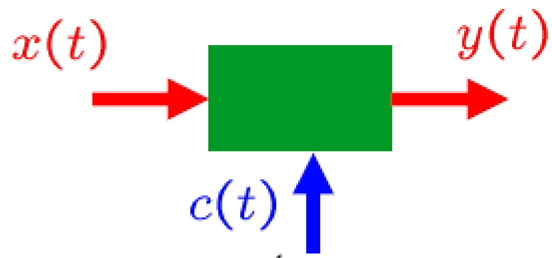
Introduction to Signals and Systems: V216

Lecture #13

Chapter 8: Communication Systems

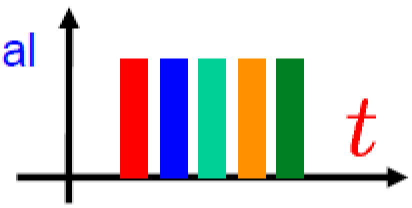
Modulation and Demodulation

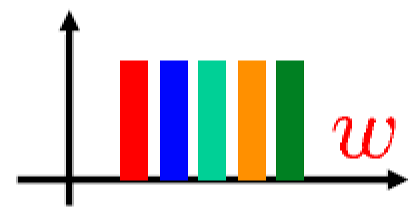
- Modulation & Demodulation:
  - M: Embedding an information-bearing signal into a second signal
  - D: Extracting the information-bearing signal
  - Methods:
    - > Amplitude Modulation (AM)
    - > Frequency Modulation (FM)



$$= A(t)\cos\big(w(t)\cdot t + \theta(t)\big)$$

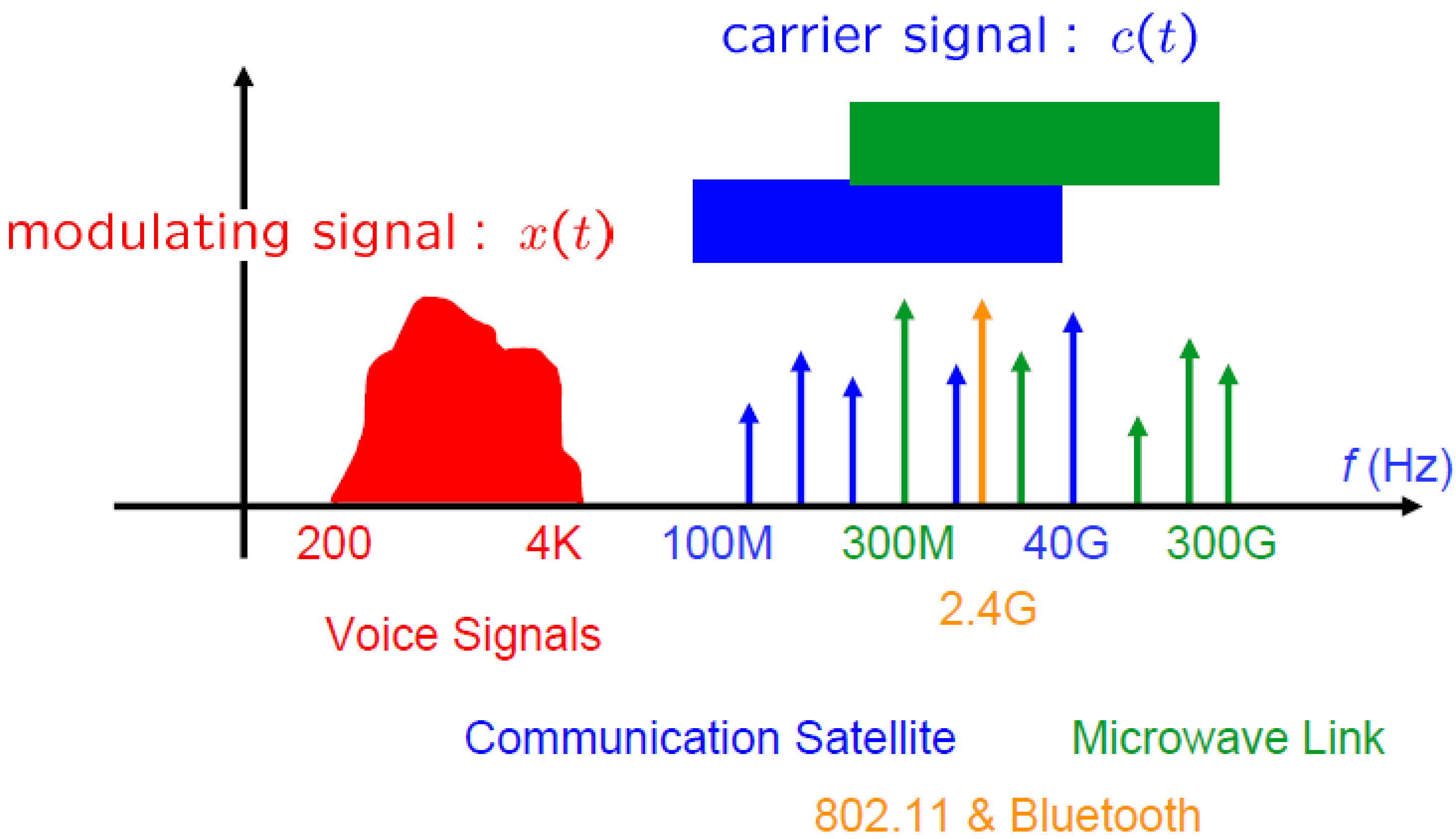
- Multiplexing & Demultiplexing:
  - Simultaneous transmission of more than one signal with overlapping spectra over the same channel
  - Methods:
    - > Time-Division Multiplexing (TDM)
    - > Frequency-Division Multiplexing (FDM)





Modulation and Demodulation

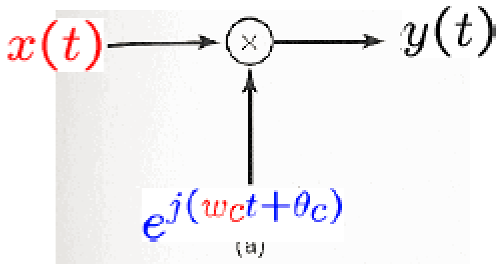
- Signal Frequency Characteristics:



modulated signal : $y(t) = x(t)\, c(t)$

AM Modulation

- AM with a Complex Exponential Carrier:

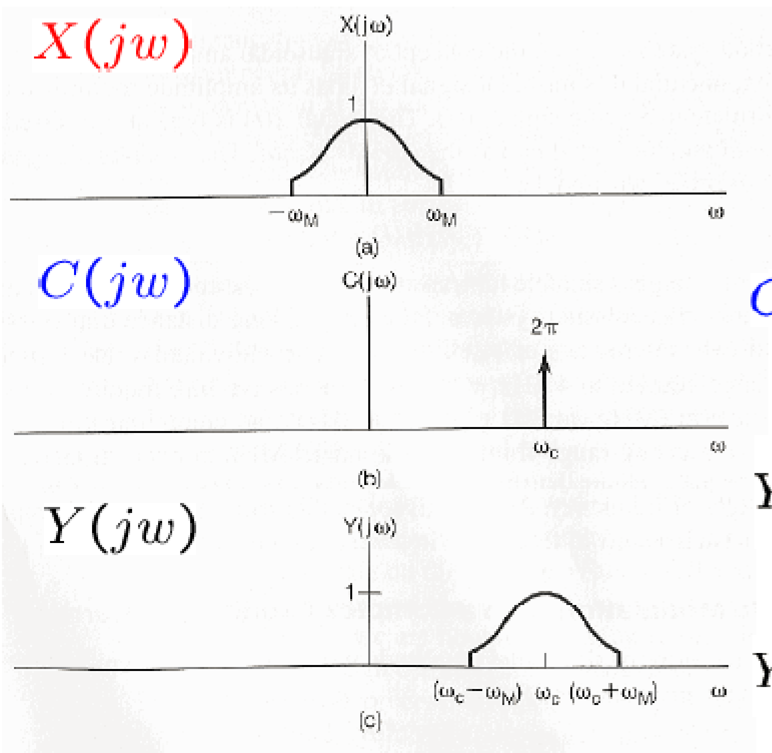


w_c : carrier frequency

$$c(t) = e^{j(w_c t+\theta_c)}$$

$$y(t) = x(t)\, c(t) \quad = x(t)\, e^{j w_c t}$$

$$\theta_c = 0$$



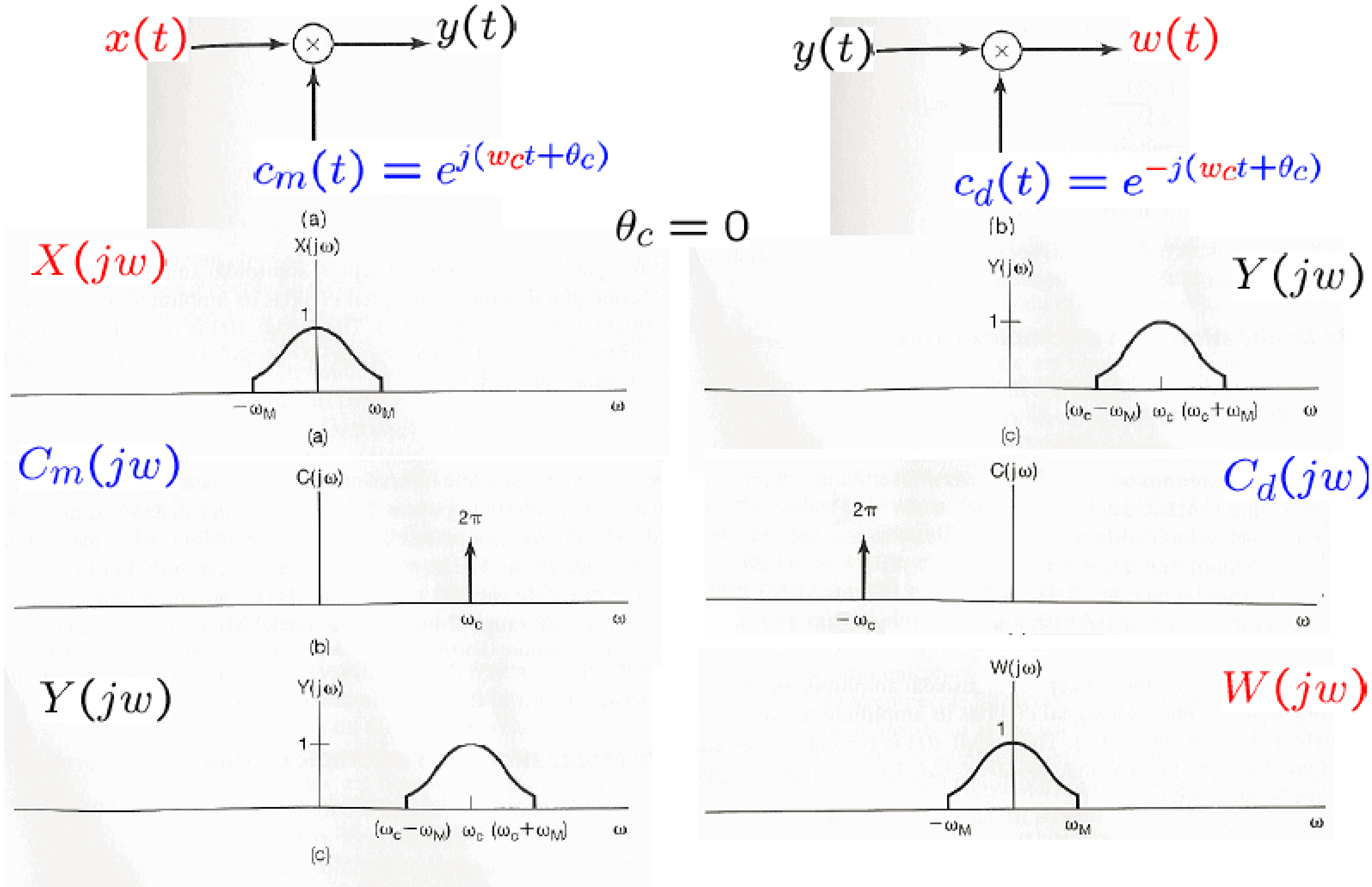
$$C(jw) = 2\pi\, \delta(w - w_c)$$

$$Y(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, C(j(w-\theta))\, d\theta$$

