## Ve215 Electric Circuits

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# Chapter 4

**Circuit Theorems** 

#### 4.1 Introduction

 Even though the nodal analysis and mesh analysis are powerful techniques for solving circuits, we are still interested in methods that can be used to simplify circuits. Series-parallel reductions and wye-delta transformations are already on our list of simplifying techniques. In this chapter, we study more.

#### 4.2 Linearity

- A linear circuit is one whose output (also called response) is linearly related to its input (also called excitation).
- The linearity property is a combination of both the homogeneity (scaling) property and the additivity property.

- Homogeneity requires that if the input is multiplied by a constant, then the output is multiplied by the same constant.
- Additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately.
- A circuit is linear if it is both <u>additive</u> and homogeneous.

## Homogeneous

- Assuming input x, output y
   If x → y, then kx → ky
   E.g., y=ax → a(kx)=k(ax)=ky
  - Example of inhomogeneity
     y=ax² → a(kx)²=ak²x²=k²(ax²)=k²y
     The output is NOT multiplied by the same constant

#### Additive

Assuming input x<sub>1</sub> produces output y<sub>1</sub>;
 input x<sub>2</sub> produces output y<sub>2</sub>

$$x_1 \rightarrow y_1, x_2 \rightarrow y_2$$
  
then  $x_1 + x_2 \rightarrow y_1 + y_2$   
E.g.,  $y = ax \rightarrow y_1 = ax_1, y_2 = ax_2 \rightarrow a(x_1 + x_2) = ax_1 + ax_2 = y_1 + y_2$ 

Counterexample

$$y=ax^2 \rightarrow y_1=ax_1^2, y_2=ax_2^2 \rightarrow a(x_1+x_2)^2=ax_1^2+2ax_1x_2+ax_2^2 \neq y_1+y_2^2$$

- Example: a diode (like a nonlinear resistor).
  - very useful nonlinear components that are widely used in communication systems, highspeed electronics, power applications, etc.
  - Solar cells, photodetectors, and lasers are also examples of diodes.

- i-v characteristics of diode
  - Diode is a two terminal nonlinear resistor whose current is exponentially related to the voltage across its terminals.
  - An analytical expression

$$\underline{i_D} = I_S(e^{q\underline{v}\underline{D}/kT}-1)$$

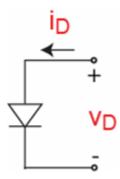
 $I_S$ : saturation current

*q*: electron charge  $(1.6 * 10^{-19}C)$ 

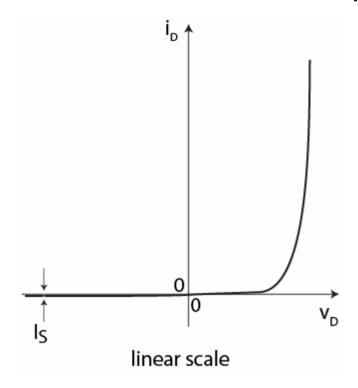
k: Boltzmann constant ( $1.38*10^{-23}J/K$ )

T: temperature (K)

kT/q: thermal voltage (26mVat300K)

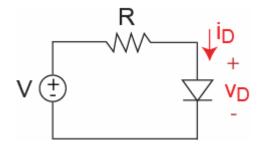


The above diode function can be plotted as:



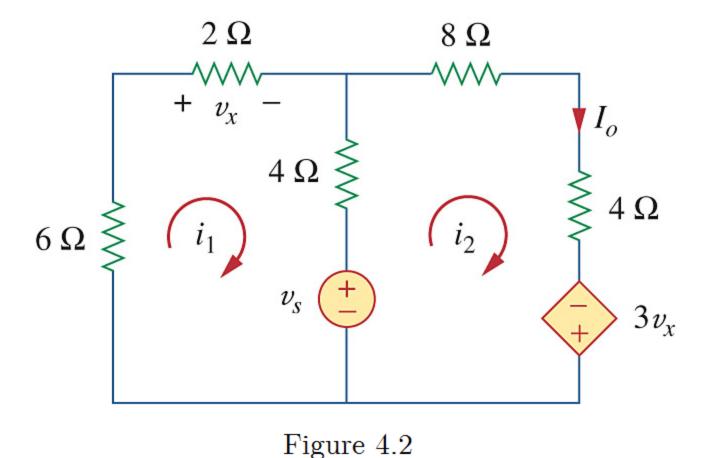
Obviously, the input-output is a nonlinear relation.

The circuit with nonlinear element

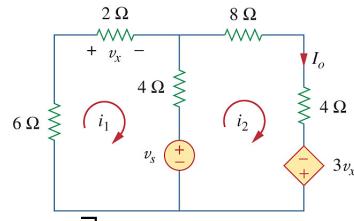


- To determine  $v_D$  and  $i_D$  for different values of V and R, we can apply KCL  $\frac{v_D-V}{R}+i_D=0$
- And using the diode relation  $\frac{v_D-V}{R}+I_S(e^{qv_D/kT}-1)=0$

**Example 4.1** For the circuit in Fig. 4.2, find  $I_o$  when  $v_s = 12$  V and  $v_s = 24$  V. This example illustrate the homogeneity property.



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#### **Solution:**

$$\begin{bmatrix} 6+2+4 & -4 \\ -4 & 4+8+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -v_s \\ 3v_x + v_s \end{bmatrix}$$

$$v_x = 2i_1$$

$$\begin{bmatrix} 6+2+4 & -4 \\ -4 & 4+8+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -v_s \\ 6i_1+v_s \end{bmatrix}$$

$$\begin{bmatrix} 6+2+4 & -4 \\ -4-6 & 4+8+4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -v_s \\ v_s \end{bmatrix}$$

$$\begin{bmatrix} 12 & -4 \\ -10 & 16 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -v_s \\ v_s \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 12 & -4 \\ -10 & 16 \end{vmatrix} = 152$$

$$\Delta_2 = \begin{vmatrix} 12 & -v_s \\ -10 & v_s \end{vmatrix} = 2v_s$$

$$I_o = i_2 = \frac{\Delta_2}{\Delta} = \frac{v_s}{76}$$

When 
$$v_s = 12 \text{ V}$$
,

$$I_o = \frac{12}{76} = \frac{3}{19} \text{ (A)}$$

When  $v_s = 24 \text{ V}$ ,

$$I_o = \frac{24}{76} = \frac{6}{19}$$
 (A)

$$v_s \rightarrow I_o$$
Homogeneous circuit
 $2v_s \rightarrow 2I_o$ 

The homogeneity property states that if  $v_s = 12 \text{ V gives } I_o = 3/19 \text{ A}$ , then  $v_s = 2 \times 12 \text{ V will give } I_o = 2 \times 3/19 \text{ A} = 6/19 \text{ A}$ .

#### 4.3 Superposition

- Superposition principle is based on additivity.
- It states that whenever a linear system is excited, or driven, by more than one independent source, the total response is the sum of the individual responses.

 The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately and adding algebraically all the contributions to find the total contribution.

- To apply the superposition principle, we must keep two things in mind:
  - We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).
  - Dependent sources are left intact because they are controlled by circuit variables.

