MA318 偏微分方程 作业一

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Ex. 1.1/6

$$\xi = x - at$$
$$\frac{\partial F(\xi)}{\partial x} = \frac{dF}{d\xi}\frac{\partial \xi}{\partial x} = F'(\xi)$$
$$\frac{\partial^2 F(\xi)}{\partial x^2} = \frac{\partial F'(\xi)}{\partial x} = \frac{dF'(\xi)}{d\xi}\frac{\partial \xi}{\partial x} = F''(\xi)$$
$$\frac{\partial F(\xi)}{\partial t} = \frac{dF}{d\xi}\frac{\partial \xi}{\partial t} = -aF'(\xi)$$
$$\frac{\partial^2 F(\xi)}{\partial t^2} = -a\frac{\partial F'(\xi)}{\partial t} = -a\frac{dF'(\xi)}{d\xi}\frac{\partial \xi}{\partial t} = a^2 F''(\xi)$$
$$\frac{\partial^2 F(\xi)}{\partial t^2} - a^2\frac{\partial^2 F(\xi)}{\partial x^2} = 0$$

同理，

$$\xi = x + at$$
$$\frac{\partial^2 G(\xi)}{\partial t^2} - a^2\frac{\partial^2 G(\xi)}{\partial x^2} = 0$$

Ex. 1.1/7

$$\frac{\partial u}{\partial t} = -t(t^2 - x^2 - y^2)^{-\frac{3}{2}}$$
$$\frac{\partial^2 u}{\partial t^2} = 3t^2(t^2 - x^2 - y^2)^{-\frac{5}{2}} - (t^2 - x^2 - y^2)^{-\frac{3}{2}} = (t^2 - x^2 - y^2)^{-\frac{5}{2}}(3t^2 - t^2 + x^2 + y^2)$$
$$\frac{\partial u}{\partial x} = x(t^2 - x^2 - y^2)^{-\frac{3}{2}}$$
$$\frac{\partial^2 u}{\partial x^2} = 3x^2(t^2 - x^2 - y^2)^{-\frac{5}{2}} + (t^2 - x^2 - y^2)^{-\frac{3}{2}} = (t^2 - x^2 - y^2)^{-\frac{5}{2}}(3x^2 + t^2 - x^2 - y^2)$$

$$\frac{\partial u}{\partial y} = y(t^2 - x^2 - y^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(t^2 - x^2 - y^2)^{-\frac{5}{2}} + (t^2 - x^2 - y^2)^{-\frac{3}{2}} = (t^2 - x^2 - y^2)^{-\frac{5}{2}}(3y^2 + t^2 - x^2 - y^2)$$

$$3t^2 - t^2 + x^2 + y^2 = (3x^2 + t^2 - x^2 - y^2) + (3y^2 + t^2 - x^2 - y^2)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Ex. 2.1/1

Ex. 2.1/2

Ex. 2.1/3