MA426 — 微分几何

Assignment 5

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3.3 例一

求旋转面的第一基本形式.

旋转面的参数方程是

$$\mathbf{r}(u,v) = (f(v)\cos u, f(v)\sin u, g(v)), \quad f(v) > 0.$$

故

$$\mathbf{r}_u(u,v) = (-f(v)\sin u, f(v)\cos u, 0),$$

$$\mathbf{r}_v(u,v) = (f'(v)\cos u, f'(v)\sin u, g'(v)),$$

$$E(u,v) = \mathbf{r}_u \cdot \mathbf{r}_u = f(v)^2\sin^2 u + f(v)^2\cos^2 u = f(v)^2,$$

$$F(u,v) = \mathbf{r}_u \cdot \mathbf{r}_v = -f(v)f'(v)\sin u\cos u + f(v)f'(v)\sin u\cos u = 0,$$

$$G(u,v) = \mathbf{r}_v \cdot \mathbf{r}_v = f'(v)^2\cos^2 u + f'(v)^2\sin^2 u + g'(v)^2 = f'(v)^2 + g'(v)^2.$$

第一基本形式为

$$I = E(u,v)(du)^2 + 2F(u,v)dudv + G(u,v)dv^2 = f(v)^2(du)^2 + (f'(v)^2 + g'(v)^2)(dv)^2.$$

习题　3.3/2

设球面的参考方程是

$$\mathbf{r} = \left(\frac{2au}{u^2+v^2+a^2}, \frac{2av}{u^2+v^2+a^2}, \frac{u^2+v^2-a^2}{u^2+v^2+a^2}\right),$$

求它的第一基本形式.

$$\mathbf{r}_u(u,v) = \left(\frac{2a\left(a^2-u^2+v^2\right)}{\left(a^2+u^2+v^2\right)^2}, -\frac{4auv}{\left(a^2+u^2+v^2\right)^2}, \frac{4a^2u}{\left(a^2+u^2+v^2\right)^2}\right),$$

$$\mathbf{r}_v(u,v) = \left(-\frac{4auv}{\left(a^2+u^2+v^2\right)^2}, \frac{2a\left(a^2+u^2-v^2\right)}{\left(a^2+u^2+v^2\right)^2}, \frac{4a^2v}{\left(a^2+u^2+v^2\right)^2}\right),$$

$$E(u,v) = \mathbf{r}_u \cdot \mathbf{r}_u = \frac{16a^4u^2}{\left(a^2+u^2+v^2\right)^4} + \frac{16a^2u^2v^2}{\left(a^2+u^2+v^2\right)^4} + \frac{4a^2\left(a^2-u^2+v^2\right)^2}{\left(a^2+u^2+v^2\right)^4} = \frac{4a^2}{\left(a^2+u^2+v^2\right)^2},$$

$$F(u,v)=\mathbf{r}_u\cdot\mathbf{r}_v=\frac{16a^4uv}{\left(a^2+u^2+v^2\right)^4}-\frac{8a^2uv\left(a^2+u^2-v^2\right)}{\left(a^2+u^2+v^2\right)^4}-\frac{8a^2uv\left(a^2-u^2+v^2\right)}{\left(a^2+u^2+v^2\right)^4}=0,$$

$$G(u,v)=\mathbf{r}_v\cdot\mathbf{r}_v=\frac{16a^4v^2}{\left(a^2+u^2+v^2\right)^4}+\frac{16a^2u^2v^2}{\left(a^2+u^2+v^2\right)^4}+\frac{4a^2\left(a^2+u^2-v^2\right)^2}{\left(a^2+u^2+v^2\right)^4}=\frac{4a^2}{\left(a^2+u^2+v^2\right)^2}.$$

第一基本形式为

$$I=E(u,v)(du)^2+2F(u,v)dudv+G(u,v)dv^2=\frac{4a^2}{\left(a^2+u^2+v^2\right)^2}(du)^2+\frac{4a^2}{\left(a^2+u^2+v^2\right)^2}(dv)^2.$$