On a New Nonlinear Image Filtering Technique

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***Abstract*—A generalization of the varying weight trimmed mean (VWTM) filter is proposed for the removal of both additive and impulse noise from corrupted images by introducing the center pixel to the original VWTM filter. Unlike many traditional nonlinear filters, one key feature of the VWTM filter is that the weights of the pixels to be averaged are not constant. They vary based on their differences to the median value. By adding the weight varying center pixel to the VWTM filtering formalism, the center pixel is therefore involved in adjusting its filtering output. Studies of the proposed filtering technique with several other methods show that the present method, in comparison, is extremely robust and efficient for removing noise from highly corrupted images.**

Keywords—α-TM filter, MTM filter, GTM filter, VWTM filter, GVWTM filter, image processing

I. Introduction

Images are often corrupted by noise that affects the performance of various signal processing techniques, data compression, and pattern recognition. The goal of noise filtering is to suppress the noise while preserving the integrity of significant visual information such as textures, edges and details. Linear local averaging filters are essentially low pass filters. Because the impulse responses of the low pass filters are spatially invariant, rapidly changing signals such as image edges and details, cannot be well preserved. Consequently, impulse noise cannot be effectively removed by linear methods; nonlinear techniques have been found to provide more satisfactory results. Some of the most popular nonlinear filters are the median filter [1], and its various generalizations [2-8], which are well known to have the required properties for edge preservation and impulse noise removal. However, the median filter is not optimal since it is typically implemented uniformly across the image. It suppresses the true signal as well as noise in many applications. In the presence of impulse noise, the median filter tends to modify pixels that are not degraded. Furthermore, it is prone to produce edge jitter when the percentage of impulse noise is large. In order to improve the performance of the median filter, two median based filters, the α-trimmed mean (α-TM) [5-6] filter and the modified trimmed mean (MTM) [6] filter, which select from the sliding window only the luminance values close to the median value, have been proposed. The selected pixels are then averaged to provide the filtering output. Although superior to the median filter in some applications, these algorithms still have problems. In general, the MTM outperforms the median filter in removing additive noise is not as effective as the median filter in removing impulse noise. The α-TM filter is generally superior to the median filter as an impulse detector, but its performance in removing impulse noise is inferior. As is shown in the test example of this paper, the α-TM filter performs even worse than the median filter when no impulse detection techniques are employed. When the level of impulse noise is high, the α-TM filter is not optimal for filtering, since the selected pixels may have a large probability of being corrupted. Removal of additive noise is not optimal for both the α-TM and MTM filters because they do not take into account the central pixel.

This paper introduces a generalized version of the α-TM filter. We call it the generalized trimmed mean (GTM) filter, which in general outperforms α-TM and MTM filters for images that are highly corrupted by impulse and additive noise. In GTM filter, a "median basket" is employed to select a predetermined number of pixels on both sides of the median value of the sorted pixels in the moving window. The values of selected pixels and the center pixel in the window are then weighted and averaged to give the filtering output.

In general practice, the averaging weights of GTM filter as well as other nonlinear filters are predetermined and are fixed throughout the filtering processing. In [13], a varying weight trimmed mean (VWTM) filter is introduced, which dynamically varies the averaging weights of the selected pixels based on their differences with the median value. This weight-adaptive filter is shown to very effective in detecting and removing the impulse noise from corrupted images.

In this study, the GTM filter and VWTM filter are combined by introducing the weight varying center pixel to the VWTM filter formalism. We call it generalized varying weight trimmed mean (GVWTM) filter. With this filter, the VWTM weight function is also applied to the center pixel. By doing so, the center pixel will contribute to the filtering output with dynamically varying weight.

