9.4: P-Values

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Definition

The **p-value** is the probability of obtaining a value of another test statistic at least as contradictory to H_0 as the value you just calculated from the available sample; this probability is calculated under the assumption that H_0 is true.

This should seem sort of complicated. We are comparing a hypothetical, unobserved test statistic, Z, and comparing it to our now nonrandom test statistic we just calculated, z.

Definition

Say you had the setup: $X_1, \ldots, X_n \overset{iid}{\sim} \text{Normal}(\mu, \sigma^2)$, with σ^2 known. And $H_0 : \mu = \mu_0$ versus $H_a : \mu > \mu_0$.

And say you calculated your test statistic $z = \frac{(\bar{x} - \mu_0)}{\sqrt{\sigma^2/n}}$ (now lowercase). Then

$$\text{p-value} = P(Z > z | \mu = \mu_0)$$

Example 9.14 on page 456

We're doing a large-sample hypothesis test here for the population mean μ. $H_0 : \mu = 2.0$ versus $H_a : \mu > 2$. They tell us that $z = \frac{\bar{x}-2.0}{s/\sqrt{n}} = 3.04$. If this was a question in one of the previous sections, we would find z_α and reject if $z > z_\alpha$.

But let's do it with a p-value now. The values that are at least as contradictory to H_0 as the test statistic is are all the numbers to the right of our test statistic. This is because $H_a : \mu > 2$.

$$\text{P-Value} = P(Z > 3.04 \text{ when } \mu = 2\ ) = .0012$$

Decision Rule

A small p-value (near 0) indicates rare-ness. Either your assumption of H_0 is absurd, or you just observed some very rare data.

If α is our pre-selected probability of type 1 error, the decision rule tells us to reject H_0 if and only if our p-value is less than α.

Summary

Old decision rule: compare z with z_α. Or compare t with $t_{n-1,\alpha}$.

New decision rule: compare p-value with α.

The p-value is based on the percentile associated with your test statistic. All of this is just two sides of the same coin. The upshot of p-values is that they're more "report-able."

People looking at a statistical analysis can just trust you on your methodology and glance at the p-value; they don't need to know about statistical distributions.

Also, if $\alpha = .1$, then a p-value of .000000001 seems to say a bit more than .08.

Summary

How to calculate p-values:

1. For an upper-tailed test: $P(Z \geq z)$ OR $P(T_{n-1} \geq t)$
2. For a lower-tailed test: $P(Z \leq z)$ OR $P(T_{n-1} \leq t)$
3. For a two-tailed test: $2 \times P(Z \leq -|z|)$ OR $2 \times P(T_{n-1} \leq -|t|)$