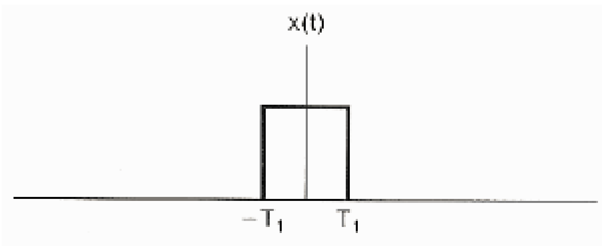
Introduction to Signals and Systems: V216

Lecture #11

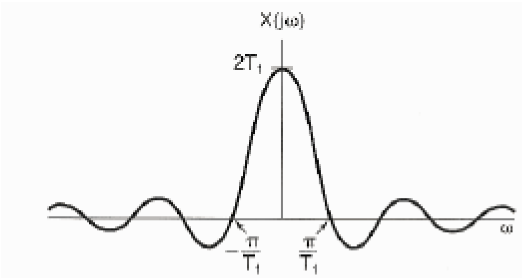
Chapter 6: Time & Frequency Characterization of Signals and Systems

Time & Frequency Characterization

- Time- & Frequency-Domain Characterization:



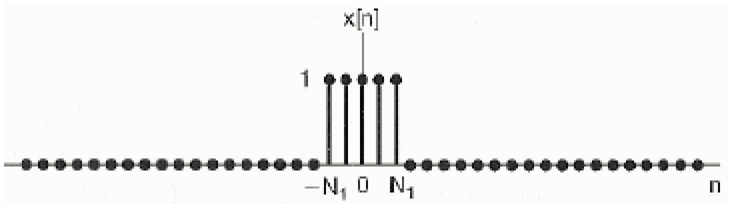
(a)



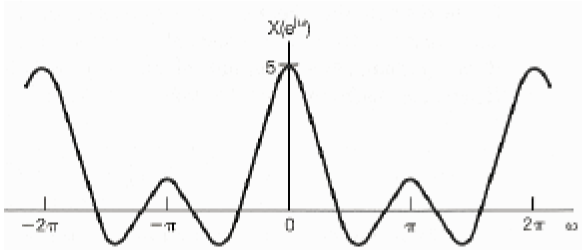
(b)

$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(jw)$$

$$h[n] \overset{\mathcal{F}}{\longleftrightarrow} H(e^{jw})$$



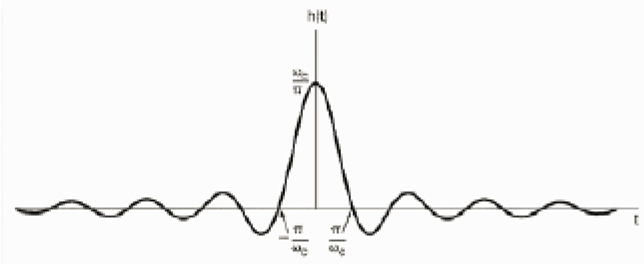
(a)

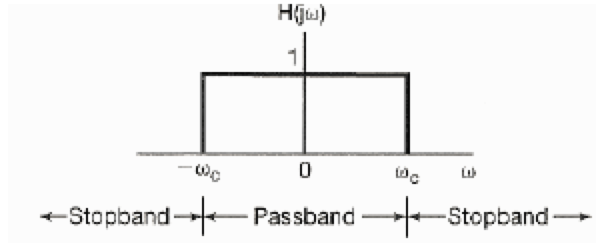


(b)

Time & Frequency Characterization

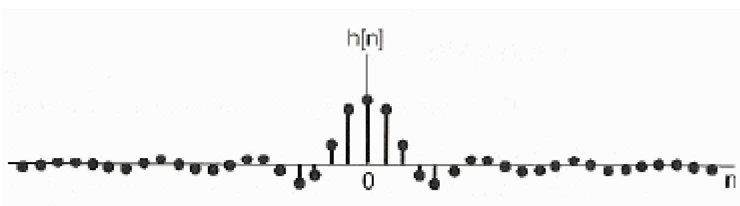
- Time- & Frequency-Domain Characterization:



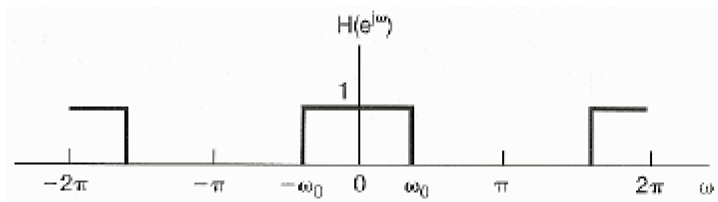


$$h(t) \stackrel{\mathcal{F}}{\longleftrightarrow} H(jw)$$

$$h[n] \stackrel{\mathcal{F}}{\longleftrightarrow} H(e^{jw})$$



(b)

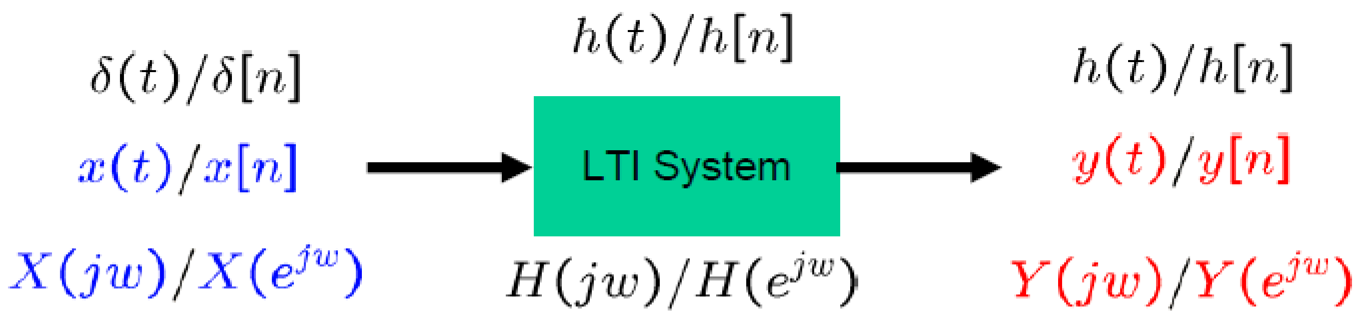


(a)

Time & Frequency Characterization

- Time- & Frequency-Domain Characterization:

$$H(jw) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt \qquad H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h[n]e^{-jwn}$$



$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(jw) \qquad h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(jw) \qquad y(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(jw)$$

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{jw}) \qquad h[n] \overset{\mathcal{F}}{\longleftrightarrow} H(e^{jw}) \qquad y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{jw})$$

$$y(t) = x(t) * h(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(jw) = X(jw)H(jw)$$

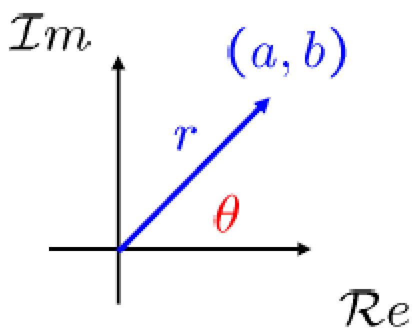
$$y[n] = x[n] * h[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{jw}) = X(e^{jw})H(e^{jw})$$

Time & Frequency Characterization

Time-Domain		Frequency-Domain
	Convolution ⟷ Transformation	
Differential or Difference Equations	System Model & Operations	Algebraic Equations
Convolution Multiplication	Techniques	Multiplication Convolution
Time-Domain Considerations	System Design	Frequency-Domain Considerations

Magnitude-Phase Representation of Fourier Transform

- Magnitude & Phase Representation:



$$a + jb \quad \Rightarrow \quad \begin{cases} r = \sqrt{a^2 + b^2} \\ \tan(\theta) = \frac{b}{a} \end{cases}$$

$$\Rightarrow \quad a + jb = re^{j\theta}$$

$$X(jw) = \mathcal{R}e\{X(jw)\} + j\ \mathcal{I}m\{X(jw)\} \qquad = \left|X(jw)\right| e^{j\measuredangle X(jw)}$$

$$X(e^{jw}) = \mathcal{R}e\left\{X(e^{jw})\right\} + j\ \mathcal{I}m\left\{X(e^{jw})\right\} \qquad = \left|X(e^{jw})\right| e^{j\measuredangle X(e^{jw})}$$

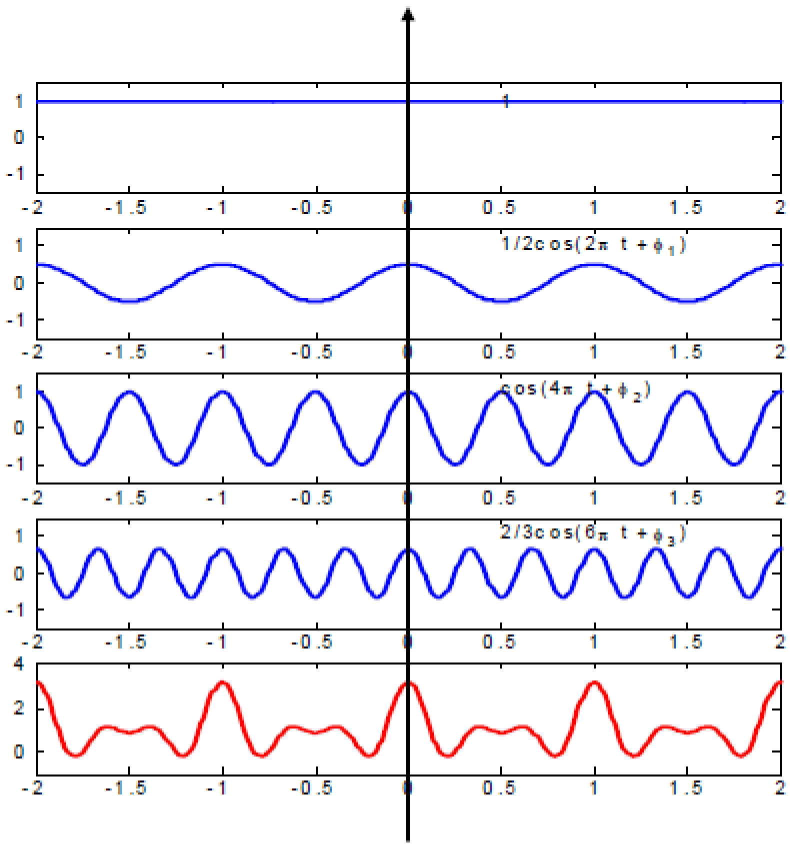
$\left|X(jw)\right|$ or $\left|X(e^{jw})\right|$: magnitude

$\measuredangle X(jw)$ or $\measuredangle X(e^{jw})$: phase angle

Magnitude-Phase Representation of Fourier Transform

- Magnitude & Phase Angle: $A\cos(w_0 t + \phi) = \frac{A}{2}\, e^{j\phi}\, e^{jw_0 t} + \frac{A}{2}\, e^{-j\phi}\, e^{-jw_0 t}$

$$x(t) = 1 + \frac{1}{2}\cos(2\pi t + \phi_1) + \cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$

