PI Calculation by Taylor Series with Multi threads

What is the Pi number?

Pi (π) is the irrational mathematical constant obtained by dividing the circumference of a circle by its diameter. The decimal places continue to infinite without following a rule. It is possible for us to find any number that comes to mind anywhere in it. Various techniques have been developed to find this number. Taylor series is one of them. Now let's examine what is the Taylor series.

Taylor series and relationship with Pi

Taylor series produce approximations for many mathematical functions and constants. According to this series, as the limit increases, the approximation to the pi number increases, and a more accurate pi number is obtained. The Taylor series used for the pi number is formulated below.

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 4\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \cdots\right)$$

Solution strategy

We will use thread to shorten the calculation time while calculating the pi number.
The number of threads and the length of the series will be entered by the user. The number of operations refers to the total number of addition and subtraction operations. The number of operations per thread is obtained by dividing the total number of operations by the number of threads. If we have 10 operations and 2 threads, we have 5 operations per thread. If we have 11 operations and 2 threads, we have 5.5 operations per thread. In this case, we either need to use an extra thread or round the operation per thread to the base number. if we choose round to base, the last thread will make extra operations than before and we will do this scenario. For example, In a 2 threaded calculation with 11 elements, there are 5 operations in the first thread and 6 operations in the second thread. We should write our program according to this logic.

Example :

Operation count 10, thread count 2

1/1 – 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + 1/13 - 1/15 + 1/17 - 1/19

|_____| |_____|

Thread 1 Thread 2

$$4 * \left(\sum_{n=0}^{4}\frac{(-1)^n}{2n+1} + \sum_{n=5}^{10}\frac{(-1)^n}{2n+1} \right) \approx \pi$$

Tests 1 : Operation count is 1.000.000, thread count increasing 1 to 64.

C:\Users\Lenovo\Desktop\pi_by_taylor_2.exe

```
Give the operation count : 1000000
Give the thread count : 64
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.07139500 second       thread count :1
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.04189200 second       thread count :2
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03337100 second       thread count :3
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02692700 second       thread count :4
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03690200 second       thread count :5
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091600 second       thread count :6
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892300 second       thread count :7
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02692600 second       thread count :8
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191400 second       thread count :9
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191400 second       thread count :10
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992100 second       thread count :11
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091700 second       thread count :12
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091800 second       thread count :13
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892200 second       thread count :14
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892200 second       thread count :15
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02792500 second       thread count :16
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991800 second       thread count :17
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892300 second       thread count :18
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992100 second       thread count :19
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892400 second       thread count :20
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991800 second       thread count :21
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091900 second       thread count :22
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991900 second       thread count :23
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02792400 second       thread count :24
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :25
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991900 second       thread count :26
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992100 second       thread count :27
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892200 second       thread count :28
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191500 second       thread count :29
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :30
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091700 second       thread count :31
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :32
```

```
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03091700 second       thread count :33
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992100 second       thread count :34
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02892200 second       thread count :35
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :36
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :37
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :38
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992100 second       thread count :39
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991900 second       thread count :40
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :41
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991900 second       thread count :42
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :43
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191400 second       thread count :44
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :45
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :46
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02992000 second       thread count :47
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.02991900 second       thread count :48
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191500 second       thread count :49
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191400 second       thread count :50
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191500 second       thread count :51
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191400 second       thread count :52
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191500 second       thread count :53
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191800 second       thread count :54
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03689700 second       thread count :55
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03291300 second       thread count :56
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191500 second       thread count :57
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03291200 second       thread count :58
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191800 second       thread count :59
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03291100 second       thread count :60
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03191200 second       thread count :61
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03391000 second       thread count :62
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03291300 second       thread count :63
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000       computation time:0.03390800 second       thread count :64
```

Test results: Approximation error is constant for every thread because operation count does not change and computation time is decreasing when the thread count is increasing. The computation time rate is increasing fastly at the first 20 threads then increasing slowly.

