notes

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R Markdown

Ideally (why, though?) the posterior predictive distribution

$$p(y_{t+1} \mid y_{1:t}) = \int p(y_{t+1} \mid y_{1:t}, \theta) p(\theta \mid y_{1:t}) d\theta = \int p(y_{t+1} \mid y_{1:t}, \theta) \frac{p(y_{1:t} \mid \theta)}{p(y_{1:t})} p(\theta) d\theta$$

Instead

$$g(y_{t+1} \mid y_{1:t}) = \int p(y_{t+1} \mid y_{1:t}, \theta) p(\theta) d\theta = \int p(y_{t+1} \mid y_{1:t}, \theta) p(\theta) d\theta$$

difference in expectations

As

$$p(y_{t+1} \mid y_{1:t}) - g(y_{t+1} \mid y_{1:t}) = \mathbb{E}_\theta \left[p(y_{t+1} \mid y_{1:t}, \theta) \left(1 - \frac{p(y_{1:t} \mid \theta)}{p(y_{1:t})} \right) \right]$$

Maybe the difference in these expectations itself can be monitored!

Let's say we're interested in some function f. Under certain conditions

$$\begin{aligned} \mathbb{E}_{p(y_{t+1}\mid y_{1:t})}[f] - \mathbb{E}_{g(y_{t+1}\mid y_{1:t})}[f] &= \int f(y_{t+1}) \mathbb{E}_\theta \left[p(y_{t+1} \mid y_{1:t}, \theta) \left(1 - \frac{p(y_{1:t} \mid \theta)}{p(y_{1:t})} \right) \right] dy_{t+1} \\ &= \mathbb{E}_\theta \left[\int f(y_{t+1}) p(y_{t+1} \mid y_{1:t}, \theta) dy_{t+1} \left(1 - \frac{p(y_{1:t} \mid \theta)}{\hat{p}(y_{1:t})} \right) \right] \end{aligned}$$

Our approach

$$\int f(y_{t+1}) p(y_{t+1} \mid y_{1:t}, \theta) dy_{t+1} = \mathbb{E}\left[f(y_{t+1}) \mid y_{1:t}, \theta \right] = \mathbb{E}\left[\mathbb{E}\{ f(y_{t+1}) \mid x_t \} \mid y_{1:t}, \theta \right] = n^{-1} \sum_{i=1}^n h(x_t^i, y_{1:t}, \theta) = g_1(\theta, y_{1:t})$$

$$\left(1 - \frac{p(y_{1:t} \mid \theta)}{\hat{p}(y_{1:t})} \right) = g_2(\theta, y_{1:t})$$