APPENDIX

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"Intelligent Locomotion Planning with Enhanced Postural Stability for Lower-Limb Exoskeletons"

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1 Inertia matrix and gravity vector values:

The elements of inertia matrix M are

$$\begin{aligned}
M_{11}&=m_1 l_{c1}^2+m_2(l_1+l_{c2})^2+m_3(l_1+l_{c3})^2+\\
&\quad m_4(l_1+l_3+l_{c4})^2+I_1+I_2+I_3+I_4\\
M_{12}&=m_2 l_{c2}(l_1+l_{c2})+m_3 l_{c3}(l_1+l_{c3})+\\
&\quad m_4(l_3+l_{c4})(l_1+l_3+l_{c4})+I_2+I_3+I_4\\
M_{13}&=m_3 l_{c3}(l_1+l_{c3})+m_4(l_3+l_{c4})(l_1+l_3+l_{c4})+\\
&\quad I_3+I_4\\
M_{14}&=m_4 l_{c4}(l_1+l_3+l_{c4})+I_4\\
M_{22}&=m_2 l_{c2}^2 m_3 l_{c3}^2+m_4(l_3+l_{c4})^2+I_2+I_3+I_4\\
M_{23}&=m_3 l_{c3}^2+m_4(l_3+l_{c4})^2+I_3+I_4\\
M_{24}&=m_4 l_{c4}(l_3+l_{c4})+I_4\\
M_{33}&=m_3 l_{c3}^2+m_4(l_3+l_{c4})^2+I_3+I_4\\
M_{34}&=m_4 l_{c4}(l_3+l_{c4})+I_4\\
M_{44}&=m_4 l_{c4}^2+I_4
\end{aligned} \tag{A1}$$

and the elements of gravity matrix G are obtained as

$$\begin{aligned}
G_{11}&=3m_1 g l_1+m_1 g l_{c1}\\
G_{12}&=G_{22}=m_2 g l_{c2}\\
G_{13}&=G_{23}=G_{33}=-m_3 g l_3-m_3 g l_{c3}\\
G_{14}&=G_{24}=G_{34}=G_{44}=-m_4 g l_{c4}
\end{aligned} \tag{A2}$$

in which g is the gravitational acceleration.

2 DCM definition

The motion of HES center of mass can be divided into the convergent (C) and divergent (D) components as

$$\begin{aligned} C &= x_{CoM} - \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \\ D &= x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \end{aligned} \tag{A3}$$

where x_{CoM} and $\dot{x}_{CoM}$ denote the position and velocity of the center of mass and α is a constant value defined in (8). Taking the time derivative of (A3) and substituting (7) in the absence of control input (τ=0), the time derivative of the divergent and convergent parts are obtained as

$$\begin{aligned} \dot{C} &= -\sqrt{\alpha} C \\ \dot{D} &= \sqrt{\alpha} D \end{aligned} \tag{A4}$$

As seen, the convergent part of the motion (C) will converge to zero without any control effort. Therefore, controlling the divergent part of the motion (D) will guarantee the convergence of x_{CoM} to its corresponding desired value. Accordingly, the DCM is defined as

$$\zeta = x_{CoM} + \frac{\dot{x}_{CoM}}{\sqrt{\alpha}} \tag{A5}$$

and the paper is focused on the control of the DCM by adjusting the control input (τ in (7)).