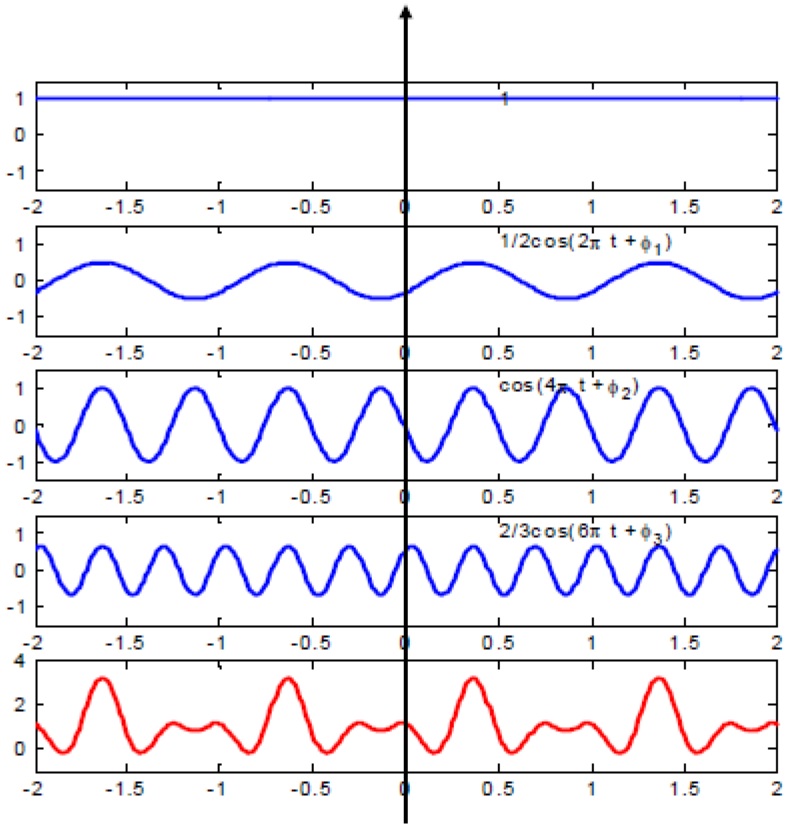


$$\begin{cases} \phi_1 = 0 \ (rad) \\ \phi_2 = 0 \ (rad) \\ \phi_3 = 0 \ (rad) \end{cases}$$

Magnitude-Phase Representation of Fourier Transform

- Magnitude & Phase Angle: $A\cos(w_0 t+\psi) = \frac{A}{2}\, e^{j\phi}\, e^{jw_0 t} + \frac{A}{2}\, e^{-j\phi}\, e^{-jw_0 t}$

$$x(t) = 1+\frac{1}{2}\cos(2\pi t+\phi_1)+\cos(4\pi t+\phi_2)+\frac{2}{3}\cos(6\pi t+\phi_3)$$

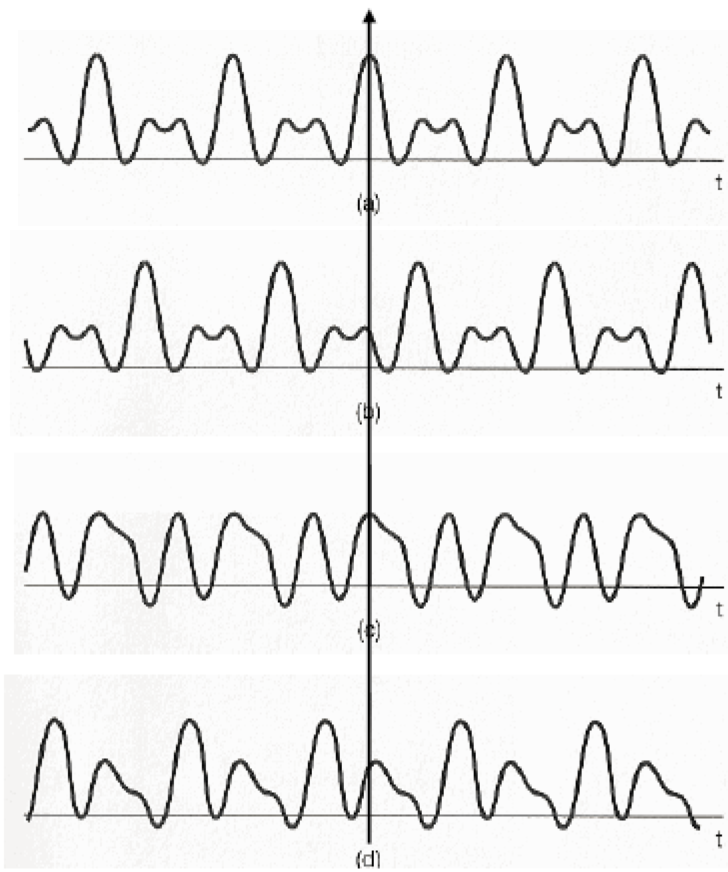


$$\begin{cases} \phi_1 = 4 \ (rad) \\ \phi_2 = 8 \ (rad) \\ \phi_3 = 12 \ (rad) \end{cases}$$

Magnitude-Phase Representation of Fourier Transform

- Magnitude & Phase Angle: $A\cos(w_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{jw_0 t} + \frac{A}{2} e^{-j\phi} e^{-jw_0 t}$

$$x(t) = 1 + \frac{1}{2}\cos(2\pi t + \phi_1) + \cos(4\pi t + \phi_2) + \frac{2}{3}\cos(6\pi t + \phi_3)$$



$$\begin{cases} \phi_1 = 0 \ (rad) \\ \phi_2 = 0 \ (rad) \\ \phi_3 = 0 \ (rad) \end{cases}$$

$$\begin{cases} \phi_1 = 4 \ (rad) \\ \phi_2 = 8 \ (rad) \\ \phi_3 = 12 \ (rad) \end{cases}$$

$$\begin{cases} \phi_1 = 6 \ (rad) \\ \phi_2 = -2.7 \ (rad) \\ \phi_3 = 0.93 \ (rad) \end{cases}$$

$$\begin{cases} \phi_1 = 1.2 \ (rad) \\ \phi_2 = 4.1 \ (rad) \\ \phi_3 = -7.02 \ (rad) \end{cases}$$

Magnitude-Phase Representation of LTI System

- Magnitude & Phase Distortions:

$$Y(jw) = X(jw)\; H(jw)$$

$$Y(e^{jw}) = X(e^{jw})\; H(e^{jw})$$

$$\Rightarrow \left|Y(jw)\right| e^{j\measuredangle Y(jw)} = \left|X(jw)\right| e^{j\measuredangle X(jw)} \left|H(jw)\right| e^{j\measuredangle H(jw)}$$

$$= \left|X(jw)\right| \left|H(jw)\right| e^{j(\measuredangle X(jw) + \measuredangle H(jw))}$$

$$\Rightarrow \begin{cases} |Y(jw)| & = |X(jw)|\; |H(jw)| \\ \measuredangle Y(jw) & = \measuredangle X(jw) + \measuredangle H(jw) \end{cases}$$

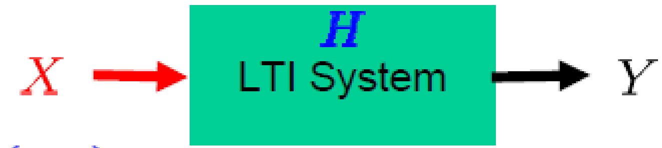
$$\Rightarrow \begin{cases} |Y(e^{jw})| & = |X(e^{jw})|\; |H(e^{jw})| \\ \measuredangle Y(e^{jw}) & = \measuredangle X(e^{jw}) + \measuredangle H(e^{jw}) \end{cases}$$

$|H(jw)|$ or $|H(e^{jw})|$: gain of the system

$\measuredangle H(jw)$ or $\measuredangle H(e^{jw})$: phase shift of the system

Magnitude-Phase Representation of LTI System

- Log-Magnitude & Bode Plots:



$$Y(jw) = X(jw)\ H(jw)$$

$$\Rightarrow \begin{cases} |Y(jw)| &= |X(jw)|\ |H(jw)| \\ \measuredangle Y(jw) &= \measuredangle X(jw) + \measuredangle H(jw) \end{cases}$$

$$\Rightarrow \log|Y(jw)| = \log|X(jw)| + \log|H(jw)|$$

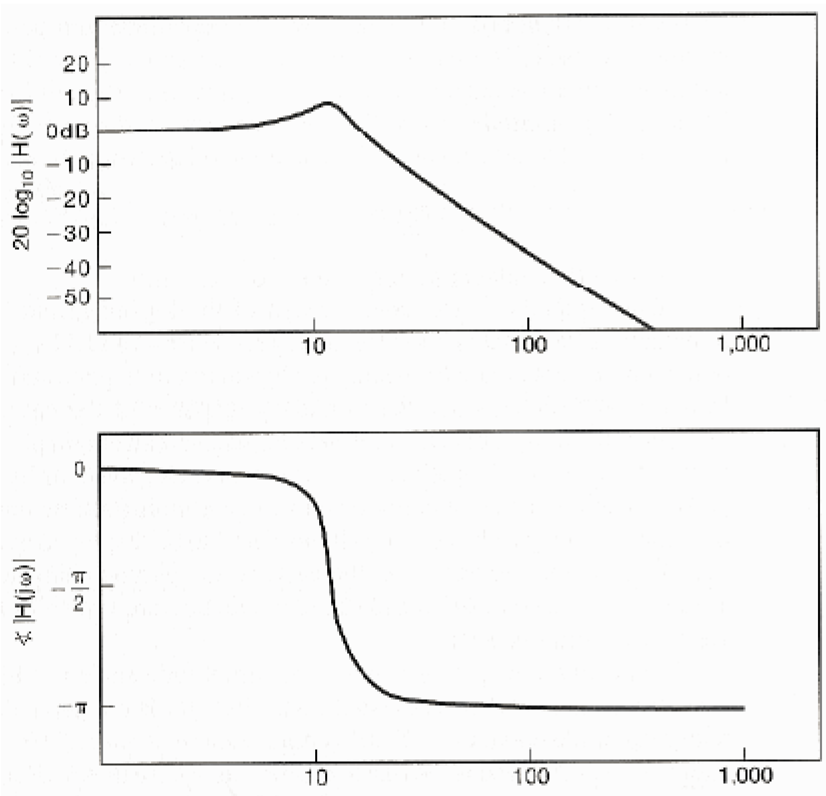
$$\Rightarrow 20\ \log_{10}|Y(jw)| = 20\ \log_{10}|X(jw)| + 20\ \log_{10}|H(jw)|$$

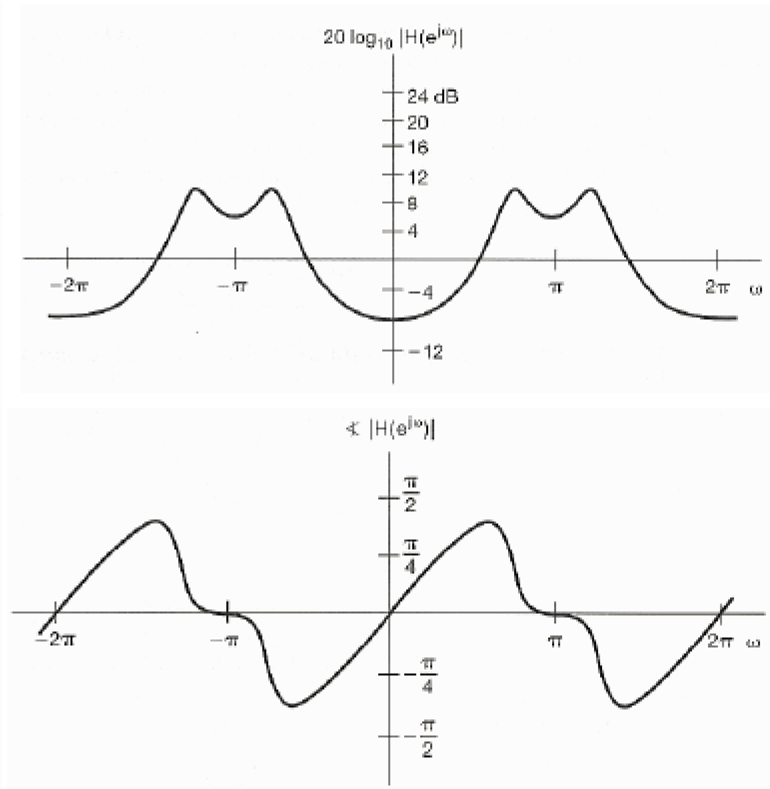
$$\Rightarrow \begin{cases} 20\ \log_{10}(1) &= 0\ \text{dB} \\ 20\ \log_{10}(10) &= 20\ \text{dB} \\ 20\ \log_{10}(0.1) &= -20\ \text{dB} \end{cases}$$

