Automated Scalable Bayesian Inference via Data Summarization

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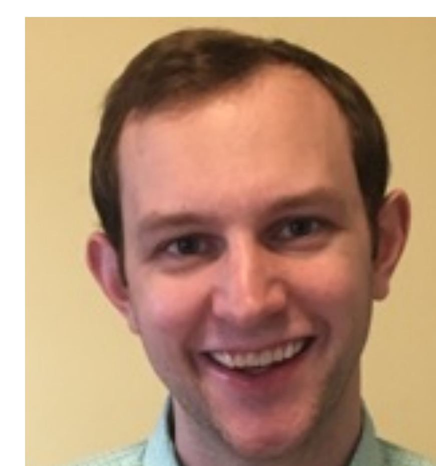
MIT

With: Trevor Campbell, Jonathan H. Huggins



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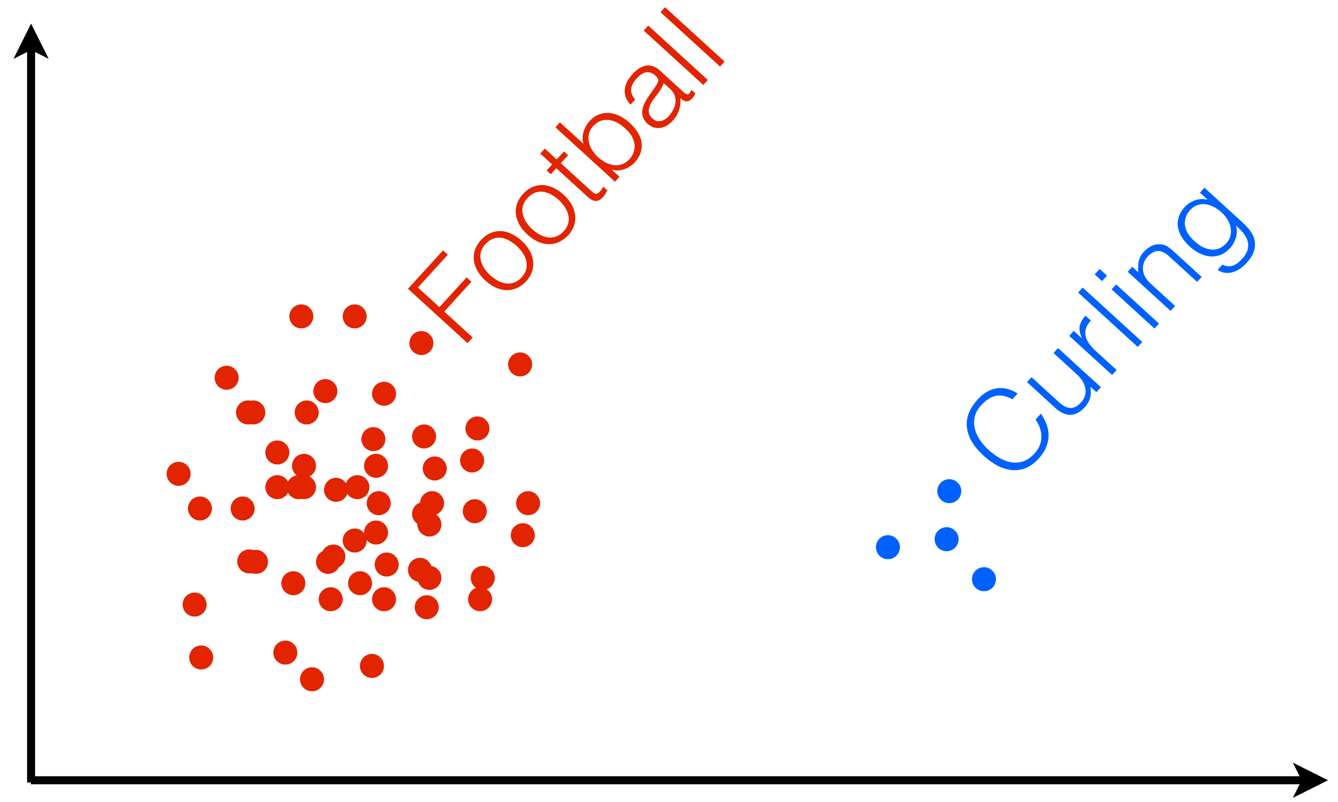

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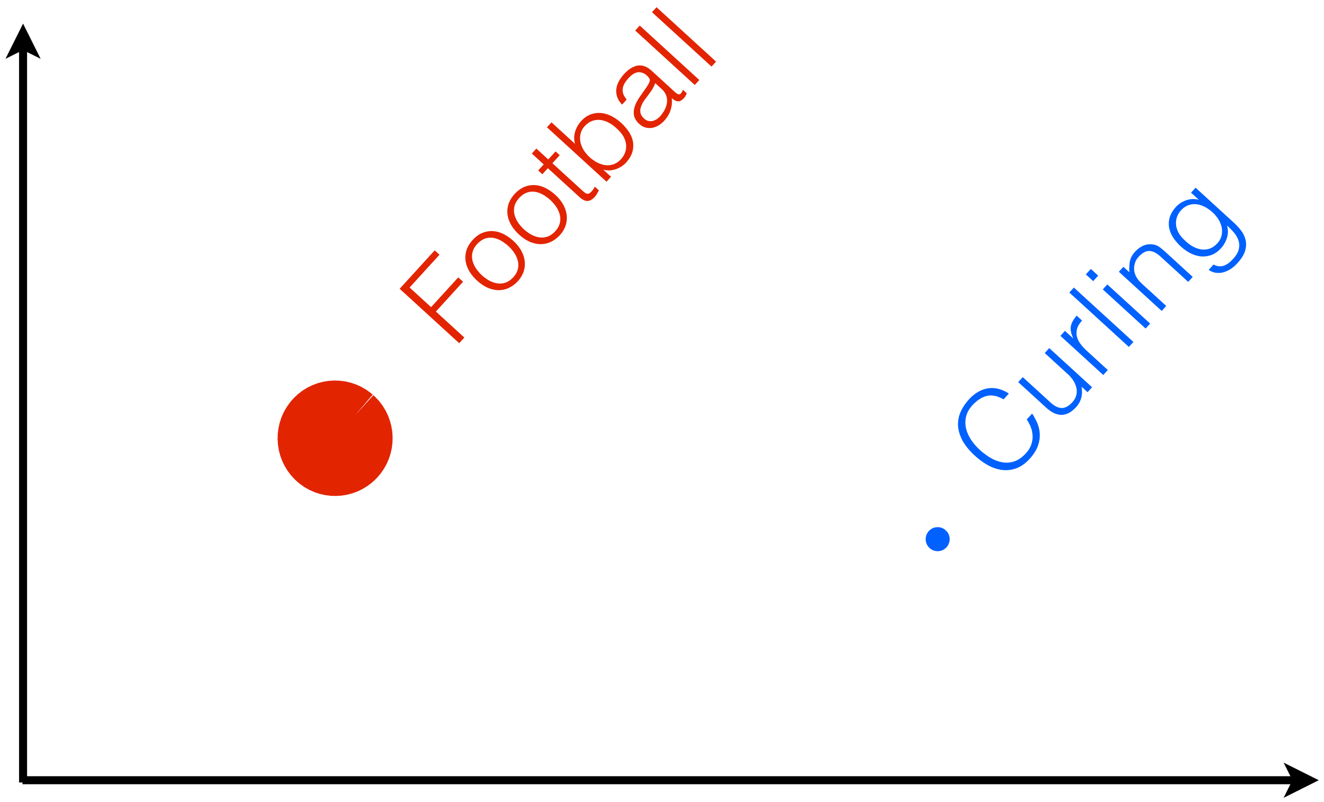

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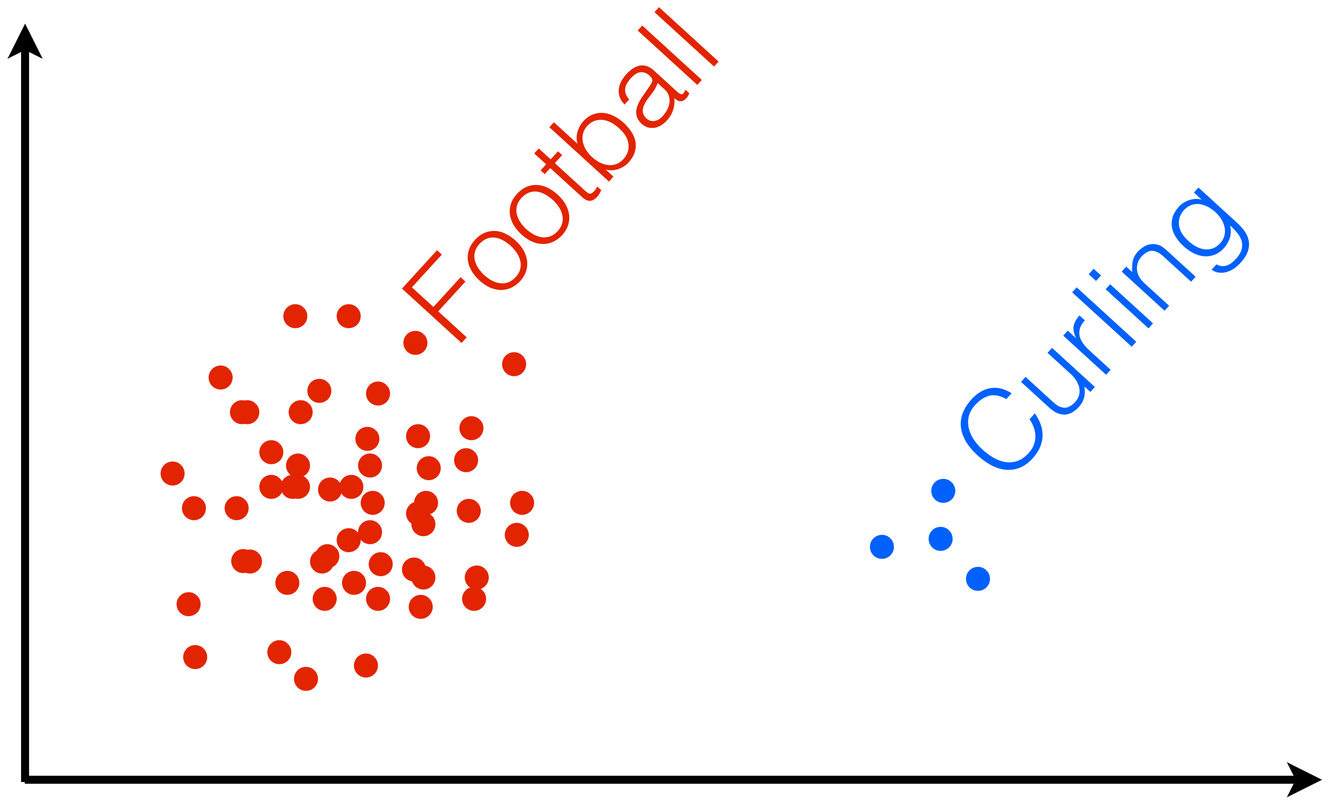

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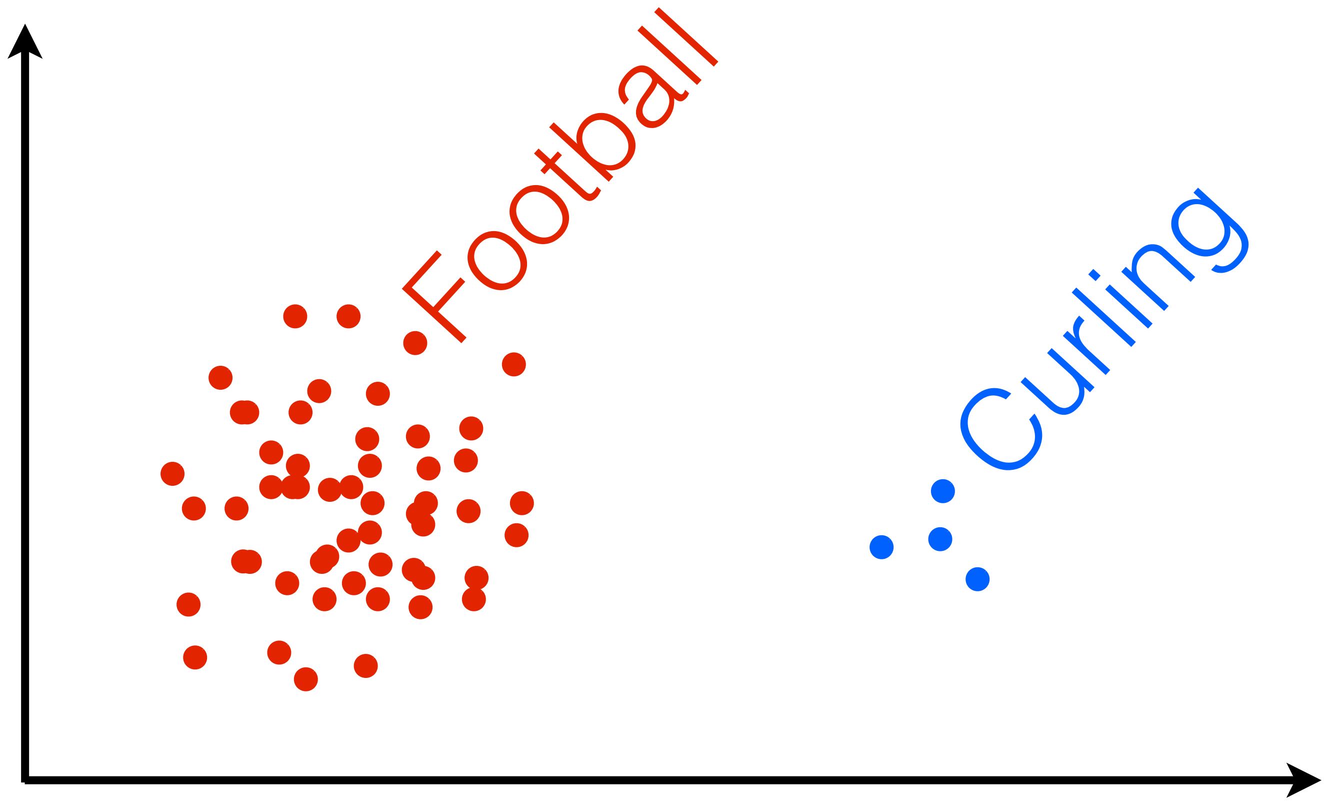

[Bădoiu, Har-Peled, Indyk 2002; Agarwal et al 2005; Feldman & Langberg 2011]

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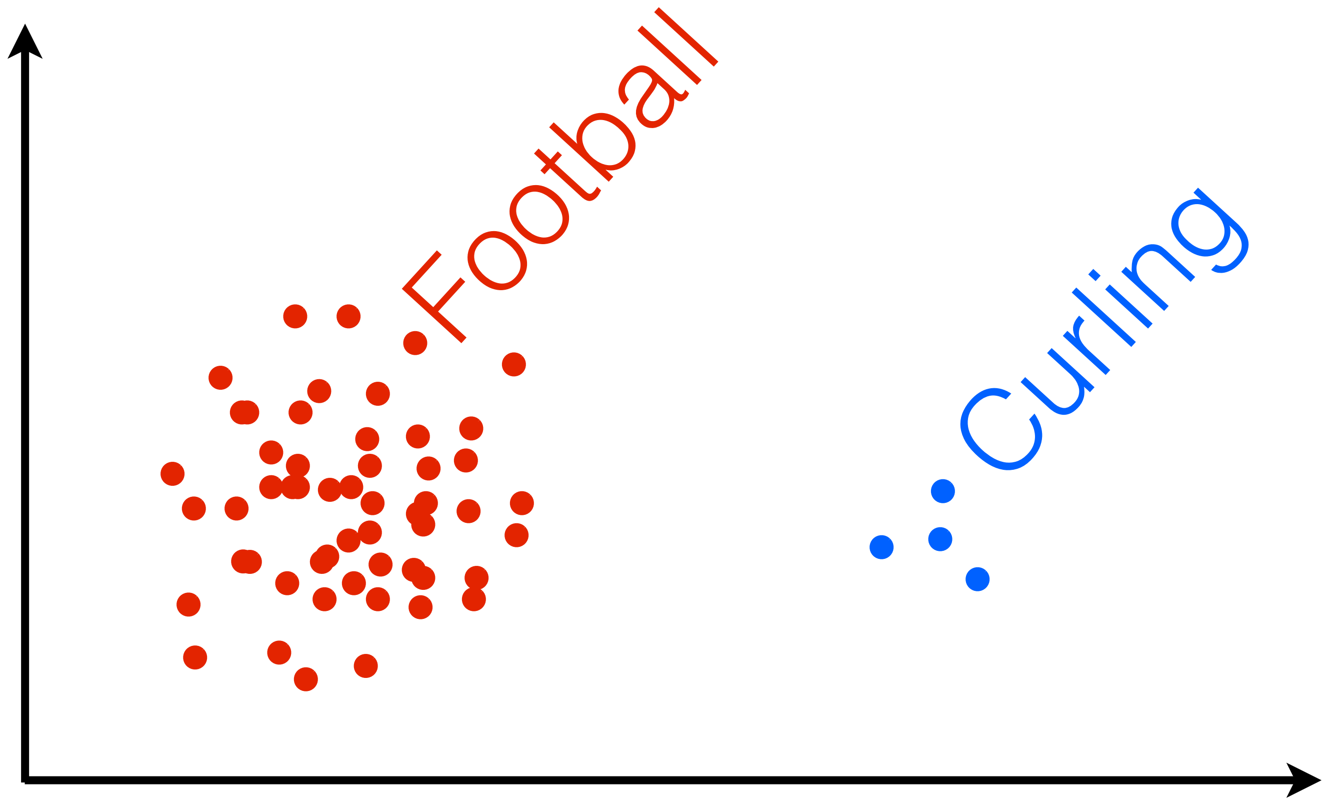

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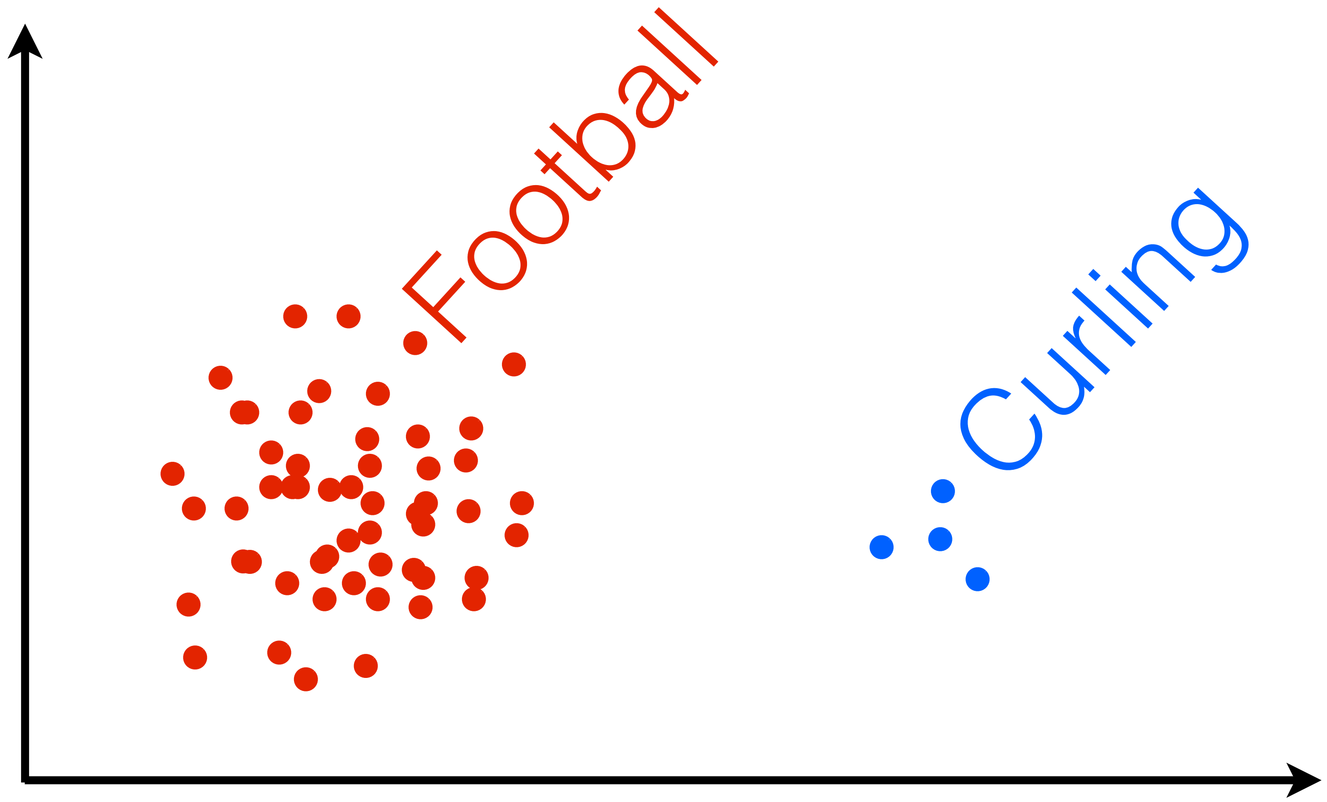

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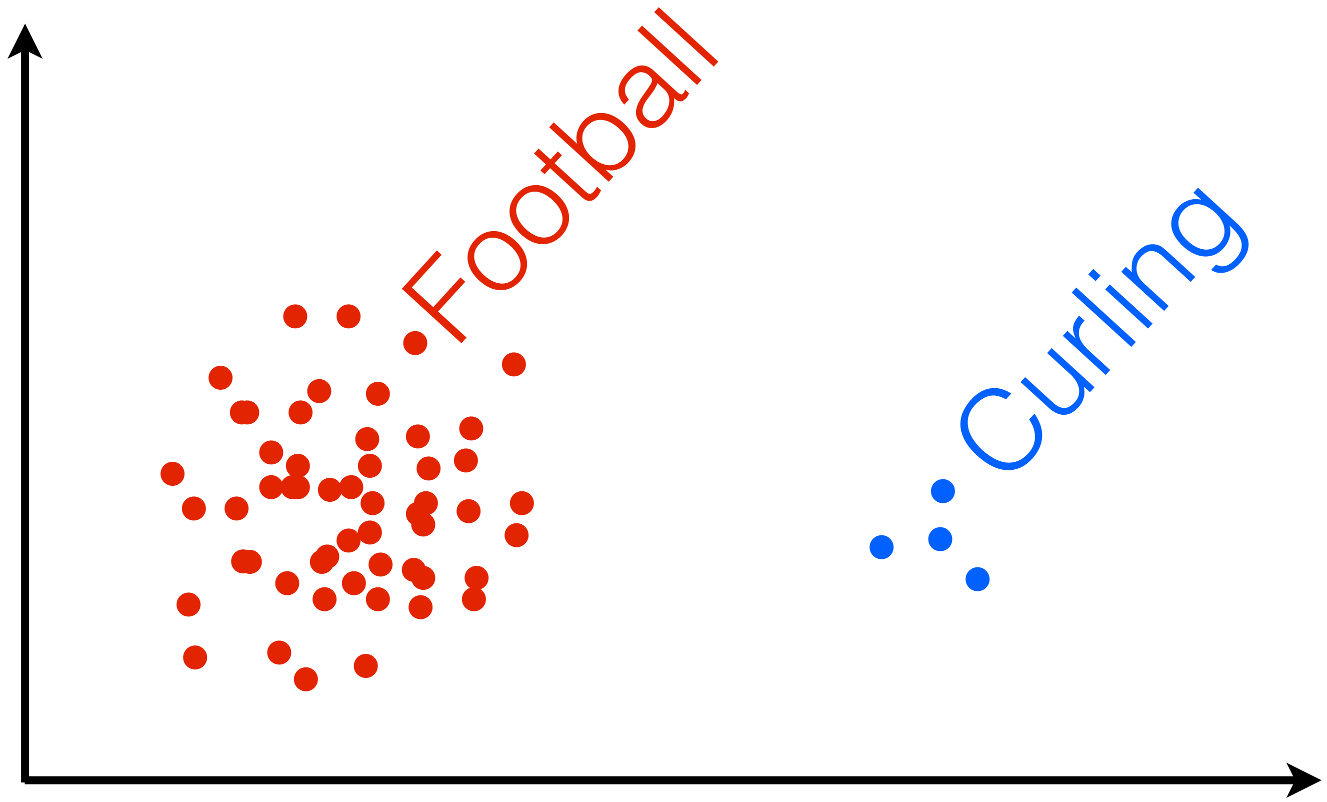

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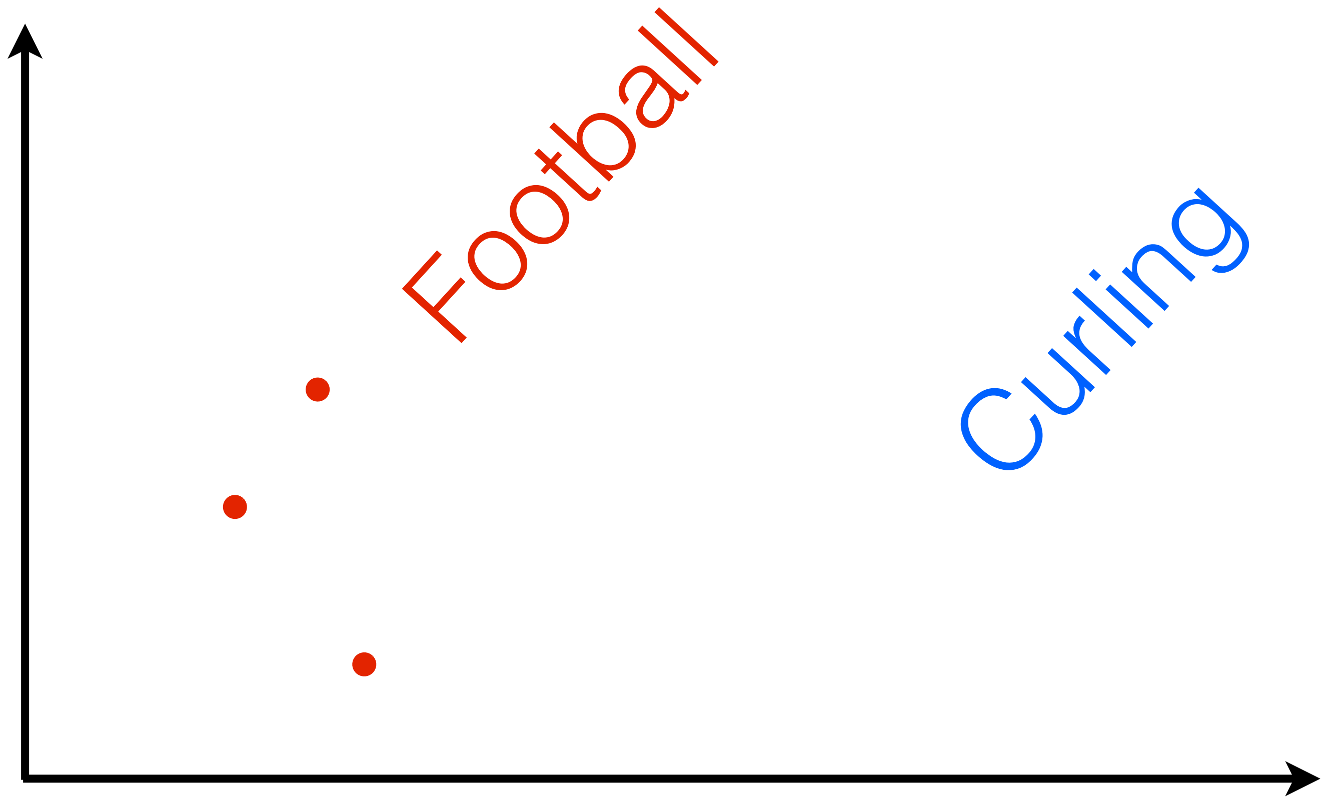

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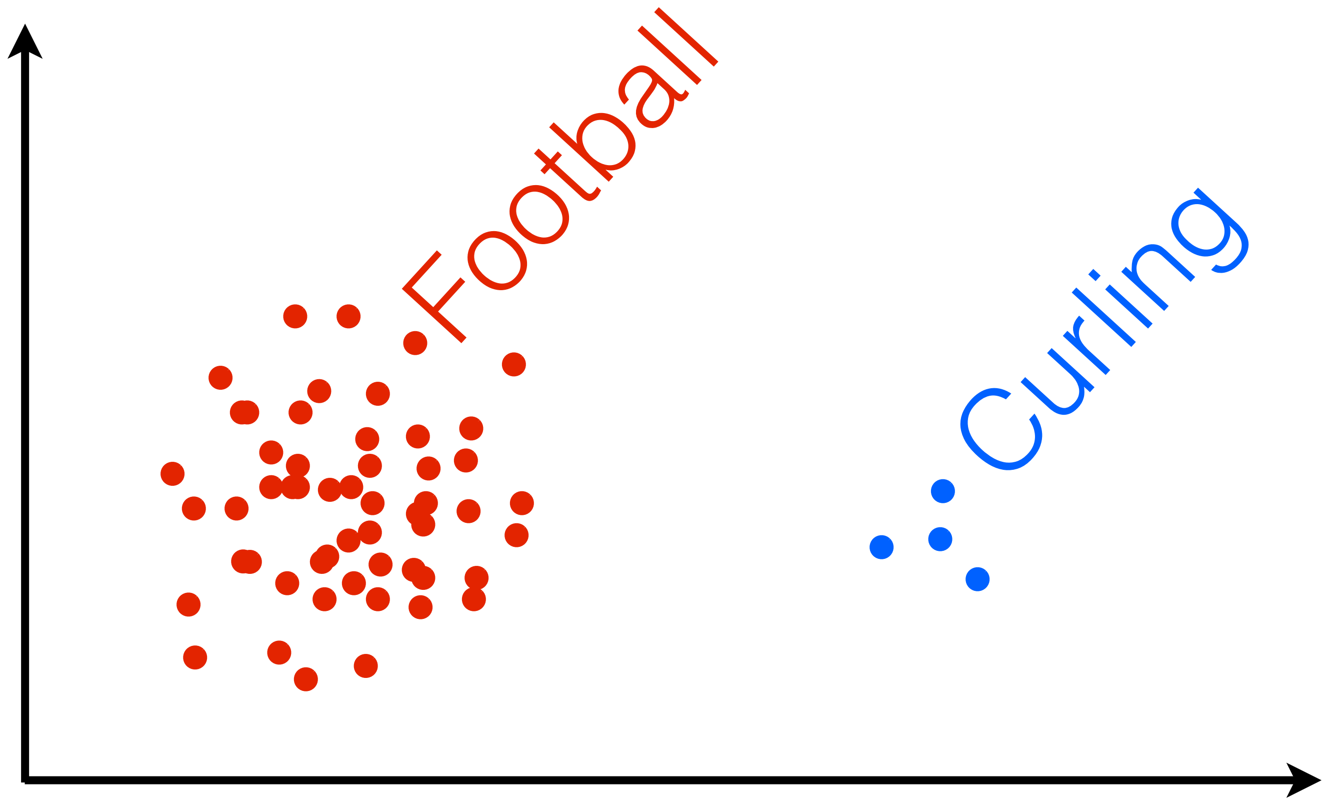

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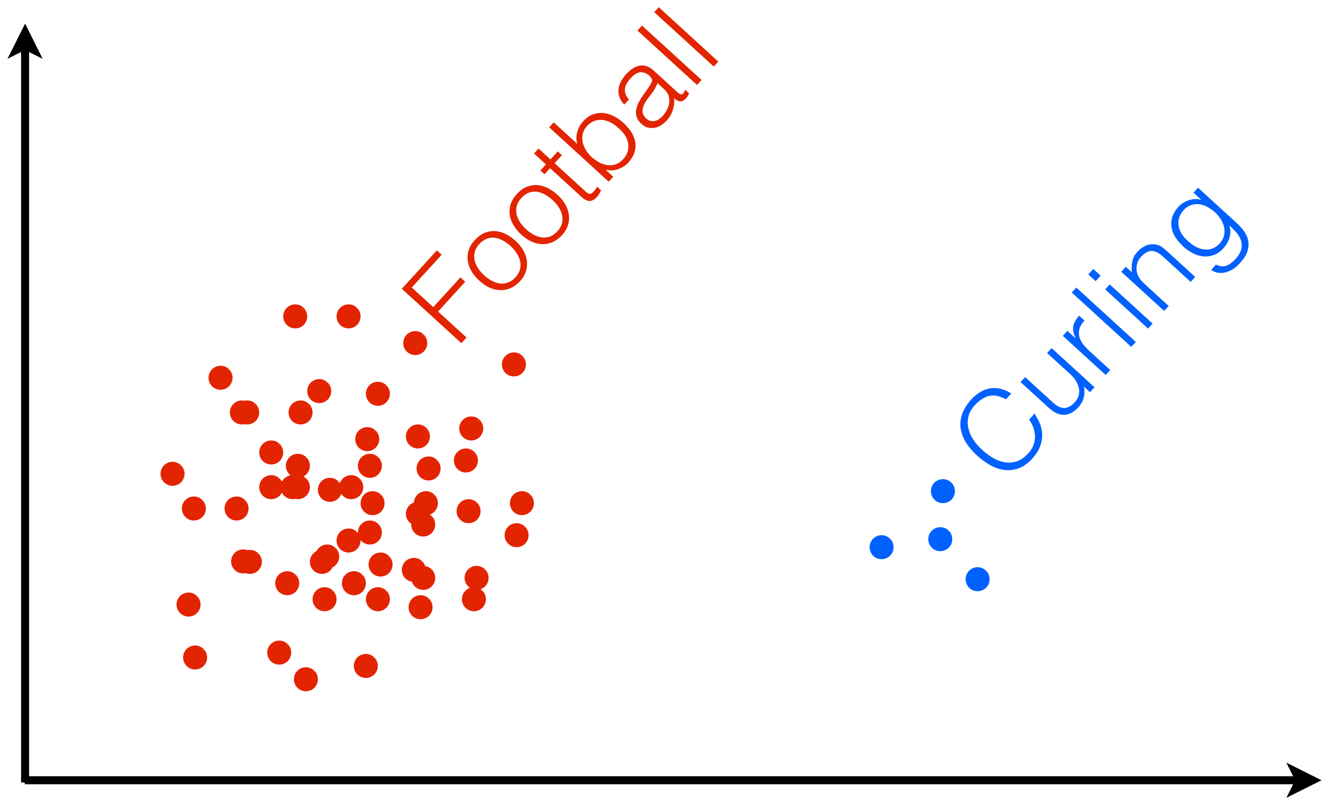

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- Cf. subsampling
- How to develop **coresets for diverse tasks/geometries**?

[Bădoiu, Har-Peled, Indyk 2002; Agarwal et al 2005; Feldman & Langberg 2011; DuMouchel et al 1999; Madigan et al 2002; Huggins, Campbell, Broderick 2016; Campbell, Broderick 2017; Campbell, Broderick 2018]

Roadmap

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- Uniform data subsampling isn’t enough
- Importance sampling for “coresets”
- Optimization for “coresets”
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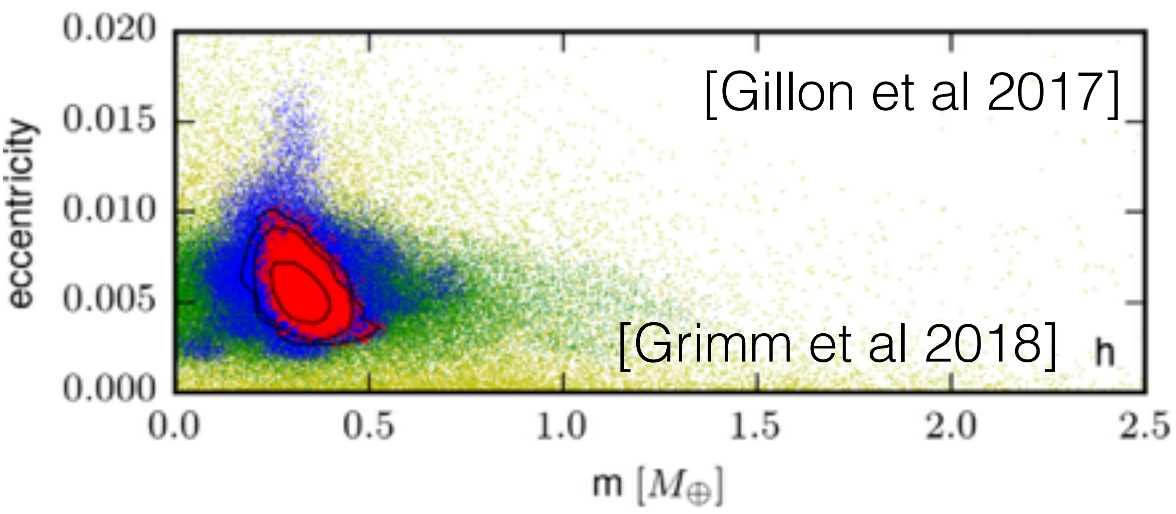

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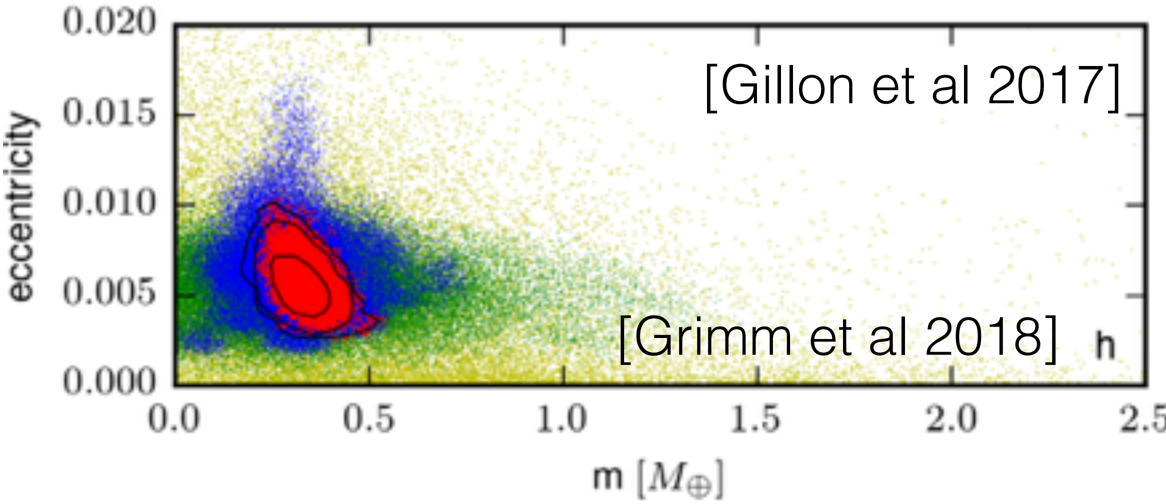

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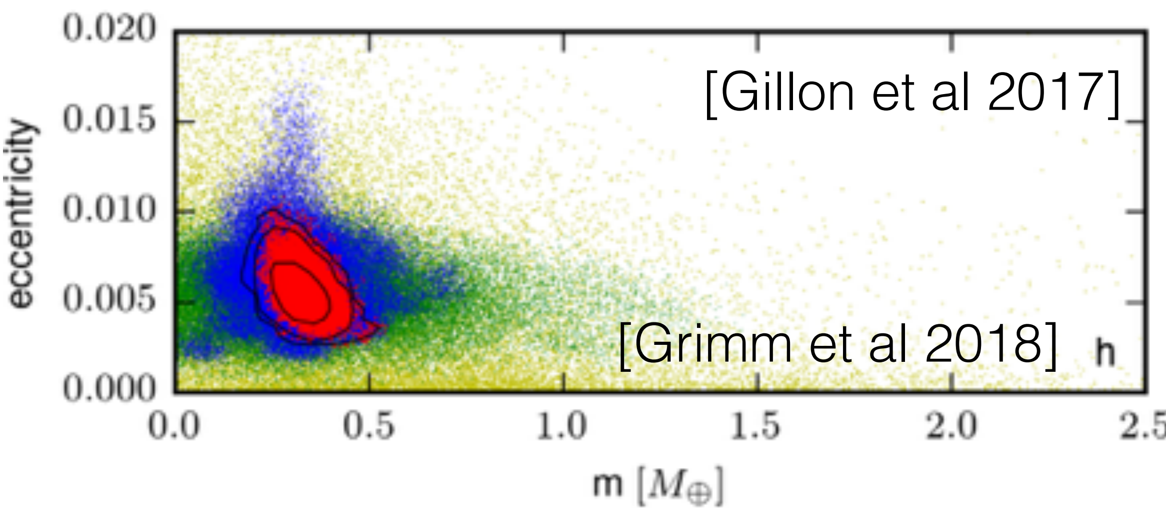

