UM–SJTU Joint Institute

Physics Laboratory
(VE215)

Laboratory Report

Exercise 3
Transient Lab

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1 Goal

1. Apply the theory you learned on the step responses in

   rst- and second- order circuits to series RC and RLC circuits, which you will build in the lab.

2. Build a series RC circuit, observe its responses to input square wave signal of varied frequency, and explain them based on the theory you learned:

   - Relate the observed capacitor voltage and resistor voltage as functions of time to your pre-lab calculations
   - Explain the changes of both output waveforms in response to the increase of the frequency of the input square wave signal
   - Explain the amplitudes of the capacitor voltage and the resistor voltage related to the amplitude of the input square wave

3. Build a series RLC circuit, observe the three types of its responses to input square wave signal, and relate them to the theory you have learned. For the under-damped/ over-damped/ critical damped response, compare the resistance in the circuit measured in the lab with the critical resistance you calculated in the pre-lab.

4. Build the simplest second-order circuit, an LC tank, and observe oscilla- tions.

2 Introduction

2.1 First-order circuits

Theoretically, the transient responses in electric circuits are described by differential equations. The circuits, whose responses obey the first-order differential equation

$$\frac{dx(t)}{dt} + \frac{1}{\tau} \cdot x(t) = f(t)$$

are called first-order circuits. Their responses are always monotonic and appear in the form of exponential function

$$x(t) = K_1 \cdot e^{-\frac{t}{\tau}} + K_2$$

A first-order circuit includes the effective resistance R and one energy-storage element, an inductor L or a capacitor C.

In an RC circuit, the time constant is

$$\tau = RC$$

In an LC circuit, the time constant is

$$\tau = \frac{L}{R}$$

The fall time of a signal is defined as the interval between the moment when the signal reaches its 90% and the moment when the signal reaches its 10% level. Note that the 10% level is reached between 2τ and 3τ . Approximately, you can assume falltime $\approx 2.2\tau$. After $t = 5\tau$, the exponent practically equals zero.

2.2 Second-order circuits

Many circuits involve two energy-storing elements, both an inductor L and a capacitor C. Such circuits require a second-order differential equation description

$$\frac{d^2x(t)}{dt^2} + 2 \cdot \alpha \cdot \frac{dx(t)}{dt} + \omega_0^2 \cdot x(t) = f(t)$$

thus they are called second-order circuits.

We will consider only second-order circuits with one inductor and one capacitor. The differential equation includes two parameters: the damping factor α and the undamped frequency ω_0 which are determined by the circuit and its components.

For example, in the series RLC circuit, which you will build and study in this lab,

$$\alpha = \frac{R}{2 \cdot L}, \text{and } \omega_0 = \frac{1}{\sqrt{\mathrm{L} \cdot \mathrm{C}}}$$

while in the parallel RLC circuit,

$$\alpha = \frac{1}{2 \cdot R \cdot C}, \text{and } \omega_0 = \frac{1}{\sqrt{\mathrm{L} \cdot \mathrm{C}}}$$

Depending on the two parameters α and ω_0, second-order circuits can exhibit three types of responses.

2.2.1 The underdamped response

If $\alpha < \omega_0$

$$x(t) = e^{-\alpha t}(K_1 \cos(\omega t) + K_2 \sin(\omega t))$$

where $\omega = \sqrt{\omega_0^2 - \alpha^2}$

The underdamped circuit response involves decaying oscillations, which may last for many periods or for less than one period, depending on the damping ratio $\delta = \frac{\alpha}{\omega_0}$, which for the series RLC circuit $\delta = \frac{R}{2L}\sqrt{LC} = \frac{R}{2} \cdot \sqrt{\frac{C}{L}}$. Varying the values of R, L, C, affects the damping ratio δ.

2.2.2 The critically damped response

If $\alpha = \omega_0$

$$x(t) = e^{-\alpha t}(K_1 + K_2 t)$$

and the circuit has the critically damped response.

The critically damped response does not involve oscillations.

For the series RLC circuits, $\alpha = \omega_0$ corresponds to $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ or $R = R_{critical} = 2\sqrt{\frac{L}{C}}$

If $L = 1mH$ and $C = 10nF$, then $R_{critical} \approx 632\Omega$.

2.2.3 The overdamped response

If $\alpha > \omega_0$

$$x(t) = K_1 \cdot e^{s_1 t} + K_2 \cdot e^{s_2 t}$$

where $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ and $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

In the series RLC circuits, the overdamped solution is obtained if the resistance is larger that the critical resistance, such that $R > R_{critical} = 2\sqrt{\frac{L}{C}}$

Notice that the larger resistance corresponds to the longer delay, and even the faster decay has a much longer fall time than the critically damped response.

One of the most interesting features of series RLC circuits is that increasing the resistance above the critical value results in much longer fall time, or longer delays of responses in digital circuits. Among all monotonic responses, the critically damped is the fastest.

3 Results and Discussion

3.1 First-order circuits

The setting of the potentiometer	Fastest circuit response	Slowest circuit response
V_{ppk} input [V]	1.13	1.13
V_{ppk} output [V]	1.13	1.13
Period T of the input [ms]	10.000	10.000
Rise Time of the Output [ms]	0.230	3.400
Fall Time of the Output [ms]	0.246	3.040

Table 1: First-order circuits.

For the fastest circuit response,

$$\tau = \frac{0.230 + 0.246}{2} \cdot \frac{1}{2.2} = 0.108\,\text{ms}$$

Theoretically,

$$\tau = RC = 0.100\,\text{ms}$$

The relative error is 8%

For the slowest circuit response,

$$\tau = \frac{3.400 + 3.040}{2} \cdot \frac{1}{2.2} = 1.464\,\text{ms}$$

Theoretically,

$$\tau = RC = 1.100\,\text{ms}$$

The relative error is 33.1%

We can find that in the slowest circuit response, the relative error is large. It is probably because if the error of the resistance and the oscilloscope.

3.2 Second-order circuits

	Resistance R_p $[k\Omega]$	Rise Time $[ms]$	Fall Time $[ms]$	Time interval Δt $[\mu s]$
Under-damped	0.000	0.00165	0.0016	5.3
	0.532	0.0018	0.0017	5.8
Critically damped	2.100	0.0029	0.0029	
Over-damped	4.460	0.0076	0.0078	
	10.000	0.0176	0.0172	

Table 2: Second-order circuits.

For the first under-damped circuit,

$$\omega = \frac{2\pi}{\Delta t} = 1.186 \times 10^9$$

Theoretically,

$$\omega = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 1.103 \times 10^9$$

The relative error is 7.5%

$$\delta = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.045$$

For the second under-damped circuit,

$$\omega = \frac{2\pi}{\Delta t} = 1.083 \times 10^9$$

Theoretically,

$$\omega = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = 1.058 \times 10^9$$

The relative error is 2.4%

$$\delta = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.286$$

Since $\delta_1 < \delta_2$, the damping in the first circuit decays more slowly.

For the critically damped circuit,

$$R_{critical} = 2\sqrt{\frac{L}{C}} = 2208.63\Omega$$

4 Conclusion

In the lab, we learned how a build a RC and RLC circuit. We also found the properties if these circuits, especially the relationship between R,L,C and the time period constant τ. We discovered δ in an under-damped RLC circuit to find how fast it decays.

5 Reference

Lab 3 Manual.

6 Pre-lab and Data sheet