Time Series

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2018-03-20

Time Series Forecasting

The Importance of Forecasting

- Governments forecast unemployment rates, interest rates, and expected revenues from income taxes for policy purposes
- Marketing executives forecast demand, sales, and consumer preferences for strategic planning
- College administrators forecast enrollments to plan for facilities and for faculty recruitment
- Retail stores forecast demand to control inventory levels, hire employees and provide training

Common Approaches to Forecasting

- Qualitative forecasting methods
  - Used when historical data are unavailable
  - Considered highly subjective and judgmental
- Quantitative forecasting methods
  - Time series or Causal
  - Use past data to predict future values

Time-Series Data

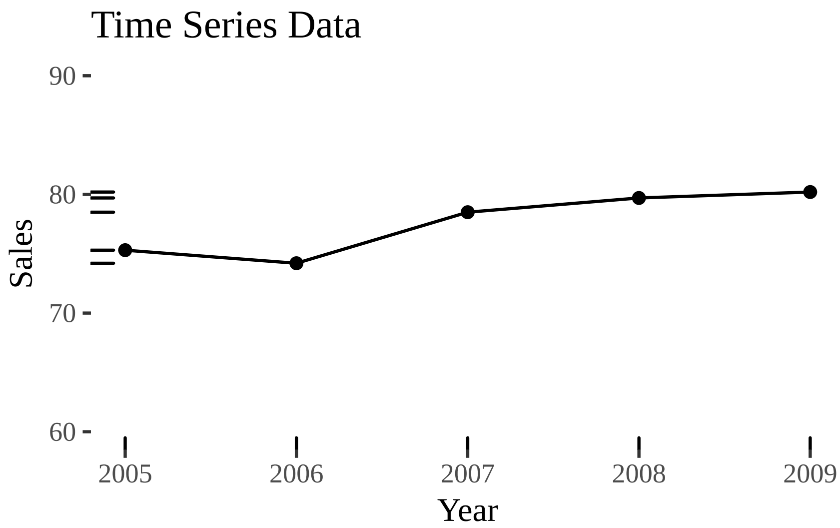
- Numerical data obtained at regular time intervals
- The time intervals can be annually, quarterly, monthly, weekly, daily, hourly, etc.

Table 1: Example

Years:	2005	2006	2007	2008	2009
Sales:	75.3	74.2	78.5	79.7	80.2

Time-Series Plot

- A time-series plot is a two-dimensional plot of time series data
  - the vertical axis measures the variable of interest
- the horizontal axis corresponds to the time periods

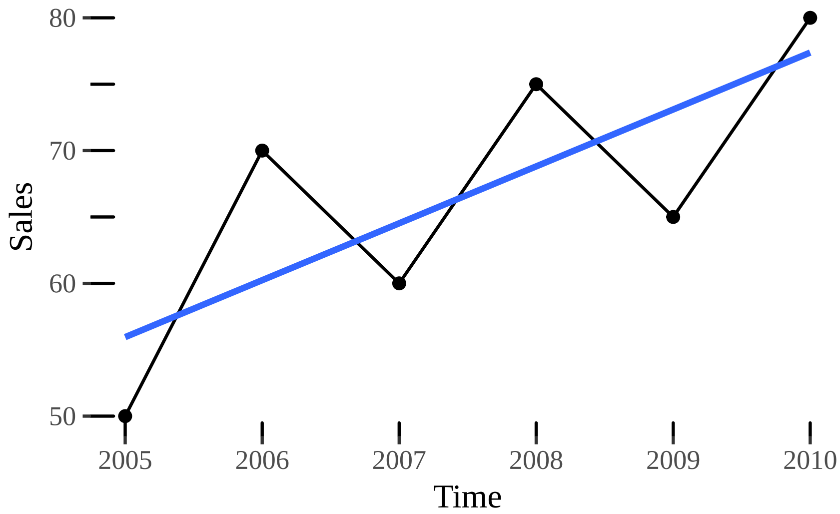


Time-Series Components

- Trend Component
  - Overall, persistent, long-term movement
- Seasonal Component
  - Regular periodic fluctuations, usually within a 12-month period
- Cyclical Component
  - Repeating swings or movements over more than one year
- Irregular Component
  - Repeating swings or movements over more than one year Erratic or residual fluctuations

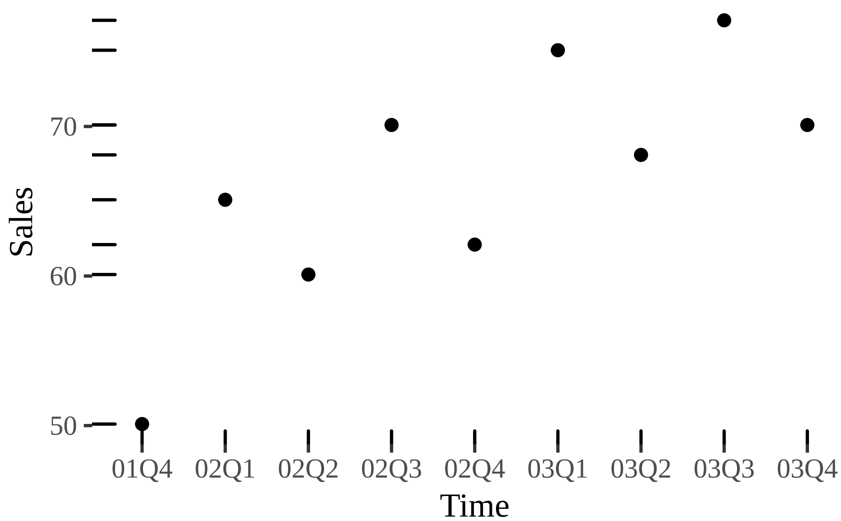
Trend Component

- Long-run increase or decrease over time (overall upward or downward movement)
- Data taken over a long period of time
- Trend can be upward or downward
- Trend can be linear or non-linear



