1 Luminosity

The intensity of collisions in a particle physics experiment is quantified through the *luminosity*. In a colliding beams experiment this is given by

$$\mathcal{L} = \frac{n_b}{2\pi}\frac{f_{\mathrm{rev}}N_1N_2}{\sigma_x\sigma_y} \tag{1}$$

when a total of n_b bunches are circulating in the collider with N_1 particles per bunch in bunches circulating in one direction and N_2 particles per bunch circulating in the opposite direction. f_{rev} is the circulation frequency of the bunches. σ_x and σ_y is the width (usually assumed to be Gaussian) of each bunch in the two directions transverse to the velocity when the bunches are in the collision region[1].

We can se from equation (1) that luminosity has the units of 1/(area×time). The conventional units to use are $\mathrm{cm}^{-2}\mathrm{s}^{-1}$. For instance, the peak luminsoity obtained by LHC in the ATLAS in 2018 was

$$\mathcal{L}_{\mathrm{peak}} = 21.0 \times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}.$$

This way of expressing the collision intensity makes it easy to calculate event rates. If the collision cross section is σ, then the event rate is

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \sigma\mathcal{L}. \tag{2}$$

This expression holds independent on whether σ is the total collision cross section or the cross section for a specific process. Since cross section is usually expressed in units of barns ($1\ \mathrm{b} = 10^{-28}\ \mathrm{m}^2$) it may be useful to express the luminosity in units of $\mathrm{b}^{-1}\mathrm{s}^{-1}$ using

$$10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} = 10^{10}\ \mathrm{b}^{-1}\mathrm{s}^{-1} = 10\ \mathrm{nb}^{-1}\mathrm{s}^{-1}.$$

1.1 Integrated luminosity

The amount of data collected over a certain time is quantified through the integrated luminosity,

$$\int \mathcal{L}\mathrm{d}t. \tag{3}$$

The integrated luminosity is given in units of inverse barn. With the amount of data collected by ATLAS, one usually have a number of fb^{-1}. Using equation (2) it is easy to calculate the expected number of events of a certain type given an integrated luminosity as long as one knows the relevant cross section.

[1] Bunches are generally squeezed to have a much smaller transverse size in the collision region than in the rest of the machine.

2 Cross section

The interaction probability in particle collisions is quantified[2] in terms of the cross section, which is measured in units of barns (1 b = 10^{-28} m^2). The cross section can be visualized as the physical cross section of the colliding particles—thus a larger cross section makes a collision/interaction more probable. However, due to the way quantum mechanics work this interpretation should only be thought of as an analogy.

With knowledge of the forces (strong, weak, electromagnetic) which mediates the interaction one can (but this is usually hard) calculate the cross section. There are tools available to make the calculation for many processes both in the Standard Model and beyond. When samples are generated for use in ATLAS analyses the cross section is calculated and made available to the analysers.

2.1 Total and partial cross sections

The total cross section (σ_{tot}) quantifies the probability for a collision to take place at all. In the case of LHC, that means that the total cross section describes

$$p + p \to \textit{anything}$$

and is of the order of 10^8 nb. Using the total cross section and the luminosity one can calculate the collision rate, i.e. number of collisions per second. Se below for details.

A partial cross section quantifies the probability for a collision to happen and for ending up in a specific final state, for instance a Higgs boson to be produced:

$$p + p \to H + X$$

Here X means anything that can be produced together with the Higgs boson. In the case of partial cross sections, the final state may be completely specified (called exclusive) or only partially specified (called inclusive) as in the example of Higgs boson production.

Partial cross sections of processes studied or searched for at LHC may often be as small as 10^{-3} nb or even lower. The many orders of magnitudes difference between total cross section and partial cross section of the process we study/search for explains the the need for both effective triggers and data analysis techniques.

[2]Note that the cross section *is not probability* which is clear from the unit, but the quantity is proportional to the probability that an interaction will take place

2.2 Event rates

The event rate in a collision experiment is calculated from the luminosity and cross section as

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \sigma\mathcal{L}$$

where σ may be either the total cross section of a partial cross section depending on which event rate we are interested in. Similarly, we can calculate the total number of events in a sample if we know the cross section and the integrated luminosity:

$$N = \sigma \int \mathcal{L}\mathrm{d}t.$$

Again this calculation may be done either with total cross section or partial cross section, depending on what we are interested in.

3 Reconstruction of decay processes

Most of the particles which are produced in a high energy particle physics collision are too short lived to reach the detector before decaying. Indeed, only a small number of particle species actually ever transverse the detector and can thus be detected[3]:

- electron
- muon
- photon
- pions (charged and neutral)
- kaons (charged and neutral)
- proton
- neutron
- neutrinoes (three species, undetectable)

All other particle species must be deduced from their decay products. For instance, we may have a Z-boson produced decaying to a muon–anti-muon pair:

$$Z \to \mu^+\mu^-.$$

[3]For each item in the list, there is also an anti-particle which also can reach the detector

In this case we will detect two muons coming from the interaction vertex (since the lifetime of the Z-boson is so short that it will decay before moving an appreciable distance). To infer that this muon–anti-muon pair actually comes from a Z-boson and not from some other process(es) we must use conservation of energy and momentum to calculate the mass of the object they came from. If this mass coincides with that of the Z-boson we can be almost certain that they come from such a decay. The mass of the decaying particle calculated this way is usually referred to as the invariant mass[4]

$$M_{\text{inv}}^2 = 2p_{T1}p_{T2}\left[\cosh(\eta_1 - \eta_2) - \cos(\phi_1 - \phi_2)\right]$$

Here p_{T1} and p_{T2} are the transverse momenta of the two particles and η and ϕ are the angles.[5] The above formula is an approximation which assumes that the masss of the detected particles are much smaller than the mass of the decaying particle. This is usually the case, so we rarely need the exact formula. In practice, the invariant mass will not be exactly equal to the mass of the decaying particle due to detector effects, but when measuring a large number of such decay we will get a distribution which is peaked at the mass of the decaying particle.

In the case of a W-boson decaying to leptons one of the decay products is a neutrino which is not detected, for instance

$$W^- \to e^- \nu_e.$$

Since we don't know the energy and momentum of the neutrino we cannot calculate the invariant mass. But since a neutrino carries energy and momentum we can infer its presence from an imbalance in the measured momentum in the event. Furthermore, in proton–proton collisions we don't know the momentum component along the beam axis before the collision. Thus we can only utilize conservation of momentum in the plane transverse to this direction. This means that in place of the invariant mass, we must use the best substitute we can get—namely the transverse mass:

$$M_{\text{T}}^2 = 2E_{T1}E_T^{\text{miss}}(1 - \cos\theta)$$

where E_T^{miss} is the magnitude of missing momentum in the transverse plane. When we measure the transverse momentum of a large number of particles from such a decay we get a distribution which is peaked below the W mass, but has an end point at that mass. Notice that transverse mass is only applicable in events where the neutrino is the only source to momentum imbalance.

[4]The calculation utilises the fact that the mass is independent on which frame of reference is used to measure the energies and momenta, thus *invariant* mass.

[5]Remember that η is strictly speaking not an angle, but derived from an angle as $\eta = -\ln(\tan(\theta/2))$.