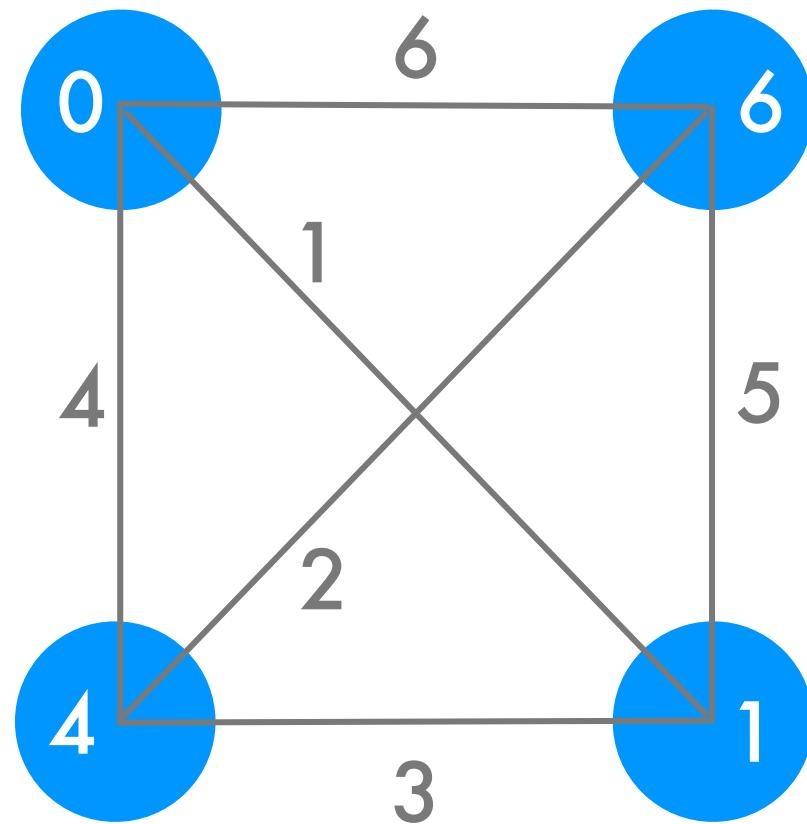


Research Frontiers - Constraints: lecture 1

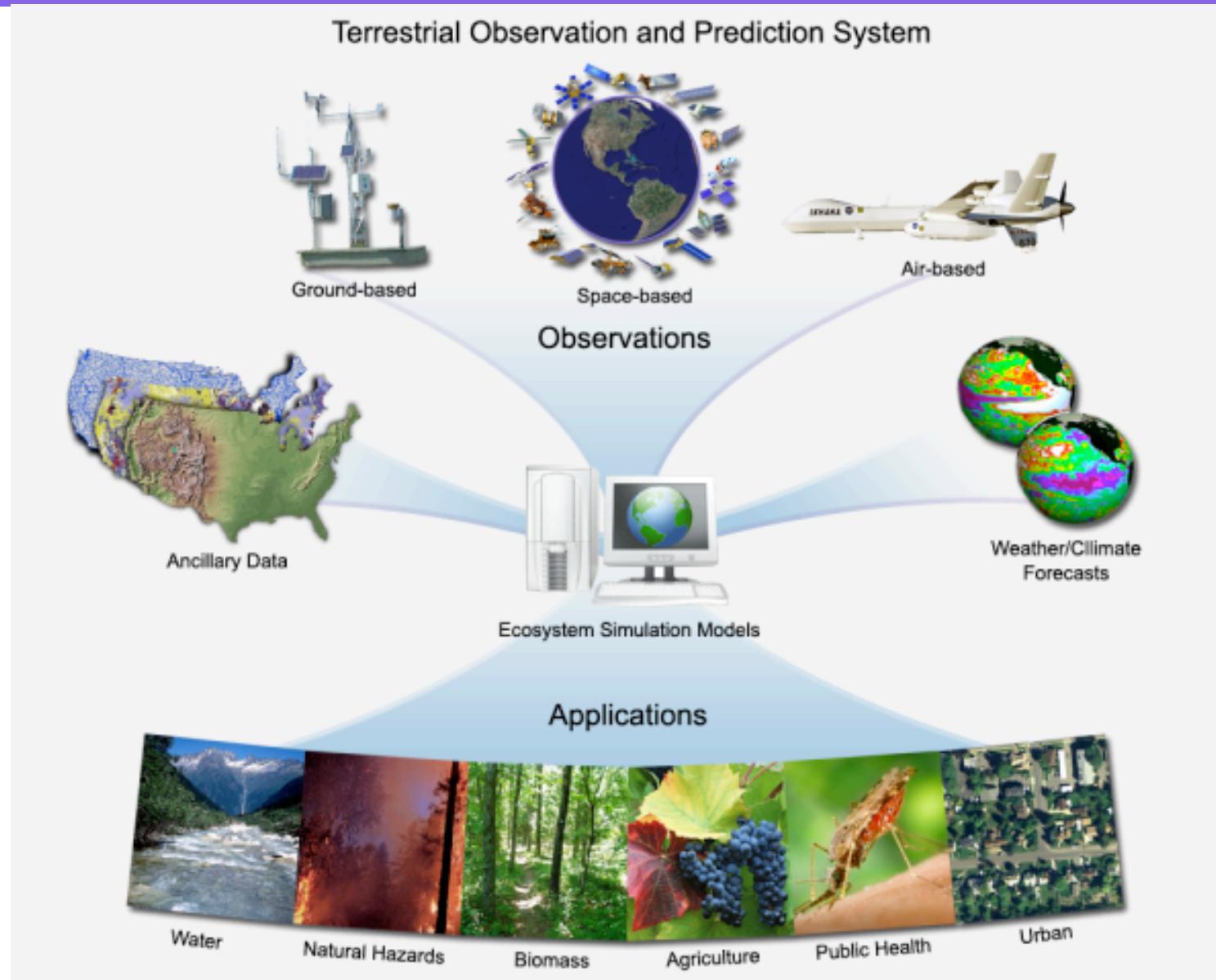
Karen Petrie
karenpetrie@computing.dundee.ac.uk

Practical Problems

Graceful Graphs

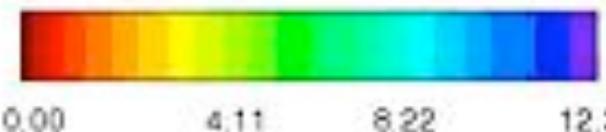
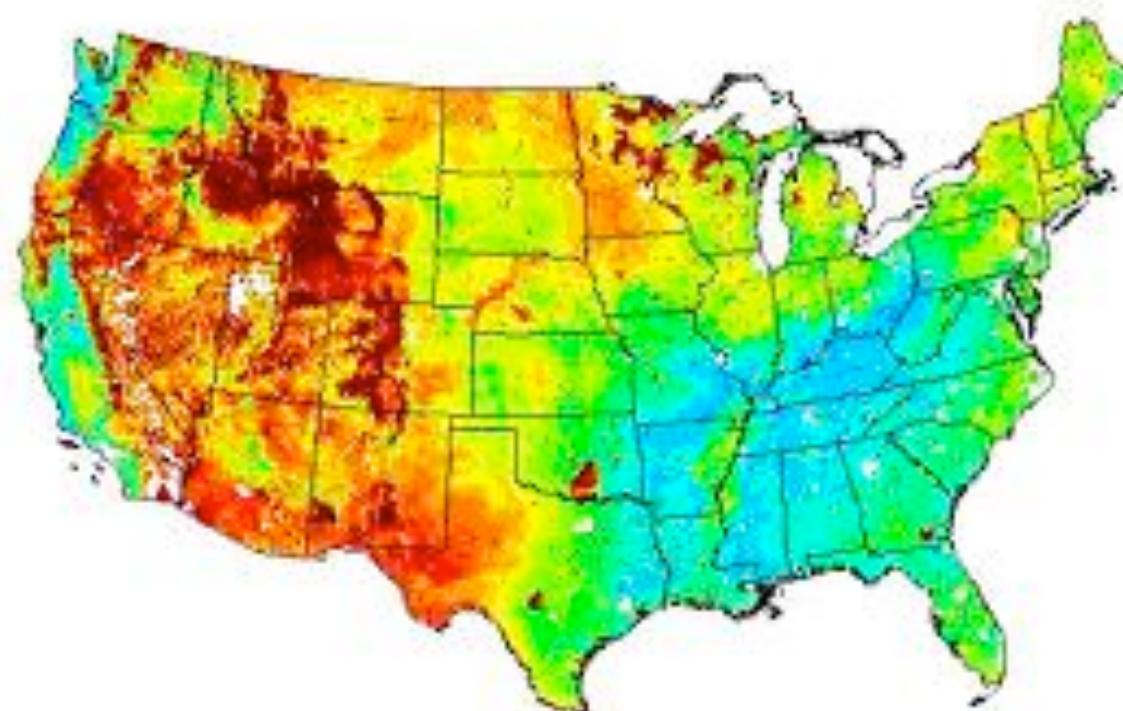


TOPS



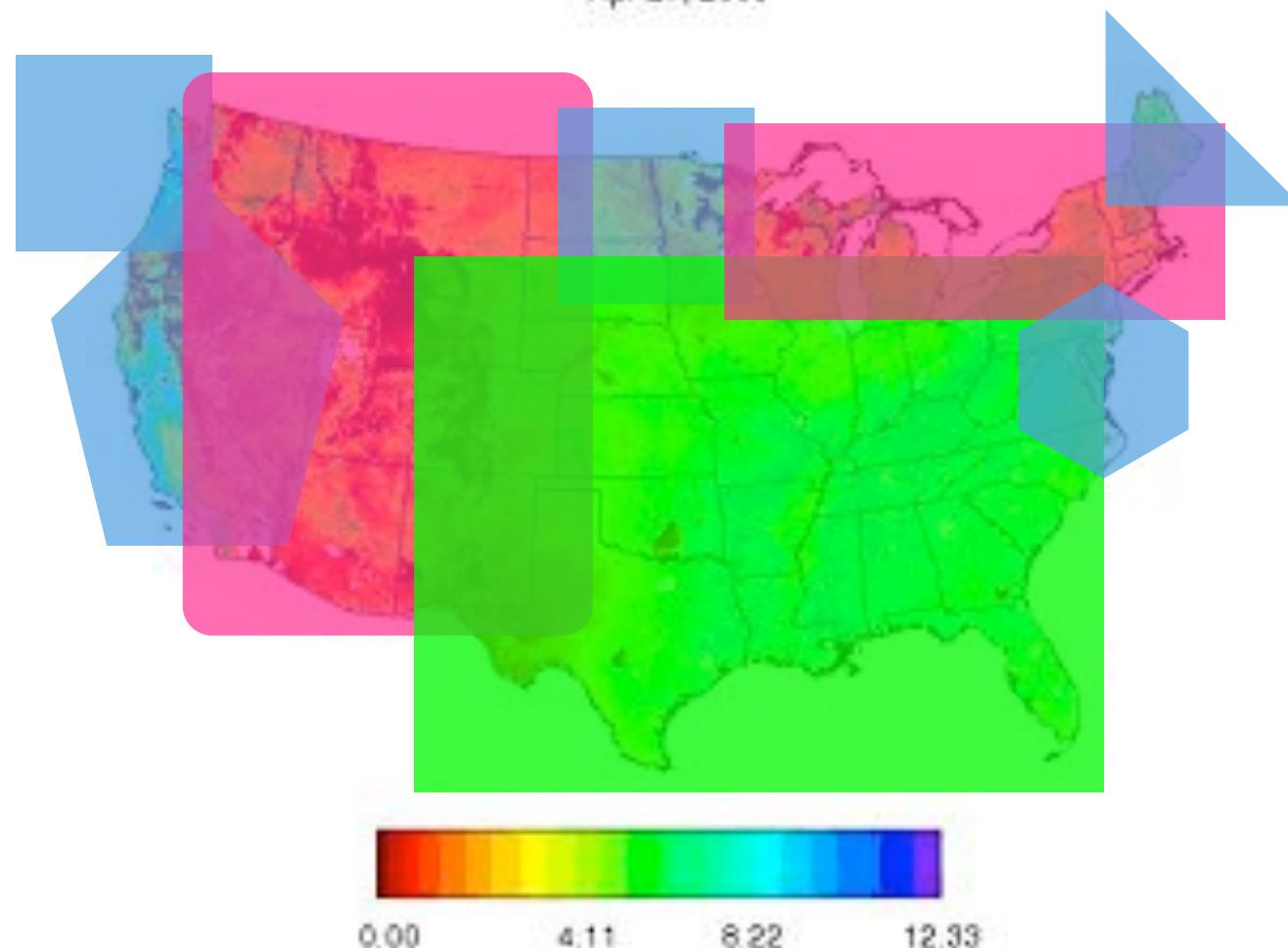
TOPS

TOPS GPP
USA - 8km
Apr 27, 2003



TOPS

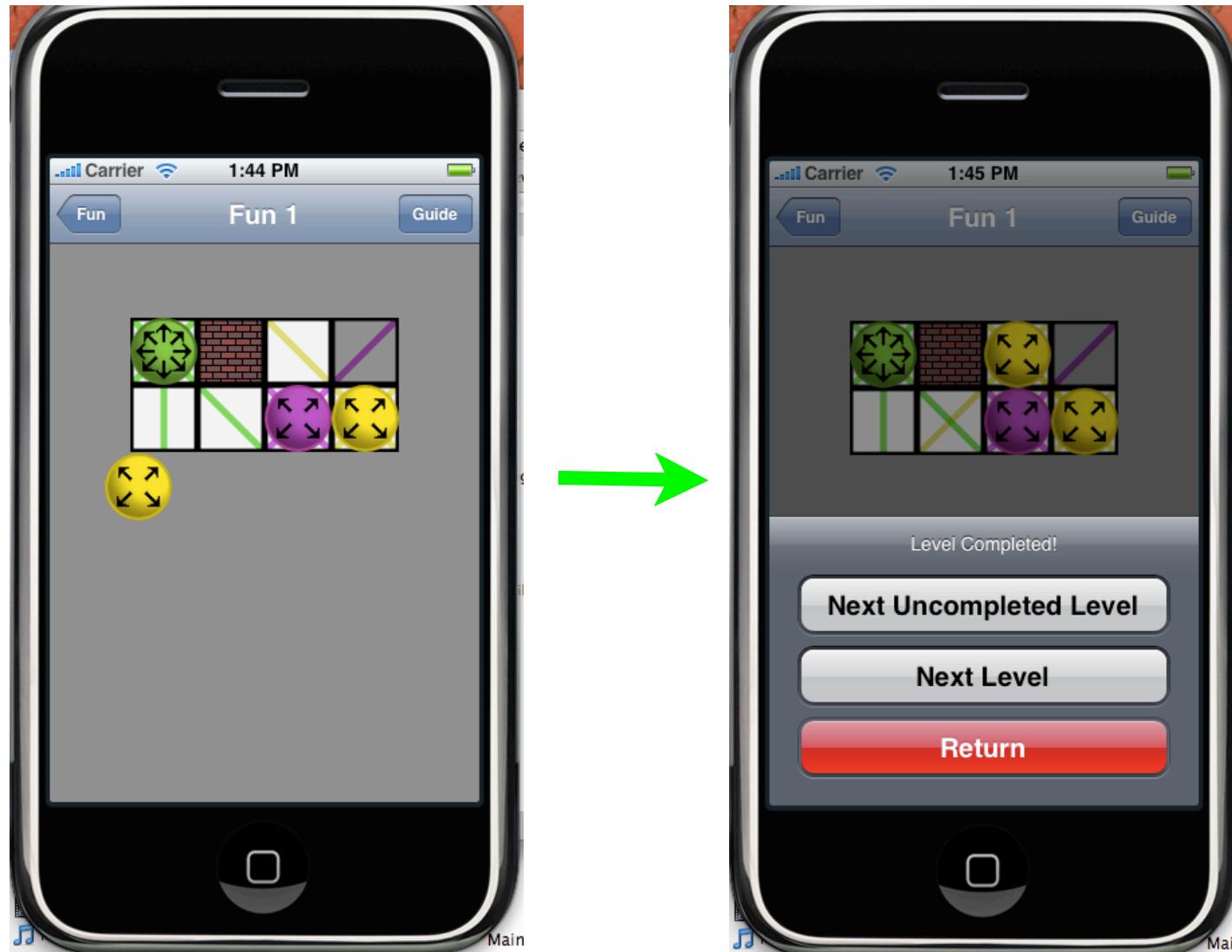
TOPS GPP
USA - 8km
Apr 27, 2003



LET systems



Combination



What is CP?

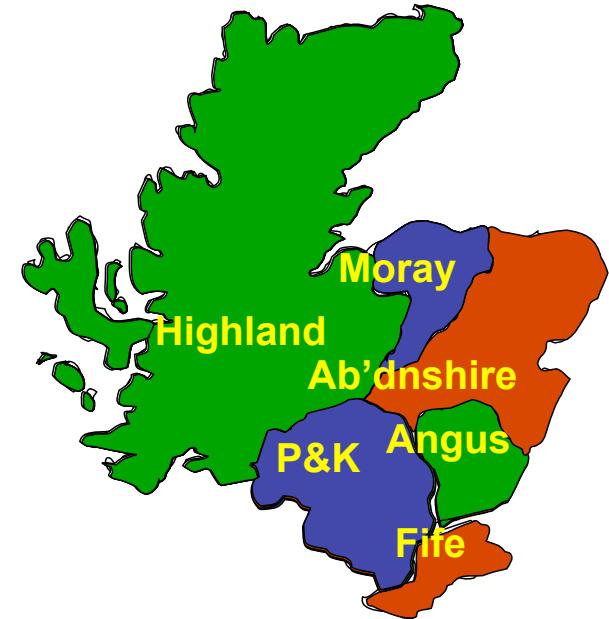
Why Constraint Programming?

- Constraint Programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it.”

[E. Freuder]

Constraints: A Natural Means of Knowledge Representation

- $x + y = 30$
- Adjacent countries on map cannot be coloured same.
- The helicopter can carry one passenger.
- Maths class must be scheduled between 9 - 11 am.



