MA319 — 偏微分方程

Assignment 1

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习题　1.1/6

对于 $F(\xi) = F(x - at)$

$$\xi = x - at, \quad \frac{\partial \xi}{\partial t} = -a, \quad \frac{\partial \xi}{\partial x} = 1,$$

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} = -aF'(\xi),$$

$$\frac{\partial^2 F}{\partial t^2} = \frac{\partial}{\partial t}[-aF'(\xi)] = \frac{\partial}{\partial \xi}[-aF'(\xi)] \cdot \frac{\partial \xi}{\partial t} = a^2 F''(\xi),$$

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} = F'(\xi),$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}[F'(\xi)] = \frac{\partial}{\partial \xi}[F'(\xi)] \cdot \frac{\partial \xi}{\partial x} = F''(\xi).$$

故

$$\frac{\partial^2 F}{\partial t^2} - a^2 \frac{\partial^2 F}{\partial x^2} = 0.$$

对于 $G(\xi) = G(x + at)$

$$\xi = x + at, \quad \frac{\partial \xi}{\partial t} = a, \quad \frac{\partial \xi}{\partial x} = 1,$$

$$\frac{\partial G}{\partial t} = \frac{\partial G}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} = aG'(\xi),$$

$$\frac{\partial^2 G}{\partial t^2} = \frac{\partial}{\partial t}[-aG'(\xi)] = \frac{\partial}{\partial \xi}[aG'(\xi)] \cdot \frac{\partial \xi}{\partial t} = a^2 G''(\xi),$$

$$\frac{\partial G}{\partial x} = \frac{\partial G}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} = G'(\xi),$$

$$\frac{\partial^2 G}{\partial x^2} = \frac{\partial}{\partial x}[G'(\xi)] = \frac{\partial}{\partial \xi}[G'(\xi)] \cdot \frac{\partial \xi}{\partial x} = G''(\xi).$$

故

$$\frac{\partial^2 G}{\partial t^2} - a^2 \frac{\partial^2 G}{\partial x^2} = 0.$$

习题　1.1/7

$$u(x, y, t) = (t^2 - x^2 - y^2)^{1/2}, \quad t^2 - x^2 - y^2 > 0.$$

一阶偏导数为

$$\frac{\partial u}{\partial t}=\frac{1}{2}(t^2-x^2-y^2)^{-1/2}\cdot 2t=\frac{t}{u},$$

$$\frac{\partial u}{\partial x}=\frac{1}{2}(t^2-x^2-y^2)^{-1/2}\cdot -2x=-\frac{x}{u},$$

$$\frac{\partial u}{\partial y}=\frac{1}{2}(t^2-x^2-y^2)^{-1/2}\cdot -2y=-\frac{y}{u}.$$

二阶偏导数为

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial}{\partial t}\frac{t}{u}=\frac{\partial t}{\partial t}\cdot\frac{1}{u}+\frac{\partial}{\partial t}\frac{1}{u}\cdot t=\frac{1}{u}-\frac{3t^2}{u^3},$$

$$\frac{\partial^2 u}{\partial x^2}=-\frac{\partial}{\partial x}\frac{x}{u}=-\frac{\partial x}{\partial x}\cdot\frac{1}{u}-\frac{\partial}{\partial x}\frac{1}{u}\cdot t=-\frac{1}{u}-\frac{3x^2}{u^3},$$

$$\frac{\partial^2 u}{\partial y^2}=-\frac{\partial}{\partial y}\frac{y}{u}=-\frac{\partial y}{\partial y}\cdot\frac{1}{u}-\frac{\partial}{\partial y}\frac{1}{u}\cdot t=-\frac{1}{u}-\frac{3y^2}{u^3}.$$

故

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=\frac{3}{u}+\frac{-3t^2+3x^2+3y^2}{u^3}=\frac{3}{u}+\frac{-3u^2}{u^3}=0,$$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}.$$