II. Generalized Variying Weight Trimmed Mean Filter

The pixels $\{I_1, I_2, \cdots, I_{m-1}, I_m, I_{m+1}, \cdots, I_N\}$ in the sliding window associated with the center pixel I_c, have been sorted in an ascending (or descending) order, with I_m being the median value. The key generalization to the median filter which is introduced in the alpha-trimmed mean (α-TM) filter [3], is to

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employ a median basket in which one collects the same predetermined number of pixels above and below the median pixel. The values of these selected pixels are then averaged to give the filtering output, A_c, as an adjusted replacement to I_c, according to:

$$A_c = \frac{1}{2L+1}\sum_{j=m-L}^{m+L} I_j \,, \qquad (1)$$

where $L = \lfloor \lambda N \rfloor$ with $0 \le \lambda \le 0.5$. When $\lambda = 0$, it becomes the median filter, and when $\lambda = 0.5$, it becomes the simple moving average. In general, the α-TM filter outperforms the median filter in detecting the impulse. However, its capability of removing the impulse noise is even worse than for the median filter when no additional procedure is employed (such as the switching scheme). When an image is corrupted by high levels of impulse noise, the α-TM filter does not perform well, since the pixels being selected for the median basket now have a large probability of being corrupted. It is therefore unreasonable for the α-TM filter to have the pixels in the basket equally weighted. Another generalization to the median filtering is the so called modified trimmed mean (MTM) filter [6]. In the MTM filter, a container, C_c, is employed to select the pixels from the sliding window centered around pixel I_c with luminance values are in the range of $[I_m - q, I_m + q]$, with q being a predetermined threshold. The mean value of the selected values in the container is taken as the filtering output.

$$T_c = \text{mean}(\{I_j\} \mid I_j \in C_c), \qquad (2)$$

Like the α-TM filter, the MTM filter can also be reduced to the median filter (at q=0) or the simple moving average filter (at q of maximum luminance value). The MTM filter is useful for removing additive noise but does not perform as well as the median filter when impulse noise removal is required, since impulse noise corrupted pixels are independent of the noise-free pixels.

There is still one problem left for the α-TM and MTM filters where additive noise removal is concerned; They do not take special account of the luminance value of the central pixel in the window. As is well known, the value of the central pixel, in general, has a larger probability of being the closest to its true value than those of all other pixels in the window. We therefore design the generalized trimmed mean (GTM) filter which improves the performance of both the α-TM and MTM filters. In the first step of the new filter, we employ the median basket to collect those pixels whose luminance values are close to the median value, in the same way as is done in the α-TM filter. The difference between our algorithm and the α-TM algorithm is that a weighted averaging is performed using the values selected for the median basket, as well as the value of the center pixel (In general, the center pixel is used in removing the additive noise.). Thus,

$$G_c = \frac{w_c I_c + \sum_{j=m-L}^{m+L} w_j I_j}{w_c + \sum_{j=m-L}^{m+L} w_j}, \qquad (3)$$

where G_c is the GTM filtering output, and w_c and w_j's are the averaging weights for the center pixel and the pixels in the median basket, respectively. When $w_c = 0$ and all w_j's are equal to each other (nonzero), the GTM filter becomes the α-TM filter.

To suppress impulse noise effectively, another improvement to the α-TM filter called varying weight trimmed mean (VWTM) filter is introduced in [13]. In VWTM filter, the pixels around I_m are weighted and averaged dynamically according to their absolute difference with the median value I_m.

$$V_c = \frac{\sum_{j=m-L}^{m+L} w(x_{jm}) I_j}{\sum_{j=m-L}^{m+L} w(x_{jm})}, \qquad (4)$$

where x_{jm} has a value in the range of [0,1], defined by

$$x_{jm} = \frac{|I_j - I_m|}{B}, \qquad (5)$$

with B being the maximum pixel value of a given type of image (e.g., B=255 for a 8-bit, gray-scale image). The weight $w(x)$ in (4) is a decreasing function in the range [0,1]. Some example weight functions are given below:

