Ve215 Electric Circuits

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Chapter 2

Basic Laws

2.1 Introduction

- In this chapter, we study some fundamental laws that govern electric circuits, known as Ohm's law and Kirchhoff's laws, and discuss some techniques commonly applied in circuit analysis.

2.2 Ohm's Law

- Materials in general have a current-resisting behavior. This physical property is known as *resistance* and is represented by the symbol *R*.
- The element used to model the current-resisting behavior of a material is the *resistor*.

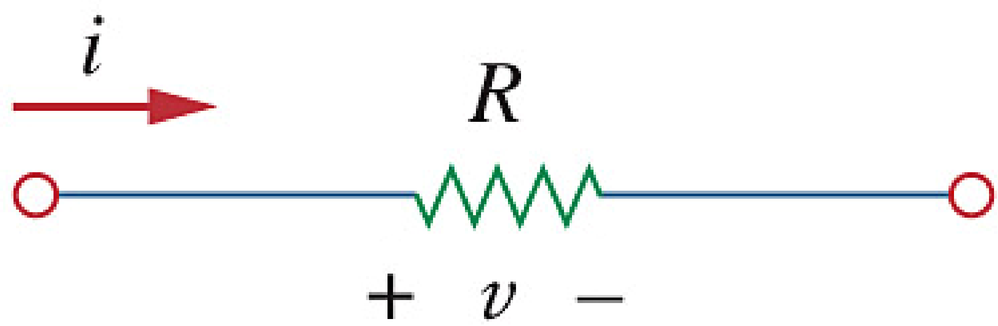
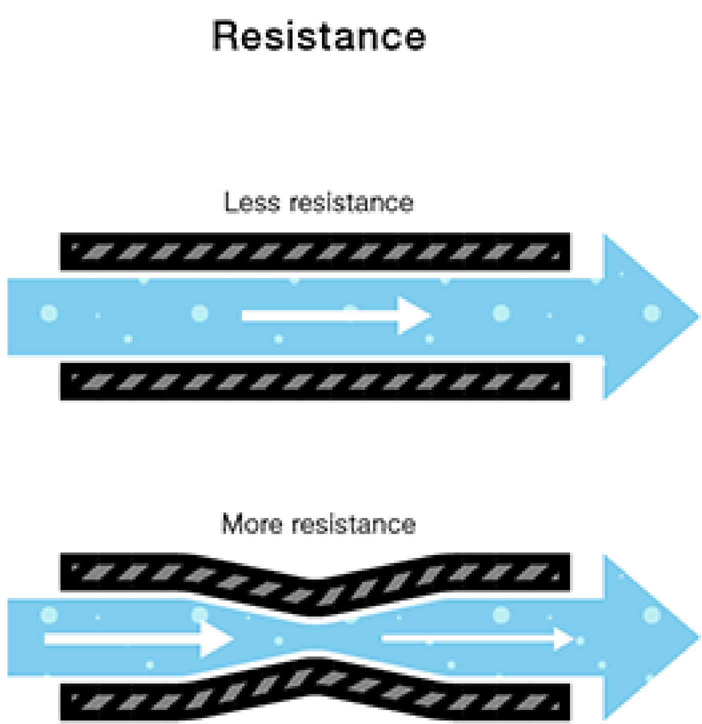
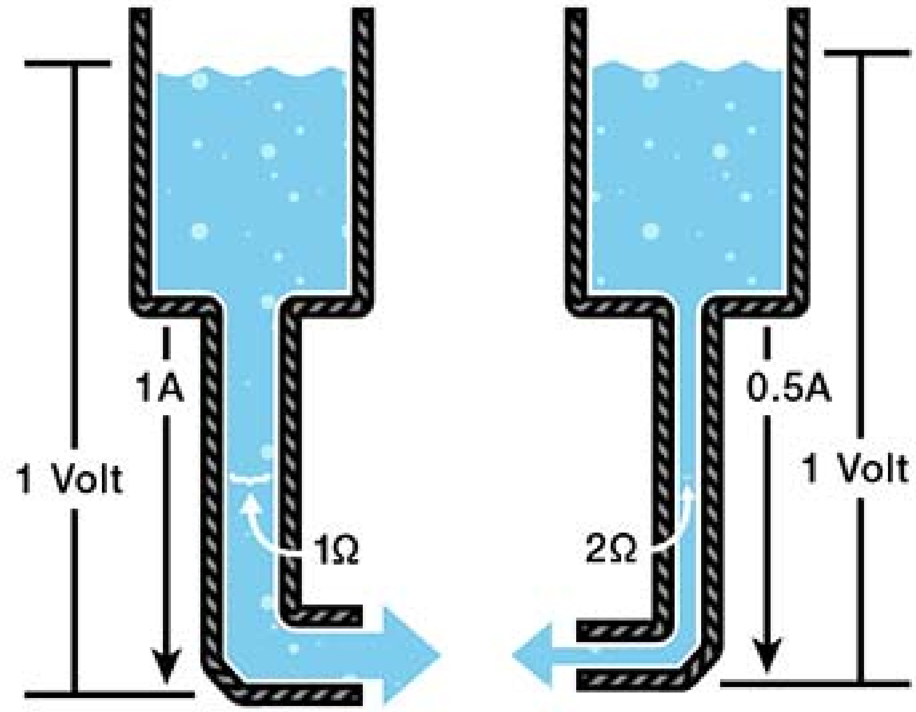


Figure 2.1(b) Circuit symbol for resistor.





Analogy of water flow and electric current

Water volume ⇔ # of electrons

Water flow ⇔ electric current

Ohm's law states that the voltage v across a resistor and the current i through the resistor are related by

$v = iR$ for PSC

Passive sign convention :
p = +vi = +(iR)i = i^2R >0
➔ Absorbing power

where R is the resistance, measured in ohms (Ω).

v=0
i can be any value

i=0
v can be any value

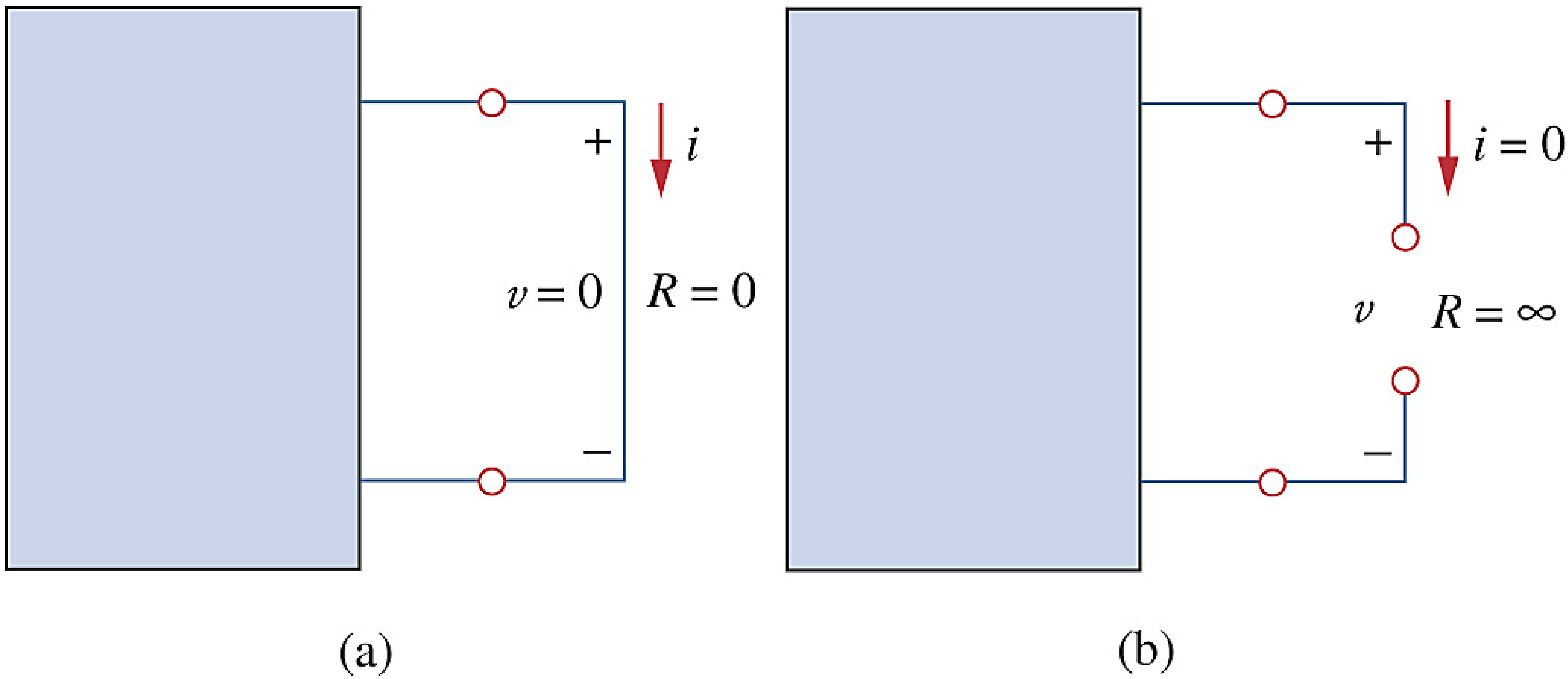


Figure 2.2 Two extreme possible values of R: (a) short circuit ($R = 0$), (b) open circuit ($R = \infty$).

- A short circuit is a circuit element with resistance approaching zero.
- An open circuit is a circuit element with resistance approaching infinity.

- A resistor is either fixed or variable.

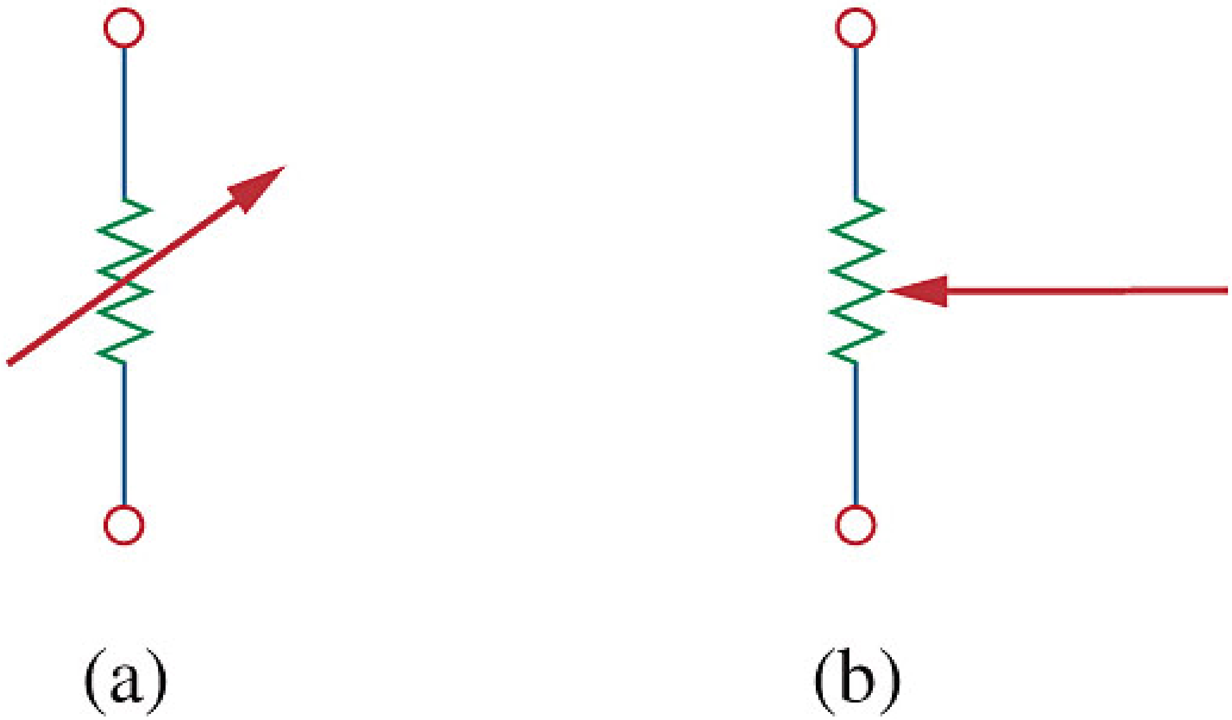


Figure 2.4 Circuit symbol for (a) a variable resistor in general, (b) a potentiometer.

