Lattice Study of Effective Field Theories for Quantum Hydrodynamics

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Motivation

- Dense matter, e.g., in nuclei and neutron star cores / crusts: high density, “low-T” environment; superfluid phases
- Early universe and heavy-ion collisions: high-T, with potentially superfluid phases
- First-principle calculations (nonzero-μ,-T QCD) still forthcoming: susceptibilities, emissivities, transport properties, entrainment, quantum turbulence(?)
- Explore viability of different effective theories for such systems: approach from “above” (astrophysical/experimental constraints) and from “below” (results from nonperturbative QCD & SM)

Quantum effective action (from "below")

consider a QFT where the ground state spontaneously breaks a ($U(1)$) symmetry...
partition function with order parameter Φ...

$$Z[J] = \int D\phi_i exp\left(iS + i\int d^4x\ J(x)\Phi(x)\right)$$

generator of connected graphs...

$$W[J] = -i \ln Z[J]$$

quantum effective action (sum of 1-particle irreducible graphs w/o external legs)...

$$\Gamma[\Phi] = W[J] - J\Phi \quad ; \quad \Phi = \frac{\delta W[J]}{\delta J} \quad ; \quad J = \frac{\delta \Gamma[\Phi]}{\delta \Phi}$$

(at tree level, all Green functions constructed from Φ)

$$\min_{\{\Phi\}}\Gamma[\Phi] = W[0] = -E_0 T$$

(D.T.Son, "Low-energy quantum effective action for relativistic superfluids," arXiv:hep-ph/0204199)

Quantum effective action (cont.)

consider the effective action, $\Gamma = \Gamma[\mu, \Phi]$, in the case...

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{3} A_\mu(x) \bar{q} \gamma^\mu q \quad ,$$

where later one sets $A_0 = \mu$, $A_i = 0$.
the effective action respects the same symmetries as the Lagrangian...

$$q \to q e^{i\alpha/3} \quad ; \quad A_\mu \to A_\mu + \partial_\mu \alpha \quad ; \quad \Gamma[A_\mu, \Phi] = \Gamma[A_\mu + \partial_\mu \alpha, \Phi e^{iM\alpha}]$$

in the case of spontaneous symmetry breaking, we integrate out (minimize w.r.t.) the amplitude $|\Phi|$...

$$\Gamma[A_\mu, \phi] = \Gamma[A_\mu + \partial_\mu \alpha, \phi + \alpha] = \min_{\{|\Phi|\}} \Gamma[A_\mu, \Phi e^{iM\phi}]$$

expanding in ϕ and its spatial derivatives (keeping only terms with $\partial^m \phi^n$, where $m \leq n$)...

$$\Gamma[A_\mu, \phi] \approx \int d^4x \ \mathcal{L}_{eff}(A_\mu, \partial_\mu \phi)$$

an expansion in $p\phi$ (valid when p is small compared to the gap energy)

(D.T.Son, arXiv:hep-ph/0204199)

Quantum effective action (cont.)

the "gauge" symmetry further restricts the action to...

$$\Gamma[A_\mu, \phi] = \int d^4x \; \mathcal{L}_{eff}(D_\mu \phi) \quad ; \quad D_\mu \phi = \partial_\mu \phi - A_\mu$$

after minimization w.r.t. ϕ...

$$\mathcal{L}_{eff}(-A_\mu) = -\epsilon_{gs}(A_\mu)$$

for $A_\mu = (\mu, 0)$, ϵ_{gs} is the ground-state energy density and $\epsilon_{gs} = -P(\mu)$ (the pressure)
we therefore have...

$$\mathcal{L}_{eff}(-A_\mu) = P\left((A_\mu A^\mu)^{1/2}\right)$$

more generally...

$$\Gamma[A_\mu, \phi] = \int d^4x \; P\left((D_\mu \phi D^\mu \phi)^{1/2}\right)$$

which at $A_\mu = (\mu, 0)$ gives...

$$\Gamma[\mu, \phi] = \int d^4x \; P\left(\sqrt{X}\right) \quad ; \quad X = (\partial_0 \phi - \mu)^2 - (\partial_i \phi)^2$$

(D.T.Son, arXiv:hep-ph/0204199)
so, given the EoS $P(\mu)$, we have our low-energy effective action $\int d^4x \; P(\sqrt{X})$.
– strictly speaking, the effective action should only be used at tree level –
so why quantize again?.. why not? let's see what happens...
besides, our EoS may come from "macroscopic" considerations...