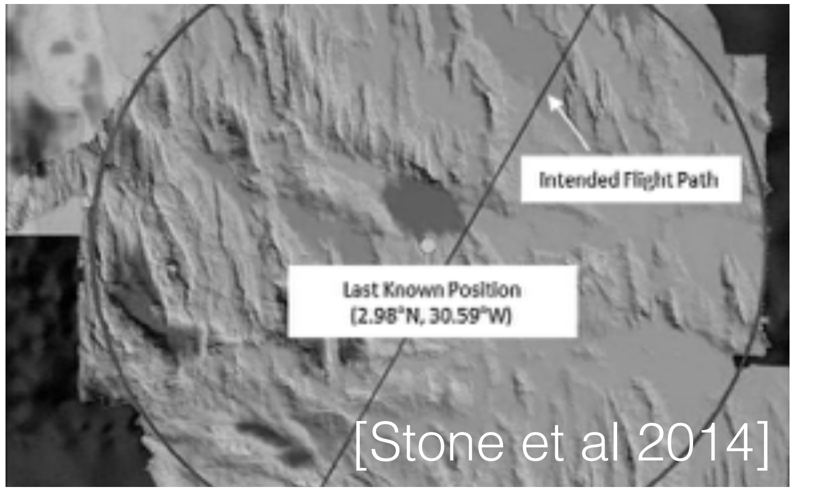

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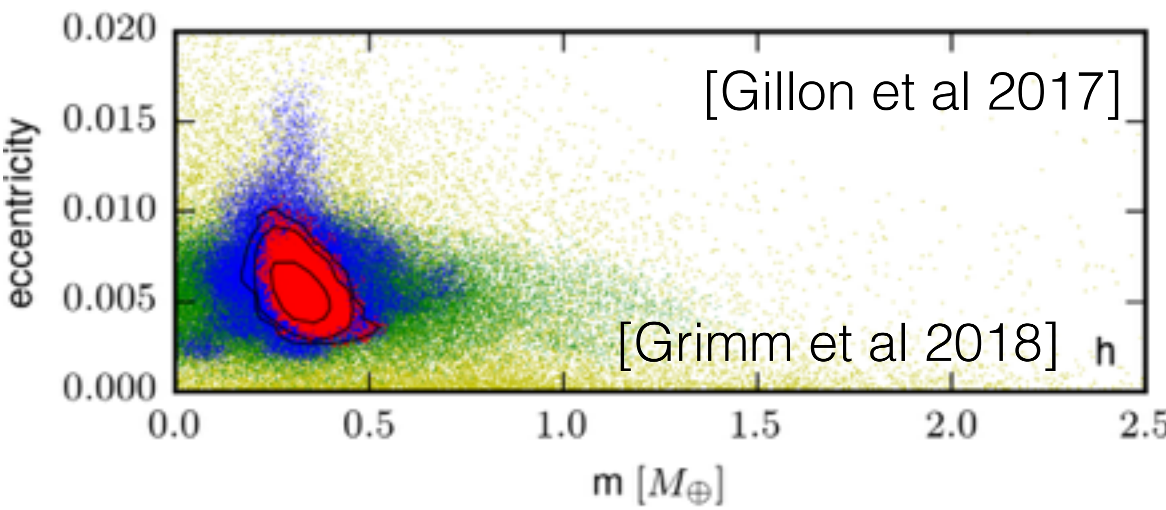

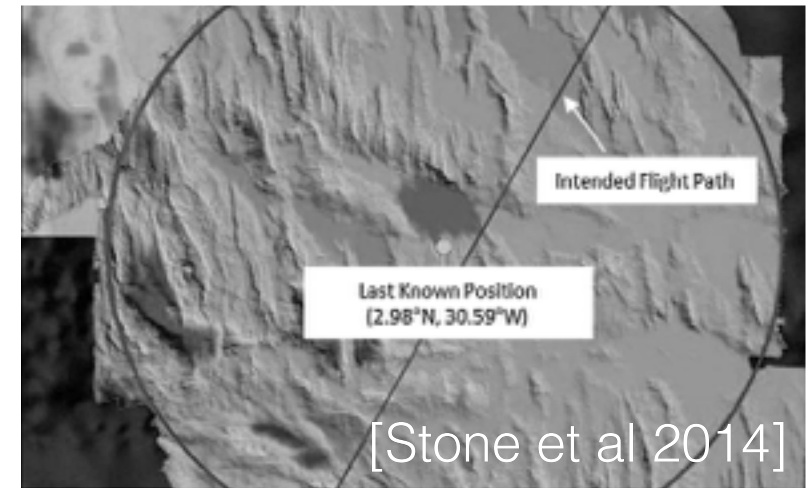

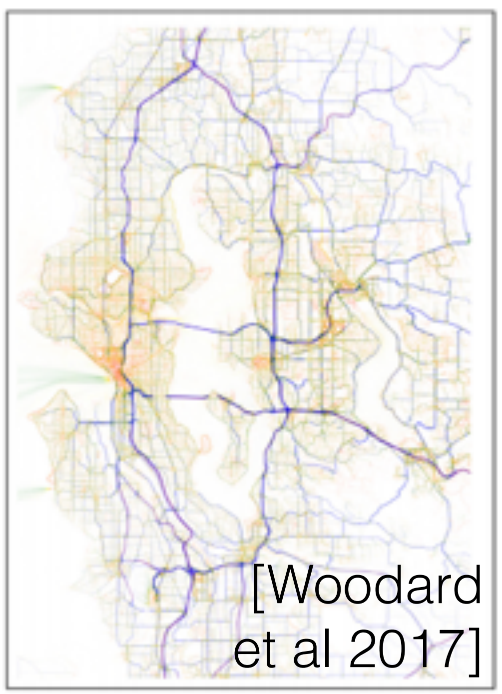

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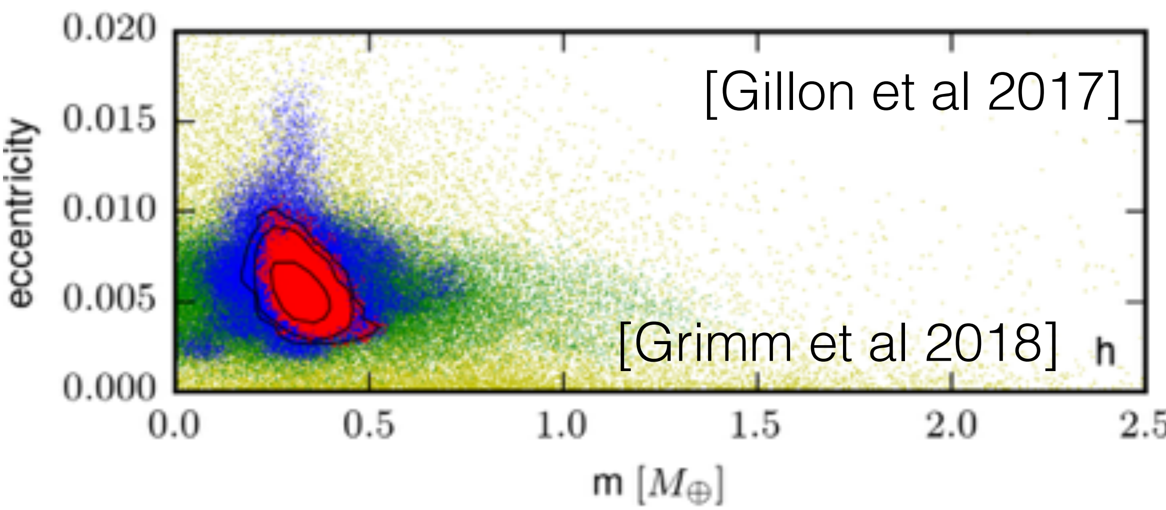

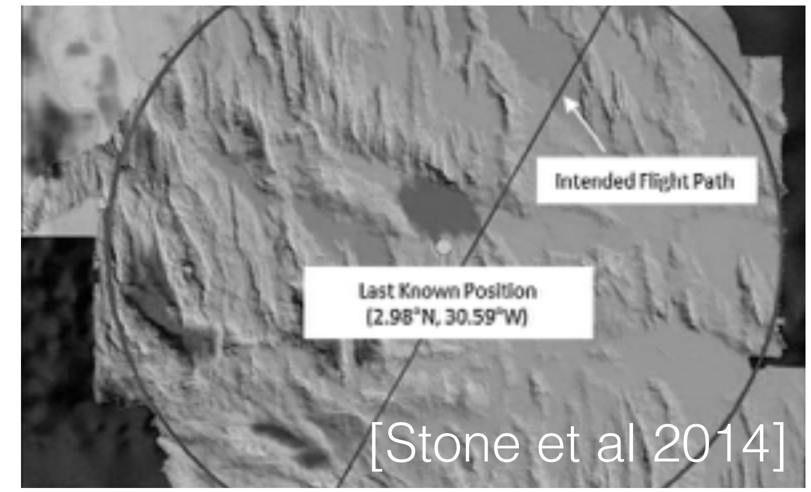

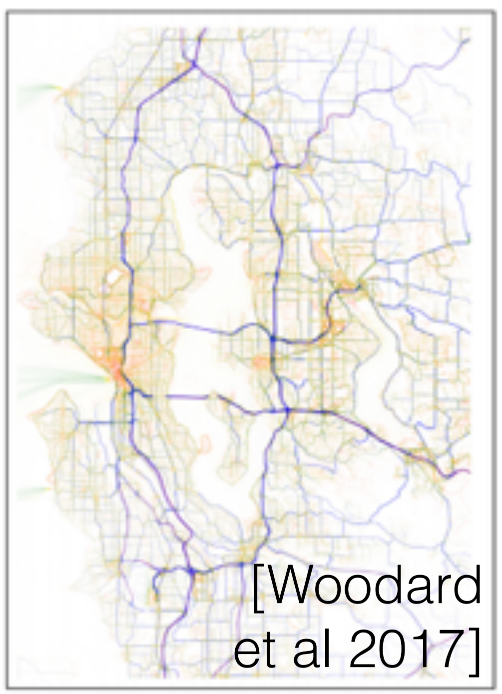

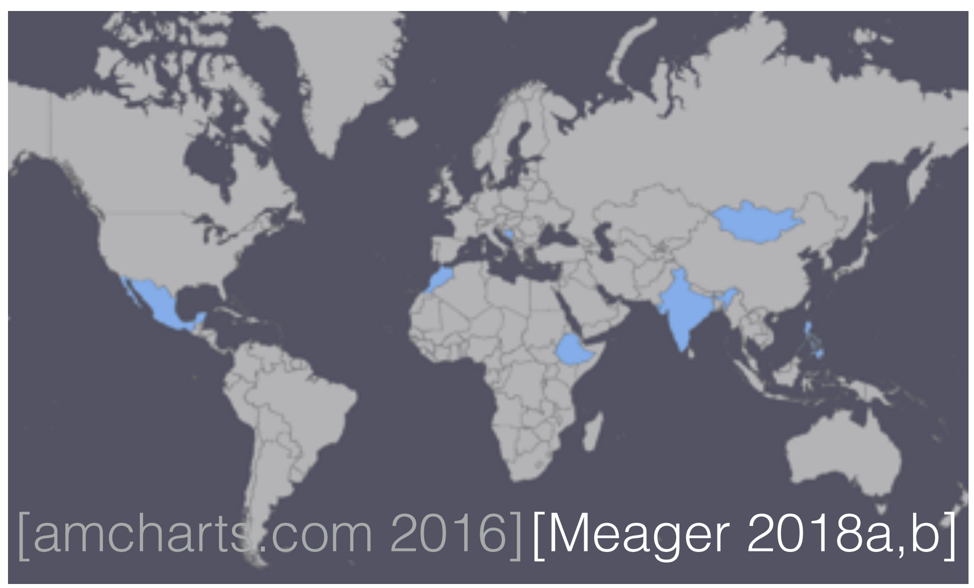

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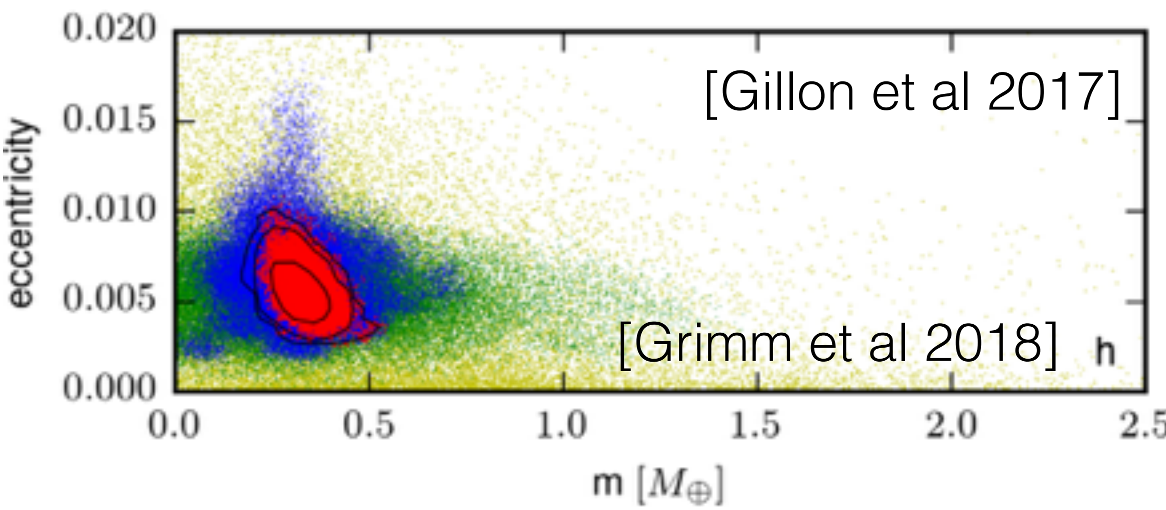

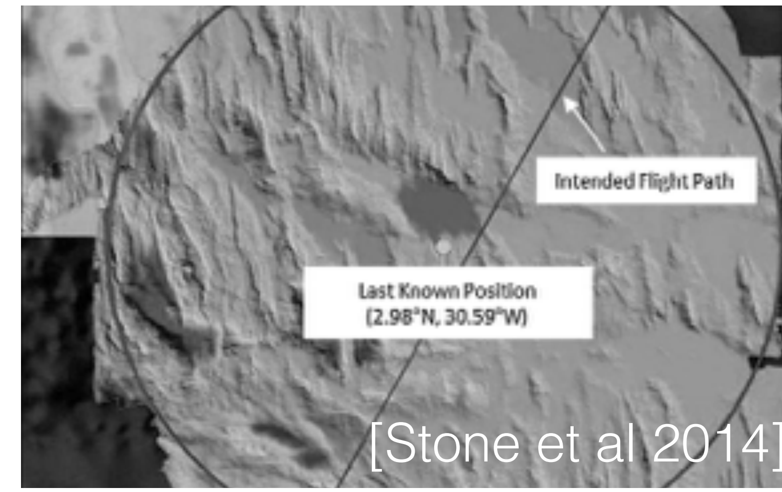

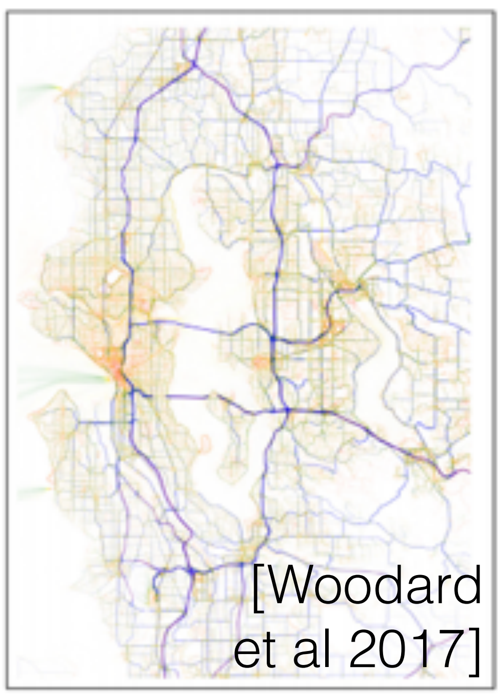

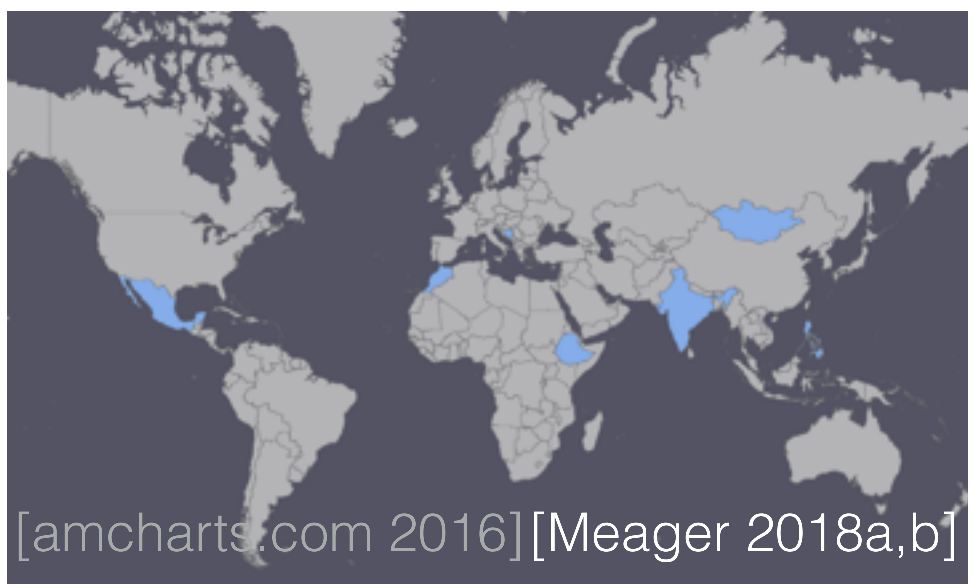

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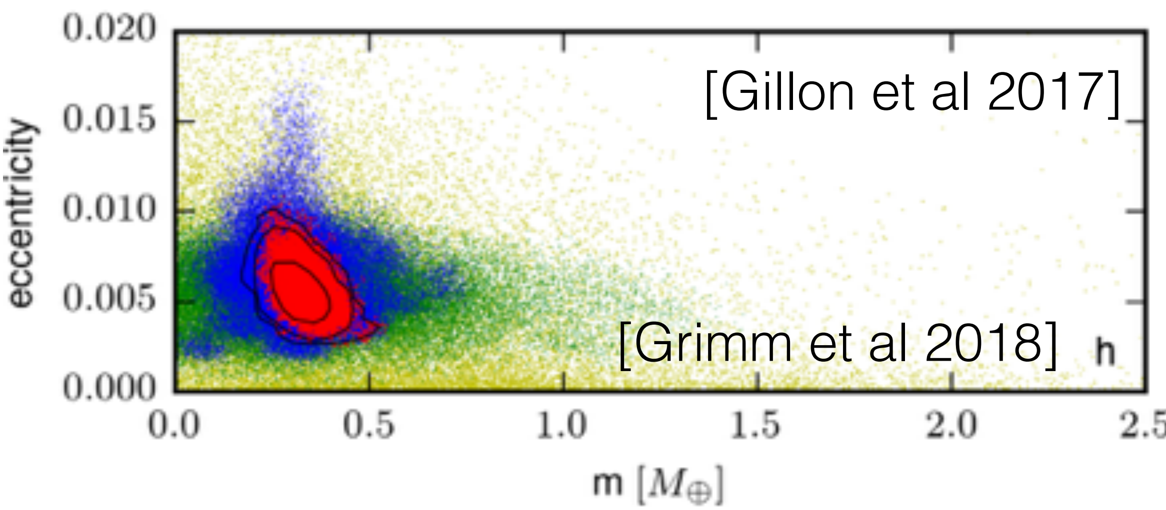
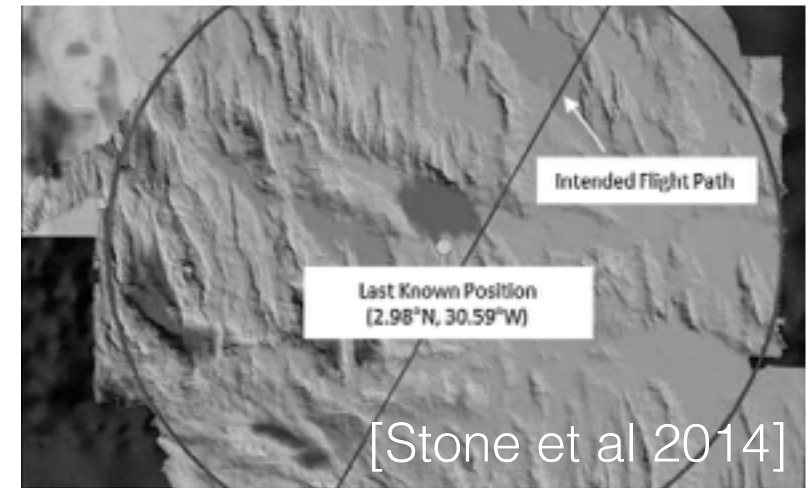
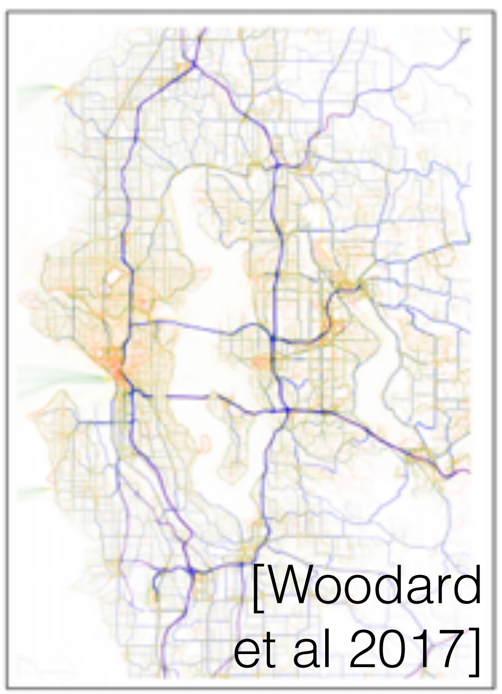
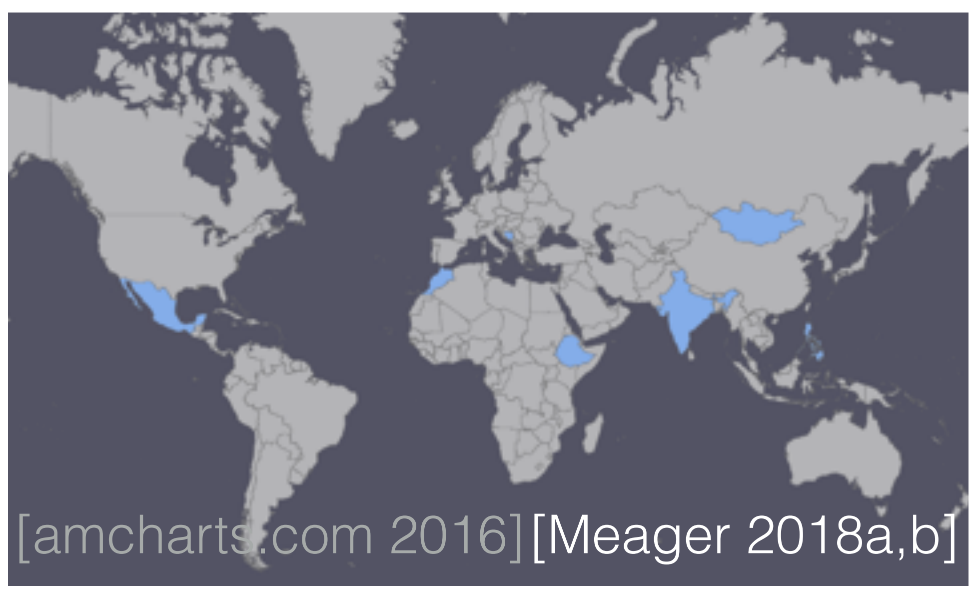


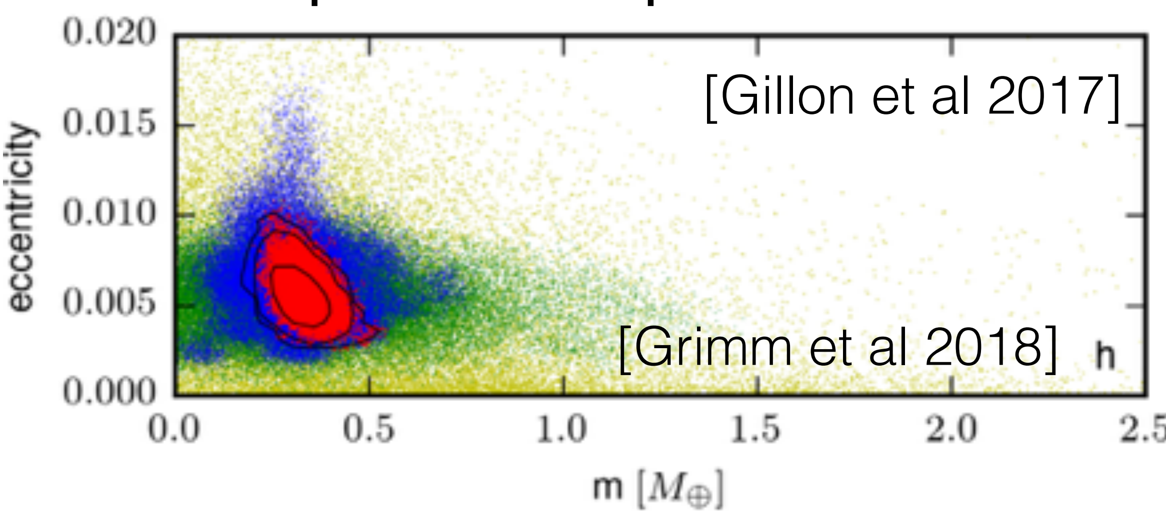


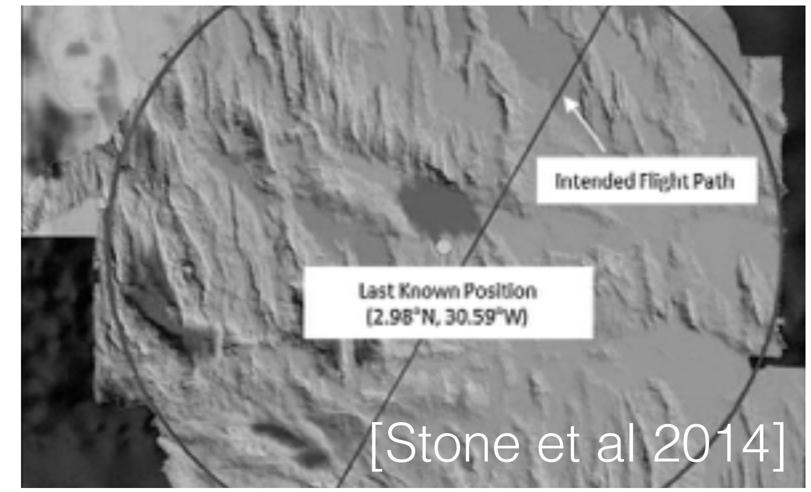


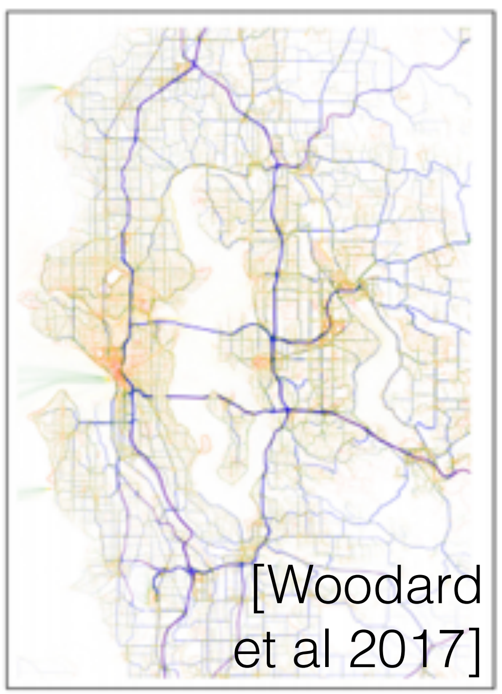


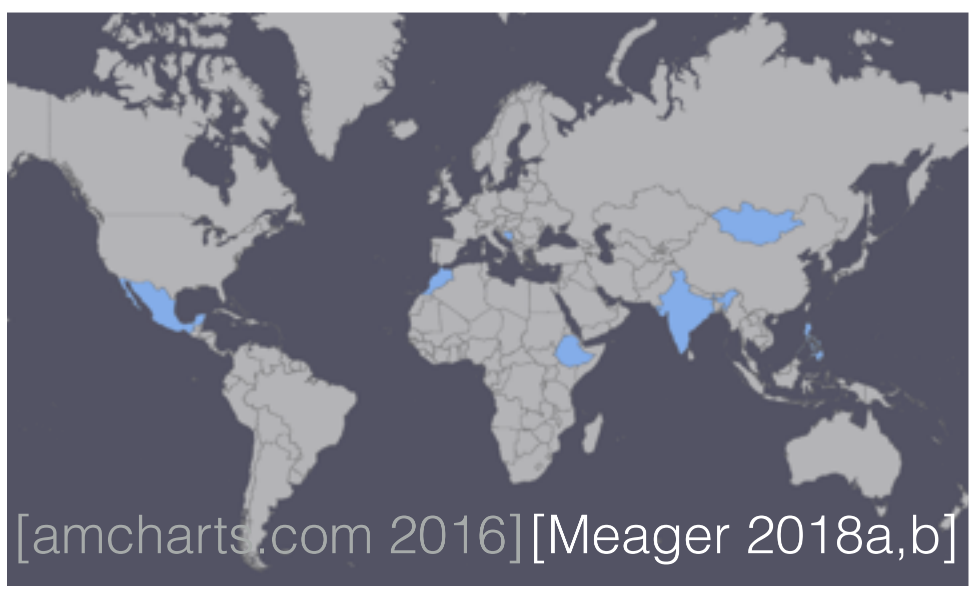
Bayesian inference

- Goals: good estimates, uncertainties; interpretable; complex; expert information









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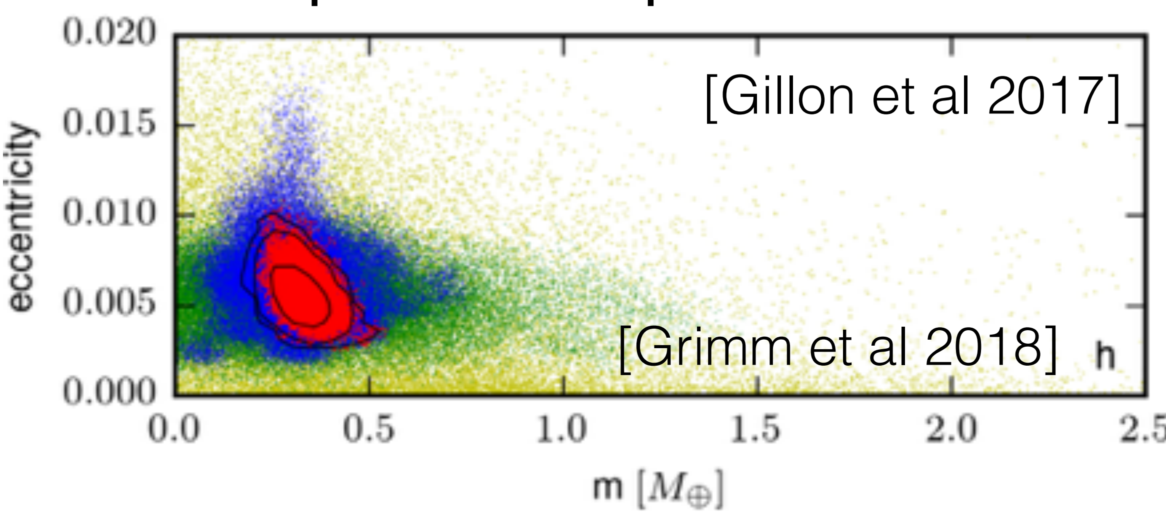

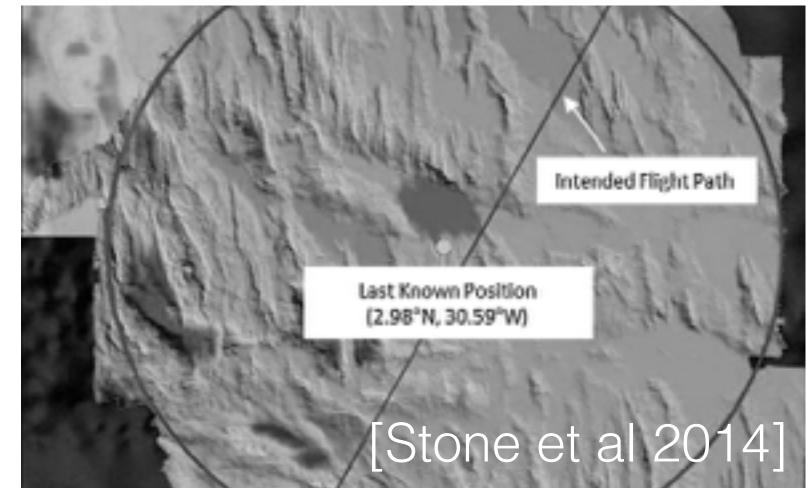

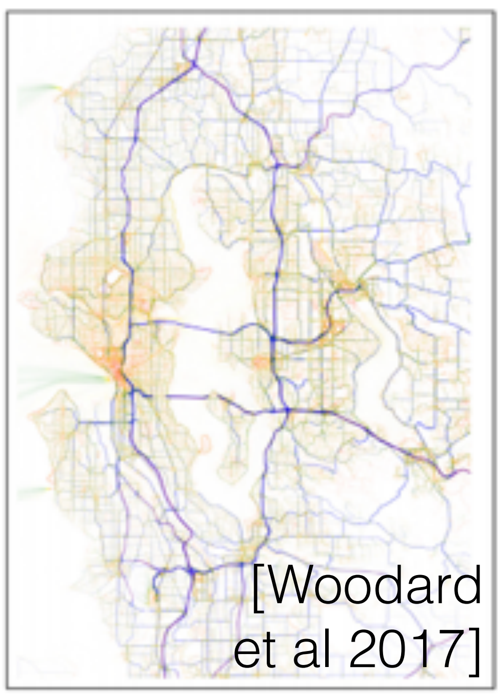

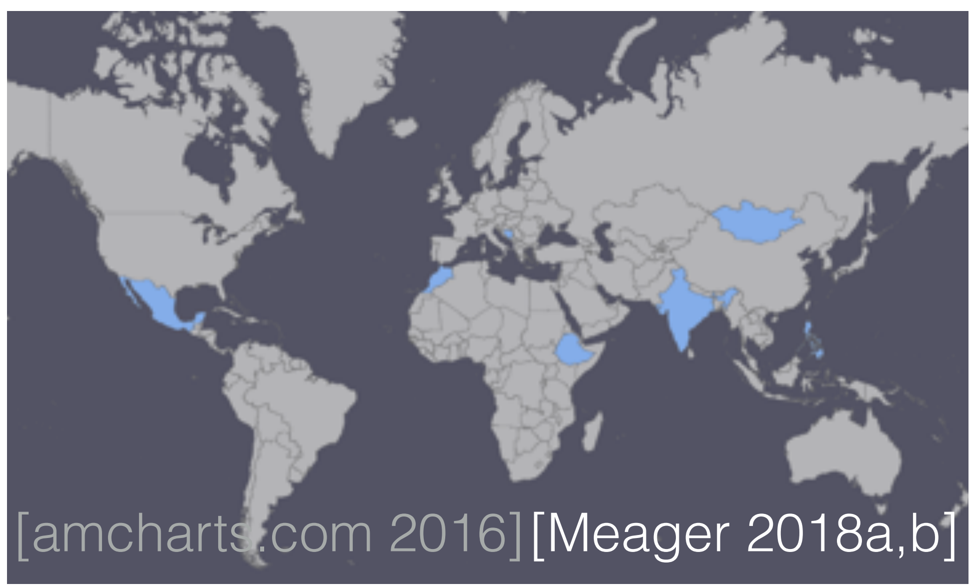

[mc-stan.org]

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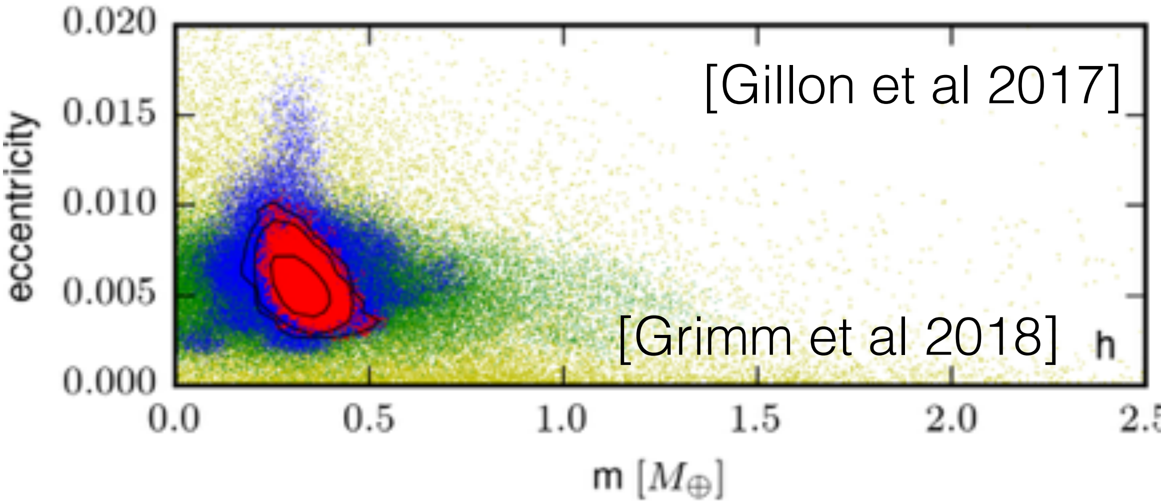
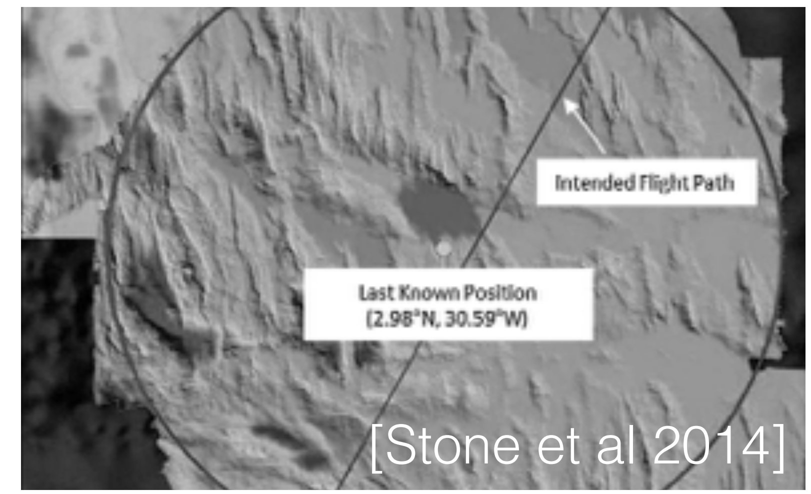
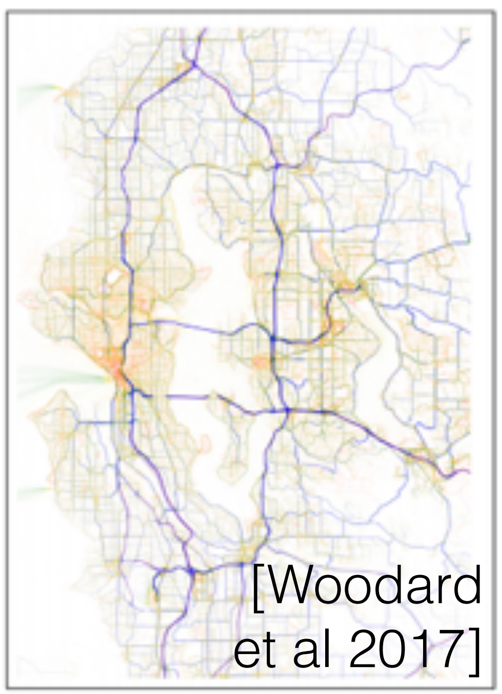


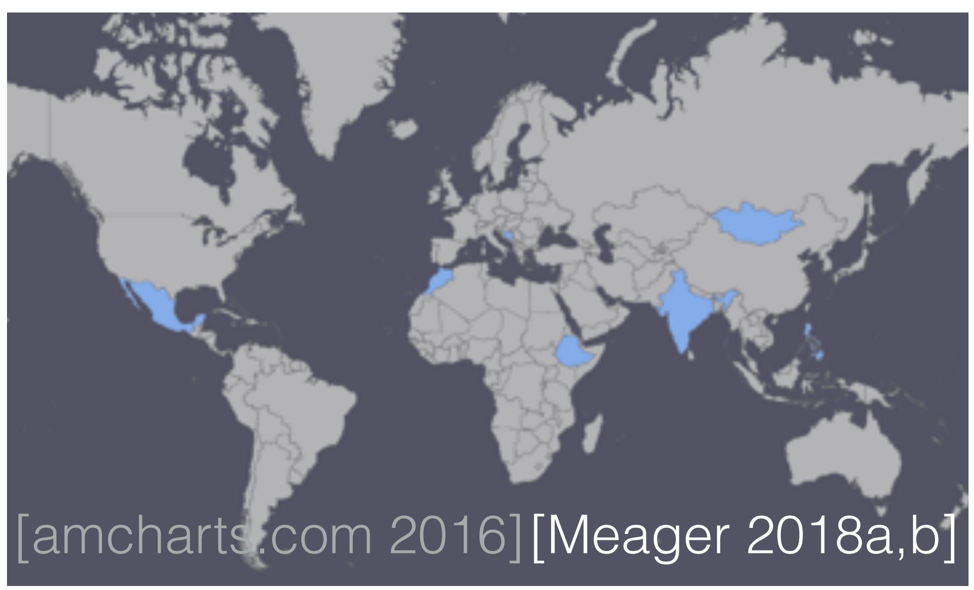
[Gillon et al 2017] [Grimm et al 2018]


[ESO/ L. Calçada/ M. Kornmesser 2017] [Abbott et al 2016a,b]



[Stone et al 2014]


[Woodard et al 2017]


[amcharts.com 2016] [Meager 2018a,b]


[Kuikka et al 2014] [Baltic Salmon Fund]


[Chati, Balakrishnan 2017] [Julian Hertzog 2016]

[mc-stan.org]