- A *linear* resistor has a constant resistance and thus its current-voltage characteristic is a straight line passing through the origin.
- The resistance of a nonlinear resistor varies with current.

v=iR, R is a const. v=i*R(i)

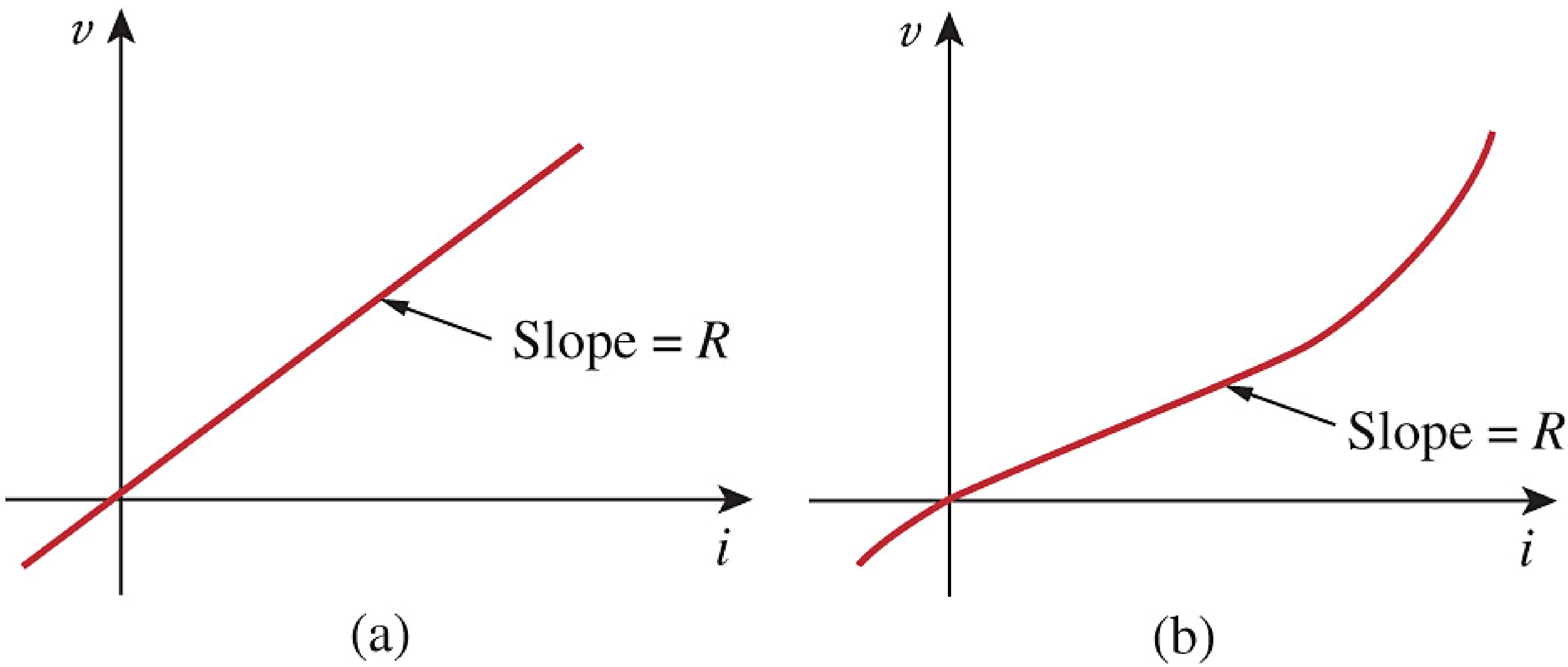


Figure 2.7 The i - v characteristic of (a) a linear resistor, (b) a nonlinear resistor.

A useful quantity in circuit analysis is the reciprocal of resistance *R*, known as conductance and denoted by *G*.

Conductance is the ability of an element to conduct electric current. It is measured in mhos (℧) or siemens (S). The word mho is ohm spelled backward.

The power dissipated by a resistor can be expressed in terms of R or G.

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = vi = \frac{i^2}{G} = v^2 G$$

2.3 Nodes, Branches, and Loops

- A circuit is also known as a *network*.
- A *branch* represents a single element such as a voltage source or a resistor. In other words, a branch represents any two-terminal element.
- A *node* is the point of connection between two or more branches.
- A *loop* is any closed path in a circuit.

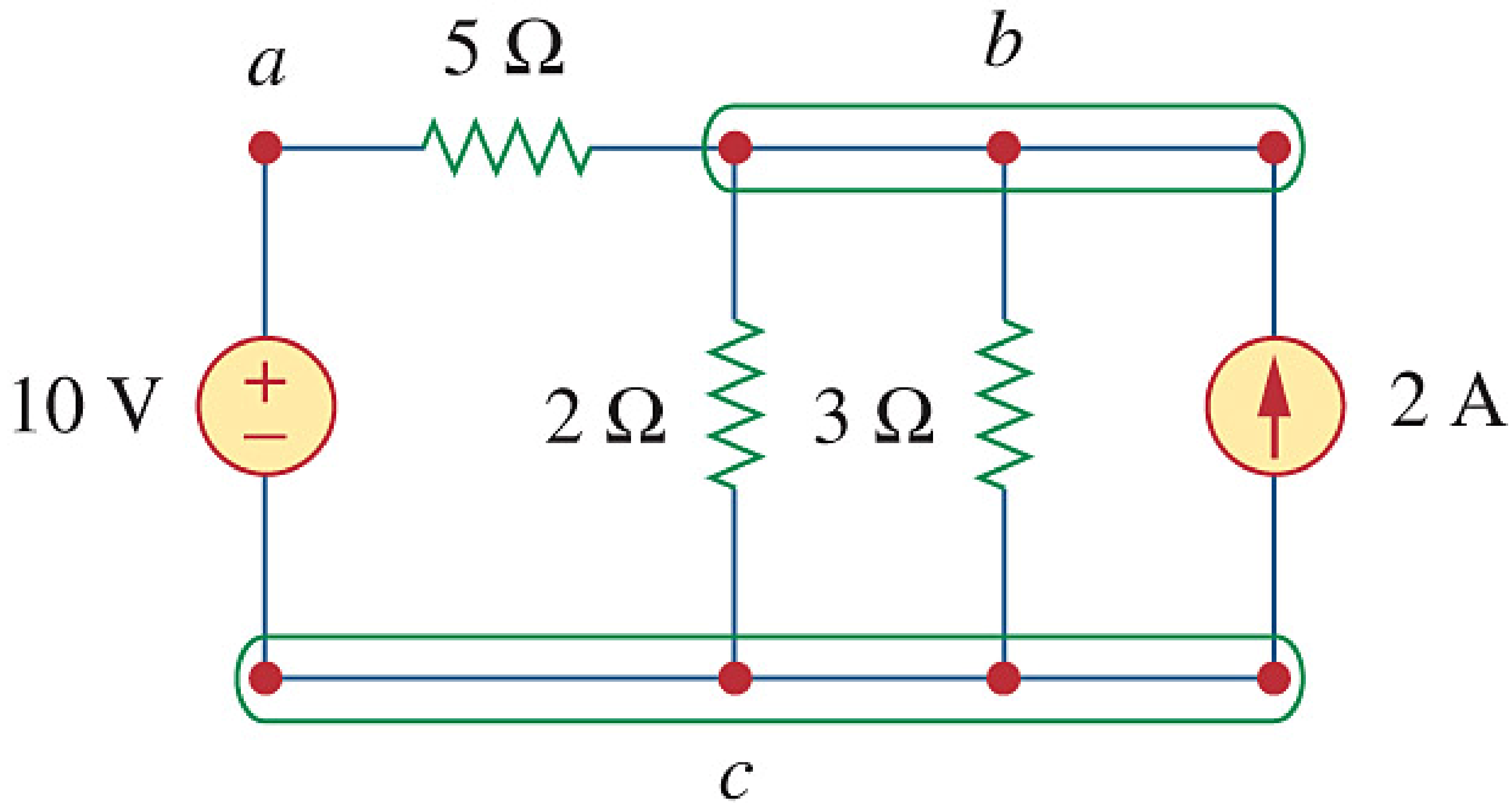


Figure 2.10 Nodes, branches, and loops.

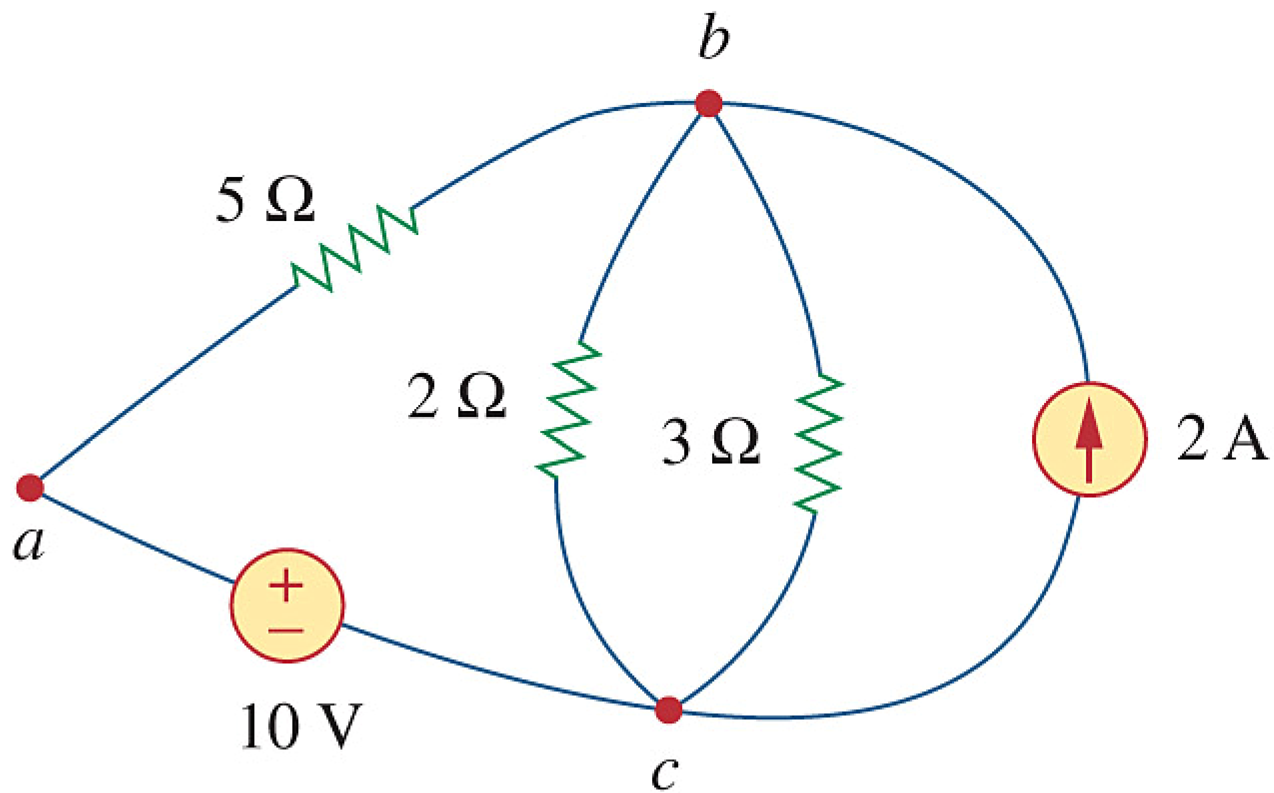


Figure 2.11 The circuit of Fig. 2.10 is redrawn.

- A *mesh* is a loop that does not enclose any other loops. (i.e., smallest loop)
- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage.

A network with b branches, n nodes, and m meshes will satisfy the fundamental theorem of network topology:

$$b = m + n - 1$$

Example 2.4 Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identity which elements are in series and which are in parallel.