# Linear circuit: homogeneous and additive

Linear circuit

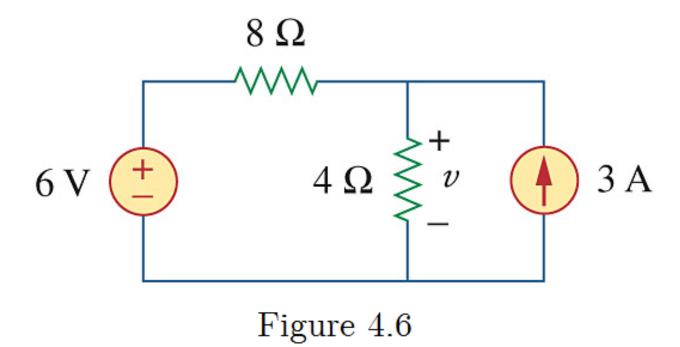
$$x_1 \rightarrow y_1; x_2 \rightarrow y_2$$

Linear circuit

 $ax_1+bx_2 \rightarrow ay_1+by_2$ 

- Superposition
  - -(1) contribution from input  $x_1$ :  $ax_1 \rightarrow ay_1$
  - (2) contribution from input  $x_2$ :  $bx_2 \rightarrow by_2$  adding algebraically all the contributions
  - → ay<sub>1</sub>+by<sub>2</sub> Linear: homogeneous and additive



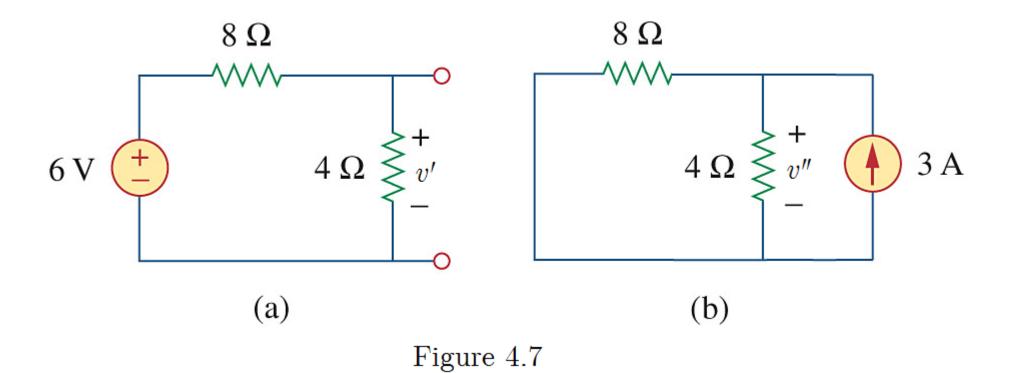


#### **Solution:**

Since there are two sources, let v = v' + v'', where v' and v'' are the contributions due to the voltage source and the current source, respectively.

Set the current source to zero, as shown in

Fig. 4.7(a), we obtain  $v' = \frac{4}{8+4} \times 6 = 2 \text{ (V)} \quad 6 \text{ V} \quad 4 \Omega \stackrel{\text{8 }\Omega}{\rightleftharpoons} \quad 3 \text{ A}$ (a)

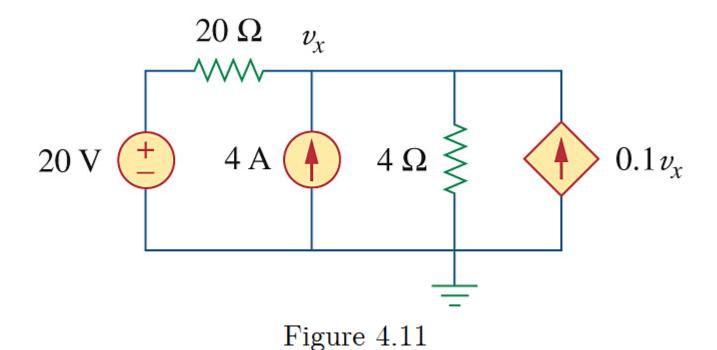


Set the voltage source to zero, as shown in Fig. 4.7(b), we obtain

$$v'' = 3 \times (4 \parallel 8) = 3 \times \frac{4 \times 8}{4 + 8} = 8 \text{ (V)}$$
Thus,
 $v = v' + v'' = 2 + 8 = 10 \text{ (V)}$ 
Figure 4.6

This example shows that suposition helps reduce a complex circuit to simpler circuits through turning off independent sources.

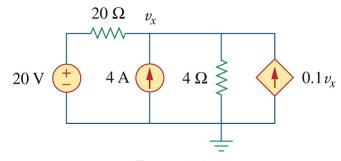
Practice Problem 4.4 Use superposition to find  $v_x$  in the circuit of Fig. 4.11.



#### **Solution:**

Let 
$$v_x = v_x' + v_x''$$

Set the voltage source to zero,



$$\frac{v_x'}{20} - 4 + \frac{v_x'}{4} - 0.1v_x' = 0 \Rightarrow v_x' = 20 \text{ (V)}$$

Set the current source to zero,

$$\frac{v_x'' - 20}{20} + \frac{v_x''}{4} = 0.1v_x'' \Rightarrow v_x'' = 5 \text{ (V)}$$

$$v_x = v_x' + v_x'' = 25 \text{ (V)}$$

$$v_x = v_x' + v_x'' = 25 \text{ (V)}$$
Input: 4A

For Practice Problem Input: 20V

#### 4.4 Source Transformation

- A source transformation is the process of replacing a voltage source in series with a resistor by a current source in parallel with a resistor, or vice versa.
- Source transformation is a tool for simplifying circuits.

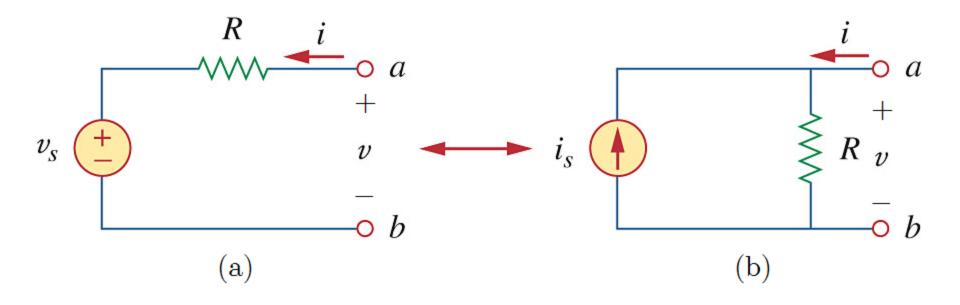
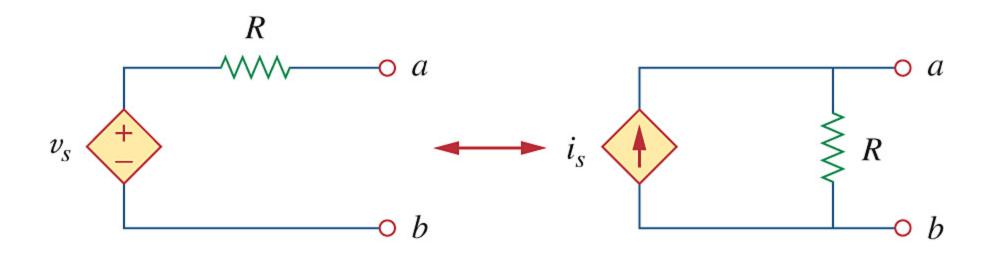


Figure 4.15 Transformation of independent sources.

$$v_s = i_s R$$
 or  $i_s = \frac{v_s}{R}$ 

 Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.



Figurev4.16 Transformation of dependent sources.