- Challenge: existing methods can be slow, tedious, unreliable

Bayesian inference

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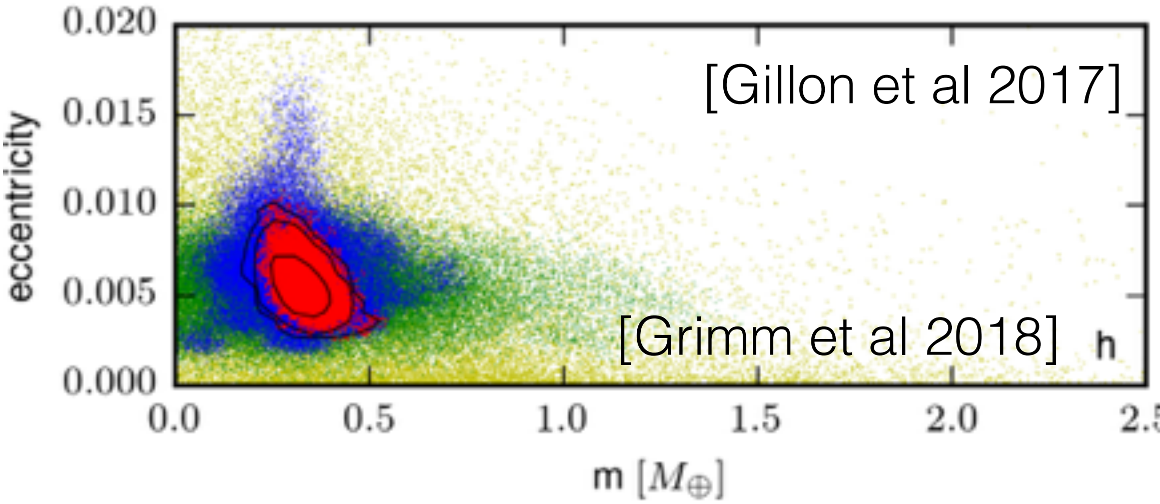

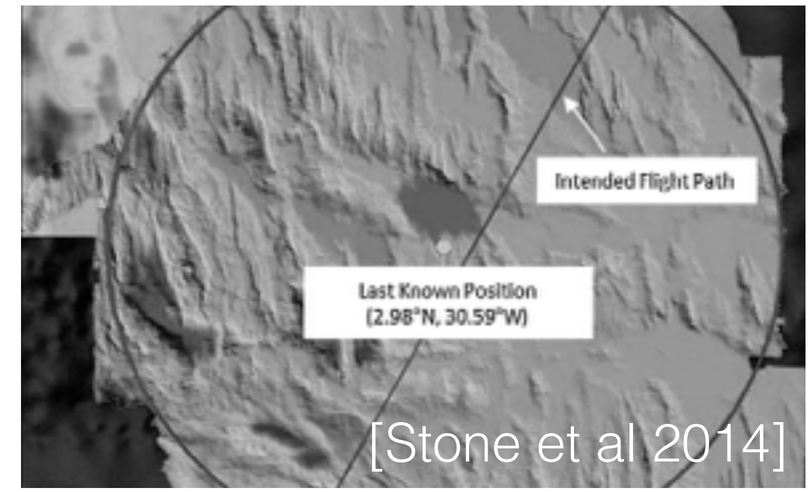

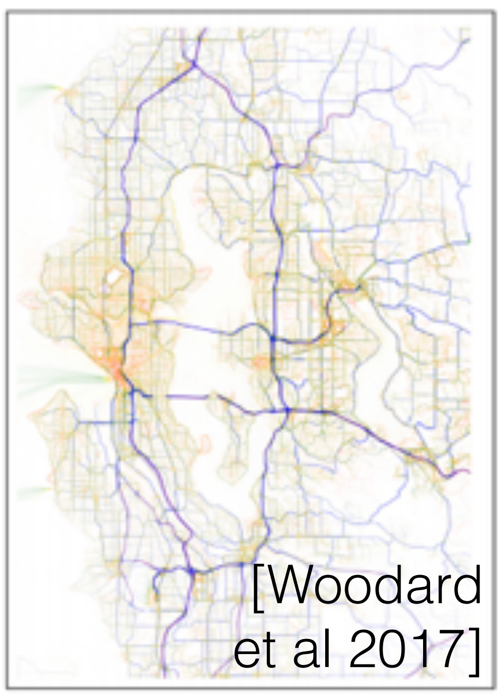

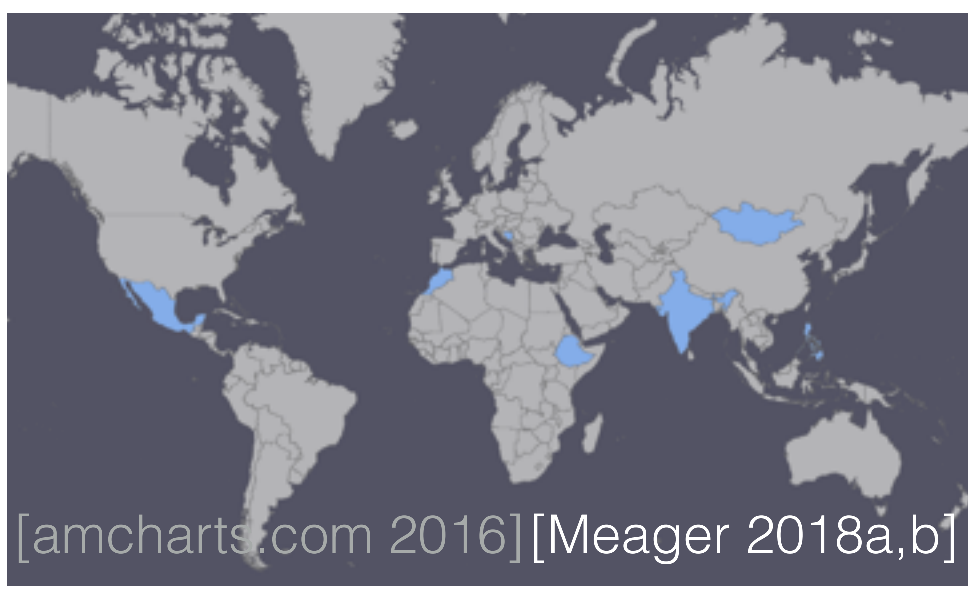

[mc-stan.org]

- Challenge: existing methods can be slow, tedious, unreliable
- Our proposal: develop *coresets* for **scalable**, **automated** algorithms with **error bounds for output quality**

Bayesian inference

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$p(\theta)$

Bayesian inference

$$p(y|\theta)p(\theta)$$

Bayesian inference $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$

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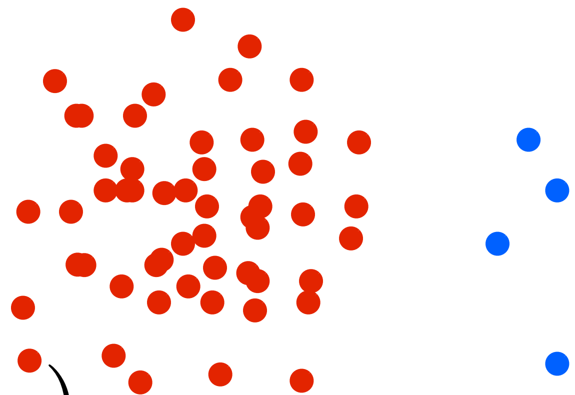

(x_n, y_n)

Bayesian inference

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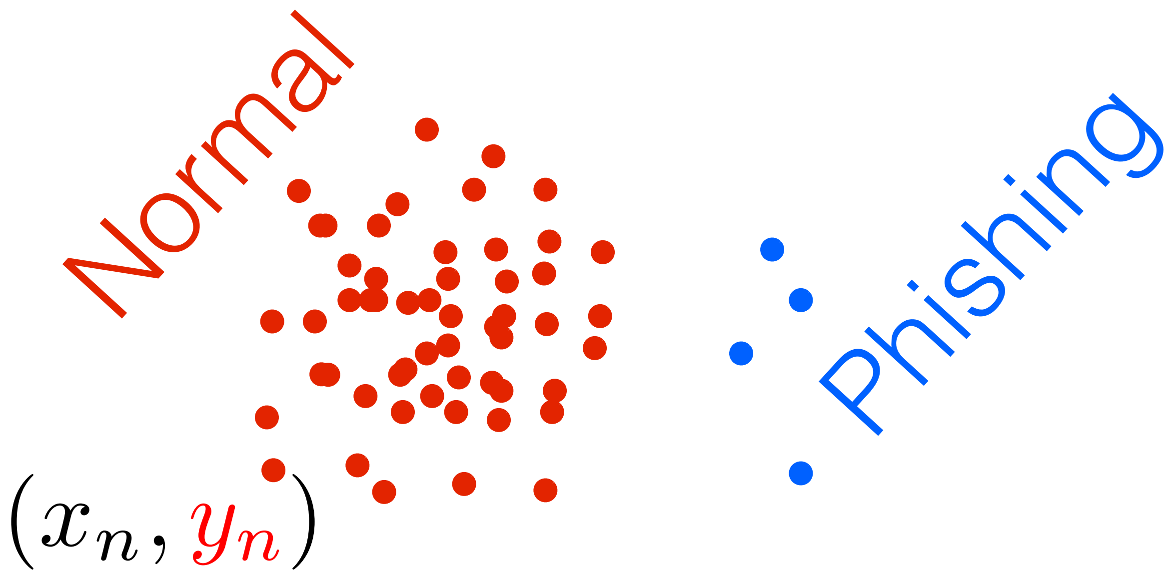

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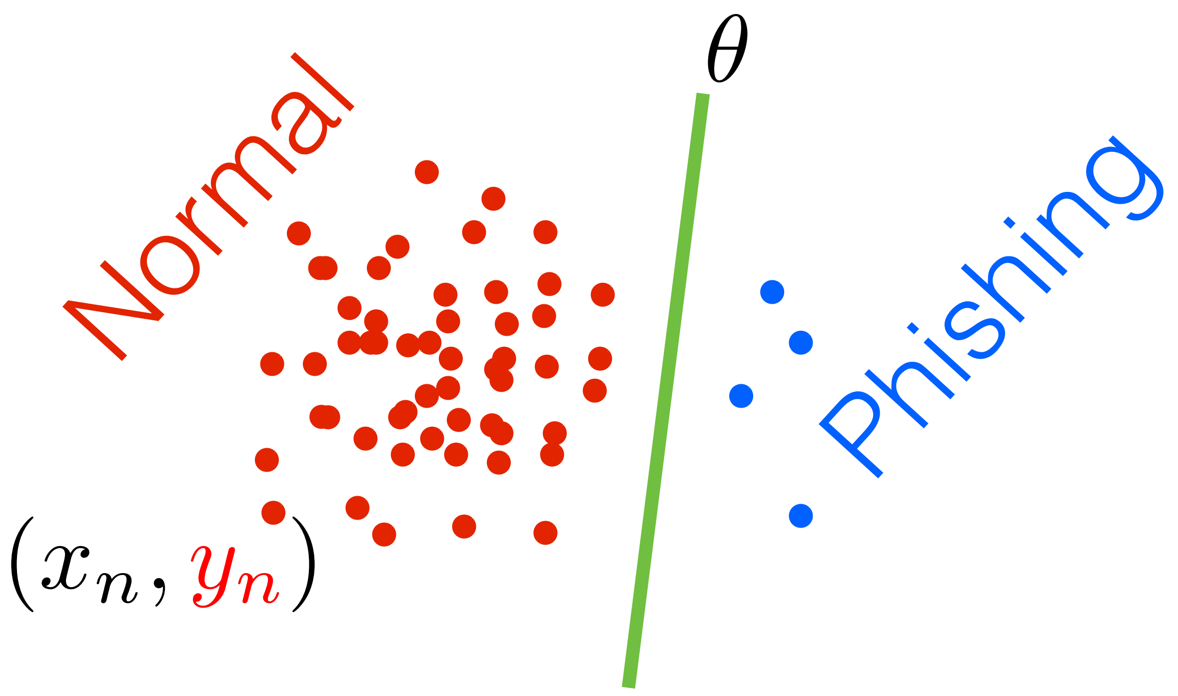

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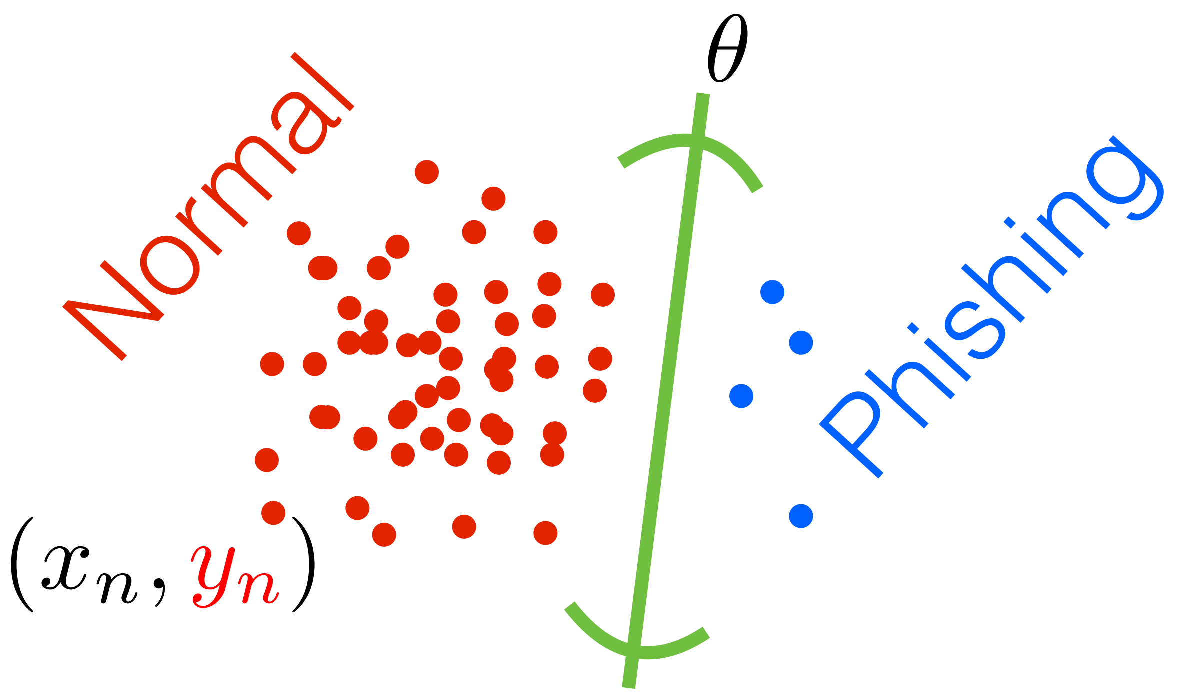

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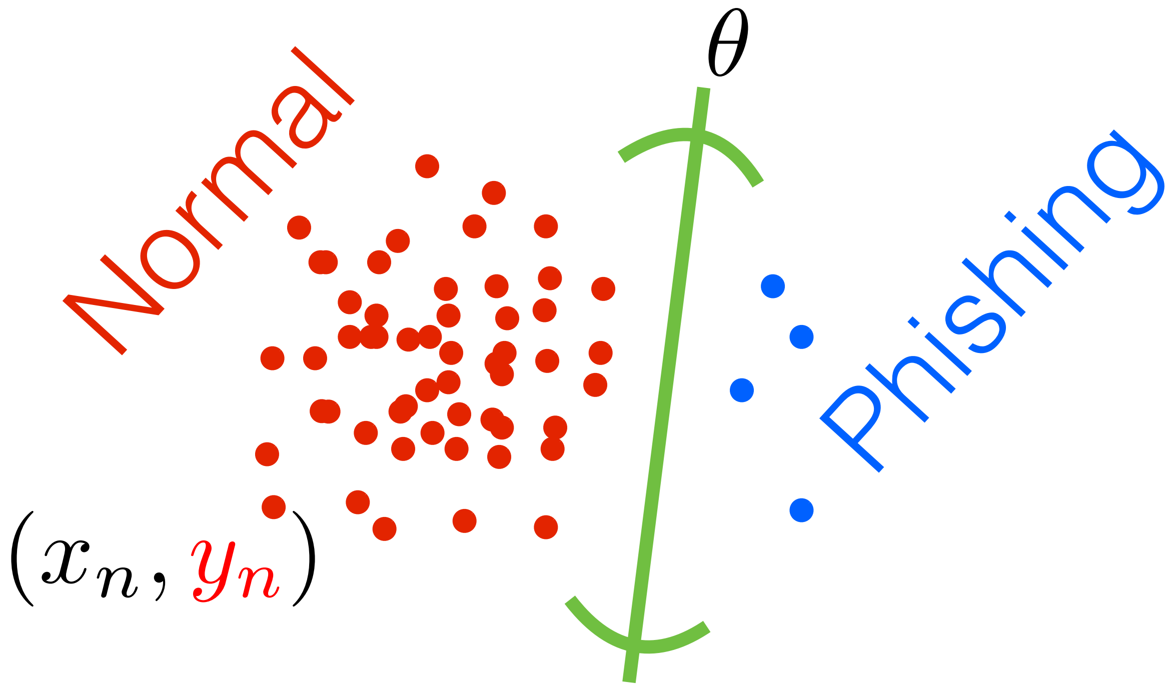

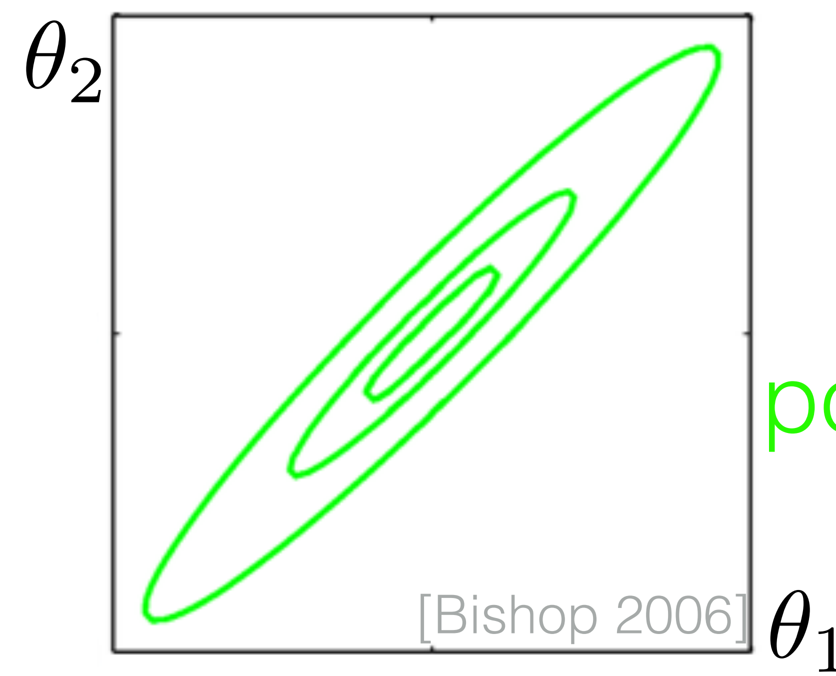

Exact posterior

Bayesian inference

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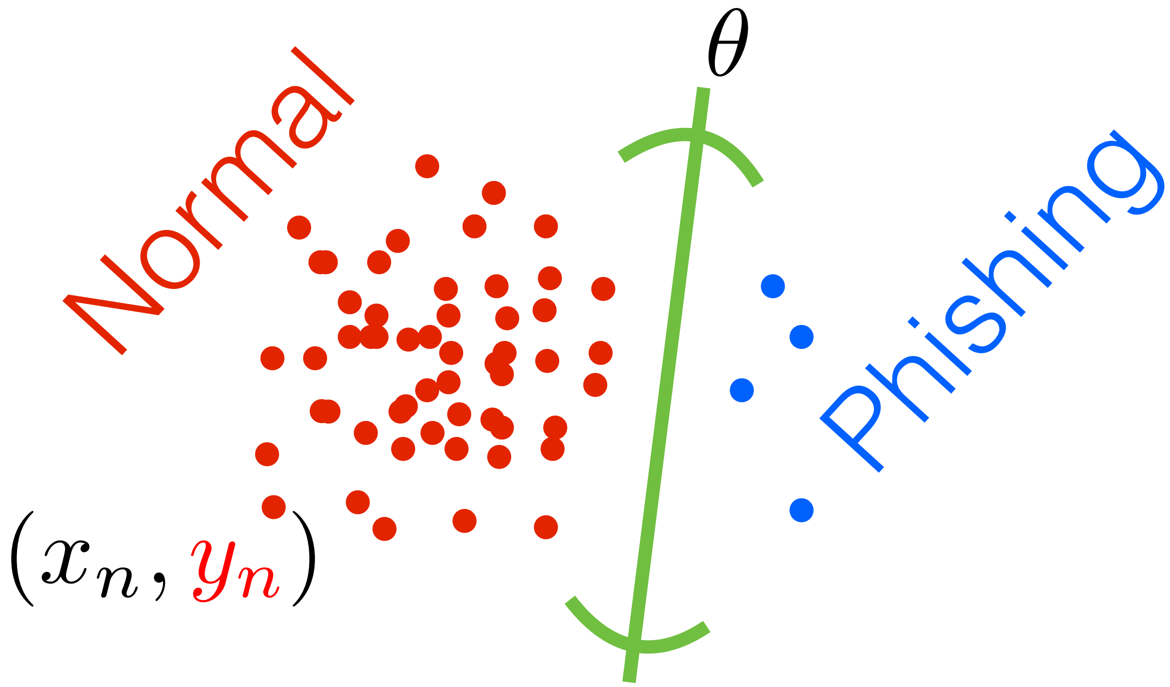

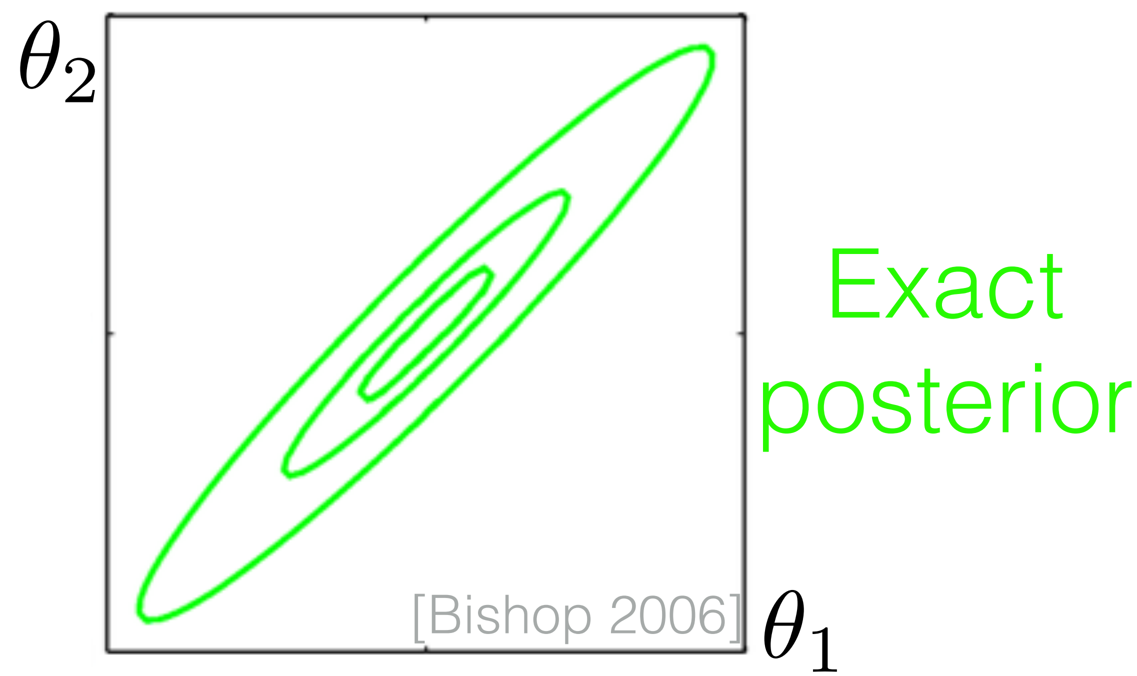

- MCMC: Eventually accurate but can be slow [Bardenet, Doucet, Holmes 2017]

Bayesian inference

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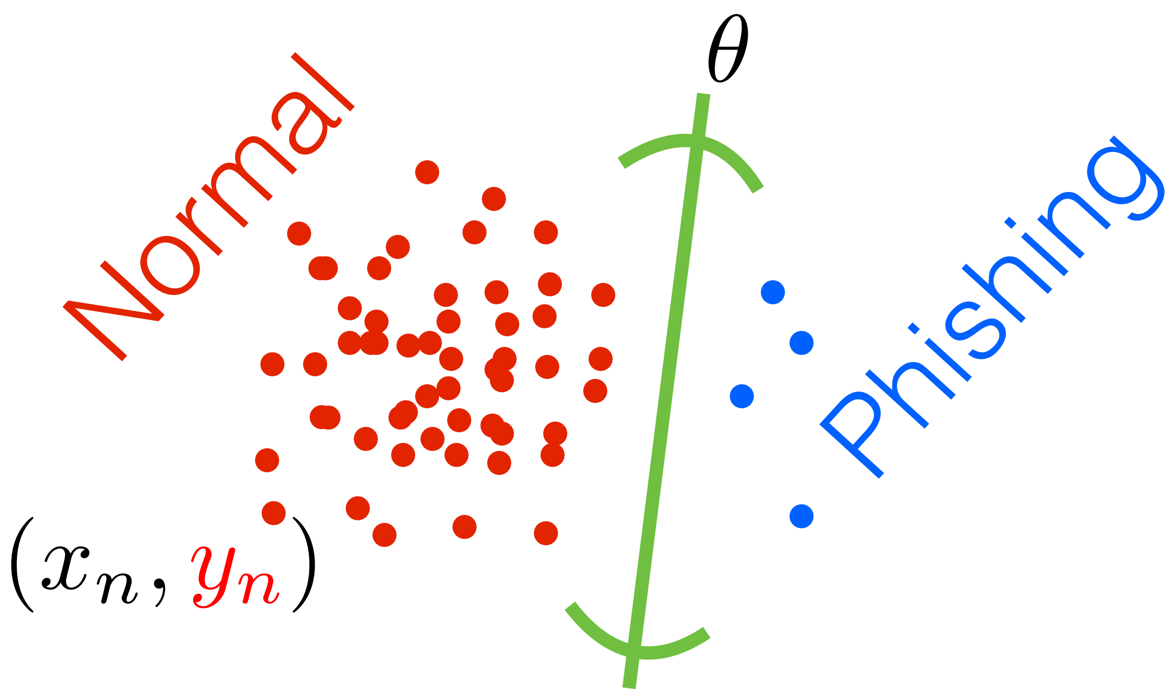

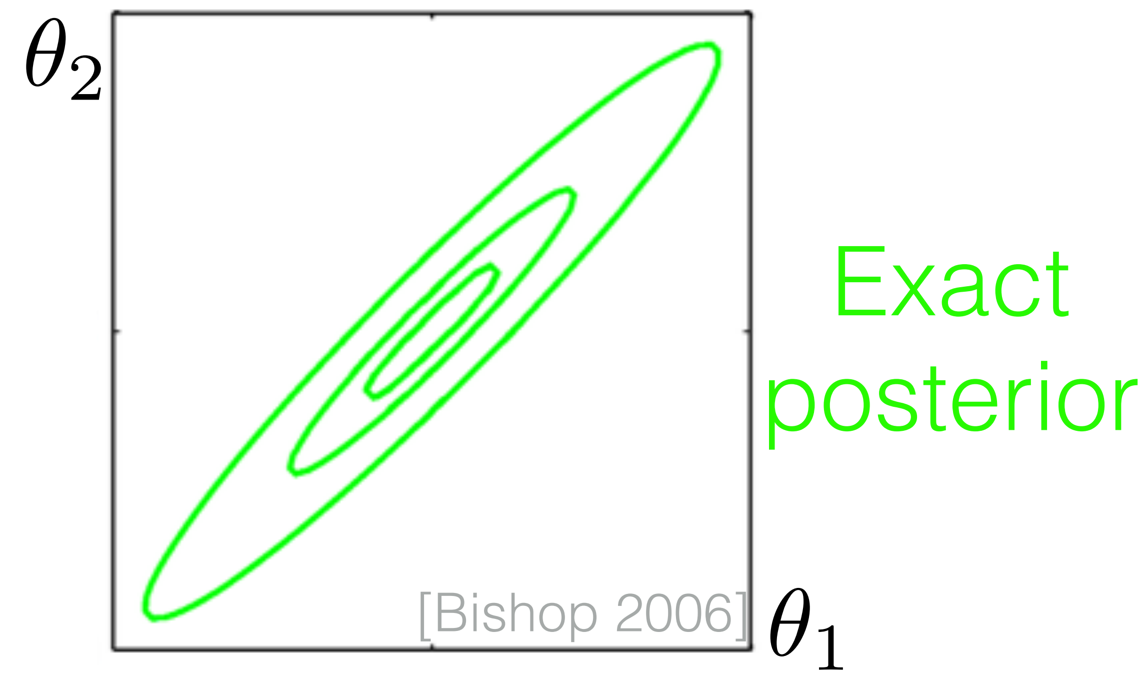

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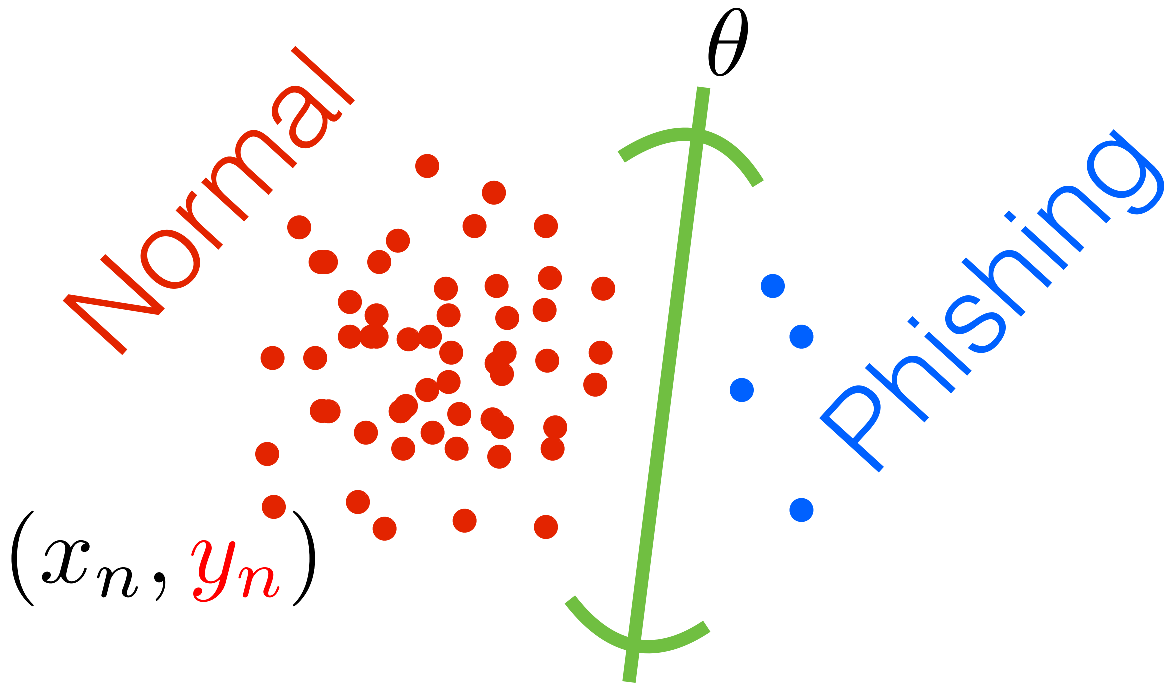

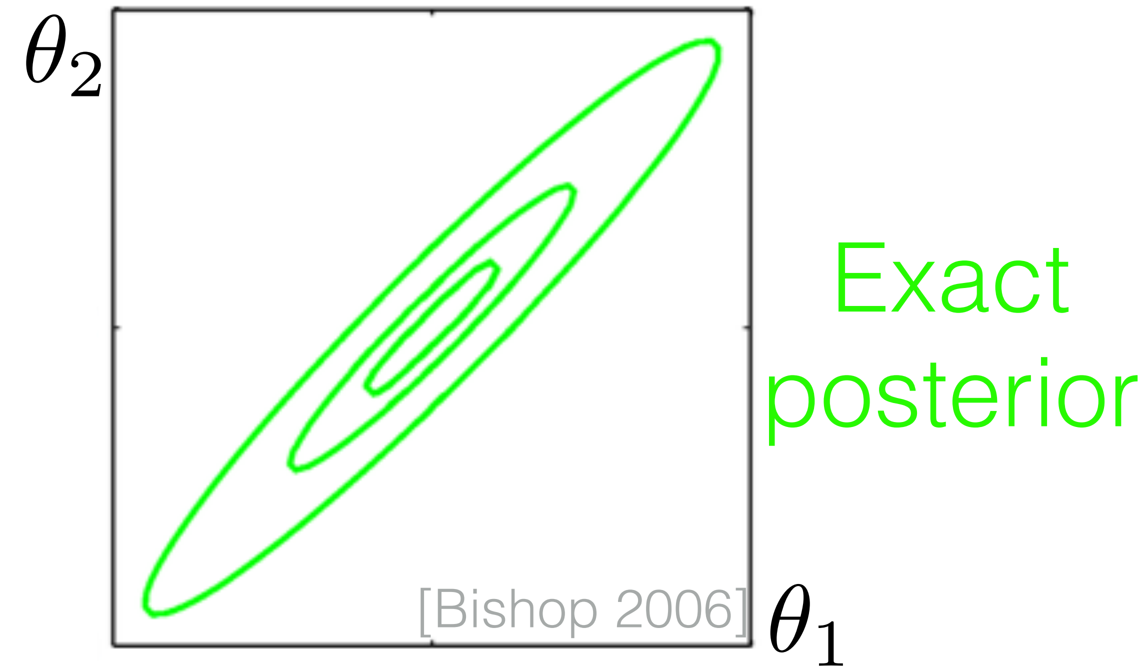

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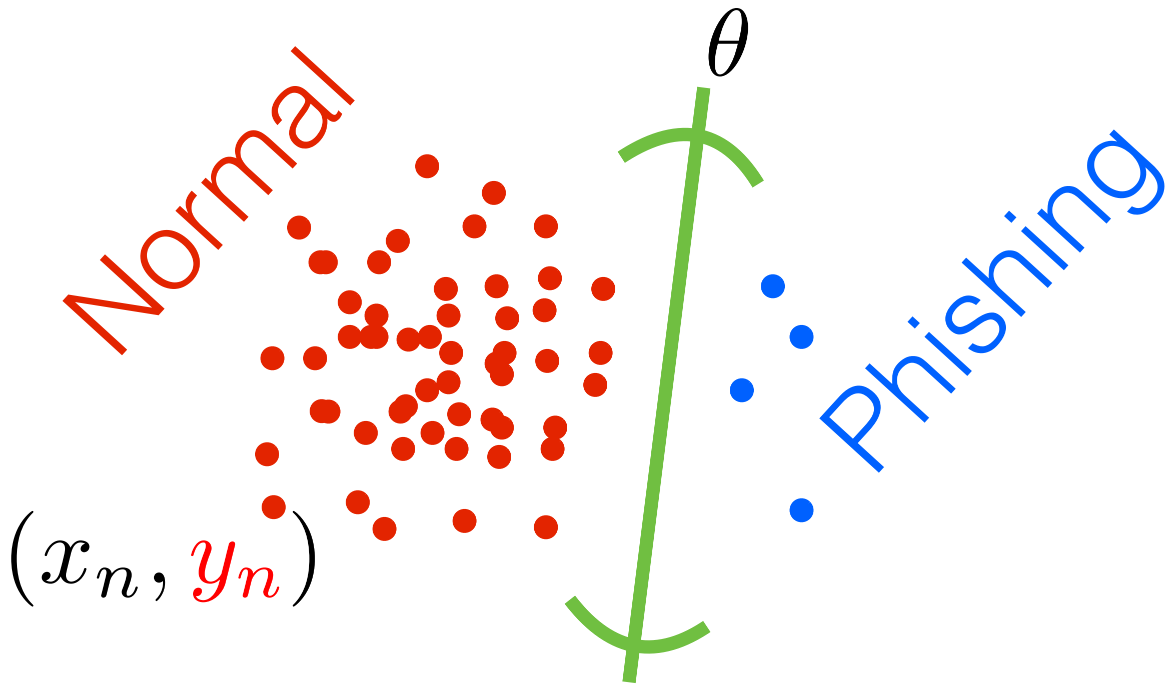

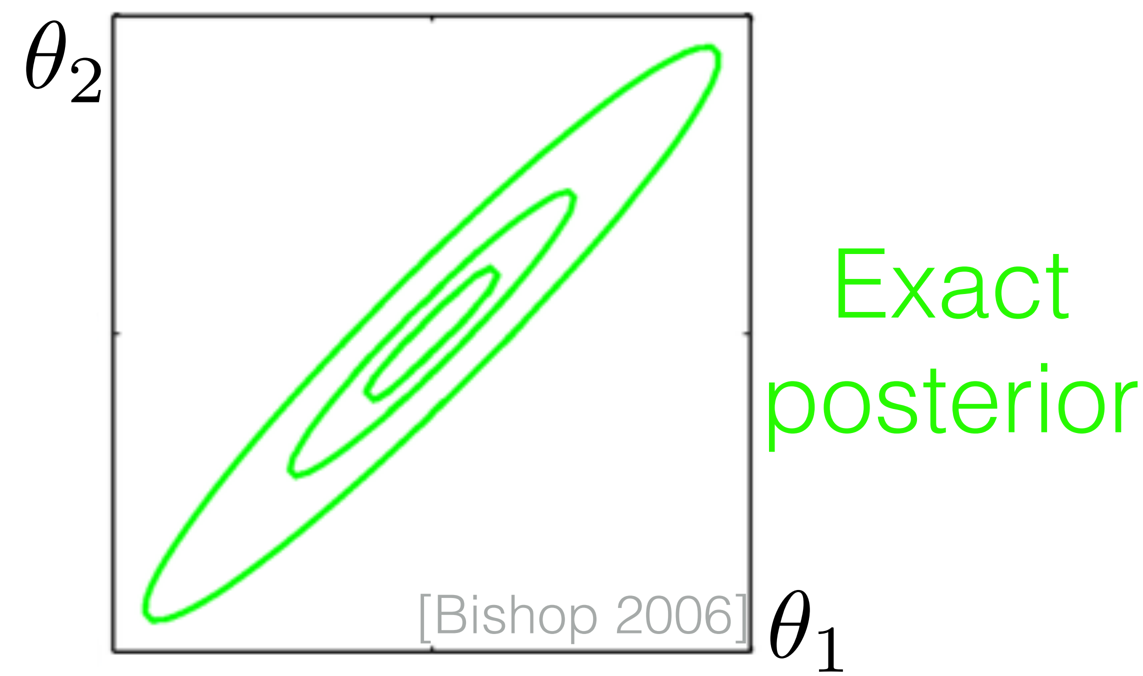

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Bayesian inference

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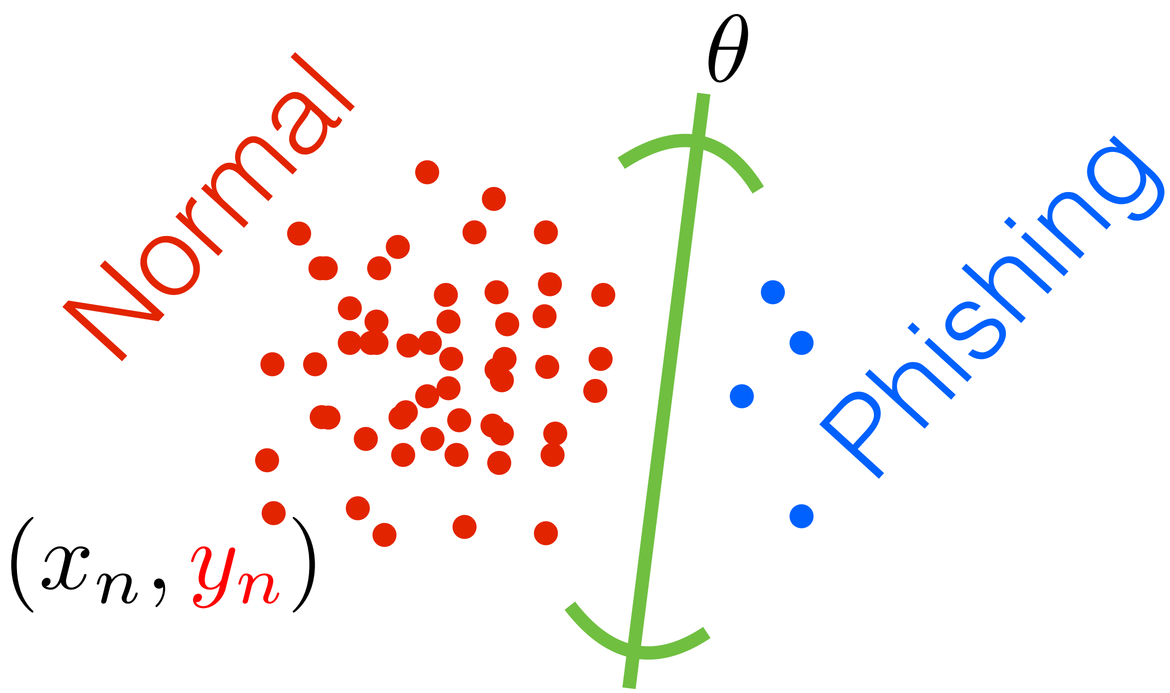

