MA426 DIFFERENTIAL GEOMETRY HOMEWORK 3

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2018 年 11 月 12 日

Ex 1.

Let $d_1 \cdot d_2 = 0$, and $|d_1| = |d_2| = 1$ to be the direction vectors, then

$$d_1 = (\cos\theta, \sin\theta), d_2 = (\sin\theta, -\cos\theta)$$

Let the principle curvatures are k_1 and k_2, the normal curvatures are

$$\kappa_1 = k_1 \cos^2\theta + k_2 \sin^2\theta$$

$$\kappa_2 = k_1 \sin^2\theta + k_2 \cos^2\theta$$

So the sum of normal curvatures is

$$\kappa_1 + \kappa_2 = k_1 + k_2$$

which is a constant.

Ex 2.

$$(x^2 + y^2 - 2)^2 + z^2 = 1$$

$$z = \sin\theta, x^2 + y^2 - 2 = \cos\theta$$

$$x = (2 + \cos\theta)\cos\gamma, y = (2 + \cos\theta)\sin\gamma$$

Let $u = \theta, v = \gamma$,

$$r = ((2 + \cos u)\cos v, (2 + \cos u)\sin v, \sin u)$$

$$r_u = (-\sin u \cos v, -\sin u \sin v, \cos u)$$

$$r_v = (-(2 + \cos u)\sin v, (2 + \cos u)\cos v, 0)$$

$$E = r_u \cdot r_u = 1, F = r_u \cdot r_v = 0, G = r_u \cdot r_v = (2 + \cos u)^2$$

$$r_{uu} = (-\cos u \cos v, -\cos u \sin v, -\sin u)$$

$$r_{vv} = (-\cos u \cos v, (\cos u + 2) \sin v, 0)$$

$$L = -1, M = 0, N = -(2 + \cos u) \cos u$$

$$H = \frac{LG - 2MF + NE}{2(EG - F^2)} = \frac{-G + N}{2G} = -\frac{1 + \cos u}{2 + \cos u}$$

$$dA = (2 + \cos u) du dv$$

$$\int_S H^2 dA = \int_0^{2\pi} \int_0^{2\pi} \frac{(1 + \cos u)^2}{2 + \cos u} du dv = \frac{4\sqrt{3}\pi^2}{3}$$