Constraint Satisfaction Problems

- A constraint satisfaction problem comprises:
 - A set of decision **variables**.
 - For each **decision** variable, a domain of potential values.
 - A set of **constraints** on the decision variables.

Decision Variables & Domains

- A decision variable corresponds to a **choice** that must be made in solving a problem.
- **Values** in the domain of a decision variable correspond to the various **options** for this choice.
- A decision variable is **assigned** a value from its domain.

CP solution

- A solution:
 - A complete assignment to the set of variables.
 - Satisfies all of the set of constraints.

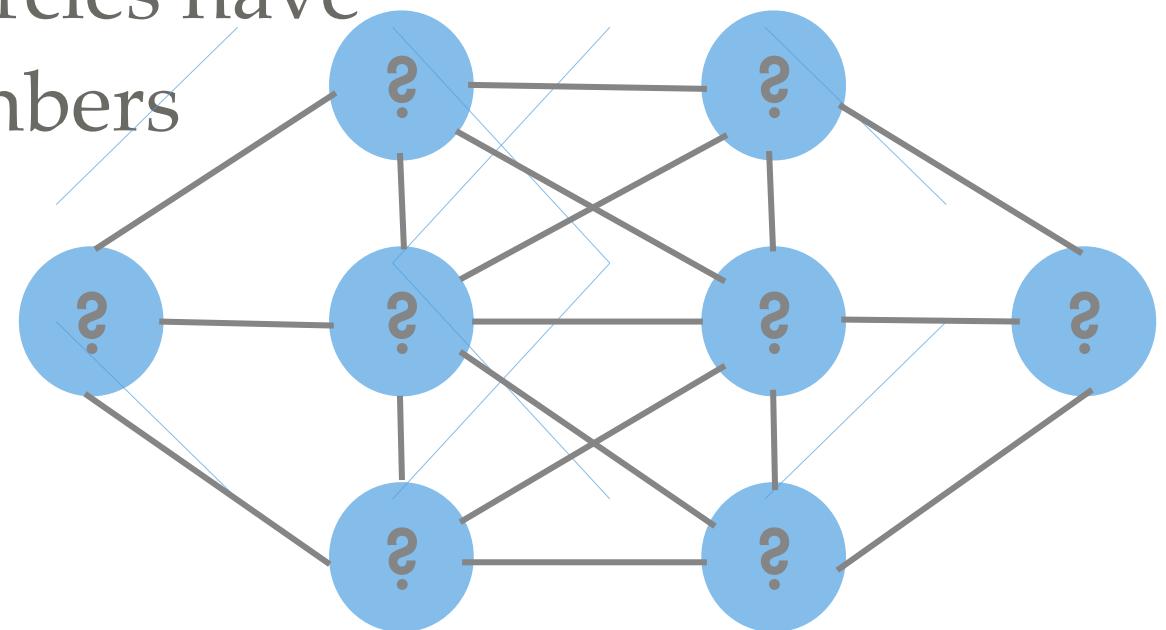
CP Objective

- You may wish to find:
 - one solution
 - all solutions
 - a good solution
 - an optimal solution

Solving a problem

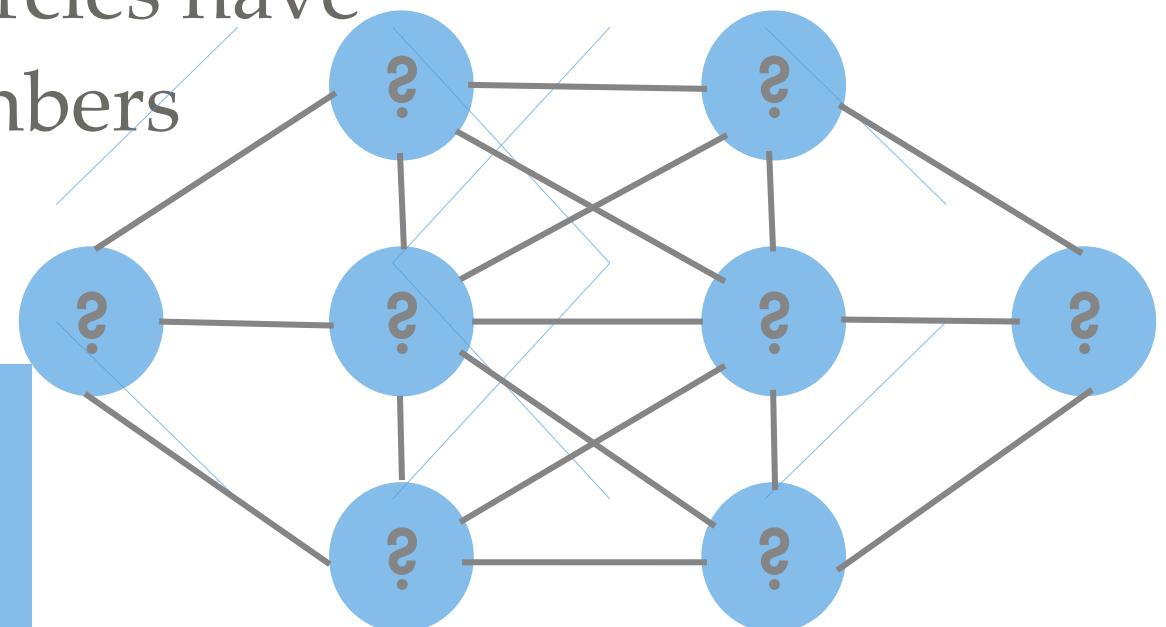
A Puzzle

- Place the numbers 1 to 8 in the circles such that:
 - Each number appears exactly once
 - No connected circles have consecutive numbers



A Puzzle

- Place the numbers 1 to 8 in the circles such that:
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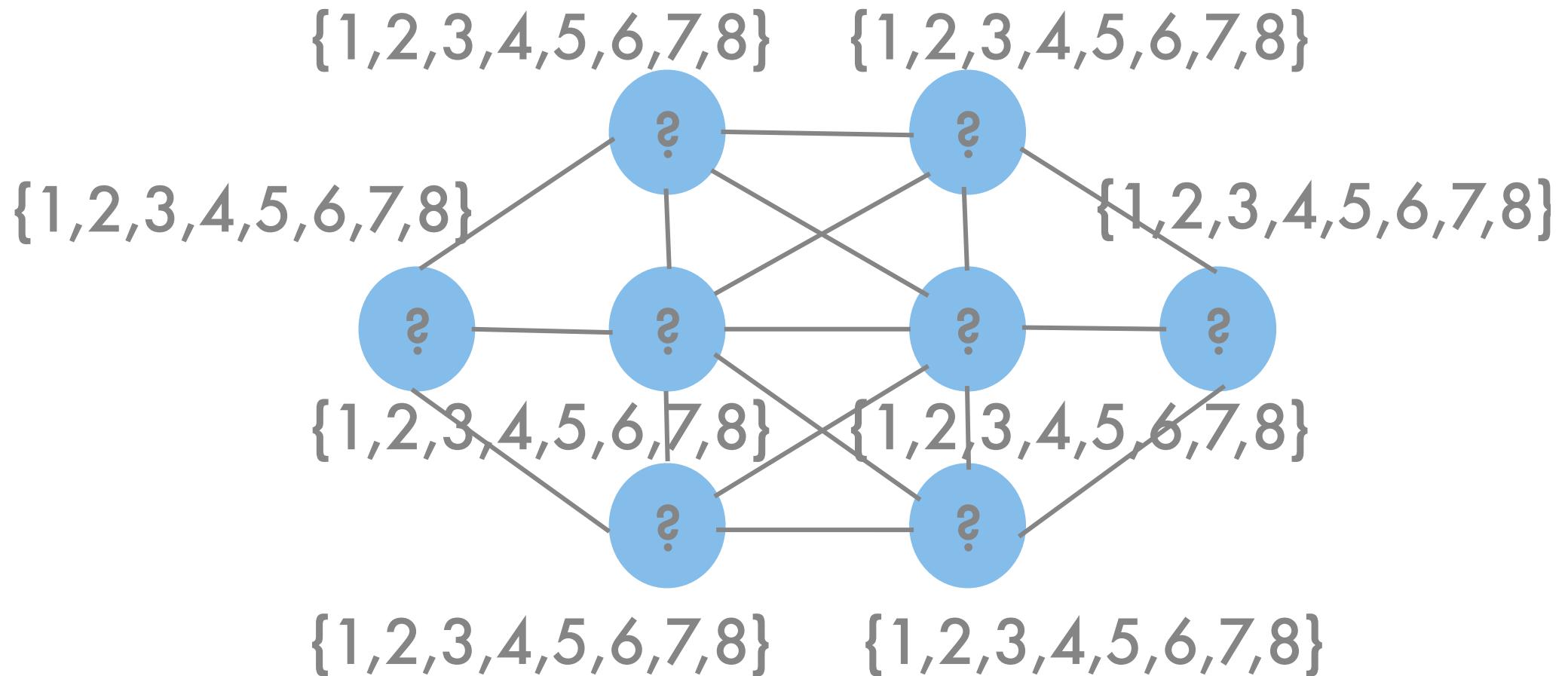


You have 5 minutes

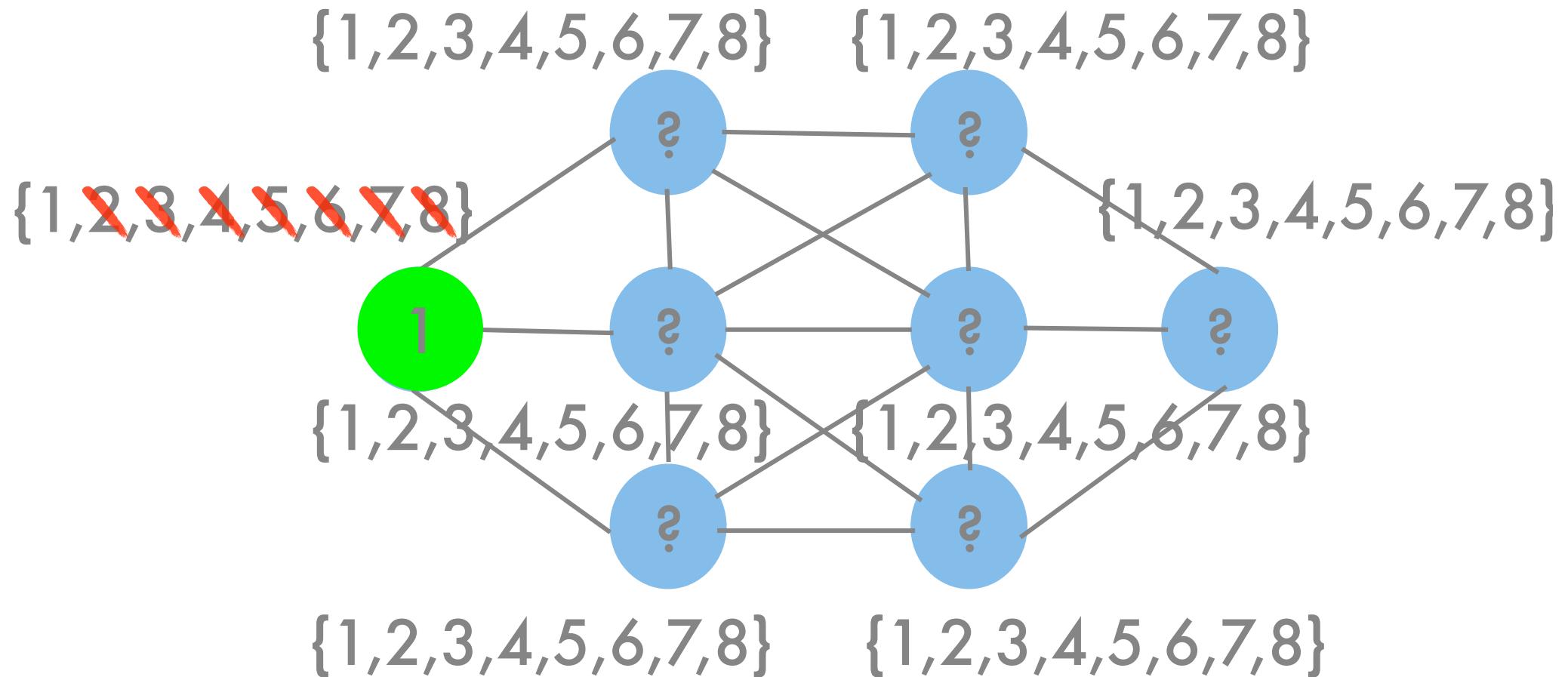
A CP model

- Each node → a variable
- $\{1, \dots, 8\}$ → values in the domain of each variable
- No consecutive numbers → a constraint
 - $|v_i - v_j| > 1$
- All values used → all-different constraint