**Example 4.6** Use source transformation to find  $v_o$  in the circuit of Fig. 4.17.

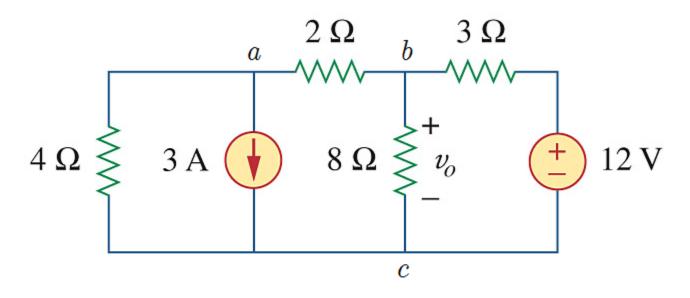


Figure 4.17

**Solution:** See Fig. 4.18. From the simpilified circuit, we have

$$v_o = 2 \times (8 \parallel 2) = 2 \times \frac{8 \times 2}{8 + 2} = 3.2 \text{ (V)}$$

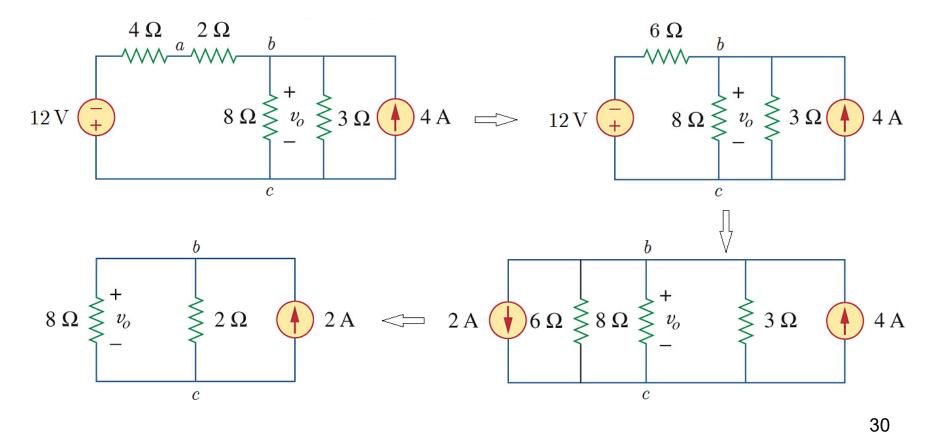
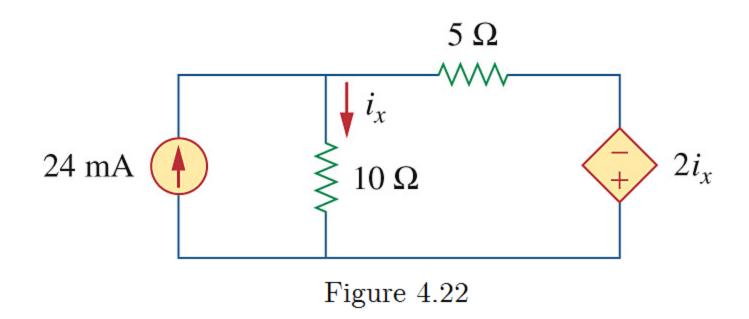


Figure 4.18.

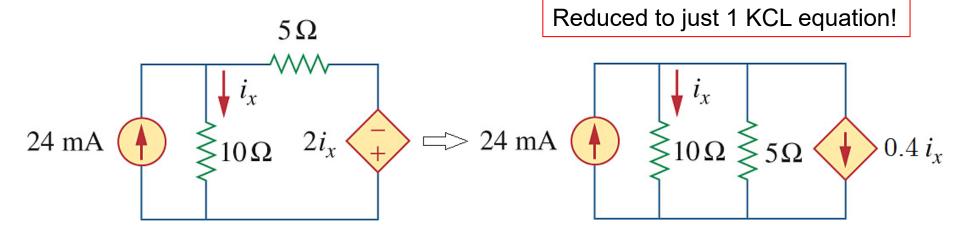
**Practice Problem** Use source transformation to find  $i_x$  in the circuit shown in Fig. 4.22.



## Solution: See the figure below. Apply KCL,

$$-24 + i_x + \frac{10i_x}{5} + 0.4i_x = 0$$

$$i_x = \frac{24}{3.4} \approx 7.06 \text{ (mA)}$$



For Practice Problem 4.7

#### 4.5 Thevenin's Theorem

 At times in circuit analysis, we want to concentrate on what happens at a specific pair of terminals. For example, when we connect a load to a linear two-terminal circuit, we are interested primarily in the voltage and current at the terminals of the load. We have little or no interest in the effect that connecting the load has on voltages or currents elsewhere in the linear two-terminal circuit.

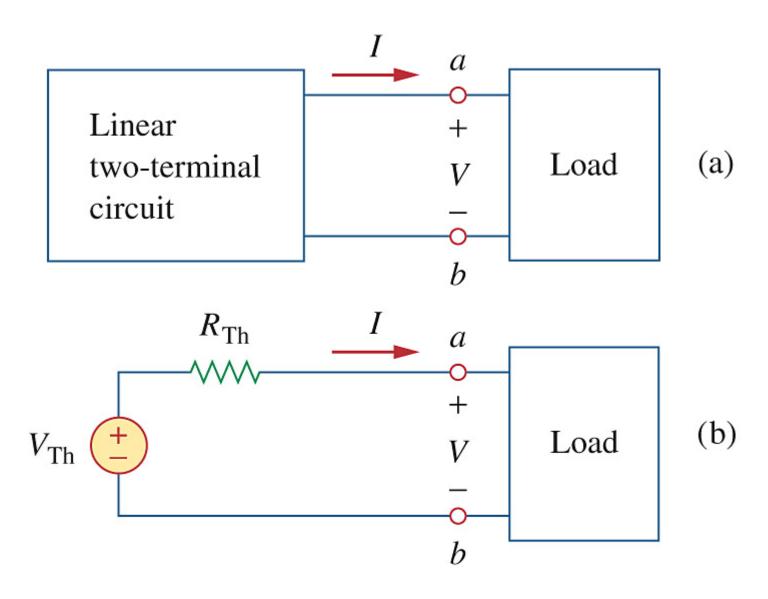


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent.

Thevenin's theorem states that a linear twoterminal circuit can be replaced by an equivalent circuit consisting of a voltage source  $V_{Th}$  in series with a resistor  $R_{Th}$ , where  $V_{Th}$  is the open-circuit voltage at the terminals and  $R_{Th}$  is the equivalent resistance at the terminals when the independent sources are turned off.

Fig. 4.24 shows the idea of finding the Thevenin voltage  $V_{Th}$  and the Thevenin resistance  $R_{Th}$ .

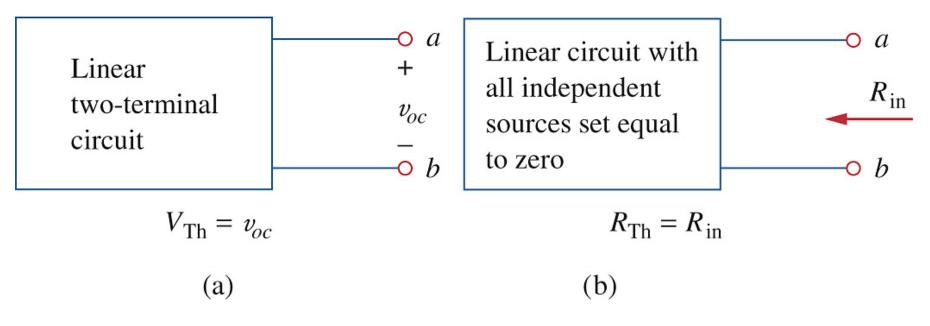
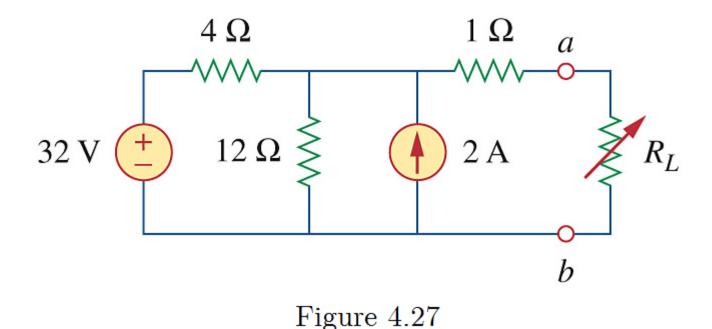


Figure 4.24 Finding  $V_{\text{Th}}$  and  $R_{\text{Th}}$ .