Ideal quantum fluids (from "above")

symmetries of fluid's comoving coordinates (ϕ^I) and conserved charge (phase ψ)...

$$\begin{aligned} \phi^I &\to \phi^I + a^I \quad , \quad a^I = \text{const} \\ \phi^I &\to R^I_J \phi^J \quad , \quad R \in SO(3) \\ \phi^I &\to \xi^I(\phi^J) \quad , \quad \det \frac{\partial \xi^I}{\partial \phi^J} = 1 \quad \text{(transverse deformations)} \\ \psi &\to \psi + a \quad , \quad a = \text{const} \end{aligned}$$

also, Poincaré invariance: construct $\mathcal{L}$ from scalar invariants.
as a consequence of these symmetries, we have the following combinations of $\partial_\mu \phi^I$ and $\partial_\mu \psi$ which can enter $\mathcal{L}$...

$$J^\mu \equiv \frac{1}{6} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{IJK} \partial_\alpha \phi^I \partial_\beta \phi^J \partial_\gamma \phi^K$$

$$b \equiv \sqrt{-J_\mu J^\mu} = \sqrt{\det \partial_\mu \phi^I \partial^\mu \phi^J} \quad ; \quad u^\mu = \frac{1}{b} J^\mu \quad \text{(note: } u^\mu \partial_\mu \phi^I = 0\text{)}$$

$$X \equiv \partial_\mu \psi \partial^\mu \psi \quad ; \quad y \equiv u^\mu \partial_\mu \psi$$

the Langrangian should be a generic function of the scalar invariants: $\mathcal{L} = F(b, X, y)$.

[S.Dubovsky et al. JHEP 0603 (2006) 025 ; S.Endlich et al. JHEP 1104 (2011) 102 ; S.Dubovsky et al. Phys.Rev. D85 (2012) 085029 ; A.Nicolis, arXiv:1108.2513]

Hydro- / thermodynamic matching

consider a small variation of the metric, $g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu}$, then...

$$\delta b = \frac{1}{2} b\, B^{-1}_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J \,\delta g^{\mu\nu} \quad ; \quad B_{IJ} \equiv \partial_\mu \phi^I \partial^\mu \phi^J \quad \text{(note: } b^2 = \det B_{IJ}\text{)}$$

similarly for the other invariants...

$$\delta X = \partial_\mu \psi \partial_\nu \psi \,\delta g^{\mu\nu} \quad ; \quad \delta y = \frac{1}{2} y (\eta_{\mu\nu} - B^{-1}_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J)\, \delta g^{\mu\nu}$$

putting these into the stress-energy tensor ($F_b = \partial F / \partial b$, etc.)...

$$\begin{aligned} T_{\mu\nu} = -2 \frac{\delta S}{\delta g^{\mu\nu}} &= (F_y y - F_b b) B^{-1}_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J + (F - F_y y)\eta_{\mu\nu} - 2F_X \partial_\mu \psi \partial_\nu \psi \\ &= (F_y y - F_b b) u_\mu u_\nu + (F - F_b b)\eta_{\mu\nu} - 2F_X \partial_\mu \psi \partial_\nu \psi \end{aligned}$$

since the flow tensor is given by $B^{-1}_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J = \eta_{\mu\nu} + u_\mu u_\nu$.
the Noether current for the phase symmetry...

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = F_y u^\mu + 2F_X \partial^\mu \psi$$

[S.Dubovsky et al. Phys.Rev. D85 (2012) 085029 ; A.Nicolis, arXiv:1108.2513]

Hydro- / thermodynamic matching (cont.)

frame choice: move with the “normal” fluid, along u^μ ; then, the mass-energy density and the number density follow from...

$$\rho \equiv T^{\mu\nu} u_\mu u_\nu = F_y y - F - 2F_X y^2$$
$$n \equiv -j^\mu u_\mu = F_y - 2F_X y$$

identifying the pressure with the isotropic tensor term in $T_{\mu\nu}$, one has...

$$p = F - F_b b$$

imposing the thermodynamic identity...

$$\rho + p = Ts + \mu n$$
$$(F_y - 2F_X y)y - F_b b = Ts + \mu(F_y - 2F_X y)$$

one arrives at the relations...

$$\mu = y \quad ; \quad s = b \quad ; \quad T = -F_b$$

for normal / superfluid relative motion...