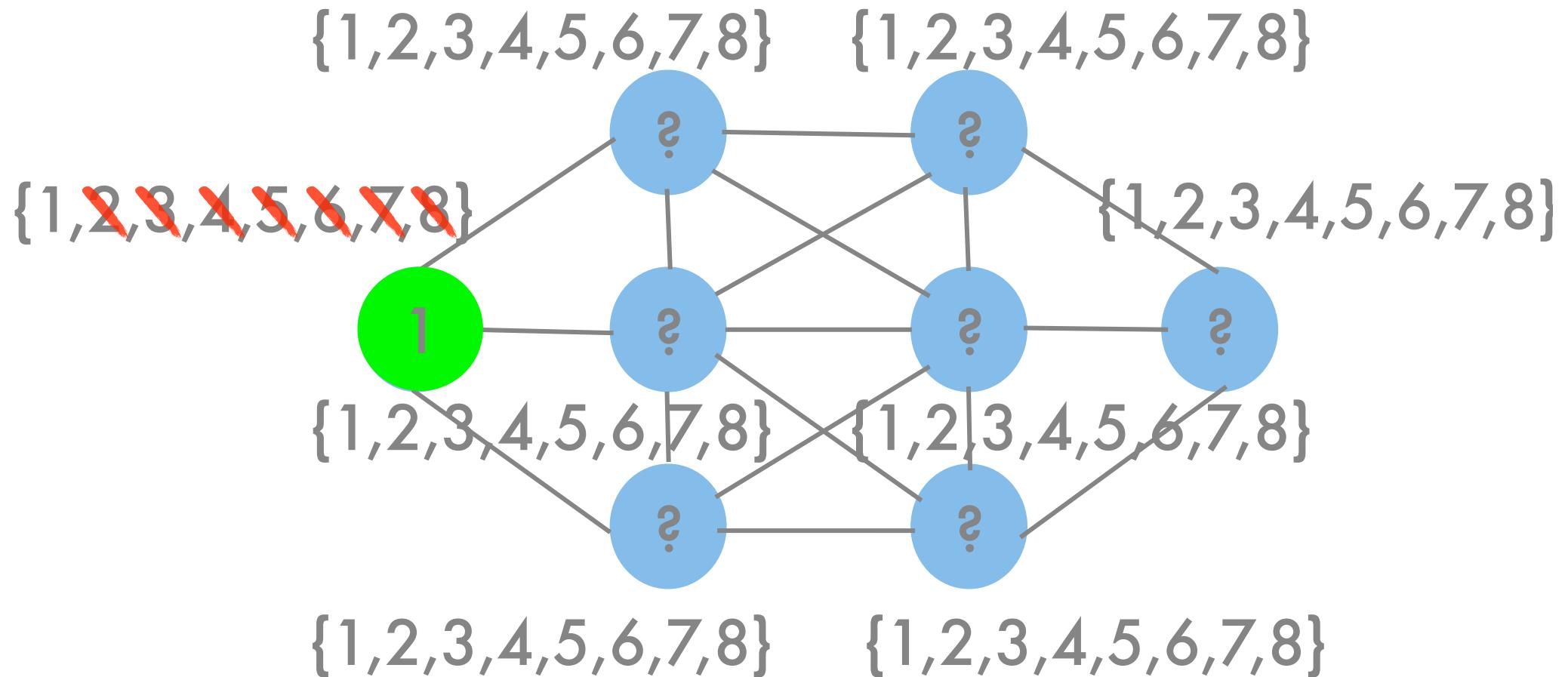
CP model in action



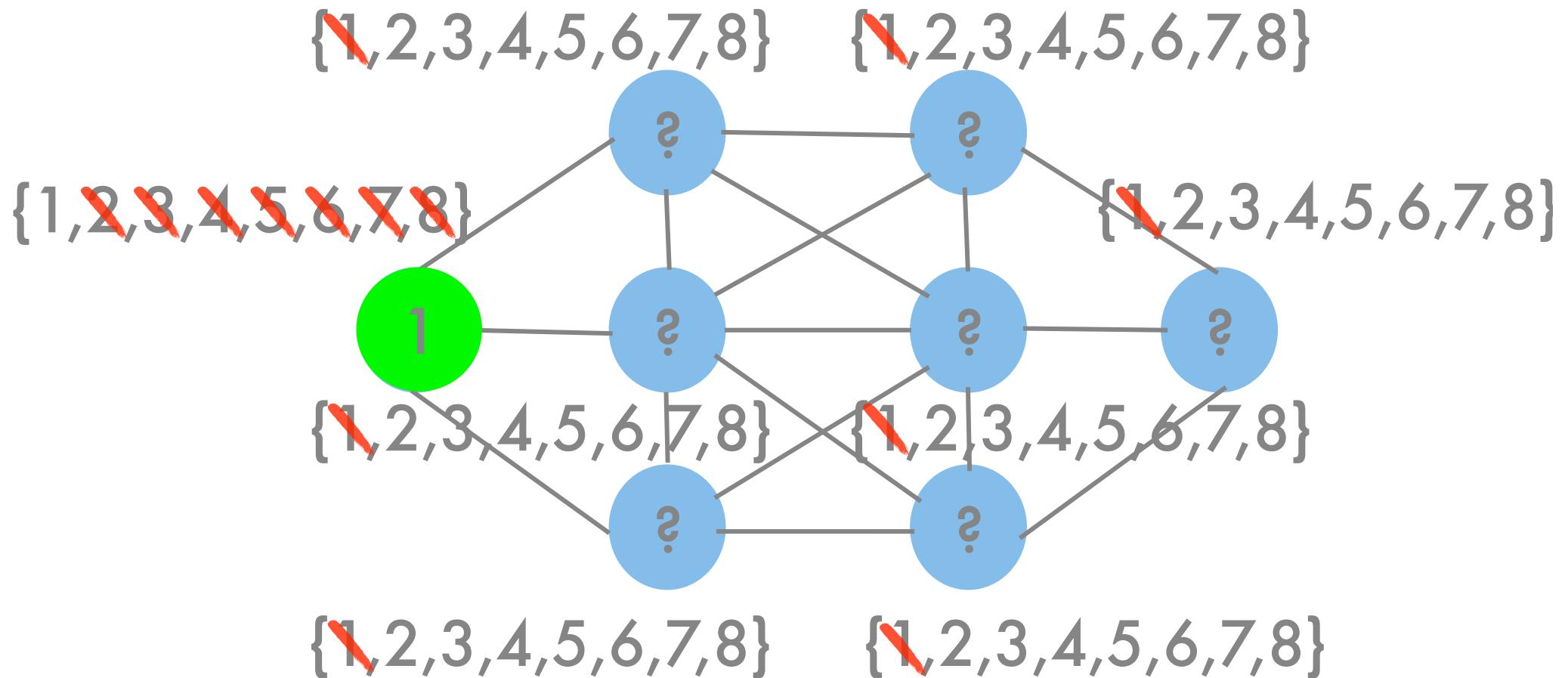
CP model in action



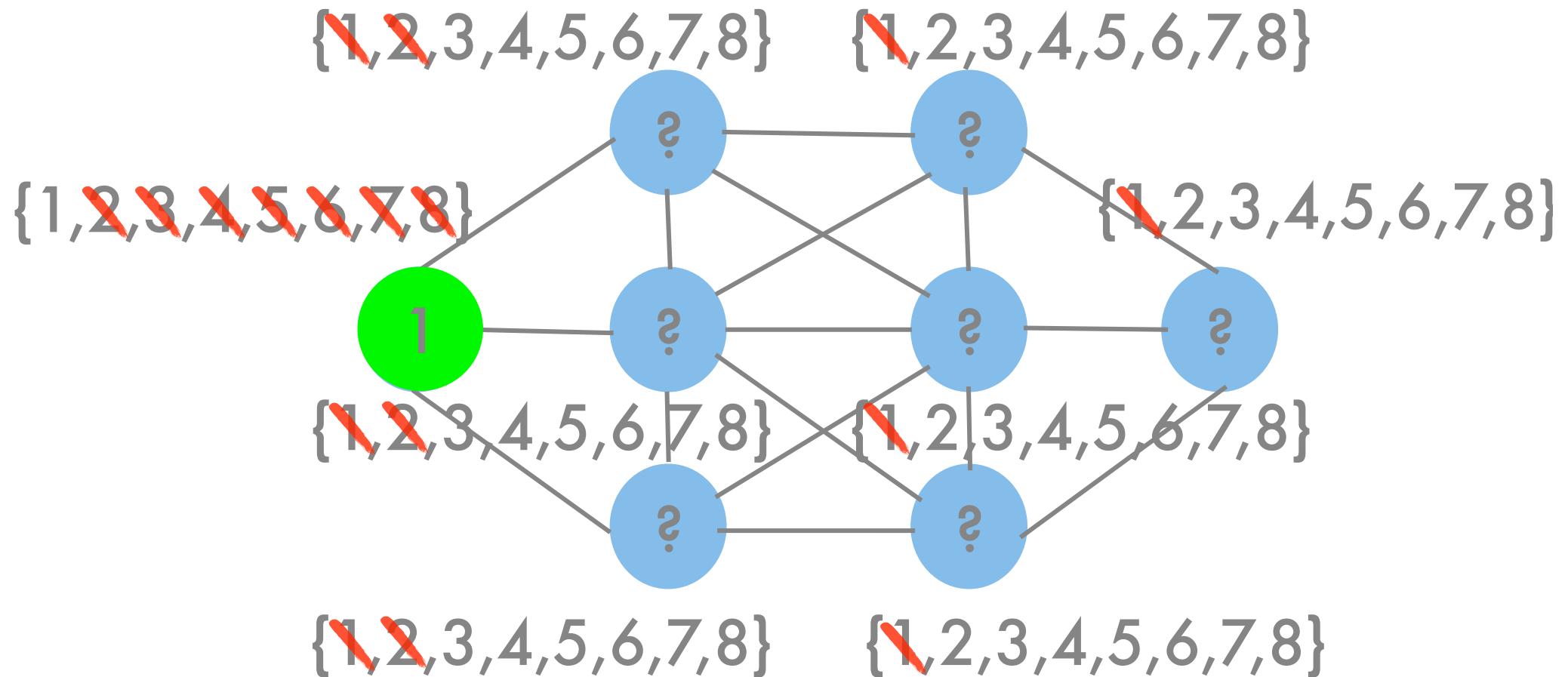
CP model in action



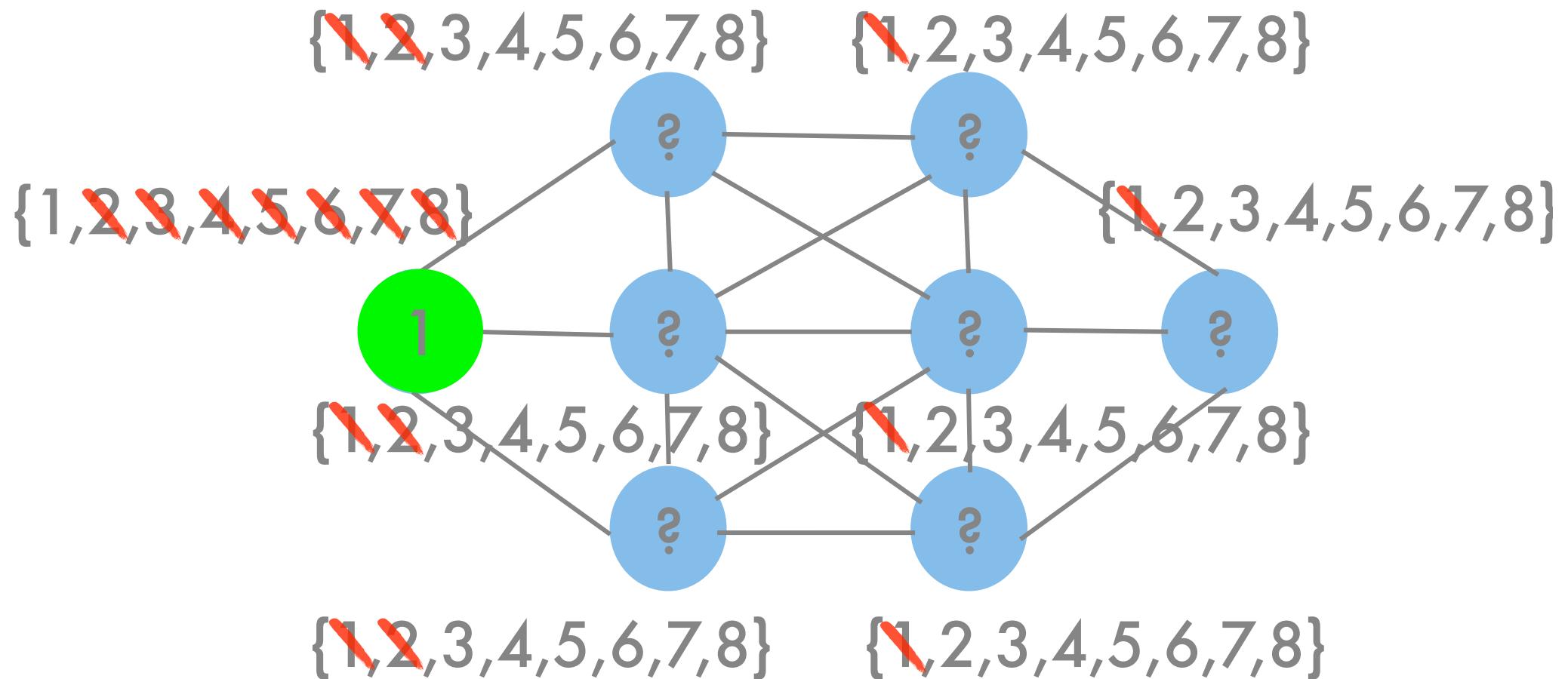
CP model in action



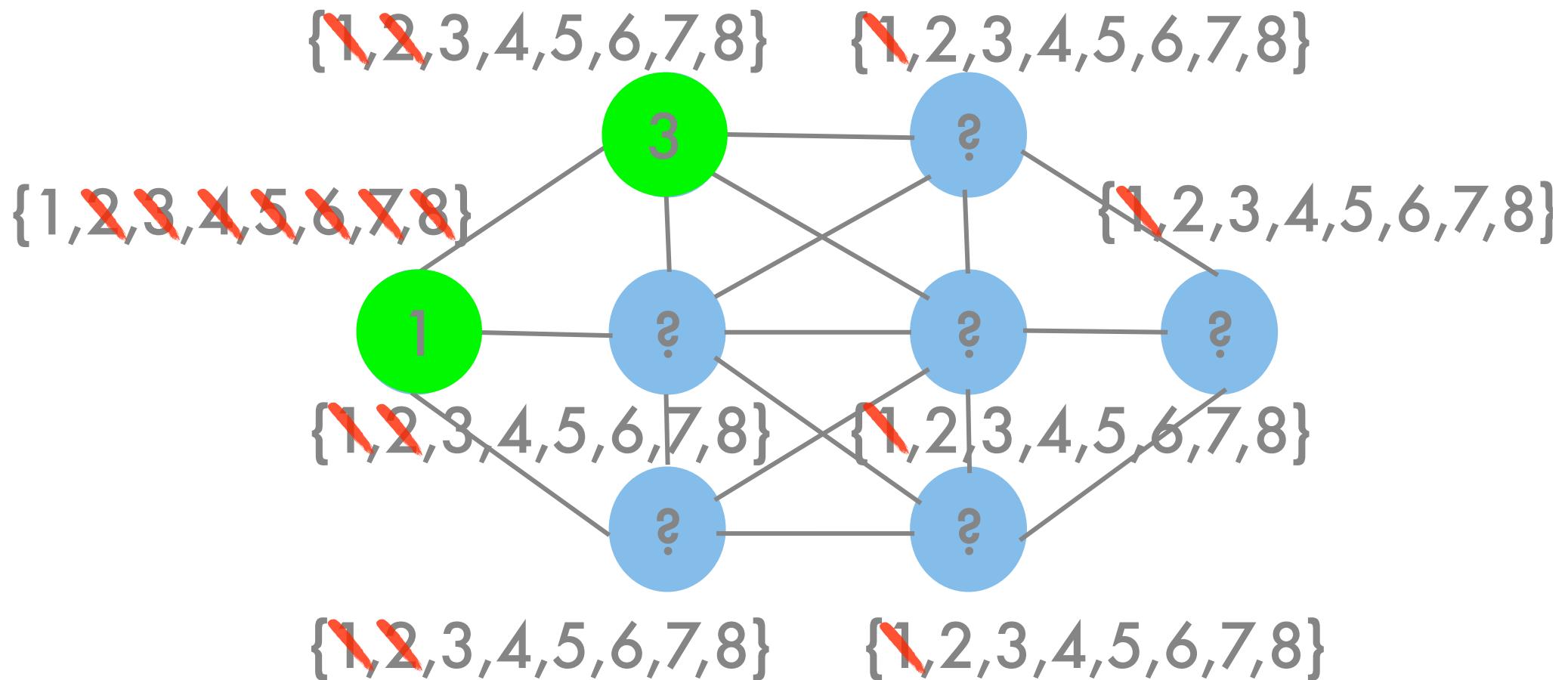
CP model in action



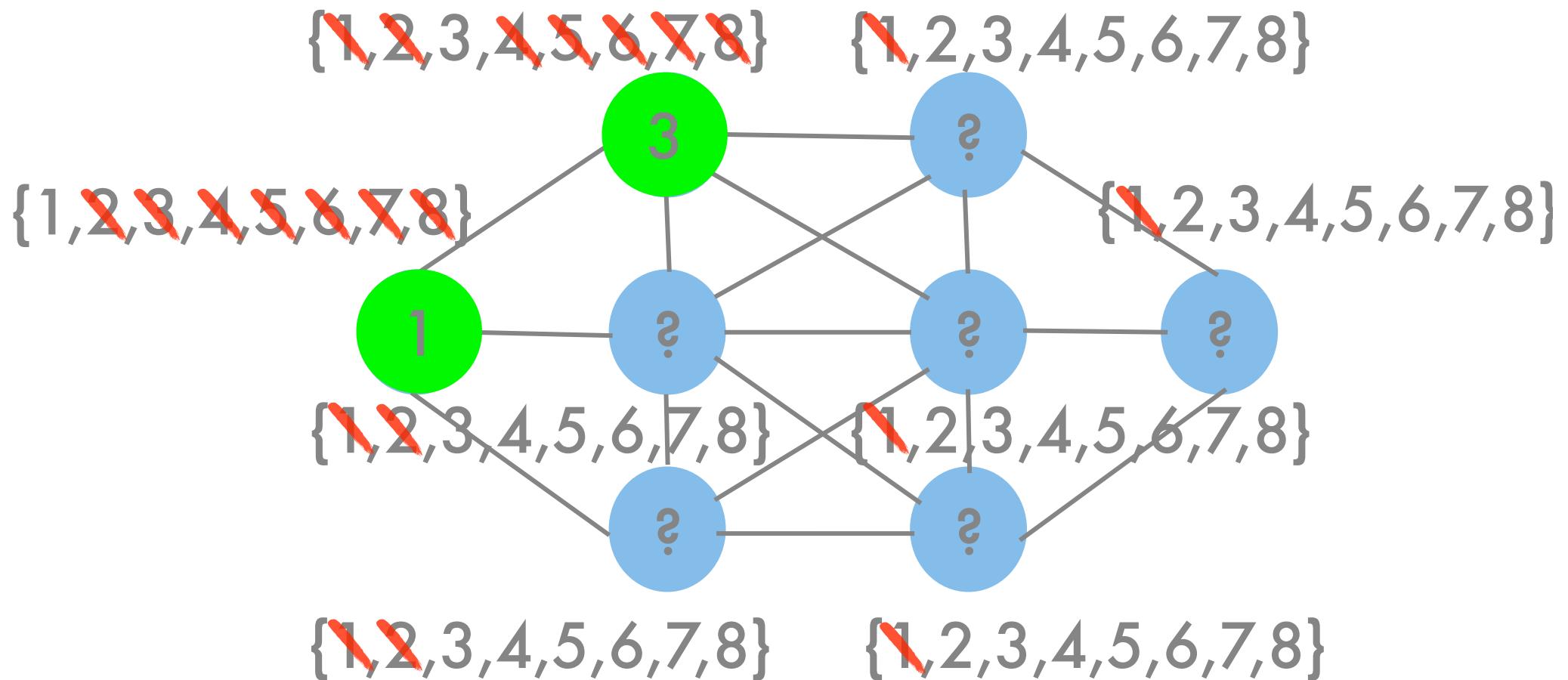
CP model in action



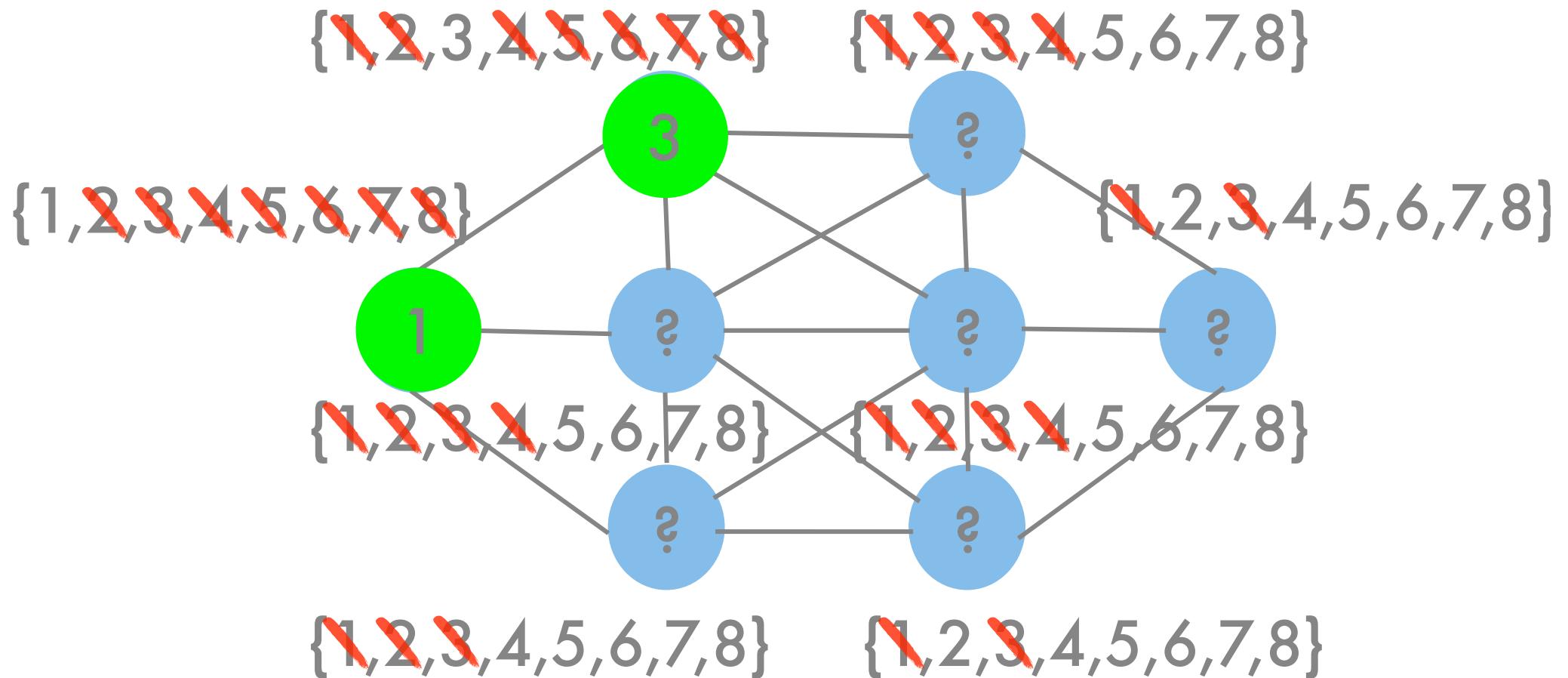
CP model in action



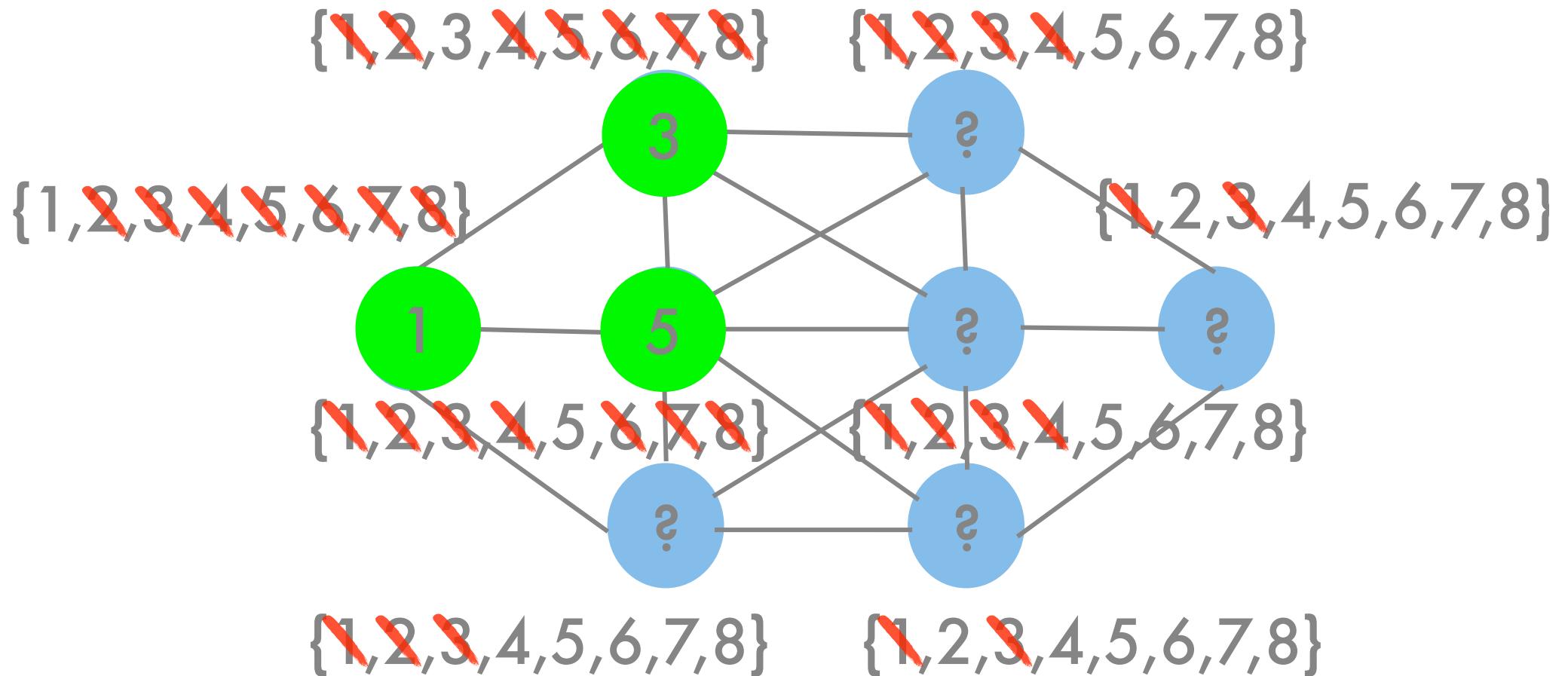
CP model in action