Test 2: Operation count is starting from 1, increasing by 10 to 1.000.000, thread count is 10

C:\Users\Lenovo\Desktop\pi_by_taylor-3.exe

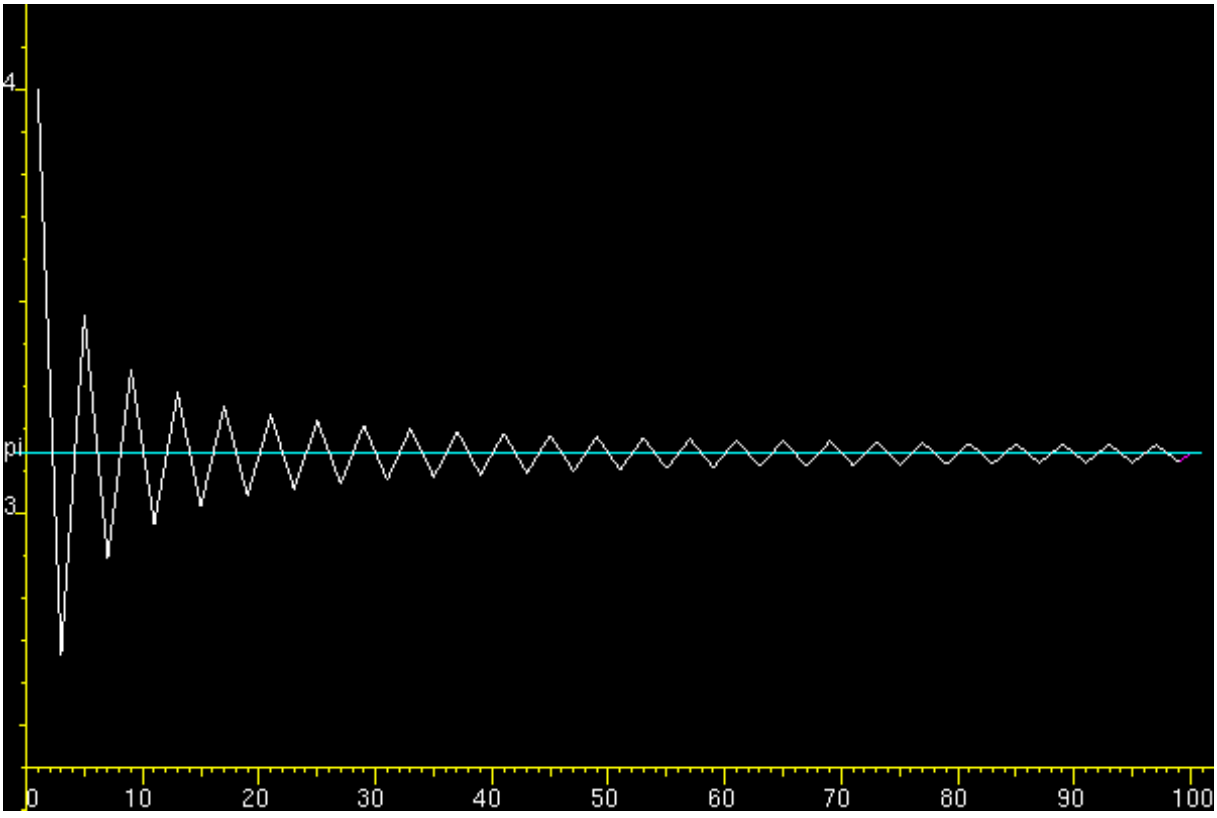
```
Give the operation count : 1000000000
Give the thread count : 10
multithreaded pi =4.0000000000   real pi =3.1415926536   approximation error=-0.8584073464       computation time:0.0019240000 second     operation  count :1    thread count:10
multithreaded pi =3.0418396189   real pi =3.1415926536   approximation error=0.0997530347        computation time:0.0000000000 second     operation  count :10   thread count:10
multithreaded pi =3.1315929036   real pi =3.1415926536   approximation error=0.0099997500        computation time:0.0009970000 second     operation  count :100  thread count:10
multithreaded pi =3.1405926538   real pi =3.1415926536   approximation error=0.0009999997        computation time:0.0000000000 second     operation  count :1000         thread count:10
multithreaded pi =3.1414926536   real pi =3.1415926536   approximation error=0.0001000000        computation time:0.0011130000 second     operation  count :10000        thread count:10
multithreaded pi =3.1415826536   real pi =3.1415926536   approximation error=0.0000100000        computation time:0.0040310000 second     operation  count :100000       thread count:10
multithreaded pi =3.1415916536   real pi =3.1415926536   approximation error=0.0000010000        computation time:0.0321980000 second     operation  count :1000000      thread count:10
multithreaded pi =3.1415925536   real pi =3.1415926536   approximation error=0.0000001000        computation time:0.2898150000 second     operation  count :10000000     thread count:10
multithreaded pi =3.1415926436   real pi =3.1415926536   approximation error=0.0000000100        computation time:3.0229140000 second     operation  count :100000000    thread count:10
multithreaded pi =3.1415926526   real pi =3.1415926536   approximation error=0.0000000010        computation time:35.9447850000 second    operation  count :1000000000   thread count:10
```