$$\partial_\mu \psi = -u_\mu y + \xi_\mu \quad ; \quad \xi^\mu = (\eta^{\mu\nu} + u^\mu u^\nu)\partial_\nu \psi \quad ; \quad X = -y^2 + \xi^2$$

finally, the Lagrangian...

$$\mathcal{L} = F(b, X, y) = p + F_b b \qquad = \qquad p - T\frac{\partial p}{\partial T} = \bar{F}(s, \xi^2, \mu)$$

[S.Dubovsky et al. Phys.Rev. D85 (2012) 085029 ; A.Nicolis, arXiv:1108.2513]

Lattice implementation

so now we would like to consider a quantized version of this (effective) theory...
recall that $B_{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ and $u^\mu = \frac{1}{b}\epsilon^{\mu\alpha\beta\gamma}\partial_\alpha\phi^1\partial_\beta\phi^2\partial_\gamma\phi^3$ and ...

$$\mathcal{L} = F(b, X, y) \quad ; \quad b = \sqrt{\det B_{IJ}} \quad ; \quad X = \partial_\mu\psi\partial^\mu\psi \quad ; \quad y = u^\mu\partial_\mu\psi \quad .$$

to avoid problems with periodic boundaries use “shifted” fields (“subtract” hydrostatic background)...

$$\pi^0 = \psi - \mu t \quad ; \quad \pi^I = \phi^I - x^I \quad \rightarrow \quad \partial_\alpha\psi = \partial_\alpha\pi^0 + \mu\delta^0_\alpha \quad ; \quad \partial_\alpha\phi^I = \partial_\alpha\pi^I + 1\delta^I_\alpha$$

fields occupy lattice sites; use centered differences for derivatives (no problem for scalars; alternatively, one could always use a “dual” BCC lattice, with field derivs defined at body centers: cf. Sadooghi & Rothe, hep-lat/9610001).
for HMC updates, one needs the variation of the action w.r.t. the local field values...

$$\begin{aligned}\frac{\delta S}{\delta\phi^I(x)} = \frac{\delta S}{\delta(\partial_\alpha\phi^J)}\frac{\delta(\partial_\alpha\phi^J)}{\delta\phi^I(x)} &= \left(\frac{\delta S}{\delta b}\frac{\delta b}{\delta(\partial_\alpha\phi^J)} + \frac{\delta S}{\delta y}\frac{\delta y}{\delta(\partial_\alpha\phi^J)}\right)\frac{\delta(\partial_\alpha\phi^J)}{\delta\phi^I(x)} \\ &= \frac{1}{2}\delta^{IJ}(F_b b - F_y y)B^{-1}_{JK}\partial_\alpha\phi^K\Big|^{x-\hat\alpha}_{x+\hat\alpha}\end{aligned}$$

$$\begin{aligned}\frac{\delta S}{\delta\psi(x)} = \frac{\delta S}{\delta(\partial_\alpha\psi)}\frac{\delta(\partial_\alpha\psi)}{\delta\psi(x)} &= \left(\frac{\delta S}{\delta X}\frac{\delta X}{\delta(\partial_\alpha\psi)} + \frac{\delta S}{\delta y}\frac{\delta y}{\delta(\partial_\alpha\psi)}\right)\frac{\delta(\partial_\alpha\psi)}{\delta\psi(x)} \\ &= \frac{1}{2}(2F_X\partial_\alpha\psi + F_y u_\alpha)\Big|^{x-\hat\alpha}_{x+\hat\alpha}\end{aligned}$$

Possible systems to consider

superfluid, relativistic, degenerate fermions [e.g., CFL quark matter: Alford, Rajagopal, & Wilczek, NPB 537 (1999) 443; D.T.Son, hep-ph/0204199]:

$$p \propto \mu^4 \quad \to \quad F(X) \propto X^2$$

relativistic, ideal gas [post heavy-ion collision fireball? G.Torrieri, PRD 85 (2012) 065006]:

$$p \propto s^{4/3} \quad \to \quad F(b) \propto b^{4/3}$$

nonzero-T, relativistic, degenerate fermions:

$$p = A_1 \mu^4 + A_2 \mu^2 T^2 \quad \to \quad F(b, y) = C_1 y^4 - C_2 (b/y)^2$$

nonzero-T, relativistic superfluid [Carter & Langlois, PRD 51 (1995) 5855]:

$$F(b, X, y) = F_0(X) - 3 \left[\frac{b^4}{c_1} \left(1 + (1 - c_1^2) \frac{y^2}{X} \right)^2 \right]^{1/3}$$

First questions

- Lattice scale: dimensionful parameter, $\mathcal{M}$ – e.g., from T_0 in $B_{IJ} = T_0^2 \partial_\mu \phi^I \partial^\mu \phi^J$ (where then $T_0^3 \sim$ entropy density) – is compared with the lattice spacing a. The continuum is approached as $a\mathcal{M} \to 0$ (ideally while keeping a ratio of dimensionful observables "physically" fixed).