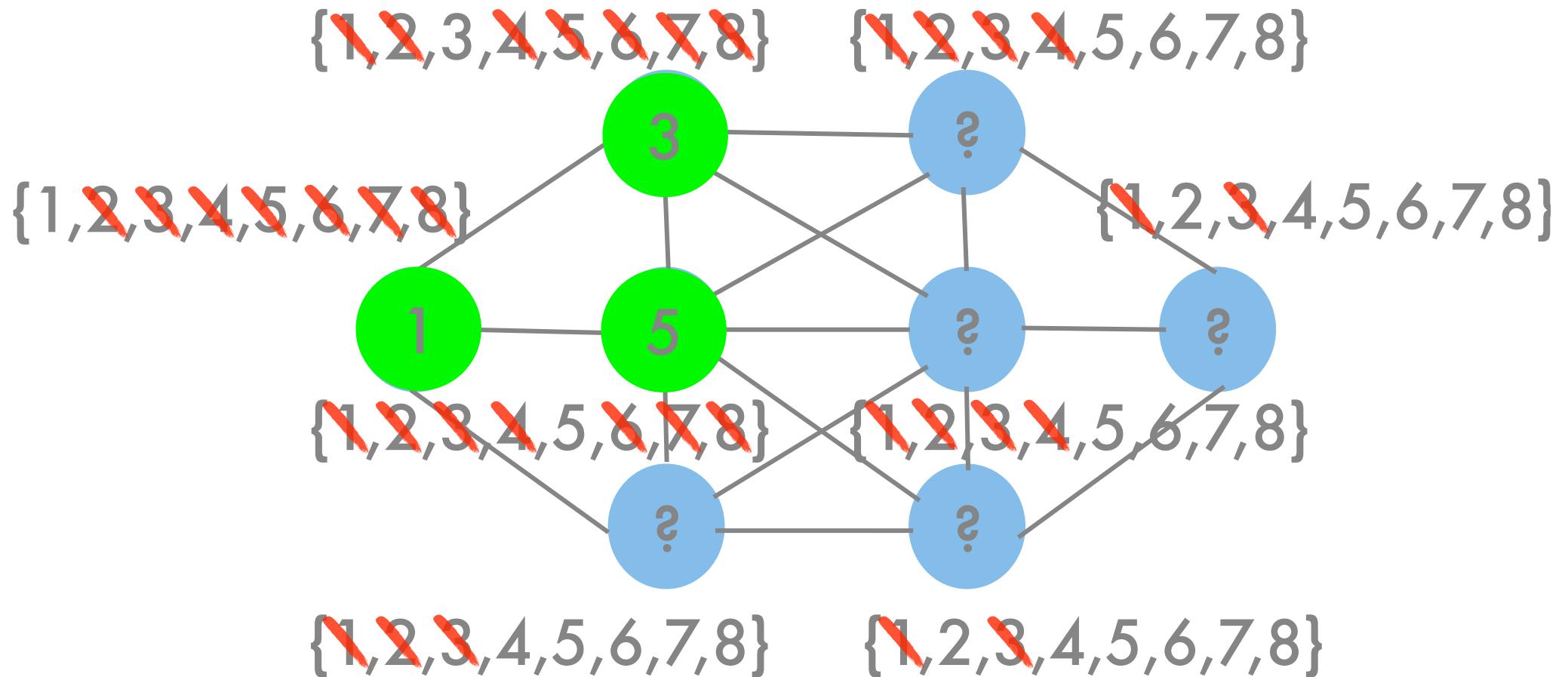
CP model in action



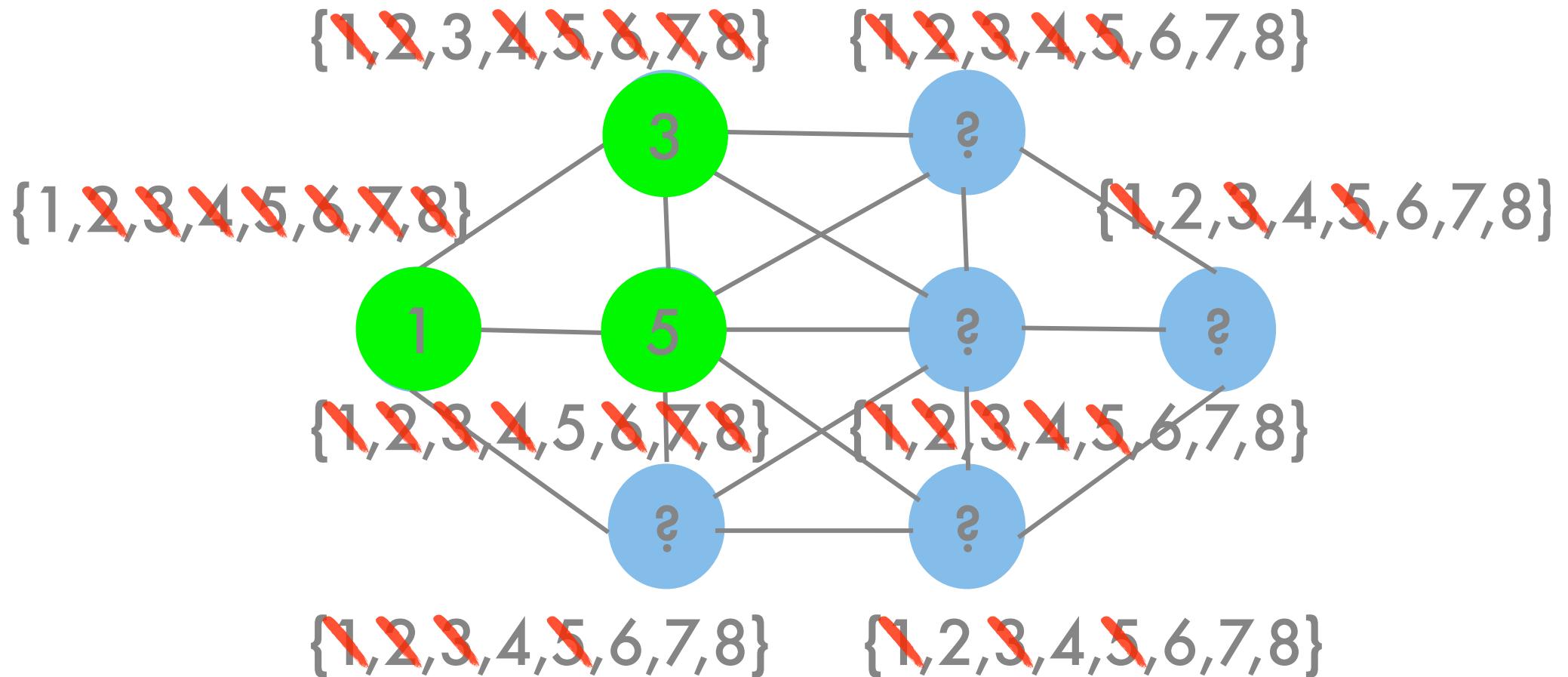
CP model in action



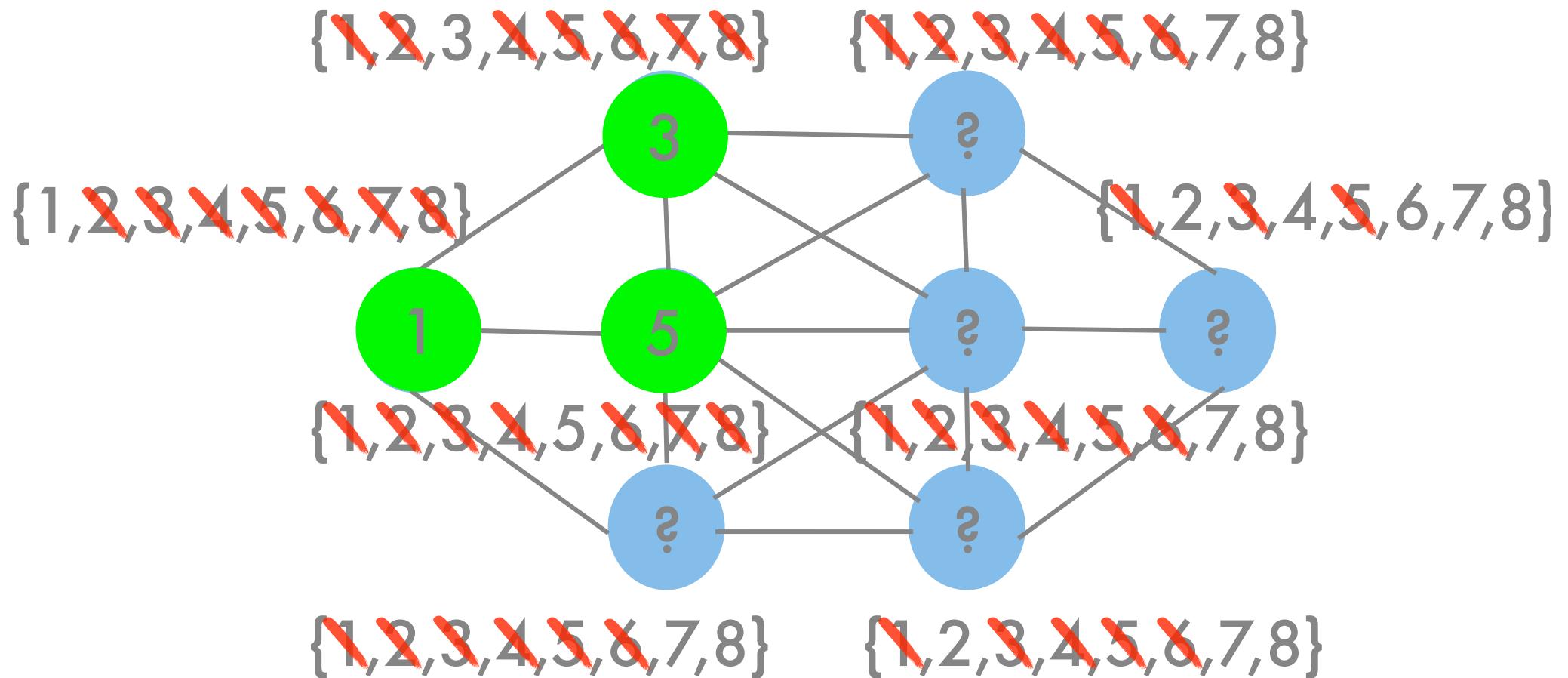
CP model in action



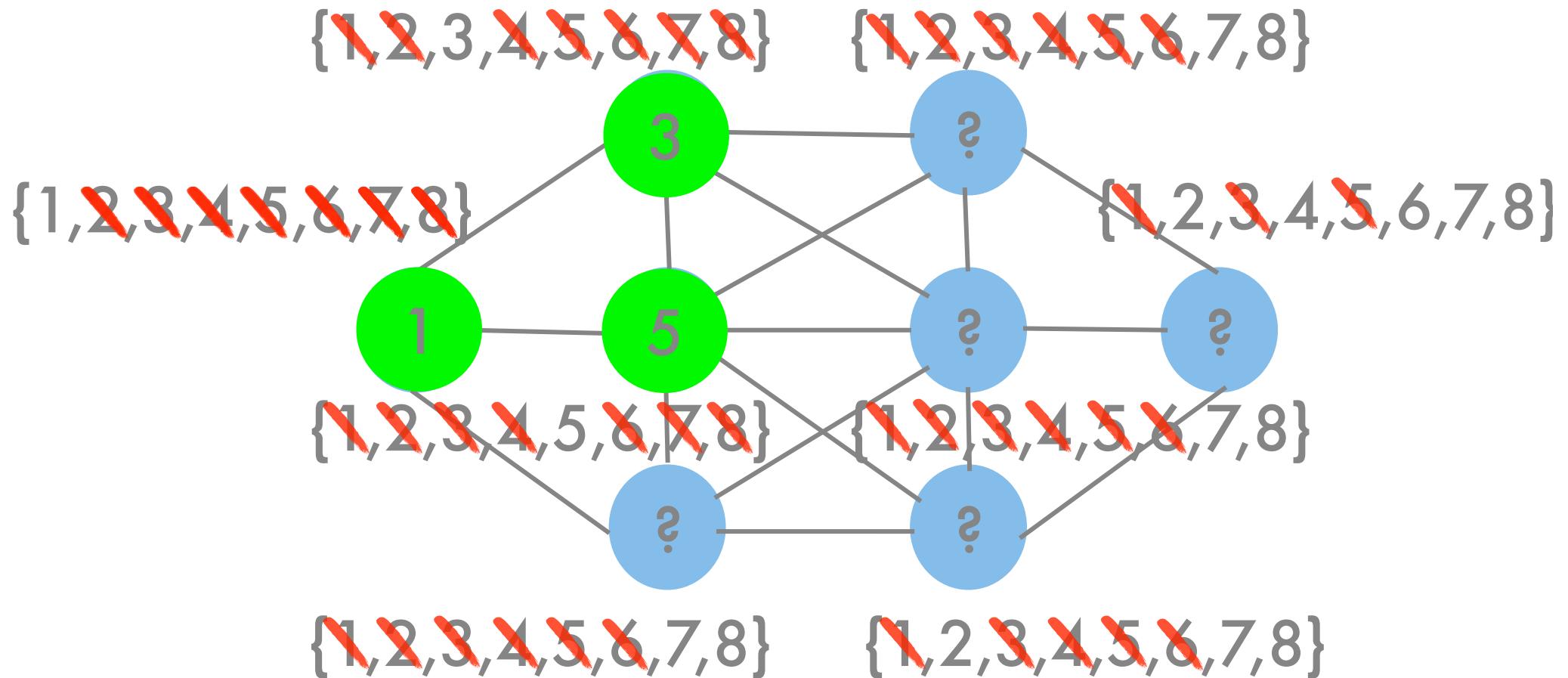
CP model in action



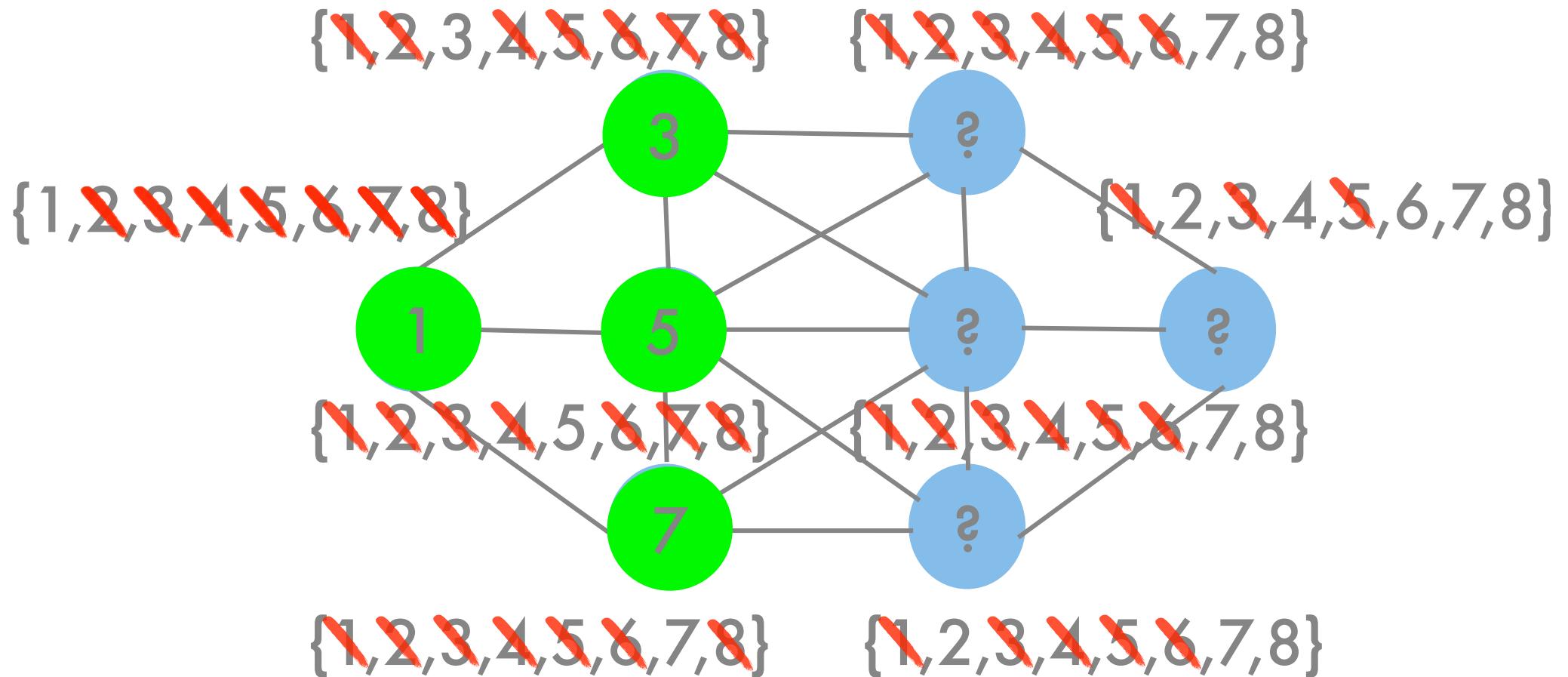
CP model in action



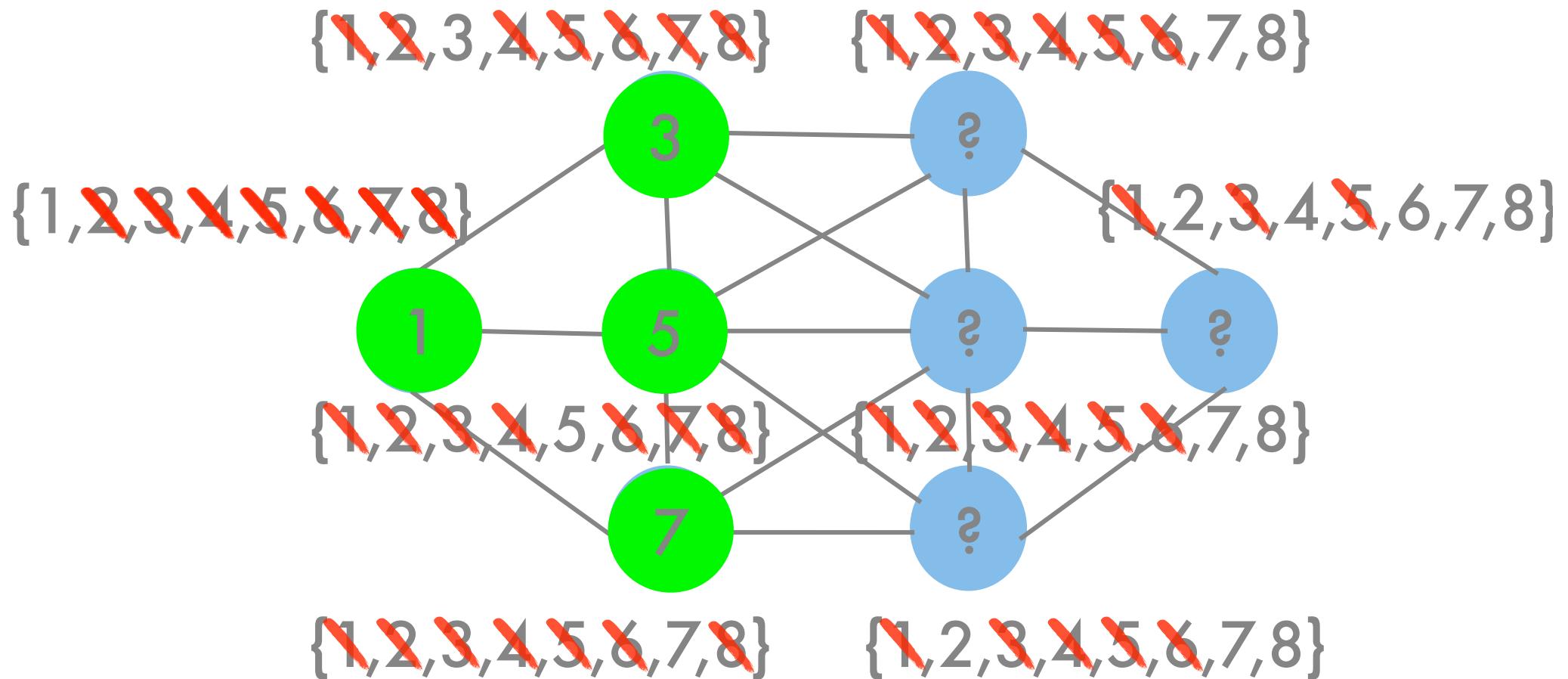
CP model in action



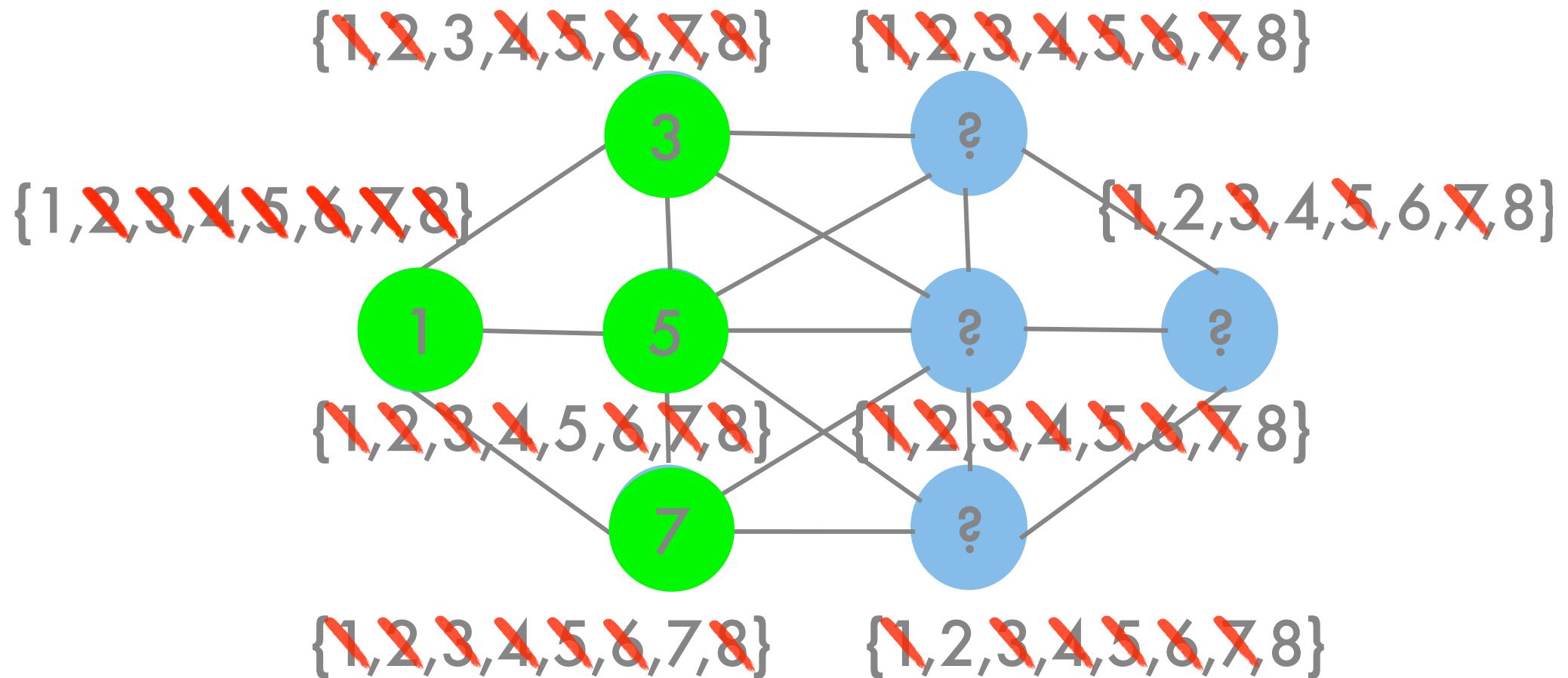
CP model in action



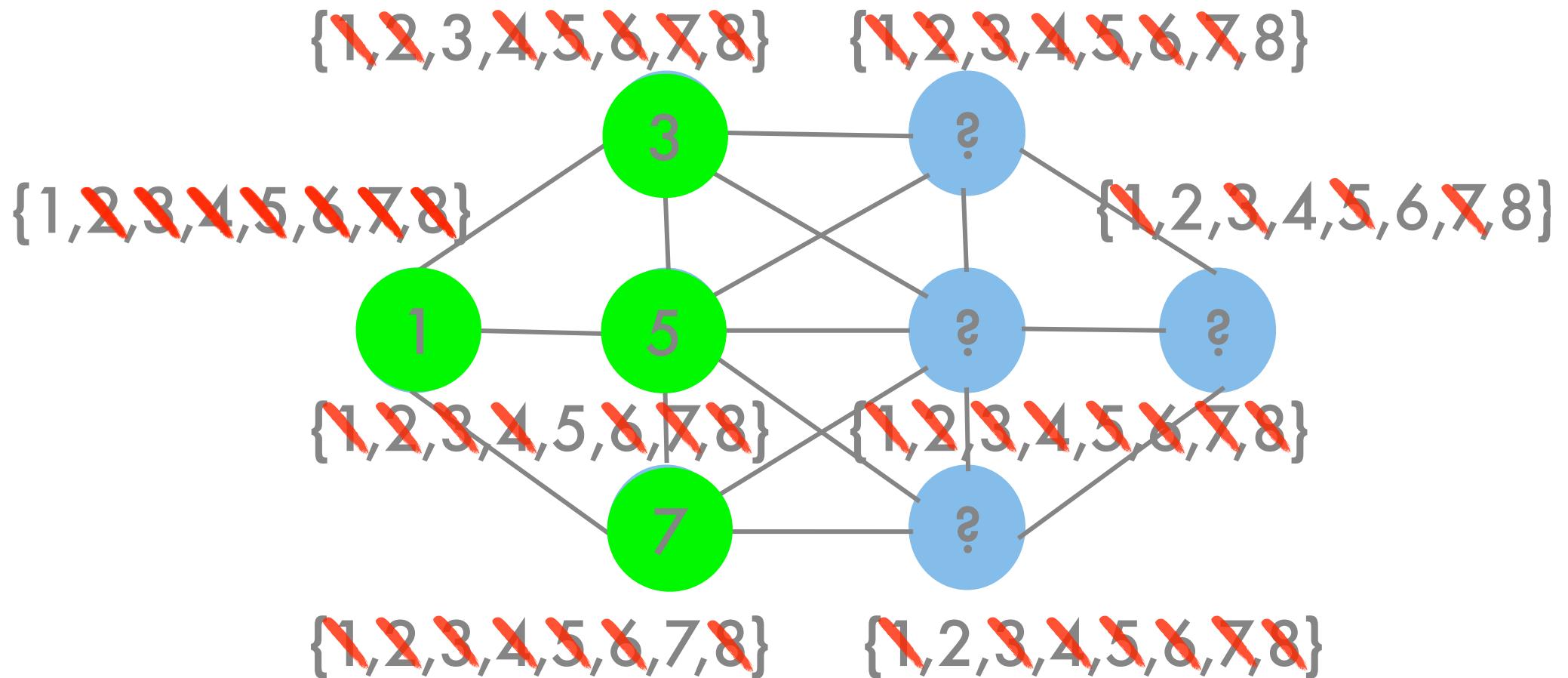
CP model in action



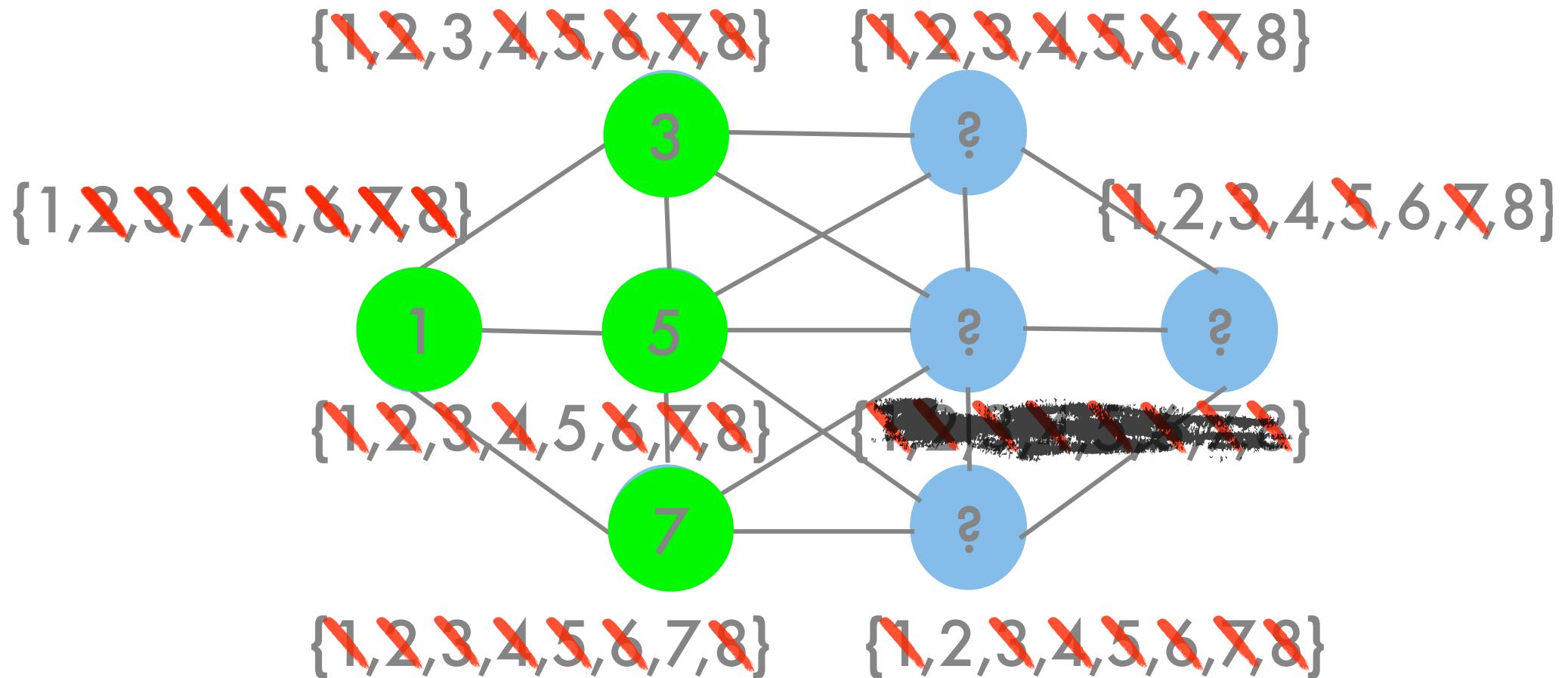
