5.3: Diagnostic Checking

Taylor

University of Virginia

Motivation

Typically, the goodness of fit of a statistical model to a set of data is judged by comparing the observed values with the corresponding predicted values obtained from the fitted model. If the fitted model is appropriate, then the residuals should behave in a manner that is consistent with the model.

Our Output

When we fit an ARMA(p,q) model, we obtain

1. estimators $\hat{\phi}, \hat{\boldsymbol{\theta}}, \hat{\sigma}^2$ (estimated from the entire dataset)
2. predicted values $\hat{X}_t(\hat{\phi}, \hat{\boldsymbol{\theta}}, \hat{\sigma}^2)$
3. predicted variances $r_{t-1}(\hat{\phi}, \hat{\boldsymbol{\theta}})$, with $\sigma^2 r_{t-1} = v_{t-1} = E[(X_t - \hat{X}_t)^2]$

NB: the predicted values are only functions of the previous data, but that function itself is estimated using future data

Definition of Residuals

The standardized residuals are

$$\hat{W}_t = \frac{(X_t - \hat{X}_t[\hat{\phi}, \hat{\boldsymbol{\theta}}, \hat{\sigma}^2]))}{\sqrt{r_{t-1}(\hat{\phi}, \hat{\boldsymbol{\theta}})}}$$

or

$$\hat{R}_t = \frac{(X_t - \hat{X}_t[\hat{\phi}, \hat{\boldsymbol{\theta}}, \hat{\sigma}^2]))}{\sqrt{\hat{\sigma}^2 r_{t-1}(\hat{\phi}, \hat{\boldsymbol{\theta}})}}$$

If the fitted model is correct, and if Z_t are WN (IID noise), then the standardized residuals are approximately WN (IID noise) with either variance σ^2 or 1, depending on which you use.

Now we can use all of the tests from Section 1.6 to check our model!