VE203 Assigment 2

Liu Yihao 515370910207

Exercise 2.1

i)

$$n + (m + 1) := \text{succ(n + m)}$$

ii)

$$2 + 2 = succ(1) + succ(1) = succ(0) + 1 + succ(0) + 1 = 1 + 1 + 1 + 1$$
$$4 = succ(3) = succ(2) + 1 = succ(1) + 1 + 1 = succ(0) + 1 + 1 + 1 = 1 + 1 + 1 + 1$$

So $2 + 2 = 4$ is proved.

iii)
(I) for $n = 1, m = 0$, $n + m = 1$, $m + n = 1$, so it is true.

(II) for $n = 1, m = k \in N$, suppose $1 + k = k + 1$.
Then for $n = 1, m = k + 1$,
$m + n = k + 1 + 1 = 1 + k + 1 = n + m$, so it is true.

(III) for $n = k \in N, m \in N$, suppose $m + k = k + m$.
Then for $n = k + 1, m \in N$,
$m + n = m + k + 1 = k + m + 1 = k + 1 + m = n + m$, so it is true.

As (I) - (III) are true, so the statement is proved.

Exercise 2.2

(I) for $n = 1, 2$, $a_1 = 3 - 2 = 1$, $a_2 = 6 + 2 = 8$, so it is true.

(II) for $n > 2$, suppose it is true

$$\begin{aligned} a_{n+1} = a_n + 2a_{n-1} &= 3 \cdot (2^{n-1} + 2 \cdot 2^{n-2}) + 2(-1)^n + 2 \cdot 2(-1)^{n-1} \\ &= 3 \cdot 2^n + 2(-1)^{n+1} \end{aligned}$$

So it is true.

As (I) and (II) are true, so the statement is proved.

Exercise 2.3

Suppose the Well-Order-Principle is false, then there exist a non-empty set $S \subset N$ which doesn't have a least element.

(I) If the set have element 0, then 0 will be a least element. So 0 is not in the set.

(II) Suppose the set have element n, and doesn't have elements in $[0, n)$, then n will be a least element and n is not in the set.
Further, we can find that $n+1$ is not in the set.

Therefore, all natural numbers are not in the set, which is contradicted with the condition that it is a non-empty set. So the Well-Order-Principle is true.

Exercise 2.4

Suppose $(1+x)^n \geqslant nx+1$

(I) for $n=0$, $(1+x)^n = 1 \geqslant 1 = nx+1$, so it is true.

(II) for $n>0$, suppose it is true,

$$(1+x)^{n+1} = (1+x)(1+x)^n \geqslant (1+x)(nx+1) = nx^2+nx+x+1$$

$$(n+1)x+1 = nx+x+1$$

Since $x>-1$,

$$nx^2+nx+x+1 >= nx+x+1$$

So it is true.

As (I) and (II) are true, so $(1+x)^n \geqslant nx+1$ is proved.
And we can simply find that $(1+x)^n \geqslant nx$ is true.

Exercise 2.5

(I) for $n=1$, $1=2^0$, so it is true.

(II) Suppose that for $n=1,2,...,n$, the statement is true,
then for $n+1$, we should consider whether it is even or odd.
When it is even, $\frac{n+1}{2} \in [1,n] \cap N$, since $\frac{n+1}{2}$ can be written as distinct powers of 2, we can write $n+1$ by adding each of the power by 1.
When it is odd, $\frac{n}{2} \in [1,n] \cap N$, since $\frac{n}{2}$ can be written as distinct powers of 2, we can write n by adding each of the power by 1. Then we can write $n+1$ by adding 2^0.
So it is true.

As (I) and (II) are true, so the statement is proved.

Exercise 2.6

Suppose $(a,b) \in S$ implies $5 \mid a+b$

(I) for $(0,0) \in S$, $5 \mid 0+0$, so it is true.

(II) for $(a,b) \in S$, suppose $5 \mid a+b$, and $(a,b) \in S$ implies $((a+2,b+3) \in S) \wedge ((a+3,b+2) \in S)$, which means for $(a+2,b+3)$ and $(a+3,b+2)$, $a+2+b+3 = a+3+b+2 = a+b+5$. Since $5 \mid a+b$, it is clear that $5 \mid a+b+5$
So it is true.

As (I) and (II) are true, $(a,b) \in S$ implies $5 \mid a+b$ is proved.

Exercise 2.7

	reflexive	symmetric	transitive
$x + y = 0$	F	T	F
$2\|(x - y)$	T	T	T
$xy = 0$	F	T	F
$x = 1$ *or* $y = 1$	F	T	F
$x = \pm y$	T	T	T
$x = 2y$	F	F	F
$xy \geqslant 0$	F	T	F
$x = 1$	F	F	T