ECSE 443 - Assigment 1

Tristan Bouchard, 260747124

Question 1) (10 Marks)

a)

```
format long
syms x;
f(x) = x*(sqrt(x) - sqrt(x-1));
X = [10;1000;1000000];

x1 = double(f(X(1)));
x2 = double(f(X(2)));
x3 = double(f(X(3)));

table = {"X:", 10, 1000, 1000000 ; "F(x)=", x1, x2, x3 }
```

```
table = 2×4 cell array
    {["X:"   ]}    {[                10]}    {[              1000]}    {[               1000000]}
    {["F(x)="]}    {[1.622776601683793]}    {[15.815343125576774]}    {[5.000001250000625e+02]}
```

c)

```
mat_val = [x1 x2 x3];
calc_val = [1.6228 15.8 0];
abs_err = calc_val - mat_val
```

```
abs_err = 1×3

10^2 ×
   0.000000233983162  -0.000153431255768  -5.000001250000625
```

```
rel_err = abs_err ./ mat_val
```

```
rel_err = 1×3

   0.000014418692125  -0.000970141808176  -1.000000000000000
```

e)

```
calc_val2 = [1.62278 15.8153 500]
```

```
calc_val2 = 1×3

10^2 ×
   0.016227800000000   0.158153000000000   5.000000000000000
```

```
abs_err2 = calc_val2 - mat_val
```

```
abs_err2 = 1×3

10^-3 ×
   0.003398316206660  -0.043125576773662  -0.125000062496383
```

```
rel_err2 = abs_err2 ./ mat_val
```

```
rel_err2 = 1×3

10^-5 ×
   0.209413680424876  -0.272681891447036  -0.025000006249272
```

Question 2) (10 Marks)

```
syms x;
f(x) = (1-cos(x))/sin(x);
res = double(f(0.007))
```

```
res =
   0.003500014291737
```

```
calc_valq2 = 0;
abs_errq2 = calc_valq2 - res
```

```
abs_errq2 =
  -0.003500014291737
```

```
rel_errq2 =  abs_errq2 / res
```

```
rel_errq2 =
    -1
```

```
abs_err3 = 0.000611087 - res
```

```
abs_err3 =
  -0.002888927291737
```

```
rel_err3 = abs_err3/res
```

```
rel_err3 =
  -0.825404427221130
```

Question 3) (**14 Marks**)

a)

```
syms x;
f(x) = -(4*exp(-4*x)*cos(6*x) + 6*exp(-4*x)*sin(6*x));
result = (f(0.5))
```

result = $-4\cos(3)\,e^{-2} - 6\,e^{-2}\sin(3)$

```
true_value = double(result)
```

```
true_value =
   0.421332562151359
```

```
f_original(x) = exp(-4*x)*cos(6*x)
```

f_original(x) = $\cos(6\,x)\,e^{-4\,x}$

```
h = 0.01;
derivative = double((f_original(0.5+h) - f_original(0.5))/(h))
```

```
derivative =
   0.438478802436531
```

c)

```
syms x;
h = 0.01;
f(x) = 1 - 4*x - 10*x^(2);
der = ((f(x + h) - f(x))/h)
```

der =

$$1000\,x^2 - 1000\,\left(x + \frac{1}{100}\right)^2 - 4$$

d)

```
% This algorithm will be used to compute the the step size h which results
% in the least error when computing the first order approximation of the
% derivative of f(x)
syms x;
f(x) = exp(-4*x)*cos(6*x);
% Define the "true" value obtained in part a)
true_val = 0.421332562151359;
% Define the value for which we are trying to compute the derivative
x = 0.5;
last_err = 100; % Set error very large for first loop
step = 0.001; % Set starting step size
cont = 1; % Set loop variable

while (cont == 1)
    % For each value of h, compute the first order approximation
    temp = ((f(x + step) - f(x))/step);
    derivative = double(temp);
    err = derivative - true_val;
    if( err >= last_err)
        % Break out of loop, error has started to increase
        cont = 0;
        return_val = [last_err, last_step, last_der];
    end
    last_der = derivative;
    last_step = step;
    last_err = err;

    step = step / 2;
end
titles = {"Computed Error", "Computed Step", "Computed Derivative"};
disp(titles), disp(return_val);
```

```
    ["Computed Error"]    ["Computed Step"]    ["Computed Derivative"]

  -0.000000001082816   0.000000001907349   0.421332561068543
```