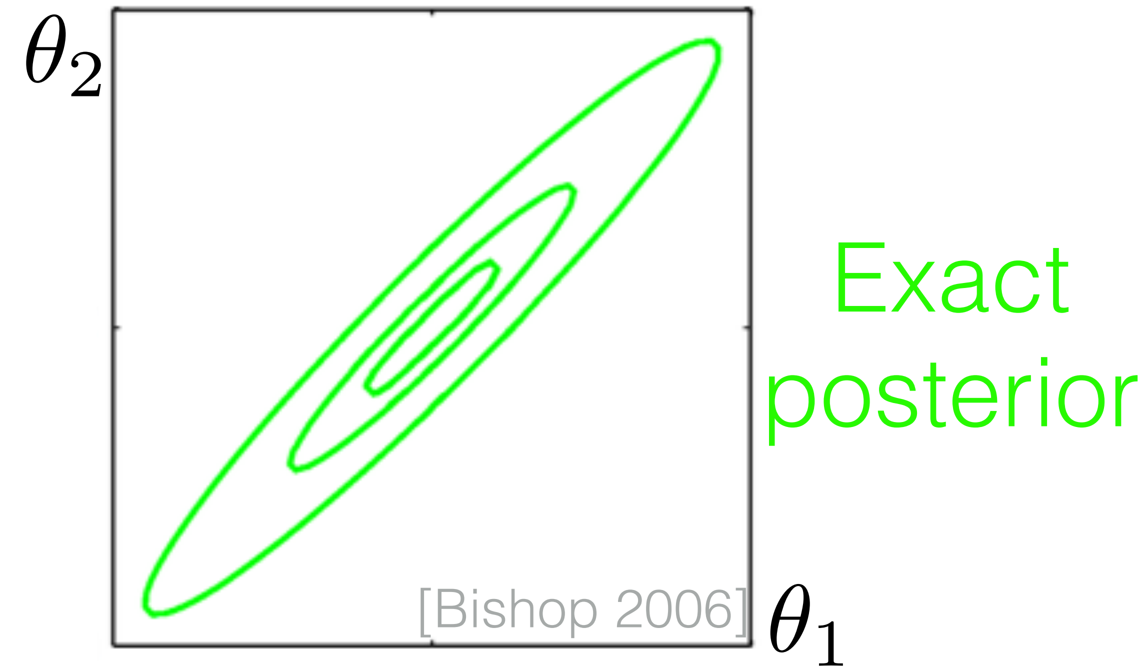

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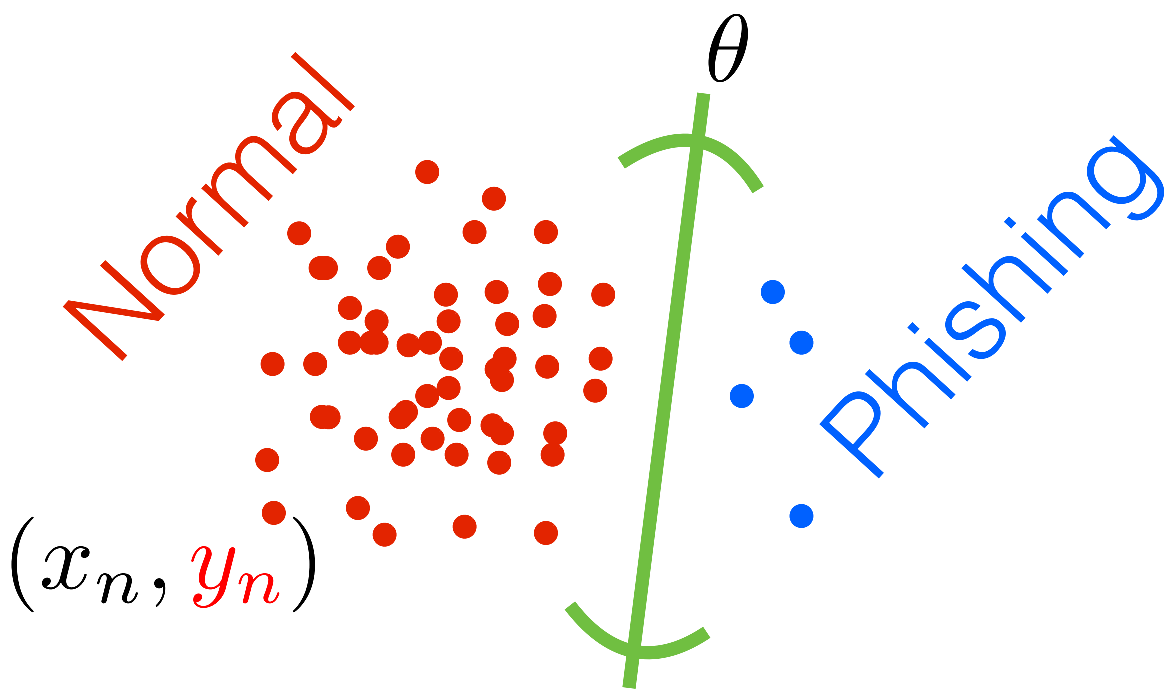

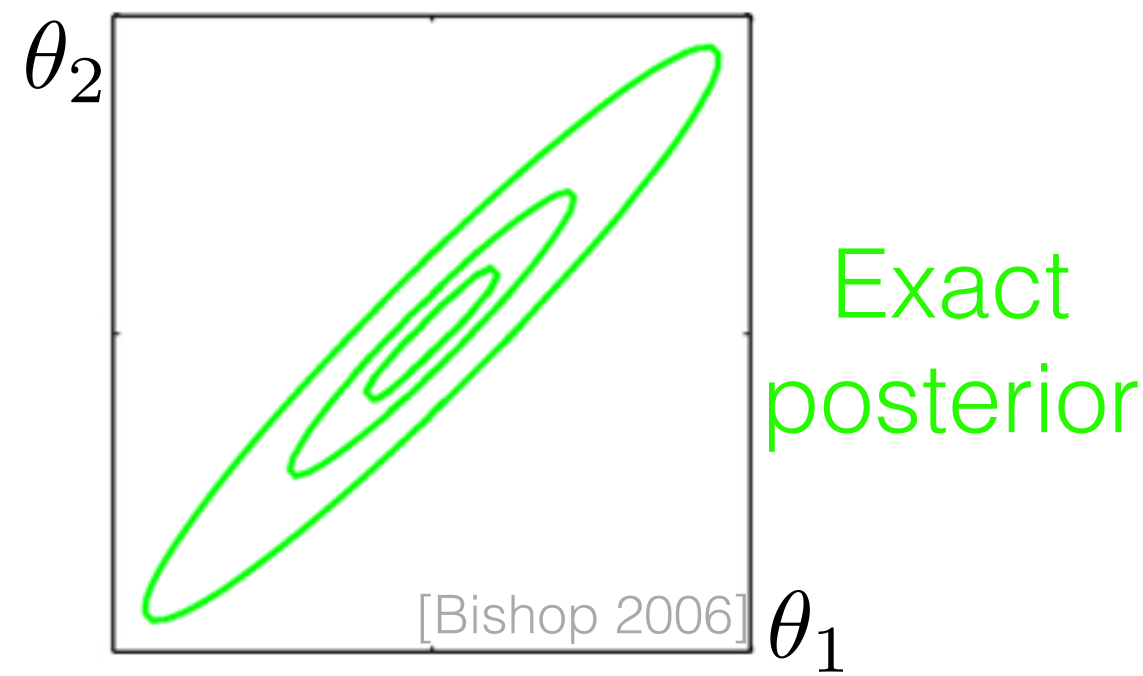

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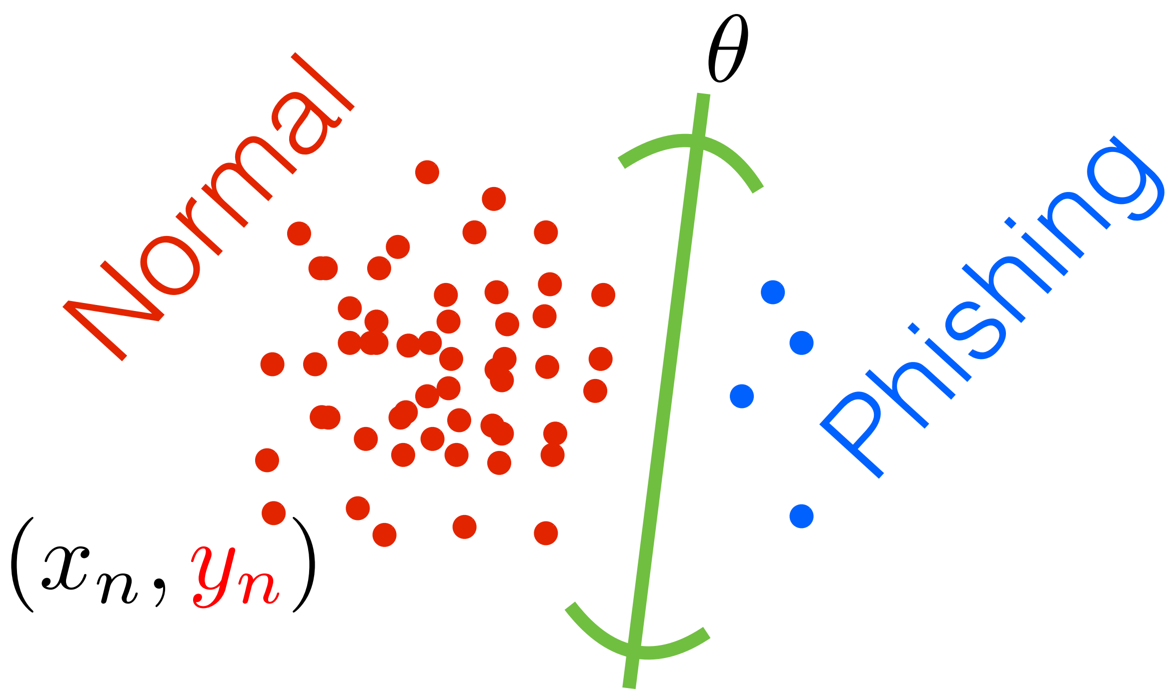

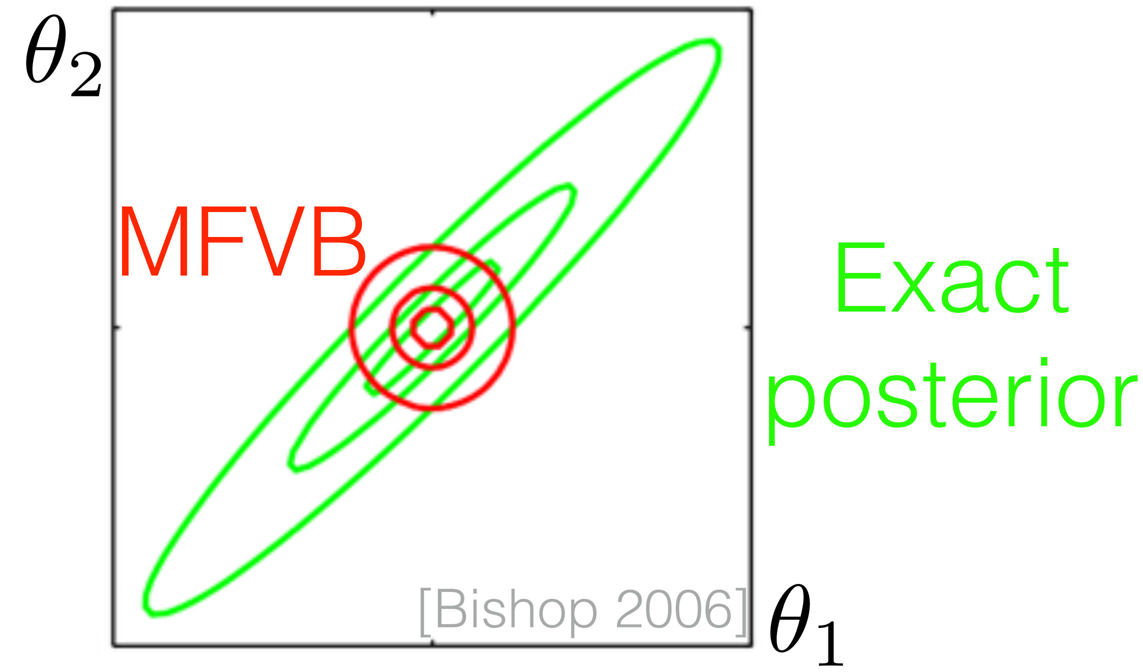

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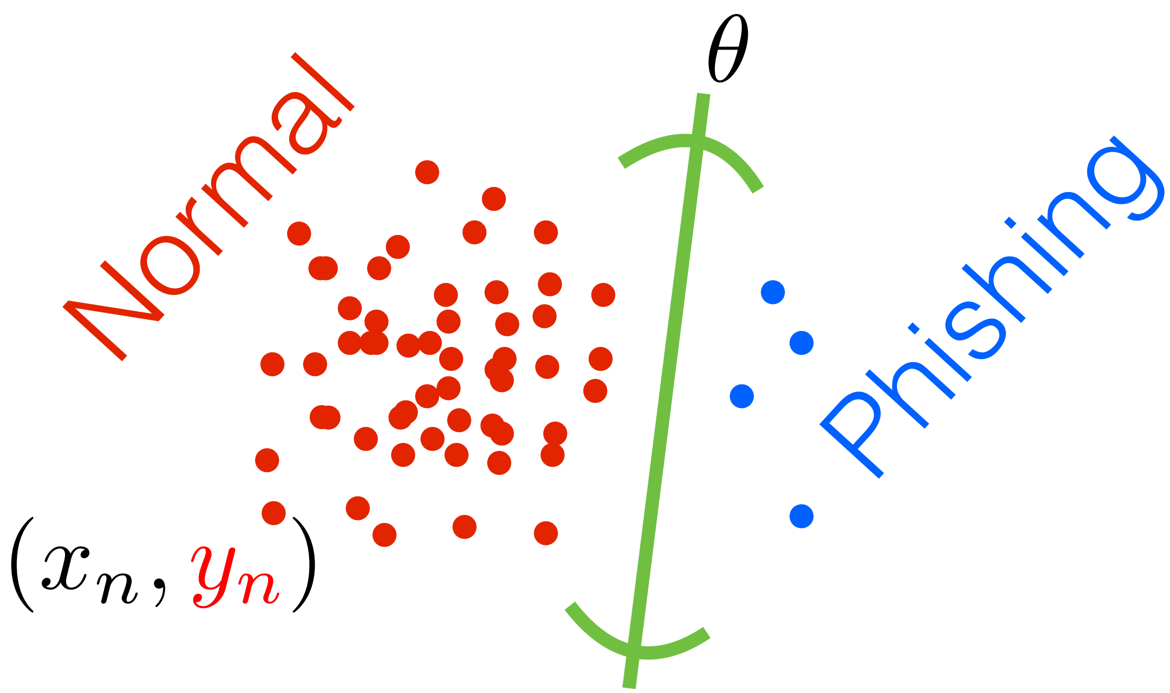

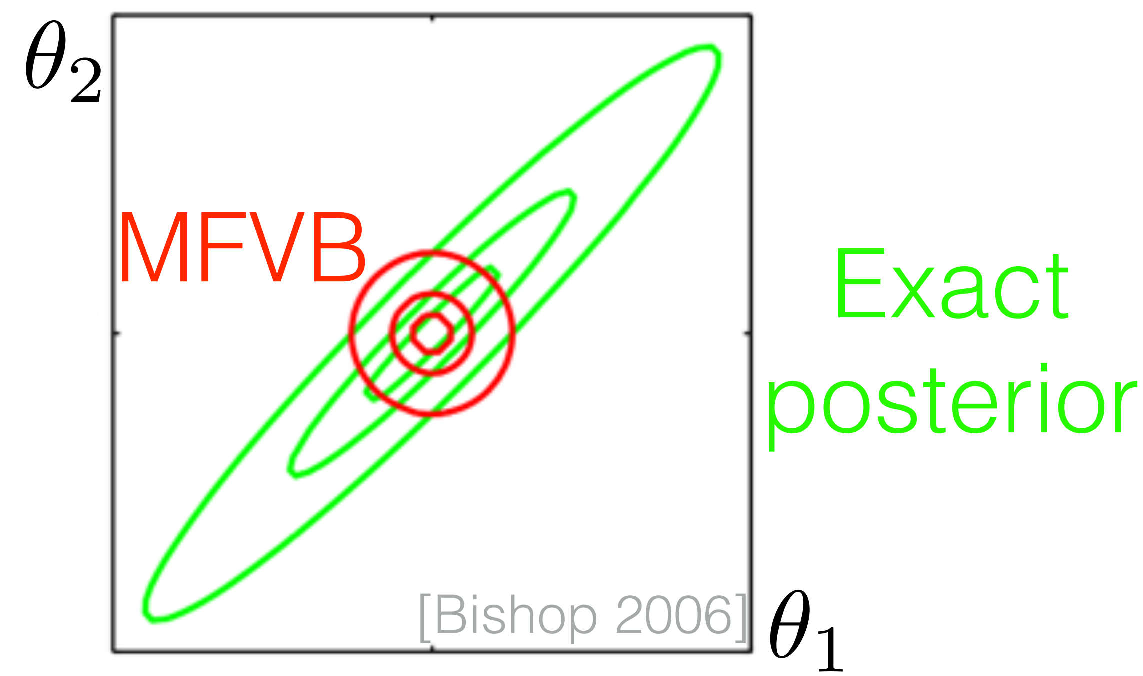

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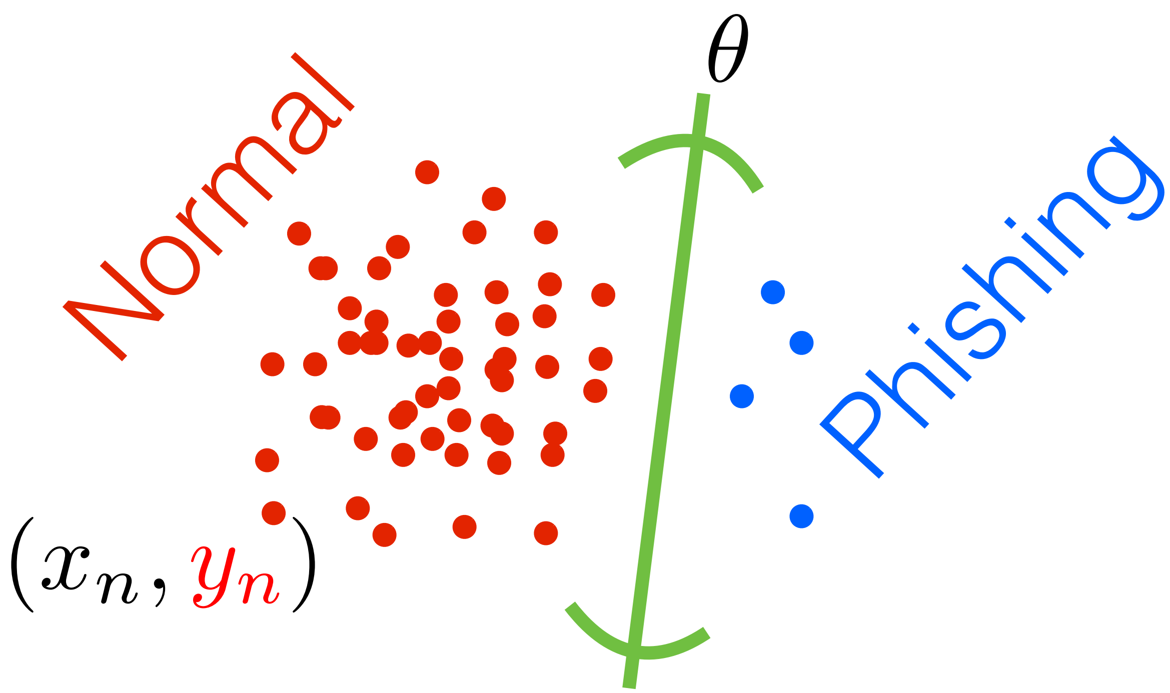

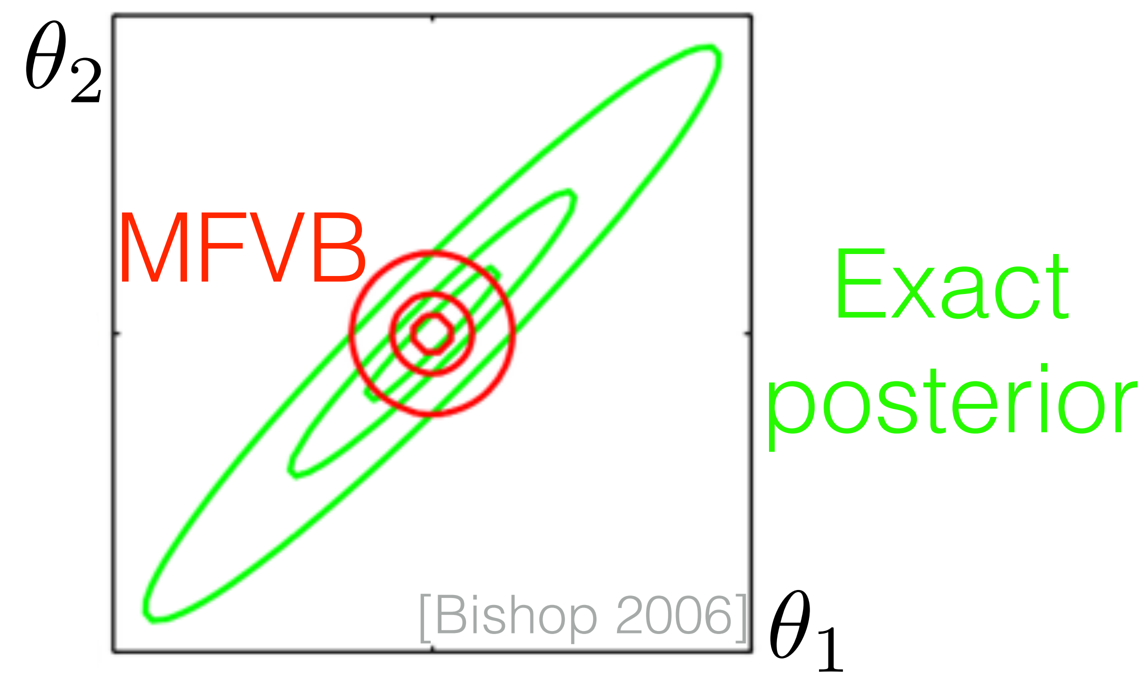

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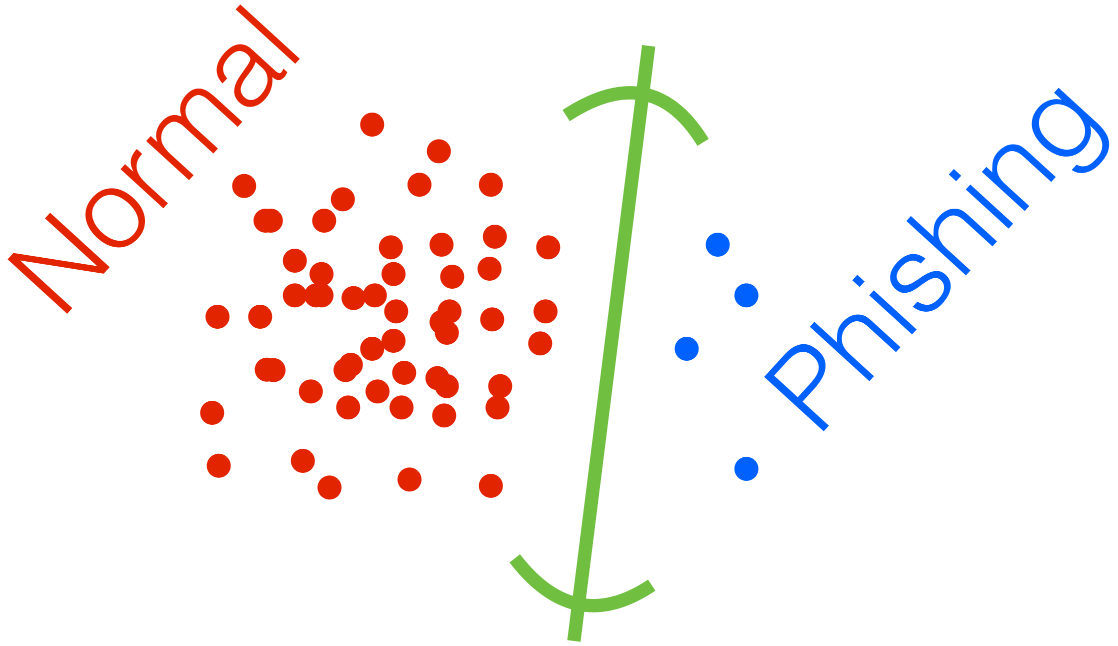
- Automation: e.g. Stan, NUTS, ADVI

[http://mc-stan.org/ ; Hoffman, Gelman 2014; Kucukelbir, Tran, Ranganath, Gelman, Blei 2017]

Bayesian coresets

- Posterior $p(\theta|y) \propto_\theta p(y|\theta)p(\theta)$



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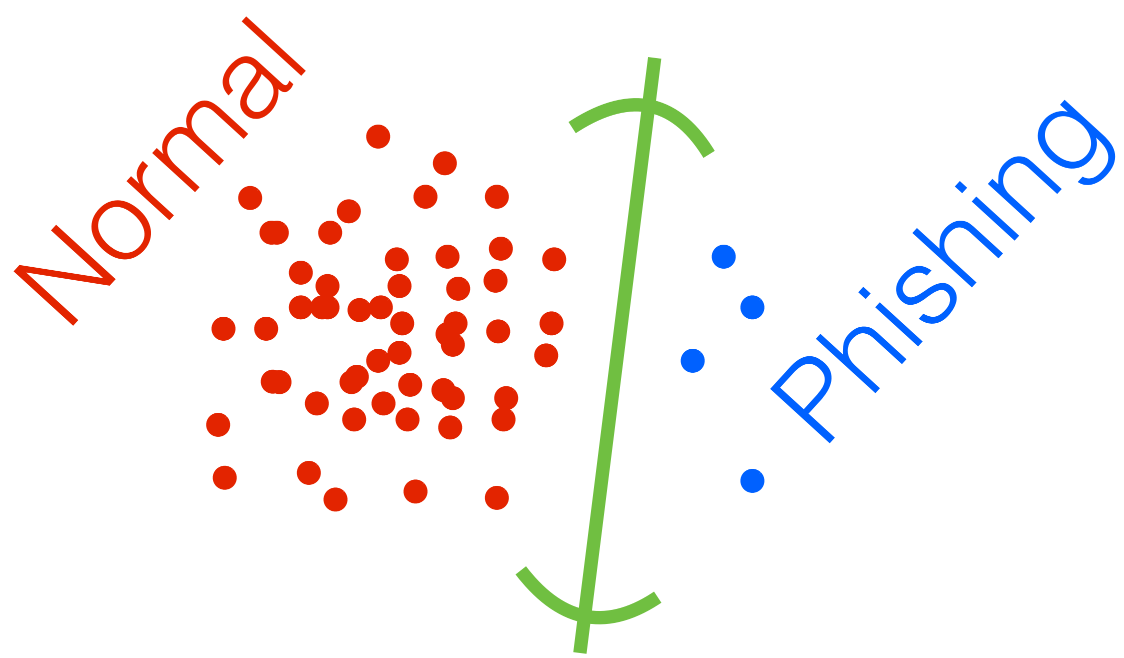

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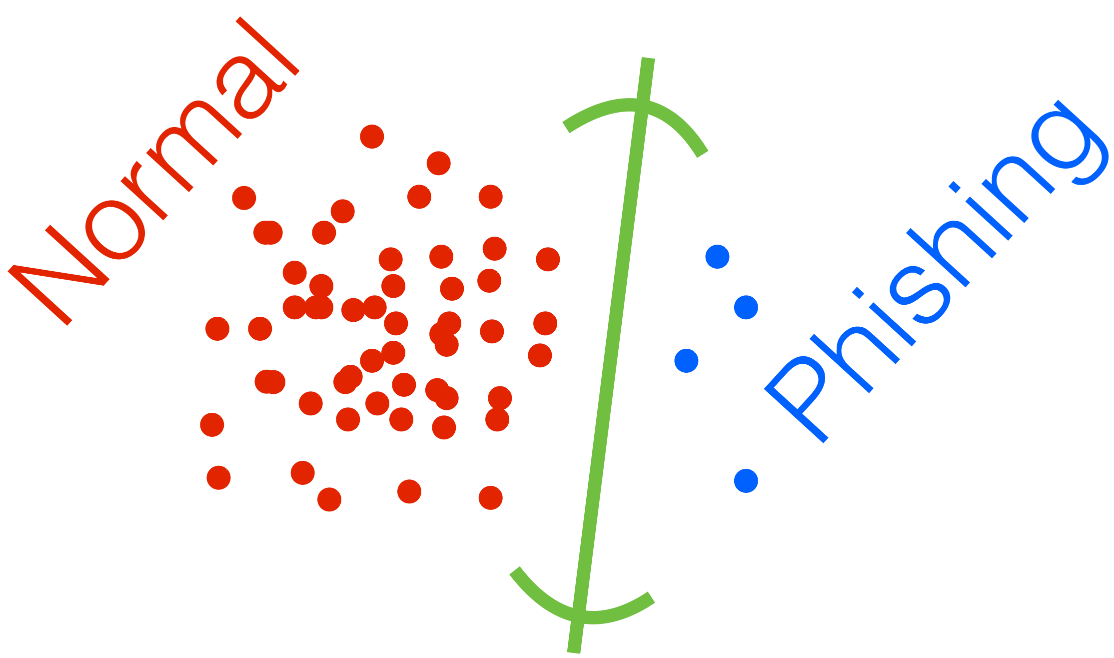

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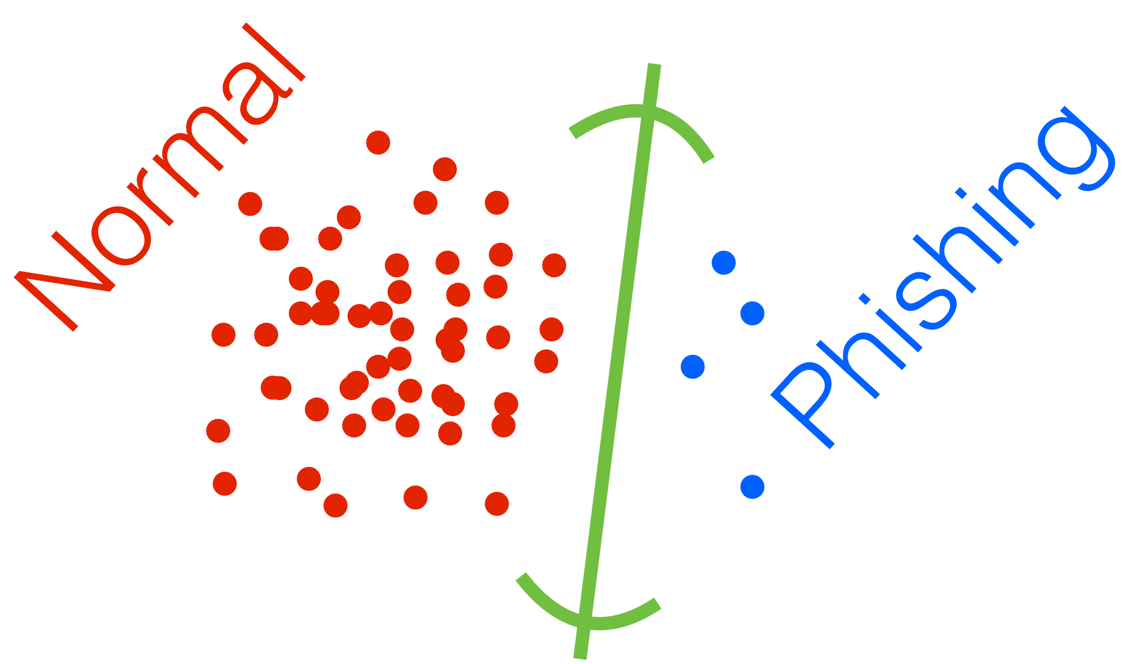
$$\|w\|_0 \ll N$$



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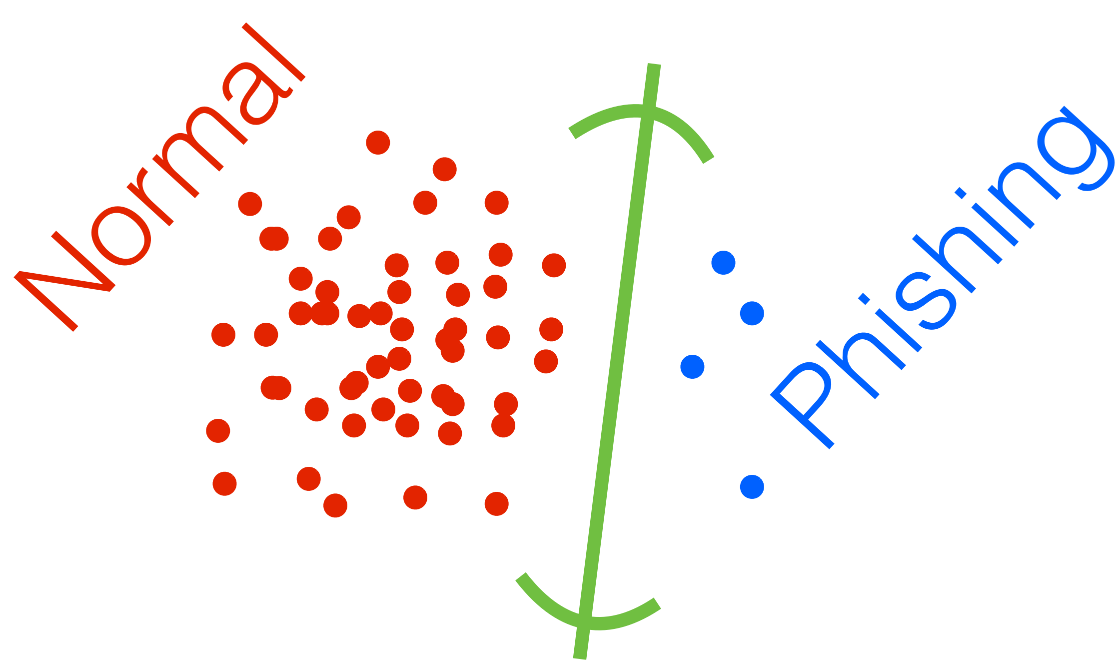

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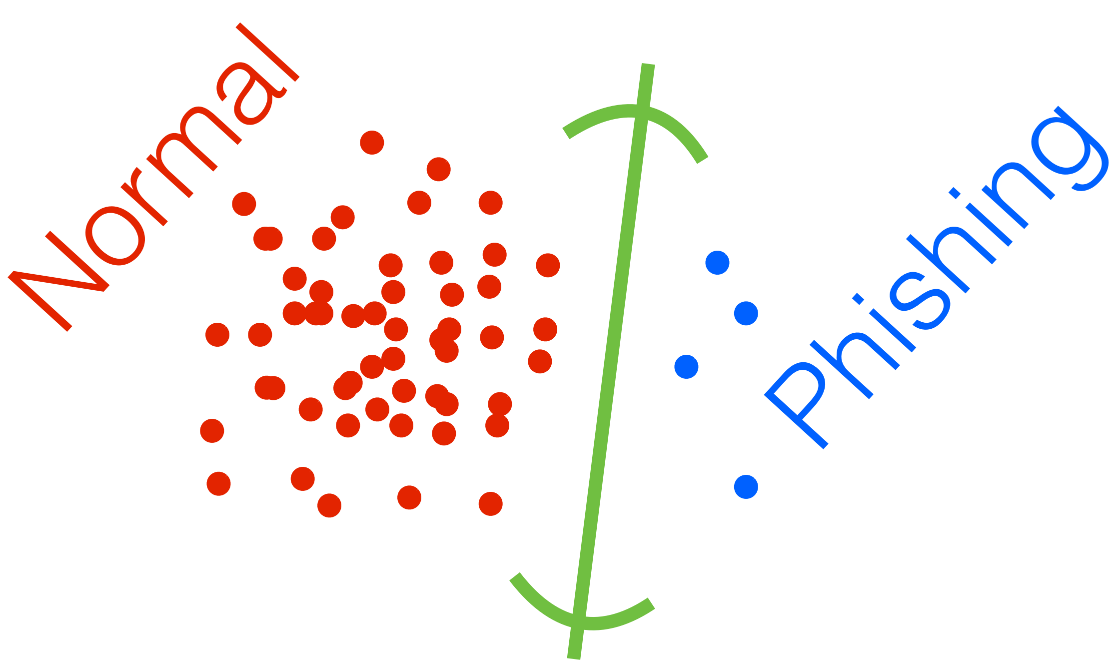

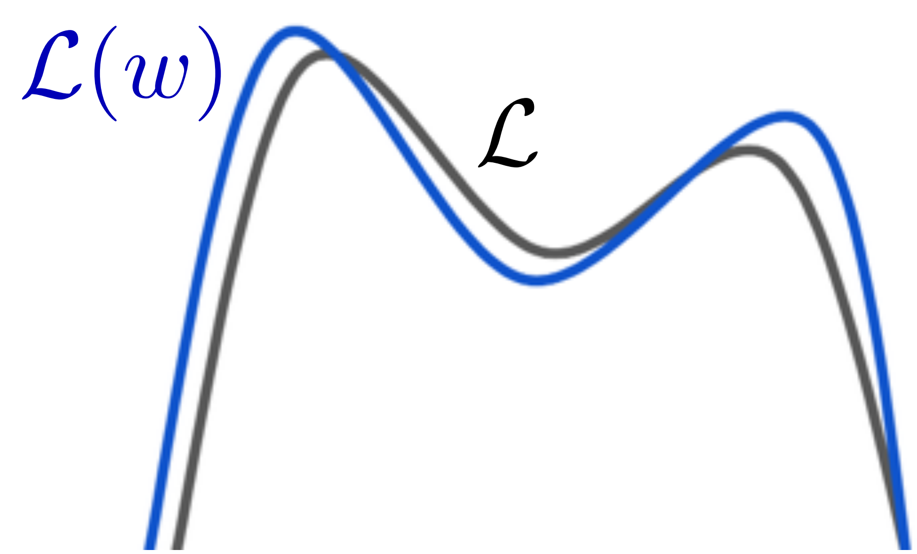

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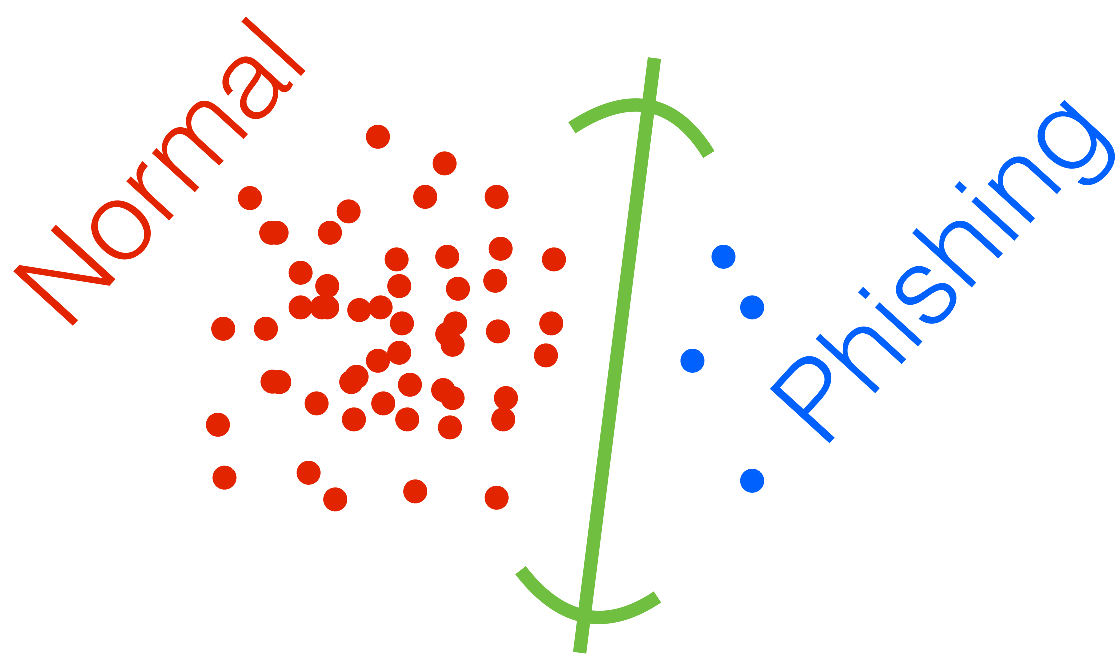