$$Y(jw) = X\big(j(w - w_c)\big)$$

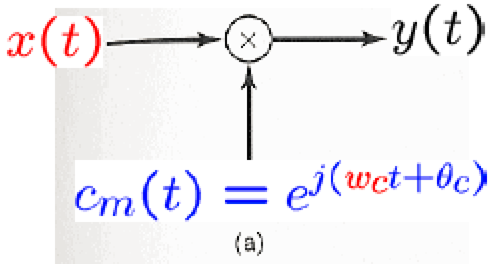
AM Modulation

- AM with a Complex Exponential Carrier:

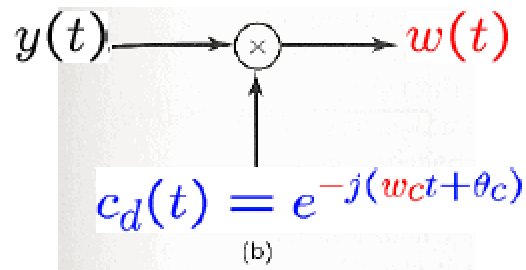


AM Modulation

- AM with a Complex Exponential Carrier:



$\theta_c = 0$



$$\begin{aligned} y(t) &= x(t)\ c_m(t) \\ &= x(t)\ e^{jw_c t} \end{aligned}$$

$$\begin{aligned} w(t) &= y(t)\ c_d(t) \\ &= y(t)\ e^{-jw_c t} \\ &= x(t)\ e^{jw_c t}\ e^{-jw_c t} \end{aligned}$$

$$\Rightarrow\ w(t) = x(t)$$

$$Y(jw) = X\left(j(w - w_c)\right)$$

$$W(jw) = Y\left(j(w + w_c)\right)$$

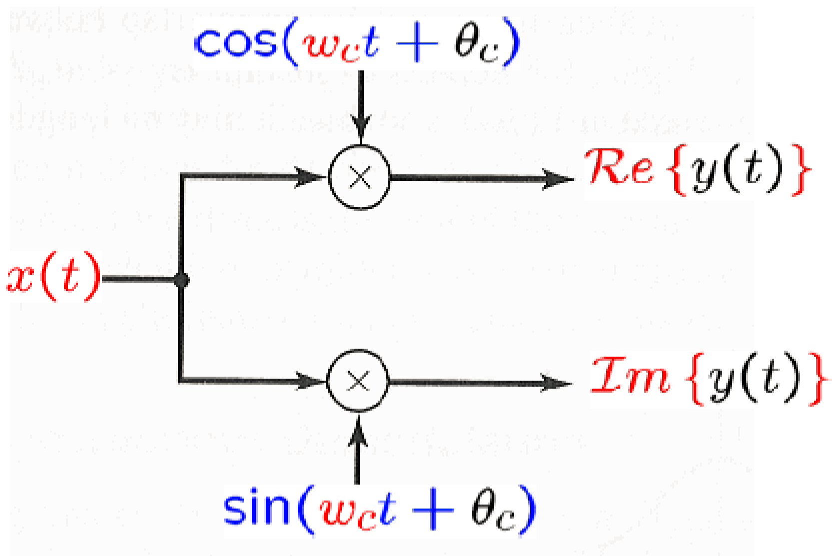
$$\Rightarrow\ W(jw) = X(jw)$$

AM Modulation

- AM with Sinusoidal Carriers:

$$c(t) = e^{jw_c t} = \cos(w_c t) + j\sin(w_c t)$$

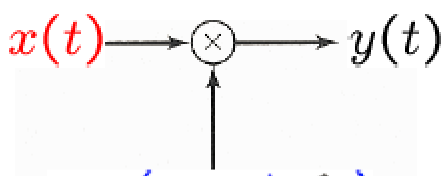
$$\Rightarrow y(t) = x(t)\cos(w_c t) + j\,x(t)\sin(w_c t)$$



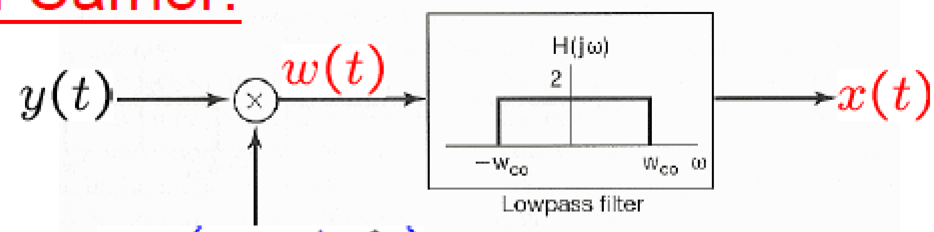
phase difference of $c_1(\cdot), c_2(\cdot)$?

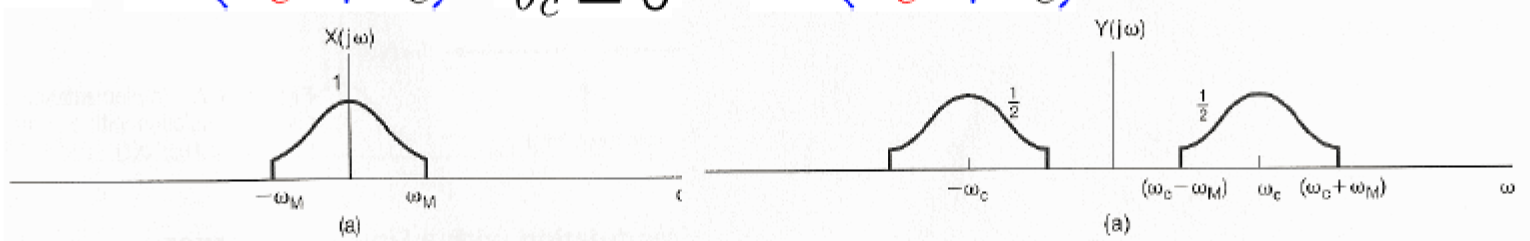
AM Modulation

- AM with a Sinusoidal Carrier:

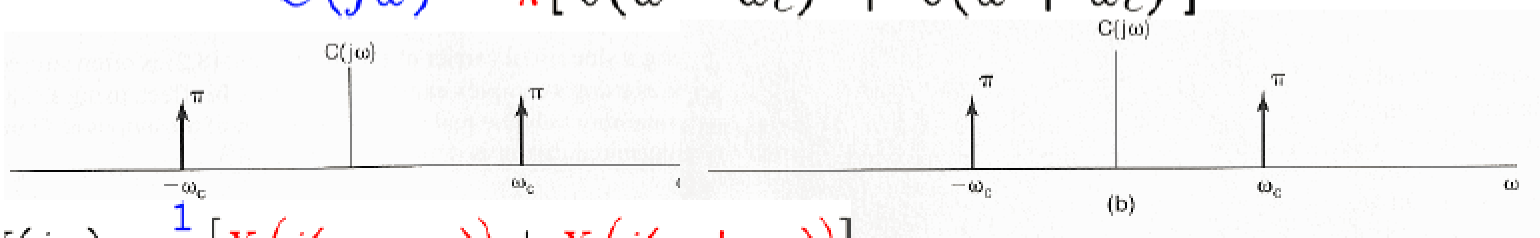


$\theta_c = 0$

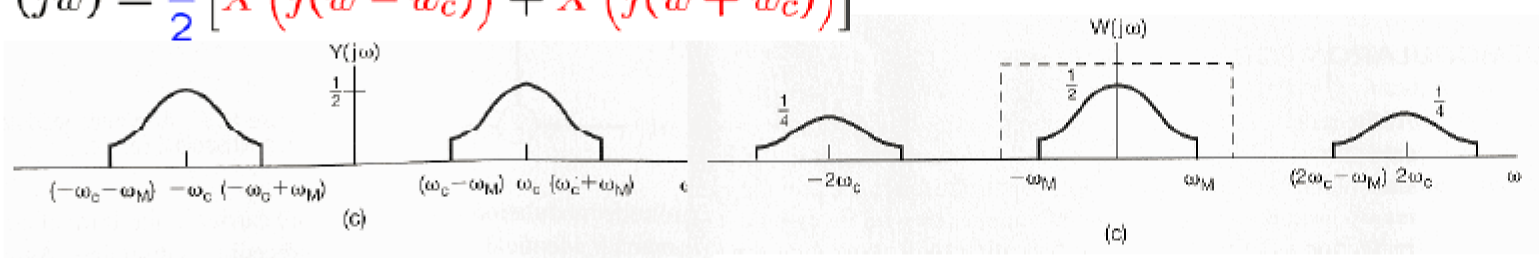




$$C(jw) = \pi[\,\delta(w - w_c) \;+\; \delta(w + w_c)\,]$$

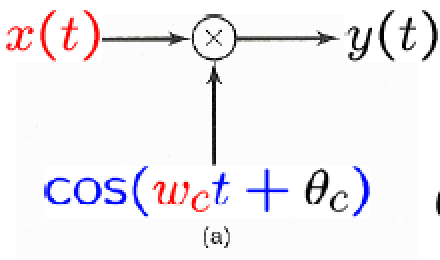


$$Y(jw) = \frac{1}{2}\left[X\left(j(w - w_c)\right) + X\left(j(w + w_c)\right)\right]$$

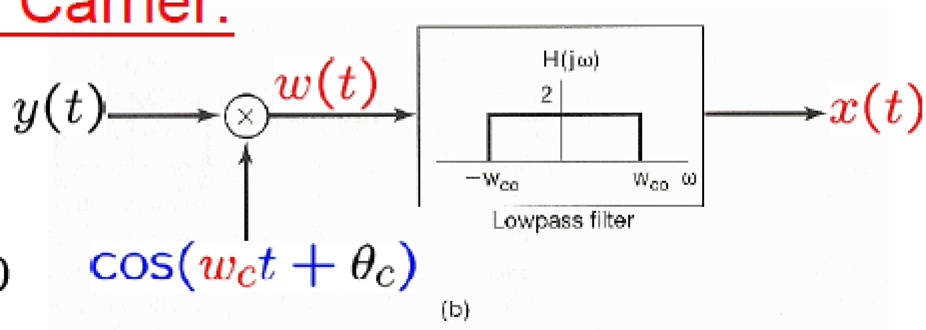


AM Modulation

- AM with a Sinusoidal Carrier:



$\theta_c = 0$



$$y(t) = x(t)\cos(w_c t)$$

$$w(t) = y(t)\cos(w_c t)$$

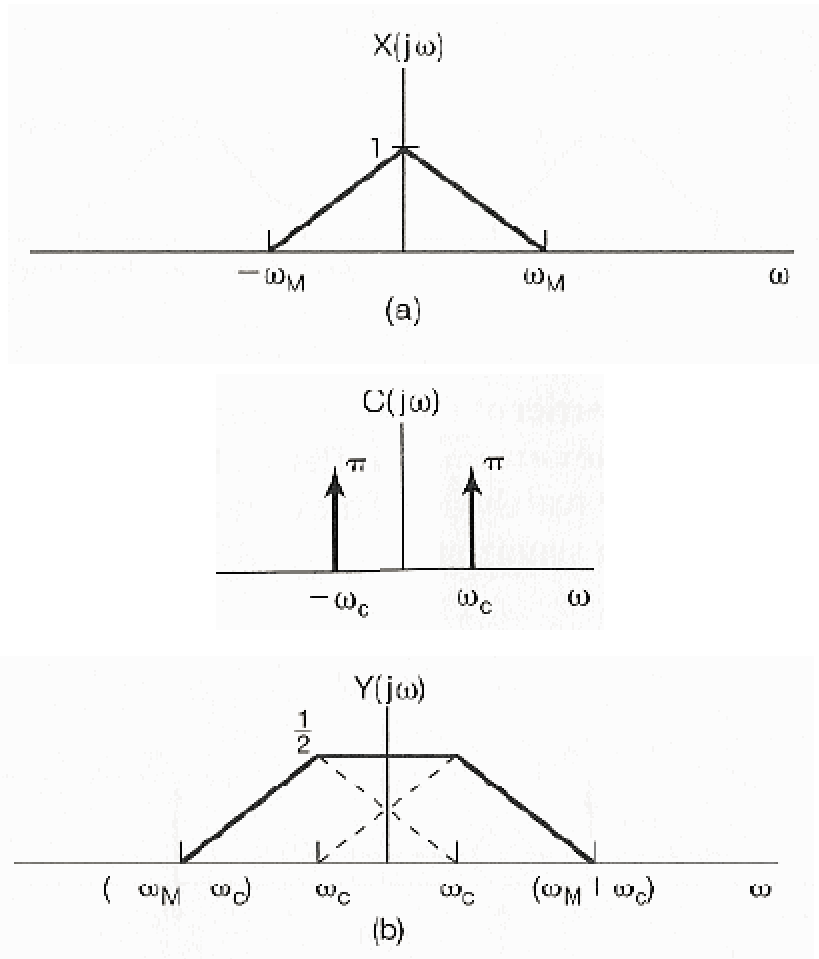
$$\Rightarrow w(t) = x(t)\cos^2(w_c t)$$

$$= x(t)\left[\frac{1}{2} + \frac{1}{2}\cos(2w_c t)\right]$$

$$= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2w_c t)$$

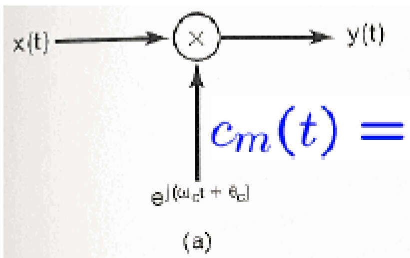
AM Modulation

- Overlapping of AM with a Sinusoidal Carrier:
  - If $w_c < w_M$,

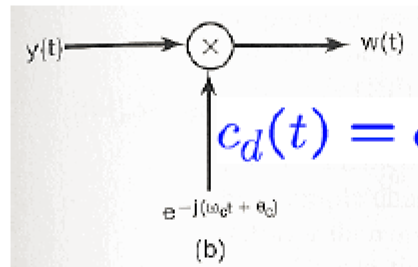


AM Demodulation

- Not Synchronized in Phase:



$$c_m(t) = e^{j(w_c t + \theta_c)}$$



$$c_d(t) = e^{-j(w_c t + \phi_c)}$$

$$\theta_c \neq \phi_c$$

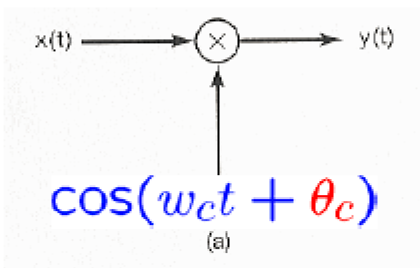
$$\begin{aligned} y(t) &= x(t)\ c_m(t) \\ &= x(t)\ e^{j(w_c t + \theta_c)} \end{aligned}$$

$$\begin{aligned} w(t) &= y(t)\ c_d(t) \\ &= y(t)\ e^{-j(w_c t + \phi_c)} \\ &= x(t)\ e^{j(\theta_c - \phi_c)} \end{aligned}$$

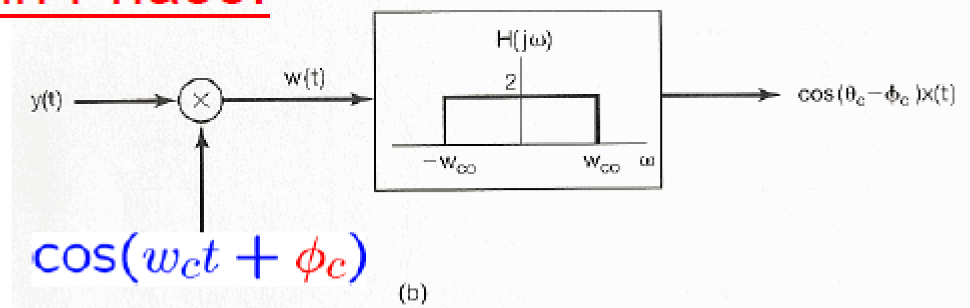
$$\Rightarrow \text{ ONLY } \quad |x(t)| = |w(t)|$$

AM Demodulation

- Not Synchronized in Phase:



(a)



(b)

$$y(t) = x(t)\cos(w_c t + \theta_c) \qquad w(t) = y(t)\cos(w_c t + \phi_c)$$

$$\Rightarrow w(t) = x(t)\cos(w_c t + \theta_c)\cos(w_c t + \phi_c)$$

$$= x(t)\left[\frac{1}{2}\cos(\theta_c - \phi_c) + \frac{1}{2}\cos(2w_c t + \theta_c + \phi_c)\right]$$

$$= \frac{1}{2}\cos(\theta_c - \phi_c)\,x(t) + \frac{1}{2}\,x(t)\,\cos(2w_c t + \theta_c + \phi_c)$$

AM Modulator for Asynchronous Modulation-Demodulation System

AM In standard AM, the carrier signal $c(t)$ has its amplitude multiplied (modulated) by the quantity

$x(t) + A$

where

$|x(t)| < A$

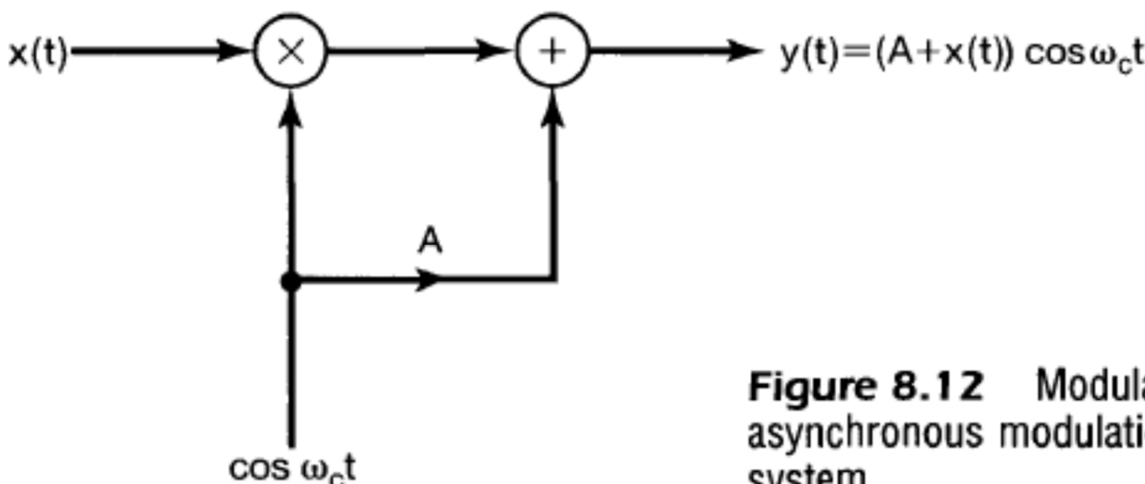
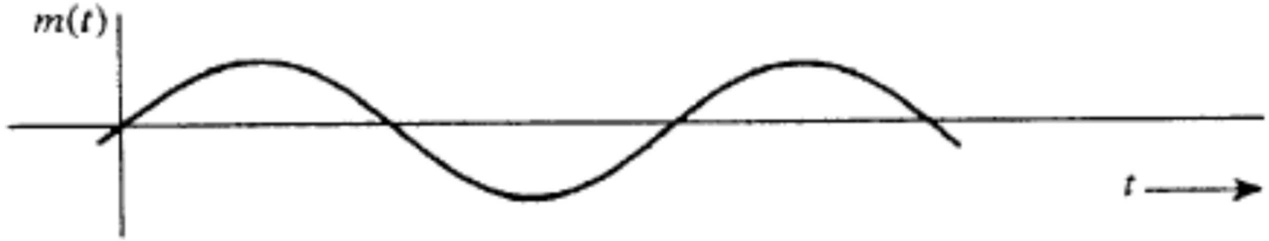
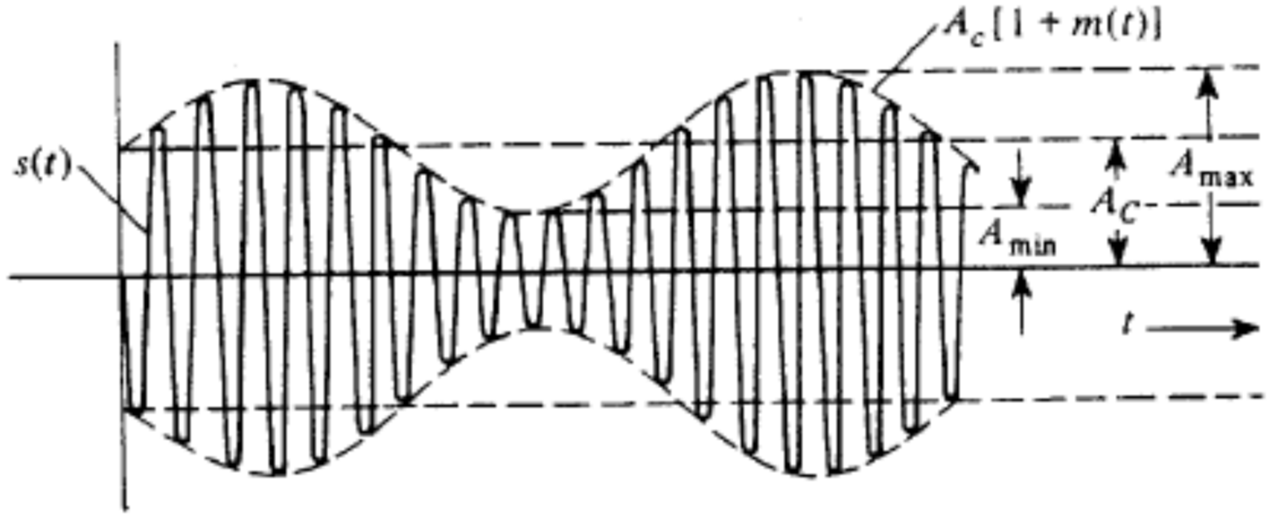


Figure 8.12 Modulator for an asynchronous modulation-demodulation system.