$$w(x) = \exp\left(-A\left(\frac{x}{x-1}\right)^2\right), \qquad (6)$$

$$w(x) = (1 - x^a)^b, \qquad (7)$$

where A, a, b are constants that controls the rate of the weight decrease with respect to the increment of the pixel difference to the median pixel. Notice from the that $w(0)$=1 and $w(1)$=0, so the median value always has the largest weight ($w(x_{mm})$=1). The larger the absolute difference between the pixel values in the median basket and the median value, the smaller the weight will be. The VWTM filter outperforms both the median filter and the α-TM filter in suppressing impulse noise while preserving edges. The median value has the least probability to be impulse noise corrupted because the impulses typically occur near the ends of the sorted pixels. However, although not corrupted, the median value may not be the optimal value to replace the center pixel value because it may differ significantly from the noise-free value. The α-TM filter itself will not perform better than the median filter when treating highly impulse noise corrupted images because corrupted pixels may also be included in the median basket for the averaging operation. In contrast, the VWTM filter can alleviate the shortcomings of both filters. The weight of the median value is the largest and the weights of other pixels in the median basket vary according to their difference from the median value. If an impulse noise corrupted pixel happens to be selected for inclusion in the median basket, its contribution to the average will be small because x_{jm} is large. In general, the weight function can assist in eliminating impulse noise while providing a well-adjusted replacement value for the center pixel I_c.

In this paper, we combine the advantages of the GTM filter (center pixel involved) and VWTM filter (dynamic varying weight) and introduce the generalized varying weight trimmed (GVWTM) filter as following:

$$F_c = \frac{(\beta + \alpha w(x_{cm}))I_c + \sum_{j=m-L}^{m+L} w(x_{jm}) I_j}{\beta + \alpha w(x_{cm}) + \sum_{j=m-L}^{m+L} w(x_{jm})}, \tag{8}$$

where α and β are additional coefficients that control the weight of the center pixel I_c. The GVWTM filter becomes VWTM filter when both α and β are 0. Therefore, it is effective in suppressing impulse noise. On the other hand, it allows the center pixel to be involved in the filtering formalism which enhances its efficiency in canceling the additive noise. Furthermore, the center pixel contribution to the filtering output is dynamically adjusted based on its difference with the median pixel.

III. Impulse Detector and the Impulse Noise Removal Iterative Method

Many algorithms have been proposed to detect and replace the impulse noise corrupted pixels [9-15]. In the present work, a simple but efficient switching scheme, similar to that used in [9], but based on the GVWTM filter, is employed to detect the impulse noise and recover the noise-free pixels. The filtering output, $I_c^{'}$, is generated according to the following algorithm:

$$I_c^{'} = \begin{cases} I_c^{(i)}, & |I_c^{(i)} - F_c| < T \\ F_c, & |I_c^{(i)} - F_c| \le T \end{cases}, \tag{9}$$

where $I_c^{(i)}$ is the initial input value and F_c is the GVWTM filtering result of the initial input value. The threshold T is chosen to characterize the absolute difference between $I_c^{(i)}$ and F_c. If the difference is larger the threshold, it implies that the pixel differs significantly from its neighbors. It is therefore identified as an impulse noise corrupted pixel, and is then replaced by F_c. If the difference is smaller than the threshold, it implies that the initial input value is similar to its statistical neighbors, and we identify it as a noise-free pixel, and it therefore retains its original value.

Iteration of the above scheme will further improve the filtering performance, especially for images that are highly corrupted by impulse noise. The iteration procedure can be depicted as

$$I_c(t) = \begin{cases} I_c^{(i)}, & |I_c^{(i)} - F_c(t)| < T \\ F_c(t), & |I_c^{(i)} - F_c(t)| \ge T \end{cases}, \tag{10}$$

where I_c (t) is the system output at time t with $I_c(0) = I_c^{(i)}$, and F_c (t) is the GVWTM filtering output at time t, given by

$$F_c(t) = \frac{(\beta + \alpha w(x_{cm} \mid t-1))I_c + \sum_{j=m-L}^{m+L} w(x_{jm} \mid t-1) I_j(t-1)}{\beta + \alpha w(x_{cm} \mid t-1) + \sum_{j=m-L}^{m+L} w(x_{jm} \mid t-1)}, \tag{11}$$

Note that it is important for the iterative procedure always to compare $F_c(t)$ with the initial input $I_c^{(i)}$ and to update the output with $I_c^{(i)}$ when their absolute difference is less than the threshold T. This switching scheme has been employed before based on the VWTM filter [13]. In [12], Zhang and Wang proposed a median-based iterative scheme, which replaces $I_c^{(i)}$ and $F_c(t)$ with $I_c(t\text{-}1)$ and median filtering output at time t, respectively in (10). Test simulations show that this iterative scheme is less efficient than the present algorithm.