V<sub>Th</sub> = open-circuit voltage of the linear circuit R<sub>Th</sub> = R<sub>eq</sub> with all independent sources off **Example 4.8** Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a - b. then find the current through  $R_L = 6, 16$ , and  $36 \Omega$ .



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### **Solution:**

Turn off all independent sources, the circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{4 + 12} + 1 = 4 (\Omega)$$

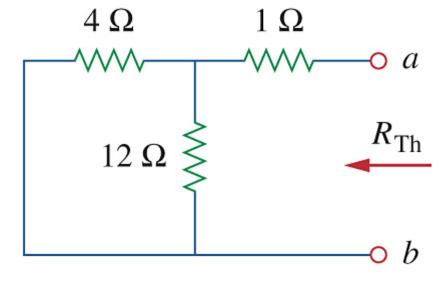
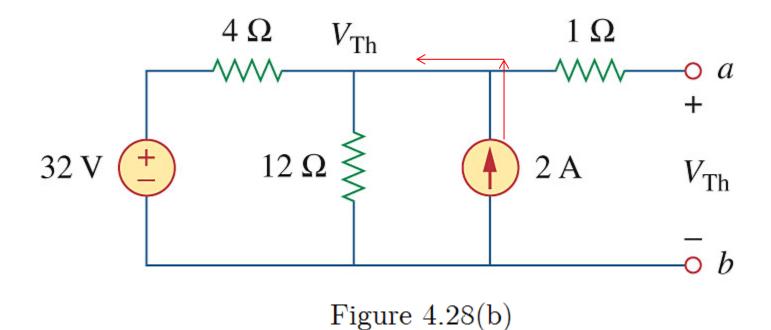


Figure 4.28(a)

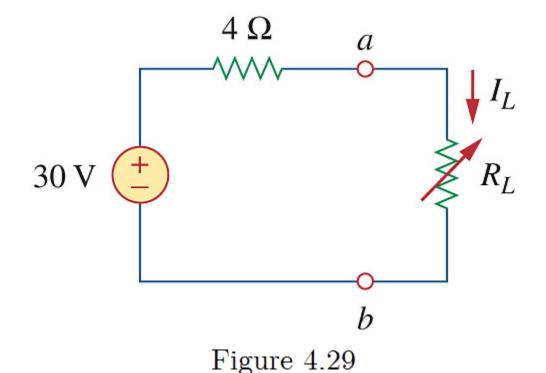
Open-circuiting  $R_L$ , the circuit becomes the one in Fig. 4.28(b).

$$\frac{V_{Th} - 32}{4} + \frac{V_{Th}}{12} - 2 = 0 \Rightarrow V_{Th} = 30 \text{ (V)}$$



The circuit in Fig. 4.27 can be replaced by the circuit shown in Fig. 4.29.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L} = 3, 1.5, 0.75 \text{ (A)}$$
  
when  $R_L = 6, 16, 26 \Omega$ .



To apply the idea in finding the Thevenin resistance  $R_{Th}$ , we need to consider two cases.

1. If the network has no dependent sources, we turn off all independent sources.  $R_{Th}$  is the equivalent resistance looking between terminals a and b, as shown in Fig. 4.24 (b).

2. If the network has dependent sources, we turn off all independent sources. And we apply a voltage source  $v_o$  (or current source  $i_a$ ) at terminals a and b and determine the resulting current  $i_o$  (or voltage  $v_o$ ). Then  $R_{Th} = v_o / i_o$ , as shown in Fig. 4.25.  $R_{Th} = R_{eq}$  with

all independent

sources off

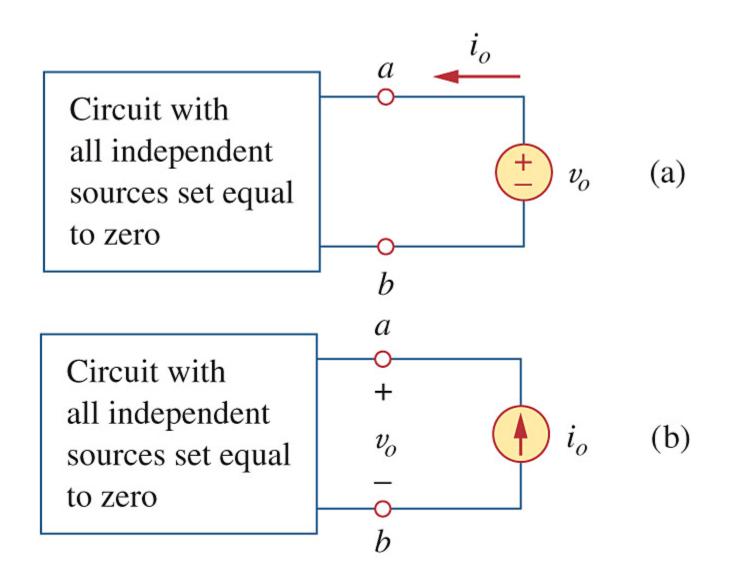
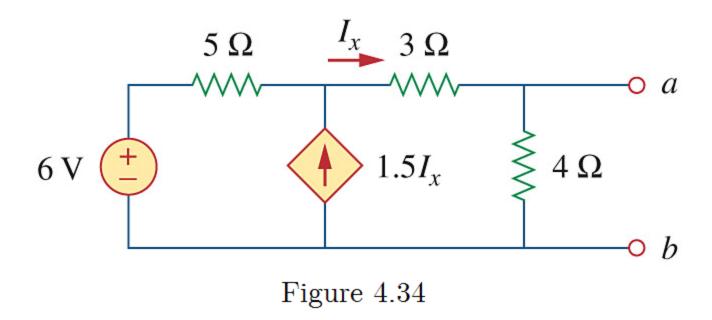


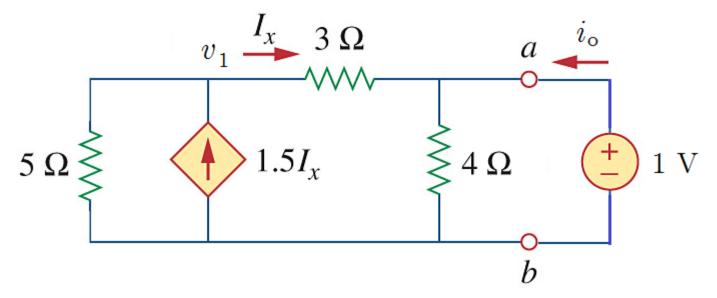
Figure 2.45 Finding  $R_{Th}$  when circuit has dependent sources.  $R_{Th} = v_o/i_o$ .

**Practice Problem 4.9** Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

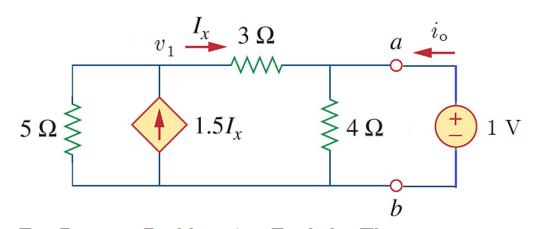


### **Solution:**

To find  $R_{Th}$ , we turn off the independent voltage source and apply a test voltage source at the terminals a - b.