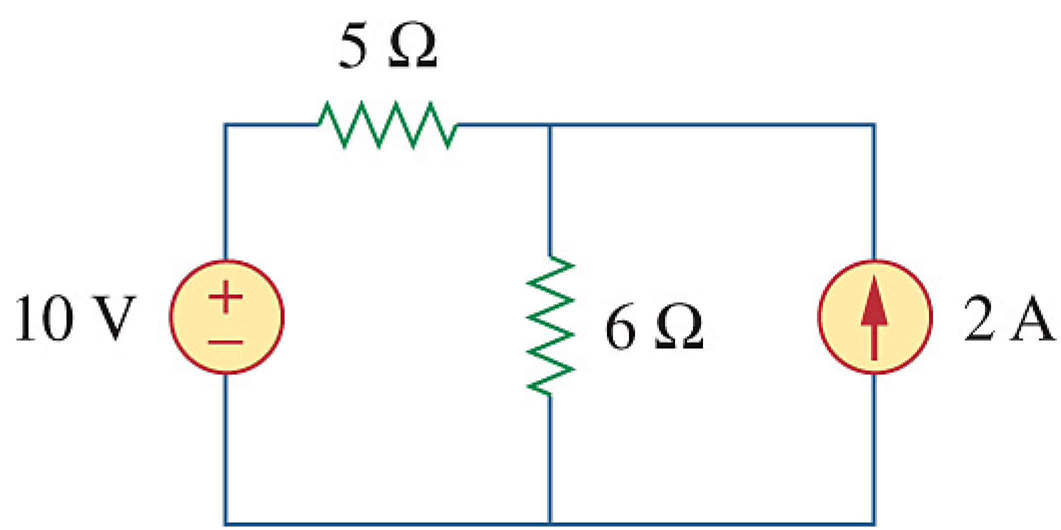


Figure 2.12

Solution :

Four branches and three nodes are identified in Fig. 2.13. ⋯

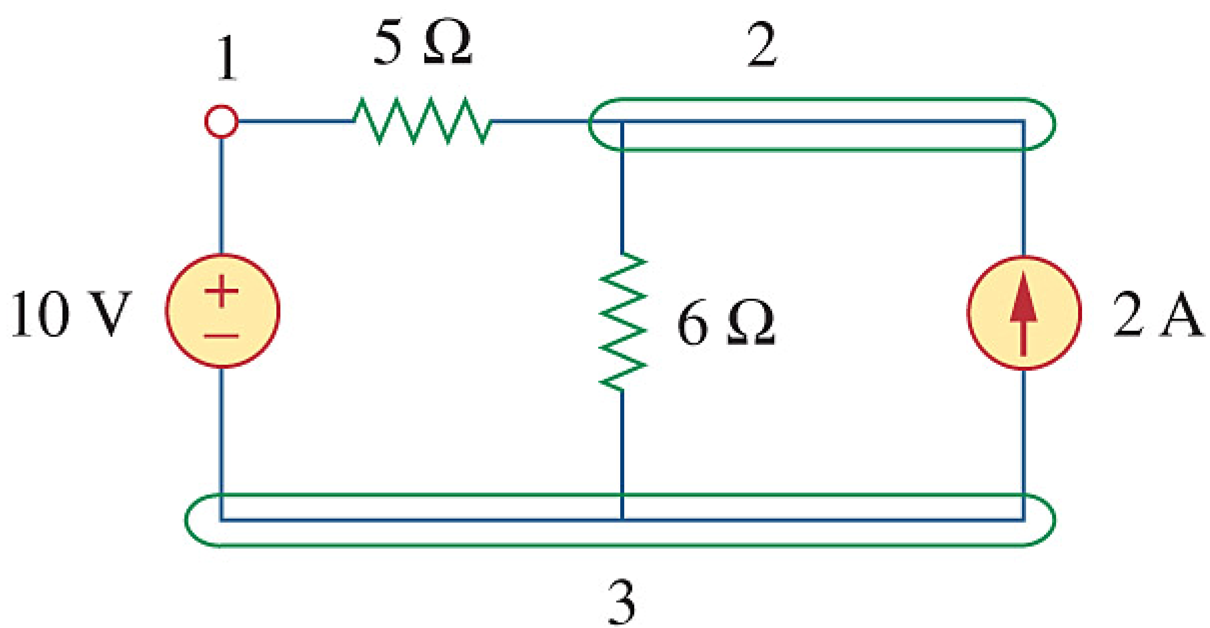


Figure 2.13

2.4 Kirchhoff's Laws

- Kirchhoff's current law (KCL) is based on the law of conservation of charge.
- It states that the algebraic sum of currents entering a node (or a closed boundary) is zero.
- In other words, the sum of the currents entering a node is equal to the sum of the currents leaving the node.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

the law of conservation of charge

KCL

$$\sum Q_n=0 \xrightarrow{d/dt} \sum i_n=0$$

$$\sum i_n=0$$
$$i_1-i_2+i_3+i_4-i_5=0$$
$$i_1+i_3+i_4=i_2+i_5$$

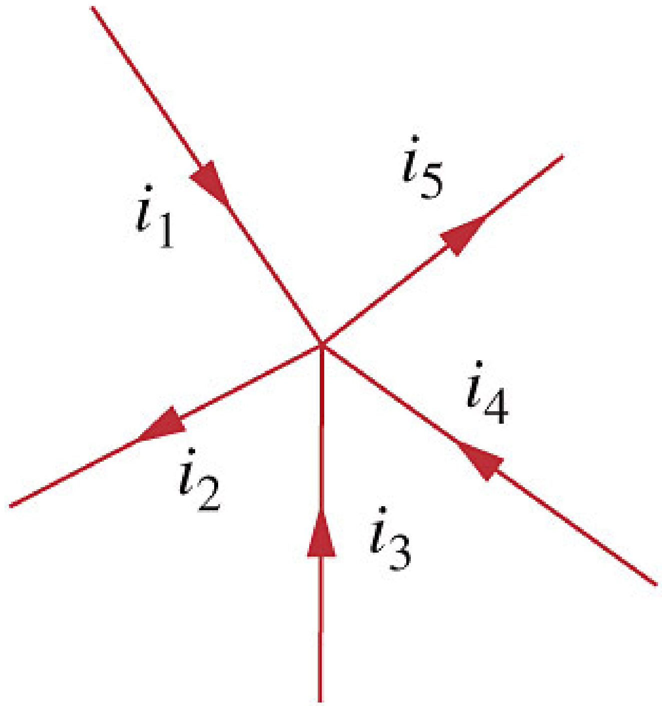


Figure 2.16 Current at a node illustrating KCL.

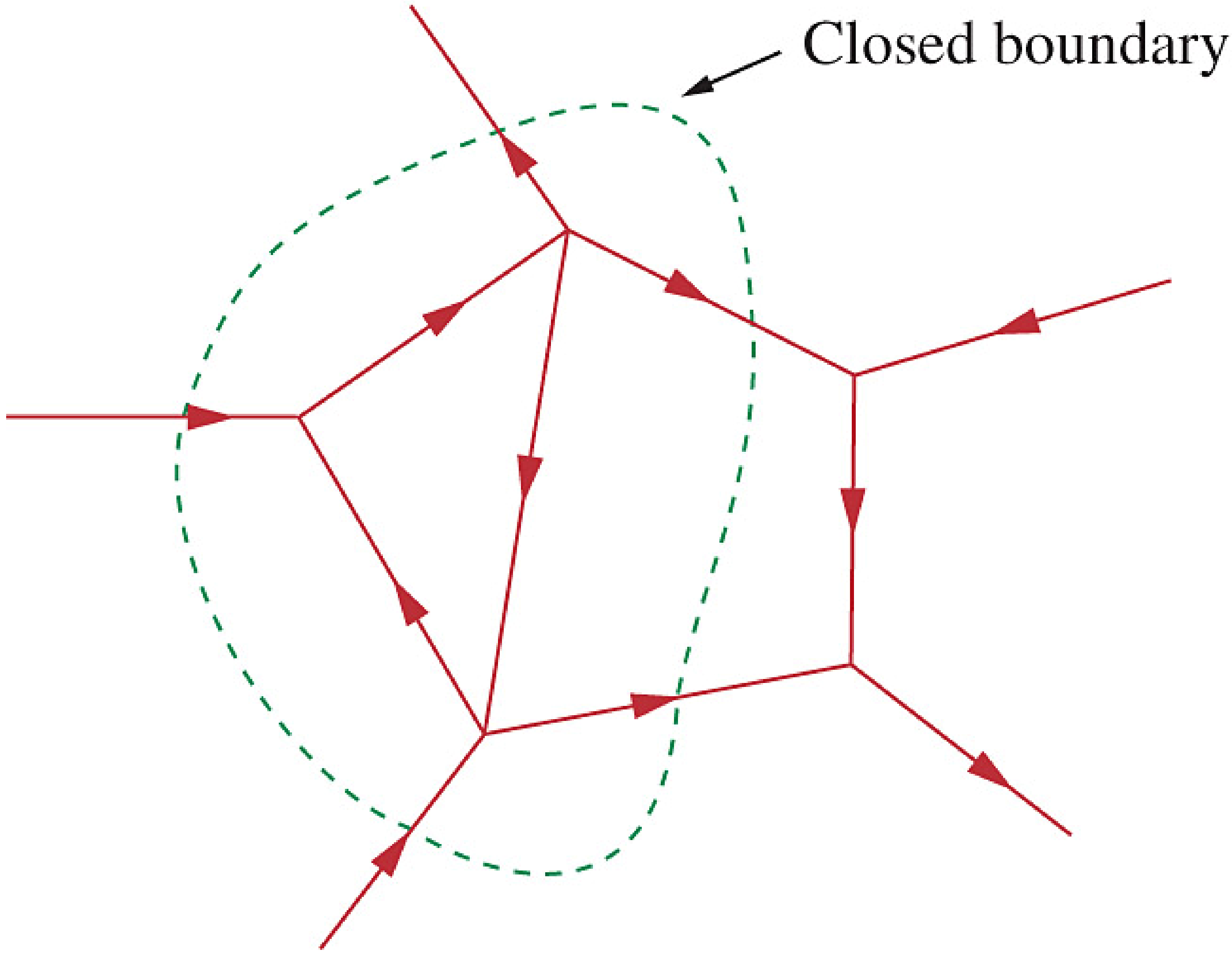
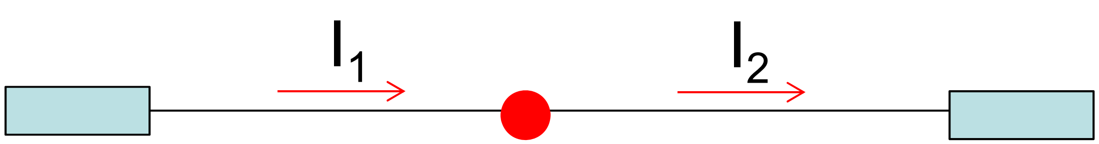


Figure 2.17 Applying KCL to a closed boundary.

- A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. See Fig. 2.18.
- A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise, KCL will be violated.



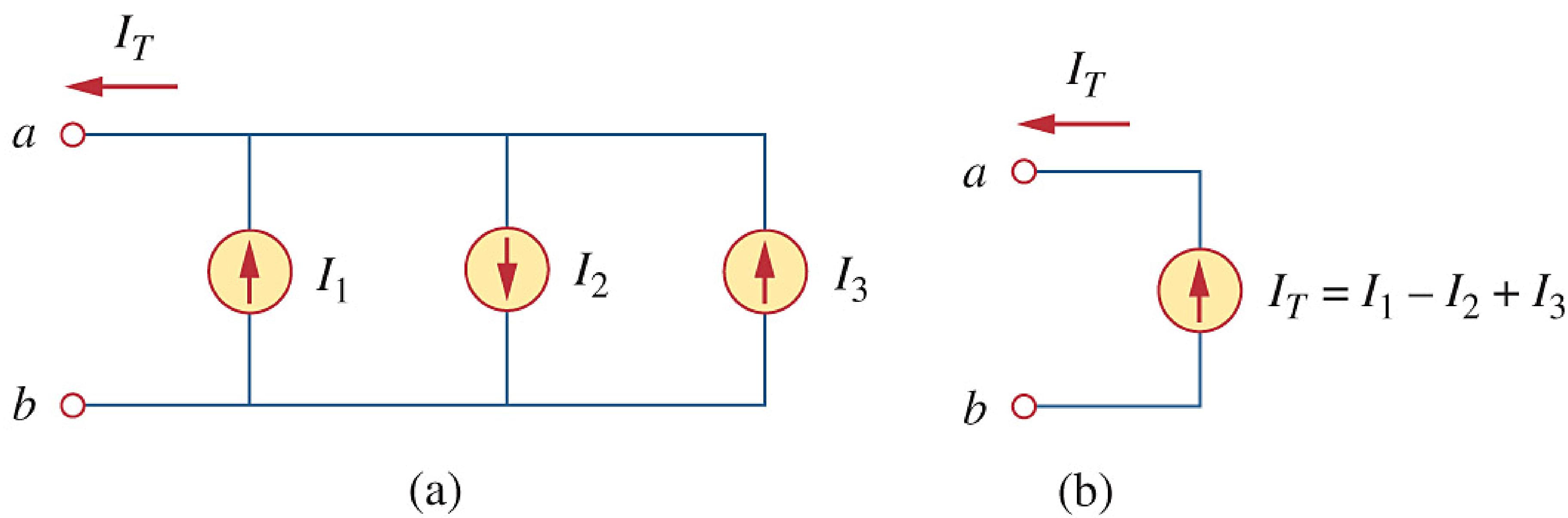


Figure 2.18 Current sources in parallel: (a) original circuit, (b) equivalent circuit. Circuits are said to be equivalent if they have the same i - v relationship at a pair of terminals.

- **Kirchhoff's voltage law (KVL)** is based on the principle of conservation of energy.
- The University Physics: *Potential* is potential energy per unit charge.

$\boxed{v=dw/dq}$

- The University Physics: The potential difference V_{ab} equals the work done by the electric force when a unit charge moves from *a* to *b*.
- The NET work done on a charged particle in a closed path is zero.