Question 4) (12 Marks)

a)

```
A  = [4 -2 -3 6; 6 -7 6.5 -6; 1 7.5 6.25 5.5; -12 22 15.5 -1]
```

```
A = 4×4
   4.000000000000000  -2.000000000000000  -3.000000000000000   6.000000000000000
   6.000000000000000  -7.000000000000000   6.500000000000000  -6.000000000000000
   1.000000000000000   7.500000000000000   6.250000000000000   5.500000000000000
 -12.000000000000000  22.000000000000000  15.500000000000000  -1.000000000000000
```

```
B_res = inv(A)*[12; -6.5; 16; 12]
```

```
B_res = 4×1
  -4.770833333333334
  -4.450757575757576
   3.757575757575756
   5.575757575757576
```

b)

```
% Gaussian elimination for matrix.
% Begin by augmenting the matrix from part a) with its solutions
B = [12; -6.5; 16; 12];
```

```
concat = [A B]
```

```
concat = 4×5
   4.000000000000000  -2.000000000000000  -3.000000000000000   6.000000000000000  12.000000000000000
   6.000000000000000  -7.000000000000000   6.500000000000000  -6.000000000000000  -6.500000000000000
   1.000000000000000   7.500000000000000   6.250000000000000   5.500000000000000  16.000000000000000
 -12.000000000000000  22.000000000000000  15.500000000000000  -1.000000000000000  12.000000000000000
```

```
% This script assumes that the matrix is a 4x4 (augmented)
% Begin iterating at the second row
for row=2:4
    % For all rows
    for j=row:4
        % Compute ratio by which to subtract first row from above row
        subs = (concat(j,row-1)/concat(row-1,row-1))*concat(row-1,:);
        concat(j, :) = concat(j,:) - subs;
    end
end
disp(concat);
```

```
   4.000000000000000  -2.000000000000000  -3.000000000000000   6.000000000000000  12.00000000000000
                   0  -4.000000000000000  11.000000000000000 -15.000000000000000 -24.50000000000000
                   0                   0  29.000000000000000 -26.000000000000000 -36.00000000000000
                   0                   0                   0   2.275862068965516  12.68965517241379
```

```
% Then, clear the rest of the rows by solving for each value of x by
% working our way up the matrix from the bottom row
% Create empty array in which to store answers
answ = zeros(1,4);
for i=1:4
    j = 5-i;
    % Solve for xi in each row
    answ(j) = (concat(j,5)-concat(j,4)*answ(4) - concat(j,3)*answ(3) - concat(j,2)*answ(2))/concat(j,j);
end
for i=1:length(answ)
    disp("x"+i + ":" + char(vpa(answ(i),15))) %, disp(answ(i));
end
```

```
x1:-4.77083333333333
x2:-4.45075757575758
x3:3.75757575757576
x4:5.57575757575758
```

c)

```
% Calculate absolute error between results in a) and b)
% Have to take transpose of answ vector to get correct dimensions.
absolute_err = transpose(answ) - B_res
```

```
absolute_err = 4×1
10^-14 ×
  -0.444089209850063
  -0.355271367880050
   0.444089209850063
   0.266453525910038
```

Question 5) (5 Marks)

a)

```
syms x;
f = exp(sin(x));
t = taylor(f,x)
```

t =

$$-\frac{x^5}{15}-\frac{x^4}{8}+\frac{x^2}{2}+x+1$$

```
f2(x)  = 1 + x + x^2/2
```

f2(x) =

$$\frac{x^2}{2}+x+1$$

b)

```
% Using the approximation in f2, compute f(0.01)
answer1 = f2(0.01);
disp(double(answer1));
```

1.010050000000000