AM Modulator for Asynchronous Modulation-Demodulation System



(a) Sinusoidal Modulating Wave



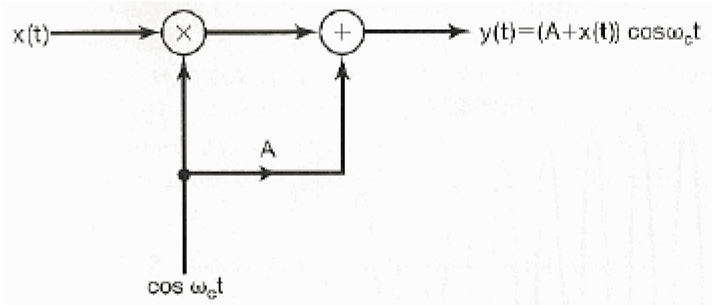
(b) Resulting AM Signal

AM signal waveform

AM Demodulation

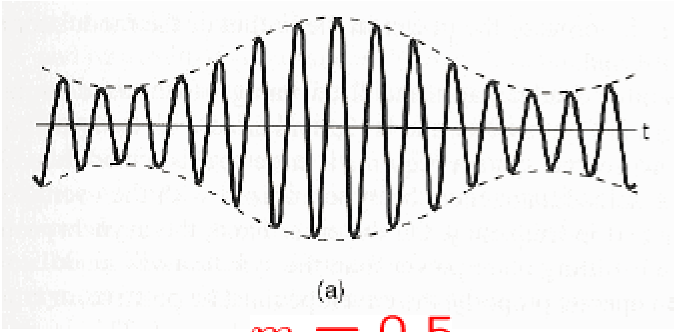
- Asynchronous Demodulation:
  - $w_c >> w_M$
  - $x(t) > 0, \quad \forall t$

    If not, $x(t) \to x(t) + A \quad > \quad 0$

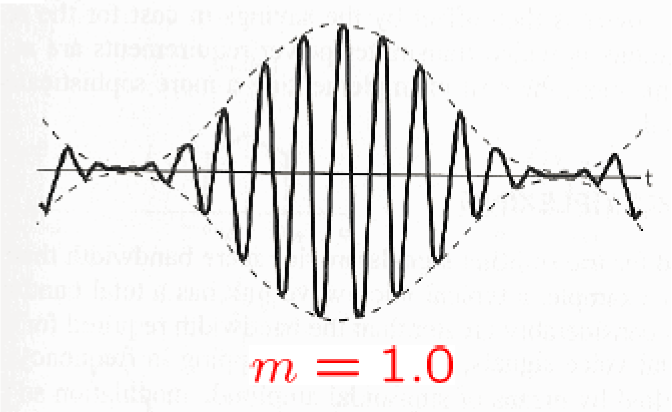


$$A \geq K, \quad |x(t)| \leq K$$

  - $\dfrac{K}{A}$: modulation index m, in %



$m = 0.5$

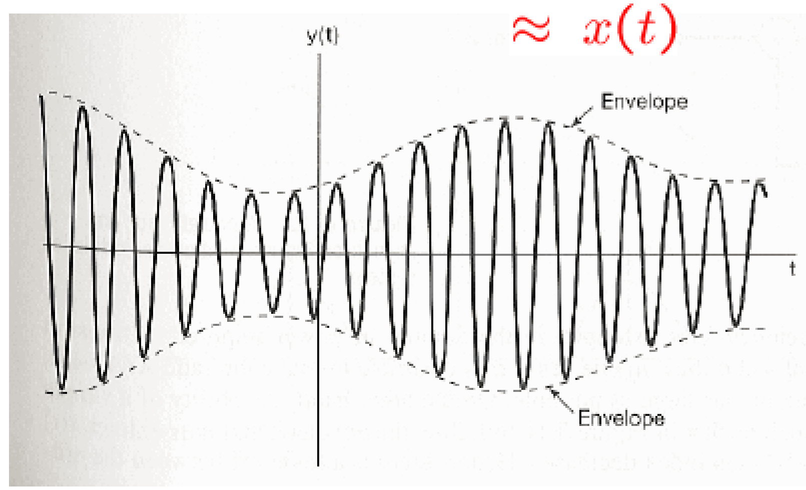


$m = 1.0$

AM Demodulation

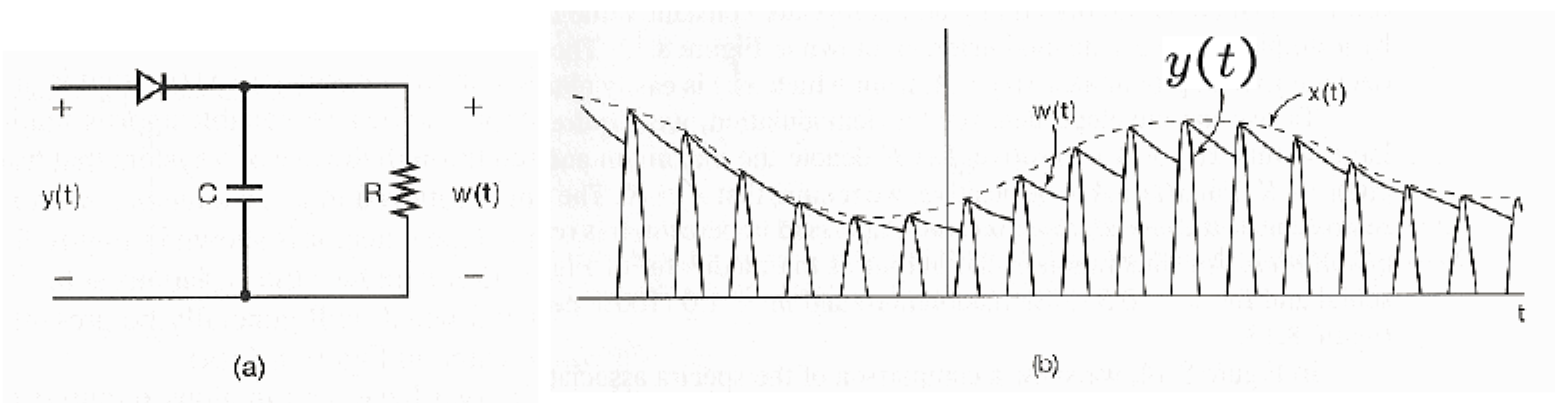
$$y(t) = x(t)\cos(w_c t + \theta_c)$$

- Asynchronous Demodulation:
  - $w_c >> w_M$
  - $x(t) > 0, \quad \forall t$



$$\approx x(t)$$

- Envelope Detector:



$$y(t)$$

AM Modulator for Asynchronous Modulation-Demodulation System

The modulated signal $y(t)$ has the general form illustrated in Fig. 8.10.

$$y(t) = [x(t) + A]\cos(\omega_c t) = x(t)\cos\omega_c t + A\cos\omega_c t$$

$$Y(j\omega) = \frac{1}{2}[X(j(\omega - \omega_c)) + X(j(\omega + \omega_c))] +$$

$$\pi A[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$

In Fig. 8.14, we show a comparison of the spectra associated with the DSB-SC signal and AM signal.

AM Modulator for Asynchronous Modulation-Demodulation System

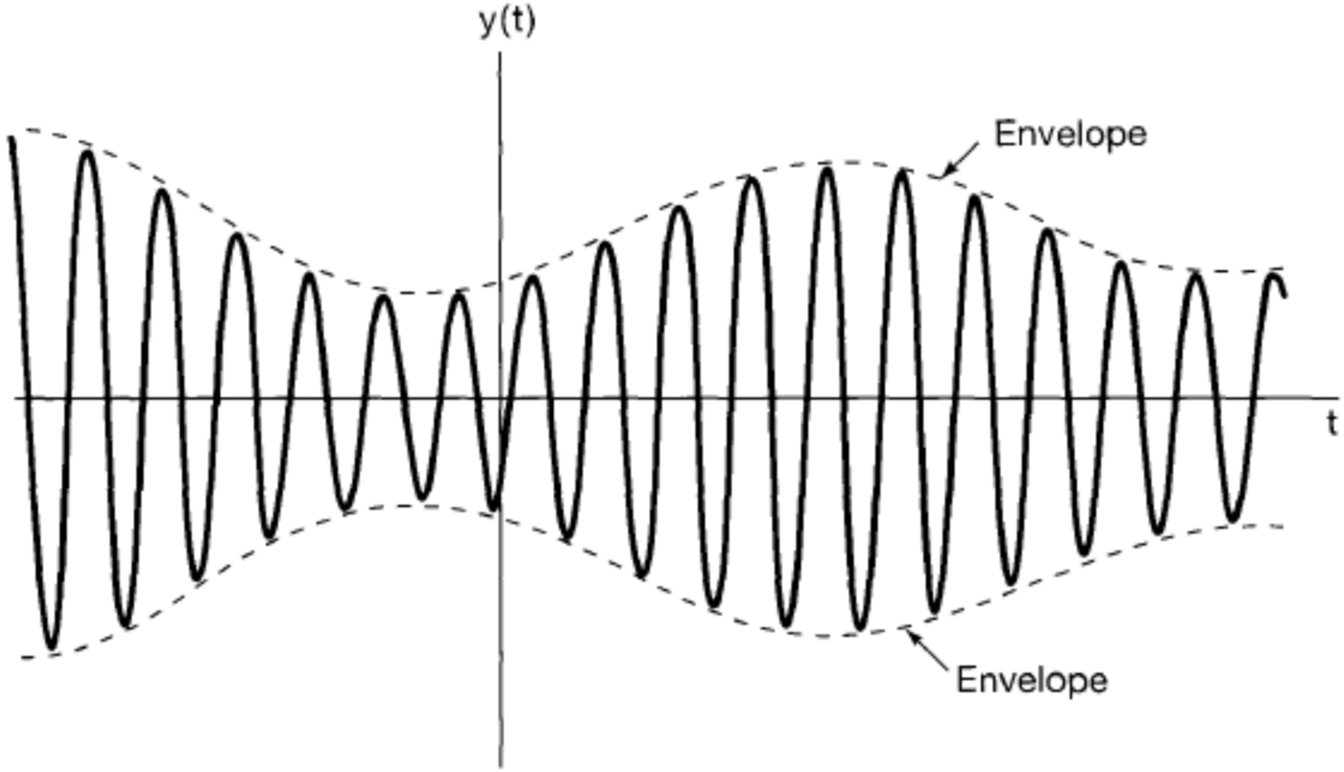


Figure 8.10 Amplitude-modulated signal. The dashed curve represents the envelope of the modulated signal.