CP model in action

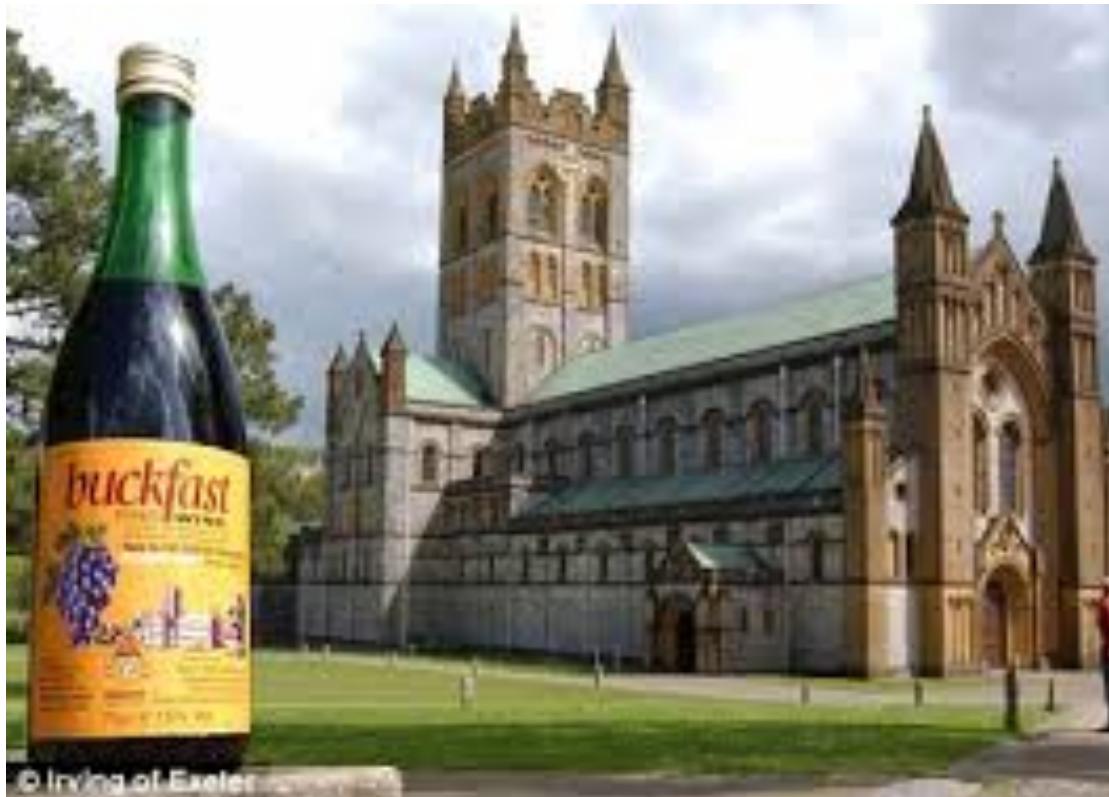


CP model in action



CP model in action





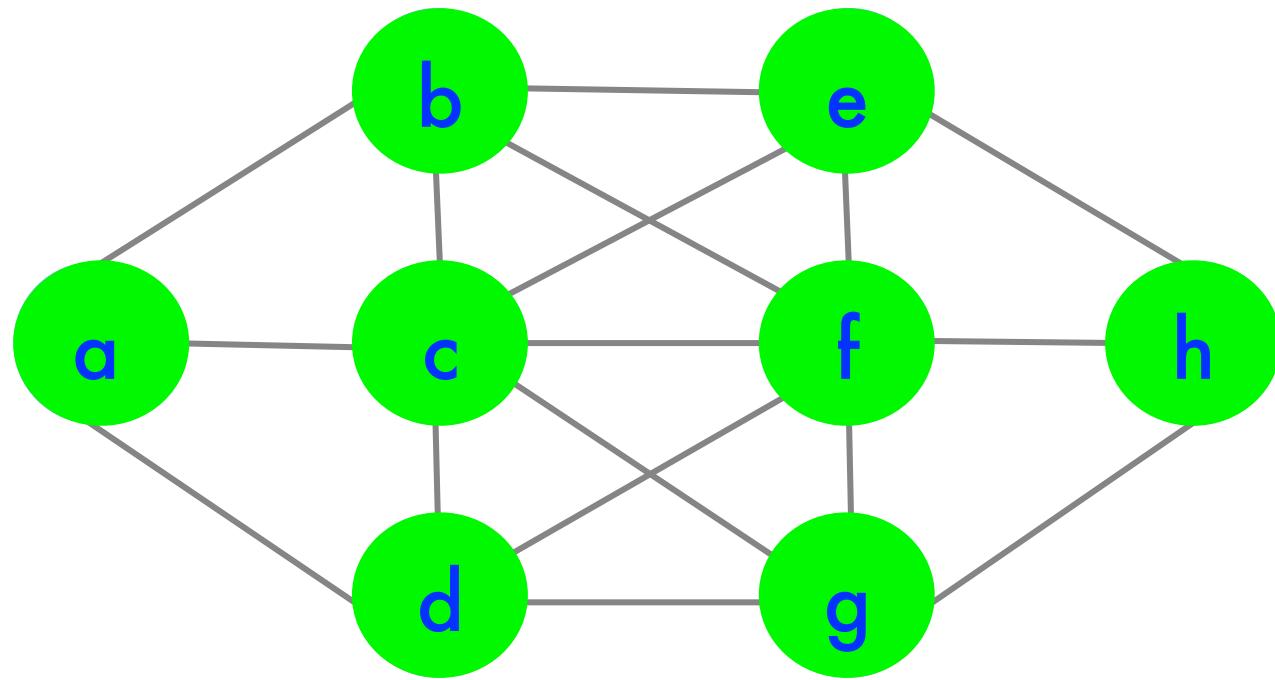
So now we
backtrack

Modelling Techniques

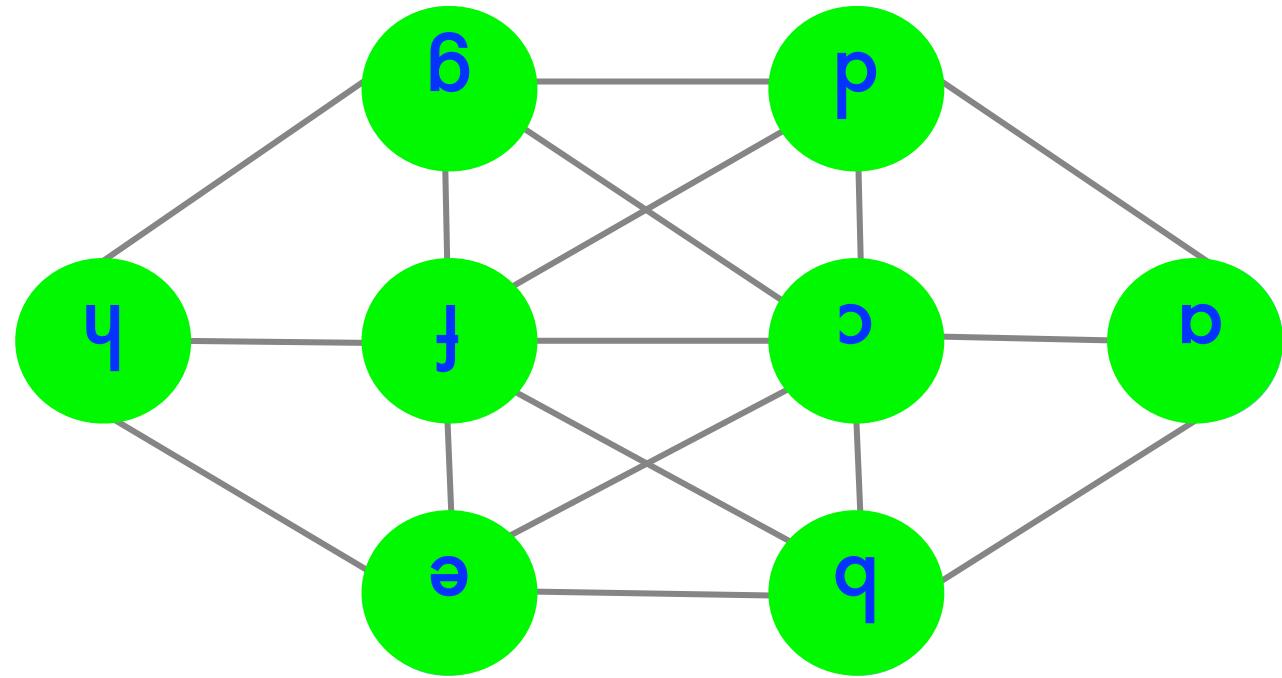
What now?

- Minion takes 104 search nodes to find the first solution.
- So what can we do to make our model better and solve this problem more efficiently?
- What do humans do?

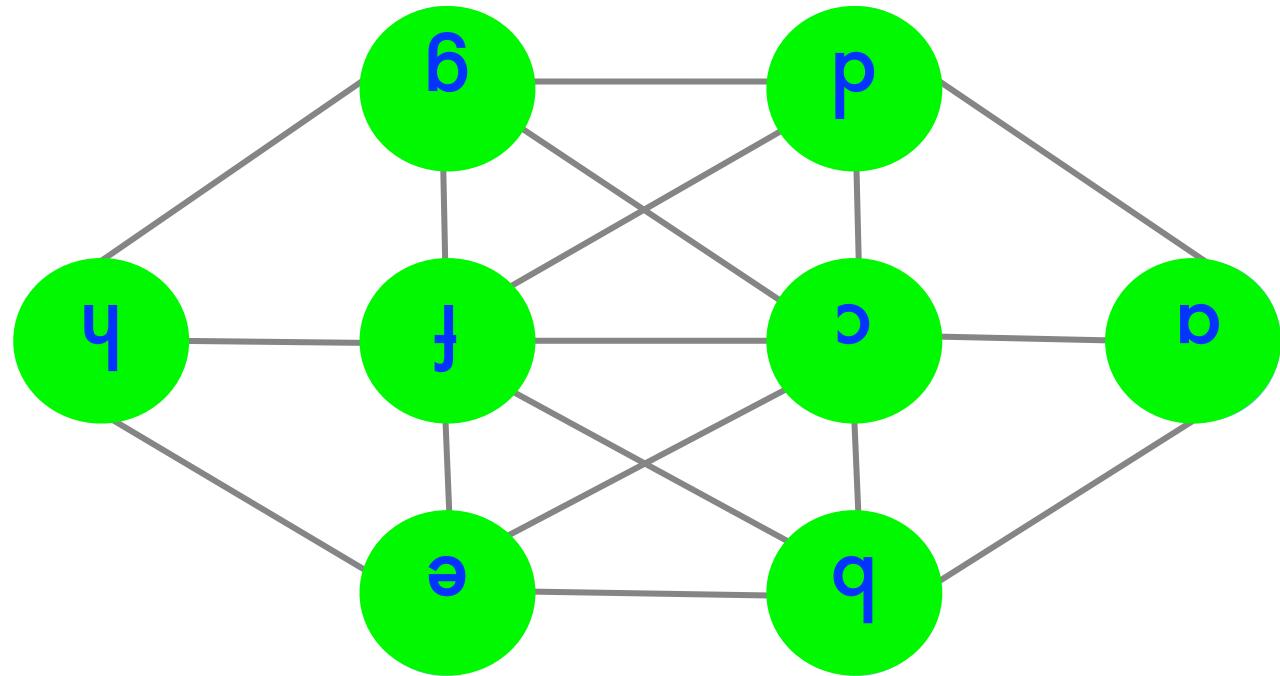
Exploit Symmetry



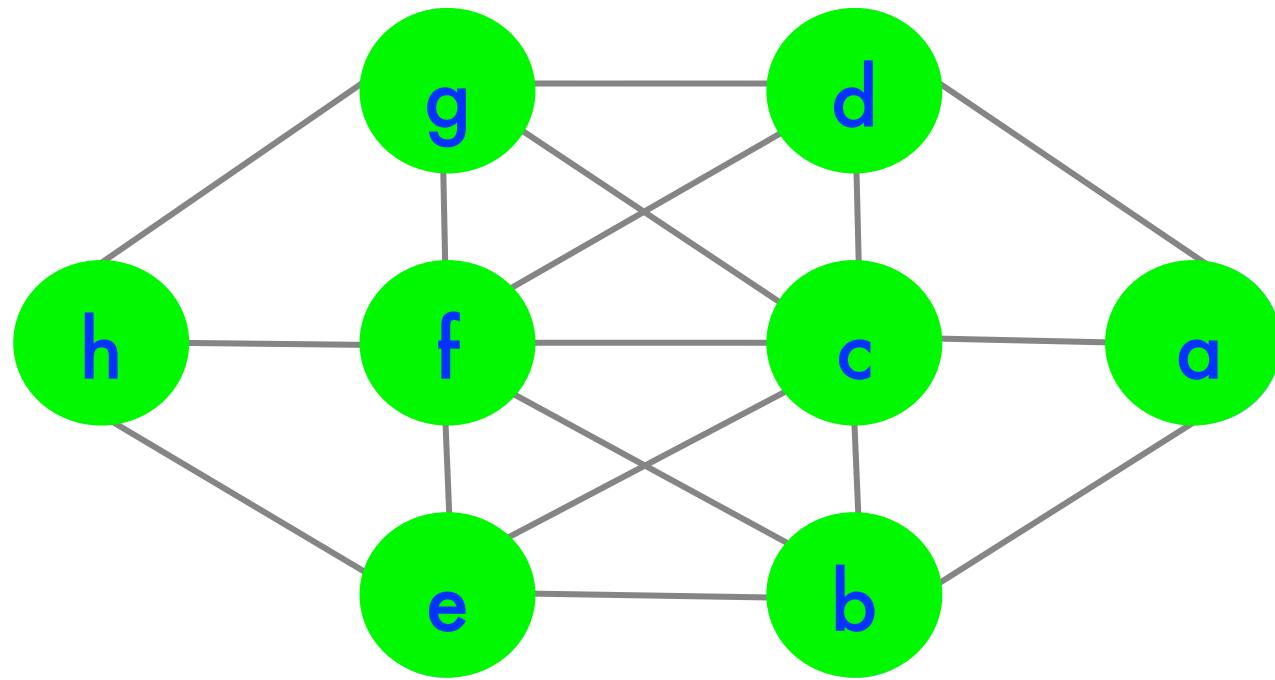