Magnitude-Phase Representation of LTI System

- Log-Magnitude & Bode Plots:

Continuous-Time Bode plot　　　Discrete-Time Bode plot

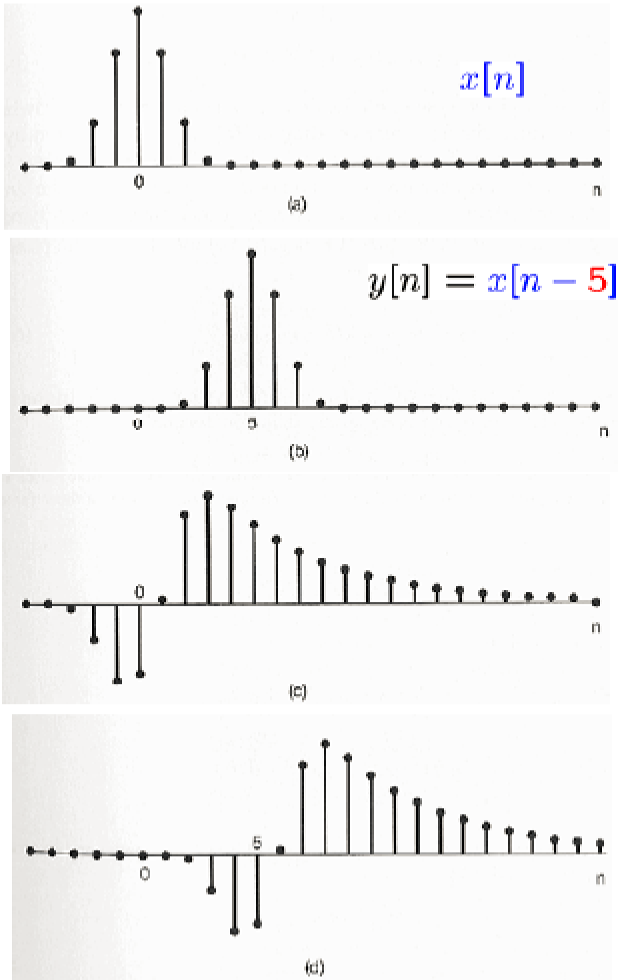




$\log(w)$, $w : 0 \leftrightarrow \infty$　　　(w), $w : -\pi \leftrightarrow \pi$

Magnitude-Phase Representation of LTI System

- Linear Phase:



$$X \rightarrow \boxed{H} \rightarrow Y$$

- $H_1(e^{jw}) = e^{-jwn_0}$

$$Y_1(e^{jw}) = H_1(e^{jw})\, X(e^{jw})$$
$$= X(e^{jw})\, e^{-jwn_0}$$
$$\Rightarrow\ y[n] = x[n - n_0]$$

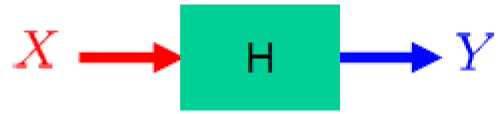
- $Y_2(e^{jw}) = H_2(e^{jw})\, X(e^{jw})$

$$H_2(e^{jw}) = e^{j\sphericalangle H_2(e^{jw})}$$

- $Y_3(e^{jw}) = H_2(e^{jw})\, H_1(e^{jw})\, X(e^{jw})$

$$H_3(e^{jw}) = H_2(e^{jw}) H_1(e^{jw})$$
$$= H_2(e^{jw}) e^{-jwn_0}$$
$$= e^{j(\sphericalangle H_2(e^{jw}) - wn_0)}$$

Magnitude-Phase Representation of LTI System



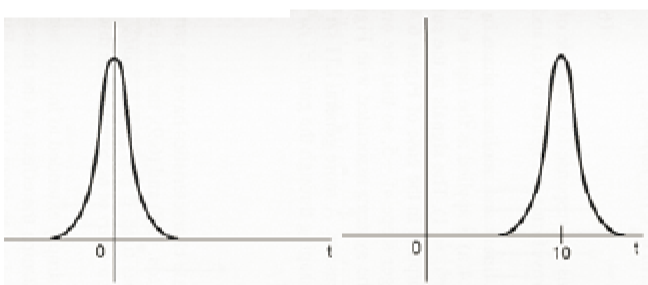
- Linear Phase:

$$H_1(jw) = e^{-jwt_0} \quad \Rightarrow \begin{cases} |H_1(jw)| = 1 \\ \sphericalangle |H_1(jw)| = -wt_0 \end{cases}$$

$$\Rightarrow y(t) = x(t - t_0)$$

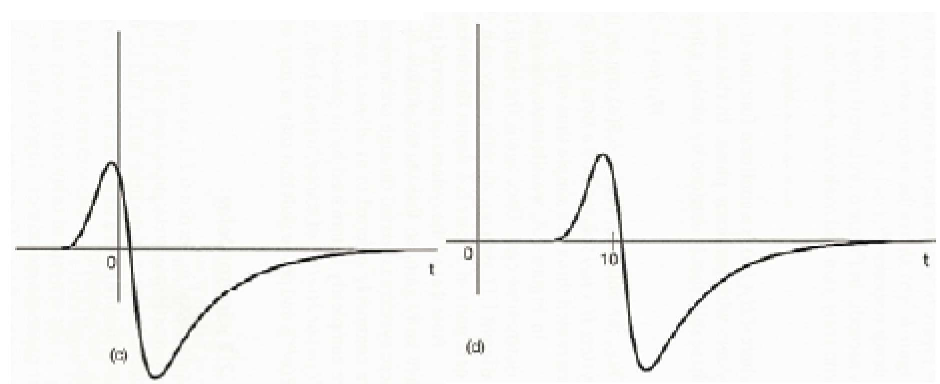
$$H_2(jw) = e^{j \sphericalangle H_2(jw)}$$

$$H_3(jw) = H_2(jw)\, H_1(jw) = H_2(jw) e^{-jwt_0} = e^{j(\sphericalangle H_2(jw) - wt_0)}$$



$x(t)$ $\quad y(t) = x(t - 10)$

$t_0 = 10$



Magnitude-Phase Representation of LTI System

- Group Delay & Phase:
  - Linear Phase & Delay:

$$H_1(jw) = e^{-jwt_0} \quad \Rightarrow \; y(t) = x(t - t_0) \quad \Rightarrow \text{delay} = t_0$$

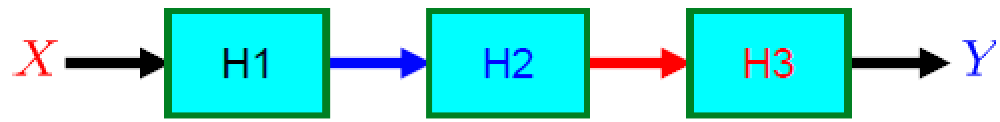
$$H_1(e^{jw}) = e^{-jwn_0} \quad \Rightarrow \; y[n] = x[n - n_0] \quad \Rightarrow \text{delay} = n_0$$

  - Nonlinear Phase & Group Delay

$$H_2(jw) = e^{j\sphericalangle H_2(jw)} \quad \Rightarrow \; \tau(w) = -\frac{d}{dw}\left\{\sphericalangle H_2(jw)\right\}$$

Magnitude-Phase Representation of LTI System

- Example 6.1:



$$H(jw) = H_1(jw)\, H_2(jw)\, H_3(jw)$$

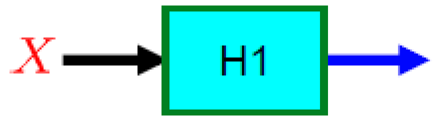
$$H_i(jw) = \frac{1 + (jw/w_i)^2 - 2j\zeta_i(w/w_i)}{1 + (jw/w_i)^2 + 2j\zeta_i(w/w_i)}$$

$$\Rightarrow \begin{cases} |H_i(jw)| &= 1 \\ \sphericalangle\, H_i(jw) &= -2\ \text{arctan}\left[\frac{2\zeta_i(w/w_i)}{1-(w/w_i)^2}\right] \end{cases}$$

$$\Rightarrow \begin{cases} |H(jw)| &= 1 \\ \sphericalangle\, H(jw) &= \sphericalangle\, H_1(jw) + \sphericalangle\, H_2(jw) + \sphericalangle\, H_3(jw) \end{cases}$$

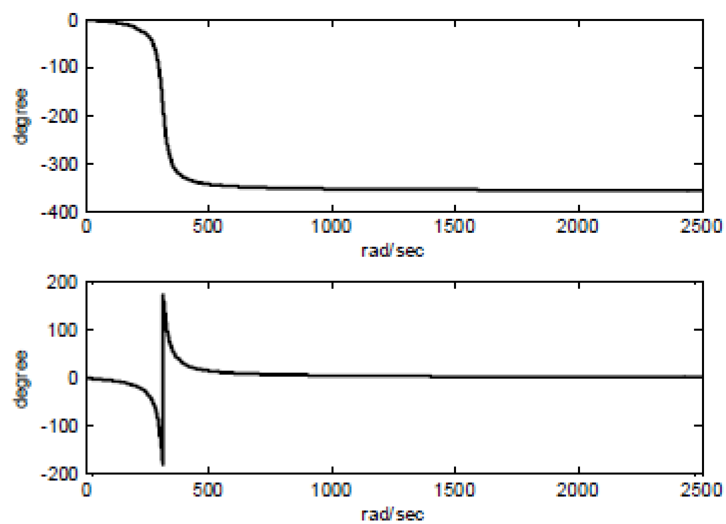
$$\Rightarrow \tau(w) = -\frac{d}{dw}\left\{\sphericalangle\, H(jw)\right\}$$

Magnitude-Phase Representation of LTI System

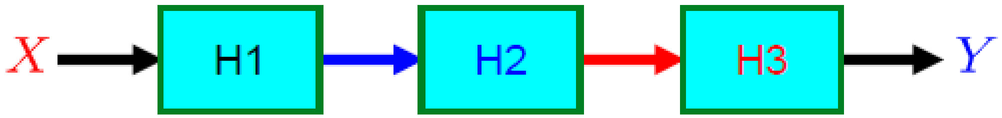


$$H_1(jw) = \frac{1 + (jw/w_1)^2 - 2j\zeta_1(w/w_1)}{1 + (jw/w_1)^2 + 2j\zeta_1(w/w_1)} \qquad \left\{ \begin{array}{lll} w_1 & = & 315 \quad \text{rad/sec} \\ \zeta_1 & = & 0.066 \\ f_1 & \approx & 50 \quad \text{Hz} \end{array} \right.$$