AM Modulator for Asynchronous Modulation-Demodulation System

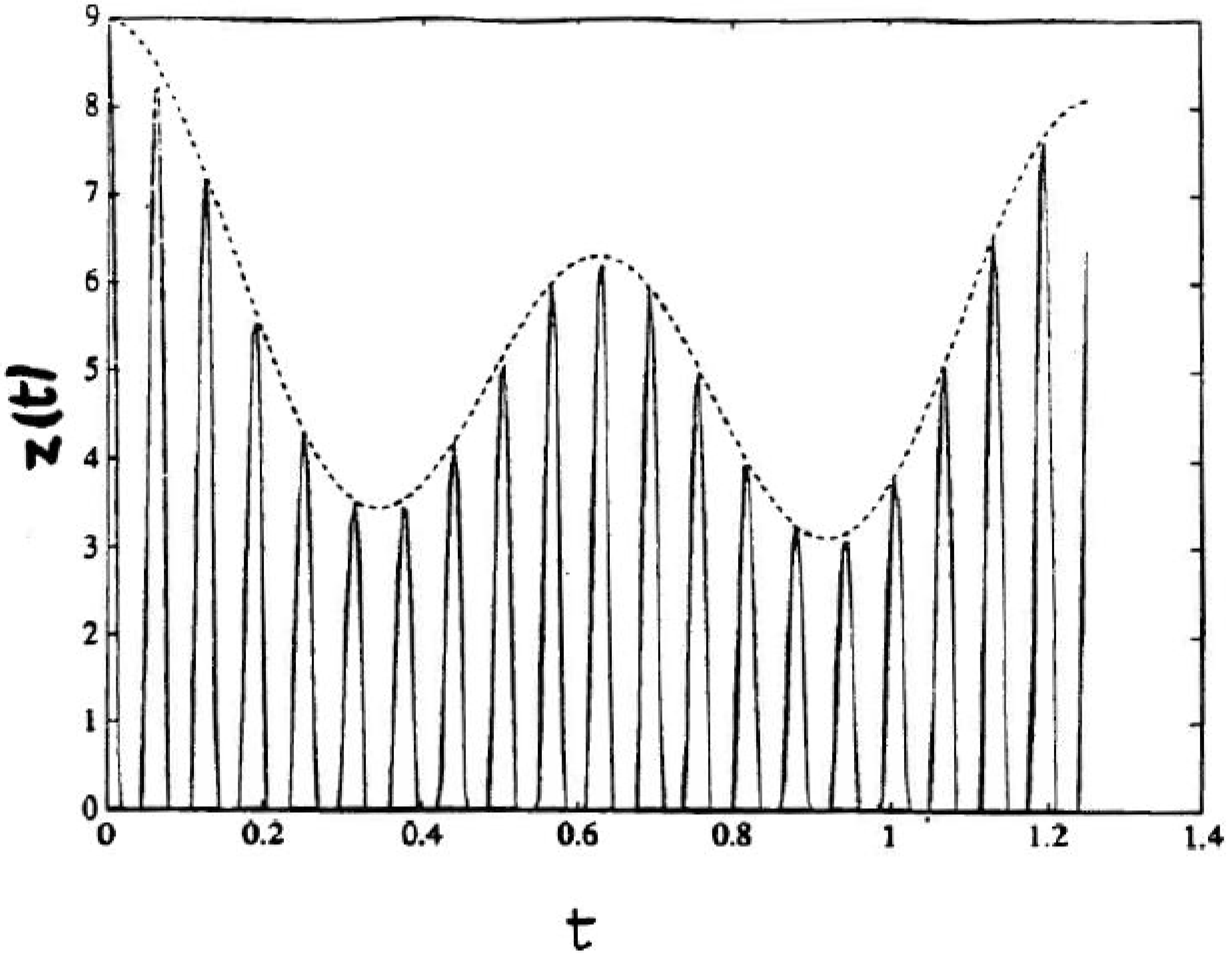
Suppose we receive the radio signal

$$y(t) = [x(t) + A]\cos(\omega_c t)$$

How can we recover $x(t)$? Lets "rectify" $y(t)$ by using a "half-wave rectifier" with the following input-output relation

$$z(t) = \begin{cases} y(t), & y(t) > 0 \\ 0, & y(t) < 0 \end{cases}$$

AM Modulator for Asynchronous Modulation-Demodulation System



AM Modulator for Asynchronous Modulation-Demodulation System

Since $[x(t)+A]>0$, we can write

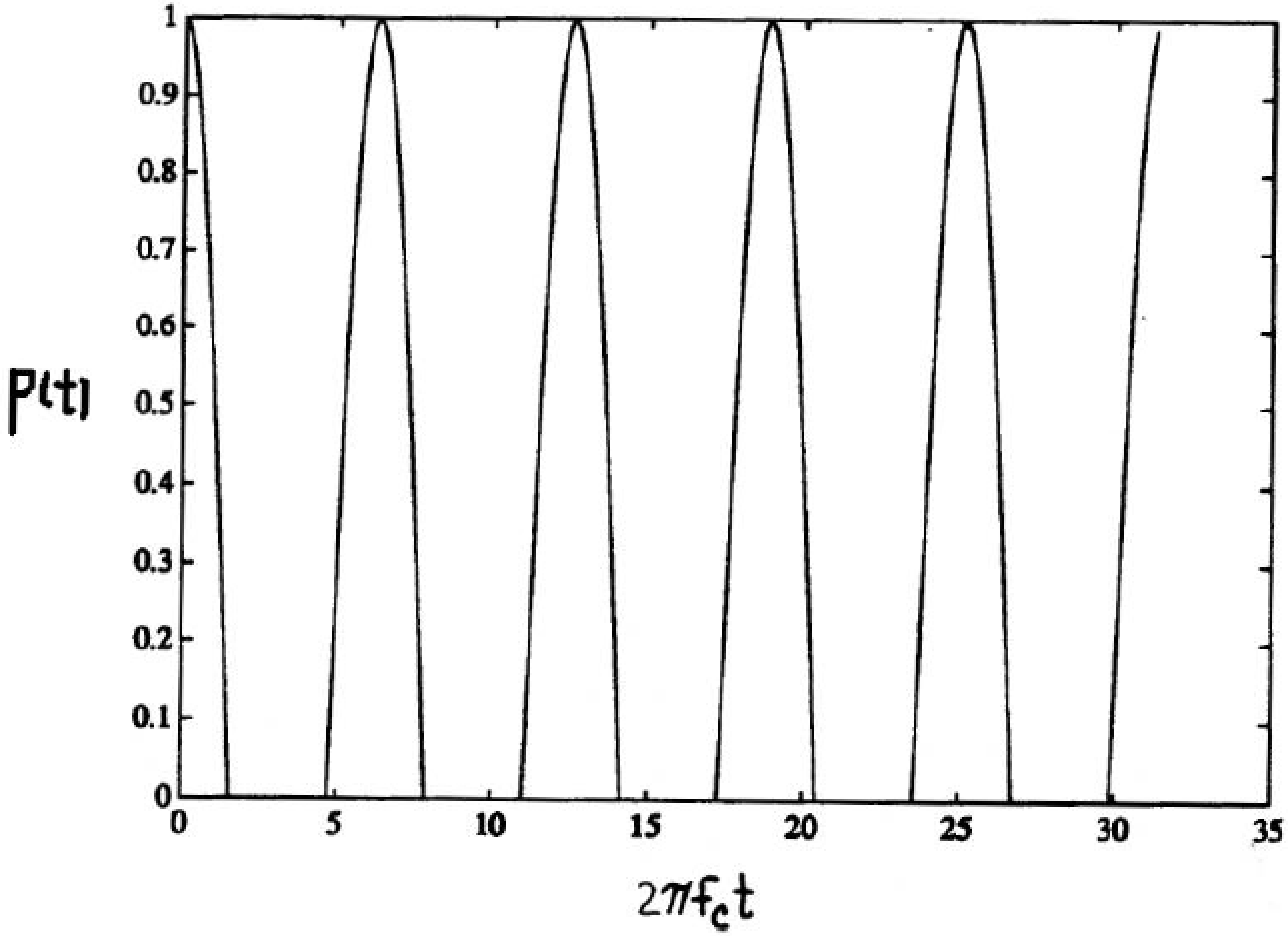
$$z(t)=\begin{cases}[x(t)+A]\cos(\omega_c t), & \cos(\omega_c t)>0\\ 0, & \cos(\omega_c t)<0\end{cases}$$

$$=[x(t)+A]p(t)$$

where

$$p(t)=\begin{cases}\cos(\omega_c t), & \cos(\omega_c t)>0\\ 0, & \cos(\omega_c t)<0\end{cases}$$

AM Modulator for Asynchronous Modulation-Demodulation System



AM Modulator for Asynchronous Modulation-Demodulation System

Note that $p(t)$ is a periodic function with $T = 2\pi / \omega_c$. Thus $p(t)$ has an FS representation