Exploit Symmetry



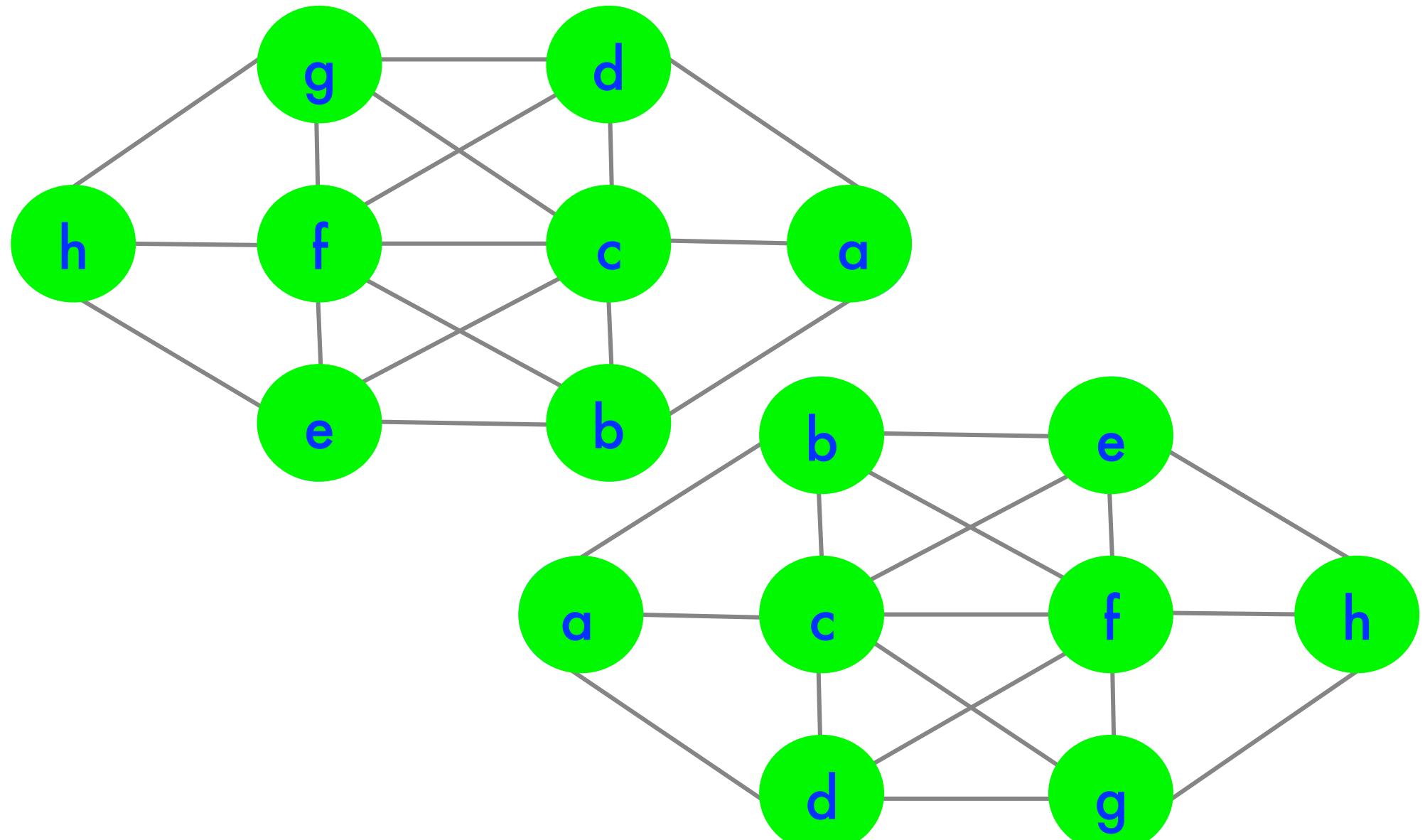
Exploit Symmetry



Exploit Symmetry



Exploit Symmetry



Why do we care about Symmetry

- Only search one symmetrically equivalent branch of the search tree:
 - cuts the search space of the CP
- Symmetry leads to large search spaces and must be eliminated
- Symmetry gives you a handle on how to cut down the search space of your problem, it can help you find a solution

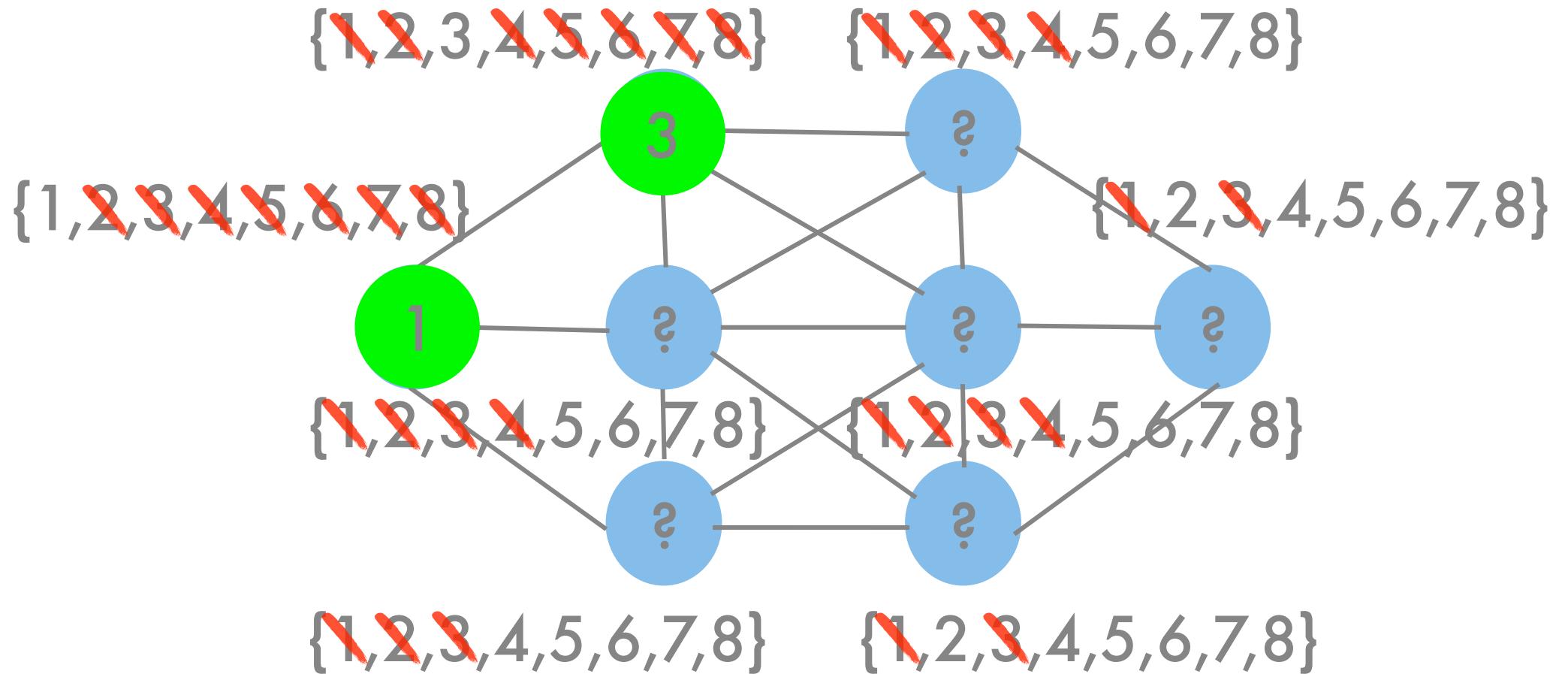
What is a symmetry?

