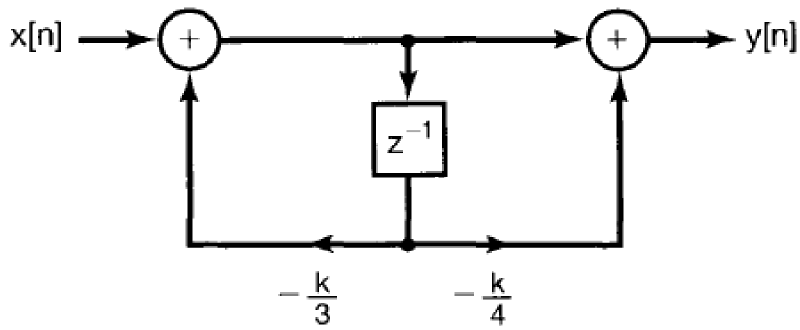
HW #8

1- Consider the digital filter structure shown in



(a) Find $H(z)$ for this causal filter. Plot the pole-zero pattern and indicate the region of convergence.
(b) For what values of the k is the system stable?
(c) Determine $y[n]$ if $k = 1$ and $x[n] = (2/3)^n$ for all n.

2- The following is known about a discrete-time LTI system with input $x[n]$ and output $y[n]$:

1. If $x[n] = (-2)^n$ for all n, then $y[n] = 0$ for all n.
2. If $x[n] = (1/2)^n u[n]$ for all n, then $y[n]$ for all n is of the form

$$y[n] = \delta[n] + a\left(\frac{1}{4}\right)^n u[n],$$

where a is a constant.

(a) Determine the value of the constant a.
(b) Determine the response $y[n]$ if the input $x[n]$ is

$$x[n] = 1, \text{ for all } n.$$

3- A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

(a) Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.

(b) Find the unit sample response of the system.

(c) You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation.

4- Consider a causal LTI system S with input $x[n]$, Calculate H(Z)

(a) How is $e_1[n]$ related to $f_1[n]$?

(b) How is $e_2[n]$ related to $f_2[n]$?

(c) Using your answers to the previous two parts as a guide, construct a direct-form block diagram for S that contains only two delay elements.

(d) Draw a cascade-form block diagram representation for S based on the observation that

$$H(z) = H_1(z)H_2(z),$$

$$H(z) = \left(\frac{1 + \frac{1}{4}z^{-1}}{1 + \frac{1}{2}z^{-1}}\right)\left(\frac{1 - 2z^{-1}}{1 - \frac{1}{4}z^{-1}}\right).$$