6.5: Seasonal ARIMA Models

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Motivation

These are useful when you have a seasonal component of period s.

Definition

SARIMA(p,d,q)×(P,D,Q)$_s$

If d and D are nonnegative integers, then $\{X_t\}$ is a **seasonal ARIMA(p,d,q)(P,D,Q) process with period** s if the differenced series $Y_t = (1-B)^d(1-B^s)^D X_t$ is a causal ARMA process defined by

$$\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t, \qquad \{Z_t\} \sim WN(0,\sigma^2),$$

where $\phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p$, $\Phi(z) = 1 - \Phi_1 z - \cdots - \Phi_P z^P$, $\theta(z) = 1 + \theta_1 z + \cdots + \theta_q z^q$, $\Theta(z) = 1 + \Theta_1 z + \cdots \Theta_Q z^Q$.

Note

Quote From Book

In Section 1.5 we discussed the classical decomposition model incorporating trend, seasonality, and random noise, namely,

$$X_t = m_t + s_t + Y_t.$$

In modeling real data it might not be reasonable to assume, as in the classical decomposition model, that the seasonal component s_t repeats itself precisely in the same way cycle after cycle. Seasonal ARIMA models allow for randomness in the seasonal pattern from one cycle to the next."

Example 1

$$\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t$$

$$X_t = U_t - .4U_{t-12}$$

Q: What are p, d, q, P, D, Q, s?

Example 1

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$$X_t = U_t - .4U_{t-12}$$

Q: What are p, d, q, P, D, Q, s?

A:

- $p = 0$
- $q = 0$
- $d = 0$
- $P = 0$
- $D = 0$
- $Q = 1$
- $s = 12$

Example 1

$$\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t$$

$$X_t - .7X_{t-12} = U_t$$

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Q: What is the ACF?

$$\rho(\pm 12) = \frac{\gamma(12)}{\gamma(0)} = \frac{-.4\sigma^2}{\sigma^2(1+.4^2)} = -.4/(1+.4^2).$$

And it equals zero everywhere else (besides at 1).

Example 2

$$\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t$$

$$X_t - .7X_{t-12} = U_t$$

Q: What is the ACF?

Example 2

$$\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t$$

$$X_t - .7X_{t-12} = U_t$$

Q: What is the ACF?

For $k \geq 1$,

$$\gamma(12k) = \text{Cov}(X_{t+12k}, X_t) = \text{Cov}(.7X_{t+12(k-1)}, X_t) = .7\gamma(12(k-1))$$

So

$$\rho(12k) = .7\rho(12[k-1])$$

as well.

Example 3

Consider the model

$$(1-\phi B)(1-\Phi B^{12})X_t = Z_t.$$

This can be rewritten as

$$X_t = \phi X_{t-1} + \Phi X_{t-12} - \phi\Phi X_{t-13} = Z_t.$$

This is an AR(13) model where a lot of the coefficients are 0, and you're only estimating three parameters.

Identification by Hand

The book suggests:

1. first find d, D, s so that $(1 - B)^d (1 - B^s)^D X_t$ is stationary
2. then look at ACF and PACF, and look out for significant bumps at lags of multiple s

AICc works, too. But you should try to narrow it down to a few models by hand, first.

See code file for example.