Decision Making

Tom Bruning

2018-04-09

Steps in Decision Making

- List Alternative Courses of Action
  - Choices or actions
- List Uncertain Events
  - Possible events or outcomes
- Determine 'Payoffs'
  - Associate a Payoff with Each Choice/Event combination
- Adopt Decision Criteria
  - Evaluate Criteria for Selecting the Best Course of Action

A Payoff Table

A payoff table shows alternatives, states of nature, and payoffs

Table 1: Payoff Table Profit in $1000s

States of Nature	Large Factory	Average Factory	Small Factory
Strong Economy	200	90	40
Stable Economy	50	120	30
Weak Economy	-120	-30	20

Opportunity Loss

Opportunity loss is the difference between an actual payoff for an action and the highest possible payoff, given a particular event

The action "Average factory" has payoff 90 for "Strong Economy". Given "Strong Economy", the choice of "Large factory" would have given a payoff of 200, or 110 higher. *Opportunity loss = 110 for this cell.*

Table 2: Opportunity Loss Table Opportunity Loss in $1000s

States of Nature	Large Factory	Average Factory	Small Factory
Strong Economy	0	110	160
Stable Economy	70	0	90
Weak Economy	140	50	0

States of Nature	Large Factory	Average Factory	Small Factory

Decision Criteria

- Maximax
  - An optimistic decision criteria
- Maximin
  - A pessimistic decision criteria
- Expected Monetary Value (EMV(j))
  - The expected profit for taking action j
- Expected Opportunity Loss (EOL(j))
  - The expected opportunity loss for taking action j
- Expected Value of Perfect Information (EVPI)
  - The expected opportunity loss from the best decision
- Return to Bayes

Maximax Solution

- Maximum payoff for **Large Factory** is 200
- Maximum payoff for **Average Factory** is 120
- Maximum payoff for **Small Factory** is 40

Maximax Decision: Build the Large Factory because 200 is the maximum
Maximin Solution

- Minimum payoff for **Large Factory** is -120
- Minimum payoff for **Average Factory** is -30
- Minimum payoff for **Small Factory* is 20

Maximin Decision: Build the Small Factory because 20 is the maximum

Expected Monetary Value Solution

Goal: Maximize expected value

The expected monetary value is the weighted average payoff, given specified probabilities for each event.

$$EMV_j = \sum_{i=1}^{N} x_{ij} P(X = x_i) \tag{1}$$

Where:
EMV_j = expected monetary value of action j
x_{ij} = payoff for action j when event i occurs
P_i = probability of event i

Table 3: Payoff Table Profit in $1000s

States of Nature	Large Factory	Average Factory	Small Factory
Strong Economy(.3)	200	90	40
Stable Economy(.5)	500	120	30
Weak Economy(.2)	-120	-30	20
EMV	61	81	31

Example:
EMV (Average factory) = (90)(.3) + (120)(.5) + (-30)(.2) = 81
Maximize expected value by choosing **Average factory**

Expected Opportunity Loss Solution

Goal: Minimize expected opportunity loss The expected opportunity loss is the weighted average loss, given specified probabilities for each event

$$EOL_j = \sum_{i=1}^{N} Lx_{ij} P(X = x_i) \qquad (2)$$

Where:
EOL_j = expected opportunity loss of action j
L_{ij} = opportunity loss for action j when event i occurs
P_i = probability of event i

Expected Opportunity Loss Solution

Goal: Minimize expected opportunity loss

Table 4: Opportunity Loss Table Opportunity Loss in $1000s

States of Nature	Large Factory	Average Factory	Small Factory
Strong Economy(.3)	0	110	160
Stable Economy(.5)	70	0	90
Weak Economy(.2	140	50	0
EOL	63	43	93

Example: EOL (Large factory) = 0(.3) + 70(.5) + (140)(.2) = 63
Minimize expected op. loss by choosing **Average factory**

Expected Opportunity Loss vs. Expected Monetary Value

- The Expected Monetary Value (EMV) and the Expected Opportunity Loss (EOL) criteria are equivalent.

- Note that in this example the expected monetary value solution and the expected opportunity loss solution both led to the choice of the average size factory.

Value of Information

Expected Value of Perfect Information, EVPI

EVPI = Expected profit under certainty – expected monetary value of the best alternative

(EVPI is equal to the expected opportunity loss from the best decision)

Expected Profit Under Certainty

Expected profit under certainty = expected value of the best decision, *given perfect information*

Examples:

- Best decision given “Strong Economy” is “Large factory” (200)
- Best decision given “Stable Economy”” is “Average factory” (120)
- Best decision given “Weak Economy” is “Small factory” (20)

Now weight these outcomes with their probabilities to find the expected value.

200(.3)+120(.5)+20(.2) = 124, which is the expected profit under certainty

Value of Information Solution (EVPI)

EVPI = Expected profit under certainty – Expected monetary value of the best decision

Recall:

- Expected profit under certainty = 124
- EMV is maximized by choosing “Average factory”, where EMV = 81

so: * EVPI = 124 – 81 = 43

(EVPI is the maximum you would be willing to spend to obtain perfect information)

Forecasting (accounting for randomness)

Suppose you are a district manager for a retail chain. You have three stores in your district that have averaged the same store sales over the past three years of $10,000,000. You know (from taking this class) that there is are random factors that are reflected in these averages. The store's past year sales were $10,500,000, $9,500,000, and $10,000,000. Your regional manager demands a 10% increase in sales from your district for the following year. How best to allocate that $3,000,000 increase?

Since tha stores have the same average sales (for the past 3 years) you could increase their quota by $1,000,000. You could increase each store sales by 10%, or you could take randomness into account and factor that randomness into the new quotas.

The following table summarizes these options.

Table 5: Summary

	District	Store 1	Store 2	Store 3
3 yr. Average Sales	$30,000,000	$10,000,000	$10,000,000	$10,000,000
Last Year Sales	$30,000,000	$10,500,000	$9,500,000	$10,000,000
10% increase	$30,000,000	$11,550,000	$10,450,000	$11,000,000
10% increase w/randomness	$30,000,000	$11,275,000	$10,725,000	$11,000,000