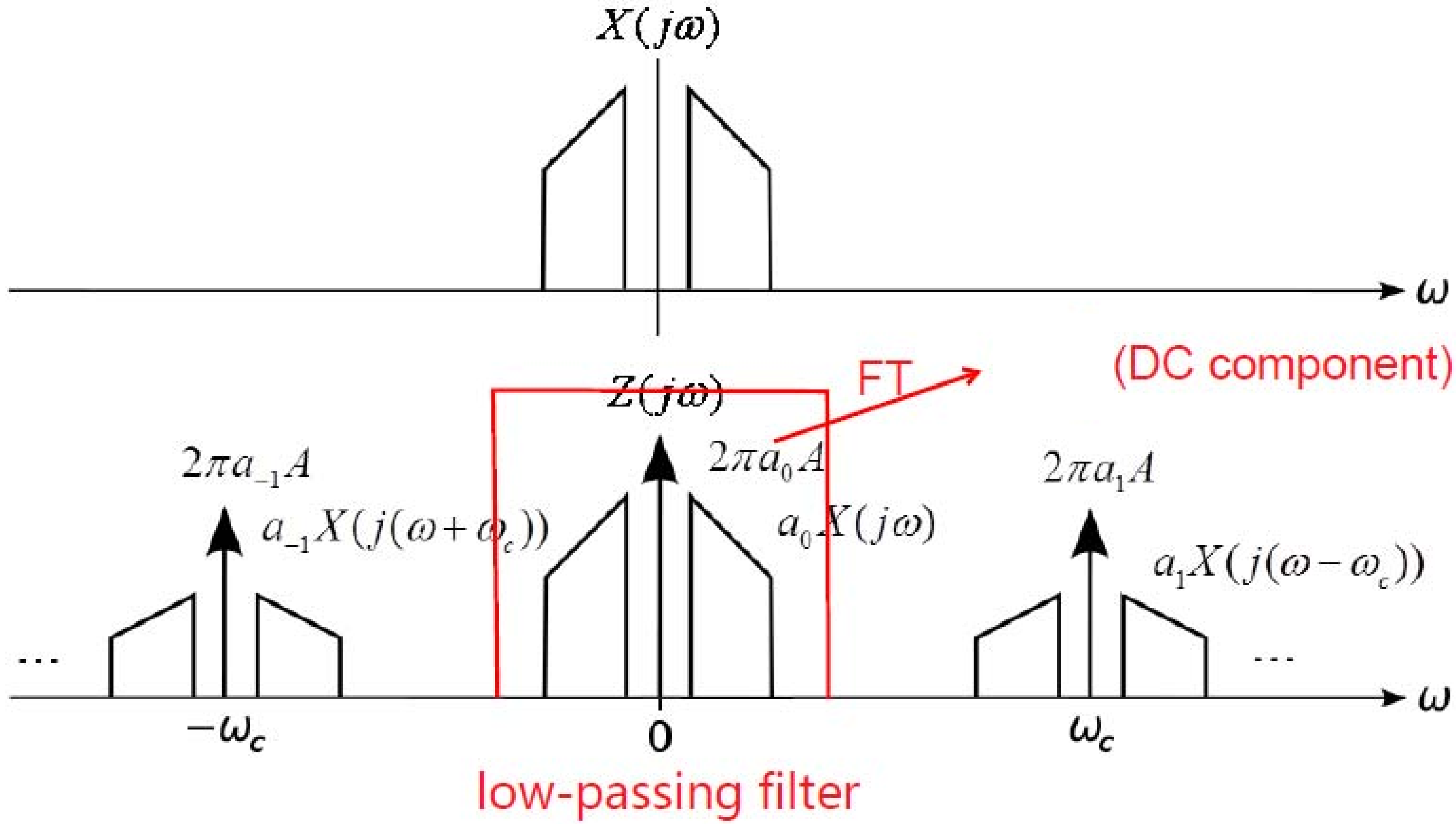
$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_c t}$$

$$z(t) = [x(t) + A]p(t) = \sum_{k=-\infty}^{\infty} a_k [x(t) + A] e^{jk\omega_c t}$$

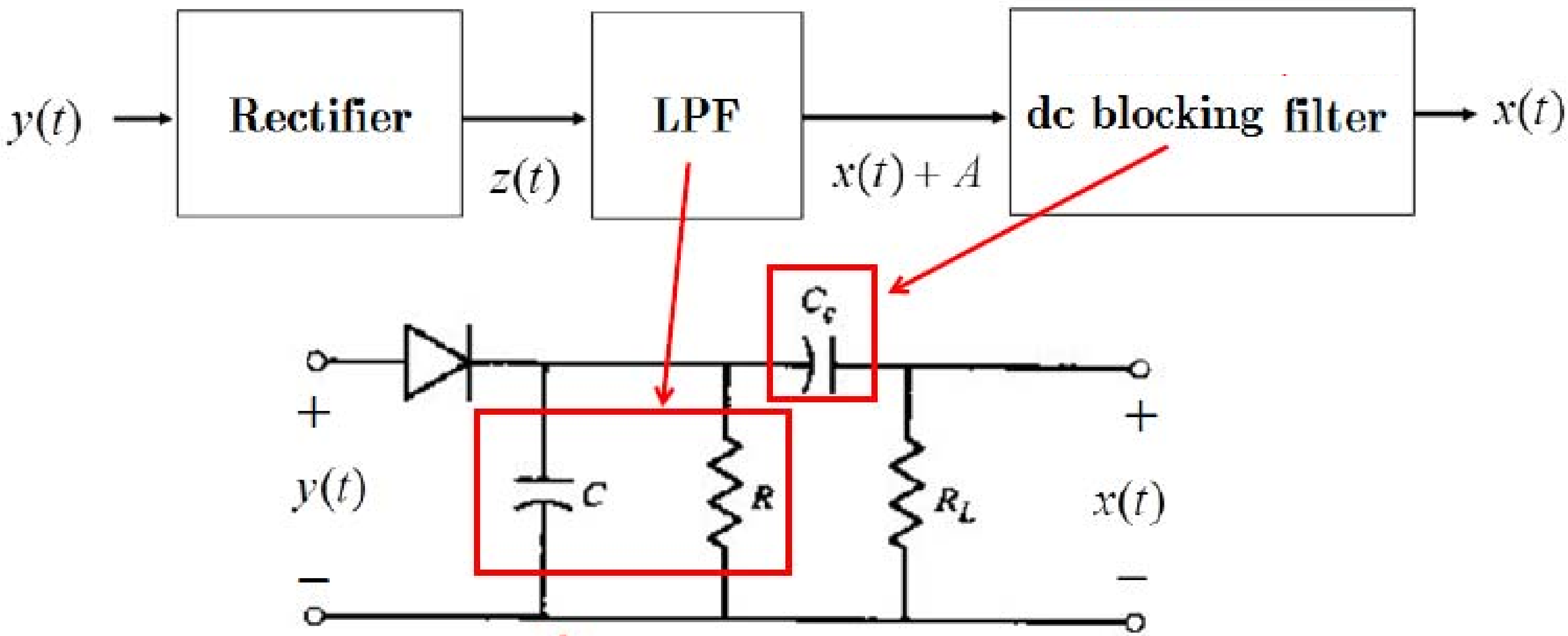
$$= \sum_{k=-\infty}^{\infty} a_k A e^{jk\omega_c t} + \sum_{k=-\infty}^{\infty} a_k x(t) e^{jk\omega_c t}$$

$$Z(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k A \delta(\omega - k\omega_c) + \sum_{k=-\infty}^{\infty} a_k X(j(\omega - k\omega_c))$$

AM Modulator for Asynchronous Modulation-Demodulation System



AM Modulator for Asynchronous Modulation-Demodulation System



AM Modulator for Asynchronous Modulation-Demodulation System

It follows from the above figure that we can recover $\left[x(t)+A\right]$ by lowpass filtering $z(t)$:

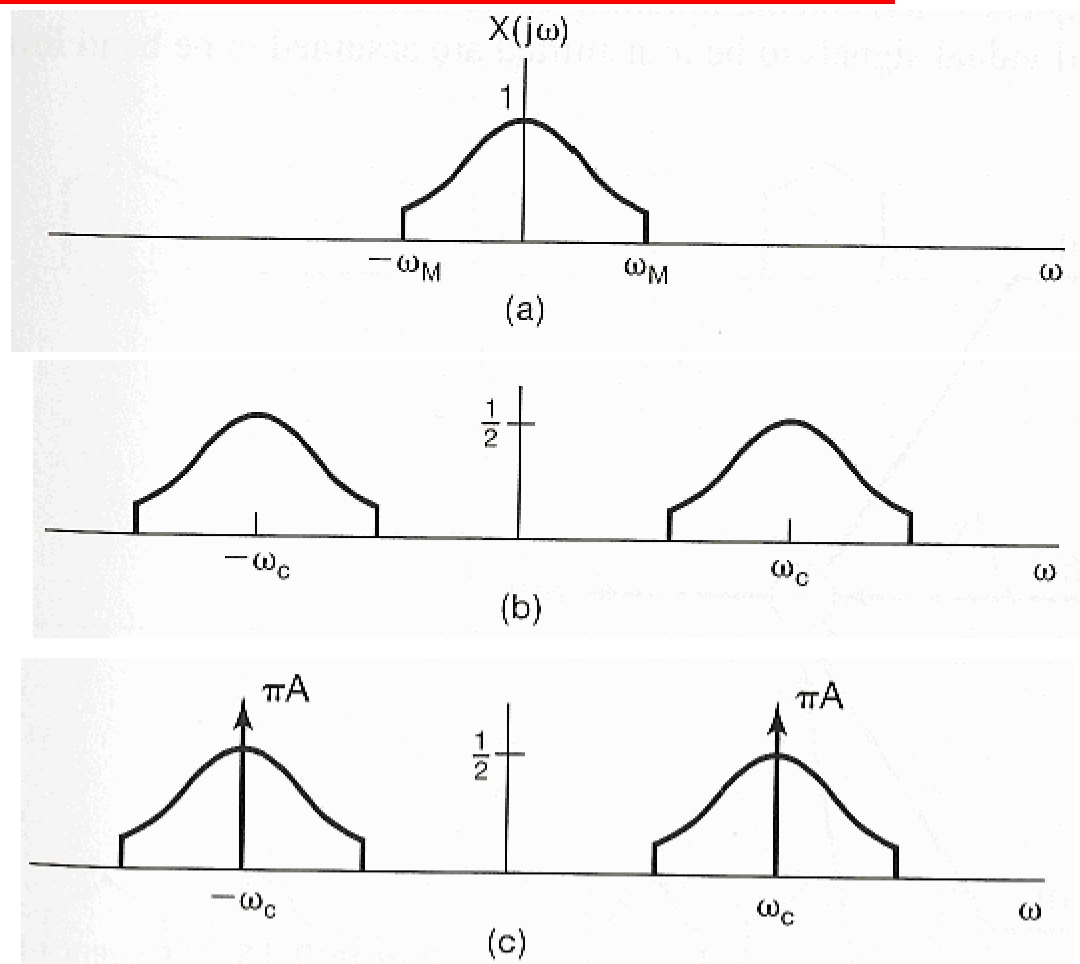
$$H(j\omega)=\begin{cases} a_0^{-1}, & |\omega|<\omega_{co} \\ 0, & |\omega|>\omega_{co} \end{cases}$$

where $\omega_M \ll \omega_{co} \ll \omega_c$.

And to recover $x(t)$ from $\left[x(t)+A\right]$, we need another filter which blocks the dc component.

