10.3: Analysis of Paired Data

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Motivation

In the last section we assumed that the two samples (the Xs and the Ys) were independent from each other. In this section we will assume they are dependent.

Assumptions

We assume that the data consists of n independently selected pairs $(X_1, Y_1), \dots, (X_n, Y_n)$, with $EX_i = \mu_1$ and $EY_i = \mu_2$. Let $D_i = X_i - Y_i$ for $i = 1, \dots, n$. Assume further that all D_is are normally distributed with μ_D and σ_D^2.

Note: since X_i and Y_i are not necessarily independent, $V(\bar{X} - \bar{Y}) \neq V(\bar{X}) + V\bar{Y})$.

Easy

So $D_1, \ldots, D_n \overset{iid}{\sim} \mathcal{N}(\mu_d, \sigma_d^2)$ where $\mu_d = \mu_1 - \mu_2$ (linearity of expectation works even if we have dependence). So

$$\frac{\bar{D} - \mu_d}{S_d/\sqrt{n}} \sim t_{n-1}$$

It's EXACTLY the same as a one-sample t-test (but we just have slightly different notation).

Easy

For the sake of completeness, our paired confidence interval is

$$\bar{d} \pm t_{\alpha/2,n-1}\frac{s_d}{\sqrt{n}}$$

and our paired hypothesis test is

1. $H_0 : \mu_d = \Delta_d$
2. $t = \frac{\bar{d}-\Delta_d}{s_d/\sqrt{n}}$
3. if $H_a : \mu_d > \Delta_0$, reject when $t > t_{\alpha,n-1}$
4. if $H_a : \mu_d < \Delta_0$, reject when $t < -t_{\alpha,n-1}$
5. if $H_a : \mu_d \neq \Delta_0$, reject when $t > t_{\alpha/2,n-1}$ or when $t < -t_{\alpha/2,n-1}$