combine like terms:

Model:

for store i

$$Y_i = X_i\beta_i + e_i \qquad e_i \sim N(0, \sigma_i^2)$$

X_i is an $n_i \times 4$ matrix where n_i is the number of observations from store i.
The first column is identically one to fit an intercept. The second is log(Price). The third is disp. The fourth is log(Price) x disp

Priors

$$\beta_i \sim N(\mu, \tau^{-1}) \qquad \sigma_i^2 \sim \frac{1}{\sigma_i^2} \text{ (Jeffrey's Prior)}$$

conditionals

$$P(\beta_i \mid Y_i, \sigma_i^2) \propto P(Y_i \mid \beta_i, \sigma_i^2)\, P(\beta_i \mid \sigma_i^2)$$

$$\propto \exp\left[\frac{-1}{2}\left([Y_i - X_i\beta_i]^T\left[\frac{1}{\sigma_i^2}I\right][Y_i - X_i\beta_i] + [\beta_i - \mu]^T[\tau][\beta_i - \mu]\right)\right]$$

$$= \exp\left[\frac{-1}{2}\left(Y_i^T\left[\frac{1}{\sigma_i^2}I\right]Y_i - 2\beta_i^T X_i^T\left[\frac{1}{\sigma_i^2}I\right]Y_i + \beta_i^T X_i^T\left[\frac{1}{\sigma_i^2}I\right]X_i\beta_i + \beta_i^T\tau\beta_i - 2\mu^T\tau\beta_i + \cancel{\mu^T\tau\mu}\right)\right].$$

can drop anything without β_i due to proportionality combining like terms:

$$\propto \exp\left[-\frac{1}{2}\left(\beta_i^T\left[X_i^T\left[\frac{1}{\sigma_i^2}I\right]X_i+\tau\right]\beta_i\right.\right.$$
$$\left.\left.-2\beta_i^T\left(X_i^T\left[\frac{1}{\sigma_i^2}I\right]Y_i+\tau\mu\right)\right)\right]$$

completing the square:

want form $[\beta_i - M_i^*]^T[K_i^*][\beta_i - M_i^*]$

$$=\beta_i^T K_i^* \beta_i - 2\beta_i^T K_i^* M_i^* + \underbrace{M_i^{T*} K_i^* M_i^*}_{I}$$

but term I is irrelevant due to proportionality

by inspection, $K_i^* = X_i^T\left[\frac{1}{\sigma_i^2}I_i\right]X_i + \tau$

$$K_i^* M_i^* = \left(X_i^T\left[\frac{1}{\sigma_i^2}I\right]Y_i + \tau\mu\right)$$

so $M_i^* = K_i^{*-1}\left(X_i^T\left[\frac{1}{\sigma_i^2}I\right]Y_i + \tau\mu\right)$

Therefore $\beta_i | Y_i, \sigma_i^2 \sim N\left(M_i^*, K_i^{*-1}\right)$

$$P(\sigma_i^2 | Y_i, \beta_i) \propto P(Y_i | \sigma_i^2, \beta_i)\, P(\sigma_i^2 | \beta_i)$$

$$\propto \exp\left[\frac{-1}{2}\left([Y_i - X_i\beta_i]^T\left[\frac{1}{\sigma_i^2}I\right][Y_i - X_i\beta_i]\right)\right]\left(\frac{1}{\sigma_i^2}\right)\left(\frac{1}{\sigma_i^2}\right)^{n_i/2}$$

$$=\frac{1}{(\sigma_i^2)^{\frac{n_i}{2}+1}}\exp\left[\frac{-1}{2\sigma_i^2}[Y_i - X_i\beta_i]^T[Y_i - X_i\beta_i]\right]$$

This is inverse Gamma w/ $a = \frac{n_i}{2}$ and

$b = \frac{1}{2}[Y_i - X_i\beta_i]^T[Y_i - X_i\beta_i]$