**McGill University Department of Electrical and Computer Engineering**

**ECSE 443 Introduction to Numerical Methods Winter 2019**

**Assignment #1 Posted: Jan 25 2019 DUE: Feb 2: 13:00**

Solve all problems. Show all your work, include all source code. All programs should be well documented, and the methods used should be clearly described. Clearly indicate the final answer. Follow instructions in the question. Printed out (hand written assignments will not be accepted). For software portions of the project the source code as well as the output of the code will be required as part of the submission. You must include all references and sources that you used. The TA's will be instructed to look for plagiarism or other forms of misconduct and if found, will report the potential misconduct.

**Question 1)** (**10 Marks**) Given the function: $f(x) = x\left(\sqrt{x} - \sqrt{(x-1)}\right)$.

a) Use MATLAB to solve for the function for $x = 10, x = 1000 \; and \; x = 1000000$. Be sure to submit the MATLAB script that was used to compute the values. Your final answer must be in the format below: (2 Marks)

| X= | 10 | 1000 | 1000000 |
|---|---|---|---|
| F(x)= | | | |

b) Using a calculator and six significant digits, (apply rounding to each step), compute the value of the function for the same values. Your final answer must be in the same format as below. Show all your work. (2 Marks)

| X= | 10 | 1000 | 1000000 |
|---|---|---|---|
| F(x)= | | | |

c) Treating the MATLAB answer as the "true" value. Compute the absolute and relative error between parts "a" and parts "b" for each value of x. In three or fewer sentences explain the source of the error. (2 Marks)

d) If we multiply the function by: $\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$, i.e. $f(x) = x\left(\sqrt{x} - \sqrt{(x-1)}\right)\frac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$, , and repeat the exercise from part b with the function above with six significant figures, (apply rounding to each step), note you should simplify the expression before computing, what is the resulting value? Your final answer must be in the same format as below. Show all your work. (2 Marks)

| X= | 10 | 1000 | 1000000 |
|---|---|---|---|
| F(x)= | | | |

e) Treating the MATLAB answer as the "true" value. Compute the absolute and relative error between parts "a" and parts "d" for each value of x. In three or fewer sentences explain how this affected the error. (2 Marks)

**Question 2)** (**10 Marks**) Given the function: $f(x) = \frac{1-\cos(x)}{\sin(x)}$.

a) Use MATLAB to solve for the function for $x = 0.007$ Be sure to submit the MATLAB script that was used to compute the values. Your final answer must be in the format below: (2 Marks)

| | |
|---|---|
| X= | 0.007 |
| F(x)= | |

b) Using a calculator and six significant digits, (apply rounding to each step), compute the value of the function for the same values. Your final answer must be in the same format as below. Show all your work. (2 Marks)

| | |
|---|---|
| X= | 0.007 |
| F(x)= | |

c) Treating the MATLAB answer as the "true" value. Compute the absolute and relative error between parts "a" and parts "b" for each value of x. In three or fewer sentences explain the source of the error. (2 Marks)

d) If we multiply the function by: $\frac{1+\cos(x)}{1+\cos(x)}$, i.e. $f(x) = \frac{1-\cos(x)}{\sin(x)}\frac{1+\cos(x)}{1+\cos(x)}$, , and repeat the exercise from part b with the function above with six significant figures, (apply rounding to each step), note you should simplify the expression before computing, what is the resulting value? Your final answer must be in the same format as below. Show all your work. (2 Marks)

| | |
|---|---|
| X= | 0.007 |
| F(x)= | |

e) Treating the MATLAB answer as the "true" value. Compute the absolute and relative error between parts "a" and parts "d" for each value of x. In three or fewer sentences explain how this affected the error. (2 Marks)

**Question 3)** (**14 Marks**) The first order approximation of a derivative is: $\frac{df(x)}{dx} \approx \frac{f(x+h)-f(x)}{h}$.

a) Compute the derivative of the function: $f(x) = e^{-4x}cos(6x)$ using calculus and solve it for the point x=0.5. Show your work and clearly indicate your final answer. (2 Marks)
b) Using MATLAB and set size of h=0.01 compute the first order approximation of the derivative at the point x=0.5. Include the MATLAB script and clearly indicate the output produced. (2 Marks)
c) Using the first three terms of the Taylor series expansion of f(x), (note that this will be the product of the first three terms of the Taylor series of f(x):i.e. the first three terms of the Taylor series for $e^{-4x}$ and the first three terms of the Taylor series for $\cos(6x)$ will be multiplied to obtain the first three terms of f(x)), compute the first order approximation of the derivative of the function with a step size of 0.01. Show your work and clearly indicate your final answer. (2 Marks)
d) Write a program that will determine the step size with the smallest absolute error (using the answer in part "a" as the true value) using double precision floating point numbers. You must provide the source code (or MATLAB script) as well as the output of the code clearly indicating the value of step size obtained. (8 Marks)

**Question 4)** (**12 Marks**) Given a system of equations:
$$\begin{matrix} 4x_1 - 2x_2 - 3x_3 + 6x_4 = 12 \\ 6x_1 - 7x_2 + 6.5x_3 - 6x_4 = -6.5 \\ x_1 + 7.5x_2 + 6.25x_3 + 5.5x_4 = 16 \\ -12x_1 + 22x_2 + 15.5x_3 - x_4 = 12 \end{matrix}.$$

a) Solve for the unknowns by inverting the matrix and solving: $[A^{-1}]\begin{bmatrix} 12 \\ -6.5 \\ 16 \\ 12 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. Note that you can use MATLAB for this. Provide the results and the work done. Clearly indicate the answer. (2 Marks)
b) Write a program that will solve for the unknowns (x1, x2, x3, x4) using ***Gaussian Elimination***. Provide the source code/MATLAB scripts as well as clearly indicating the solution and the output of the code. (8 Marks)
c) Using the answer from part "a" as the true value compute the largest absolute error between the result from part "a" and the result from part b. (2 Marks)

**Question 5) (5 Marks)**

Write a program to perform the operations in parts a and b.

a) Use the Taylor series about x=0 to estimate $f(x) = e^{\sin(x)}$ to an error of $x^4$, (i.e. all terms in th expression will be of the order $x^3$ or lower order). Show all your work and justify your answer. (1 Mark)
b) Using the series in part "a" to estimate $f(0.01)$. Show all your work and justify your answer (1 Mark)
c) Using your calculator compute $f(0.01) directly\ using\ e^{\sin(0.01)}$. Show all your work and justify your answer (1 Mark)
d) Compare the results of parts "b" and "c". Briefly comment on any differences. (1 Mark)