our model

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R Markdown

Stochastic vol model with leverage

formulation one

$$
\begin{aligned}
y_t &= z_{1t} \exp[x_t/2] \\
x_{t+1} &= \mu + \phi(x_t - \mu) + \sigma z_{2t}
\end{aligned}
$$

$$
\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)
$$

$$
x_1 \sim \mathcal{N}(\mu, \sigma^2/(1-\phi^2))
$$

formulation two

This rewriting is detailed in PARTICLE FILTERS FOR INFERENCE OF HIGH-DIMENSIONAL MULTIVARIATE STOCHASTIC VOLATILITY MODELS WITH CROSS-LEVERAGE EFFECTS, although they go beyond to deail with multivariate models

$$
\begin{bmatrix} y_t \\ x_{t+1} \end{bmatrix} \Bigg| x_t \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ \mu + \phi(x_t - \mu) \end{bmatrix}, \begin{bmatrix} \exp[x_t] & \rho\sigma \exp[x_t/2] \\ \rho\sigma \exp[x_t/2] & \sigma^2 \end{bmatrix} \right)
$$

We can factor this into the product of

$$
y_t \mid x_t \sim \mathcal{N}(0, \exp[x_t])
$$

and

$$
x_{t+1} \mid y_t, x_t \sim \mathcal{N}(\mu + \phi(x_t - \mu) + \rho\sigma \exp[-x_t/2] y_t, \sigma^2(1-\rho^2))
$$

sources:

continuous time introduction: - The Pricing of Options on Assets with Stochastic Volatilities JOHN HULL and ALAN WHITE*

discrete time analysis:

- The estimation of an asymmetric stochastic volatility model for asset returns.
- Gibbs samplign: BUGS for a Bayesian analysis of stochastic volatility models