

## MA426 — 微分几何

### Assignment 6

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## 习题 4.1/1

求下列曲线的第二基本形式:

(1)  $\mathbf{r} = (a \cos \varphi \cos \theta, a \cos \varphi \sin \theta, b \sin \varphi)$  (椭球面);

(1)

在椭球面中  $a, b > 0, 0 < \varphi < \pi, 0 < \theta < 2\pi$ .

$$\mathbf{r}_\varphi = (-a \sin \varphi \cos \theta, -a \sin \varphi \sin \theta, b \cos \varphi),$$

$$\mathbf{r}_\theta = (-a \cos \varphi \sin \theta, a \cos \varphi \cos \theta, 0),$$

$$\mathbf{r}_\varphi \times \mathbf{r}_\theta = (-ab \cos^2 \varphi \cos \theta, -ab \cos^2 \varphi \sin \theta, -a^2 \cos \varphi \sin \varphi),$$

$$|\mathbf{r}_\varphi \times \mathbf{r}_\theta| = a \cos \varphi \sqrt{b^2 \cos^2 \varphi \cos^2 \theta + b^2 \cos^2 \varphi \sin^2 \theta + a^2 \sin^2 \varphi} = a \cos \varphi \sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi},$$

$$\mathbf{n} = \frac{\mathbf{r}_\varphi \times \mathbf{r}_\theta}{|\mathbf{r}_\varphi \times \mathbf{r}_\theta|} = \frac{1}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} (-b \cos \varphi \cos \theta, -b \cos \varphi \sin \theta, -a \sin \varphi),$$

$$\mathbf{r}_{\varphi\varphi} = (-a \cos \varphi \cos \theta, -a \cos \varphi \sin \theta, -b \sin \varphi),$$

$$\mathbf{r}_{\varphi\theta} = (a \sin \varphi \sin \theta, -a \sin \varphi \cos \theta, 0),$$

$$\mathbf{r}_{\theta\theta} = (-a \cos \varphi \cos \theta, -a \cos \varphi \sin \theta, 0),$$

$$L = \mathbf{r}_{\varphi\varphi} \cdot \mathbf{n} = \frac{ab \cos^2 \varphi \cos^2 \theta + ab \cos^2 \varphi \sin^2 \theta + ab \sin^2 \varphi}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} = \frac{ab}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}},$$

$$M = \mathbf{r}_{\varphi\theta} \cdot \mathbf{n} = \frac{-ab \sin \varphi \cos \varphi \sin \theta \cos \theta + ab \sin \varphi \cos \varphi \sin \theta \cos \theta}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} = 0,$$

$$N = \mathbf{r}_{\theta\theta} \cdot \mathbf{n} = \frac{ab \cos^2 \varphi \cos^2 \theta + ab \cos^2 \varphi \sin^2 \theta}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} = \frac{ab \cos^2 \varphi}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}}.$$

第二基本形式为

$$II = L(d\varphi^2) + 2Md\varphi d\theta + N(d\theta)^2 = \frac{ab(d\varphi^2) + ab \cos^2 \varphi (d\theta)^2}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}}.$$

## 习题 4.1/3