Test results: At every operation, approximation error is decreasing, we get a more accurate pi number and computation time is increasing.

To see approximation I set the thread count 1 increasing by operation count.

```
Seç C:\Users\Lenovo\Desktop\pi_by_taylor-3.exe
Give the operation count : 1000000
Give the thread count : 1
multithreaded pi =4.0000000000   real pi =3.1415926536   approximation error=-0.8584073464       computation time:0.0010 second   operation  count :1    thread count:1
multithreaded pi =2.6666666667   real pi =3.1415926536   approximation error=0.4749259869        computation time:0.0003 second   operation  count :2    thread count:1
multithreaded pi =3.4666666667   real pi =3.1415926536   approximation error=-0.3250740131       computation time:0.0003 second   operation  count :3    thread count:1
multithreaded pi =2.8952380952   real pi =3.1415926536   approximation error=0.2463545584        computation time:0.0003 second   operation  count :4    thread count:1
multithreaded pi =3.3396825397   real pi =3.1415926536   approximation error=-0.1980898861       computation time:0.0000 second   operation  count :5    thread count:1
multithreaded pi =2.9760461760   real pi =3.1415926536   approximation error=0.1655464775        computation time:0.0000 second   operation  count :6    thread count:1
multithreaded pi =3.2837384837   real pi =3.1415926536   approximation error=-0.1421458301       computation time:0.0000 second   operation  count :7    thread count:1
multithreaded pi =3.0170718171   real pi =3.1415926536   approximation error=0.1245208365        computation time:0.0000 second   operation  count :8    thread count:1
multithreaded pi =3.2523659347   real pi =3.1415926536   approximation error=-0.1107732811       computation time:0.0010 second   operation  count :9    thread count:1
multithreaded pi =3.0418396189   real pi =3.1415926536   approximation error=0.0997530347        computation time:0.0000 second   operation  count :10   thread count:1
multithreaded pi =3.2323158094   real pi =3.1415926536   approximation error=-0.0907231558       computation time:0.0000 second   operation  count :11   thread count:1
multithreaded pi =3.0584027659   real pi =3.1415926536   approximation error=0.0831898877        computation time:0.0000 second   operation  count :12   thread count:1
multithreaded pi =3.2184027659   real pi =3.1415926536   approximation error=-0.0768101123       computation time:0.0000 second   operation  count :13   thread count:1
multithreaded pi =3.0702546178   real pi =3.1415926536   approximation error=0.0713380358        computation time:0.0000 second   operation  count :14   thread count:1
multithreaded pi =3.2081856523   real pi =3.1415926536   approximation error=-0.0665929987       computation time:0.0010 second   operation  count :15   thread count:1
multithreaded pi =3.0791533942   real pi =3.1415926536   approximation error=0.0624392594        computation time:0.0000 second   operation  count :16   thread count:1
multithreaded pi =3.2003655154   real pi =3.1415926536   approximation error=-0.0587728618       computation time:0.0000 second   operation  count :17   thread count:1
multithreaded pi =3.0860798011   real pi =3.1415926536   approximation error=0.0555128525        computation time:0.0000 second   operation  count :18   thread count:1
multithreaded pi =3.1941879092   real pi =3.1415926536   approximation error=-0.0525952556       computation time:0.0000 second   operation  count :19   thread count:1