- **Kirchhoff’s voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- In other words, the sum of voltage drops is equal to the sum of voltage rises.

Mathematically, KVL implies that

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of branches in the loop and v_m is the mth voltage drop (or rise) in the loop.

We can start with any branch and go around the loop either clockwise or counterclockwise.

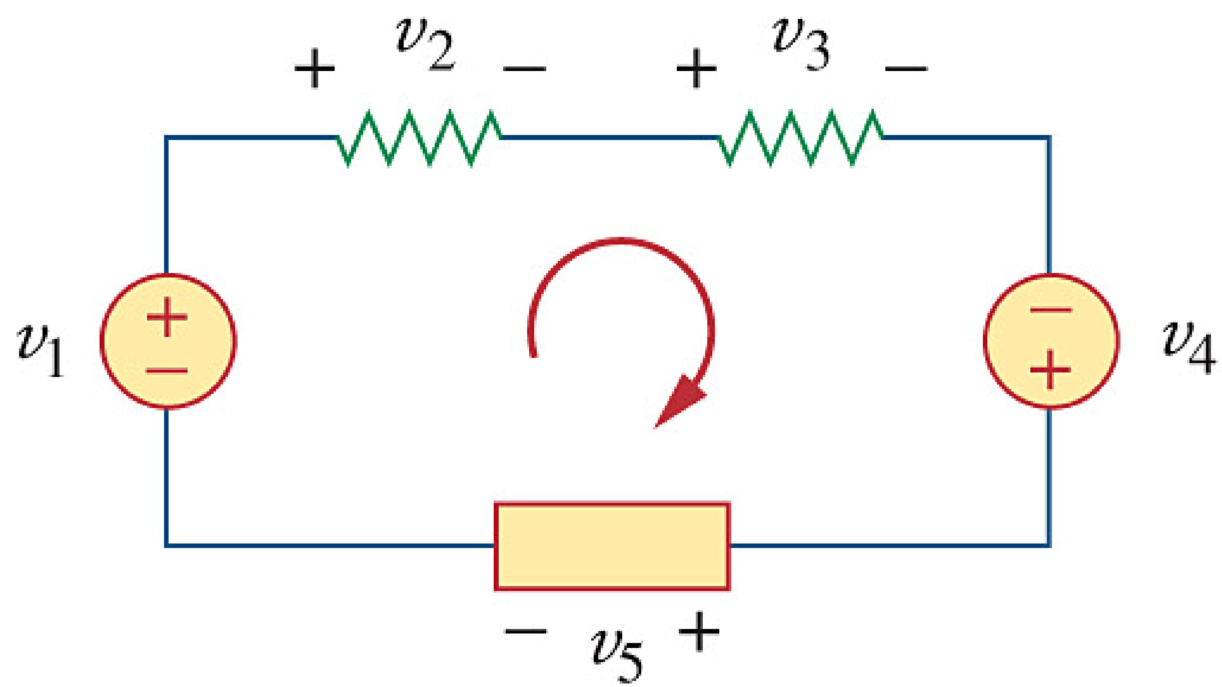
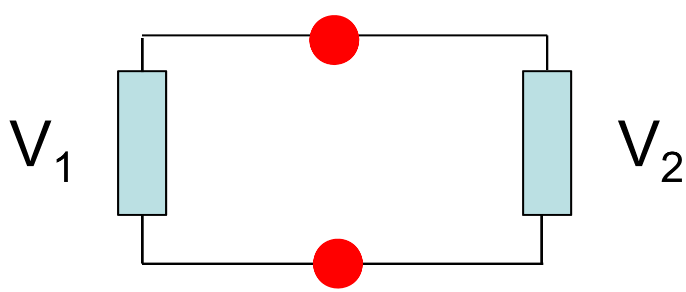


Figure 2.19 A single-loop circuit illustrating KVL.

$$-v_1+v_2+v_3-v_4+v_5 = 0$$

- A simple application of KVL is combining voltage sources in series. The combined voltage is the algebraic sum of the voltages supplied by the individual sources. See Fig. 2.20.
- A circuit cannot contain two different voltages, V_1 and V_2, in parallel, unless $V_1 = V_2$; otherwise, KVL will be violated.



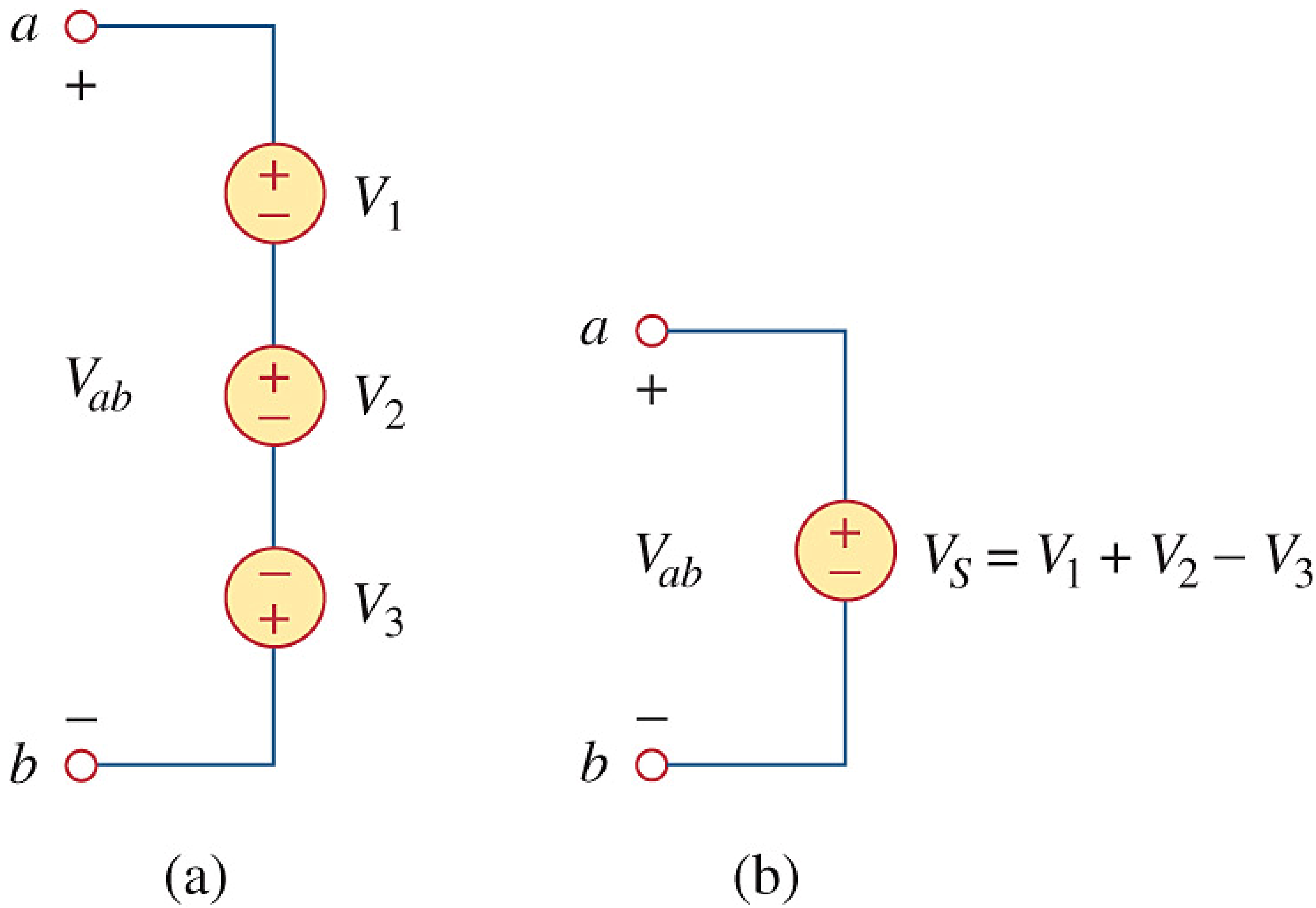


Figure 2.20 Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Practice Problem 2.5 Find v_1 and v_2 in the circuit of Fig. 2.22.

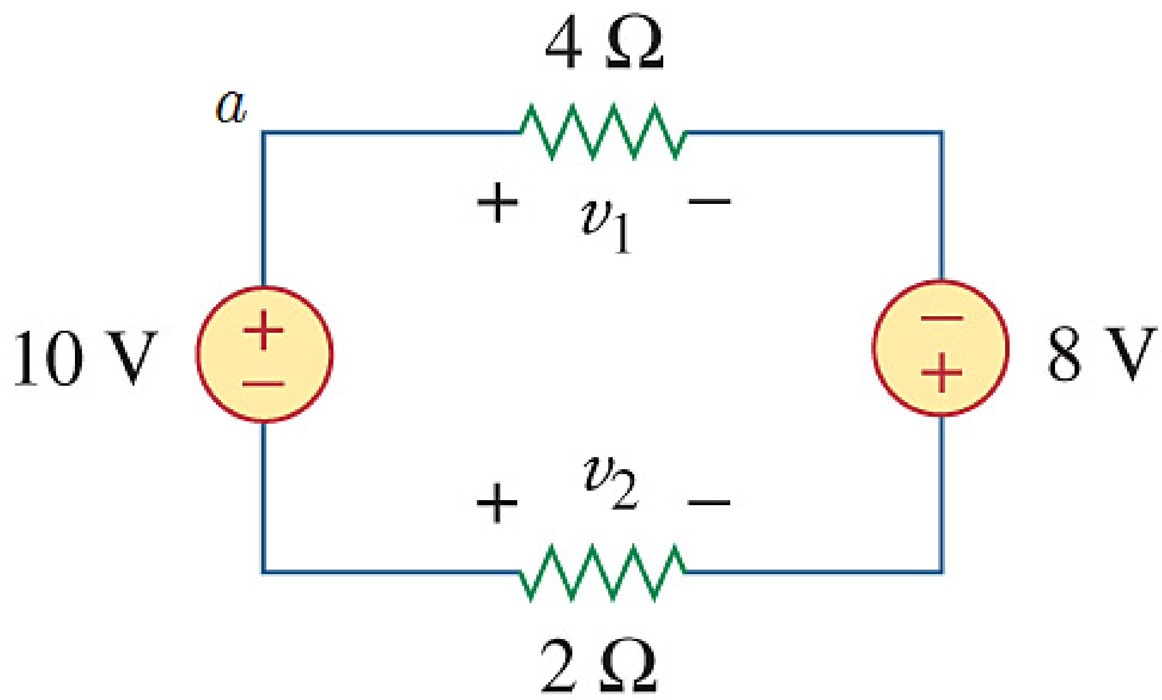


Figure 2.22

Solution : Apply KVL (Choose node a as the strating point, trace the loop clockwise, and assign a positive sign to a voltage drop)

$$v_1 - 8 - v_2 - 10 = 0 \qquad (1)$$

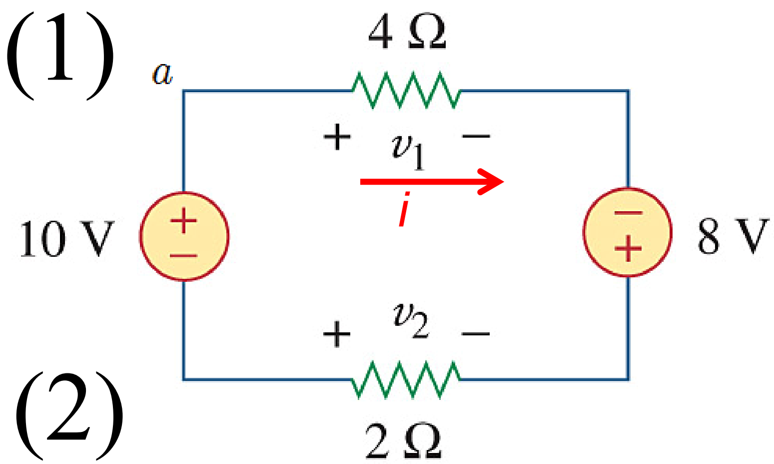


Figure 2.22

From Ohm's law,

$$i = \frac{v_1}{4} = -\frac{v_2}{2} \Rightarrow v_1 = -2v_2 \qquad (2)$$

Slove the simultaneous equations, we have $v_1 = 12$ V and $v_2 = -6$ V.

Example 2.6 Determine v_o and i in the circuit shown in Fig. 2.23(a).