- Two view points:
 - Symmetries are defined by the solutions.
 - Symmetries are defined by the constraints
- Symmetry Definitions for Constraint Satisfaction Problem by K. E. Petrie with D. Cohen, P. Jeavons, C. A. Jefferson and B.M. Smith. Constraints, Vol. 11,

Getting rid of Symmetry

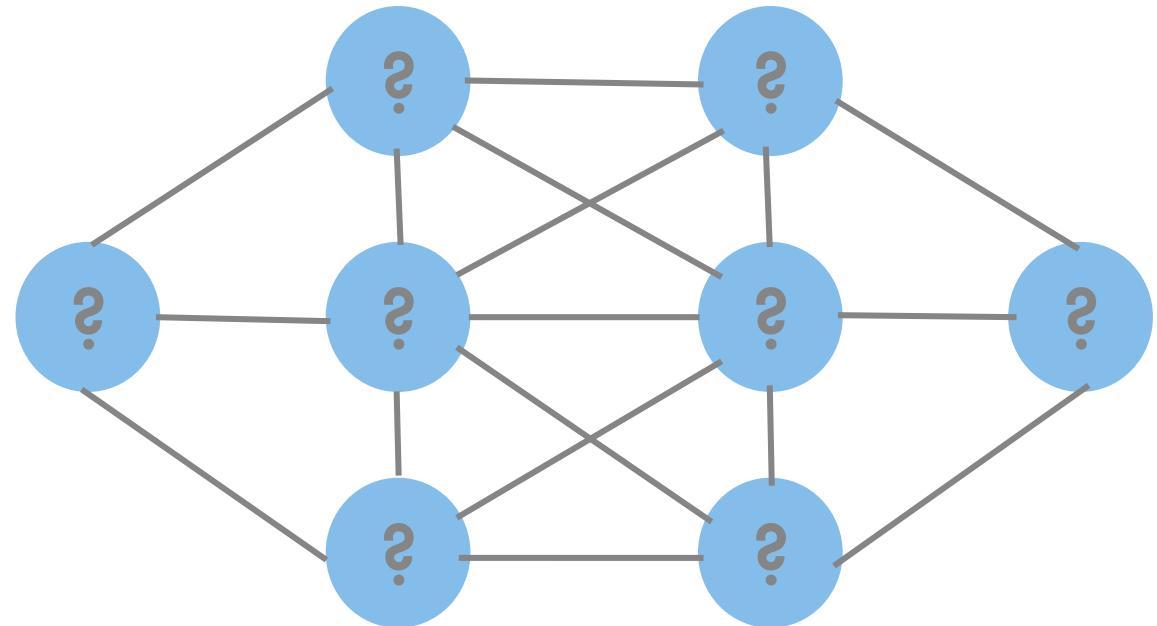
- Static Methods
 - Remodel problem
 - Add constraints
- Dynamic methods
 - nasty group theory
- Handbook of Constraints for more info by K. E. Petrie with I. P. Gent and J.-F. Puget

Think about ordering

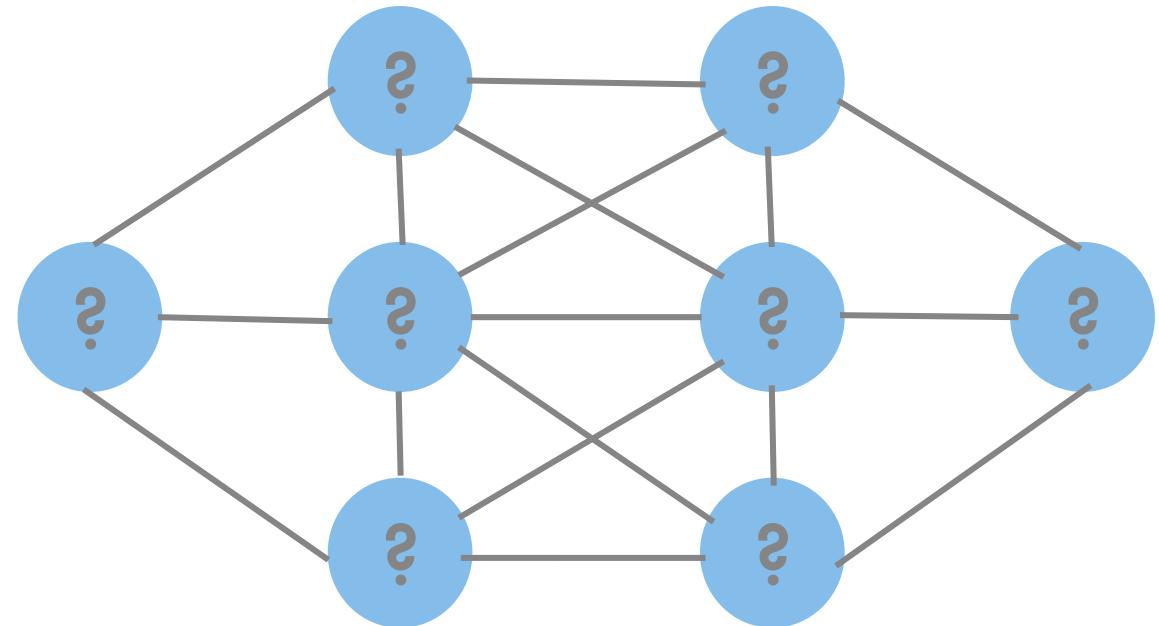


Is there a better Variable or Value Ordering?

Anything else?

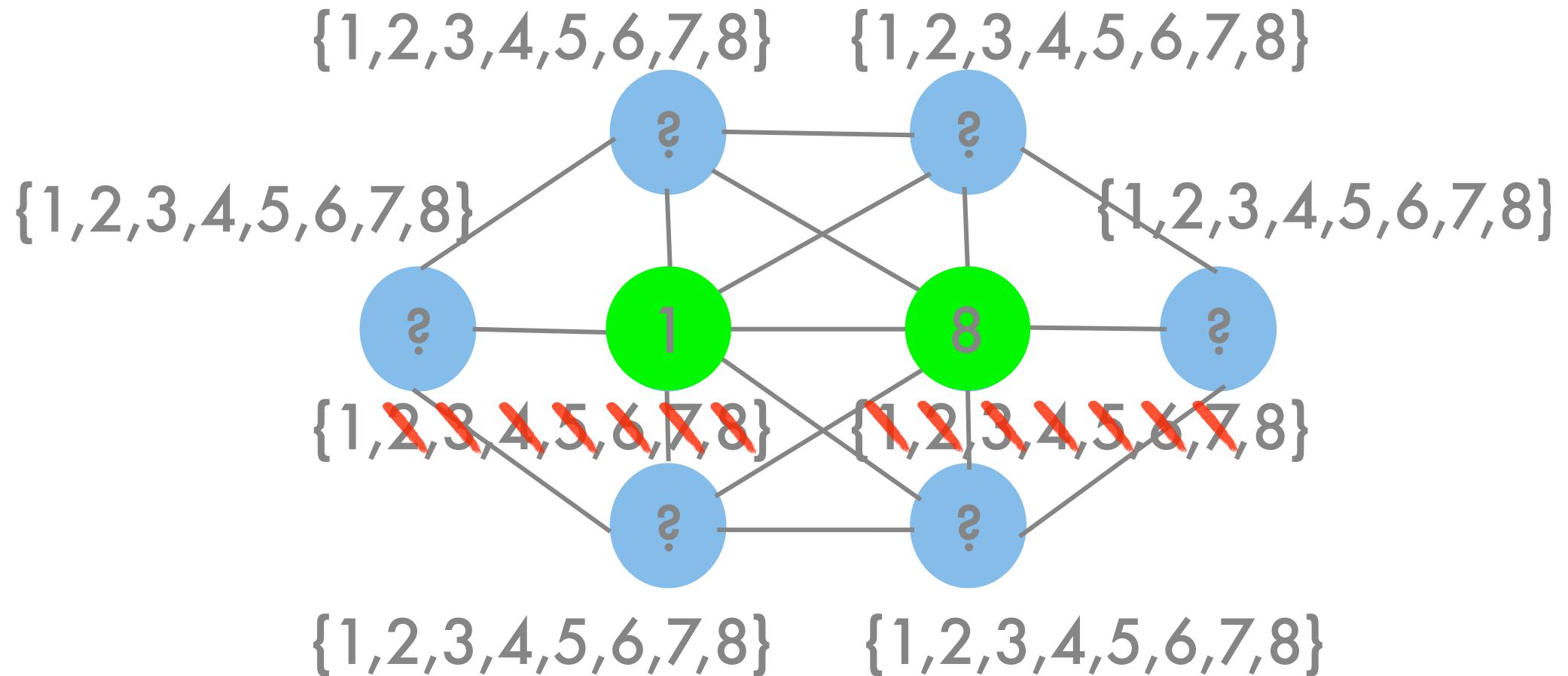


Anything else?

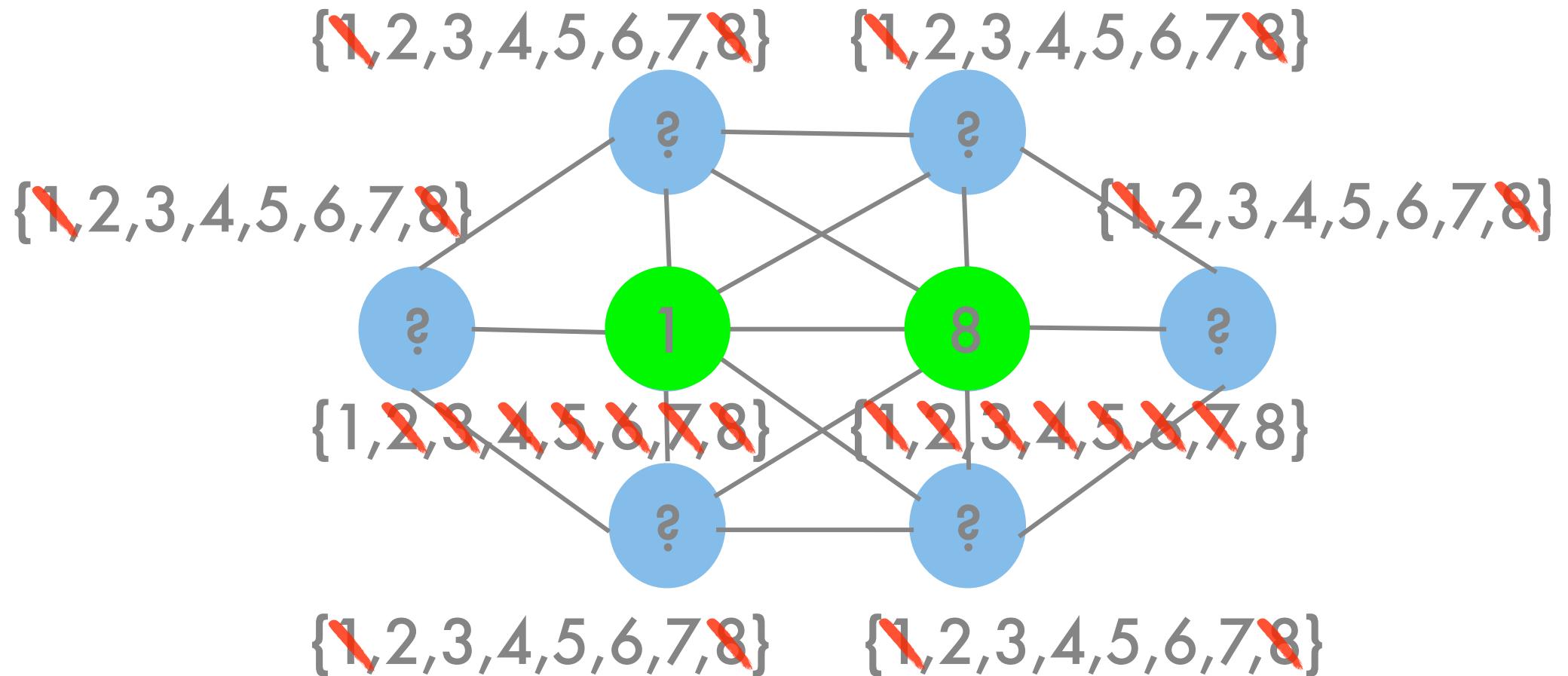


- What else did you notice?
 - The middle variables are very tightly constrained so they can only be 1 and 8.

