7.1: Historical Overview

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Definitions

- P_t: the closing price on day/week/month t of stock/stock index
- $X_t = \log P_t$: log-price...observed paths look very much like those of a random walk
- $Z_t = \nabla X_t$: the log return
- $100Z_t$: percentage returns
- $h_t = \text{Var}(Z_t | Z_{1:t-1})$: the conditional variance aka volatility

Motivation

Z_t sometimes looks roughly stationary. The ACF rarely gives any clear or consistent ARMA model recommendations. Also, if we fit some ARMA model, h_t is independent of t and independent of $Z_{1:t-1}$[1]. This assumption is often violated.

We would like models that take into account stylized features that appear such as **tail-heaviness, asymmetry, volatility clustering and serial dependence without correlation**. We introduce AutoRegressive Conditional Heteroscedasticity (ARCH) models, Generalized AutoRegressive Conditional Heteroscedasticity (GARCH), and stochastic volatility (SVOL) models in the next slide.

[1] homework question

Definitions

ARCH(p)

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2.$$

$\alpha_0 > 0,\ a_i \geq 0,\ p \in \mathbb{N}$

- volatility increases if we have observed big movements
- setting a_i to 0 gives us white noise model
- $\{e_t\}$ is sometimes but not always assumed to be Normal

Definitions

GARCH(p,q)

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}.$$

$\alpha_0 > 0,\ a_i \geq 0,\ \beta_j \geq 0,\ p \in \mathbb{N}$

- now volatility is a function of its own past values, in addition to the past observations
- $\{e_t\}$ may or may not be normal

Definitions

SVOL

$$Z_t = \sqrt{h_t} e_t, \qquad \{e_t\} \sim IID(0,1)$$
$$\ln h_t = \gamma_0 + \gamma_1 \ln h_{t-1} + \eta_t, \qquad \{\eta_t\} \sim IID(0,\sigma^2).$$

where $\{\eta_t\}$ and $\{e_t\}$ are independent.

- log-volatility is an AR(1) process.
- γ_1 is usually around .95.
- more difficult to estimate (can't even evaluate likelihood)