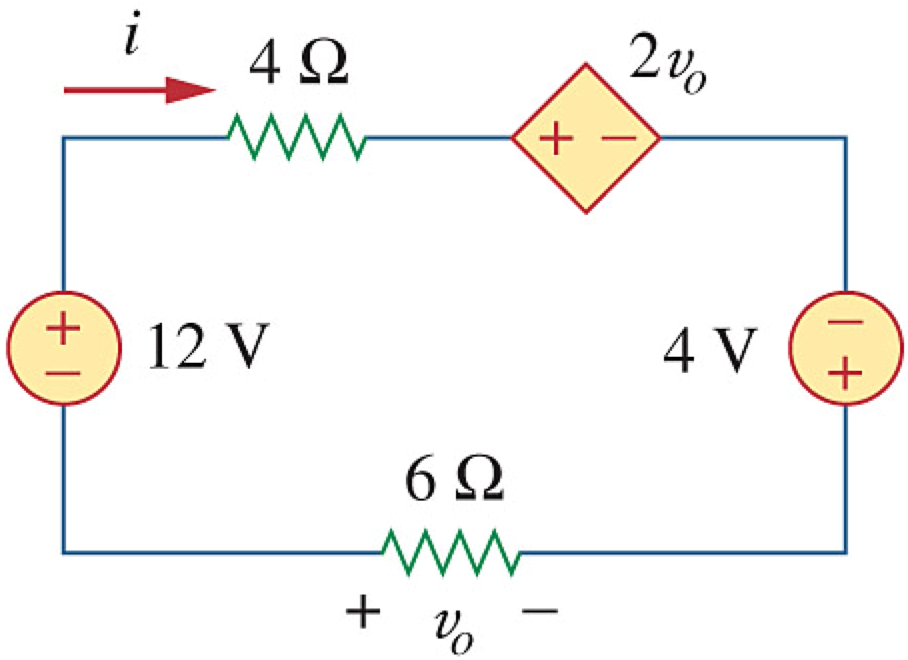


Figure 2.23 (a)

Answer : 48 V, −8 A.

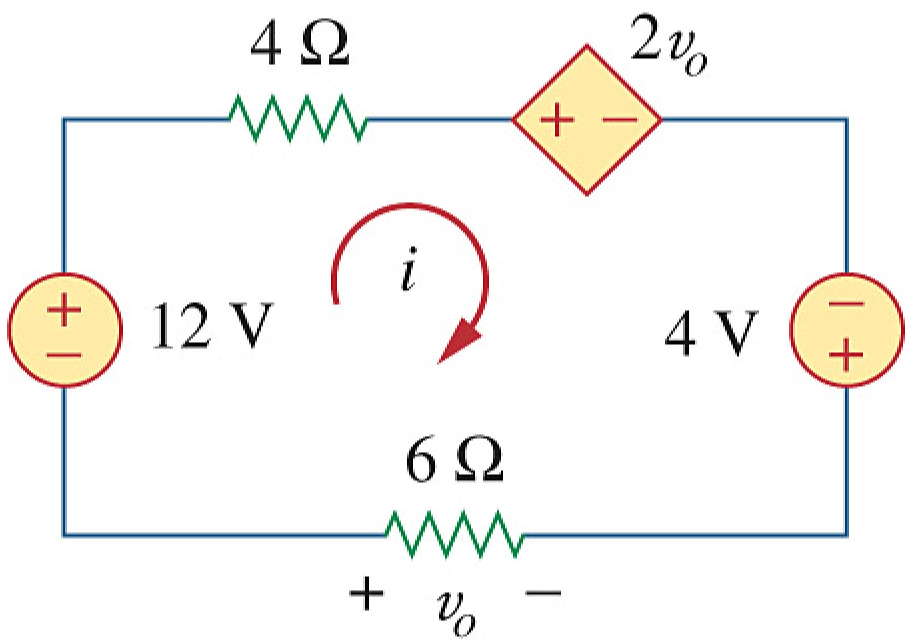


Figure 2.23 (b)

Series connection ➔ KVL

$-12+4i+2v_o-4-v_o=0$

$v_o=6(-i)$

$-12+4i-12i-4+6i=0$

$-2i=16$

$i=-8A$

$v_o=48V$

Practice Problem 2.7 Find v_o and i_o in the circuit of Fig. 2.26.

Answer : 8V, 4A.

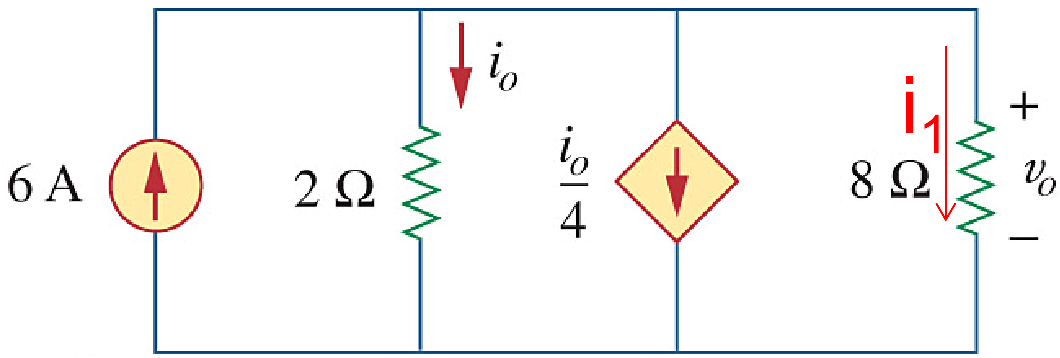


Figure 2.26

Parallel connection → KCL

$i_1 = v_o/8$; $v_o = 2i_o$

$6 = i_o + i_o/4 + i_1$

$6 = i_o + i_o/4 + i_o/4$

$3i_o/2 = 6$

$i_o = 4A$

$v_o = 8V$

Example 2.8 Find currents and voltages in the circuit shown in Fig. 2.27(a).

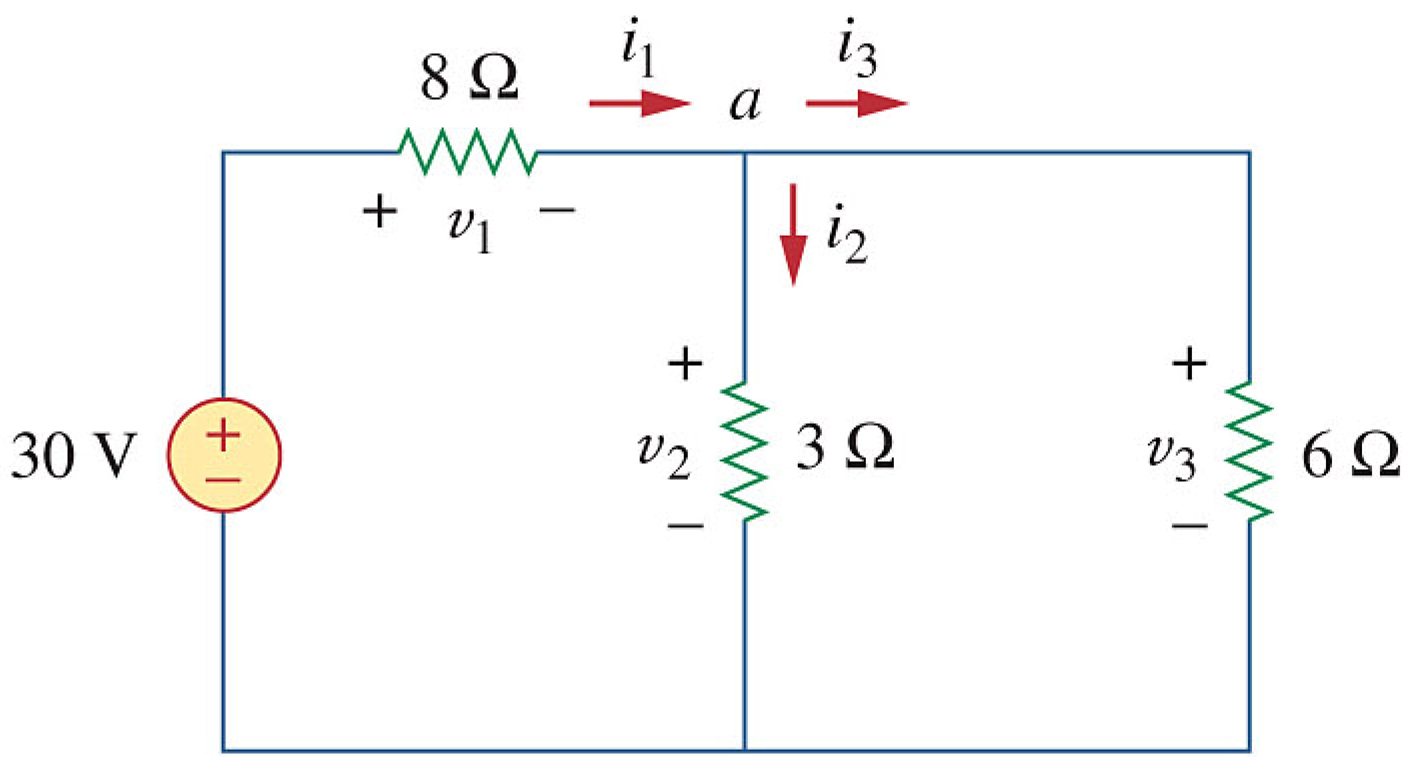


Figure 2.27(a)

Answer : $i_1 = 3$ A, $i_2 = 2$ A, $i_3 = 1$ A,

$v_1 = 24$ V, $v_2 = v_3 = 6$ V.

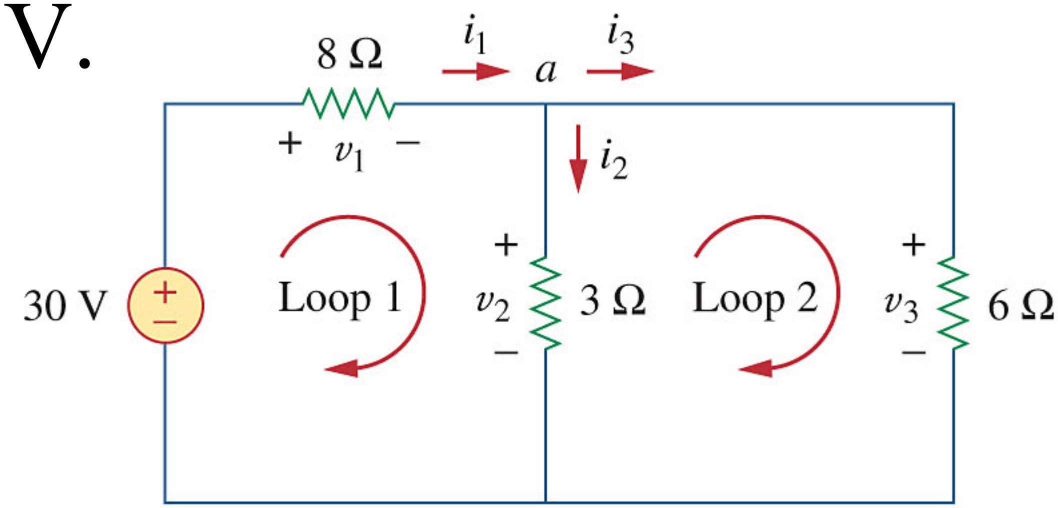


Figure 2.27 (b)

KVL for loop 1: $-30+v_1+v_2=0$ … (1)

KVL for loop 2: $-v_2+v_3=0 \rightarrow v_2=v_3$ … (2)

KCL at node a: $i_1=i_2+i_3$

Ohm's law: $v_1/8=v_2/3+v_3/6$ … (3)

$v_1+v_2=30$; $v_1/8=V_2/2$ or $V_1=4V_2$

$5v_2=30$, $v_2=6V$…

2.5 Series Resistors and Voltage Division

- The equivalent resistance of *N* resistors connected in series is the sum of their individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

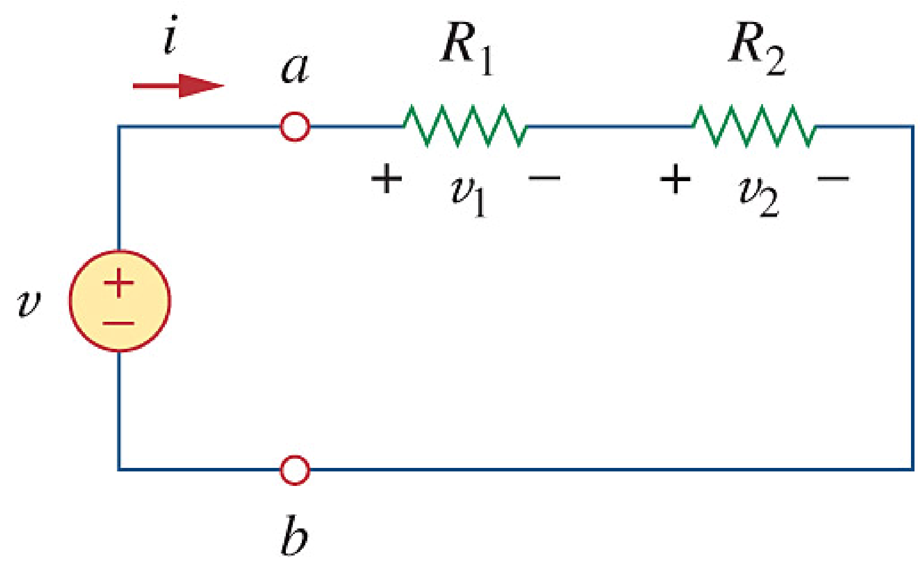


Figure 2.29 A single-loop circuit with two resistors in series.

