2.5: Independence

Taylor

University of Virginia

Motivation

When we were talking about conditional probability before we usually talked about examples where either $P(A) > P(A|B)$ or $P(A) < P(A|B)$. Quite often we'll assume that this isn't true when we're putting together statistical models, though.

Definition

Two events A and B are **independent** if $P(A|B) = P(A)$.

Two events A and B are **dependent** if $P(A|B) \neq P(A)$

Motivation

The following are equivalent (you should check all of these):

1. $P(A|B) = P(A)$
2. $P(A|B') = P(A)$
3. $P(A'|B) = P(A')$
4. $P(A'|B') = P(A')$

Motivation

The following are equivalent (you should check all of these):

1. $P(A|B) = P(A)$
2. $P(A|B') = P(A)$
3. $P(A'|B) = P(A')$
4. $P(A'|B') = P(A')$

Also, these are too

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$

Another definition

Another definition:

Events A and B are **independent** if

$$P(A \cap B) = P(A)P(B)$$

this one is less intuitive but more general; it holds when we're talking about events that might have probability 0. It also extends more easily to when we talk about independence between more than two things at a time.

Definition

$A_1, \dots A_n$ are **mutually independent** if for every $k = 1, 2, \dots, n$ and every subset of indices you can make with such a k $i_1, \dots, i_k$

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \times \cdots \times P(A_{i_k})$$

Definition

$A_1, \ldots A_n$ are **mutually independent** if for every $k = 1, 2, \ldots, n$ and every subset of indices you can make with such a k $i_1, \ldots, i_k$

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \times \cdots \times P(A_{i_k})$$

Note that this is stronger than pairwise independence...

Example

Let's try number 70 on page 89.

Example

Let's try number 70 on page 89.

Let T_1 and T_2 be the blood types for the first and second persons. Then $P(\{T_1 = O\} \cap \{T_2 = 0\}) = P(\{T_1 = O\})P(\{T_2 = O\}) = .44^2 = .1936$

Example

Let's try number 70 on page 89.

Let T_1 and T_2 be the blood types for the first and second persons. Then $P(\{T_1 = O\} \cap \{T_2 = 0\}) = P(\{T_1 = O\})P(\{T_2 = O\}) = .44^2 = .1936$

$$\begin{aligned}
P(\text{both match}) &= P(\text{both O or both A or both B or both AB}) \\
&= P\left[(\{T_1 = O\} \cap \{T_2 = O\}) \cup \cdots\right] \\
&= P(\{T_1 = O\} \cap \{T_2 = O\}) + \cdots \\
&= P(\{T_1 = O\})P(\{T_2 = O\}) + \cdots \\
&= .42^2 + .1^2 + .04^2 + .44^2 \\
&= .3816
\end{aligned}$$