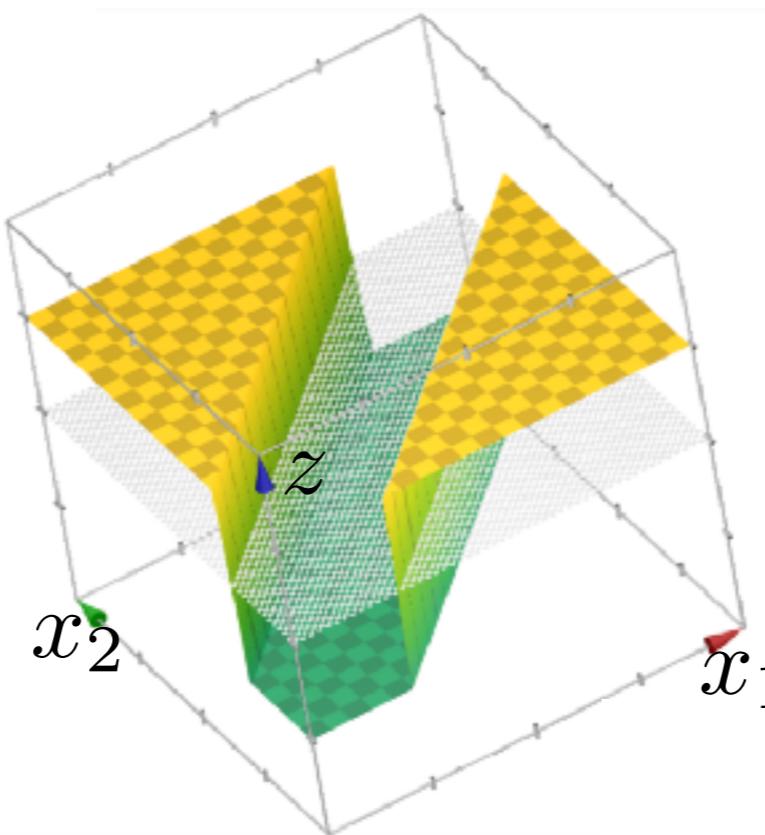


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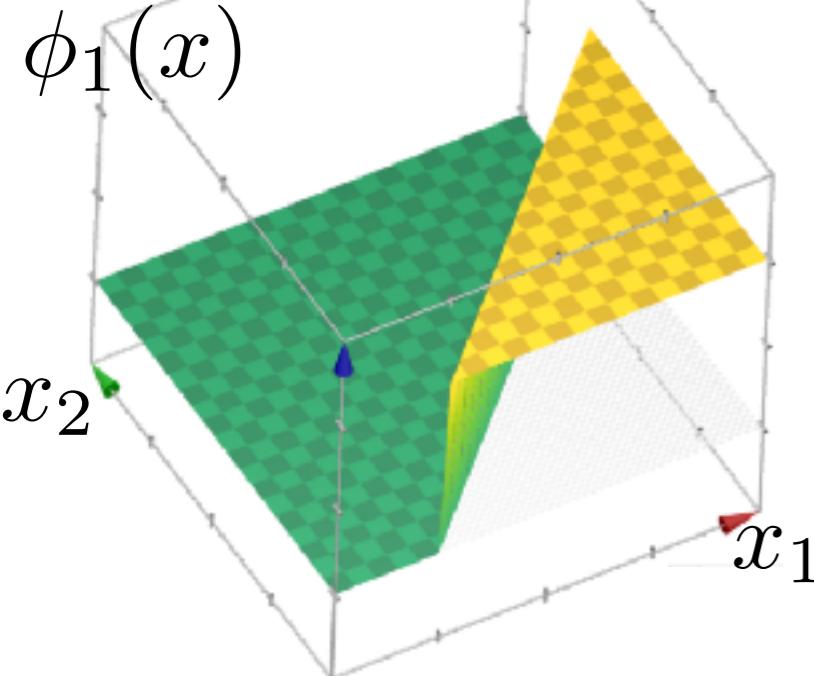


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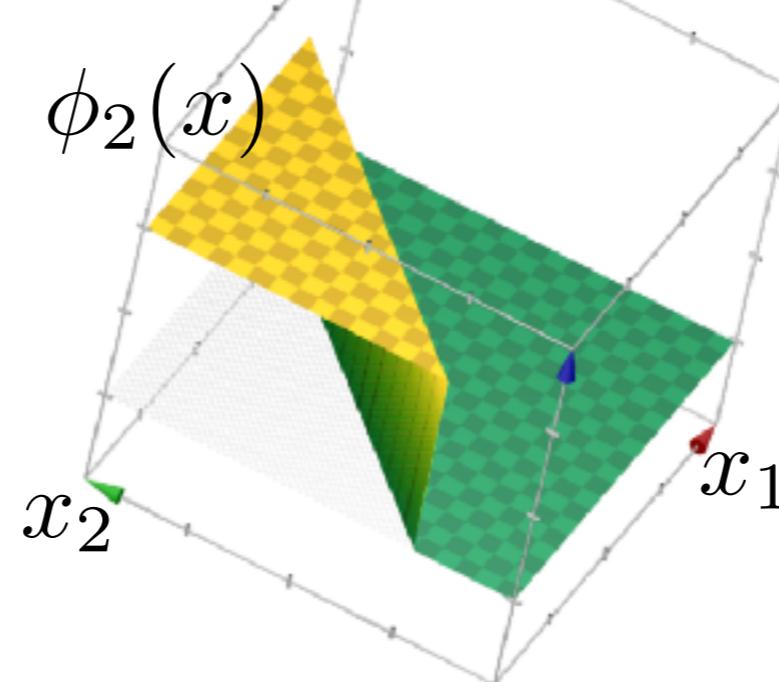


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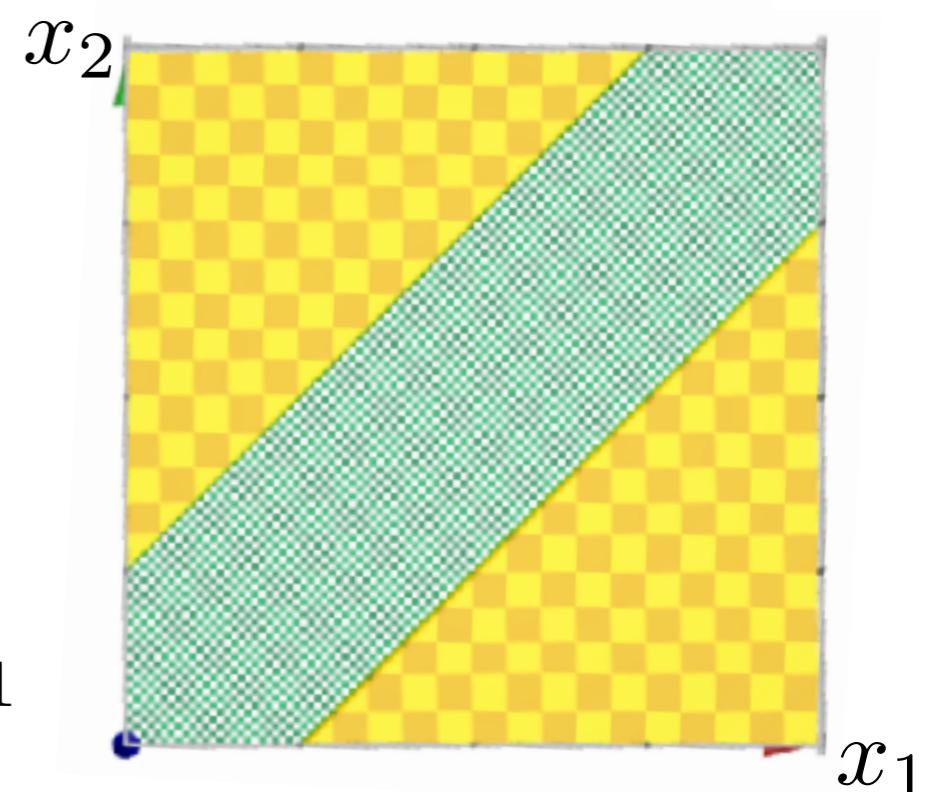
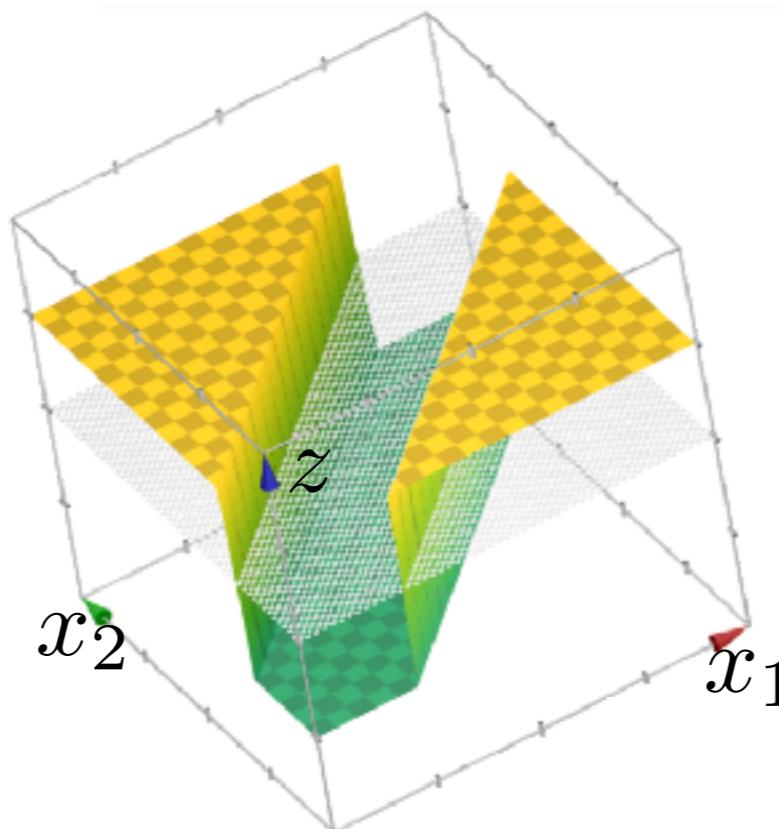
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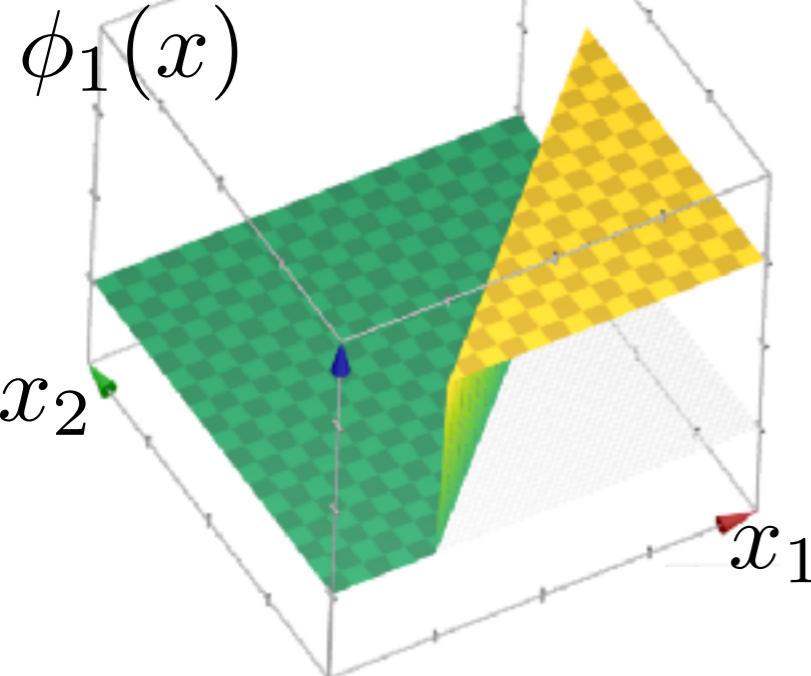


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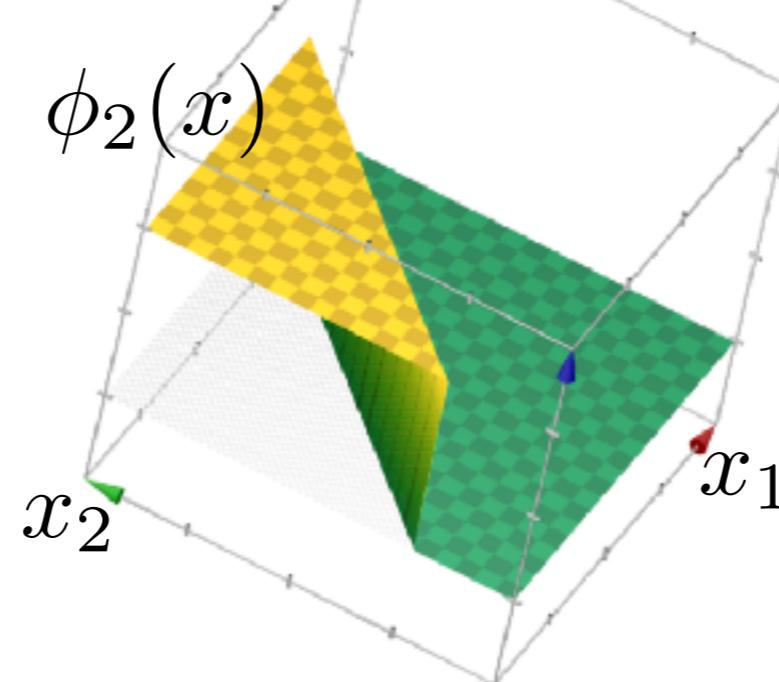


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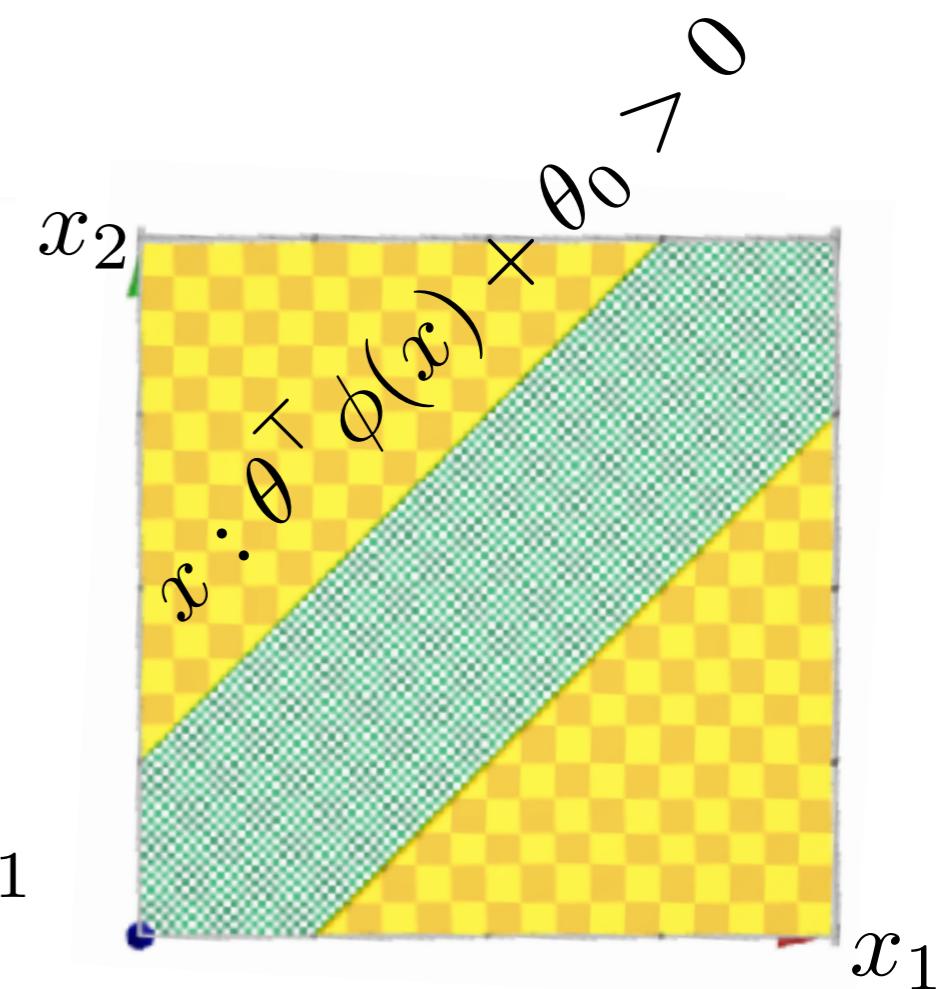
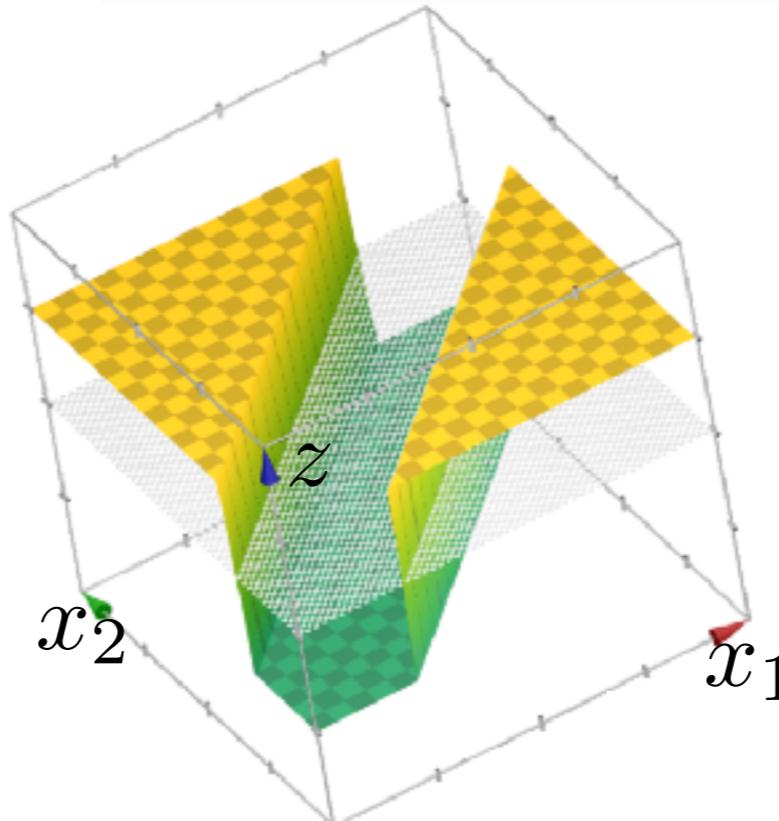
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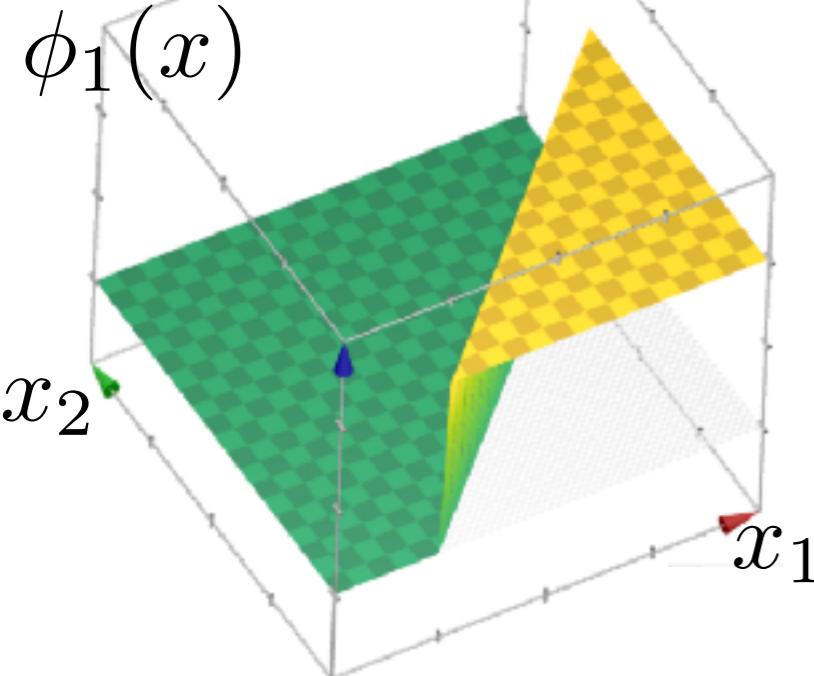


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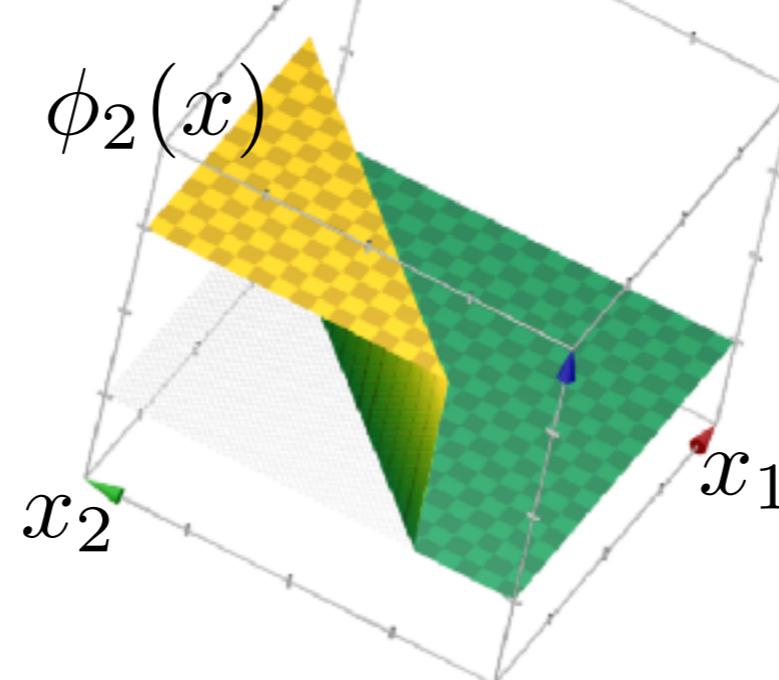


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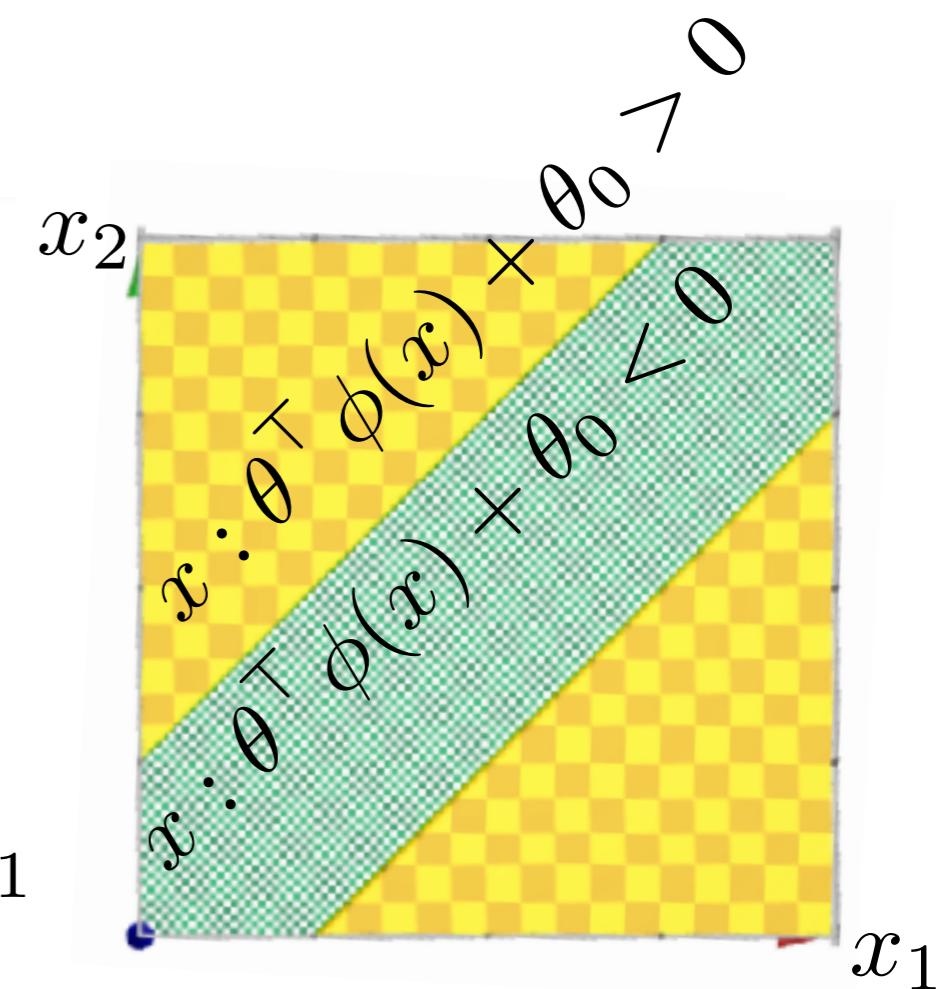
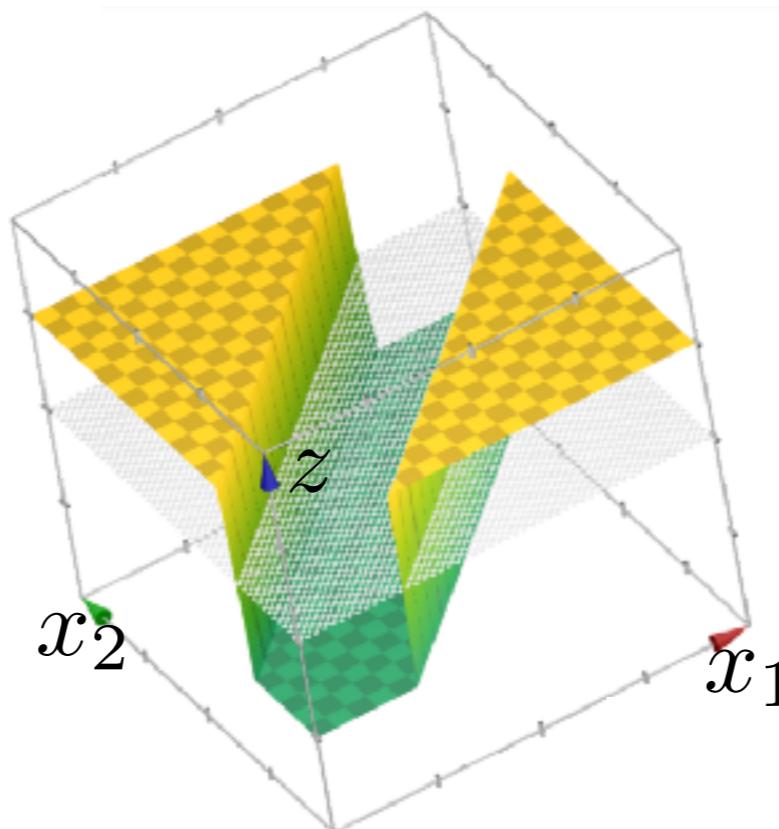
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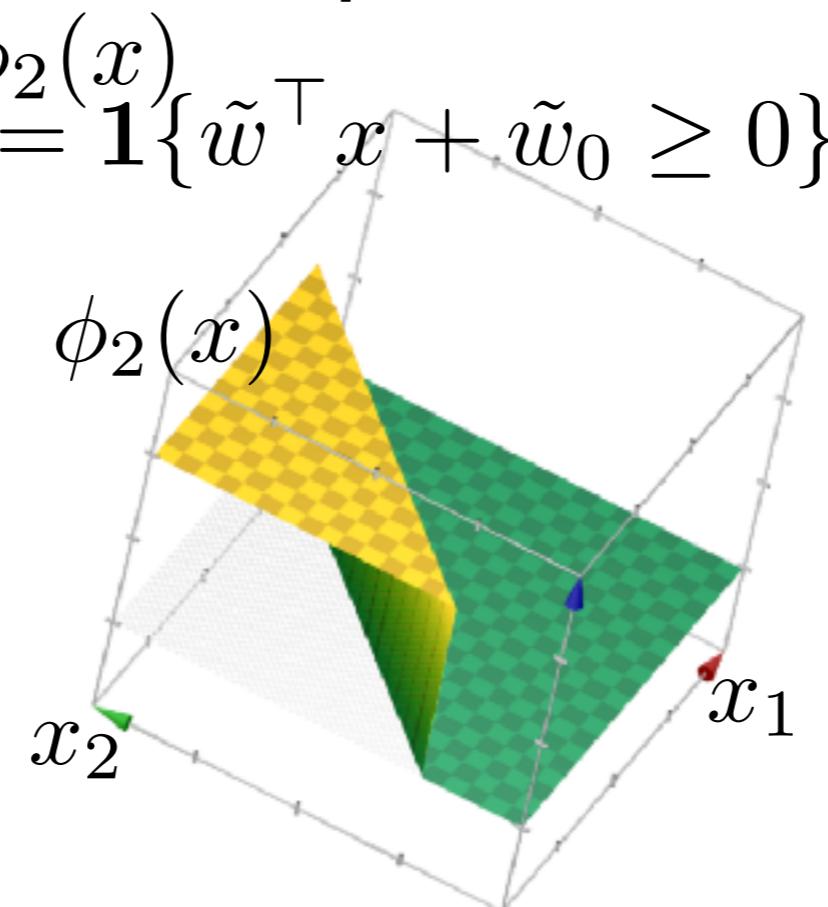
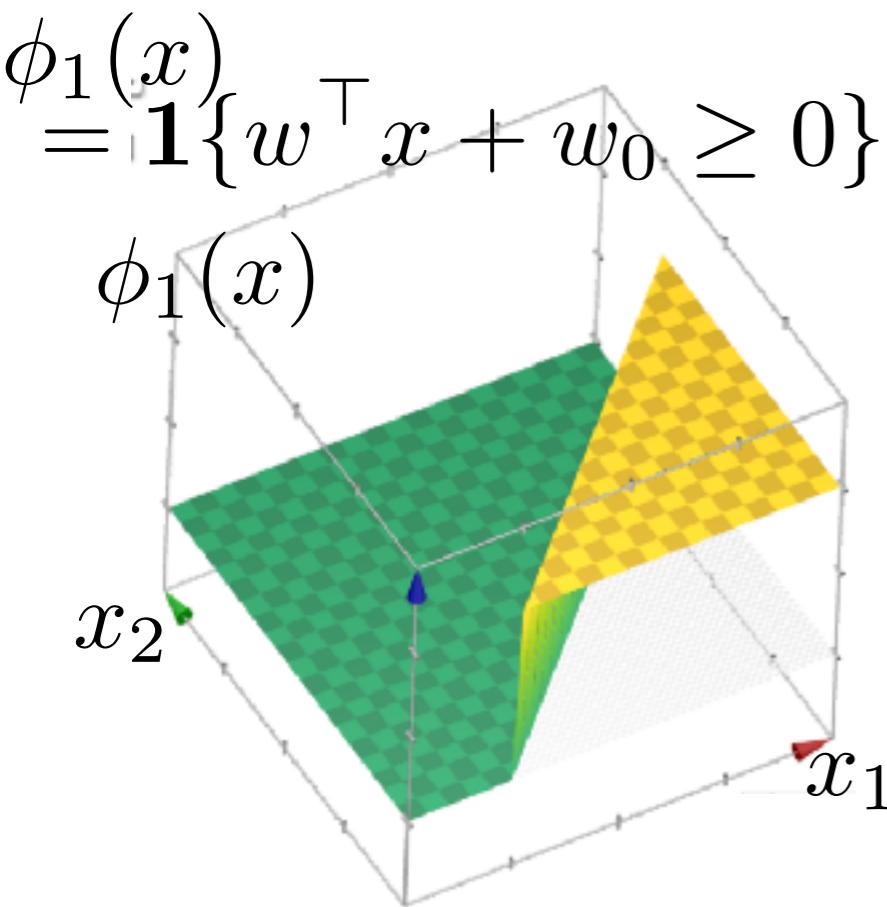
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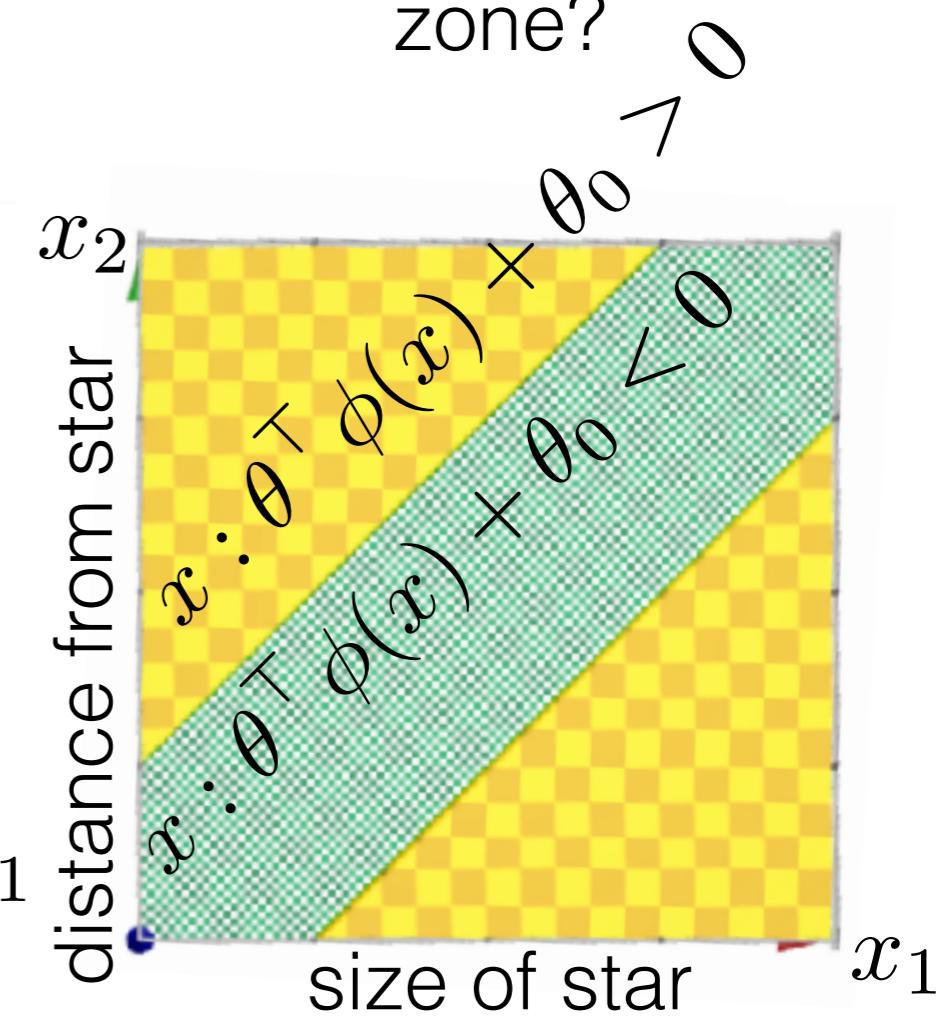
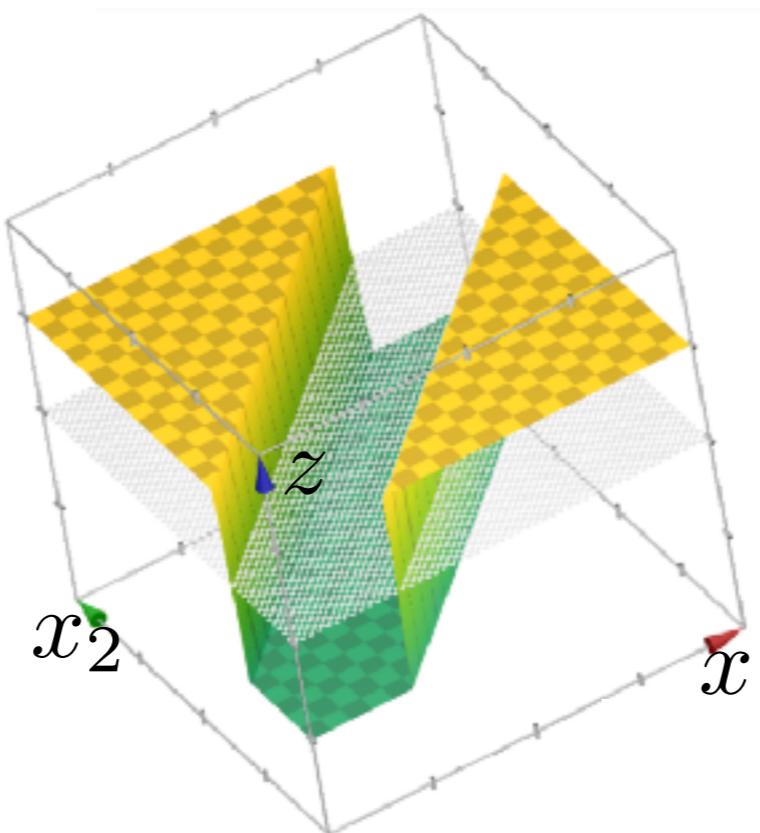


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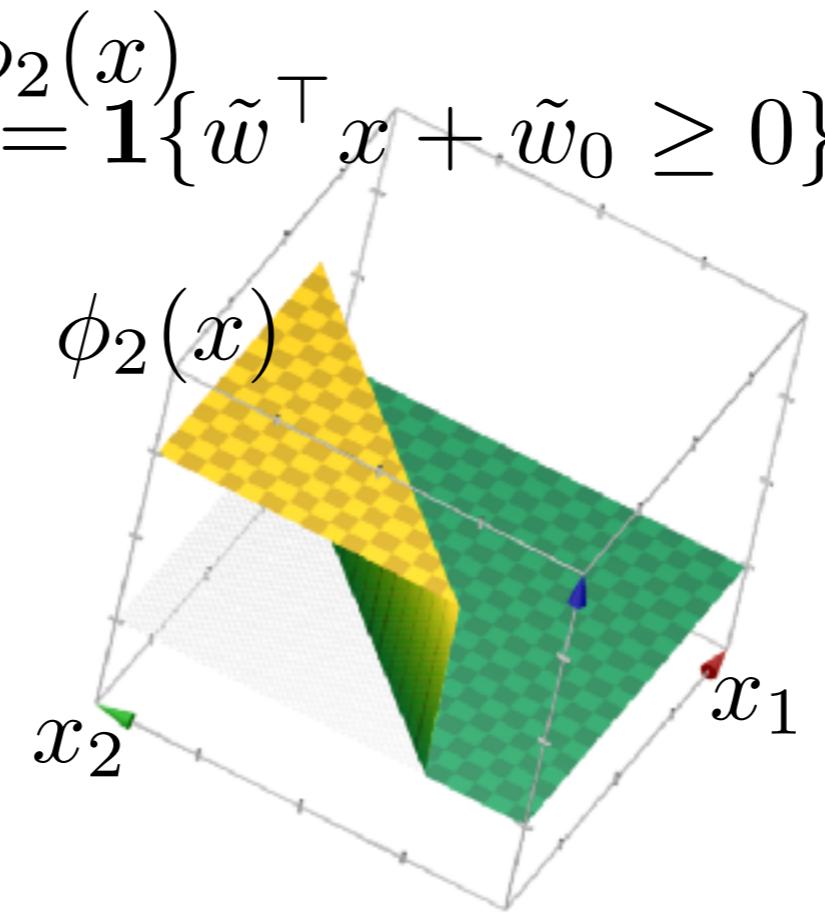
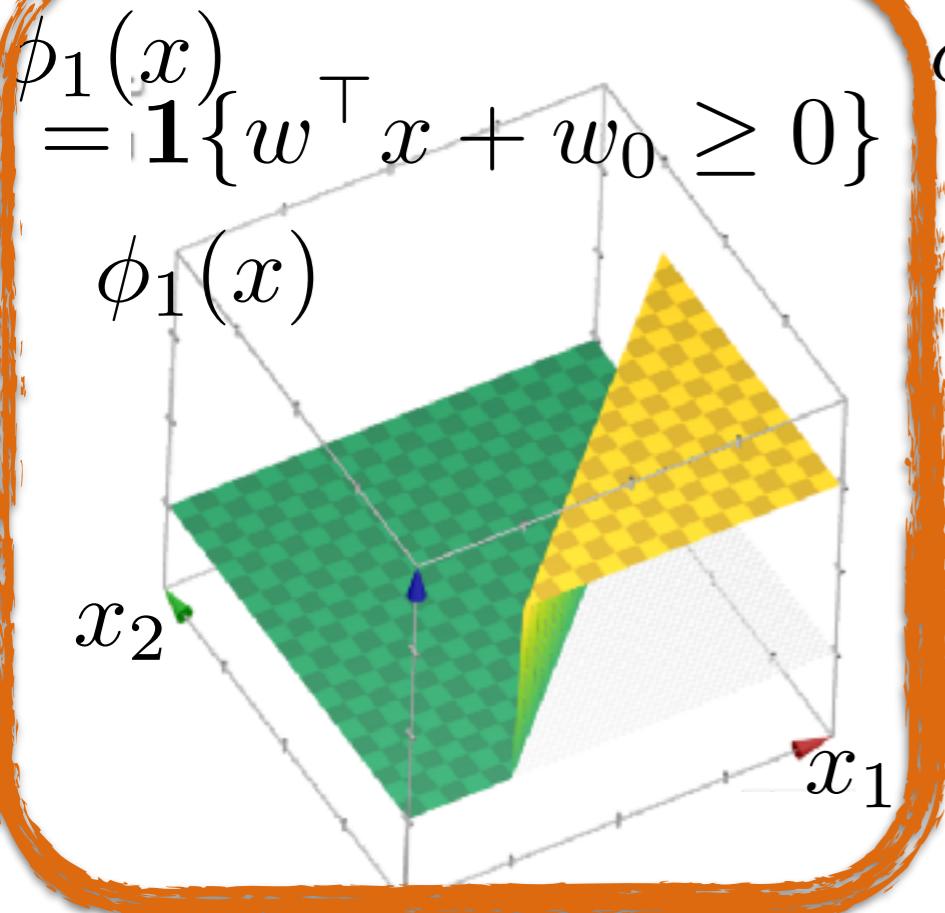


Is an exoplanet in  
the habitable  
zone?

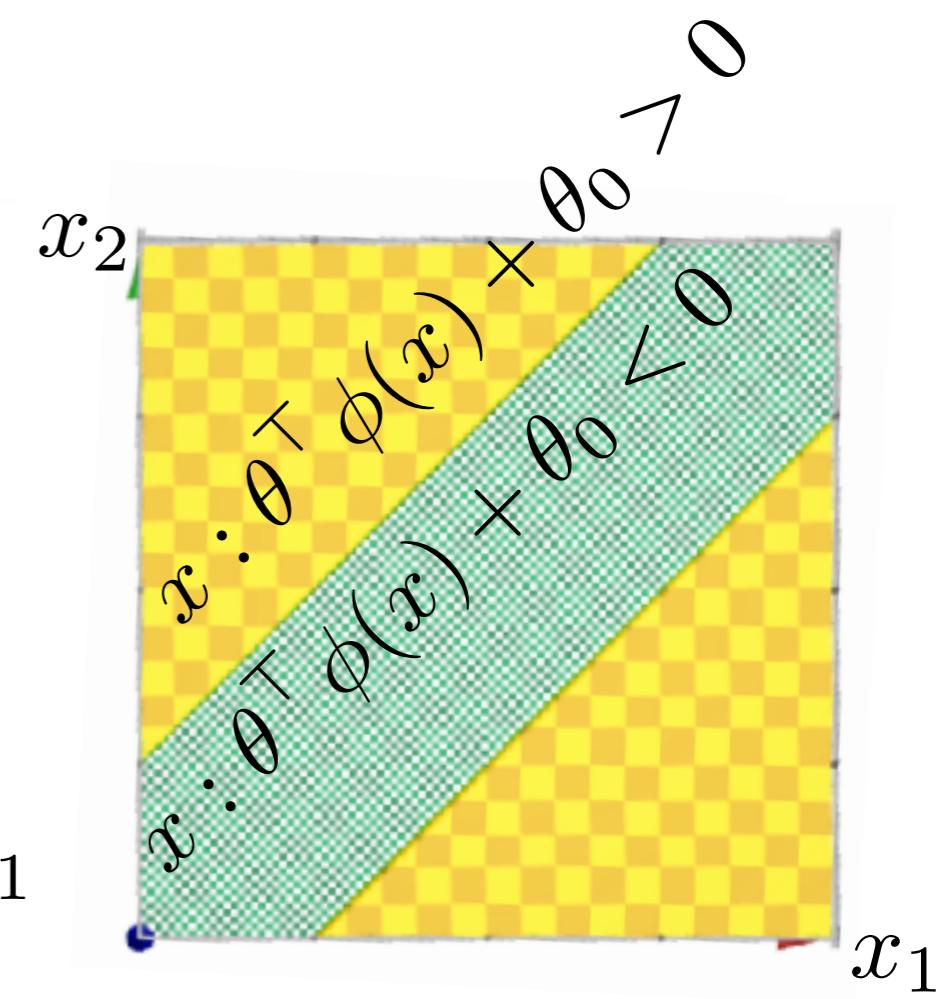
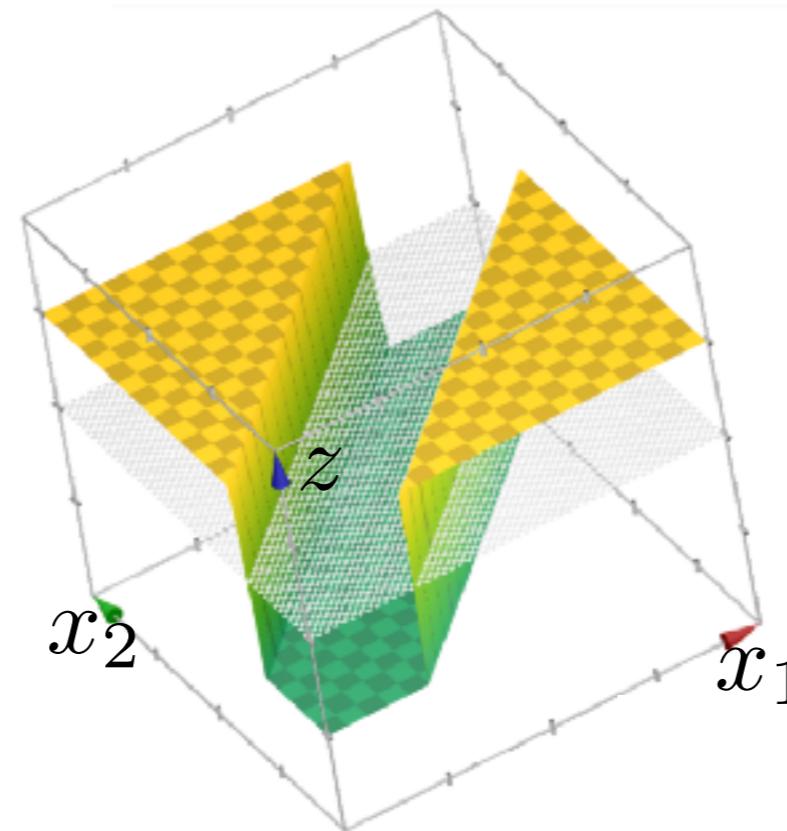
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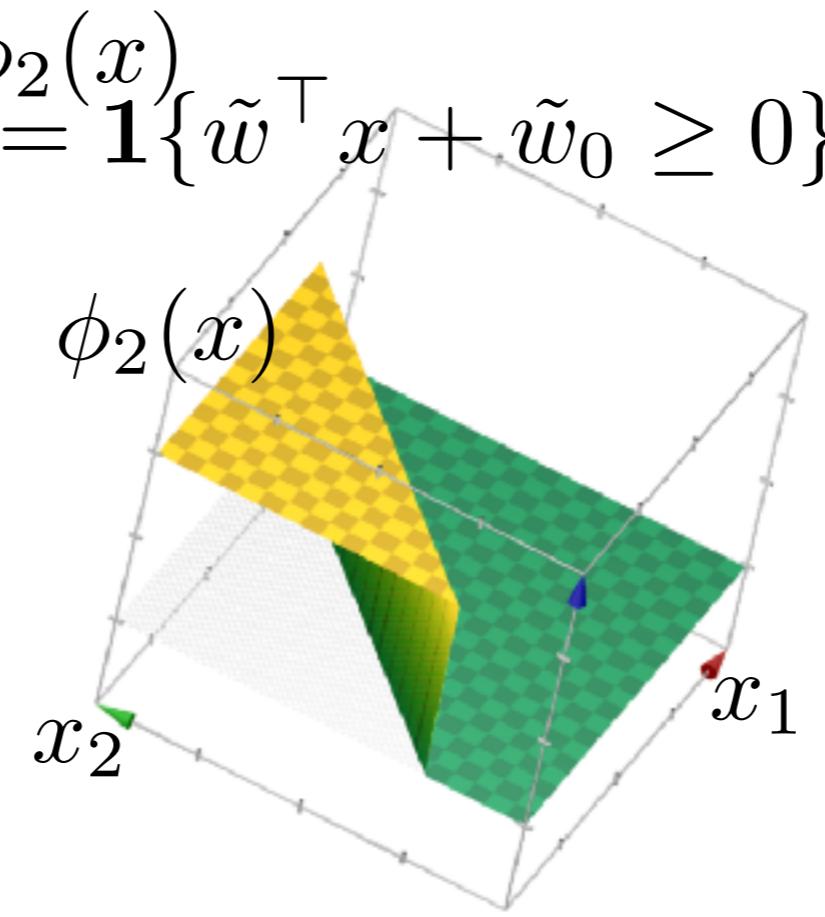
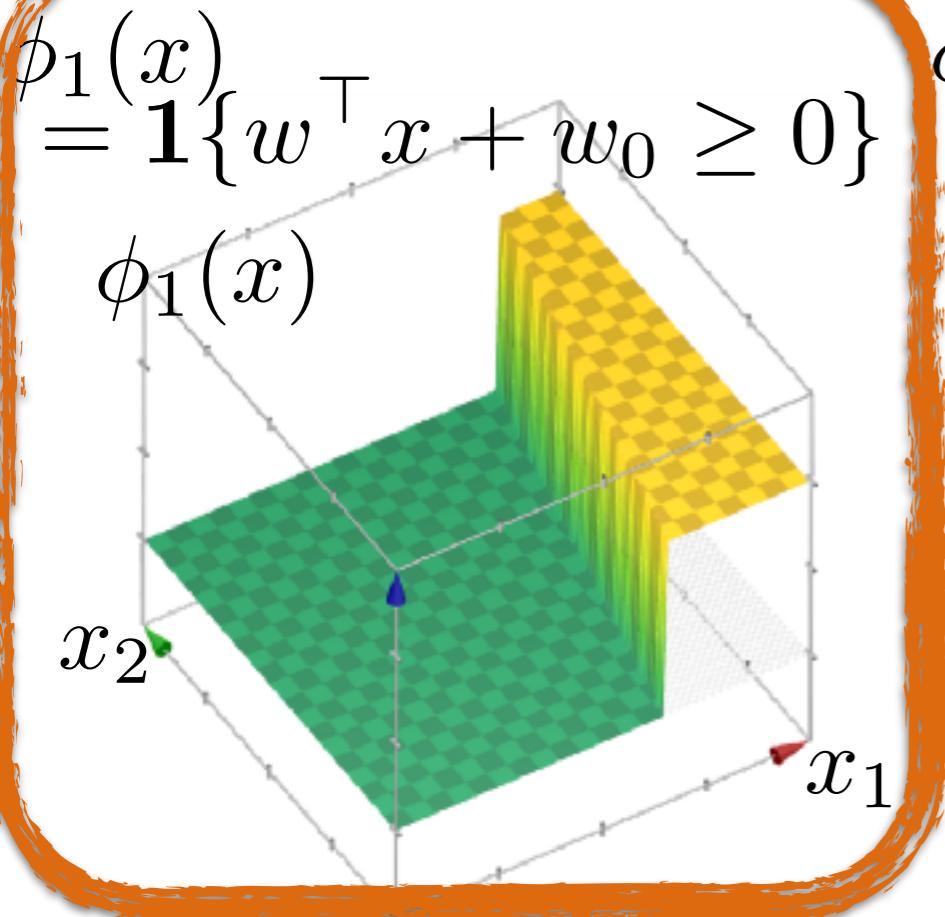
# New features: step functions!



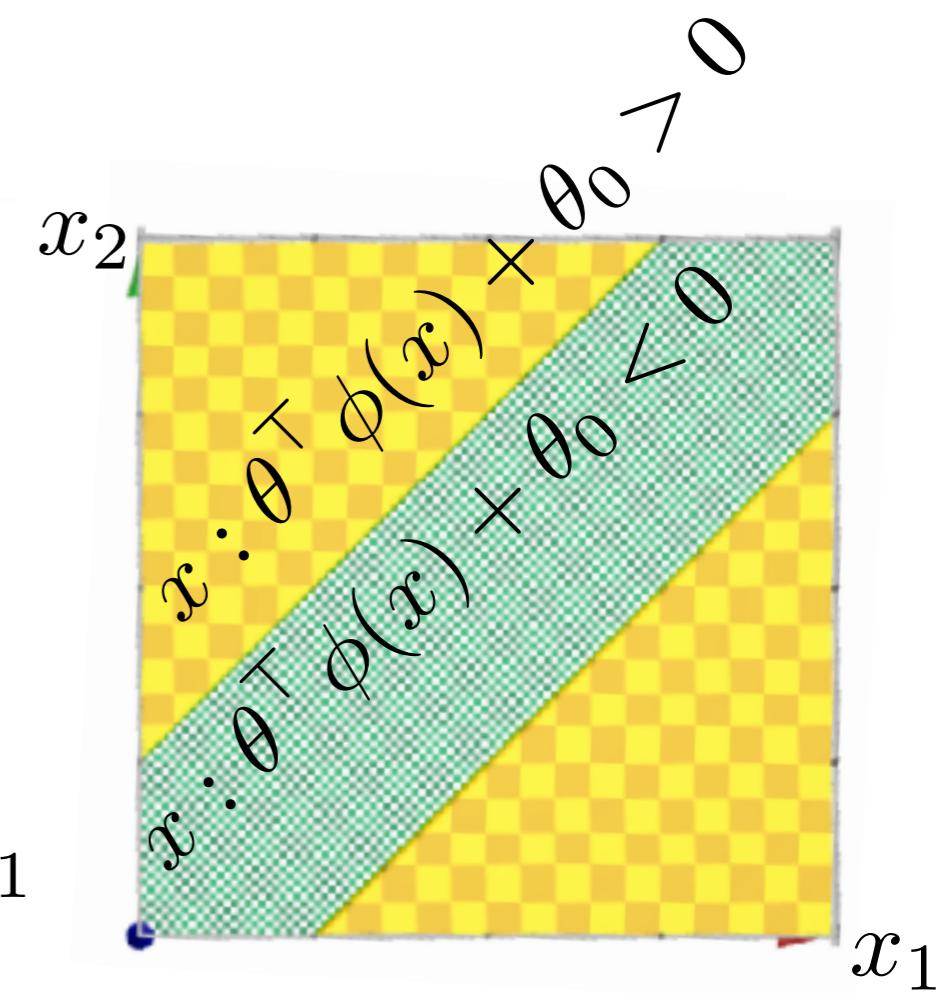
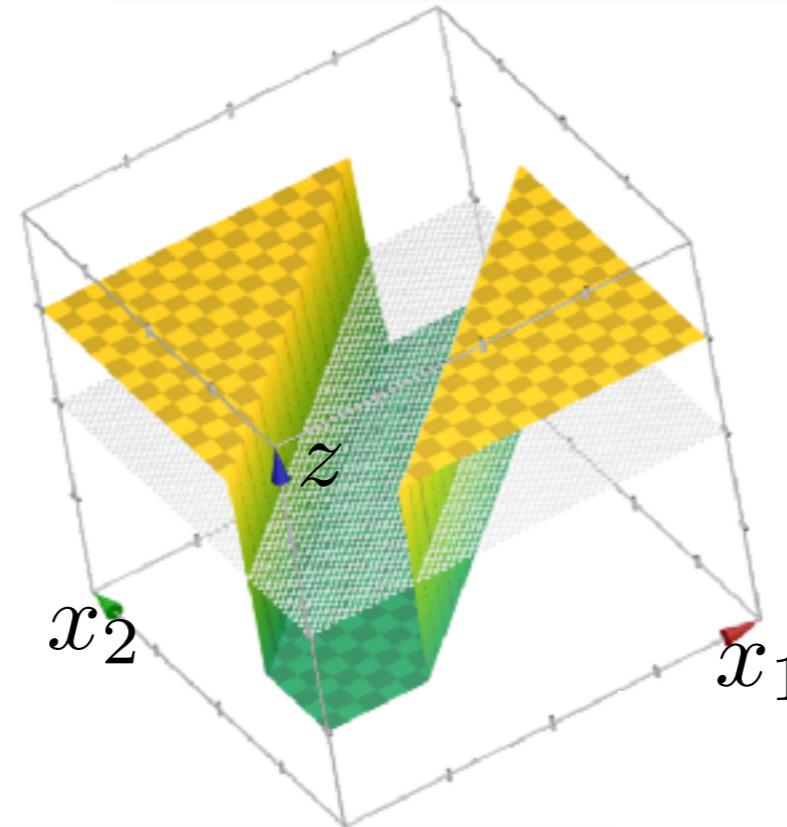
$$\begin{aligned}z &= \theta^\top \phi(x) + \theta_0 \\&= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\&\quad + \theta_0 \\&= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\&\quad + (-0.5)\end{aligned}$$



# New features: step functions!

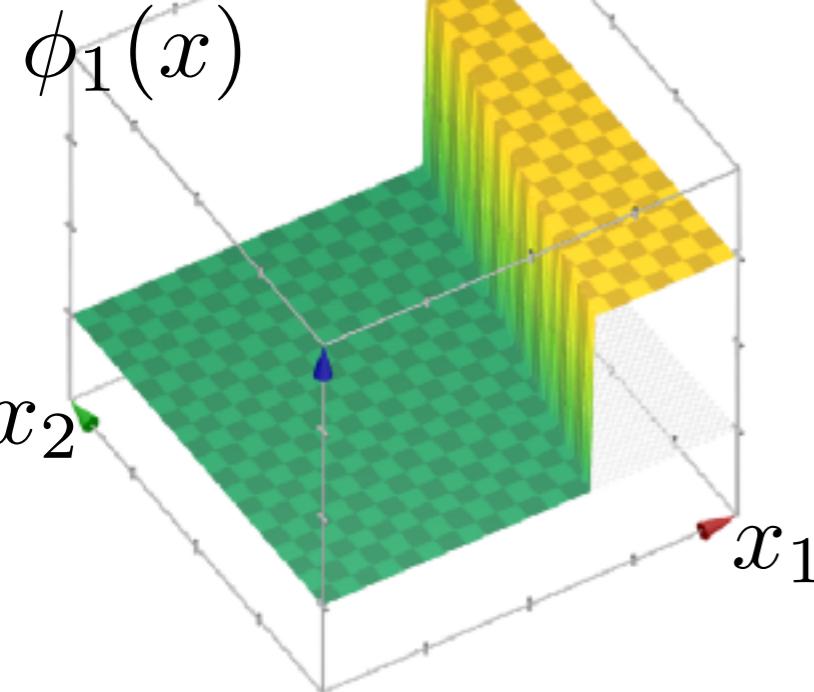


$$\begin{aligned}z &= \theta^\top \phi(x) + \theta_0 \\&= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\&\quad + \theta_0 \\&= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\&\quad + (-0.5)\end{aligned}$$

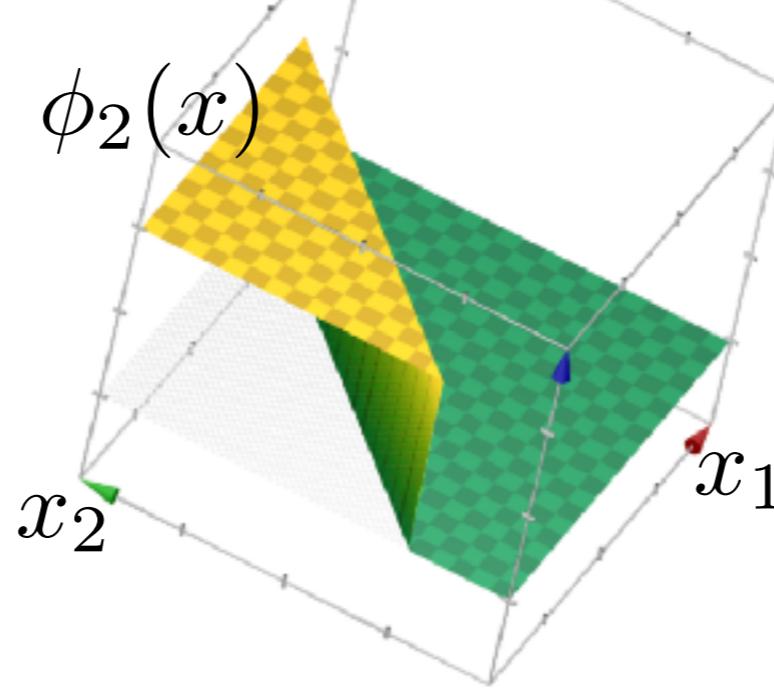


# New features: step functions!

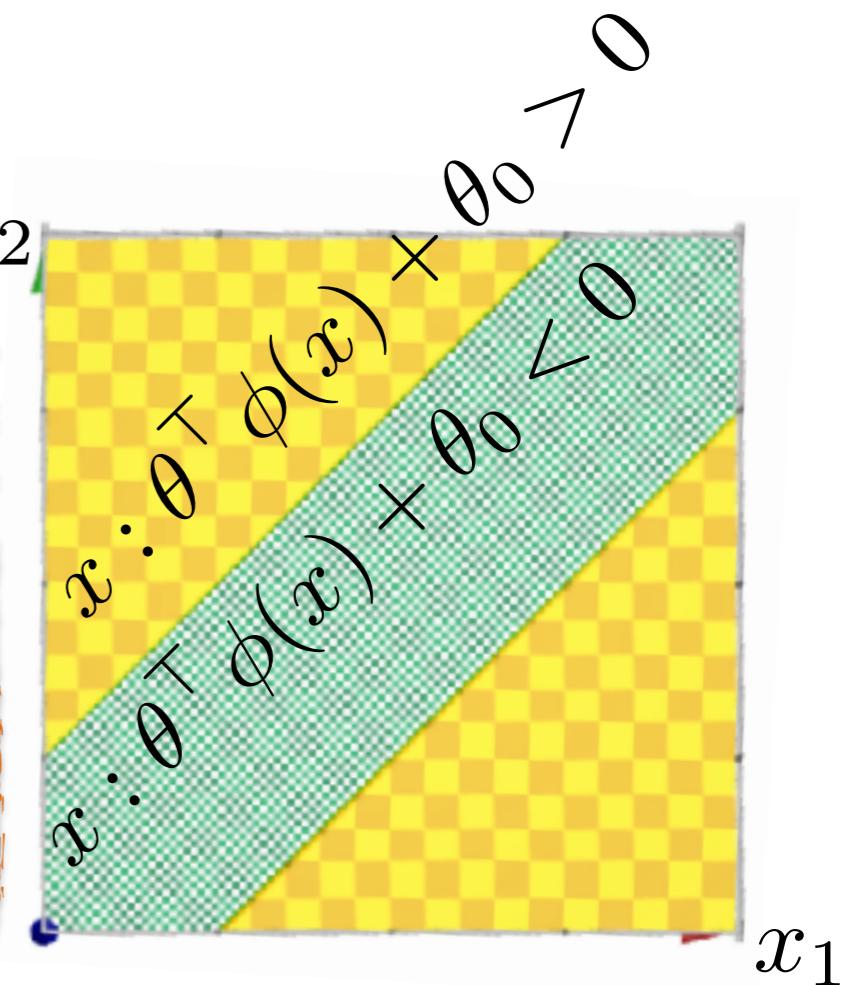
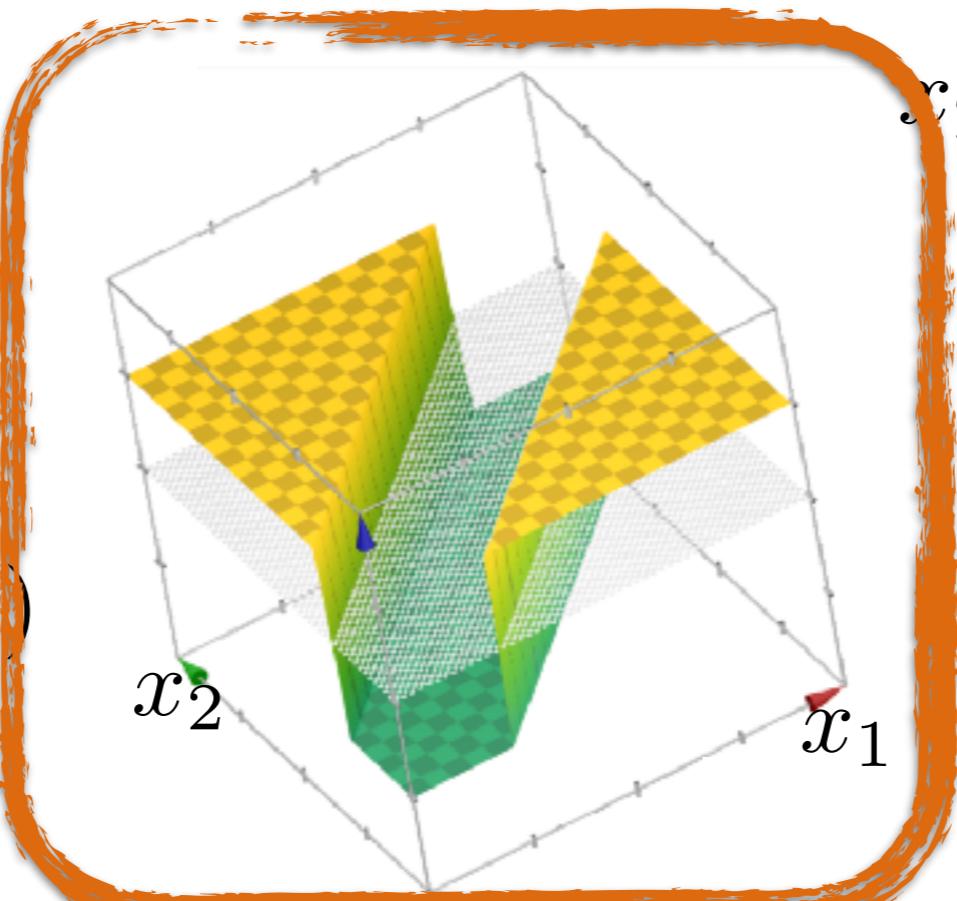
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

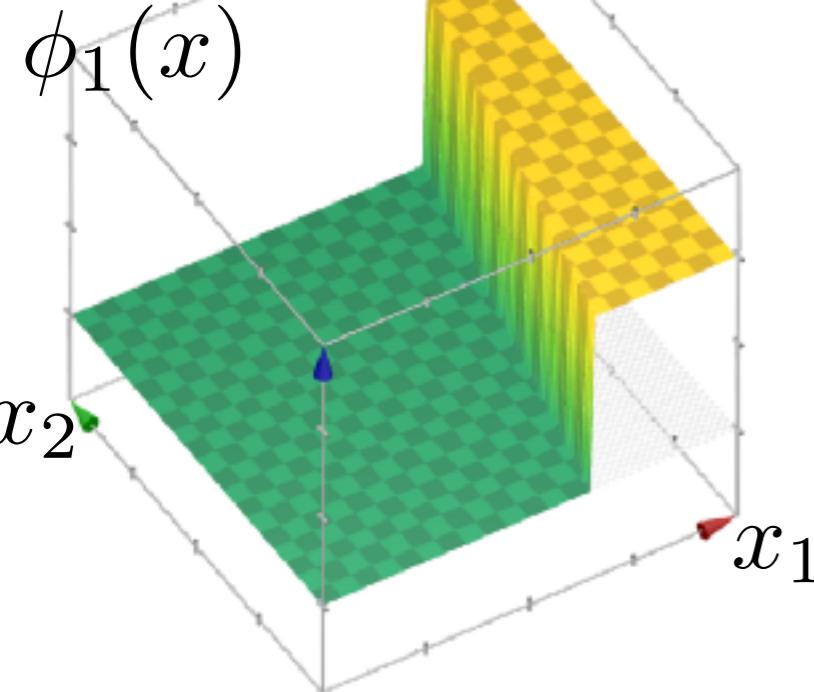


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

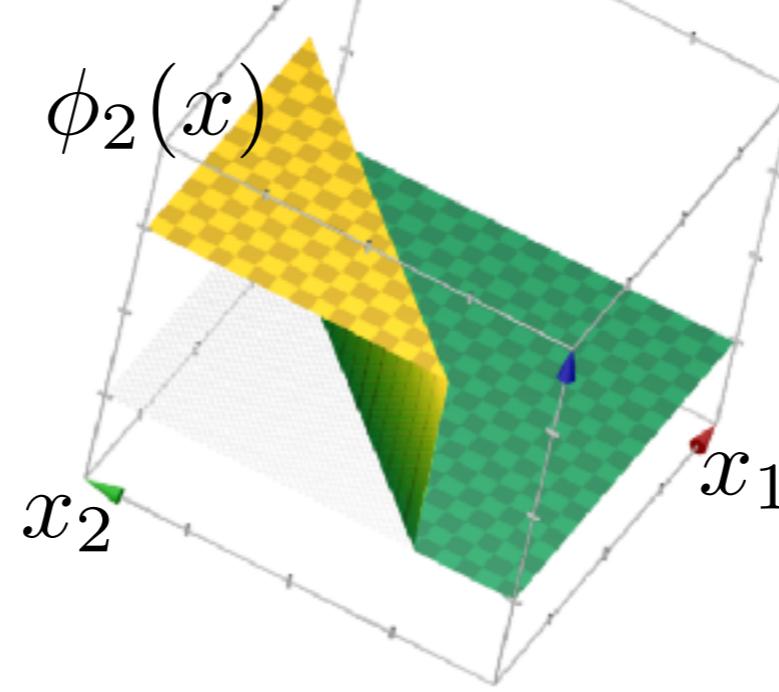


# New features: step functions!

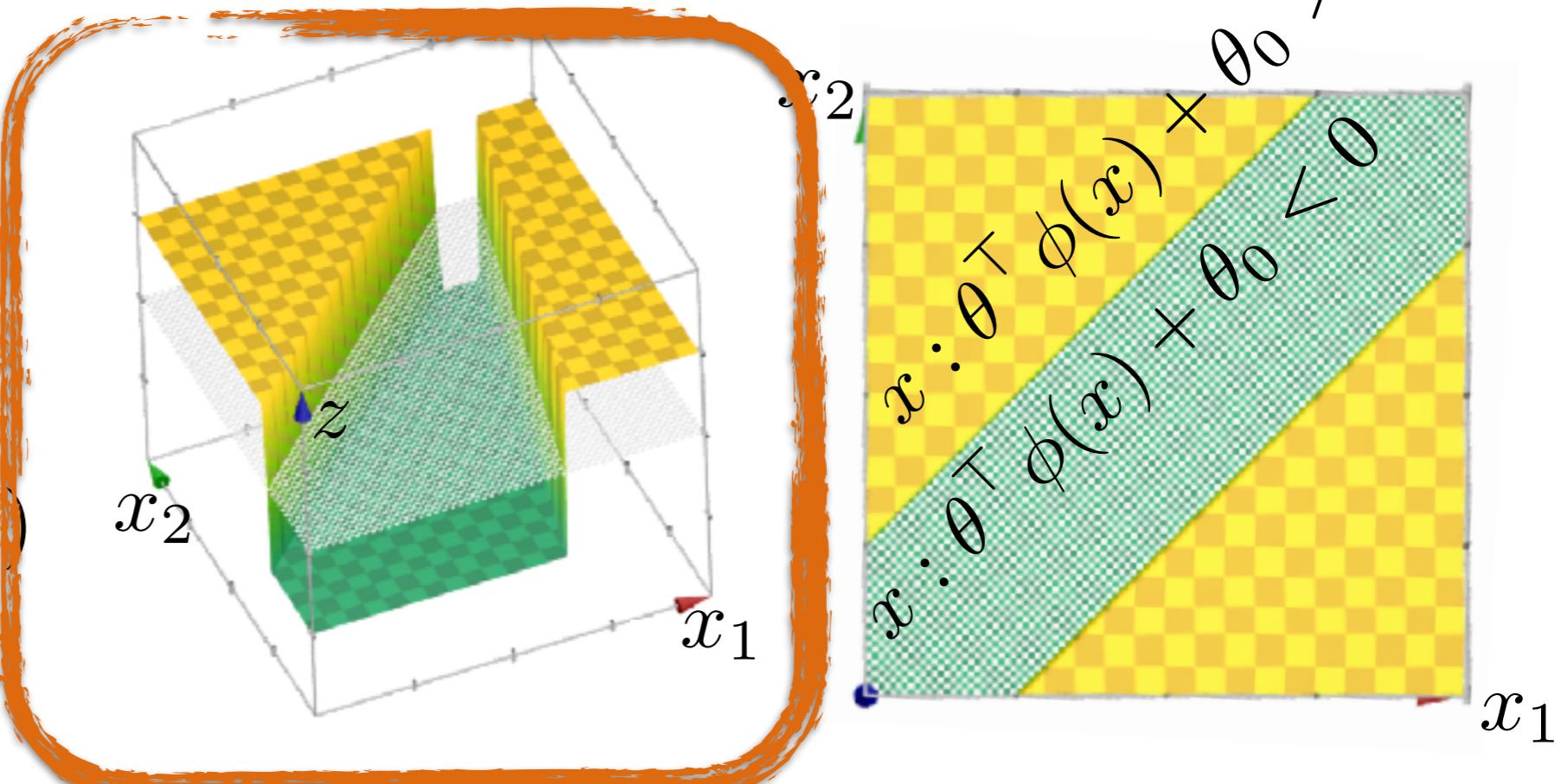
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

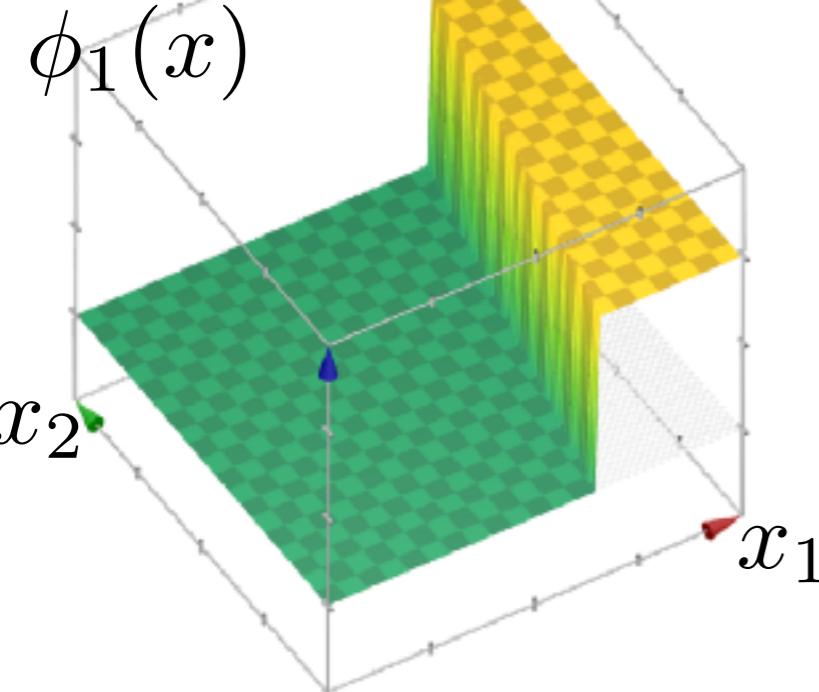


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

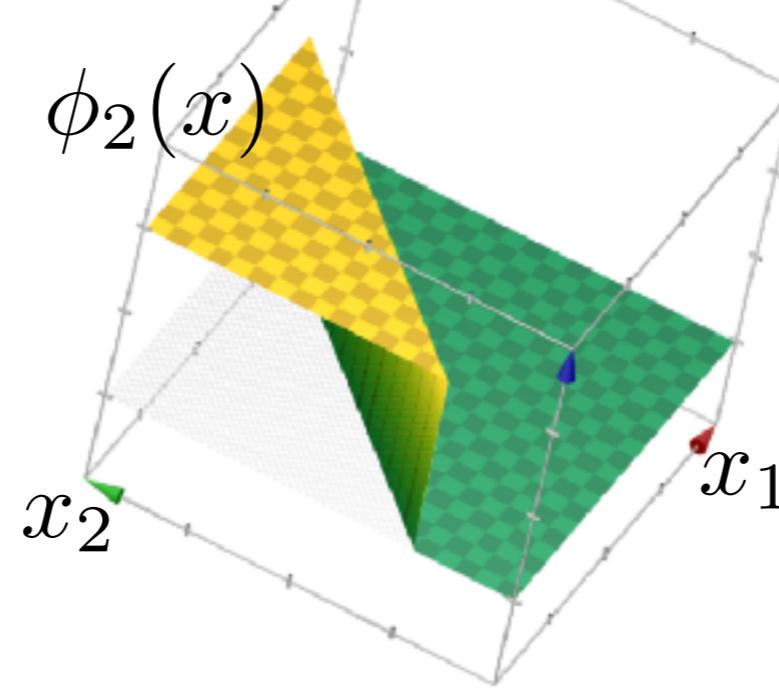


# New features: step functions!

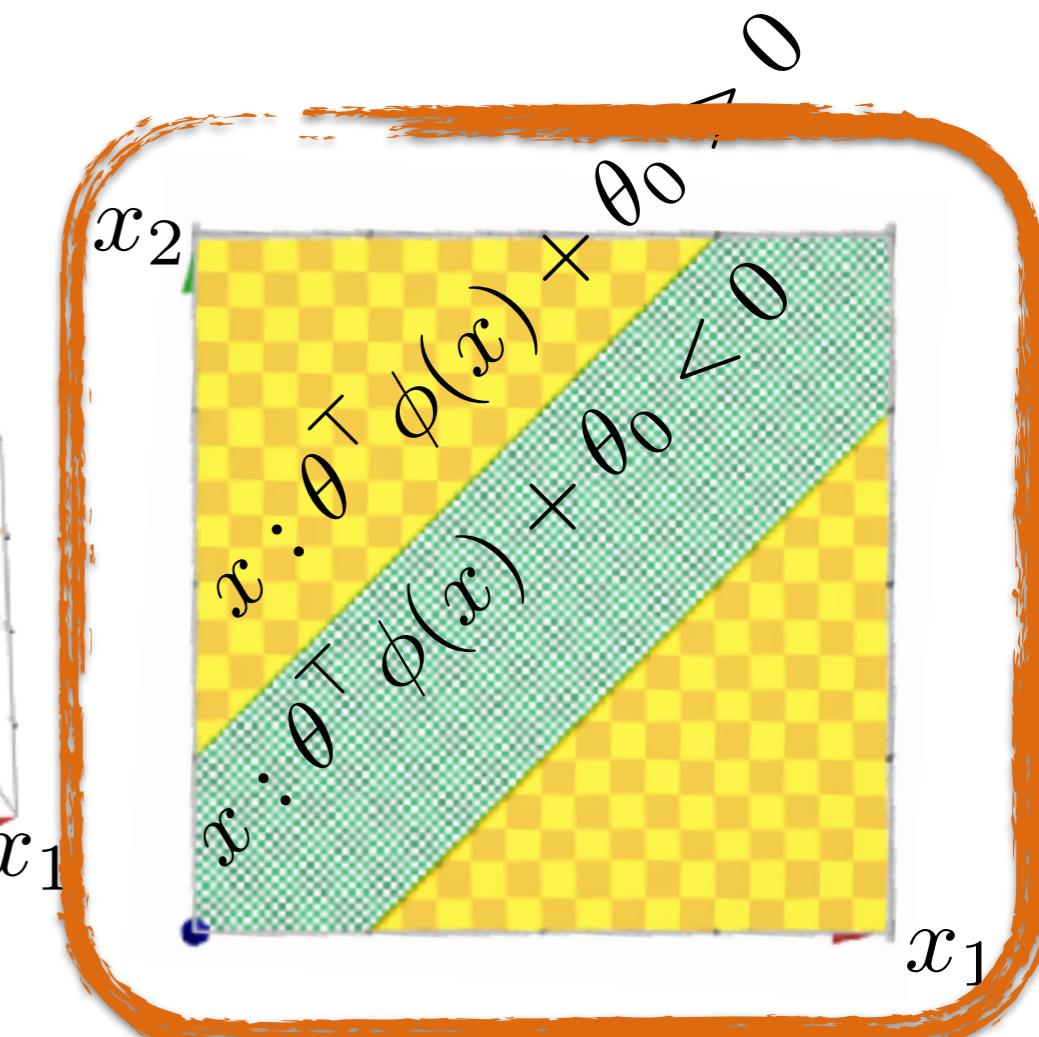
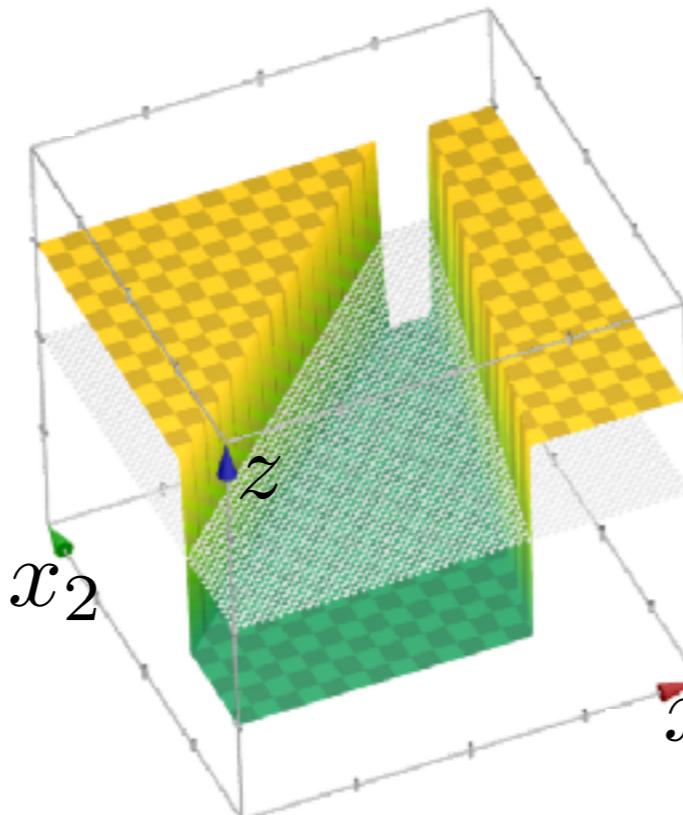
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

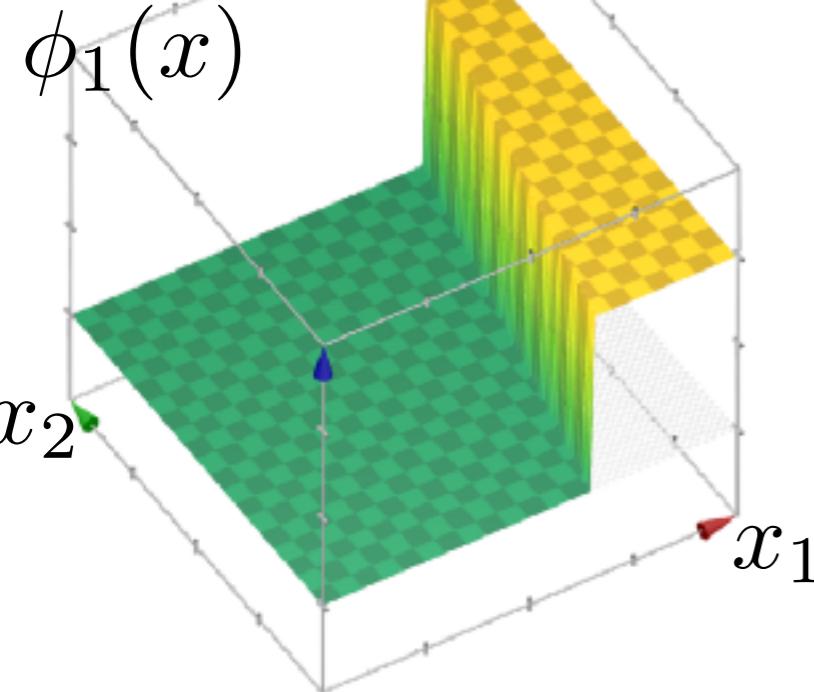


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

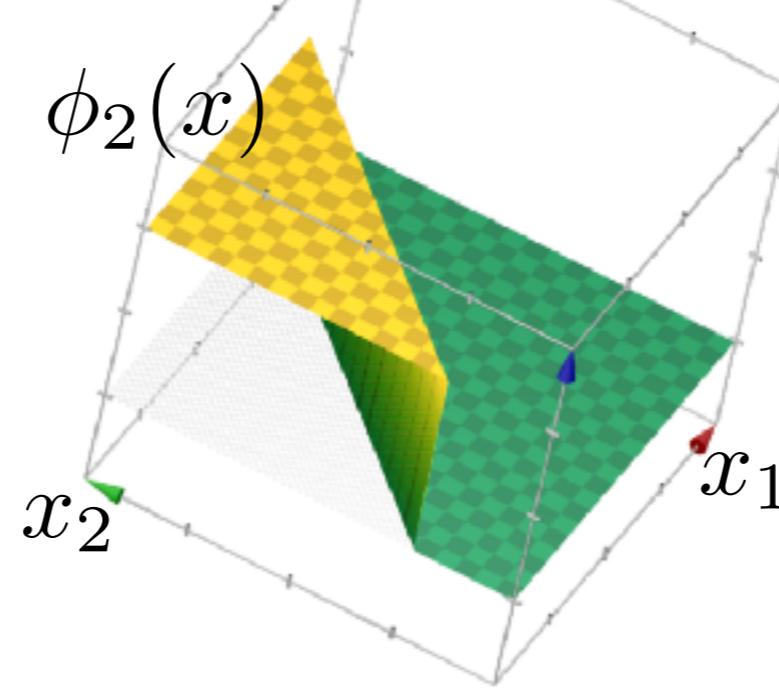


# New features: step functions!

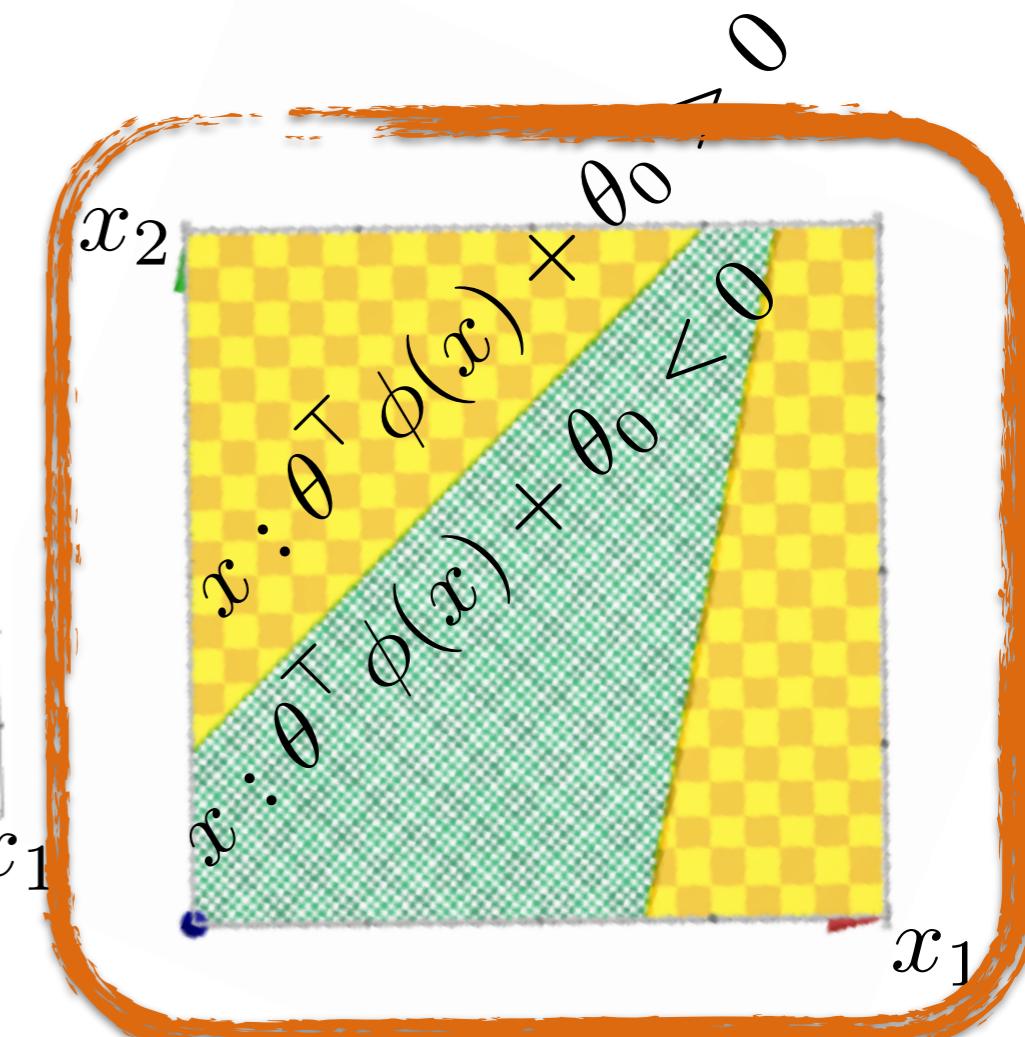
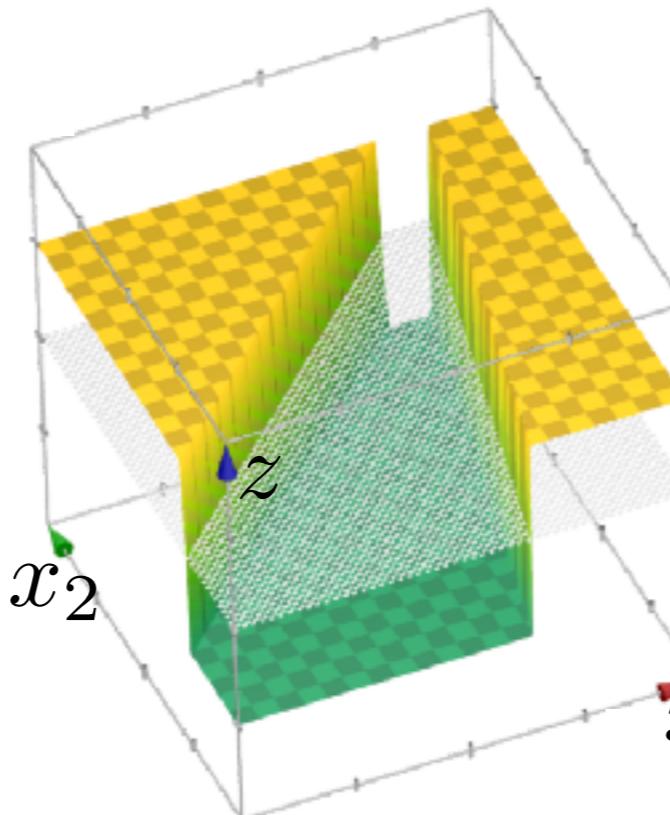
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



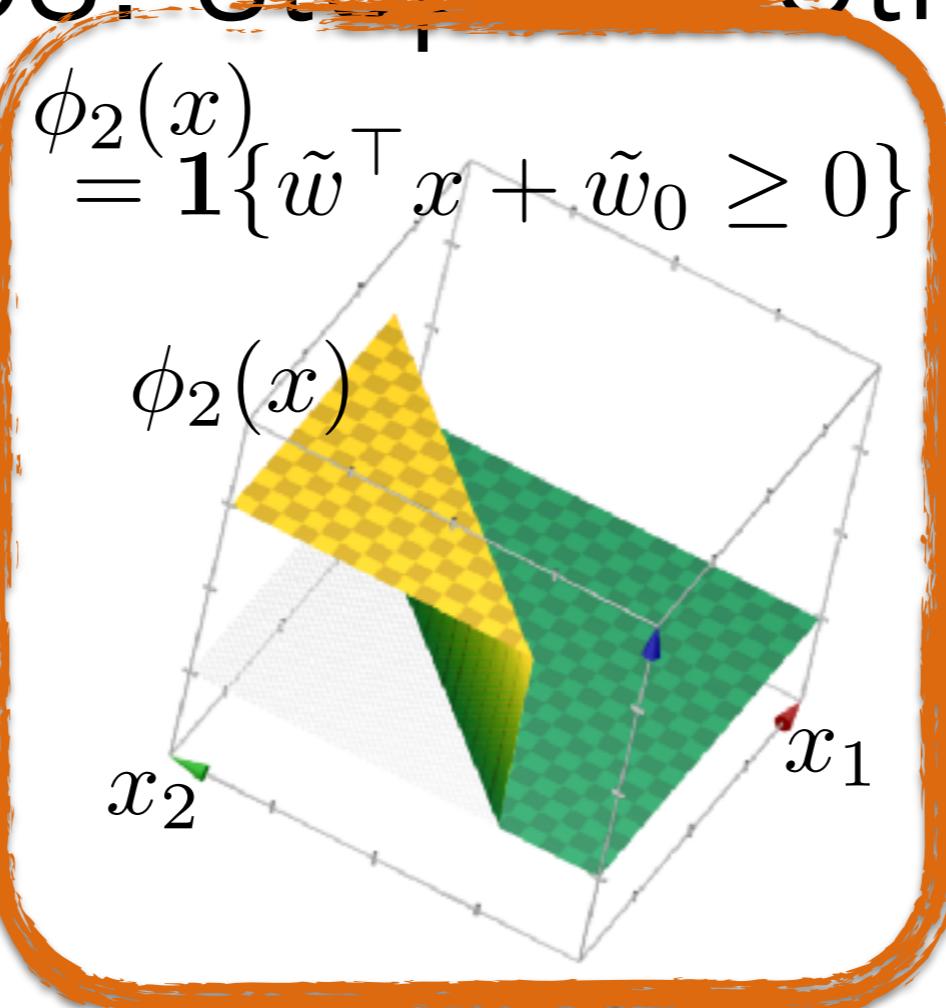
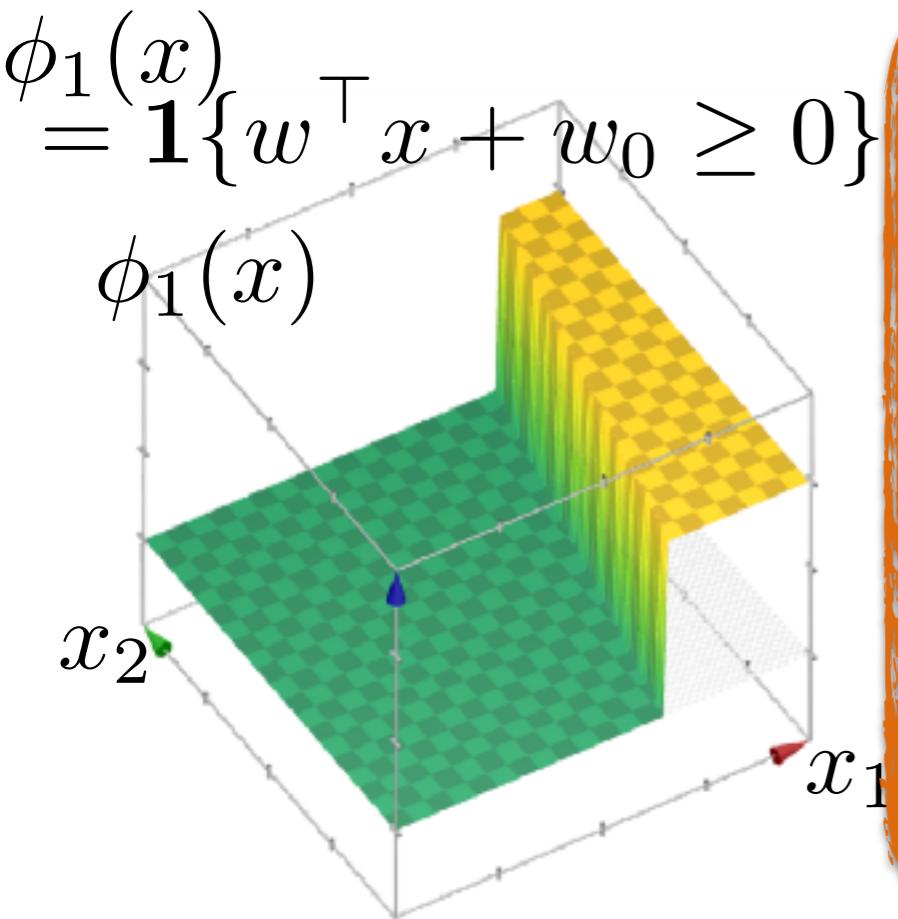
$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



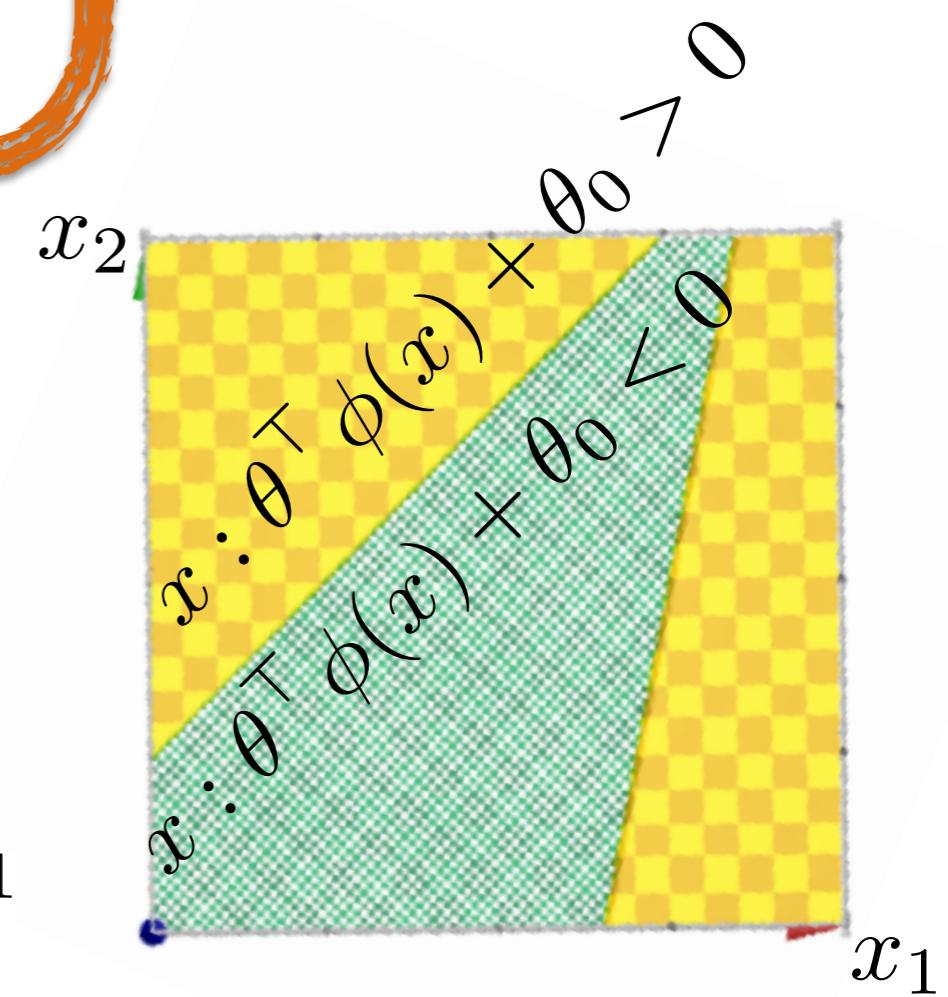
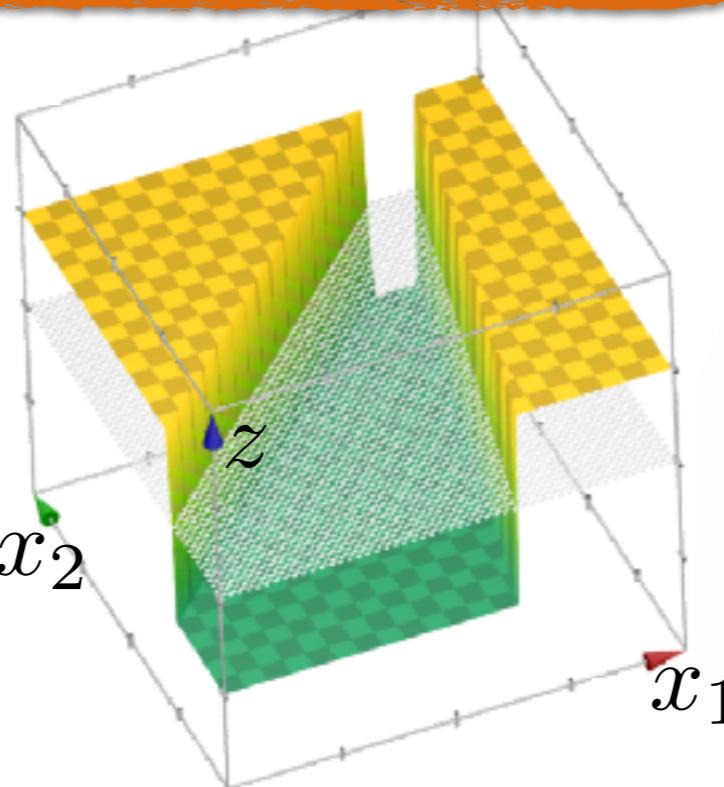
$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$



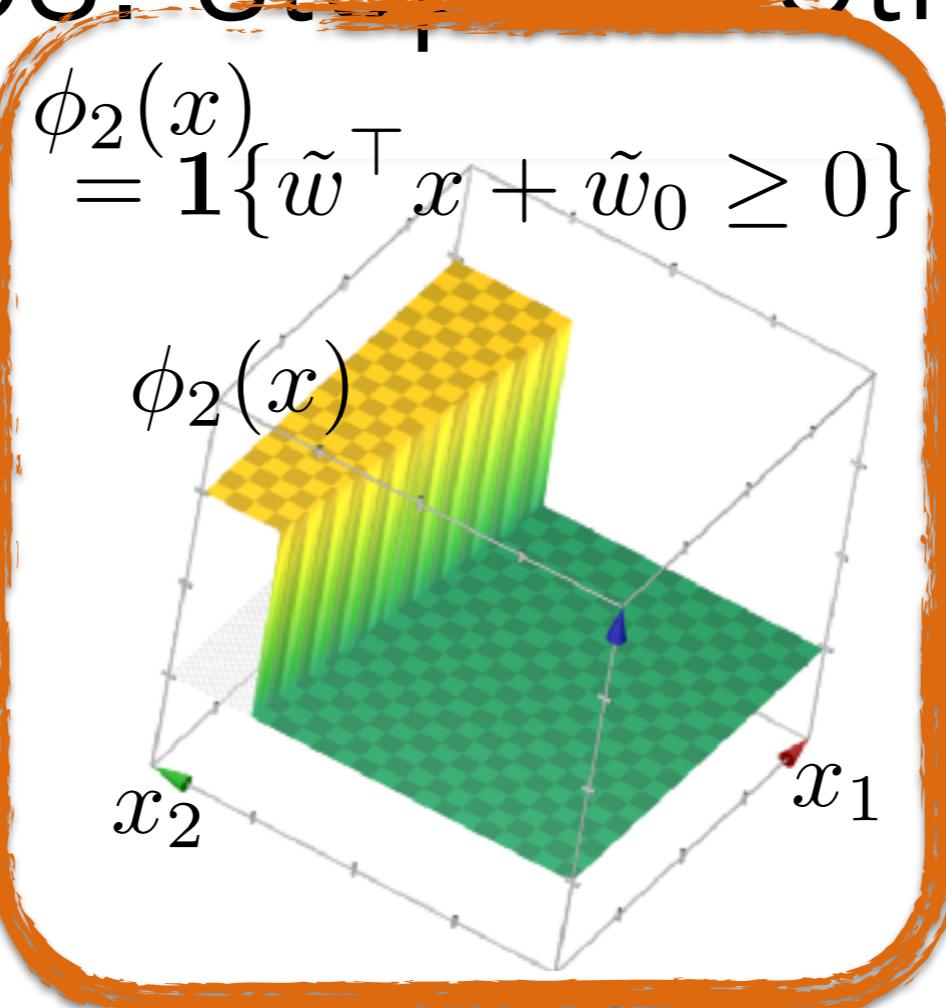
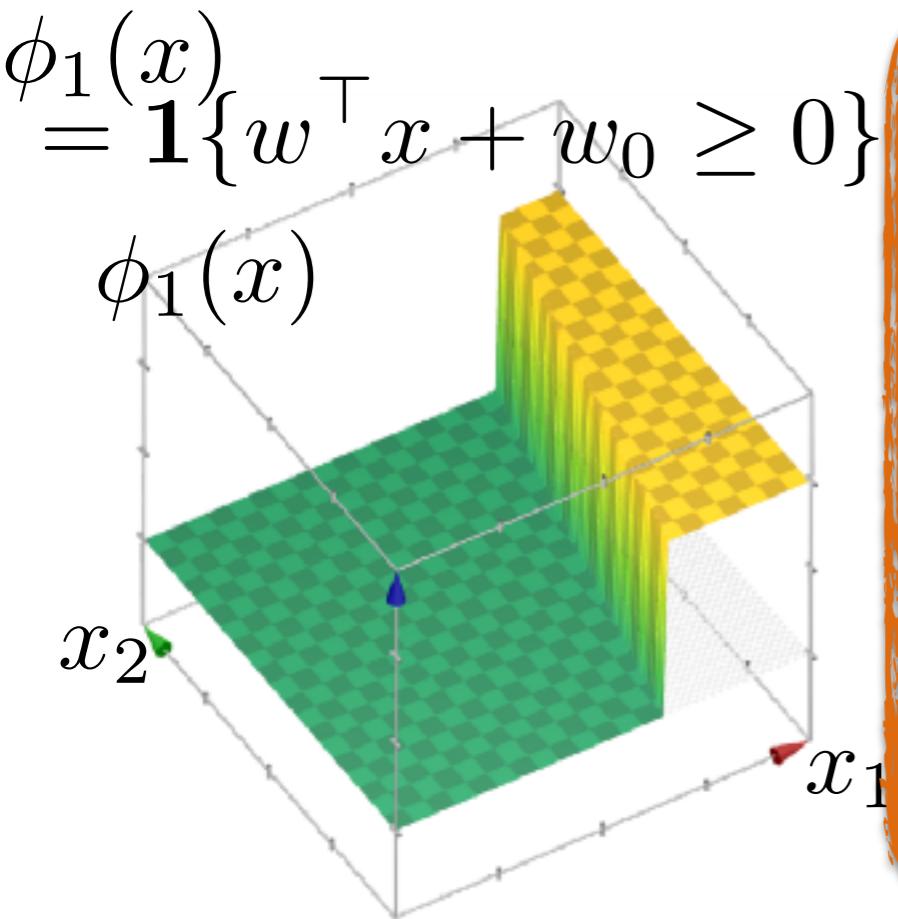
# New features: step functions!



