DIFFERENTIAL GEOMETRY HOMEWORK 2

Each problem is worth 10 points.

1. Show the area of the surface given by a C^2 function $f(x, y) : \Omega \to \mathbb{R}$ is

$$\iint_\Omega \sqrt{1 + f_x^2 + f_y^2} dxdy.$$

2. (Tchebyshef net) The coordinate curves of a parametrized surface $\mathbb{X}(u, v)$ constitute a Tchebyshef net if the lengths of the opposite sides of any quadrilateral formed by them are equal. Show that a necessary and sufficient condition for this

$$\frac{\partial E}{\partial v} = \frac{\partial G}{\partial u} = 0.$$

3. Compute the mean curvature and the Gauss curvature of the graph $z = x^3$ in $\mathbb{R}^3$.

4. (Ruled surfaces) A one-parameter family of lines can be given by $\{\alpha(t), \omega(t)\}$, where for each t, $\alpha(t)$ is a point on the line and $\omega(t)$ represents the direction vector of this line, then we can define a parametrized surface as

$$\mathbb{X}(t, v) = \alpha(t) + v\omega(t), t, v \in \mathbb{R}.$$

This is called a ruled surface. Compute its Gauss curvature.

5. (Catenoid) Rotate the curve $y = a \cosh(z/a)$ around z-axis, we get a surface of revolution. Compute its mean curvature.

Date: Due 10/22.