AM Demodulation

- Synchronous & Asynchronous Demodulation:

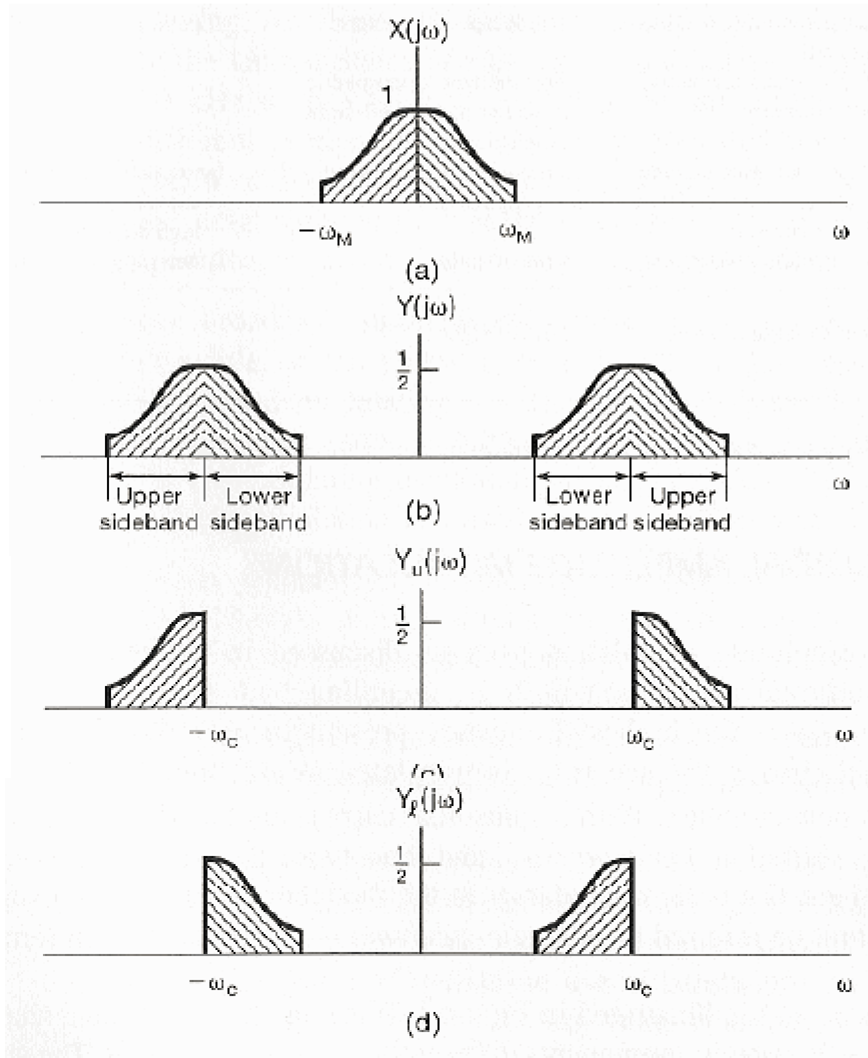


$x(t)\ \cos(w_c t)$

$[x(t) + A]\ \cos(w_c t)$

Single-Sideband Sinusoidal Amplitude Modulation

- SSB Modulation:

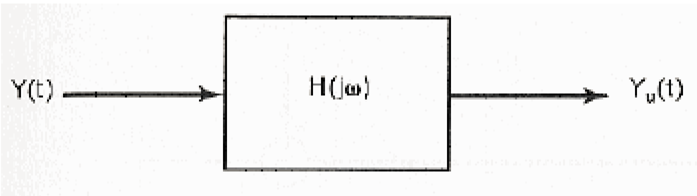


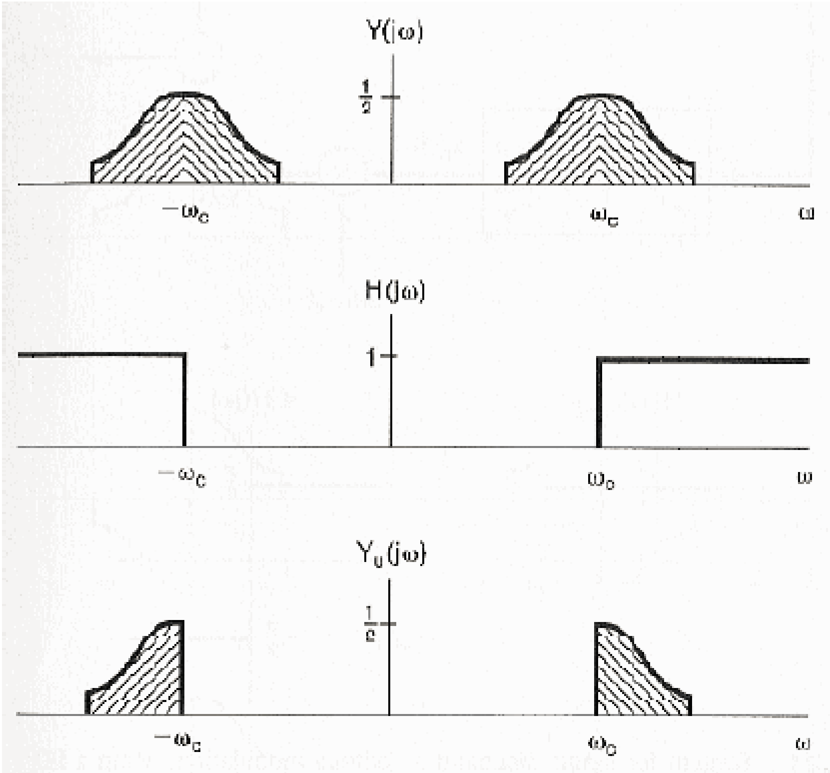
upper sidebands

lower sidebands

Single-Sideband Sinusoidal Amplitude Modulation

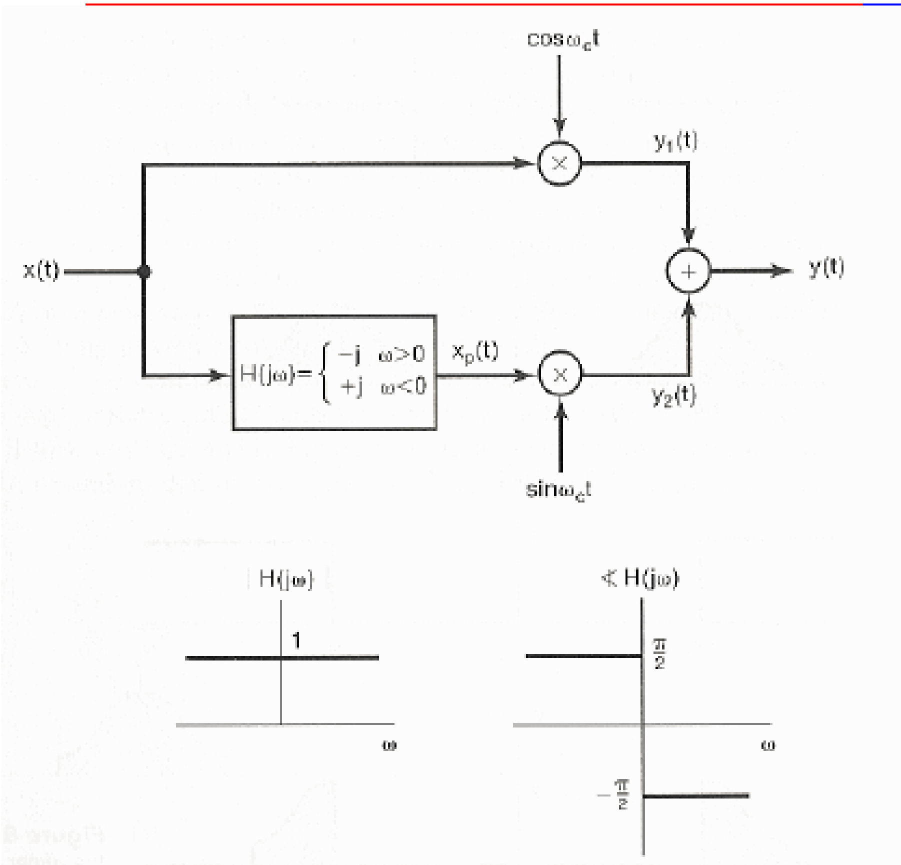
- Retain Upper Sidebands Using Ideal Highpass Filter

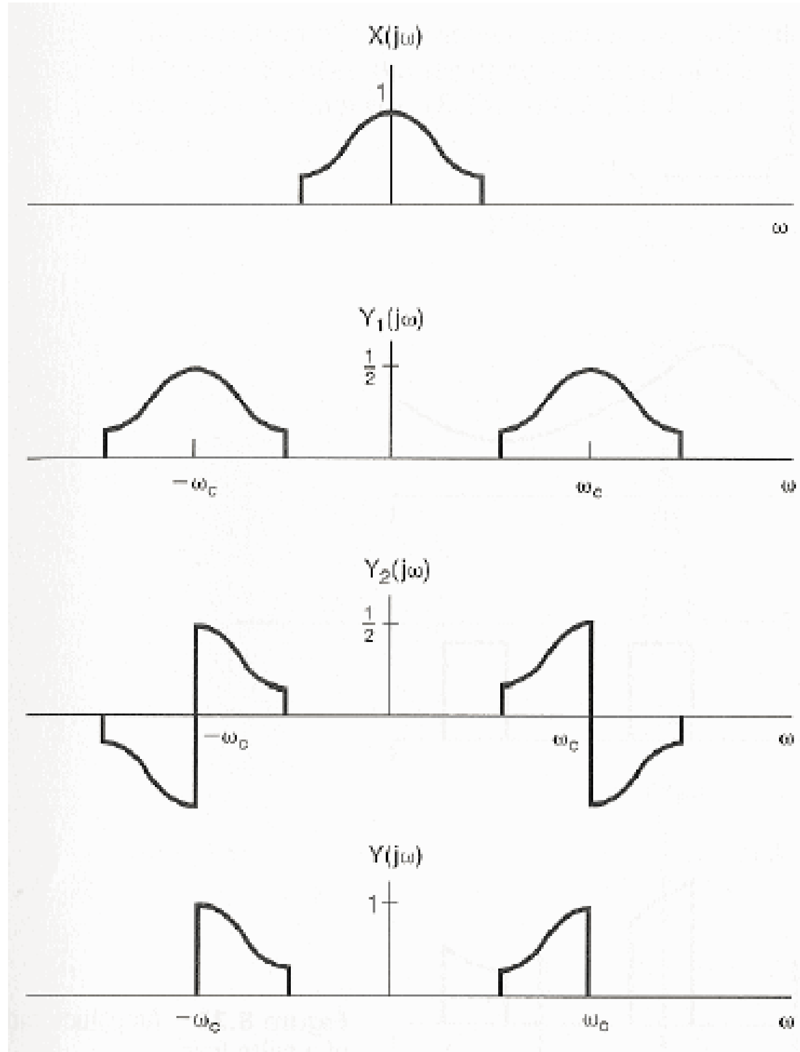




Single-Sideband Sinusoidal Amplitude Modulation

- Retain Lower Sidebands Using Phase-Shift Network





- Retain Upper Sidebands

$$H(jw) = \begin{cases} j, & w > 0 \\ -j, & w < 0 \end{cases}$$

Single-Sideband Sinusoidal Amplitude Modulation

$$X_p(j\omega) = -j\,\mathrm{sgn}(\omega)X(j\omega)$$

$$= -j\left[u(\omega) - u(-\omega)\right]X(j\omega)$$

$$= -jX(j\omega)u(\omega) + jX(j\omega)u(-\omega)$$

$$X_+(j\omega) \triangleq X(j\omega)u(\omega),\ X_-(j\omega) \triangleq X(j\omega)u(-\omega)$$

$$X(j\omega) = X_+(j\omega) + X_-(j\omega)$$

$$X_p(j\omega) = -jX_+(j\omega) + jX_-(j\omega)$$

Single-Sideband Sinusoidal Amplitude Modulation

$$y_1(t) = x(t)\cos(\omega_c t) \leftrightarrow$$

$$Y_1(j\omega) = \frac{1}{2}[X(j(\omega - \omega_c)) + X(j(\omega + \omega_c))]$$

$$= \frac{1}{2}[X_+(j(\omega - \omega_c)) + X_-(j(\omega - \omega_c))$$

$$+ X_+(j(\omega + \omega_c)) + X_-(j(\omega + \omega_c))]$$

Single-Sideband Sinusoidal Amplitude Modulation

$$y_2(t) = x_p(t)\sin(\omega_c t) \leftrightarrow$$

$$Y_2(j\omega) = \frac{1}{2j}[X_p(j(\omega-\omega_c)) - X_p(j(\omega+\omega_c))]$$