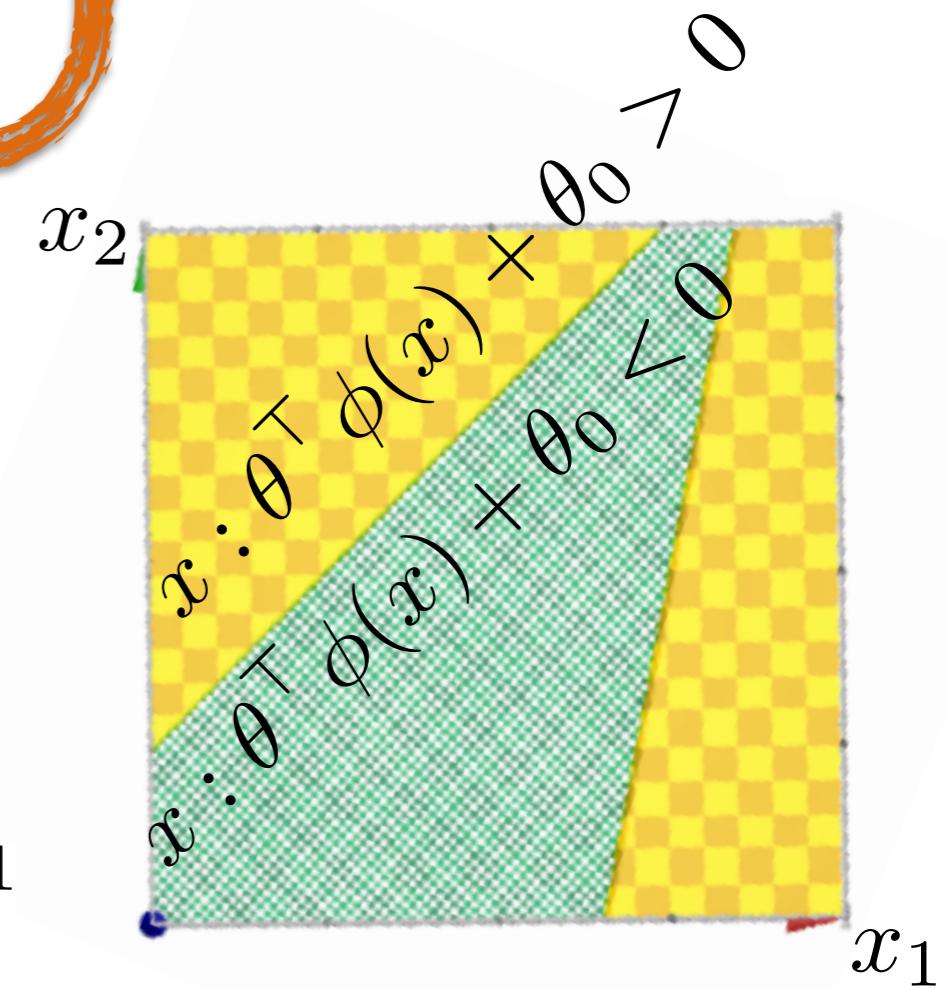
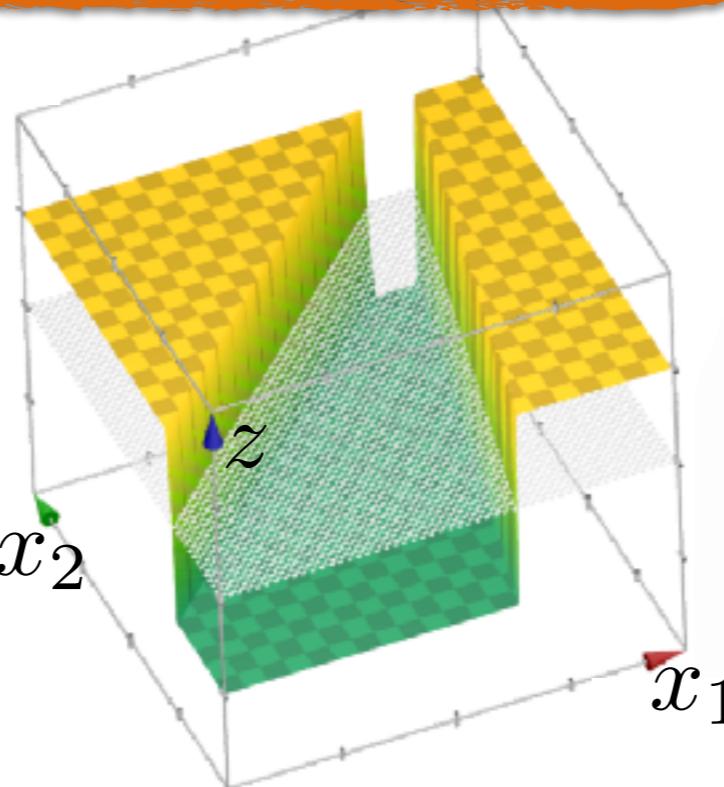
$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$



# New features: step functions!

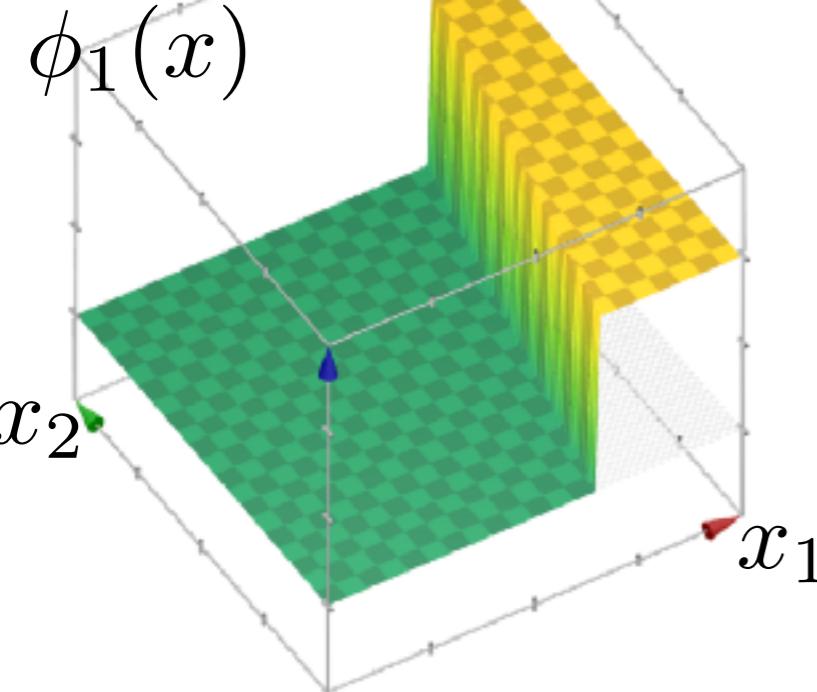


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

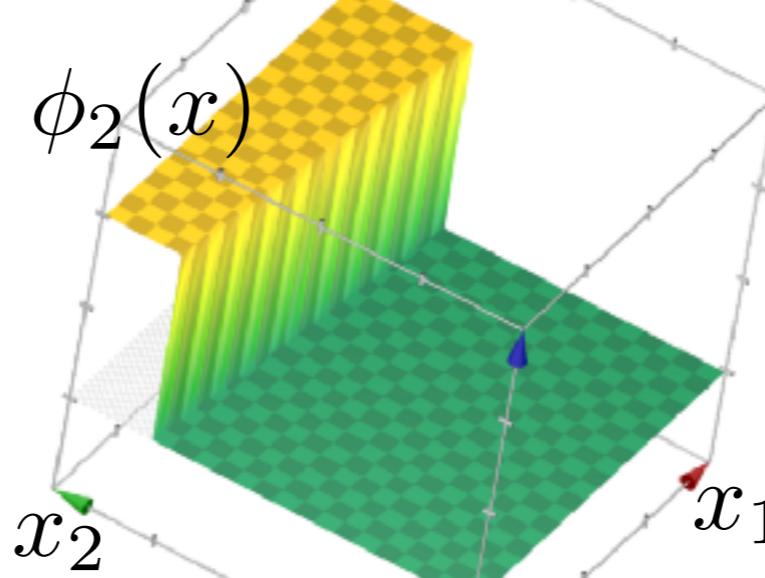


# New features: step functions!

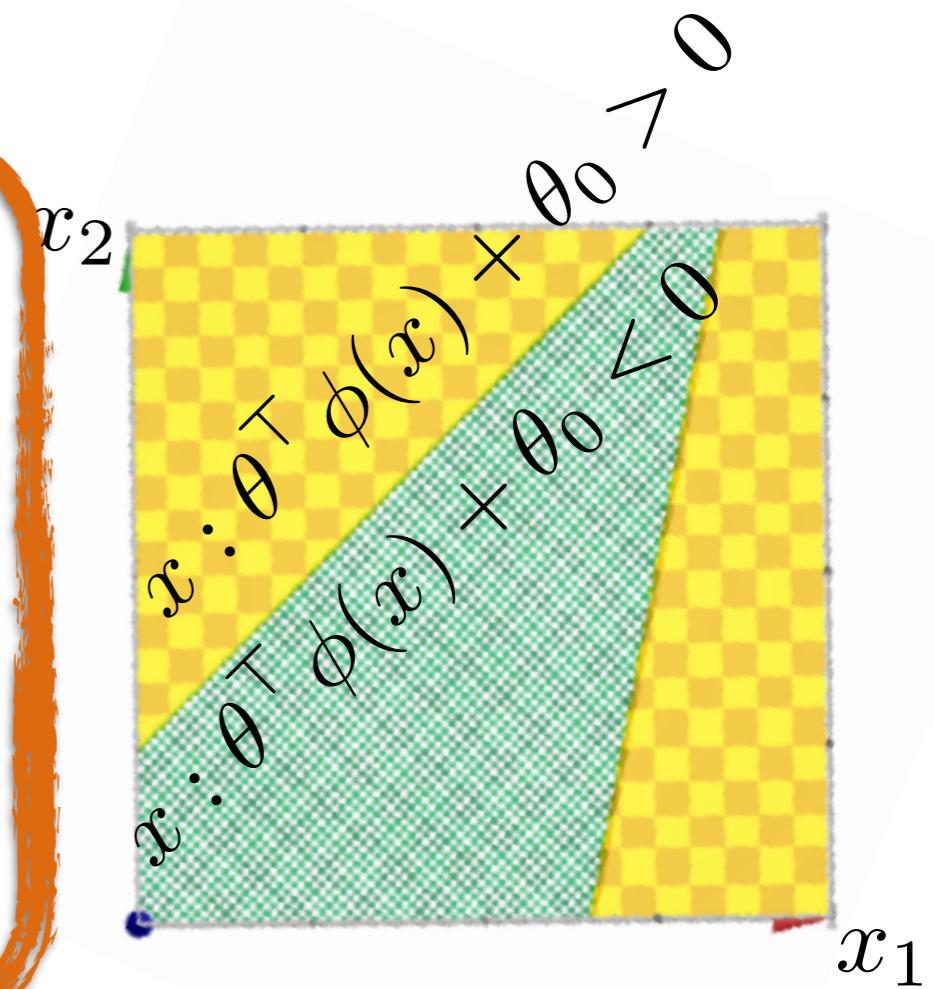
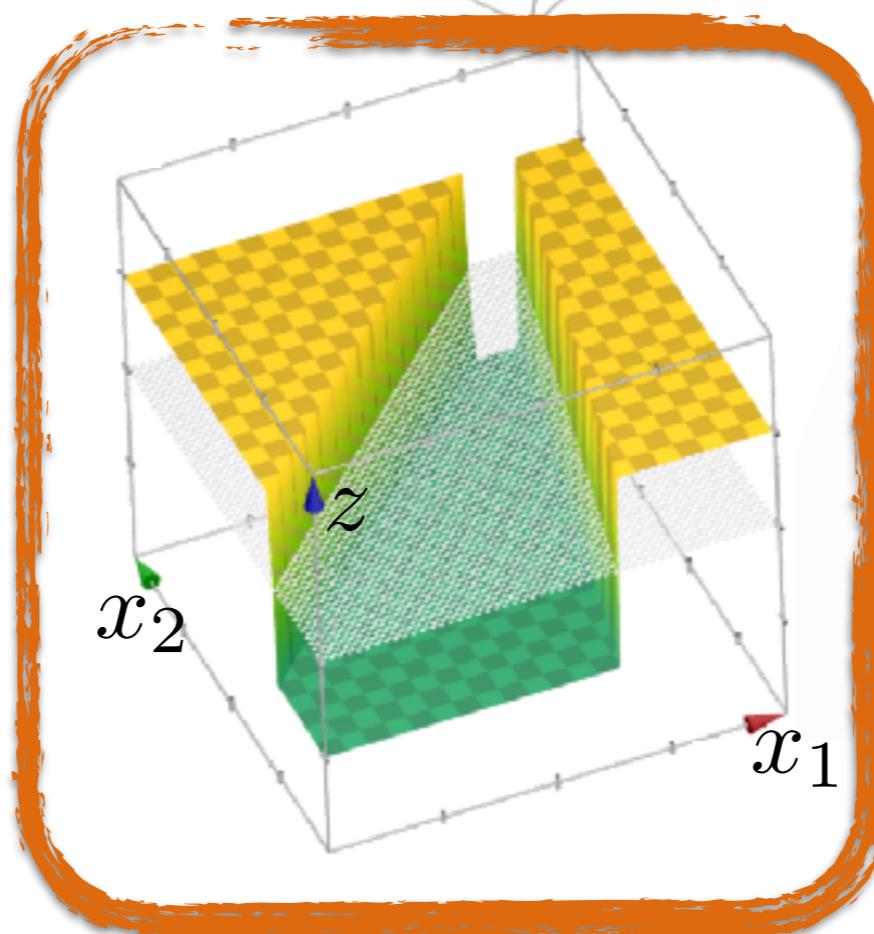
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

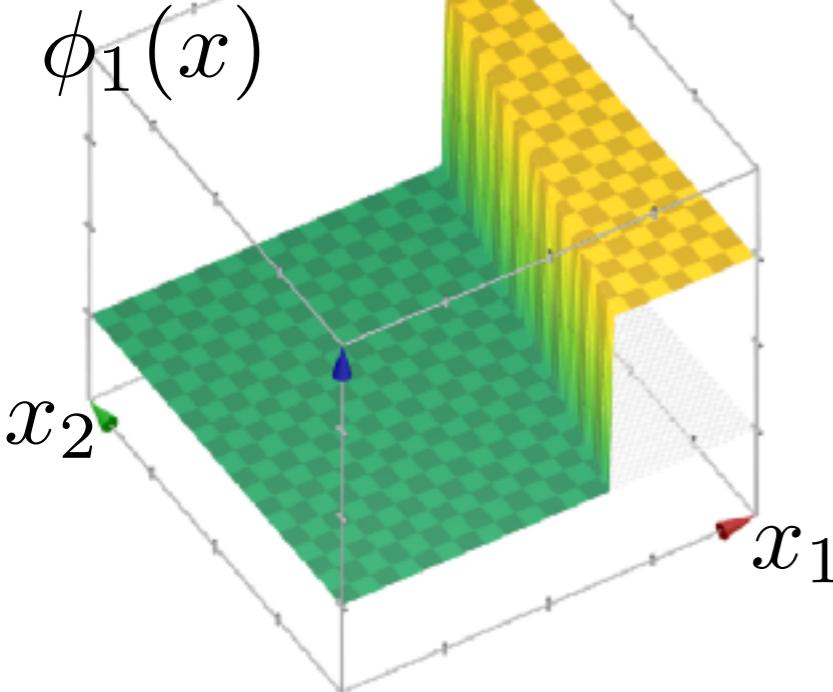


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

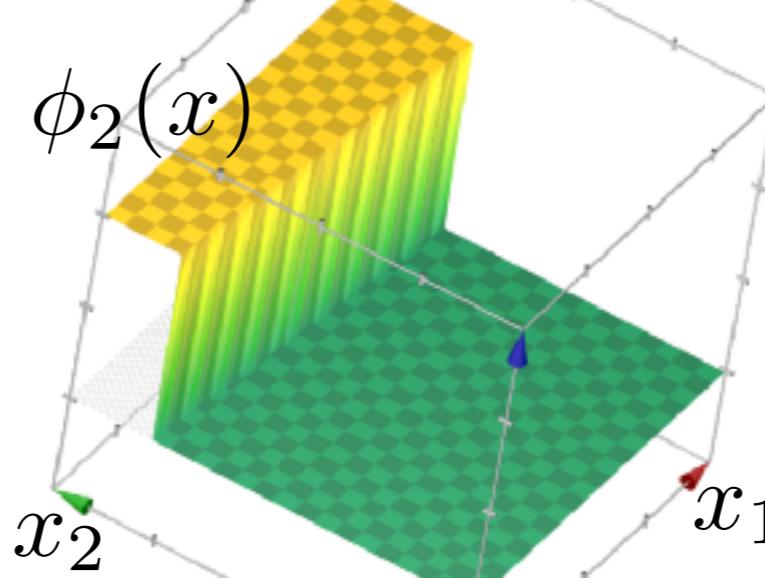


# New features: step functions!

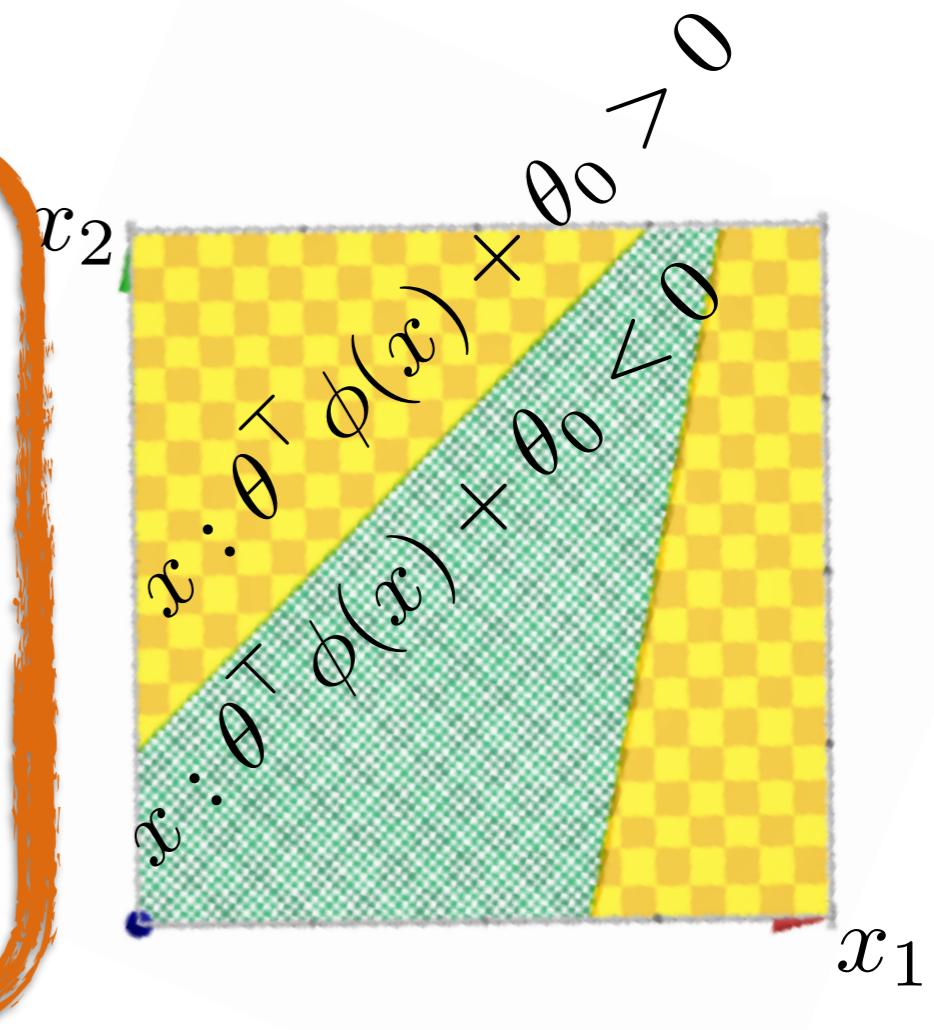
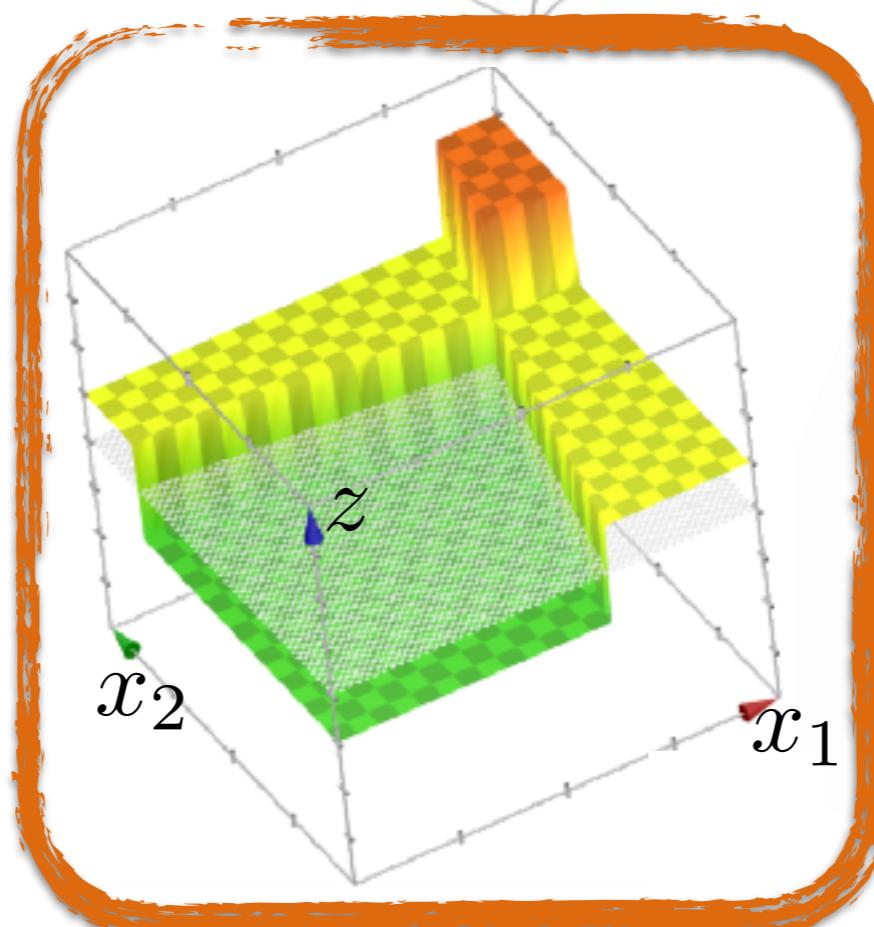
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

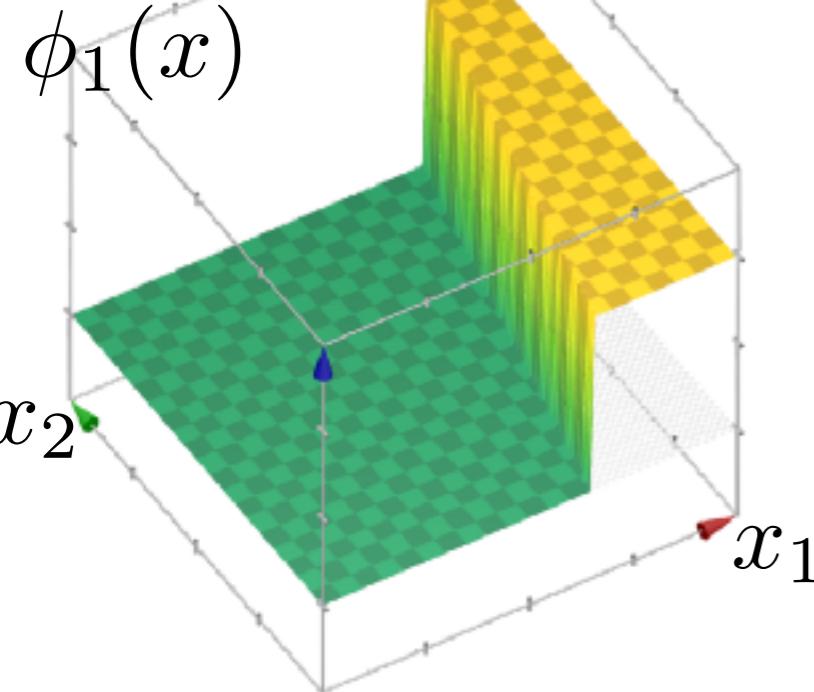


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

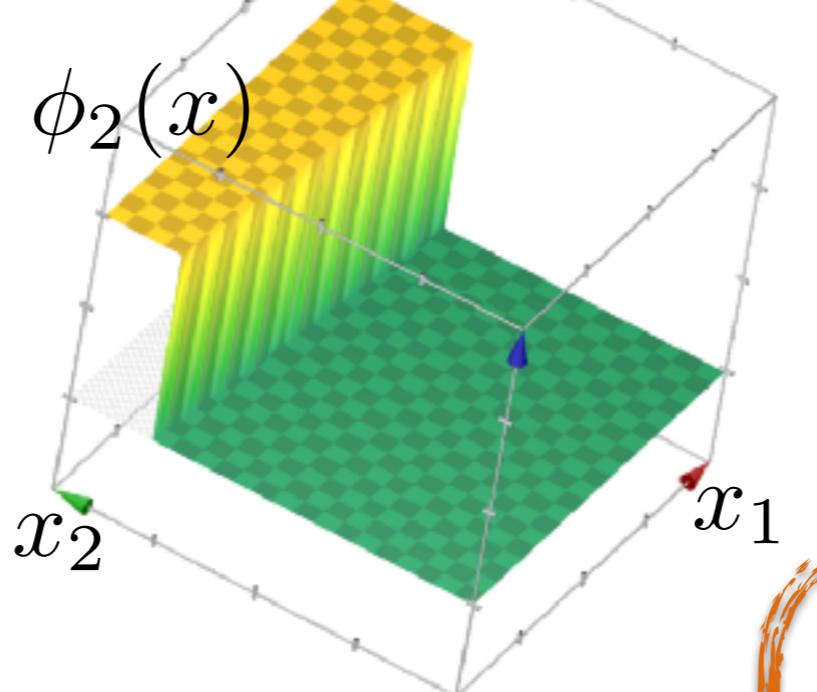


# New features: step functions!

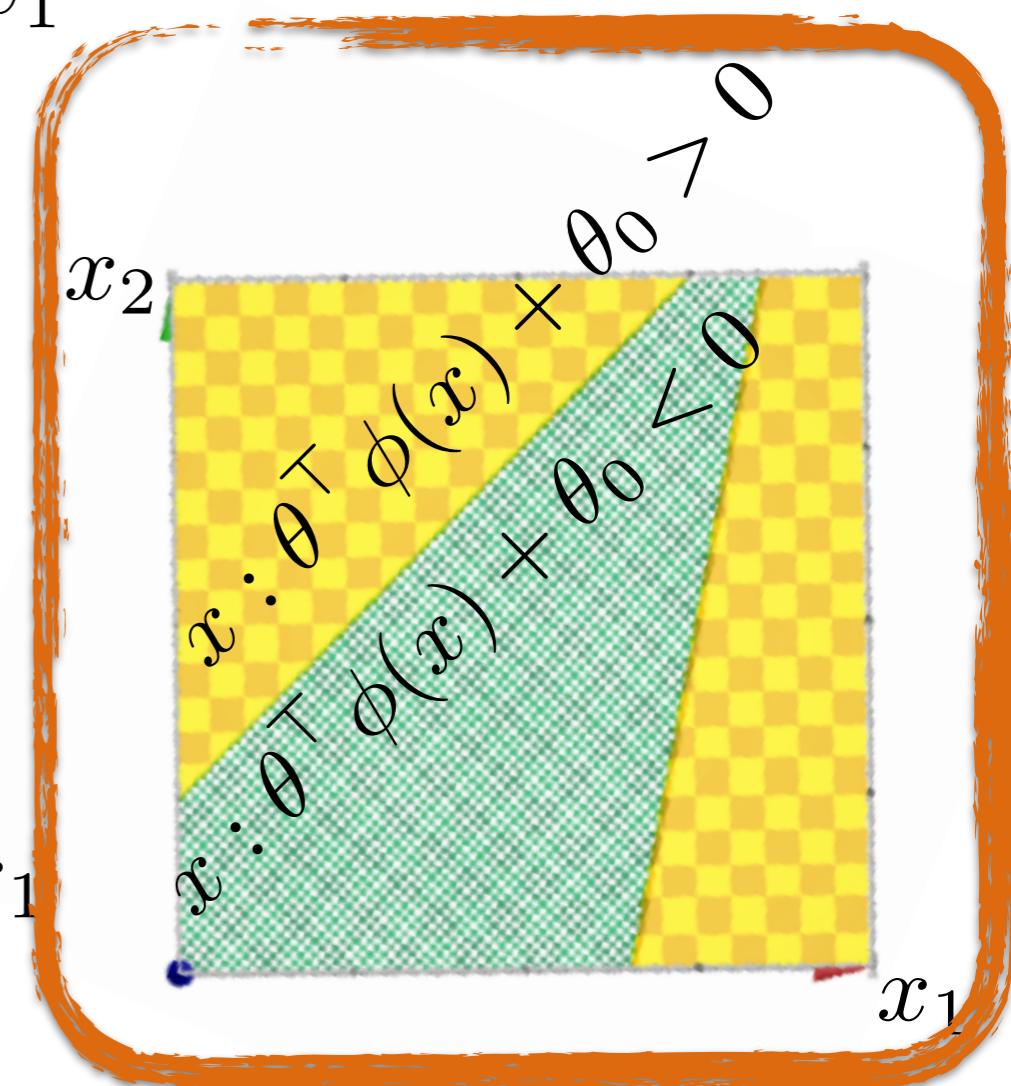
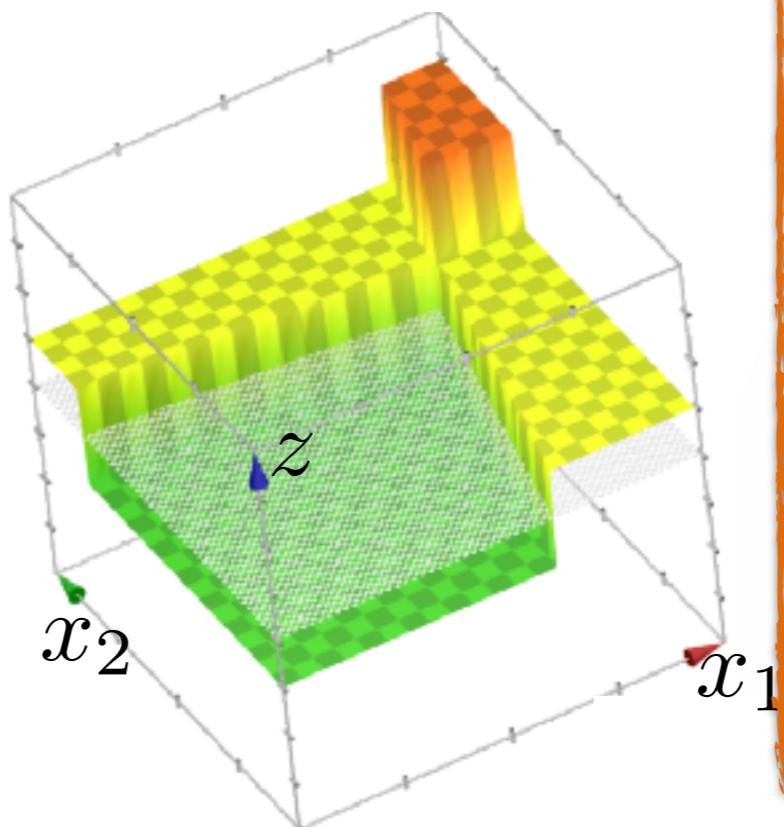
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

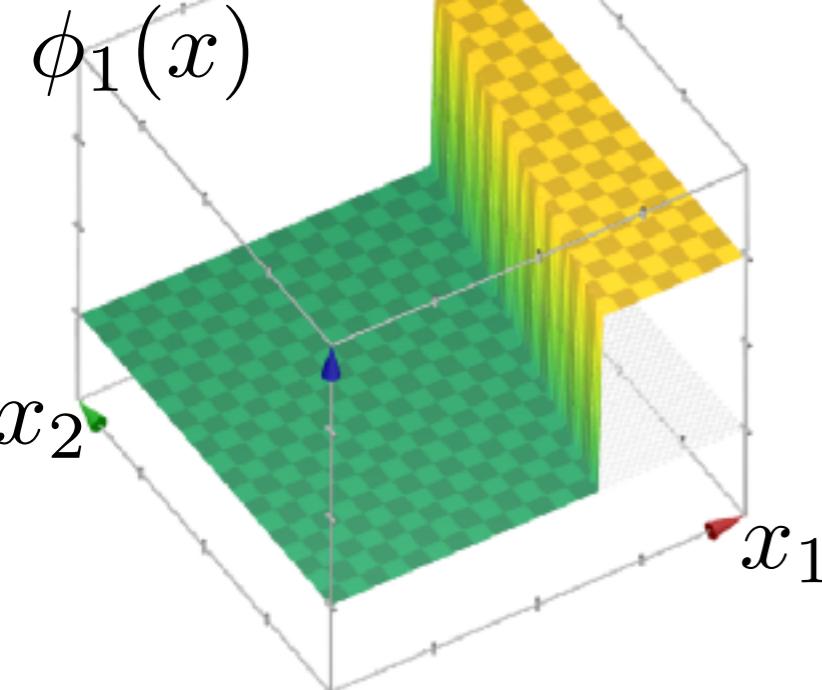


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

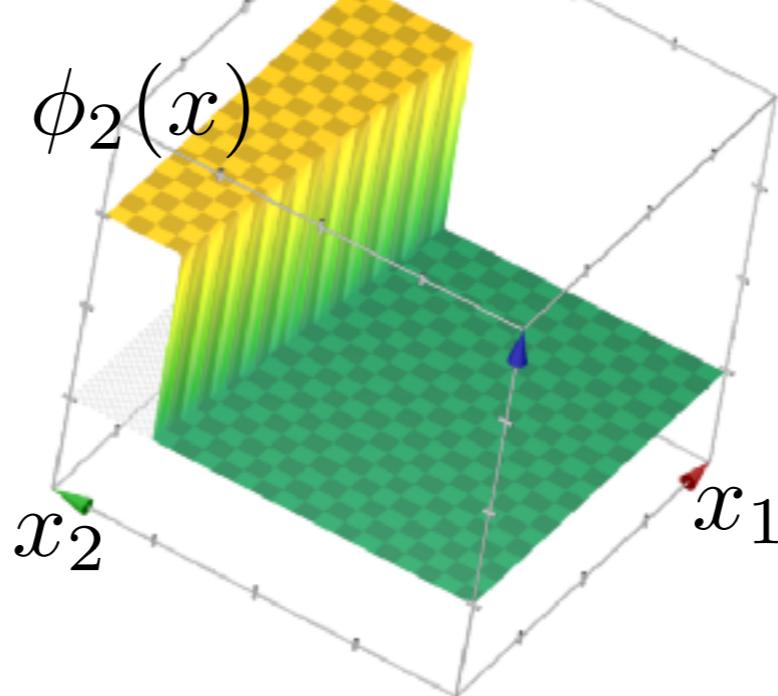


# New features: step functions!

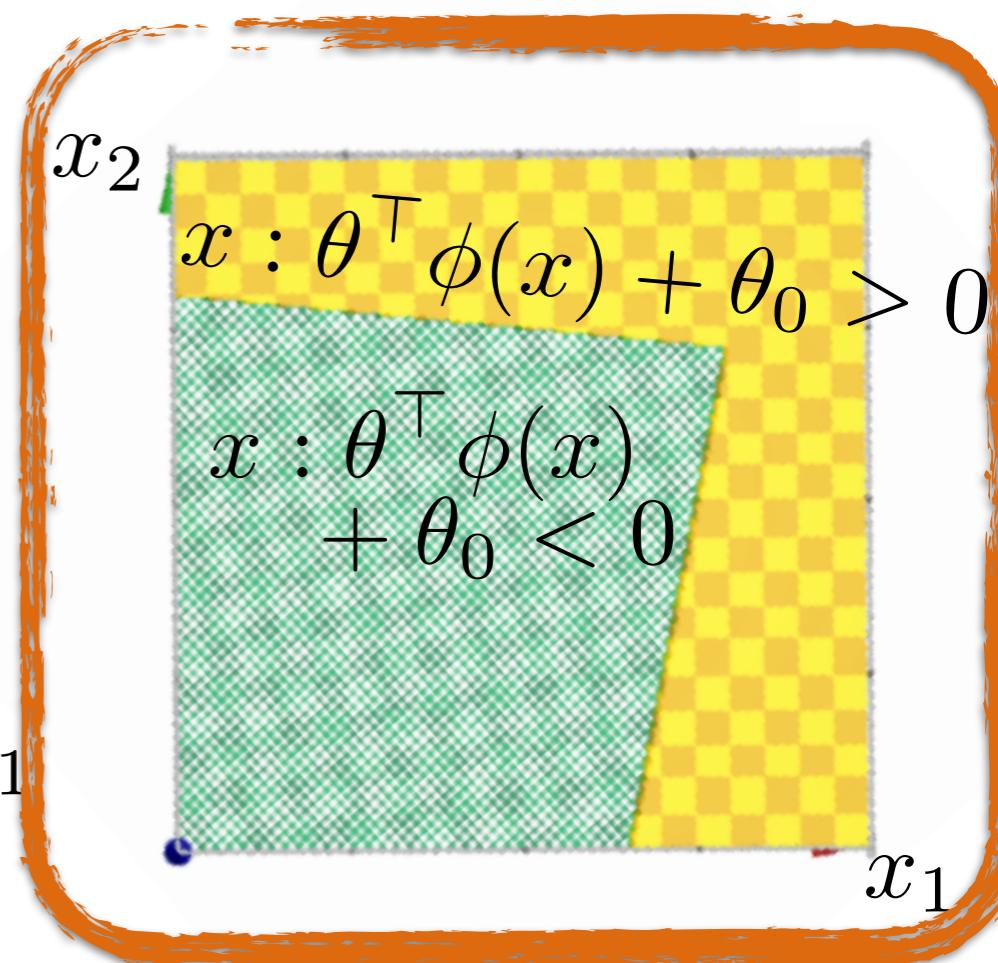
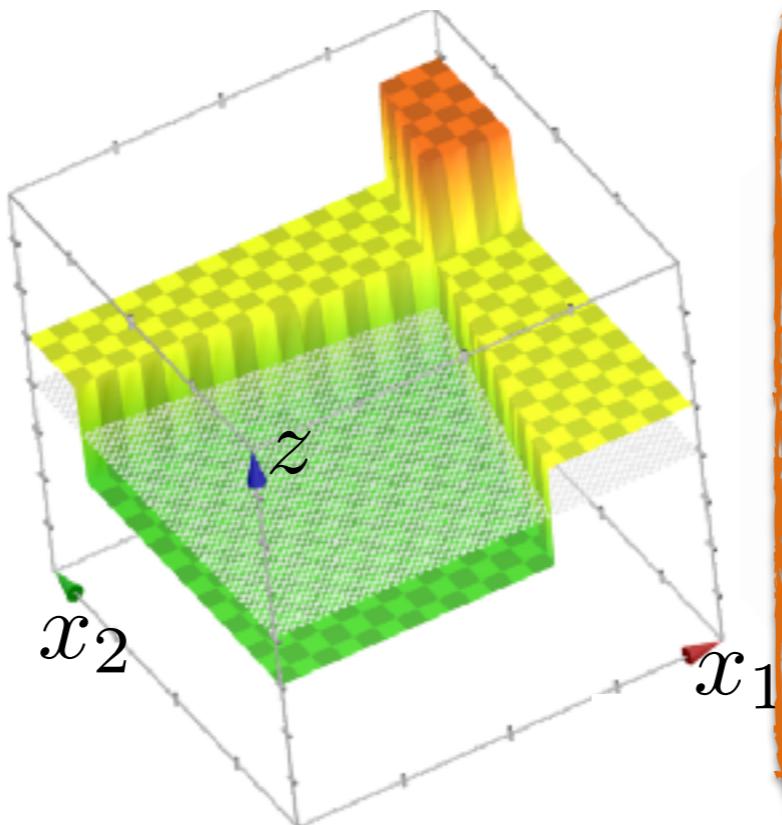
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

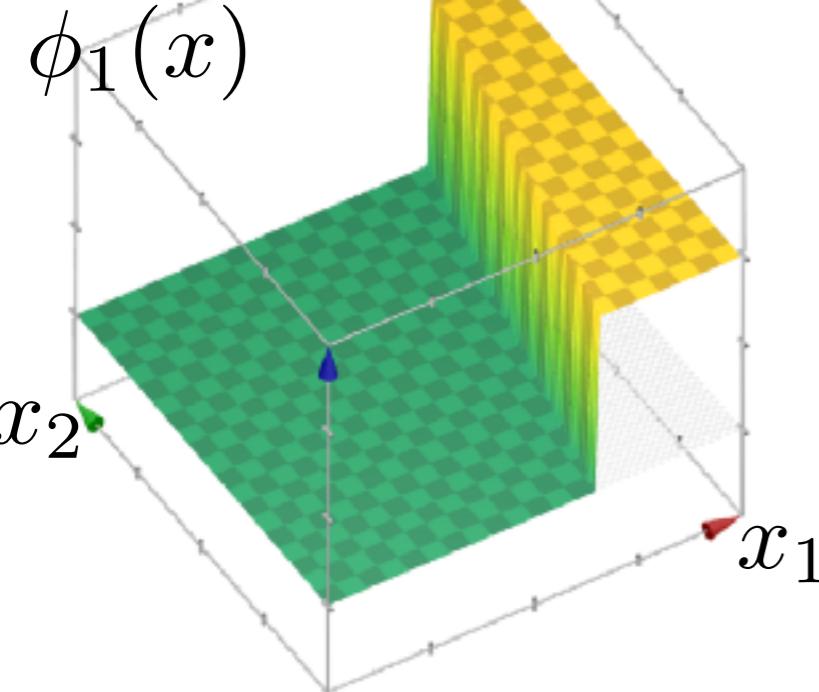


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

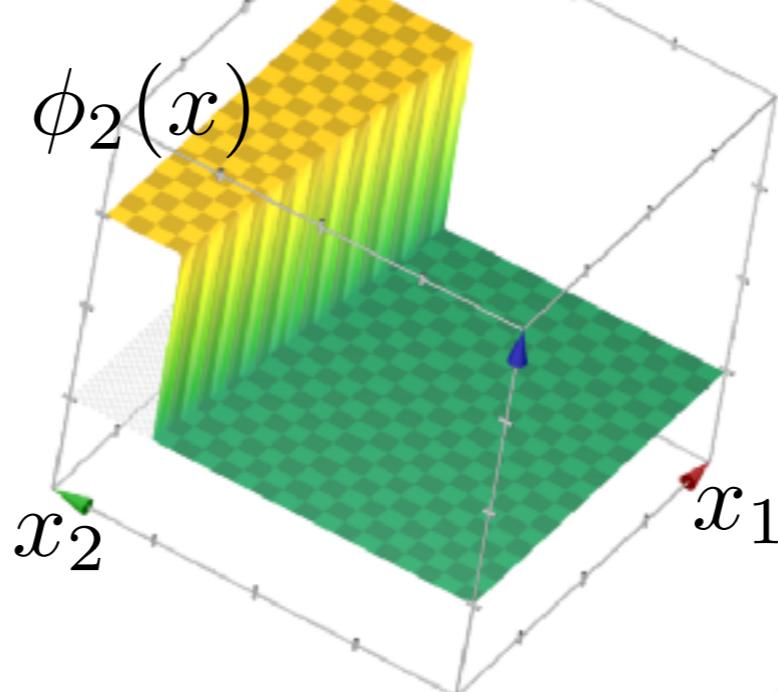


# New features: step functions!

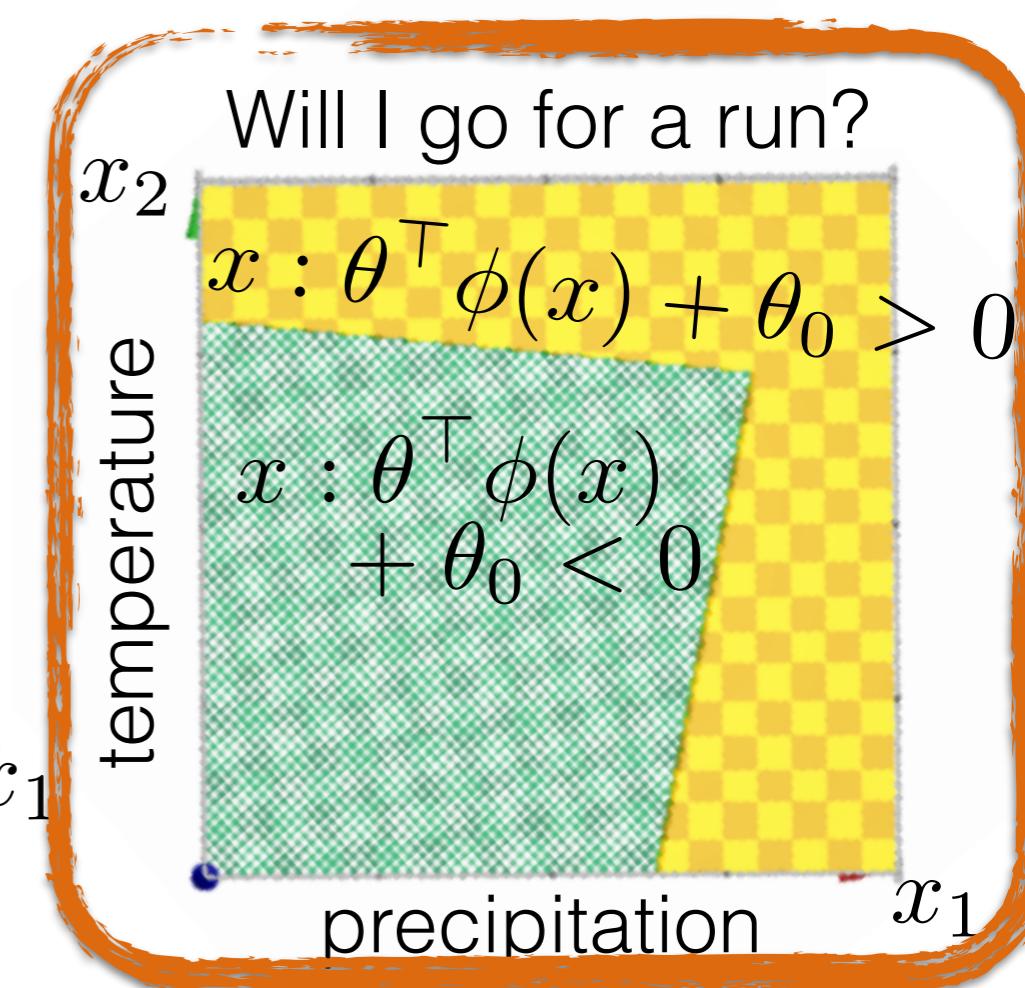
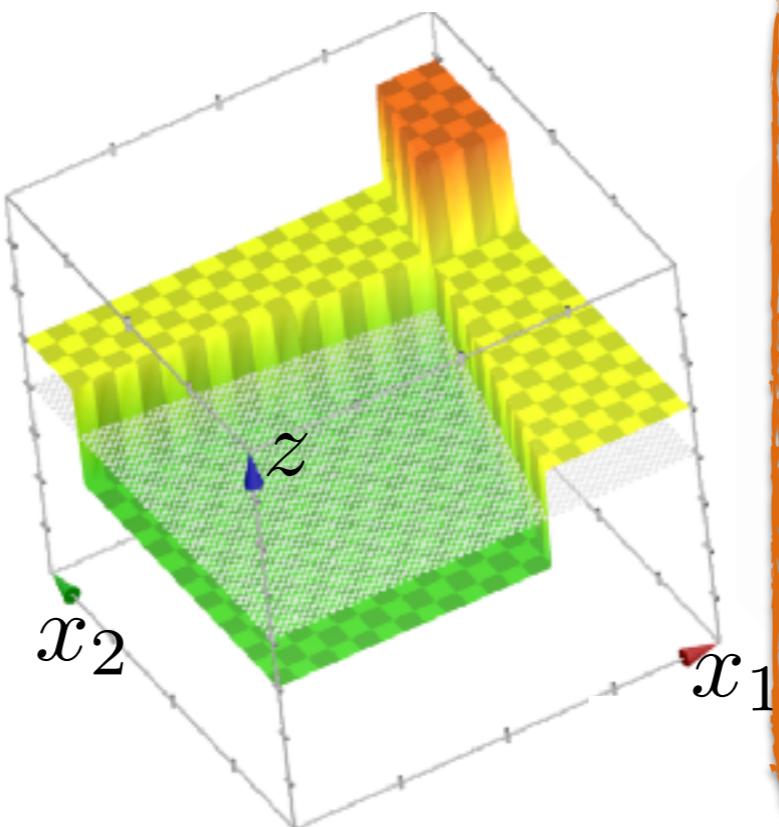
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$



Will I go for a run?

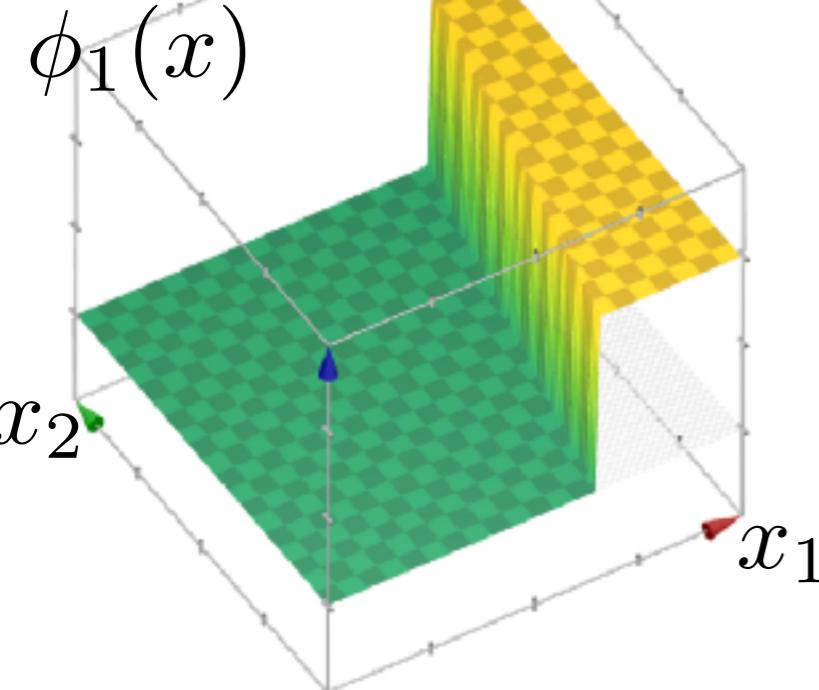
$$x : \theta^\top \phi(x) + \theta_0 > 0$$

$$x : \theta^\top \phi(x) + \theta_0 < 0$$

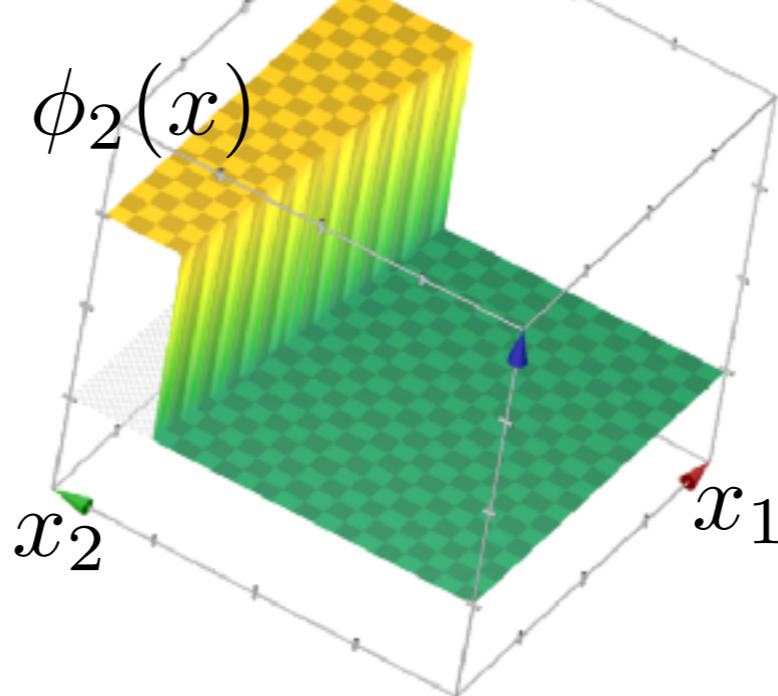
temperature  
precipitation

# New features: step functions!

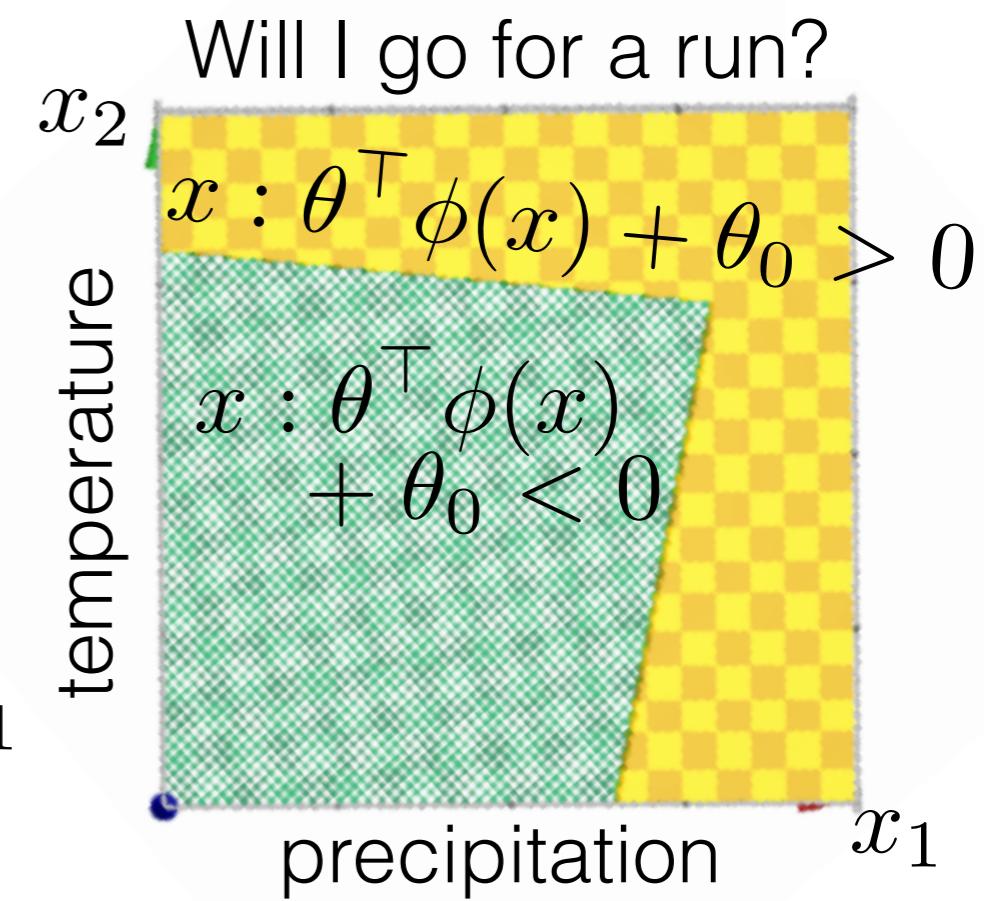
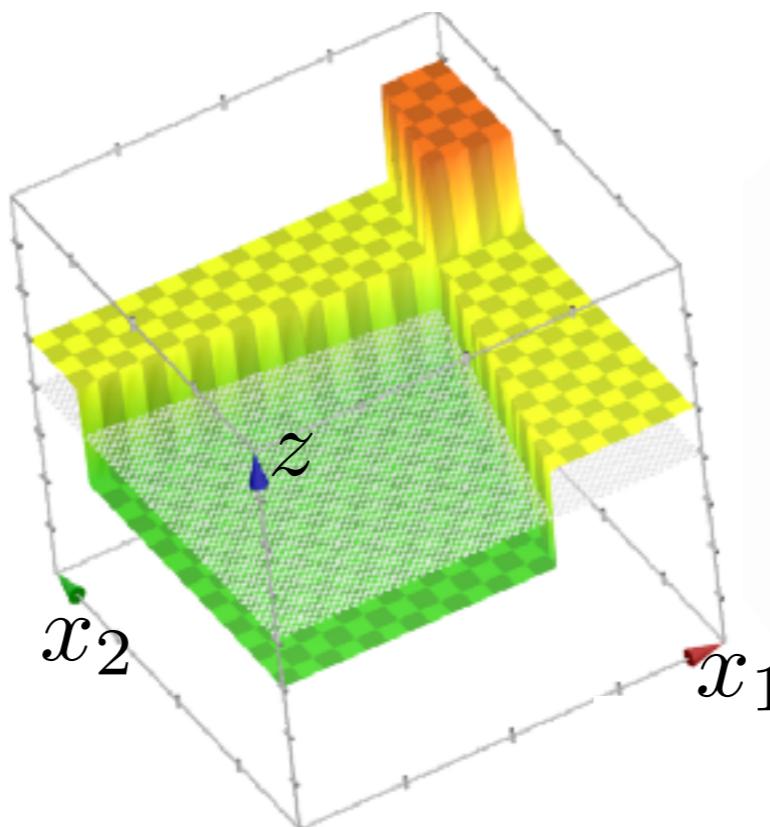
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

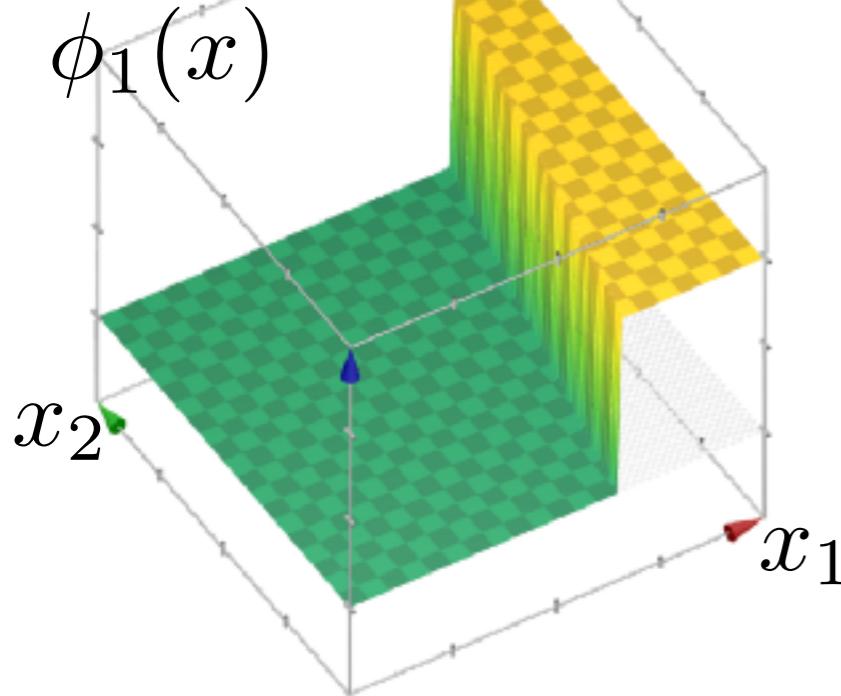


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

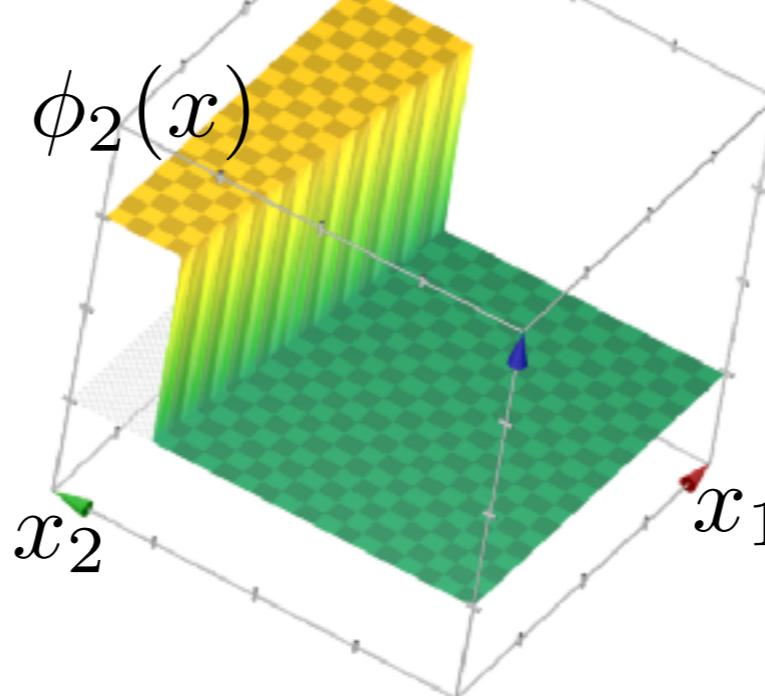


# New features: step functions!

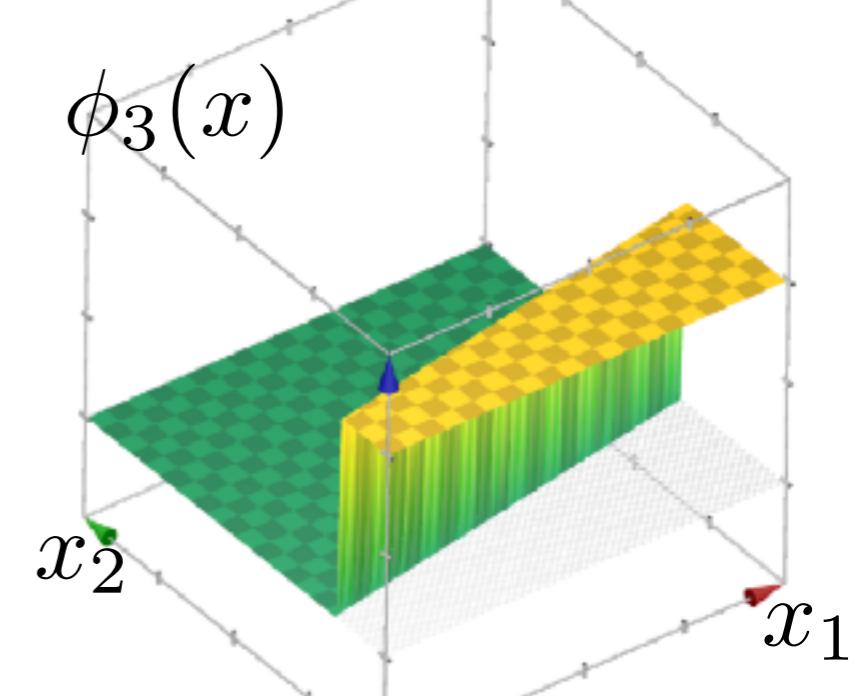
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



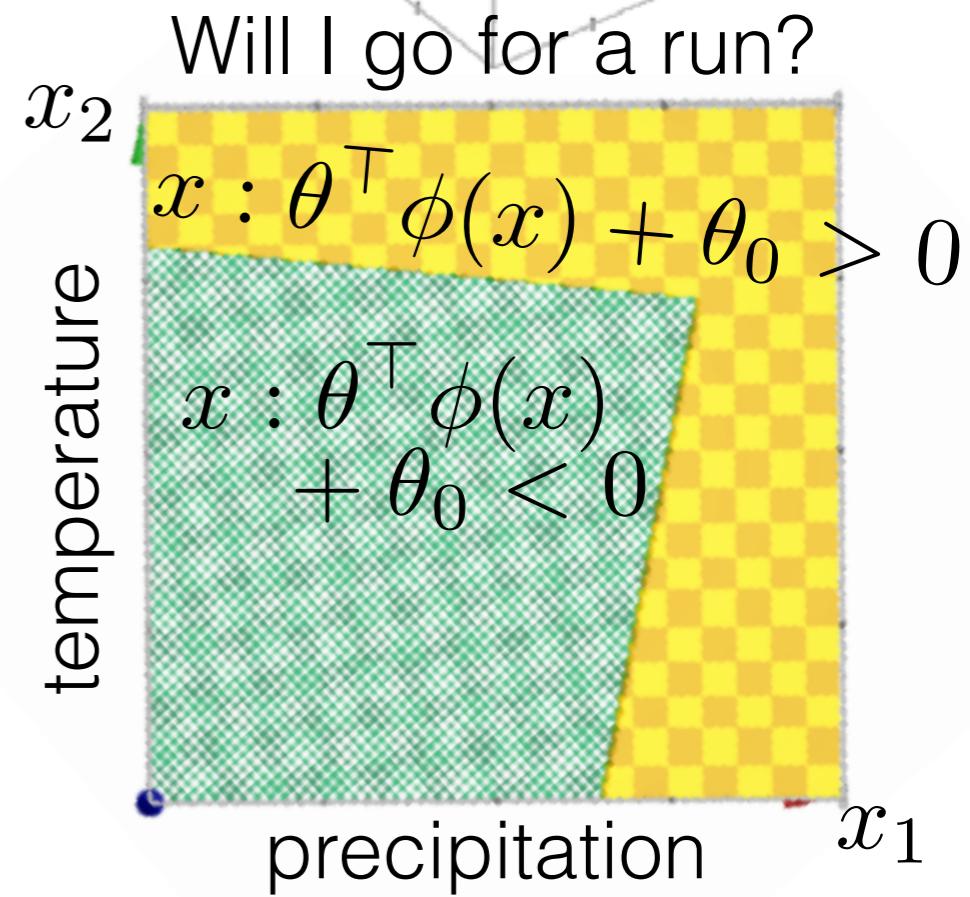
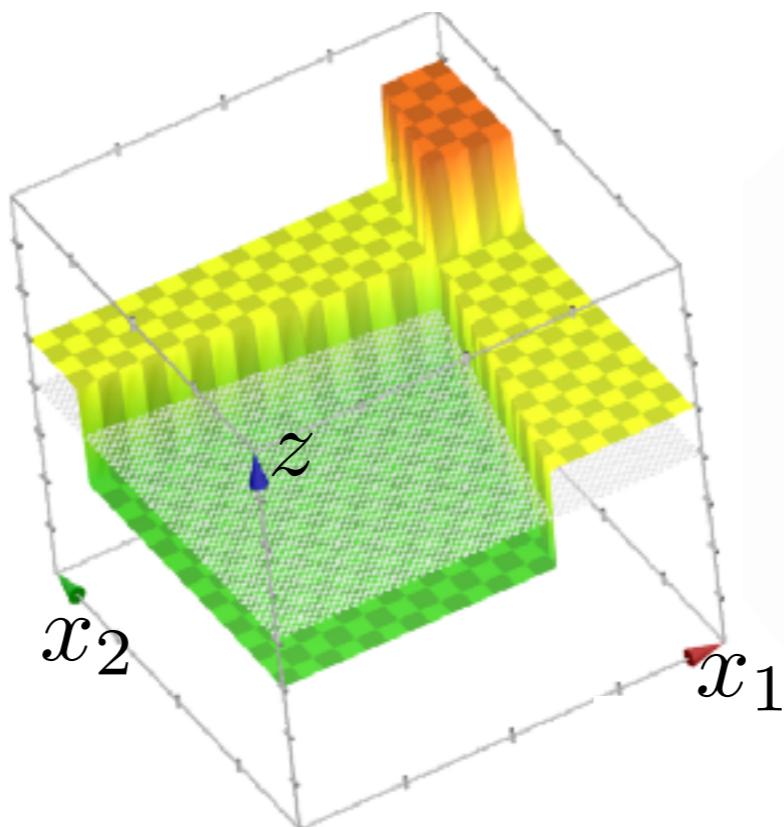
$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



$$\phi_3(x) = \mathbf{1}\{\tilde{\tilde{w}}^\top x + \tilde{\tilde{w}}_0 \geq 0\}$$

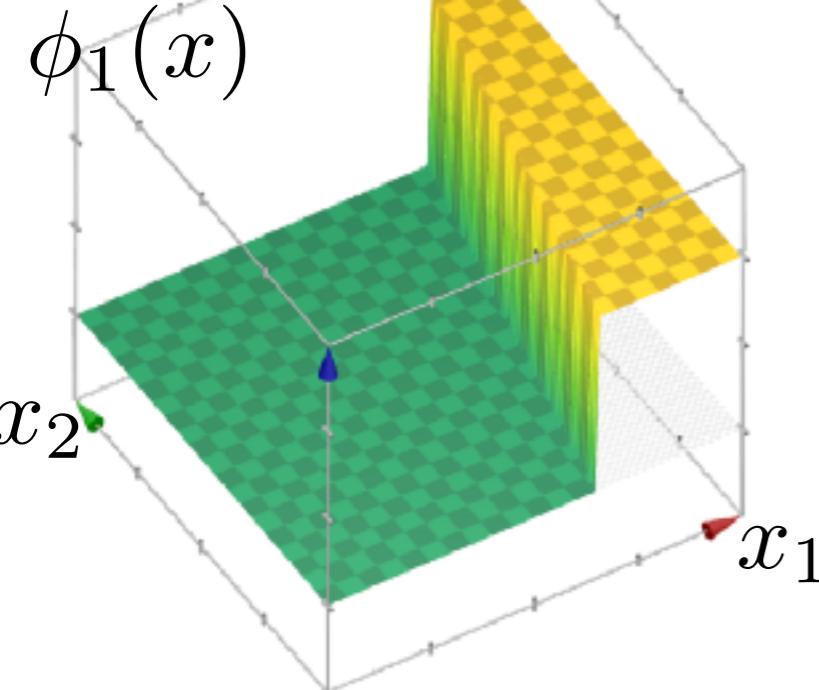


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

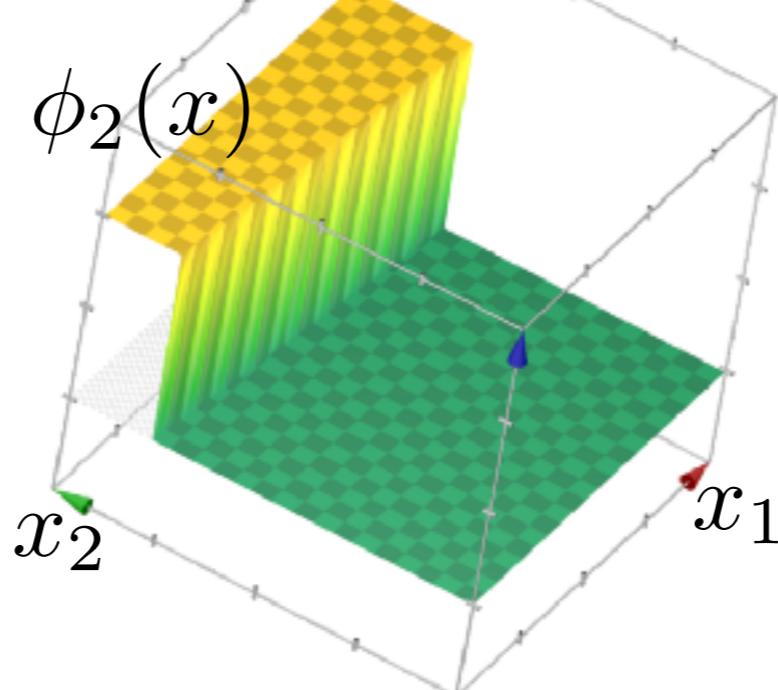


# New features: step functions!

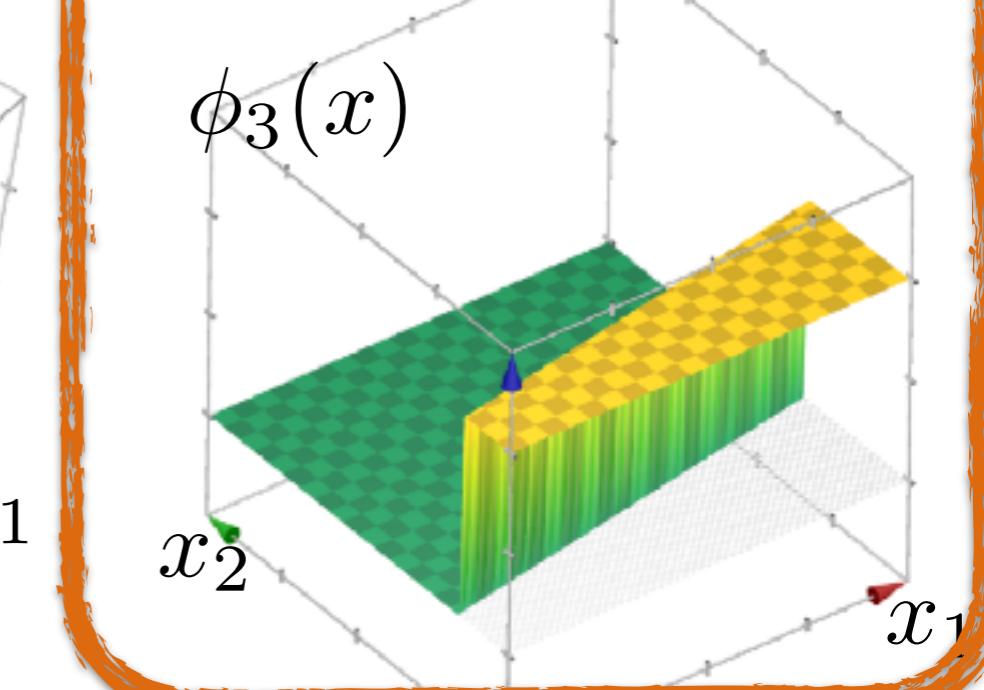
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$

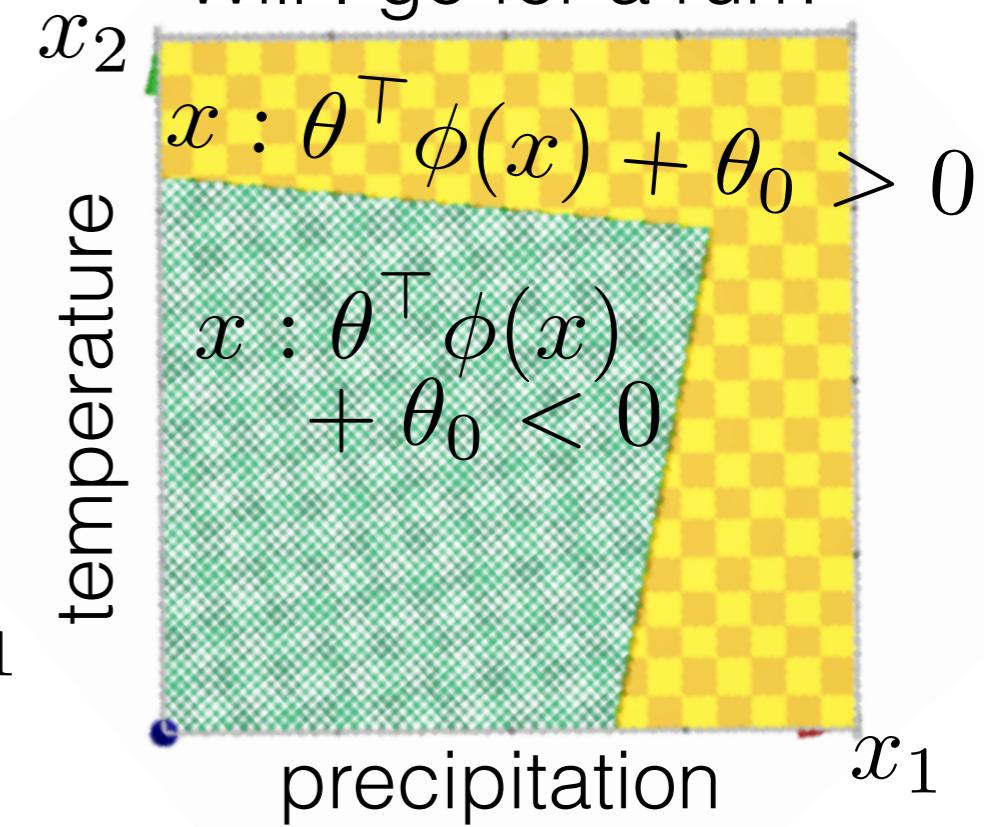
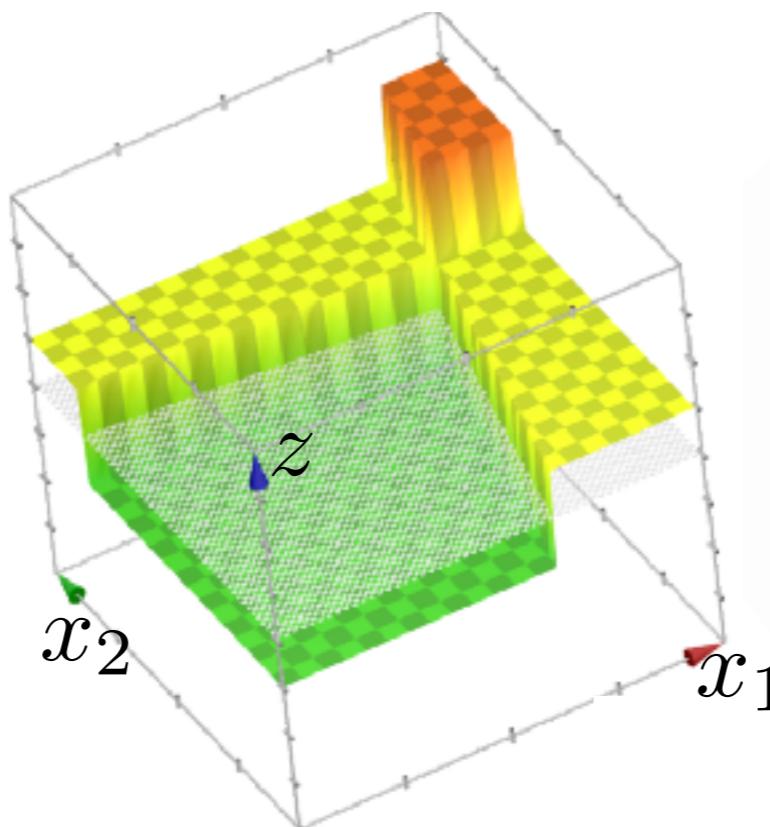


$$\phi_3(x) = \mathbf{1}\{\tilde{\tilde{w}}^\top x + \tilde{\tilde{w}}_0 \geq 0\}$$



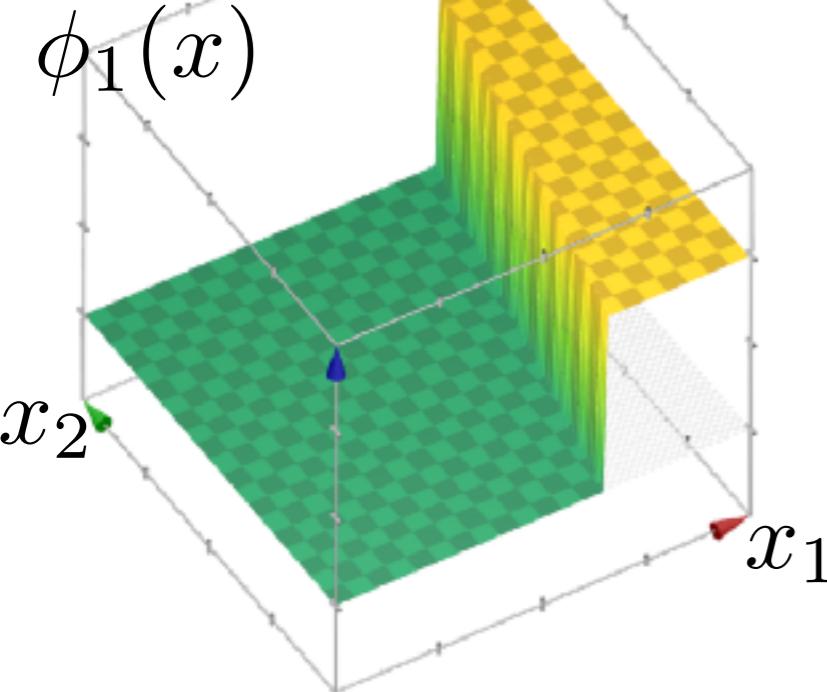
Will I go for a run?

$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

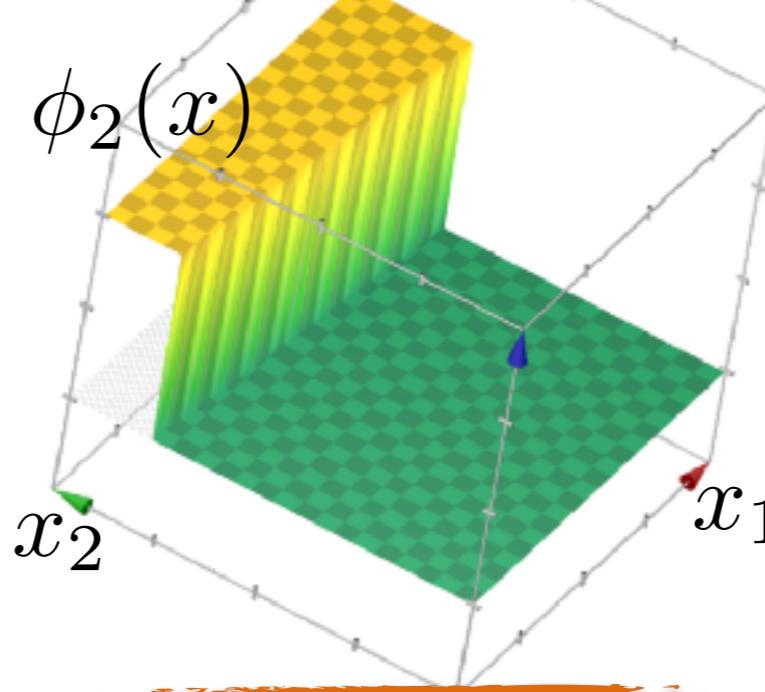


# New features: step functions!

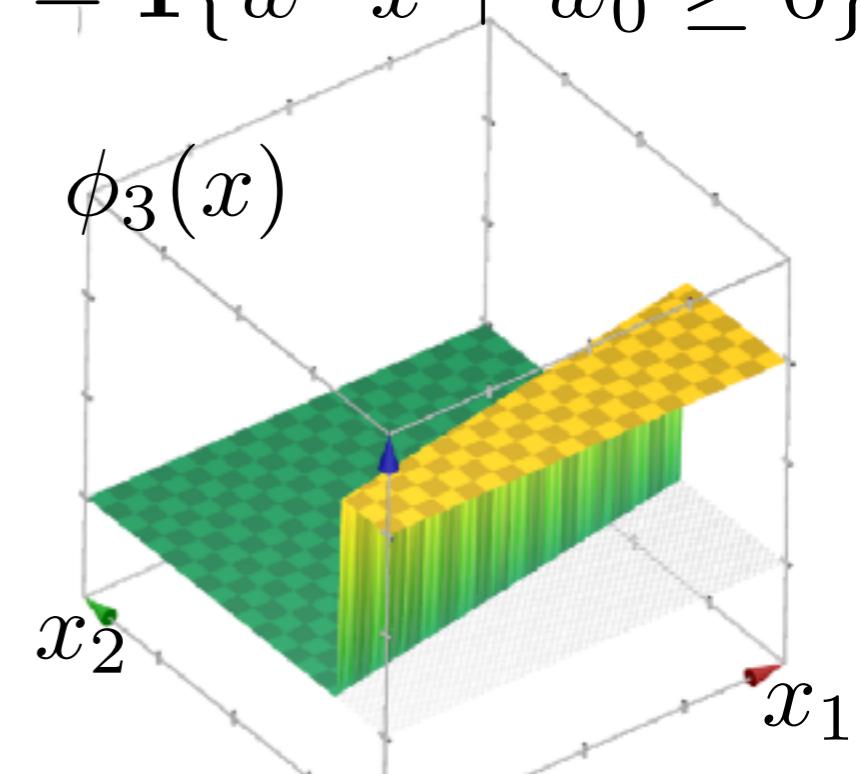
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



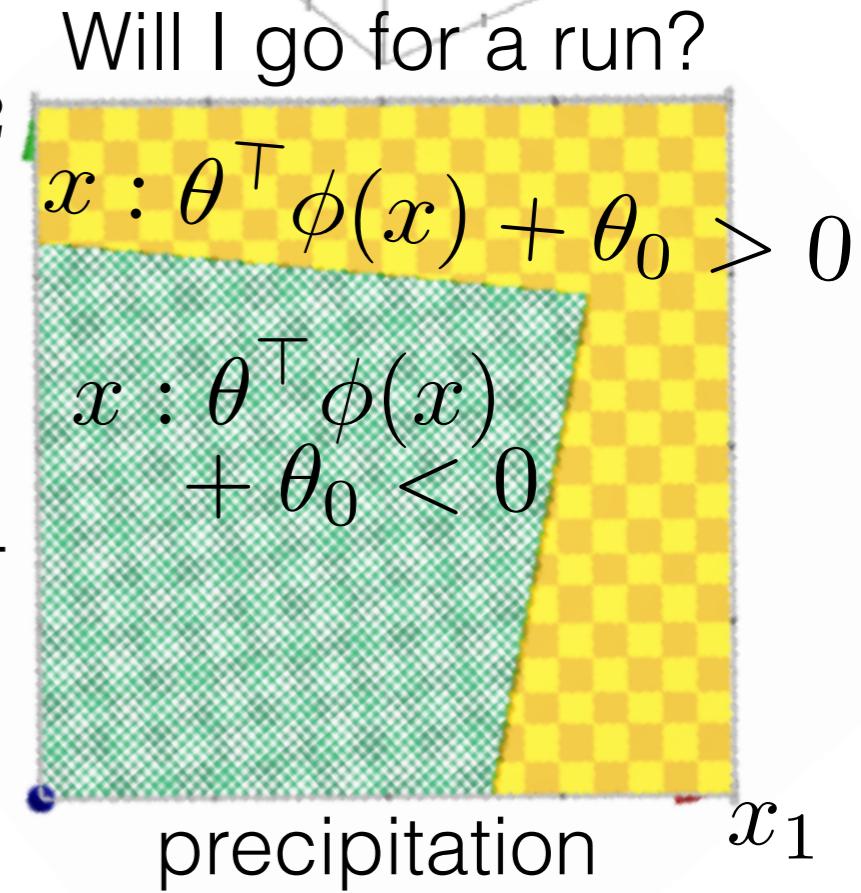
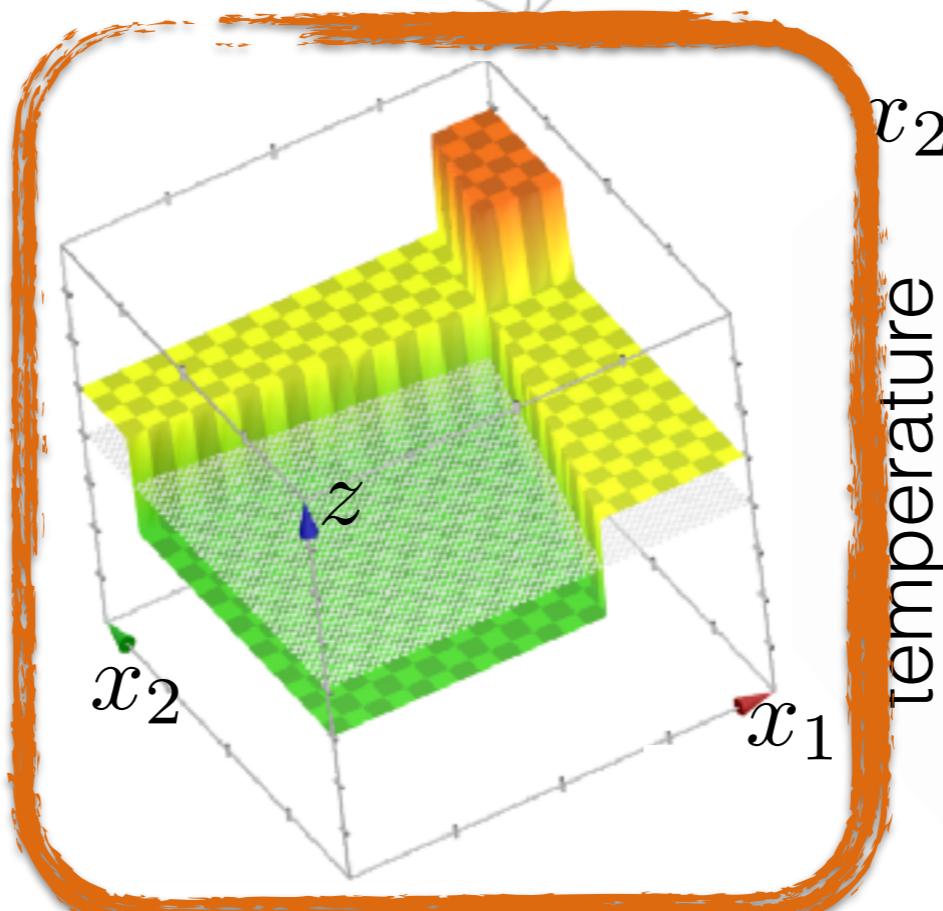
$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



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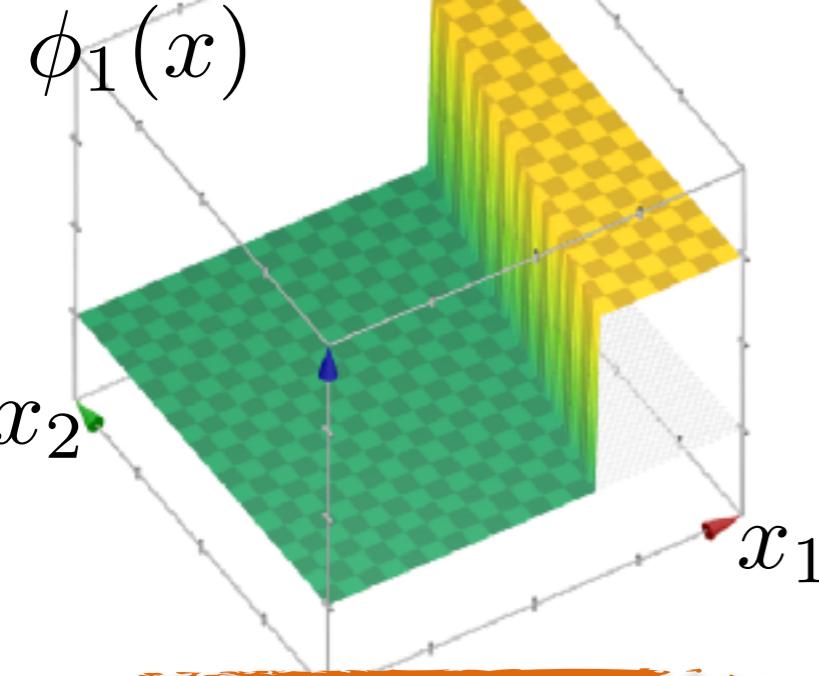


$$\begin{aligned} z &= \theta^\top \phi(x) + \theta_0 \\ &= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) \\ &\quad + \theta_0 \\ &= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) \\ &\quad + (-0.5) \end{aligned}$$

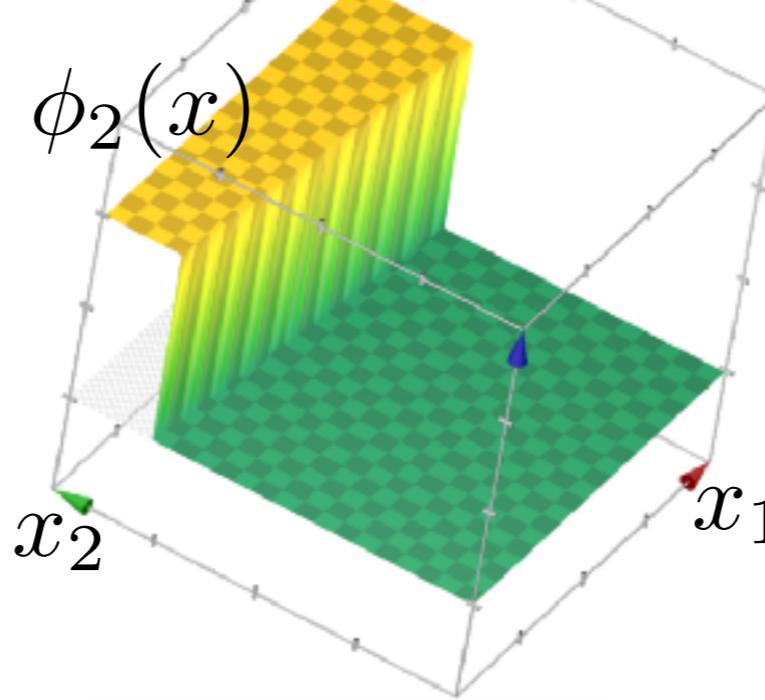


# New features: step functions!

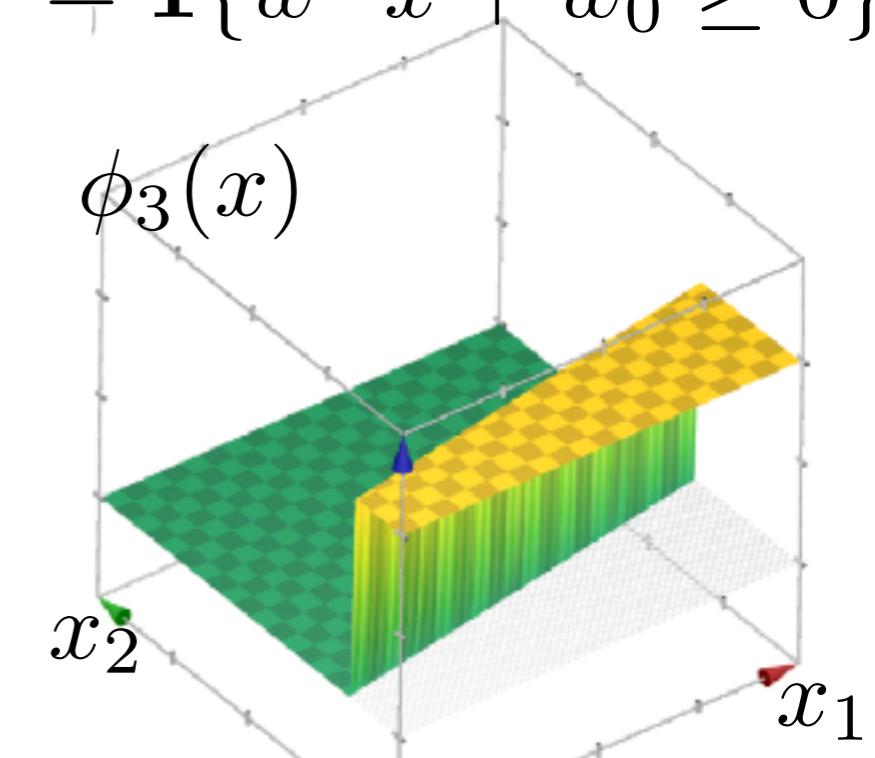
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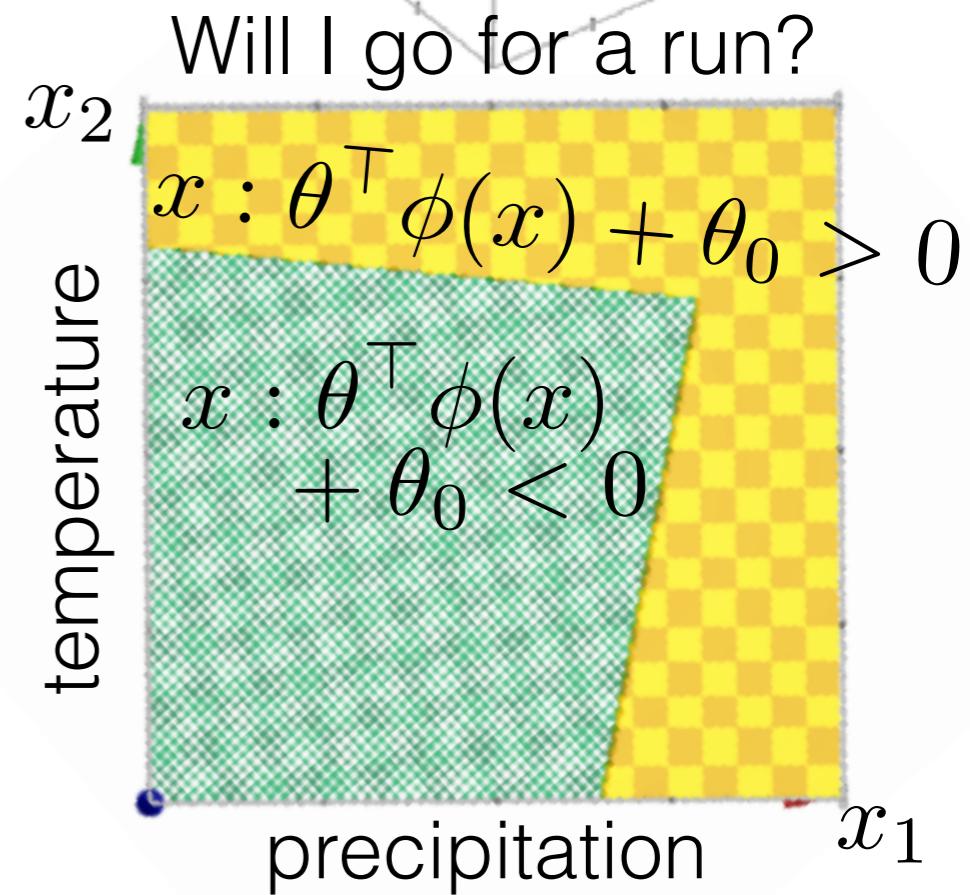
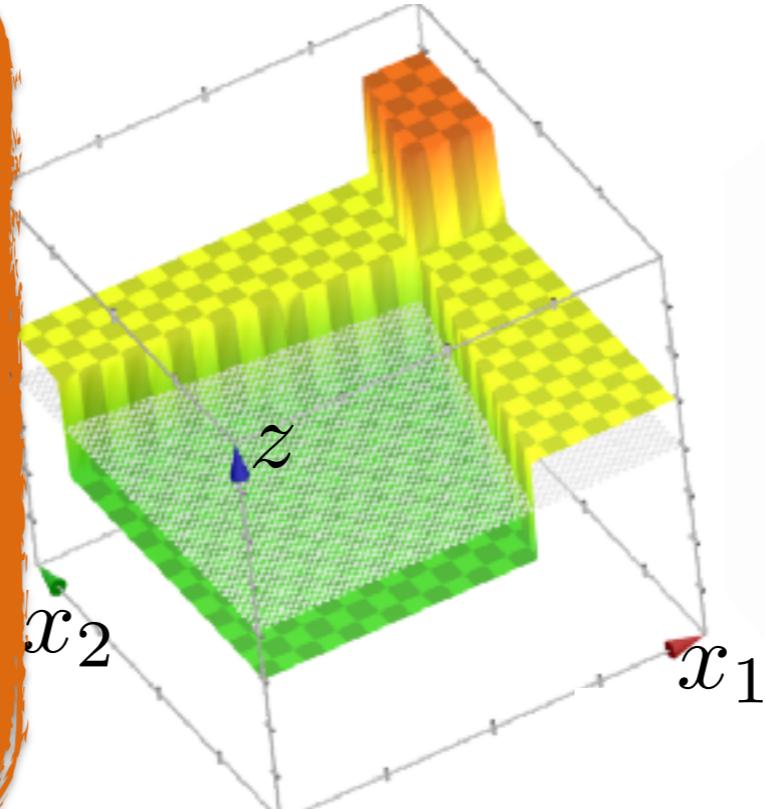
$$\phi_2(x) = \mathbf{1}\{\tilde{w}^\top x + \tilde{w}_0 \geq 0\}$$



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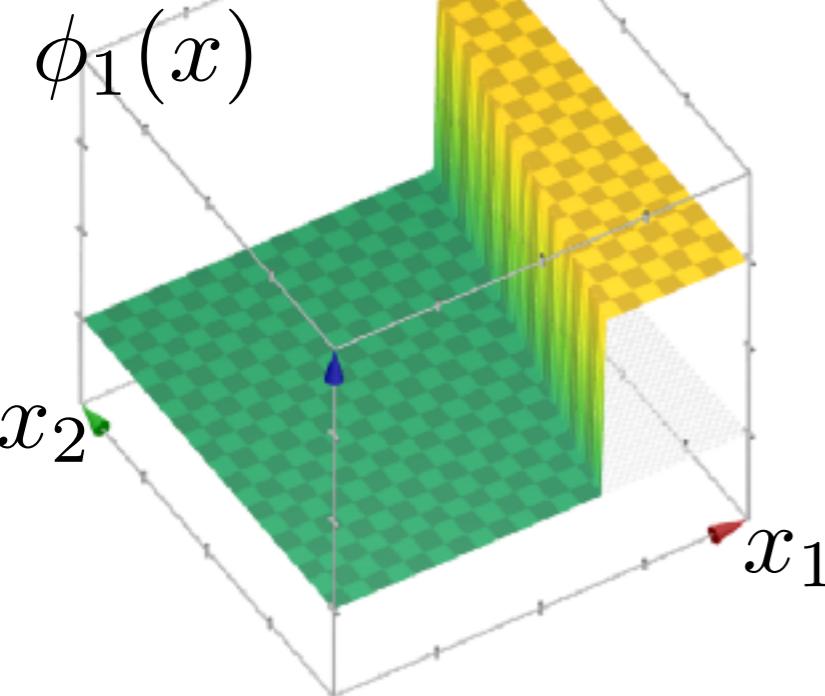


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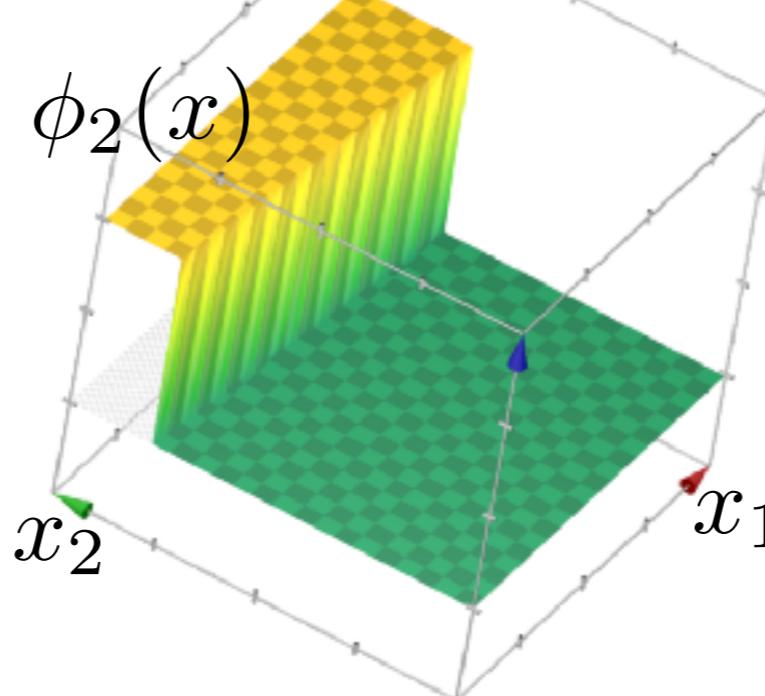


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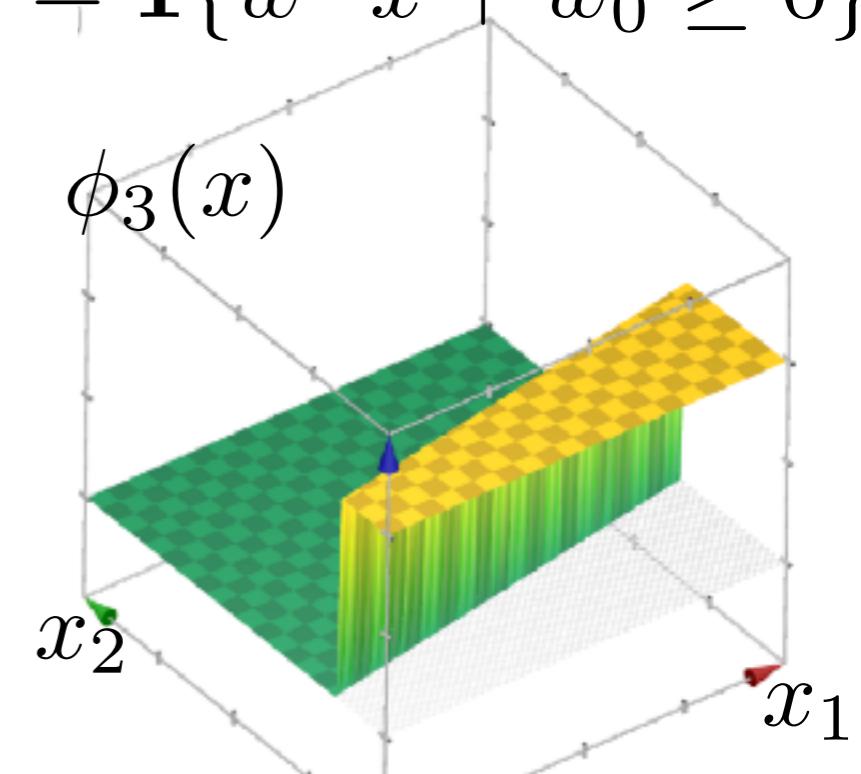
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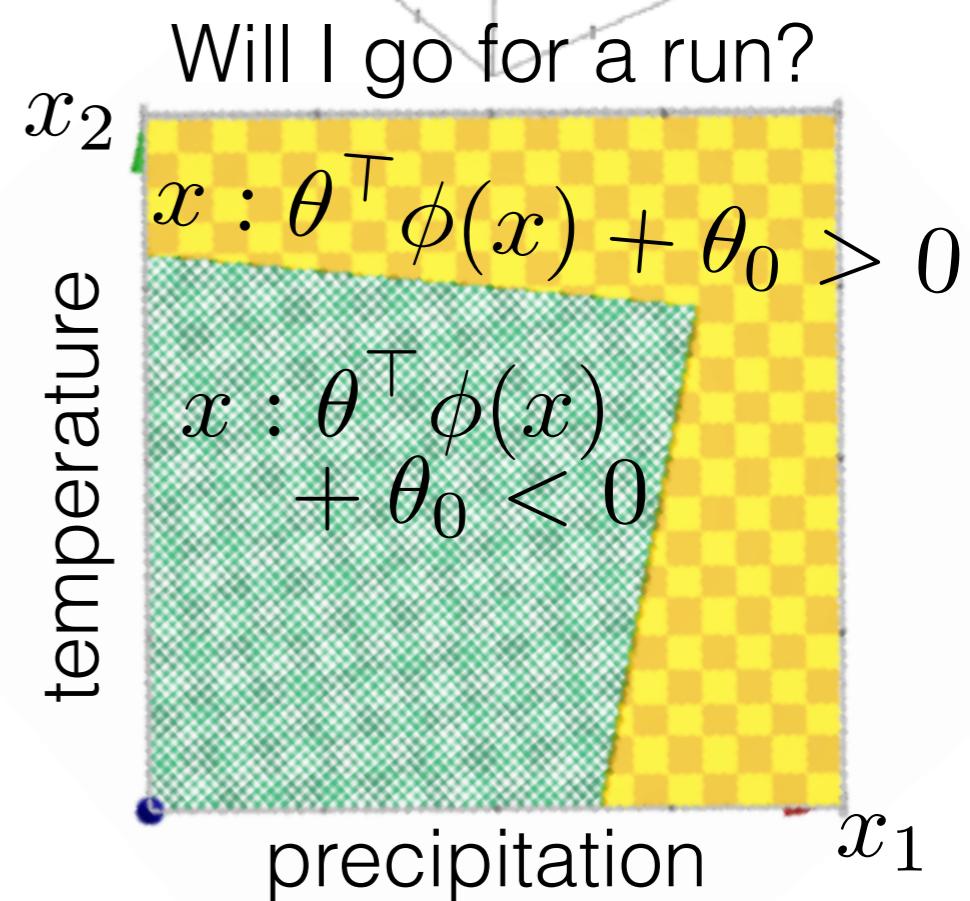
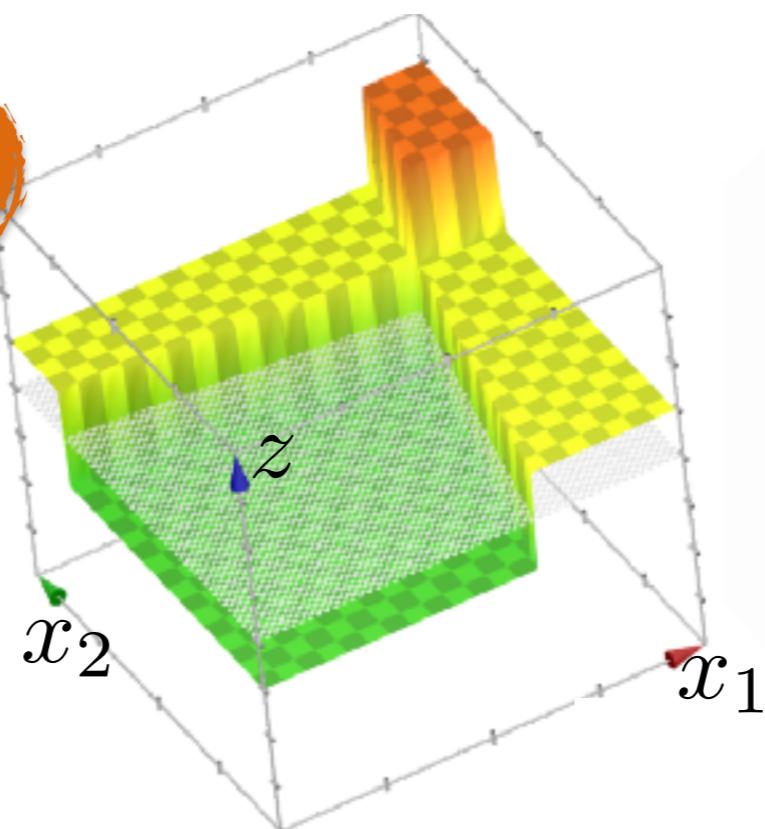
$\text{z} = \theta^\top \phi(x) + \theta_0$

$= \theta_1 \phi_1(x) + \theta_2 \phi_2(x)$

$+ \theta_0$

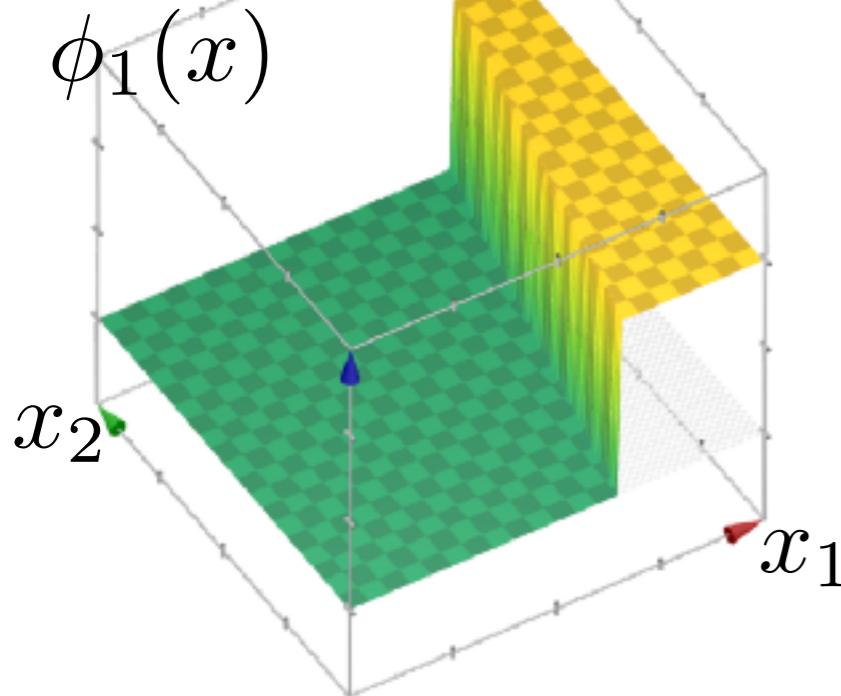
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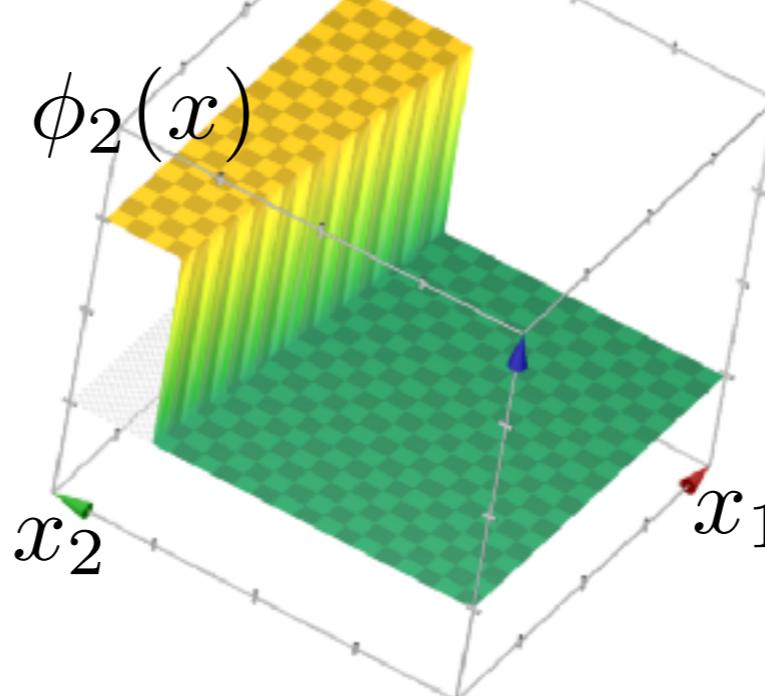


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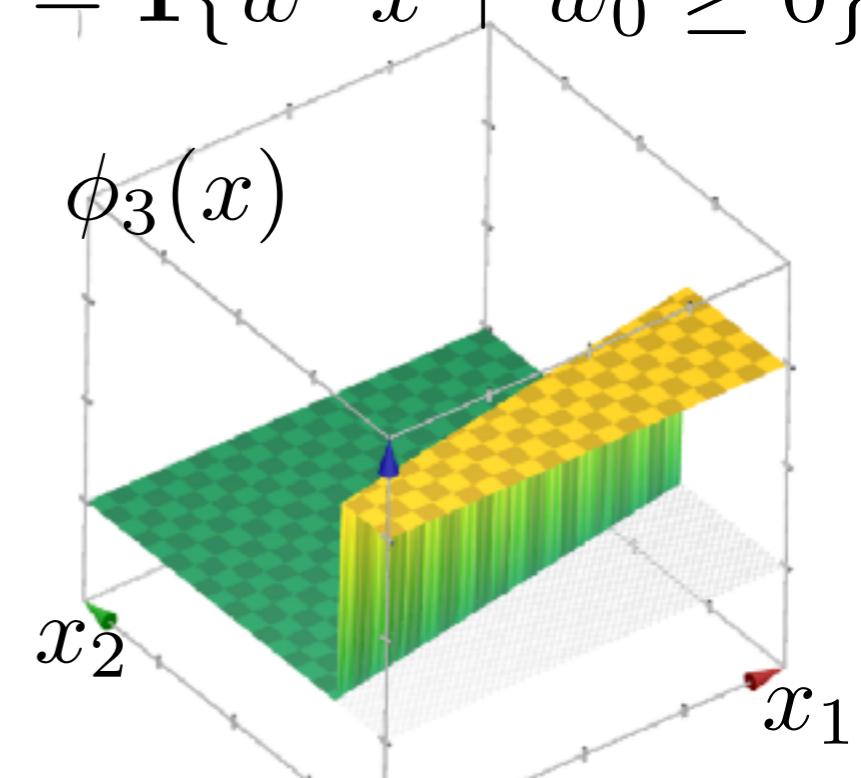
$$\phi_1(x) = \mathbf{1}\{w^\top x + w_0 \geq 0\}$$



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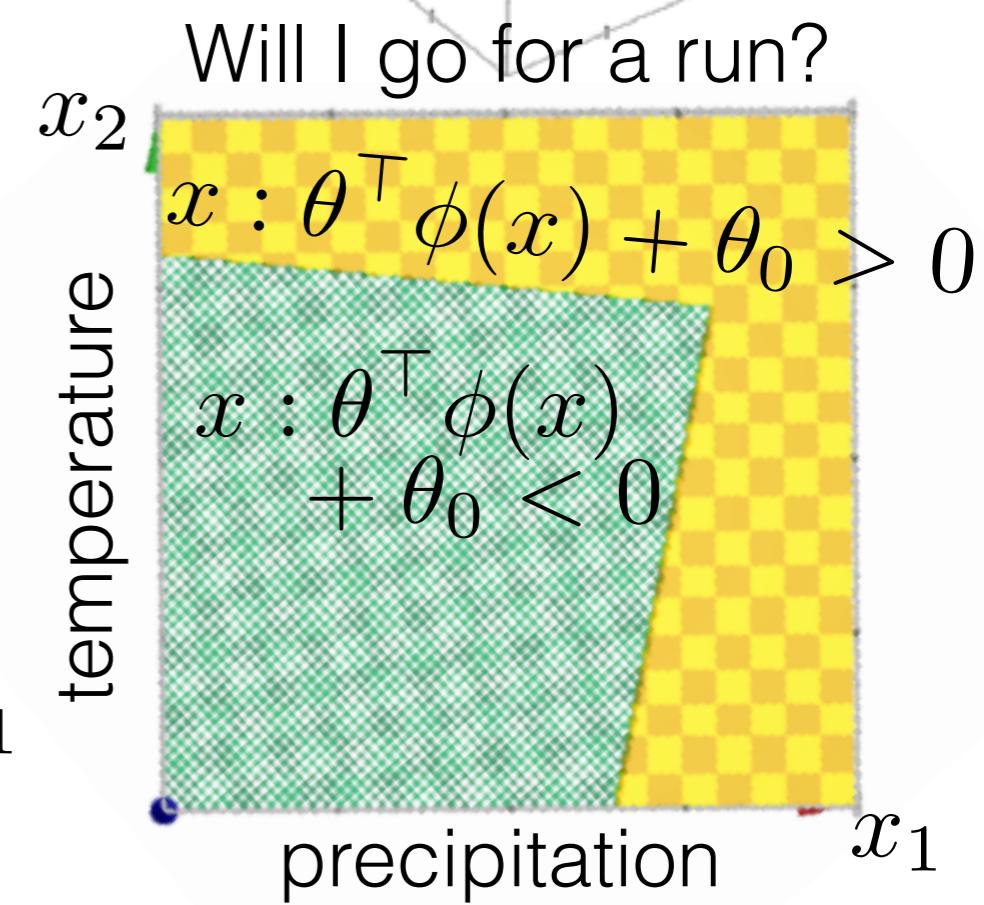
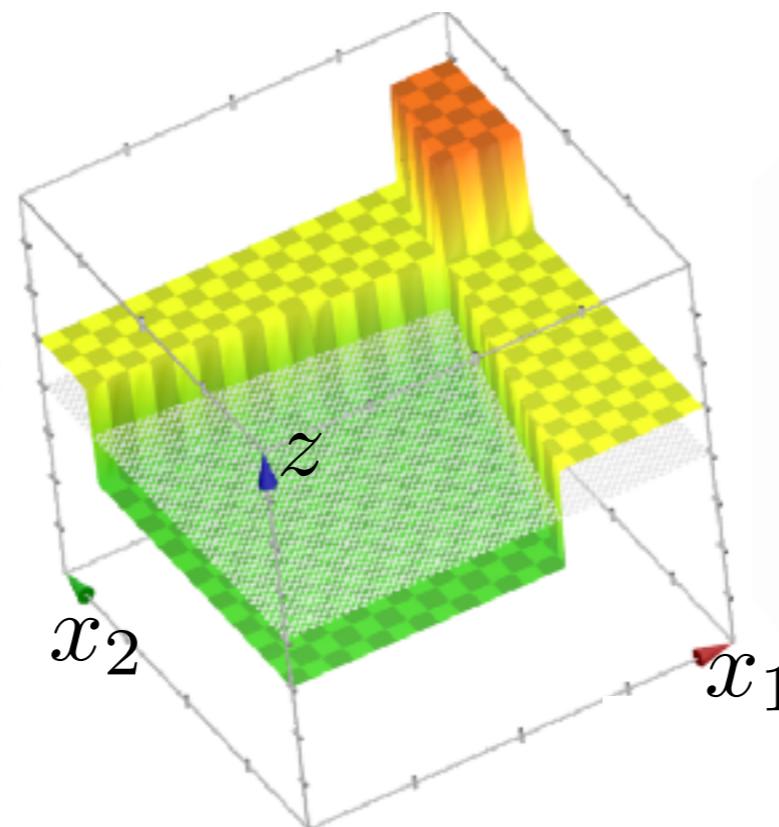
$z = \theta^\top \phi(x) + \theta_0$

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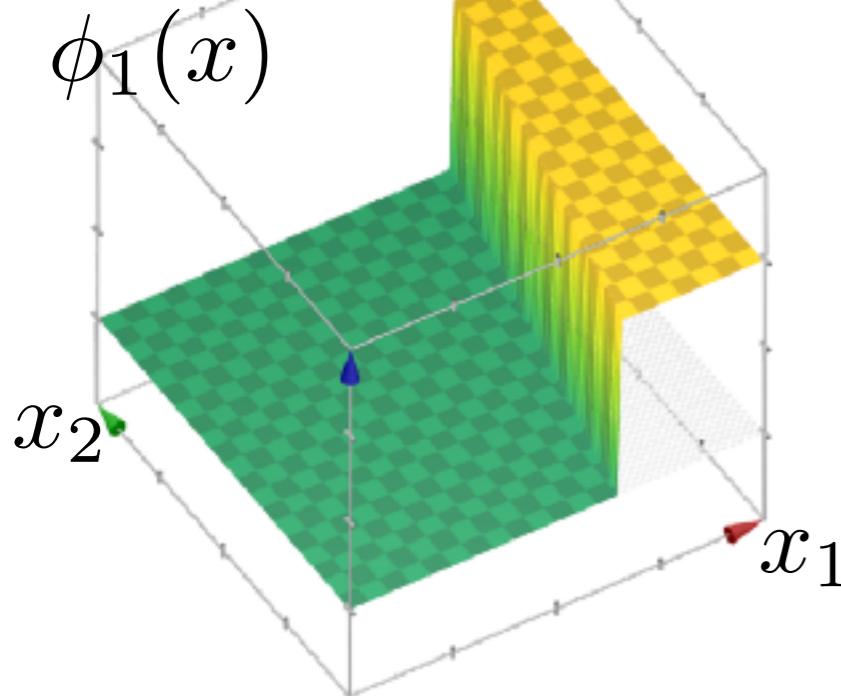
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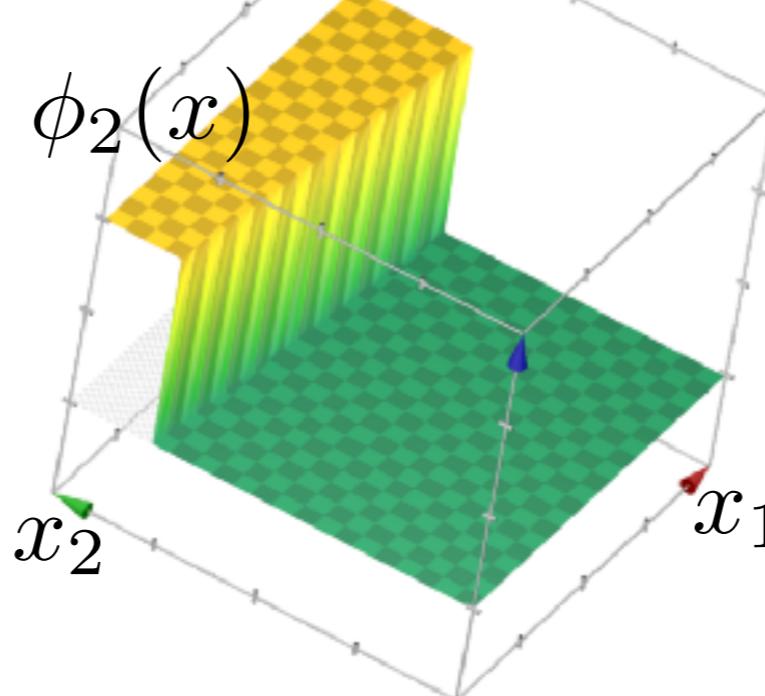


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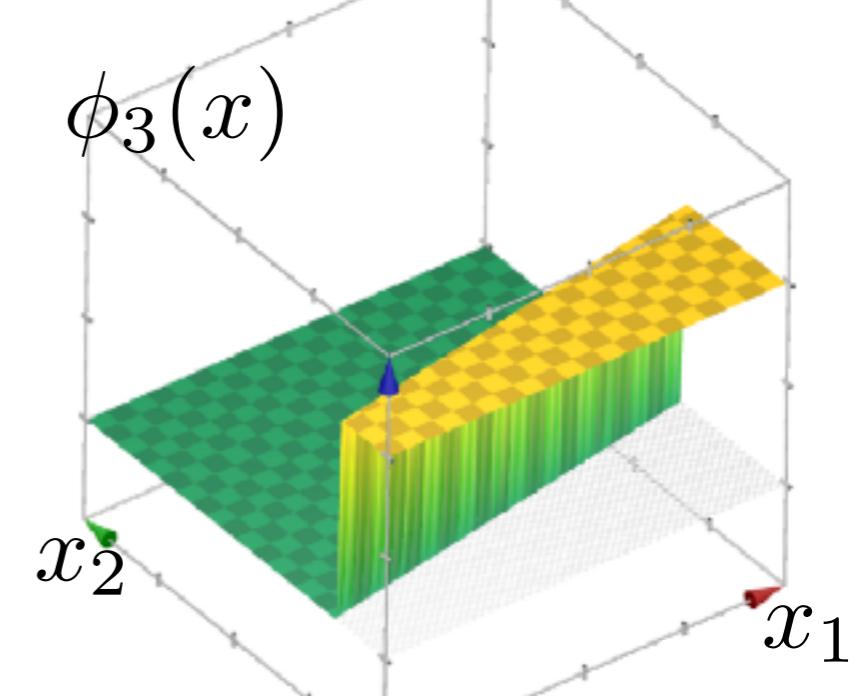
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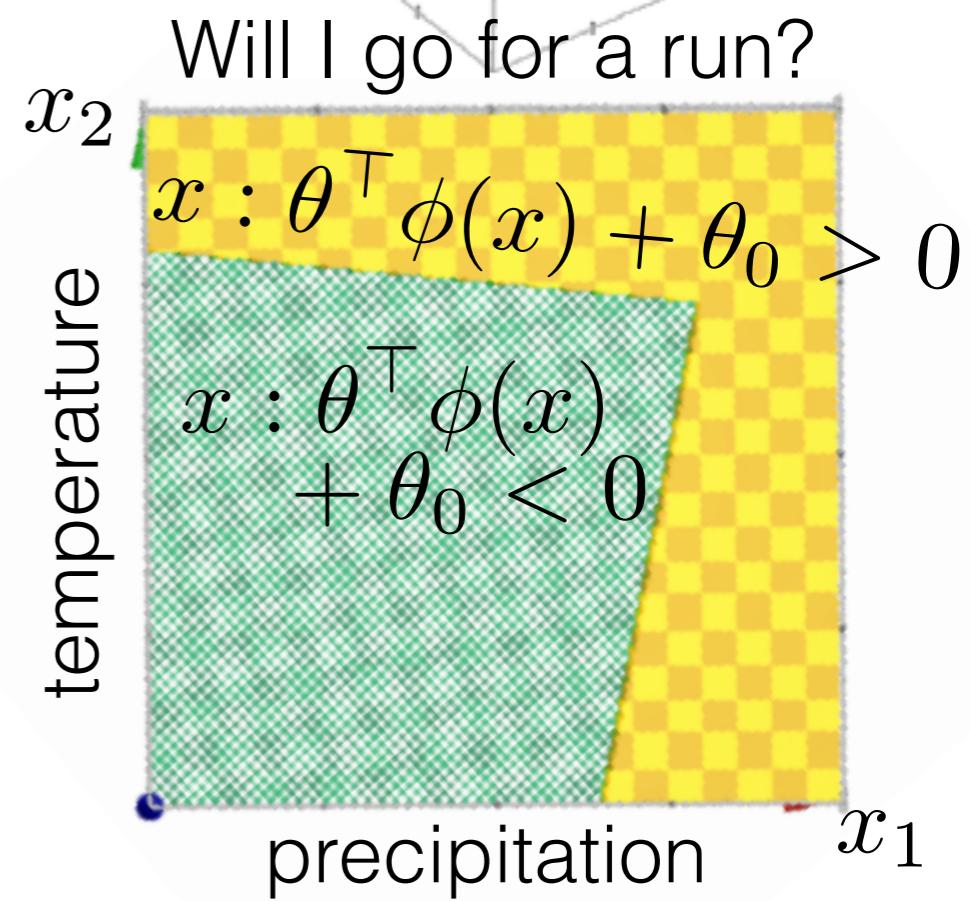
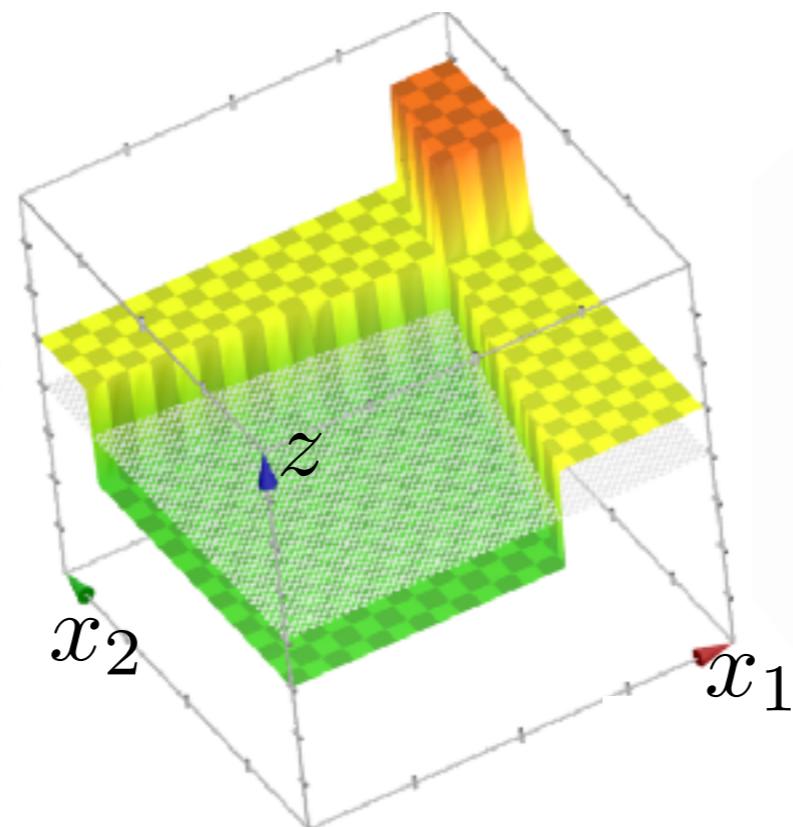
$$\phi_3(x) = \mathbf{1}\{\tilde{\tilde{w}}^\top x + \tilde{\tilde{w}}_0 \geq 0\}$$



$z = \theta^\top \phi(x) + \theta_0$

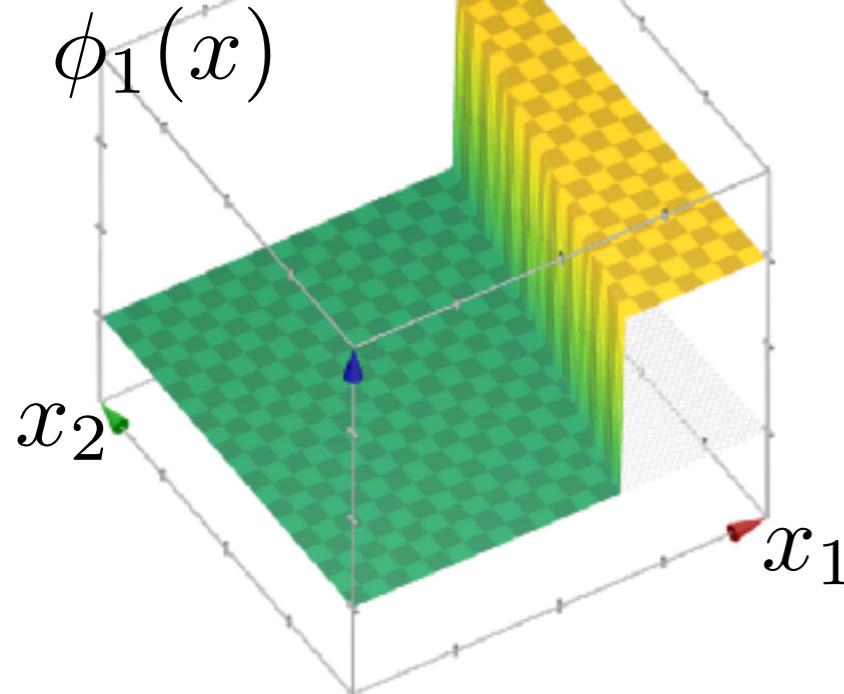
$$= \theta_1 \phi_1(x) + \theta_2 \phi_2(x) + \theta_3 \phi_3(x) + \theta_0$$

$$= 1 \cdot \phi_1(x) + 1 \cdot \phi_2(x) + (-0.5)$$

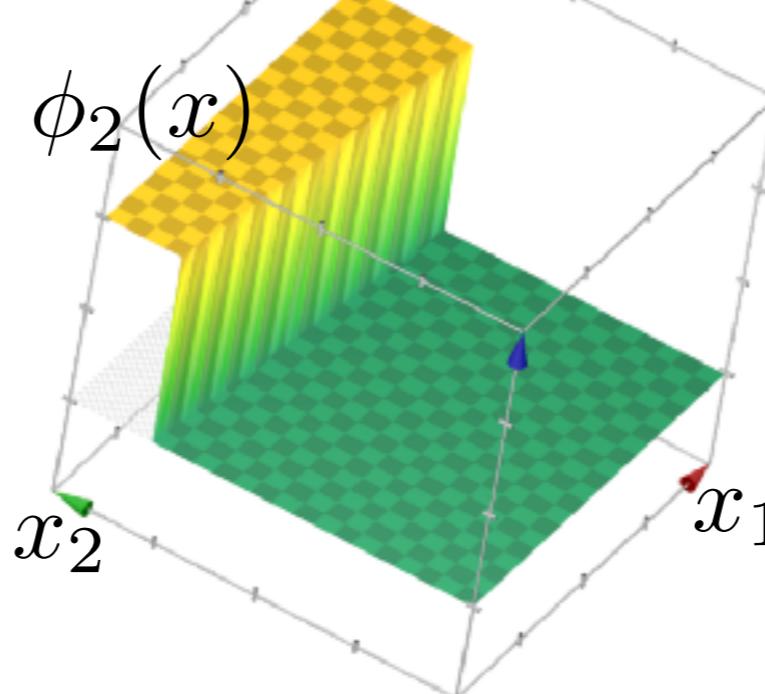


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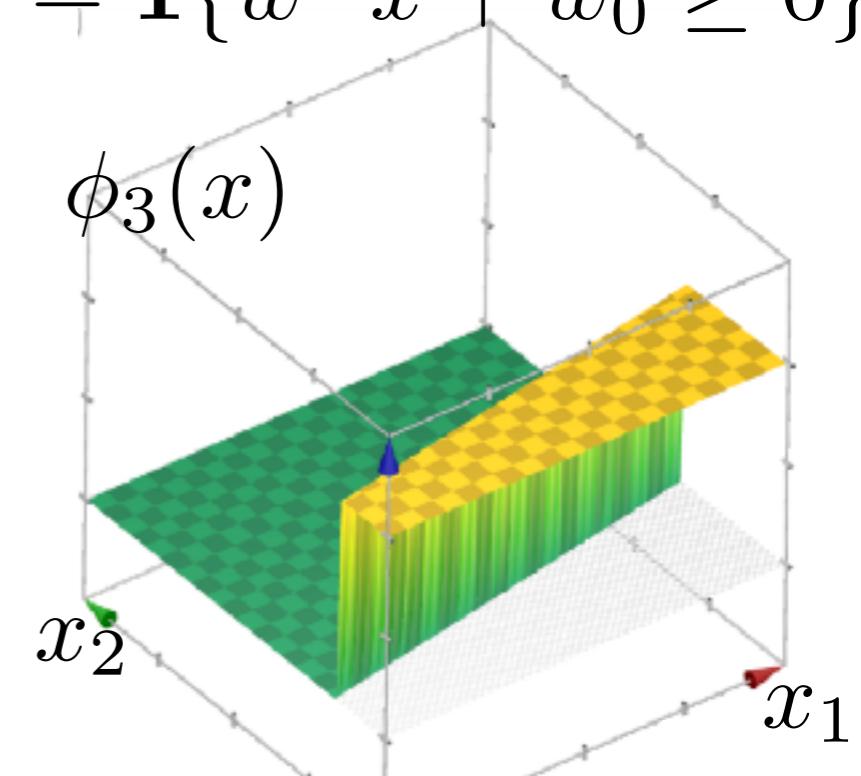
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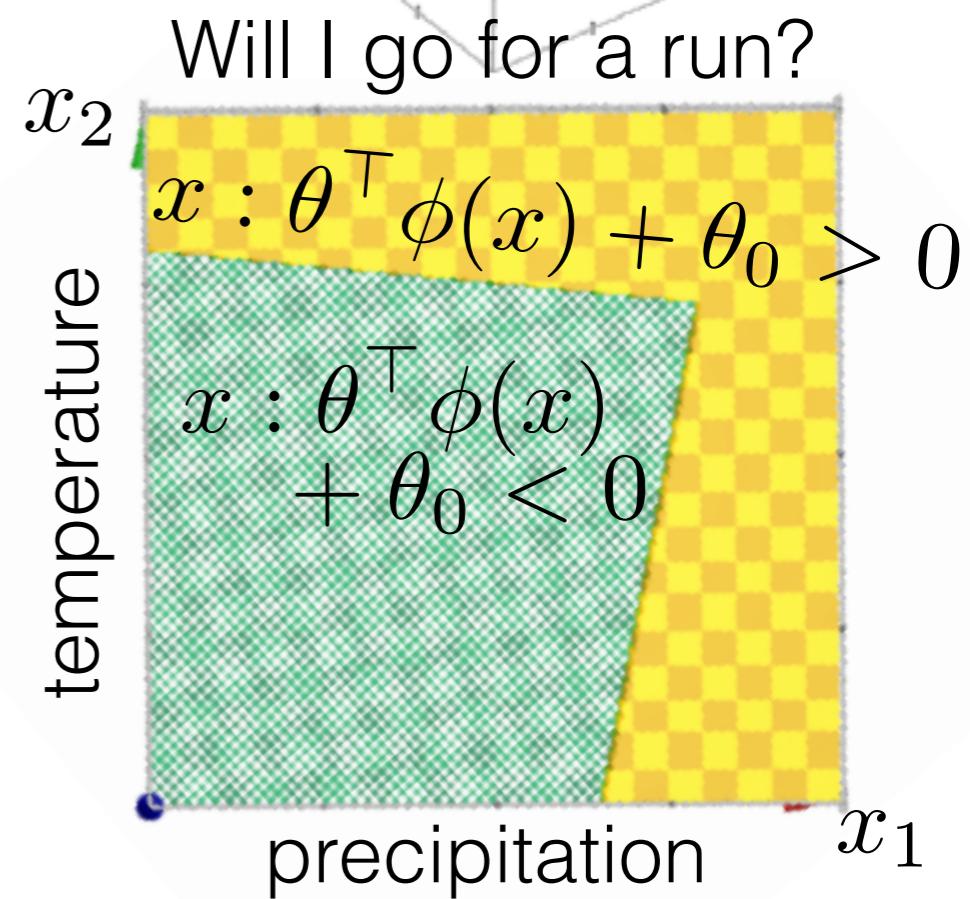
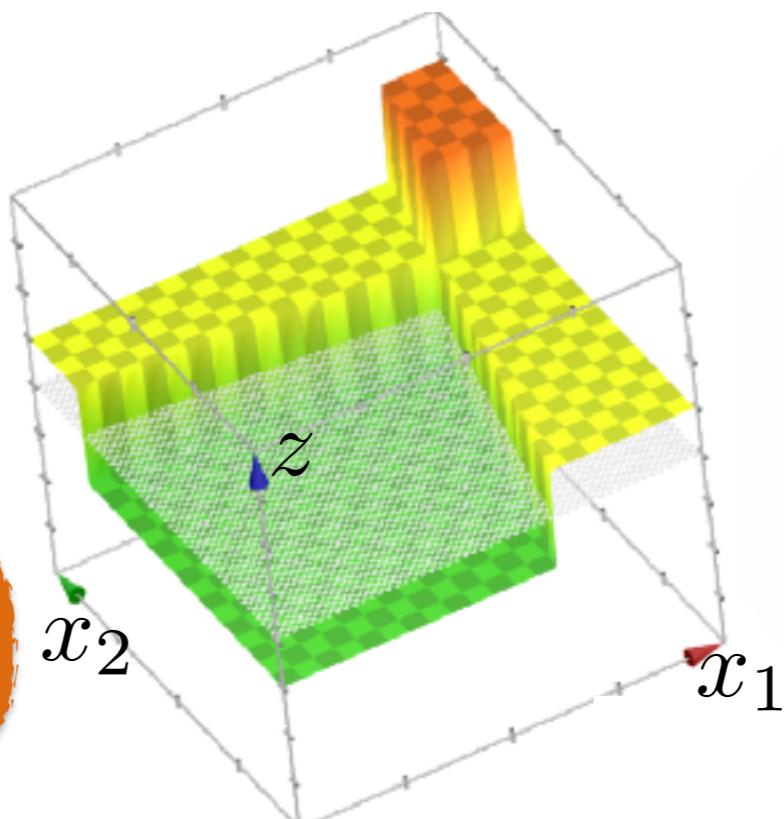
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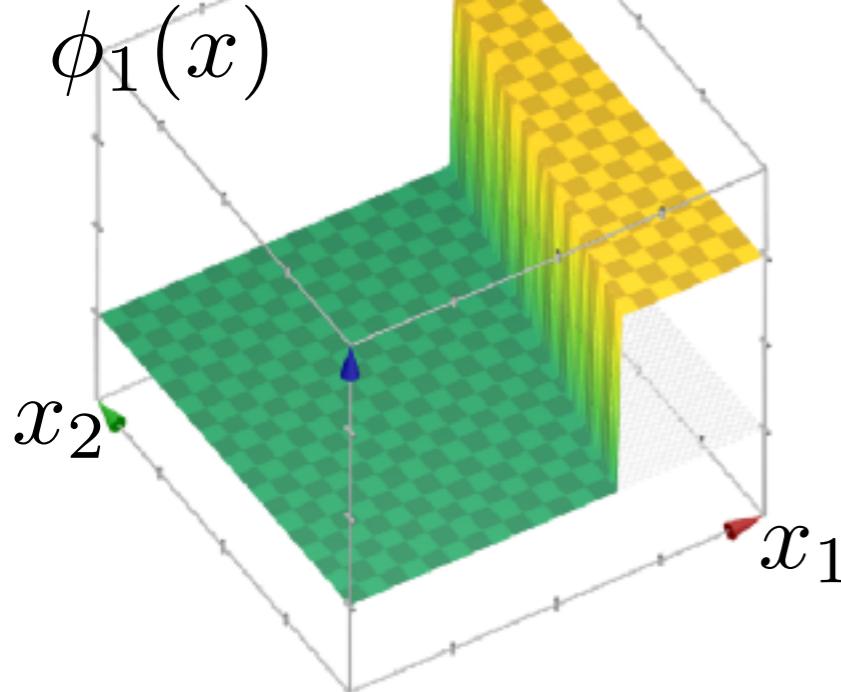


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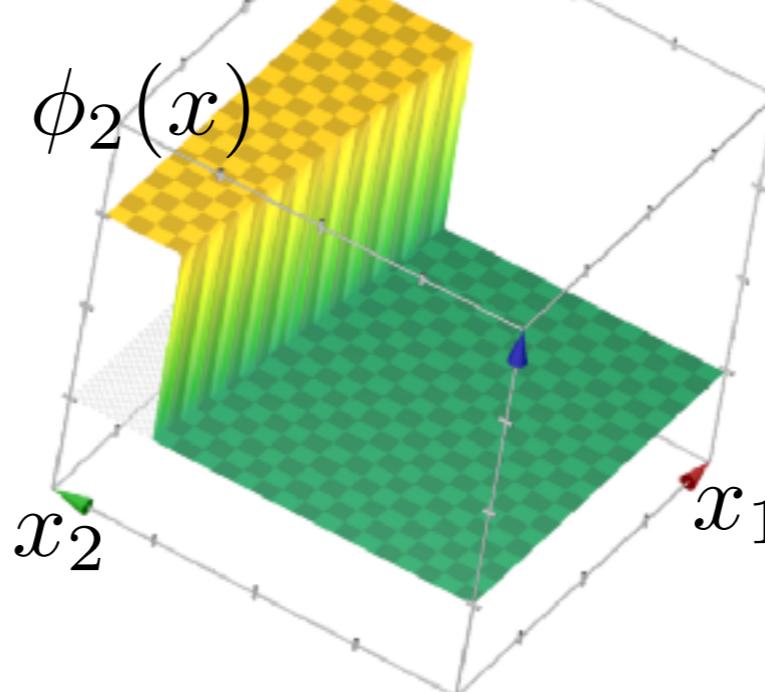


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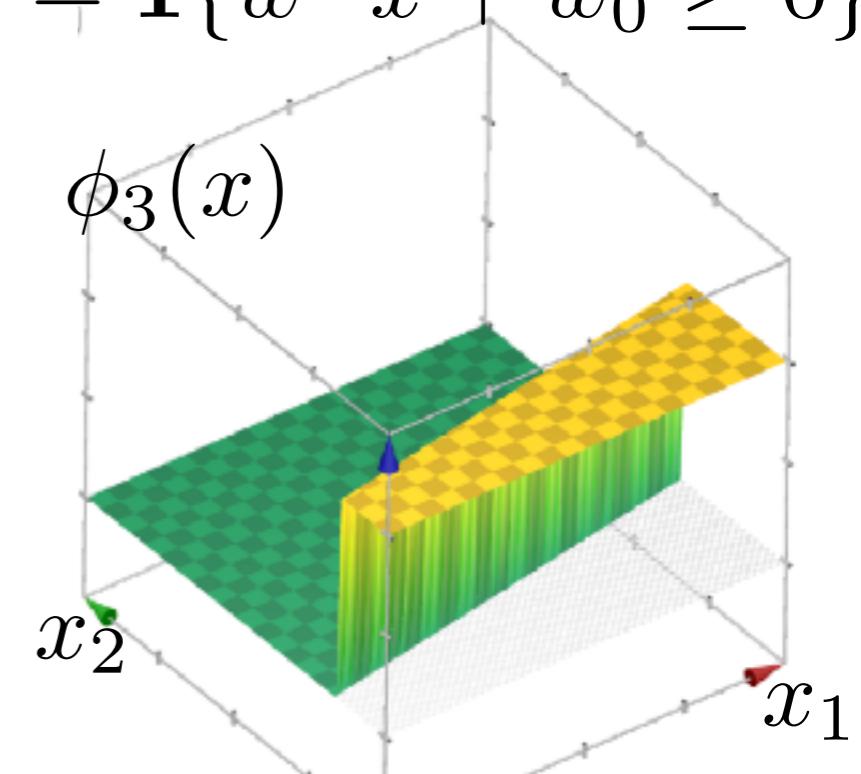
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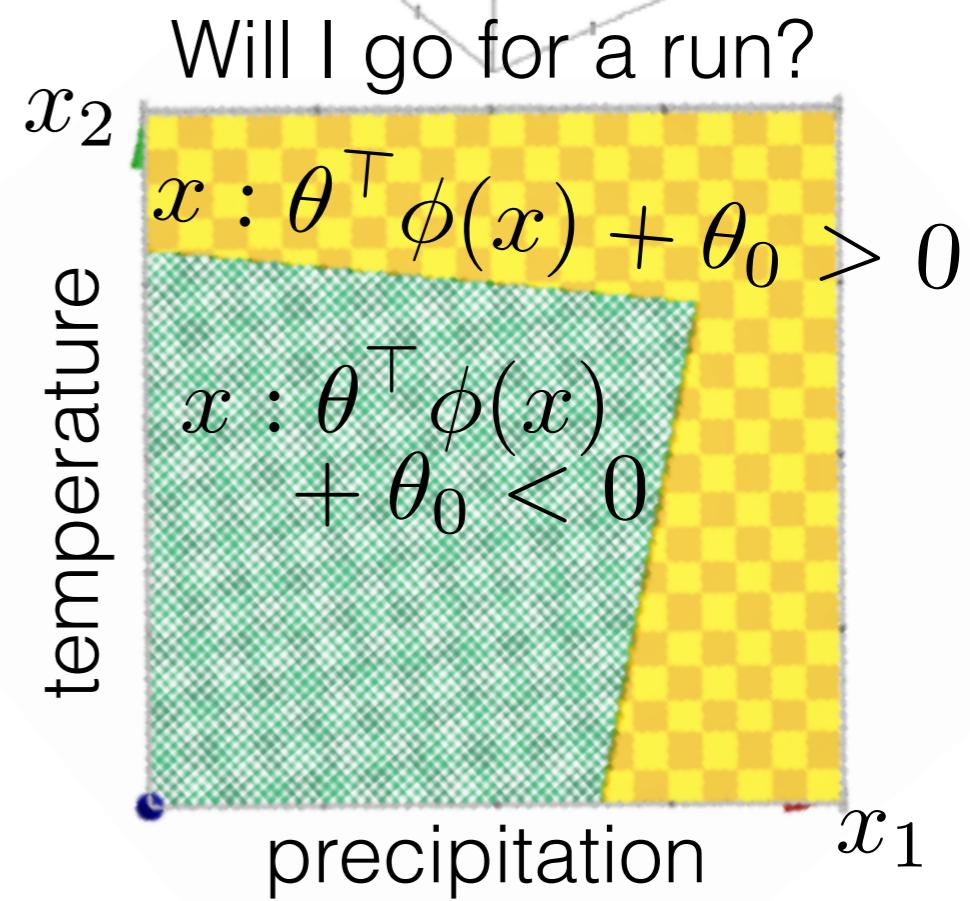
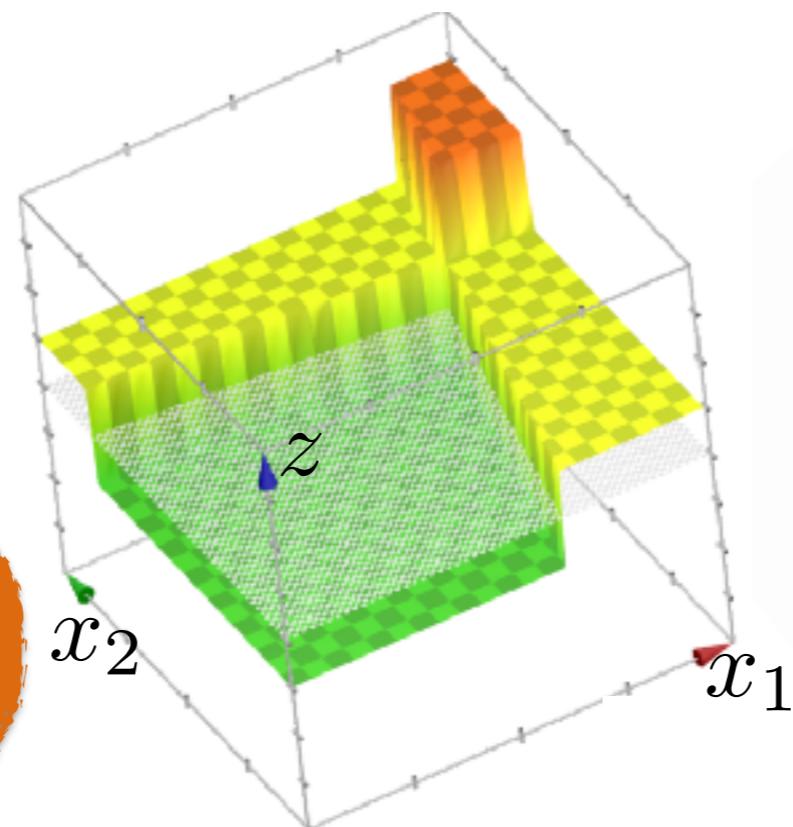
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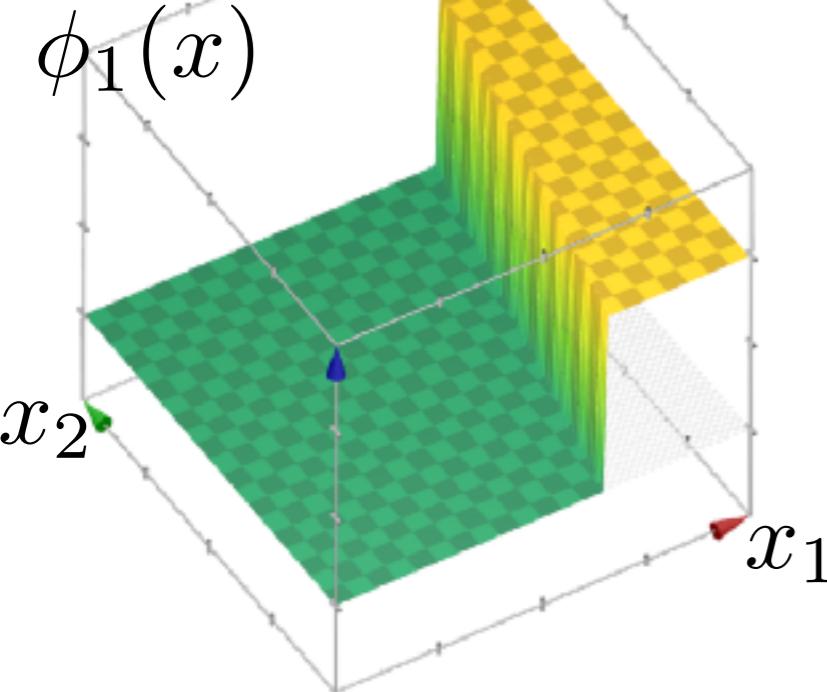


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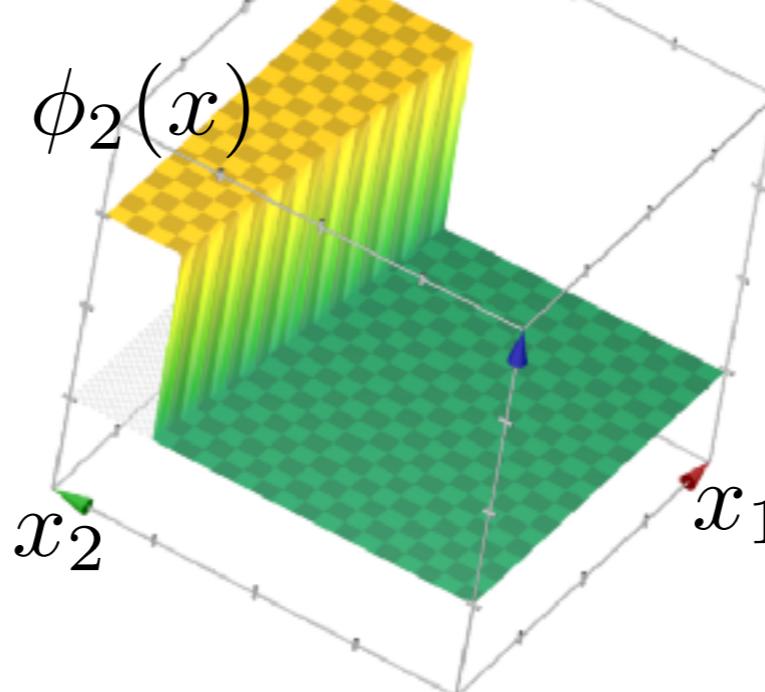


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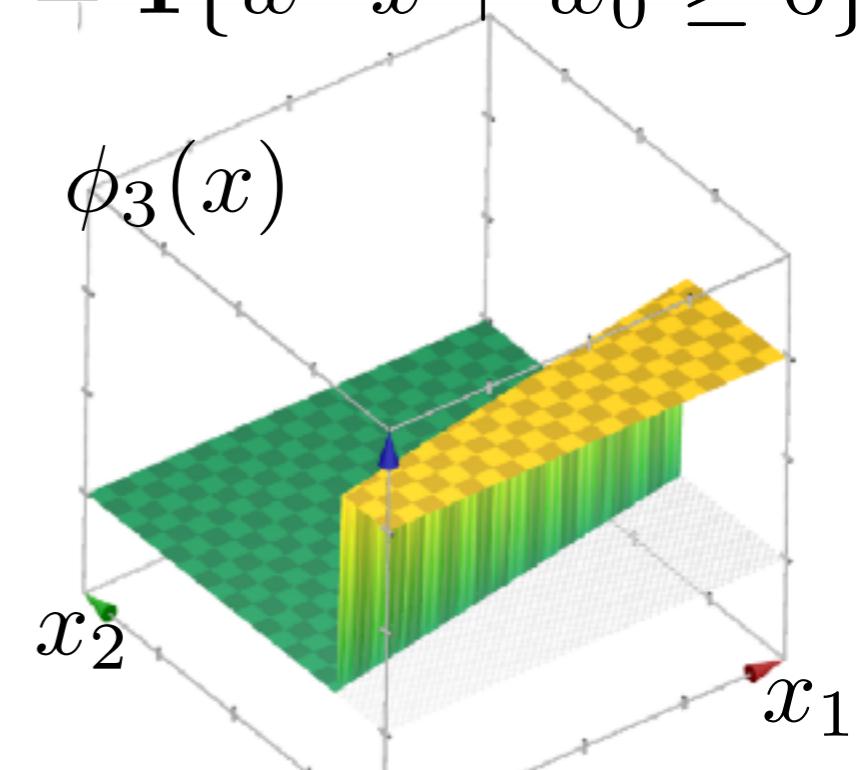
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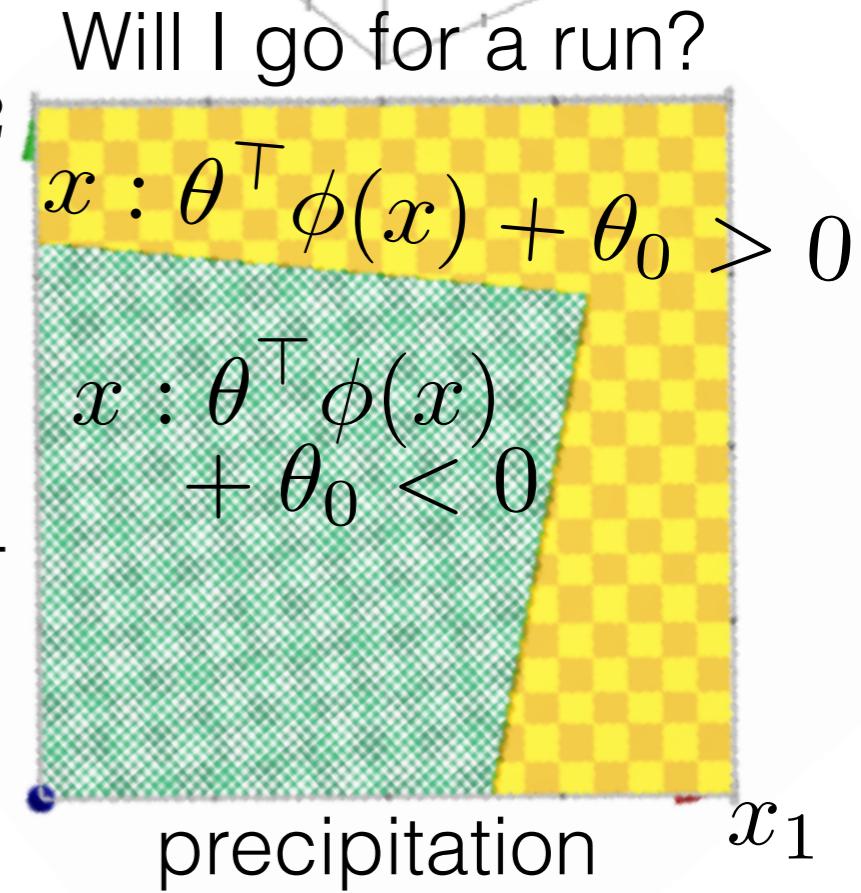
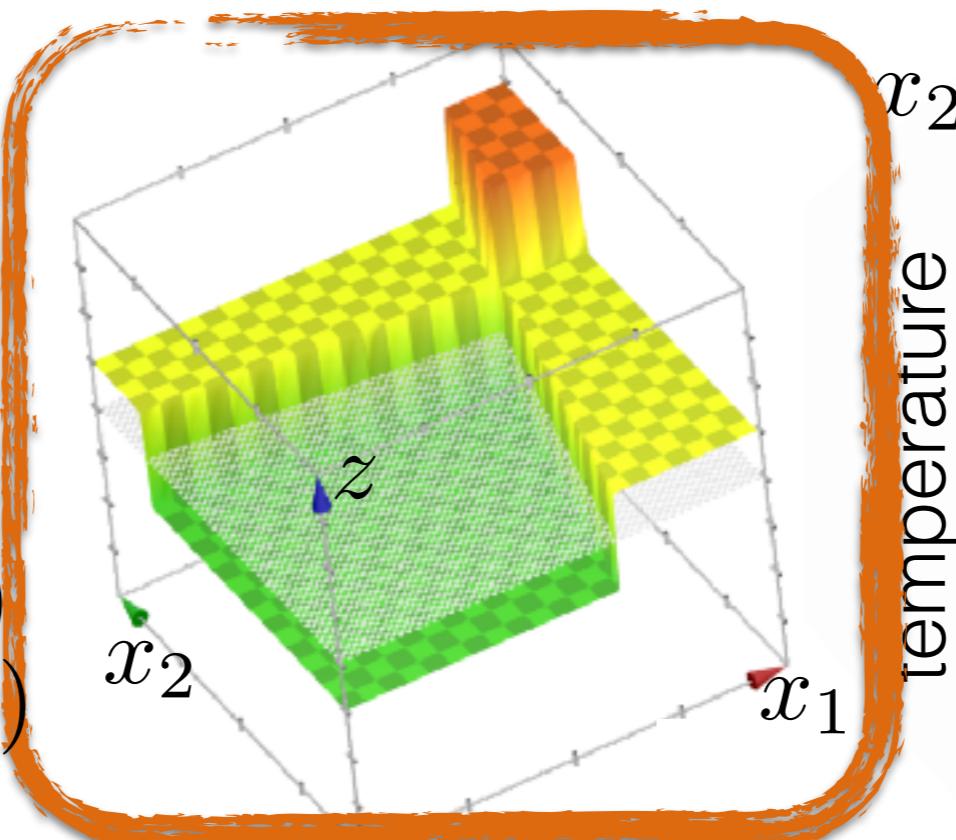
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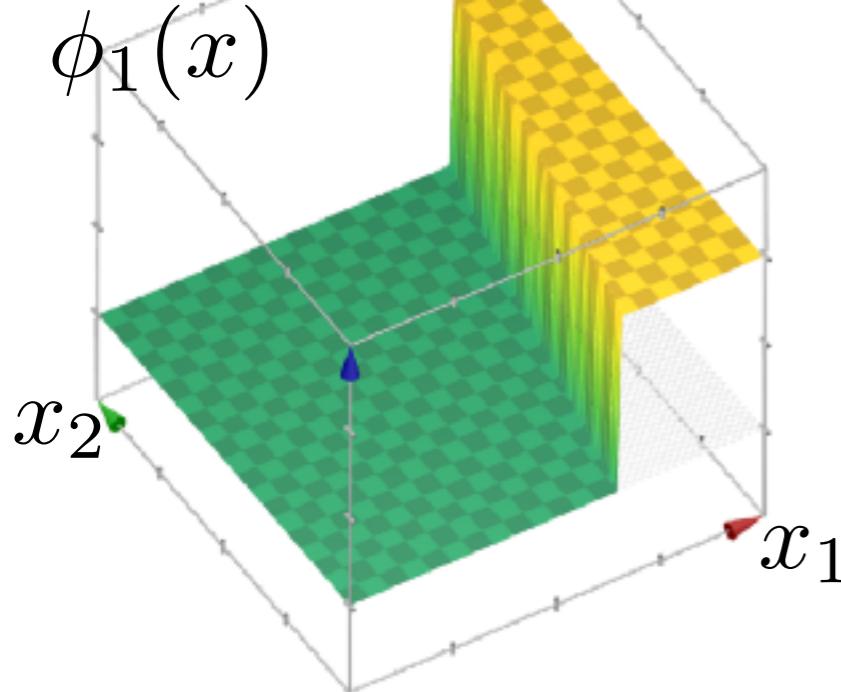


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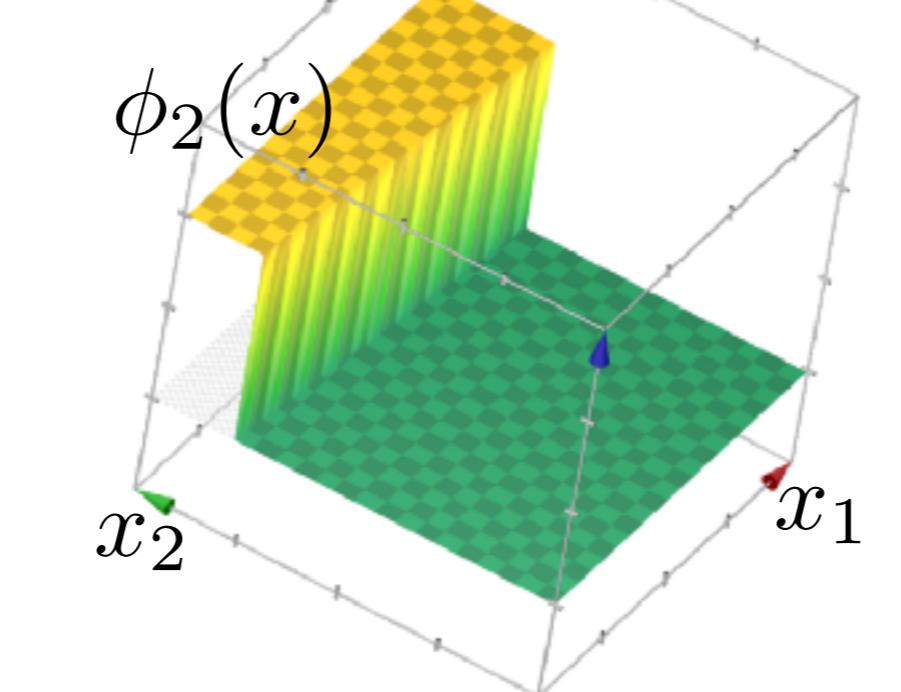


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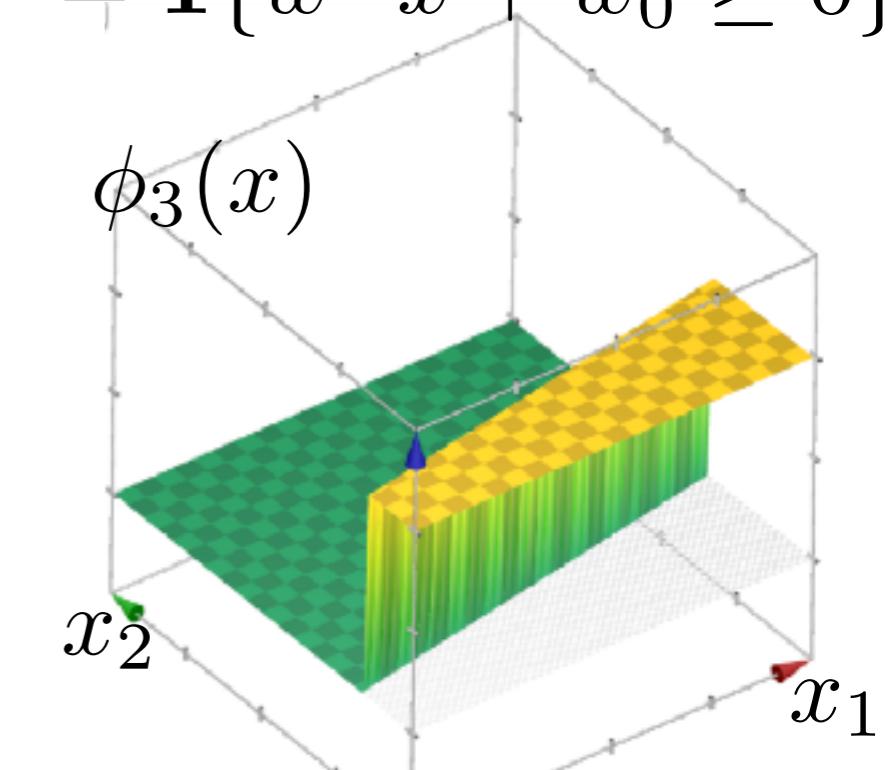
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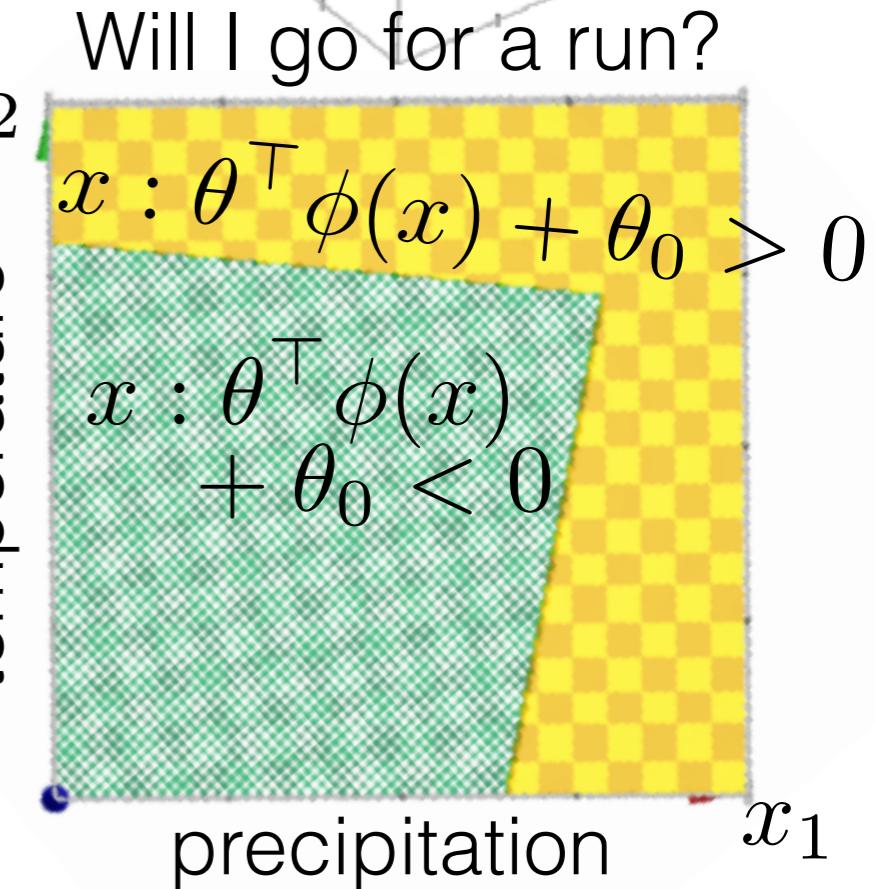
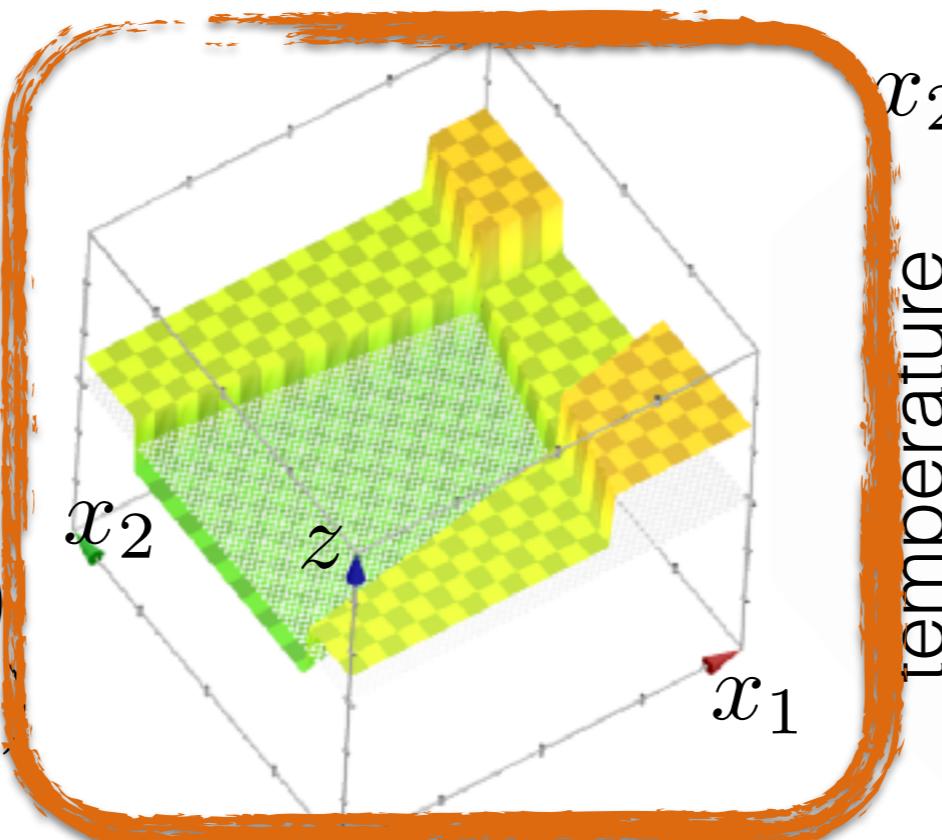
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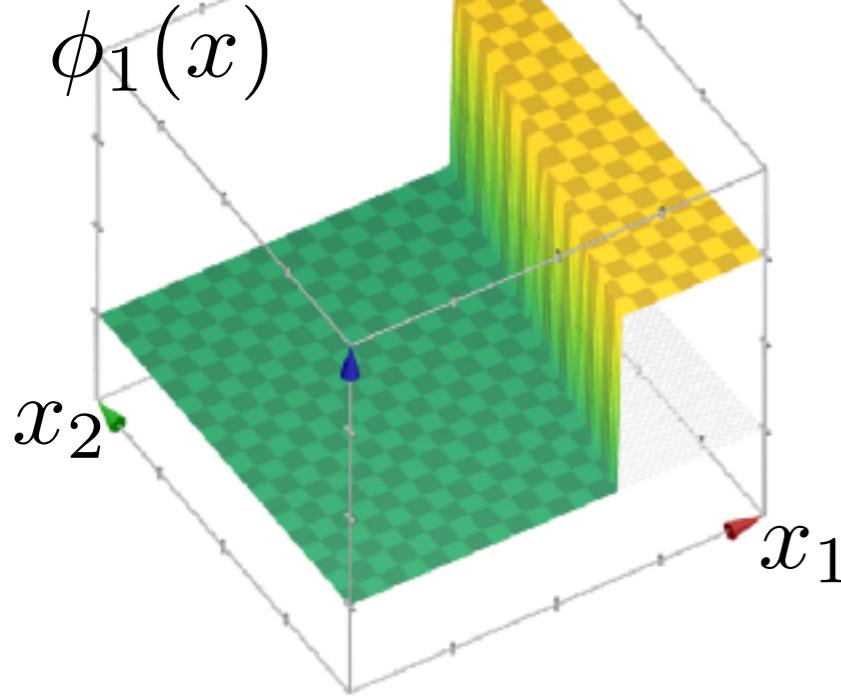


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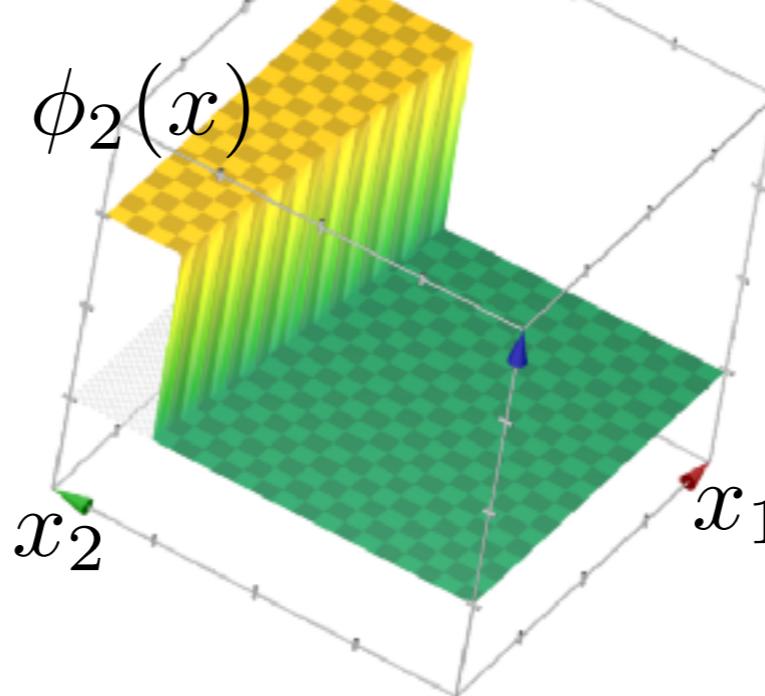


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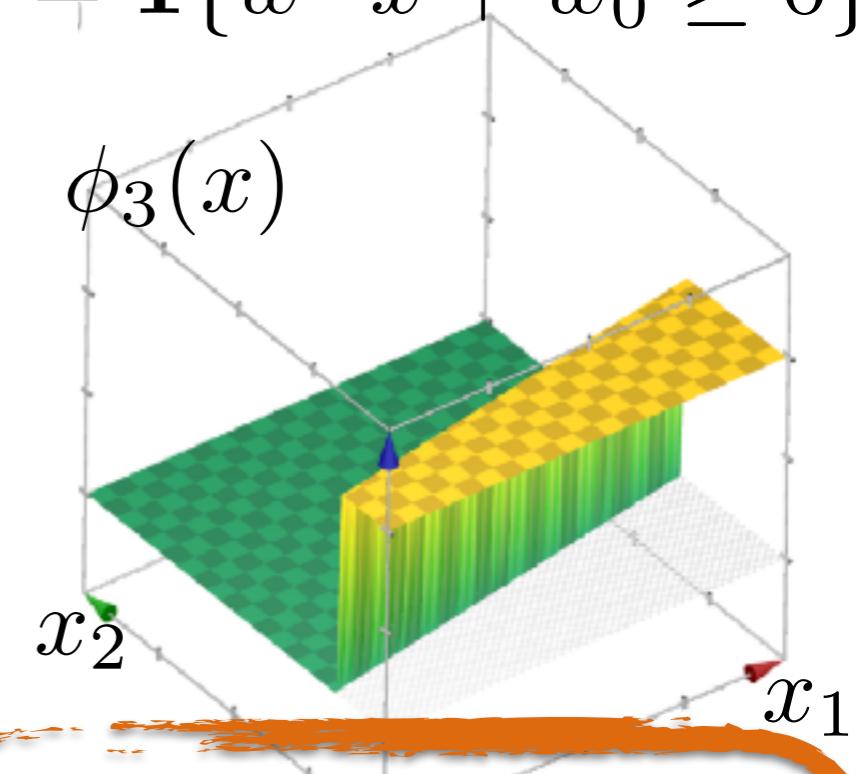
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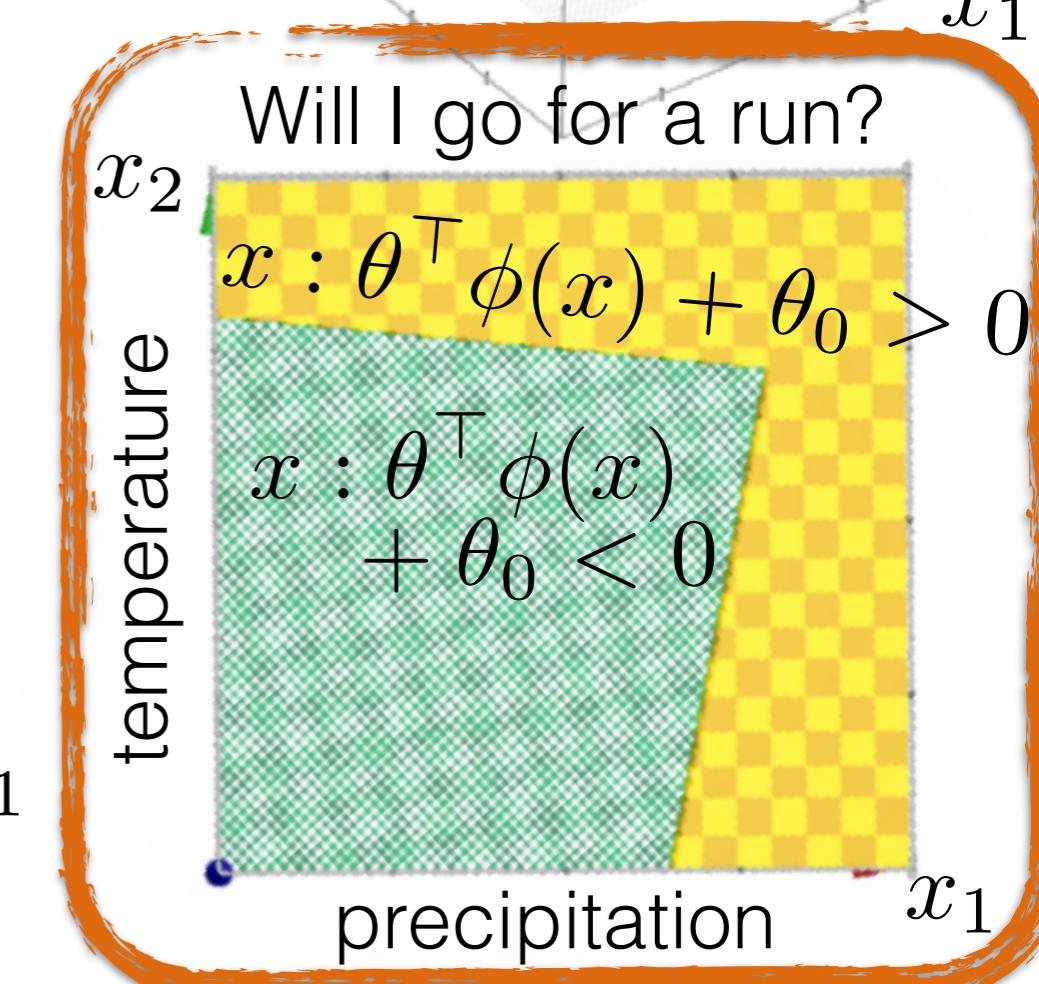
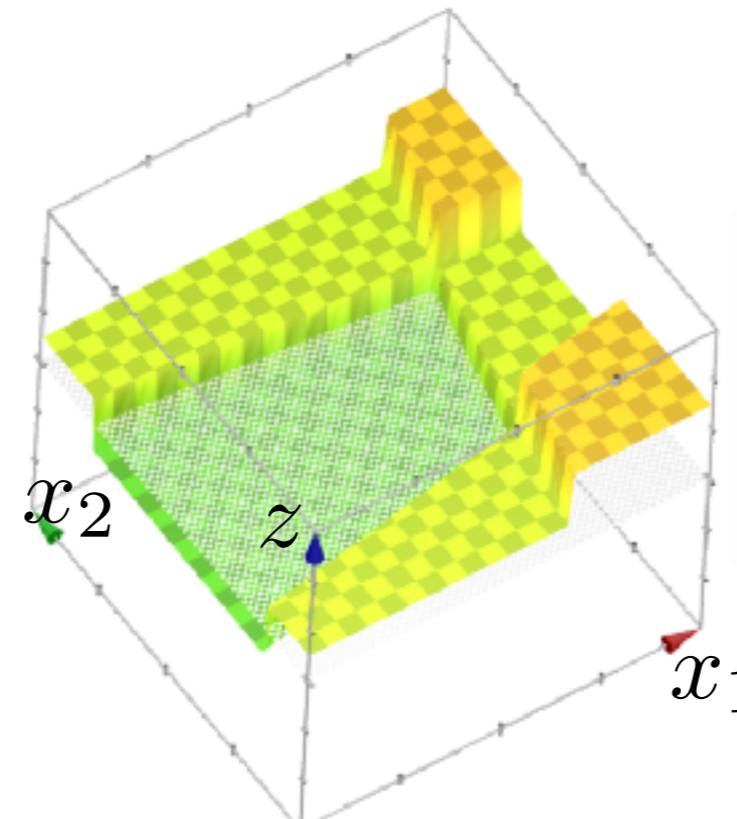
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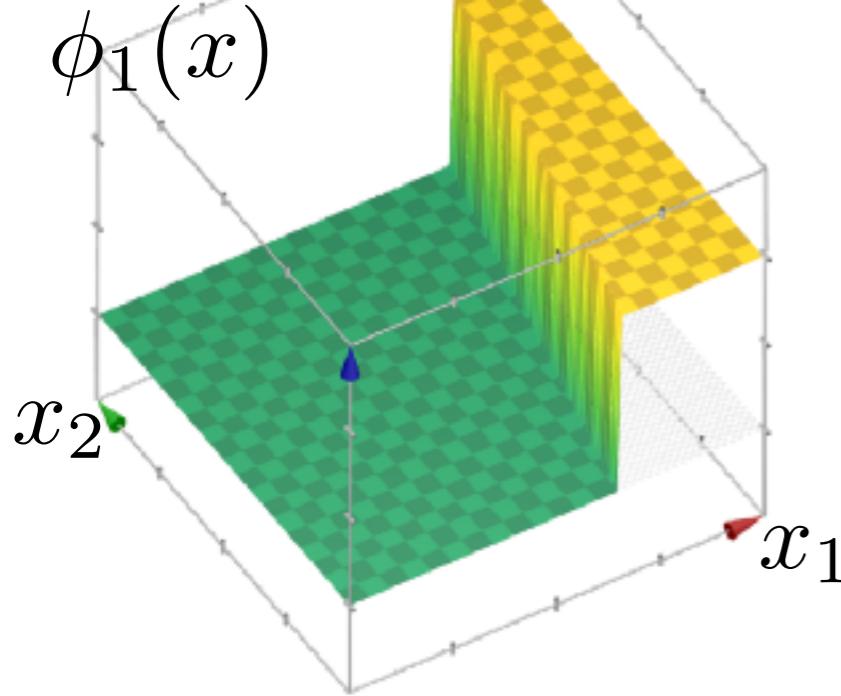


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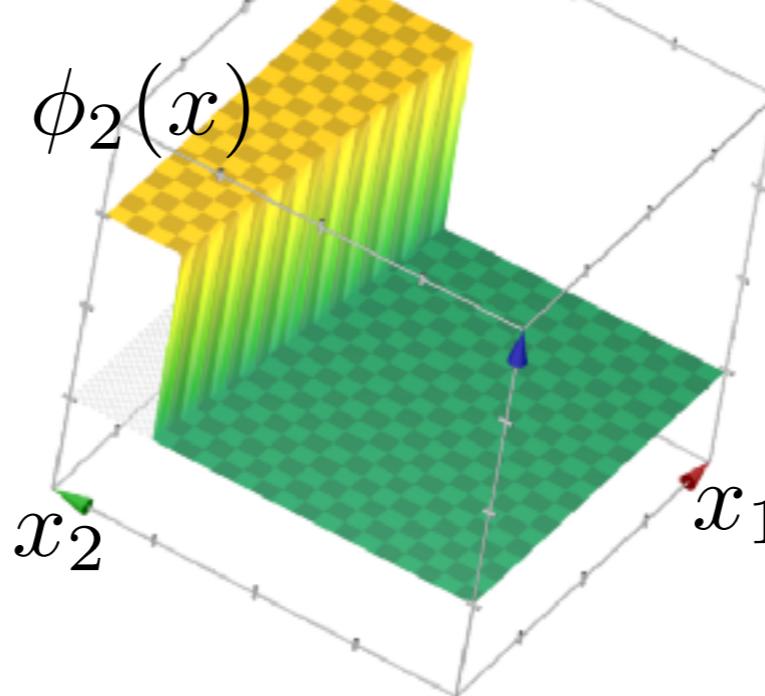


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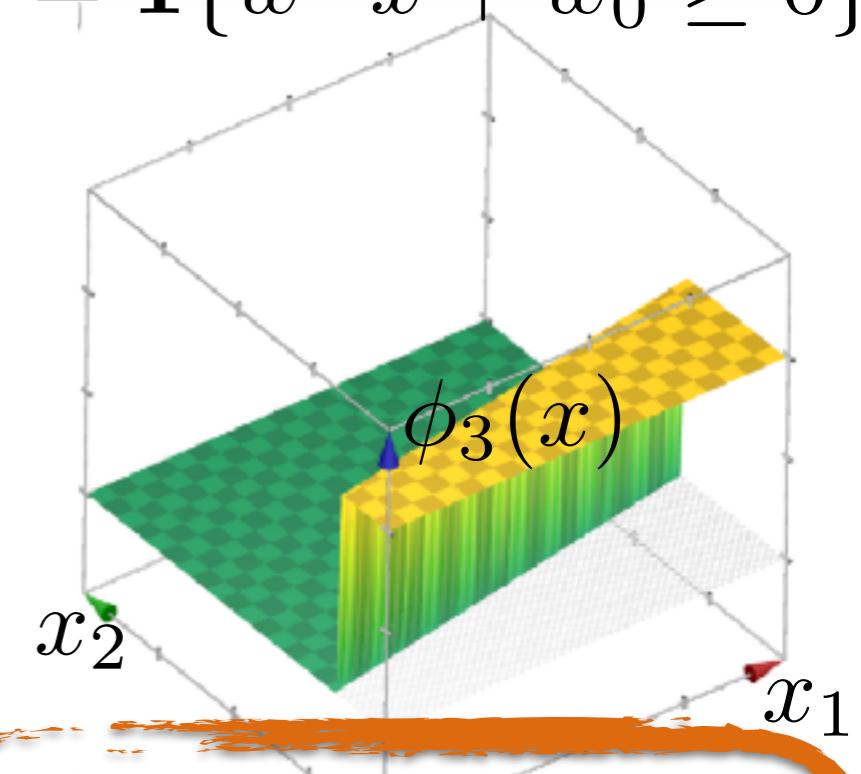
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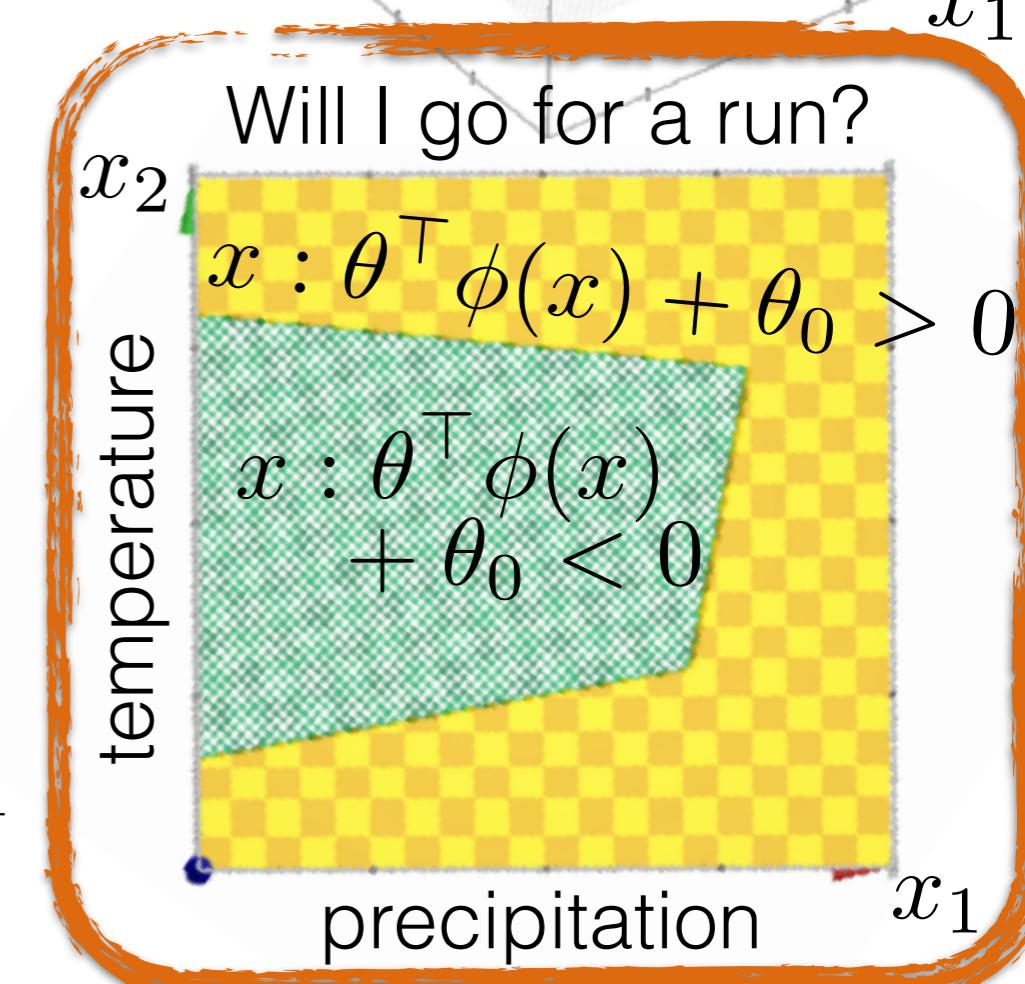
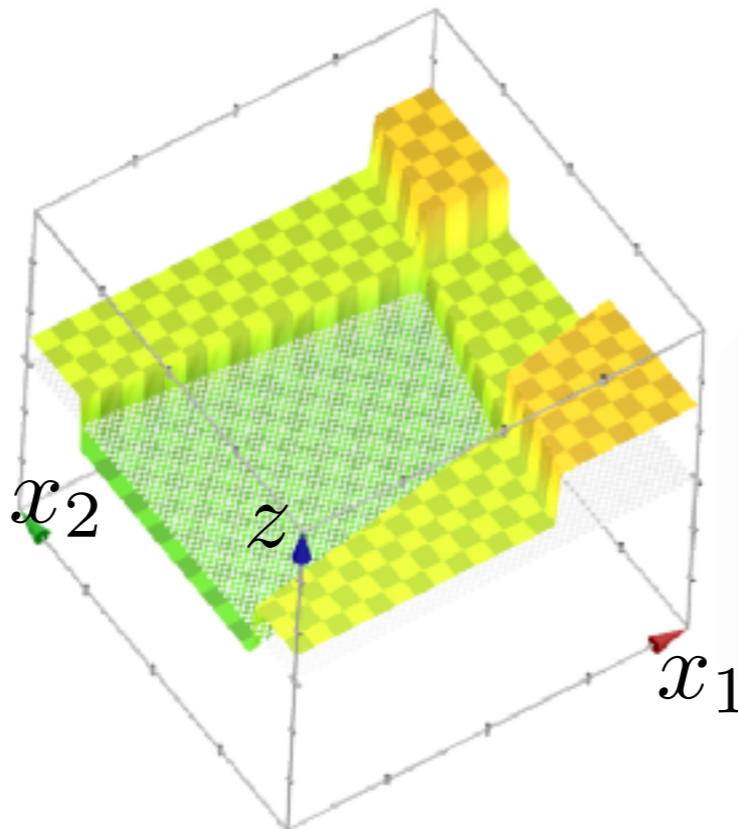
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# Let's get some new notation

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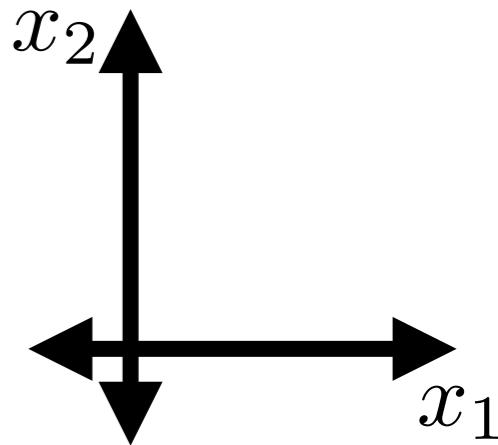
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  - Input  $x$  (a data point)

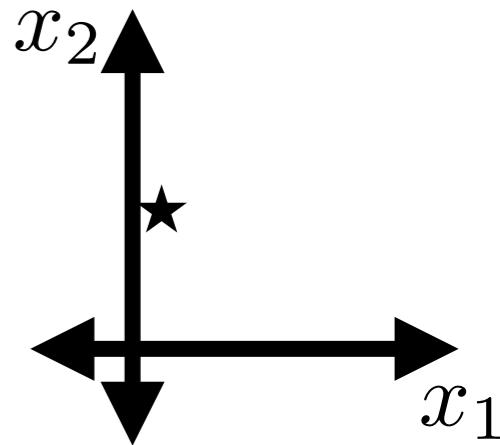
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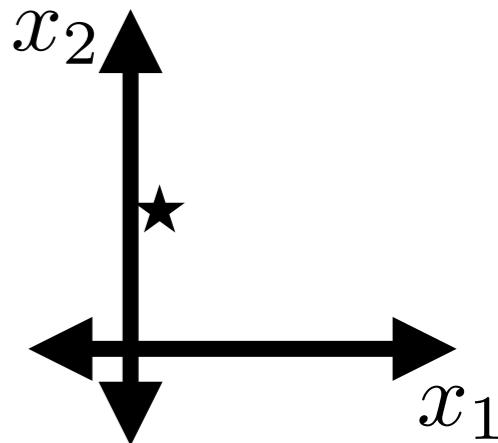
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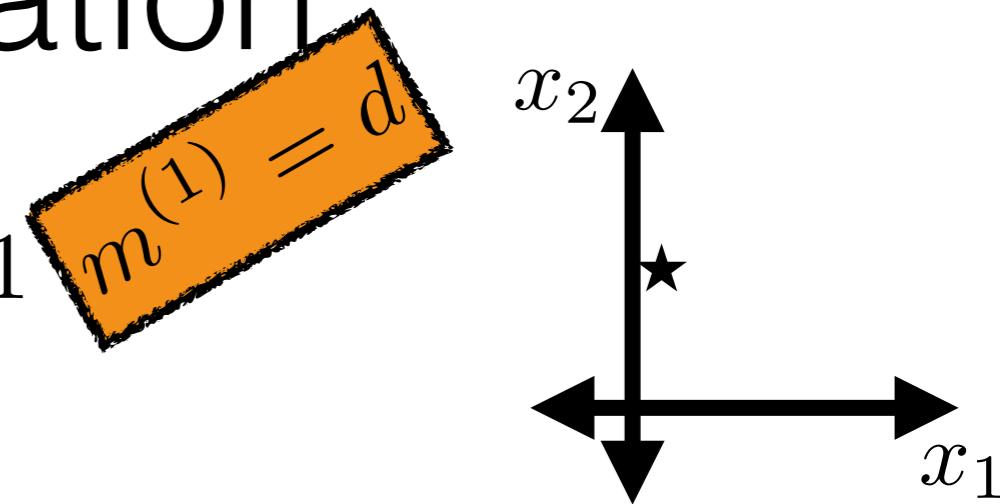
# Let's get some new notation

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  - Input  $x$  (a data point): size  $m^{(1)} \times 1$



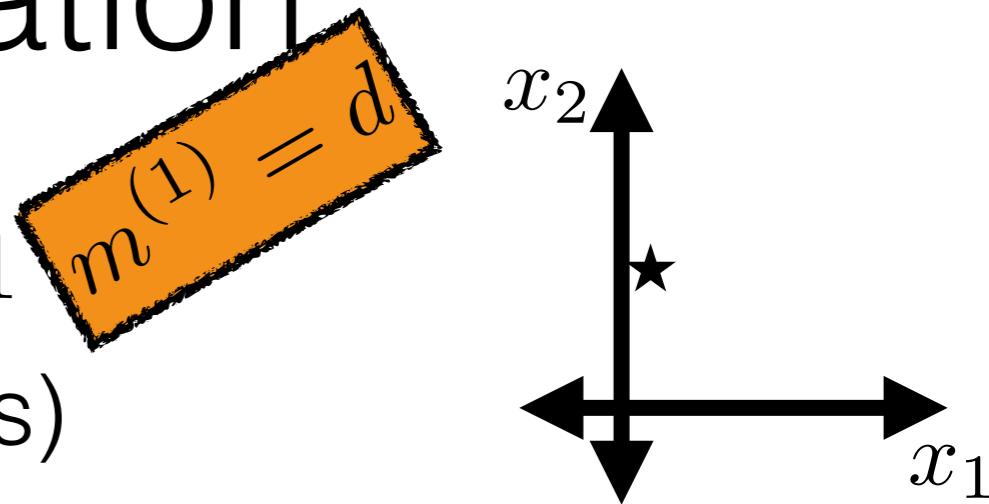
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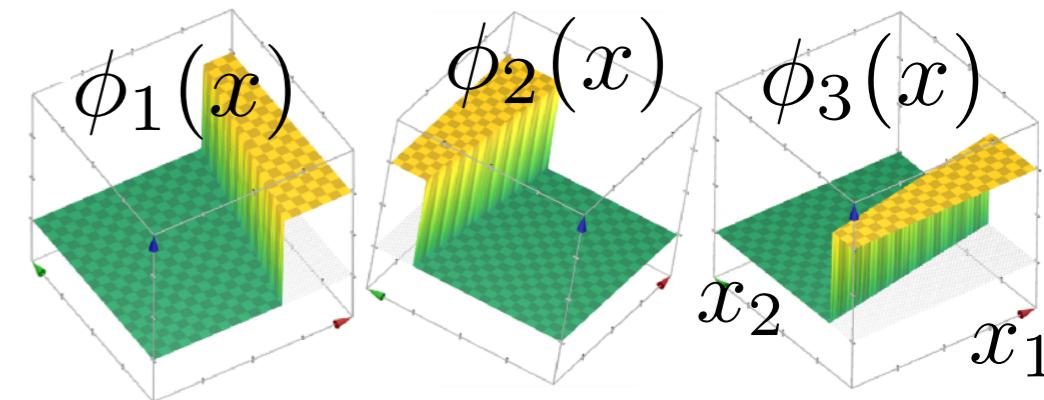
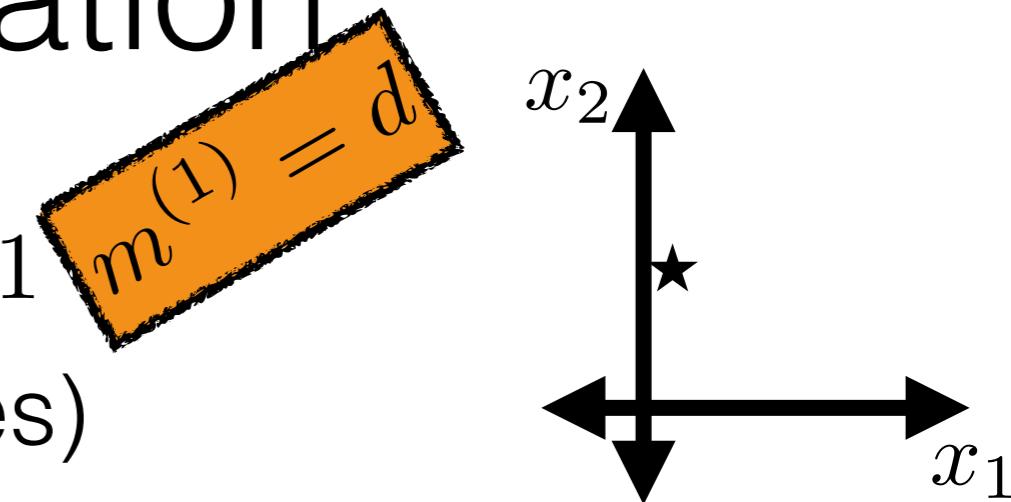
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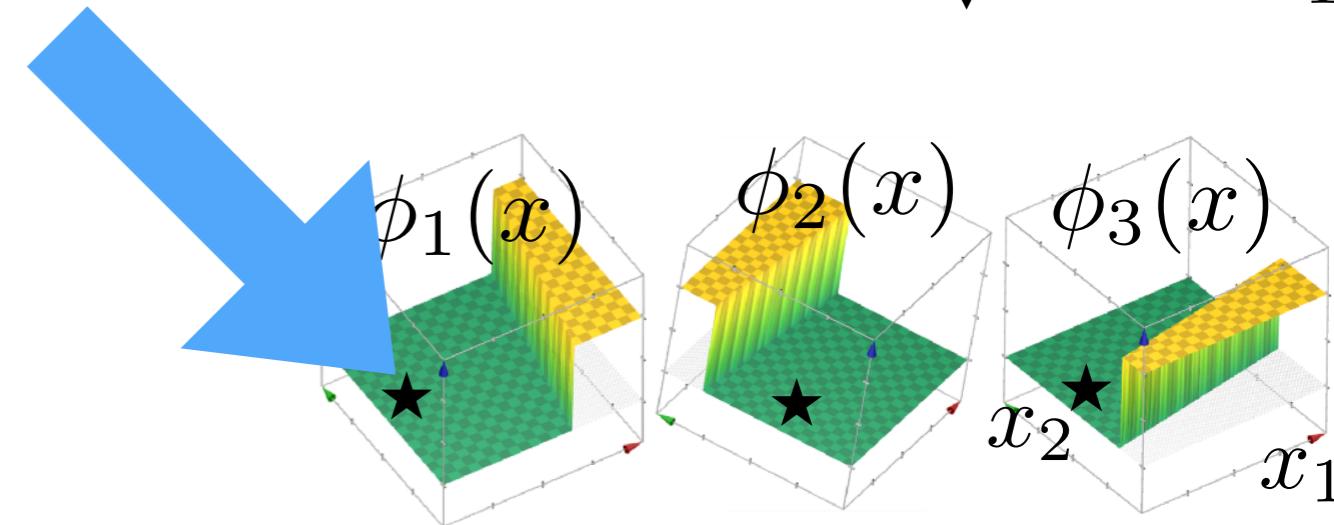
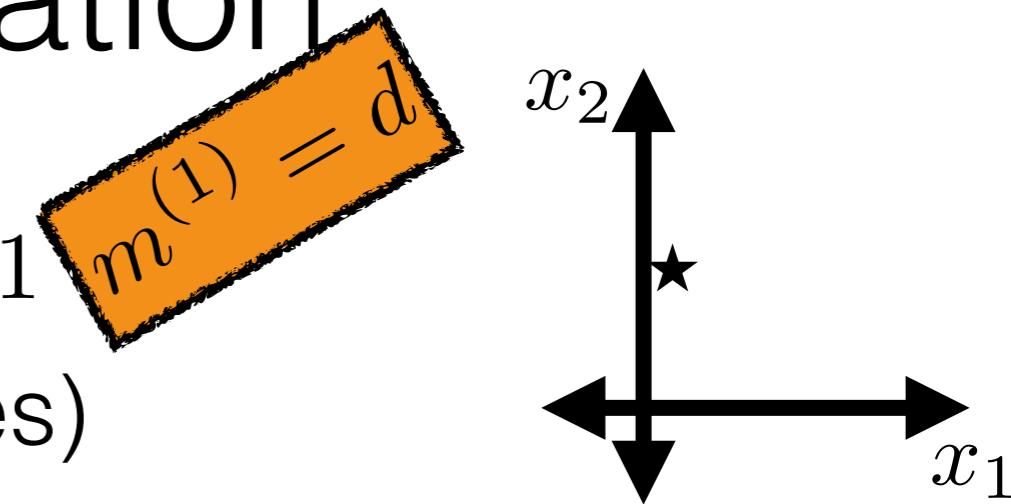
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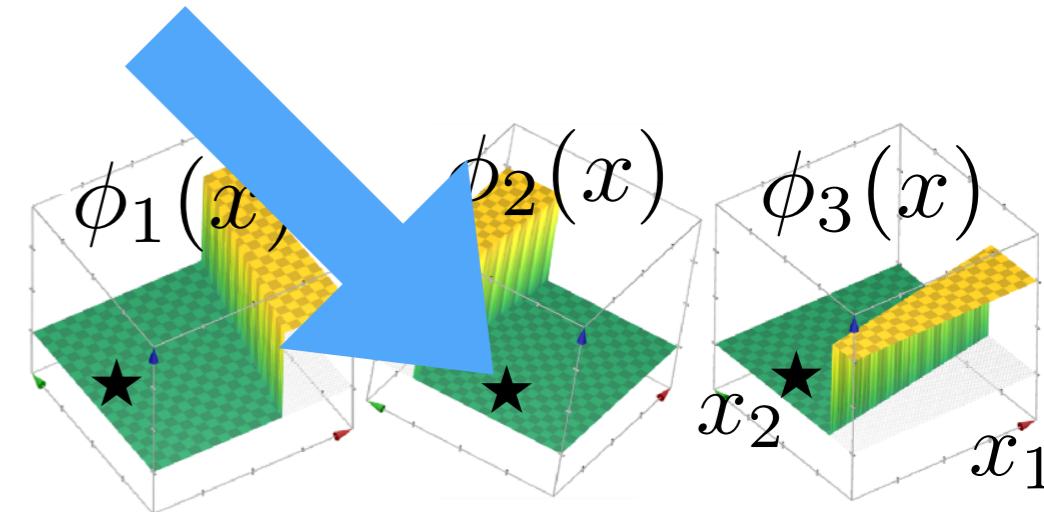
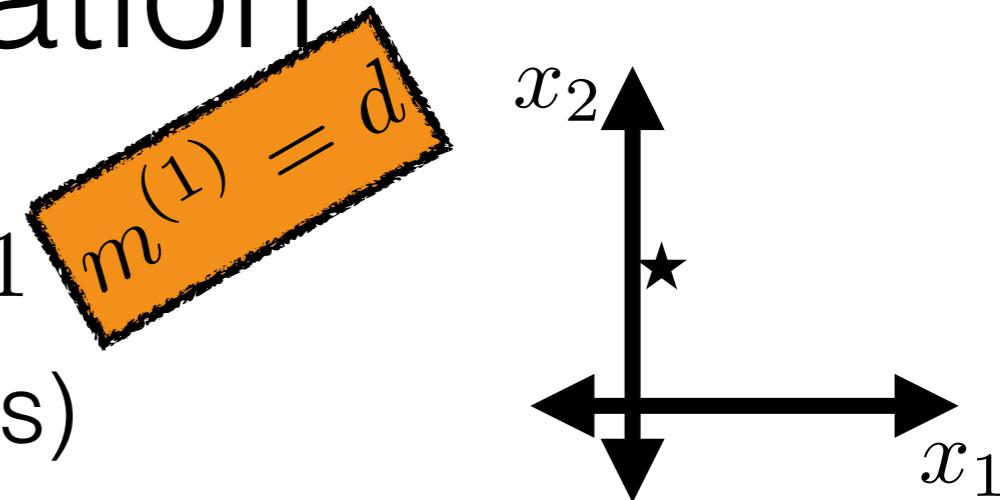
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of feature values)



# Let's get some new notation

- 1st layer, constructing the features:

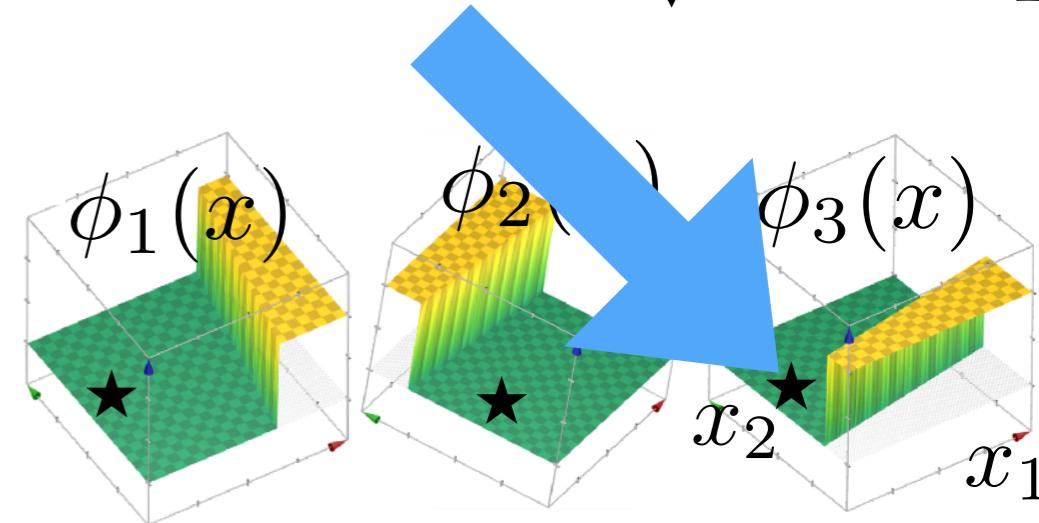
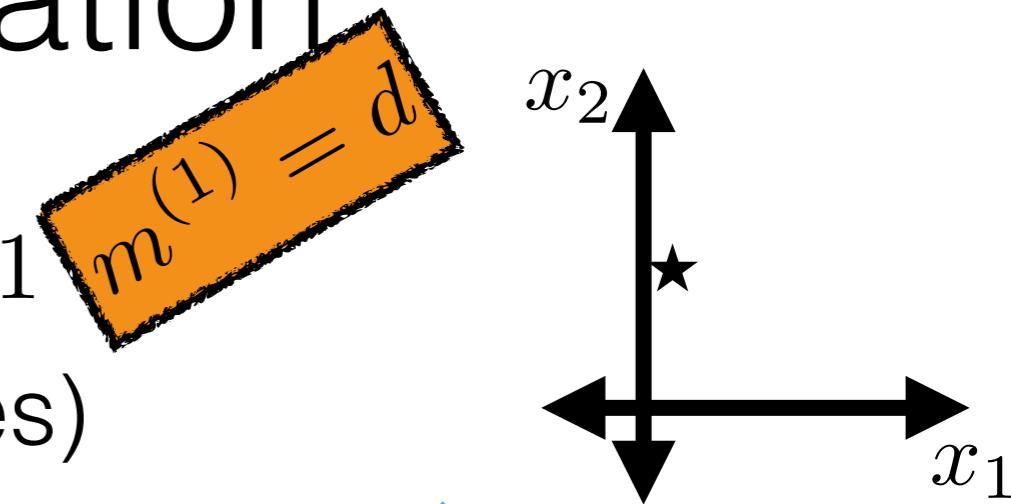
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of feature values)



# Let's get some new notation

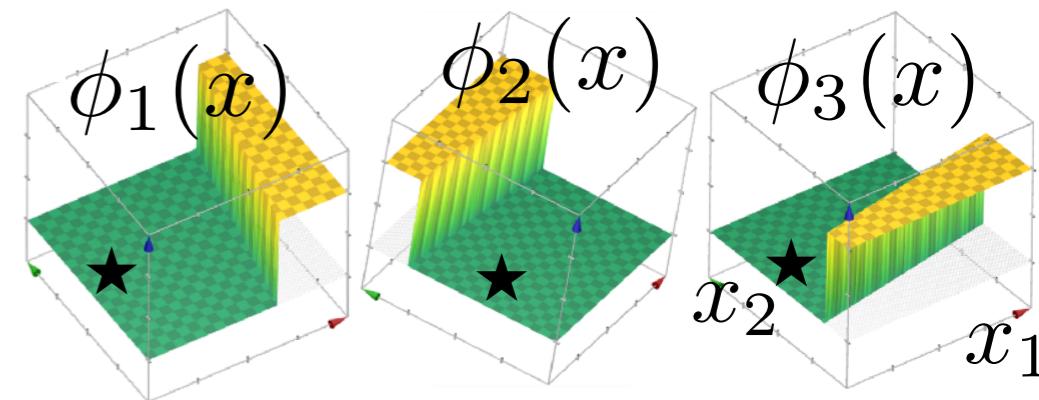
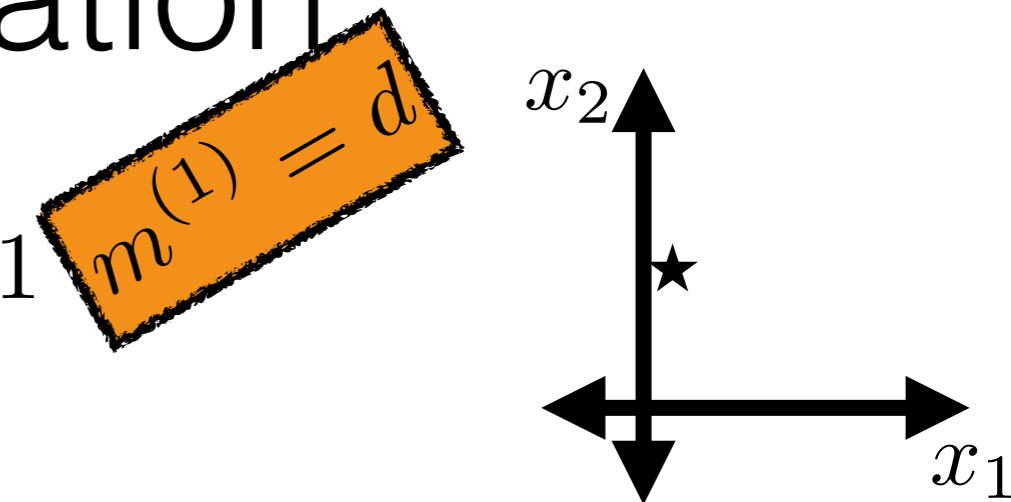
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of feature values)



# Let's get some new notation

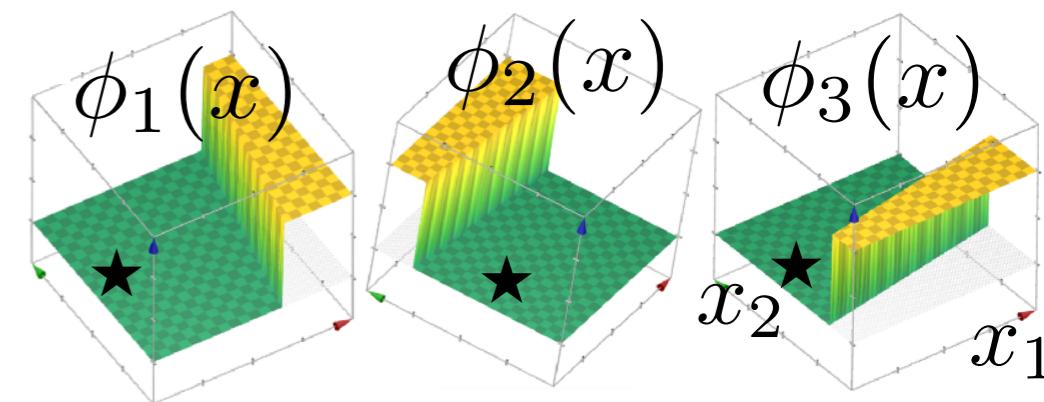
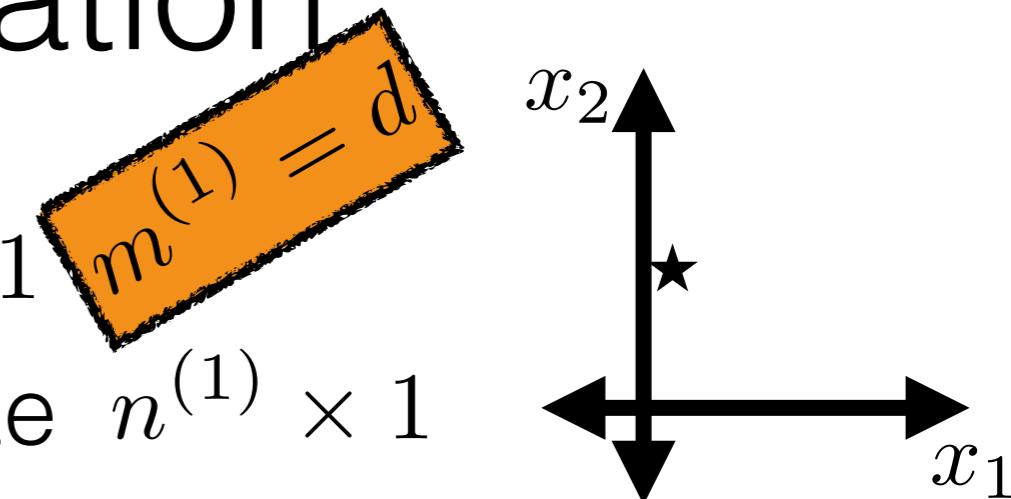
- 1st layer, constructing the features:
  - Input  $x$  (a data point): size  $m^{(1)} \times 1$
  - Output  $A^{(1)}$  (vector of features)



# Let's get some new notation

- 1st layer, constructing the features:

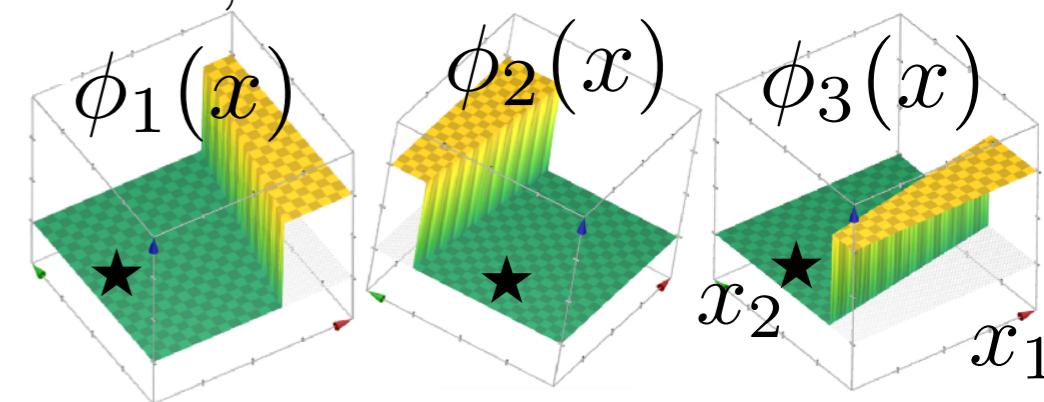
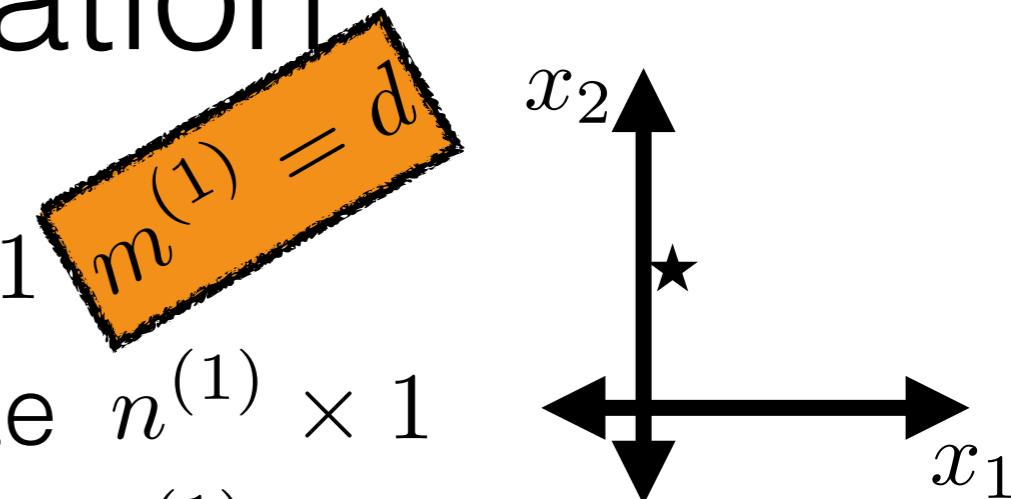
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$



# Let's get some new notation

- 1st layer, constructing the features:

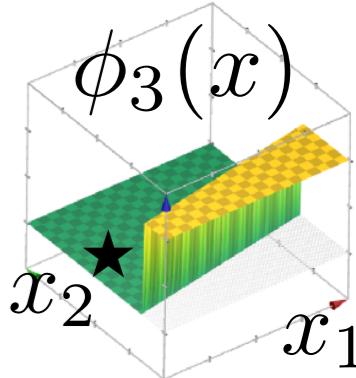
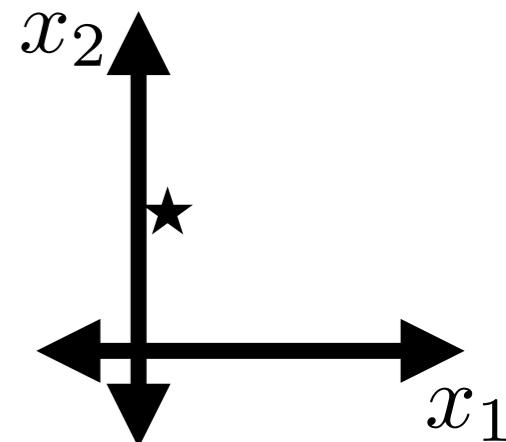
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$



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- 1st layer, constructing the features:
  - Input  $x$  (a data point): size  $m^{(1)} \times 1$
  - Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
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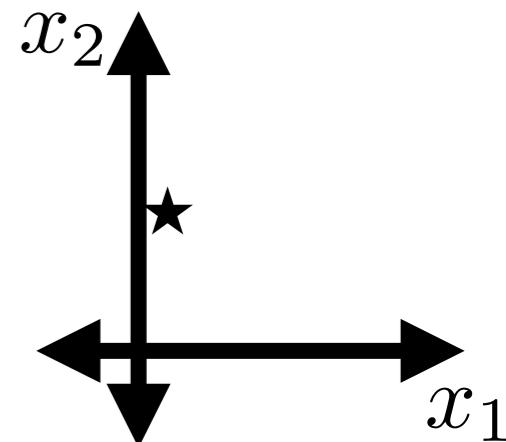
$$m^{(1)} = d$$



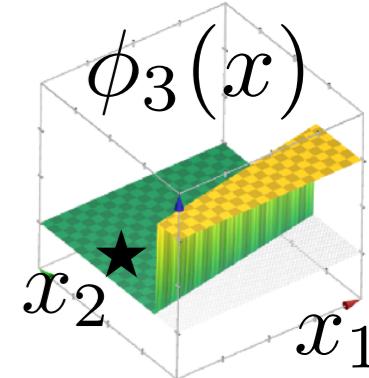
# Let's get some new notation

- 1st layer, constructing the features:
  - Input  $x$  (a data point): size  $m^{(1)} \times 1$
  - Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
  - The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$

$$m^{(1)} = d$$



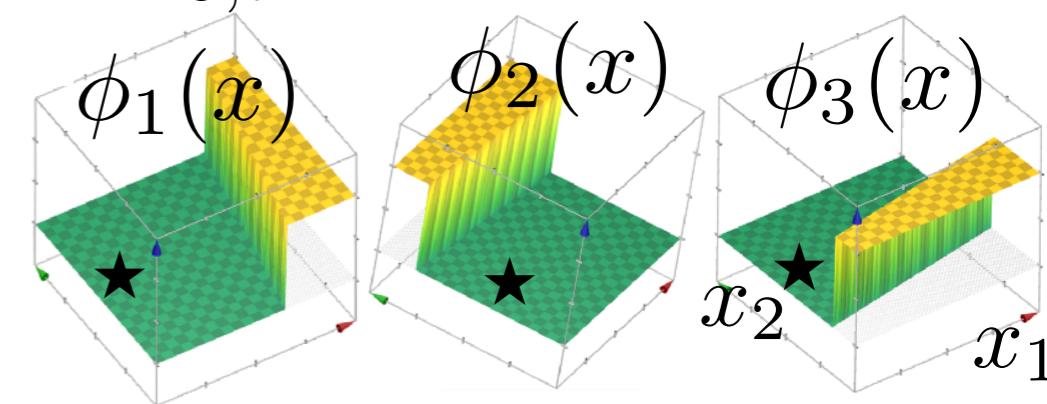
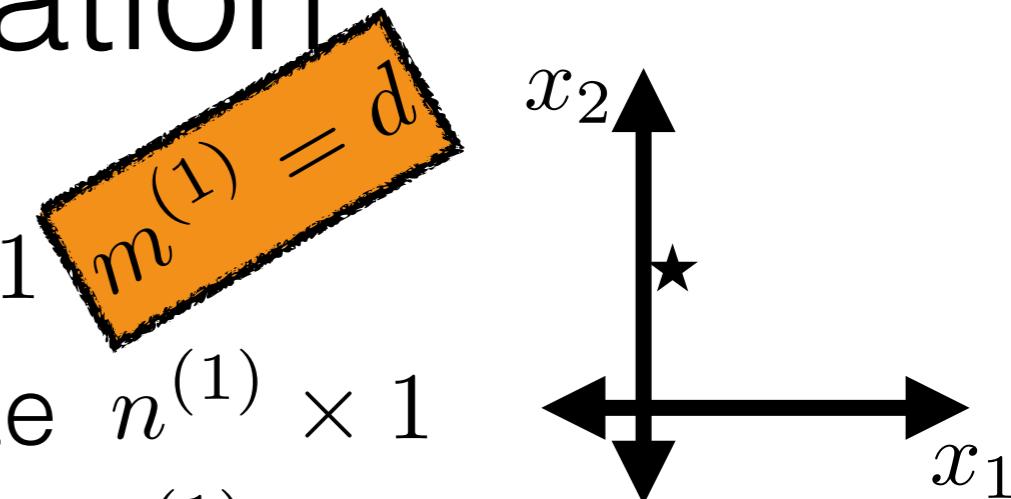
$$A^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$$



# Let's get some new notation

- 1st layer, constructing the features:

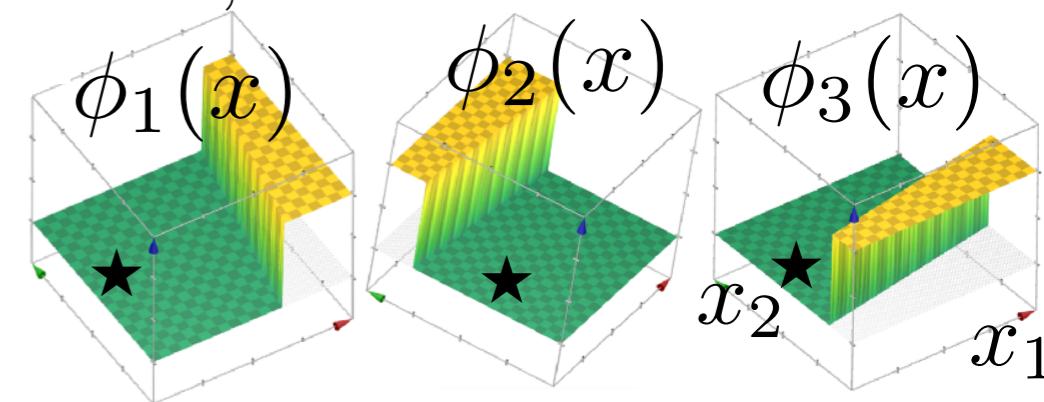
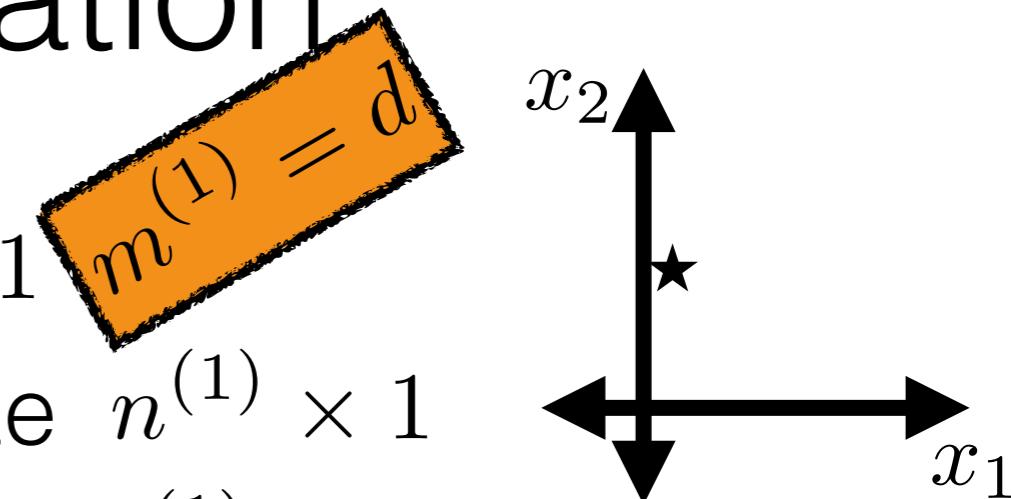
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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# Let's get some new notation

- 1st layer, constructing the features:

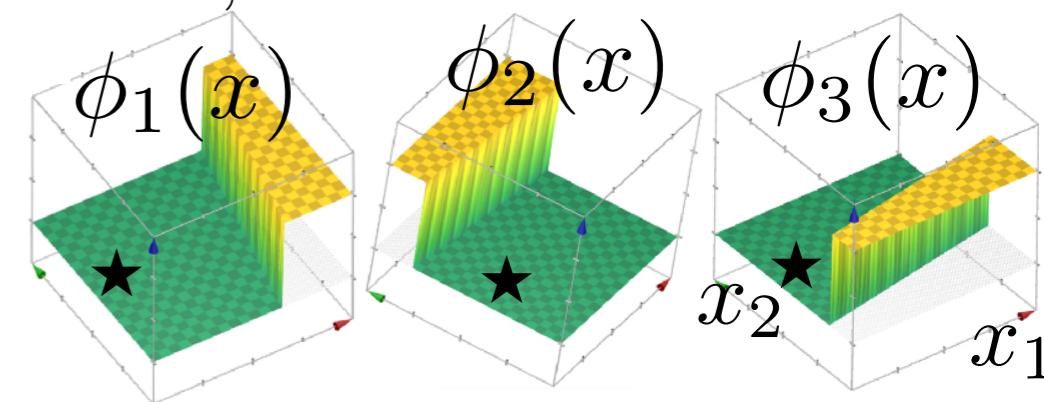
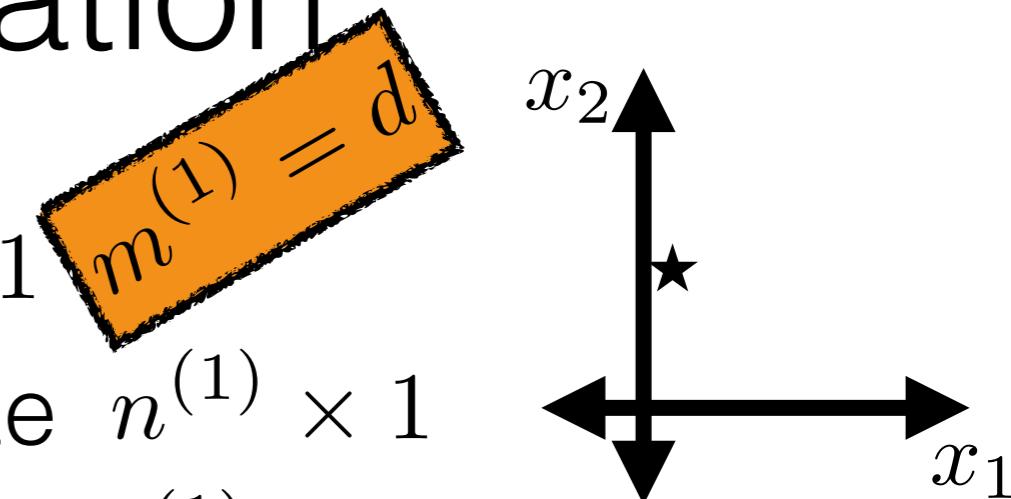
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:



# Let's get some new notation

- 1st layer, constructing the features:

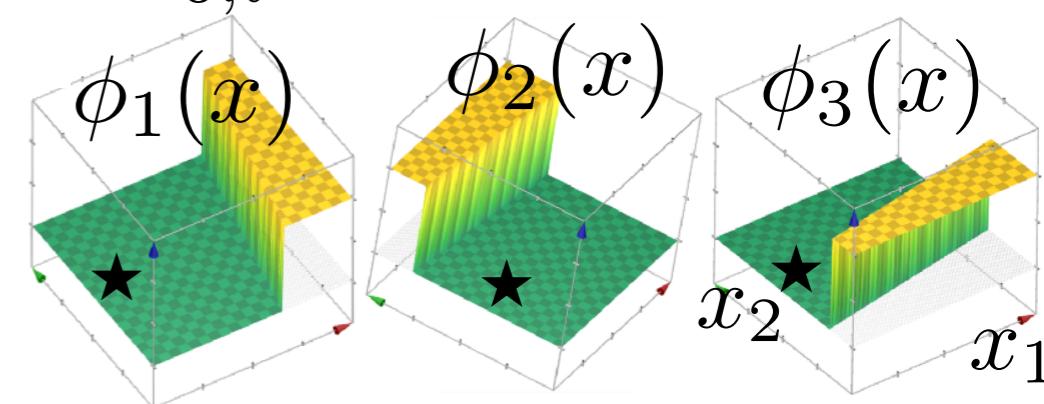
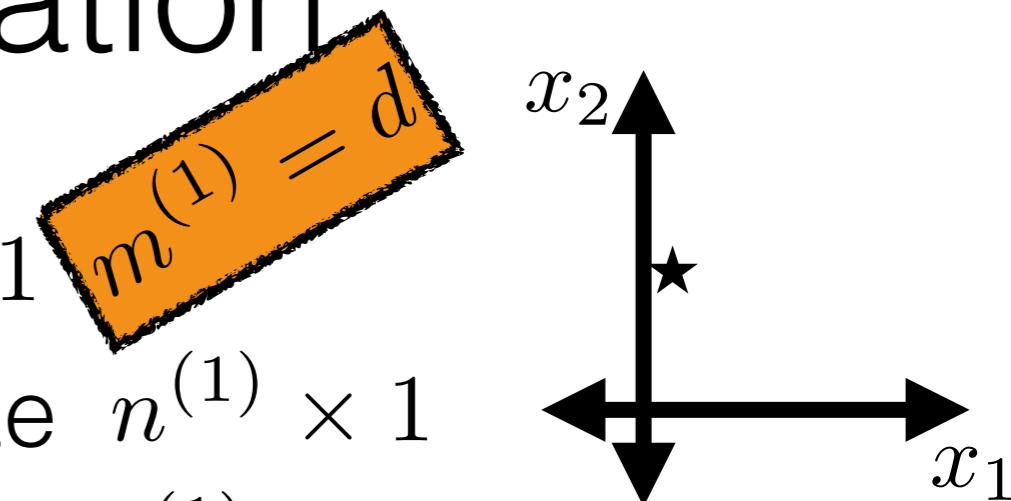
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
  - All the features at once:
    - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$



# Let's get some new notation

- 1st layer, constructing the features:

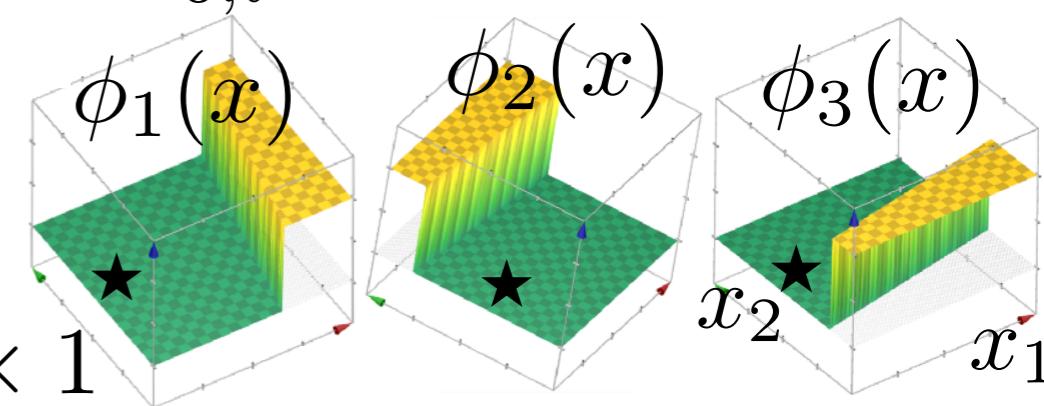
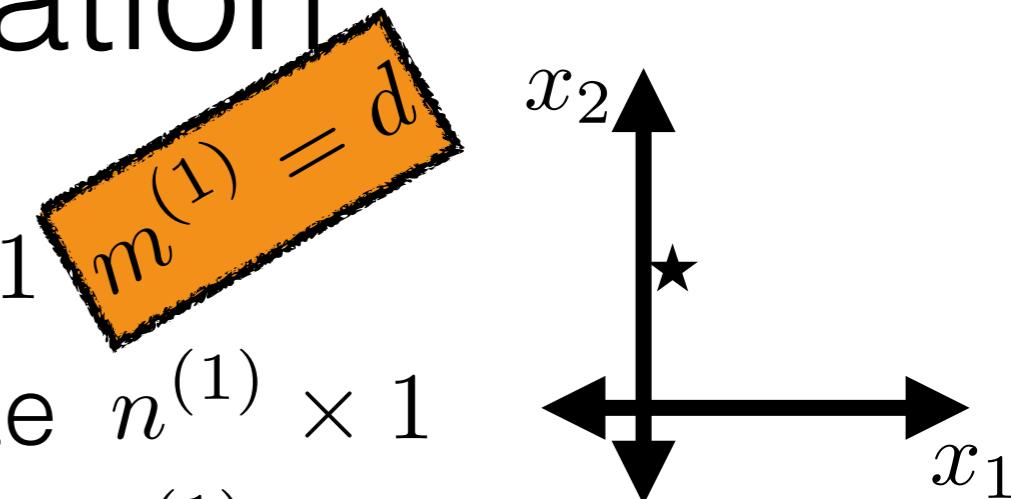
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
  - All the features at once:
    - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
    - $W^{(1)} : m^{(1)} \times n^{(1)}$



# Let's get some new notation

- 1st layer, constructing the features:

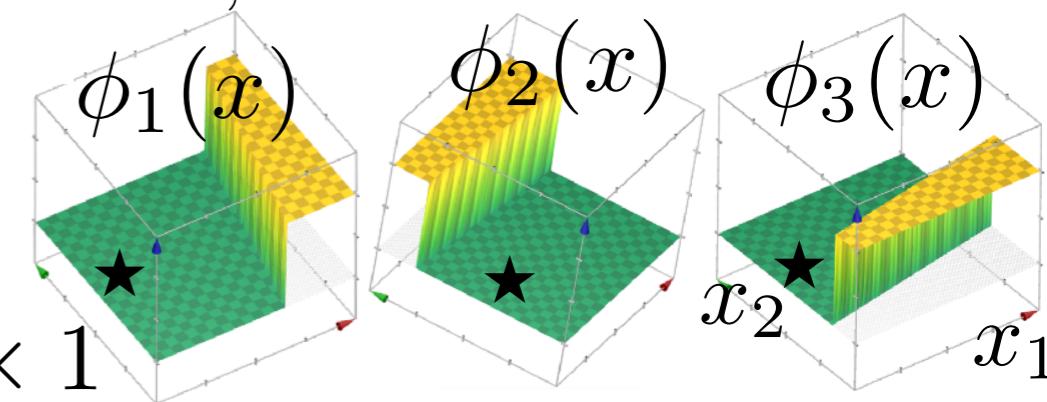
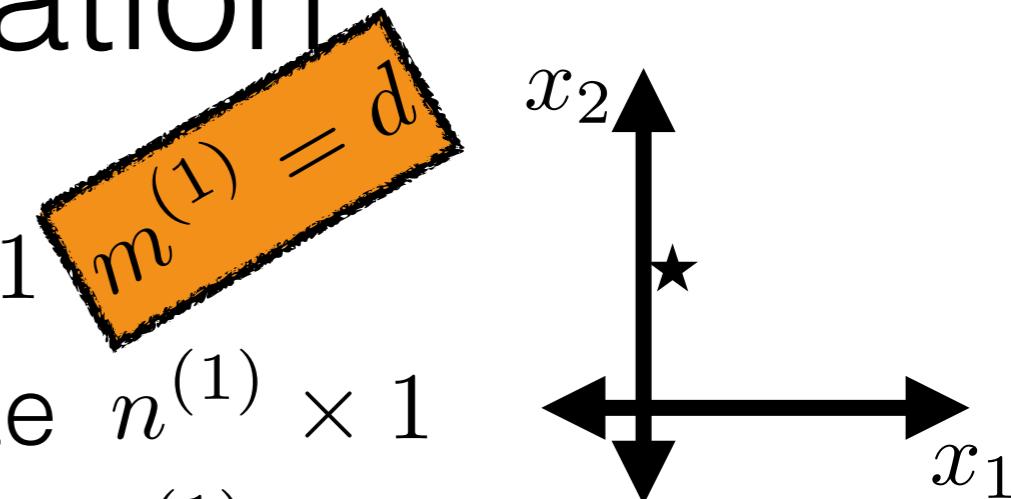
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:
  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



# Let's get some new notation

- 1st layer, constructing the features:

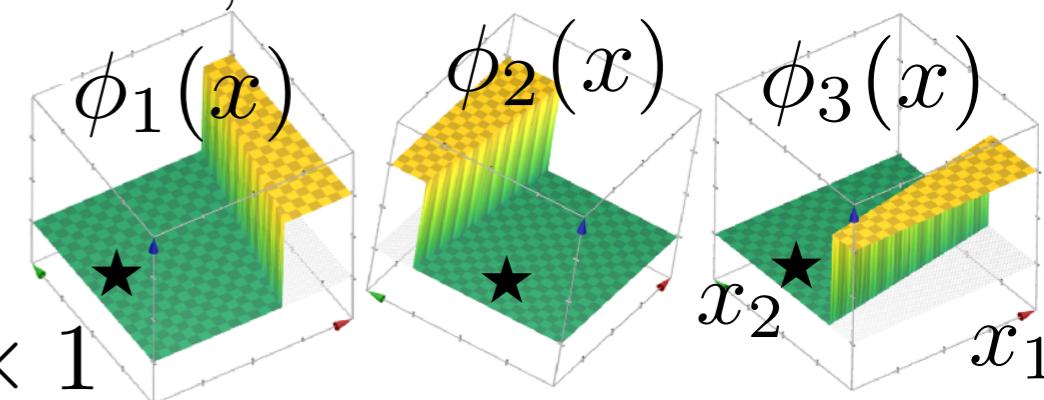
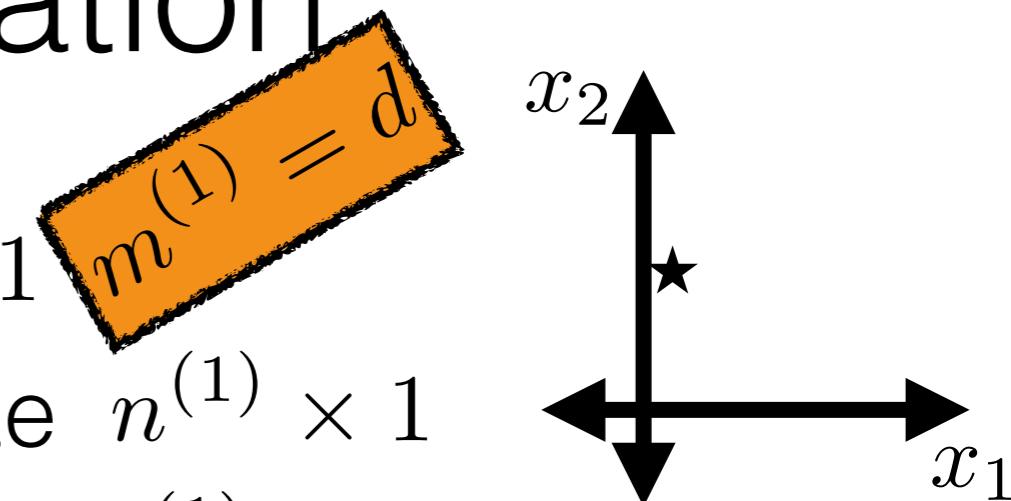
- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



# Let's get some new notation

- 1st layer, constructing the features:

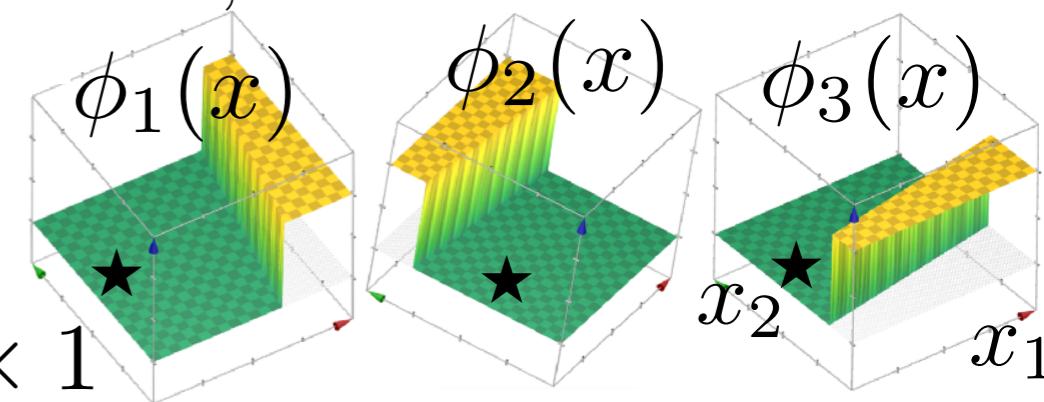
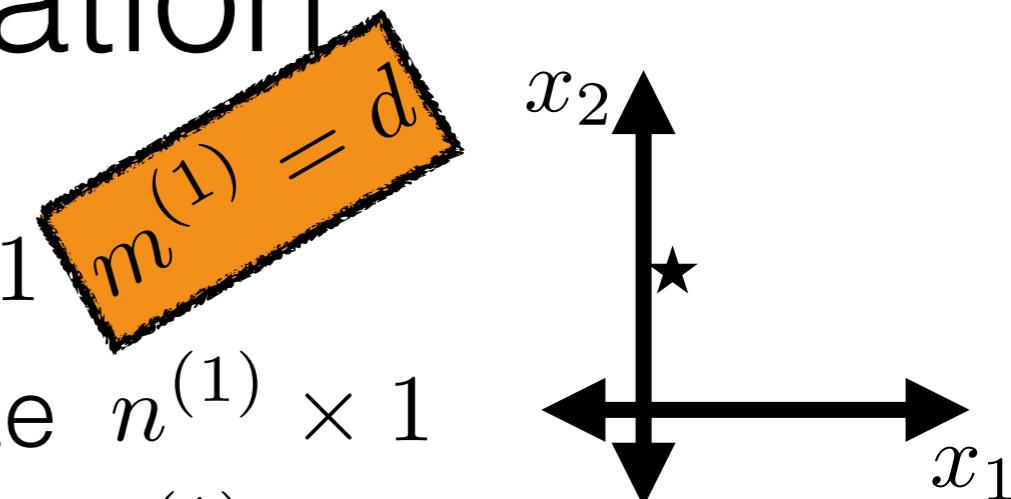
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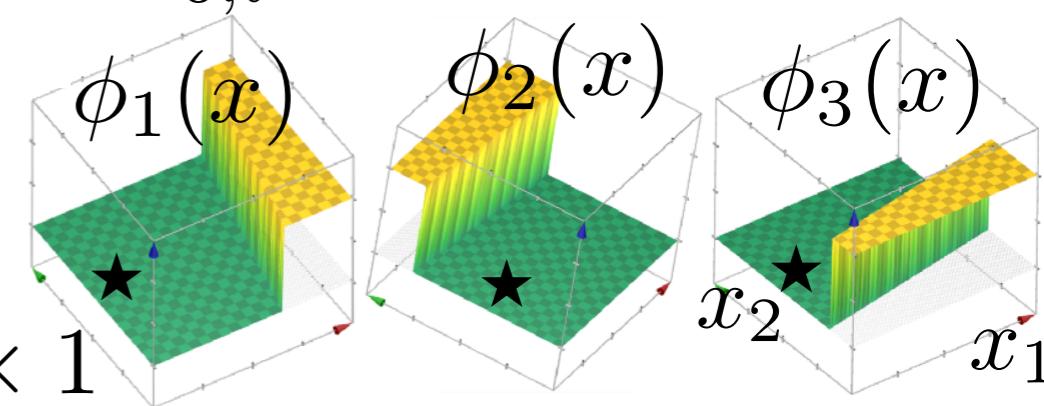
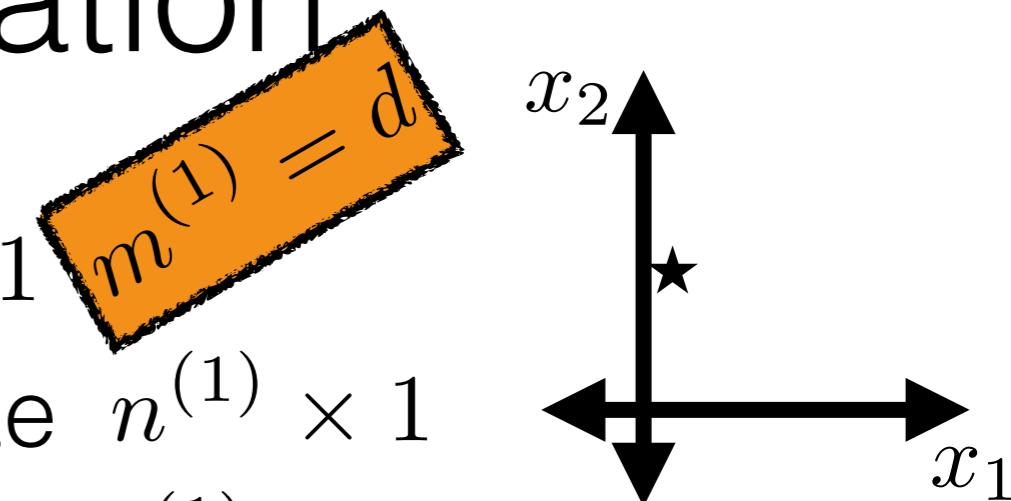


$f^{(1)}$  is applied componentwise!

# Let's get some new notation

- 1st layer, constructing the features:

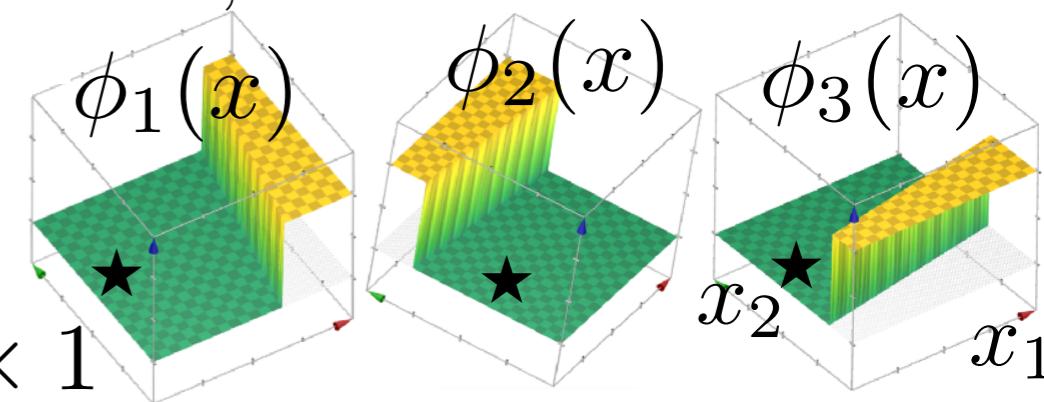
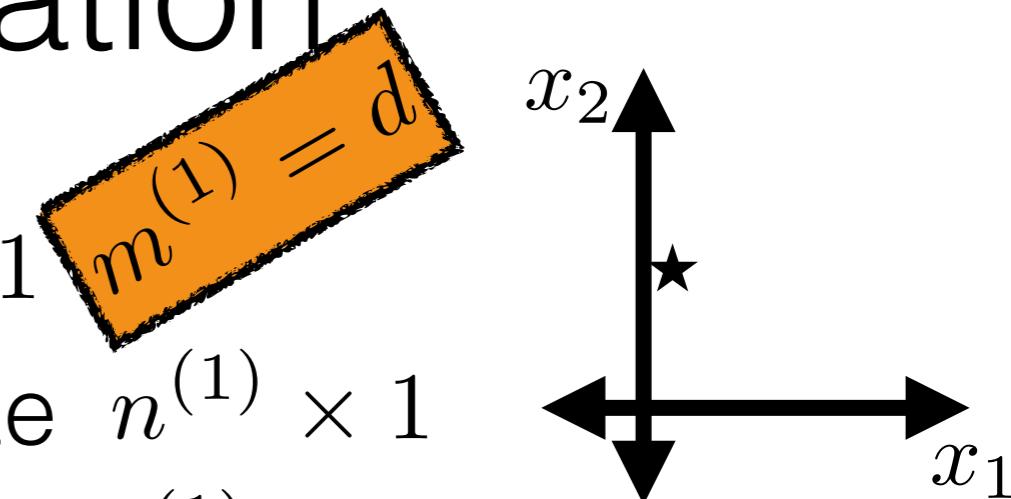
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  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



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- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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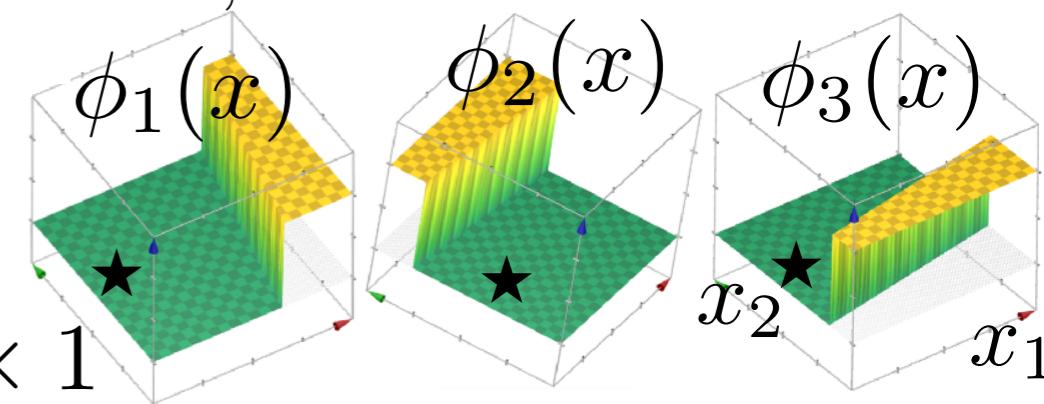
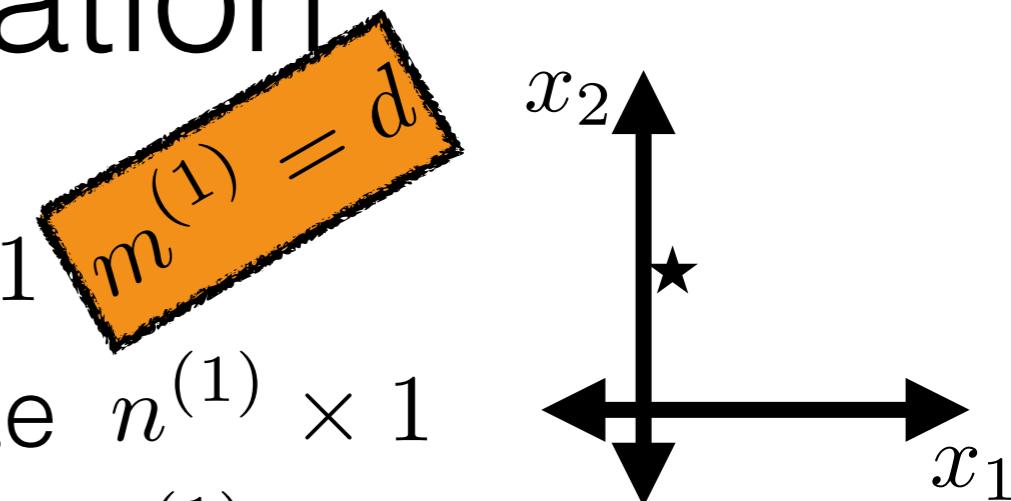


- 2nd layer, assigning a label (or labels):

# Let's get some new notation

- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:
  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$

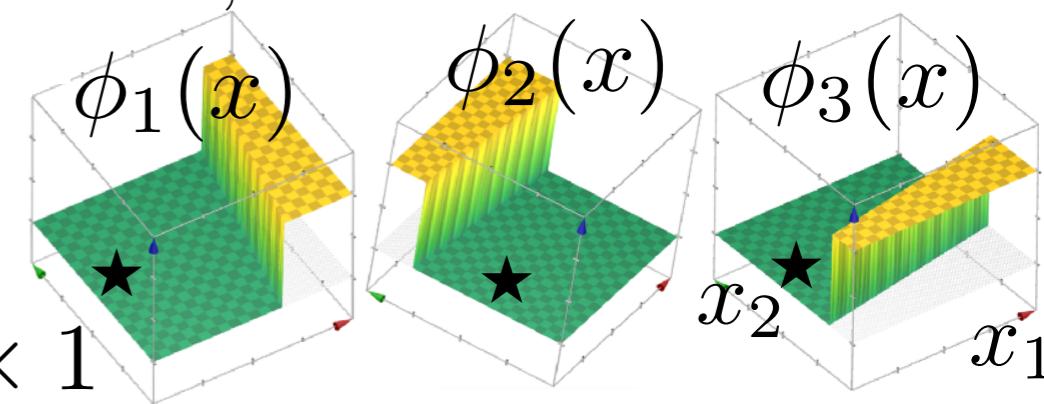
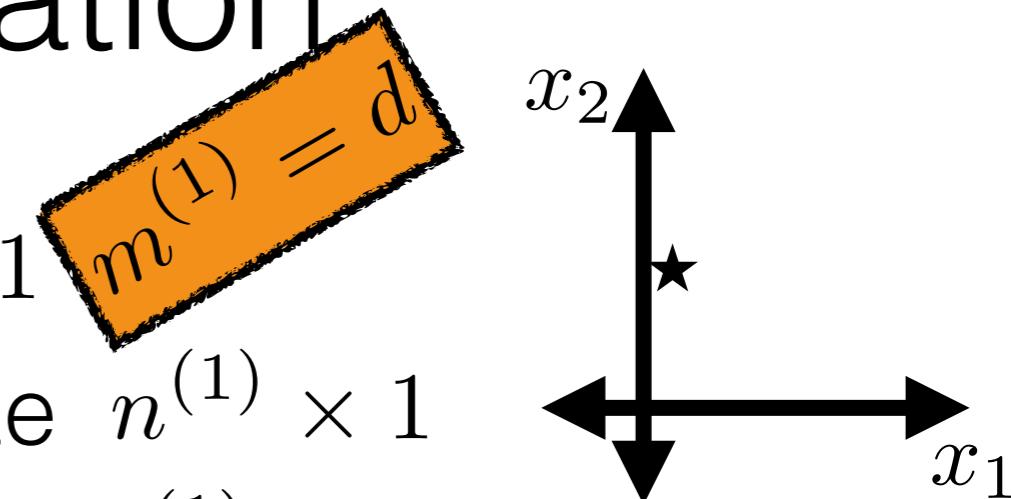


- 2nd layer, assigning a label (or labels):
  - Input (the features)

# Let's get some new notation

- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:
  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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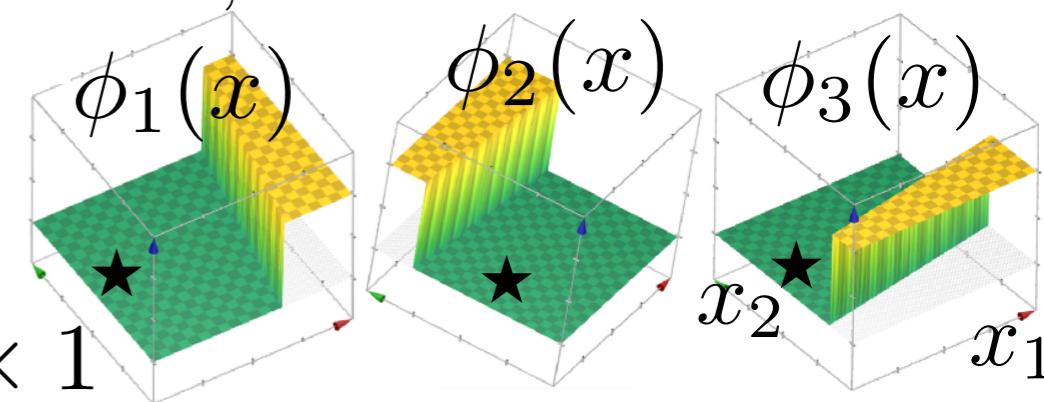
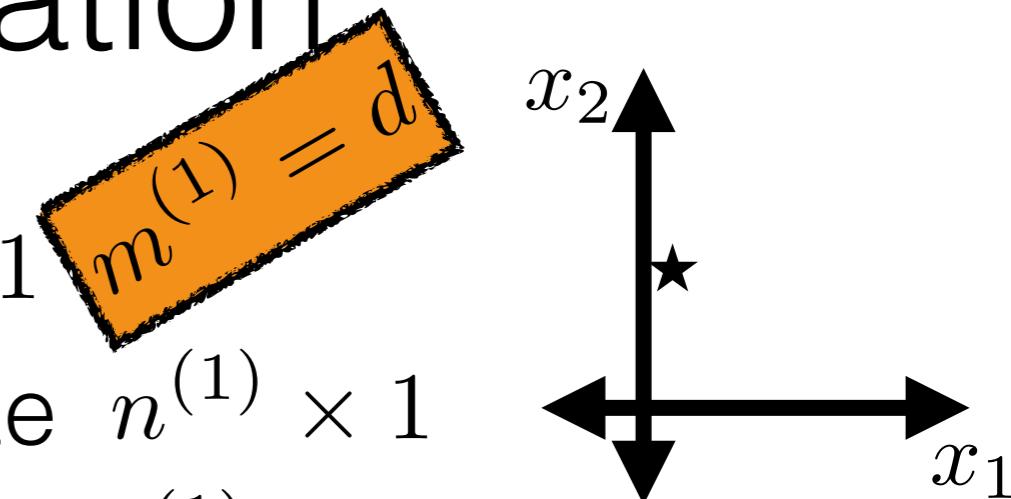


- 2nd layer, assigning a label (or labels):
  - Input (the features): size  $m^{(2)} \times 1$

# Let's get some new notation

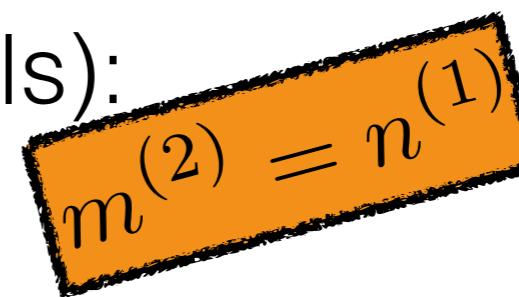
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:
  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



- 2nd layer, assigning a label (or labels):

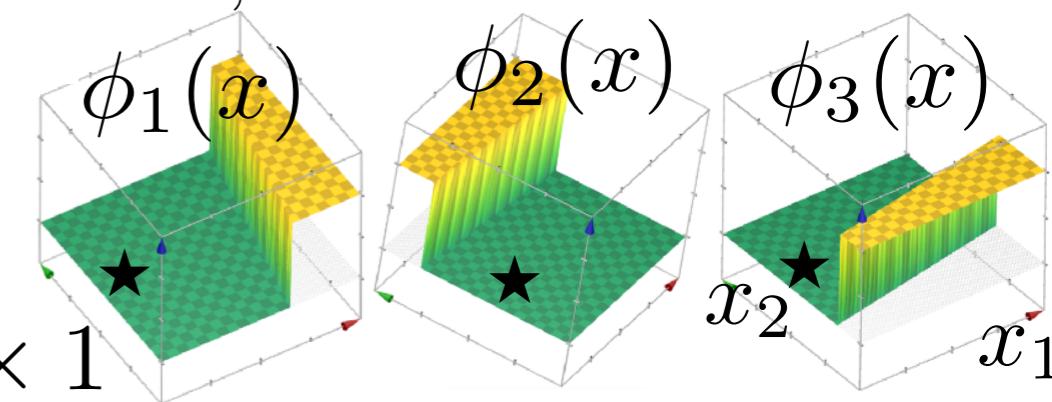
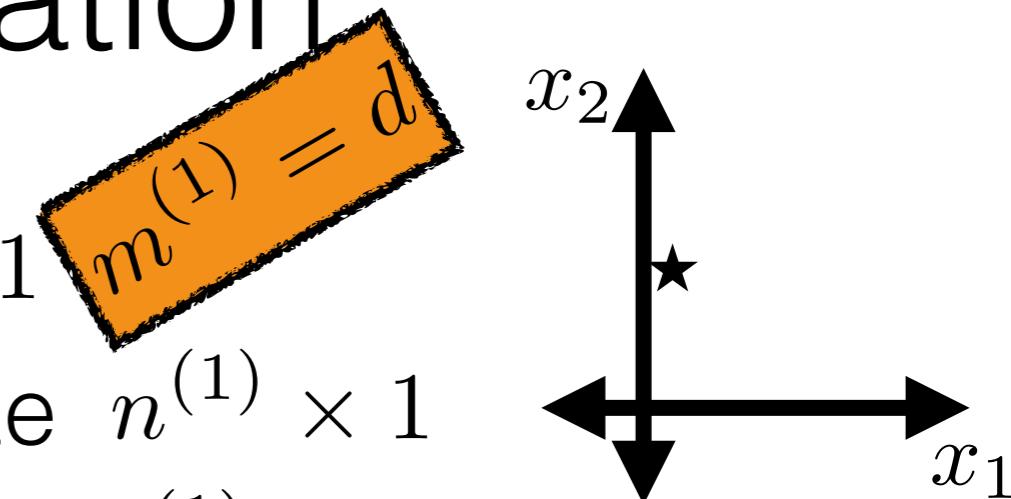
- Input (the features): size  $m^{(2)} \times 1$



# Let's get some new notation

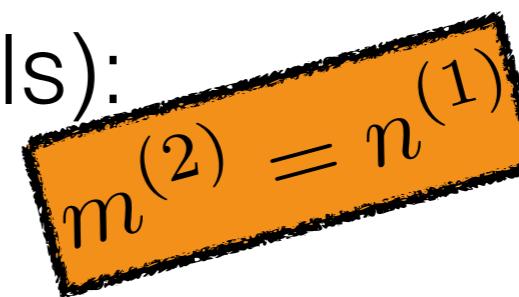
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
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- 2nd layer, assigning a label (or labels):

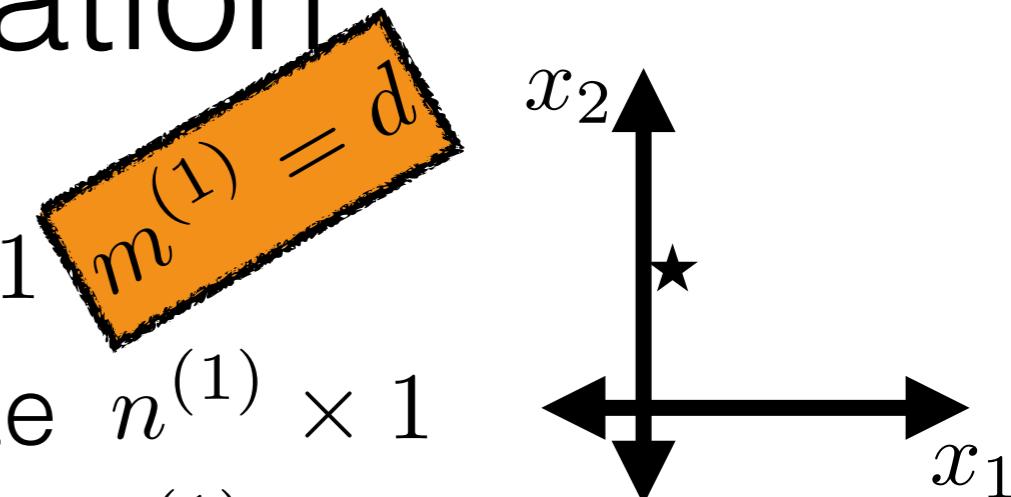
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels)



# Let's get some new notation

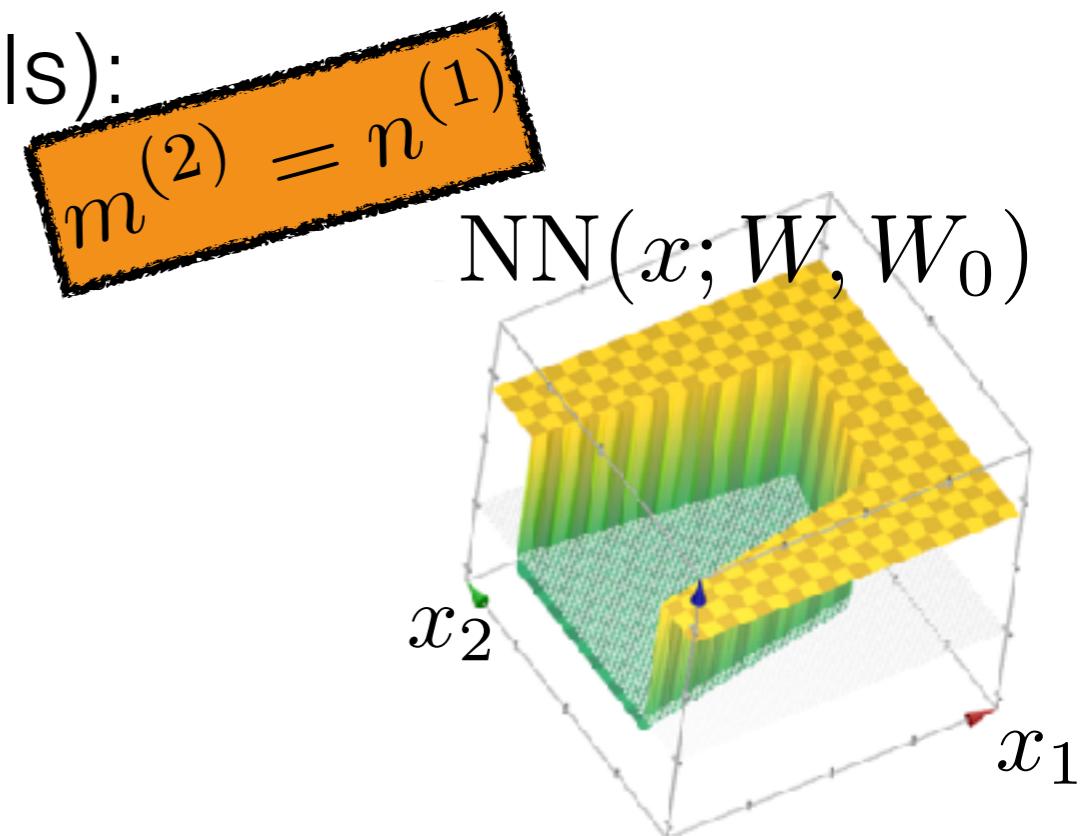
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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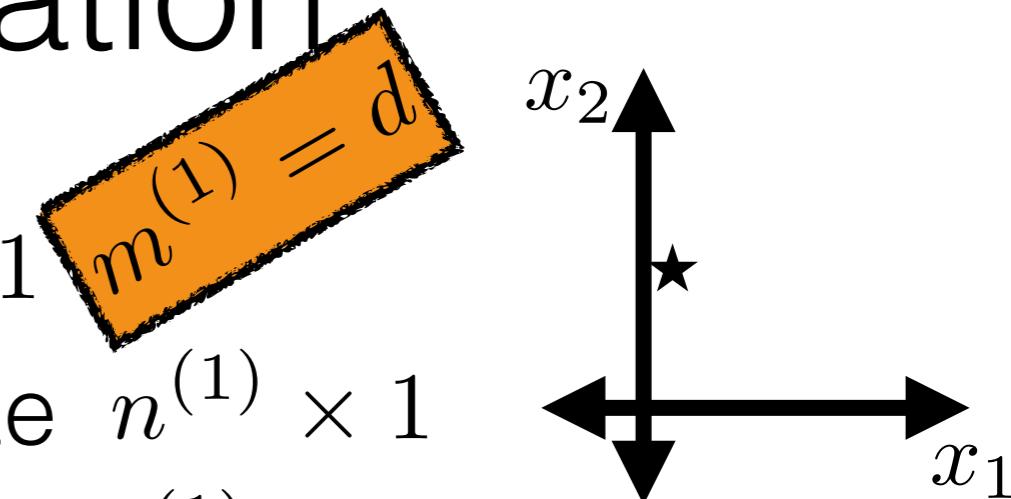
- Input (the features): size  $m^{(2)} \times 1$
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# Let's get some new notation

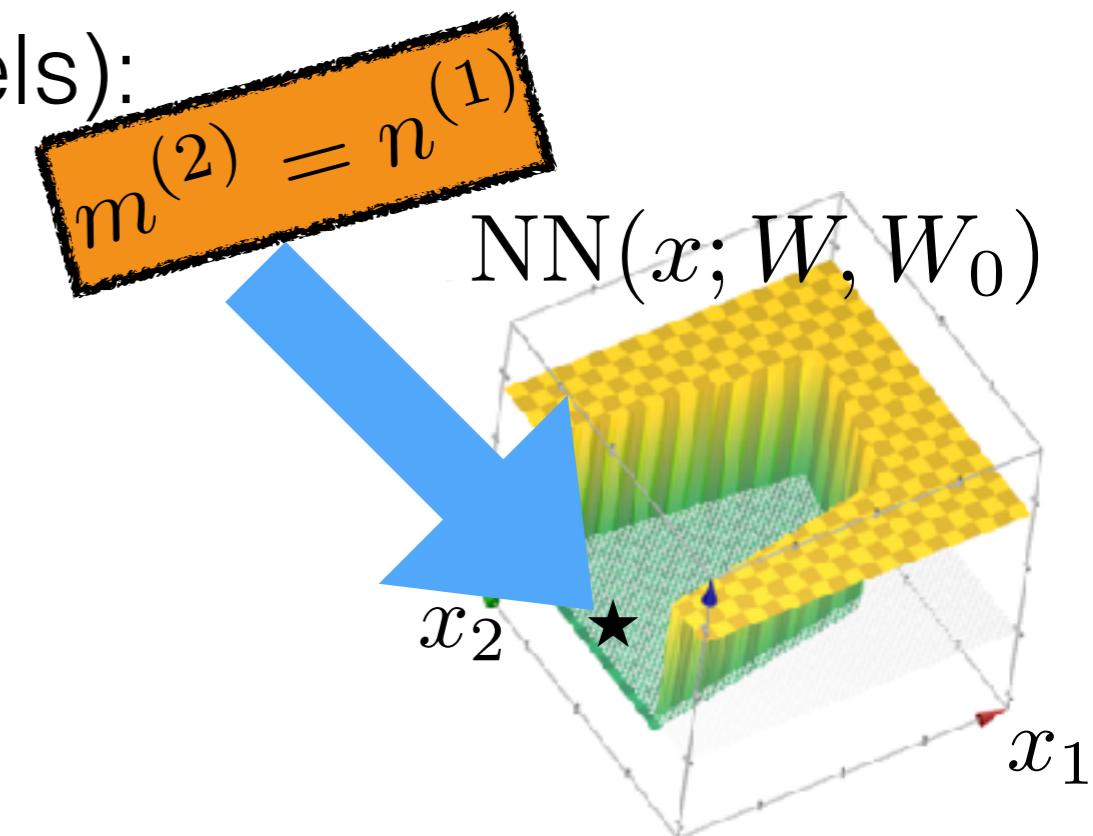
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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- 2nd layer, assigning a label (or labels):

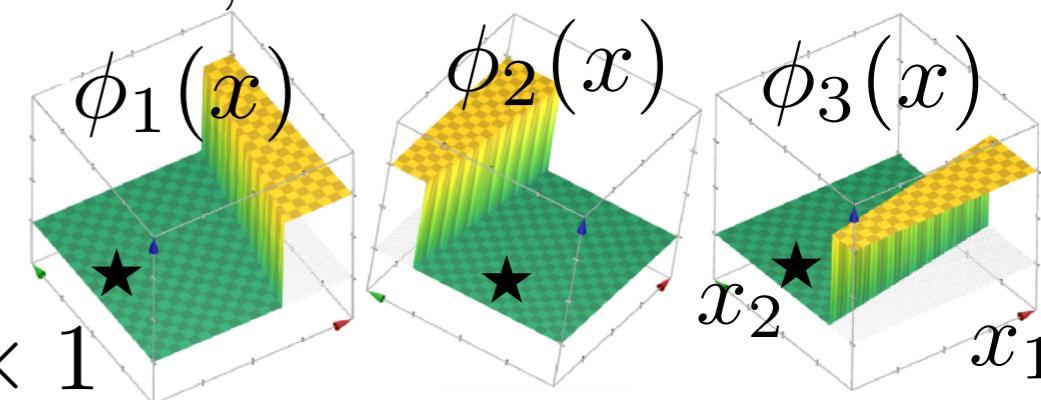
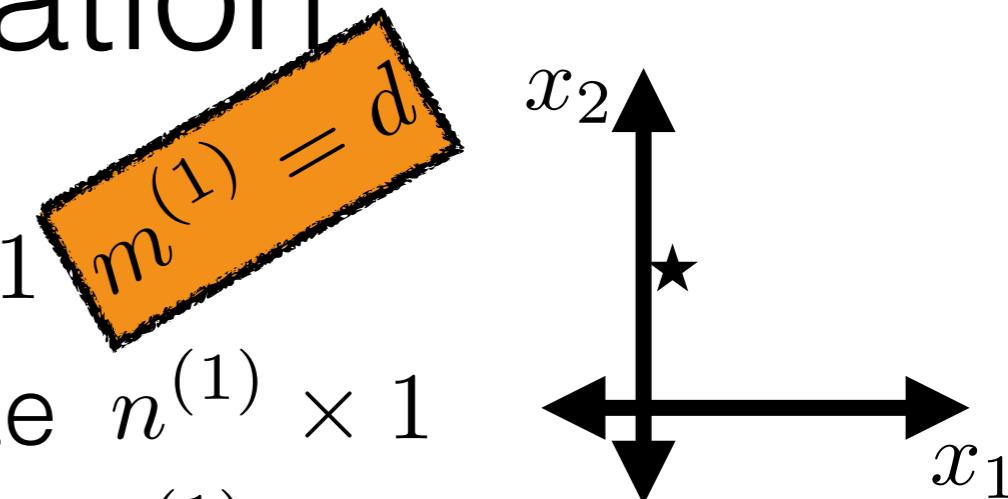
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels)



# Let's get some new notation

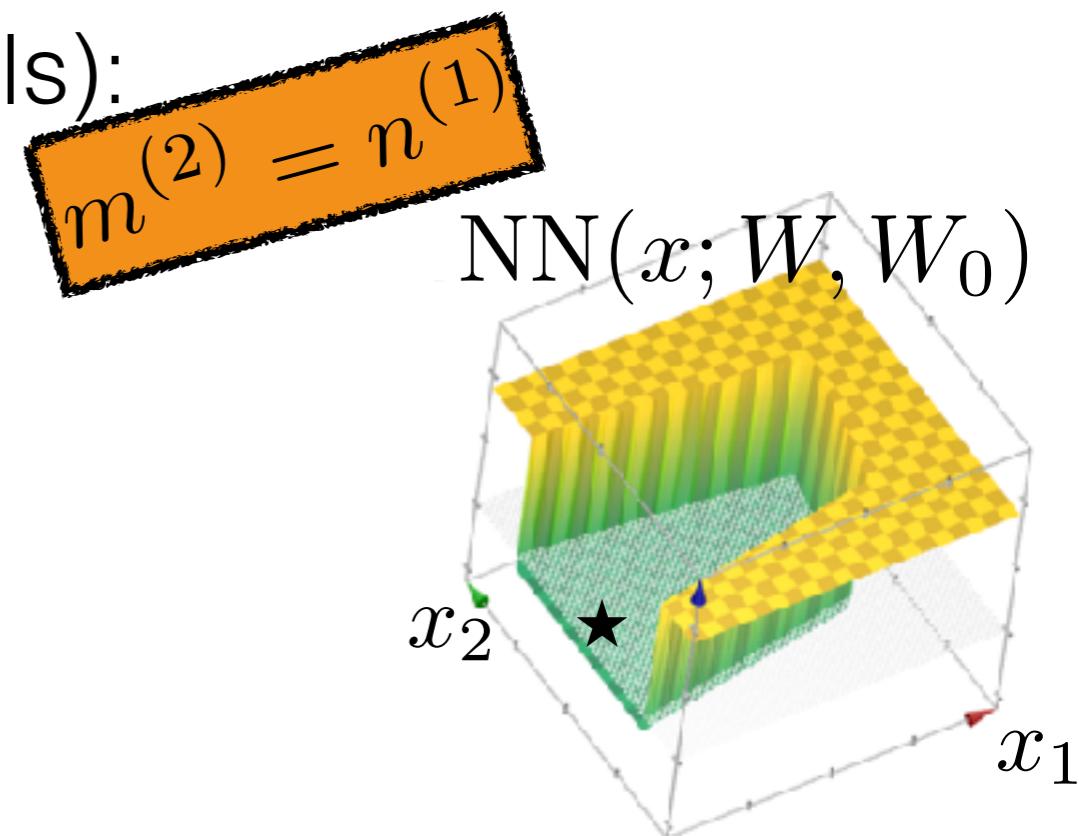
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
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  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



- 2nd layer, assigning a label (or labels):

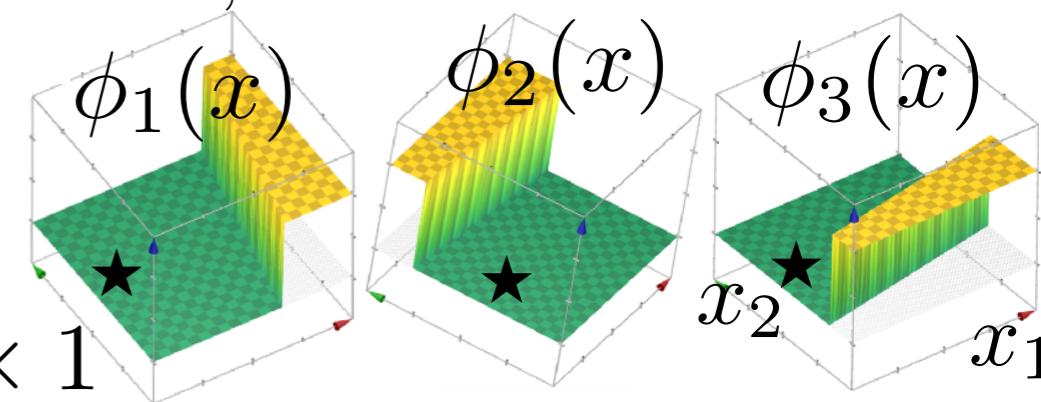
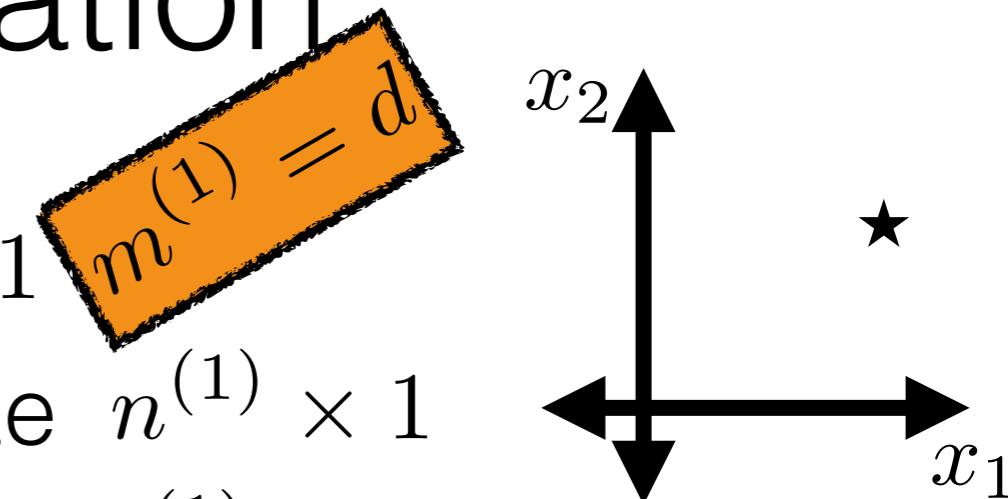
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels)



# Let's get some new notation

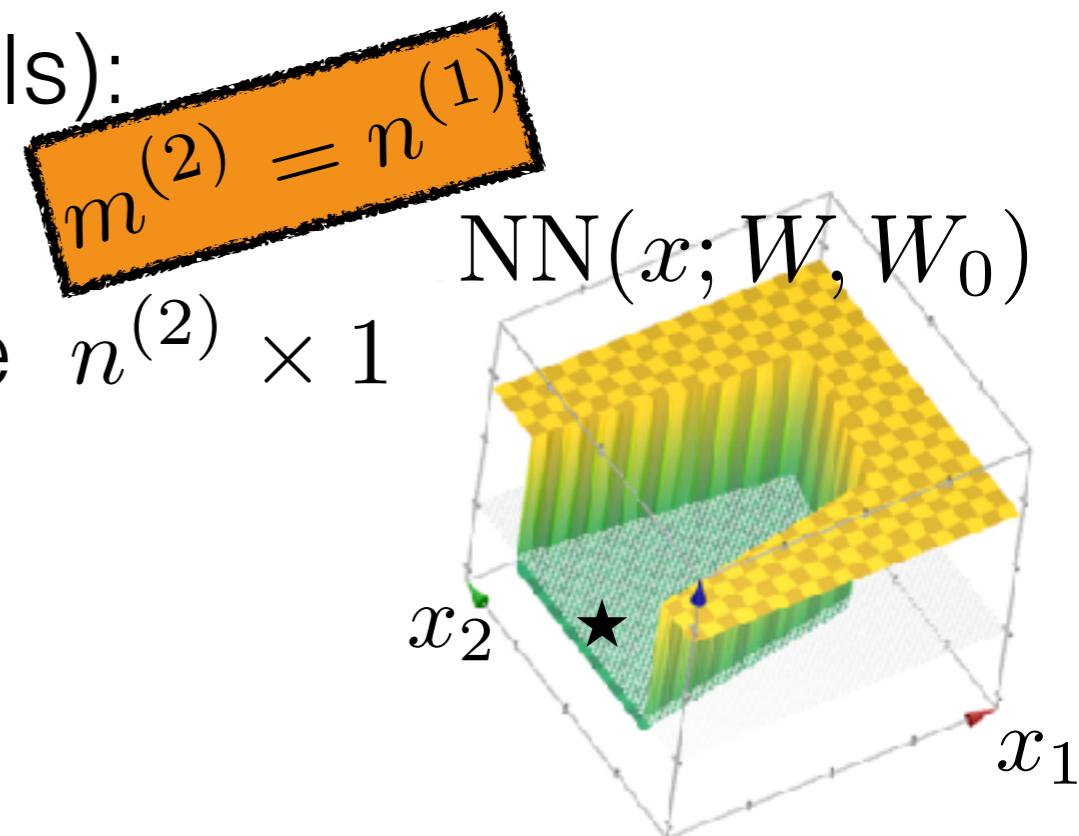
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$
- All the features at once:
  - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
  - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



- 2nd layer, assigning a label (or labels):

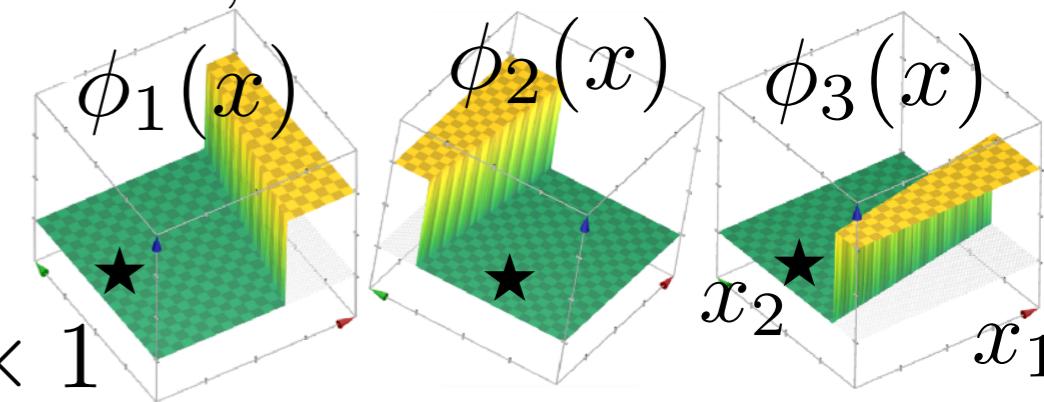
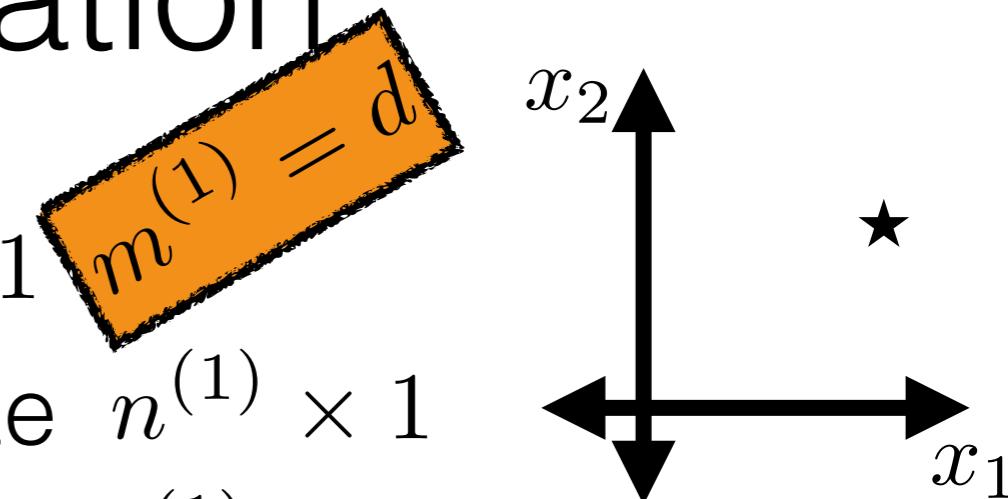
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels): size  $n^{(2)} \times 1$



# Let's get some new notation

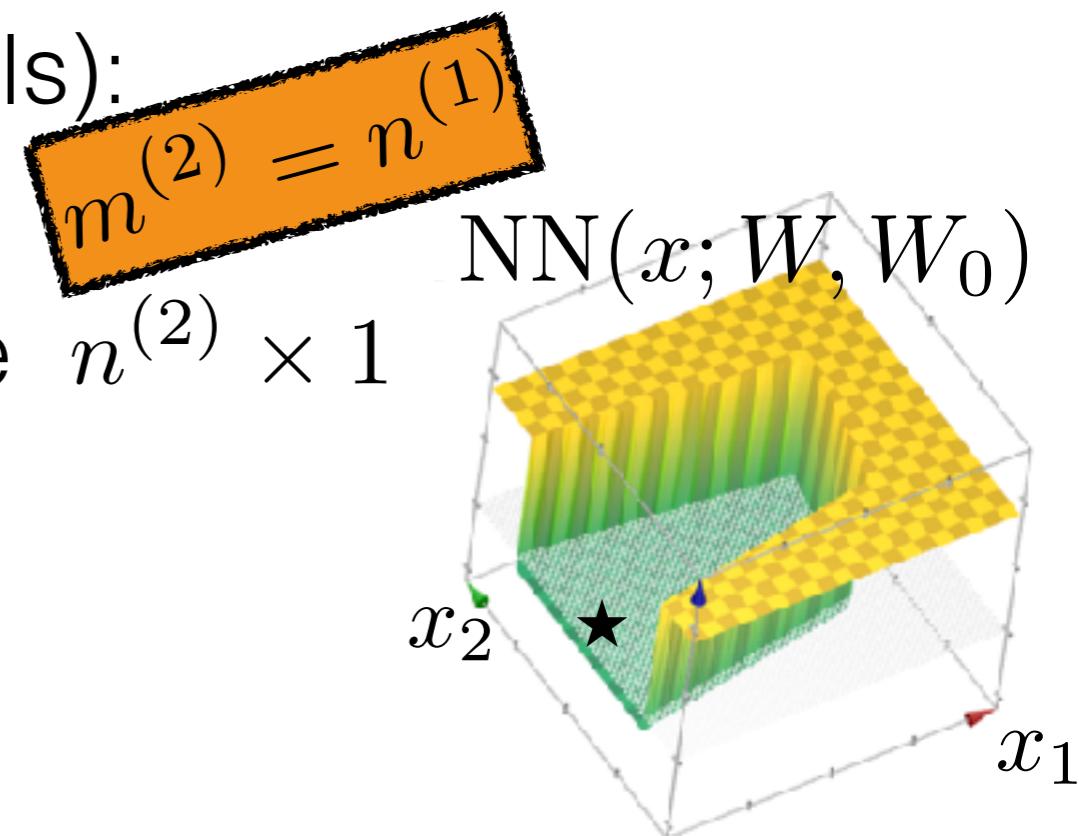
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
  - All the features at once:
    - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
    - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



- 2nd layer, assigning a label (or labels):

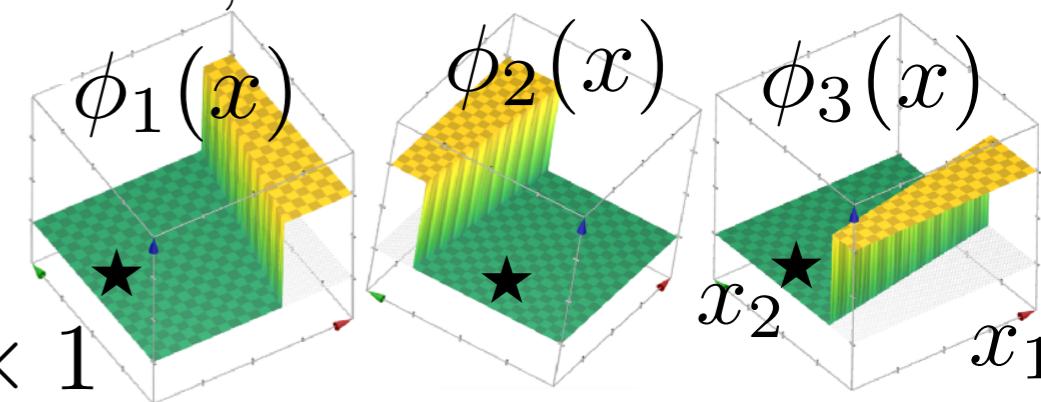
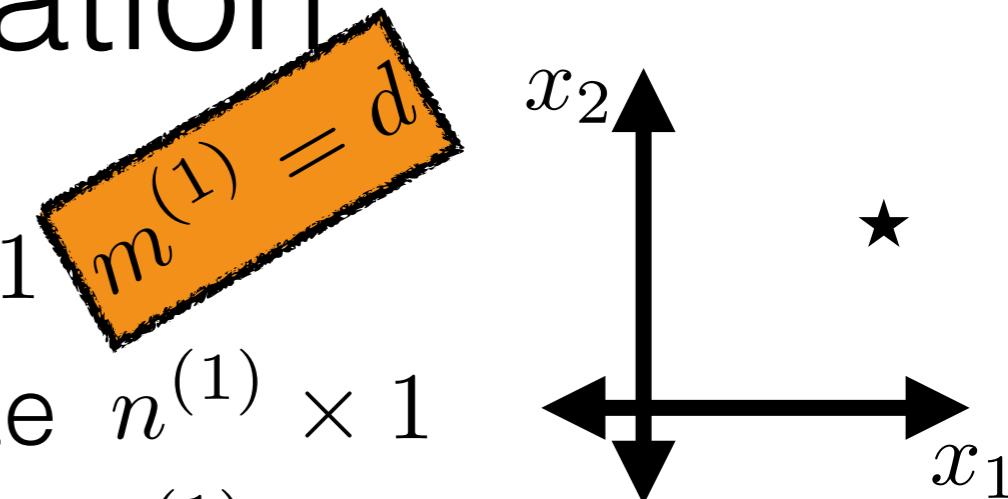
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels): size  $n^{(2)} \times 1$
- The  $i$ th label:



# Let's get some new notation

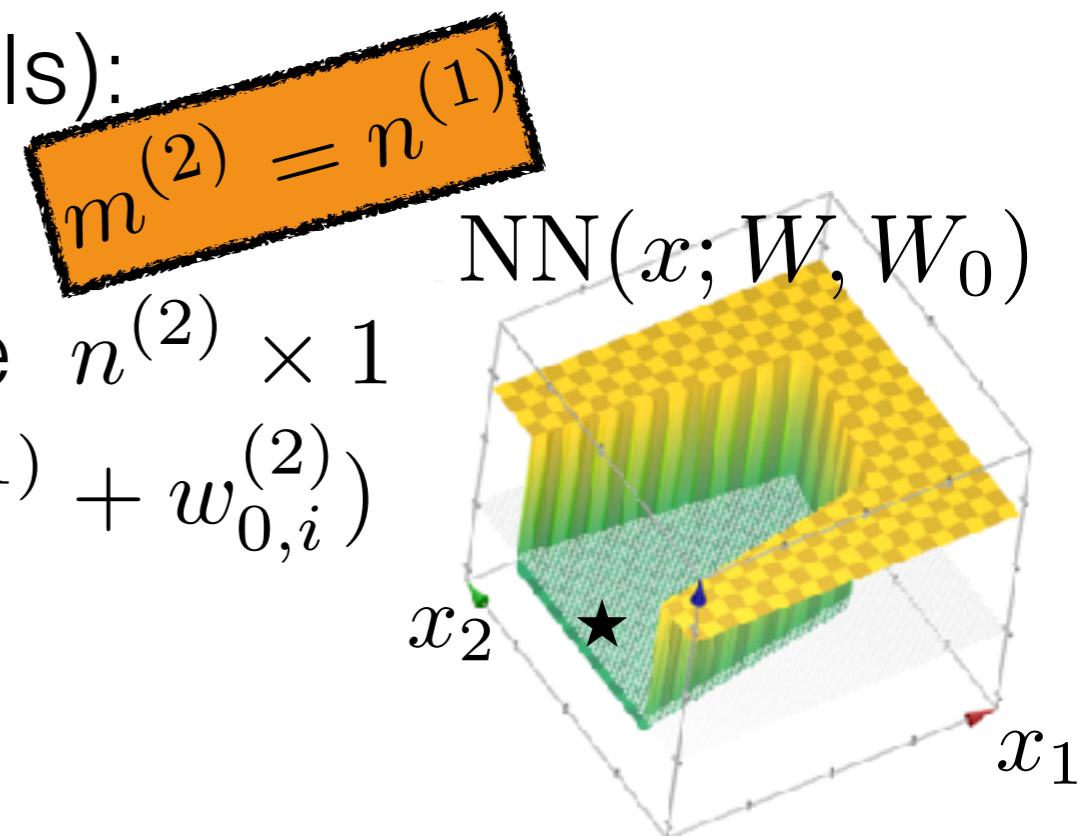
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
  - All the features at once:
    - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
    - $W^{(1)} : m^{(1)} \times n^{(1)}; W_0^{(1)} : n^{(1)} \times 1$



- 2nd layer, assigning a label (or labels):

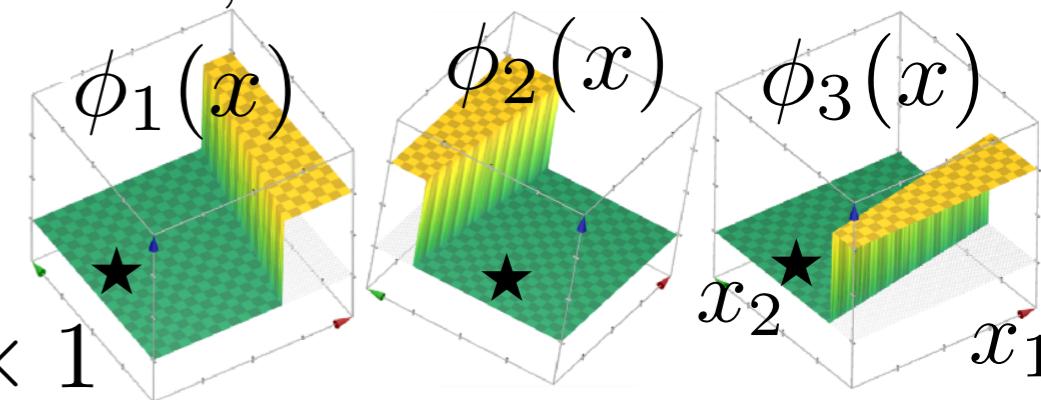
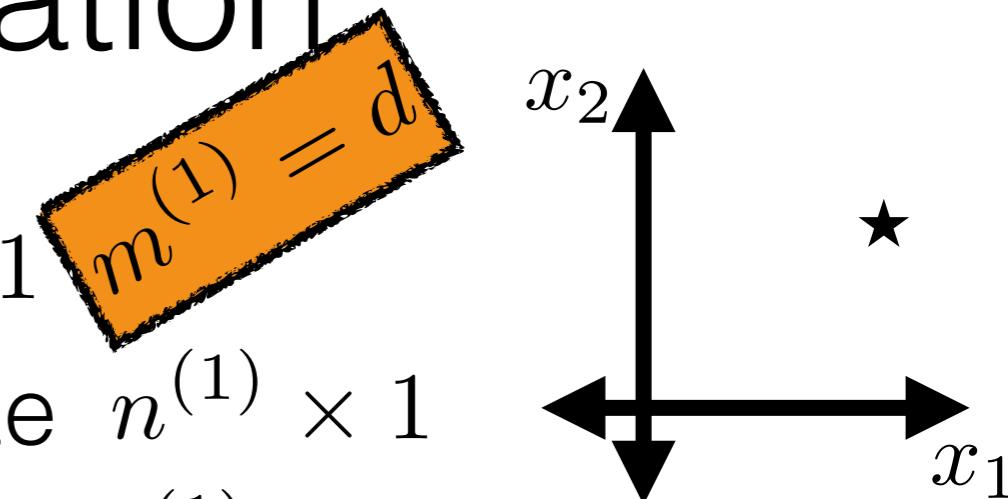
- Input (the features): size  $m^{(2)} \times 1$
- Output  $A^{(2)}$  (vector of labels): size  $n^{(2)} \times 1$
- The  $i$ th label:  $A_i^{(2)} = f^{(2)}(w_i^{(2)\top} A^{(1)} + w_{0,i}^{(2)})$



# Let's get some new notation

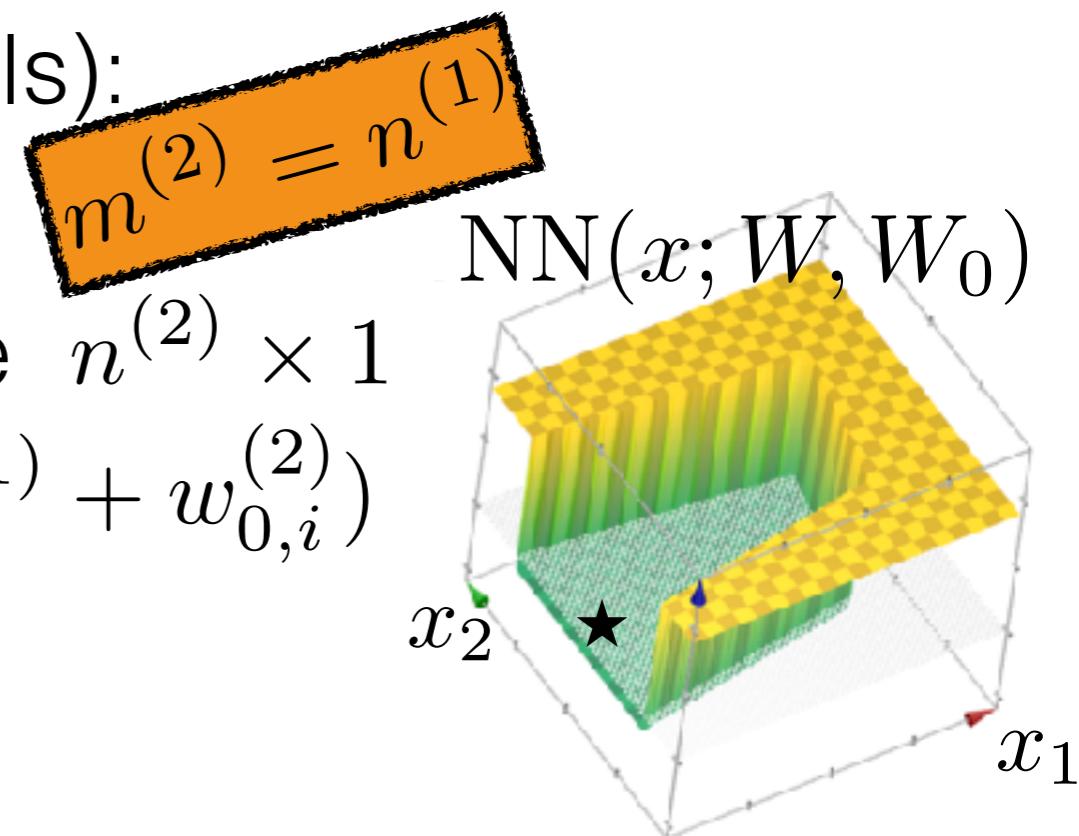
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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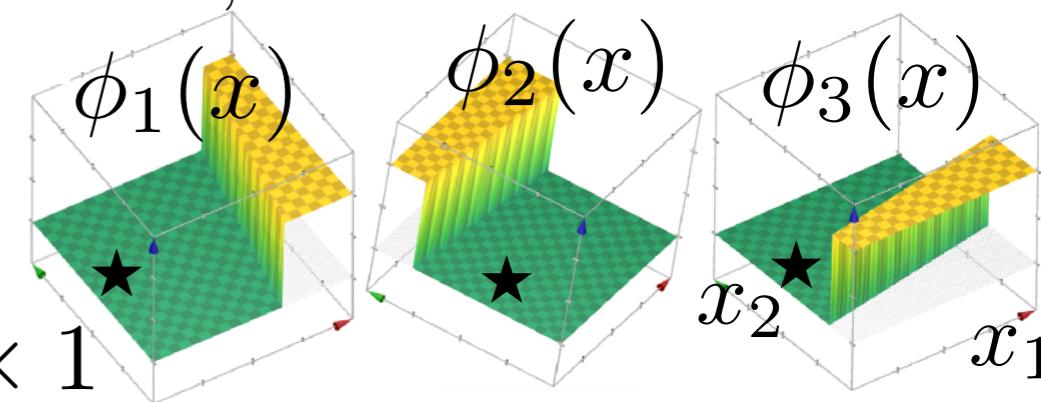
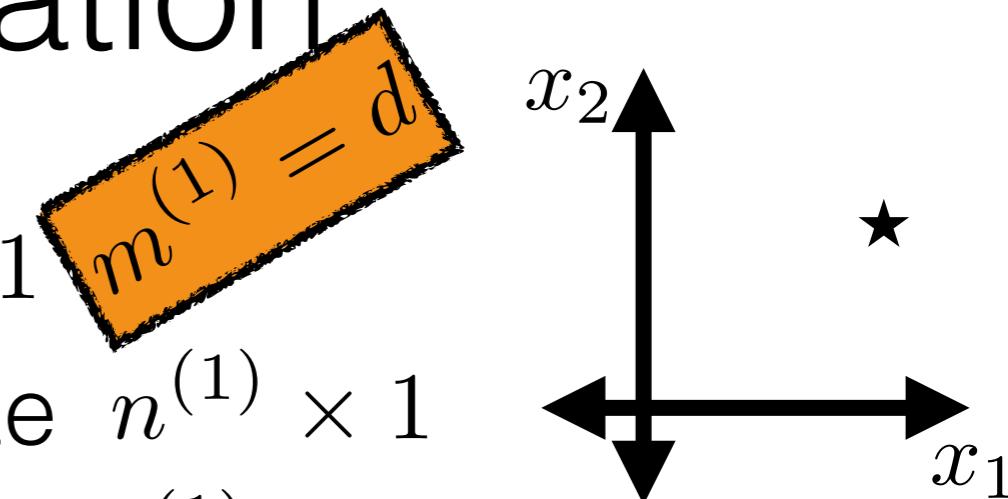
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  - All:



# Let's get some new notation

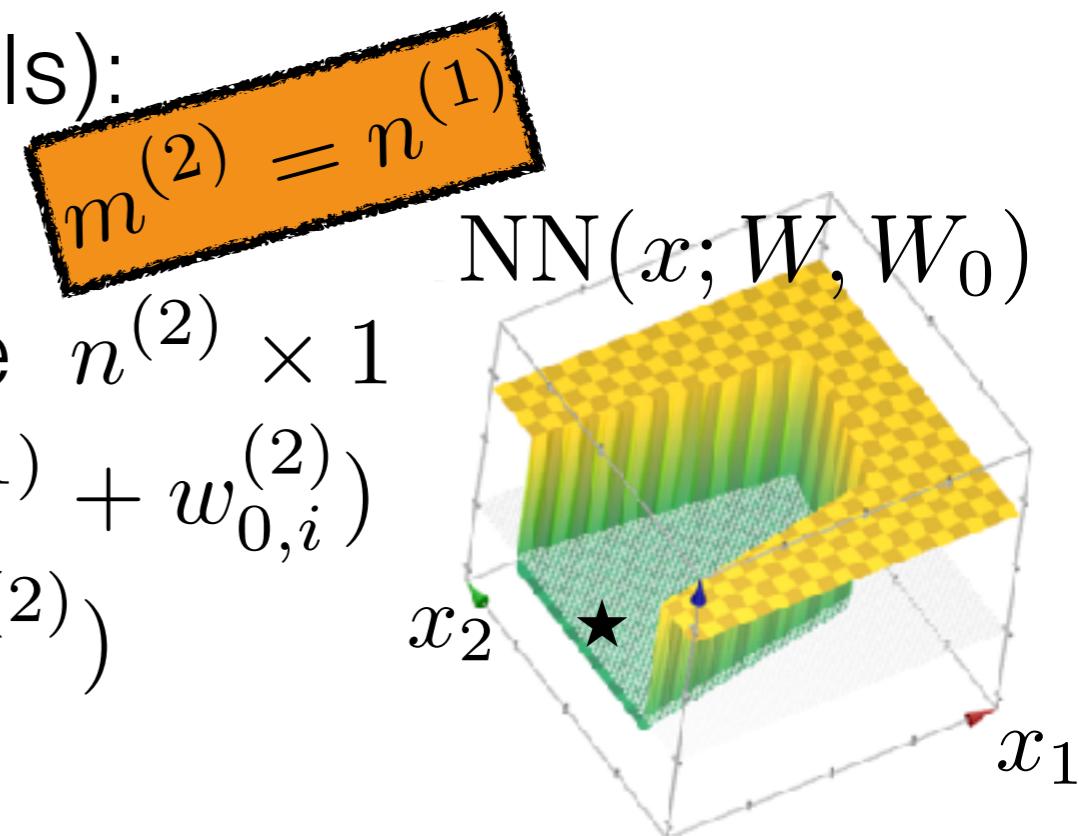
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
- Output  $A^{(1)}$  (vector of features): size  $n^{(1)} \times 1$
- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
  - All the features at once:
    - $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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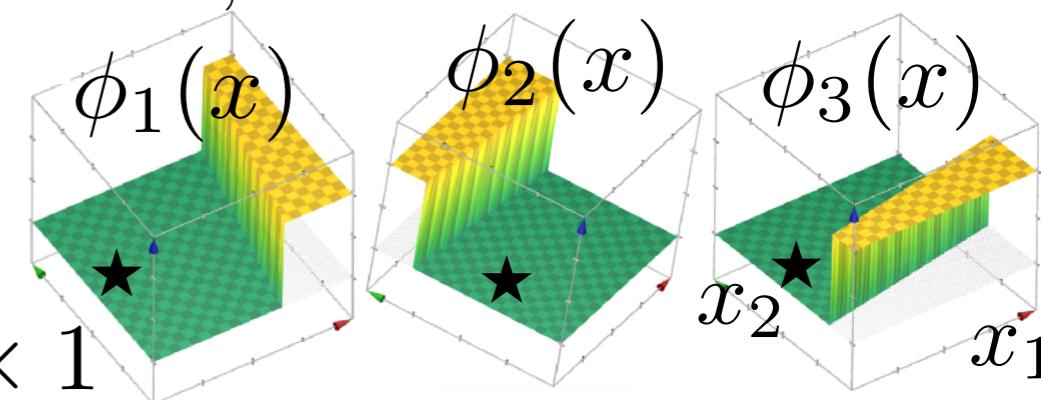
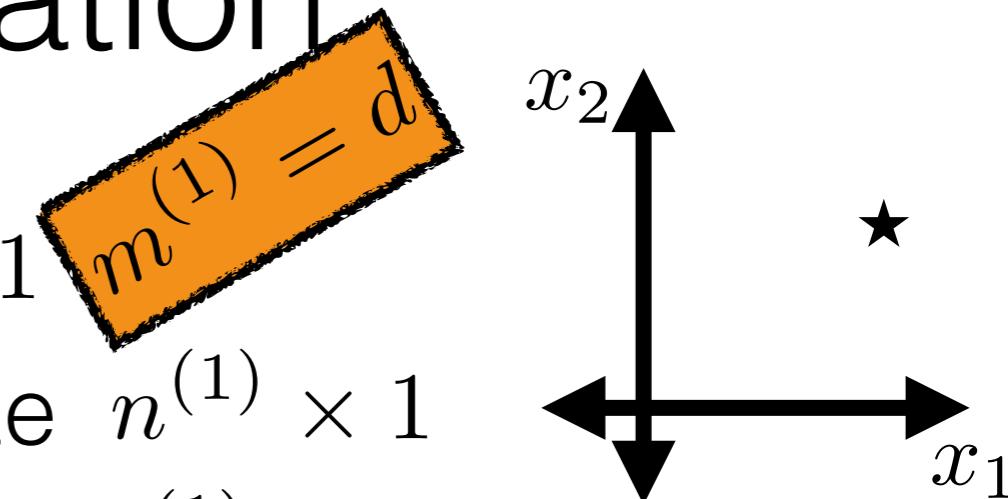
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# Let's get some new notation

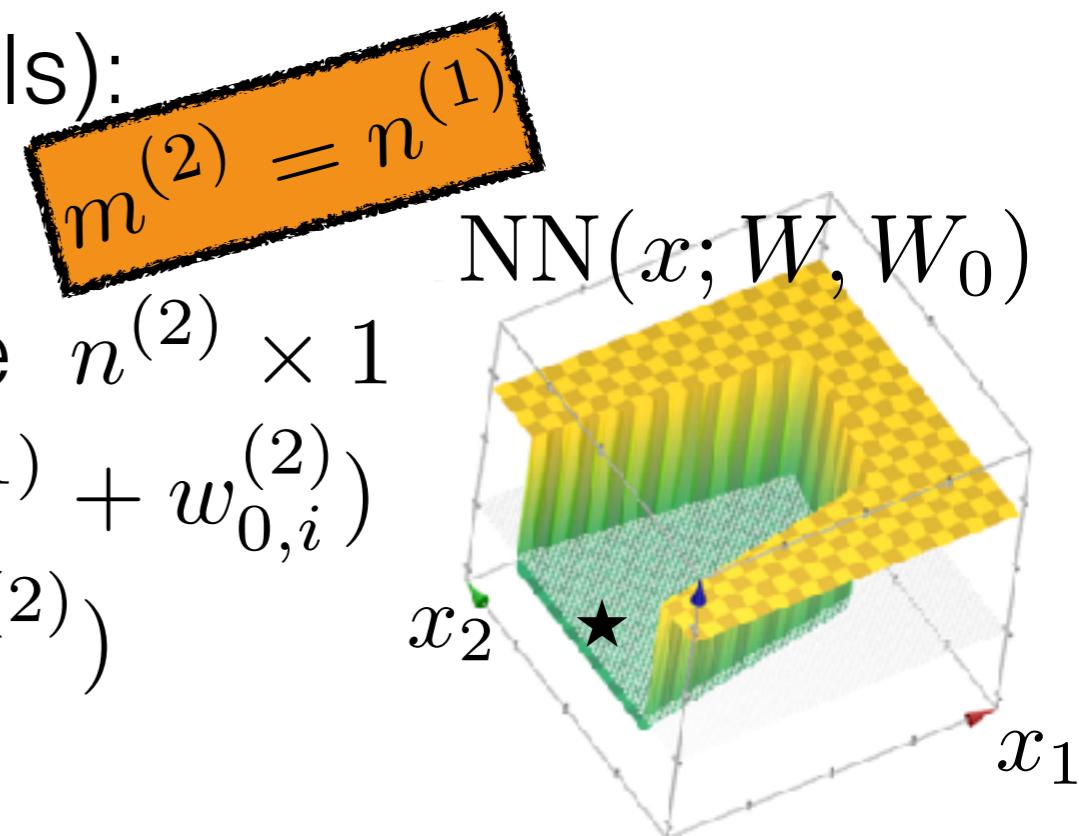
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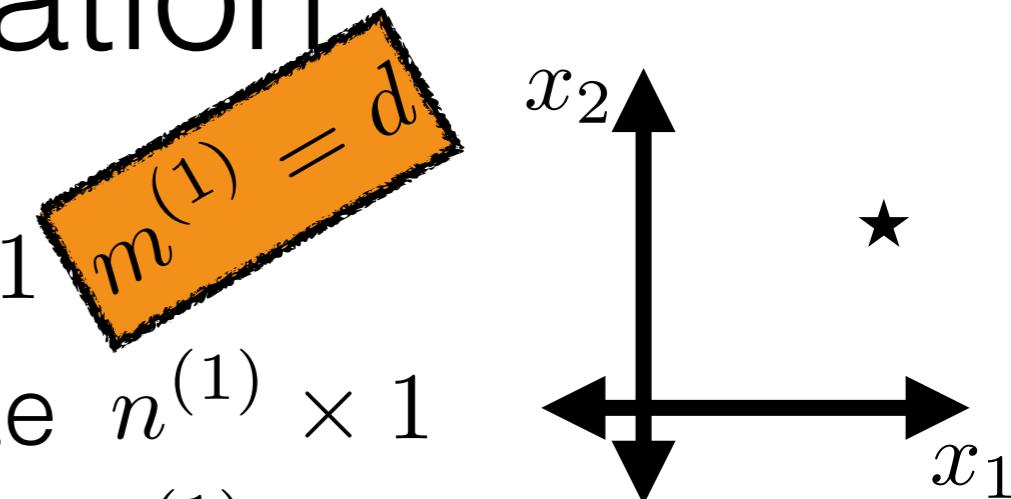
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  - All:  $A^{(2)} = f^{(2)}(W^{(2)\top} A^{(1)} + W_0^{(2)})$
  - $W^{(2)} : m^{(2)} \times n^{(2)}$



# Let's get some new notation

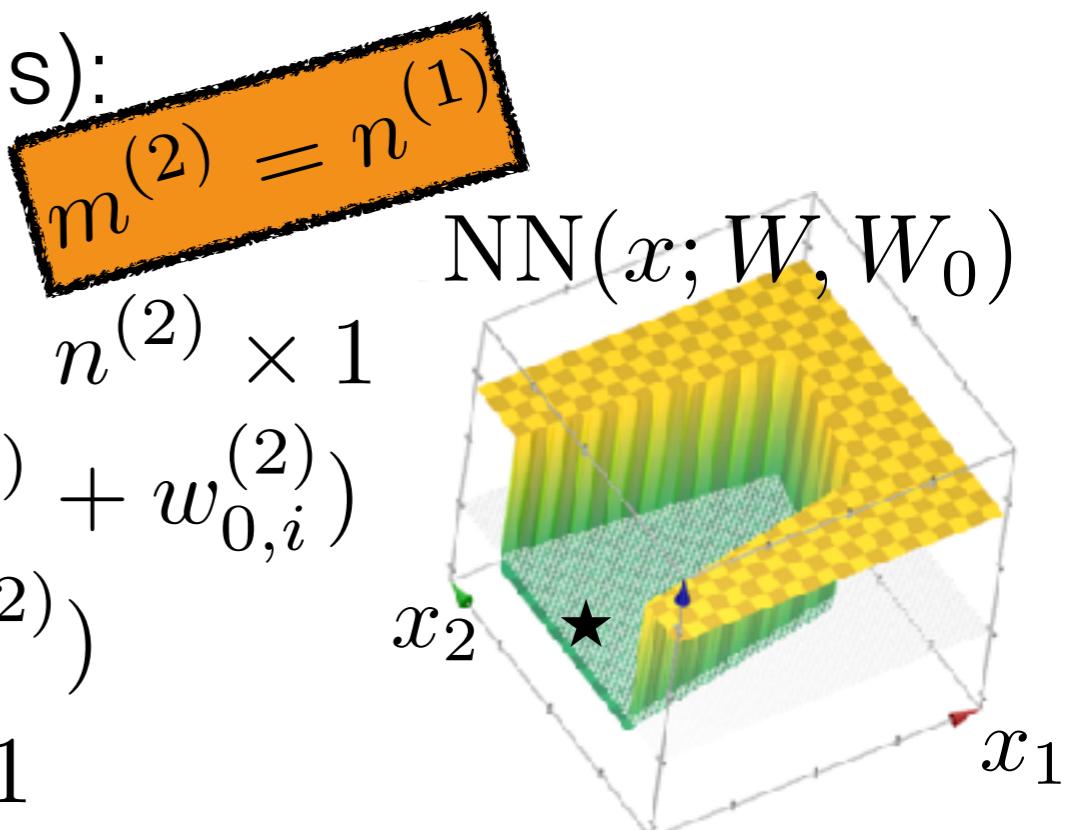
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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- The  $i$ th feature:  $A_i^{(1)} = f^{(1)}(w_i^{(1)\top} x + w_{0,i}^{(1)})$ 
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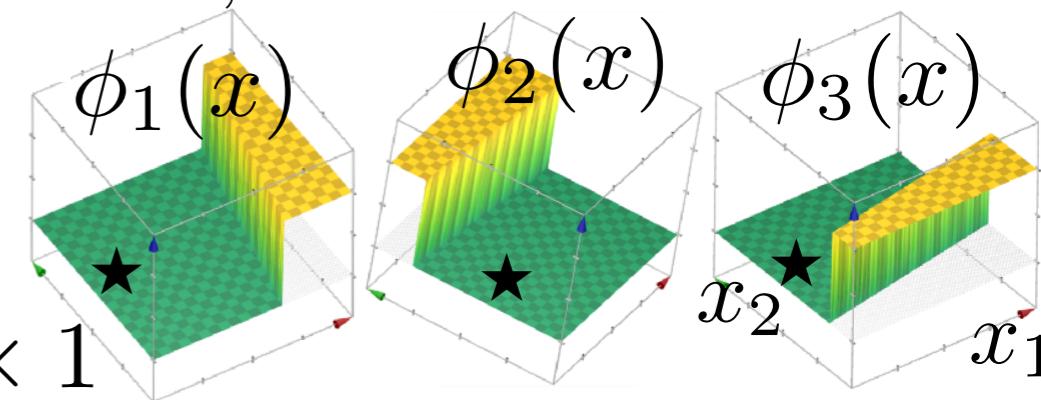
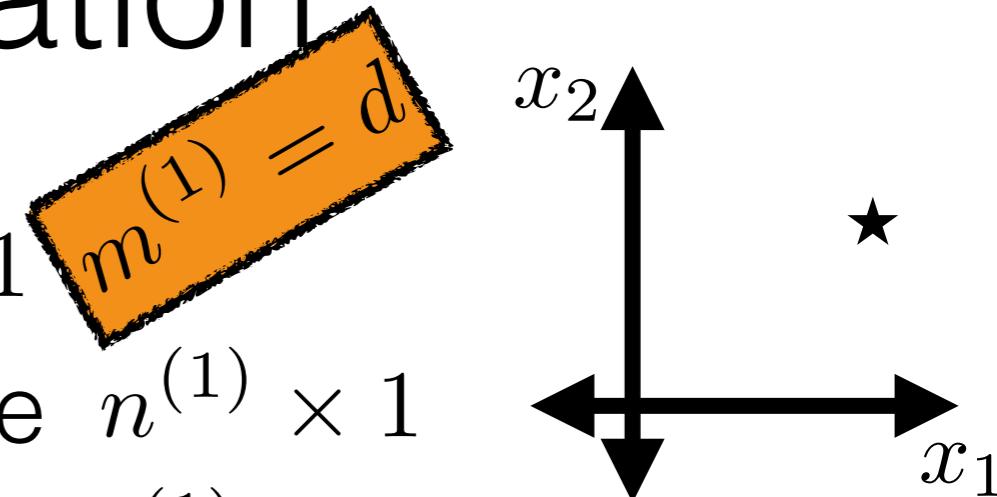
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# Let's get some new notation

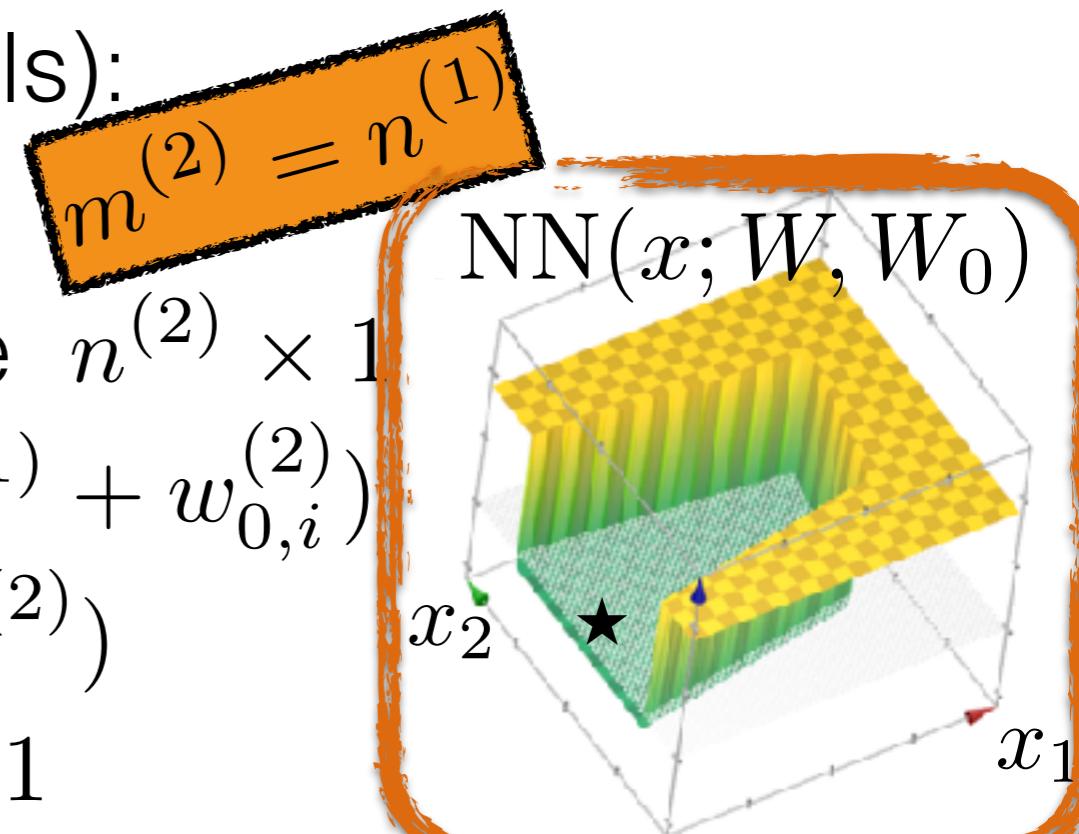
- 1st layer, constructing the features:

- Input  $x$  (a data point): size  $m^{(1)} \times 1$
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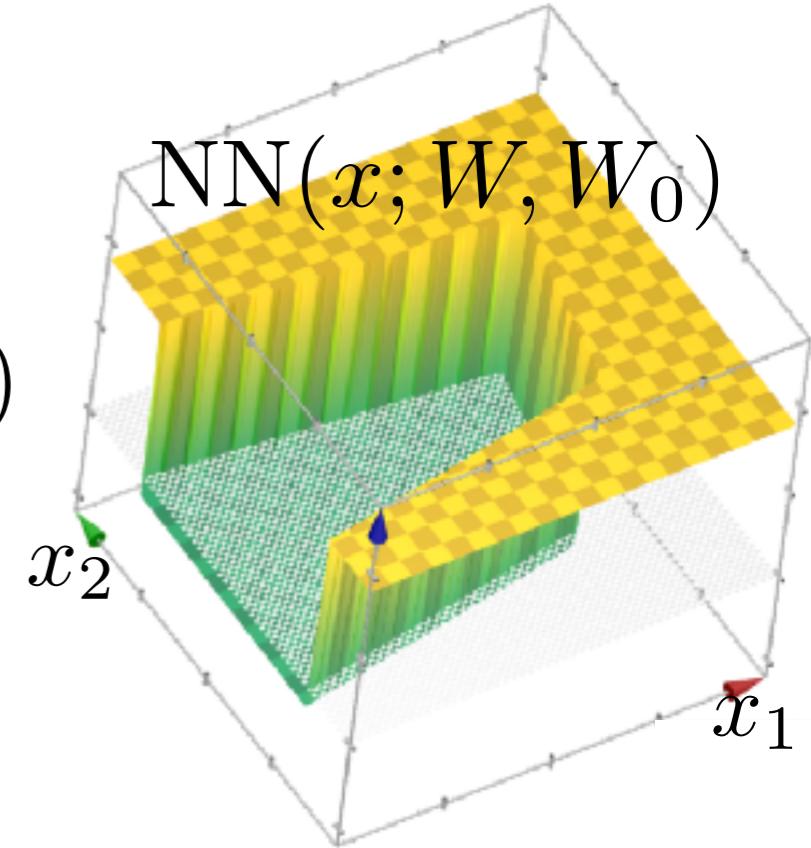
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- 2nd layer:  $A^{(2)} = f^{(2)}(W^{(2)\top}A^{(1)} + W_0^{(2)})$

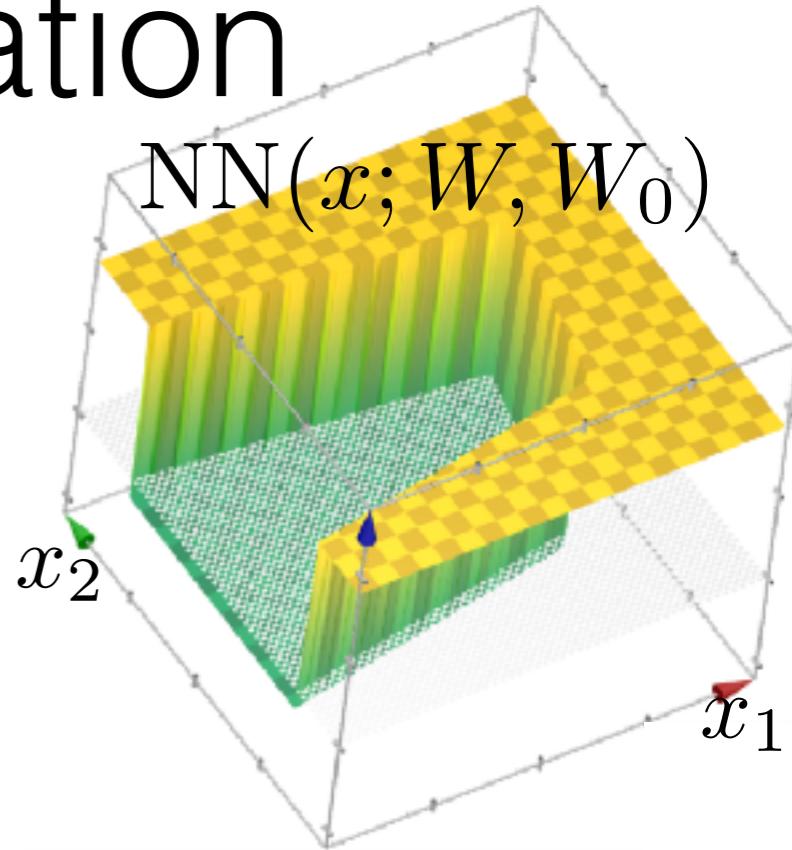
- 1st layer:  $A^{(1)} = f^{(1)}(W^{(1)\top}x + W_0^{(1)})$
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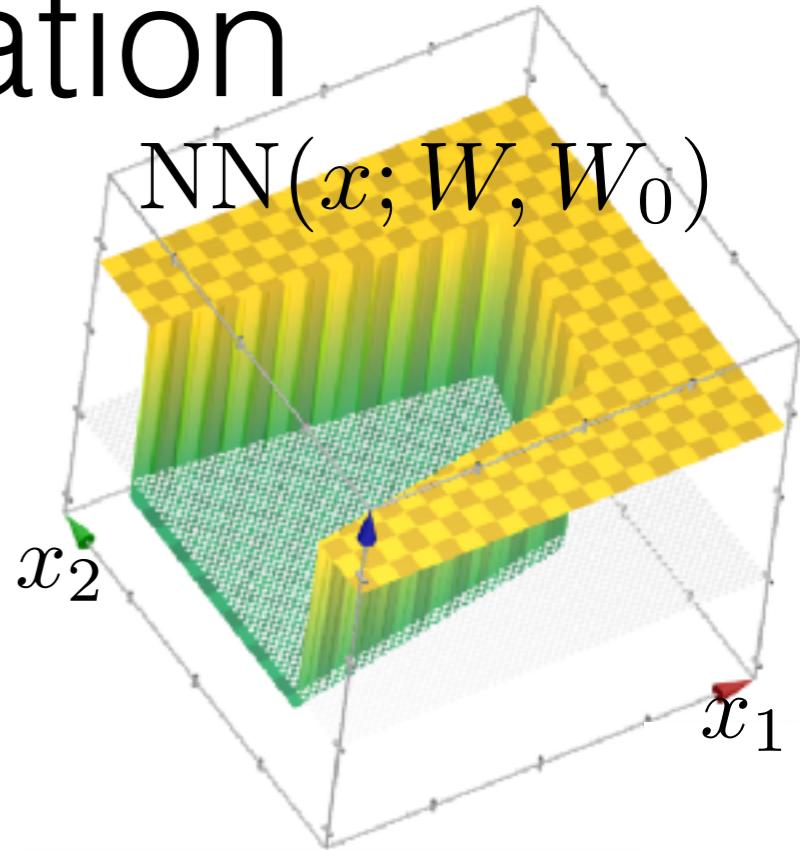
# Function graph representation

- 1st layer:  $A^{(1)} = f^{(1)}(W^{(1)\top} x + W_0^{(1)})$
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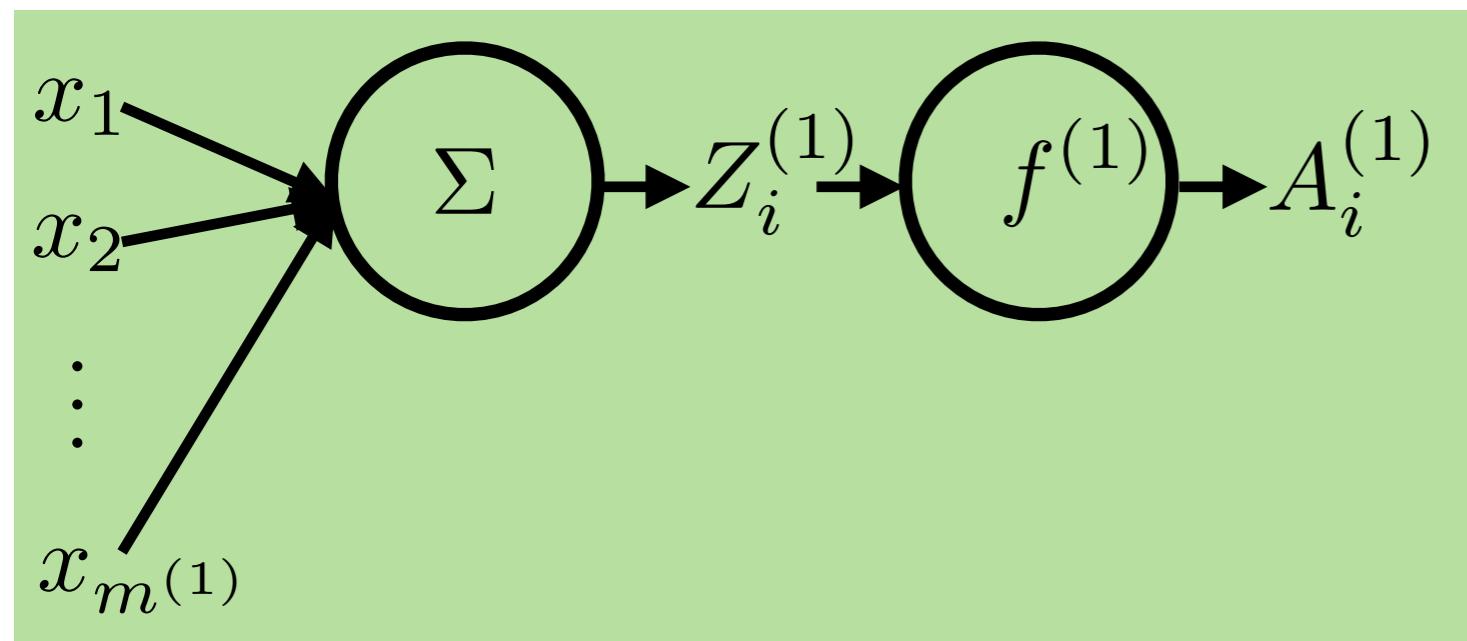
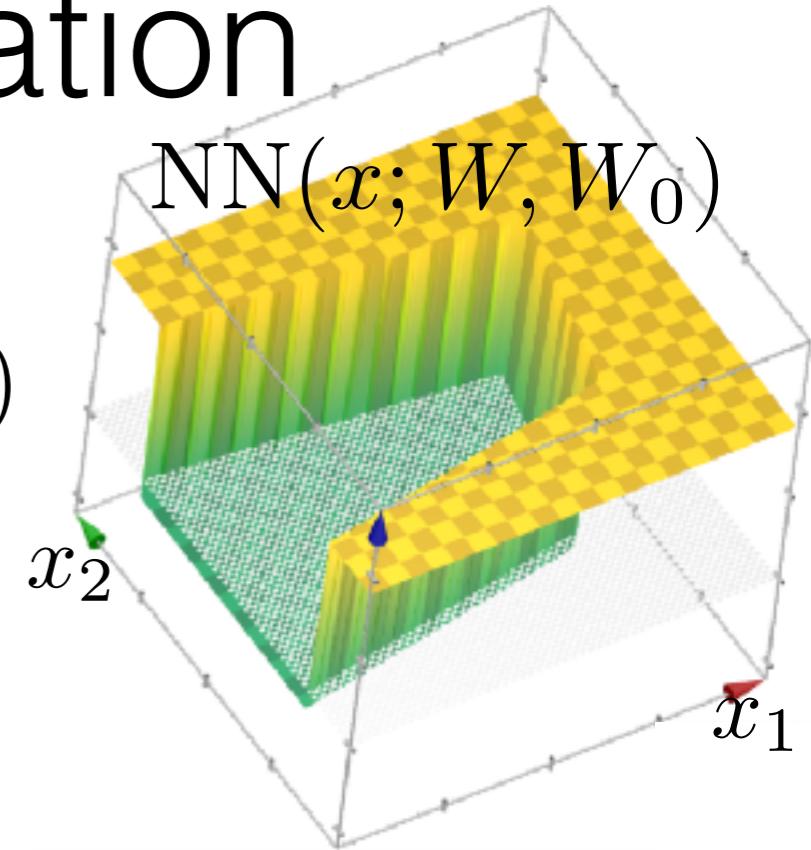
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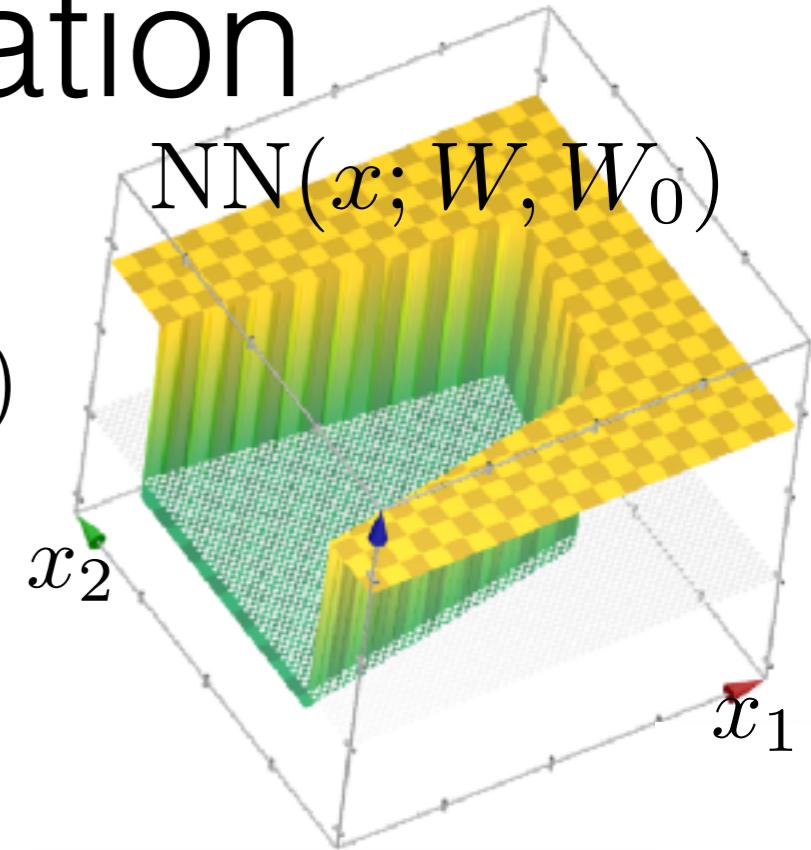
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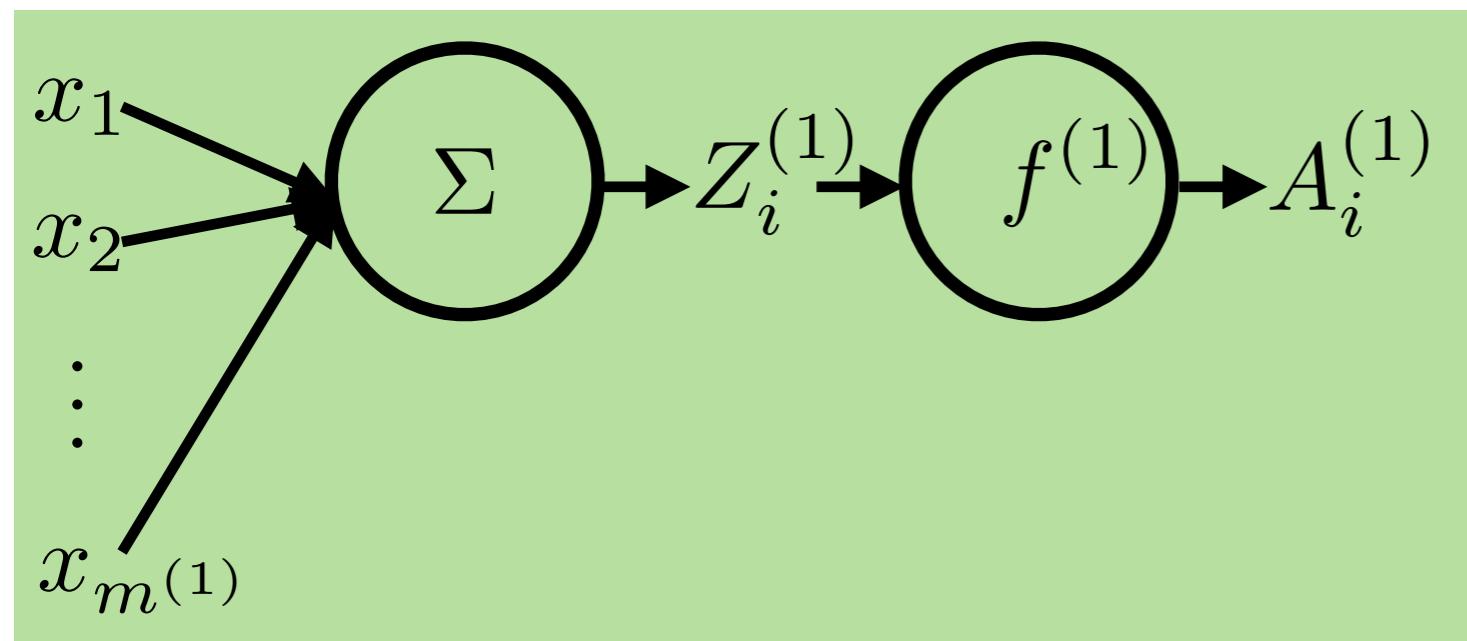


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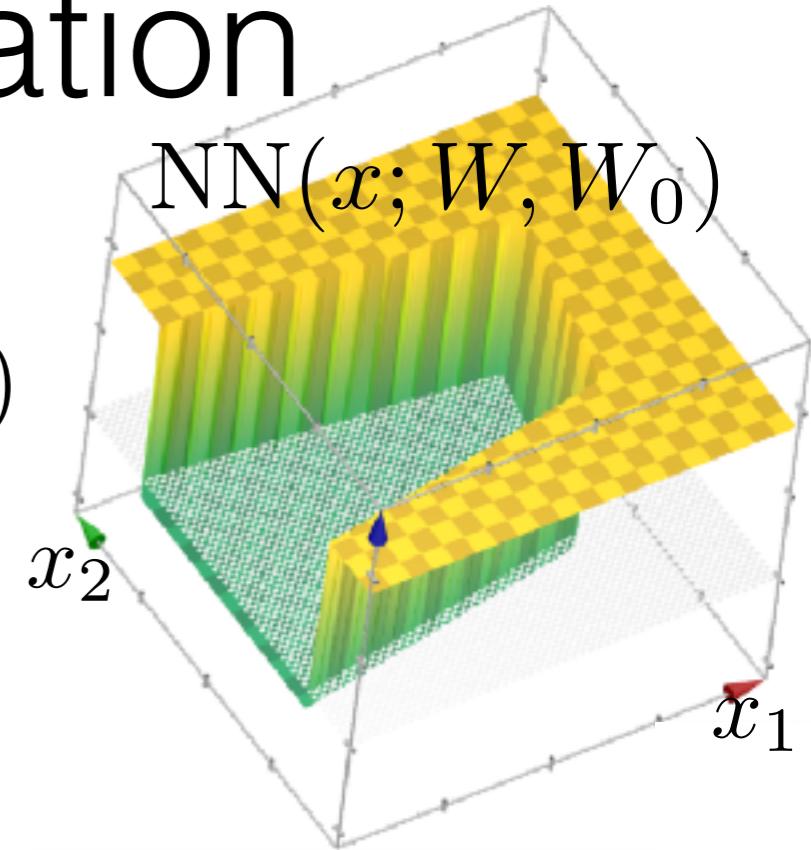


- Circle: function evaluation

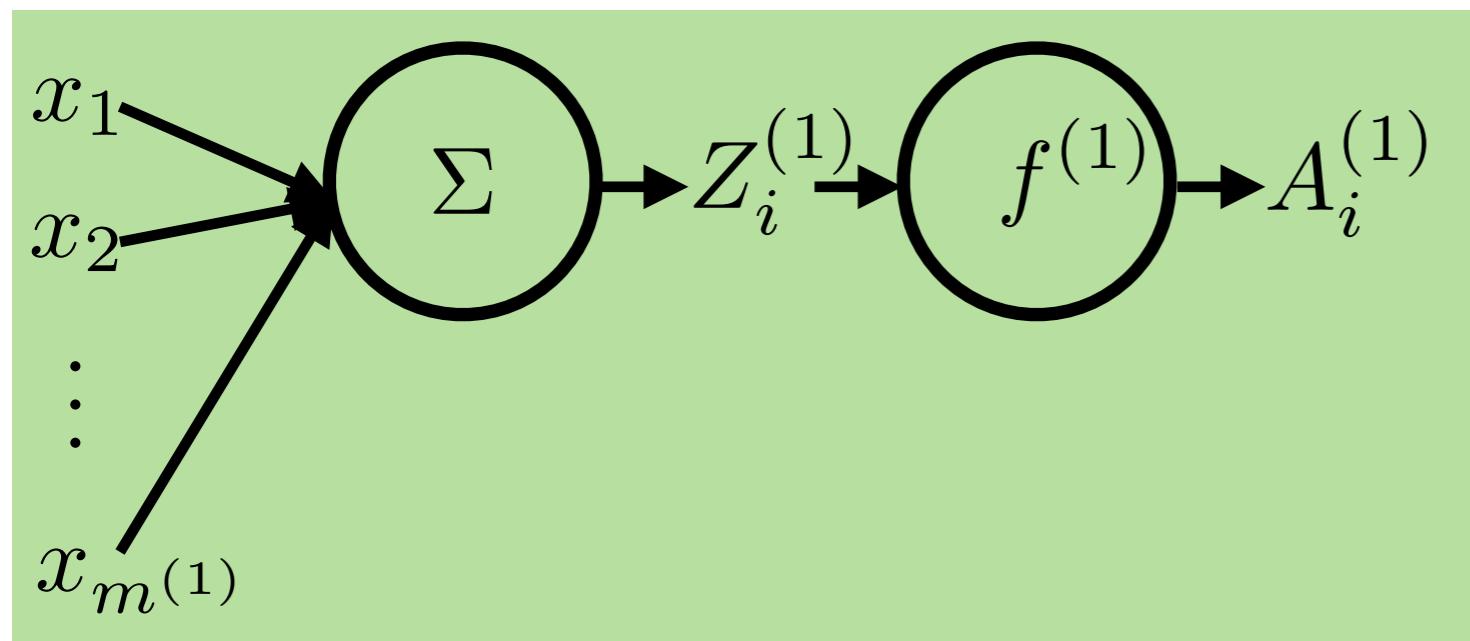


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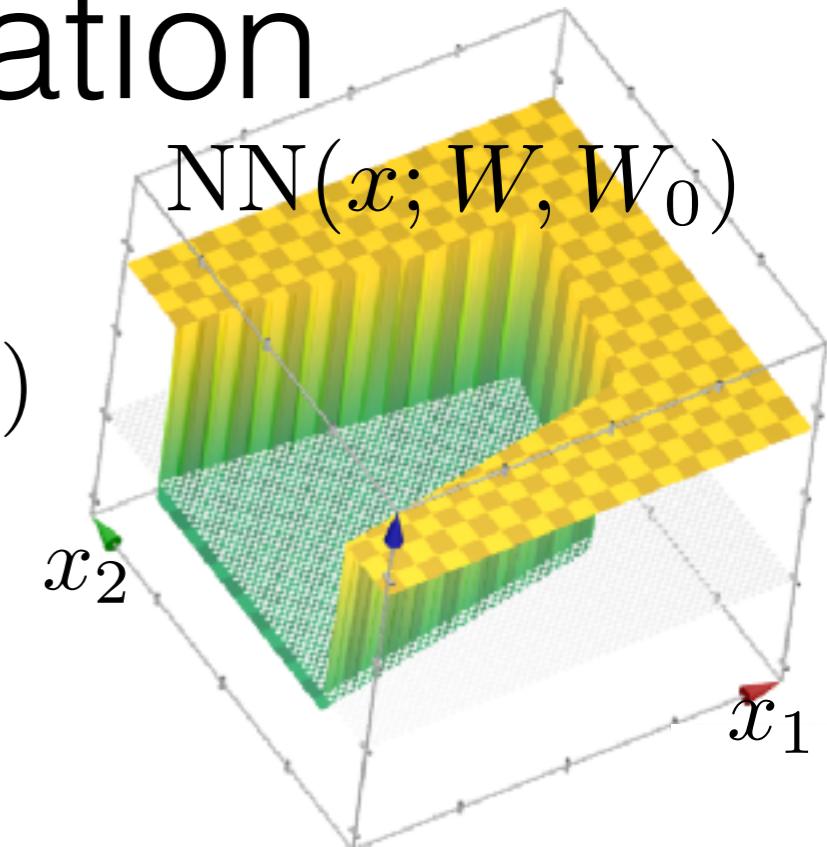
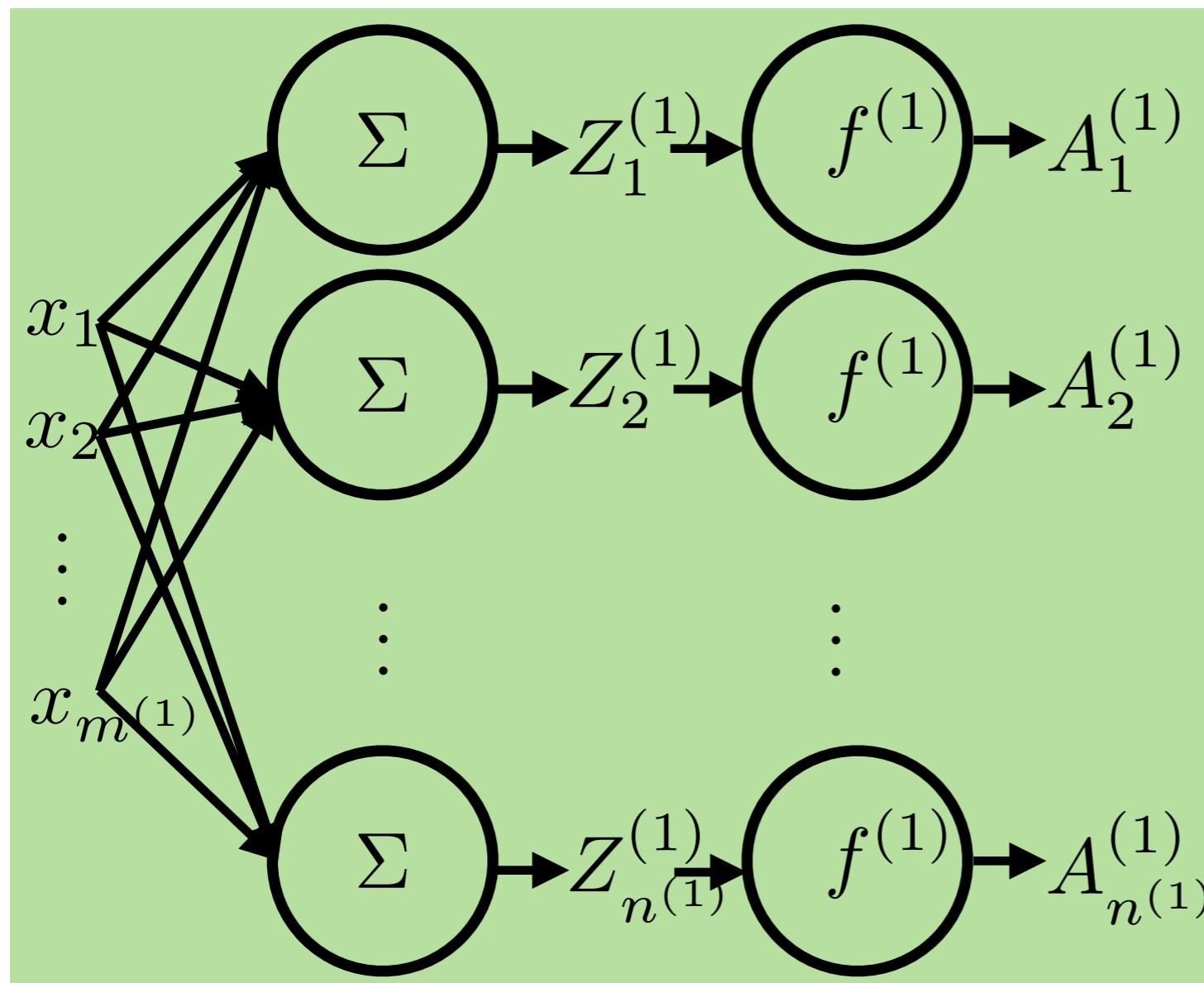


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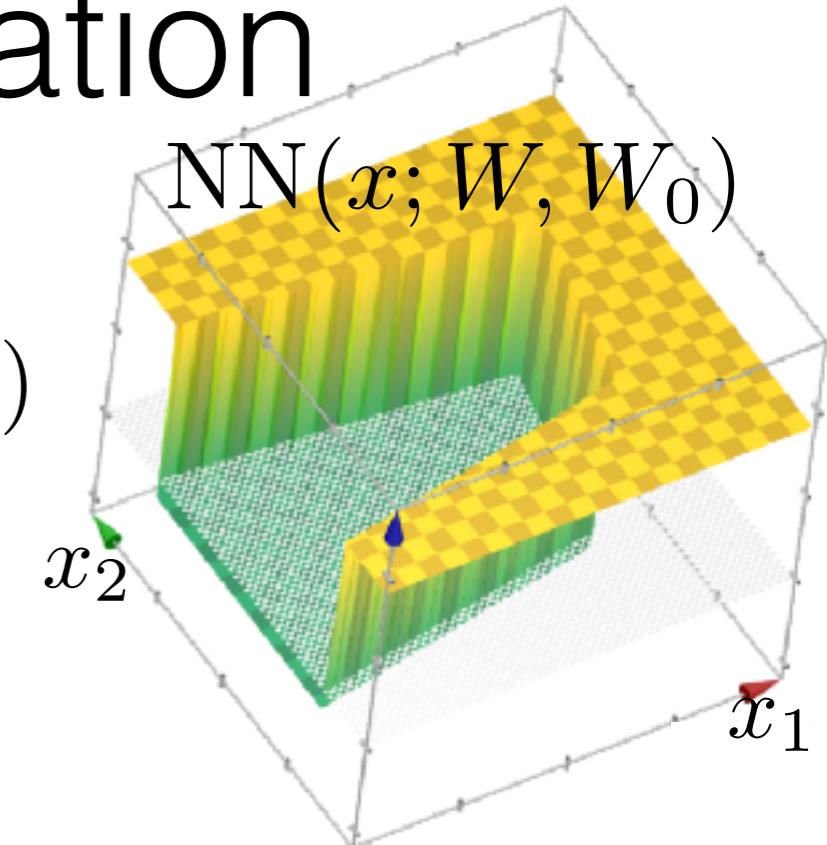
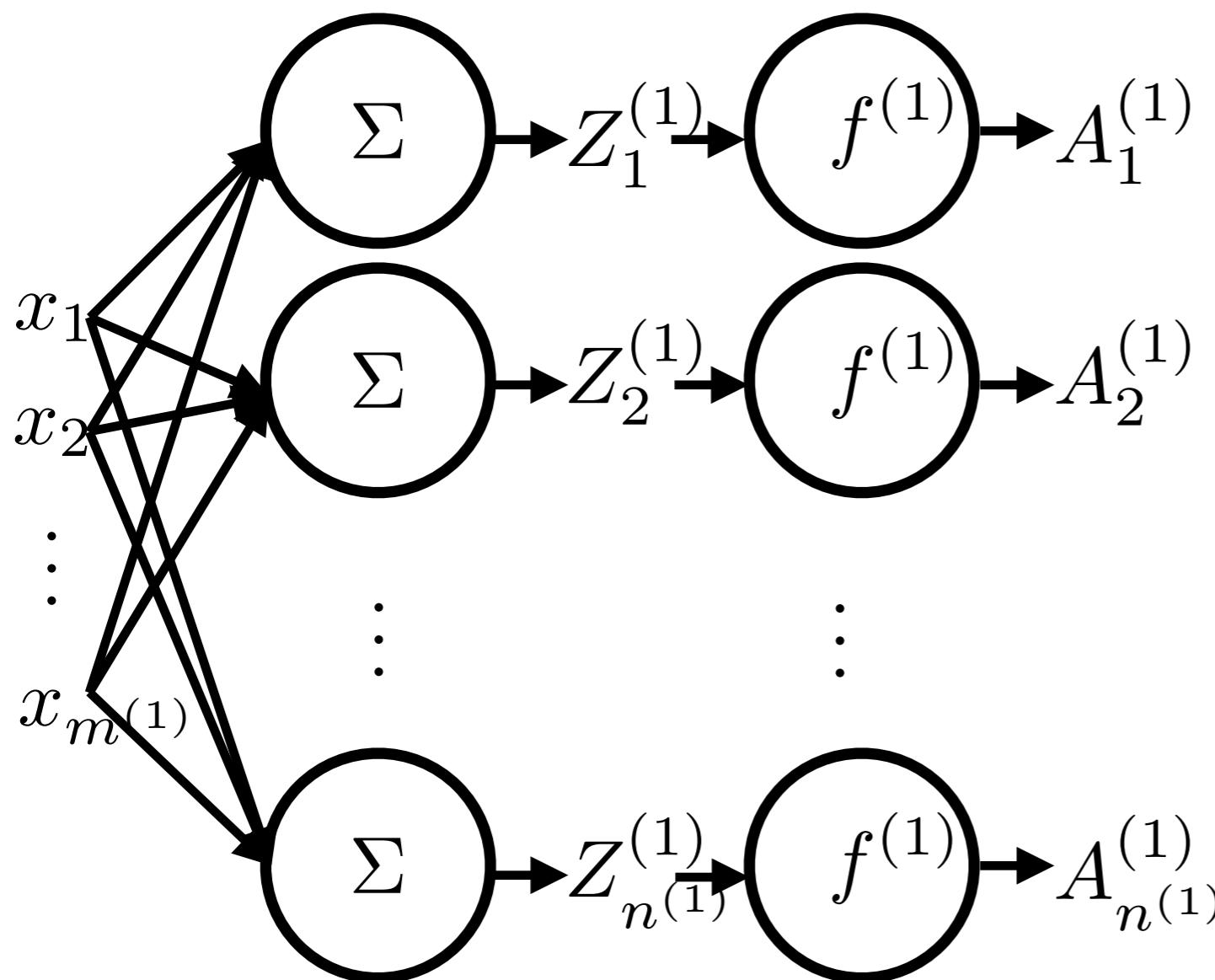
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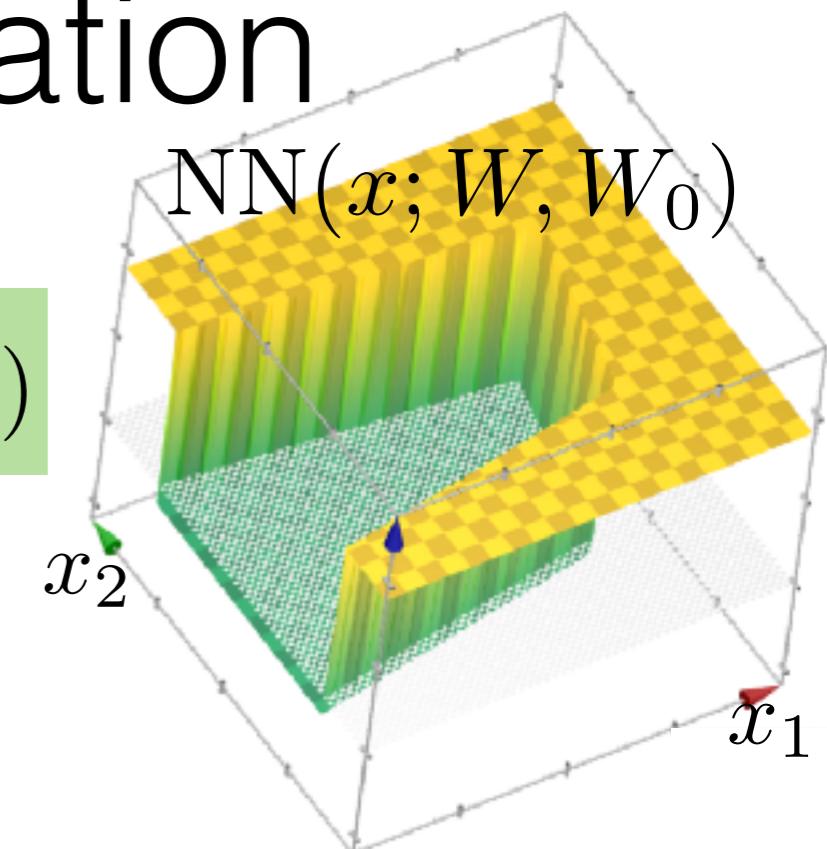
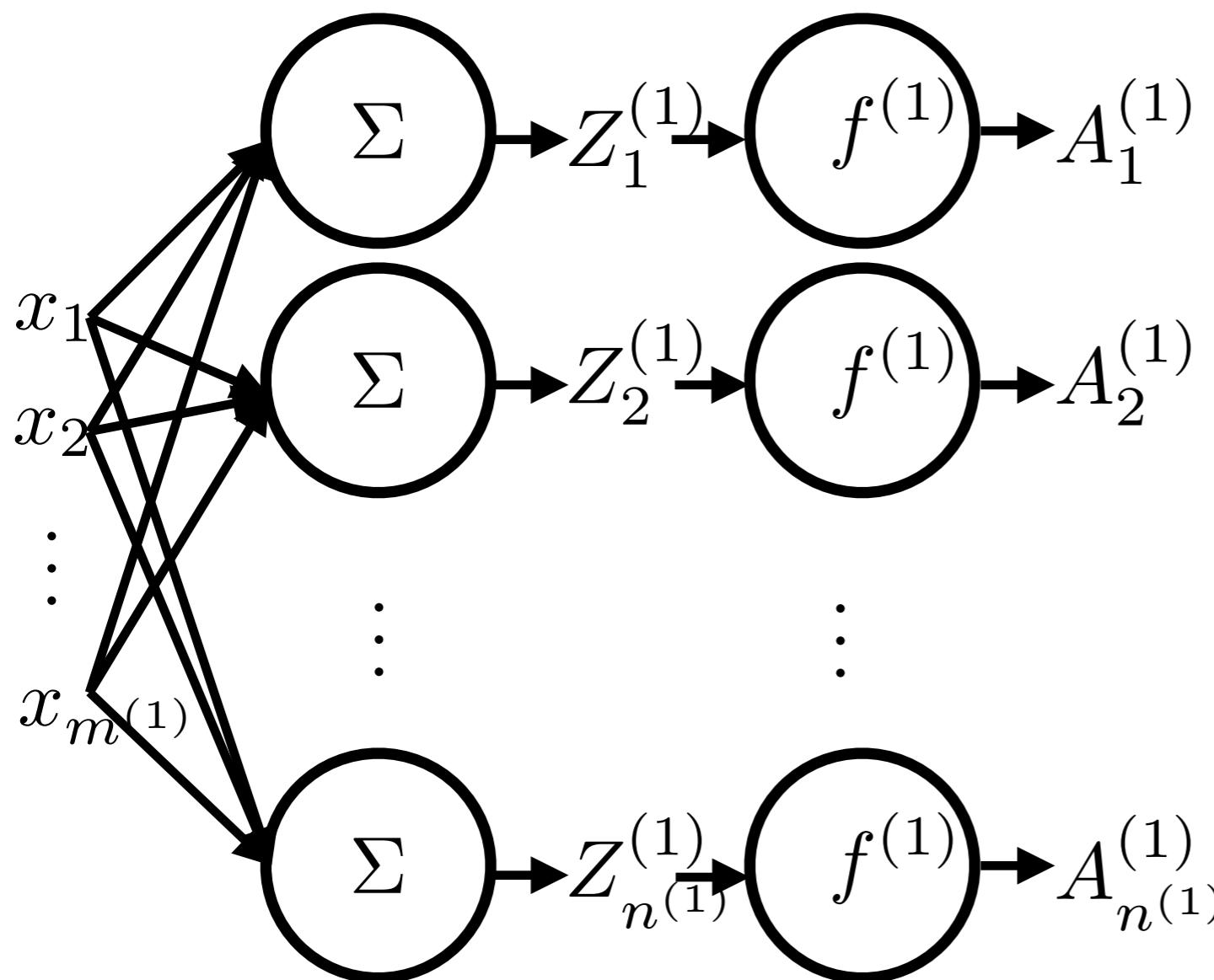
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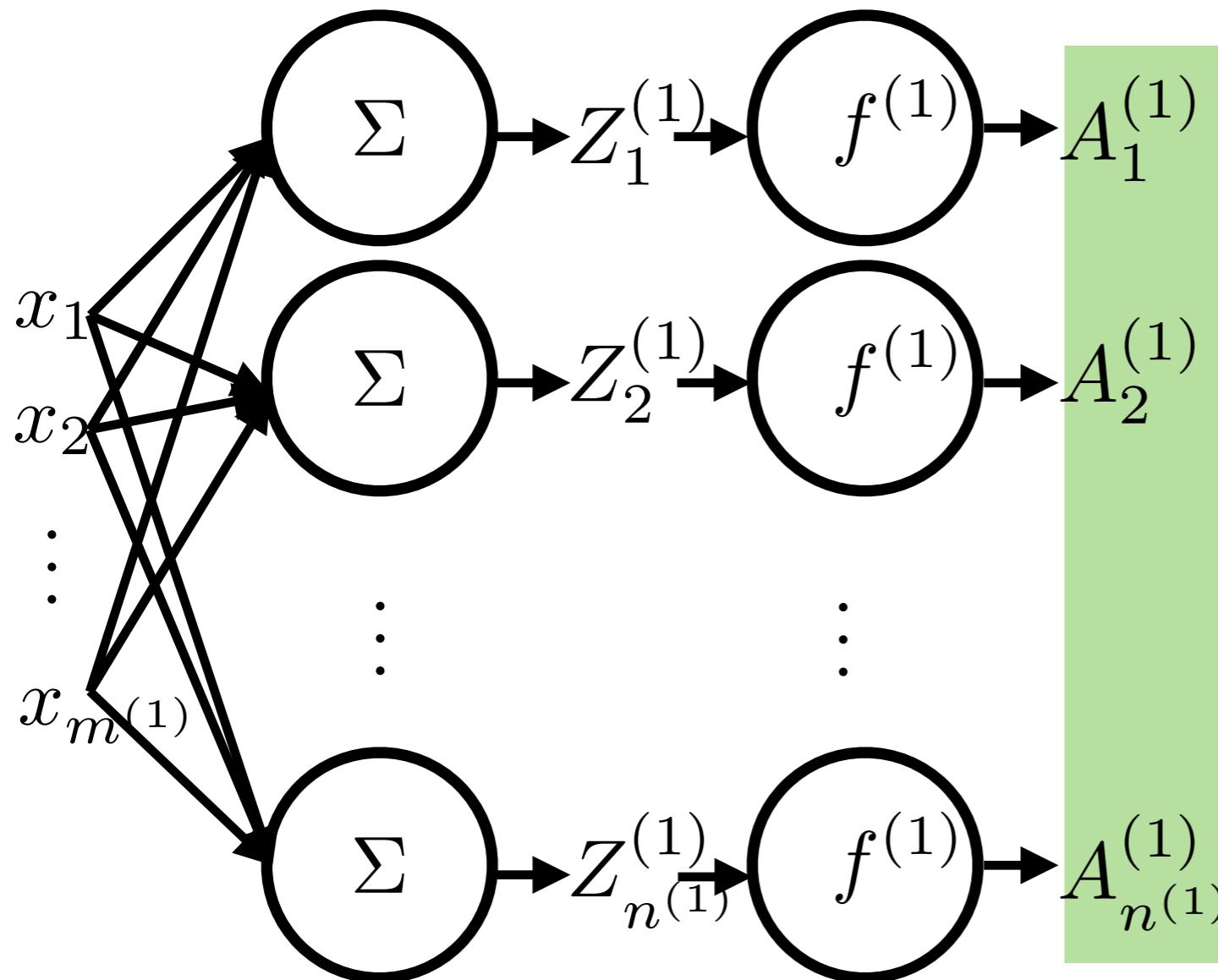
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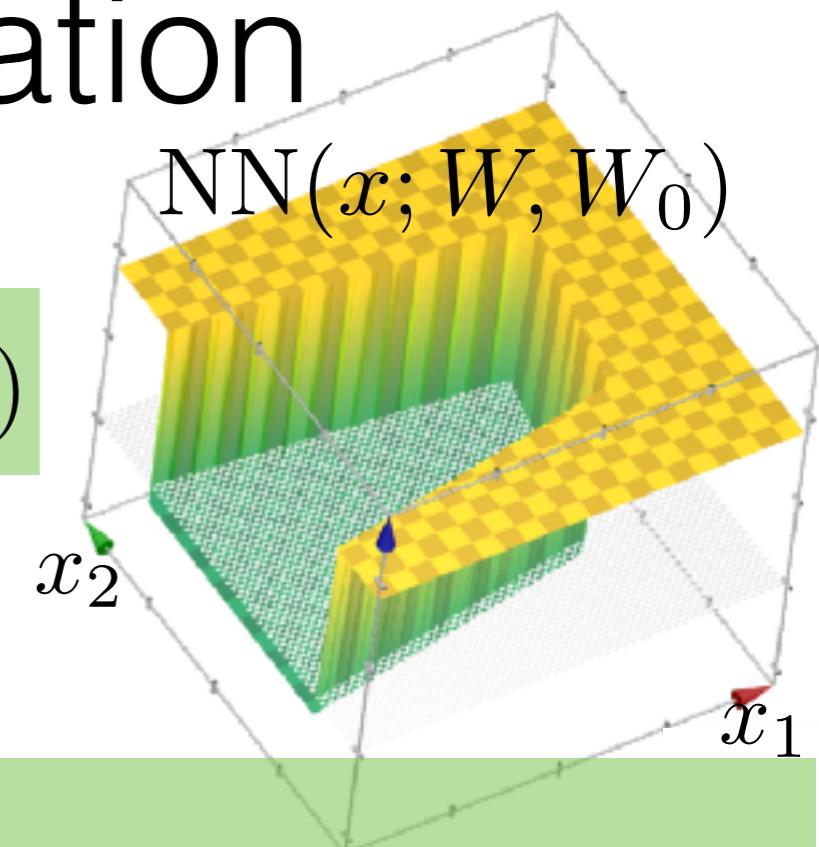
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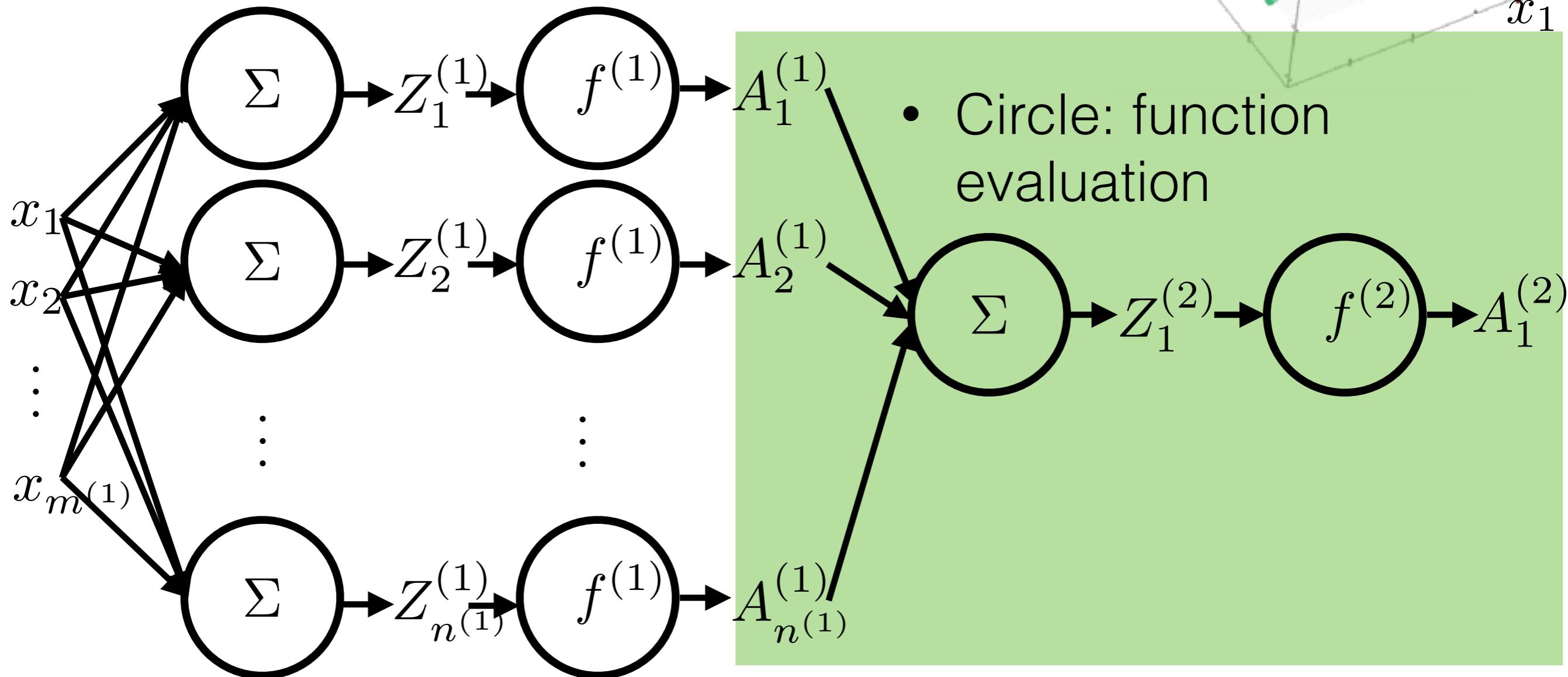
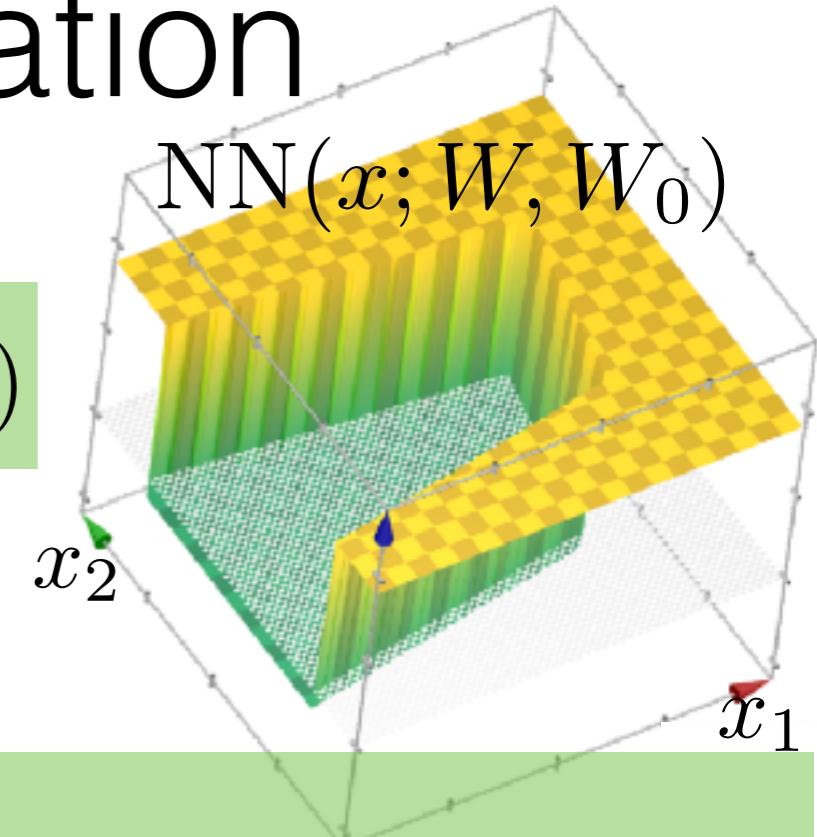


- Circle: function evaluation



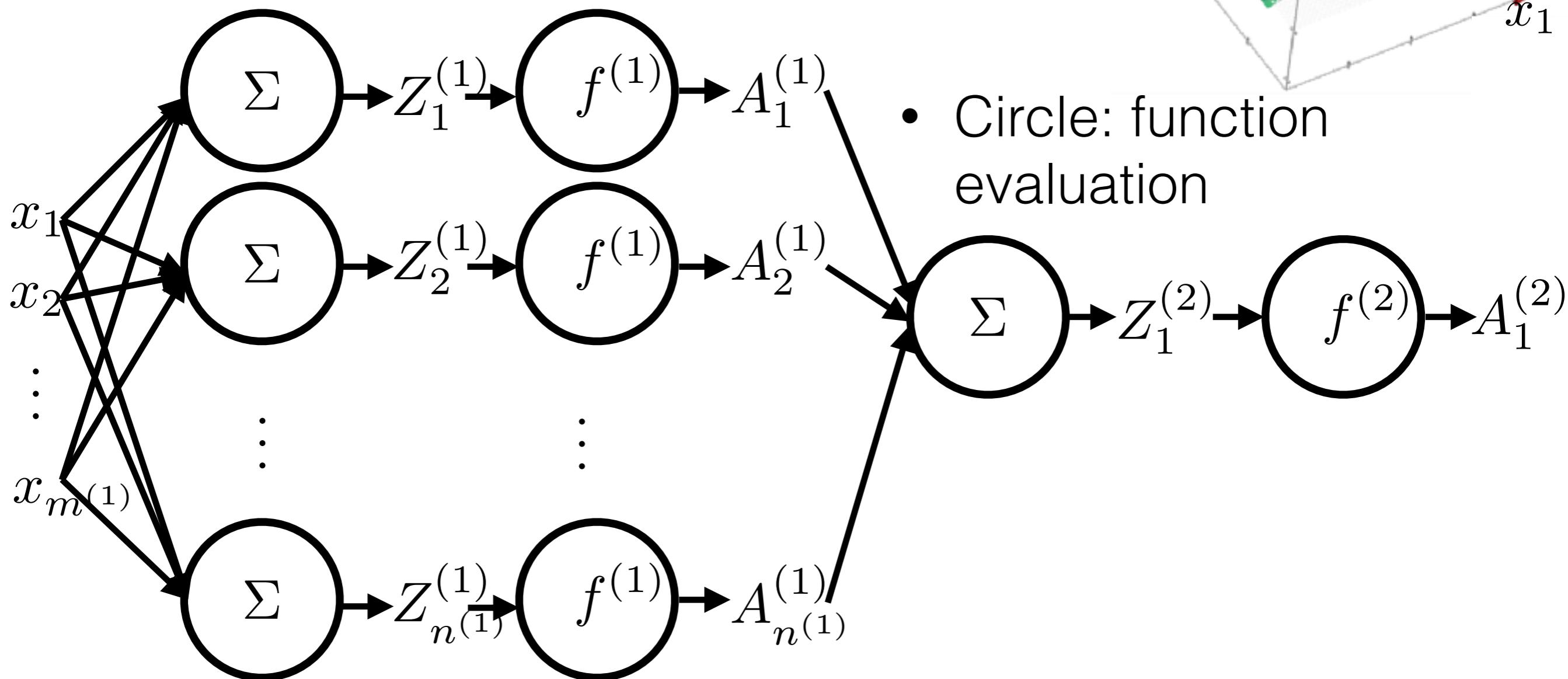
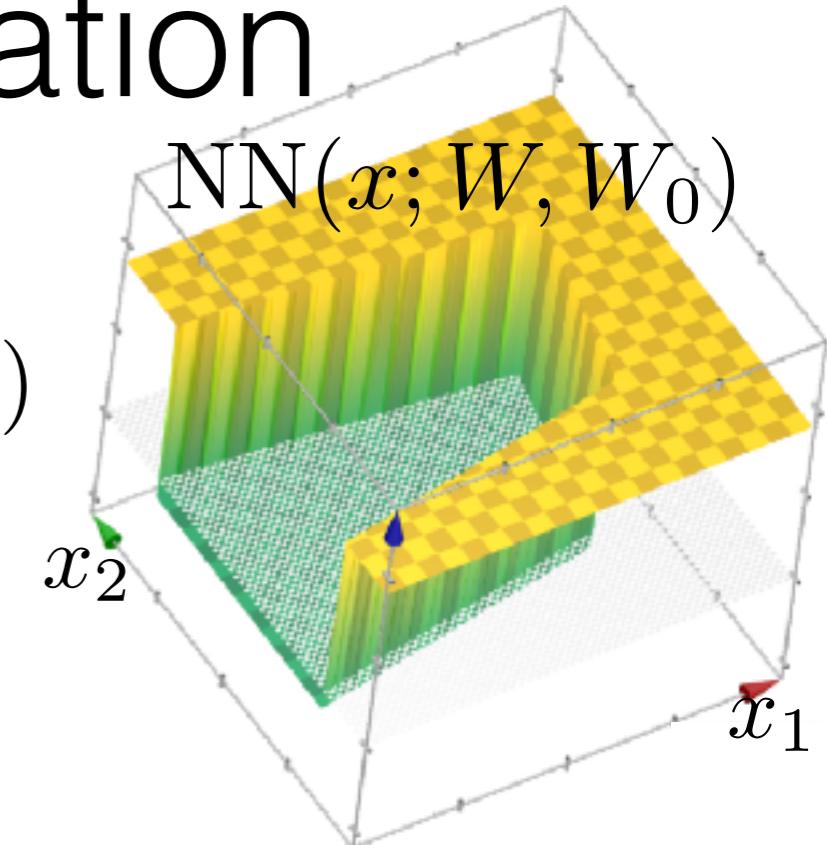
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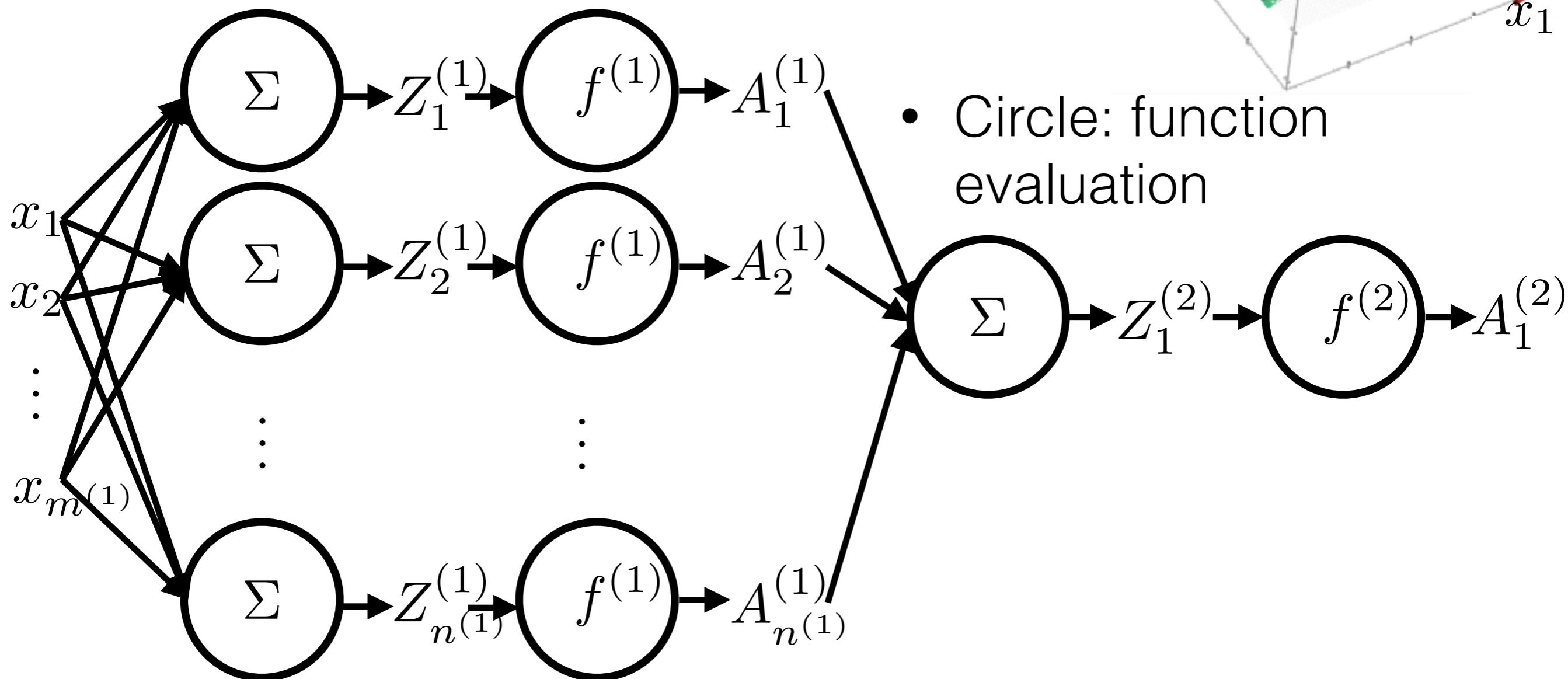
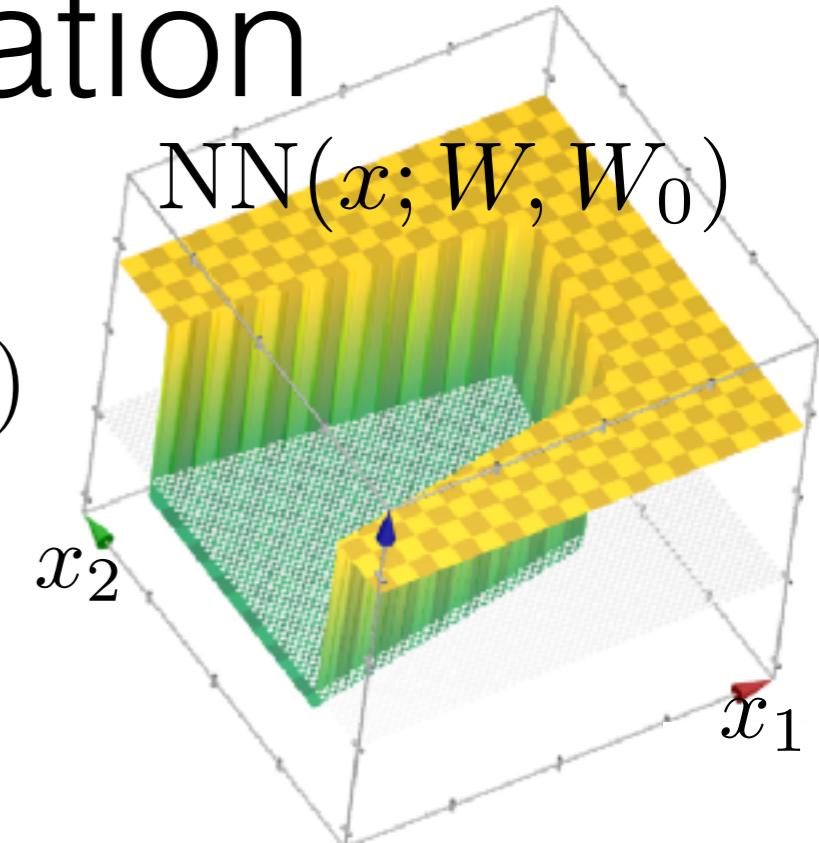
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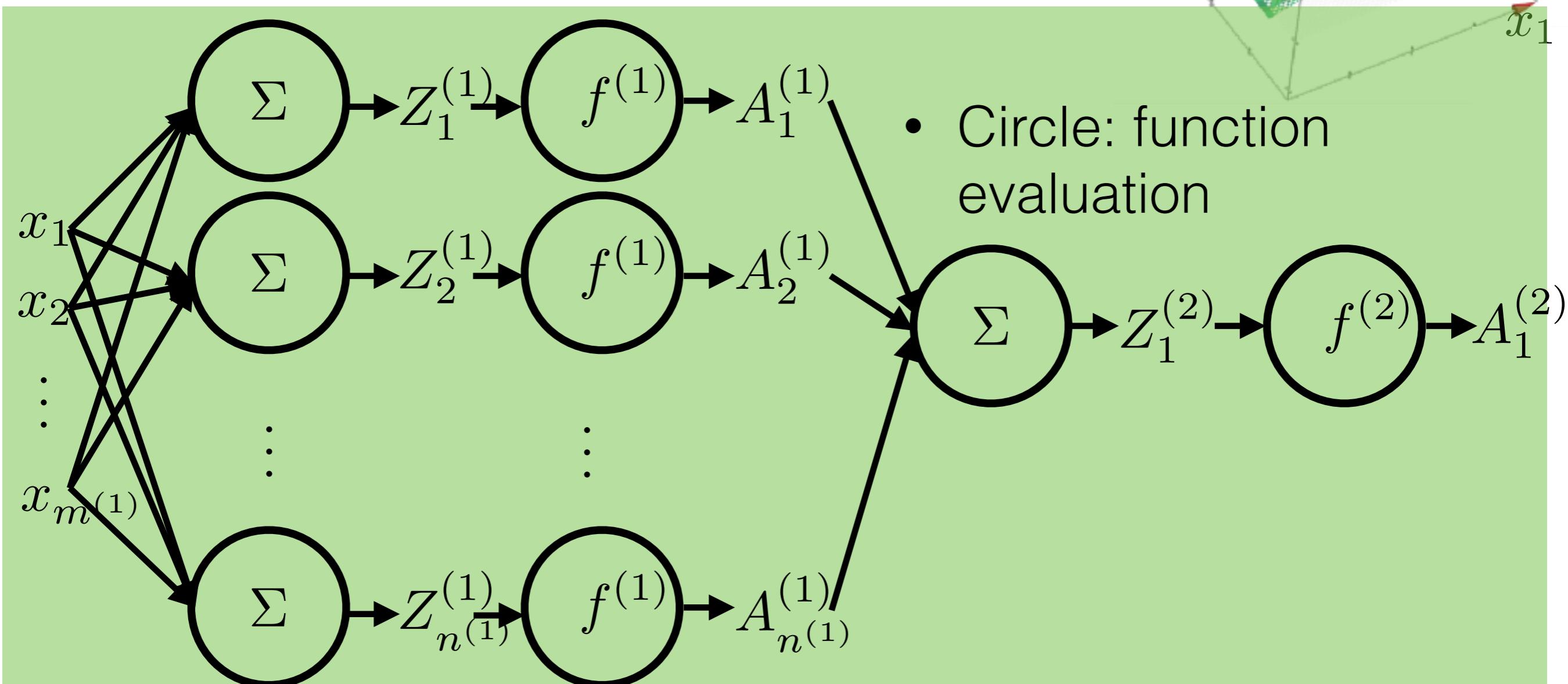
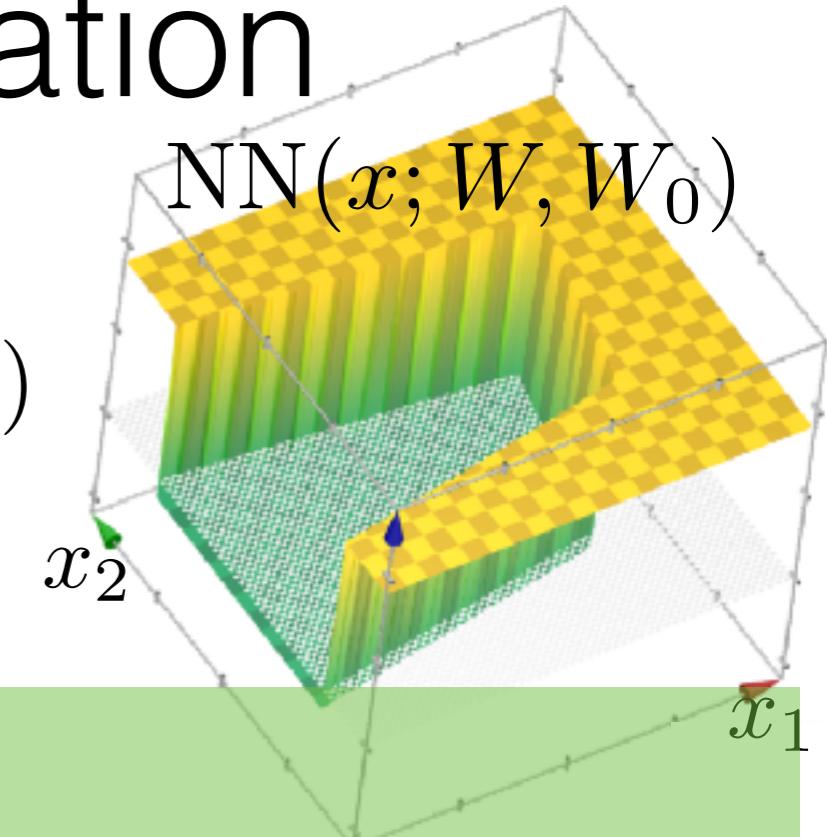
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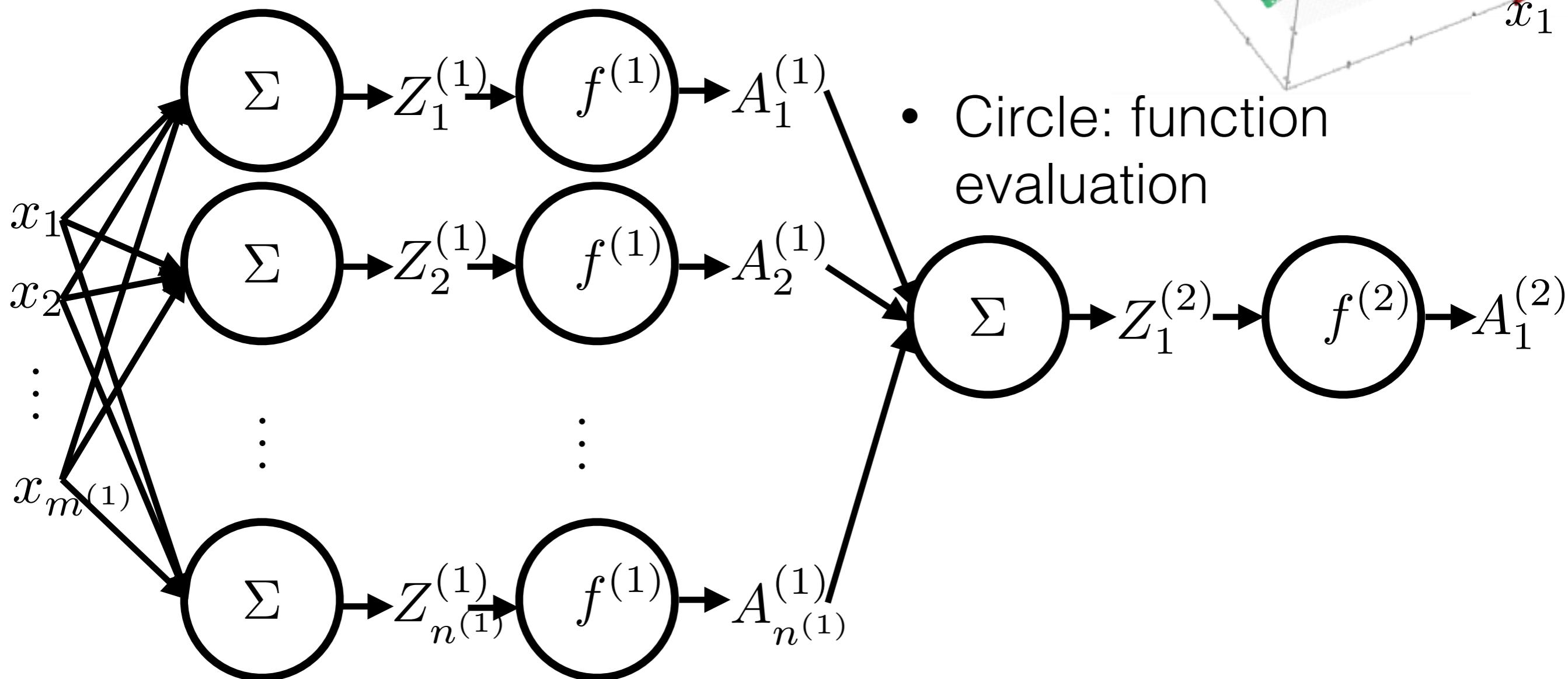
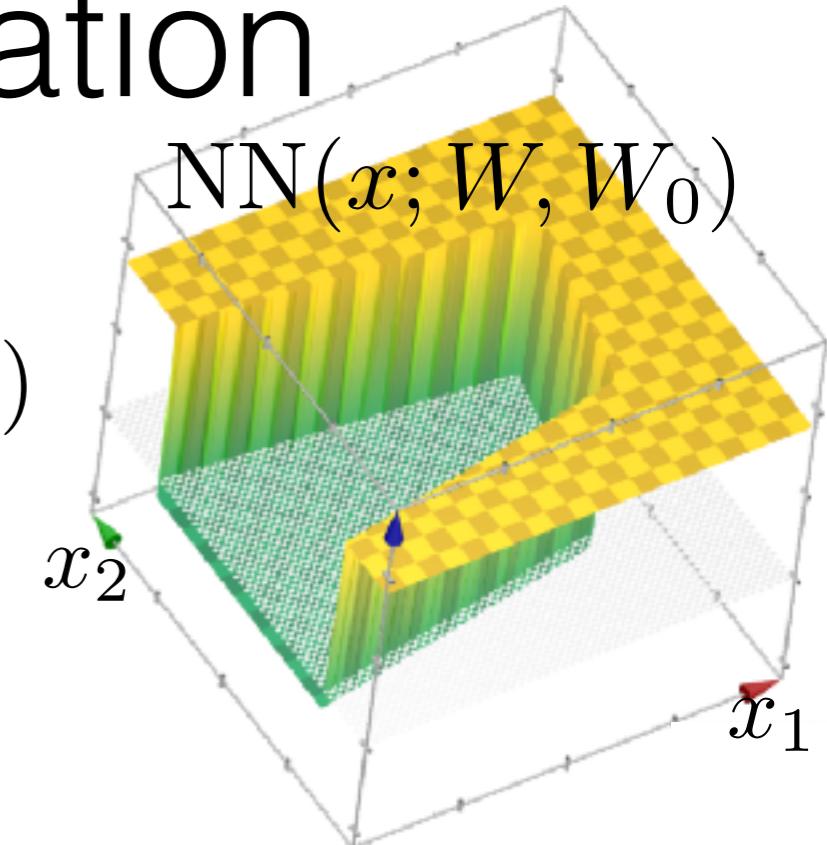
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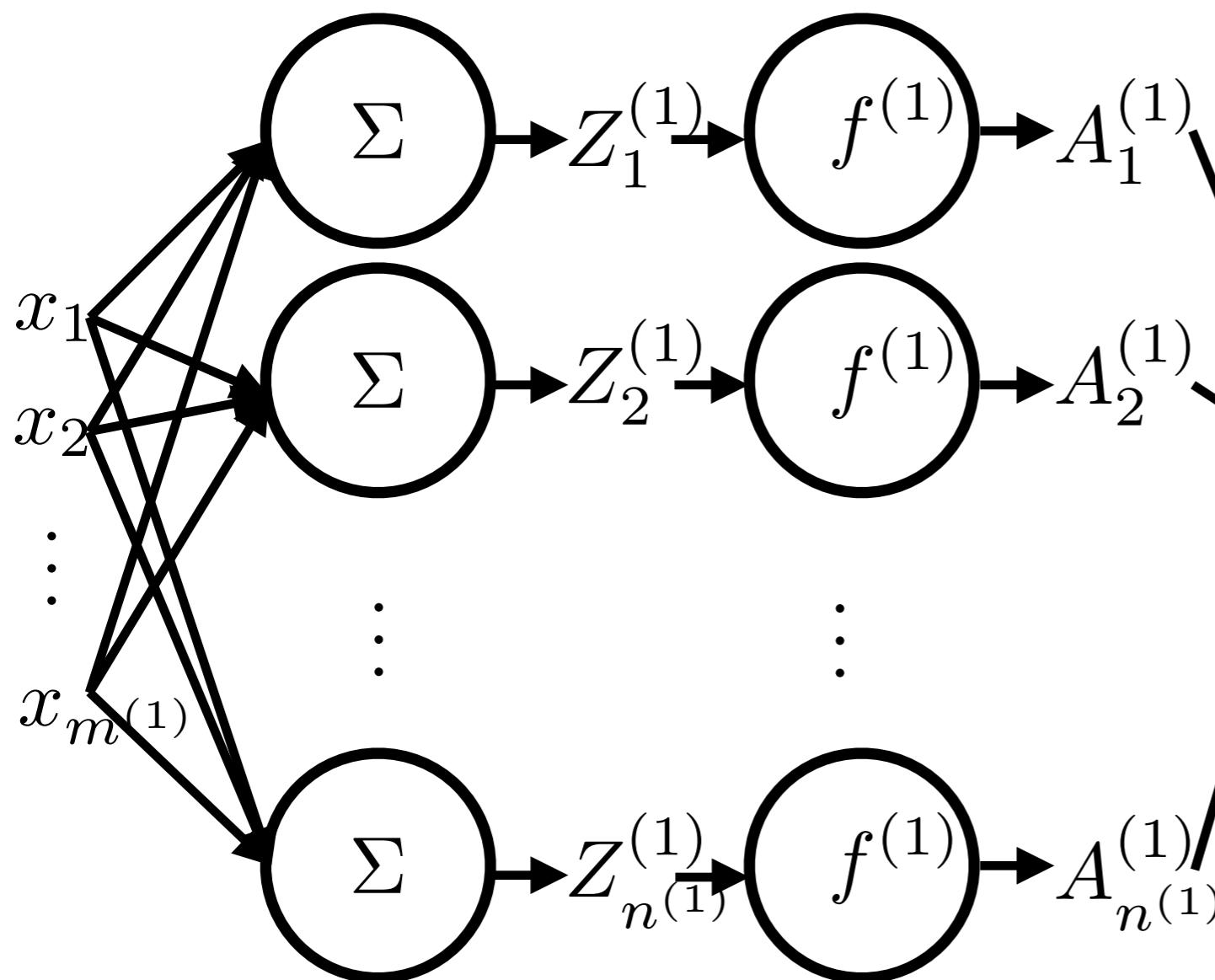
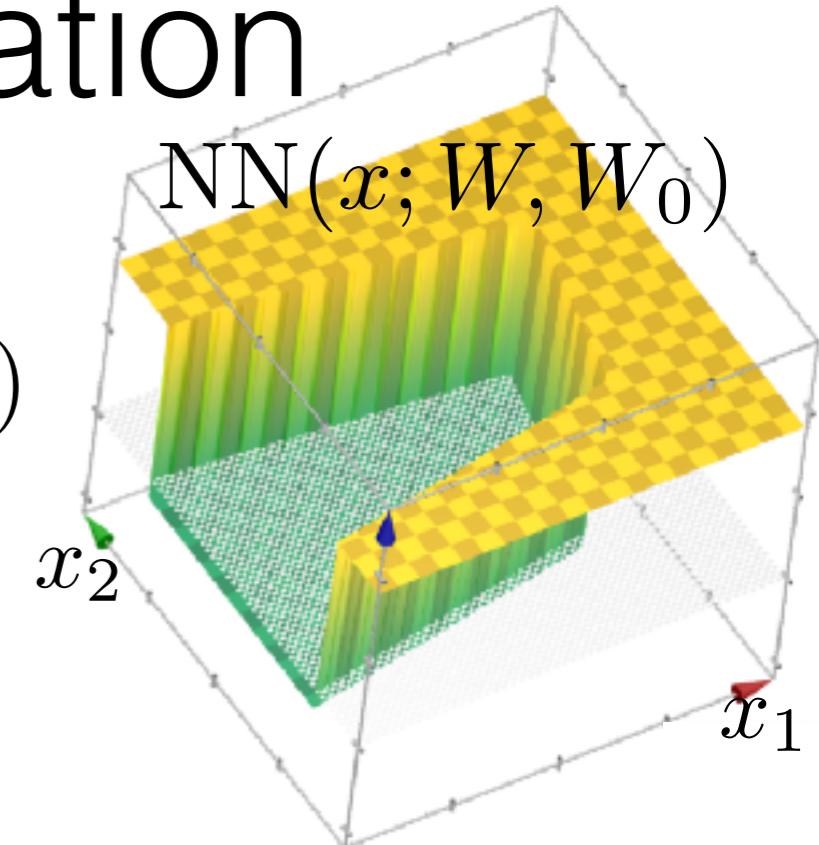
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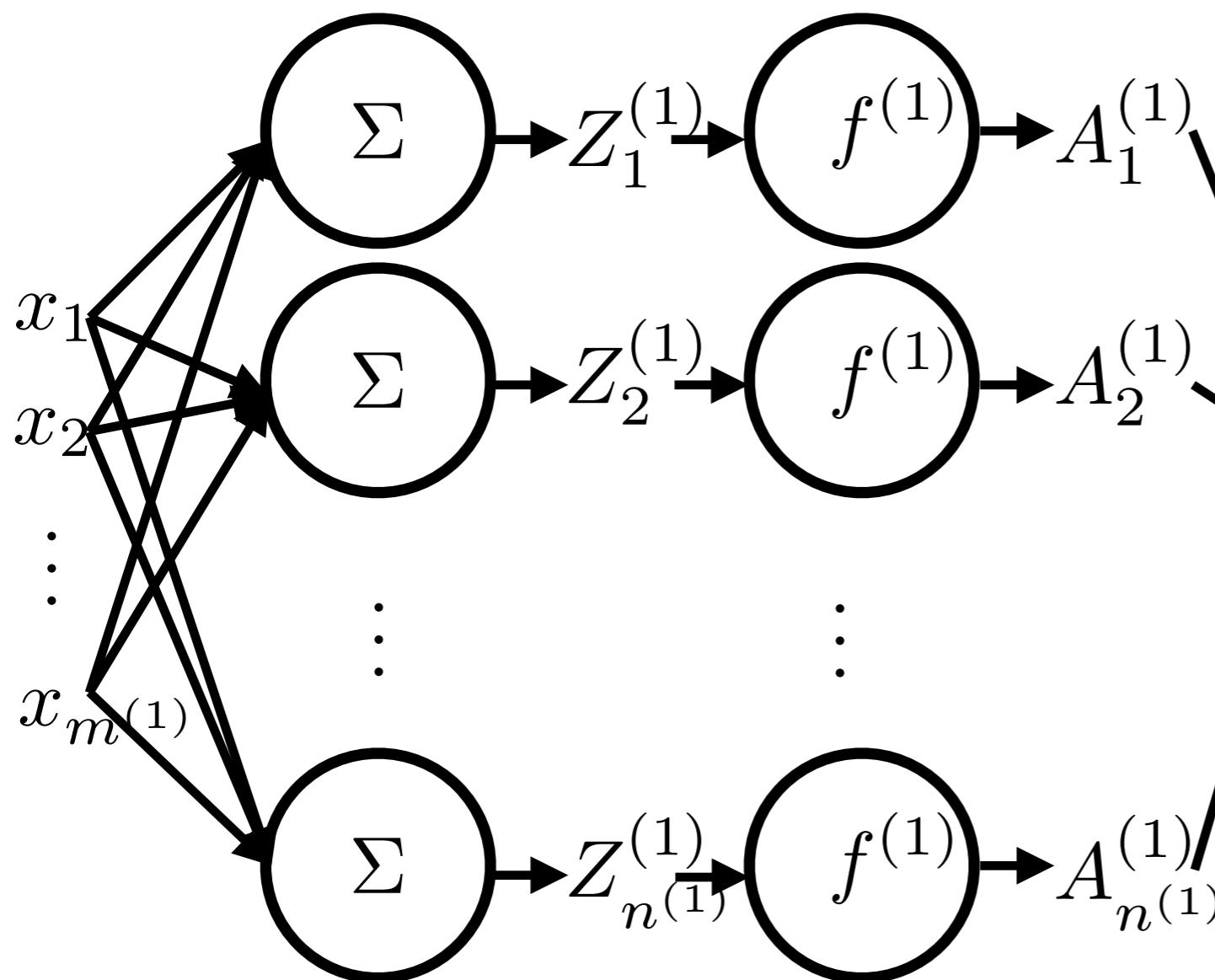
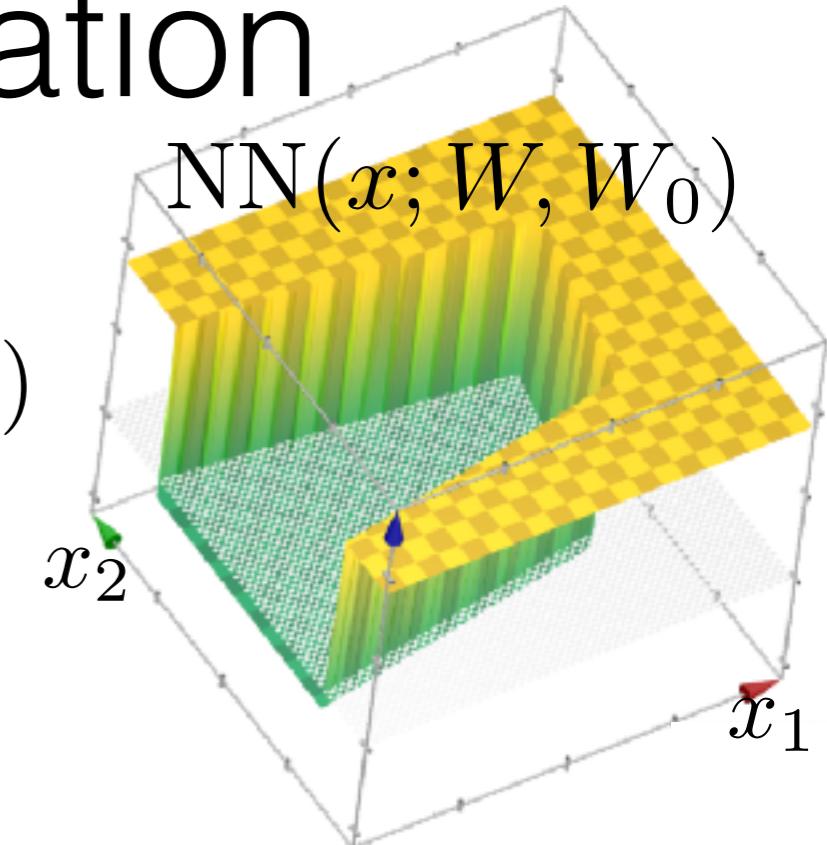
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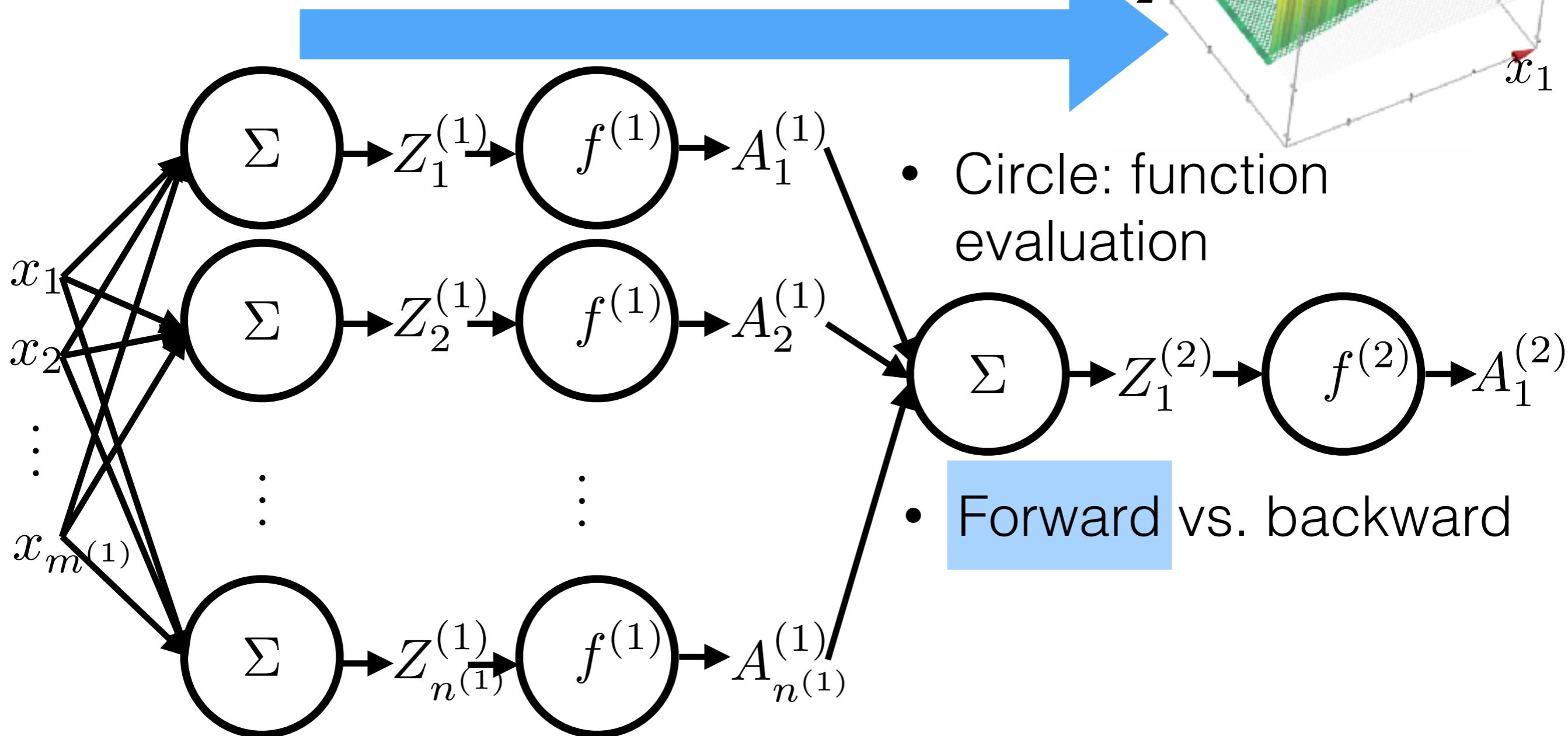
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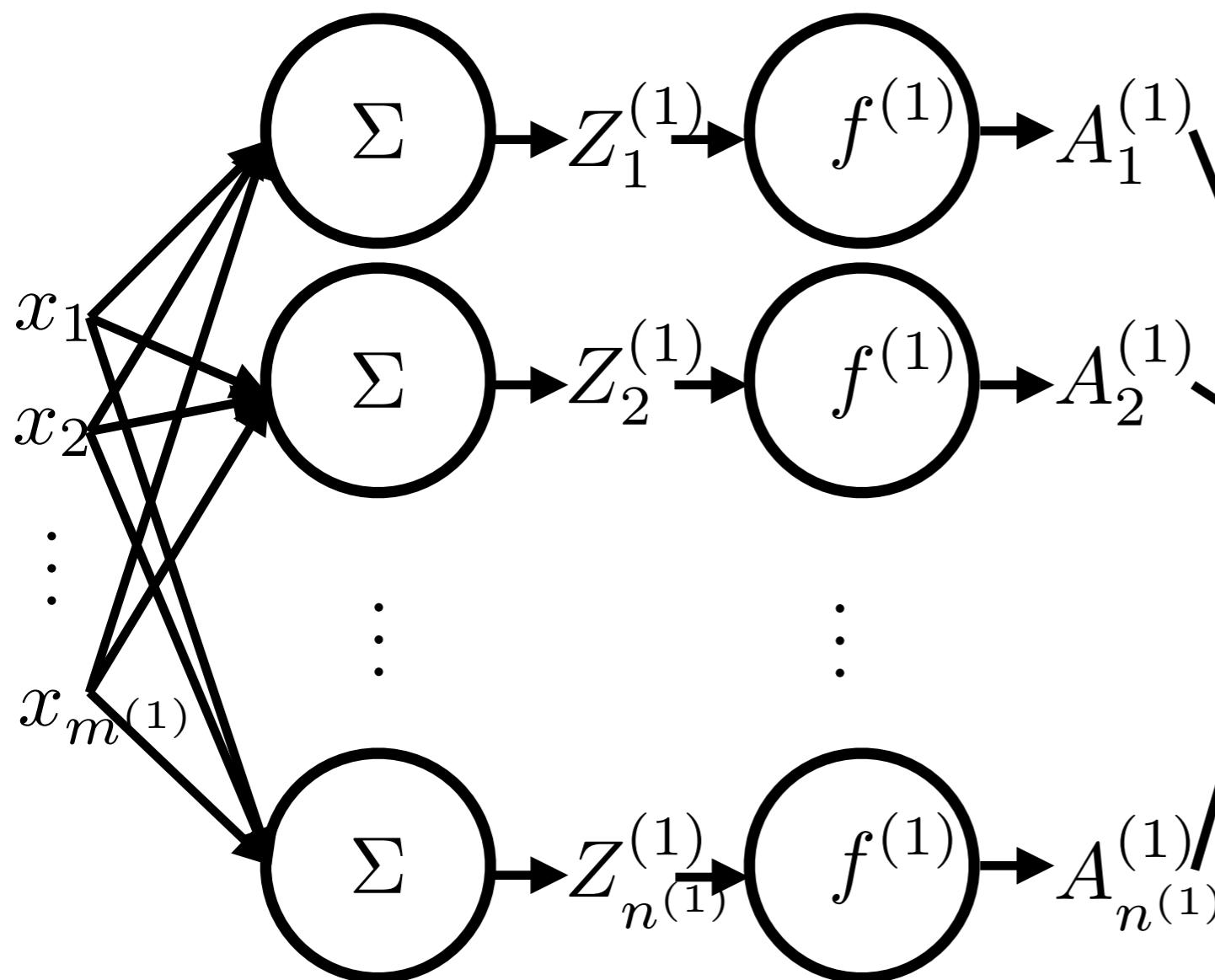
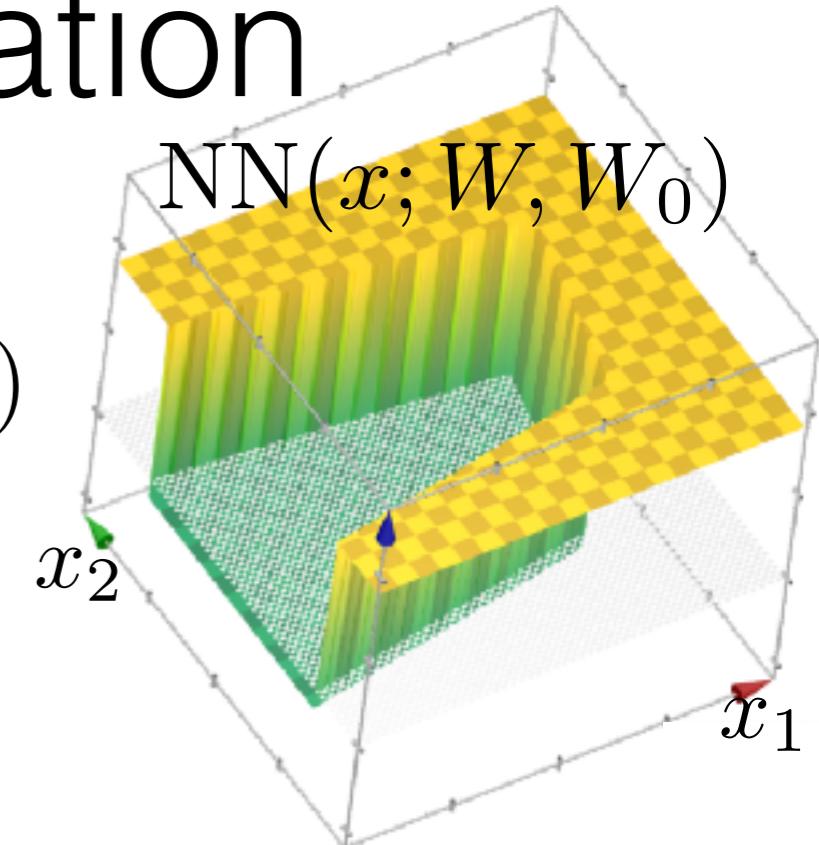
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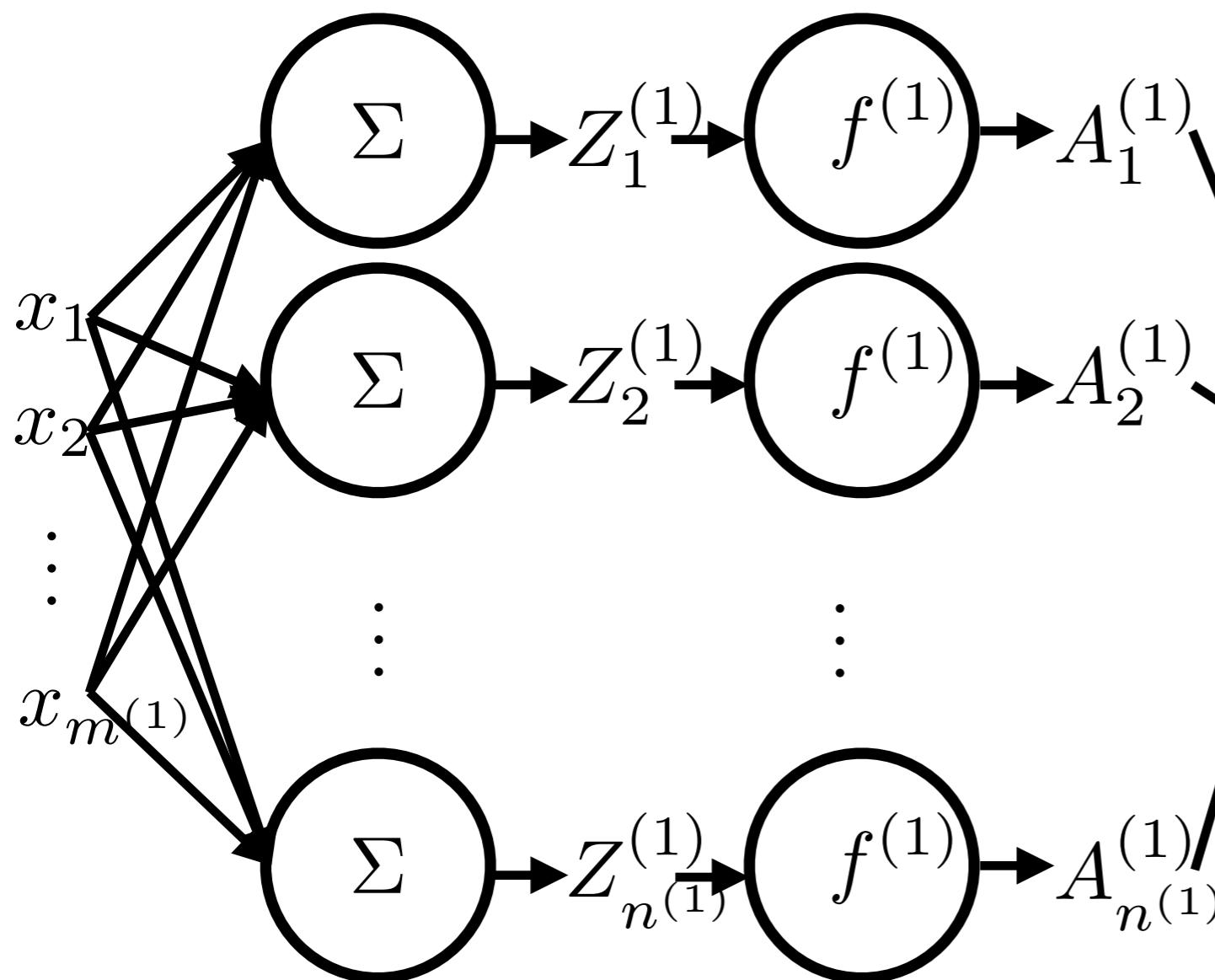
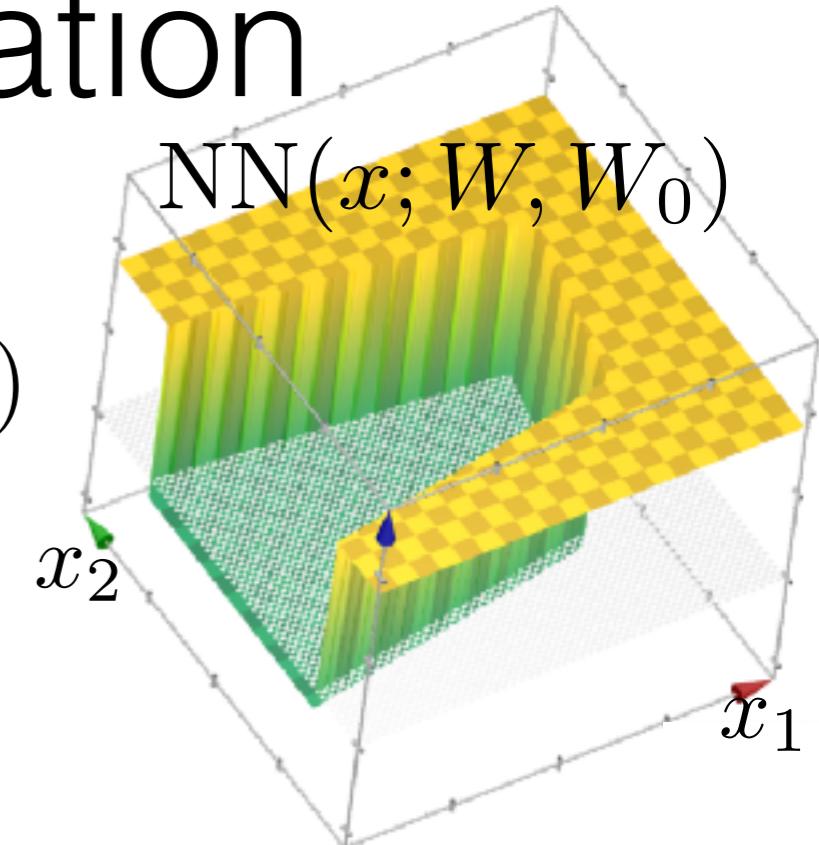
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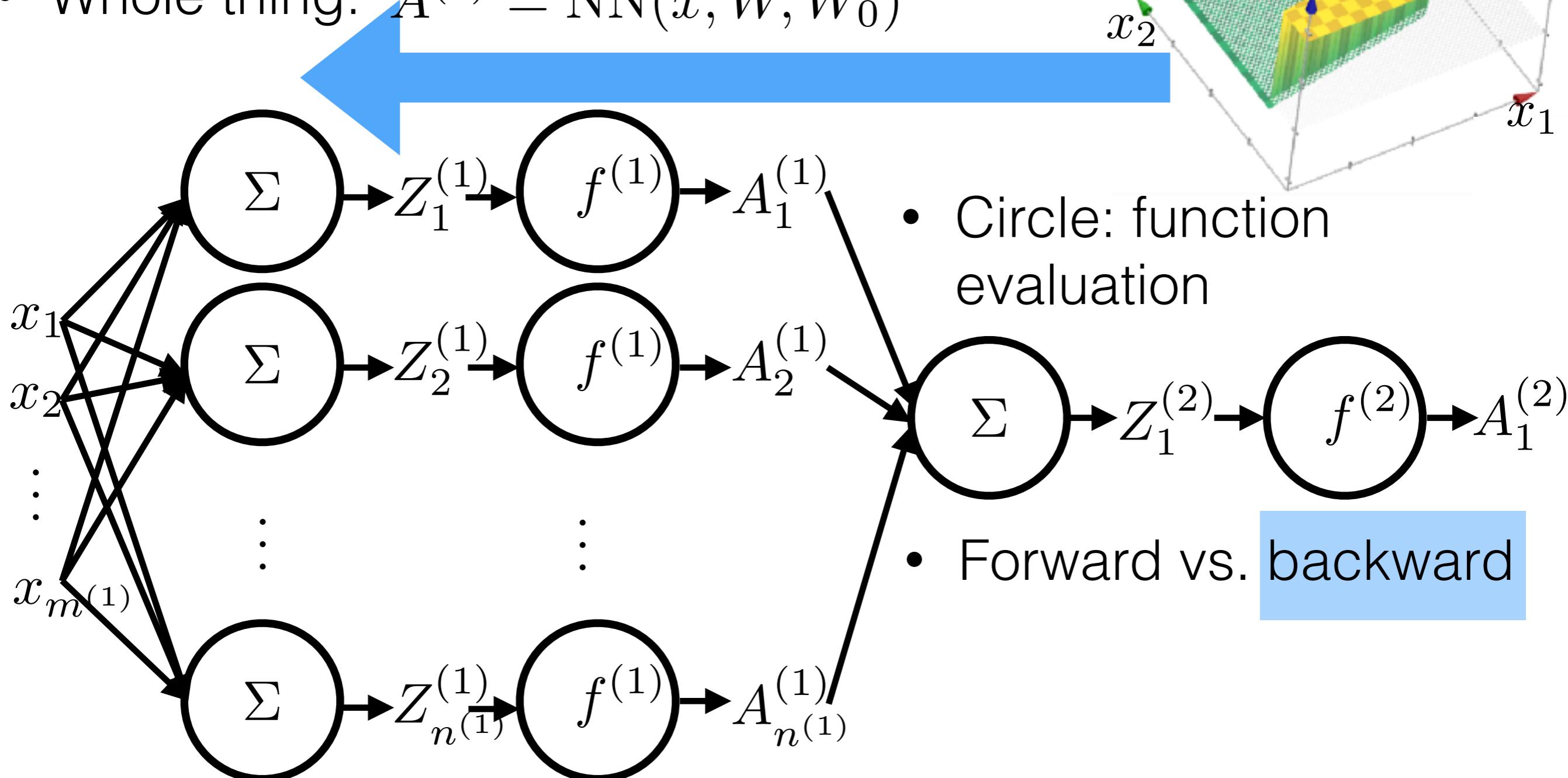
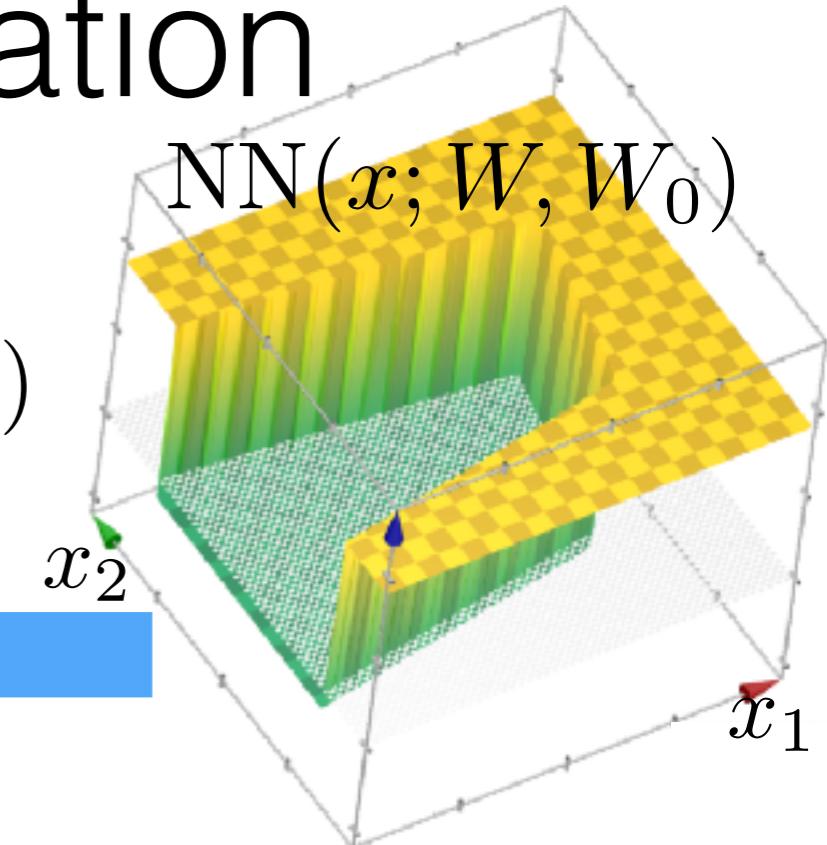
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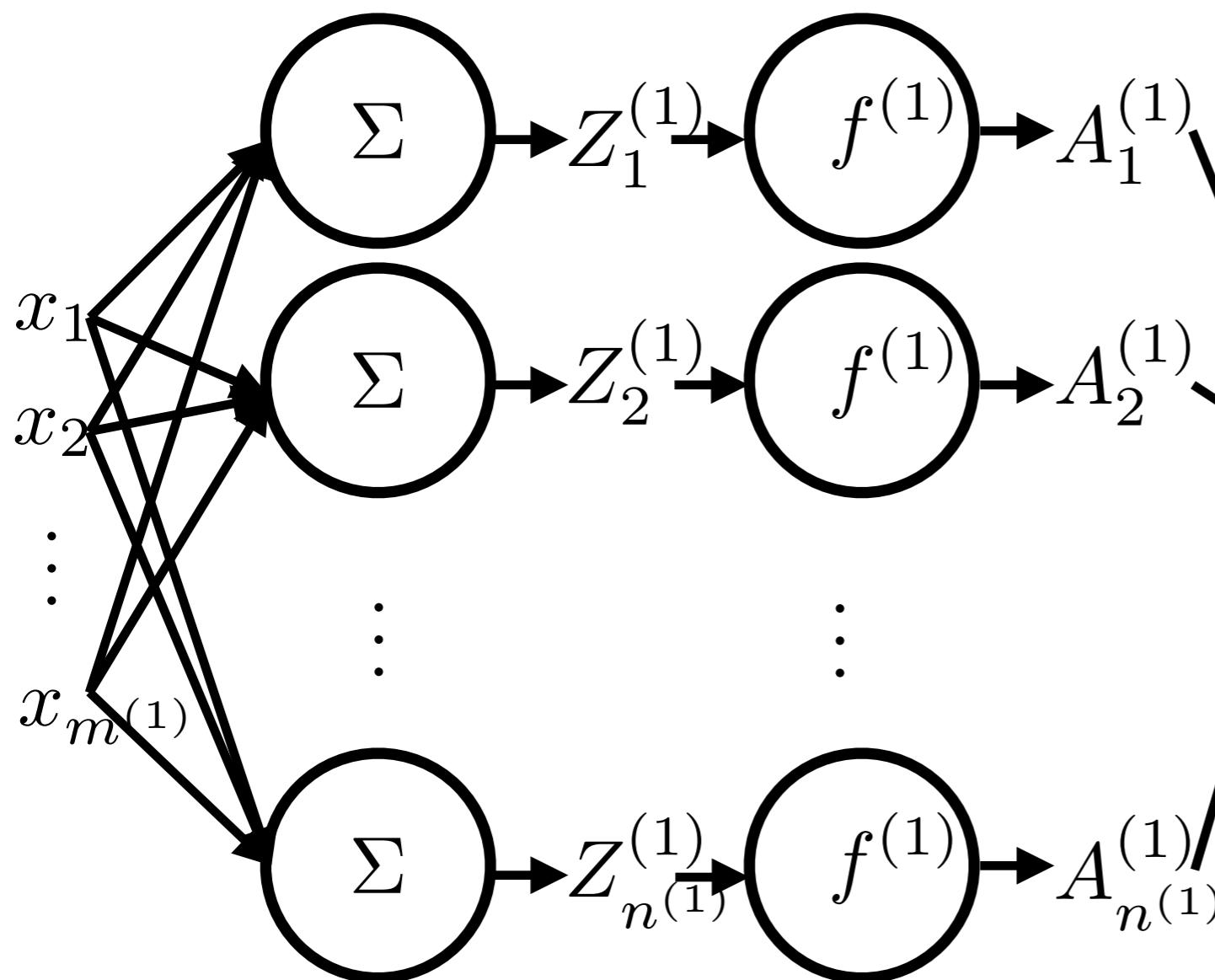
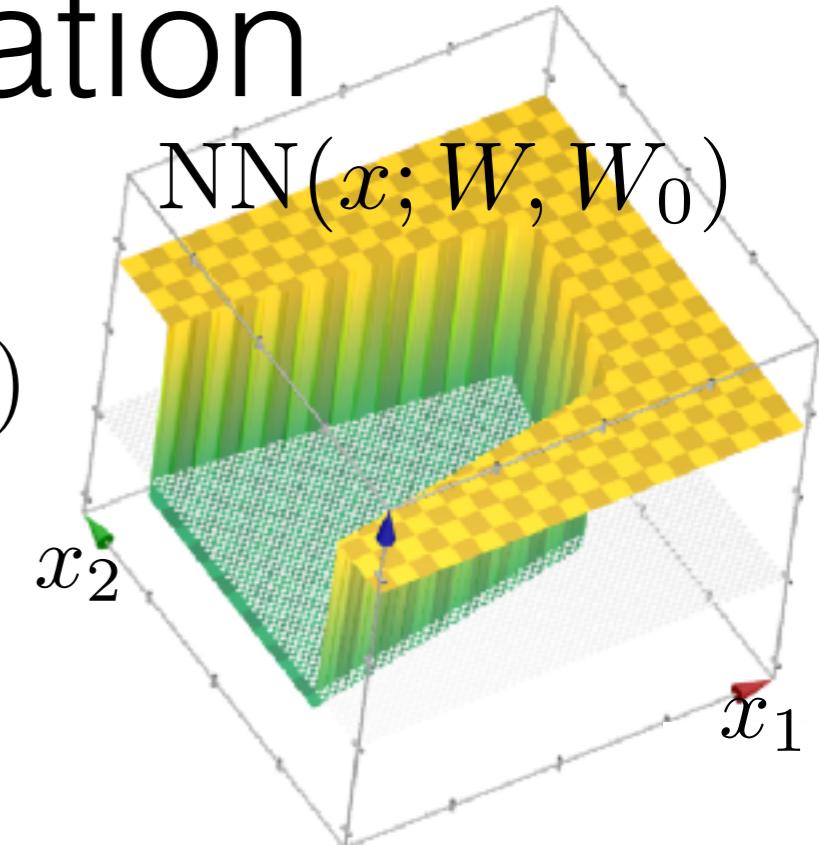
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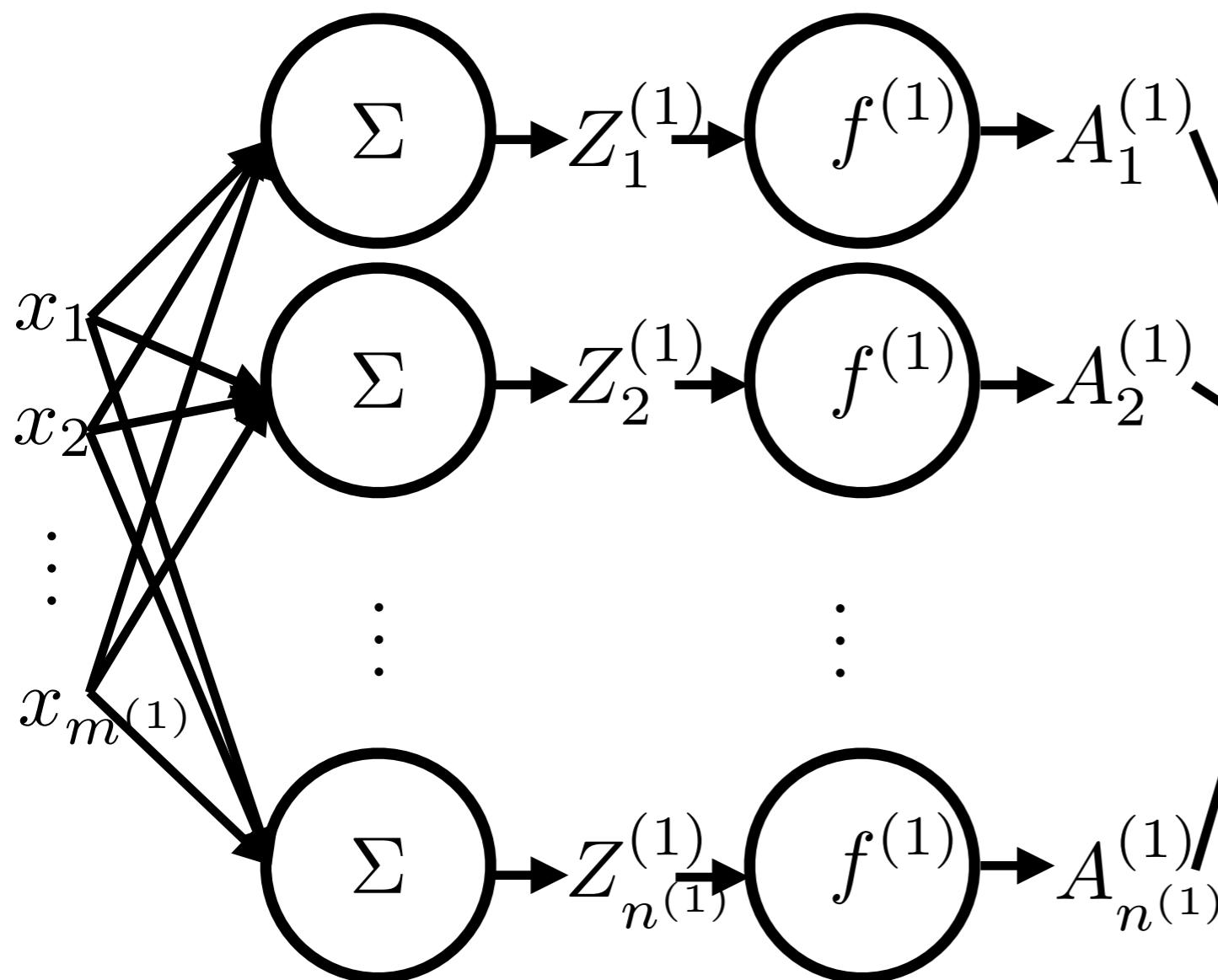
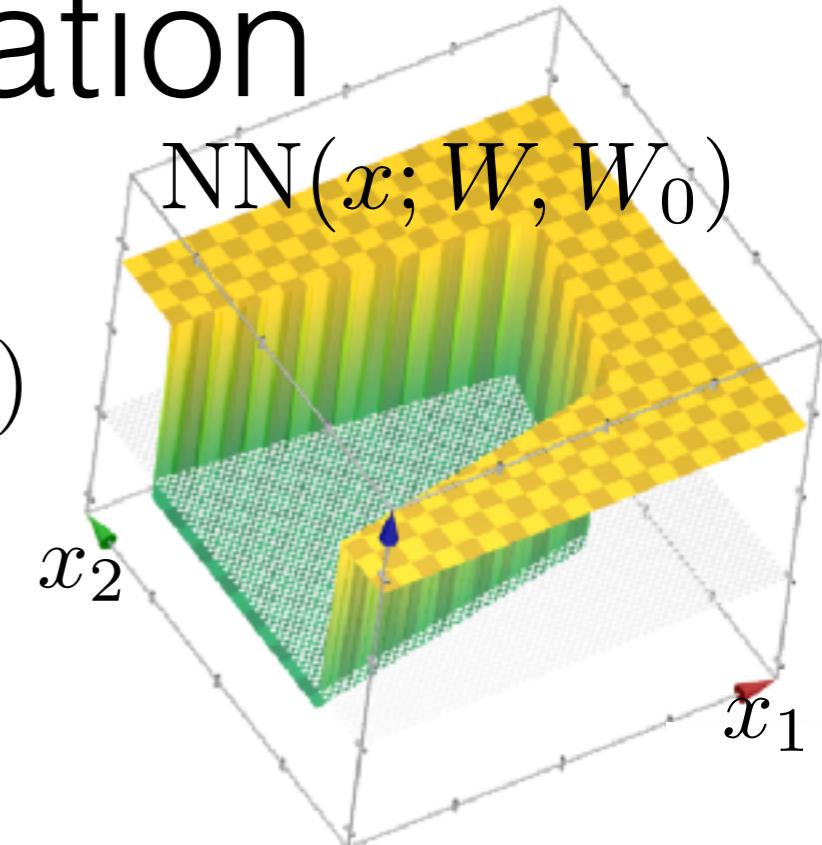
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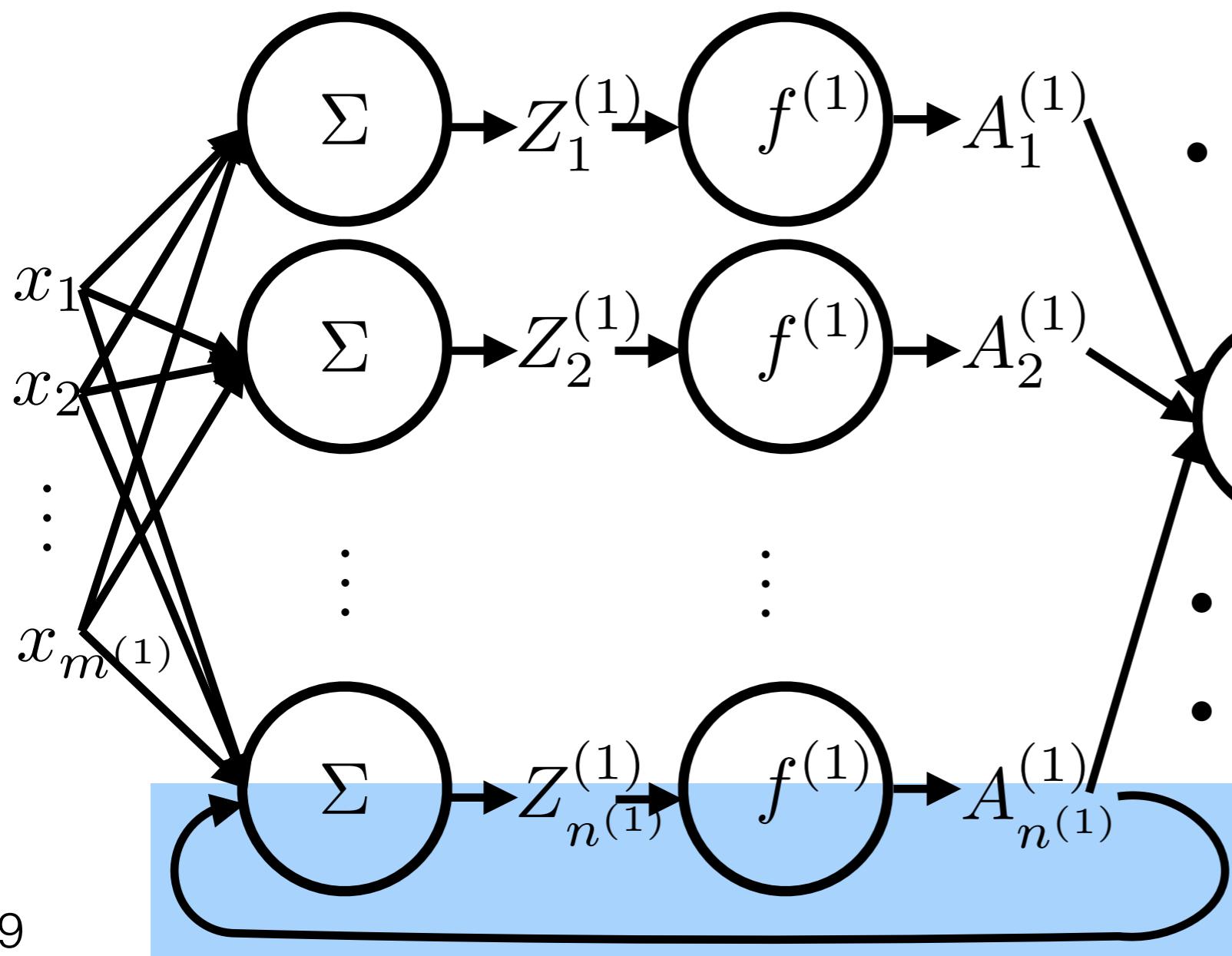
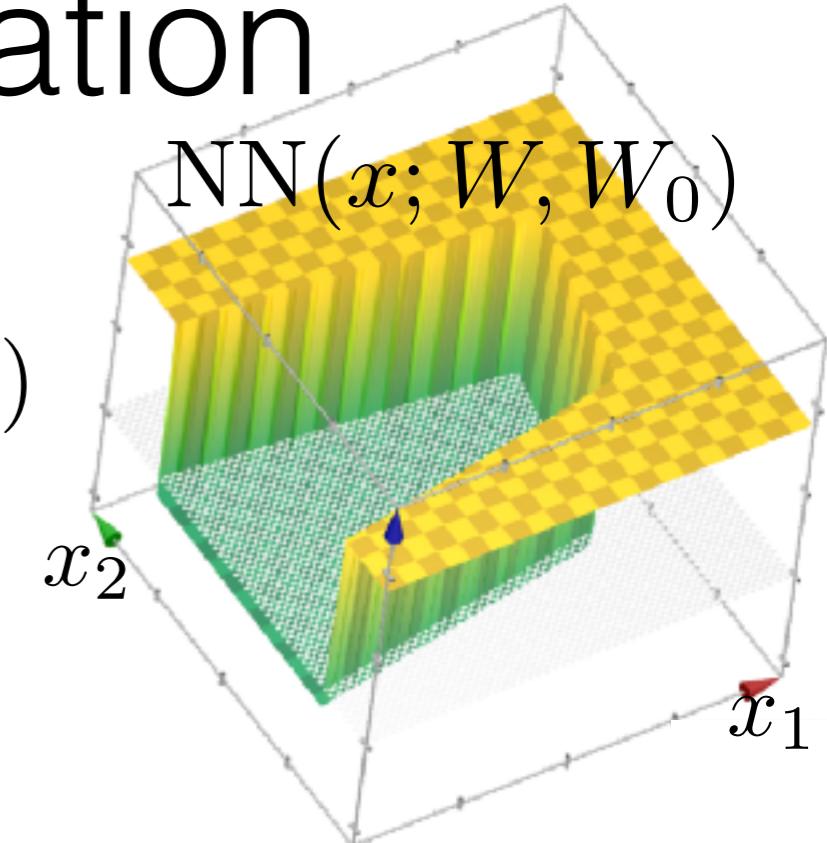
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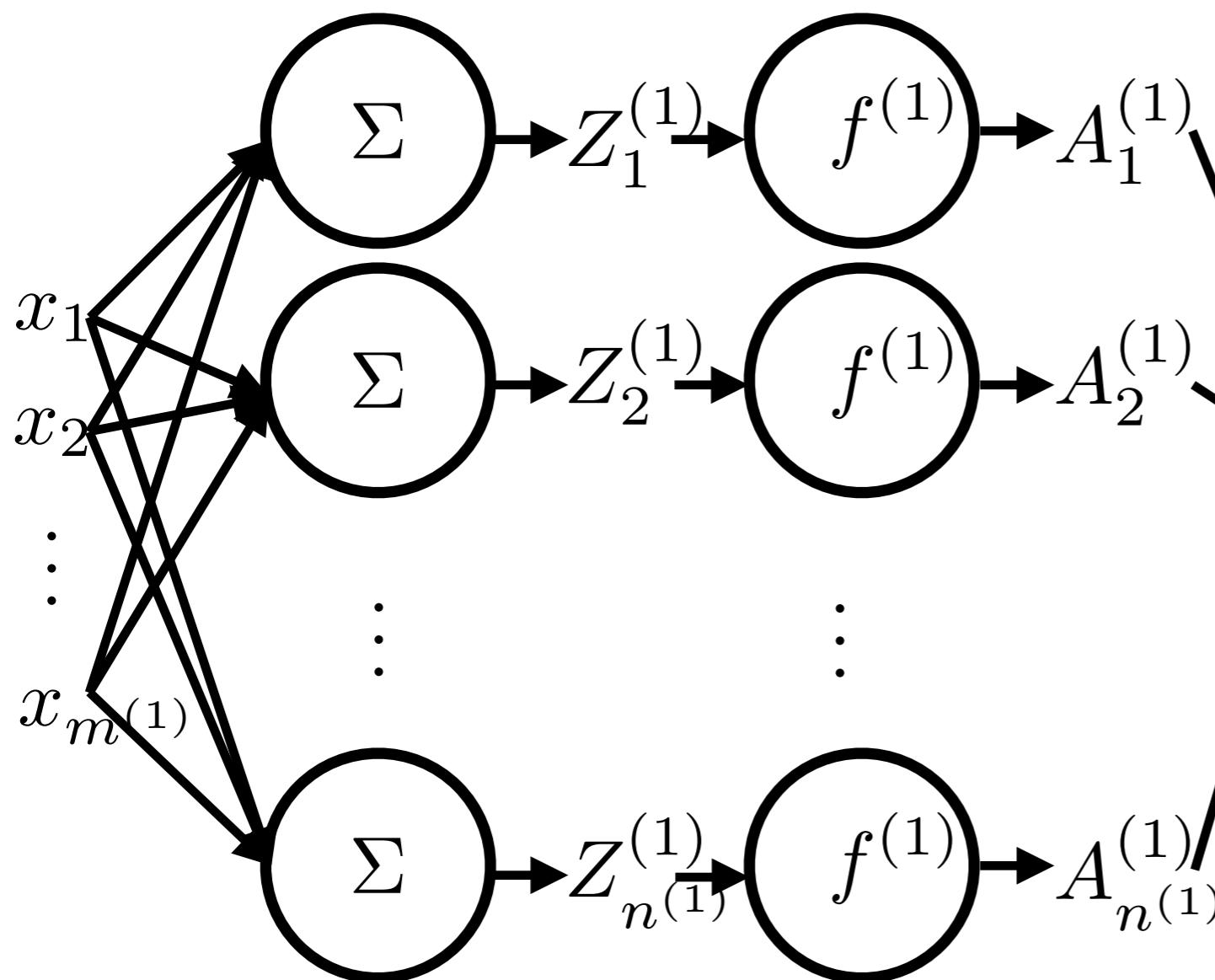
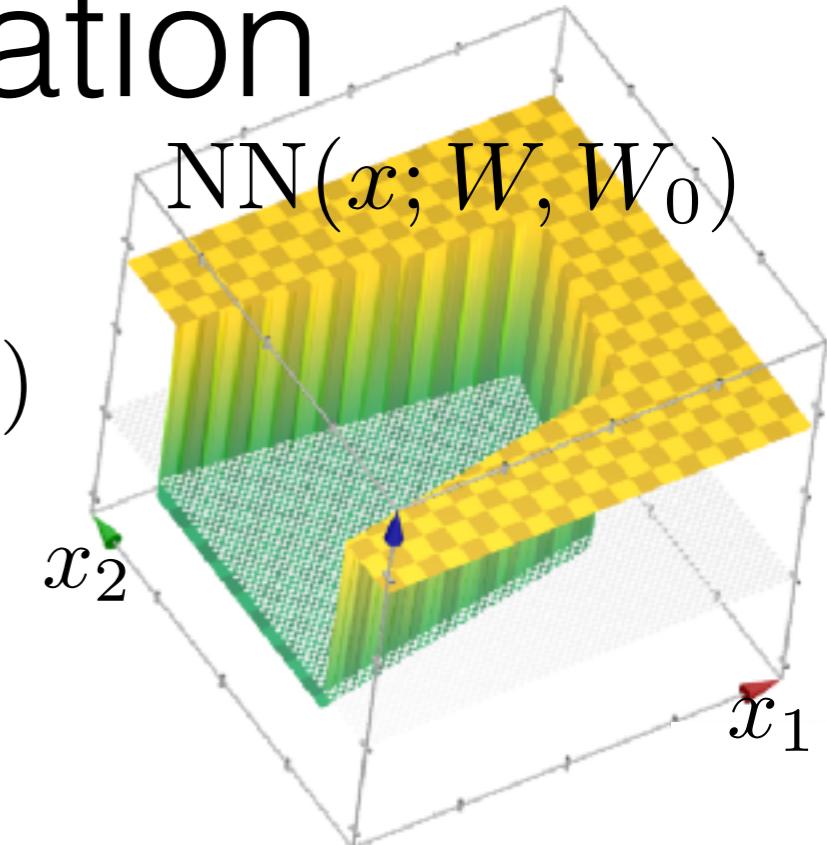
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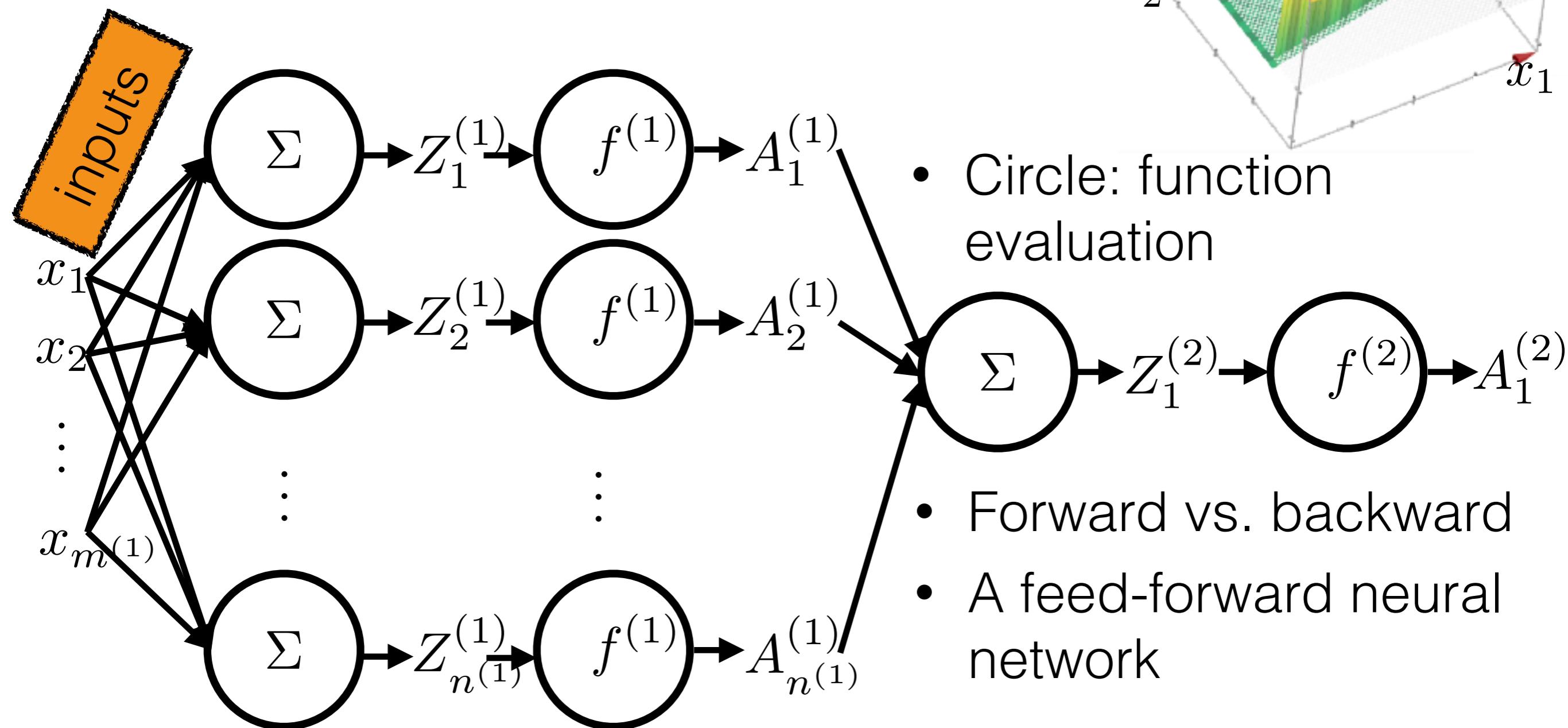
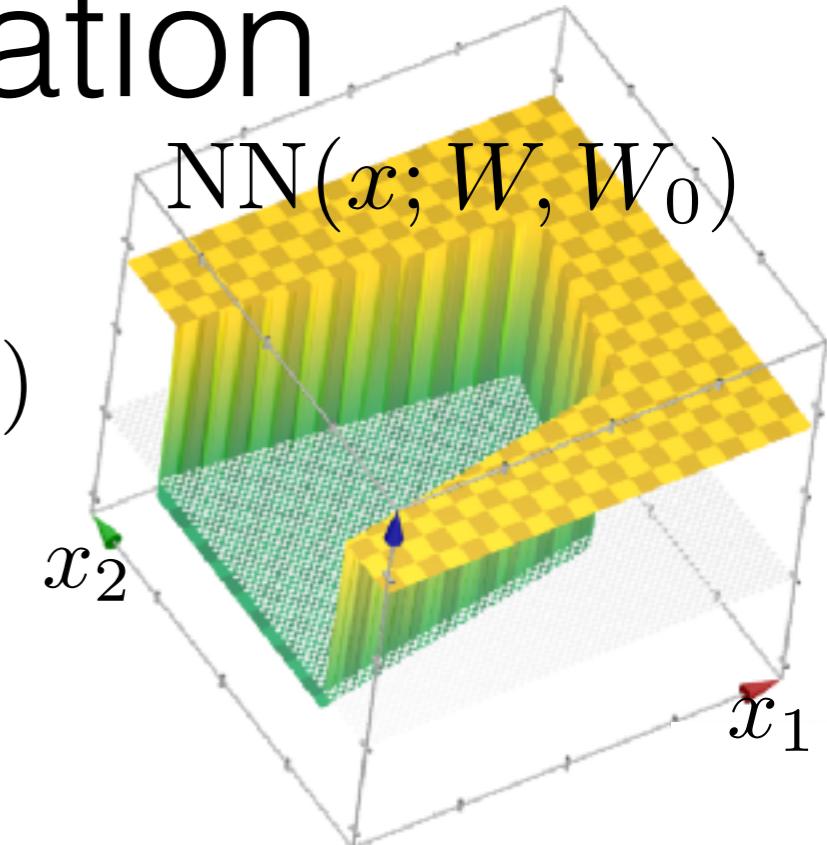
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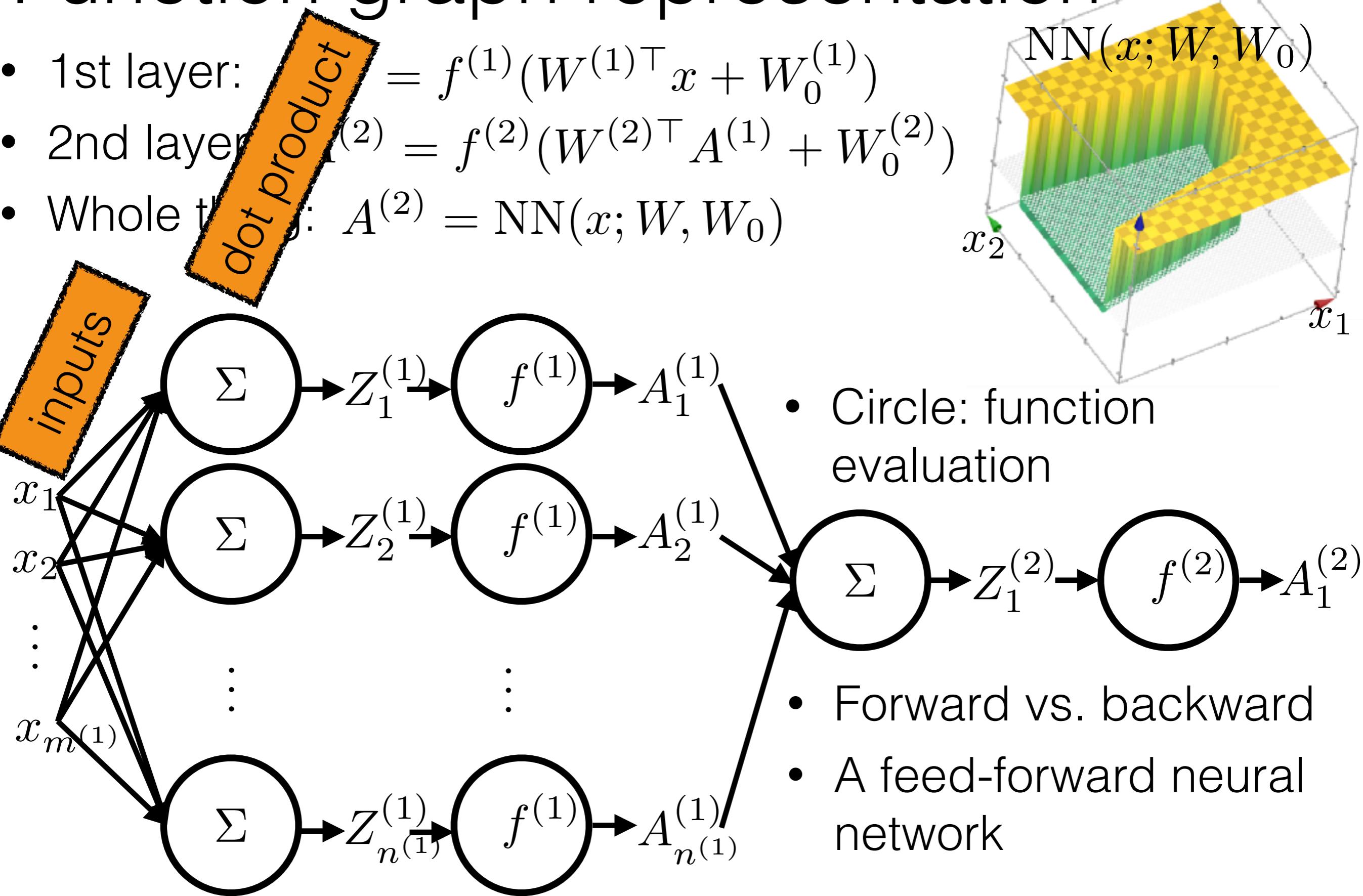
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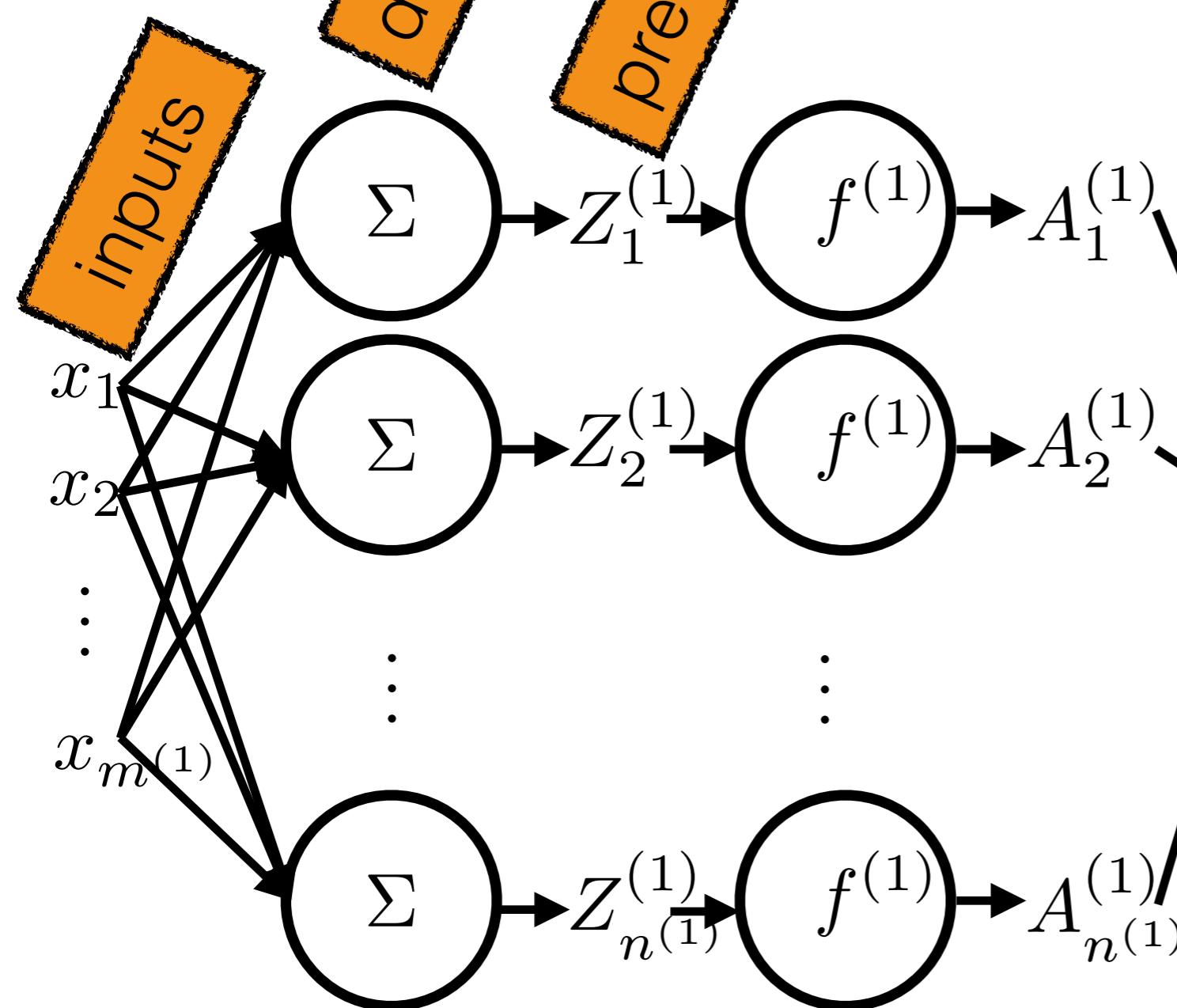
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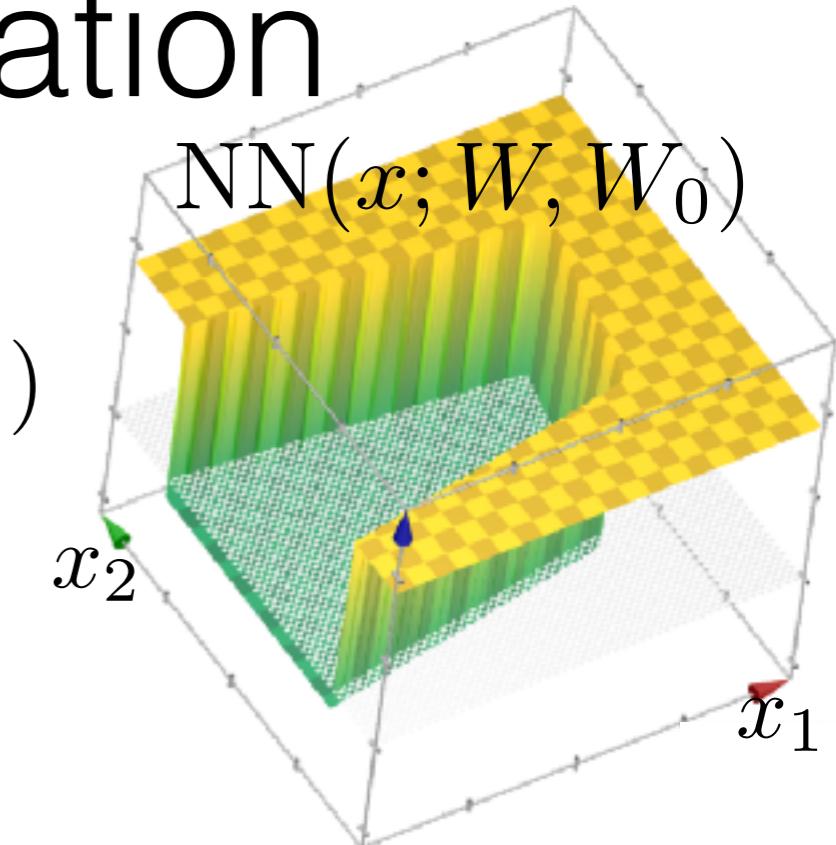


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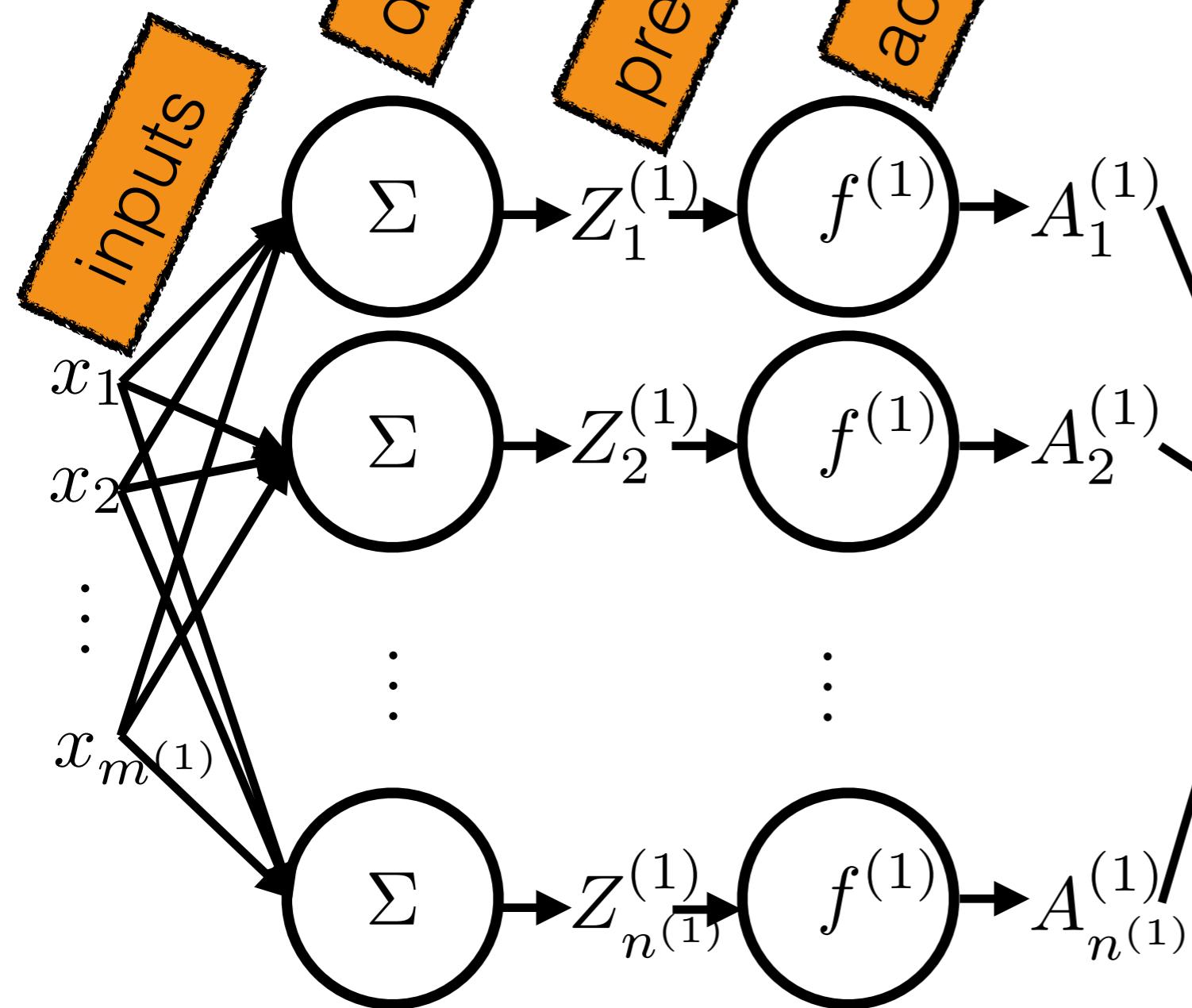


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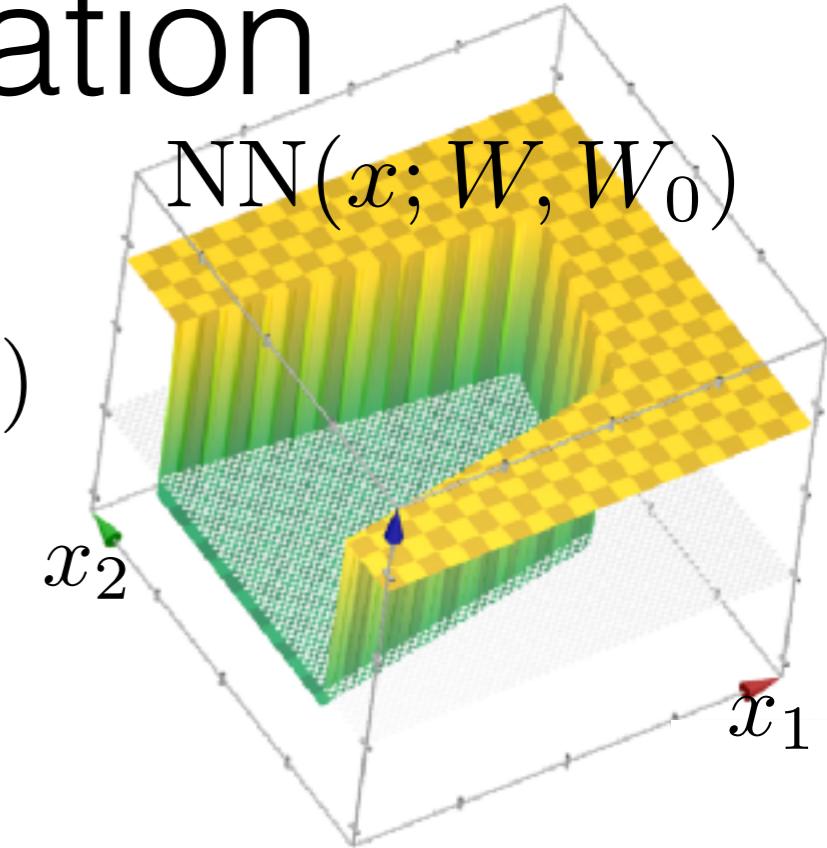


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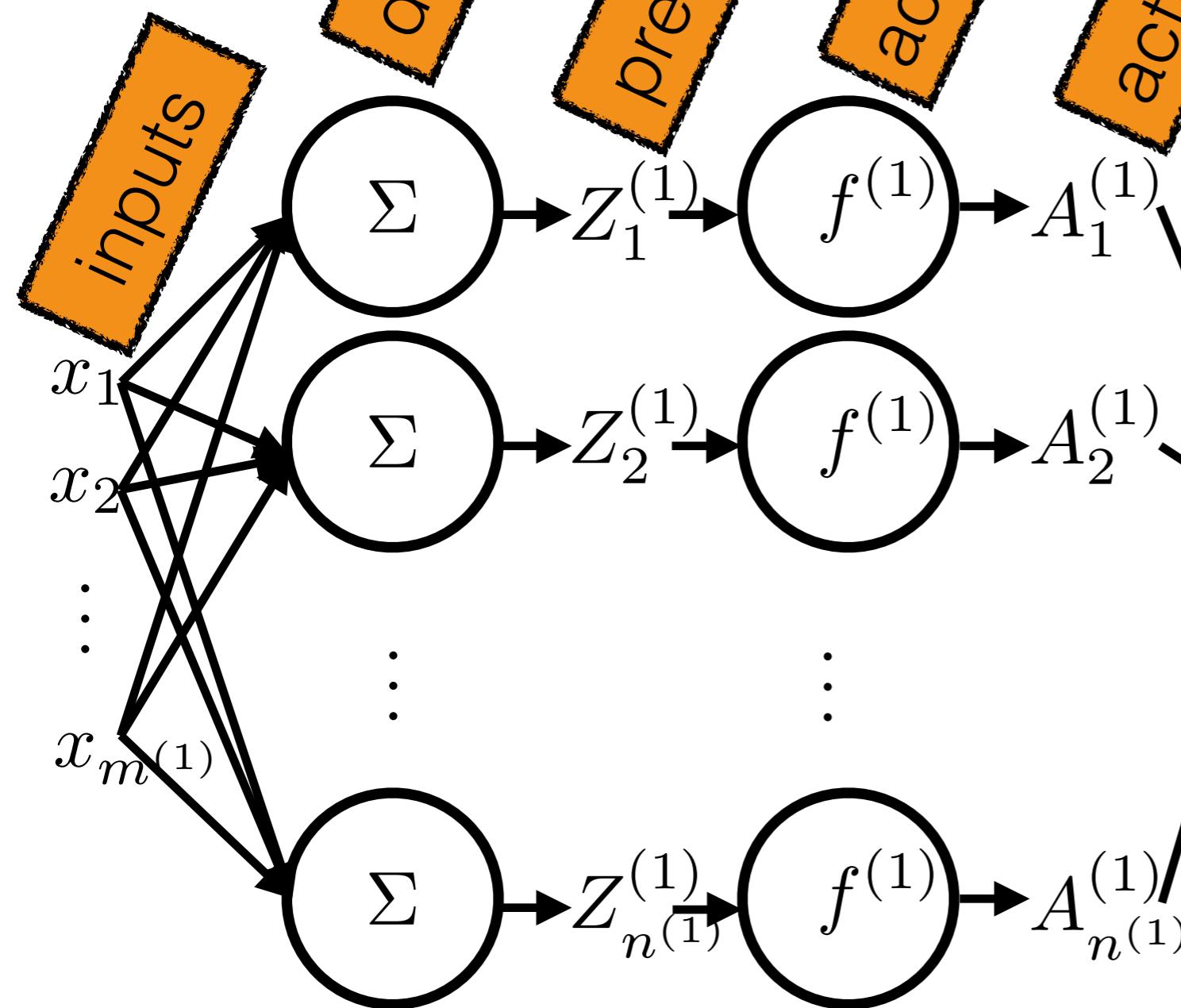
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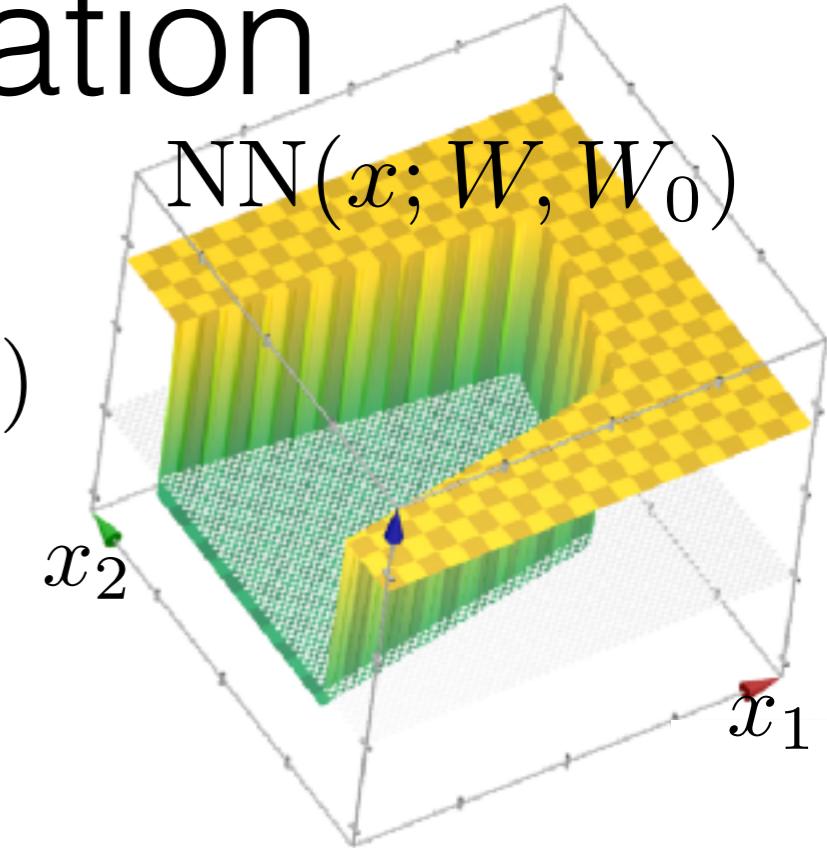
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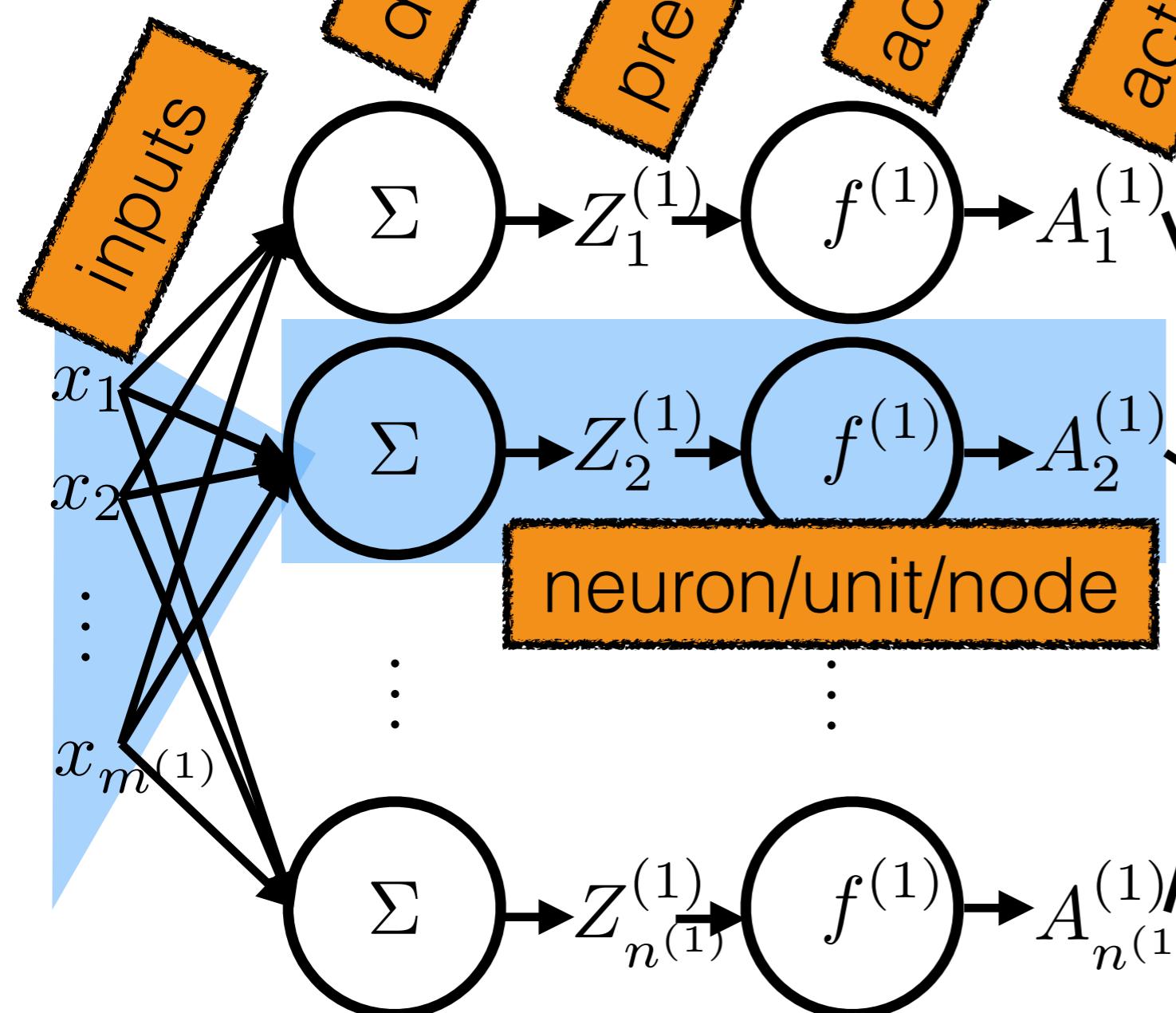
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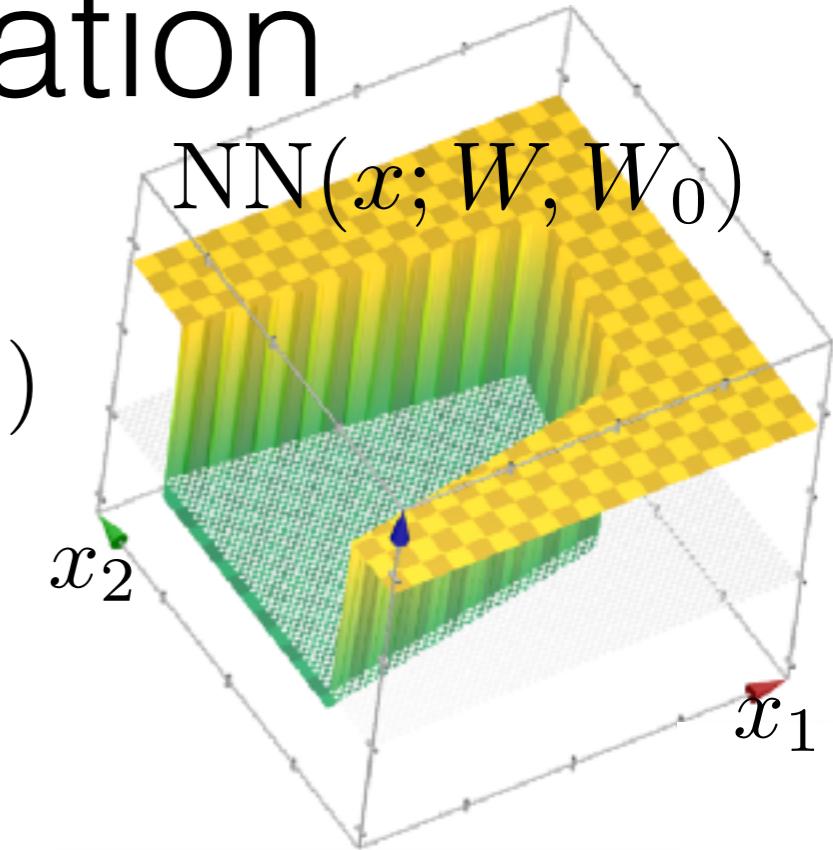
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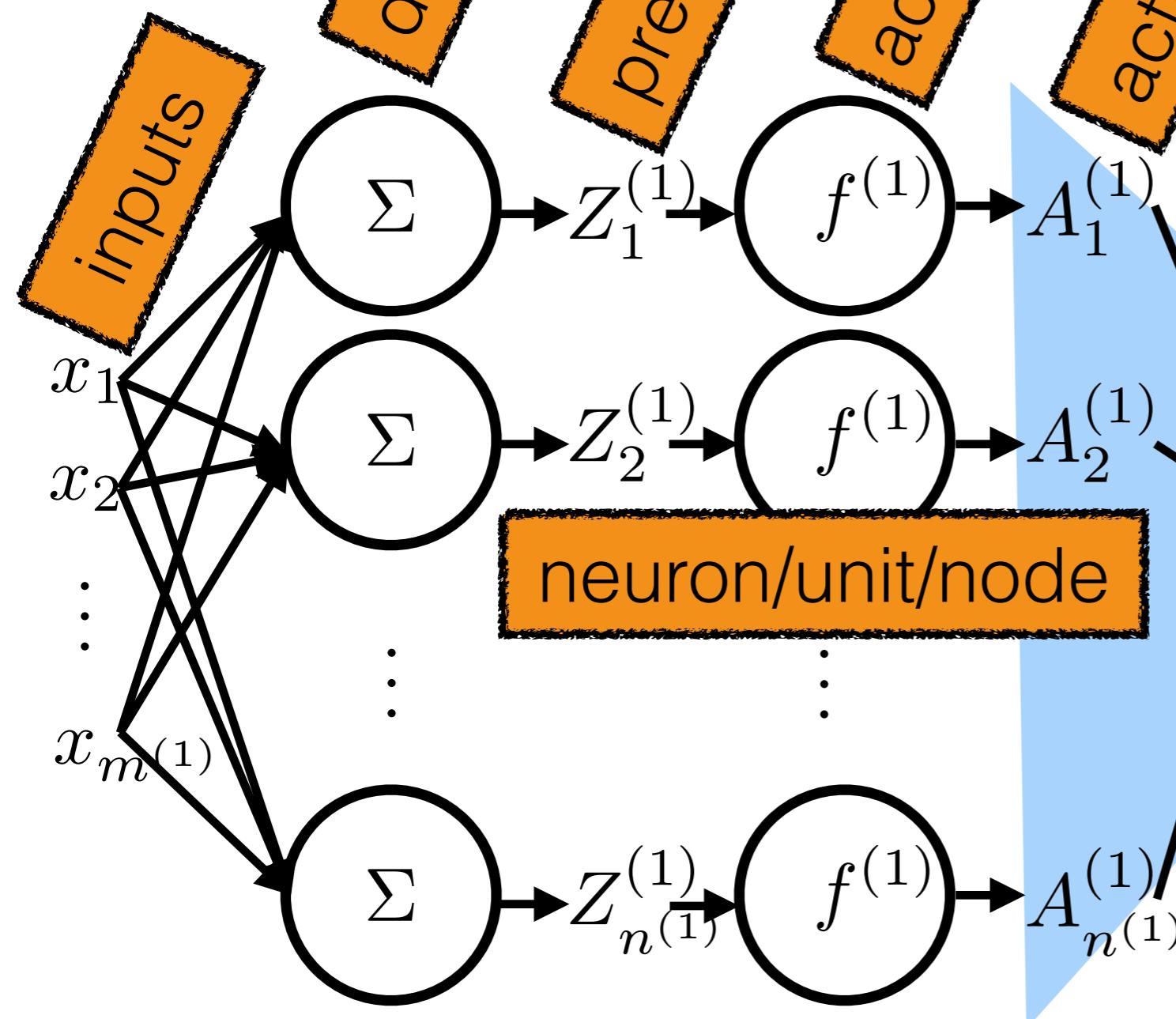
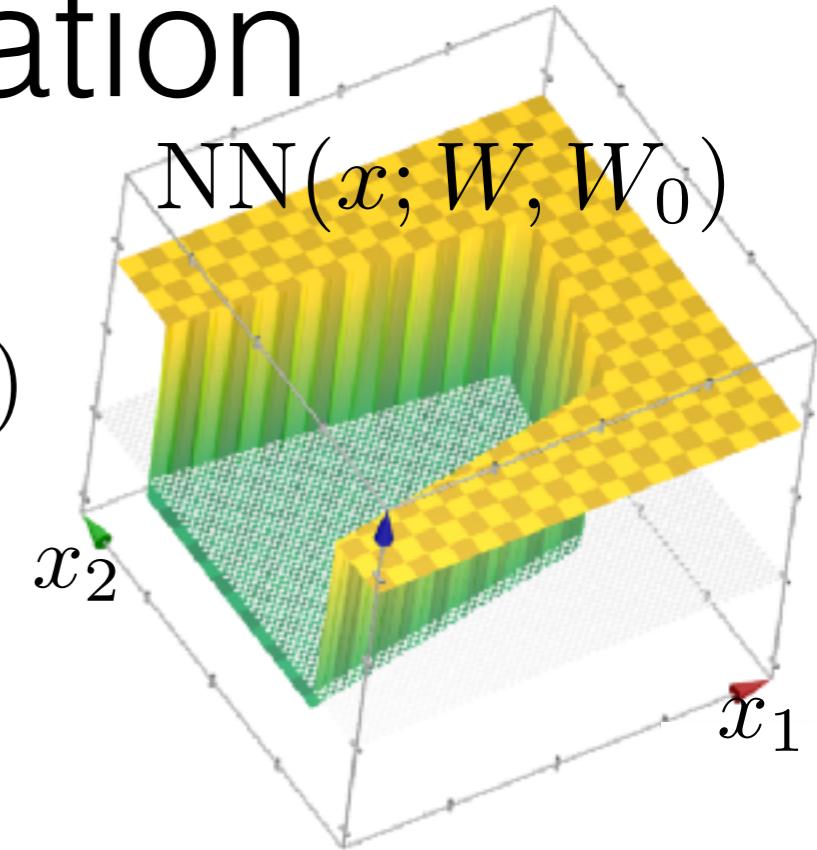
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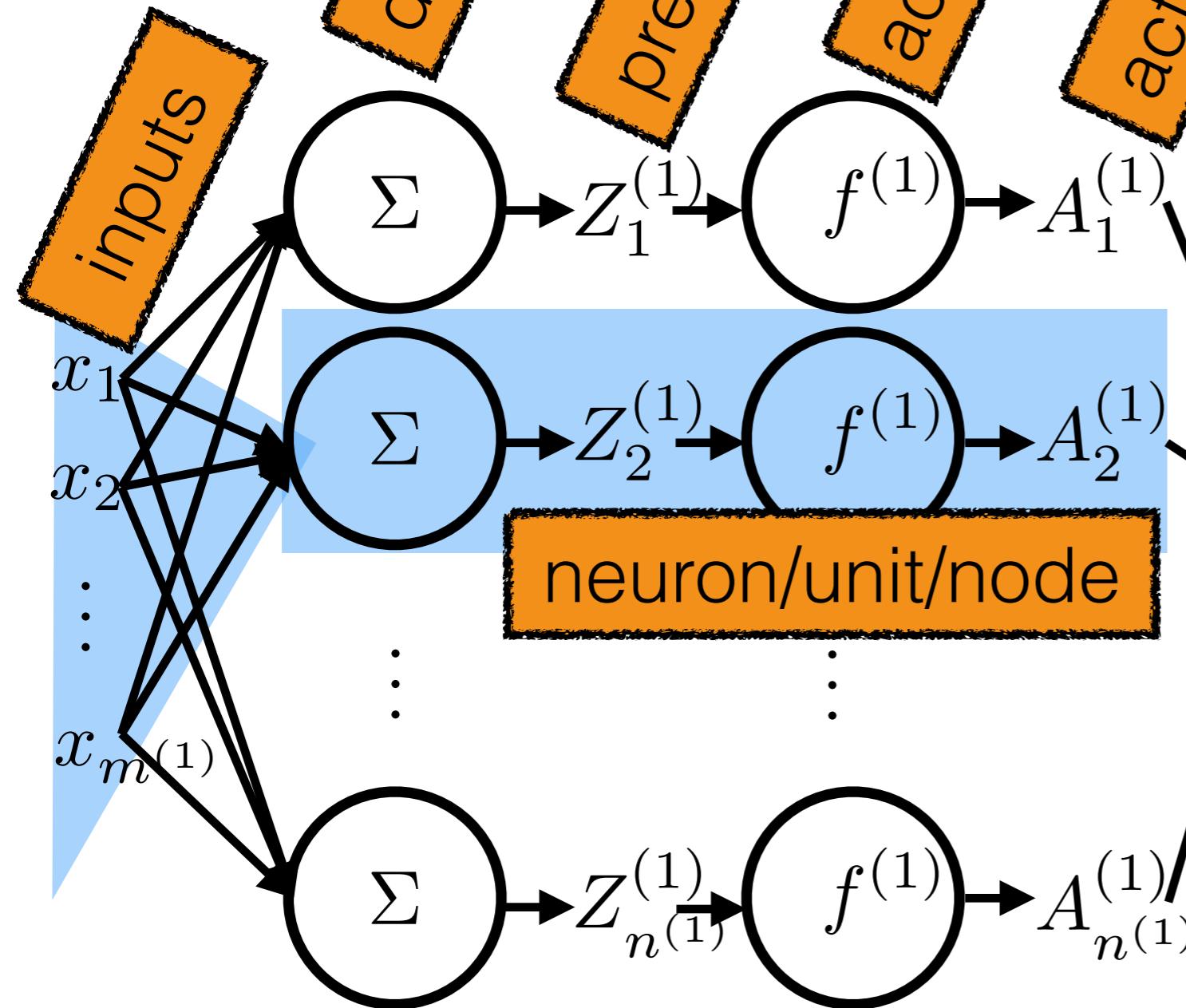


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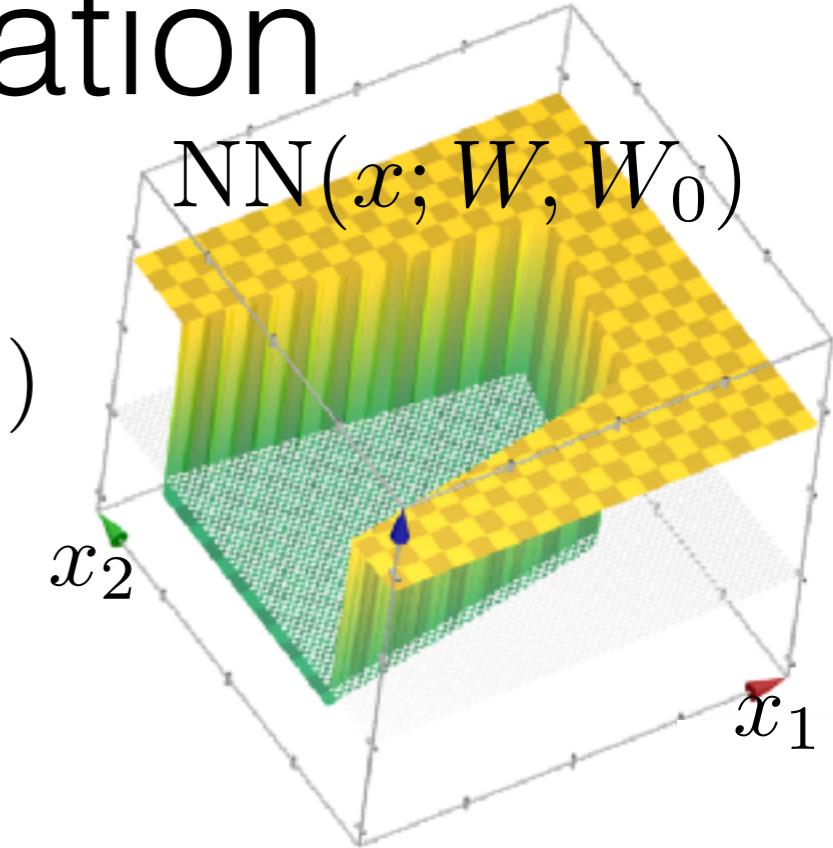
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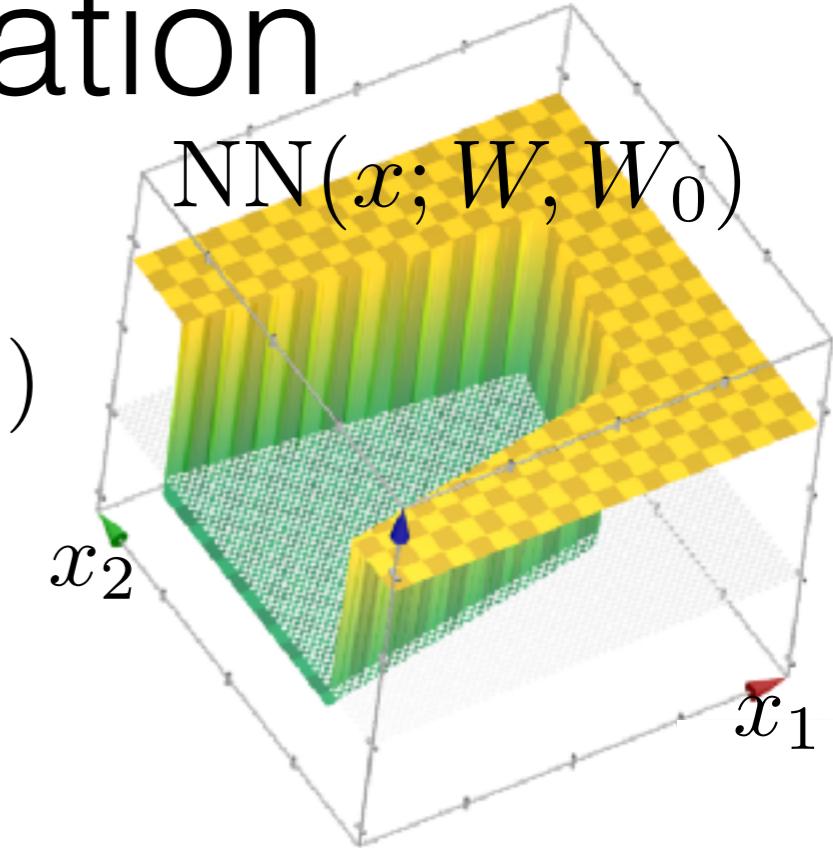
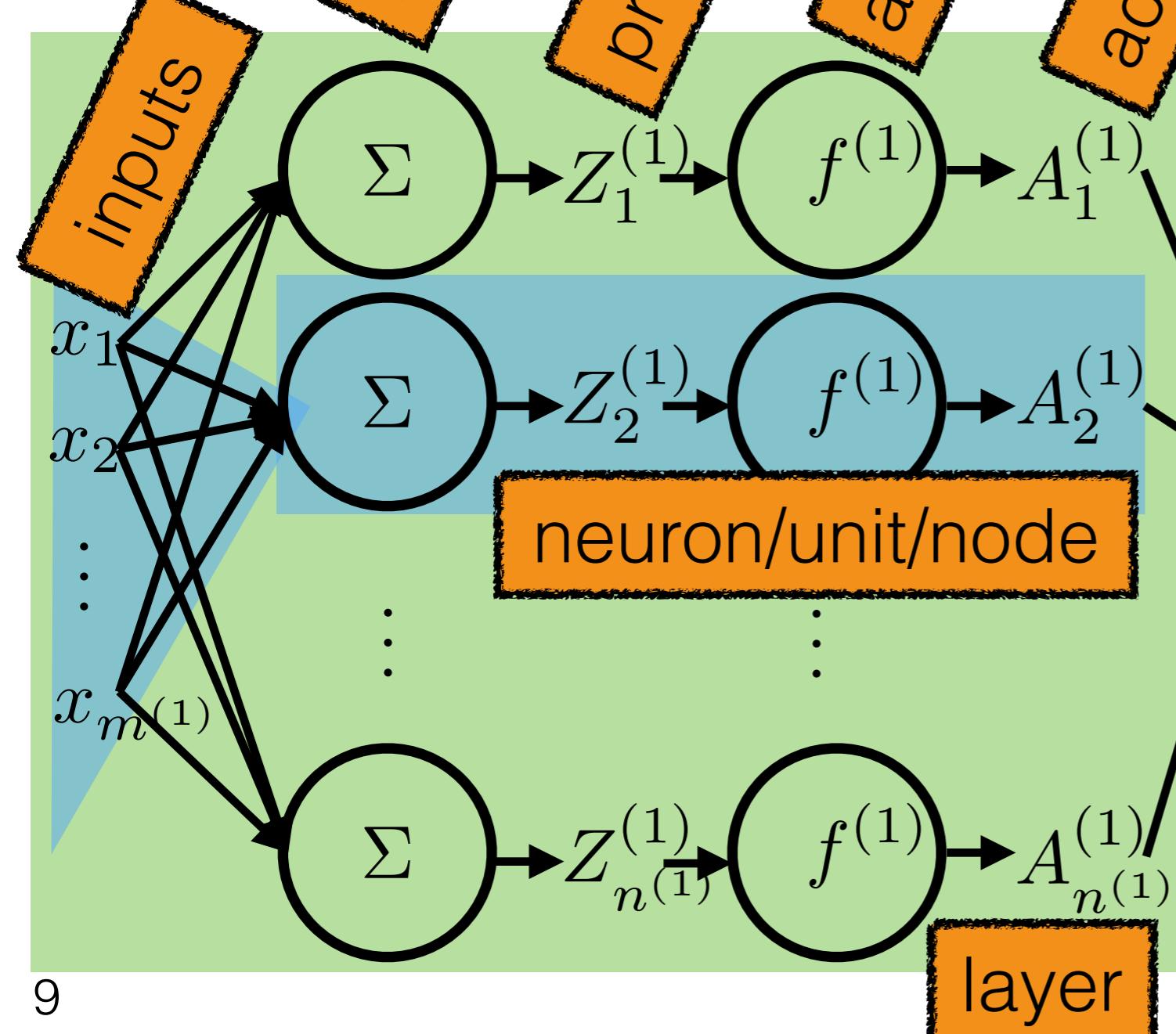


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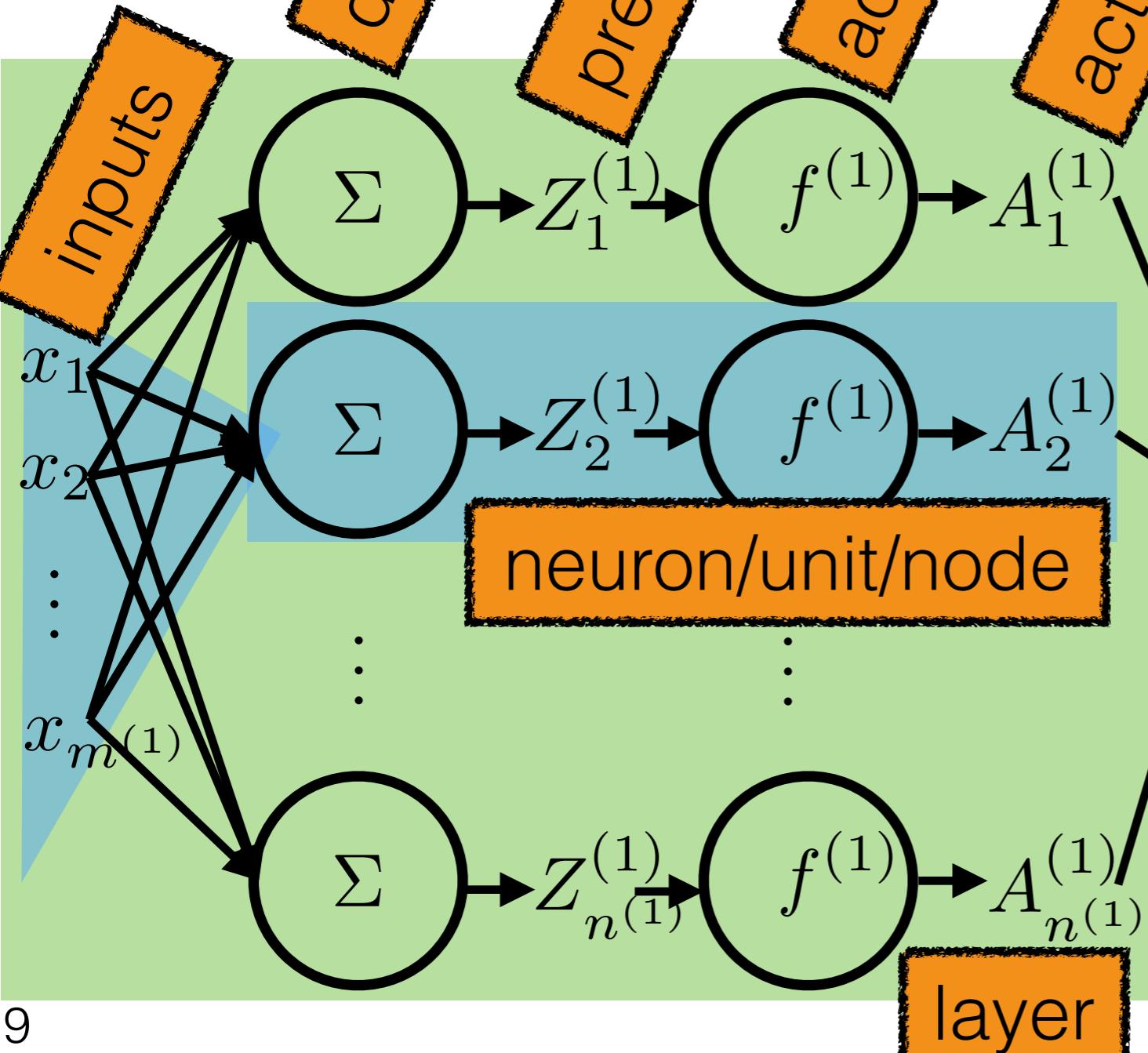
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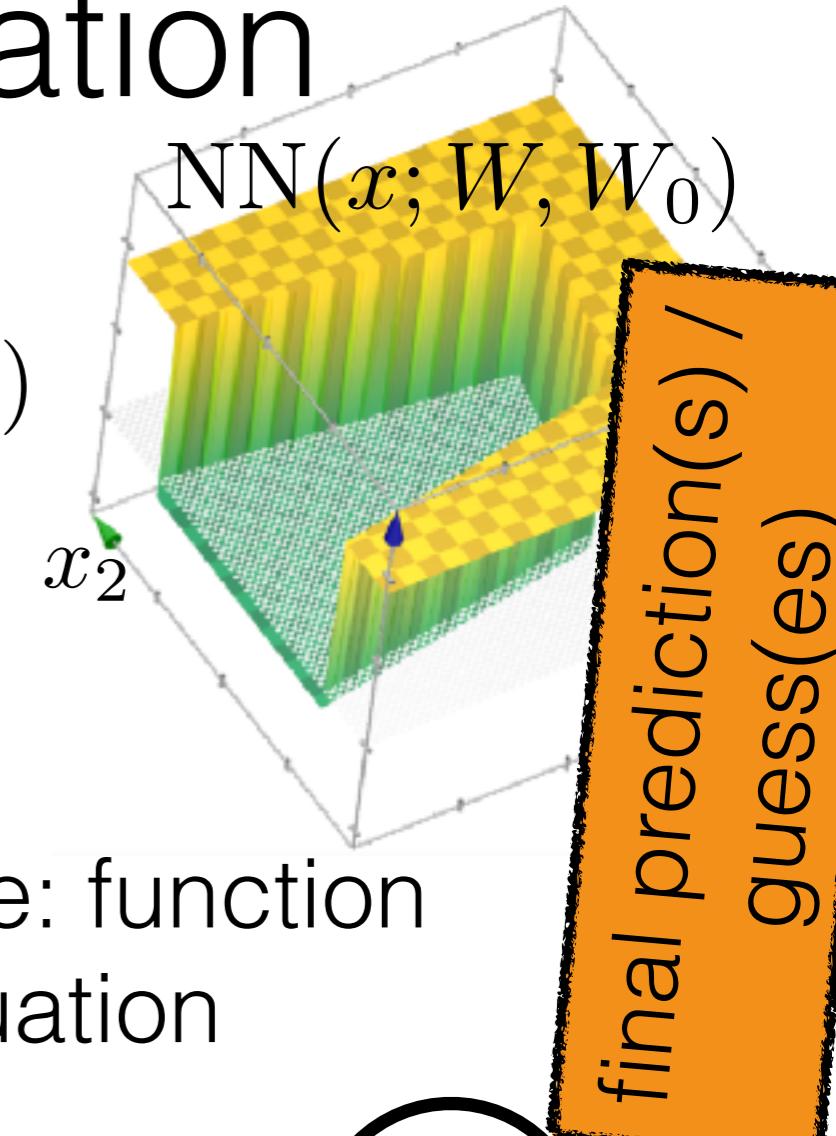
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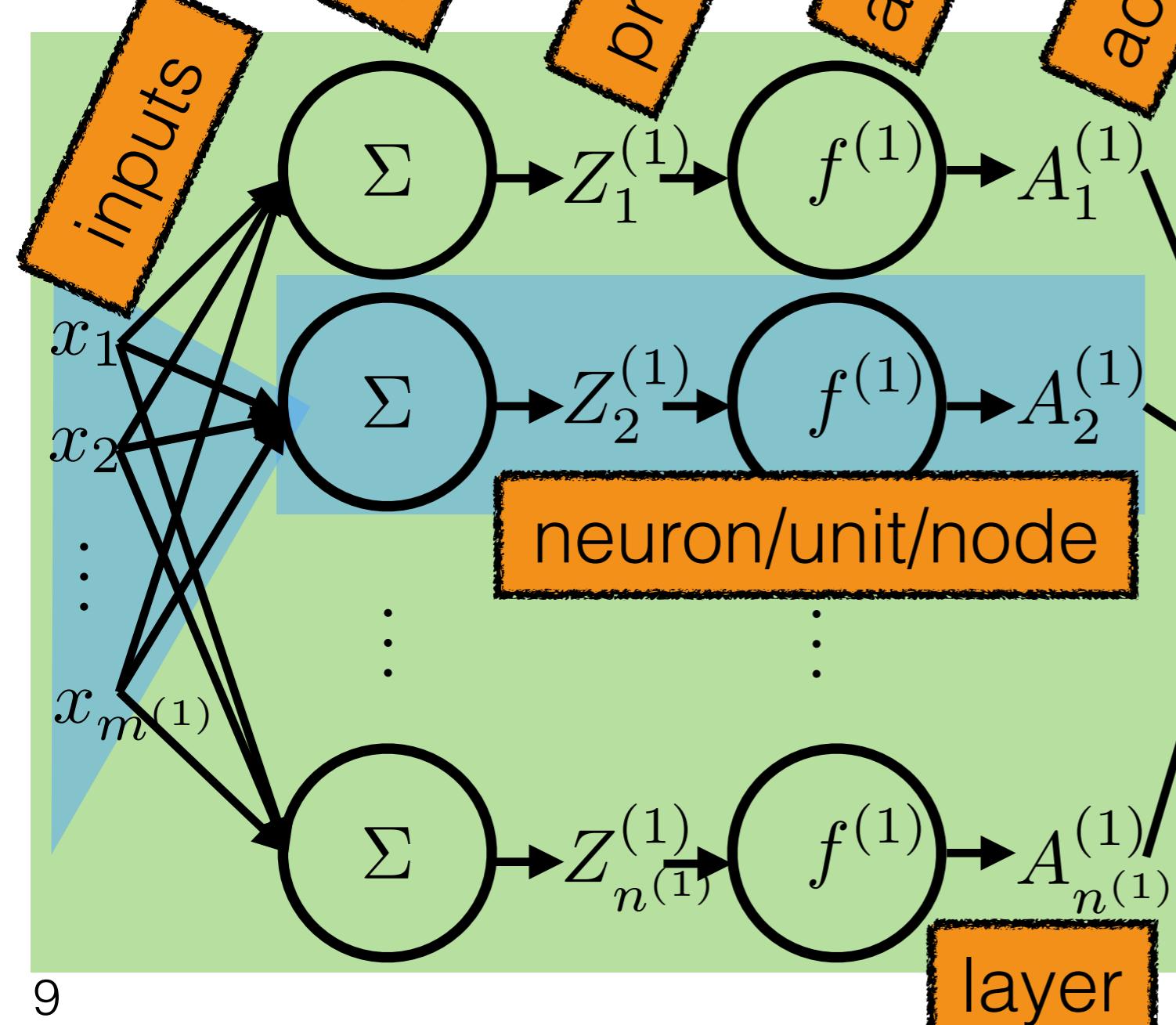


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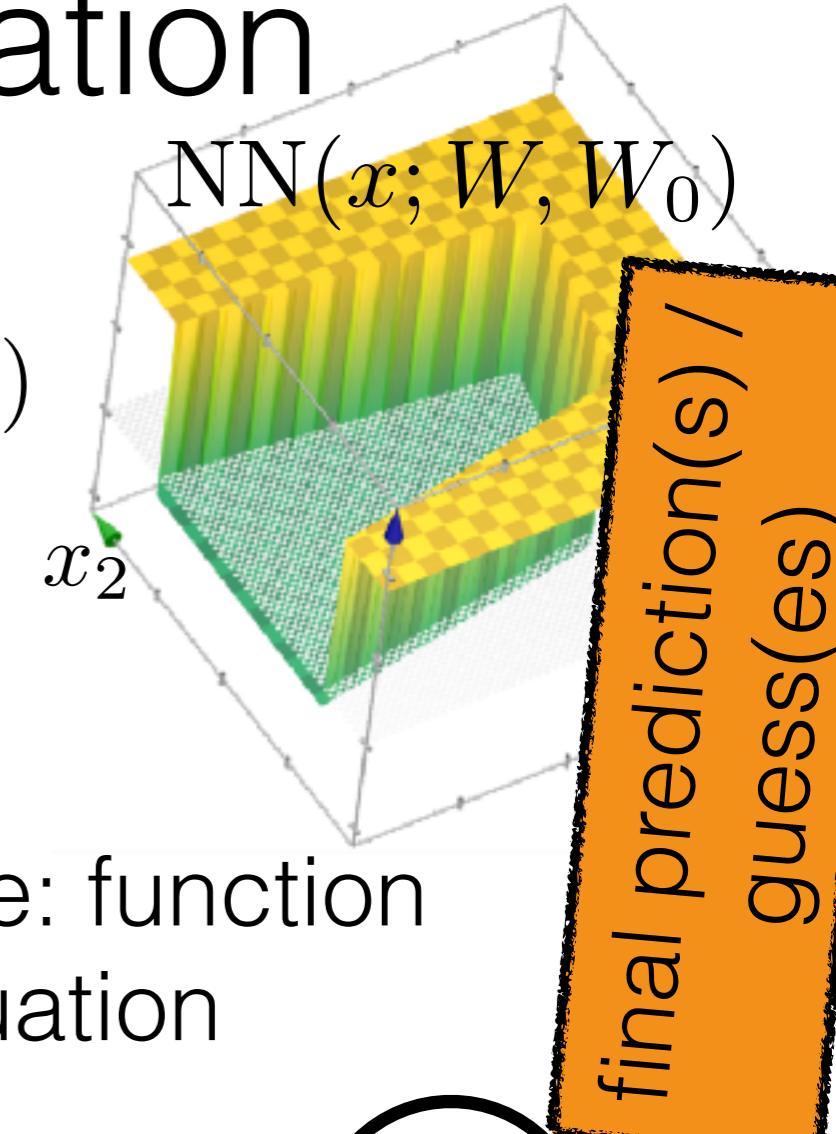


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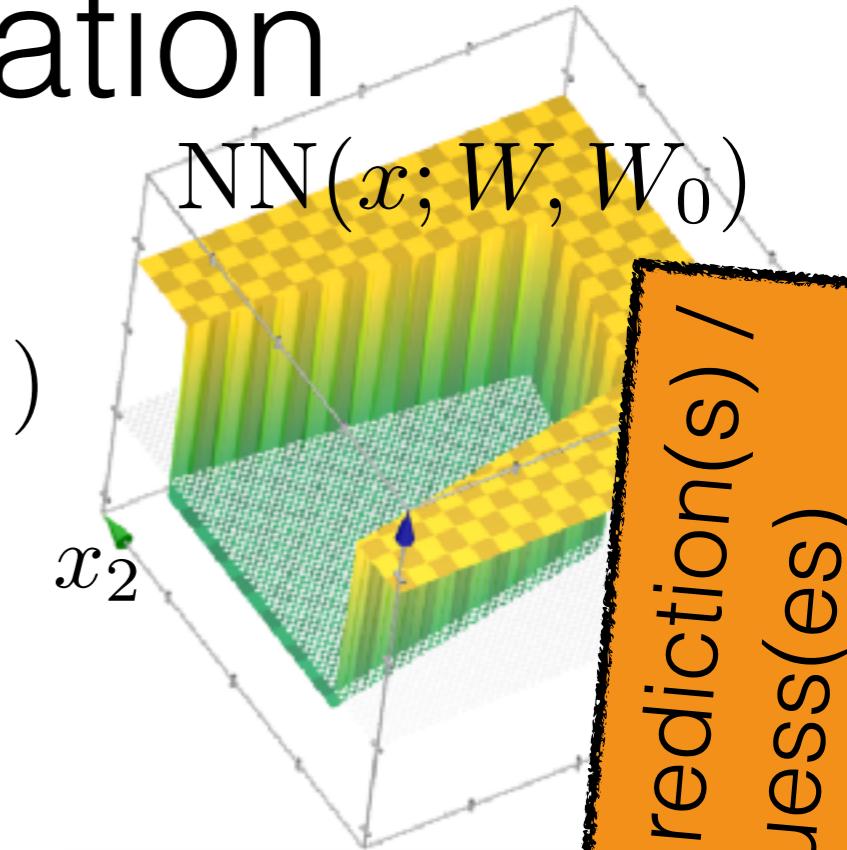
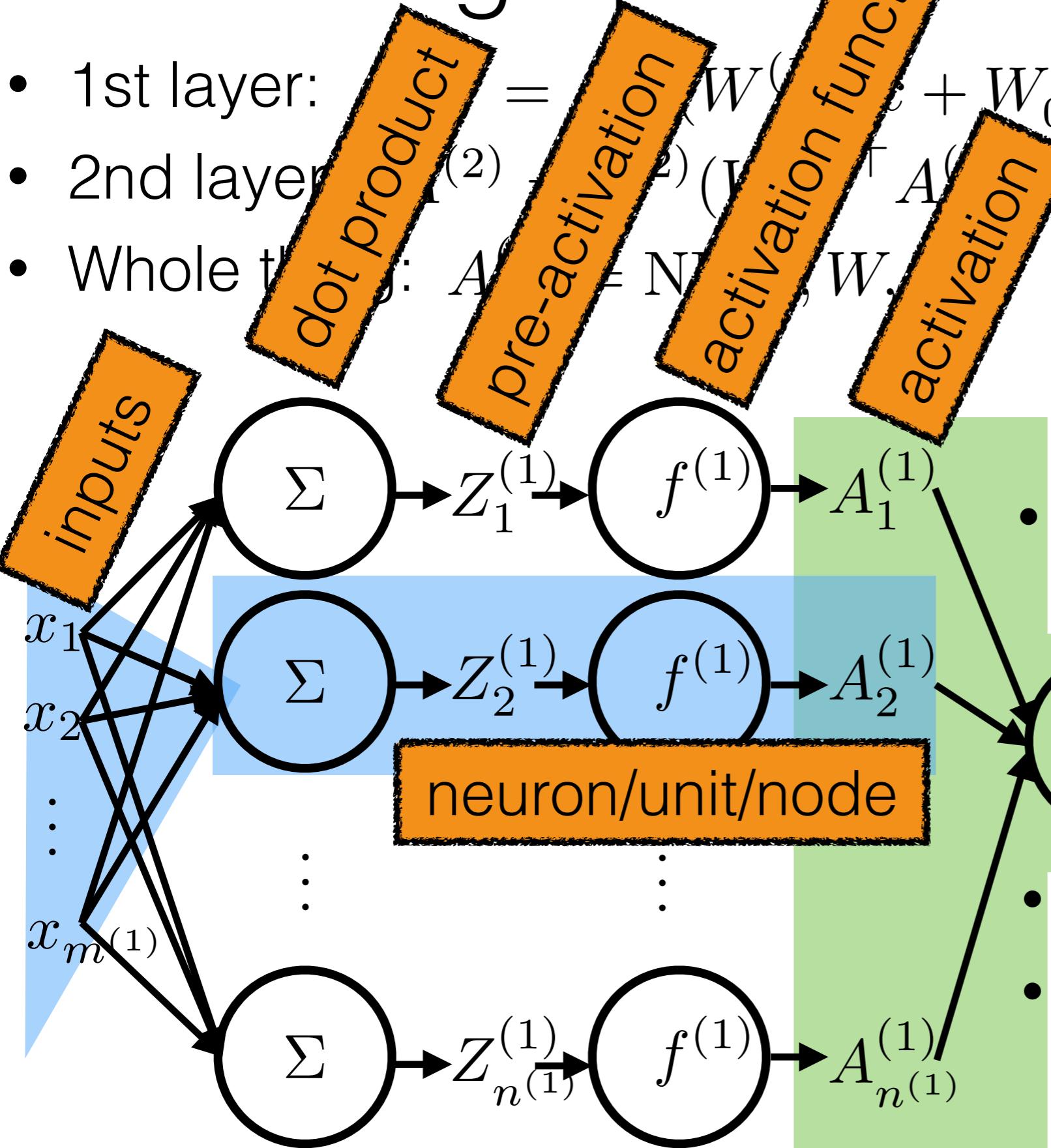


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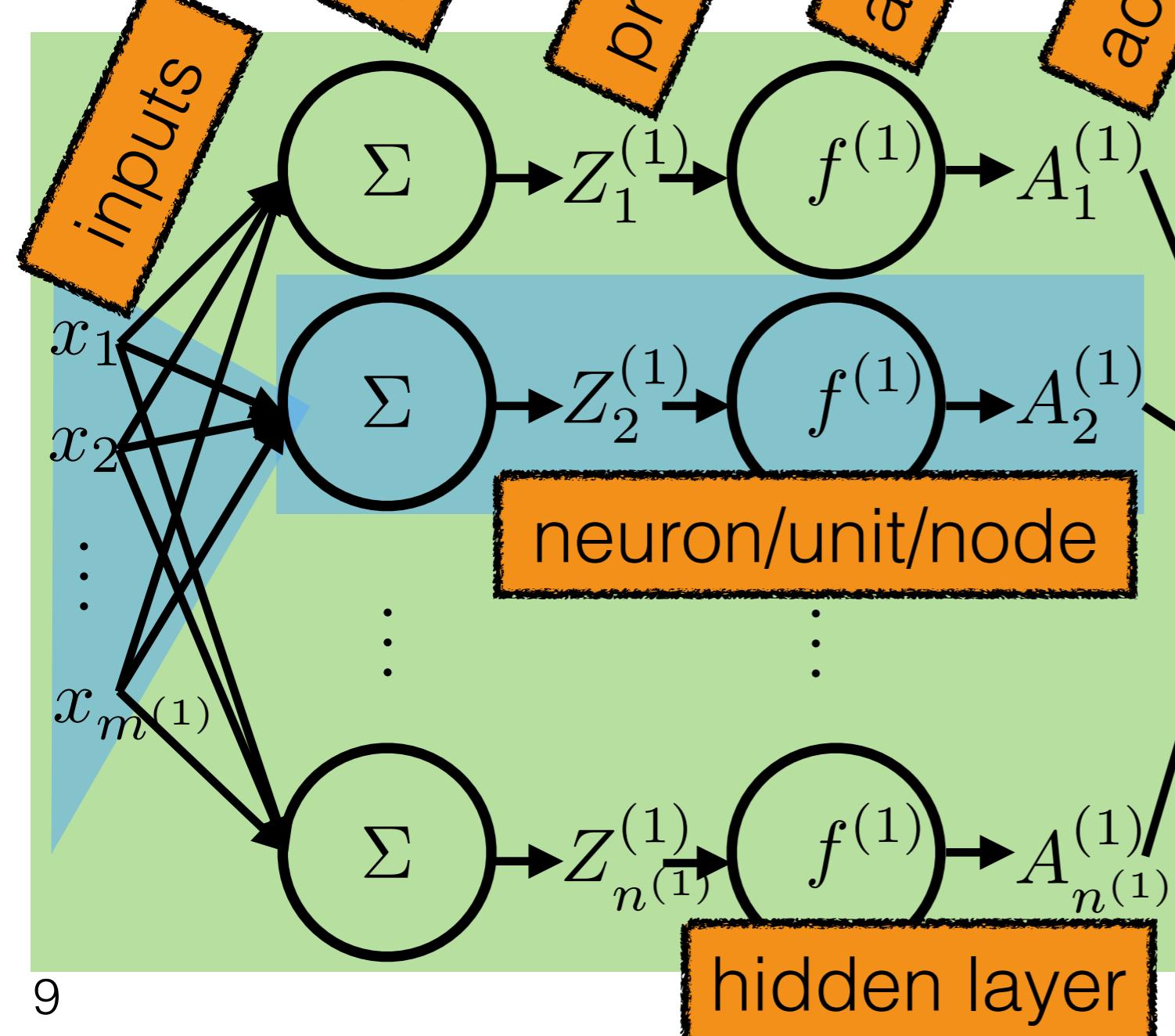
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Issues:

- Derivatives are zero (or undefined) if we use the step function activation, so (S)GD won't do what we want
- What if I want to do regression?
- What if I want to use NLL loss?

# Different activation functions

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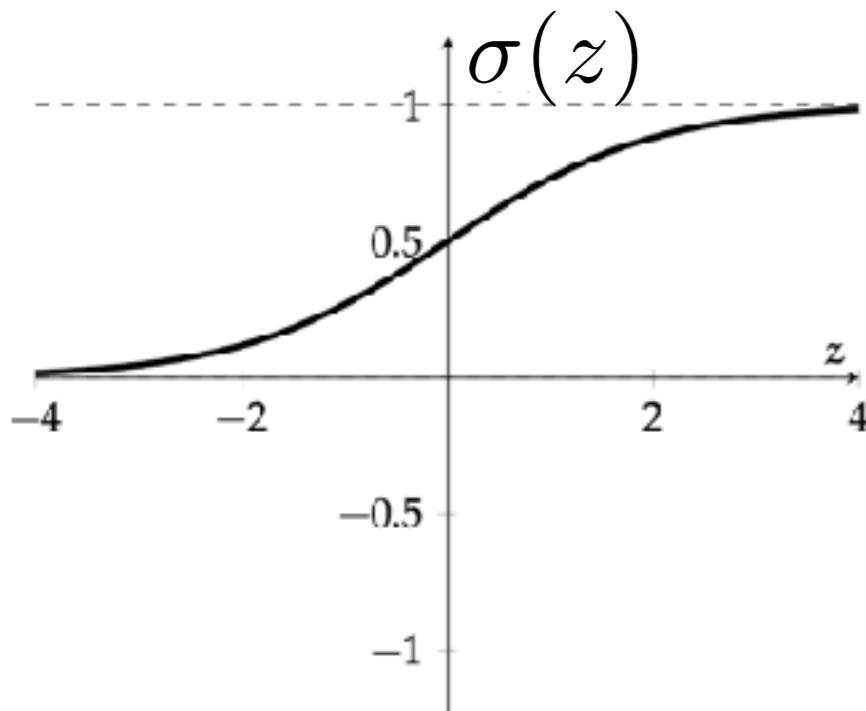
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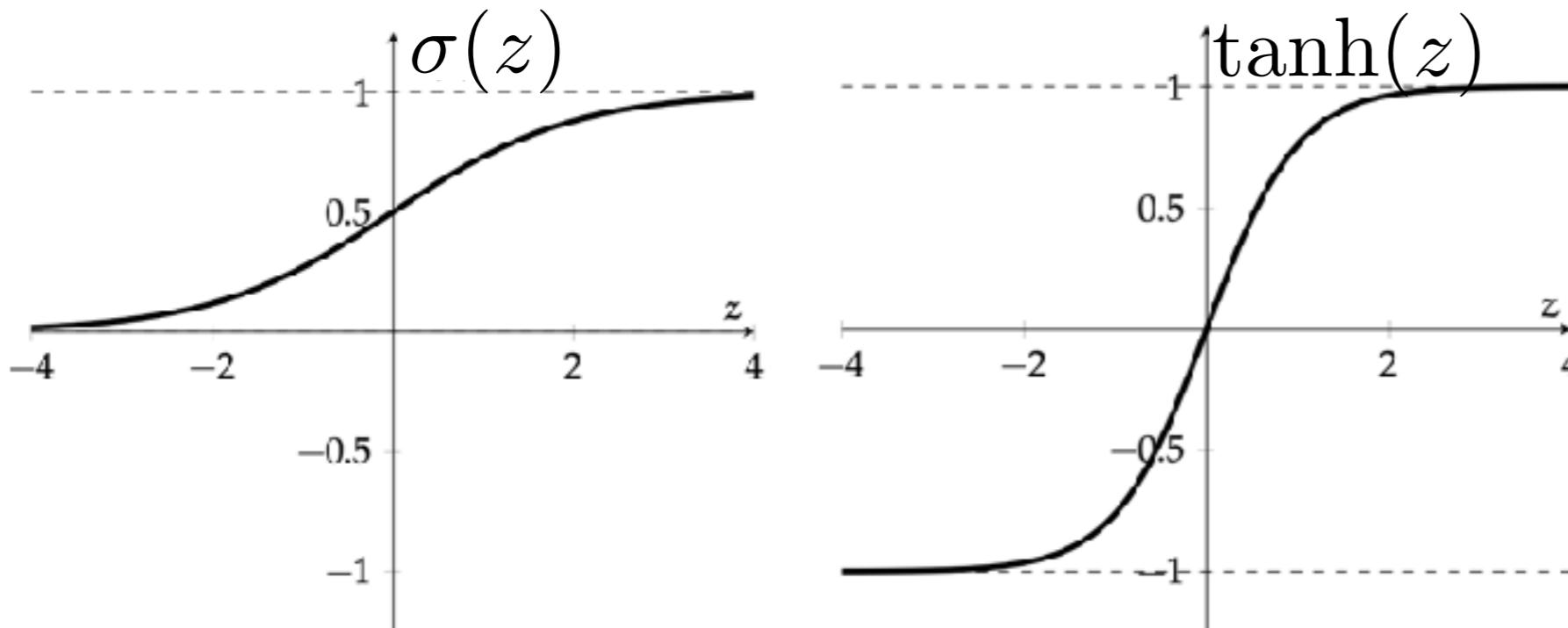
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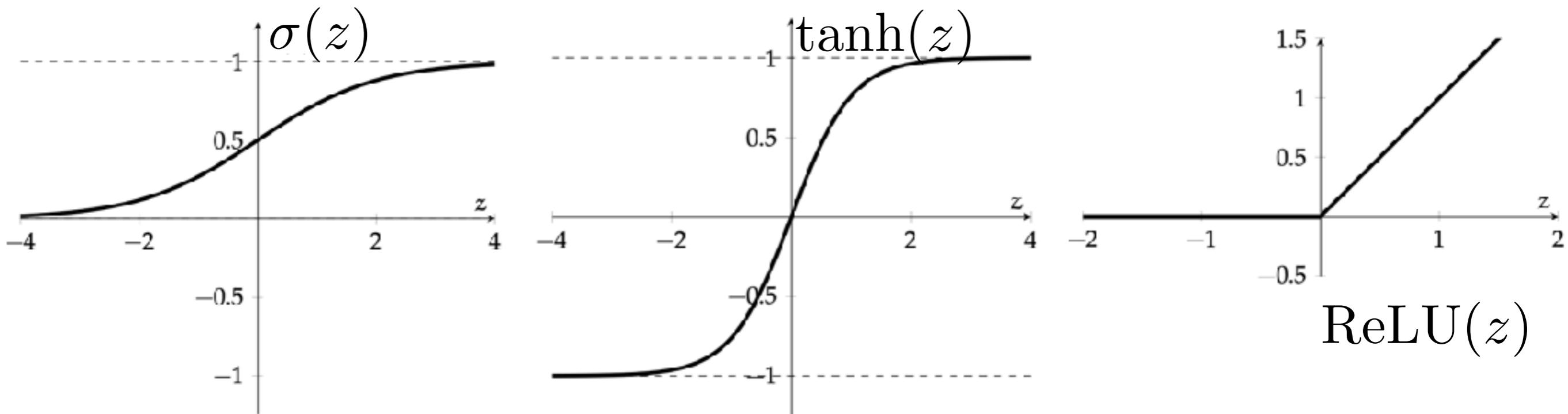
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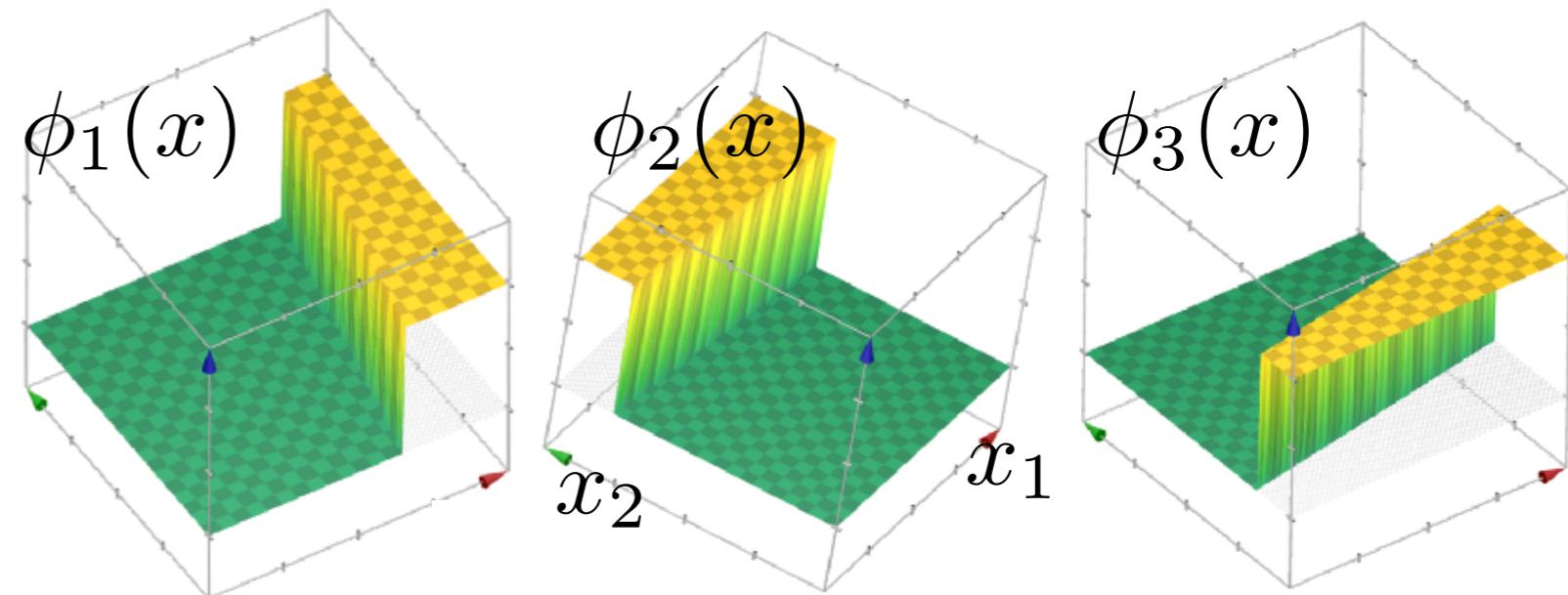
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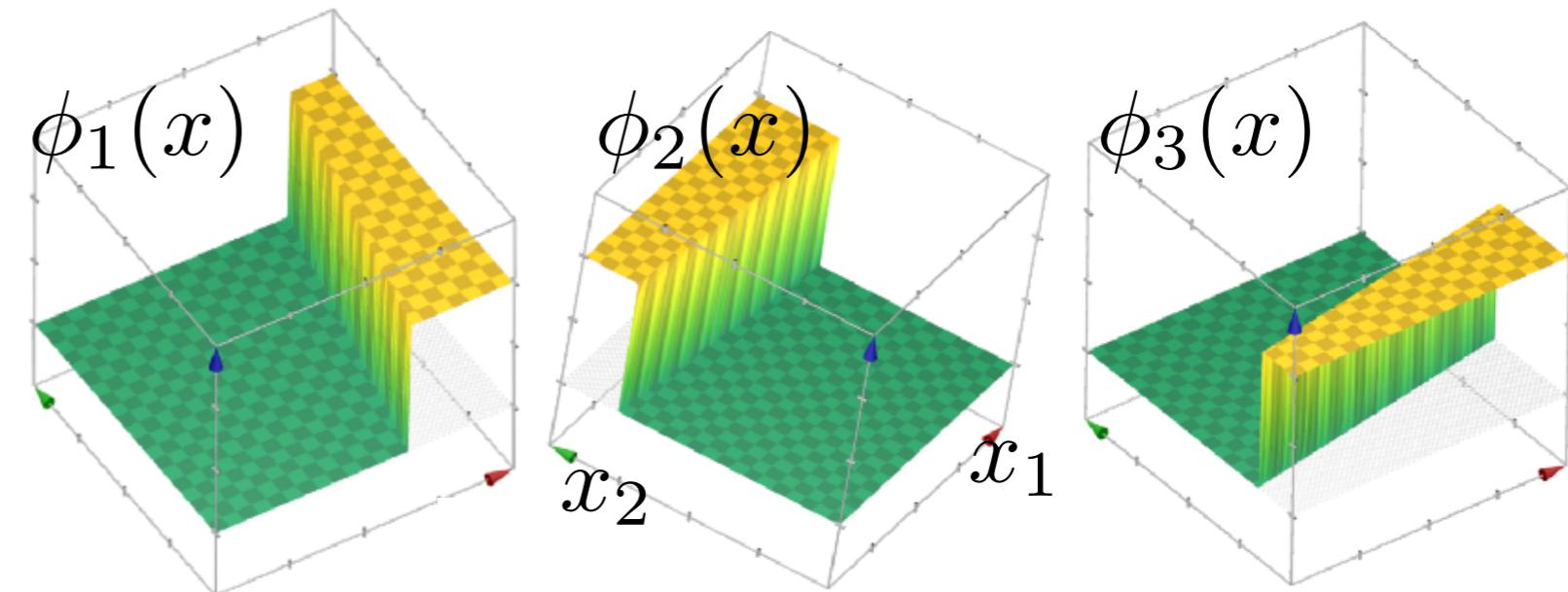
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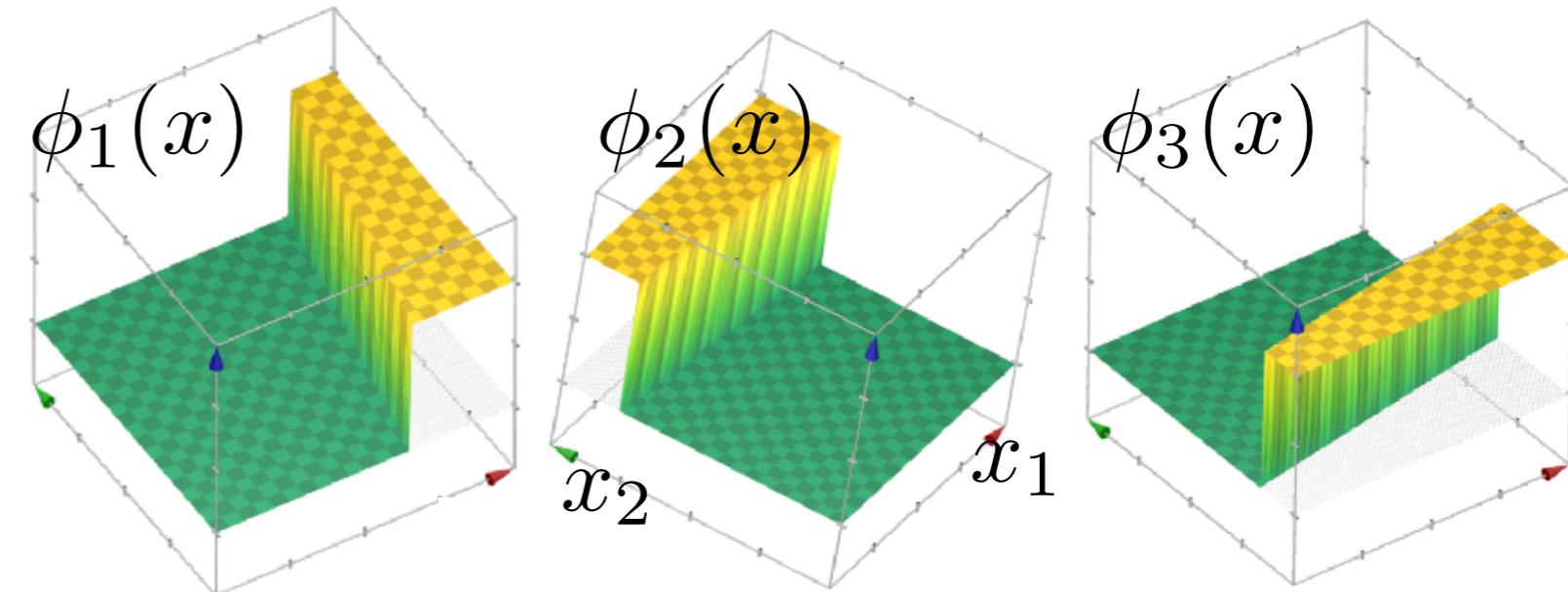


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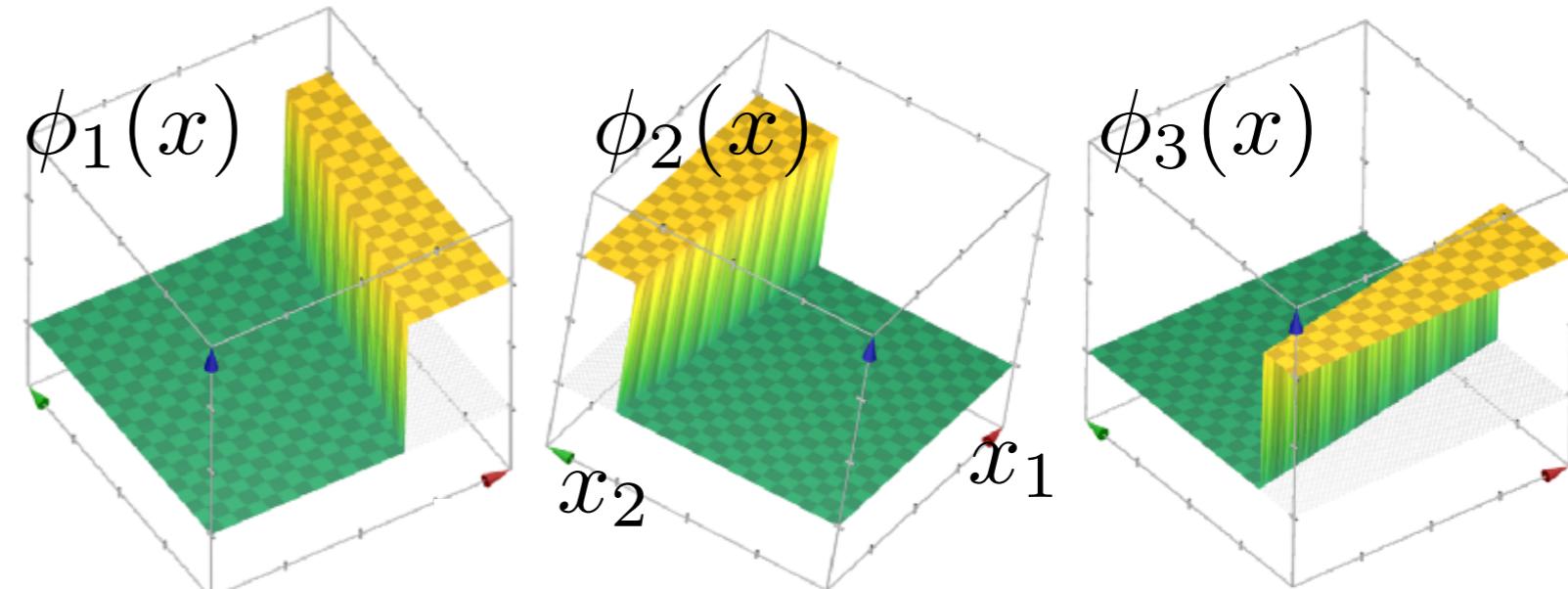
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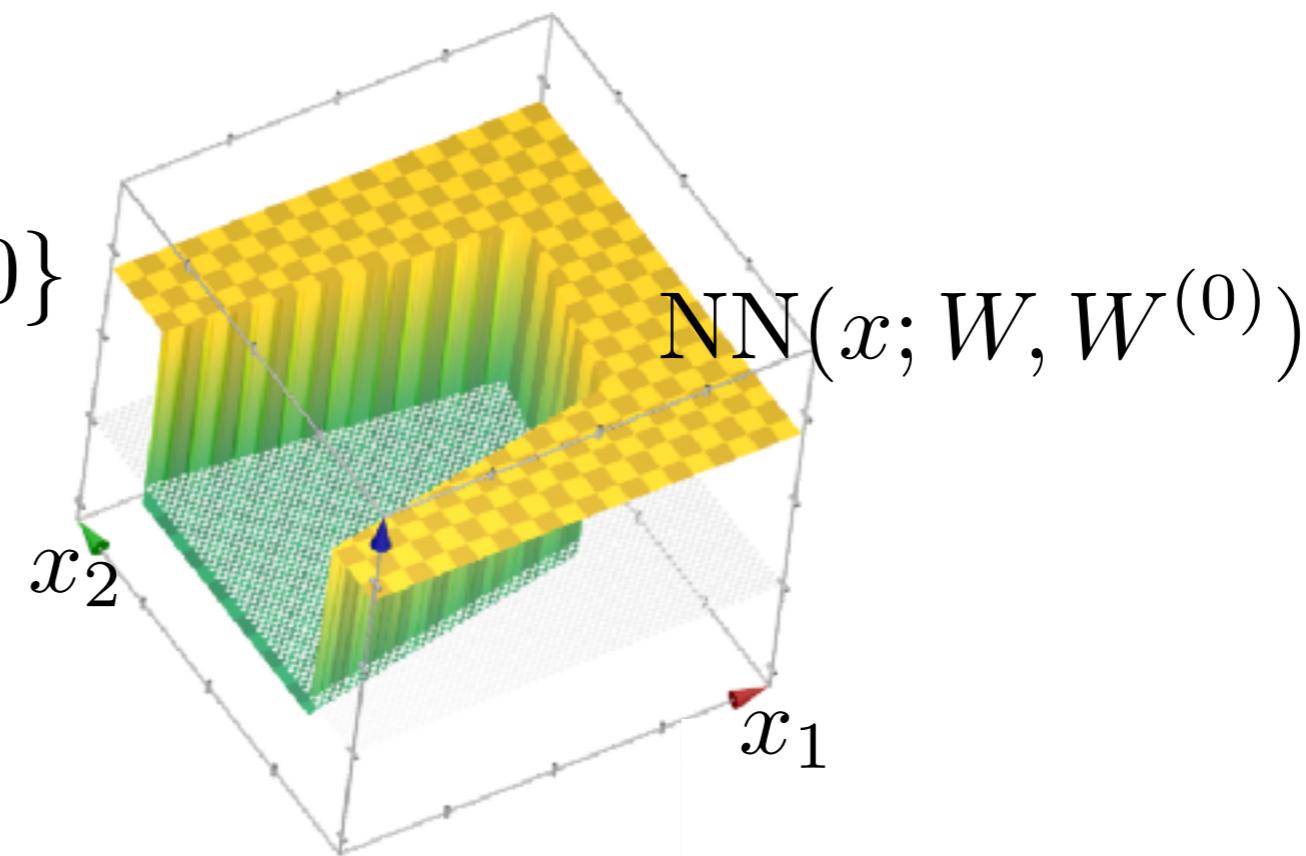
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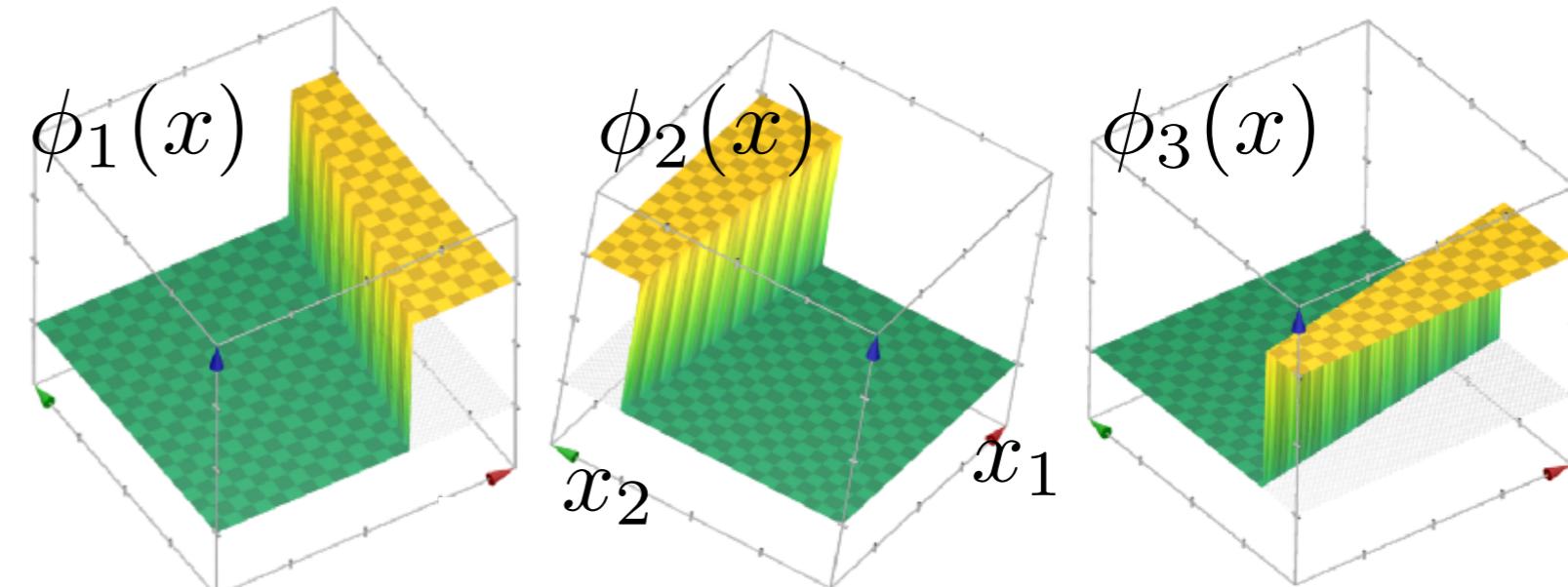


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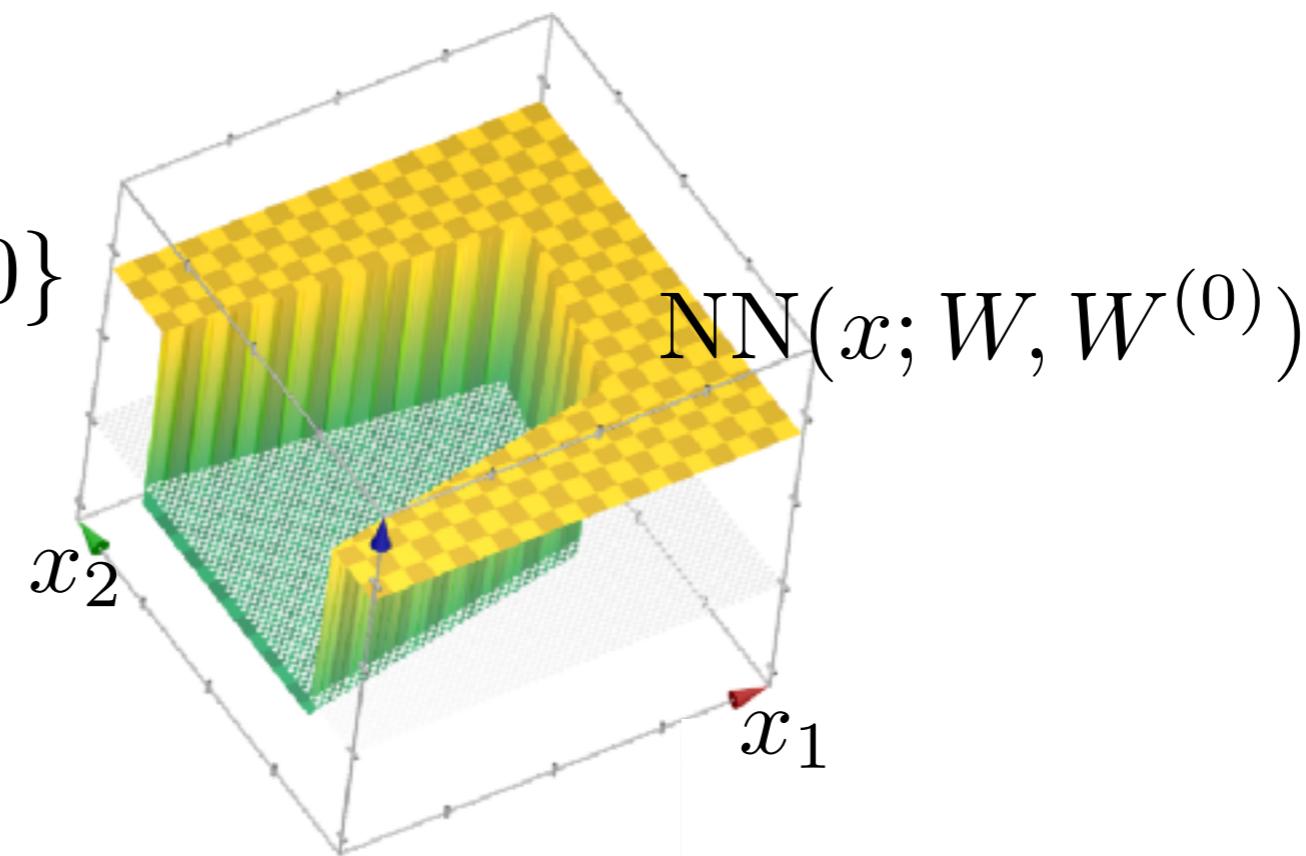
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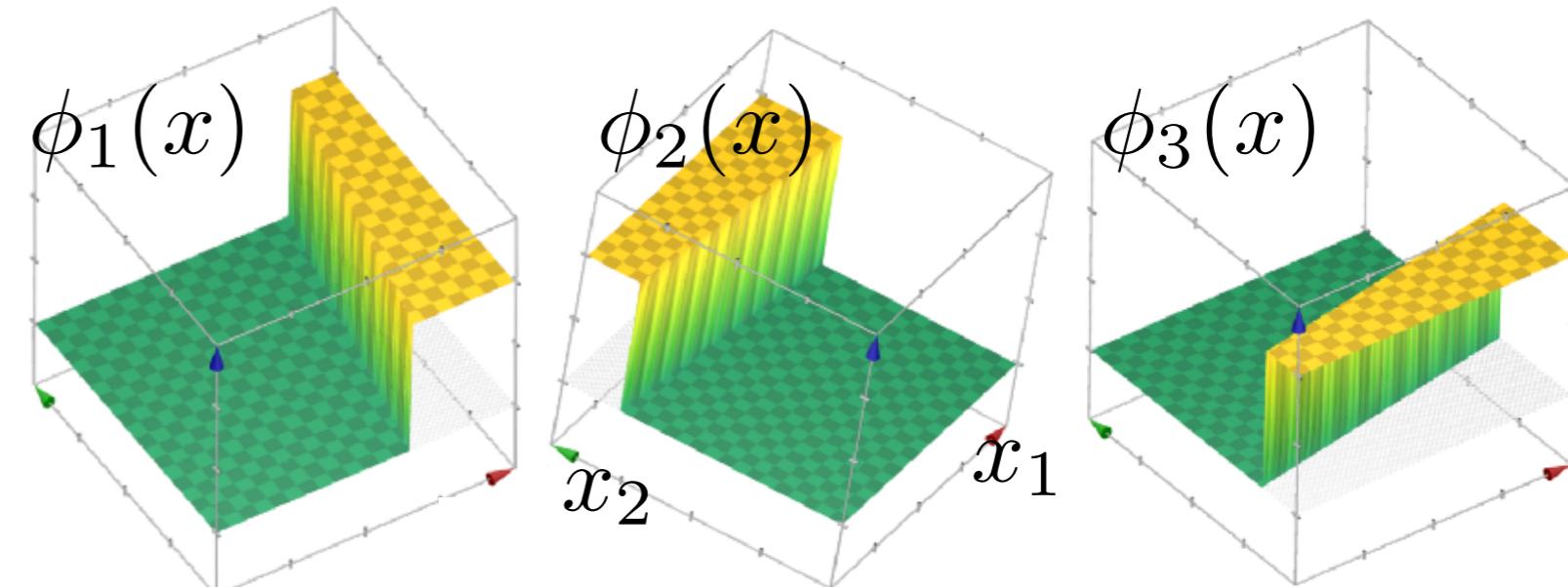


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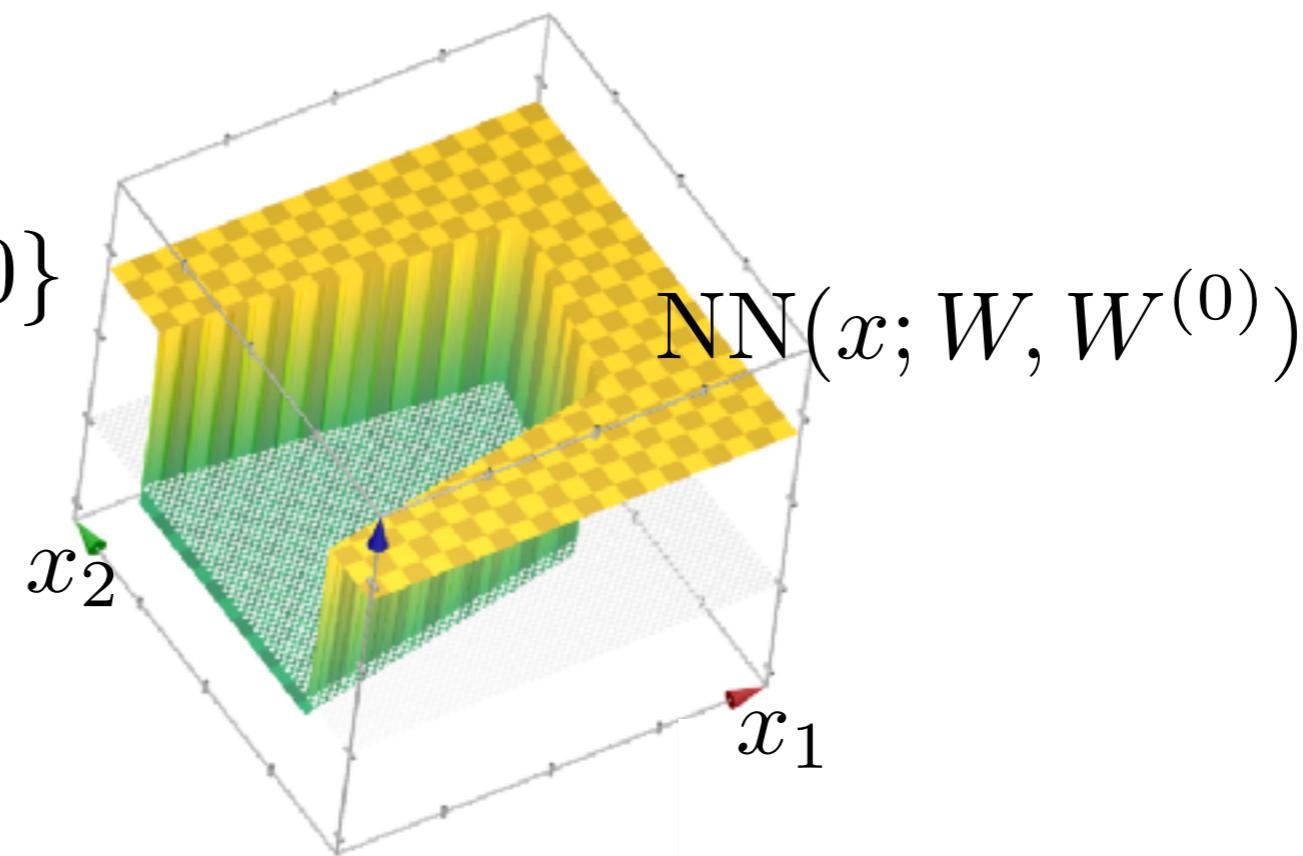
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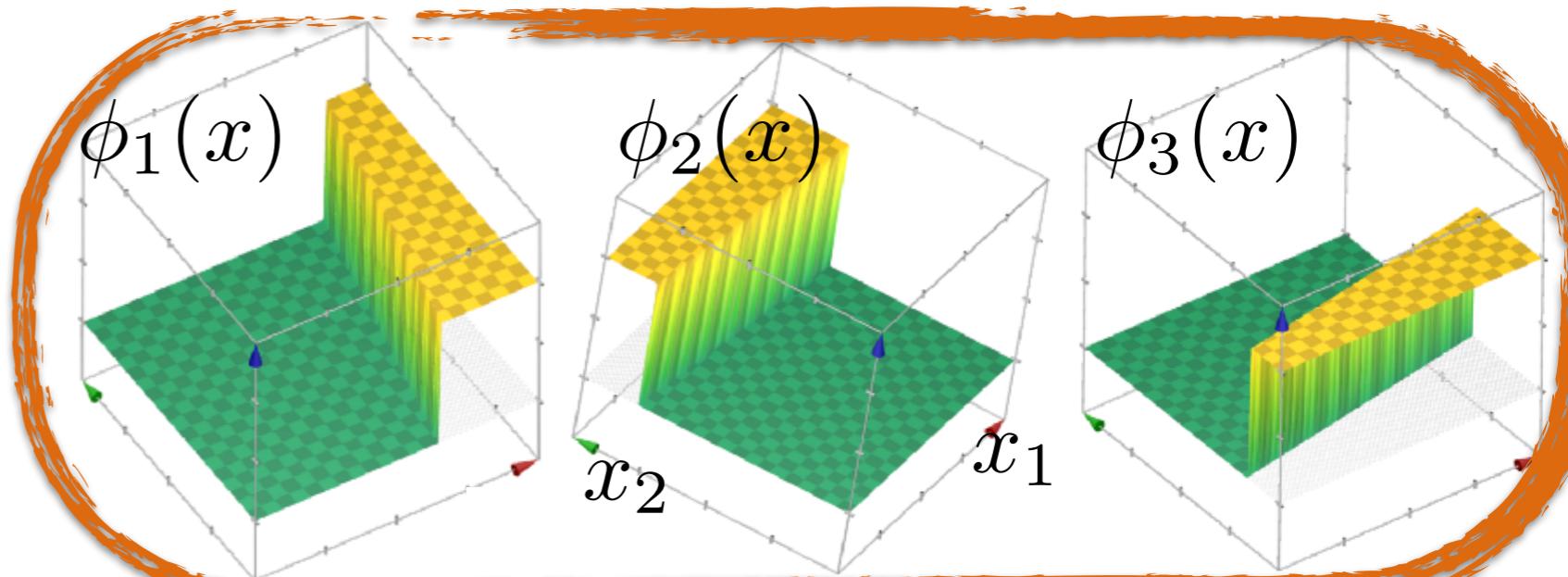
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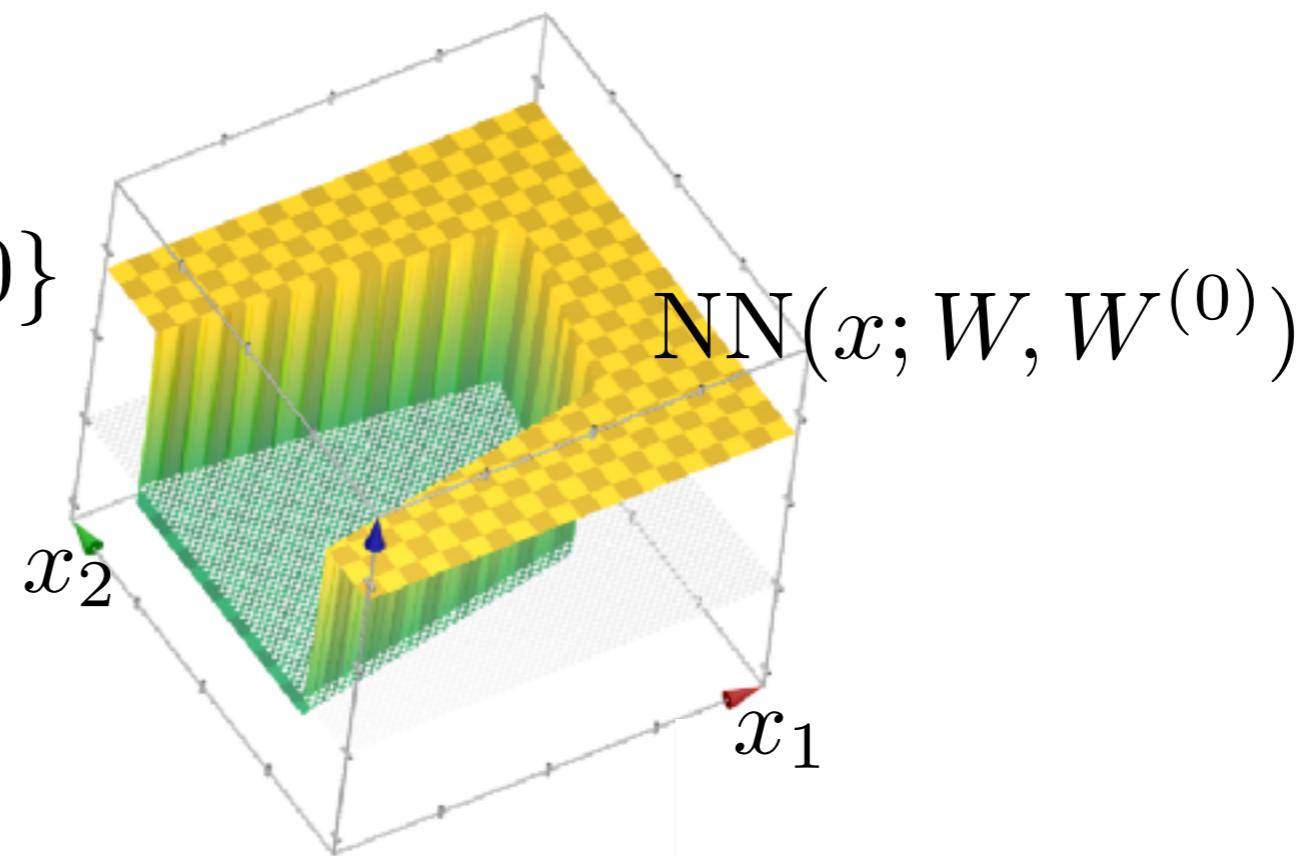
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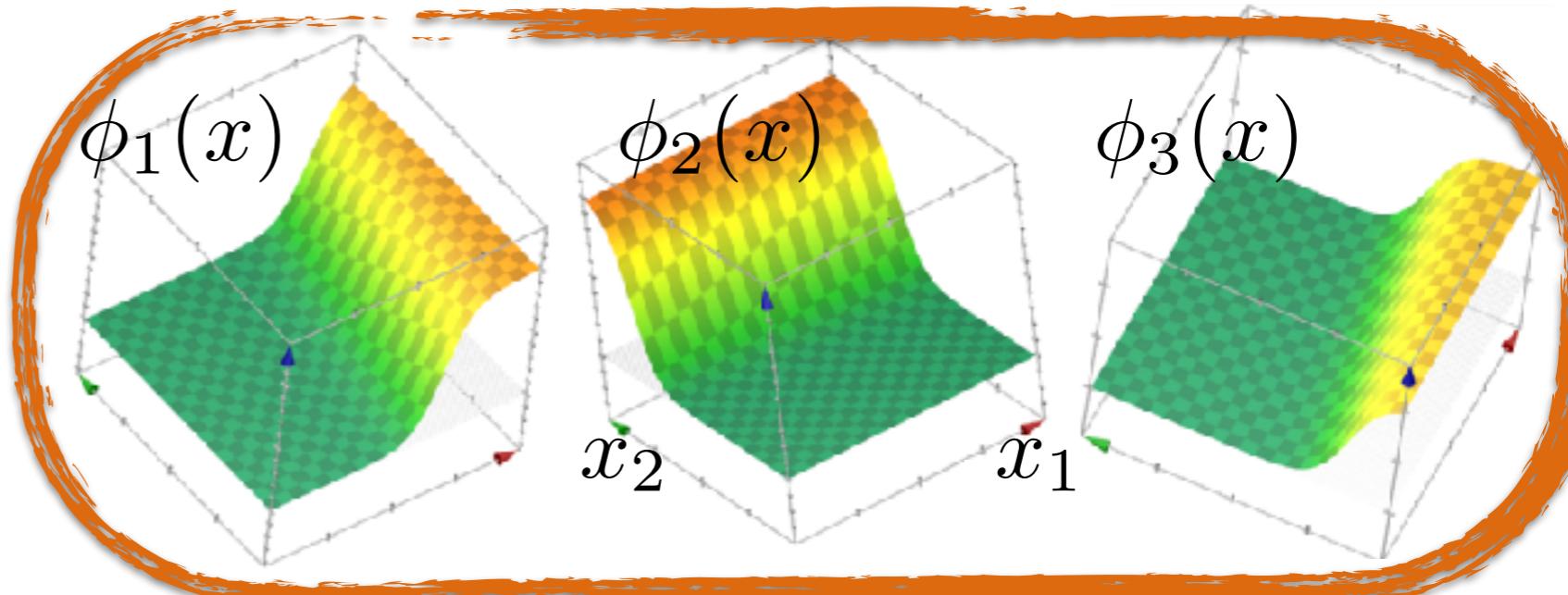
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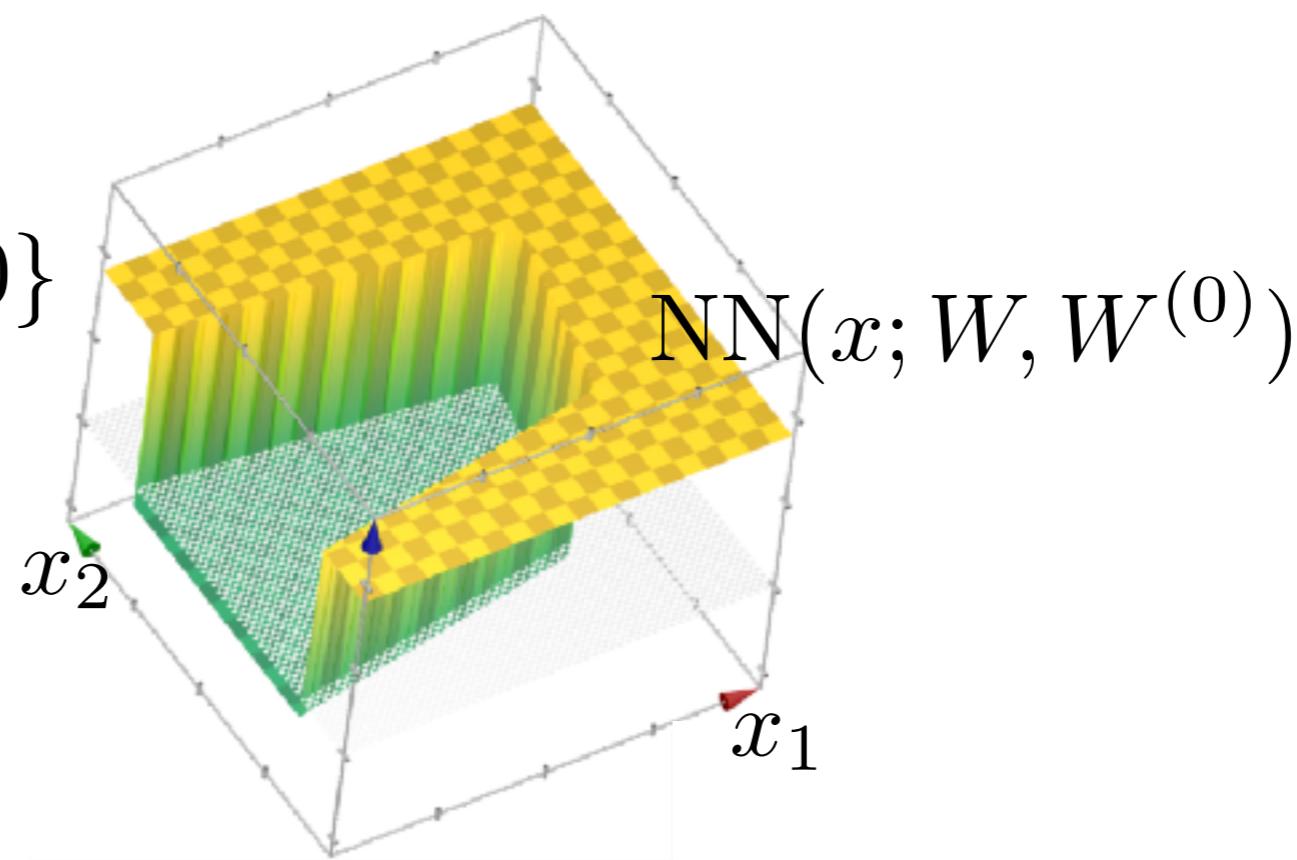
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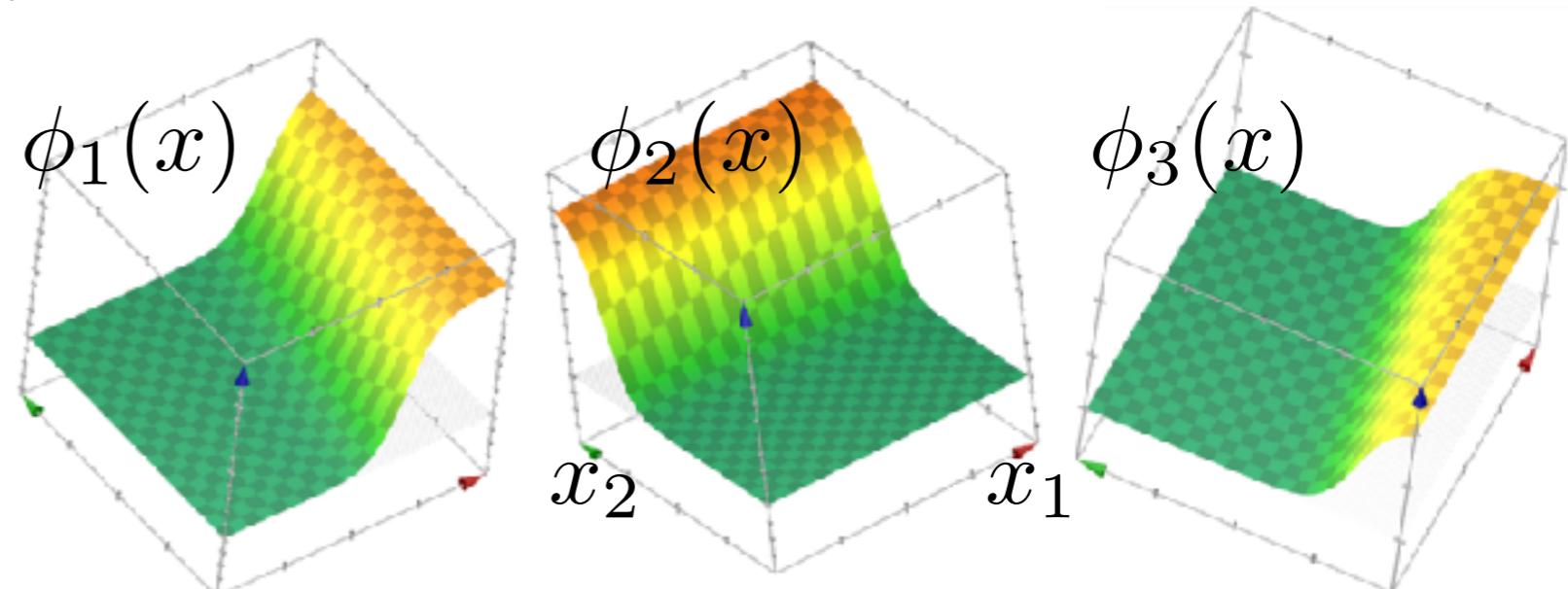


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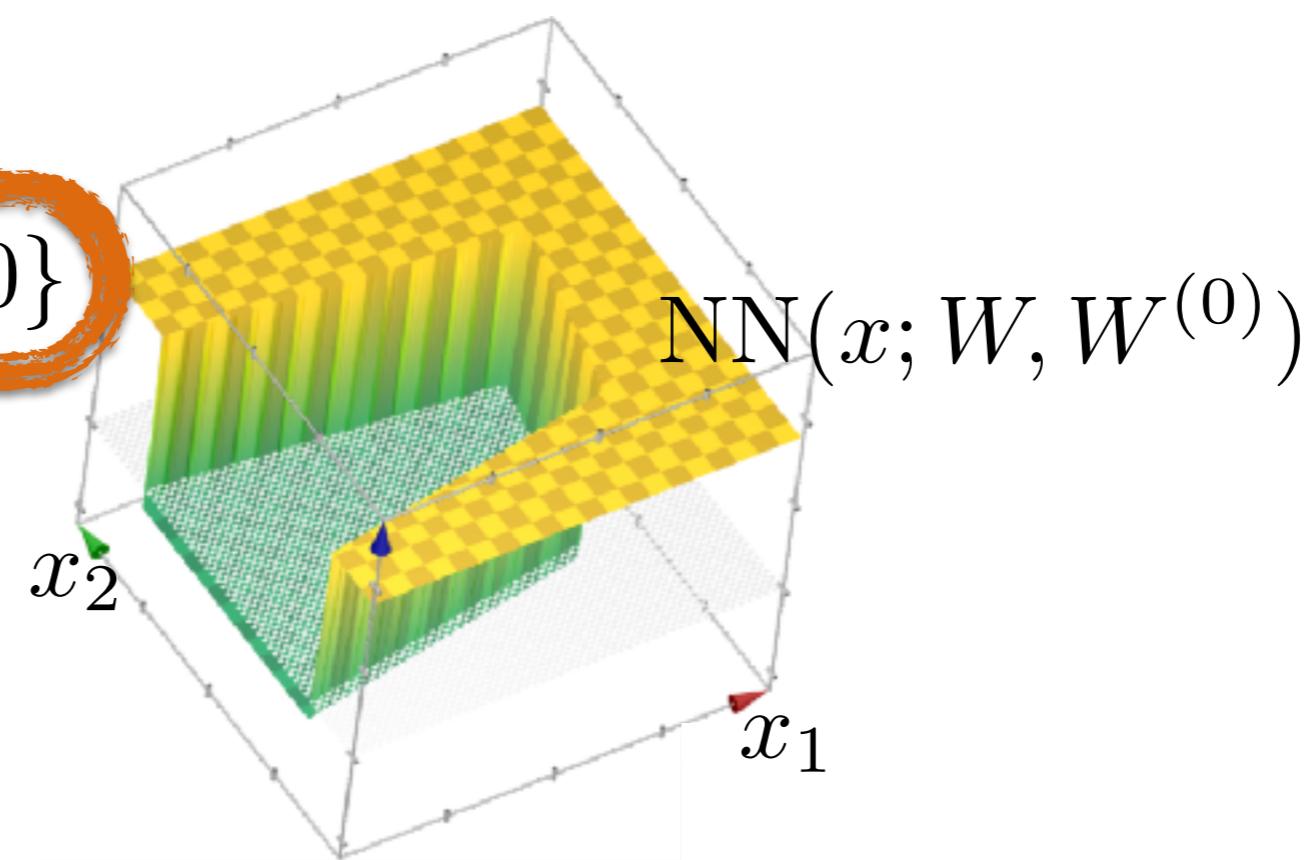
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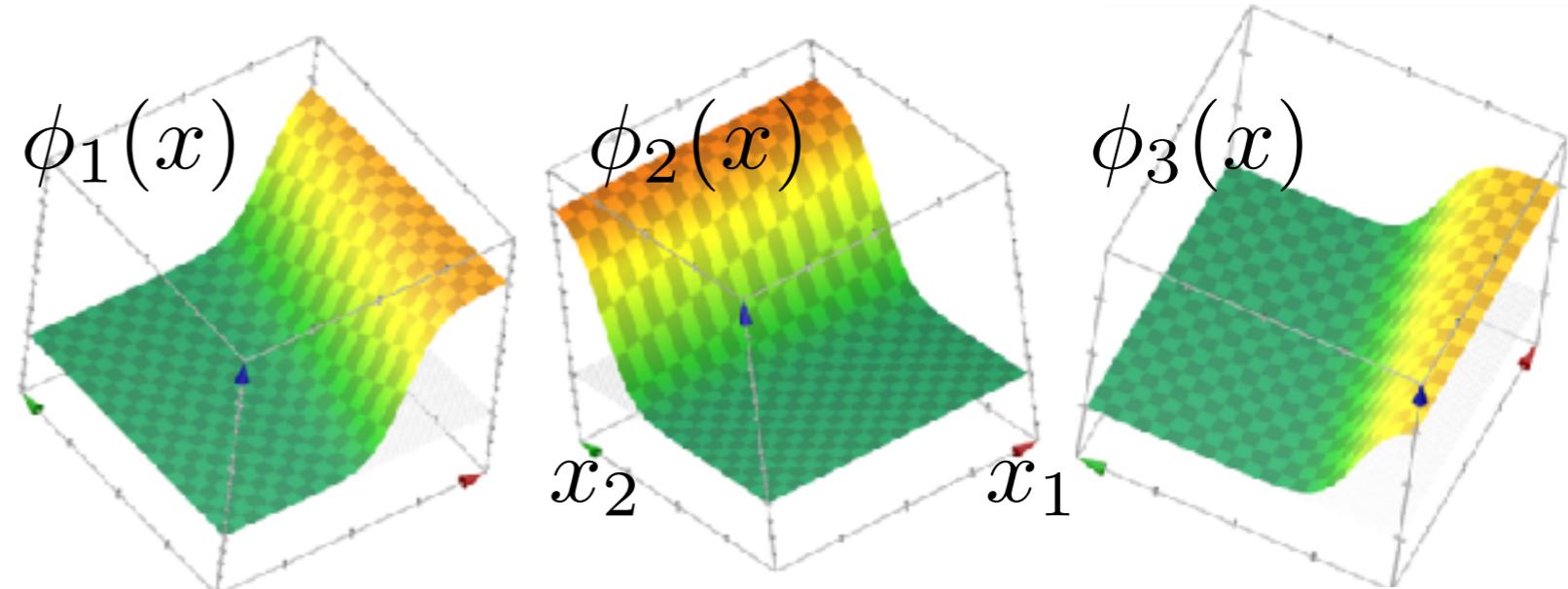
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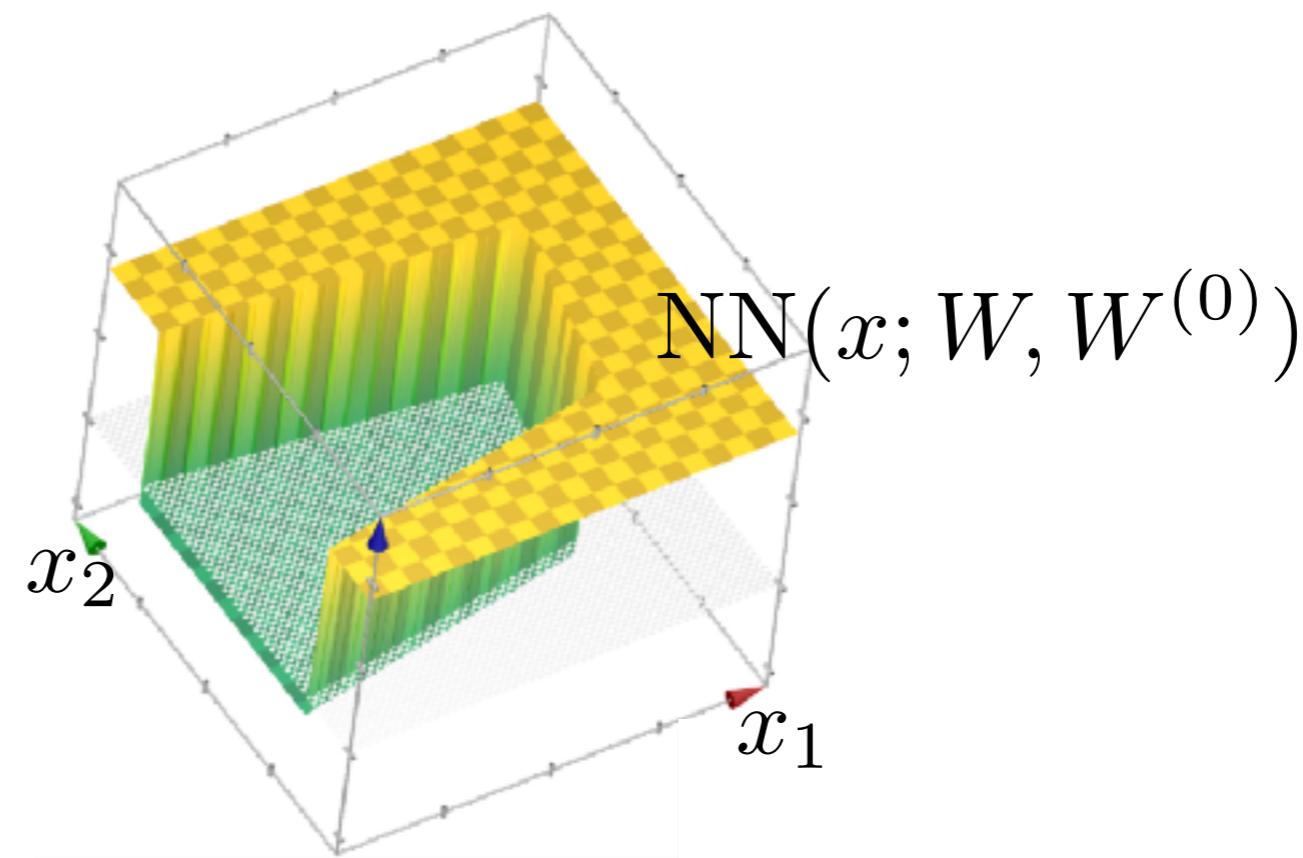
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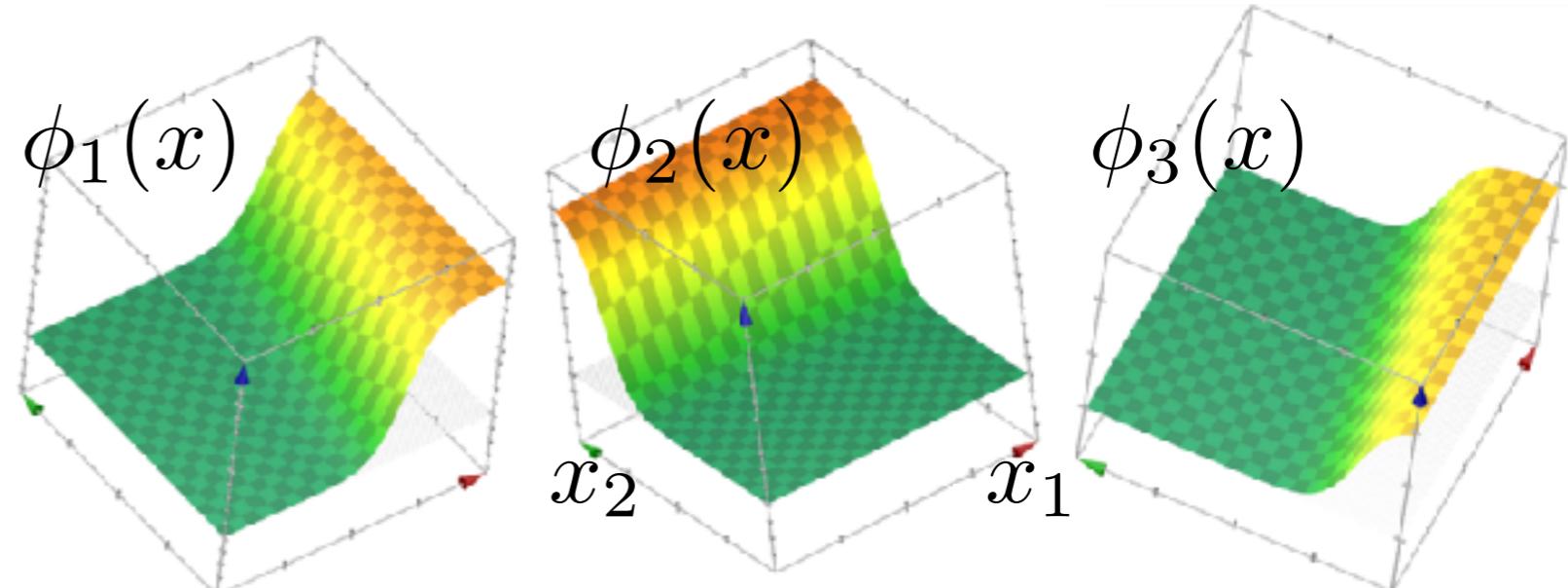
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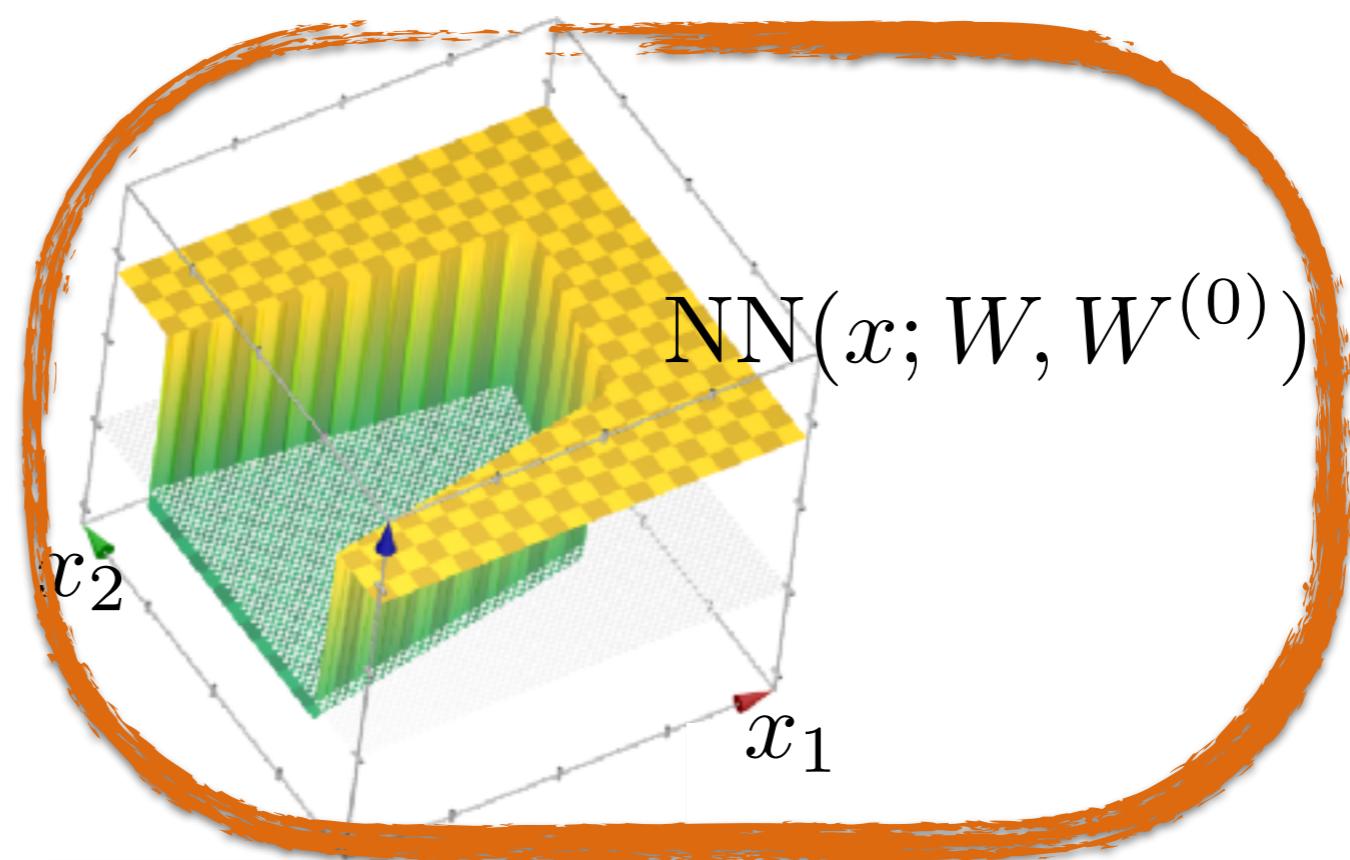
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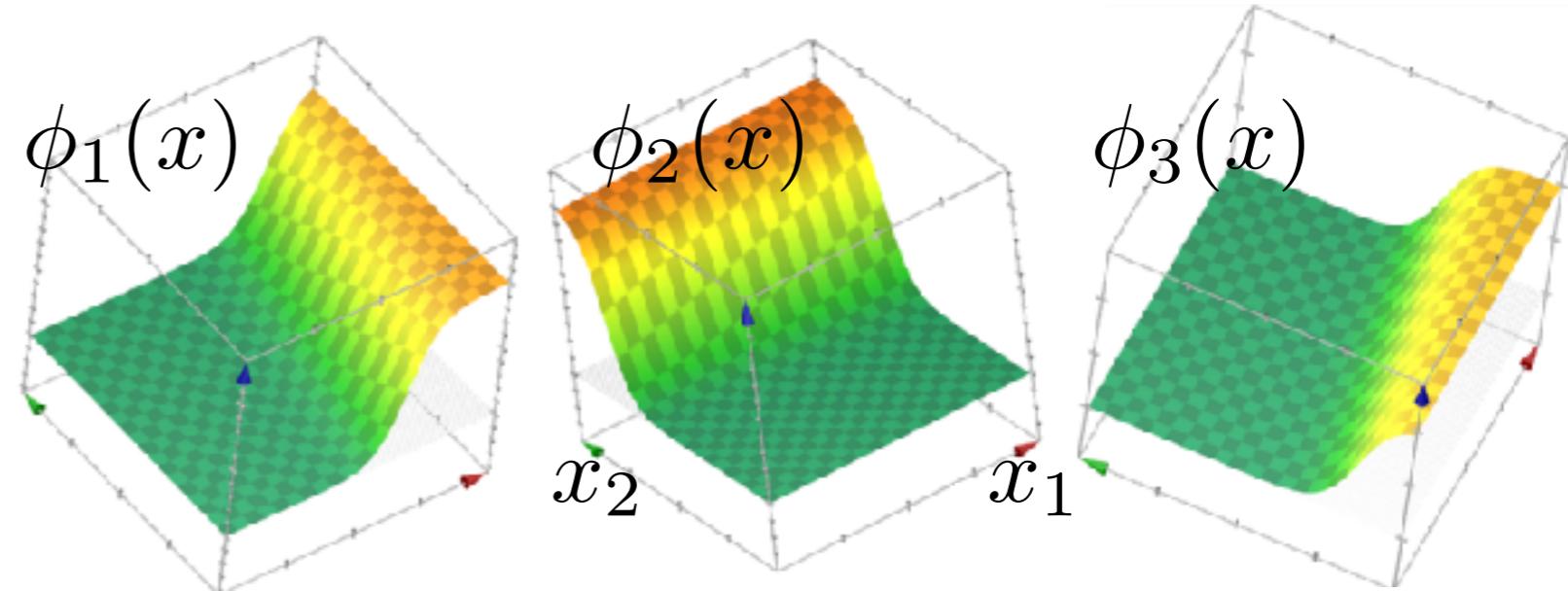


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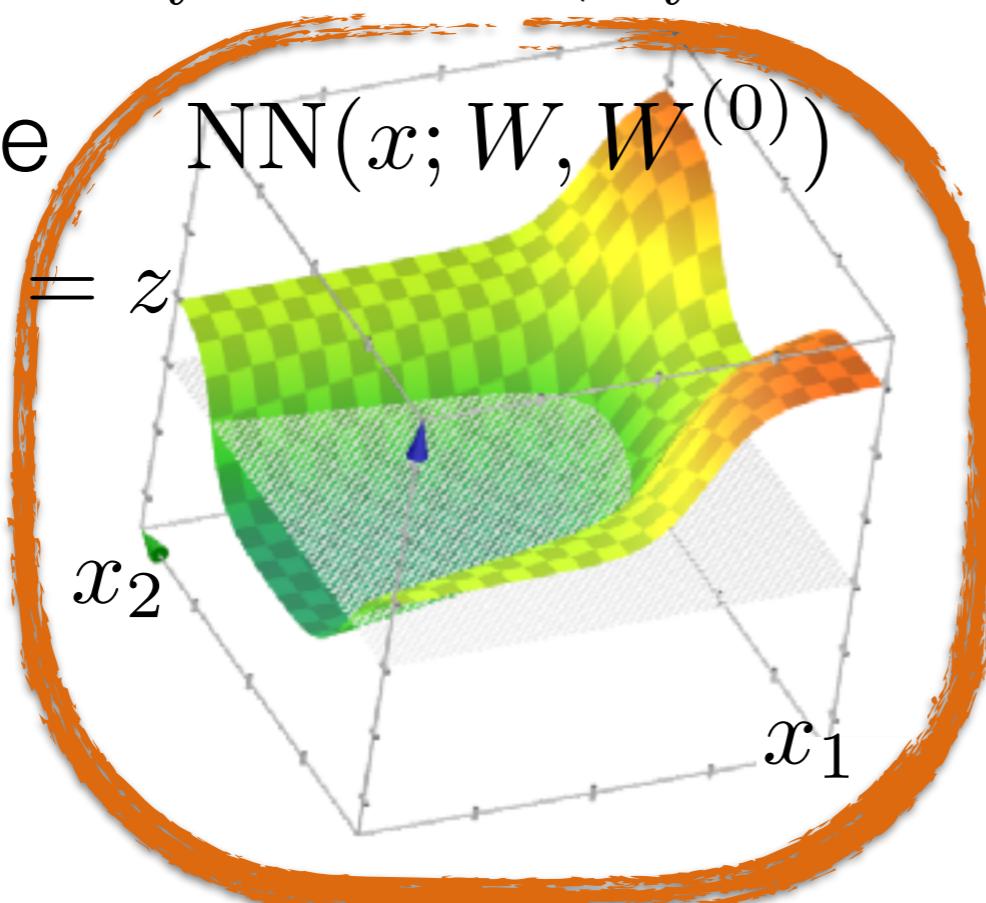
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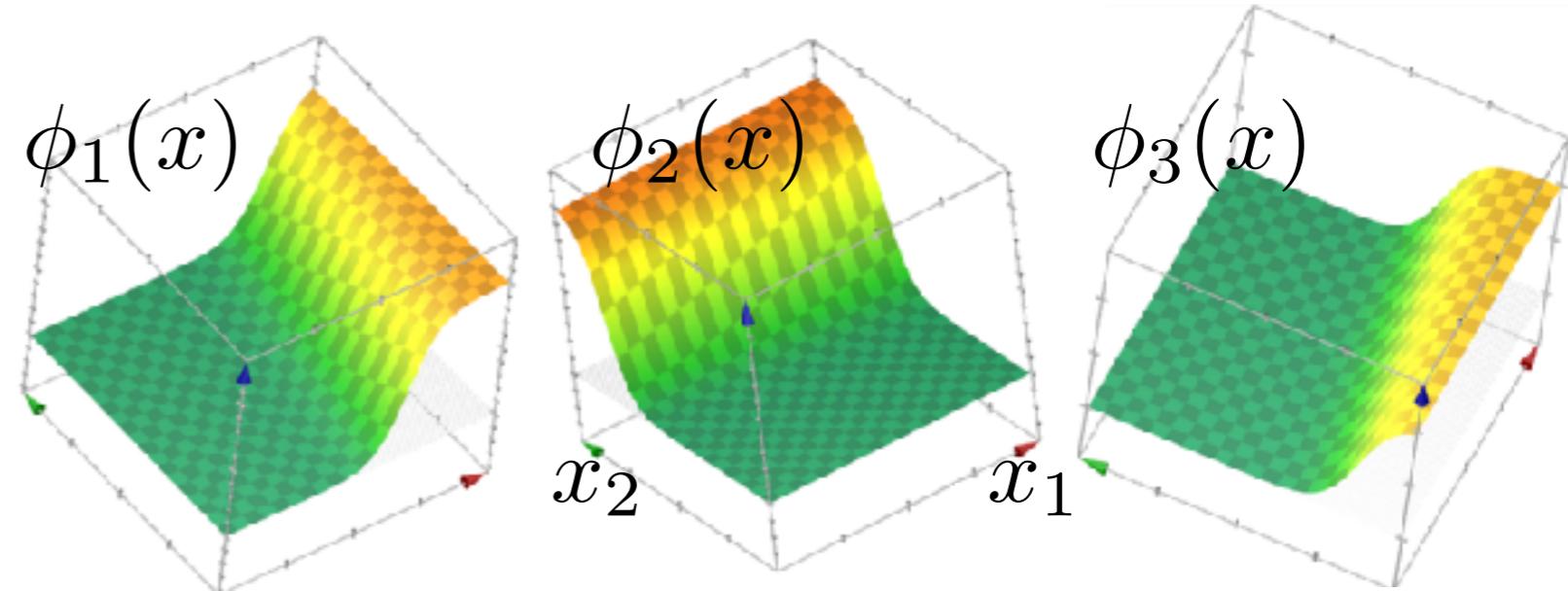


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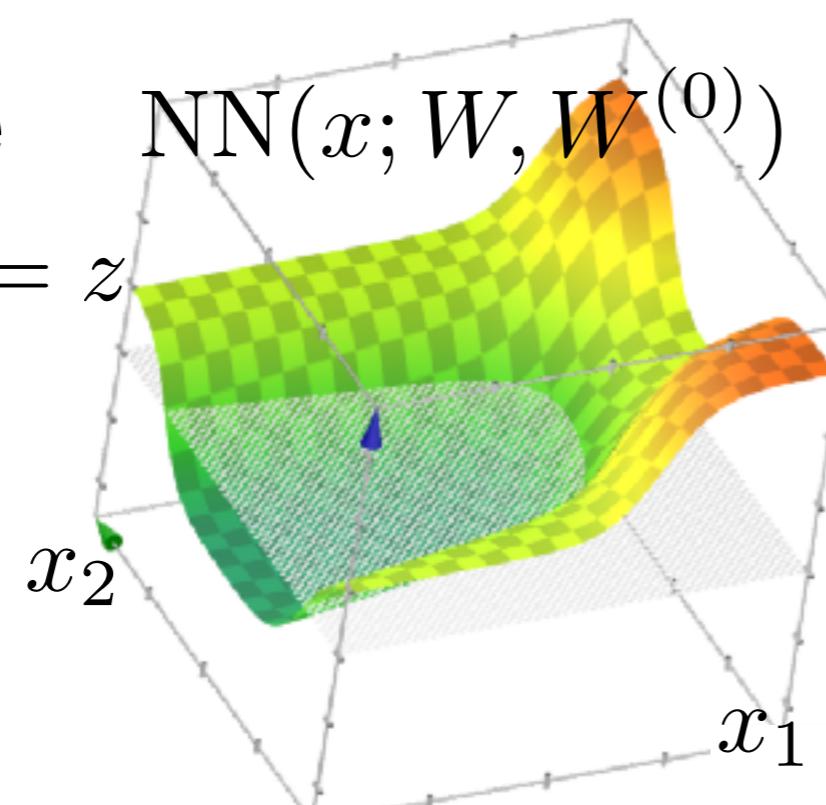
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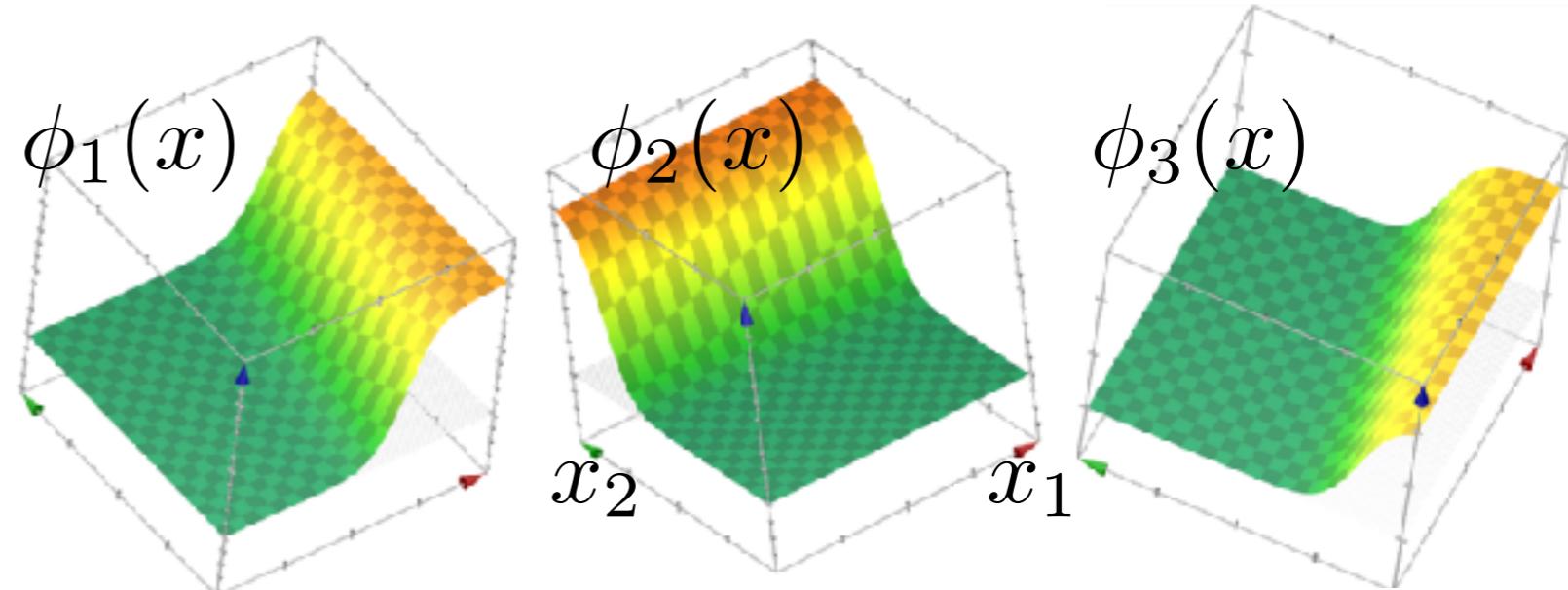


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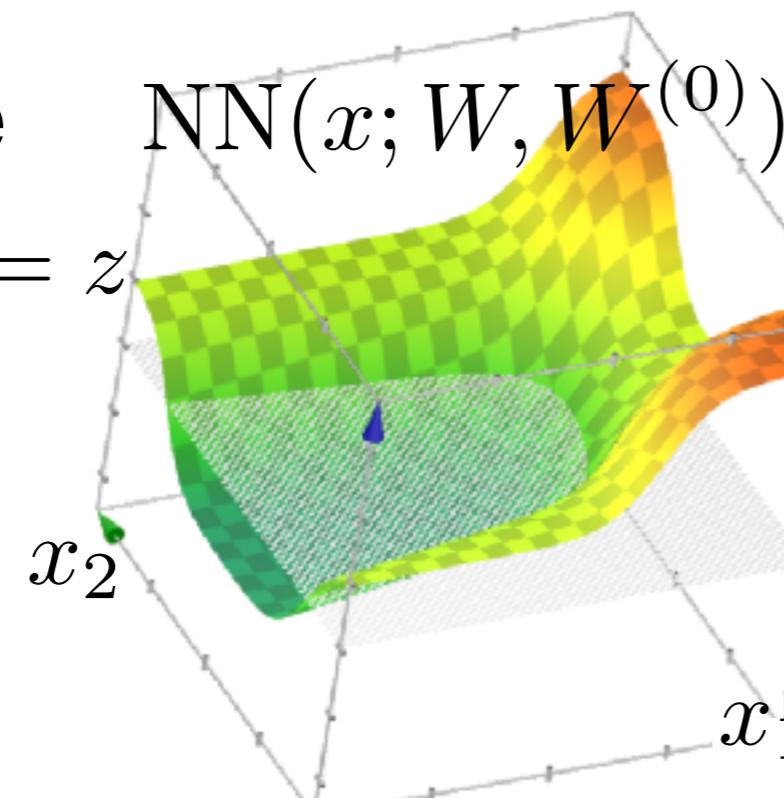


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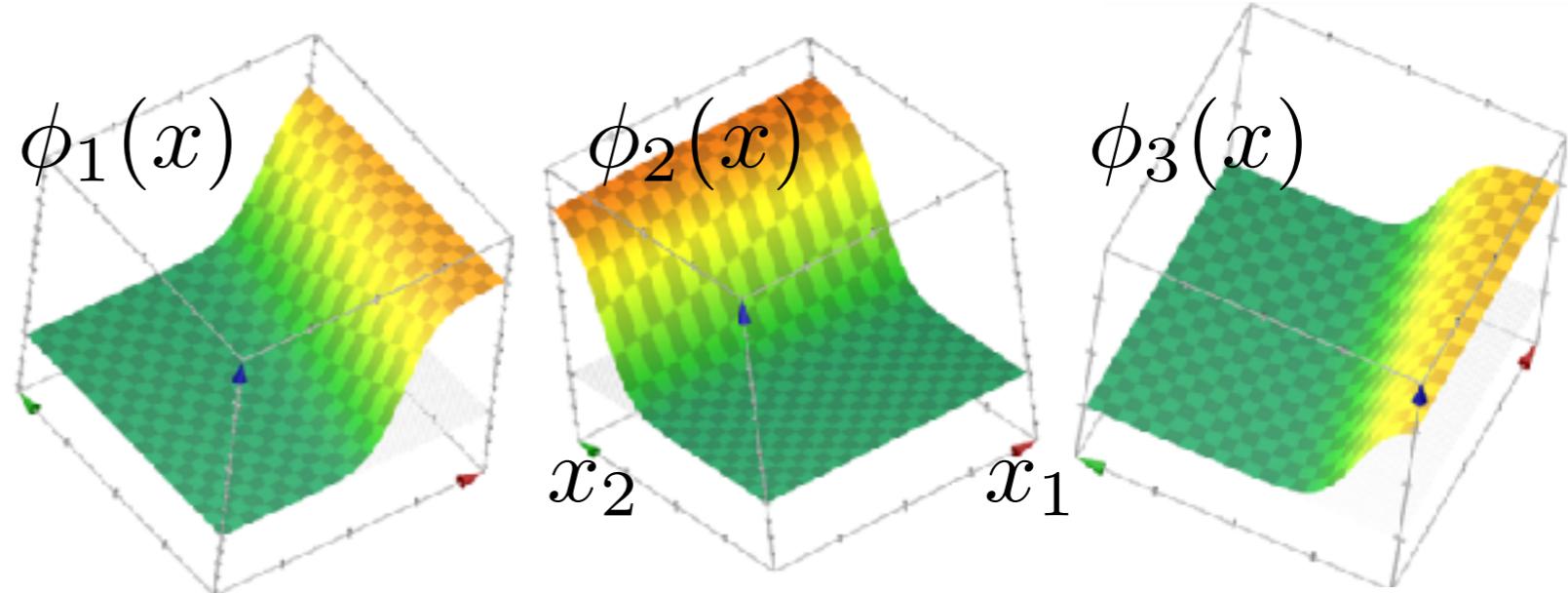
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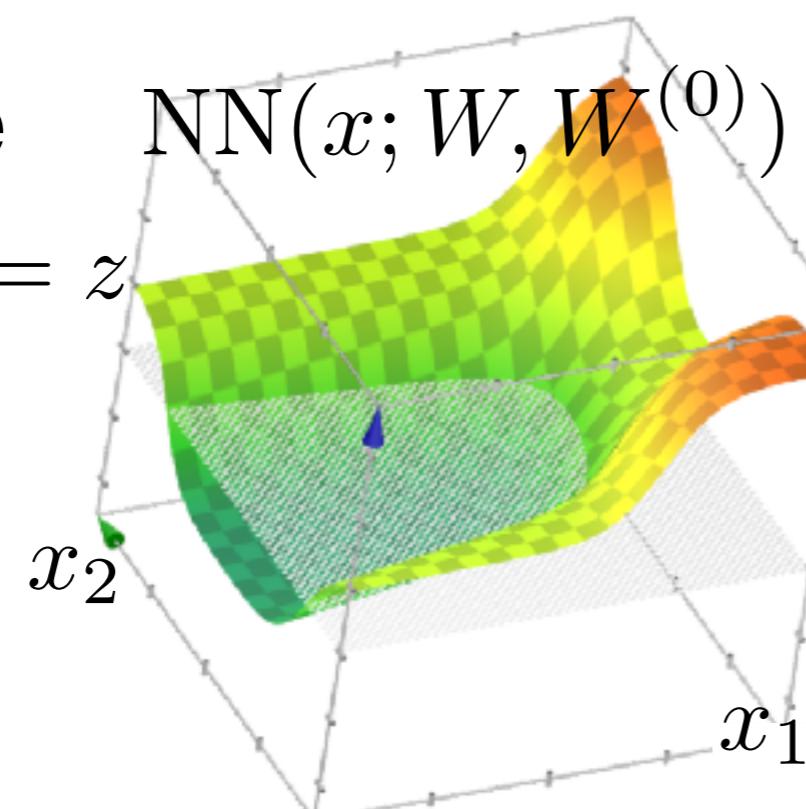


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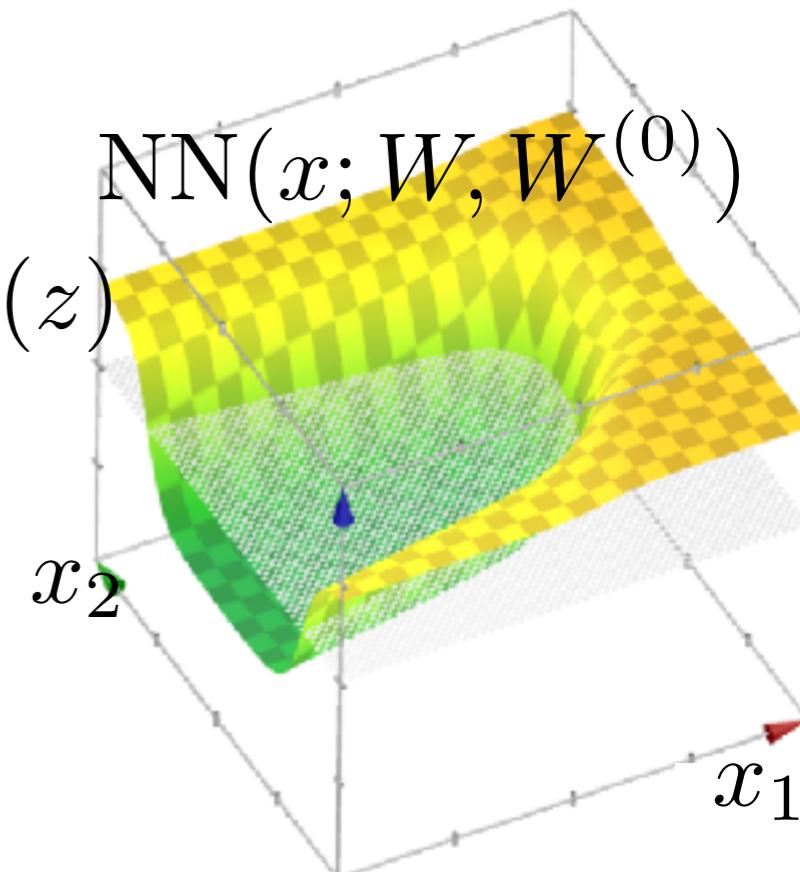
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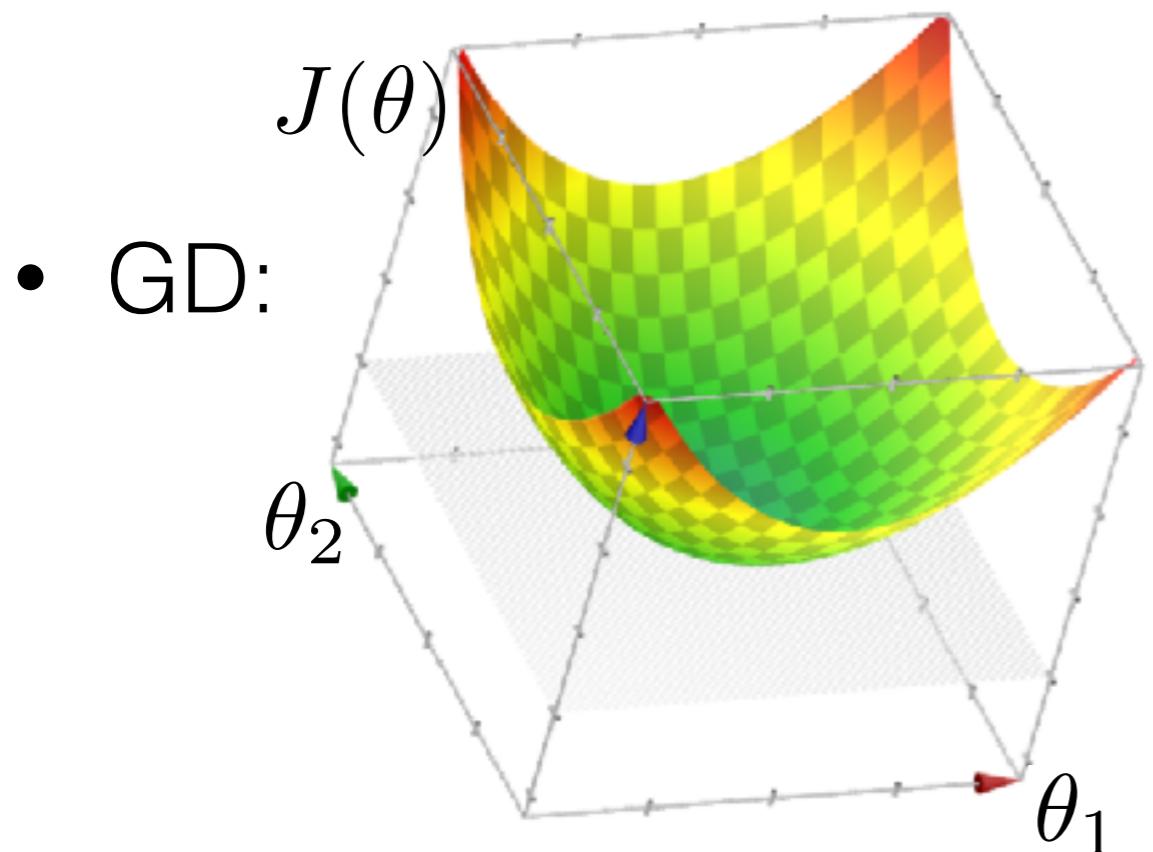
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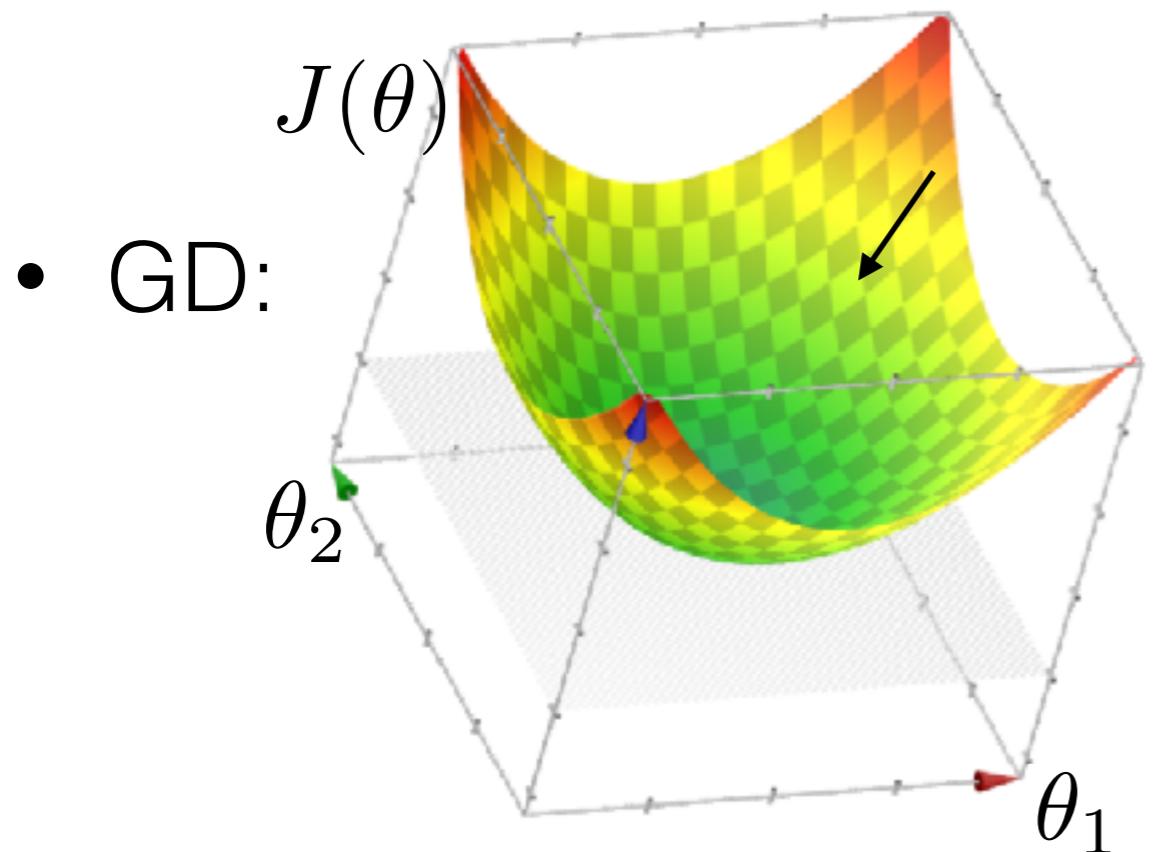
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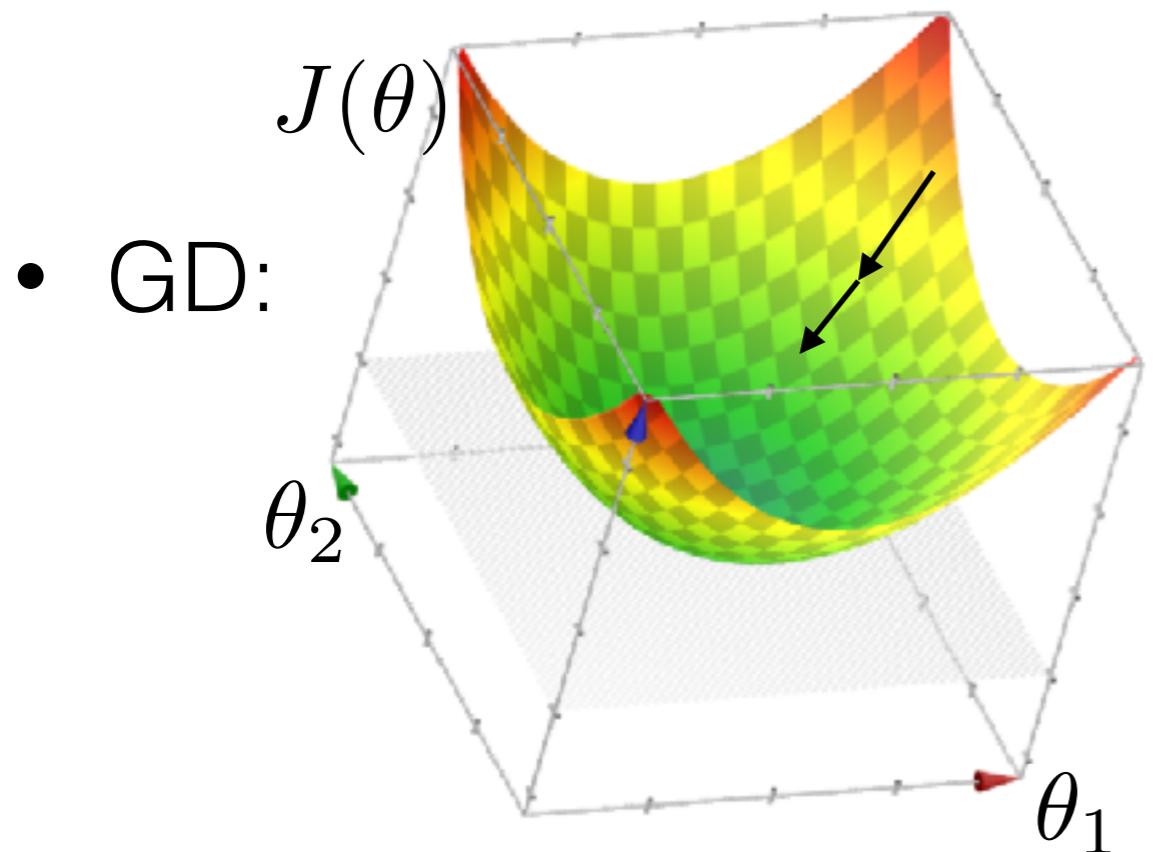
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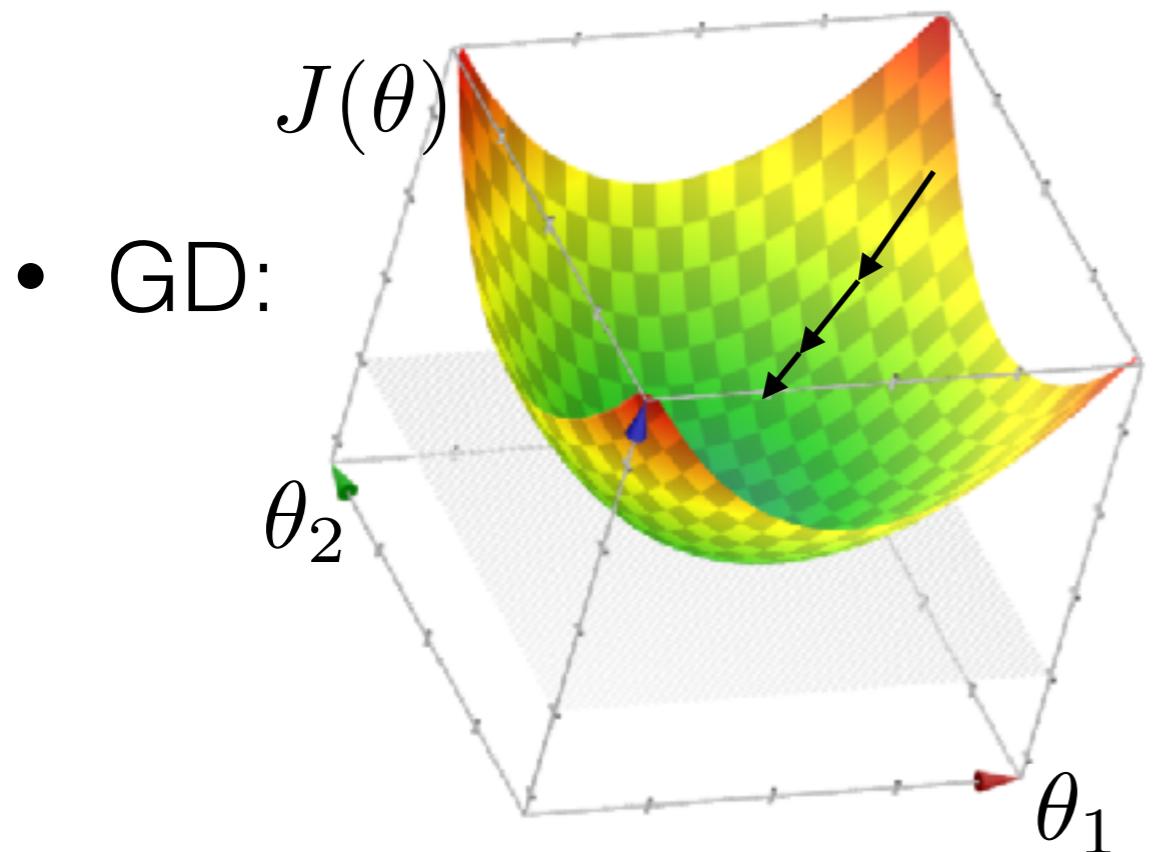
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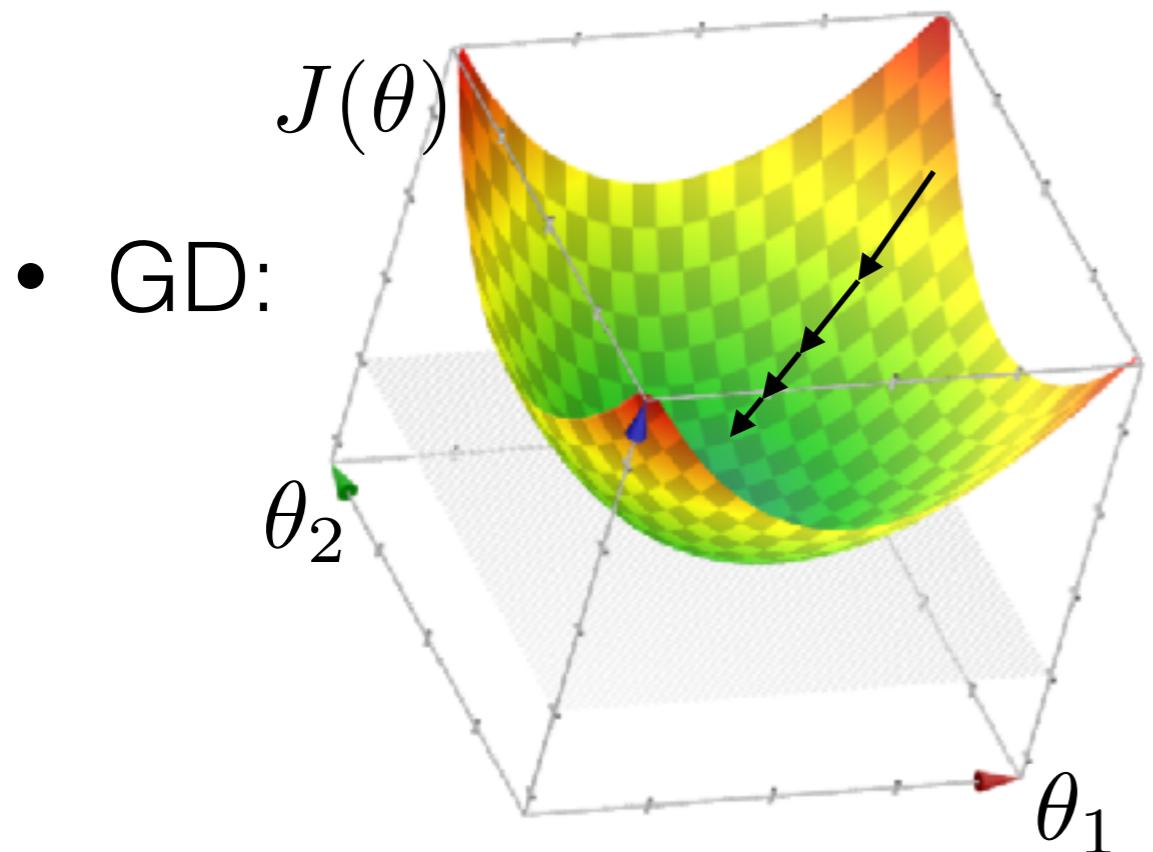
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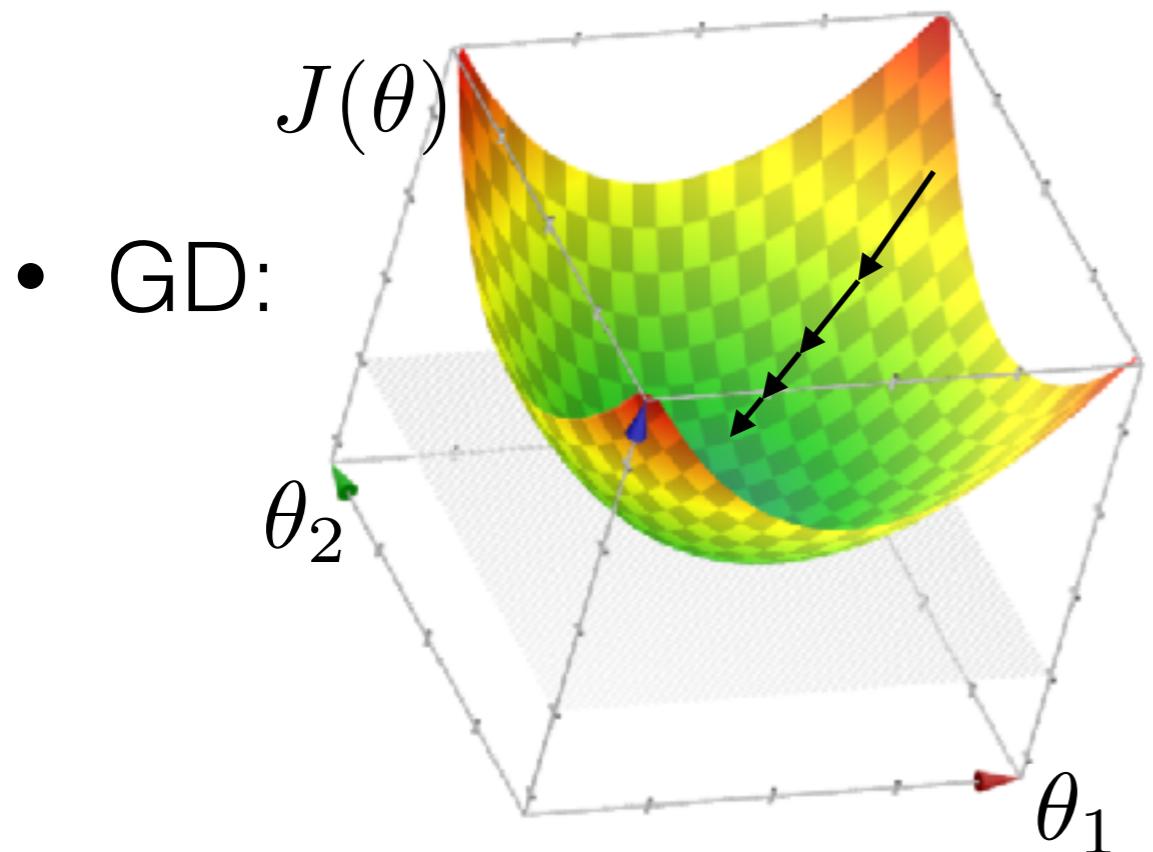
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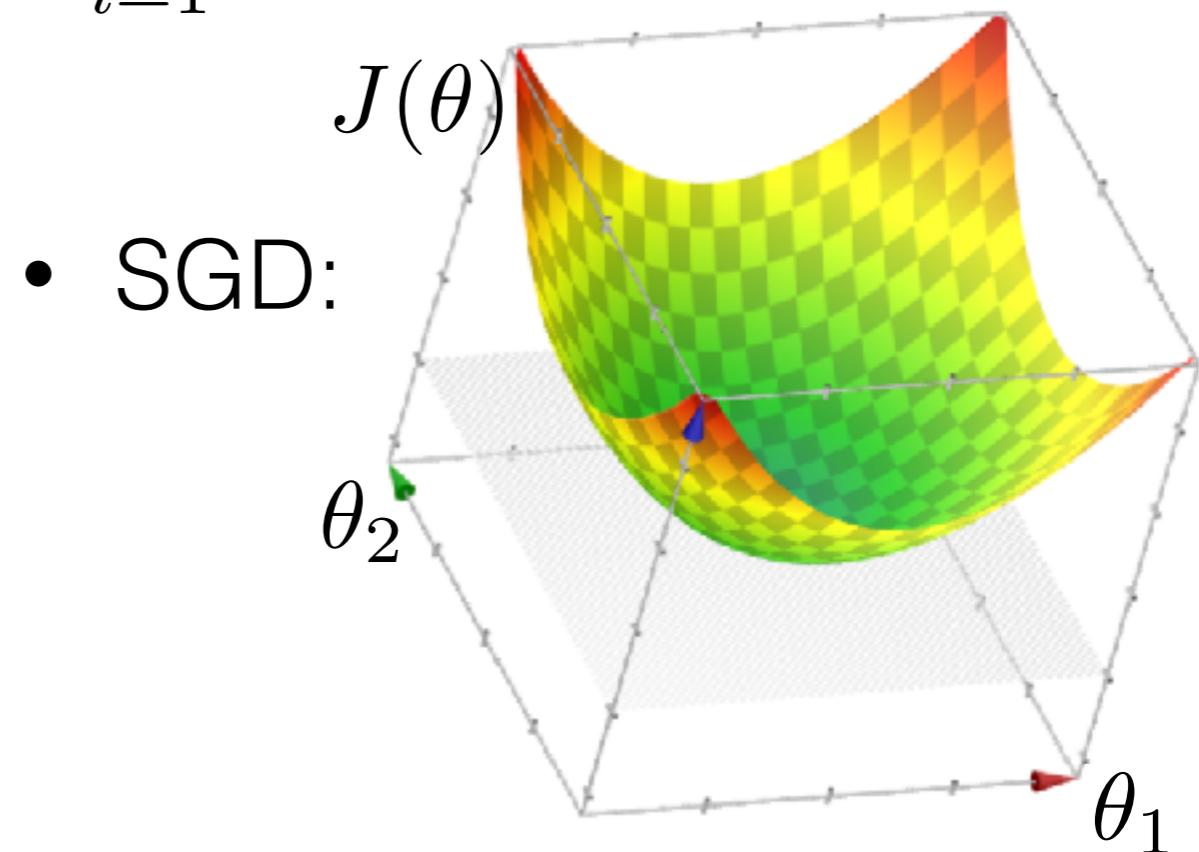
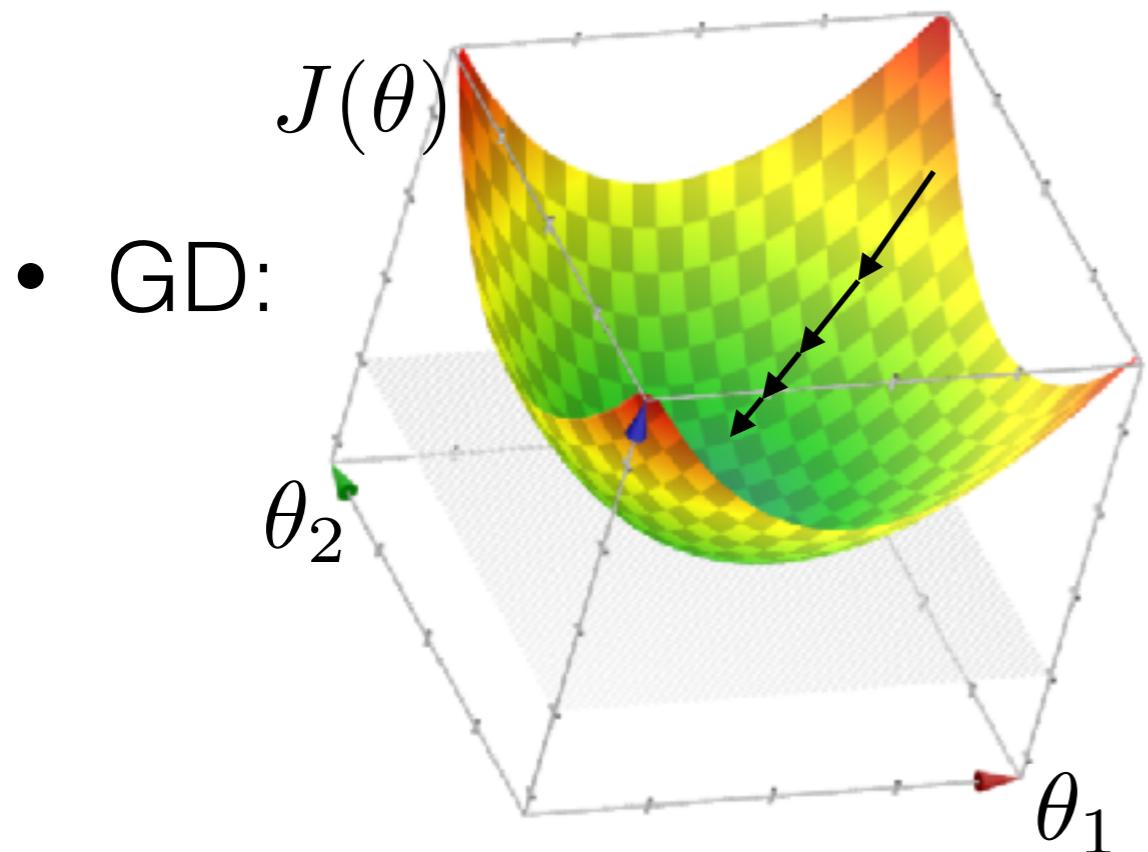
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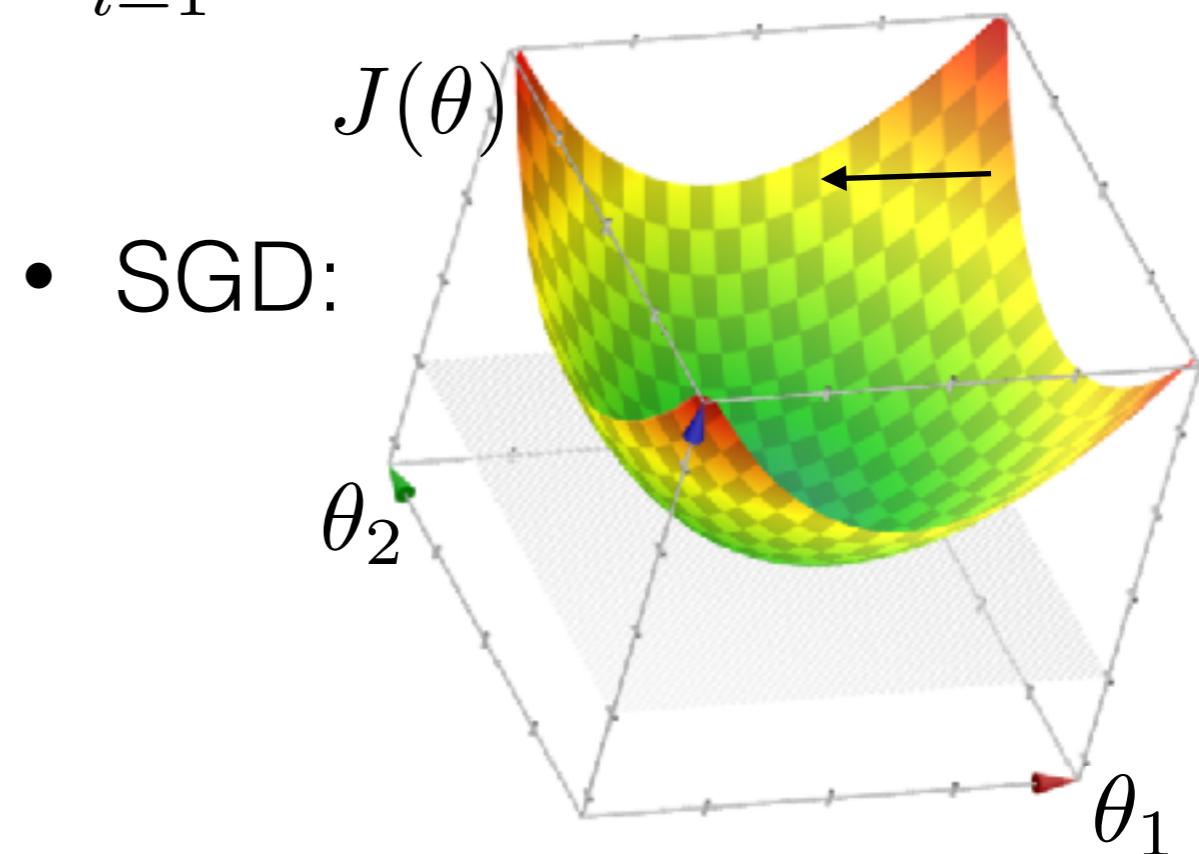
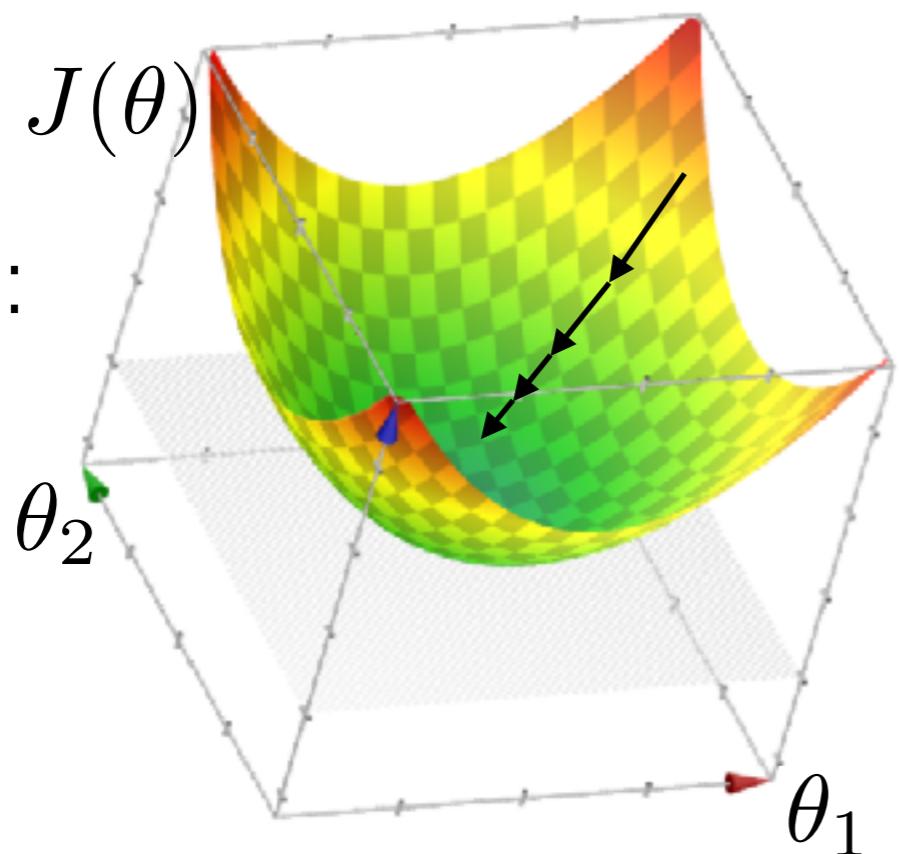
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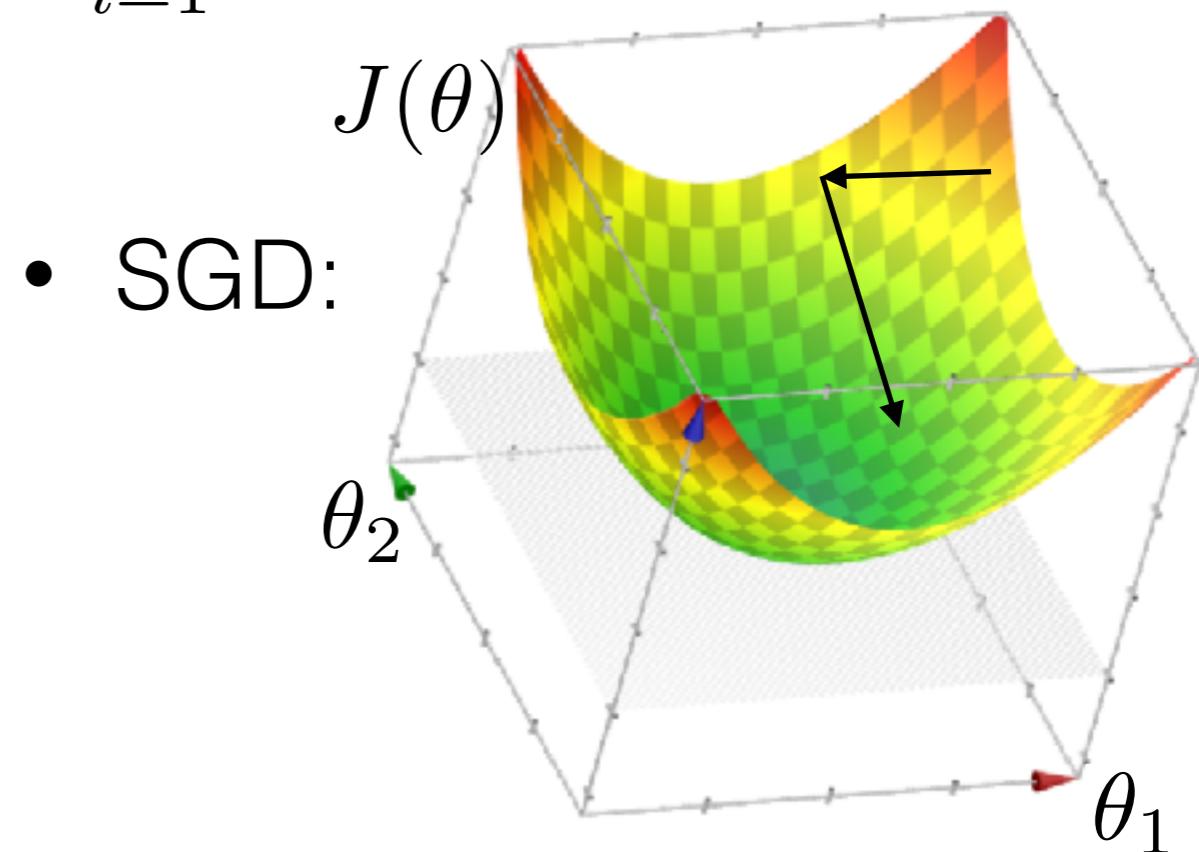
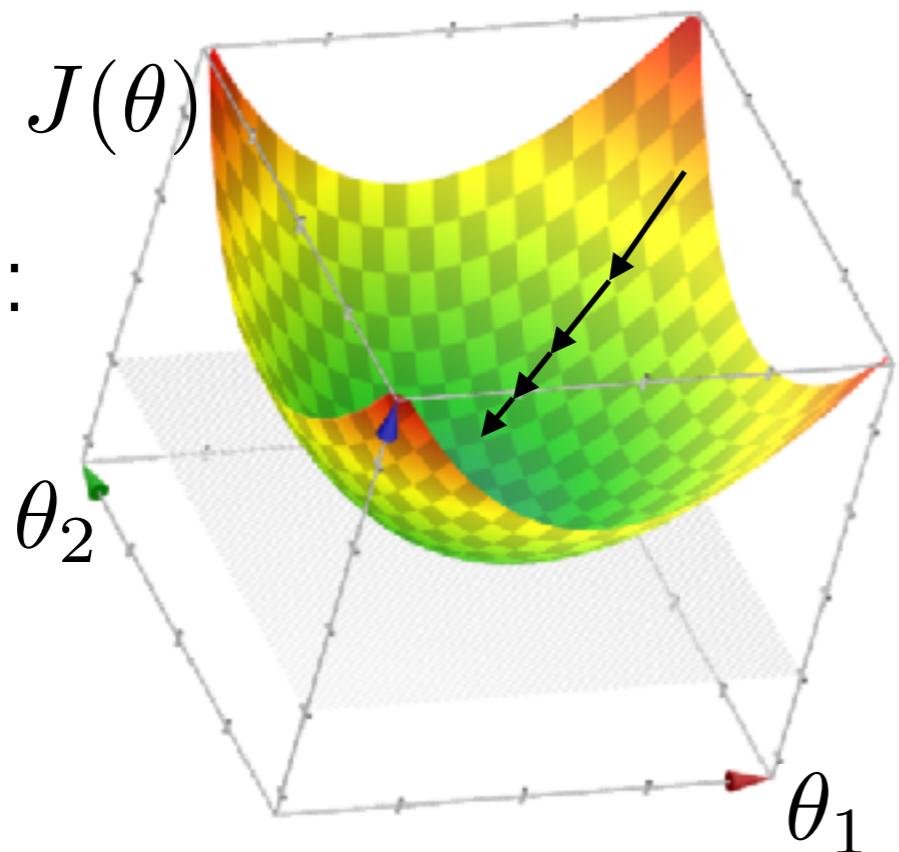
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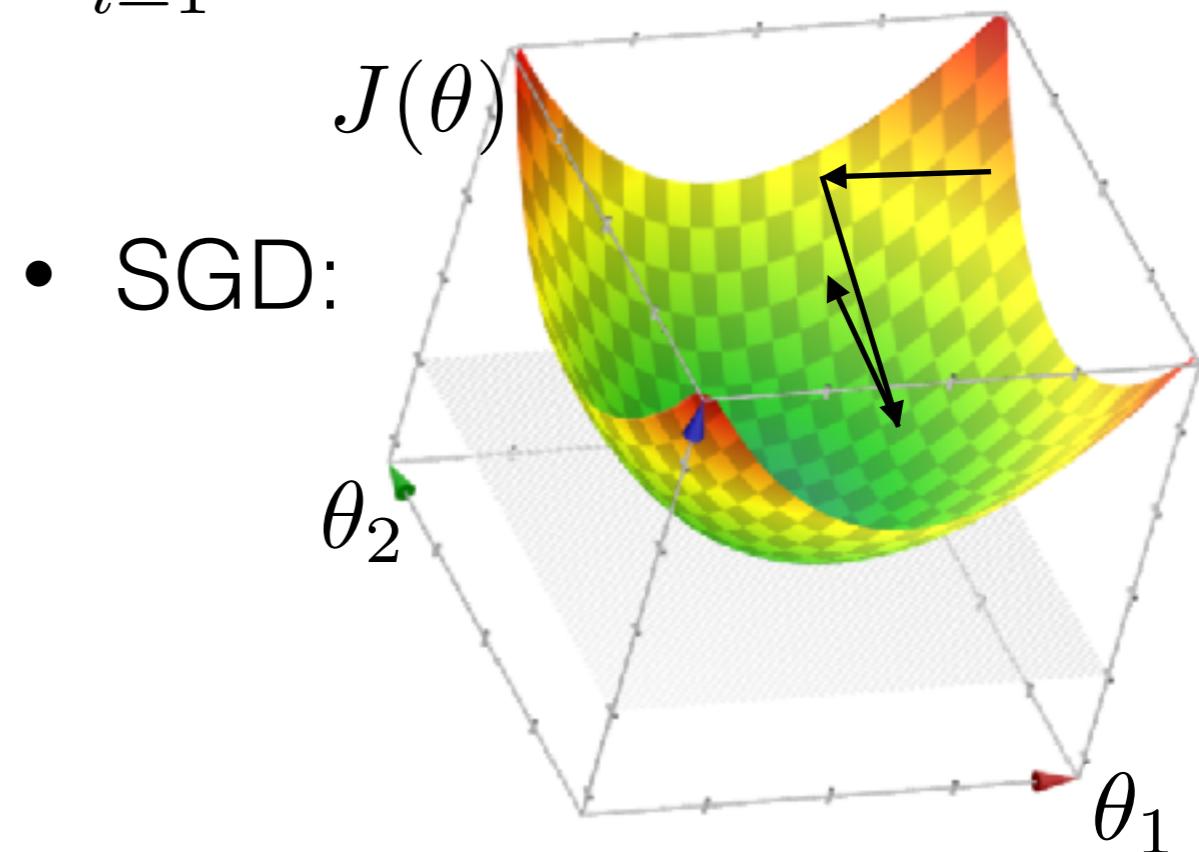
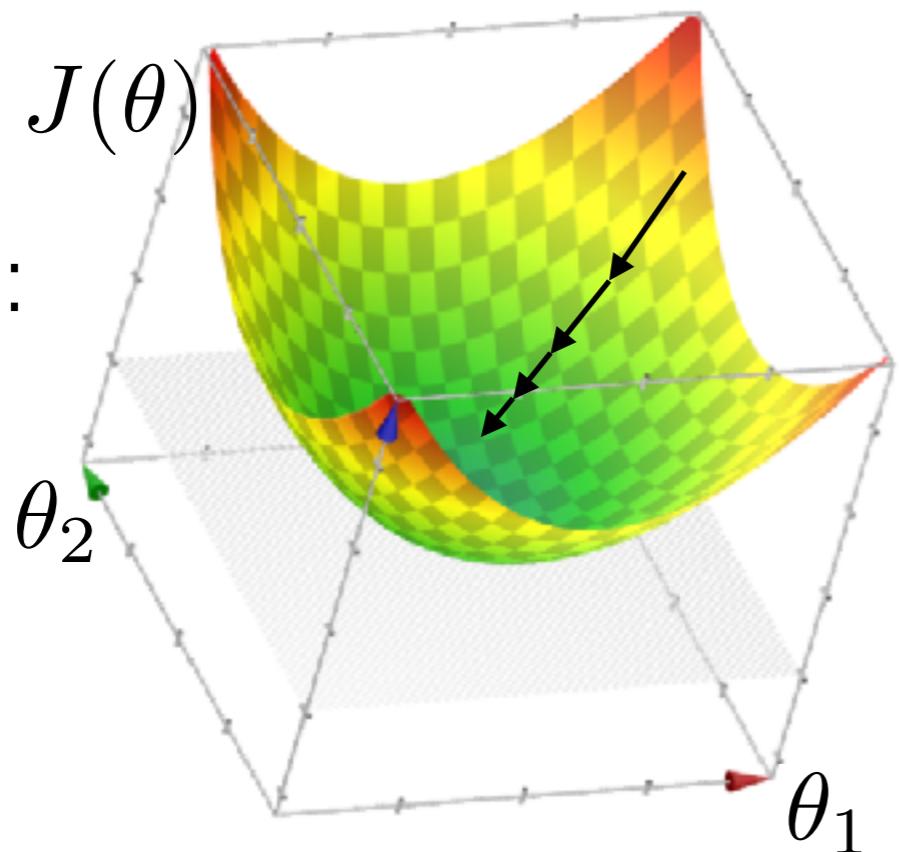
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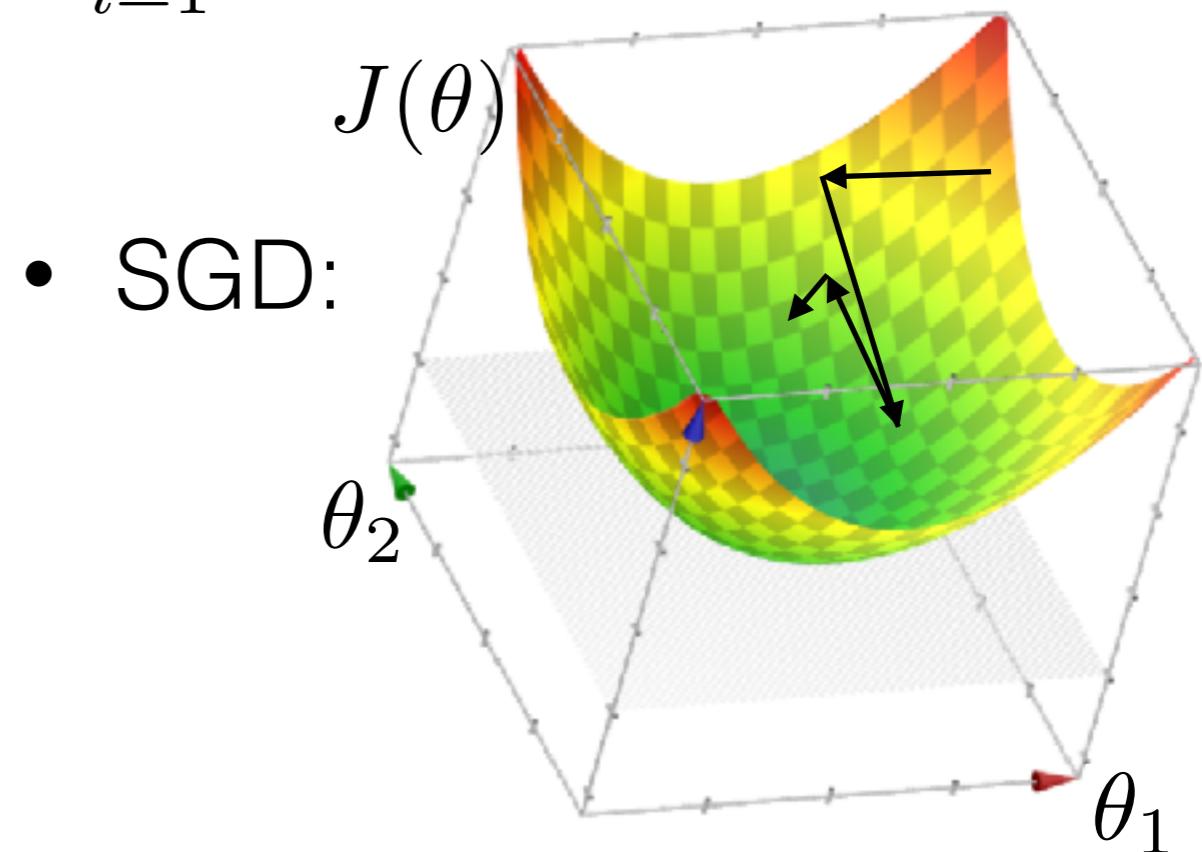
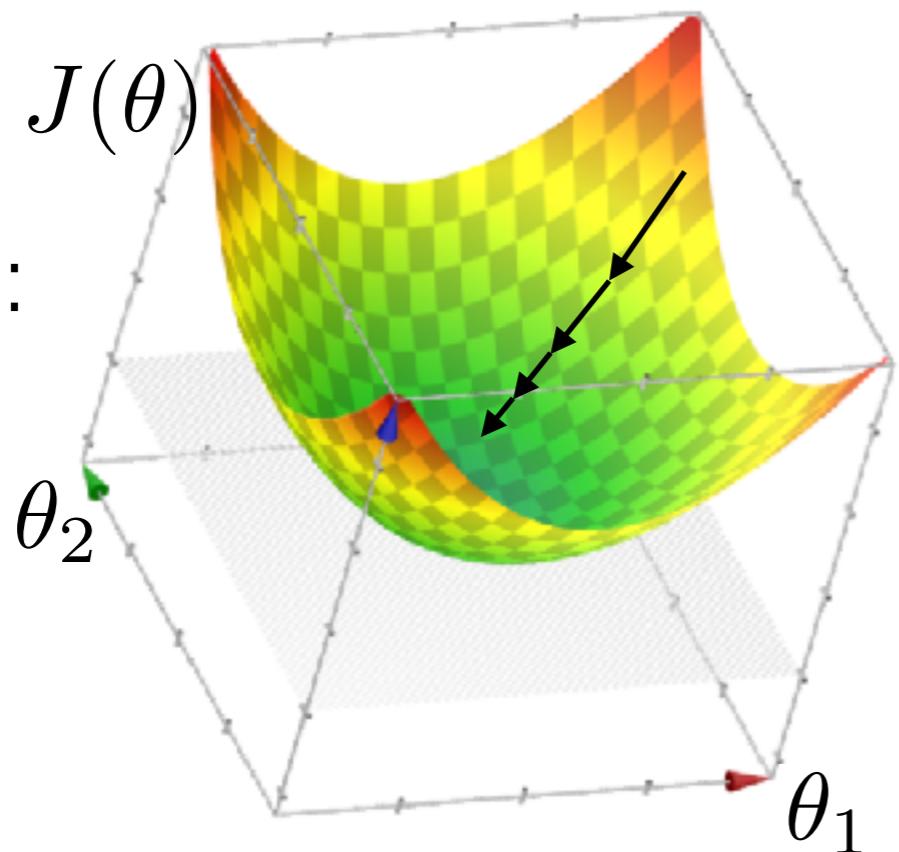
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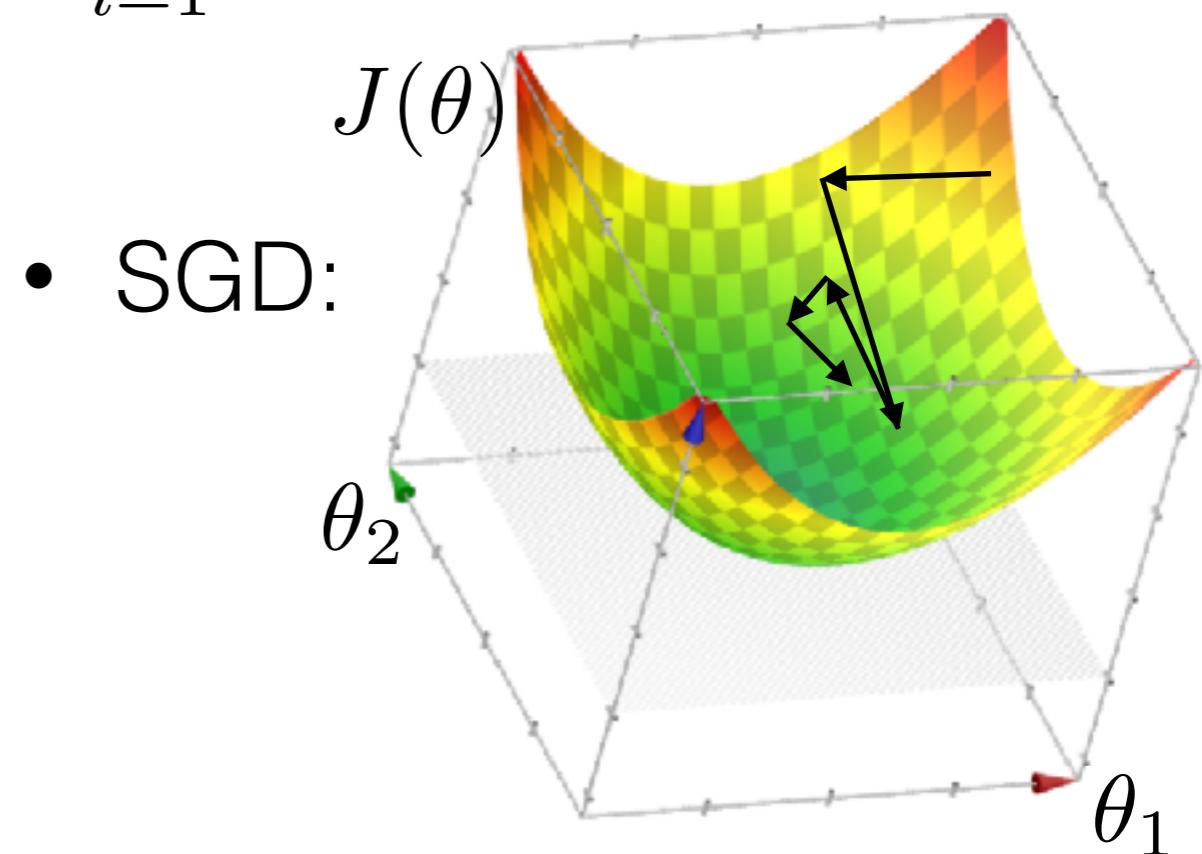
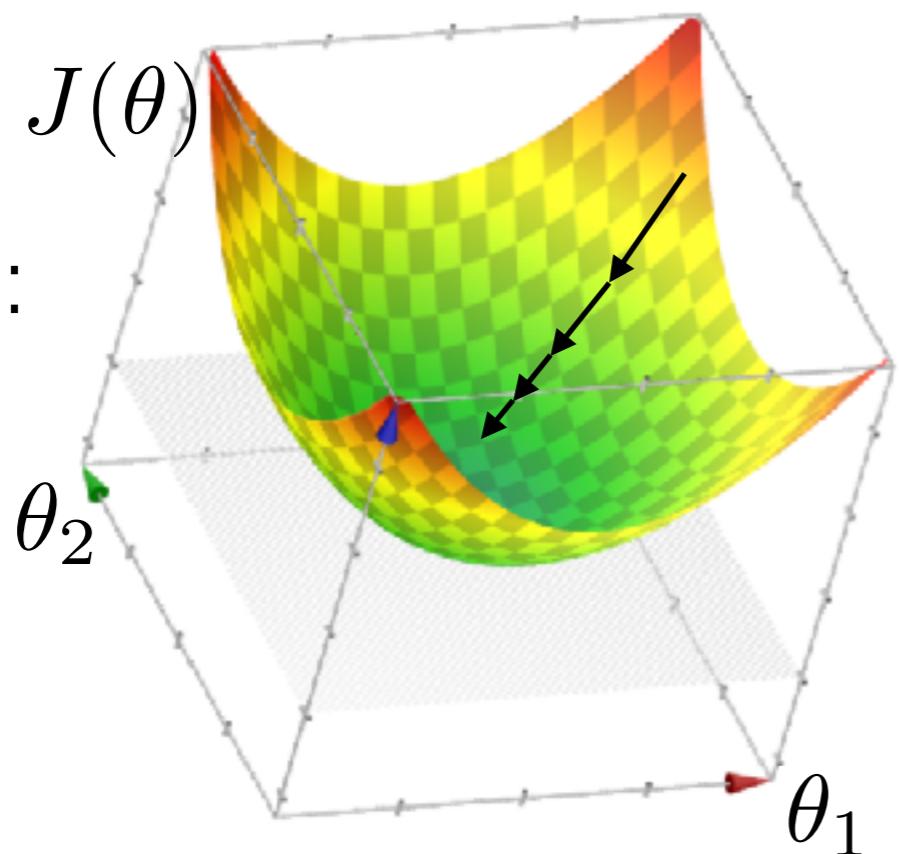
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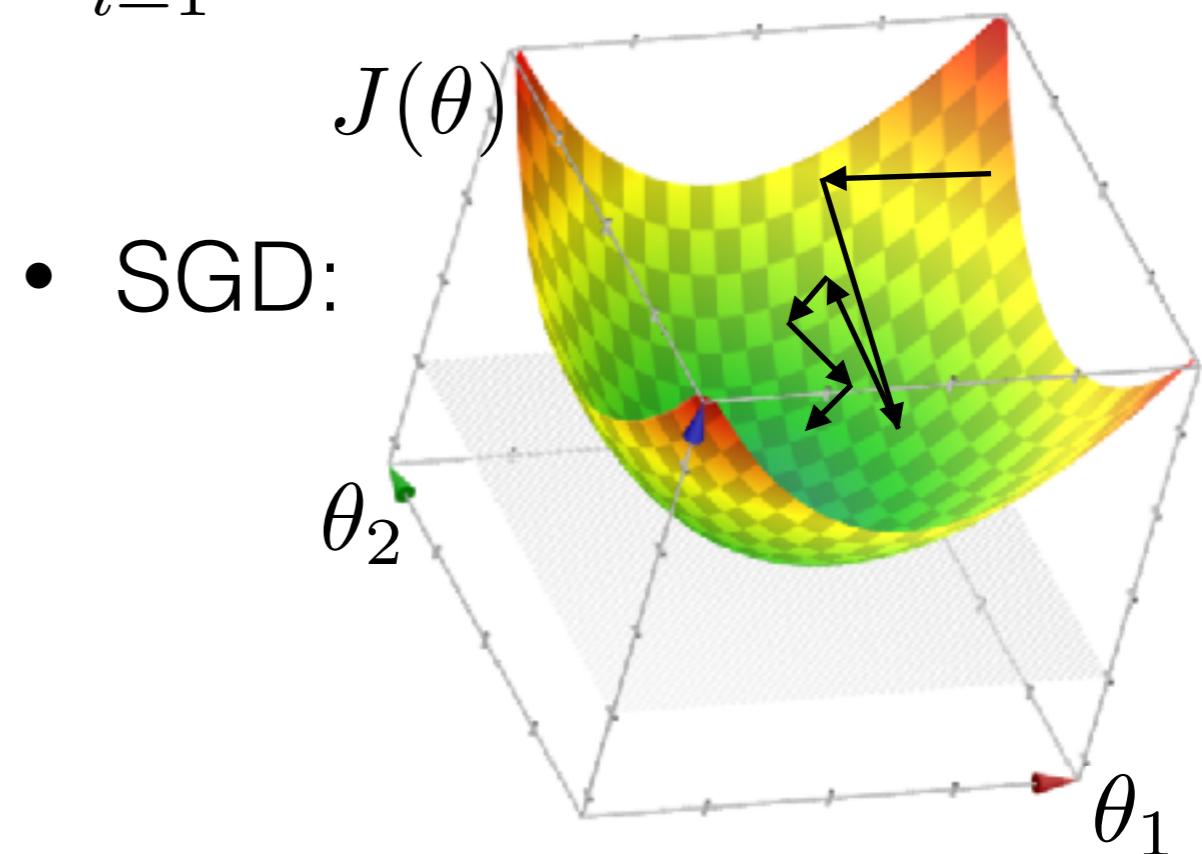
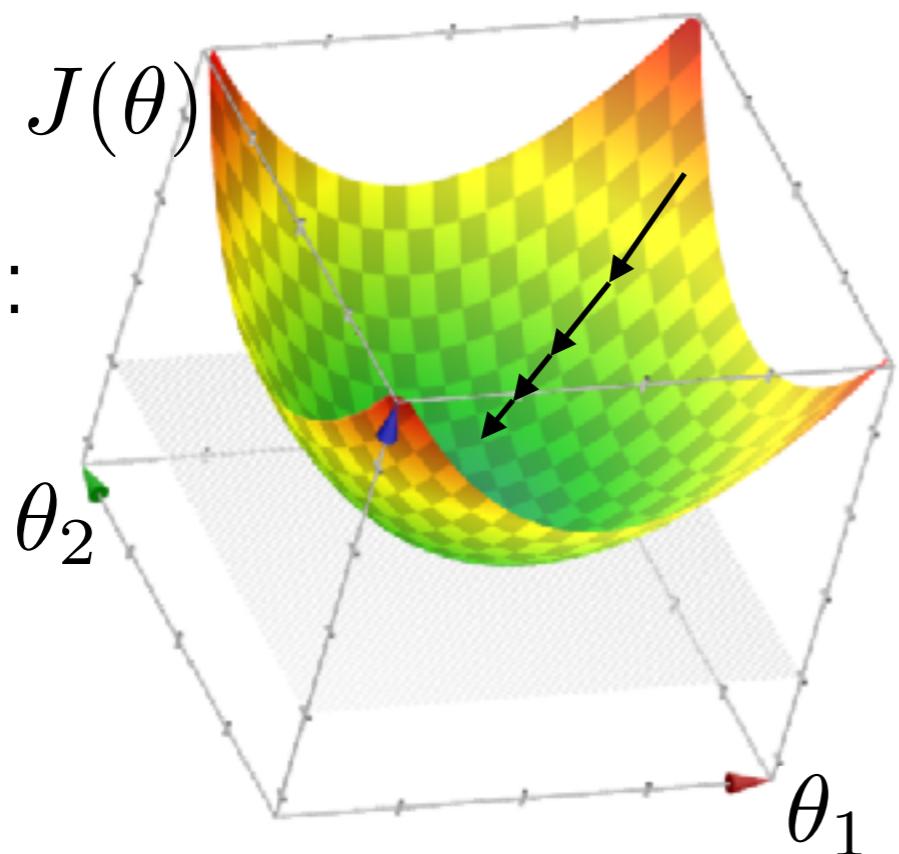
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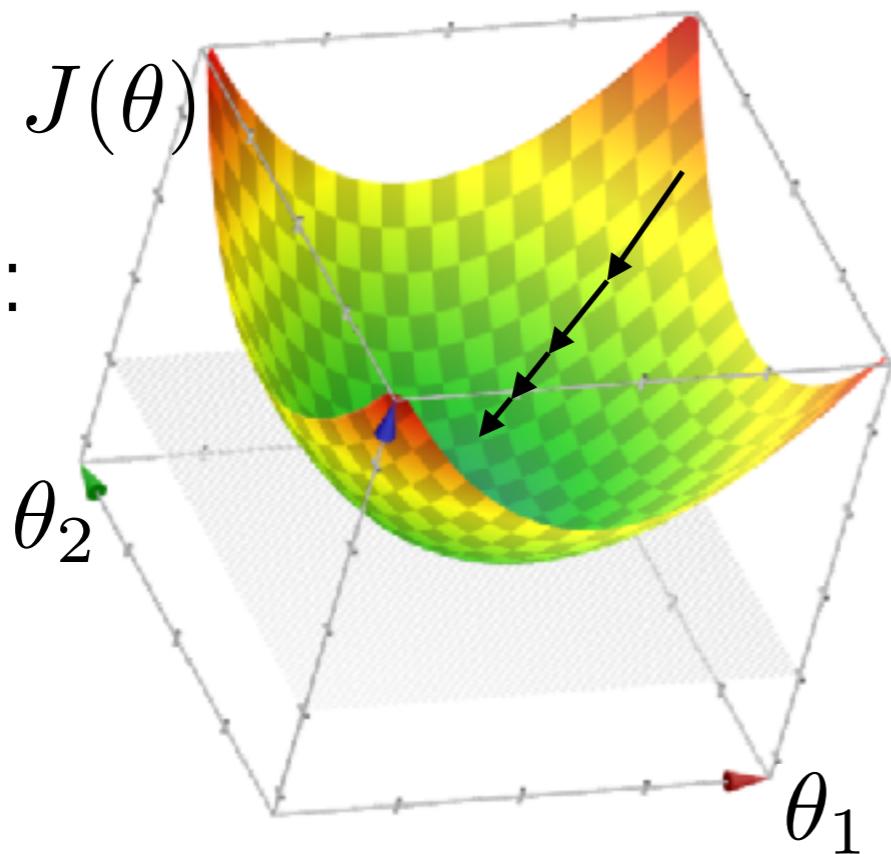
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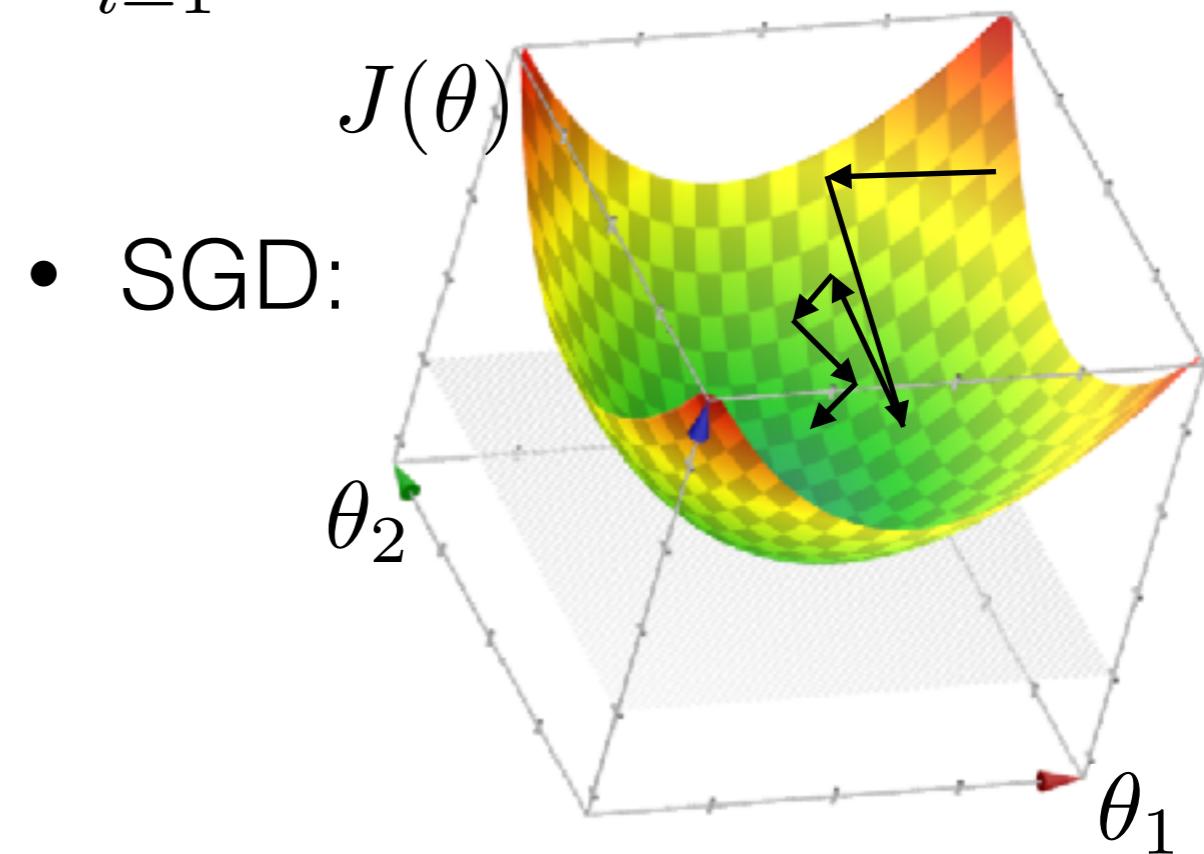


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- GD:

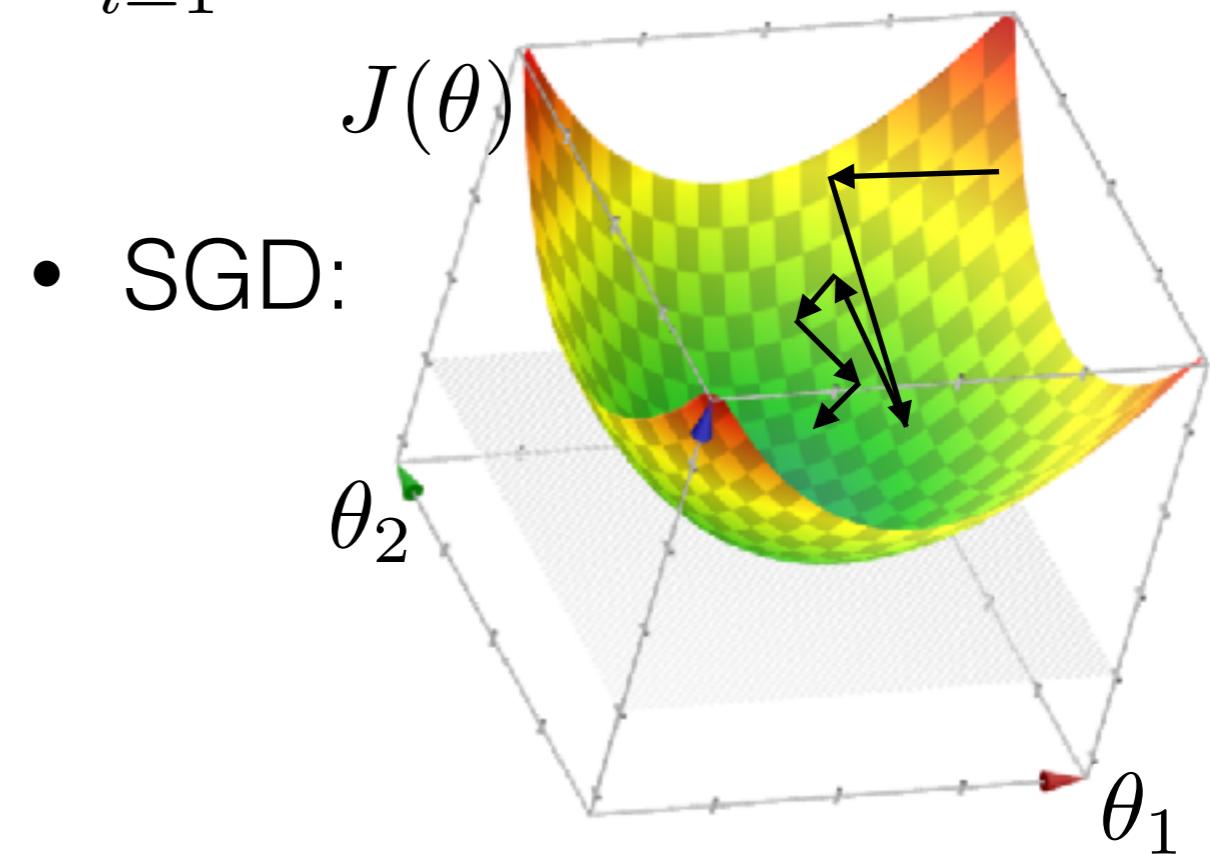
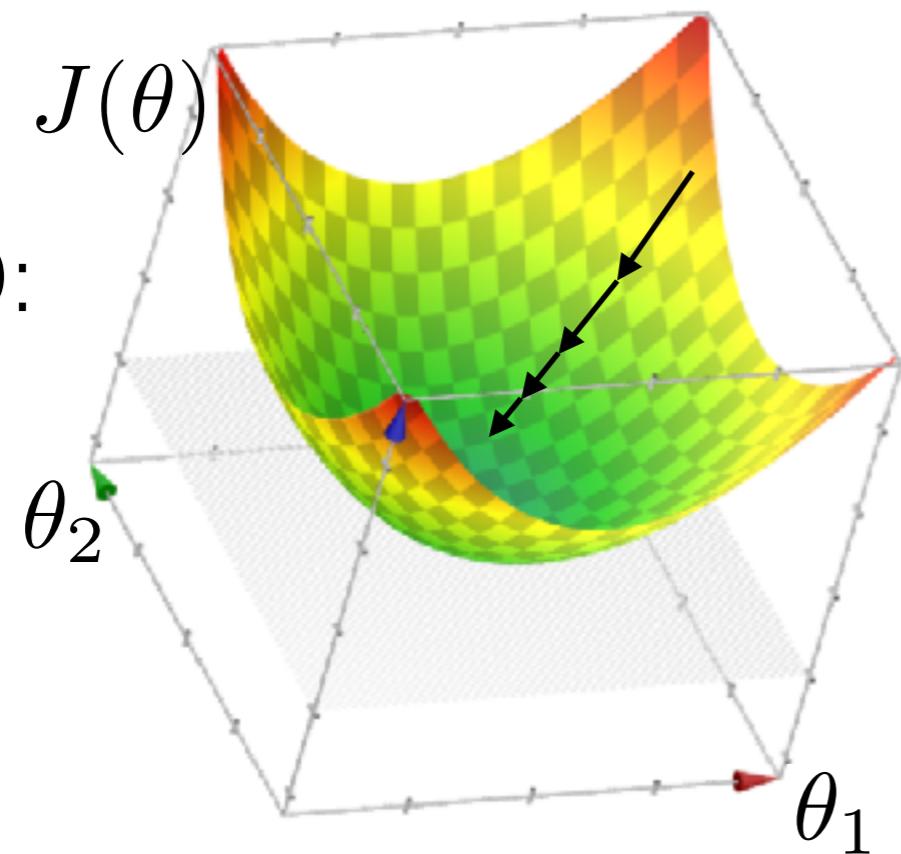


- SGD:

- **Theorem:** (Roughly) if the objective is nice and convex, GD and SGD perform well

# Learning the parameters

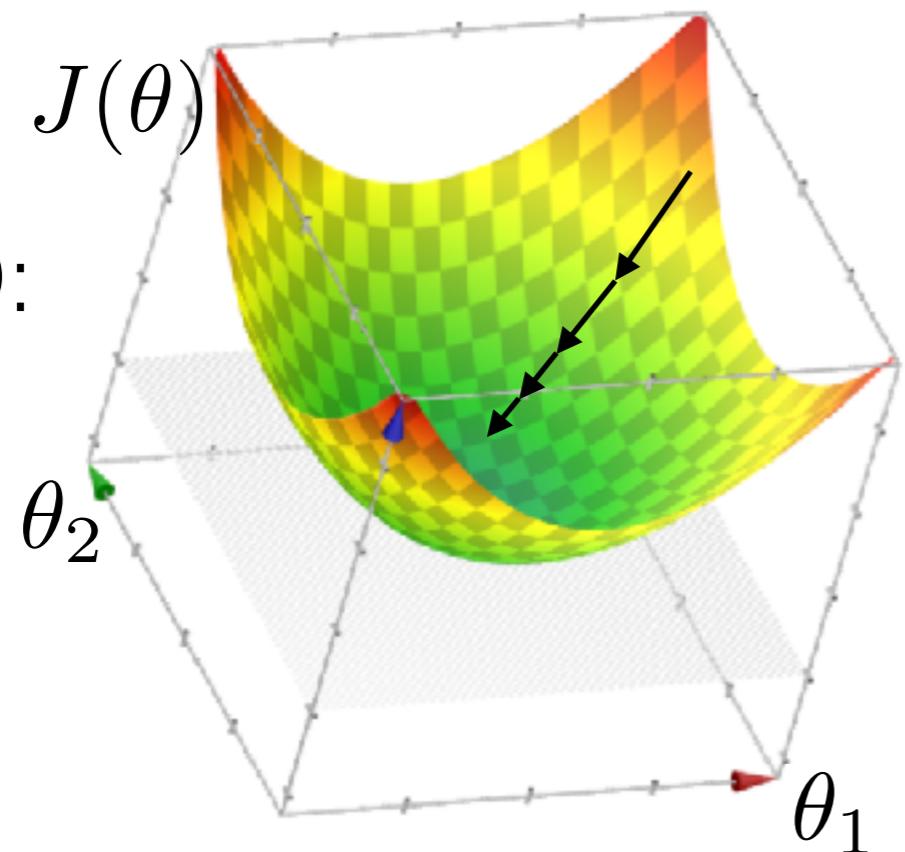
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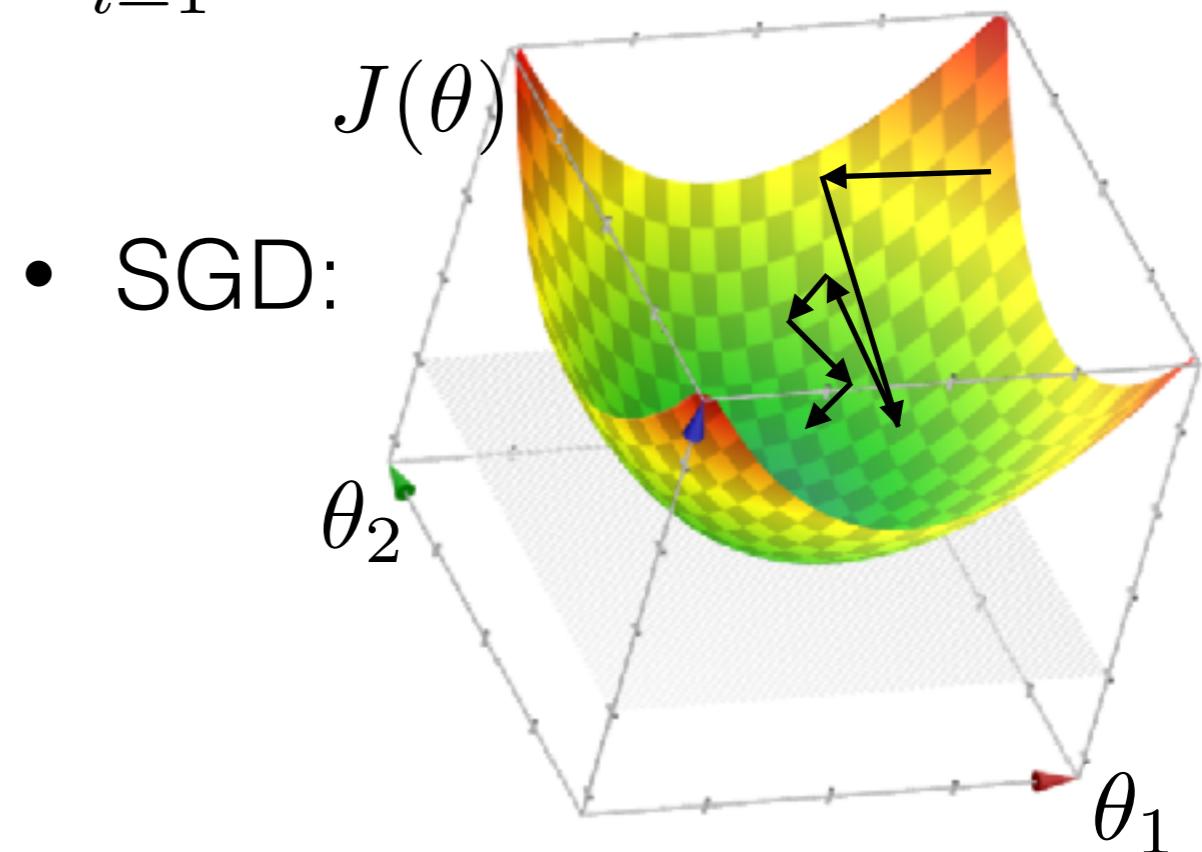
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- **Big challenge:** the NN objective is a (very) non-convex function of the parameters (except in e.g. 1 layer)

# Learning the parameters

- Objective:  $J(W, W_0) = \frac{1}{n_{\text{data}}} \sum_{i=1}^{n_{\text{data}}} L(h(x^{(i)}; W, W_0), y^{(i)})$



- GD:



- SGD:

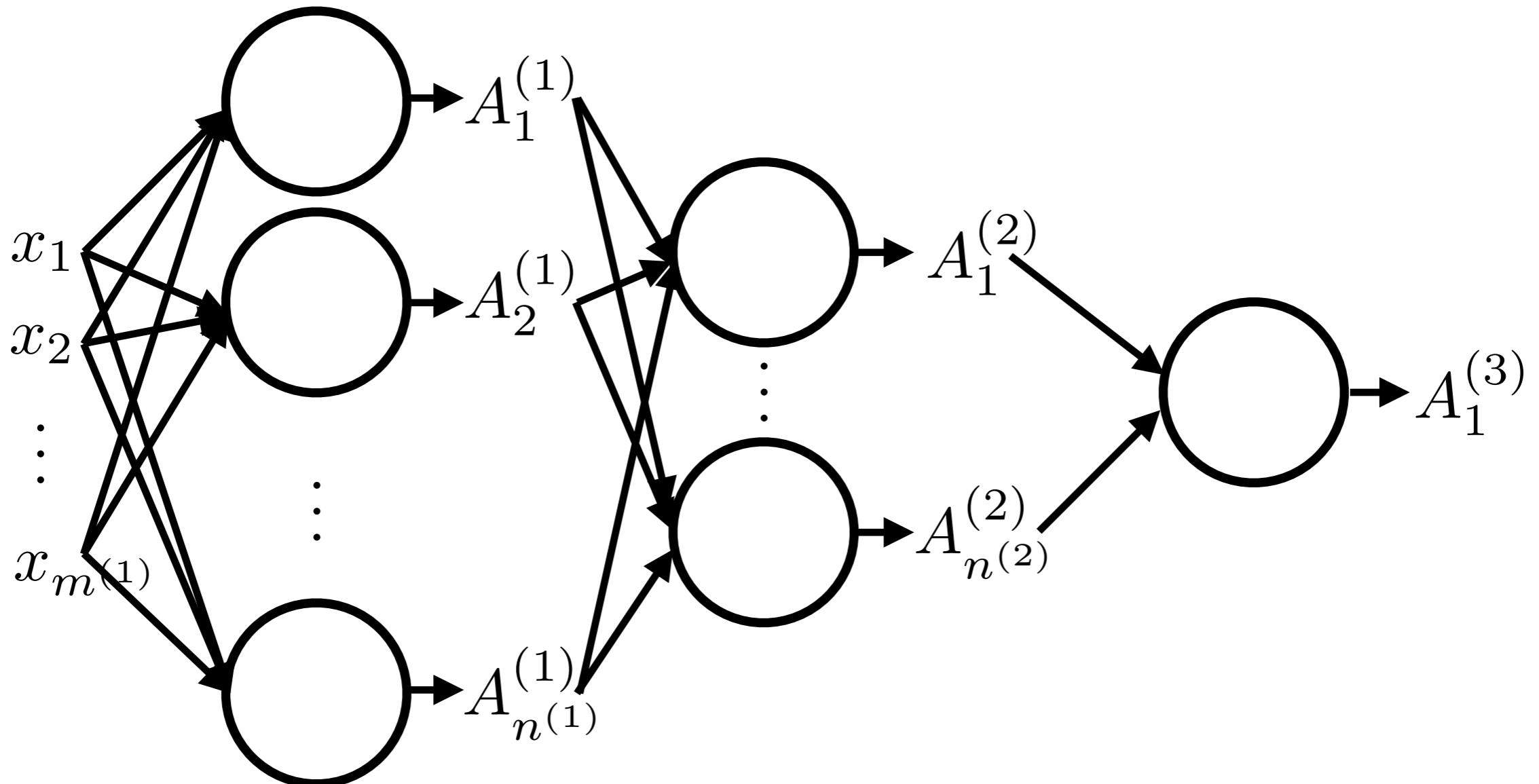
- **Theorem:** (Roughly) if the objective is nice and convex, GD and SGD perform well
- **Big challenge:** the NN objective is a (very) non-convex function of the parameters (except in e.g. 1 layer)
- Huge bag of tricks to optimize / regularize

# More layers!

- Why stop at 2 layers?

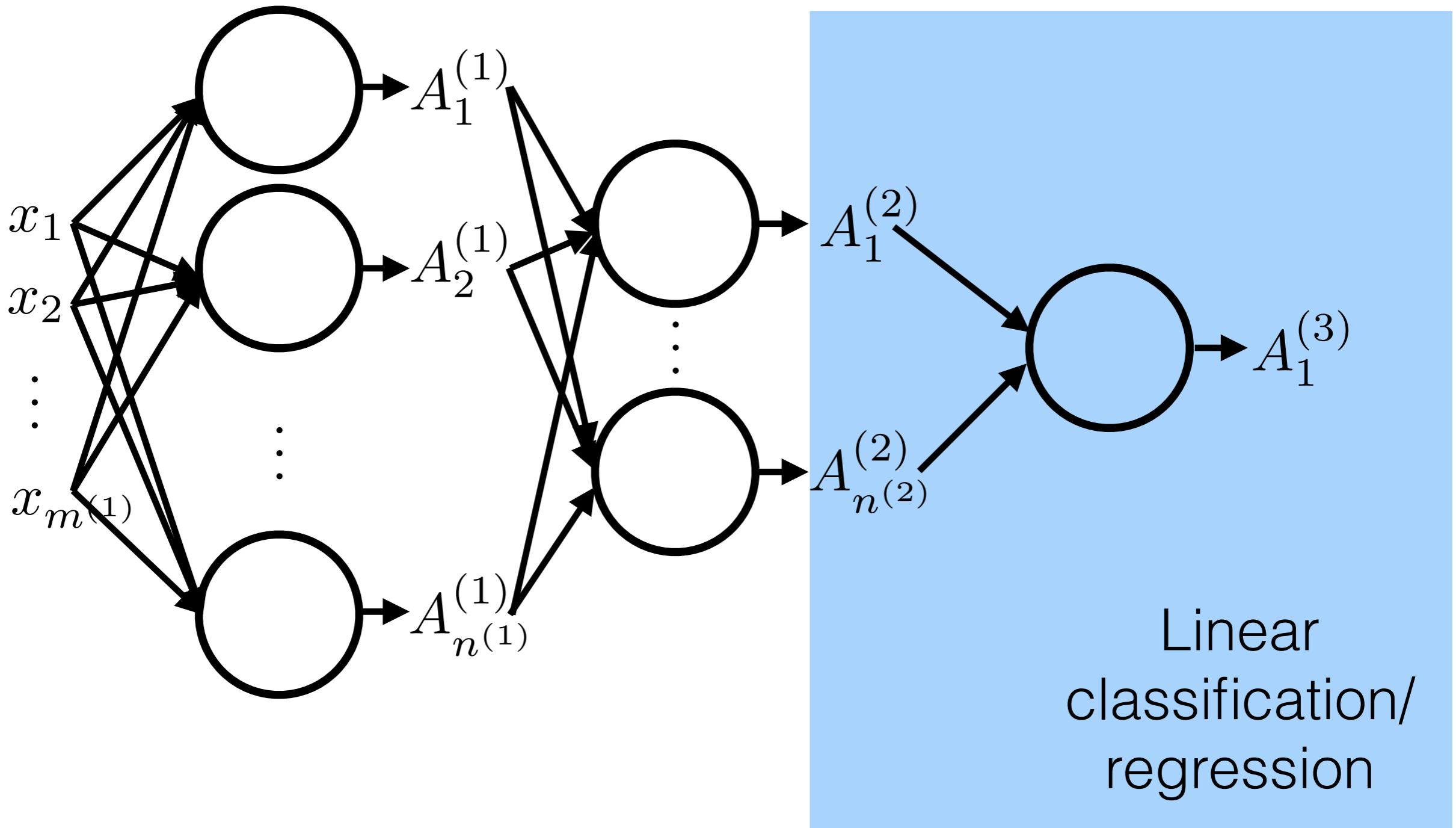
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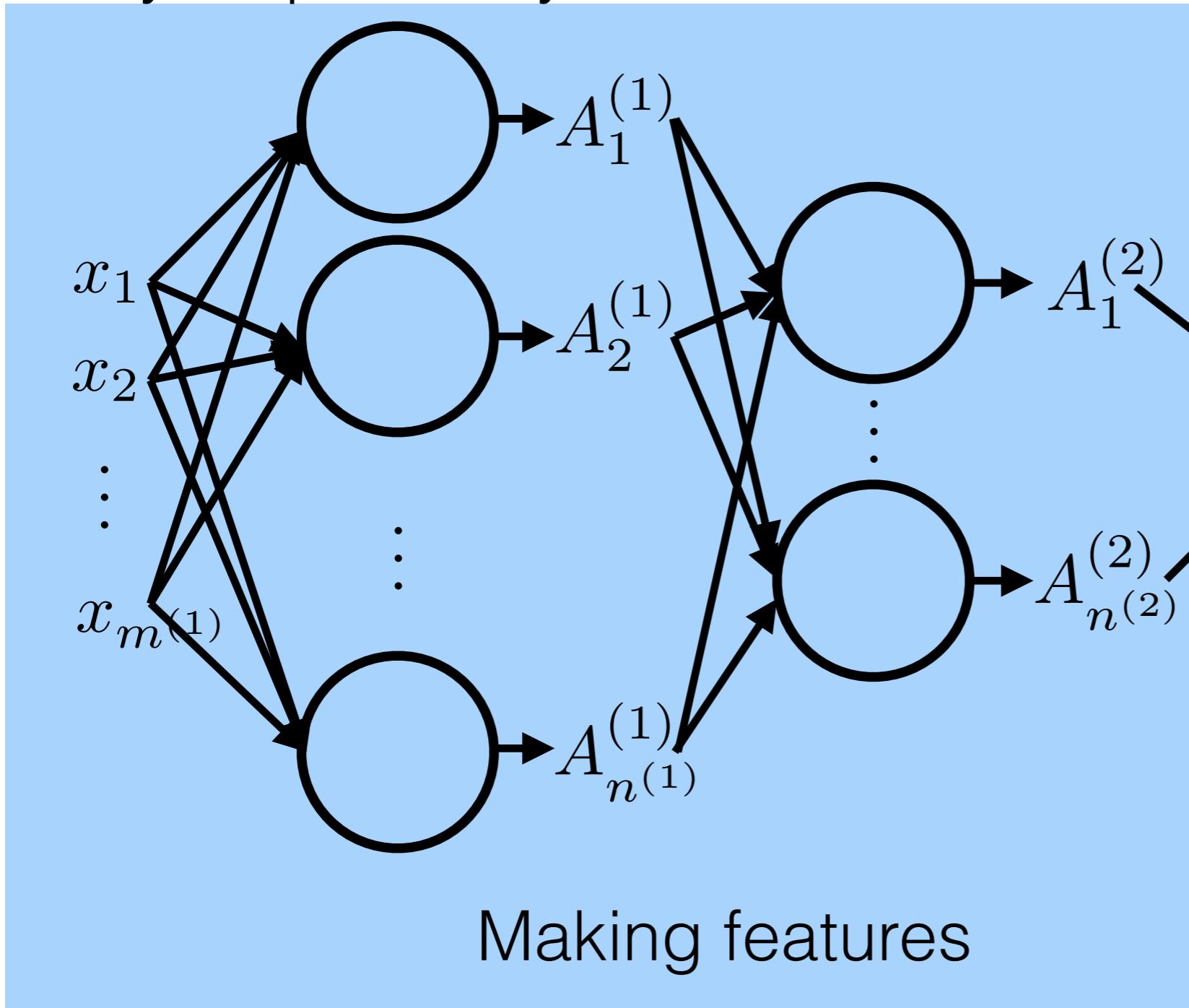
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# More layers!

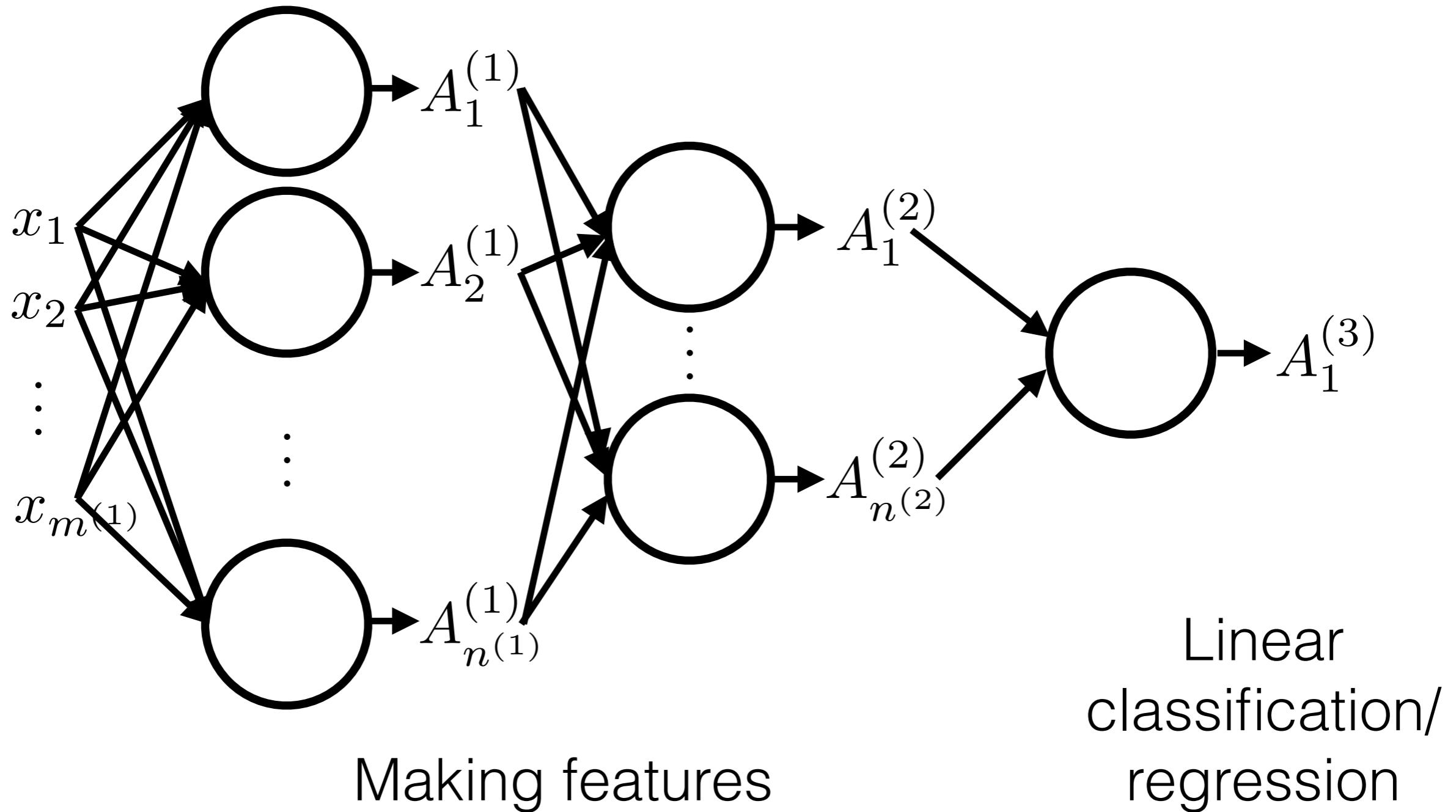
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Linear  
classification/  
regression

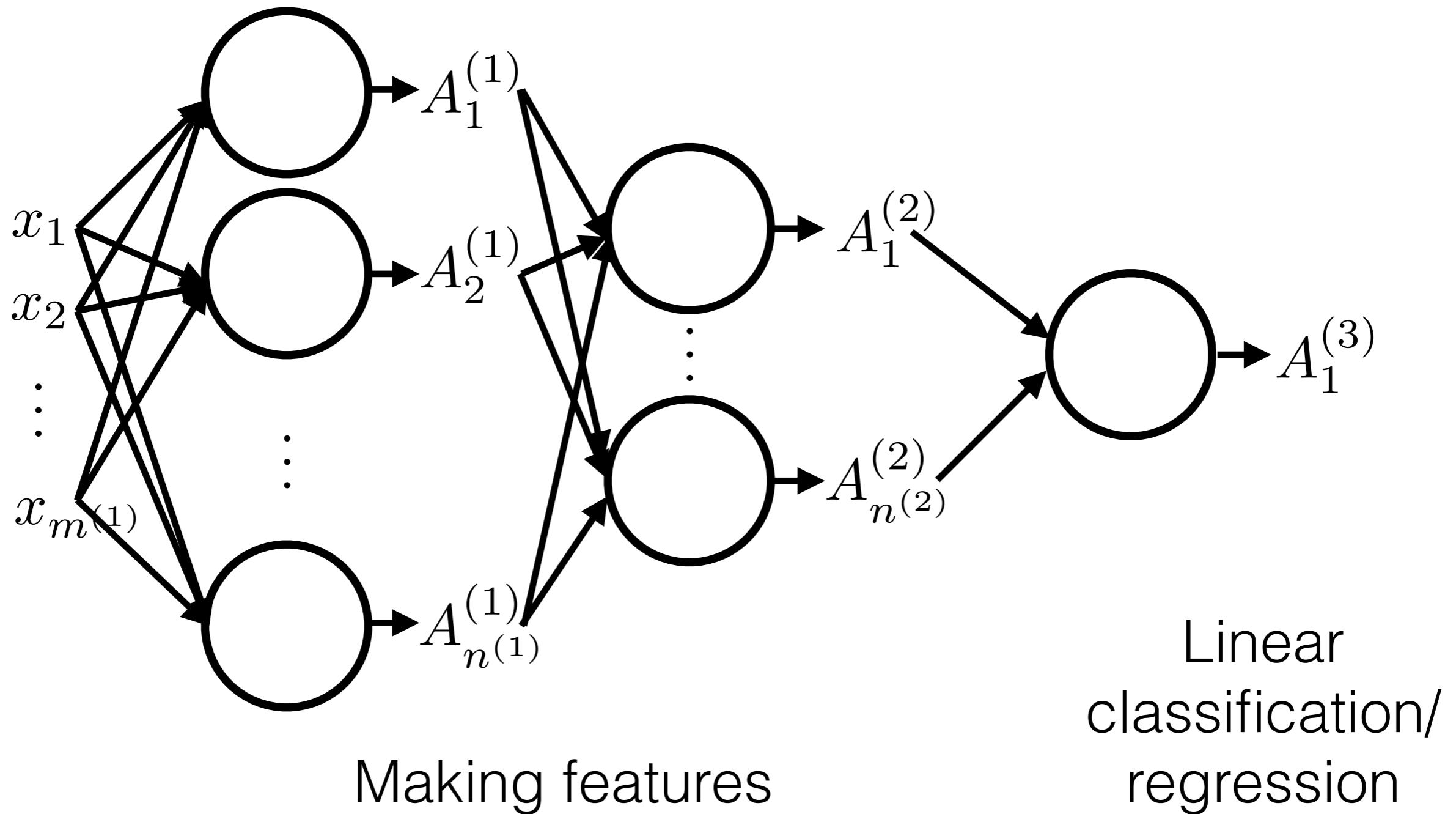
# More layers!

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# More layers!

- Why stop at 2 layers?



- Just one layer: linear classification/regression with default features