IV. Numerical Experiments

The standard 8-bit, gray-scale "Lena" image (size 512×512) is used as an example to test the usefulness of the new filtering technique. The performance of the GVWTM filter is compared against the standard median, α-TM, and MTM filters. For the GVWTM filter, $\beta = 0$ and weight function (6) with A=2 are used for all the simulations. All algorithms are implemented using a 3×3 window. A 3-entry median basket is used for all α-TM, MTM, and GVWTM filters. We first degraded the Lena image with additive Gaussian noise (Fig. 1). The peak signal-to-noise ratio (PSNR) of the corrupted image is 22.17 dB. Fig. 2 shows the PSNR of GVWTM filtered image as a function of iterative times with various α and Fig. 3 shows the optimal performing PSNR as a function of α.

(a) (b)

Fig. 1. (a) Original image; (b) Image corrupted with Gaussian noise (PSNR=22.17 dB)

As one can see from Fig. 2 and 3, GVWTM filtering with smaller α usually converges faster with iterations but yields a less optimal PSNR, and the PSNR performance tends to degrade faster after optimally performing the iteration. In contrast, GVWTM filtering with larger α usually converges slower with iterations but yields more optimal PSNR and the

PSNR performance tends to be stabilized for more iterations after the optimal performing one. This example shows the importance of involving the center pixel in the filtering formalism.

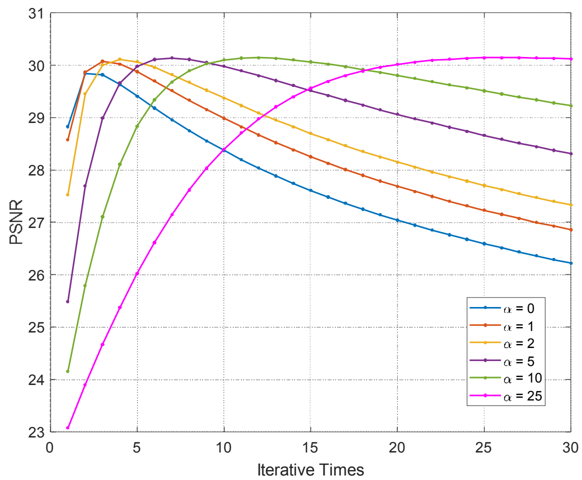

Fig. 2. GVWTM Filtering PSNR as a function of the iterative times with various α

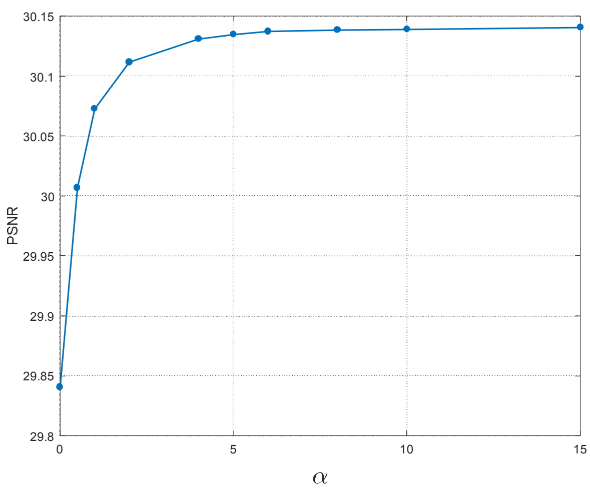

Fig. 3. GVWTM Filtering optimal PSNR as a function of the α

Next, the performance of the GVWTM filter on the images corrupted with 2 different amounts of additive of Gaussian noise is compared with other methods and the result is shown in Table I. For the GVWTM filter, α is set to 20 for noisy image 1, and 40 for noisy image 2. Clearly, the performance of the GVWTM filter is better than other filters. The greater the noise, the larger the α is needed for better performance.