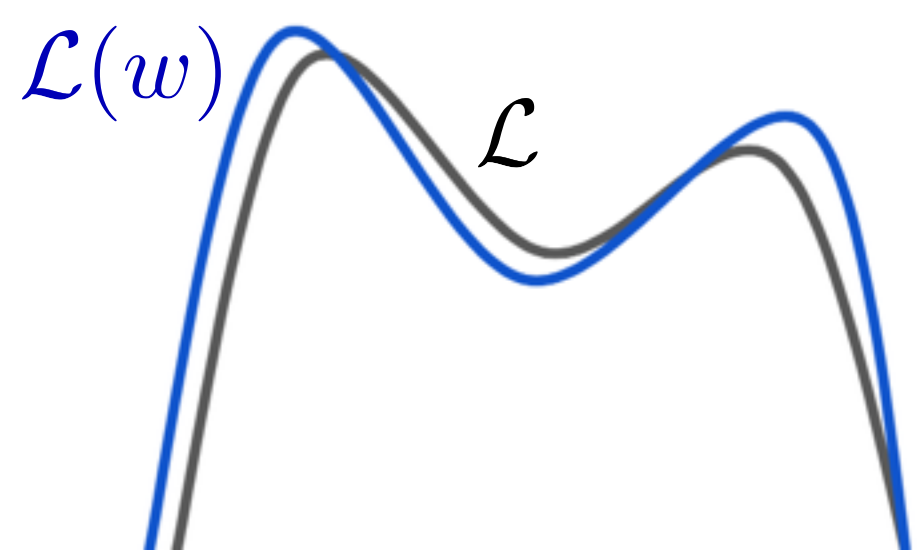

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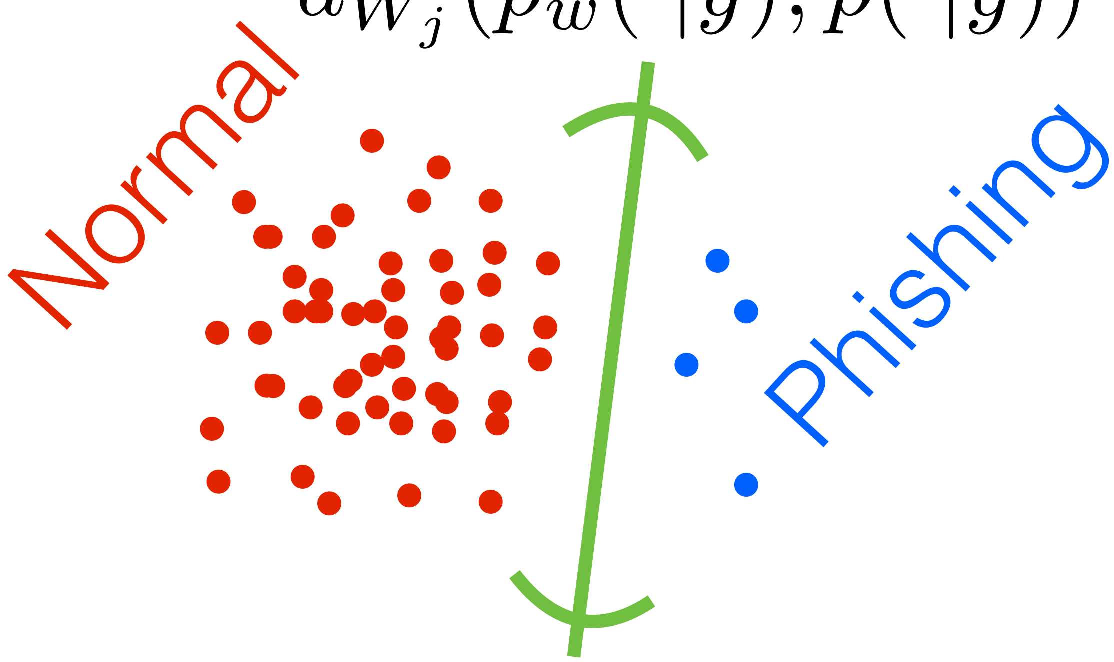
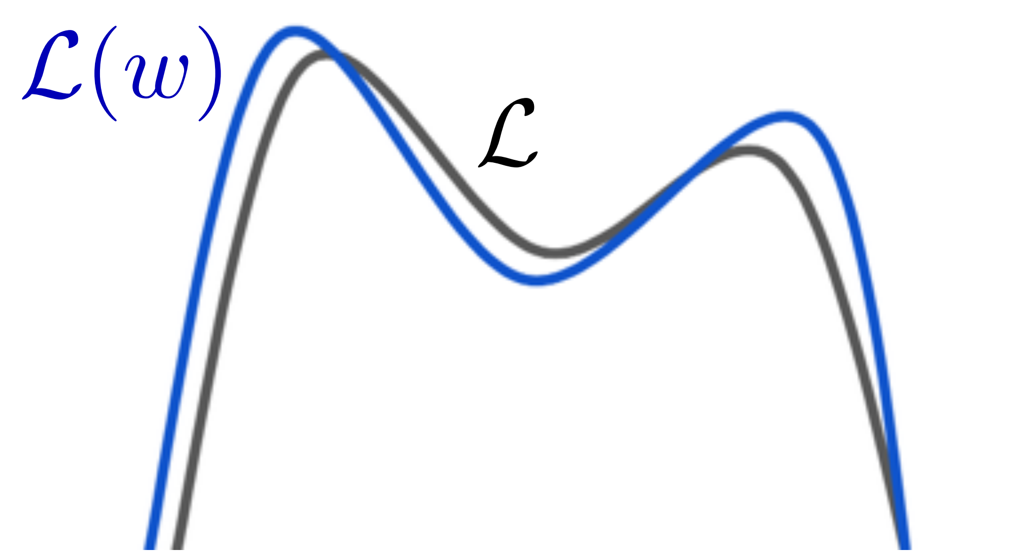
  - Approximate posterior close in Wasserstein distance $d_{W_j}(p_w(\cdot|y), p(\cdot|y)) \le C_j \|\mathcal{L}(w) - \mathcal{L}\|_{\mathrm{WFID}}, j \in \{1, 2\}$





[Huggins, Kasprzak, Campbell, Broderick, forthcoming]

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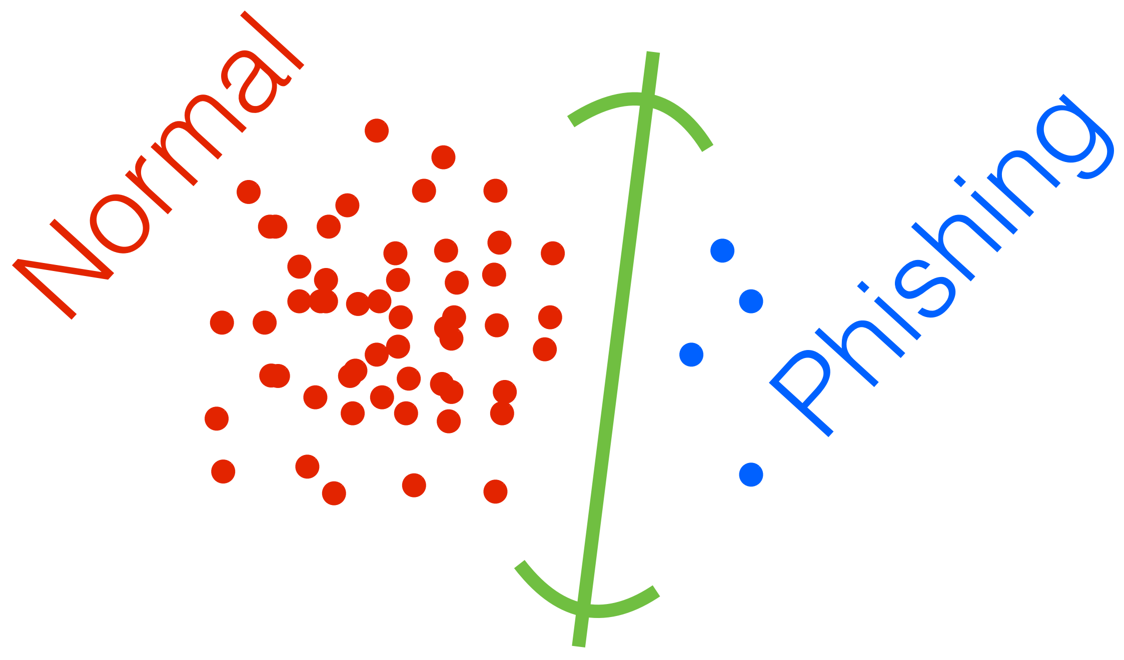

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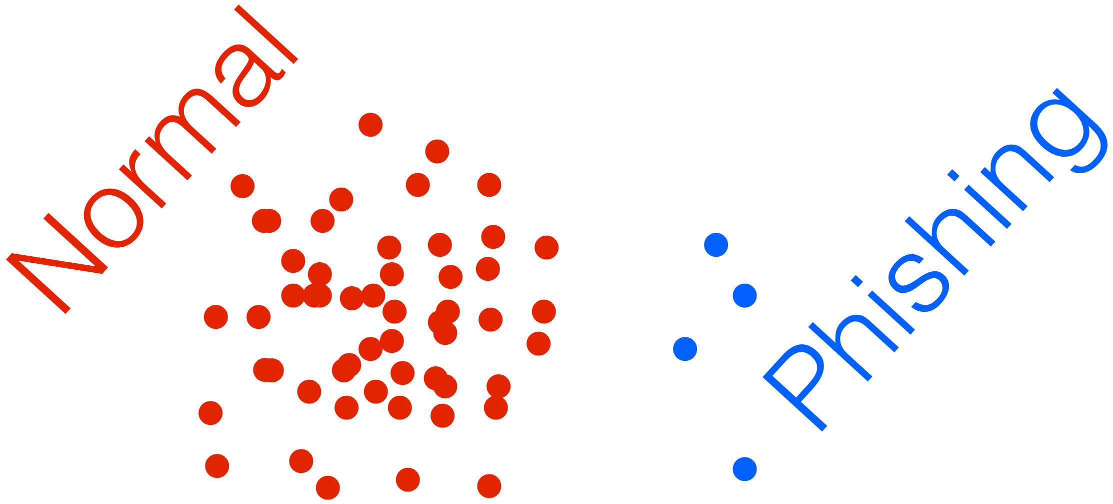

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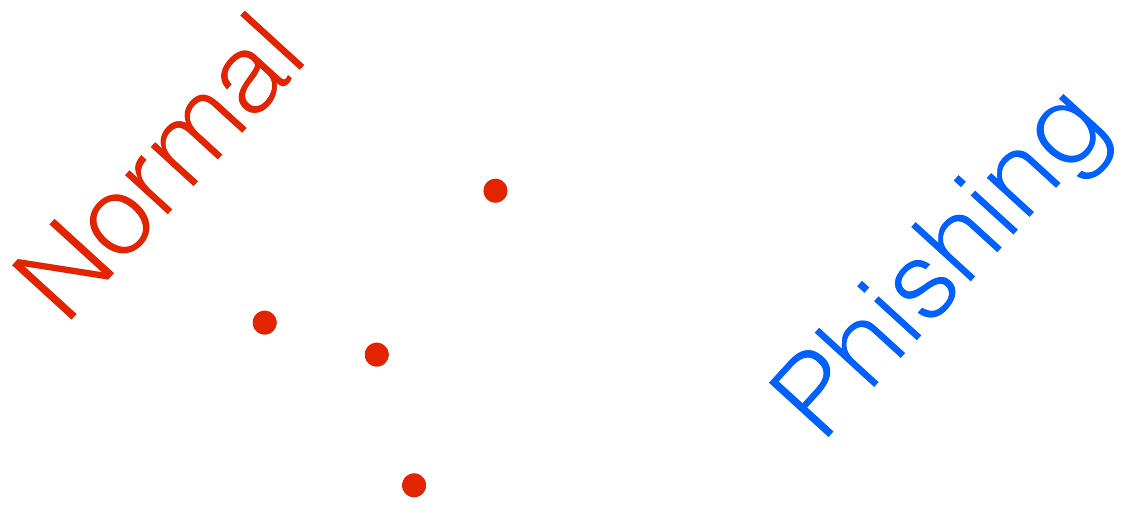

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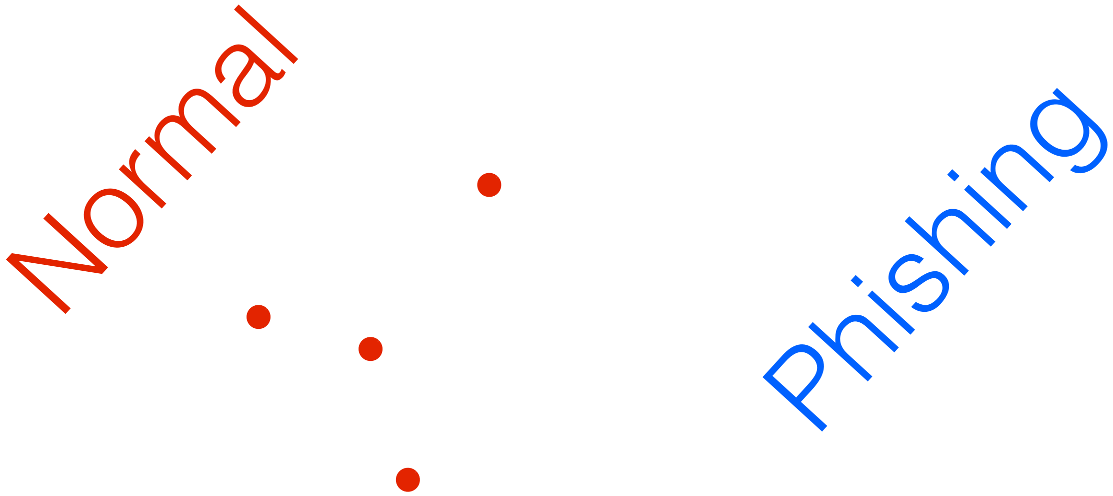

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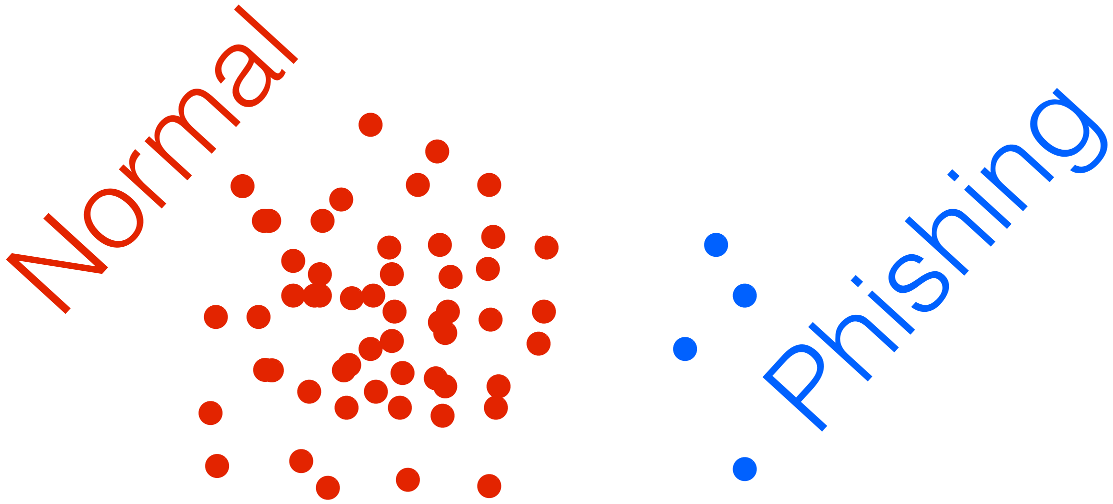

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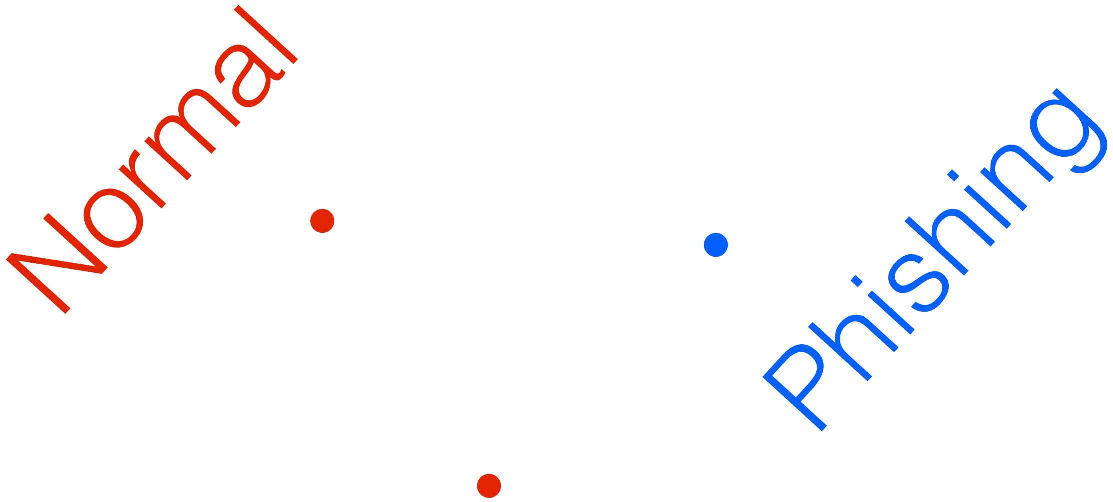

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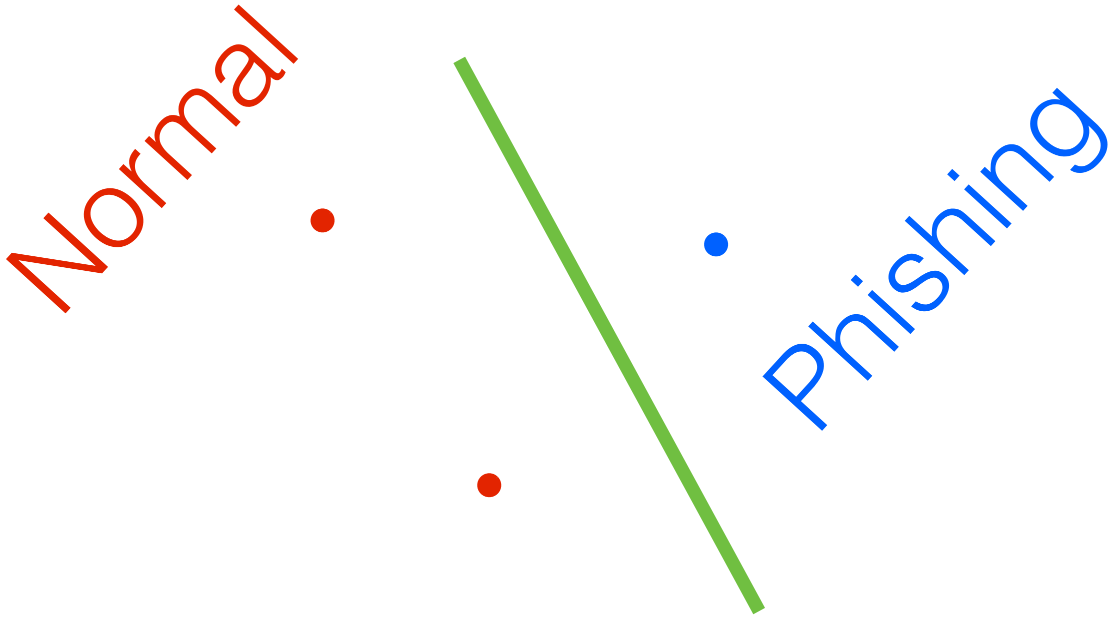

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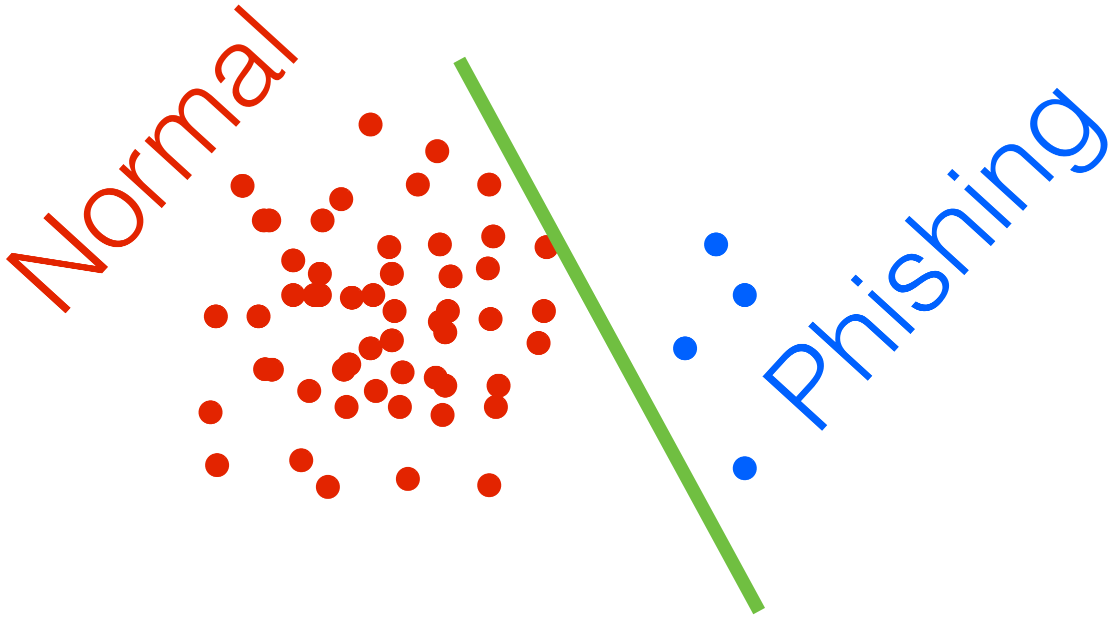

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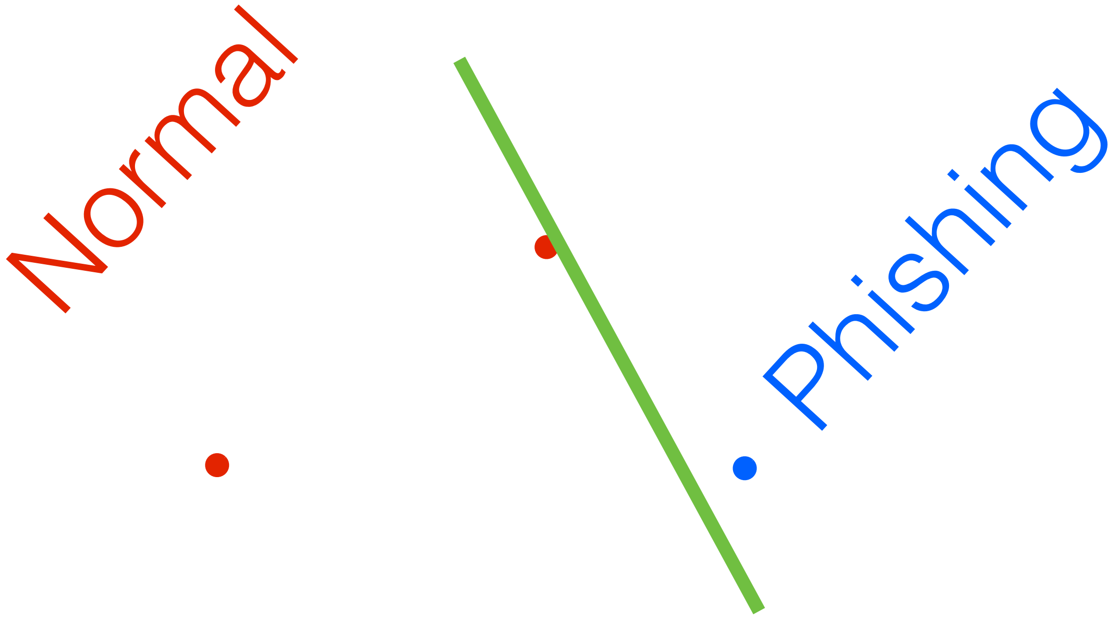

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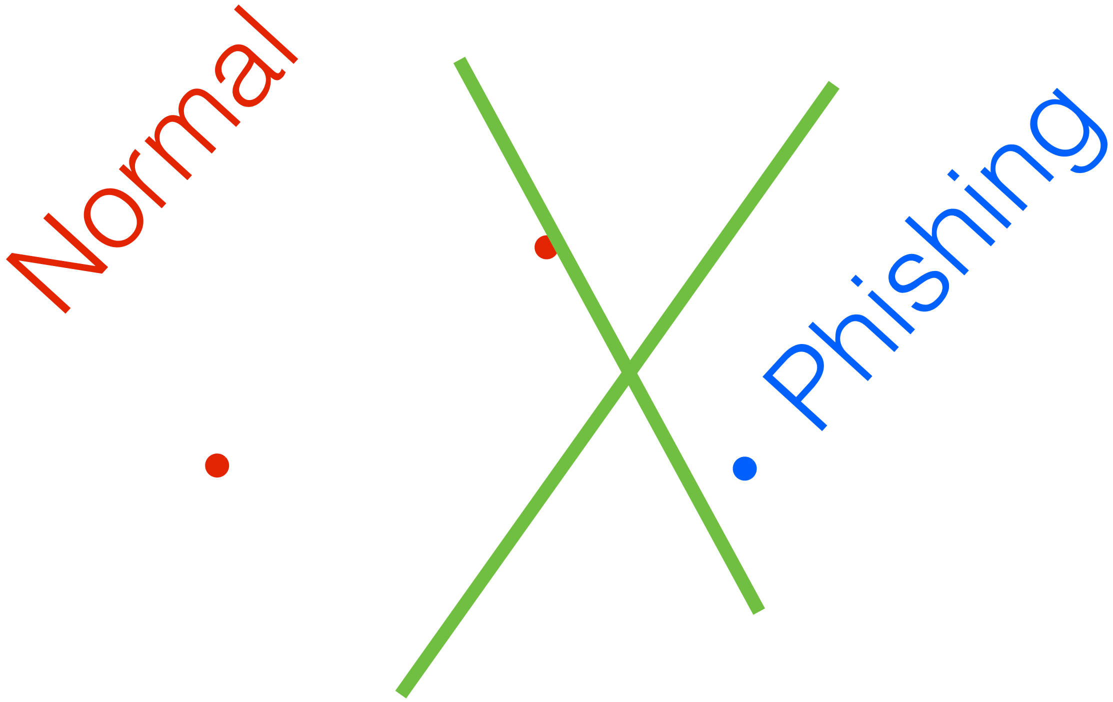

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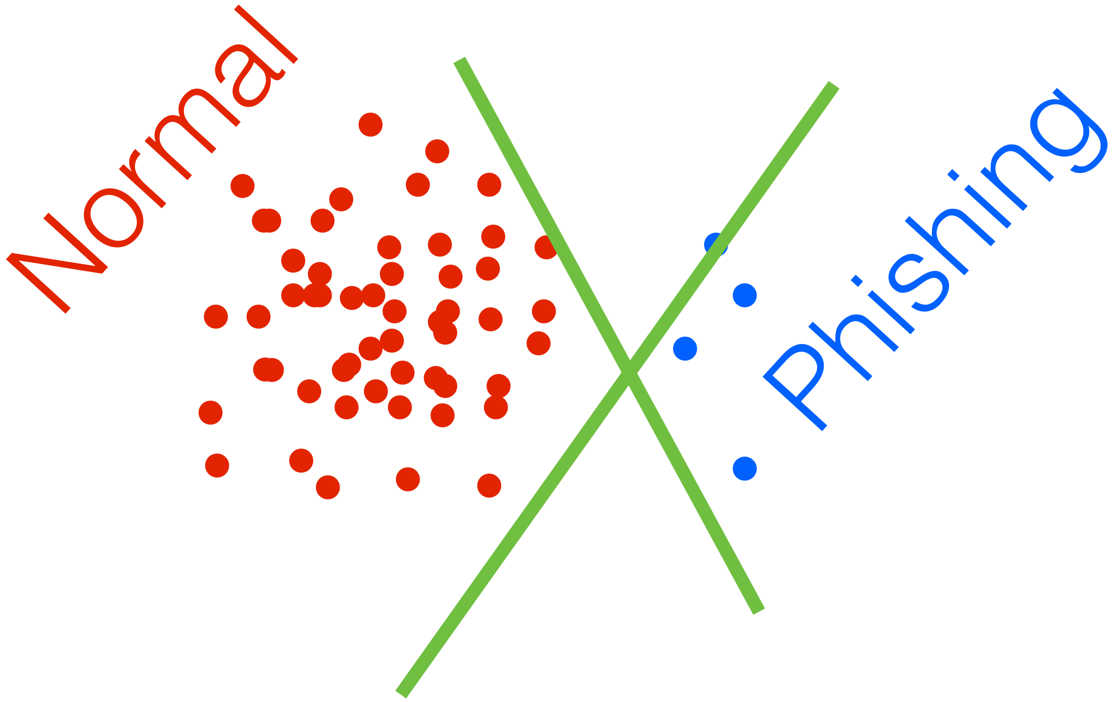

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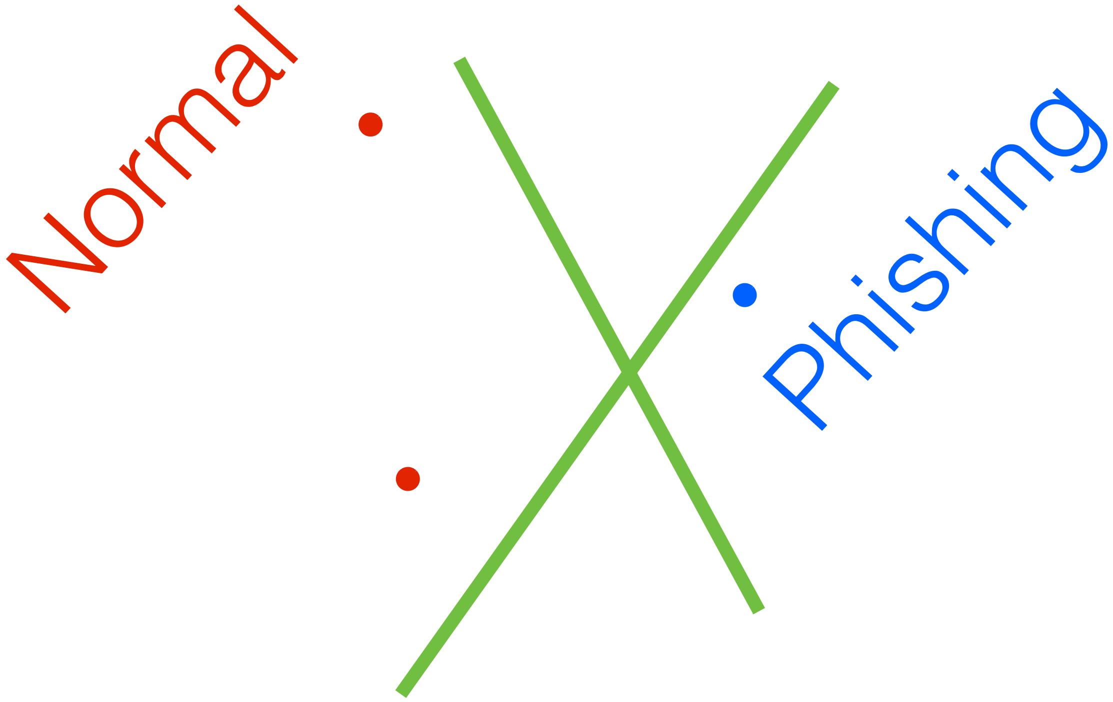

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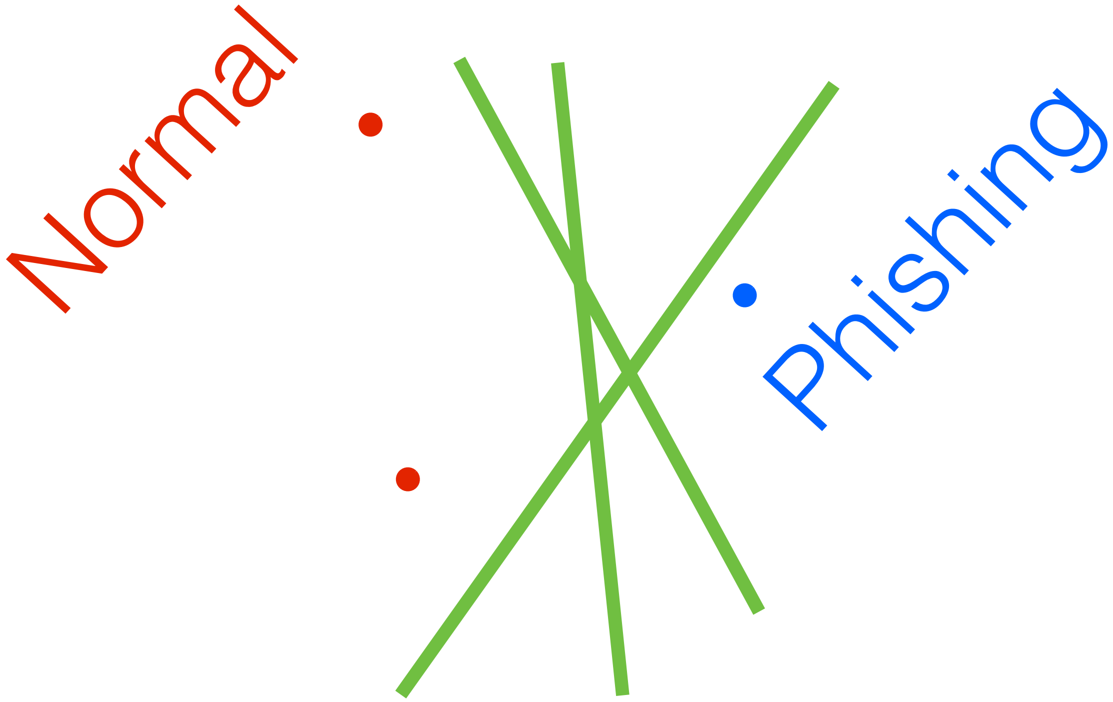

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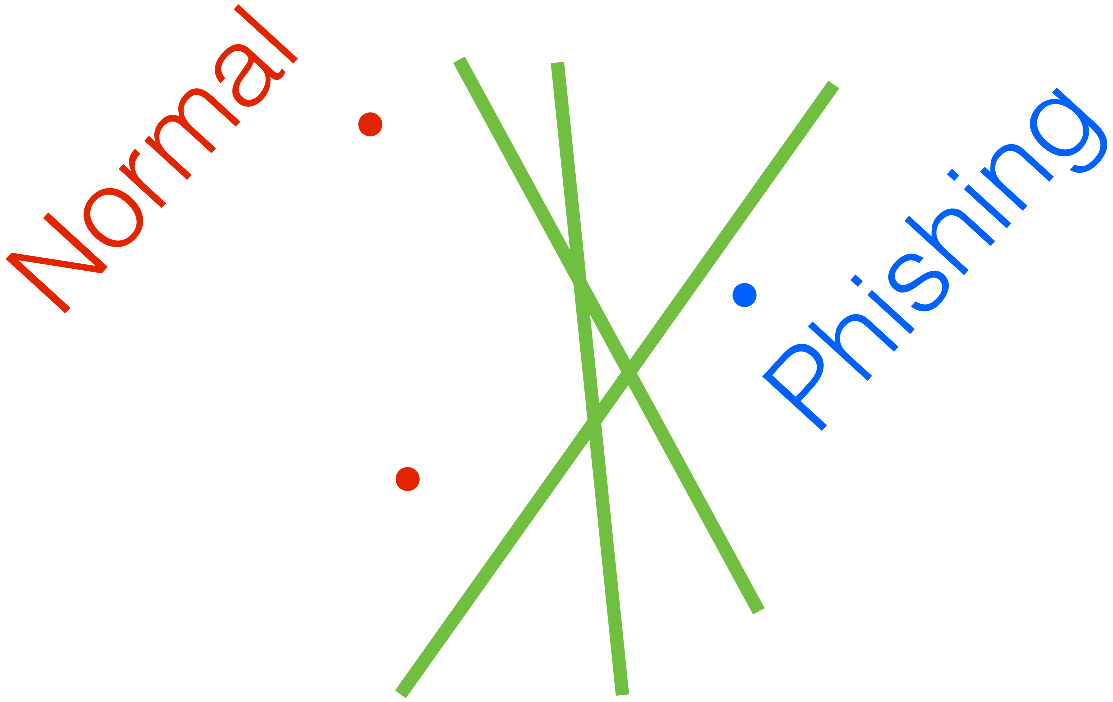

- Might miss important data
- Noisy estimates

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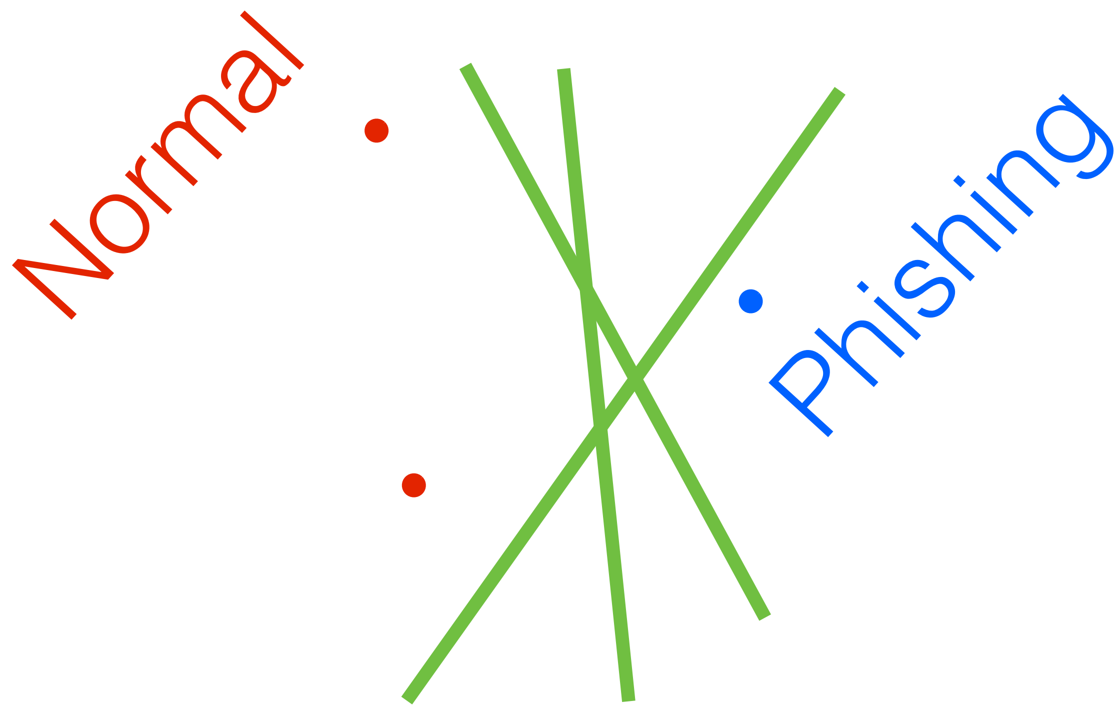

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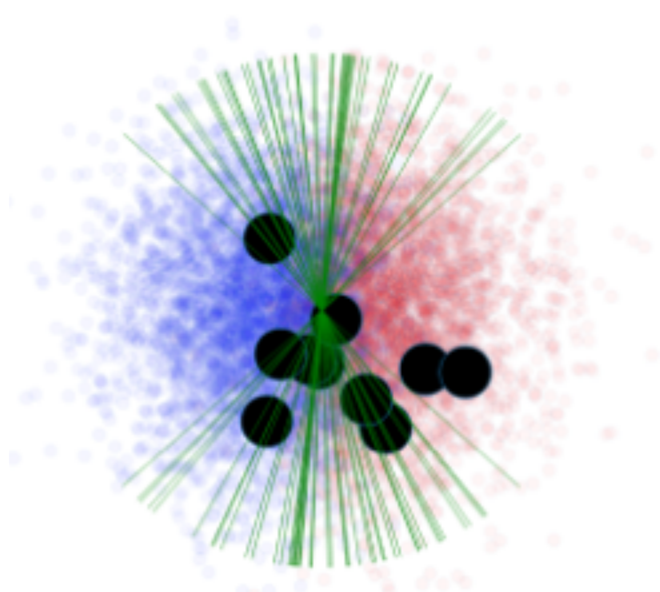
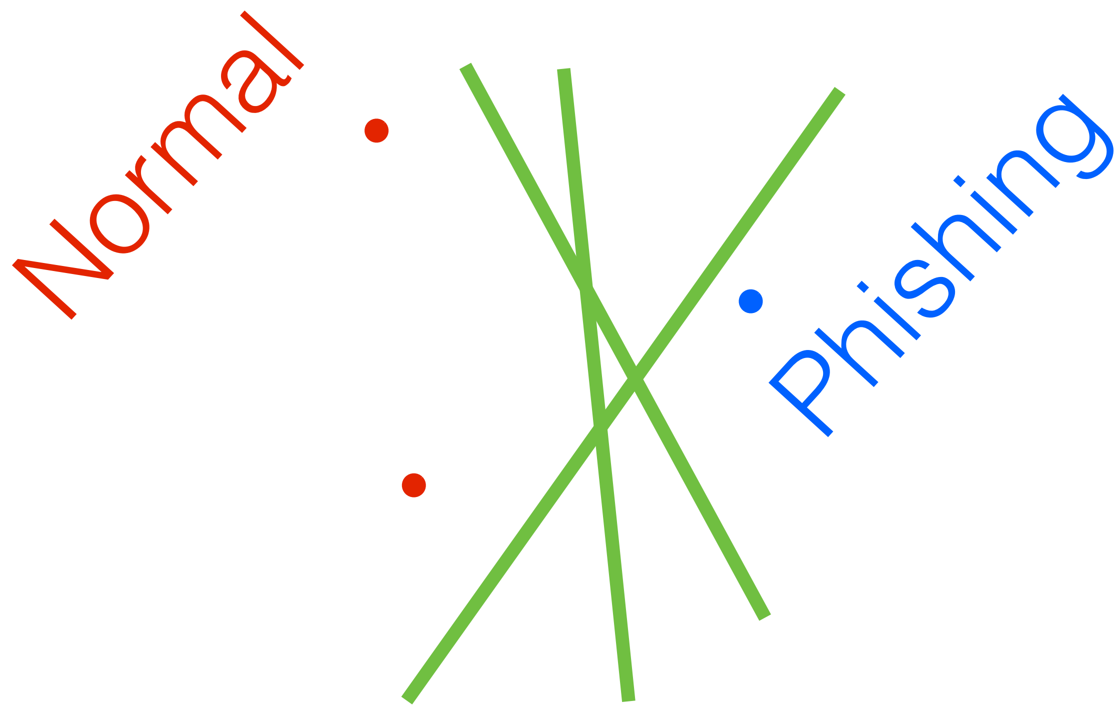


