8.1: Multivariate Time Series Examples

Taylor

University of Virginia

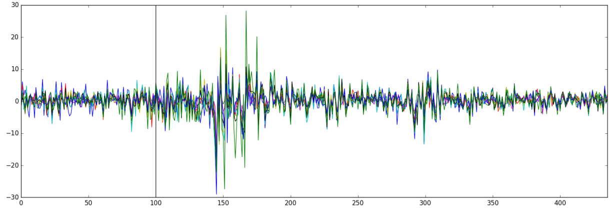
Motivation

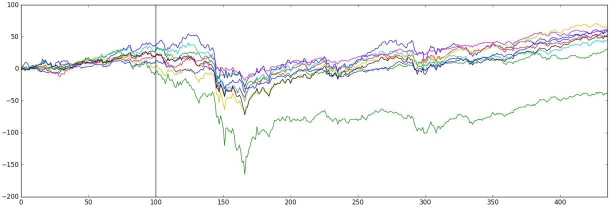
$\mathbf{X}_t$ is a vector-valued time series now. We usually consider the bivariate case for clarity, although most of what we learn will apply to higher dimensions.

Each time series $\{X_{t,1}\}$ and $\{X_{t,2}\}$ can be considered separately with the techniques we learned before. Although, this does not take into account the relationship they have between each other.

Motivation

Weekly log-returns and their cumulative sums.





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Sector SPDR ETFs 2005/12/23-2014/5/1.

Means and Covariances

The mean vector is

$$\boldsymbol{\mu}_t = E[\mathbf{X}_t] = \begin{bmatrix} EX_{t,1} \\ EX_{t,2} \end{bmatrix}$$

and the covariance matrices are

$$\Gamma(t+h,t) = \mathrm{Cov}(\mathbf{X}_{t+h}, \mathbf{X}_t) = \begin{bmatrix} \mathrm{Cov}(X_{t+h,1}, X_{t,1}) & \mathrm{Cov}(X_{t+h,1}, X_{t,2}) \\ \mathrm{Cov}(X_{t+h,2}, X_{t,1}) & \mathrm{Cov}(X_{t+h,2}, X_{t,2}) \end{bmatrix}$$

for each h

Means and Covariances

weak stationarity

A bivariate series $\mathbf{X}_t$ is said to be **weakly stationary** if $\boldsymbol{\mu}_t$ and $\Gamma(t+h,t)$ are independent of t.

If that's the case, we will write $\boldsymbol{\mu}$ and $\Gamma(h)$ for these quantities.

Notice that

$$\gamma_{12}(h) = \text{Cov}(X_{t+h,1}, X_{t,2}) = \text{Cov}(X_{t,2}, X_{t+h,1}) = \gamma_{21}(-h)$$

so these matrices are not symmetric!

Means and Covariances (and correlations!)

Also, we can define the correlation matrices

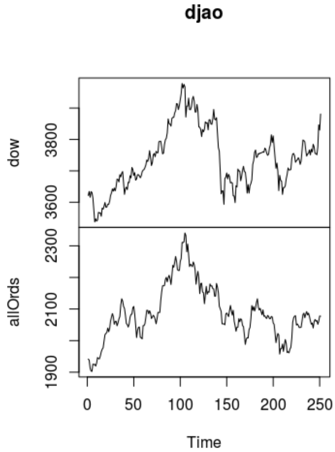
$$R(h) = \begin{bmatrix} \rho_{11}(h) & \cdots & \rho_{1m}(h) \\ \vdots & \ddots & \vdots \\ \rho_{m1}(h) & \cdots & \rho_{mm}(h) \end{bmatrix}$$

where $\rho_{ij}(h) = \frac{\gamma_{ij}(h)}{\sqrt{\gamma_{ii}(0)\gamma_{jj}(0)}}$

Pictures

Example: Dow Jones and the All Ordinaries Index (Australia)

See 8.1.R for more details



Pictures

Example: Dow Jones and the All Ordinaries Index (Australia)

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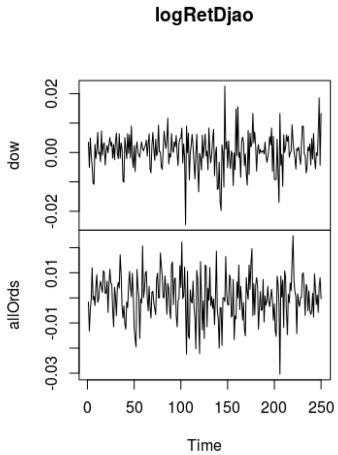

Pictures

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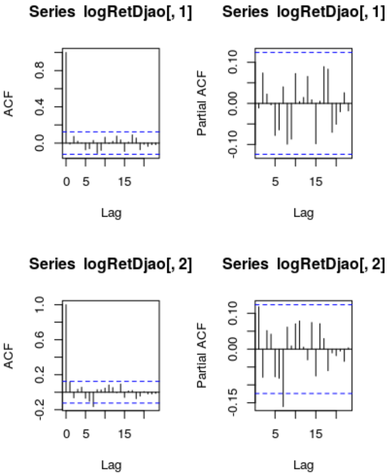
Univariate summaries:



Pictures

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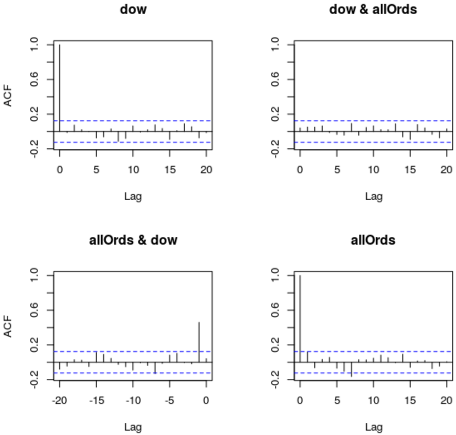

Pictures

Example: Dow Jones and the All Ordinaries Index (Australia)

See 8.1.R for more details. $\hat{\rho}_{12}(-1) = .46$