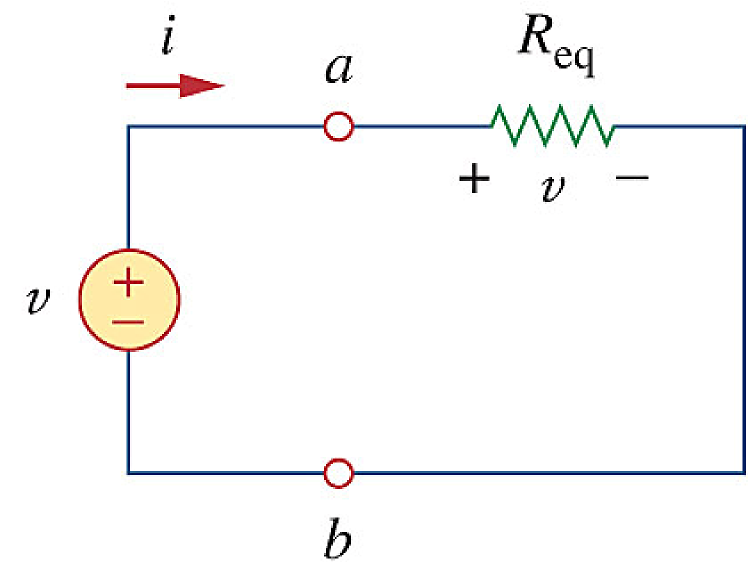


Figure 2.30 Equivalent circuit of the Fig. 2.29 circuit.

$$v = v_1+v_2 = iR_1+iR_2 = i \times (R_1+R_2)$$

$$v = i \times R_{eq}$$

$$R_{eq} = R_1+R_2$$

The voltage across each resistor is

$$v_n = \frac{R_n}{\sum_{n=1}^{N} R_n} v, \quad n = 1, 2, \ldots, N$$

In series ➔ same $i = v/\sum R_n$
$v_n = iR_n = (v/\sum R_n)R_n = (R_n/\sum R_n)v$

Notice that the source voltage *v* is divided among the resistors in direct proportion to their resistances. This is called the *principle of voltage division*, and the the circuit in Fig. 2.29 is called a *voltage divider*.

- The equivalent conductance of *N* resistors connected in parallel is the sum of their individual conductances.

$$G_{eq} = G_1 + G_2 + \cdots + G_N = \sum_{n=1}^{N} G_n$$

i.e.,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

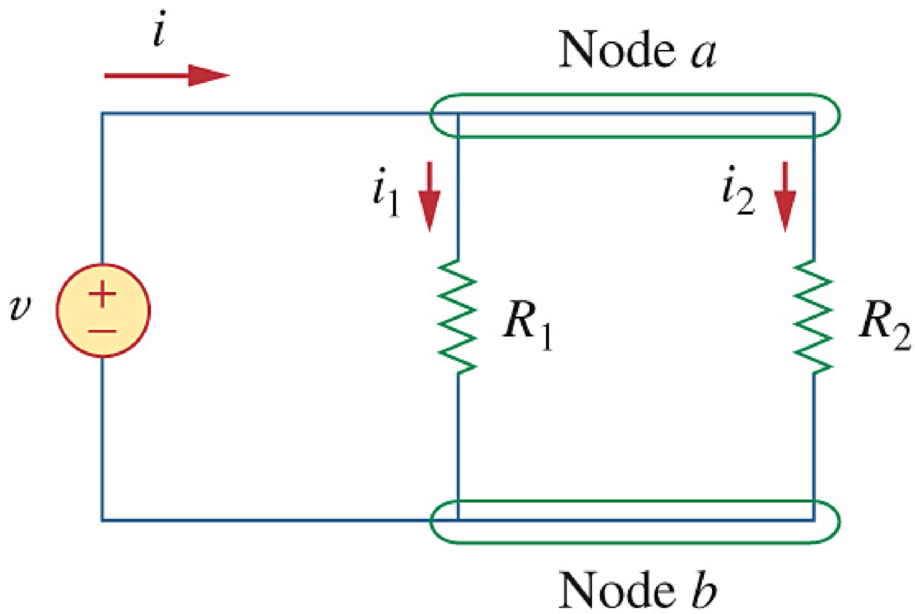


Figure 2.31 Two resistors in parallel

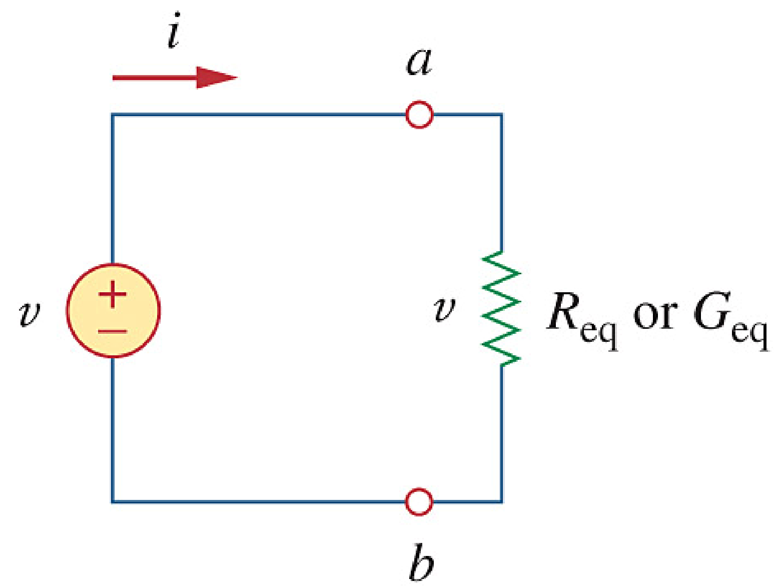


Figure 2.32 Equivalent circuit to Fig. 2.31

$$i = i_1 + i_2 = v/R_1 + v/R_2 = v \times (1/R_1 + 1/R_2)$$

$$I = v \times (1/R_{eq})$$

$$1/R_{eq} = 1/R_1 + 1/R_2$$

The current through each resistor is

$$i_n = \frac{G_n}{\sum_{n=1}^{N} G_n} i, \quad n = 1, 2, \ldots, N$$

In parallel ➔ same $v = i/\sum G_n$
$i_n = vG_n = (i/\sum G_n)Gn = (G_n/\sum G_n)i$

Notice that the source current i is divided among the resistors in direct proportion to their conductances. This is called the *principle of current division*, and the the circuit in Fig. 2.31 is called a *current divider*.

Example 2.9 Find R_{eq} for the circuit shown in Fig. 2.34.

Answer : 14.4 Ω.

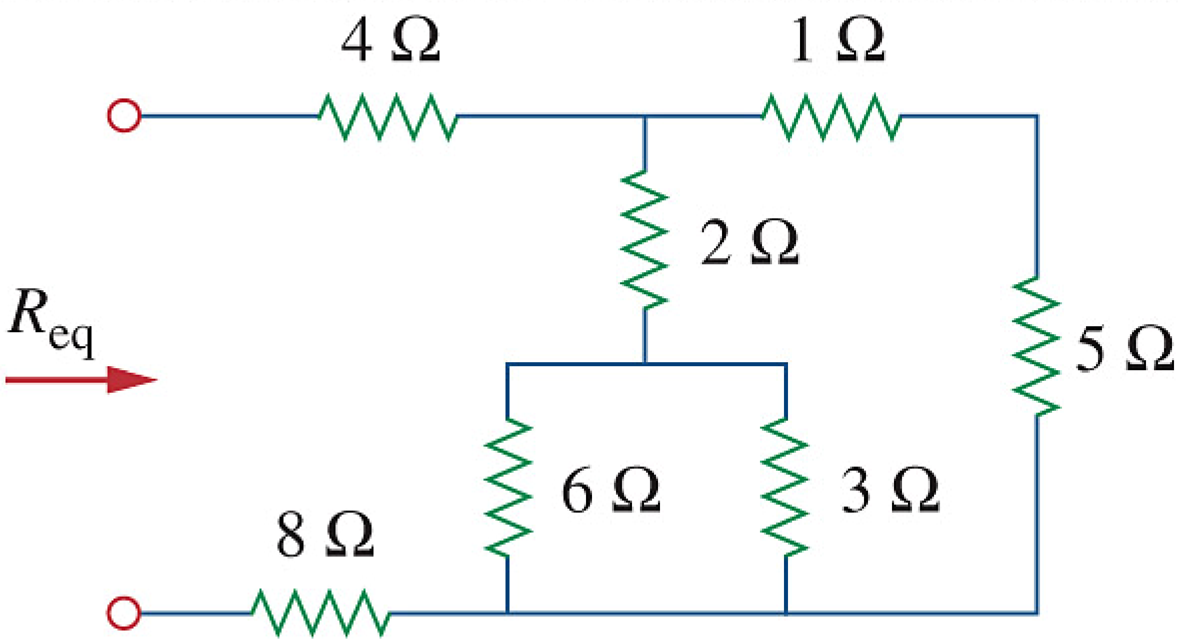


Figure 2.34

6||3=2

2+2=4

5+1=6

4||6=2.4

8+2.4+4=14.4

Example 2.10 Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

Answer : 11.2 Ω.

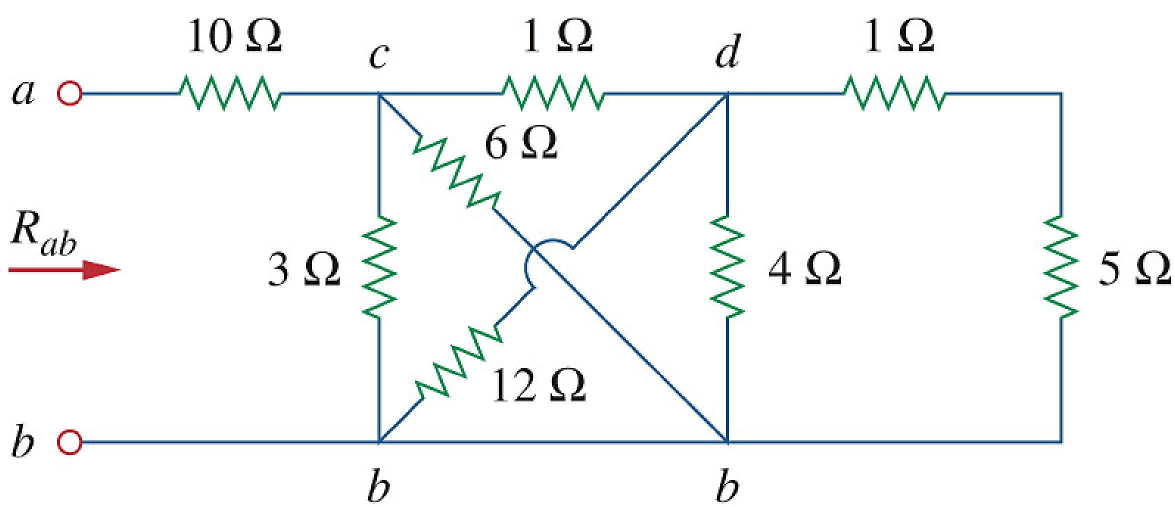


Figure 2.37

6||3=2

12||4=3

(1+5)||3=2

(1+2)||2=1.2

10+1.2=11.2

2.7 Wye-Delta Transformations

- Situations often arise in circuit analysis when resistors are neither in parallel nor in series.

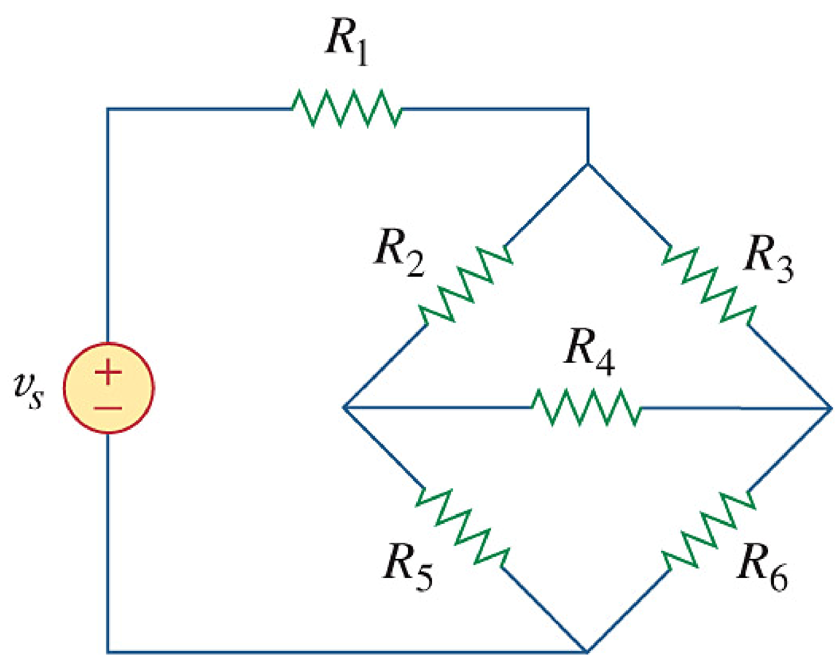


Figure 2.46 The bridge network.