multithreaded pi =3.0916238067   real pi =3.1415926536   approximation error=0.0499688469        computation time:0.0000 second   operation  count :20   thread count:1
multithreaded pi =3.1891847823   real pi =3.1415926536   approximation error=-0.0475921287       computation time:0.0000 second   operation  count :21   thread count:1
multithreaded pi =3.0961615265   real pi =3.1415926536   approximation error=0.0454311271        computation time:0.0000 second   operation  count :22   thread count:1
multithreaded pi =3.1850504154   real pi =3.1415926536   approximation error=-0.0434577618       computation time:0.0000 second   operation  count :23   thread count:1
multithreaded pi =3.0999440324   real pi =3.1415926536   approximation error=0.0416486212        computation time:0.0000 second   operation  count :24   thread count:1
multithreaded pi =3.1815766854   real pi =3.1415926536   approximation error=-0.0399840318       computation time:0.0000 second   operation  count :25   thread count:1
multithreaded pi =3.1031453129   real pi =3.1415926536   approximation error=0.0384473407        computation time:0.0000 second   operation  count :26   thread count:1
multithreaded pi =3.1786170110   real pi =3.1415926536   approximation error=-0.0370243574       computation time:0.0000 second   operation  count :27   thread count:1
multithreaded pi =3.1058897383   real pi =3.1415926536   approximation error=0.0357029153        computation time:0.0000 second   operation  count :28   thread count:1
multithreaded pi =3.1760651769   real pi =3.1415926536   approximation error=-0.0344725233       computation time:0.0000 second   operation  count :29   thread count:1
multithreaded pi =3.1082685667   real pi =3.1415926536   approximation error=0.0333240869        computation time:0.0000 second   operation  count :30   thread count:1
multithreaded pi =3.1738423372   real pi =3.1415926536   approximation error=-0.0322496836       computation time:0.0000 second   operation  count :31   thread count:1
multithreaded pi =3.1103502737   real pi =3.1415926536   approximation error=0.0312423799        computation time:0.0000 second   operation  count :32   thread count:1
multithreaded pi =3.1718887352   real pi =3.1415926536   approximation error=-0.0302960816       computation time:0.0000 second   operation  count :33   thread count:1
multithreaded pi =3.1121872427   real pi =3.1415926536   approximation error=0.0294054109        computation time:0.0010 second   operation  count :34   thread count:1
multithreaded pi =3.1701582572   real pi =3.1415926536   approximation error=-0.0285656036       computation time:0.0000 second   operation  count :35   thread count:1
multithreaded pi =3.1138202290   real pi =3.1415926536   approximation error=0.0277724246        computation time:0.0000 second   operation  count :36   thread count:1
multithreaded pi =3.1686147496   real pi =3.1415926536   approximation error=-0.0270220960       computation time:0.0000 second   operation  count :37   thread count:1
multithreaded pi =3.1152814162   real pi =3.1415926536   approximation error=0.0263112374        computation time:0.0000 second   operation  count :38   thread count:1
multithreaded pi =3.1672294682   real pi =3.1415926536   approximation error=-0.0256368146       computation time:0.0000 second   operation  count :39   thread count:1
multithreaded pi =3.1165965568   real pi =3.1415926536   approximation error=0.0249960968        computation time:0.0010 second   operation  count :40   thread count:1
```