Finally, the performance of the GVWTM filter is compared with other methods when impulse noise is present. The results are shown in Table II with the Lena images corrupted with 6 different amounts of Impulse noise (salt and pepper noise). For GVWTM filter without impulse detection and switching, α is set to 1 for the first 3 lightly corrupted images, and 0.2 for last 3 heavily corrupted images. For GVWTM filter with impulse detection and switching, α is set to 0.2 for the first 3 lightly corrupted images, and 0 for the last 3 heavily corrupted images. From Table II, one can see that reasonable center pixel weights can improve the filtering performance of lightly impulse corrupted images. The greater the noise, the smaller the α is needed for better performance. This is easy to understand because impulse noise corrupted pixels are usually not correlated with their original pixels. Another thing one can see from Table II is that the GVWTM filter performs better than other methods no matter whether impulse detection and switching scheme is used or not. Fig. 4 shows the image corrupted with 40% impulse noise and the GVWTM filtered image.

TABLE I. COMPARATIVE FILTERING RESULTS FOR LENA IMAGE CORRUPTED WITH DIFFERENT AMOUNT OF GAUSSIAN NOISE

Algorithm	**Noisy Image 1**			**Noisy Image 2**		
	PSNR	***MSE***	***MAE***	***PSNR***	***MSE***	***MAE***
No Denoising	22.17	394.4	15.97	18.82	853.1	23.71
Median	29.38	75.09	6.26	27.60	113.0	7.72
α-TMF	29.84	67.46	6.07	28.13	100.1	7.04
MTMF	29.91	66.46	5.79	28.23	97.74	7.39
GVWTMF	30.14	62.95	5.83	28.37	94.59	7.11

*All results are implemented recursively for optimal PSNR performance. The threshold used for the MTMF algorithm is optimized to q=65 for image 1 and q = 100 for image 2. The PSNR unit is in dB.

TABLE II. COMPARITIVE FILTERING PERFORMANCE IN PSNR FOR LENA IMAGE CORRUPTED WITH VARIOUS IMPULSE NOISE LEVELS

Algorithm	**Impulse Noise Amount**					
	15%	***20%***	***25%***	***30%***	***35%***	***40%***
No Denoising	13.53	12.28	11.34	10.55	9.87	9.29
Median	32.24	31.29	30.77	29.90	29.40	28.65
α-TM	32.16	31.07	30.30	29.38	28.39	27.50
GVWTM	32.89	31.70	31.03	30.19	29.62	28.88
Median*	29.97	29.55	29.24	28.74	28.36	28.07
α-TM*	36.22	34.54	33.67	32.65	31.72	30.61
GVWTM*	36.47	34.83	33.98	33.13	32.34	31.37

*Switching scheme. The switching scheme used for median and α-TM filters are the one described in this paper. All methods are implemented recursively for optimal PSNR performance. The PSNR unit is in dB.

Fig. 4. Image restoration from 40% impulse noise corrupted image. (a) Noisy Image, PSNR=9.29 dB; (b) GVWTM-switch filtering result, PSNR=31.37 dB

V. CONCLUSIONS

In this paper, we present a new nonlinear filtering algorithm that can be used to suppress both additive and impulse noise corrupted images effectively. The GTM filter introduces the center pixel to the weighted and averaged filtering formalism, and the GVWTM filter makes the weight of the center pixel vary based on its luminance value compared with that of the median pixel. For images corrupted with additive noise, larger center pixel weights with more iterations can usually yield better filtering performance. For images corrupted with impulse noise, a small center pixel weight can also give improved performance. Simulations show that the GVWTM filter is robust and efficient, and its performance compares favorably with many other nonlinear filtering techniques.

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