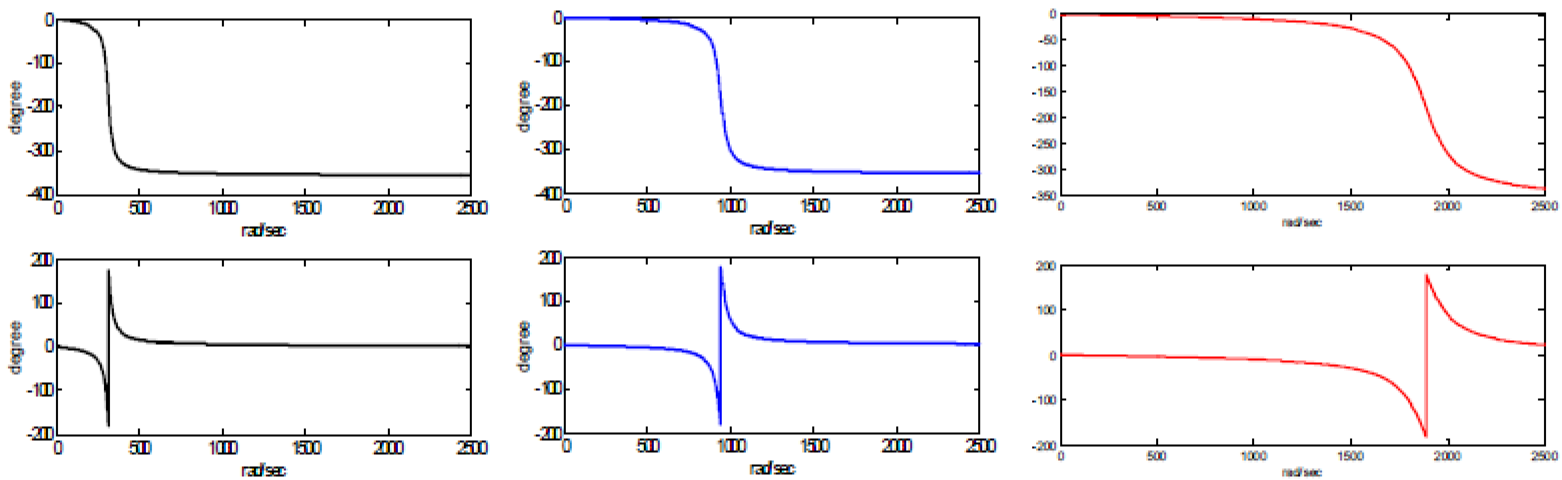
$$\Rightarrow \left\{ \begin{array}{lll} |H_1(jw)| & = & 1 \\ \measuredangle H_1(jw) & = & -2 \ \arctan\left[\frac{2\zeta_1(w/w_1)}{1-(w/w_1)^2}\right] \end{array} \right.$$



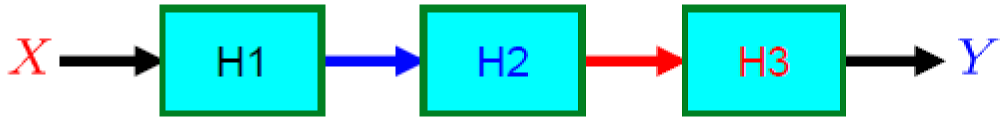
Magnitude-Phase Representation of LTI System



$$\begin{cases} w_1 = 315 \text{ rad/sec} \\ w_2 = 943 \text{ rad/sec} \\ w_3 = 1888 \text{ rad/sec} \end{cases} \qquad \begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$



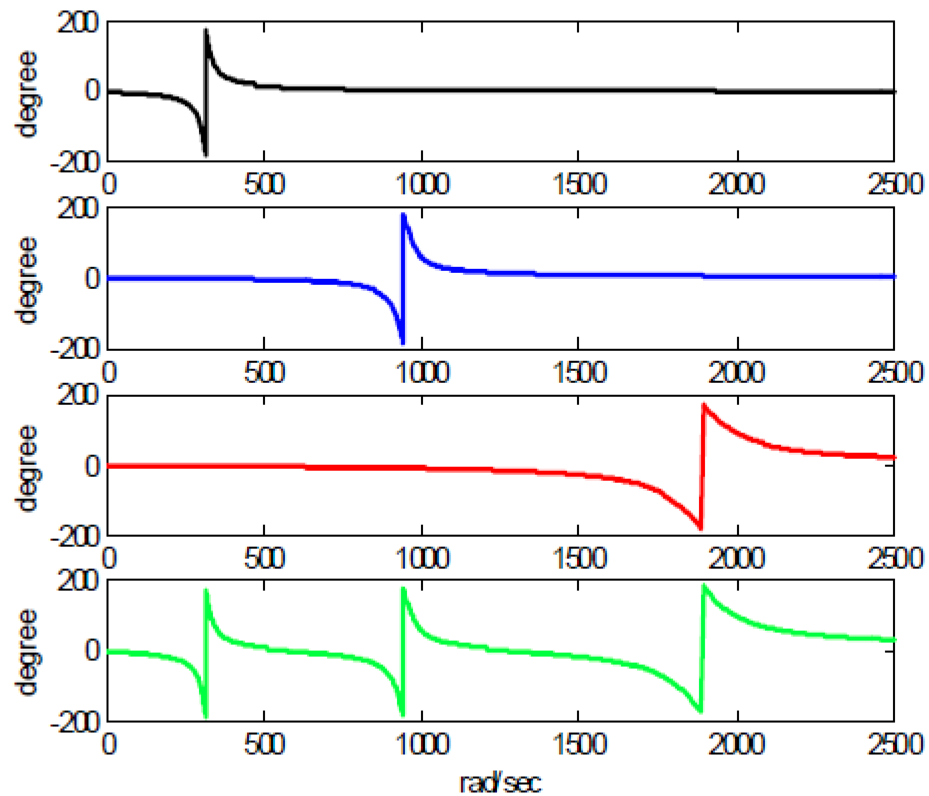
Magnitude-Phase Representation of LTI System



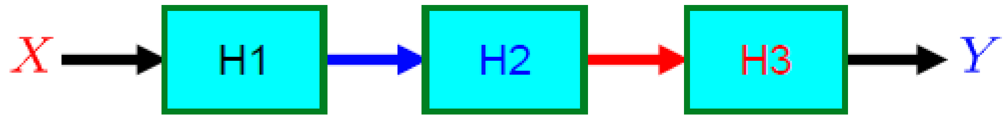
$$\begin{cases} |H(jw)| &= 1 \\ \sphericalangle H(jw) &= \sphericalangle H_1(jw) + \sphericalangle H_2(jw) + \sphericalangle H_3(jw) \end{cases}$$

$$\begin{cases} w_1 &= 315 \quad \text{rad/sec} \\ w_2 &= 943 \quad \text{rad/sec} \\ w_3 &= 1888 \quad \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 &= 0.066 \\ \zeta_2 &= 0.033 \\ \zeta_3 &= 0.058 \end{cases}$$



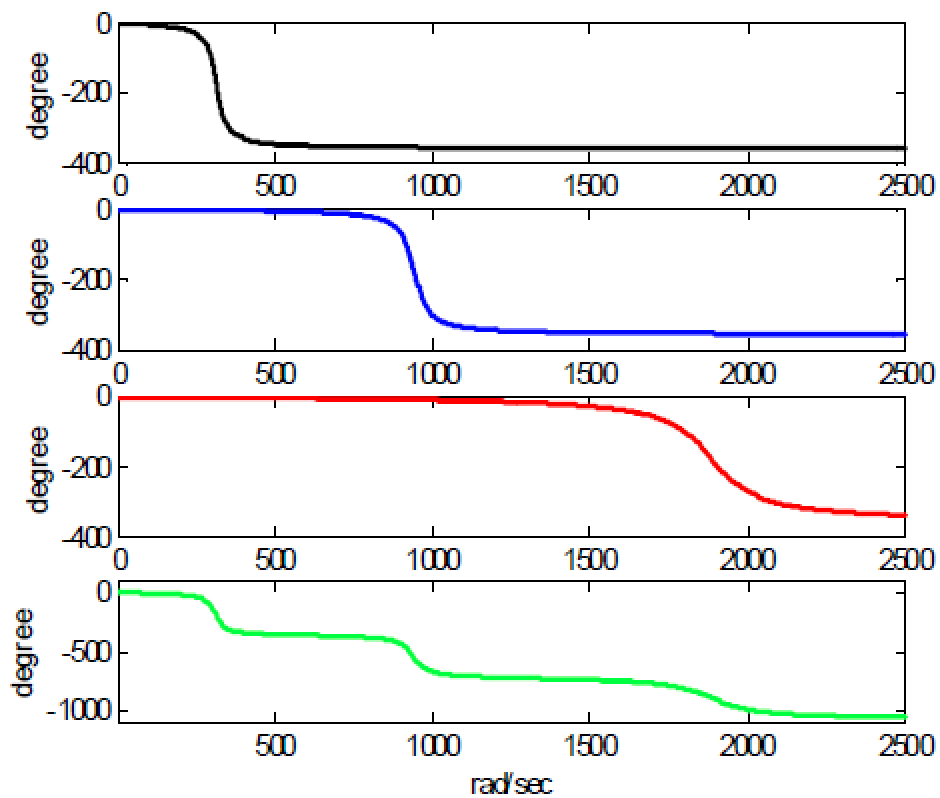
Magnitude-Phase Representation of LTI System



$$\begin{cases} |H(jw)| &= 1 \\ \measuredangle H(jw) &= \measuredangle H_1(jw) + \measuredangle H_2(jw) + \measuredangle H_3(jw) \end{cases}$$

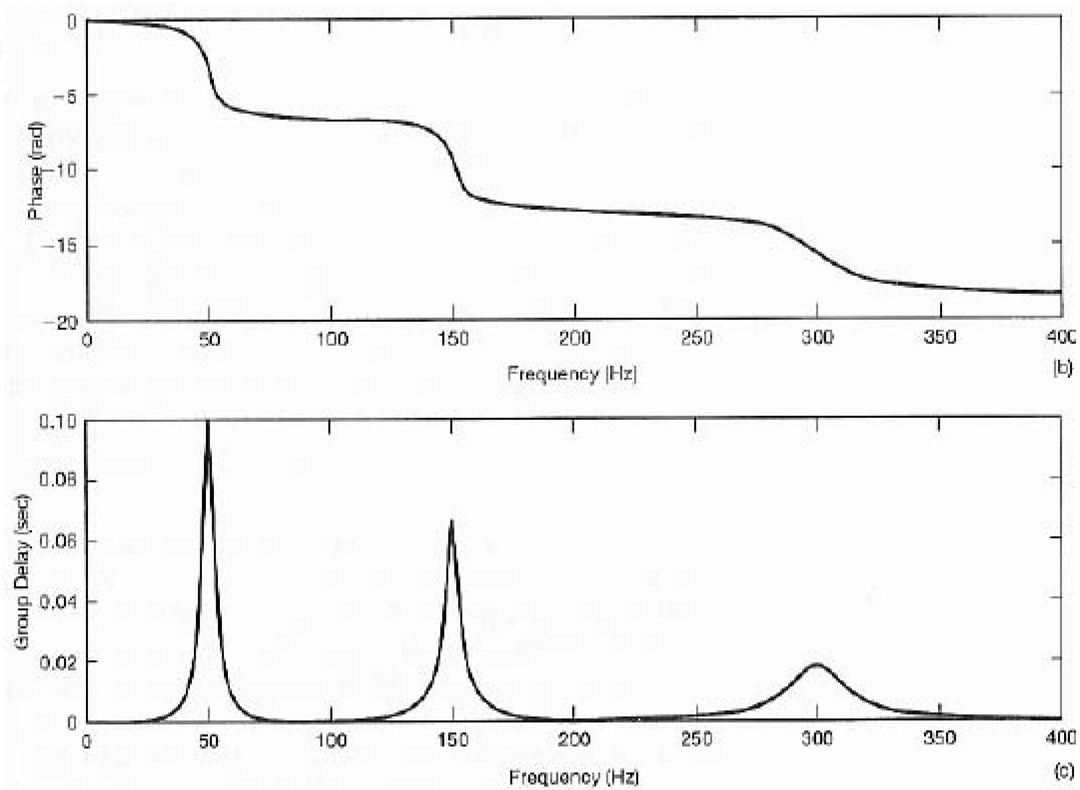
$$\begin{cases} w_1 = 315 \ \text{rad/sec} \\ w_2 = 943 \ \text{rad/sec} \\ w_3 = 1888 \ \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$



