

## 2.2: Axioms, Interpretations and Properties of Probability

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# Motivation

The sets are basically questions we can ask about something random...but how do we talk about the chances that those things will occur? A: we need a function that takes events as input, and as output gives us a number between 0 and 1. This is our probability function  $P(\cdot)$ .

# Properties of $P$

Here are the rules any probability function has to satisfy;

- 1 For any event  $A$ ,  $P(A) \geq 0$
- 2  $P(S) = 1$
- 3 let  $A_1, A_2, \dots$  be a countably infinite collection of disjoint sets; then
$$P(A_1 \cup A_2 \cup \dots) = \sum_{i=1}^{\infty} P(A_i)$$

# Derived Properties

Here are some things you can make sure are true using only the stuff from the previous slide:

- 1  $P(\emptyset) = 0$
- 2  $P(\bigcup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$  if  $A_1, \dots, A_n$  are all disjoint
- 3  $P(A') = 1 - P(A)$  for any  $A$
- 4 if  $A \subset B$  then  $P(A) \leq P(B)$
- 5 (in particular) for any  $A$ ,  $P(A) \leq 1$
- 6 for any  $A$  and  $B$  (not necessarily disjoint),  
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Keep in mind that we are talking about a general  $P(\cdot)$ . In practice, you don't know it. However, every possibility must satisfy all these rules we have talked about.