For Practice Problem 4.9: Find the Thevenin resistance



$$\begin{cases} I_x = \frac{v_1 - 1}{3} \\ v = \frac{v_1 - 1}{3} \end{cases}$$

For Practice Problem 4.9: Find the Thevenin resistance 
$$\rightarrow I = -2 (A)$$

$$\frac{v_1}{5} + I_x = 1.5I_x$$

$$i_o = -I_x + \frac{1}{4} = \frac{9}{4}$$
 (A)

$$R_{Th} = \frac{1}{i_o} = \frac{4}{9} \approx 0.44 \ (\Omega)$$

 $V_{Th}$  is the terminal voltage  $V_{ab} = 4I_x$ .

$$\begin{cases} I_{x} = \frac{v'_{1}}{3+4} \\ \frac{v'_{1}-6}{5} + I_{x} = 1.5I_{x} \end{cases} \Rightarrow I_{x} = \frac{4}{3} \text{ (A)}$$

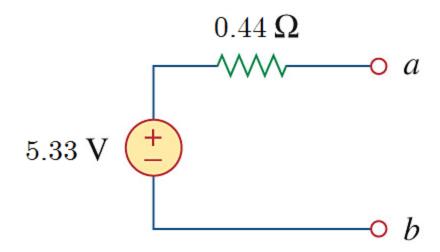
$$V_{Th} = 4I_x = \frac{16}{3} \approx 5.33 \text{ (V)}$$

$$5 \Omega \quad v_1' \quad I_x \quad 3 \Omega \quad V_{Th} \quad a$$

$$V \quad + \quad 1.5I_x \quad 4 \Omega$$

For Practice Problem 4.9: Find the Thevenin voltage.

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For Practice Problem 4.9: The Thevenin Equivalent.

It often occurs that  $R_{Th}$  takes a negative value. In this case, the negative resistance implies that the circuit is supplying power. This is possible in a circuit with dependent sources. Example 4.10 will illustrates this.

**Example 4.10** Determine the Thevenin equivalent of the circuit in Fig. 4.35(a) at terminals a - b.

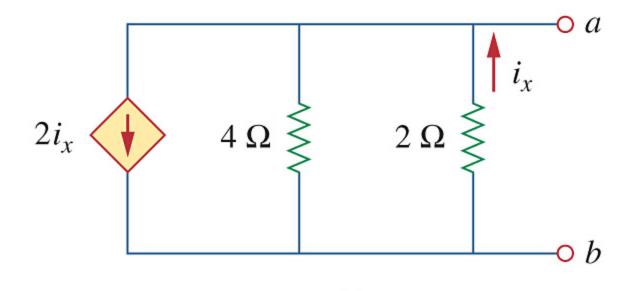


Figure 4.35(a)

## **Solution:**

$$\begin{cases} v_o = -2i_x \\ 2i_x + \frac{v_o}{4} = i_x + i_o \end{cases} \Rightarrow v_o = -4i_o$$

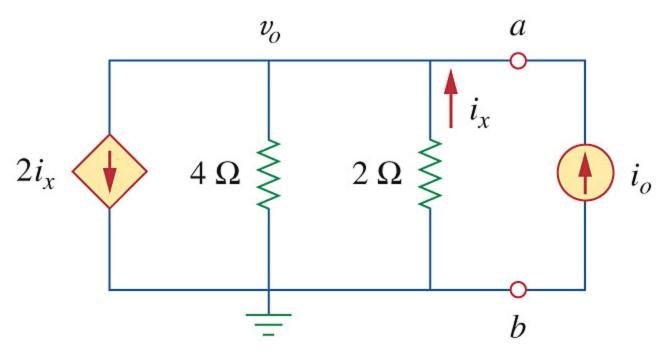


Figure 4.35(b)

$$R_{Th} = \frac{v_o}{i_o} = -4 \ (\Omega)$$

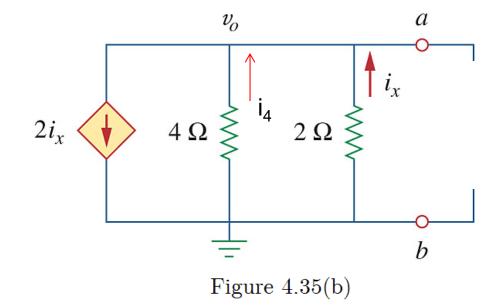
$$V_{Th} = 0$$

The negative value of  $R_{Th}$  tells us that the circuit is supplying power. Of course, the resistors in the circuit cannot supply power; it is the dependent source that supplies the power.

$$V_{Th} = 0$$

Method 1:  $V_{Th} = A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2}$ Where  $v_{s1}$ ,  $v_{s2}$ ,  $i_{s1}$ , and  $i_{s2}$  are independent sources Thus, no independent sources  $\rightarrow V_{Th} = 0$ 

Method2:  $V_{Th} = V_{oc}$   $2i_x=i_x+i_4$   $i_4=i_x$   $v_{oc}=-4i_x=-2i_x$   $i_x=0$  $V_{oc}=0$ 



Norton's theorem states that a linear twoterminal circuit can be replaced by an equivalent circuit consisting of a currrent source  $I_N$  in parallel with a resistor  $R_N$ , where  $I_N$  is the short-circuit current through the terminals and  $R_N$  is the equivalent resistance at the terminals when the independent sources are turned off.

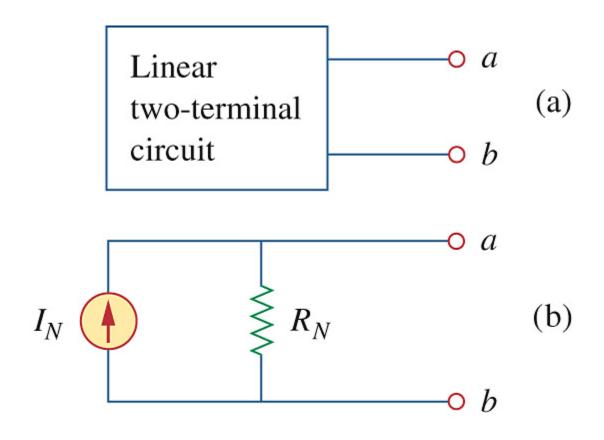
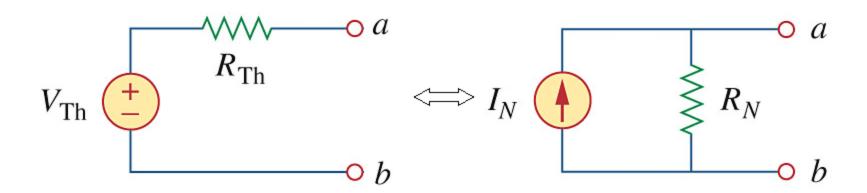


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

A Norton equivalent circuit consists of an independent current source in parallel with the Norton equivalent resistance. We can derive it from a Thevenin equivalent circuit simply by making a source transformation.



From what we know about the source transformation, the Thevenin and Norton resistances are equal and the Norton current equals the Thevenin voltage divided by the Thevenin resistance; that is,

$$R_N = R_{Th}$$

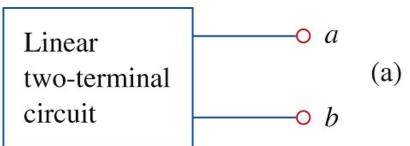
$$I_N = \frac{V_{Th}}{R_{Th}}$$

It is evident that the short-circuit current in Fig. 4.37(b) is  $I_N$ . This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are

equivalent. Thus,

$$I_N = i_{sc}$$

shown in Fig. 4.38.



