Chapter 10-1

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6 Steps in Hypothesis Testing (Review)

- Planning the test

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Determine the critical values that divide the rejection and nonrejection regions

- Run the test

5. Collect data and compute the value of the test statistic
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, do not reject the null hypothesis H0. If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem.

Two Sample Tests

Goal: Test hypothesis or form a confidence interval for the difference between two population means, $\mu_1 – \mu_2$

- The point estimate for the difference is: $\bar{X}_1 – \bar{X}_2$

Difference Between Two Means: Independent Samples

- Different data sources
  - Unrelated
  - Independent

Sample selected from one population has no effect on the sample selected from the other population.

Population means, independent samples

Hypothesis Tests for Two Population Means

Two Population Means, Independent Samples

- Lower-tail test:

$H_0 : \mu_1 \geq \mu_2$
$H_1 : \mu_1 1 < \mu_2$
i.e.,
$H_0 : \mu_1 – \mu_2 \geq 0$
$H_1 : \mu_1 – \mu_2 < 0$
So: Reject H_0 if $t_{STAT} < -t_a$

- Upper-tail test:

$H_0 : \mu_1 \leq \mu_2$
$H_1 : \mu_1 1 > \mu_2$
i.e.,
$H_0 : \mu_1 – \mu_2 \leq 0$
$H_1 : \mu_1 – \mu_2 > 0$ So: Reject H_0 if $t_{STAT} > t_a$

- Two-tail test:

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 1 \neq \mu_2$
i.e.,
$H_0 : \mu_1 – \mu_2 = 0$
$H_1 : \mu_1 – \mu_2 \neq 0$
So: Reject H_0 if $t_{STAT} < -t_{\alpha/2}$ or $t_{STAT} > t_{\alpha/2}$

Hypothesis tests for μ_1 - μ_2 with σ_1 and σ_2 unknown and assumed equal

Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed or both sample sizes are at least 30
- Population variances are unknown but assumed equal

The pooled variance is:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} \qquad (1)$$

The test statistic is:
$t_{stat} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 = \mu_2)}{\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})}}$
Where $t_S TAT$ has d.f. $= (n_1 + n_2 – 2)$

Pooled-Variance t Test Example

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	21.00	25.00
Sample mean	3.27	2.53
Sample std dev	1.30	1.16

Assuming both populations are approximately normal with equal variances, is there a difference in mean yield ($\alpha = 0.05$)?

$H_0 : \mu_1 - \mu_2 = 0 \;\; i.e. \;\; (\mu_1 = \mu_2)$

$H_1 : \mu_1 - \mu_2 \neq 0 \;\; i.e. \;\; (\mu_1 \neq \mu_2)$

$S_p^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{(n_1-1)+(n_2-1)} = \frac{(21-1)1.30^2+(25-1)1.16^2}{(21-1)+(25-1)} = 1.5021$

$t_{stat} = \frac{(\bar{X}_1-(\bar{X}_2)-(\mu_1=\mu_2}{\sqrt{S_p^2(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{(3.27-2.53)-0}{\sqrt{1.5021(\frac{1}{21}+\frac{1}{25})}} = 2.040$

Note: You assume $\mu_1 - \mu_2 = 0$ since you are trying to reject H_0

$\alpha = 0.05$ df = 21 + 25 - 2 = 44 Critical Values: $t = \pm 2.0154$ (from the student t table)

Test Statistic = 2.040

Decision: Reject $H_0 \;\; at \;\; \alpha = 0.05$

Conclusion: There is evidence of a difference in means.

Pooled-Variance t Test Example: Confidence Interval for $\mu_1 - \mu_2$

Since we rejected H_0 can we be 95% confident that $\mu_{NYSE} > \mu_{NASDAQ}$?

95% Confidence Interval for $\mu_{NYSE} - \mu_{NASDAQ}$

$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})} = (3.27-2.53) \pm 2.0154\sqrt{1.5021(\frac{1}{21} + \frac{1}{25})} =$ $0.74 \pm 2.0154(0.3628) = (0.009, 1.471)$

Since 0 is less than the entire interval, we can be 95% confident that $\mu_{NYSE} > \mu_{NASDAQ}$