MA426 — 微分几何

Assignment 2

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习题 2.5/3

(1)

$$\mathbf{r}(t) = (a(3t - t^3), 3at^2, a(3t + t^3)),$$

$$\mathbf{r}'(t) = (a(3 - 3t^2), 6at, a(3 + 3t^2)),$$

$$\mathbf{r}''(t) = (-6at, 6a, 6at),$$

$$\mathbf{r}'''(t) = (-6a, 0, 6a),$$

$$|\mathbf{r}'(t)| = \sqrt{a^2(3 - 3t)^2 + (6at)^2 + a^2(3 + 3t)^2} = 3\sqrt{2}|a|(t^2 + 1),$$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = (18a^2(t^2 - 1), -36a^2t, 18a^2(t^2 + 1)),$$

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = 18a^2\sqrt{(t^2 - 1)^2 + (-2t)^2 + (t^2 + 1)^2} = 18\sqrt{2}a^2(t^2 + 1),$$

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{18\sqrt{2}a^2(t^2 + 1)}{[3\sqrt{2}|a|(t^2 + 1)]^3} = \frac{1}{3|a|(t^2 + 1)^2},$$

$$\tau(t) = \frac{(\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}'''(t))}{|\mathbf{r}'(t) \times \mathbf{r}''(t)|^2} = 18a^2 \cdot \frac{(t^2 - 1, -2t, t^2 + 1) \cdot (-6a, 0, 6a)}{[18\sqrt{2}a^2(t^2 + 1)]^2} = \frac{1}{3a(t^2 + 1)^2}.$$

(2)

$$\mathbf{r}(t) = (t, t^2/2a, t^3/6a^2),$$

$$\mathbf{r}'(t) = (1, t/a, t^2/2a),$$

$$\mathbf{r}''(t) = (0, 1/a, t/a^2),$$

$$\mathbf{r}'''(t) = (0, 0, 1/a^2),$$

$$|\mathbf{r}'(t)| = \sqrt{1 + (t/a)^2 + (t^2/2a)^2} = \frac{2a^2 + t^2}{2a^2},$$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = (t^2/2a^3, -t/a^2, 1/a),$$

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{(t^2/2a^3)^2 + (-t/a^2)^2 + (1/a)^2} = \frac{2a^2 + t^2}{2|a|^3},$$

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\frac{2a^2 + t^2}{2|a|^3}}{[\frac{2a^2 + t^2}{2a^2}]^3} = \frac{4|a|^3}{(2a^2 + t^2)^2},$$

$$\tau(t) = \frac{(\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}'''(t))}{|\mathbf{r}'(t) \times \mathbf{r}''(t)|^2} = \frac{(t^2/2a^3, -t/a^2, 1/a) \cdot (0, 0, 1/a^2)}{[\frac{2a^2+t^2}{2|a|^3}]^2} = \frac{4a^3}{(2a^2 + t^2)^2}.$$

(3)

$$\mathbf{r}(t) = (t + a^2/t, t - a^2/t, 2a \log(t/a),$$

$$\mathbf{r}'(t) = (1 - a^2/t^2, 1 + a^2/t^2, 2a/t),$$

$$\mathbf{r}''(t) = (2a^2/t^3, -2a^2/t^3, -2a/t^2),$$

$$\mathbf{r}'''(t) = (-6a^2/t^4, 6a^2/t^4, 4a/t^3),$$

$$|\mathbf{r}'(t)| = \sqrt{(1 - a^2/t^2)^2 + (1 + a^2/t^2)^2 + (2a/t)^2} = \frac{\sqrt{2}(a^2 + t^2)}{t^2},$$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = (2a^3/t^4 - 2a/t^2, 2a^3/t^4 + 2a/t^2, -4a^2/t^3),$$

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{(2a^3/t^4 - 2a/t^2)^2 + (2a^3/t^4 + 2a/t^2)^2 + (-4a^2/t^3)^2} = \frac{2\sqrt{2}a(a^2 + t^2)}{t^4},$$

$$\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \frac{\frac{2\sqrt{2}a(a^2+t^2)}{t^4}}{[\frac{\sqrt{2}(a^2+t^2)}{t^2}]^3} = \frac{at^2}{(a^2 + t^2)^2},$$

$$\tau(t) = \frac{(\mathbf{r}'(t), \mathbf{r}''(t), \mathbf{r}'''(t))}{|\mathbf{r}'(t) \times \mathbf{r}''(t)|^2} = \frac{(\frac{2a^3}{t^4} - \frac{2a}{t^2}, \frac{2a^3}{t^4} + \frac{2a}{t^2}, -\frac{4a^2}{t^3}) \cdot (-\frac{6a^2}{t^4}, \frac{6a^2}{t^4}, \frac{4a}{t^3})}{[\frac{2\sqrt{2}a(a^2+t^2)}{t^4}]^2} = \frac{at^2}{(a^2 + t^2)^2}.$$