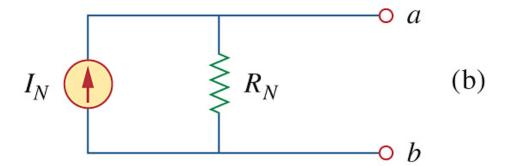


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

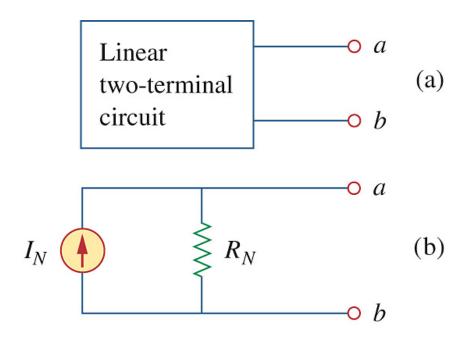


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

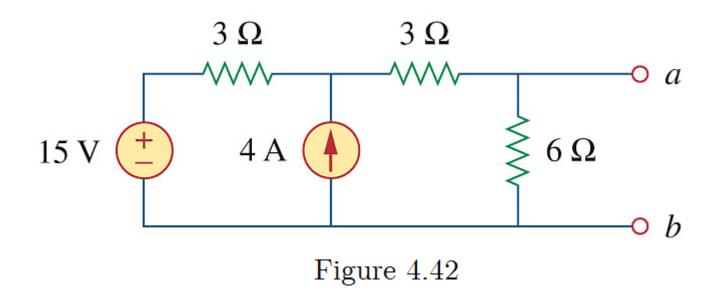
From  $I_N = V_{Th} / R_{Th}$ , we have

$$R_{Th} = \frac{V_{Th}}{I_N}$$

But  $V_{Th} = v_{oc}$  and  $I_N = i_{sc}$ , so

$$R_{Th} = \frac{v_{oc}}{i_{sc}}$$

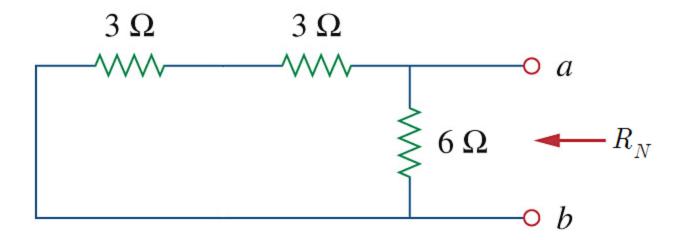
Thus the Thevenin or Norton resistance is the ratio of the open-circuit voltage to the short-circuit current. **Practice Problem 4.11** Find the Norton equivalent circuit for the circuit in Fig. 4.42, at terminals *a - b*.



### **Solution:**

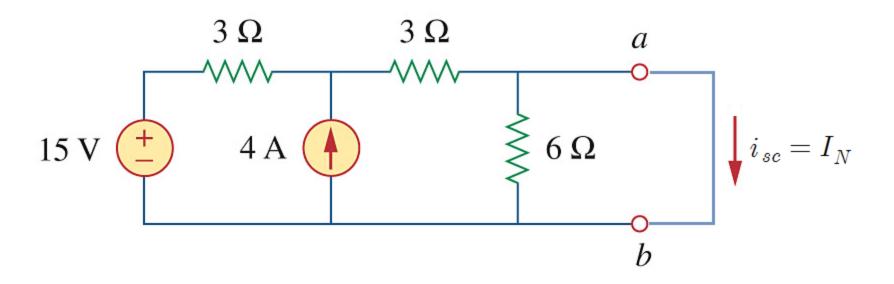
Turn off the voltage and current sources, the Norton resistance is

$$R_N = (3+3) \parallel 6 = 6 \parallel 6 = 3 \ (\Omega)$$

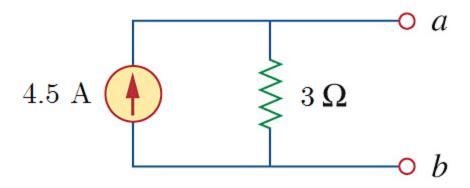


Short-circuit terminals a and b, ignore the 6- $\Omega$  resistor, apply superposition principle,

$$I_N = I_N' + I_N'' = \frac{15}{3+3} + 4 \times \frac{3}{3+3} = \frac{9}{2} = 4.5 \text{ (A)}$$



# The Norton equivalent is shown below.



## 4.8 Maximum Power Transfer

- In many practical situations, a circuit is designed to provide power to a load.
- I. Maximum power efficiency

  Power utility systems are concerned with the generation, transmission, and distribution of large quantities of electric power. This type of systems emphasizes the efficiency of the power transfer.

- II. Maximum power transfer
  In communication and instrumentation
  systems, the amount of power being
  transferred is small, so the efficiency is
  not a primary concern. It is often desirable
  to transmit as much of power as possible
  to the load. (E.g., a cell phone)
- We now consider maximum power transfer in systems with the aid of the circuit shown in Fig. 4.48.

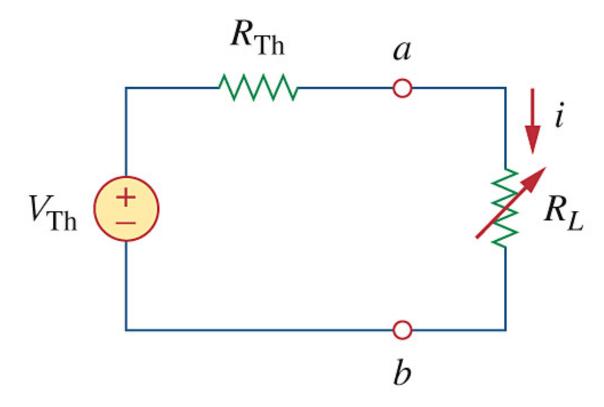
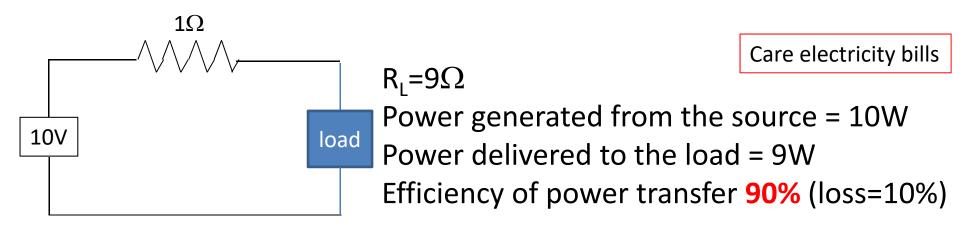


Figure 4.48 The circuit used for maximum power transfer.

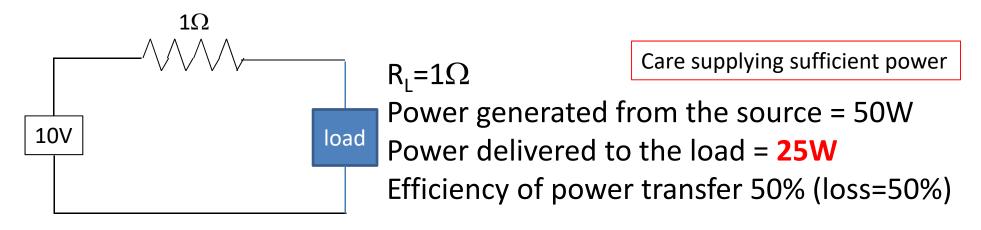
Circuit types	I. Power utility systems	II. Communication & instrumentation systems
Power scale	Large quantities of electric power (generation, etc.)	Small amount of power is being transferred
Primary concern of power	Efficiency of the power transfer	Transmit as much of power as possible to the load

## Assuming the same source for easy comparison

I. Power utility systems



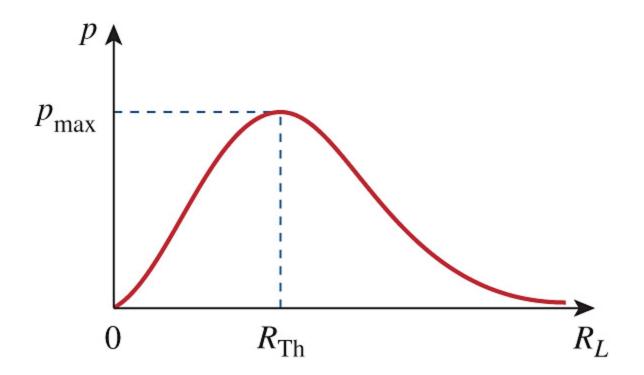
II. Communication & instrumentation systems



The power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

as sketched in Fig. 4.49.