- How do we combine resistors when the resistors are neither in parallel nor in series?
- Many circuits of the type shown in Fig 2.46 can be simplified by using three-terminal networks.
- These are the wye or tee network shown in Fig. 2.47 and the delta or pi network shown in Fig. 2.48.

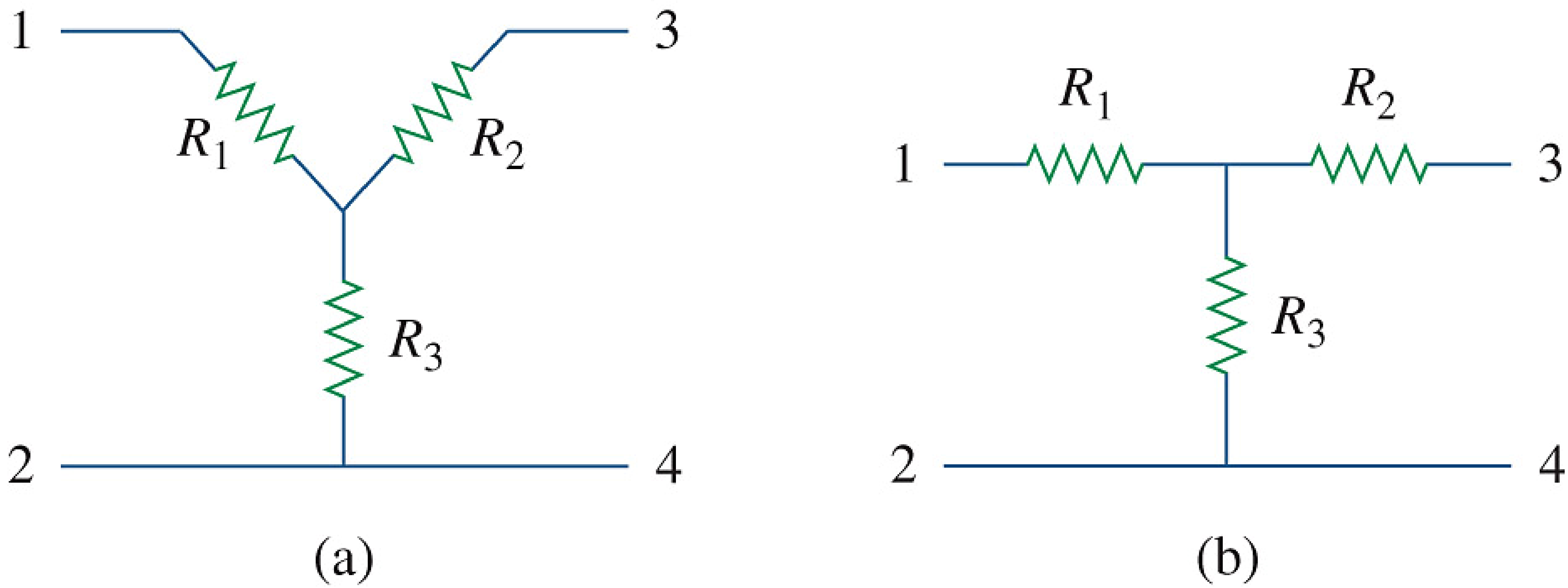


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

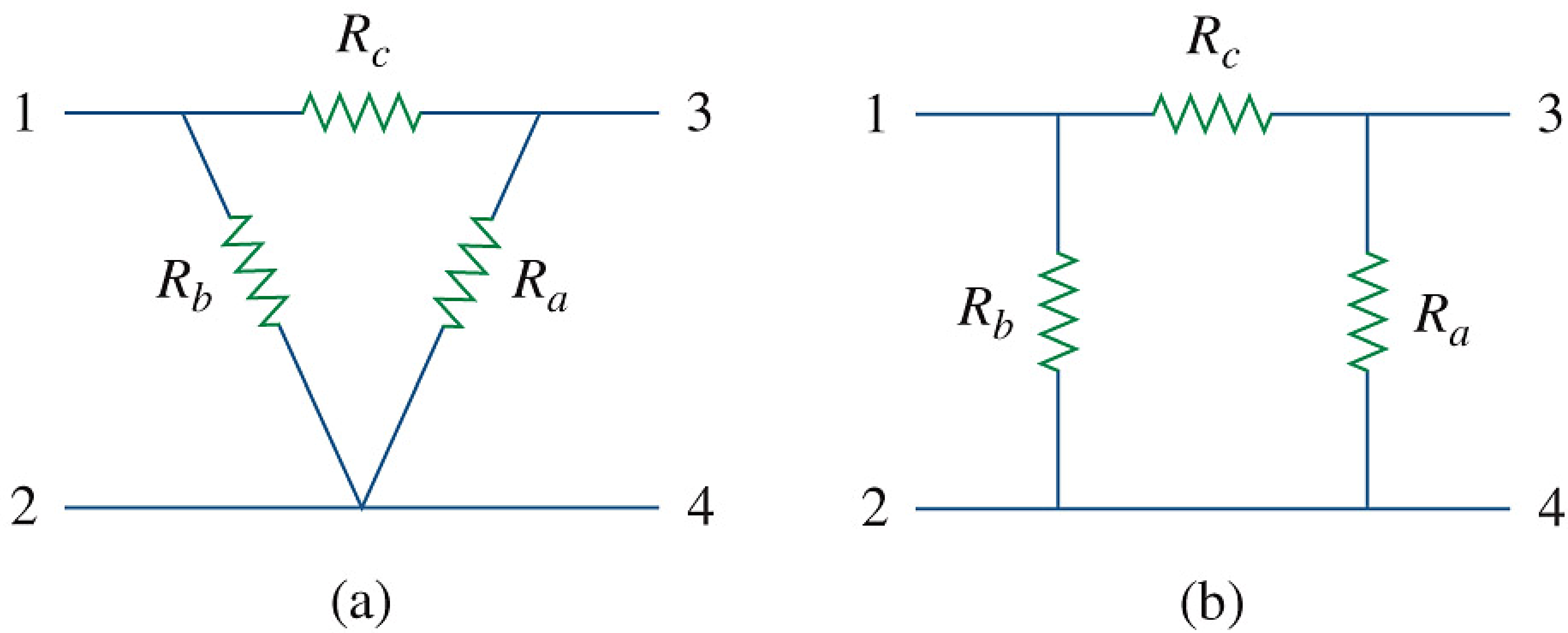


Figure 2.48 Two forms of the same network: (a) $\mathbf{\Delta}$, (b) $\mathbf{\Pi}$.

- The networks occur by themselves or as part of a larger network. Our major interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network. There are two types of transformation:
  - Delta to wye conversion
  - Wye to delta conversion

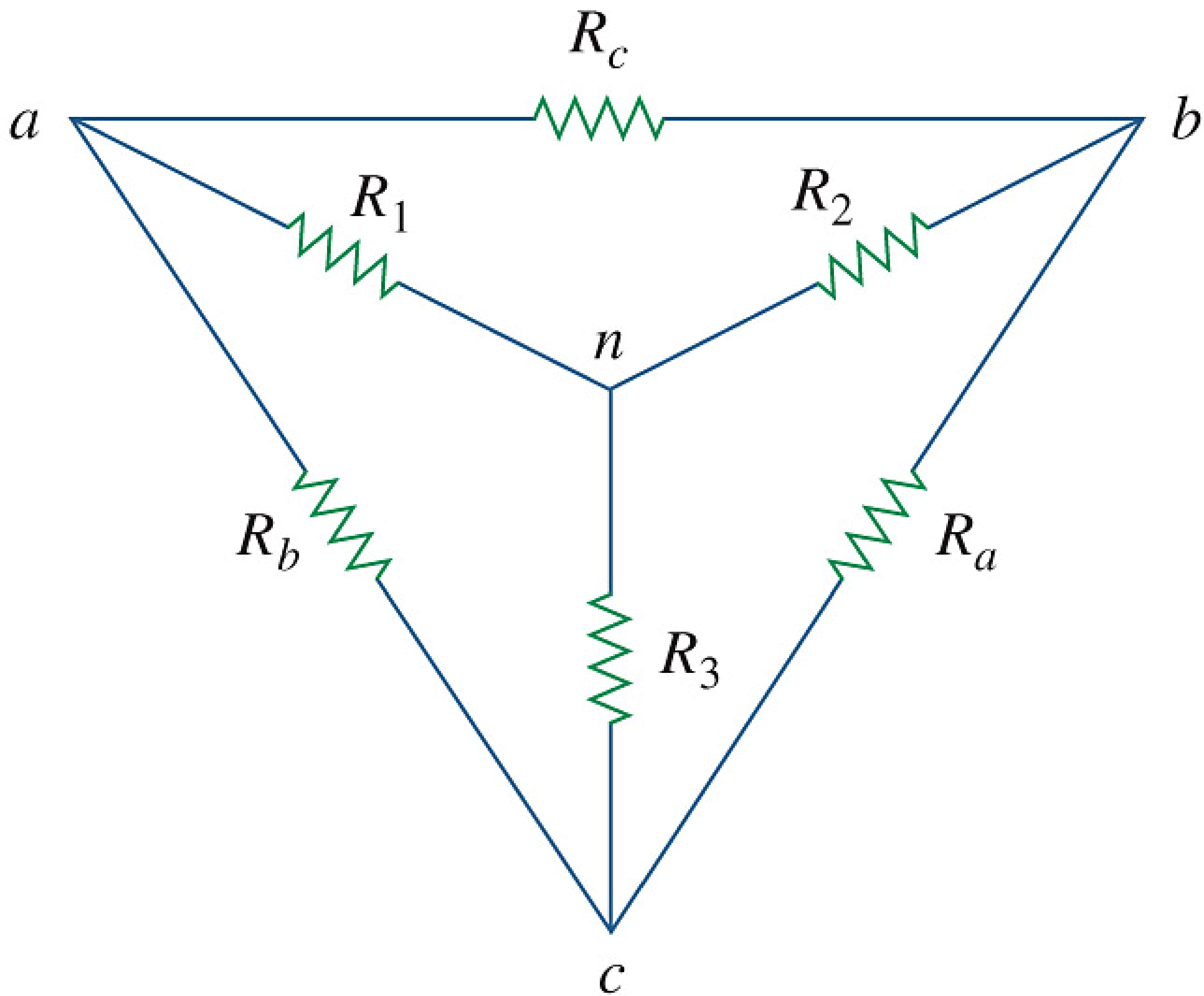


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Delta to Wye Conversion Each resistance in the Y network is the product of the resistances in the two adjacent Δ branches, divided by the sum of the three Δ resistances.

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\ R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\ R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c} \end{cases}$$

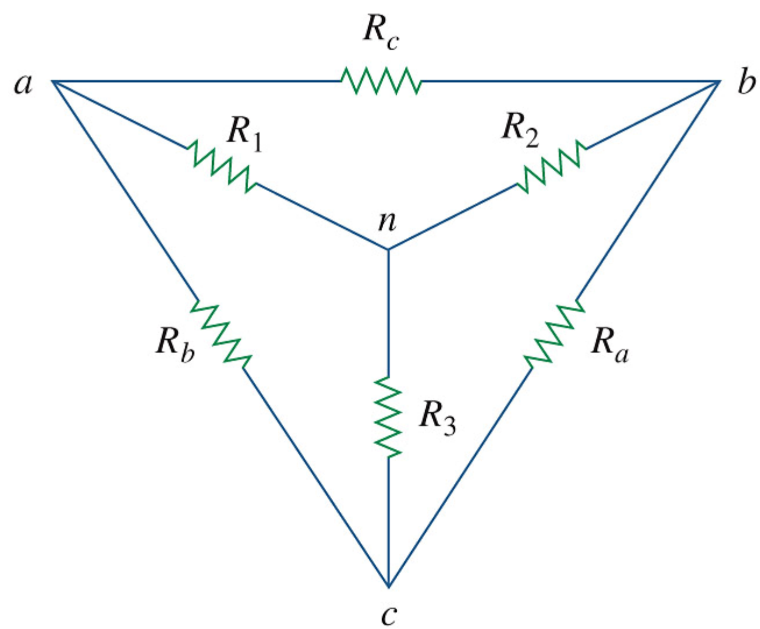


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Wye to Delta Conversion Each resistance in the Δ network is the sum of all possible products of Y resistances taken two at a time, divided by the opposite Y resistance.