- Study the "vacuum": decrease $a\mathcal{M}$ and observe the behavior of $\langle \mathcal{O} \rangle$ (normalized by an appropriate power of $\mathcal{M}$ or T_0):
  – if $\lim_{a\to 0} \langle \mathcal{O} \rangle \to \langle \mathcal{O} \rangle_0$, then $\langle \mathcal{O} \rangle$ is stable
  – if $\lim_{a\to 0} \langle \mathcal{O} \rangle / \langle \mathcal{O} \rangle_0 \sim f(b)$, the vacuum is non-trivial, but "well-behaved"
  – if $\lim_{a\to 0} \langle \mathcal{O} \rangle / \langle \mathcal{O} \rangle_0 \sim a^{-\alpha}$ or $\sim e^{\alpha/a}$ for a universal value of α, the theory is renormalizable (a is needed for an overall scale, but dimensionless ratios are independent of it)
  – if $\lim_{a\to 0} \langle \mathcal{O} \rangle / \langle \mathcal{O} \rangle_0 \sim a^{-\alpha}$ or $\sim e^{\alpha/a}$ for α's that are $\langle \mathcal{O} \rangle$-specific, the theory is trivial

Observables

– scalars and their correlations: e.g., $\langle s \rangle$, $\langle s(x)s(x') \rangle$, $\langle \rho \rangle$, $\langle \rho(x)\rho(x') \rangle$, $\langle p \rangle$, $\langle p(x)p(x') \rangle$, etc.
– "scalar perturbation":

$$\mathcal{S} = T_0^4 \frac{\langle F_b b \rangle}{\langle T^\mu_\mu \rangle} ,$$

the backreaction on the EoS of an ensemble of quantum sound-waves.

- note that in the hydrostatic limit...

$$T^\mu_\mu = \rho - 2p = -3F + 2F_b b$$

the divergence of this as $a \to 0$ could signal the triviality of the vacuum.

– a closer look at the temperature and entropy:

$$s_{micro} = gb \quad \to \quad \langle T \rangle = -\langle F_b \rangle / g \quad \leftrightarrow \quad \langle T \rangle = (N_t a)^{-1} \quad \to \quad \frac{s_{sound}}{s_{micro}} = \frac{d(T \ln Z_{sound})/dT}{T_0^3 b} = \frac{1}{g} ,$$

where g is the microscopic degeneracy: look for divergences as $a \to 0$ at fixed g.

More observables

– flow tensor:

$$\langle \Omega_{\mu\nu} \rangle = \langle g_{\mu\nu} + u_\mu u_\nu \rangle = \left\langle B_{IJ}^{-1} \partial_\mu \phi^I \partial_\nu \phi^J \right\rangle \quad \left[= \langle g_{\mu\nu} + \partial_\mu \psi \partial_\nu \psi / X \rangle \quad \text{(superfluid)} \right]$$

and its two-point correlation function, $\langle \Omega_{\alpha\beta}(x) \Omega_{\mu\nu}(x') \rangle$, both of which may reveal turbulence seeded by quantum fluctuations.

– stress-energy tensor:

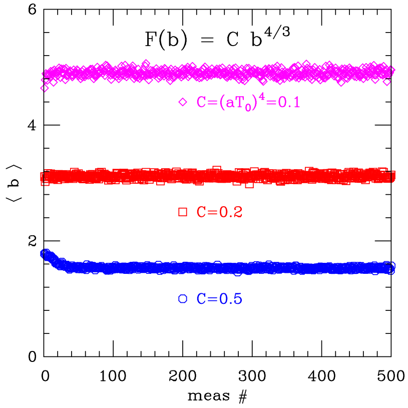
$$\langle T_{\mu\nu} \rangle = \langle (p+\rho) u_\mu u_\nu - p g_{\mu\nu} \rangle = \left\langle (F_y y - F_b b) B_{IJ}^{-1} \partial_\mu \phi^I \partial_\nu \phi^J + (F - F_y y) g_{\mu\nu} - 2F_X \partial_\mu \psi \partial_\nu \psi \right\rangle$$