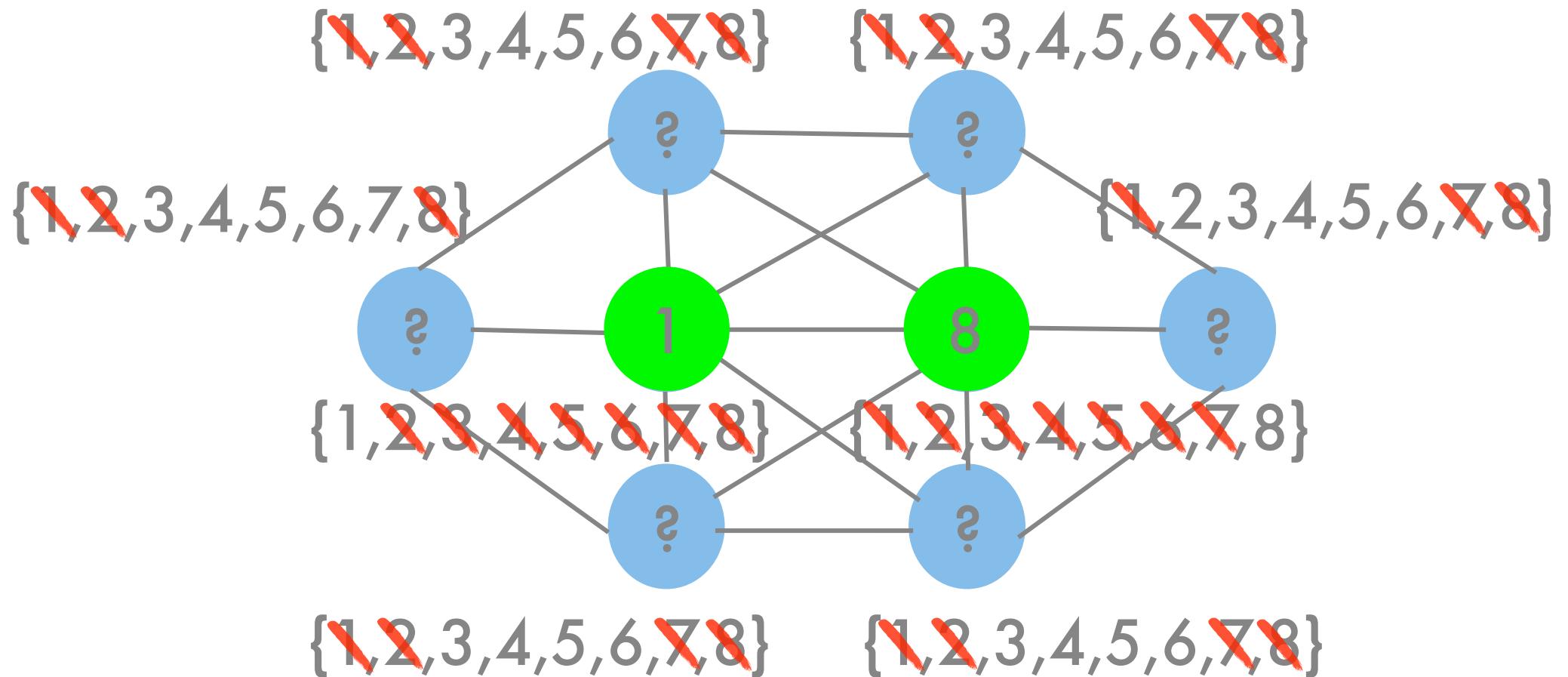
Improved model in action



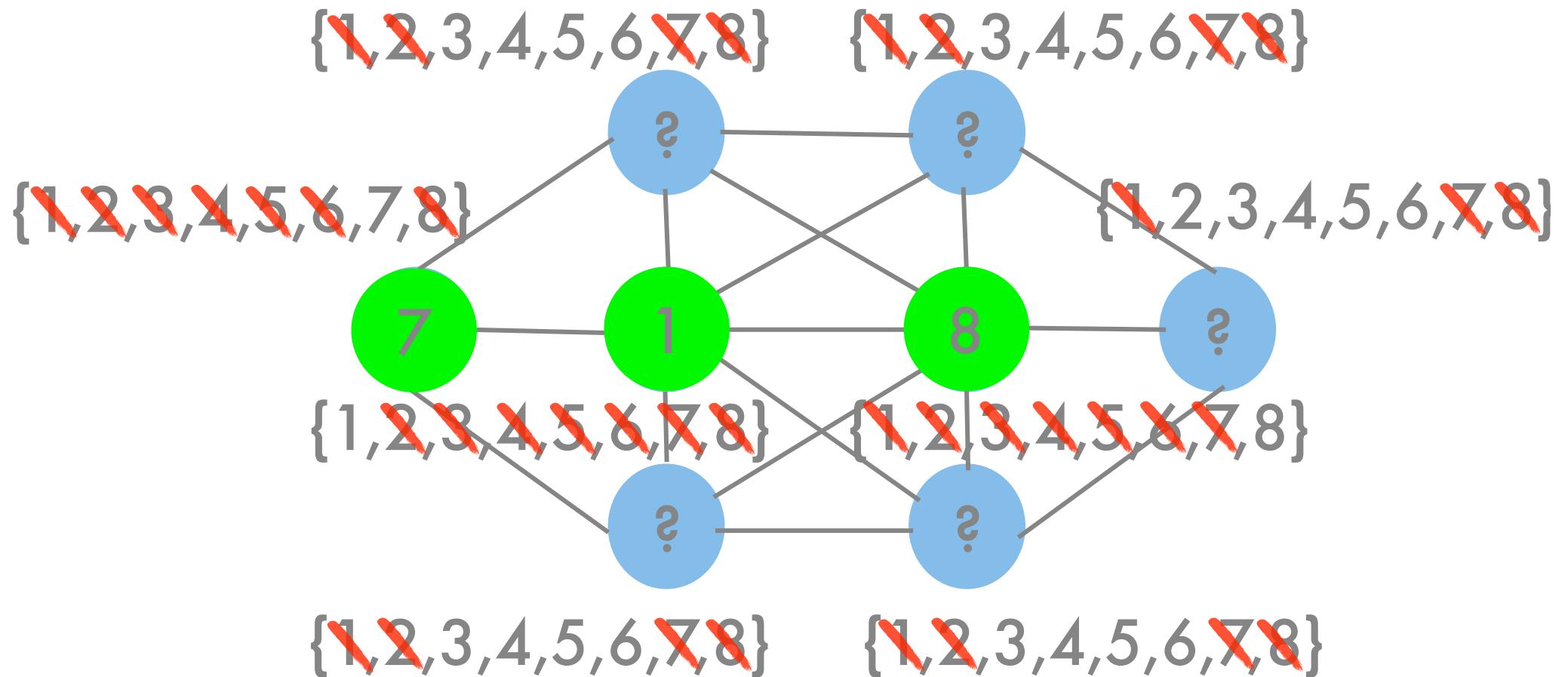
Improved model in action



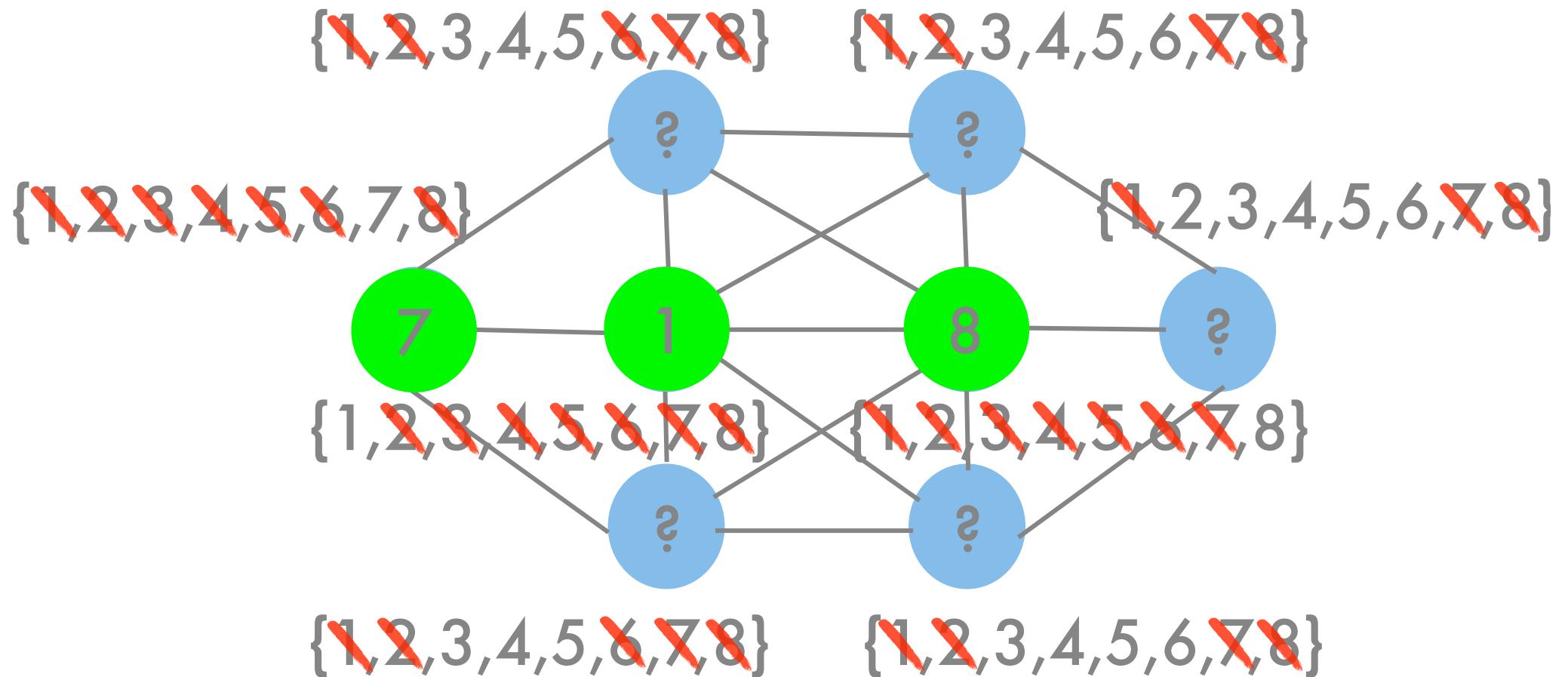
Improved model in action



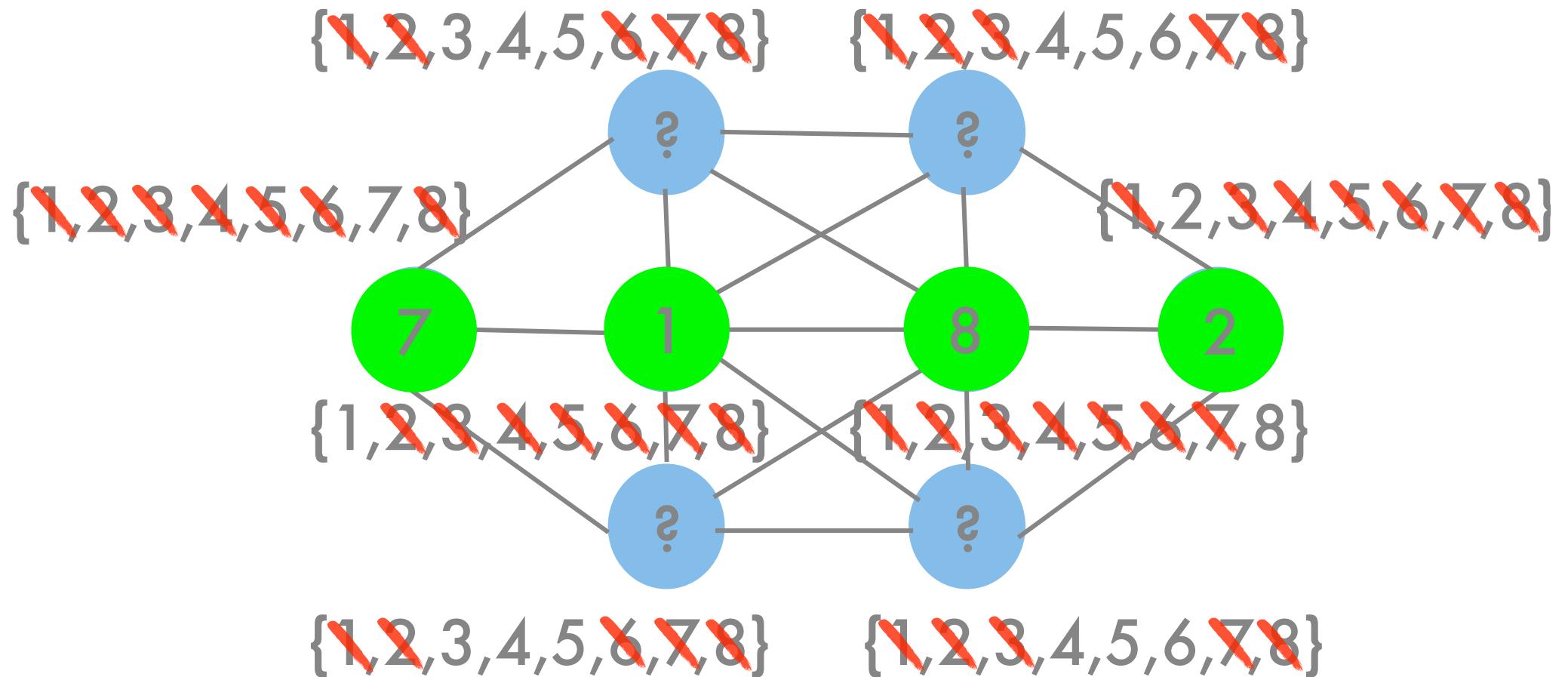
Improved model in action



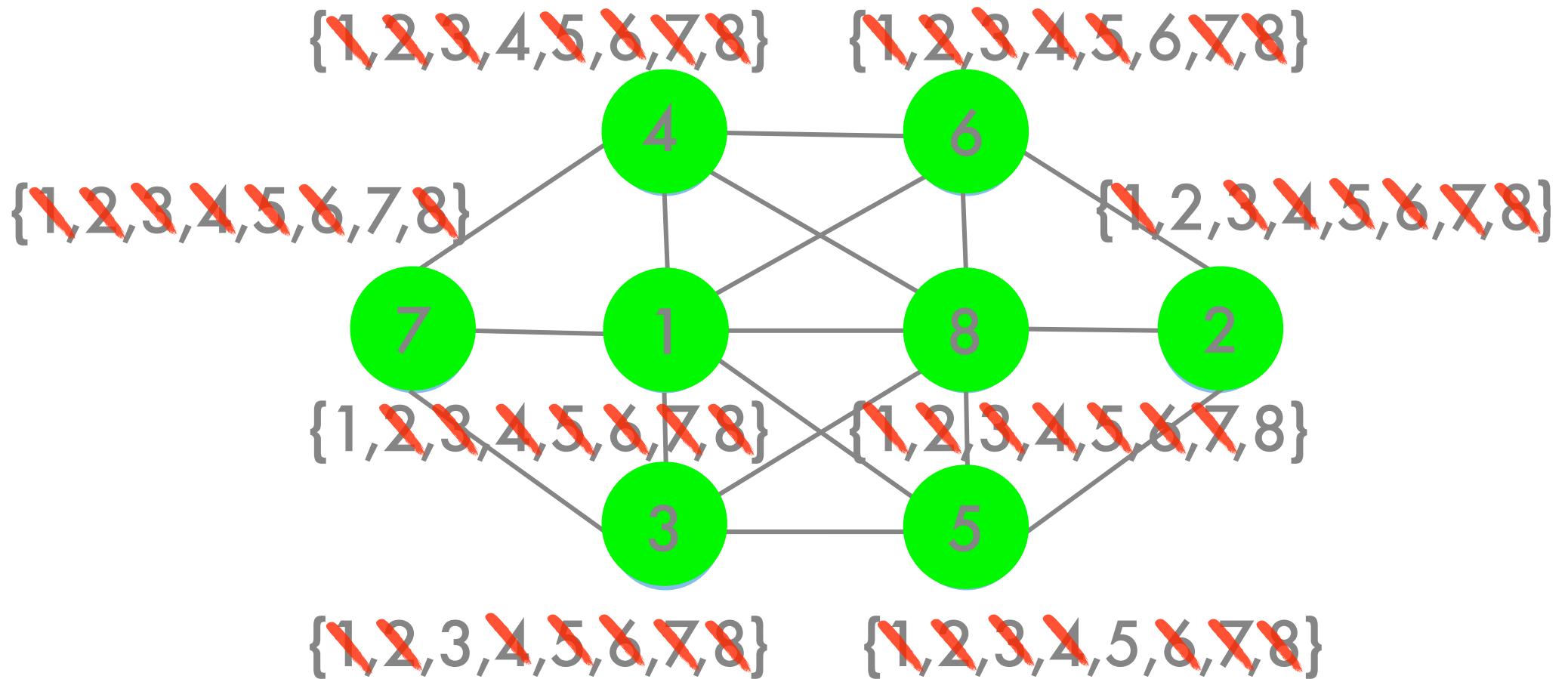
Improved model in action



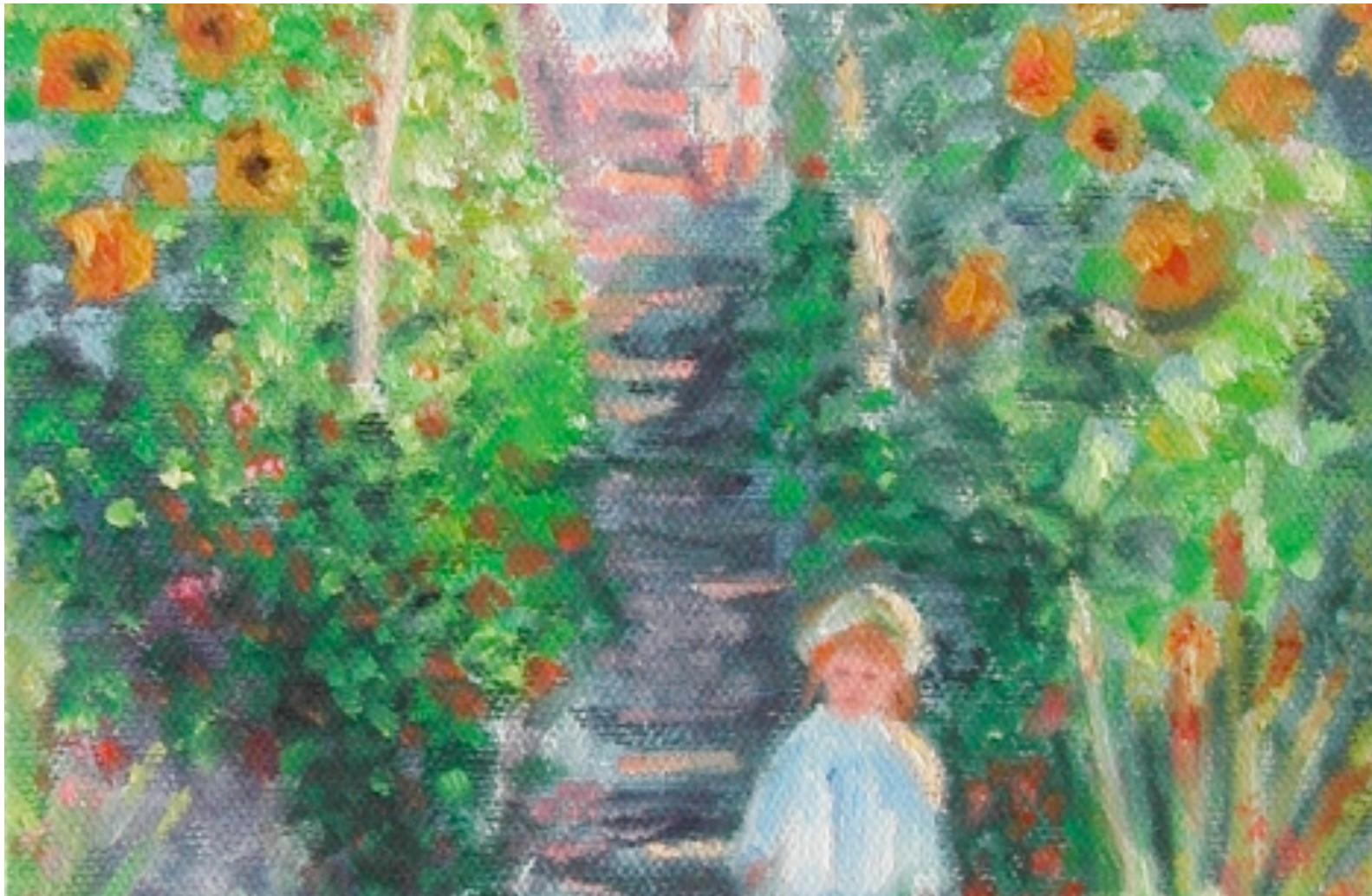
Improved model in action



Improved model in action



Minion can solve this model in 1 search node



Modelling = Art



Modelling = Power

Conclusion

- CP is useful and Fun
- Modelling a problem as a CP is still more than an art of a science
 - But we are working on that!
- The flexibility that the modelling process offers means that we can solve a large variety of interesting problems
- The focus of this course is to learn modelling.