3.5: The Binomial Probability Distribution

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Motivation

Many experiments conform either exactly or approximately to the following list of requirements:

1. The experiment consists of a sequence of n *trials* (n is nonrandom)
2. Each trial results in one of two outcomes (success or failure)
3. Trials are independent of one another
4. The probability of a success is the same for all trials

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An experiment for which these conditions are satisfied is called a **binomial experiment**

A **binomial rv** is the random variable denoting the number of successes for such an experiment

Definition

For a binomial rv, the pmf is

$$p(x; n, p) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n$$

and the cdf is

$$F_X(x) = P(X \le x) = \sum_{k=0}^{x} p(k; n, p)$$

Example

A basketball player is a 70% free-throw shooter. He has to make both of two shots to win the game for his team. What is the probability he makes one shot? What is the probability he doesn't win the game for his team?

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$$P(\text{he makes one}) = p(1; 2, .7)$$

$$P(\text{loses}) = P(X < 2) = P(X \leq 1) = p(0; 2, .7) + p(1; 2, .7)$$

Mean, Variance and MGF

A few things about the binomial distribution

1. if $n = 1$, it's called a Bernoulli distribution/rv
2. in general, $EX = np$ and $V(X) = np(1-p)$
3. also, $M_X(t) = (1 - p + pe^t)^n$

proofs are left as exercise