Magnitude-Phase Representation of LTI System

$$\tau(w) = -\frac{d}{dw}\left\{\measuredangle H(jw)\right\}$$



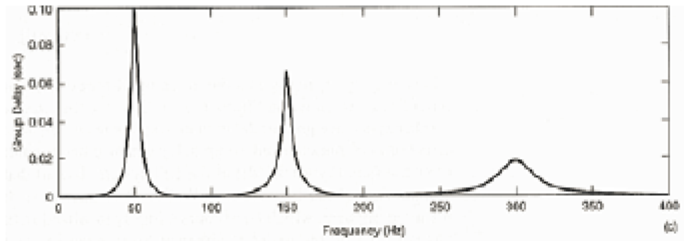
$$\begin{cases} w_1 = 315 \text{ rad/sec} \\ w_2 = 943 \text{ rad/sec} \\ w_3 = 1888 \text{ rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 = 0.066 \\ \zeta_2 = 0.033 \\ \zeta_3 = 0.058 \end{cases}$$

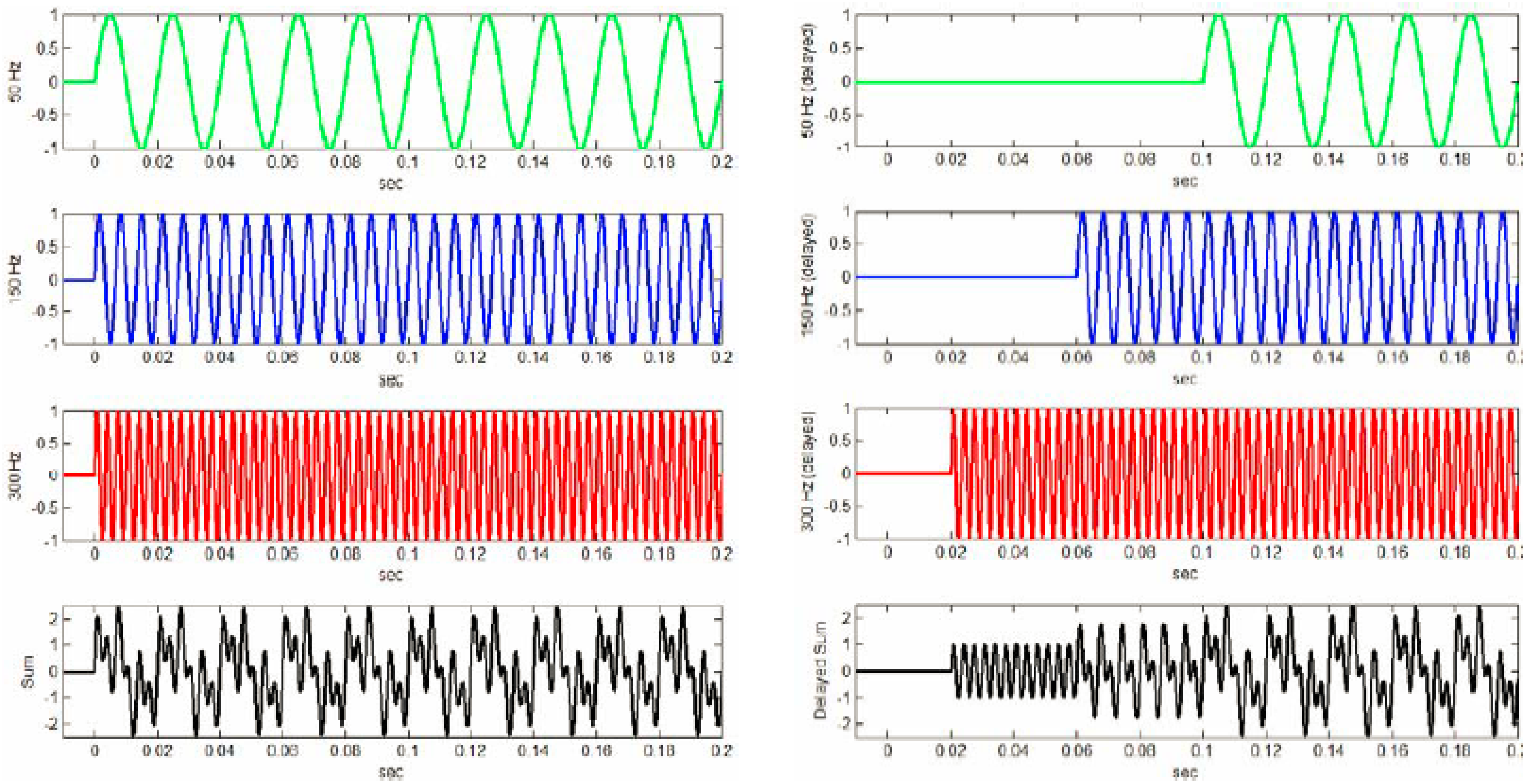
$$w_i = 2\pi f_i$$

$$\begin{cases} f_1 \approx 50 \text{ Hz} \\ f_2 \approx 150 \text{ Hz} \\ f_3 \approx 300 \text{ Hz} \end{cases}$$

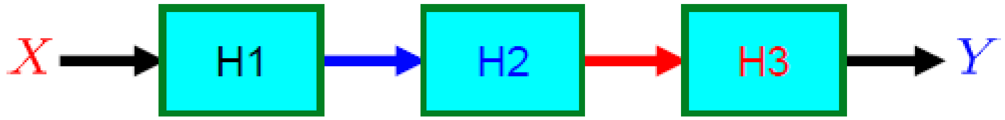
Magnitude-Phase Representation of LTI System



$$x(t) = x_1(t) + x_2(t) + x_3(t) \qquad y(t) = y_1(t) + y_2(t) + y_3(t)$$



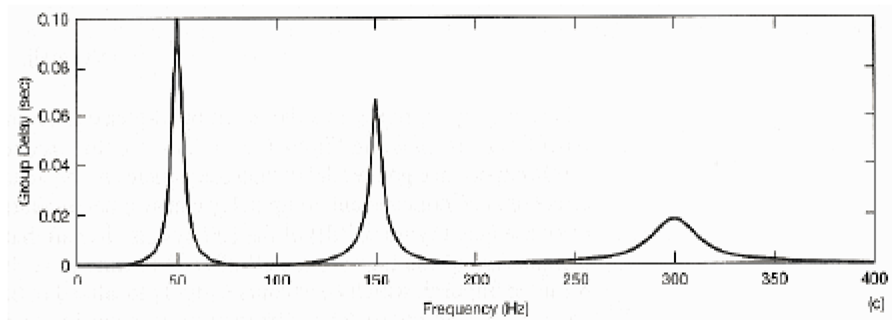
Magnitude-Phase Representation of LTI System



$$x(t) = \delta(t)$$

$$X(jw) = 1, \ \forall w$$

$$\begin{cases} |H(jw)| & = 1 \\ \measuredangle H(jw) & = \measuredangle H_1(jw) + \measuredangle H_2(jw) + \measuredangle H_3(jw) \end{cases}$$

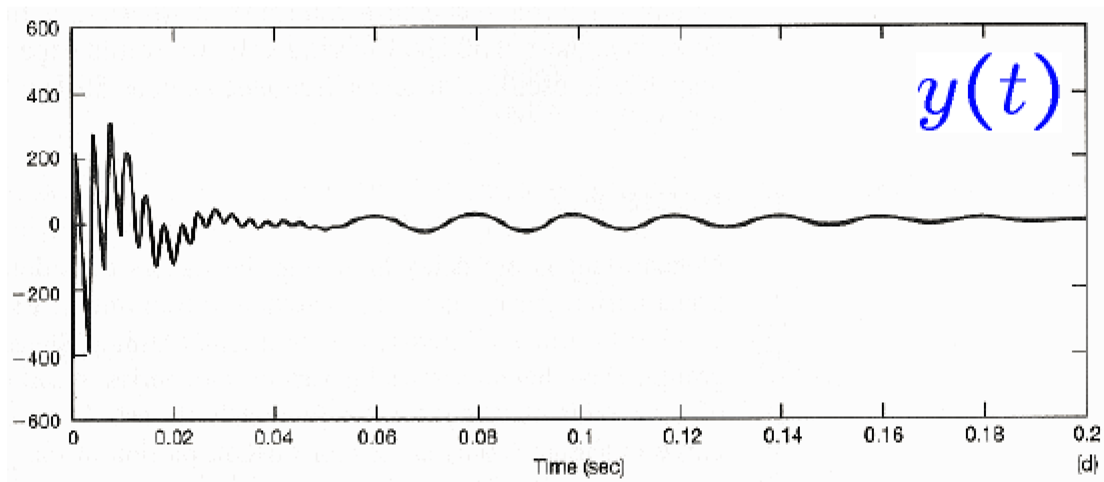


$$\begin{cases} w_1 & = 315 & \text{rad/sec} \\ w_2 & = 943 & \text{rad/sec} \\ w_3 & = 1888 & \text{rad/sec} \end{cases}$$

$$\begin{cases} \zeta_1 & = 0.066 \\ \zeta_2 & = 0.033 \\ \zeta_3 & = 0.058 \end{cases}$$

$$w_i = 2\pi f_i$$

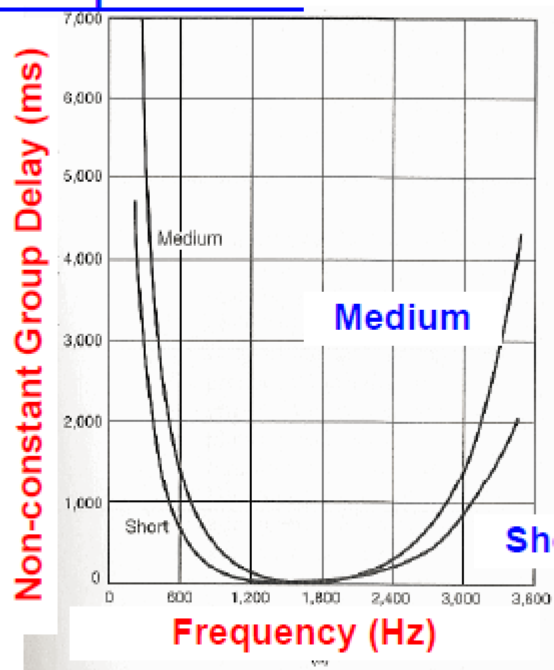
$$\begin{cases} f_1 & \approx 50 & \text{Hz} \\ f_2 & \approx 150 & \text{Hz} \\ f_3 & \approx 300 & \text{Hz} \end{cases}$$