The maximum power theorem states that the maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load  $(R_L = R_{Th})$ .

### **Proof:** Let

$$\frac{dp}{dR_L} = V_{Th}^2 \frac{R_{Th} - R_L}{(R_{Th} + R_L)^3} = 0$$

We have  $R_L = R_{Th}$ .

$$\left. \frac{d^{2}p}{dR_{L}^{2}} \right|_{R_{L}=R_{Th}} = V_{Th}^{2} \left. \frac{2R_{L} - 4R_{Th}}{(R_{Th} + R_{L})^{4}} \right|_{R_{L}=R_{Th}} = -\frac{V_{Th}^{2}}{8R_{Th}^{3}}$$

$$\left. \frac{d^2 p}{dR_L^2} \right|_{R_L = R_{Th}} < 0 \text{ implies that at } R_L = R_{Th}, p \text{ takes}$$

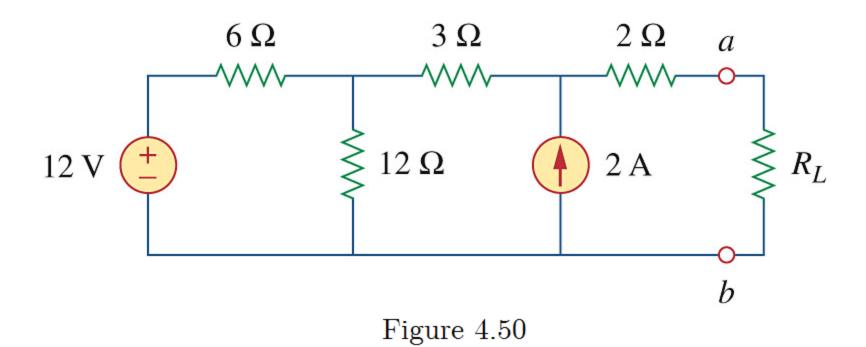
the maximum value.

The maximum power transferred is

$$p_{\text{max}} = \frac{V_{Th}^2}{4R_{Th}}$$

It should be noted that delivering the maximum power to the load results in significant internal losses.

**Example 4.13** Find the value of  $R_L$  for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.



## **Solution:**

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{6 + 12} = 9 \ (\Omega)$$

For maximum power transfer,

$$R_{L} = R_{Th} = 9 \Omega$$

$$6 \Omega \qquad 3 \Omega \qquad 2 \Omega$$

$$R_{Th} = 12 \Omega$$

$$V_{Th} = 12 \times \frac{12}{6+12} + 2 \times (3+6 \parallel 12) = 22 \text{ (V)}$$

The maximum power transferrd

$$p_{\text{max}} = \frac{V_{Th}^{2}}{4R_{Th}} = \frac{22^{2}}{4 \times 9} = \frac{121}{9} \approx 13.44 \text{ (W)}$$

$$\begin{array}{c} 6 \Omega \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{c} 3 \Omega \\ 0 \\ 0 \end{array} \begin{array}{c} 2 \Omega \\ 0 \\ 0 \end{array}$$

## **Source Modeling (Section 4.10)**

A practical voltage source is modeled by an ideal voltage source  $v_s$  in series with an internal (or a source) resistance  $R_s$ , a practical current source is modeled by an ideal current source  $i_s$  in parallel with an internal resistance  $R_p$ , as shown in Fig. 4.58.

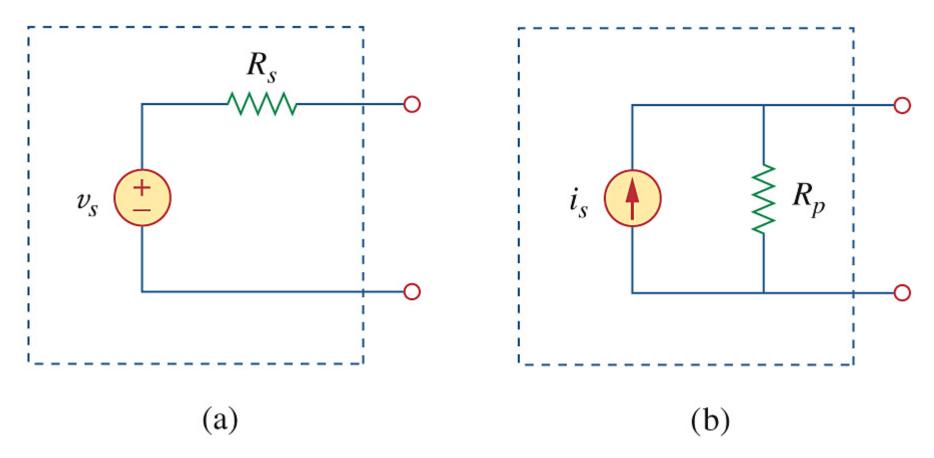
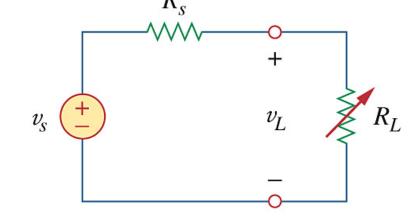


Figure 4.58 (a) Practical voltage source, (b) practical current source.

In Fig. 4.59(a), a practical voltage source is connected to a variable load. The load

voltage is

$$v_L = \frac{R_L}{R_s + R_L} v_s$$



This equation tells us that (1)  $v_L$  will be constant if  $R_s = 0$  or at least  $R_s << R_L$ . In other words, the smaller  $R_s$  is compared with  $R_L$ , the closer the voltage source is to being ideal.

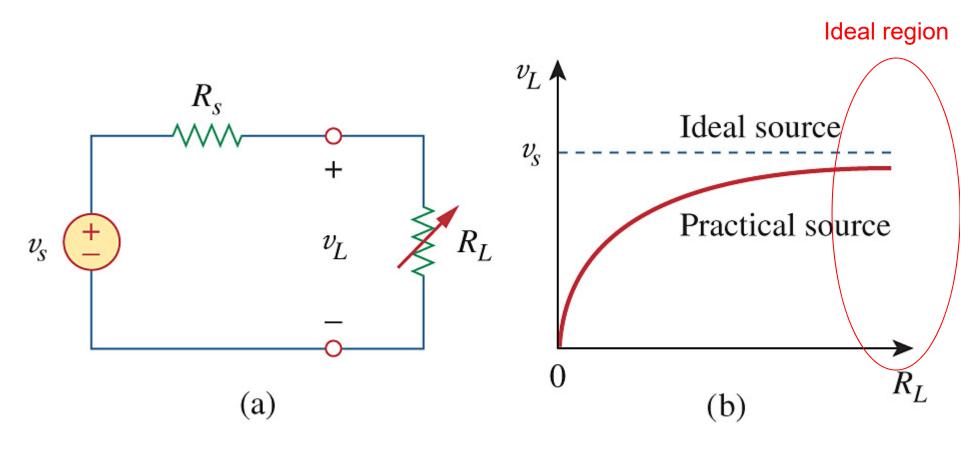
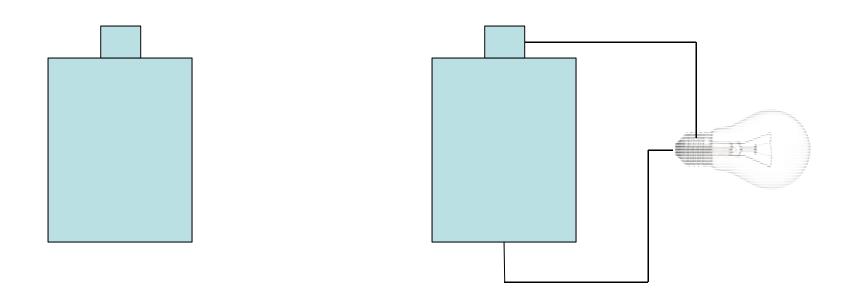