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

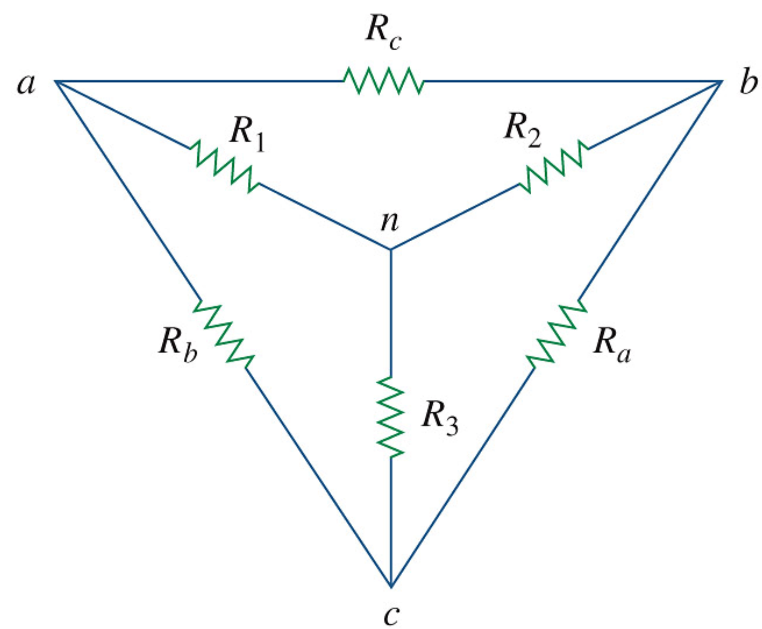


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Example 2.14 Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

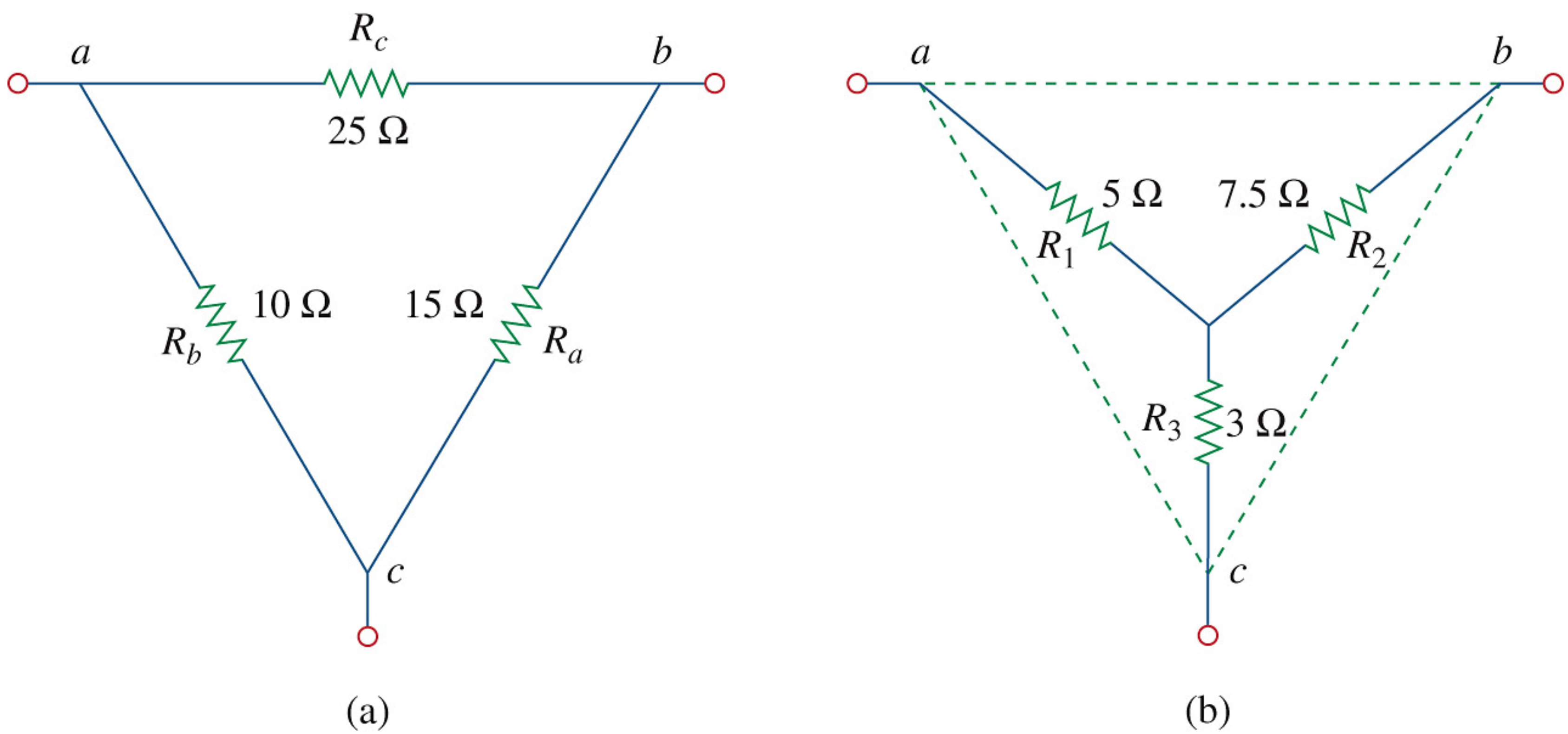


Figure 2.50

Practice Problem 2.14 Transform the wye network in Fig. 2.51 to a delta network.

Answer : $R_a = 140\ \Omega,\ R_b = 70\ \Omega,\ R_c = 35\ \Omega.$

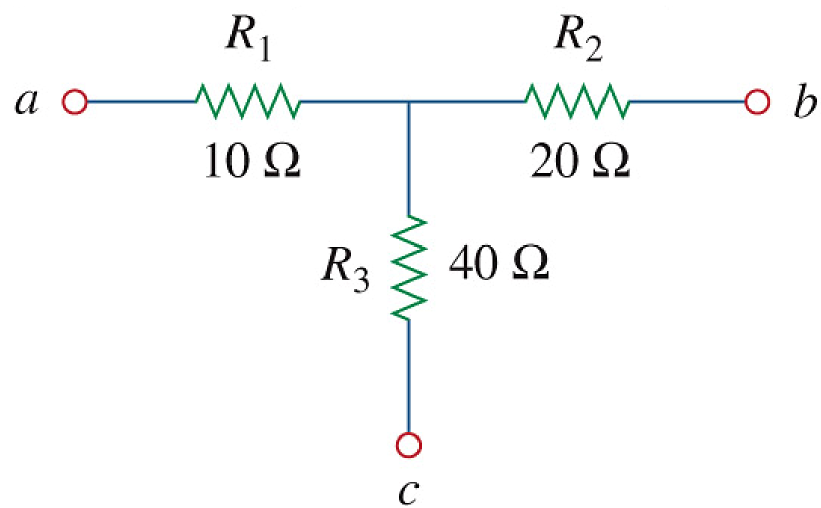


Figure 2.51

$R_a = R_{bc} = (R_1R_2 + R_2R_3 + R_3R_1)/R_1$
$= (200+800+400)/10 = 140$ ohm

Practice Problem 2.15 For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer : $R_{ab} = 40\ \Omega,\ i = 2.5\ \text{A}.$

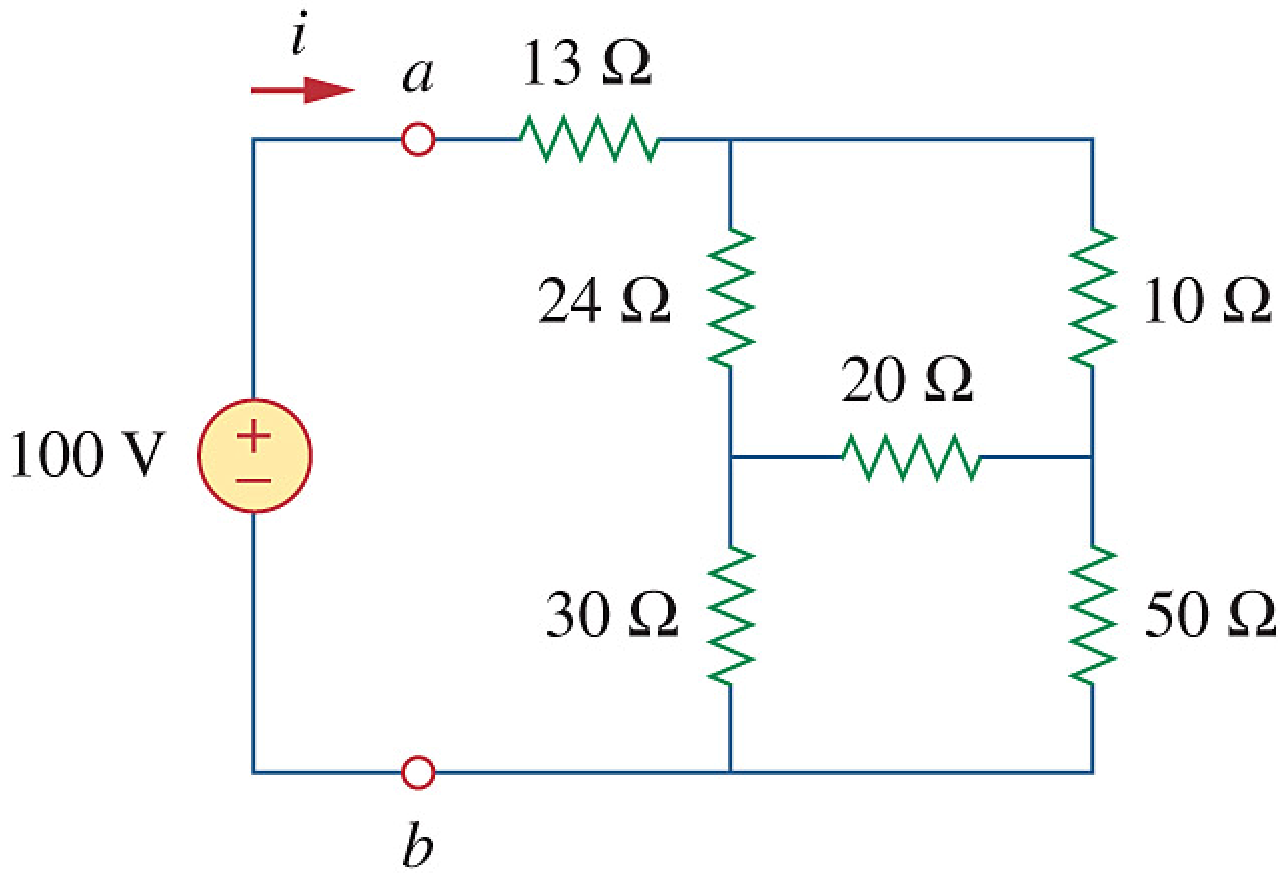


Figure 2.54

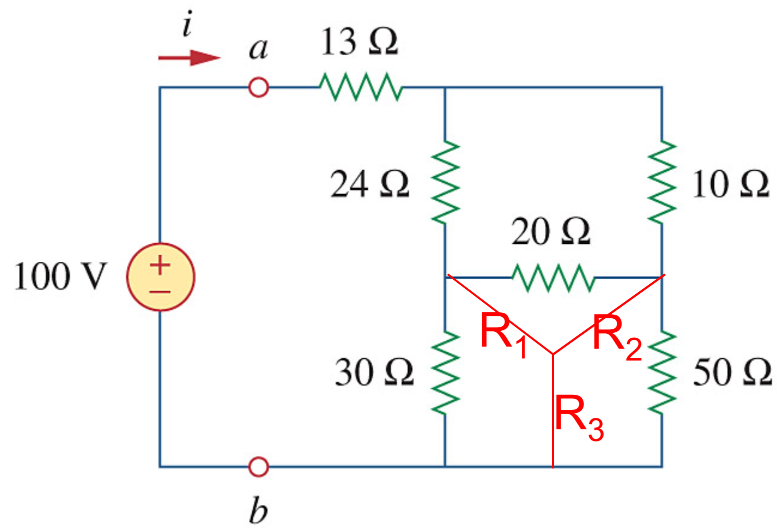


Figure 2.54

-Delta to Y, either convert the upper delta or the lower delta.

-For example, the lower delta (easier…)

R_1=600/100=6; R_2=10; R_3=15

15+(30||20)+13=40 ohm

i=V/R_{eq}=100/40=2.5A