McGill University Department of Electrical and Computer Engineering

ECSE 443 Introduction to Numerical Methods Winter 2019

Final Project Posted: March 27, 16:00 DUE: April 12, 17:00

Solve all problems. Show all your work, include all source code. All programs should be well documented, and the methods used should be clearly described. Clearly indicate the final answer. Follow instructions in the question. The assignment must be written using a word processor such as Microsoft word. Hand written submissions will not be accepted. For software portions of the project the source code as well as the output of the code will be required as part of the submission. You must include all references and sources that you used. The TA's will be instructed to look for plagiarism or other forms of misconduct and if found, will report the potential misconduct. For all these problems you are to write your own codes and develop your own answers. Using pre-made functions or sharing code/answers will be treated as misconduct. For the project all questions have equal value.

Question 1) Evaluate the integral $\int_0^{\infty} \frac{1}{e^x+e^{-x}} dx$ using Simpson's rule. You answer should match the true value within 4 decimal places (error < 0.0001).

Question 2) Evaluate the integral $\int_0^{1} \frac{x^3}{\sqrt{1-x^2}} dx$ using Mid-point integral method with h=0.01.

Question 3) Verify Gauss' Law $\left(\int_L AdL = \iint_S \bar{D} \cdot d\bar{S}\right)$ using numerical integration, (you may select the algorithm used), for the following geometry. The charge distribution "A" is an infinite line of charge on the z-axis with charge density 1X.Y micro-Coulombs per meter (where X and Y are the last two digits of your ID number) and the Gaussian surface enclosing the charge is a sphere radius 2 meters centered on the origin. The step size, i.e. the segment width, for the line integral is 0.01 meters and you should choose the number of segments so that the two integrals agree within 10% of each other.

Question 4) A system has a second order response modeled as: $A\frac{d^2f(t)}{dt^2} + B\frac{df(t)}{dt} + Cy(t) = x(t)$, where the output is y(t) and the input is x(t) and the constants A, B, C are to be determined. Your team performed some measurements of the output when a unit step was applied and obtained the following data:

Time (sec)	0	0.5	0.75	1.00	1.25	1.50	1.75	
Output	1	0.1160	-0.1084	-0.0409	0.0052	0.0067	0.009	

With this data determine the values of the coefficients A, B, C using numerical techniques and engineering principles. Clearly indicate any reasonable assumptions you have made.

Question 5) Solve the following set of equations $\begin{bmatrix} 3 & -5 & 47 & 20 \\ 11 & 16 & 17 & 10 \\ 56 & 22 & 11 & -18 \\ 17 & 66 & -12 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 18 \\ 26 \\ 34 \\ 82 \end{bmatrix}$ using a program/script that employs LU factorization and verify that your answer solves the equations. Compute the square error of your solution. Clearly indicate the method and answer.

Question 6) Repeat the problem in question #5 using a program/script that uses fixed point iteration. Compute the square error of your solution. Clearly indicate the method and answer.

Question 7) Using the secant method find the smallest positive root, (other than zero), of: $f(x) = \cos(x)\cosh(x) - 1$. The relative error should be on the order of: 10^{-4}.

Question 8) The measured data was taken at equally spaced intervals. The data was expected to result in a smooth function, like a second-degree polynomial. However, there appears to be a significant error in one of the data points. Find this point and "correct" it.

X	1	2	3	4	5	6	7	8
F(x)	0.812	0.642	0.691	0.893	1.454	2.164	3.092	4.24

Question 9) Given the system modeled as: $\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 4y(t) = 0$ with $y(0) = 2$ and $\frac{dy(0)}{dt} = 0.$ Compare the solutions obtained from a program programs/ scripts using Euler and a program/script using Runge-Kutta-4 method. Both methods should use step size h=0.1 and the solutions are in the range of t=0 to t=5 (inclusively).

Question 10) Write a program/script to use second order Runge-Kutta method to solve the following ODE: $\frac{dy}{dx} = -1.2y + 7e^{-0.3x}$ for x=0 to x=2 with a step size of h=0.1 and an initial condition of y(x=0)=3. Plot the resulting solution.

Question 11) Solve the boundary value problem using a program/script that applied the finite difference method. $\frac{d^2y(t)}{dt^2} + \frac{1}{4}\frac{dy(t)}{dt} = 8$ with the boundary conditions of y(0)=0 and y(10)=0. Use $\Delta x = 1$. Plot on the same axis your solution and the exact solution.

Question 12) Solve the boundary value problem using a program/script that applied the shooting method. $\frac{d^2y(t)}{dt^2} + \frac{1}{4}\frac{dy(t)}{dt} = 8$ with the boundary conditions of y(0)=0 and y(10)=0. Use $\Delta x = 1$. Plot on the same axis your solution and the exact solution.