Figure 4.59 (a) Practical voltage connected to a load  $R_L$ , (b) load voltage decreases as  $R_L$  decreases.

(2) When the voltage source is open-circuited,  $v_{oc} = v_s$ . Thus,  $v_s$  may be regarded as the *unloaded source voltage*. The connection of  $R_L$  causes the terminal voltage to drop in magnitude; this is known as the *loading effect*.



Same or different voltages?

To find  $v_s$  and  $R_s$ , we follow the procedure illustrated in Fig. 4.61.

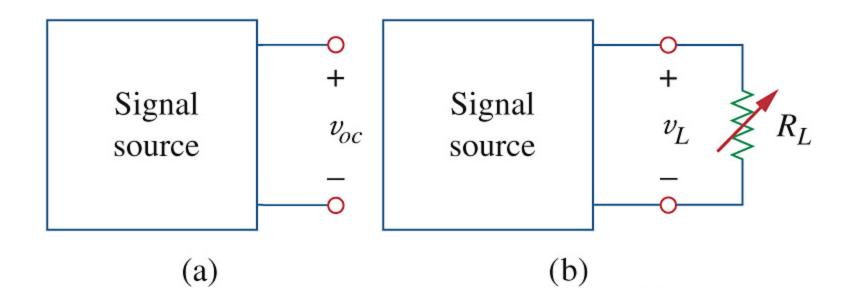


Figure 4.61 (a) Measuring the open-circuit voltage, (b) measuring the load voltage.

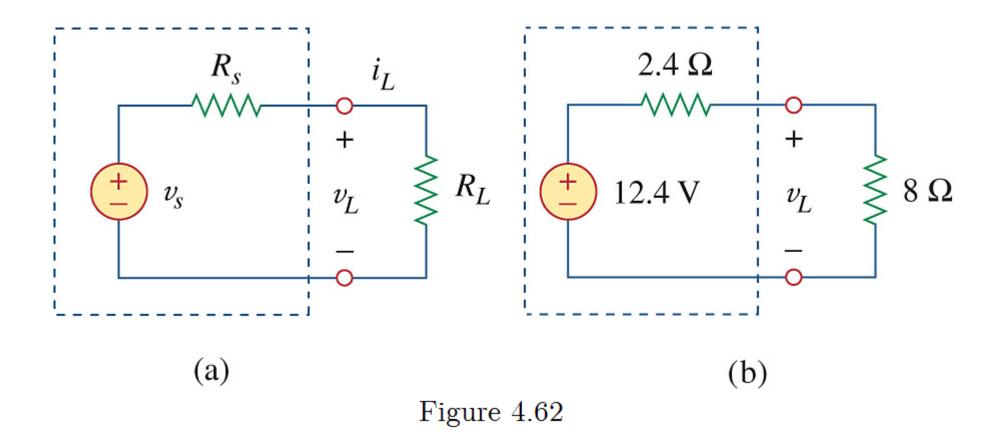
First, we measure the open-circuit voltage  $v_{oc}$  and set

$$v_s = v_{oc}$$

Then, we connect a variable load  $R_L$  across the terminals. We adjust  $R_L$  until  $v_L = v_{oc} / 2$ . At this point, we disconnect  $R_L$  and measure it. We set

$$R_{s} = R_{L}$$

**Example 4.16** The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V. (a) Calculate the source voltage  $v_s$  and internal resistance  $R_s$ . (b) determine the voltage when an  $8-\Omega$  load is connected to the source.



## **Solution:**

(a) 
$$v_s = v_{oc} = 12.4 \text{ V}$$

$$p_L = \frac{v_L^2}{R_L} \Longrightarrow R_L = \frac{v_L^2}{p_L} = \frac{12^2}{2} = 72 \; (\Omega)$$
 Figure 4.62

$$v_L = v_s \frac{R_L}{R_s + R_L} \Rightarrow R_s = R_L \left(\frac{v_s}{v_L} - 1\right) = 72 \times$$

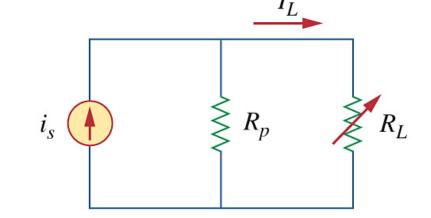
$$\left(\frac{12.4}{12}-1\right)=2.4\ (\Omega)$$

(b) 
$$v_L = v_s \frac{R_L}{R_s + R_L} = 12.4 \times \frac{8}{2.4 + 8} \approx 9.54 \text{ (V)}$$

In Fig. 4.60(a), a practical current source is connected to a variable load. The load current

is

$$i_L = \frac{R_p}{R_p + R_L} i_s$$



This equation tells us that (1)  $i_L$  will be constant if  $R_p = \infty$  or at least  $R_p >> R_L$ . In other words, the larger  $R_p$  is compared with  $R_L$ , the closer the current source is to being ideal.

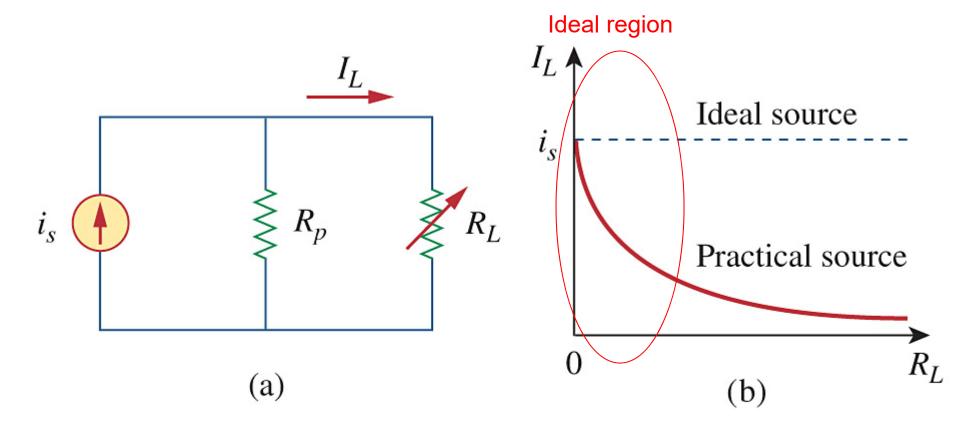


Figure 4.60 (a) practical current connected to a load  $R_L$ , (b) load current decreases as  $R_L$  increases.

(2) When the current source is short-circuited,  $i_{sc} = i_s$ . Thus,  $i_s$  may be regarded as the *unloaded source current*. The connection of  $R_L$  causes the terminal current to drop in magnitude; this is known as the *loading effect*.