$M = 10$

Uniform subsampling revisited



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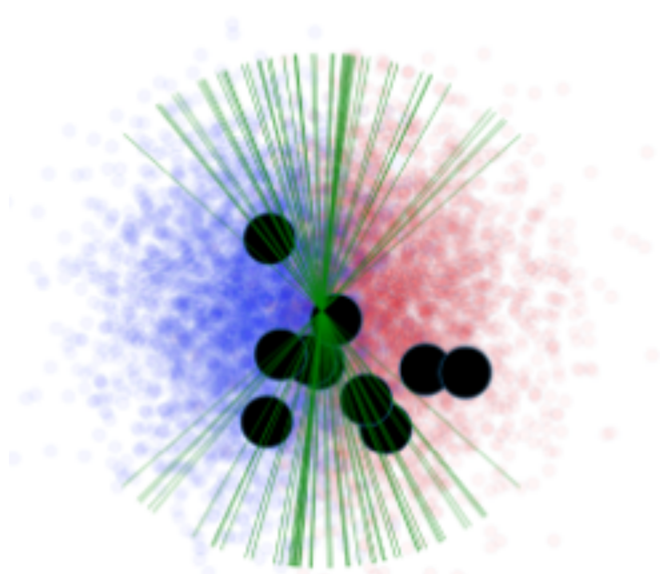

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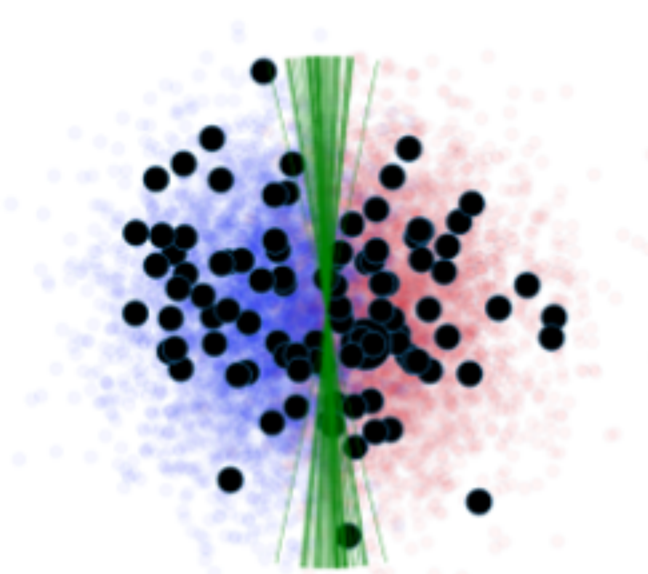
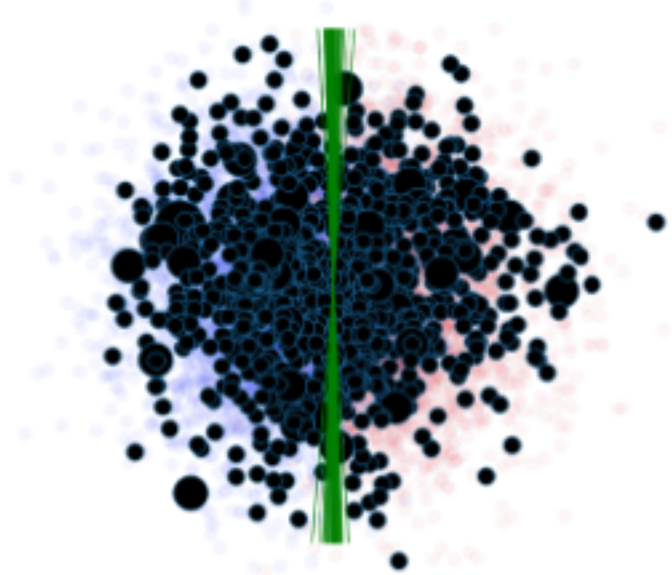


$M = 100$



$M = 1000$

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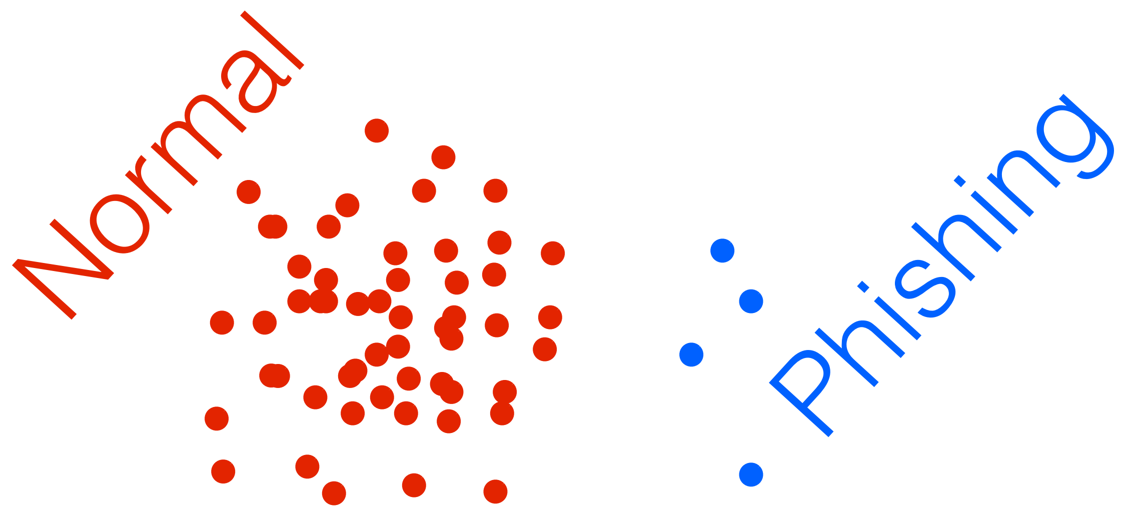

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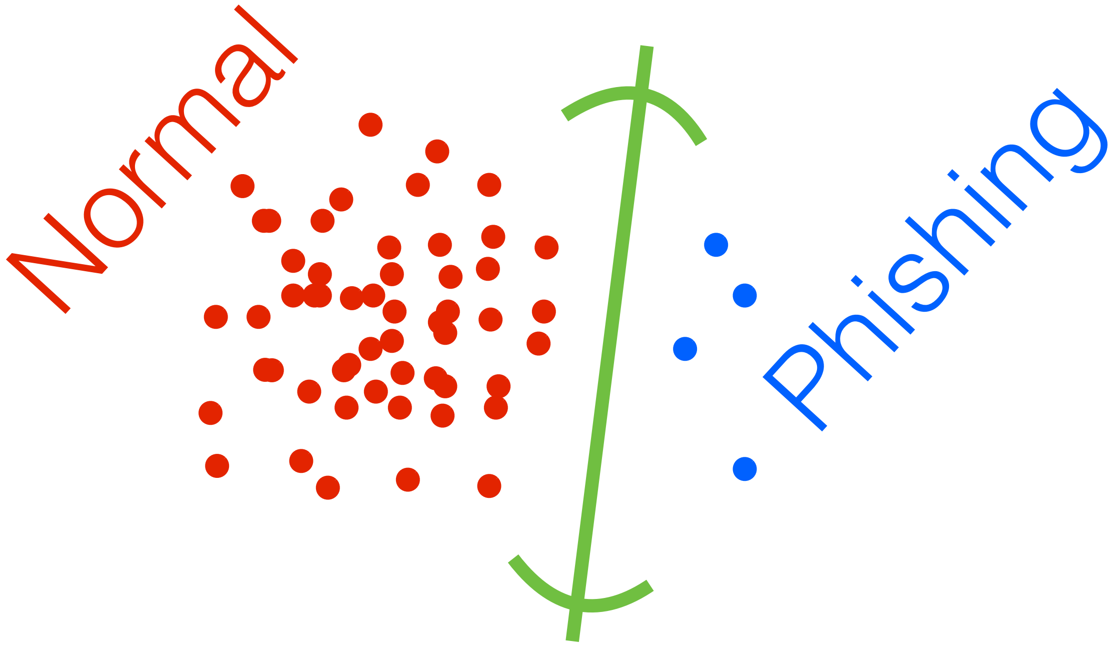

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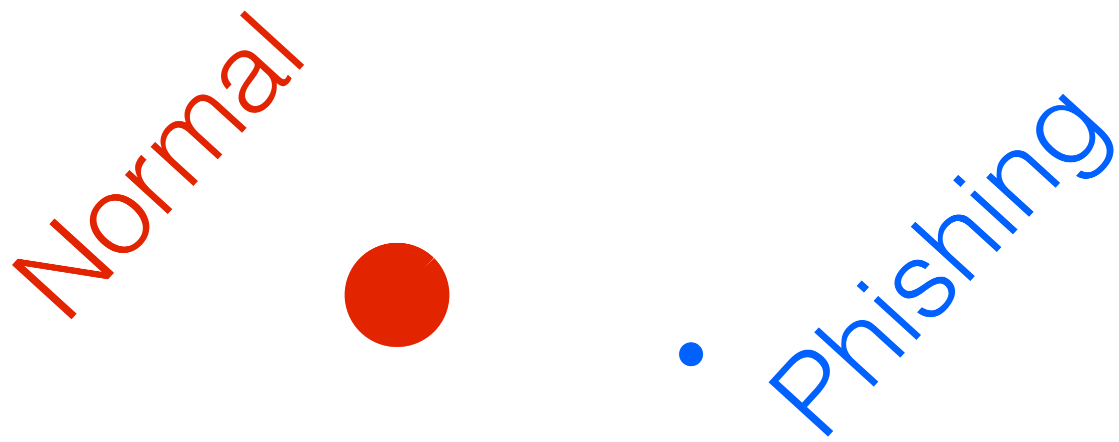

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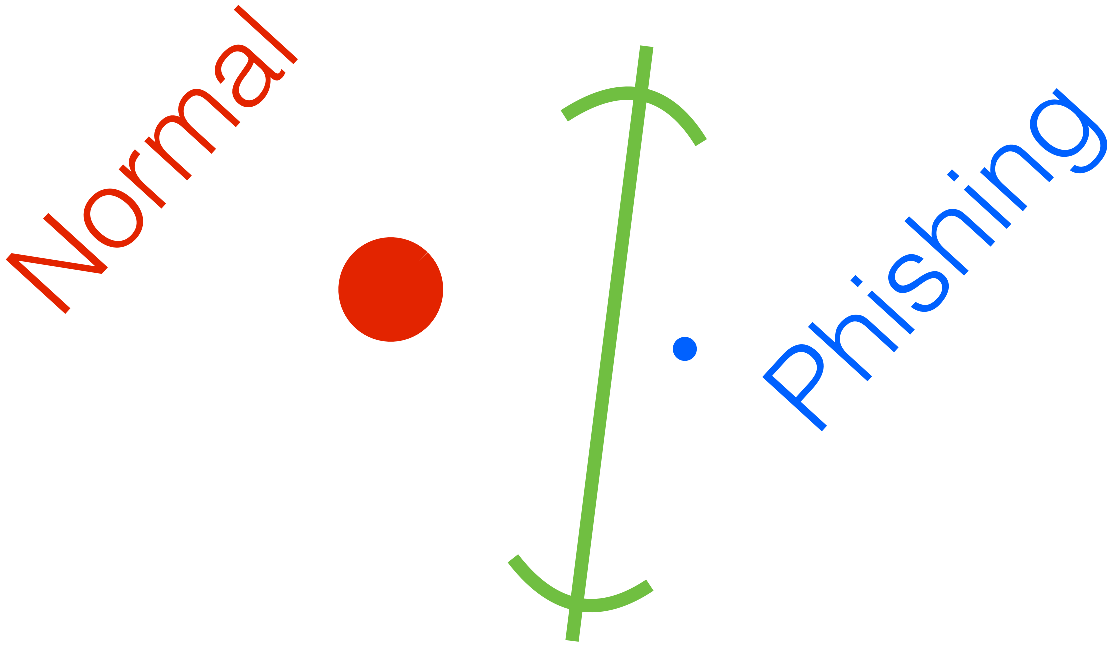

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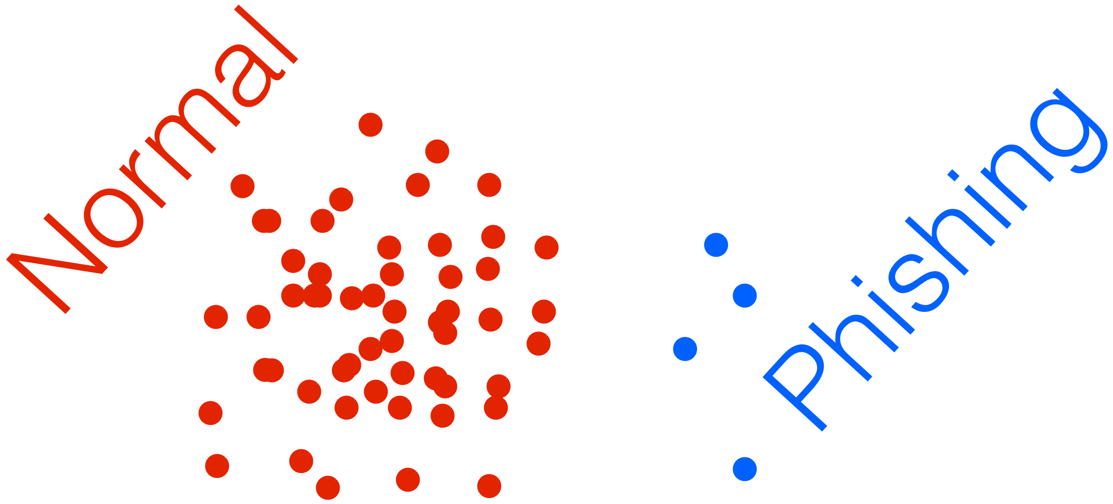

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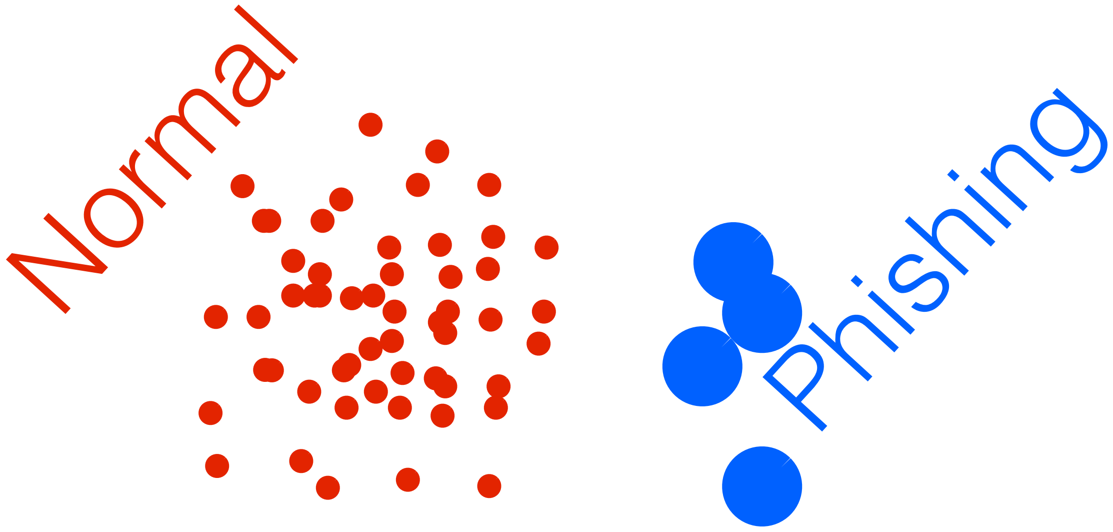

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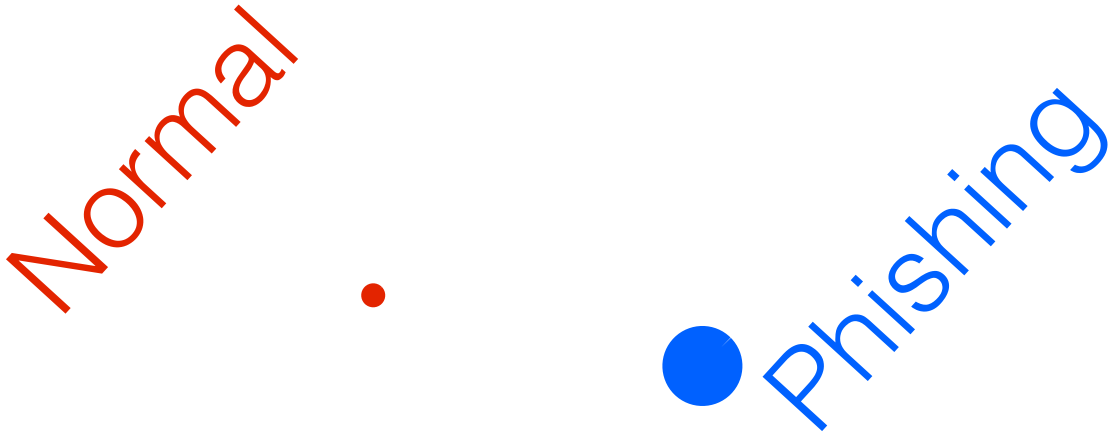

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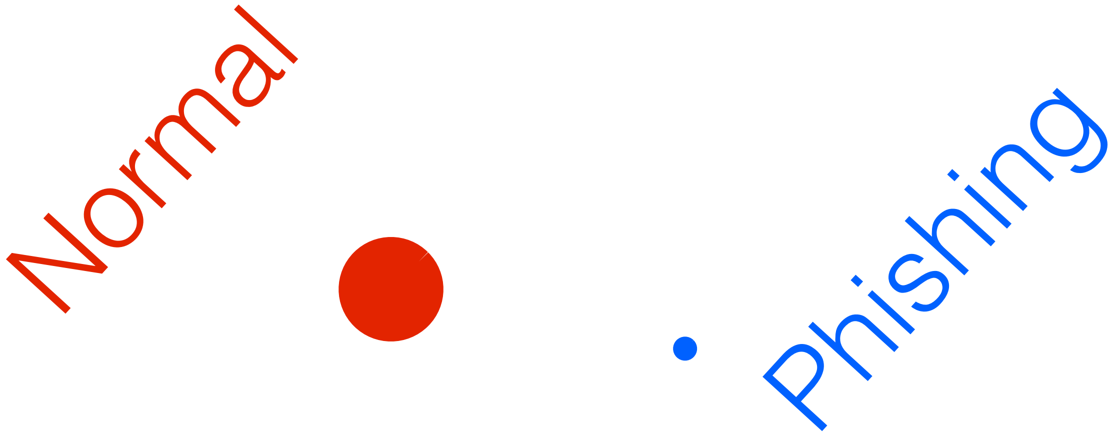

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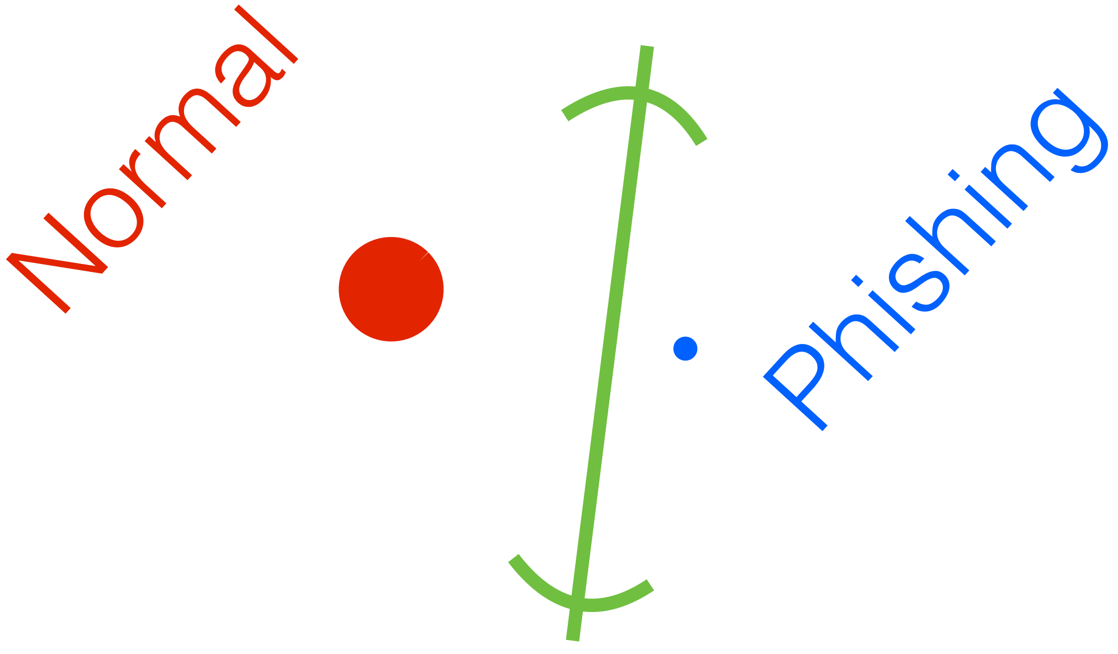

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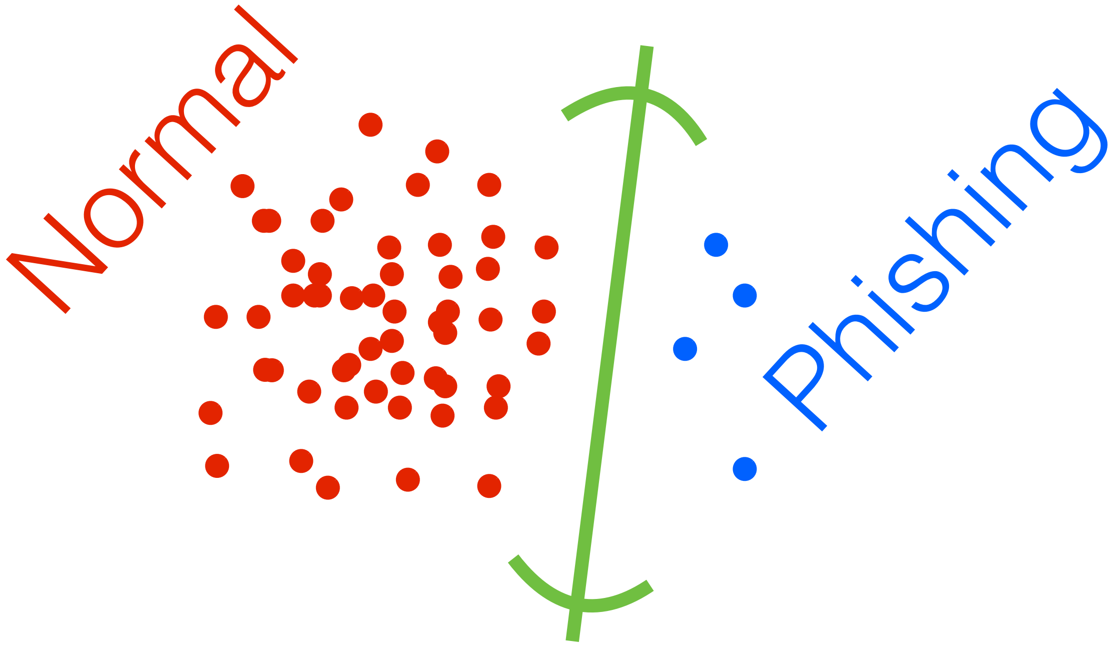

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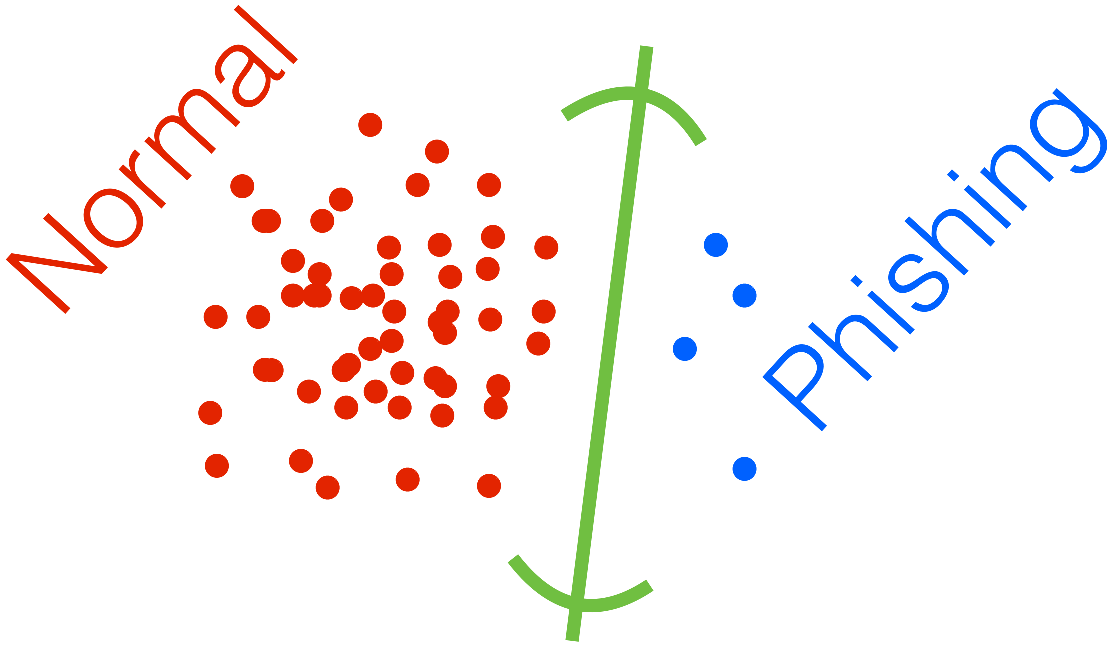
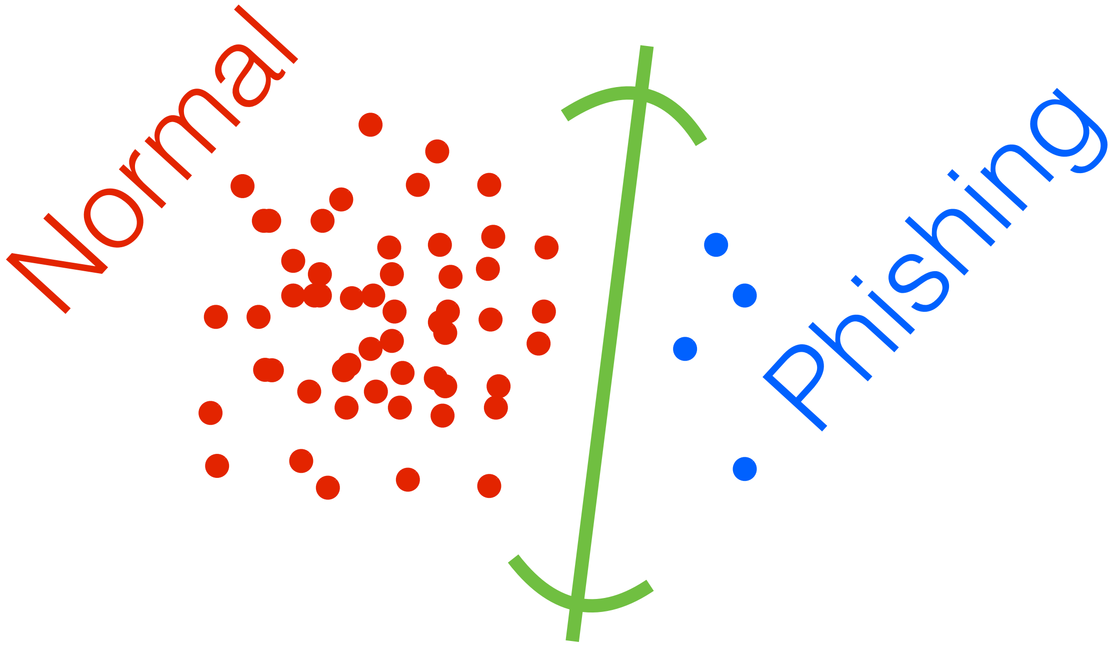


$$\sigma_n \propto \|\mathcal{L}_n\|$$

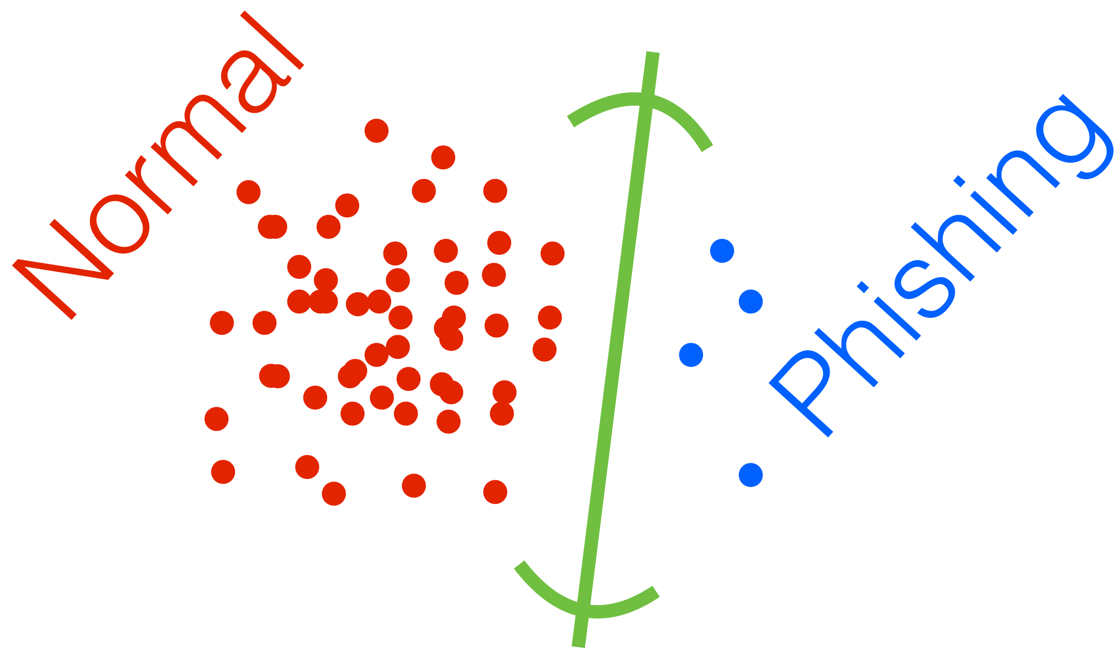
Importance sampling



$$\sigma := \sum_{n=1}^{N} \|\mathcal{L}_n\|$$

$$\sigma_n := \|\mathcal{L}_n\|/\sigma$$

Importance sampling



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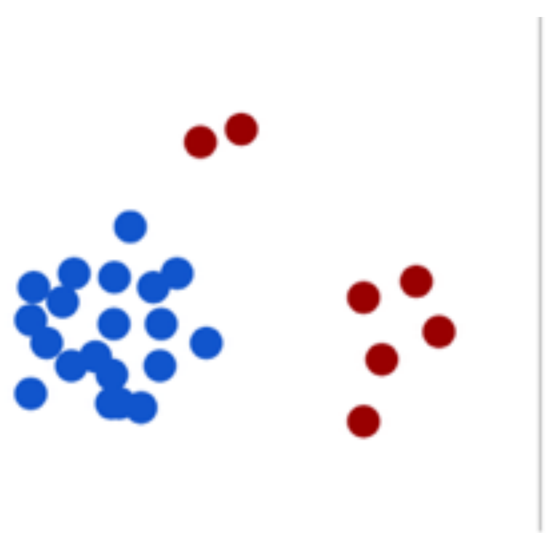
1. data



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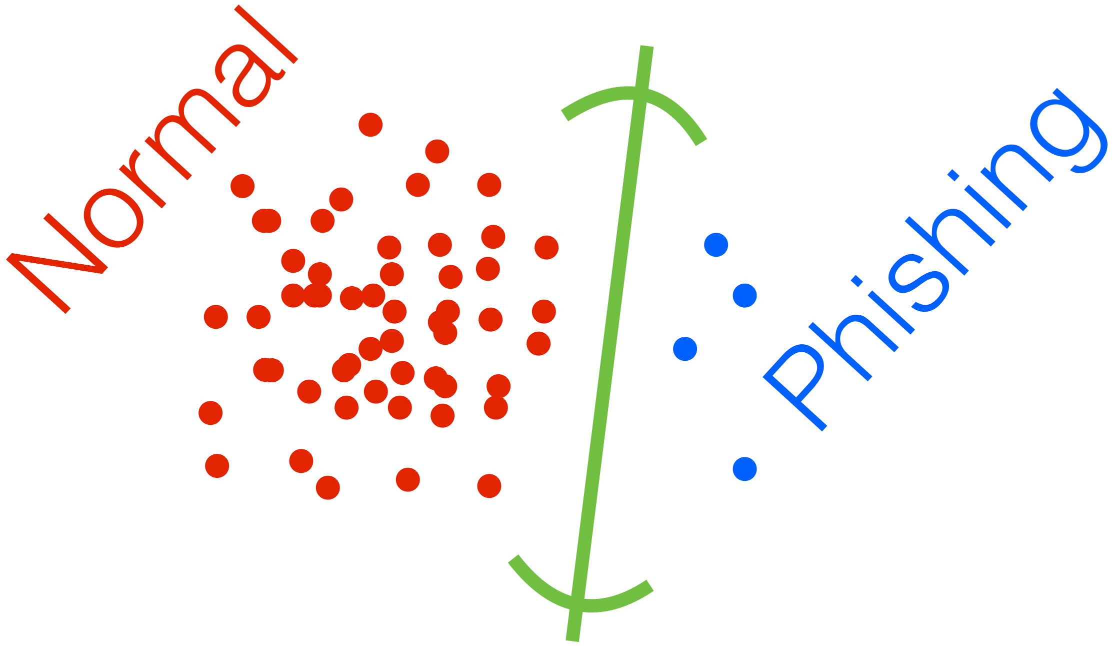

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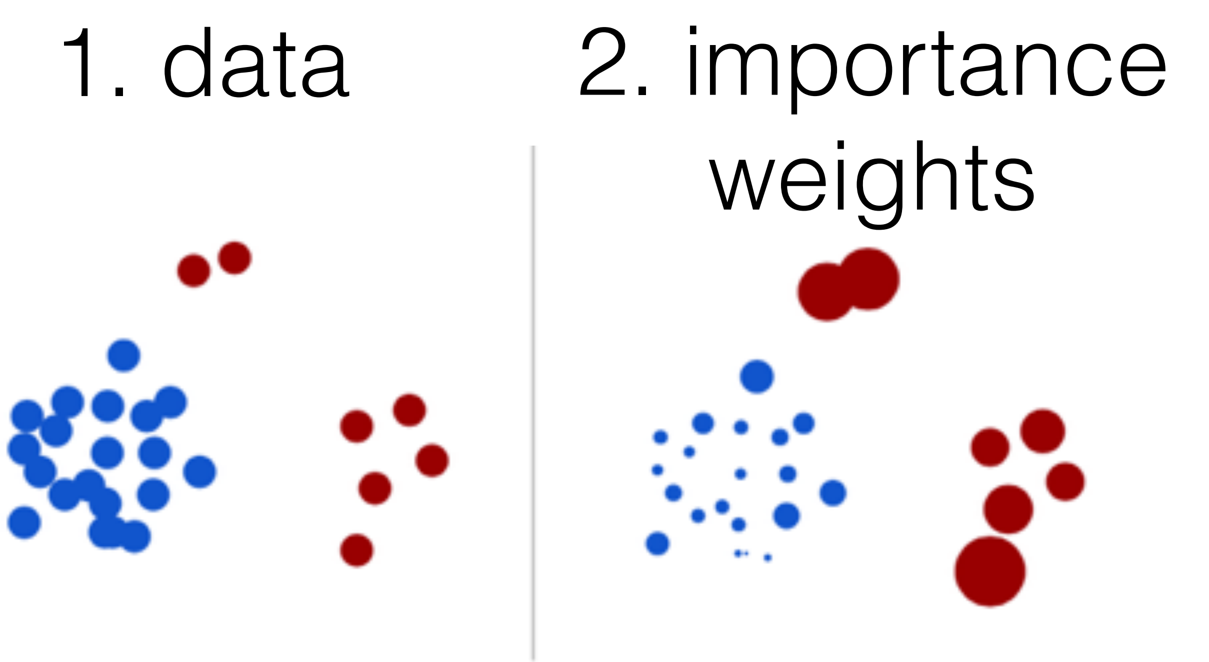

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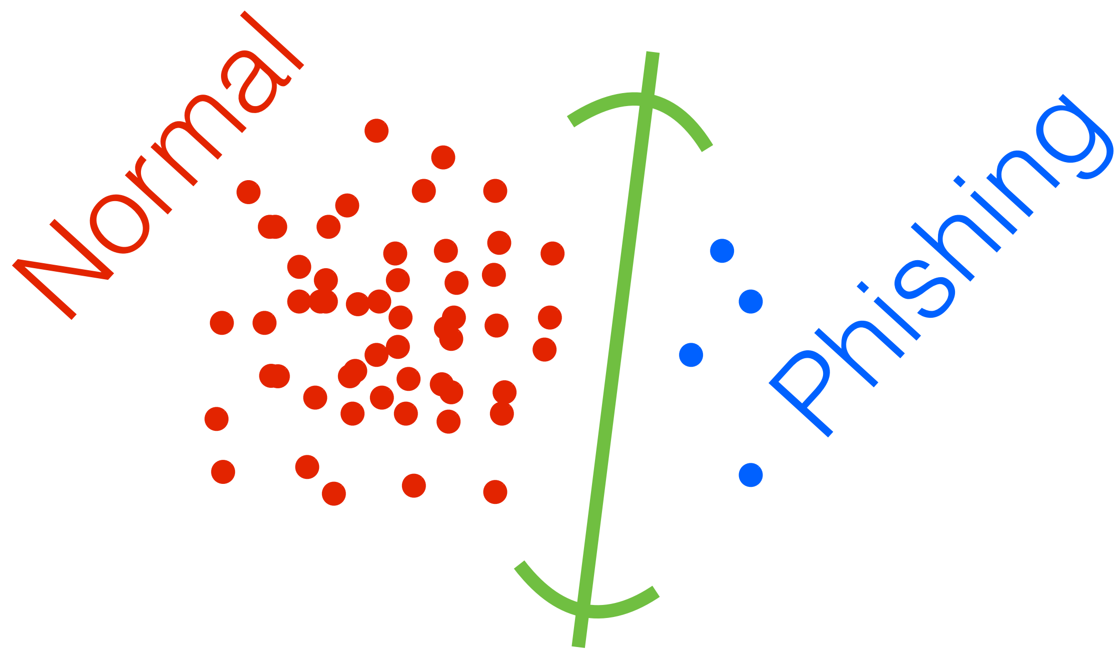

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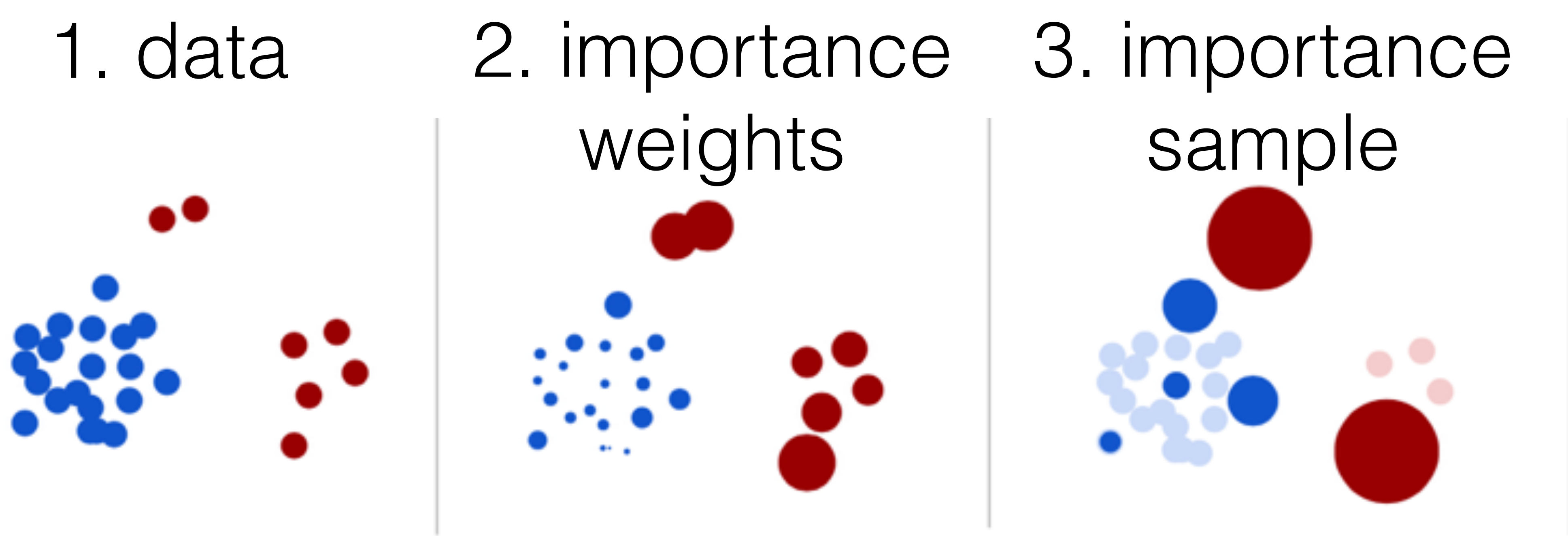