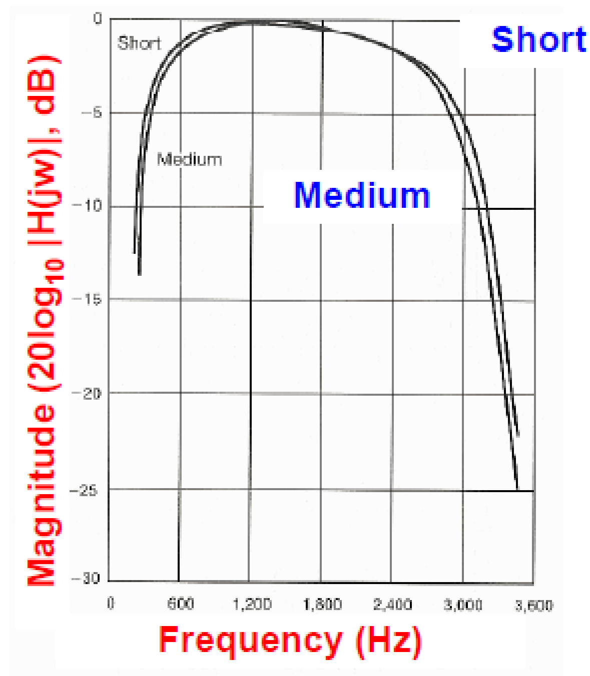


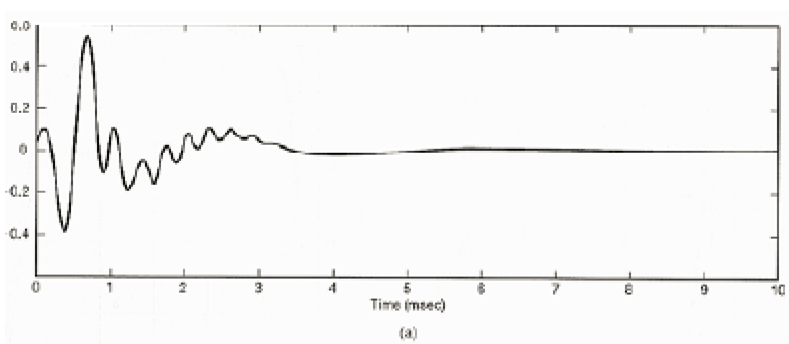
Magnitude-Phase Representation of LTI System

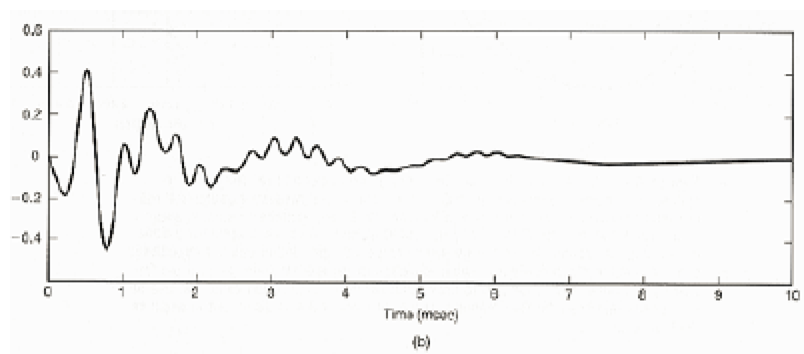
- Example 6.2:

Analog Transmission Performance on the Switched Telecommunication Networks (AT&T/Bell)









Time-Domain Properties of Ideal Frequency-Selective Filters

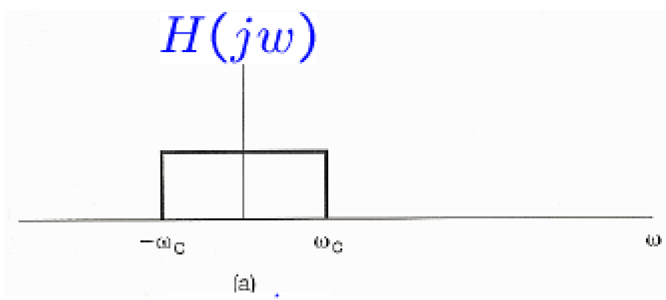
- <u>Ideal Lowpass Filters:</u>

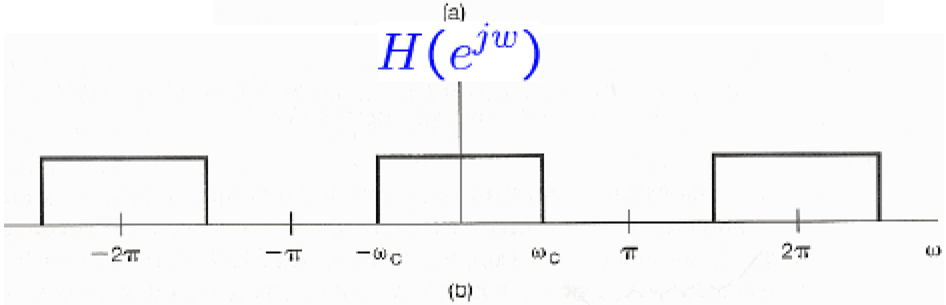
$$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}$$

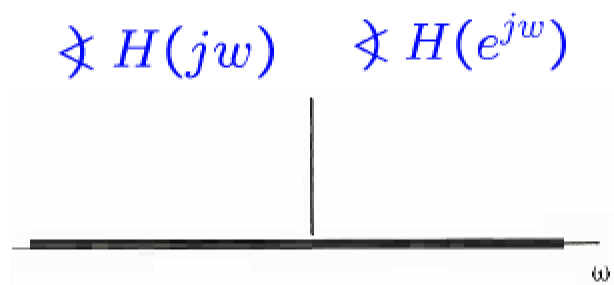
–unit gain

$$H(e^{jw}) = \begin{cases} 1, & |w| \le w_c \\ 0, & w_c < |w| \le \pi \end{cases}$$

–zero phase distortion







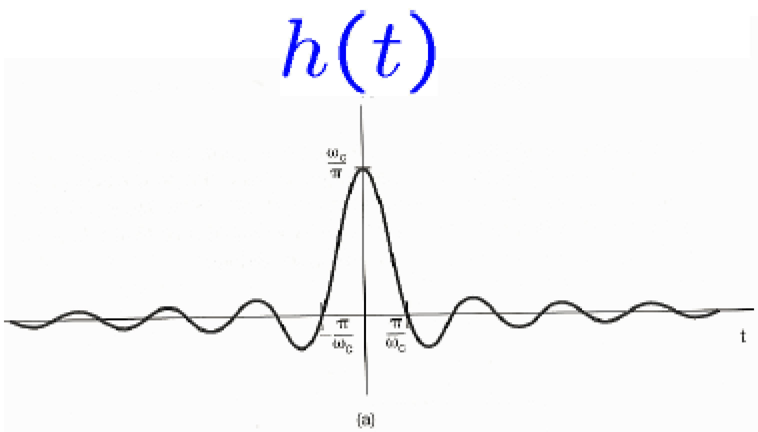
Time-Domain Properties of Ideal Frequency-Selective Filters

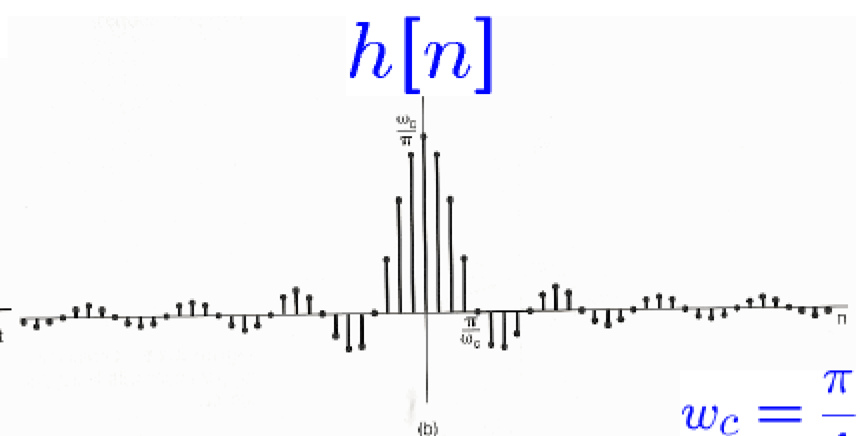
- Ideal Lowpass Filters:

$$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}$$

$$H(e^{jw}) = \begin{cases} 1, & |w| \le w_c \\ 0, & w_c < |w| \le \pi \end{cases}$$

$$\Rightarrow \begin{cases} h(t) = \dfrac{\sin w_c t}{\pi t} \\ h[n] = \dfrac{\sin w_c n}{\pi n} \end{cases}$$

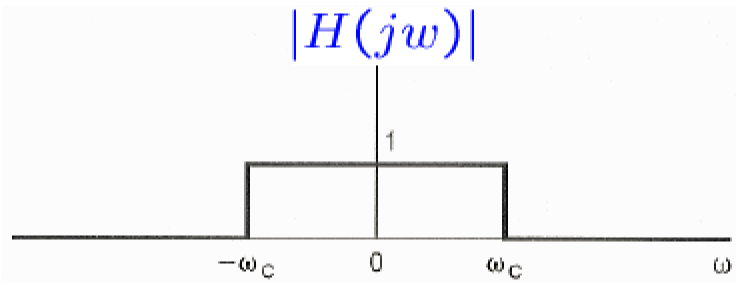


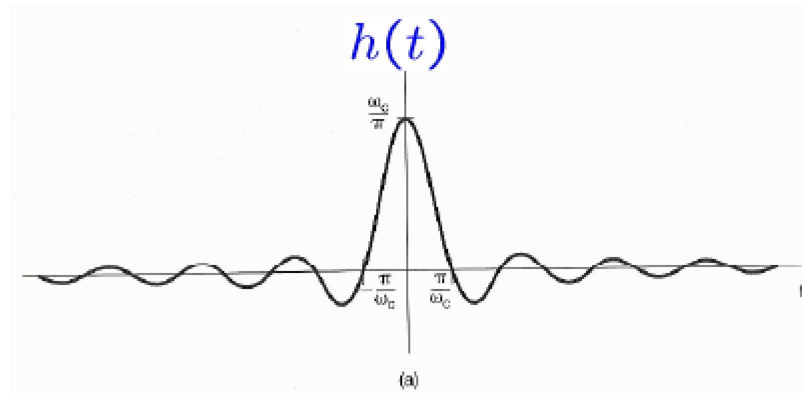


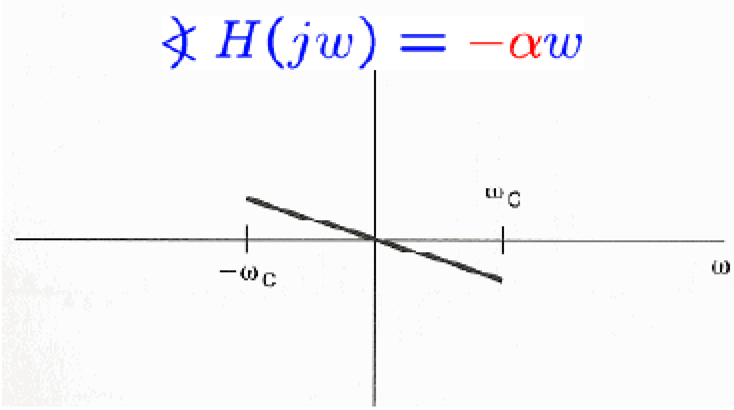
$w_c = \dfrac{\pi}{4}$

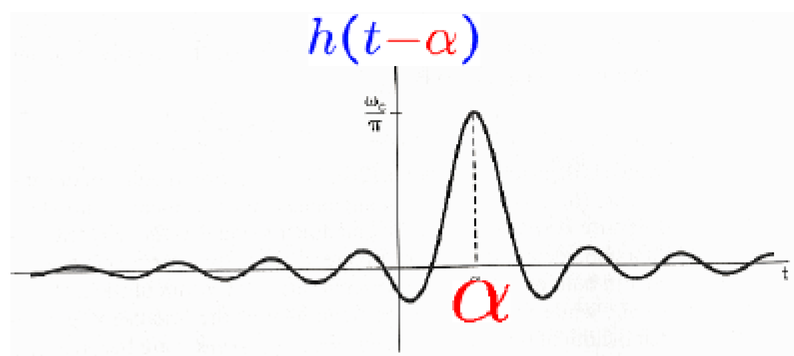
Time-Domain Properties of Ideal Frequency-Selective Filters