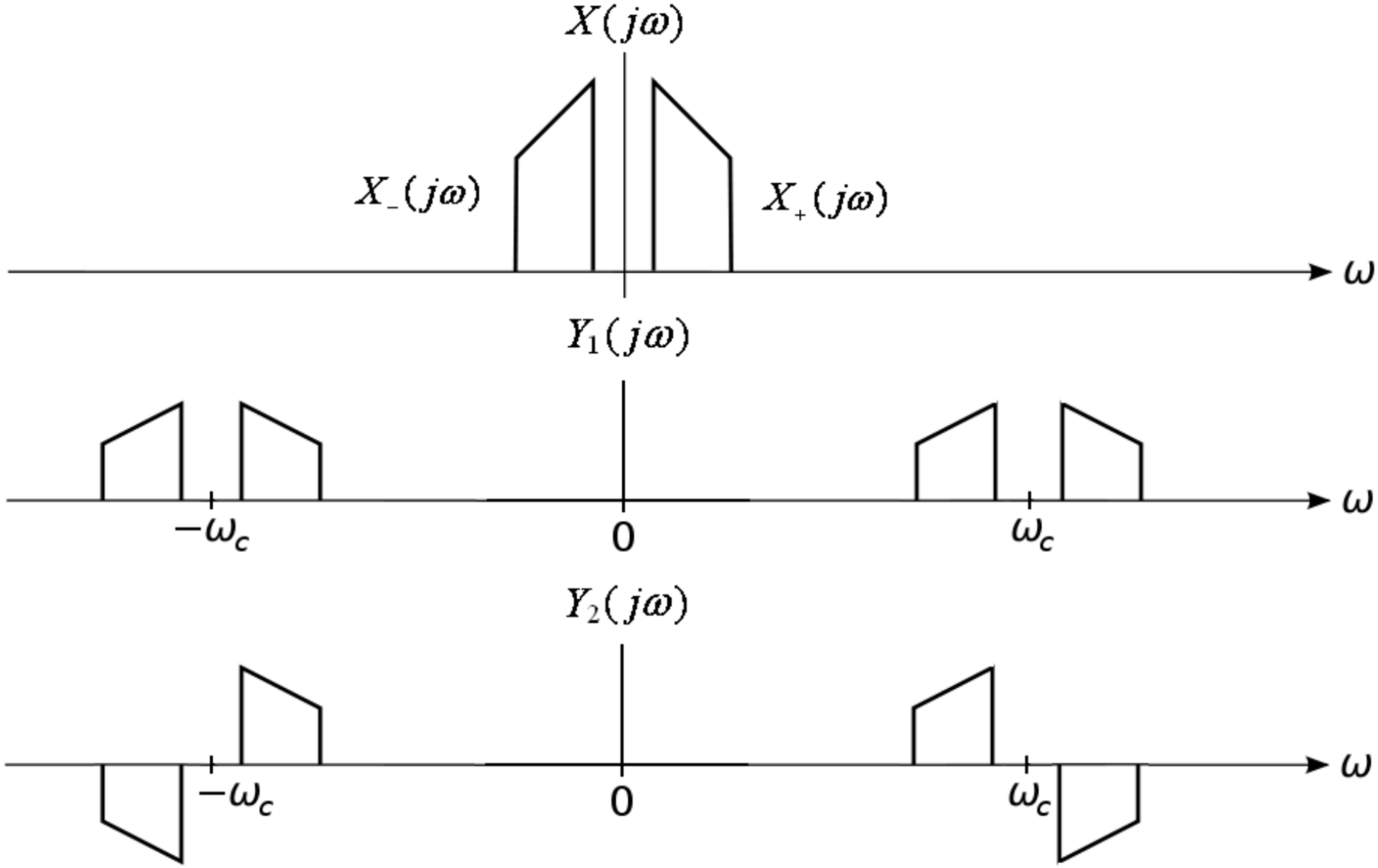
$$= \frac{1}{2j}[-jX_+(j(\omega-\omega_c)) + jX_-(j(\omega-\omega_c))$$

$$-(-jX_+(j(\omega+\omega_c)) + jX_-(j(\omega+\omega_c)))]$$

$$= \frac{1}{2}[-X_+(j(\omega-\omega_c)) + X_-(j(\omega-\omega_c))$$

$$+X_+(j(\omega+\omega_c)) - X_-(j(\omega+\omega_c))]$$

Single-Sideband Sinusoidal Amplitude Modulation



Single-Sideband Sinusoidal Amplitude Modulation

$$y(t) = y_1(t) \pm y_2(t) \leftrightarrow$$

$$Y(j\omega) = Y_1(j\omega) \pm Y_2(j\omega)$$

$$Y_{USB}(j\omega) = Y_1(j\omega) + Y_2(j\omega) = \frac{1}{2}[X_+(j(\omega - \omega_c)) +$$

$$X_-(j(\omega - \omega_c)) + X_+(j(\omega + \omega_c)) + X_-(j(\omega + \omega_c))] +$$

$$\frac{1}{2}[-X_+(j(\omega - \omega_c)) + X_-(j(\omega - \omega_c)) + X_+(j(\omega + \omega_c))$$

$$-X_-(j(\omega + \omega_c))]$$

$$= X_-(j(\omega - \omega_c) + X_+(j(\omega + \omega_c))$$

Single-Sideband Sinusoidal Amplitude Modulation

$$Y_{LSB}(j\omega) = Y_1(j\omega) - Y_2(j\omega) = \frac{1}{2}[X_+(j(\omega-\omega_c)) +$$

$$X_-(j(\omega-\omega_c)) + X_+(j(\omega+\omega_c)) + X_-(j(\omega+\omega_c))] -$$

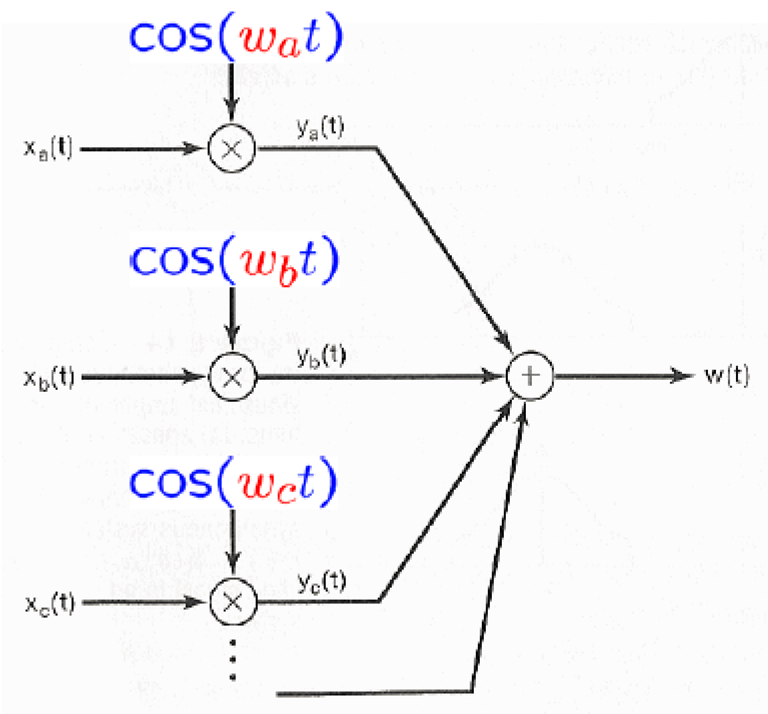
$$\frac{1}{2}[-X_+(j(\omega-\omega_c)) + X_-(j(\omega-\omega_c)) + X_+(j(\omega+\omega_c))$$

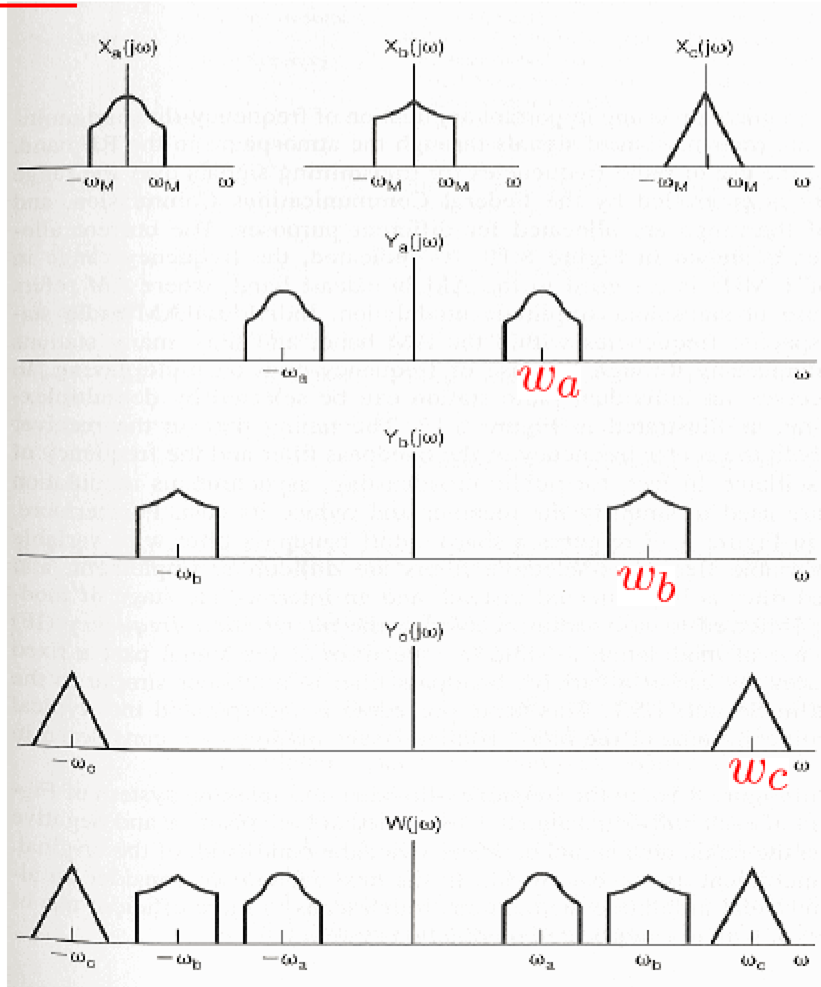
$$-X_-(j(\omega+\omega_c))]$$

$$= X_+(j(\omega-\omega_c) + X_-(j(\omega+\omega_c))$$

FDM

- FDM Using Sinusoidal AM:





FDM

- Demultiplexing and Demodulation: