8.4: Confidence Intervals for the Variance and Standard Deviation of a Normal Population

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Motivation

We have the same set up as the last chapter (that is, a normal random sample). Now instead of being concerned about μ, we're concerned about σ or σ^2.

The set-up

Recall that if $X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, \sigma^2)$, then

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}.$$

Now let

$$\chi^2_{\alpha,\nu}$$

denote the $(1-\alpha)100$th percentile of the χ^2_ν distribution.

It is important to remember that the χ^2_ν distribution isn't symmetric, so $\chi^2_{1-\alpha,\nu} \neq -\chi^2_{\alpha,\nu}$. We have to find two quantiles per problem this time.

Example

But the idea is the same as in previous chapters:

$$P\left(\chi^2_{1-\alpha/2,n-1} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{\alpha/2,n-1}\right) = 1-\alpha$$

or

$$P\left(\sigma^2\chi^2_{1-\alpha/2,n-1} \leq (n-1)S^2 \leq \sigma^2\chi^2_{\alpha/2,n-1}\right) = 1-\alpha$$

or

$$P\left(\sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}} \text{ and } \sigma^2 \geq \frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}}\right) = 1-\alpha$$

So our $(1-\alpha)100$th percentile for σ^2 is $\left[\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}\right]$