MA319 — 偏微分方程

Assignment 8

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求解热传导方程 (3.17) 的柯西问题, 已知

(1) $u|_{t=0} = \sin x$,

(2) 用延拓法求解半有界直线上的热传导方程 (3.17), 假设

$$\begin{cases} u(x,0) = \varphi(x) \quad (0 < x < \infty), \\ u(0,t) = 0. \end{cases}$$

(i)

初值条件为

$$\varphi(x) = u|_{t=0} = \sin x.$$

故

$$\begin{aligned}
u(x,t) &= \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} \varphi(\xi) e^{-\frac{(x-\xi)^2}{4a^2 t}} \, d\xi \\
&= \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} \varphi(x + 2a\sqrt{t}\eta) e^{-\eta^2} \, d(x + 2a\sqrt{t}\eta) \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \sin\left(x + 2a\sqrt{t}\eta\right) e^{-\eta^2} \, d\eta \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[\sin x \cos\left(2a\sqrt{t}\eta\right) + \cos x \sin\left(2a\sqrt{t}\eta\right)\right] e^{-\eta^2} \, d\eta \\
&= \frac{\sin x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \cos\left(2a\sqrt{t}\eta\right) e^{-\eta^2} \, d\eta \\
&= \frac{\sin x}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \left[e^{-\eta^2 + i2a\sqrt{t}\eta} + e^{-\eta^2 - i2a\sqrt{t}\eta}\right] d\eta \\
&= \frac{\sin x}{2\sqrt{\pi}} e^{-a^2 t} \left[\int_{-\infty}^{\infty} e^{-(\eta - ia\sqrt{t})^2} \, d\eta + \int_{-\infty}^{\infty} e^{-(\eta + ia\sqrt{t})^2} \, d\eta\right] \\
&= \frac{\sin x}{\sqrt{\pi}} e^{-a^2 t} \int_{-\infty}^{\infty} e^{-\chi^2} \, d\chi \\
&= e^{-a^2 t} \sin x.
\end{aligned}$$

(2)

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