Seasonal Component

- Short-term regular wave-like patterns
- Observed within 1 year
- Often monthly or quarterly Sales



Cyclical Component

- Long-term wave-like patterns
- Regularly occur but may vary in length
- Often measured peak to peak or trough to trough

Irregular Component

- Unpredictable, random, “residual” fluctuations
- Due to random variations of Nature

- Accidents or unusual events
- "Noise" in the time series

Does Your Time Series Have A Trend Component?

- A time-series plot should help you to answer this question.
- Often it helps if you "smooth" the time-series data to help answer this question.
- Two popular smoothing methods are moving averages and exponential smoothing.

Smoothing Methods

- Moving Averages
  - Calculate moving averages to get an overall impression of the pattern of movement over time
  - Averages of consecutive time-series values for a chosen period of length L.
- Exponential Smoothing
  - A weighted moving average

Moving Averages

- Used for smoothing
- A series of arithmetic means over time
- Result dependent upon choice of L (length of period for computing means)
- Last moving average of length L can be extrapolated one period into future for a short term forecast
- Examples:
  - For a 5 year moving average, L = 5
  - For a 7 year moving average, L = 7
  - Etc.
- Example: Five-year moving average
  - First average: $MA(5) = \frac{Y_1+Y_2+Y_3+Y_4+Y_5}{5}$
  - Second average:$MA(5) = \frac{Y_2+Y_3+Y_4+Y_5+Y_6}{5}$
  - etc.
- The 5-year moving average smoothes the data and makes it easier to see the underlying trend

Exponential Smoothing

- Used for smoothing and short term forecasting (one period into the future)
- A weighted moving average
  - Weights decline exponentially
  - Most recent observation is given the highest weight
- The weight (smoothing coefficient) is W
  - Subjectively chosen
  - Ranges from 0 to 1
  - Smaller W gives more smoothing, larger W gives less smoothing
- The weight is:
  - Close to 0 for smoothing out unwanted cyclical and irregular components
  - Close to 1 for forecasting

Exponential Smoothing Model

- $E_1 = Y_1$
- $E_i = WE_i + (1 - W)E_{i-1}$ For i = 2, 3, 4, ... where: Ei = exponentially smoothed value for period i
  E_{i-1} = exponentially smoothed value already computed for period i - 1
  Y_i = observed value in period i
  W = weight (smoothing coefficient), 0 < W < 1

- Suppose we use weight W = 0.2

Time Period (i)	Sales (Y_i)	Forecast from prior period (E_{i-1})	Exponentially Smoothed Value for this period (E_i)
1	23	--	23
2	40	23	(.2)(40)+(.8)(23)=26.4
3	25	26.4	(.2)(25)+(.8)(26.4)=26.12
4	27	26.12	(.2)(27)+(.8)(26.12)=26.296
5	32	26.296	(.2)(32)+(.8)(26.296)=27.437
6	48	27.437	(.2)(48)+(.8)(27.437)=31.549
7	33	31.549	(.2)(48)+(.8)(31.549)=31.840
8	37	31.840	(.2)(33)+(.8)(31.840)=32.872
9	37	32.872	(.2)(37)+(.8)(32.872)=33.697
10	50	33.697	(.2)(50)+(.8)(33.697)=36.958
etc.	etc.	etc.	etc.

$E_1 = Y_1$ since no prior information exists

$E_i = WY_i + (1-W)E_{i-1}$

Forecasting Time Period i + 1

The smoothed value in the current period (i) is used as the forecast value for next period (i + 1) :
- $\hat{Y_{i+1}} = E_i$

There Are Three Popular Methods For Trend-Based Forecasting

- Linear Trends
  - Linear Trend Model
- Non Linear Trends
  - Quadratic Trend Model
  - Exponential Trend Forecasting

Linear Trend Forecasting

Estimate a trend line using regression analysis Use time (X) as the independent variable:

- $\hat{Y} = b_0 + b_1 X$

In least squares linear, non-linear, and exponential modeling, time periods are numbered starting with 0 and increasing by 1 for each time period.

Nonlinear Trend Forecasting

- A nonlinear regression model can be used when the time series exhibits a nonlinear trend
- Compare adj. r^2 and standard error to that of linear model to see if this is an improvement

Choosing A Forecasting Model

- Perform a residual analysis
  - Eliminate a model that shows a pattern or trend
- Measure magnitude of residual error using squared differences and select the model with the smallest value
- Measure magnitude of residual error using absolute differences and select the model with the smallest value
- Use simplest model
  - Principle of parsimony

Principal of Parsimony

- Suppose two or more models provide a good fit for the data
- Select the simplest model

Pitfalls in Time-Series Analysis

- Assuming the mechanism that governs the time series behavior in the past will still hold in the future
- Using mechanical extrapolation of the trend to forecast the future without considering personal judgments, business experiences, changing technologies, and habits, etc.