Approximation error plots below graphic, as we see more operation make better pi number.

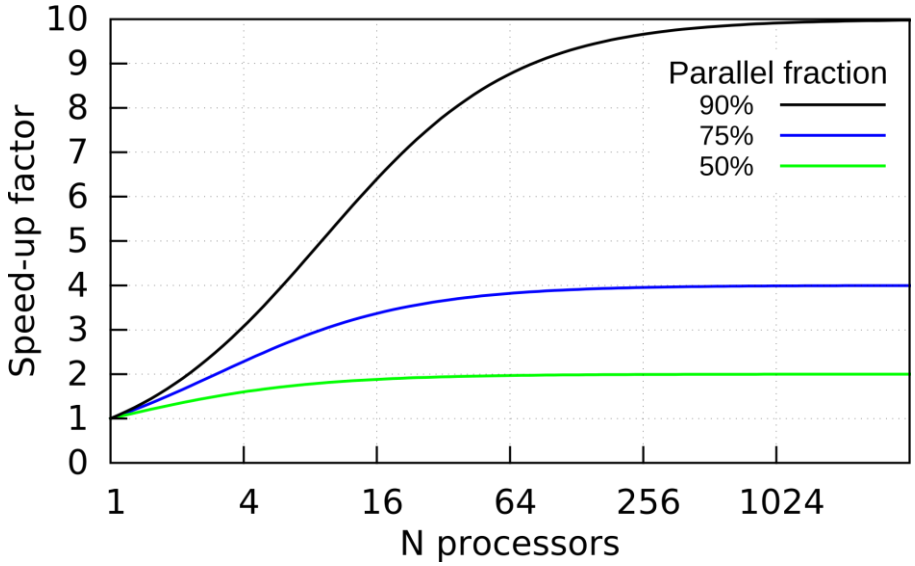


Conclusion

- According to the test results, the calculation time is shortened when the thread count increases from 1 to 64 for 1,000,000 operations.

- In addition, as the number of threads increases, the time between computation times decreases. 0.071 seconds with 1 thread and 0.041 seconds with 2 threads. While the computation time difference between 1 and 2 threads is about 2 times (1,73), this rate decreases when the number of threads increases. Speed is inceasig fastly in first 20 threads, then increasing slowly.

- If the number of threads constant and the operation counts are increased, the observed computation difference is more explicit. According to this, the use of threads in large amounts of operations greatly shortens the time. However, increasing the number of threads does not much affect after a certain number of times. This situation is related to Amdahl's law.



Hardware and software environments

Operating system: Windows 10
Editor: DEVC++
CPU: Intel i7-7500U
2.7 -2.9 GHz, 4 core
RAM:12 Gb DDR4
Harddisk:256 Gb SSD

Code

main.c

```c
#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <pthread.h>
#include <sys/time.h>

int op_count;
double range;
int NUM_THREADS;
clock_t start, end;
double cpu_time_used;
double pi_sum = 0;
double PI = 3.14159265358979323846264338327 9;

void *threadFunc(void *parg) {

    int thread_id = *((int*) parg);
    int end;
    int start;
    double part_sum = 0;

    start = (thread_id * (int) range);
    end = (thread_id + 1) * (int) range;

    if (thread_id + 1 == NUM_THREADS) {
        end = op_count;
    }

    int x;
    for (x = start; x < end; x++) {
        part_sum += 4 * (pow(-1, x) / (2 * x + 1));
    }

    pi_sum += part_sum;

    return 0;
}
```

```c
int main(void) {

    printf("Give the operation count : ");
    scanf("%d", &op_count);
    printf("Give the thread count \n: ");
    scanf("%d", &NUM_THREADS);
    int thread_range=NUM_THREADS;
    int q;
  //-------------------------------------------------------------------------
    for (q =1; q<=thread_range; ++q){

    NUM_THREADS = q;

    range = (double) op_count / (double) NUM_THREADS;

    if (range > floor(range)) {
        range = floor(range);
    }

    struct timeval begin, end;
    gettimeofday(&begin, 0);

  //-------------------------------------------------------------------------
    int i;
    pthread_t tid[NUM_THREADS];
    for (i = 0; i < NUM_THREADS; i++) {
        int *thread_id;
        thread_id = (int *) malloc(sizeof (int));
        *thread_id = i;
        pthread_create(&tid[i], NULL, threadFunc, (void *) thread_id);
    }
    for (i = 0; i < NUM_THREADS; i++) {
        pthread_join(tid[i], NULL);
    }
  //-------------------------------------------------------------------------

    gettimeofday(&end, 0);
    long seconds = end.tv_sec - begin.tv_sec;
    long microseconds = end.tv_usec - begin.tv_usec;
    double elapsed = seconds + microseconds*1e-6;

    printf("multithreaded pi =%.10f\treal pi =%.10f\tapproximation error=%.10f\t computation time:%.8f second\t  thread count :%d\t\n" ,
    pi_sum, PI, PI - pi_sum, elapsed,q);
    pi_sum=0;
    cpu_time_used =0;
  }
    return 0;
}
```

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