the two-point correlation function, $\langle T_{\alpha\beta}(x) T_{\mu\nu}(x') \rangle$, may be used to determine the amount of “quantum viscosity” [G.Torrieri, PRD 85 (2012) 065006]:

$$\eta = \frac{\beta}{20} \lim_{\omega \to 0} \lim_{\vec{q} \to 0} \int d^3\vec{x} dt \, e^{-i\vec{q}\cdot\vec{x} + i\omega t} \langle \pi_{lm}(t, \vec{x}) \pi_{lm}(0, \vec{0}) \rangle \quad ,$$

where $\pi_{lm}(x) = T_{lm}(x) - \frac{1}{3}\delta_{lm} T_i^i(x)$.

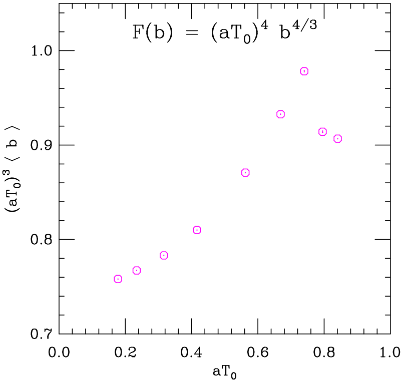
First results (RIG)



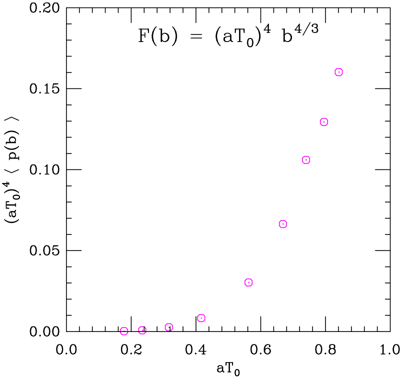
8^4 lattice
entropy density MC history

First results (RIG)

entropy density vs lattice spacing × micro scale:

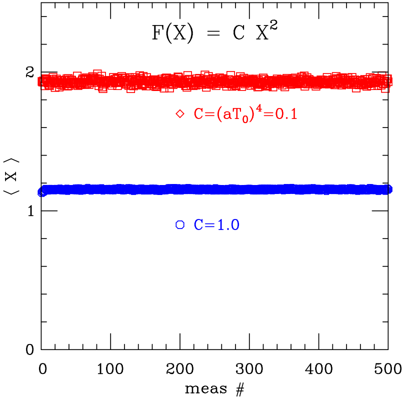


pressure vs lattice spacing × micro scale:

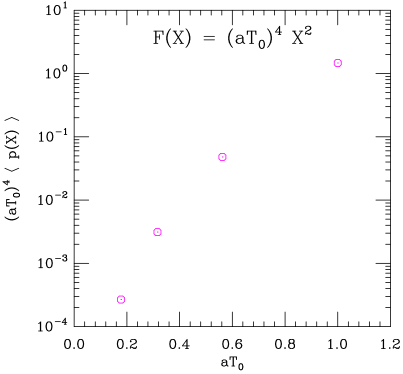


First results (RDFS)

8^4 lattice, $a\mu = 1$, $\langle X \rangle = \langle \partial_\mu \psi \partial^\mu \psi \rangle$ MC history:



pressure vs lattice spacing $\times$ micro scale:



Conclusions / Outlook

- Lagrangian hydrodynamics as an EFT [A.Nicolis, arXiv:1108.2513]; we've decided (unwisely?) to quantize it...
- working HMC code for purely normal fluid, $\mathcal{L} = F(b)$, and pure superfluid, $\mathcal{L} = F(X)$, cases; two-fluid case, $\mathcal{L} = F(b, X, y)$, requires further testing
- first results with ideal relativistic gas, $F(b) = (T_0^3 b)^{4/3}$, and relativistic, degenerate-fermion superfluid, $F(X) = (\mu_0^2 X)^2$: first look at some scalar invariants

- near future: "scalar perturbation", flow-tensor and stress-energy-tensor correlators, "micro vs macro" entropy and temperature
- several volumes: $L \to \infty$
- two-fluid systems: dense, low-temperature fermions
- might lessons learned here be used in (together with) lattice QCD simulations? doublers? sign problem?
- include dissipative terms in the EFT? [S.Endlich et al, arXiv:1211.6461]

"Experience, n. The wisdom that enables us to recognize as an undesirable old acquaintance the folly that we have already embraced." –Ambrose Bierce, The Devil's Dictionary