DIFFERENTIAL GEOMETRY HOMEWORK 3

Each problem is worth 10 points.

1. Show that the sum of the normal curvature at p for any pair of orthogonal directions is constant.

2. (Willmore energy) Let S be the torus obtained by rotating the circle $(y-2)^2 + z^2 = 1$ in $y - z$ plane around z-axis, compute $\int_S H^2 dA$.

3. Describe the region of the unit sphere covered by the images of the Gauss map of the following surfaces:
- $x^2 + y^2 - z^2 = 1$.
- $x^2 + y^2 = \cosh^2 z$.

4. (Local convexity) A surface is locally convex at p if there exists a neighborhood of p which is contained in one side of T_pS.
- Prove that if S is locally convex at p then $K(p) \geq 0$;
- Show that the converse of above is not true,i.e., $K \geq 0$ does not imply lcoal convexity. A counterexample would be the graph of $f(x, y) = x^3(1+y^2)$, show it is not locally convex at $(0, 0)$.

5. Determine all umbilical points of the spheroid
$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1.$$

Date: Due 11/12.