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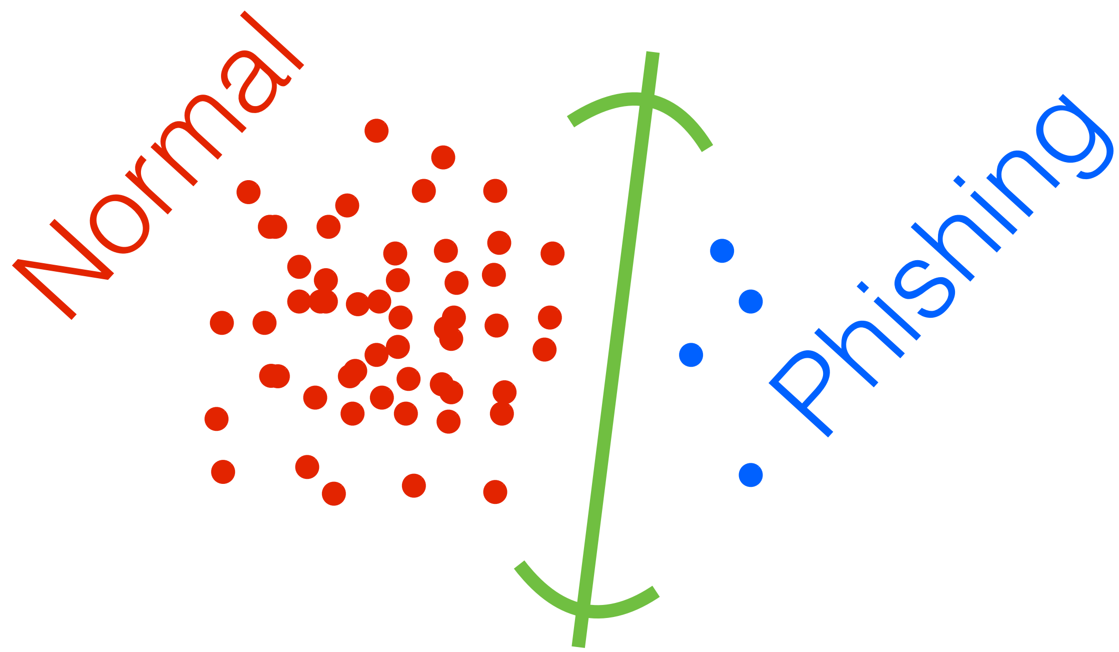

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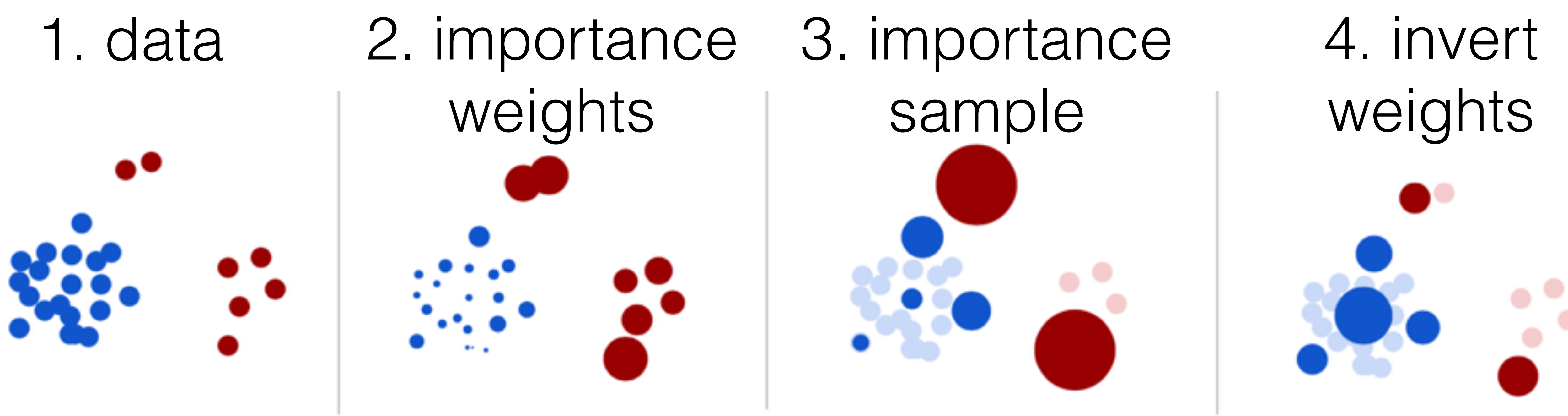

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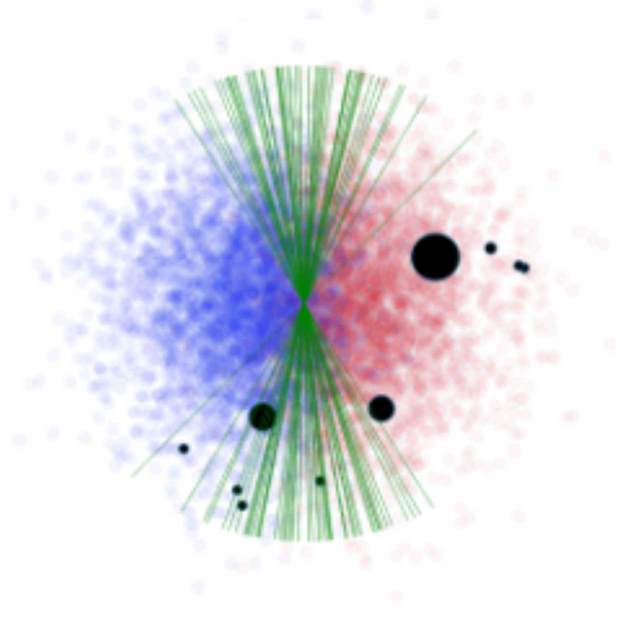

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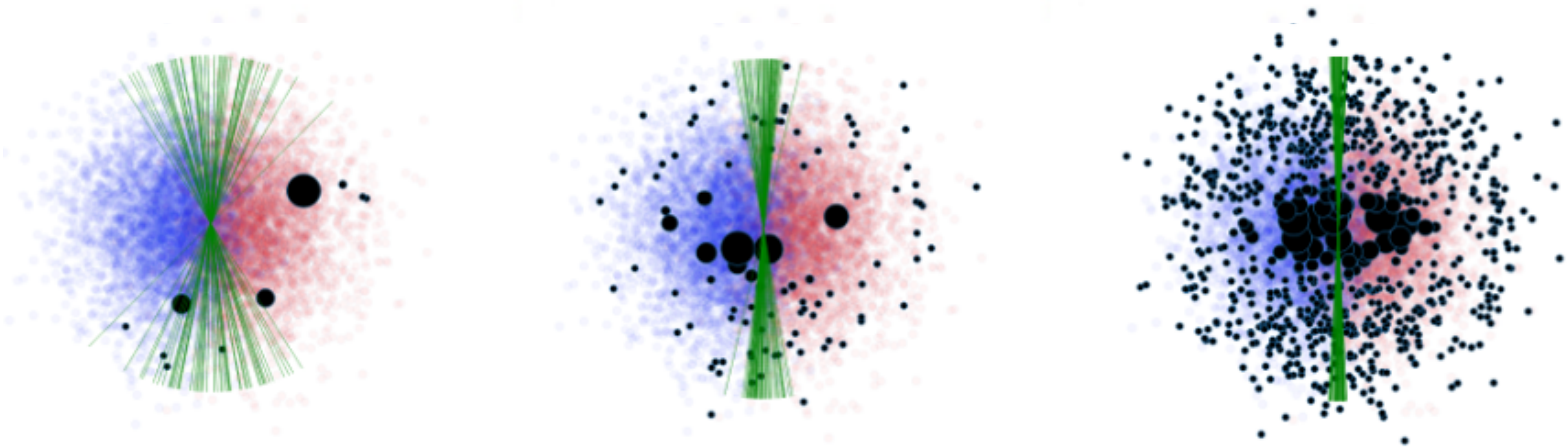

M = 10 *M* = 100 *M* = 1000

Hilbert coresets

- Want a good coreset: $\min_{w\in\mathbb{R}^N} \|\mathcal{L}(w) - \mathcal{L}\|$

  $\text{s.t. } w \geq 0, \|w\|_0 \leq M$

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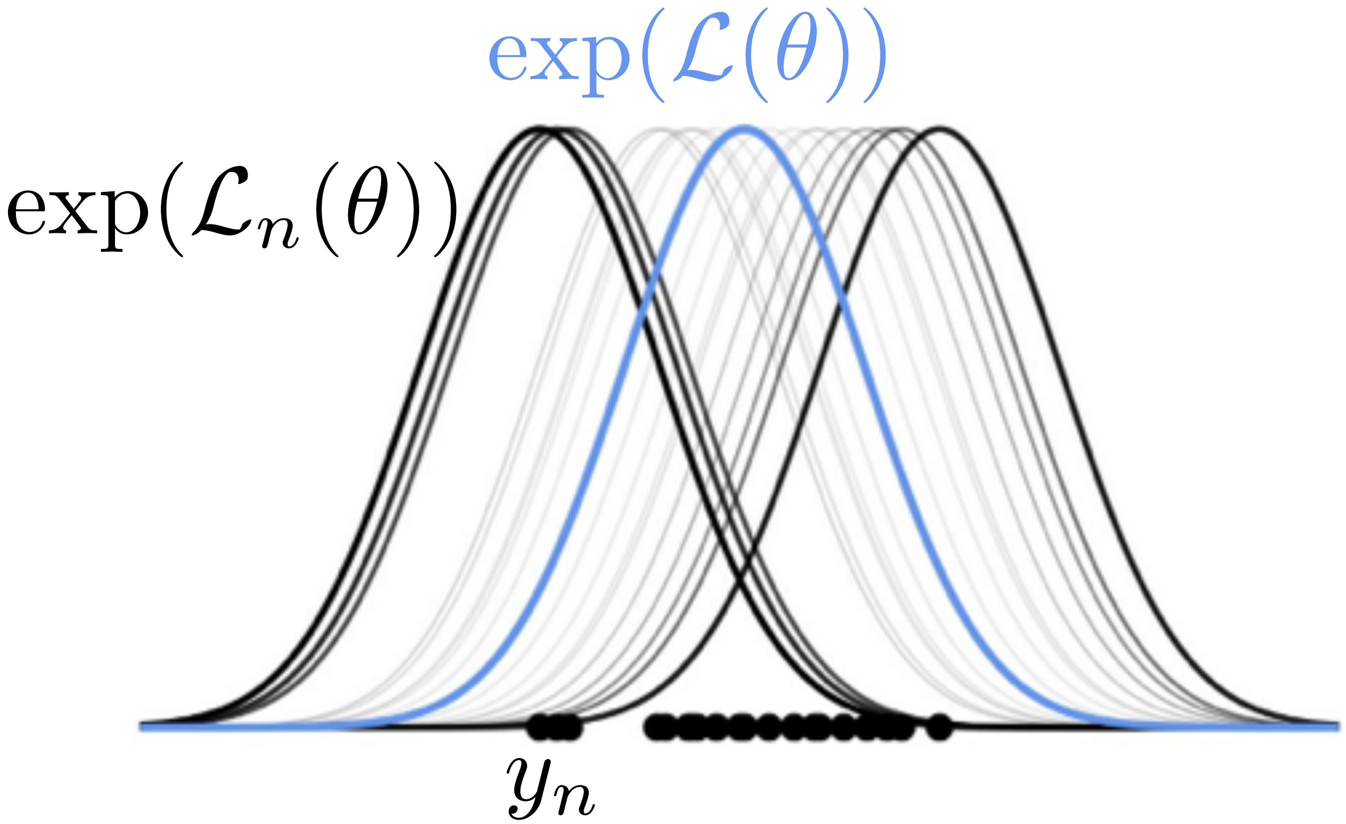

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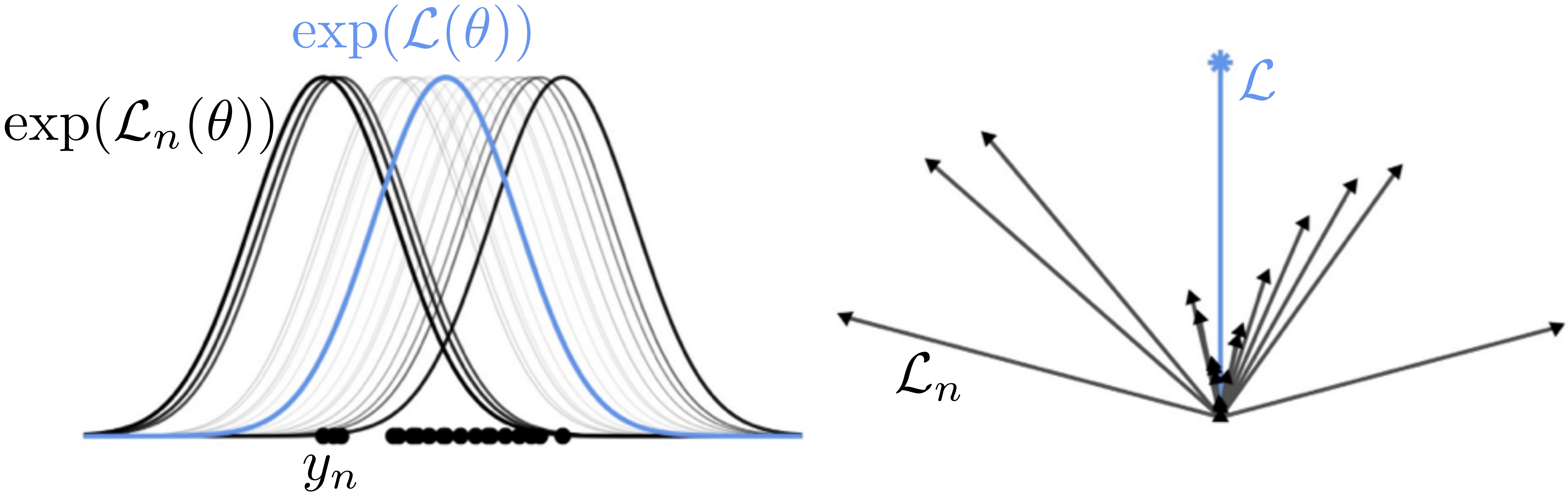

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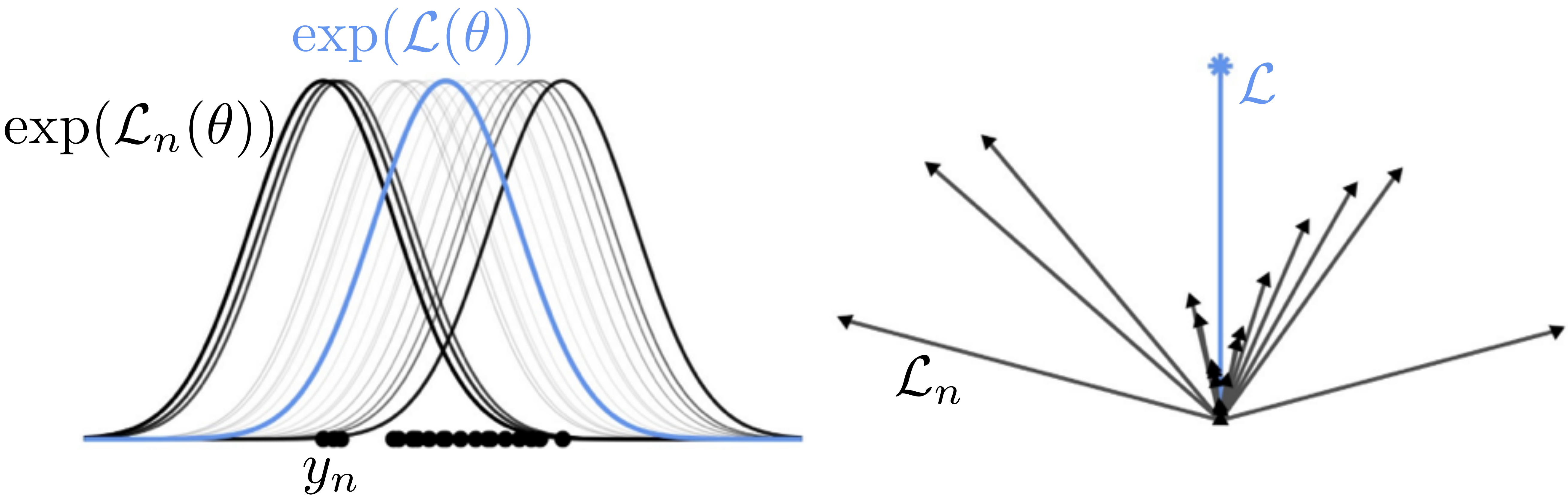

- need to consider (residual) error direction

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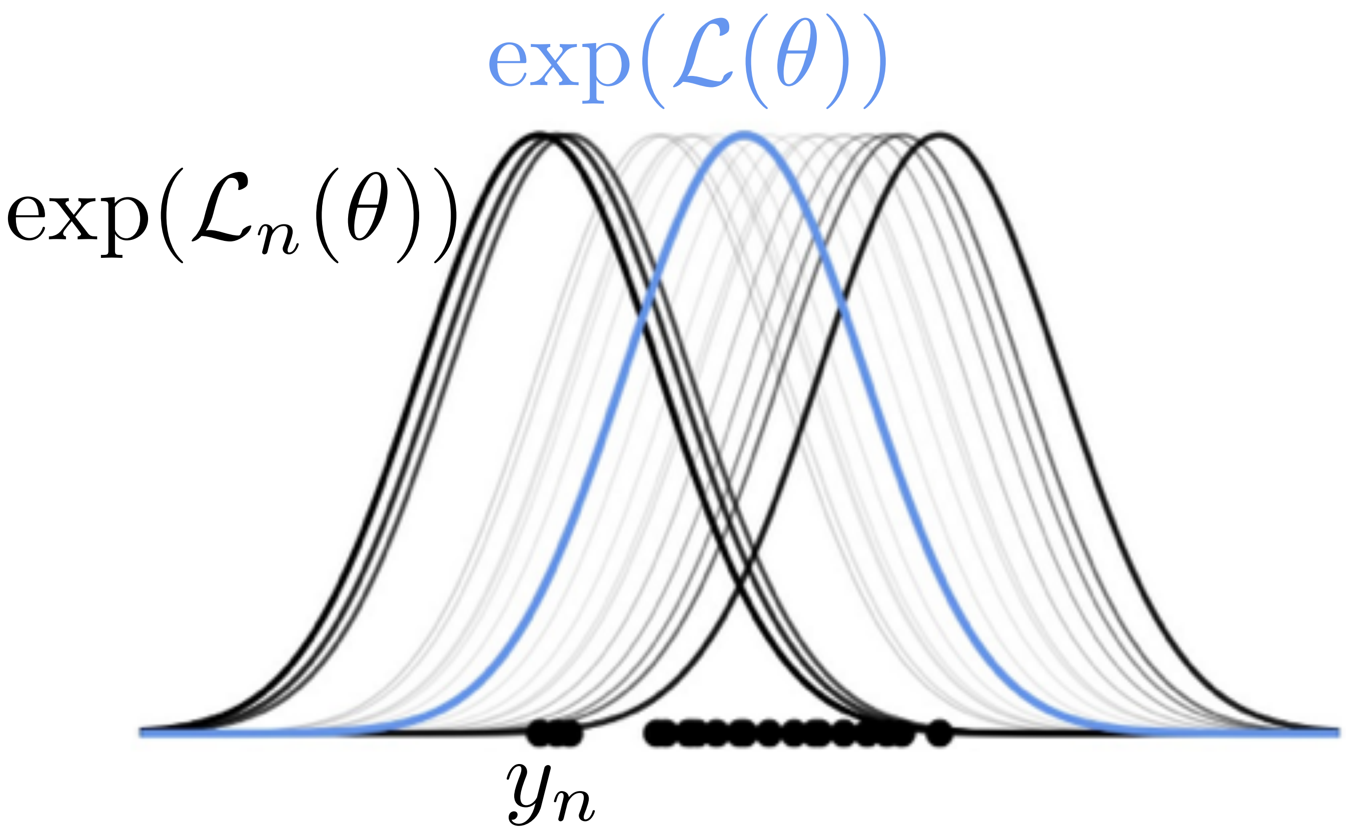

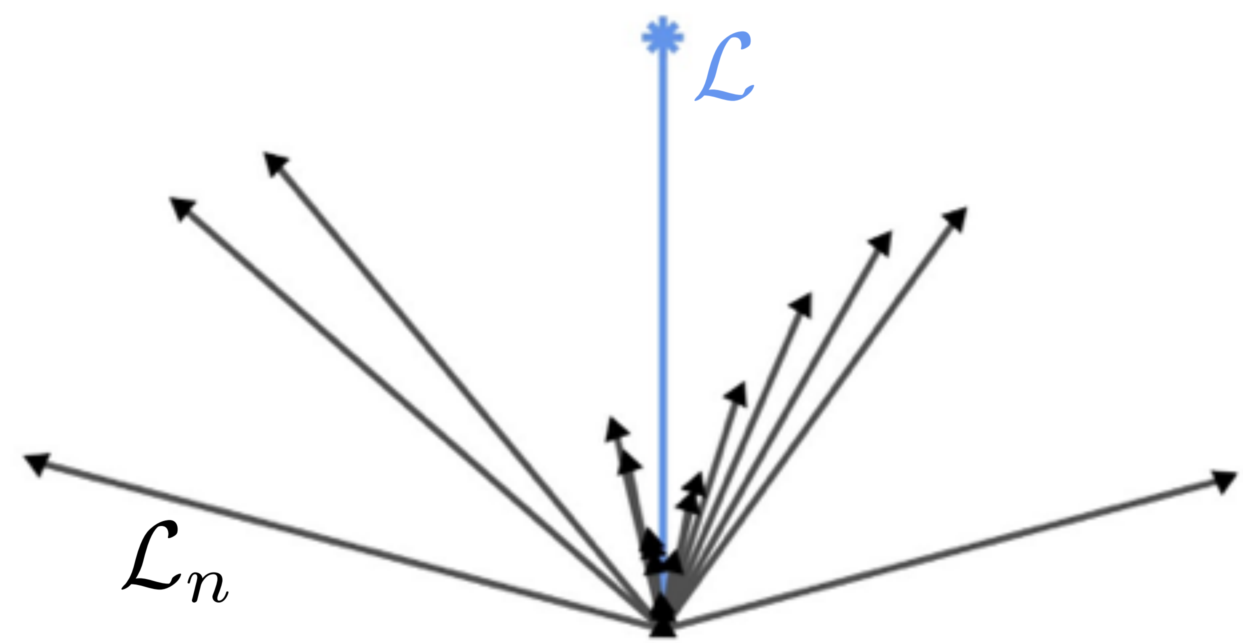

- need to consider (residual) error direction
- sparse optimization

Roadmap

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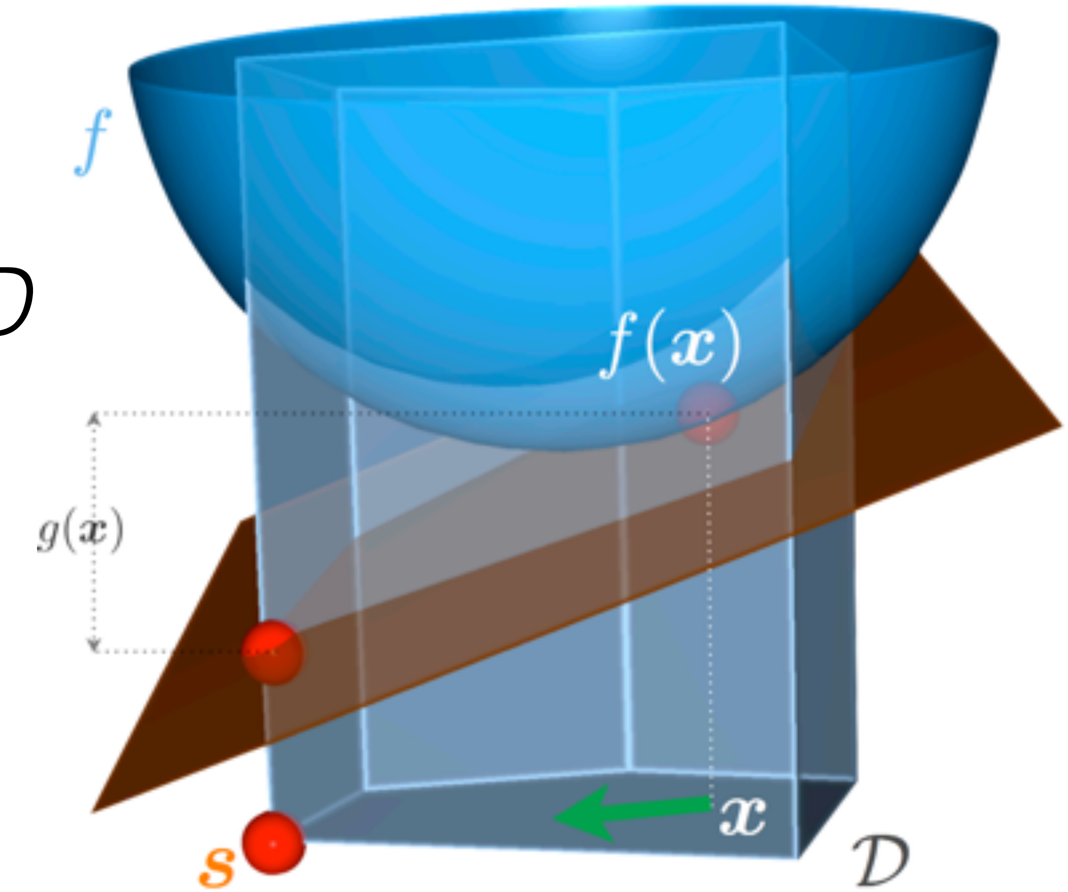
Frank-Wolfe

Convex optimization on a polytope *D*



[Jaggi 2013]

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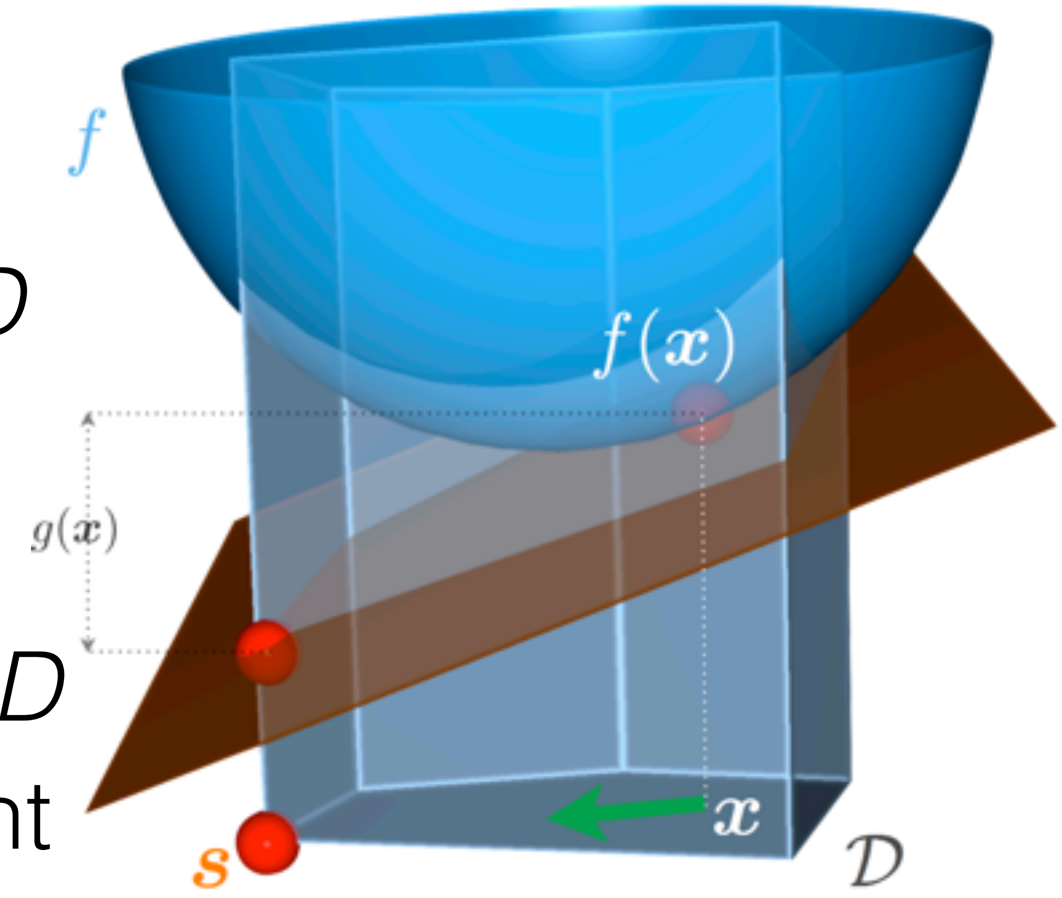

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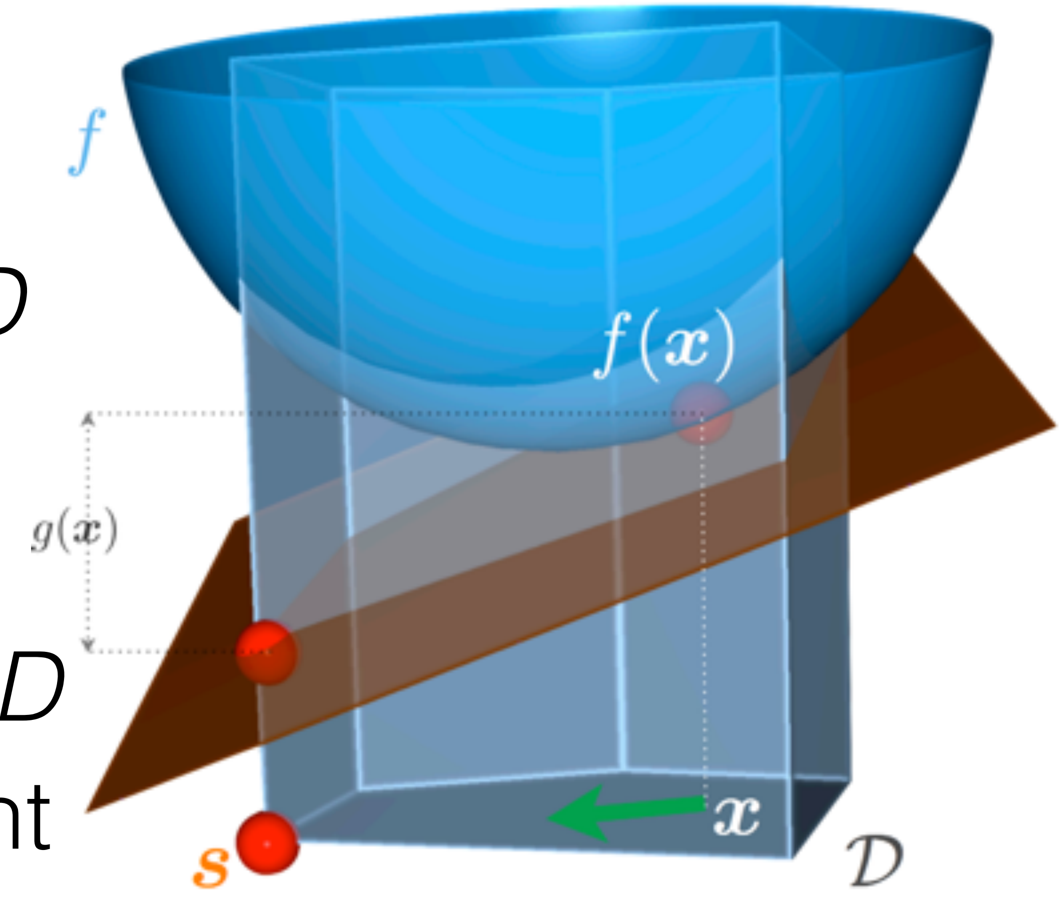

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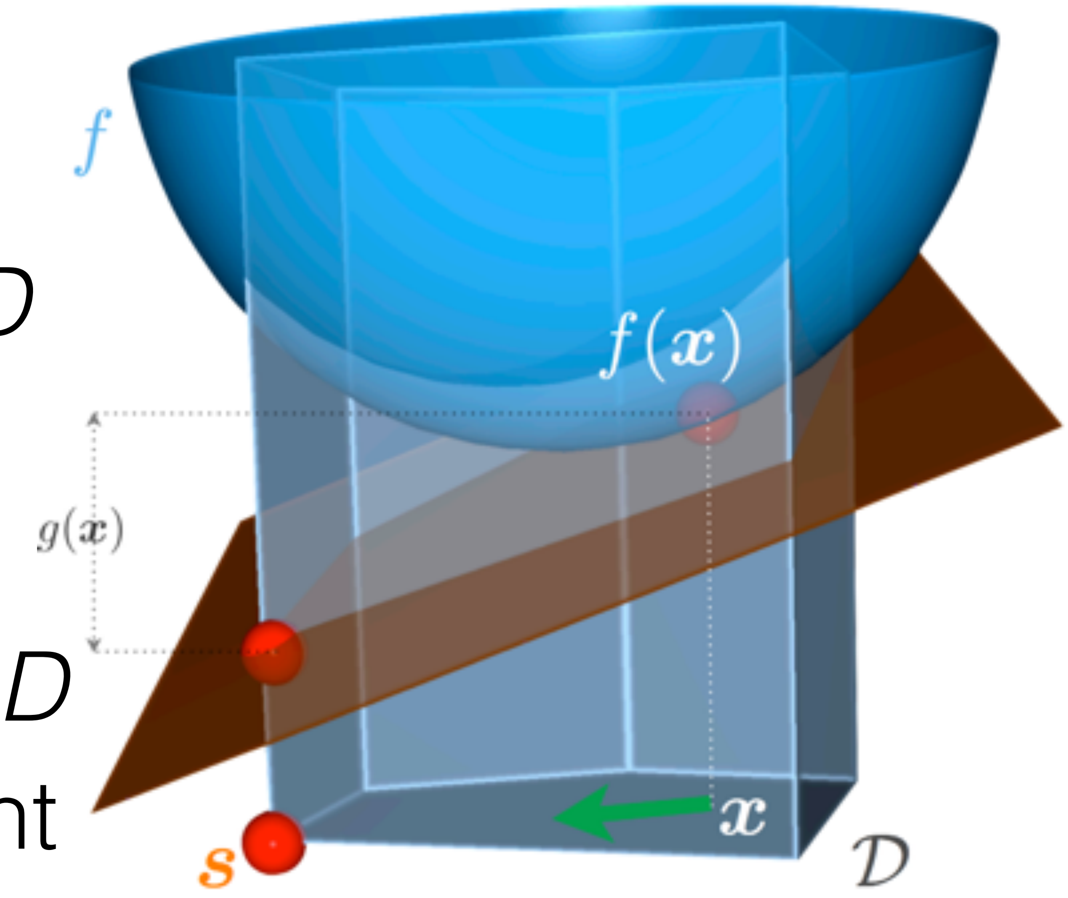

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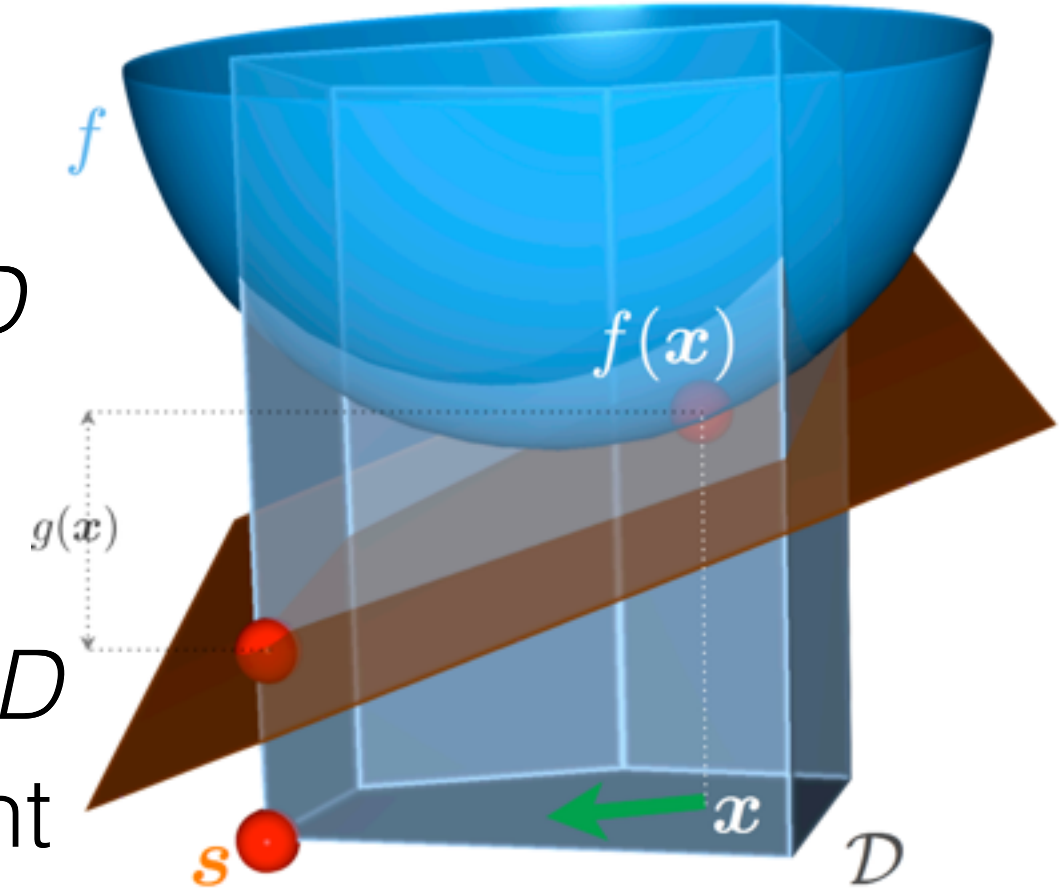

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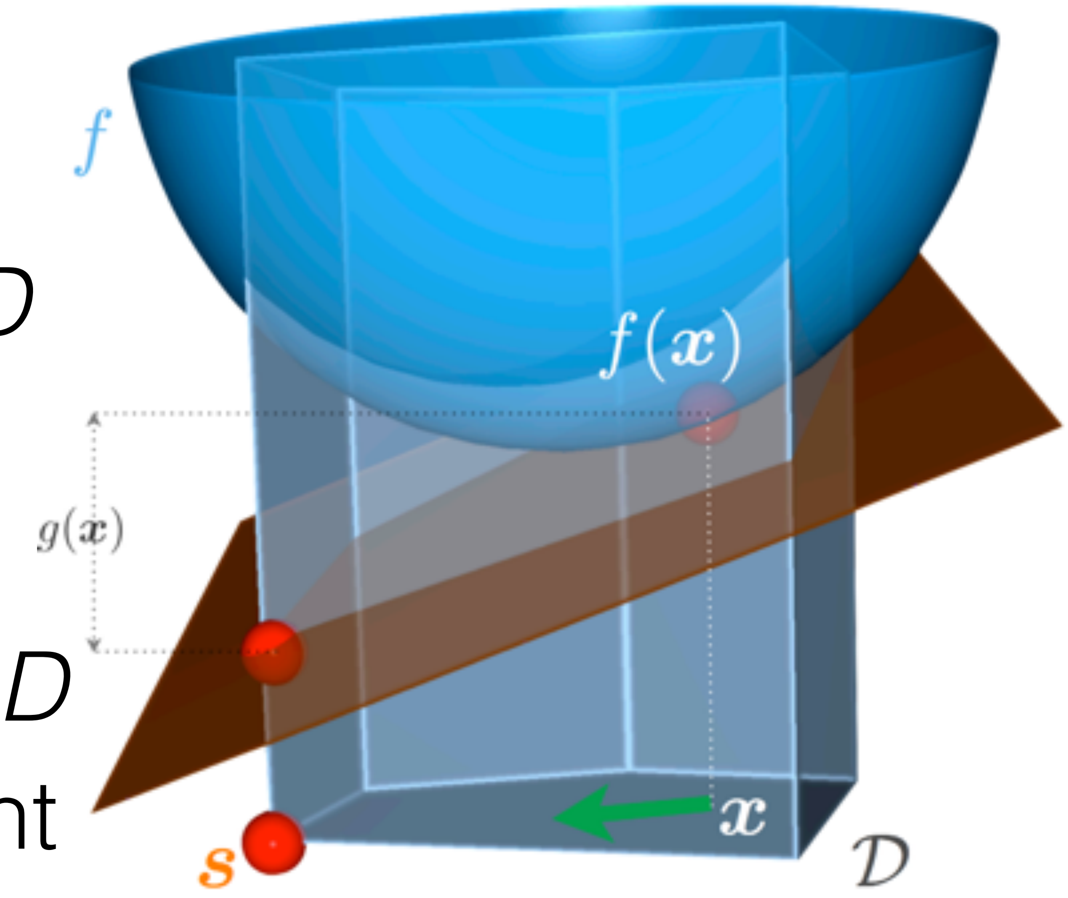

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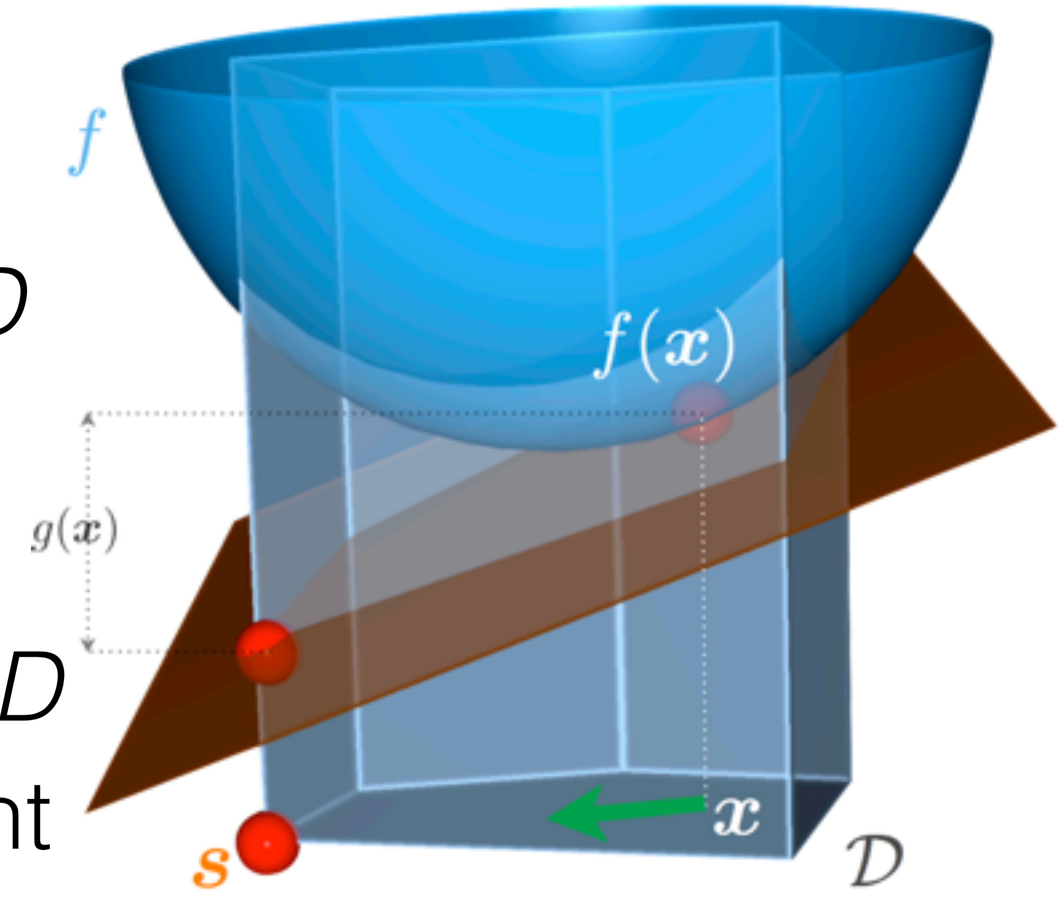

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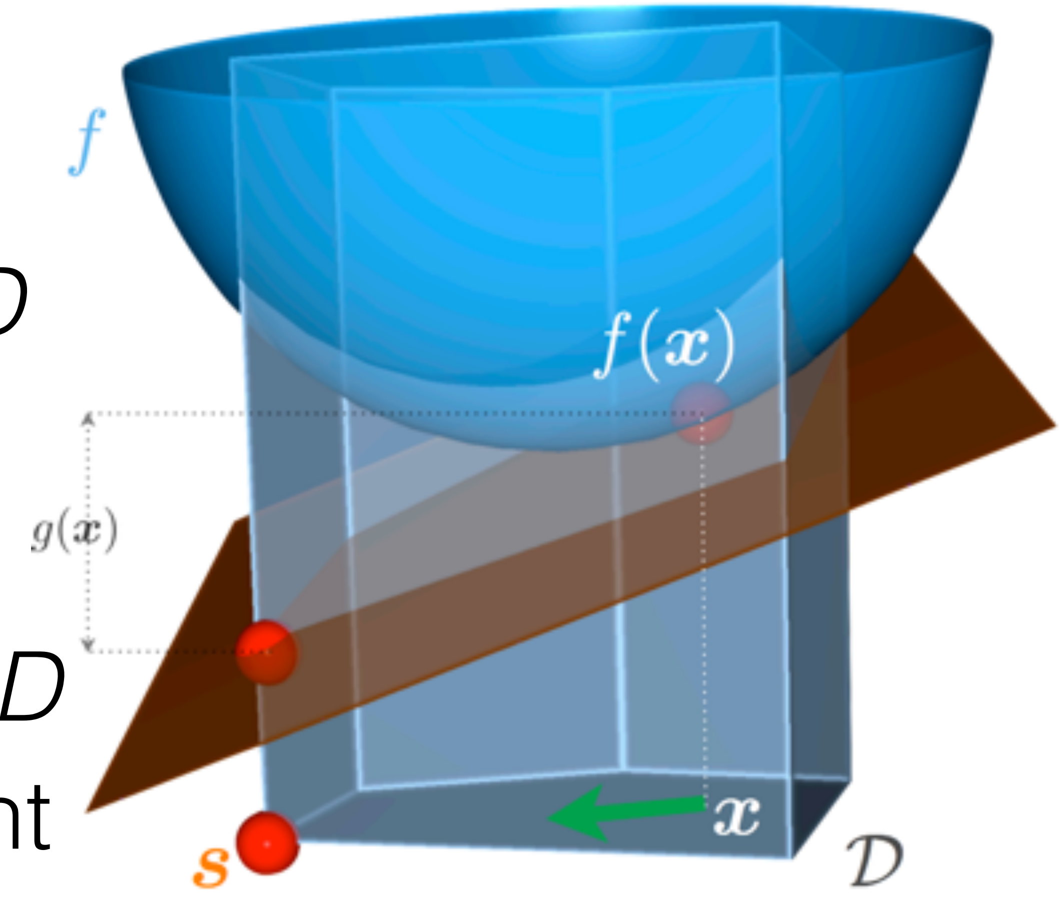

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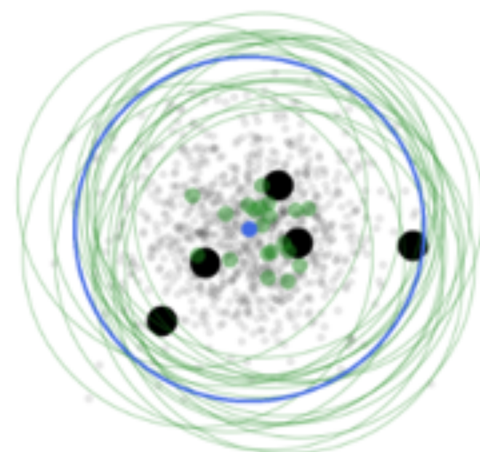
Gaussian model (simulated)

- 10K pts; norms, inference: closed-form

Uniform subsampling



$M = 5$

Gaussian model (simulated)

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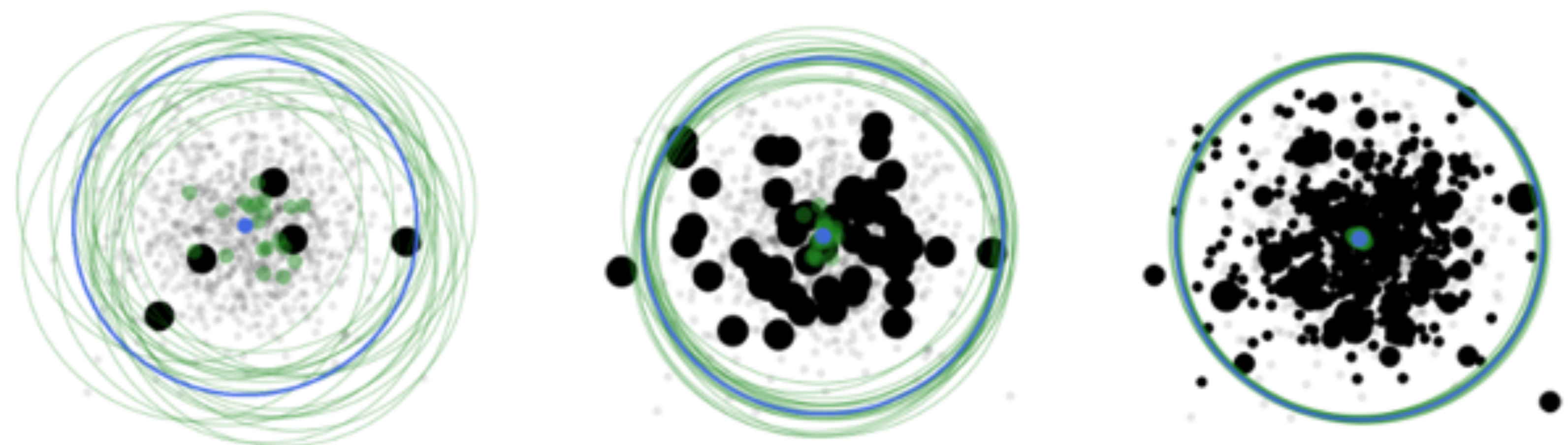

$M = 5$ $M = 50$ $M = 500$

Gaussian model (simulated)

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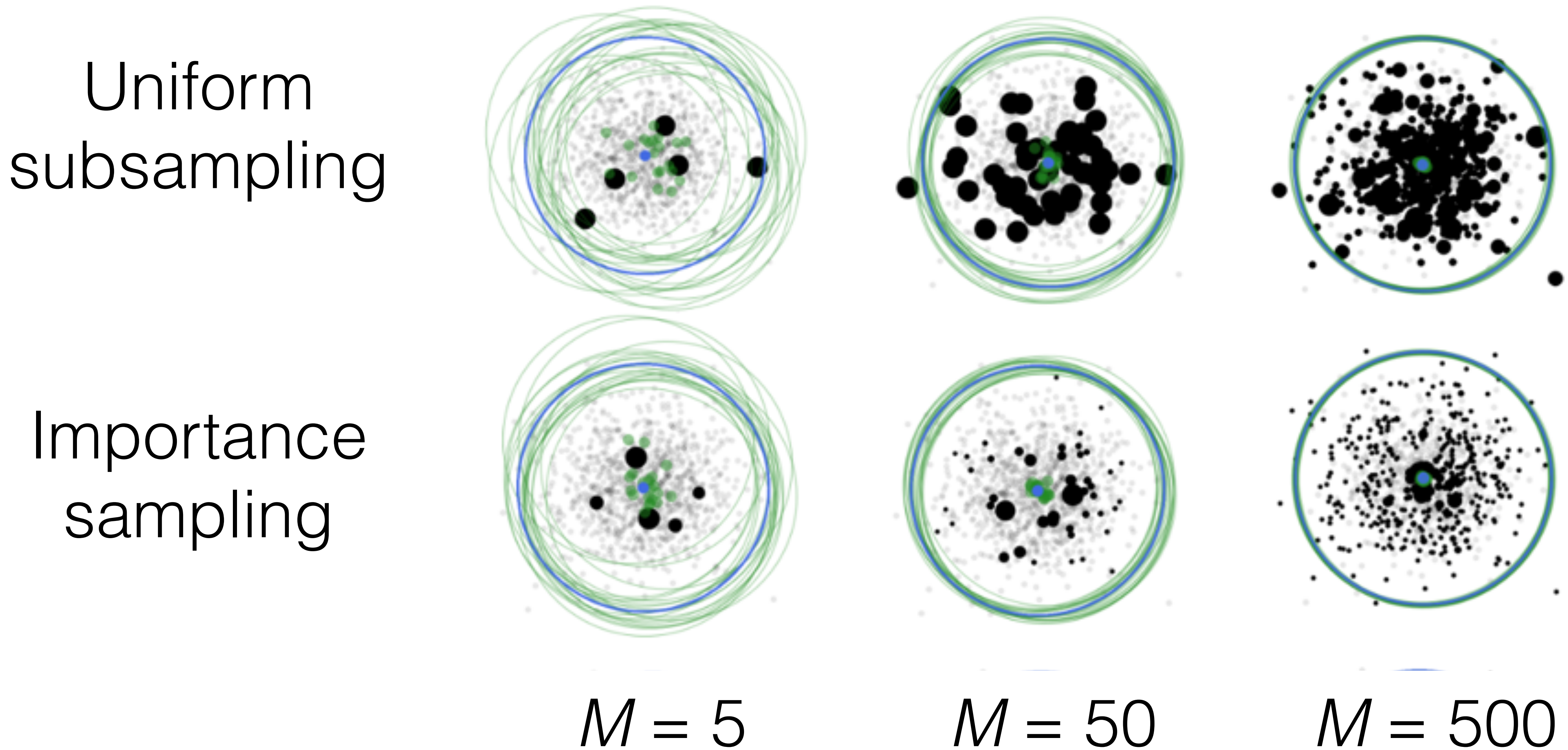

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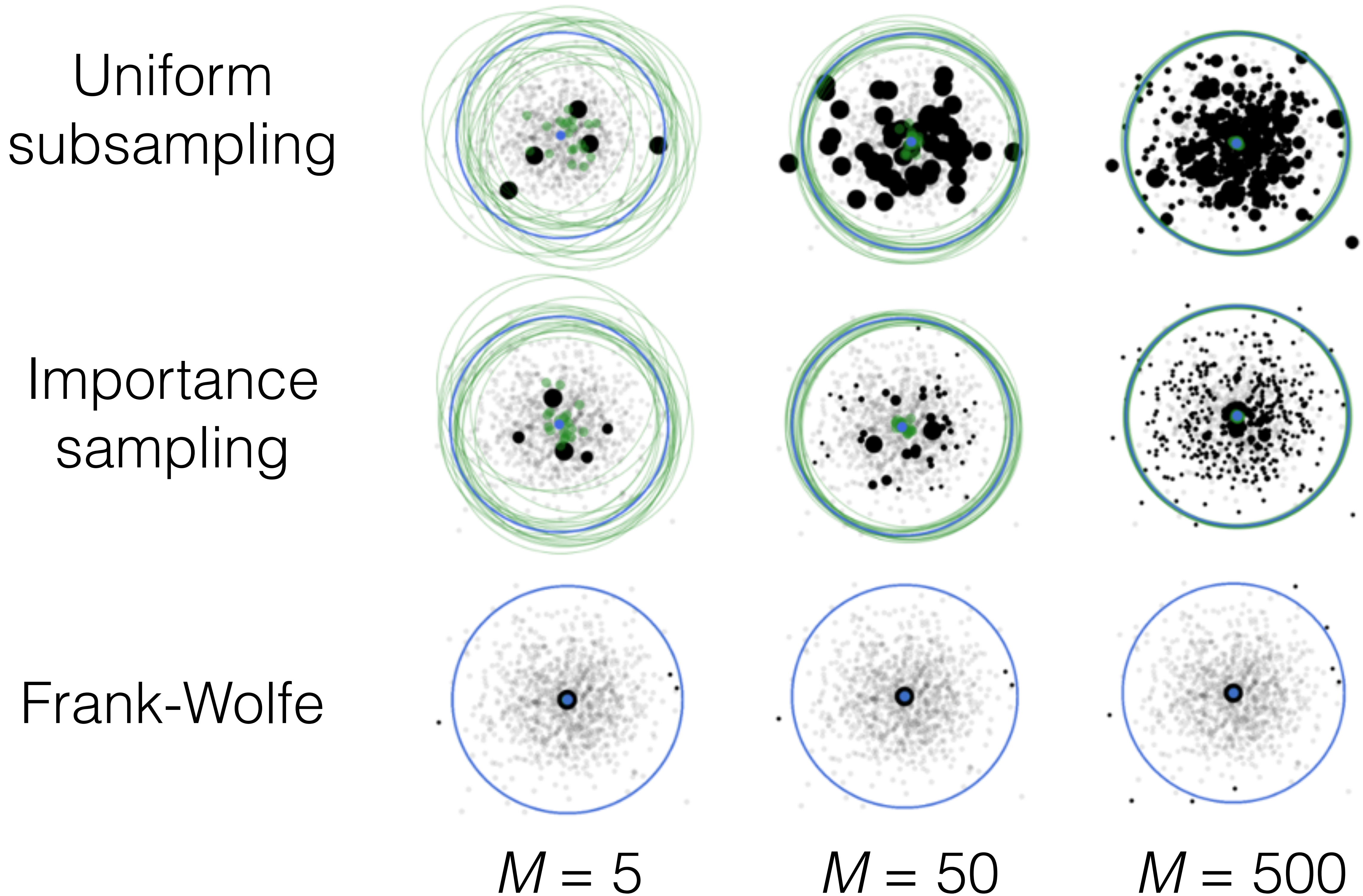

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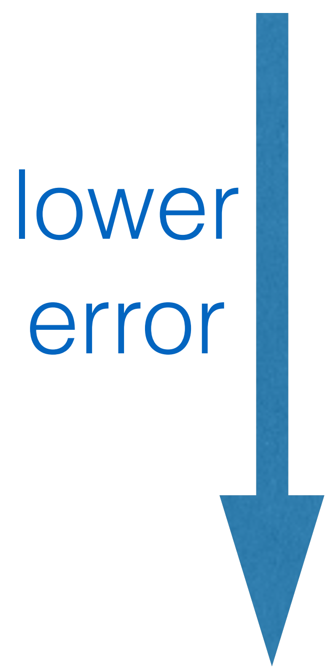
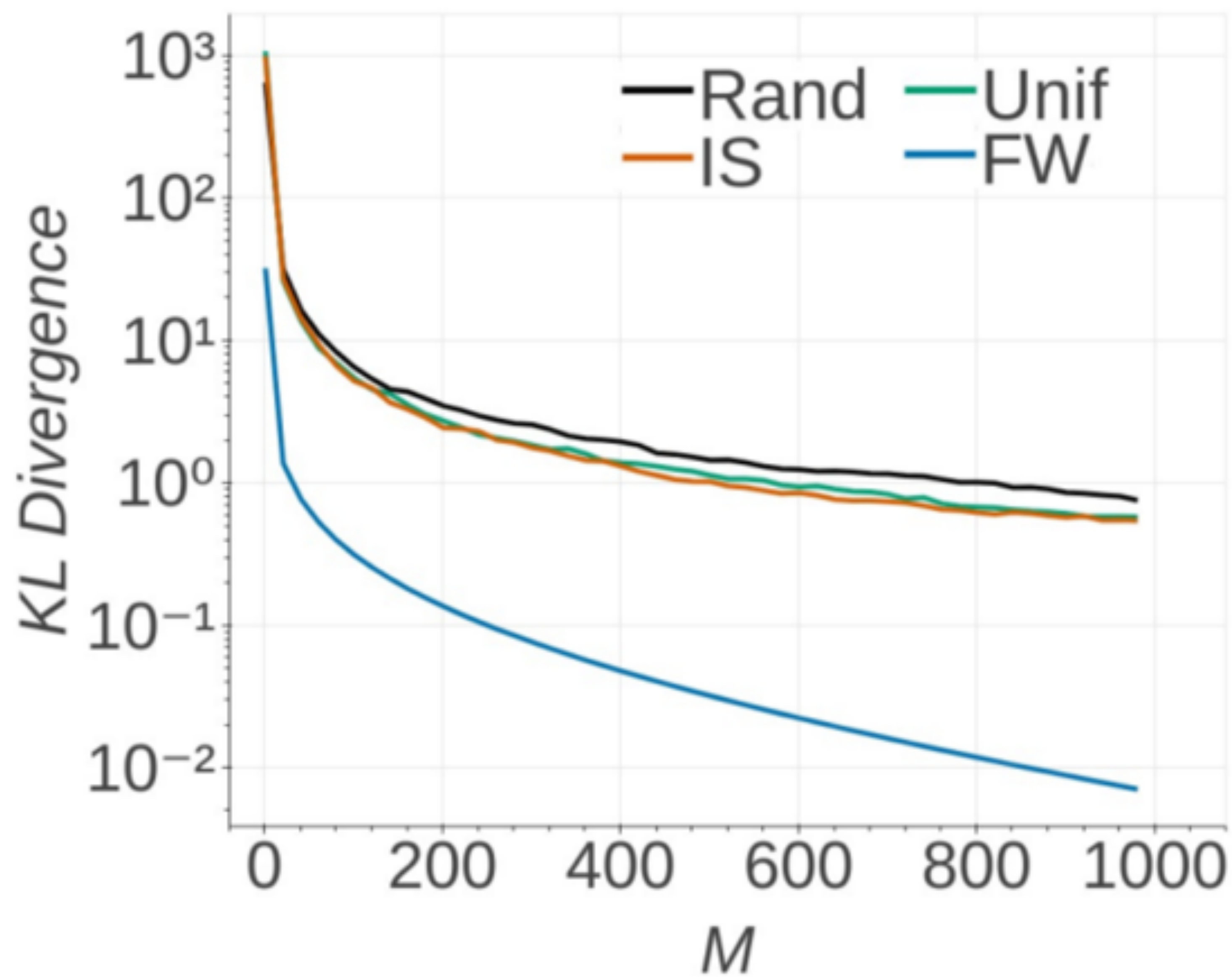
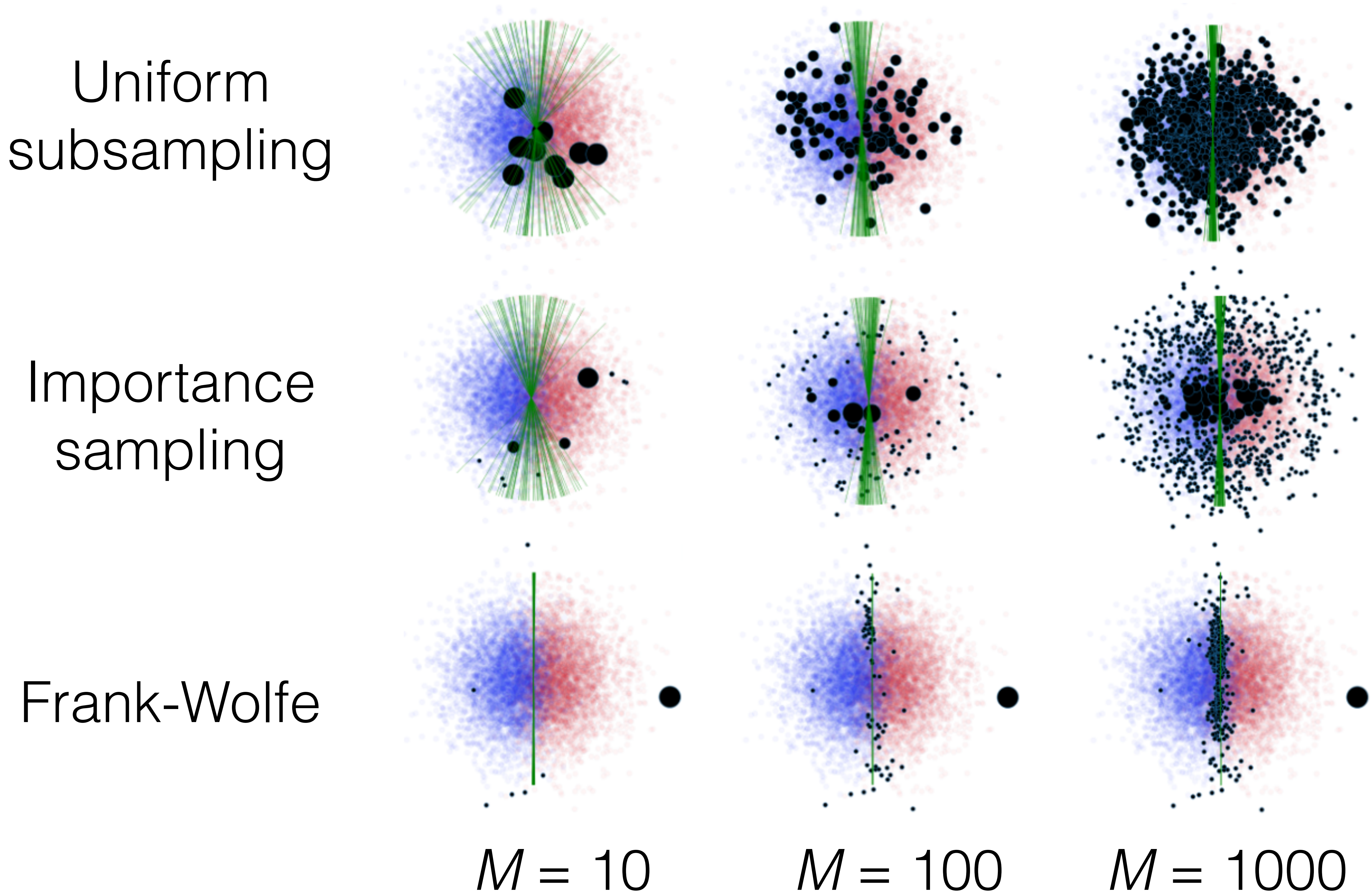




Logistic regression (simulated)

- 10K pts; general inference



Poisson regression (simulated)

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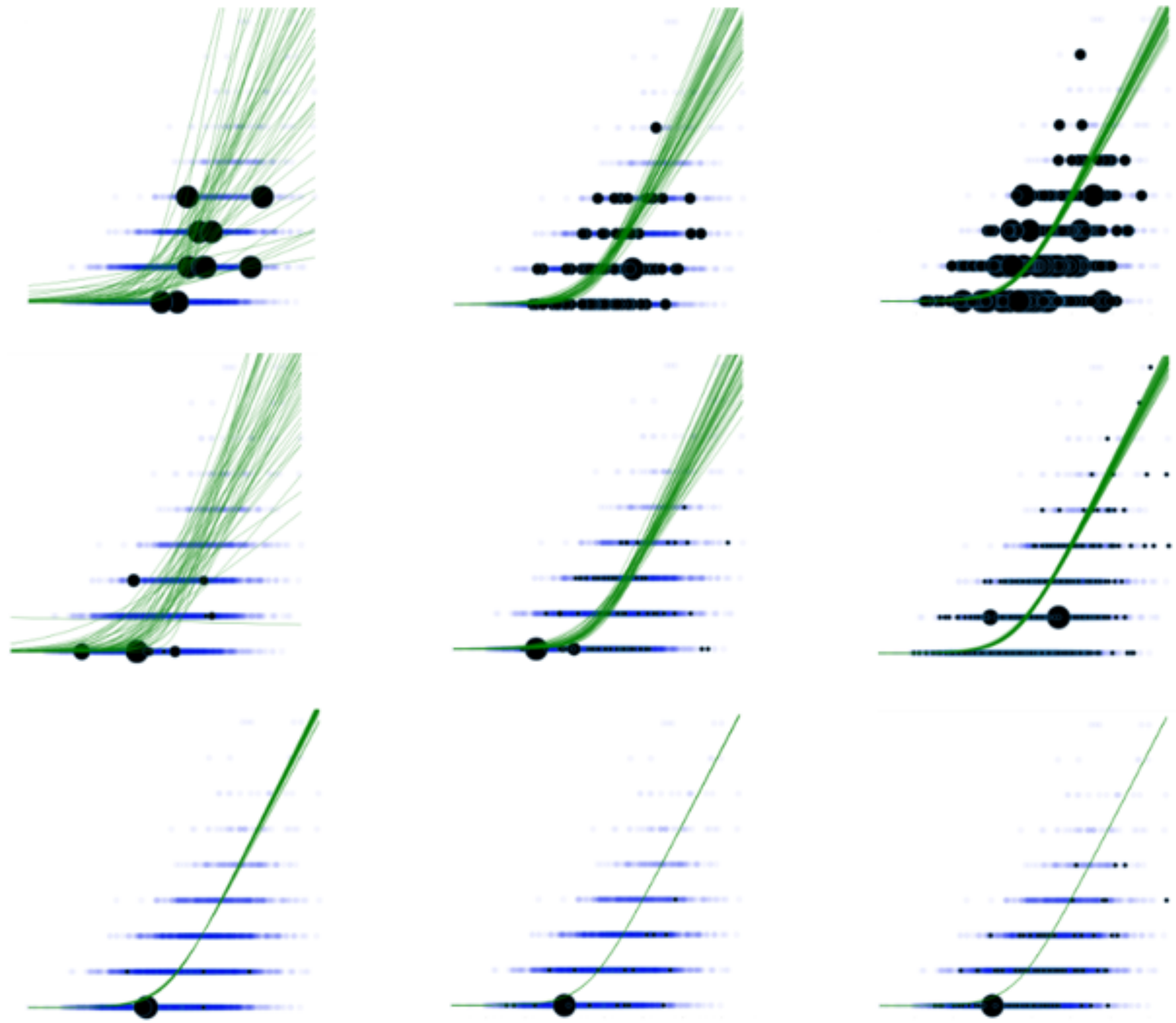
Uniform subsampling

Importance sampling

Frank-Wolfe



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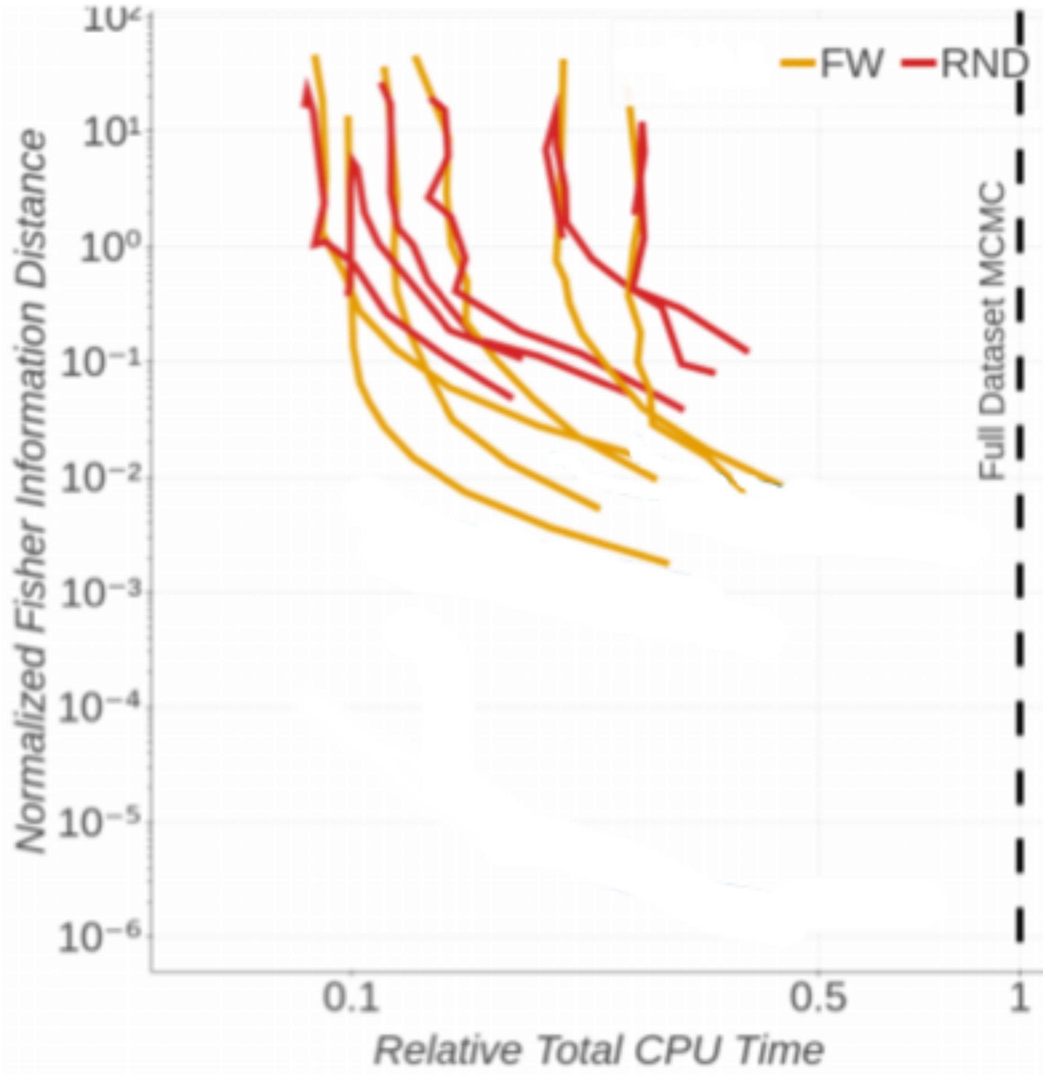
Real data experiments

lower error



less total time

Uniform subsampling

Frank Wolfe coresets

Data sets include:

- Phishing
- Chemical reactivity
- Bicycle trips
- Airport delays

[Campbell, Broderick 2017]

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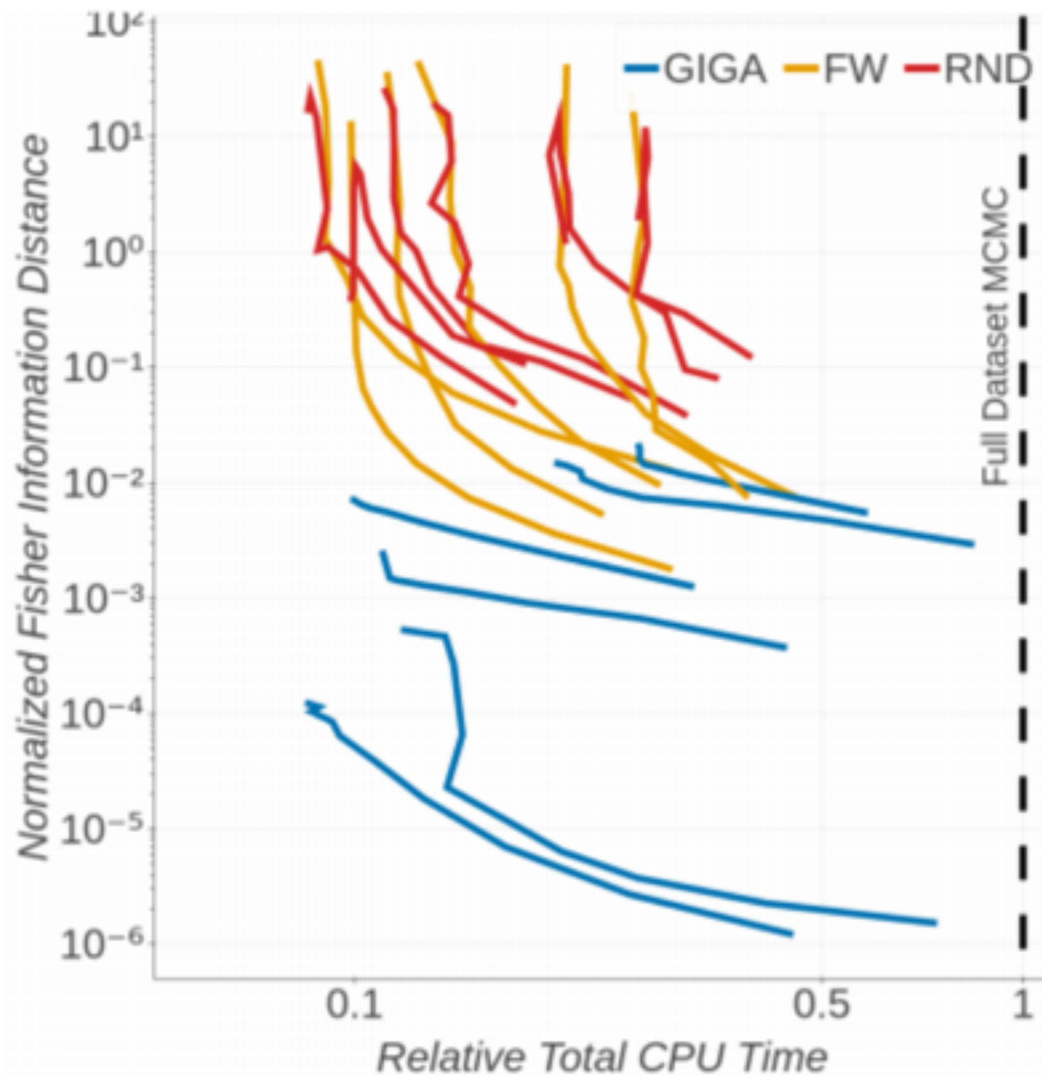

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GIGA coresets

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[Campbell, Broderick 2017, 2018]

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- Our proposal: (polynomial) *approximate* sufficient statistics

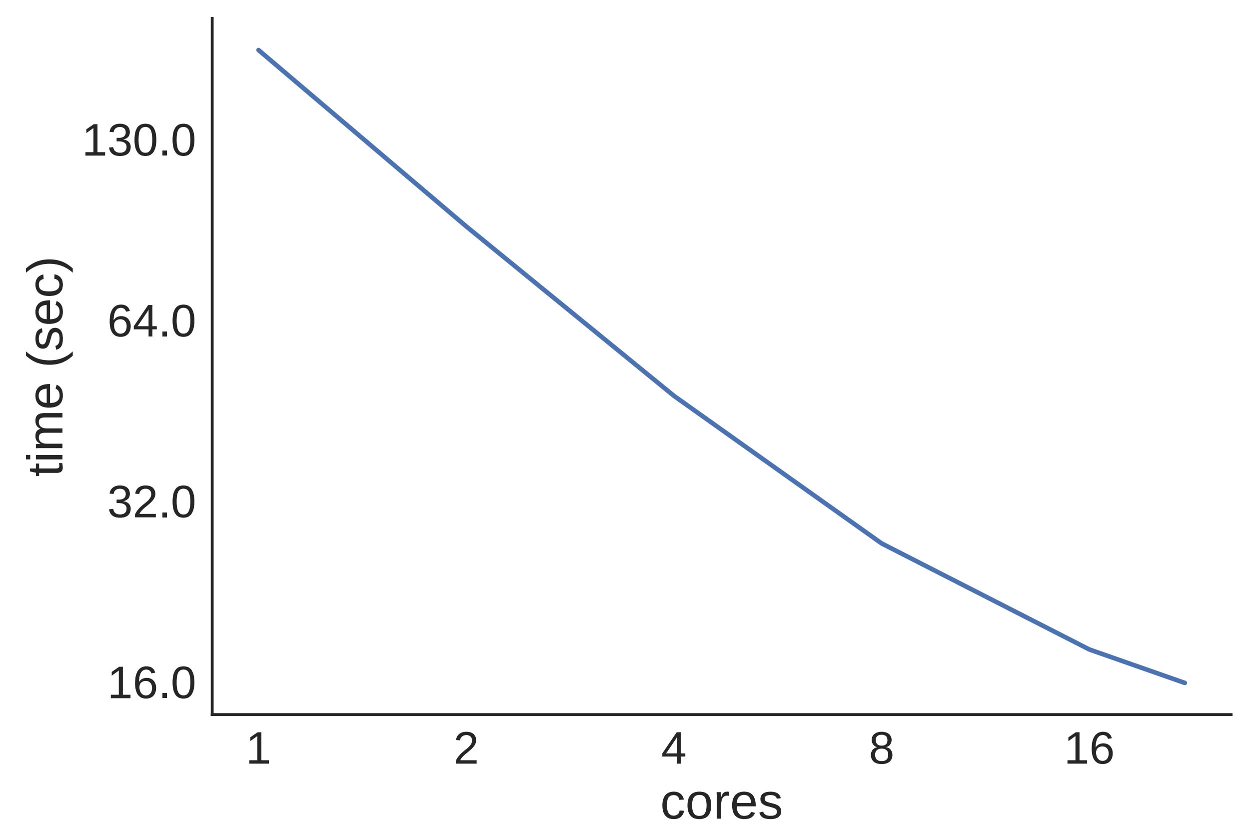
Data summarization

Criteo Labs > Algorithms > Criteo Releases its New Dataset

Criteo Releases its New Dataset

By: CriteoLabs / 31 Mar 2015

- 6M data points, 1000 features
- Streaming, distributed; minimal communication
- 22 cores, 16 sec
- Finite-data guarantees on Wasserstein distance to exact posterior



[Huggins, Adams, Broderick 2017]

Conclusions

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  - Get more accurate with more computation investment
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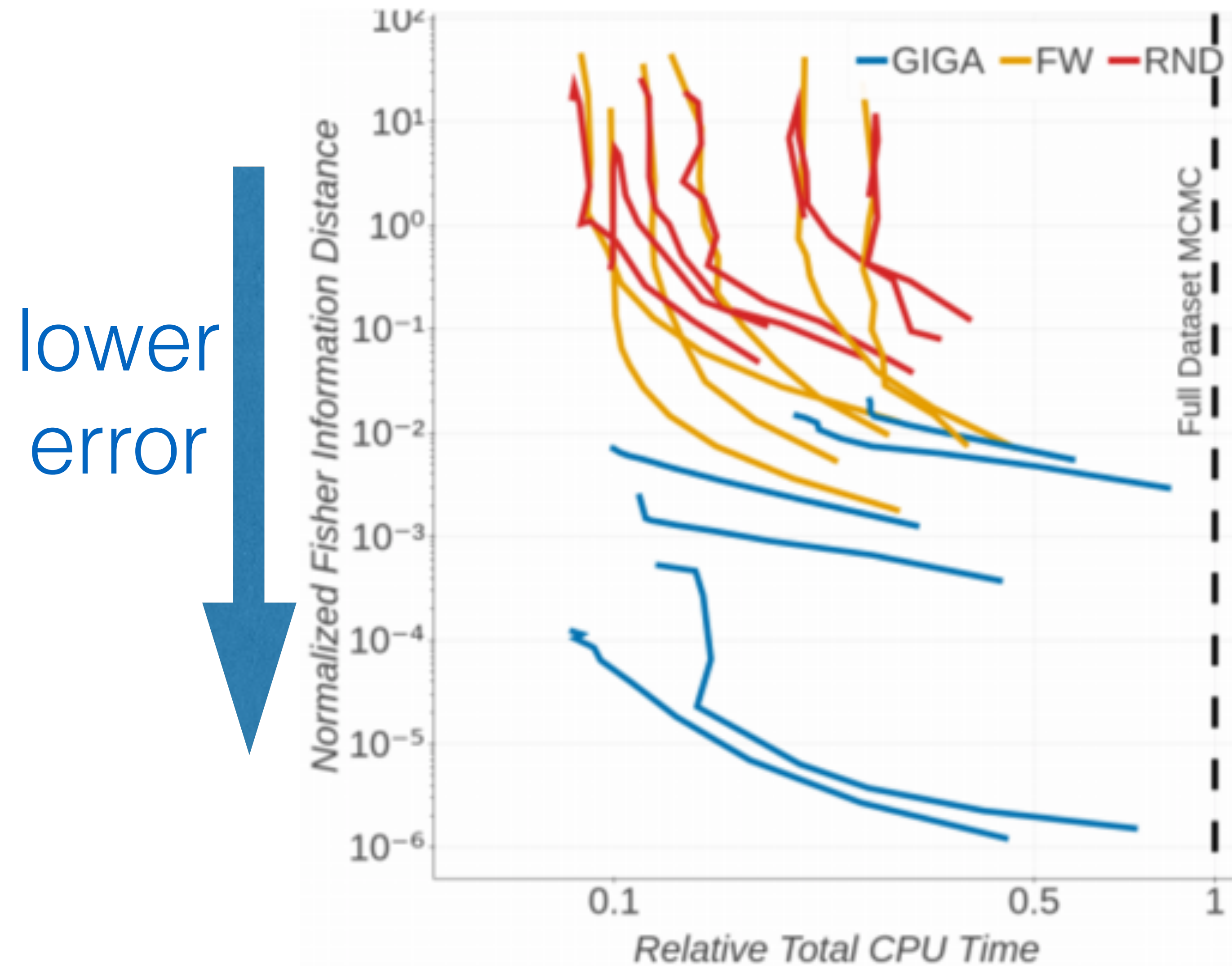

[Campbell, Broderick 2018]

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