- Ideal Lowpass Filters with Linear Phase:





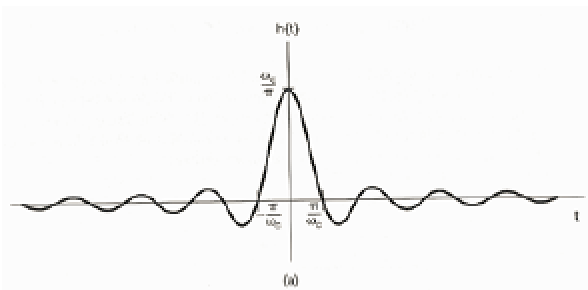


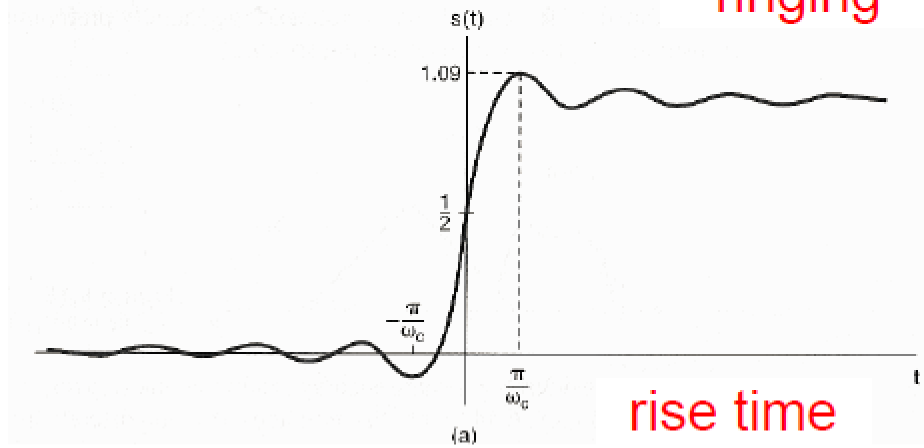


Time-Domain Properties of Ideal Frequency-Selective Filters

- Step Response of Ideal Lowpass Filters:

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

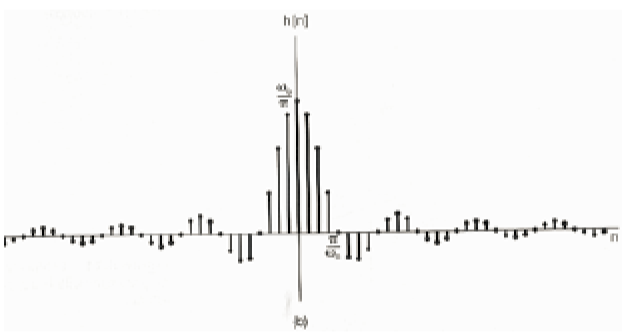


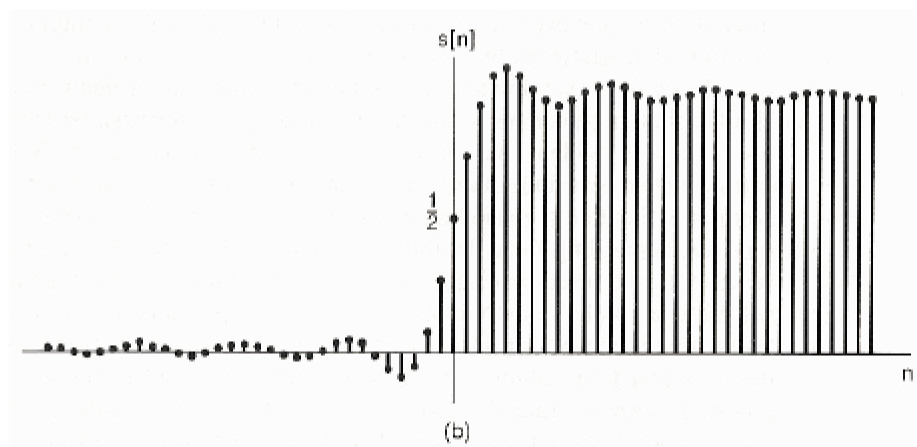


ringing

rise time

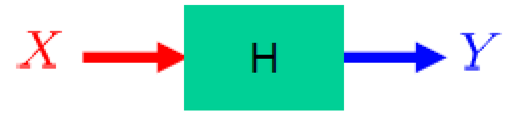
$$s[n] = \sum_{m=-\infty}^{n} h[m]$$

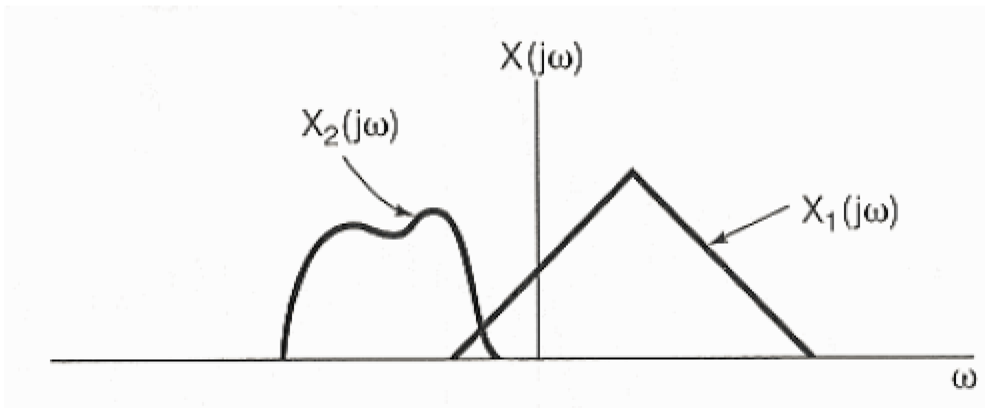


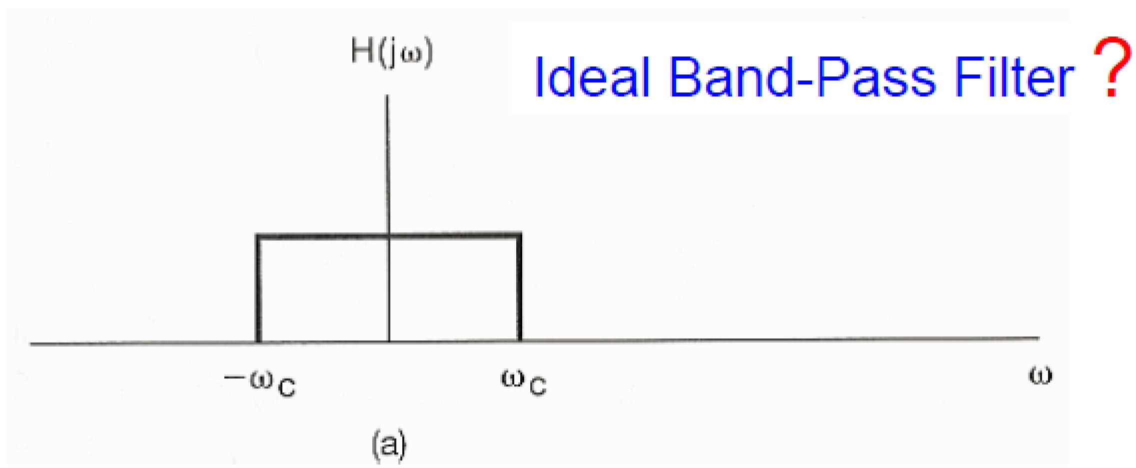


Time-Domain and Frequency-Domain Aspects of Non-ideal Filters

- Overlapping Spectra:

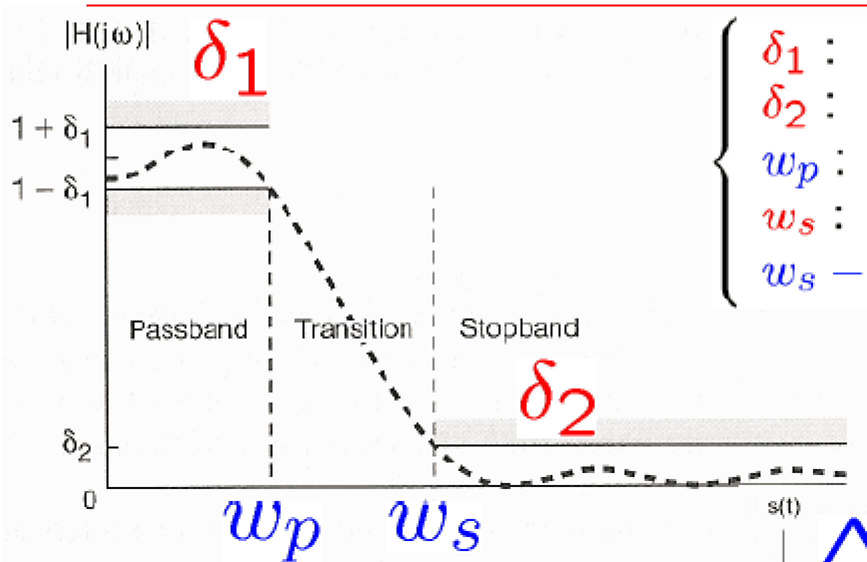




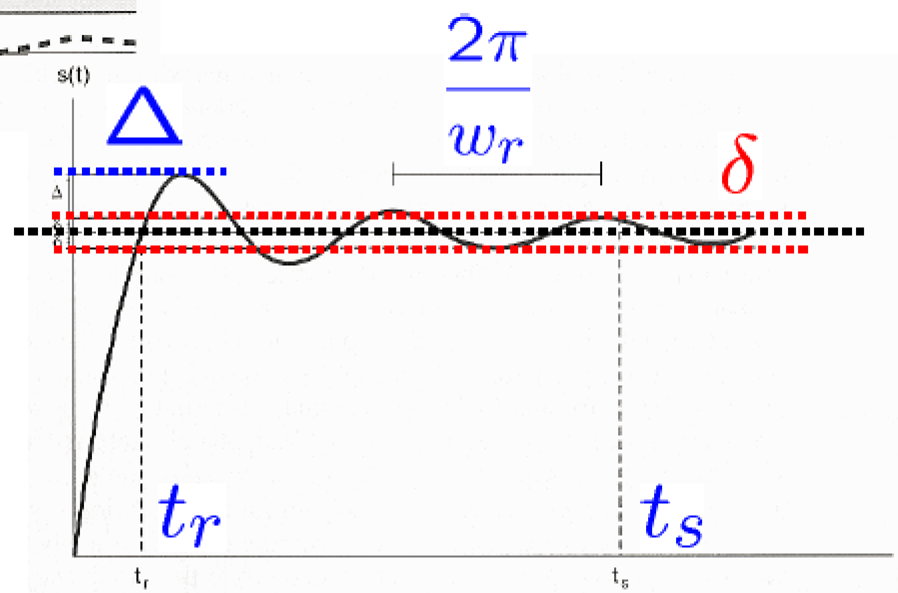


Time-Domain and Frequency-Domain Aspects of Non-ideal Filters

- Desired Filter Characteristics:



- δ_1 : allowable passband ripple
- δ_2 : allowable stopband ripple
- w_p : passband edge
- w_s : stopband edge
- $w_s - w_p$: transition band

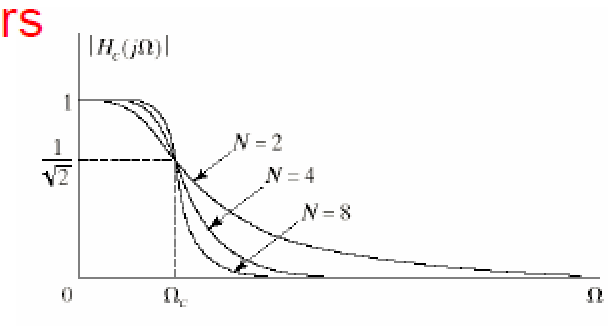


- Δ : overshoot
- δ : steady-state error
- w_r : ringing frequency
- t_r : rise time
- t_s : settling time

Time-Domain and Frequency-Domain Aspects of Non-ideal Filters

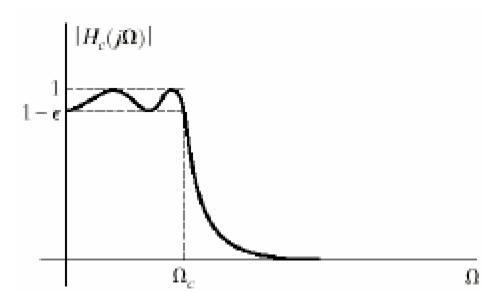
- Example 6.3: Two Frequently Used Filters:
  - Butterworth, Chebyshev, Elliptic filters

$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$



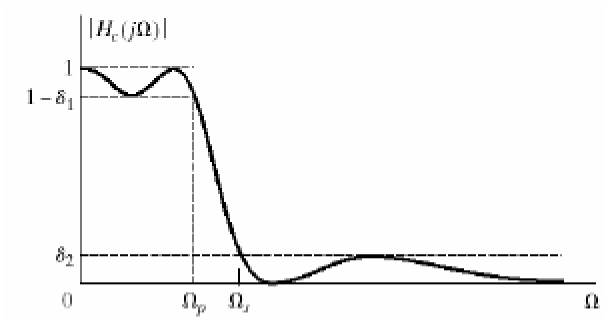
$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\epsilon^2 V_N^2\left(\frac{\Omega}{\Omega_c}\right)}$$

$$V_N(x) = \cos\left(N\cos^{-1}x\right)$$



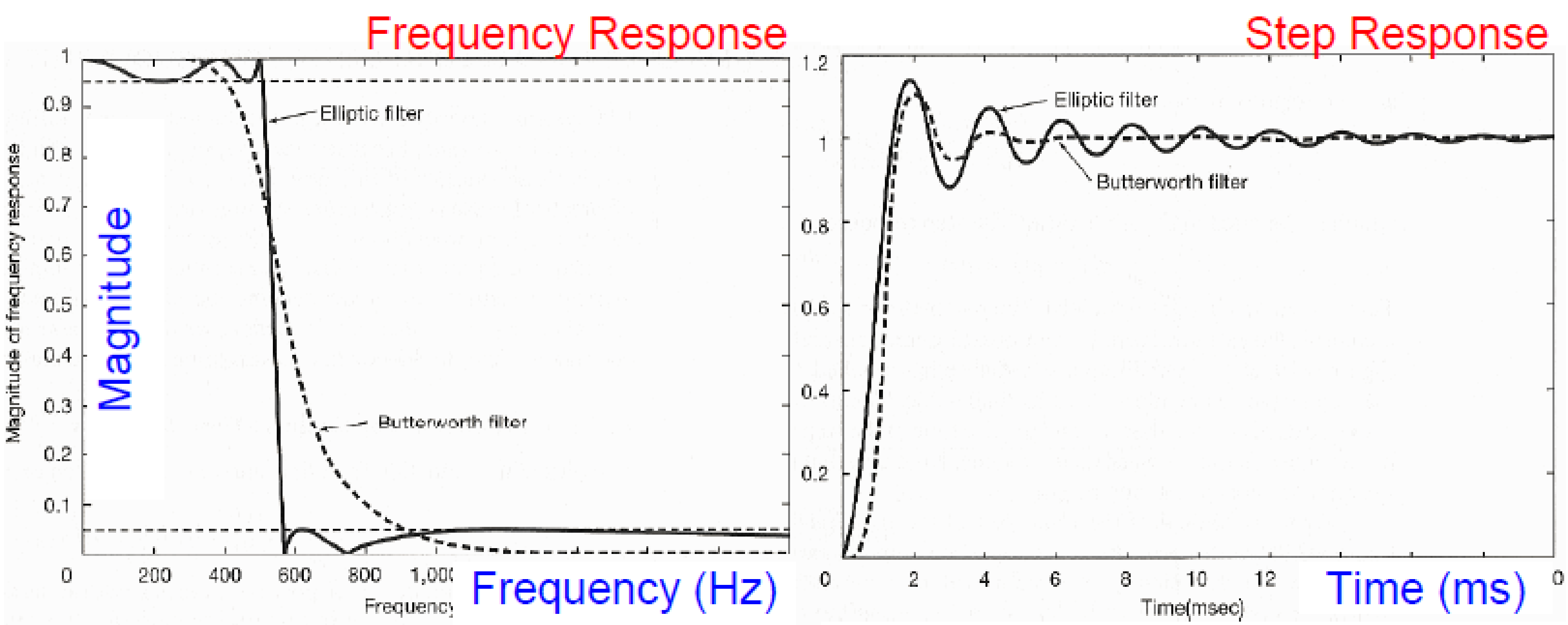
$$\left|H_c(j\Omega)\right|^2 = \frac{1}{1+\epsilon^2 U_N^2(\Omega)}$$

$U_N(x)$: Jacabian elliptic function



Time-Domain and Frequency-Domain Aspects of Non-ideal Filters

- Example 6.3: Two Frequently Used Filters:
  - Butterworth filter
  - Elliptic filter
    - Fifth-order rational frequency response
    - Cutoff frequency = 500 Hz



Analysis of an Automobile Suspension System

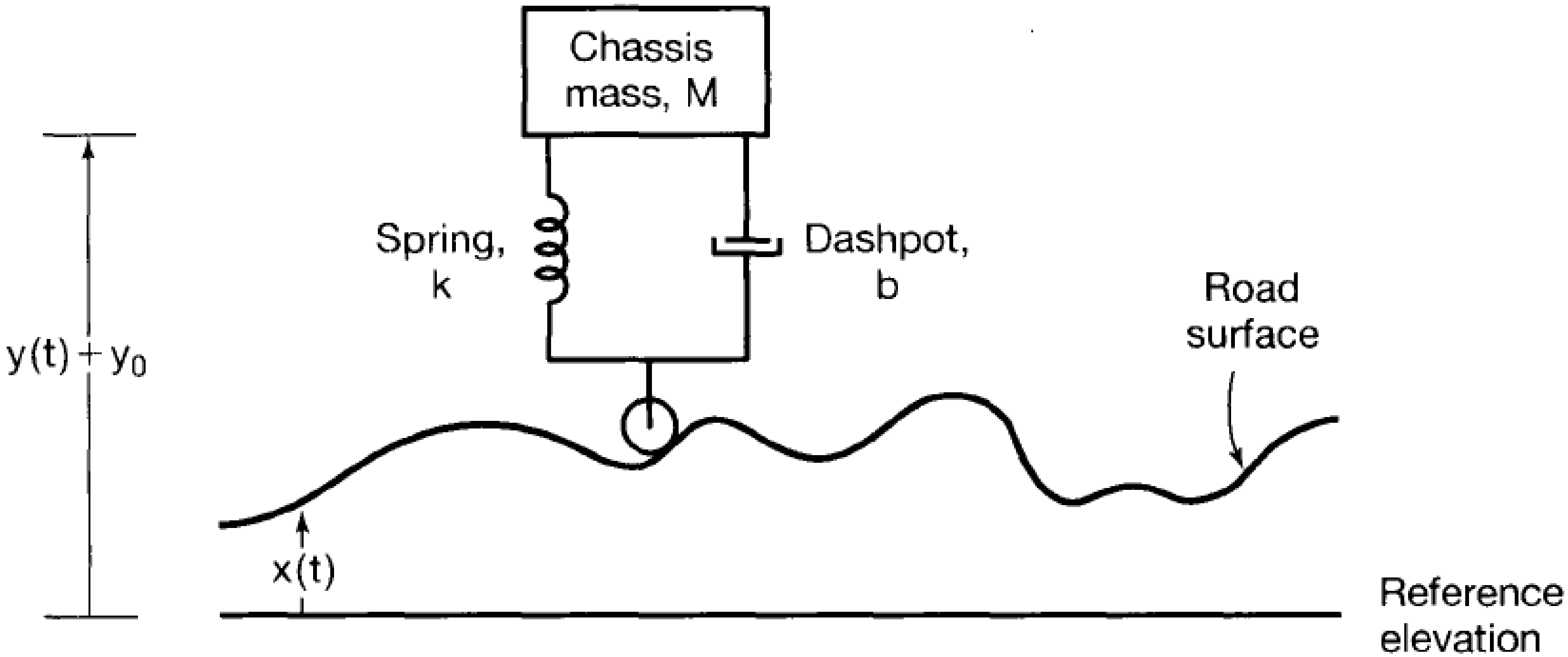


Figure 6.32 Diagrammatic representation of an automotive suspension system. Here, y_0 represents the distance between the chassis and the road surface when the automobile is at rest, $y(t) + y_0$ the position of the chassis above the reference elevation, and $x(t)$ the elevation of the road above the reference elevation.

Analysis of an Automobile Suspension System

The differential equation governing the motion of the chassis is then

$$M\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + ky(t) = kx(t) + b\frac{dx(t)}{dt},$$

where M is the mass of the chassis and k and b are the spring and shock absorber constants, respectively. The frequency response of the system is

$$H(j\omega) = \frac{k + bj\omega}{(j\omega)^2M + b(j\omega) + k},$$

or

$$H(j\omega) = \frac{\omega_n^2 + 2\zeta\omega_n(j\omega)}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2},$$

$$\omega_n = \sqrt{\frac{k}{M}} \quad \text{and} \quad 2\zeta\omega_n = \frac{b}{M}.$$

Analysis of an Automobile Suspension System

The denominator of $H(j\omega)$ can be factored to yield

$$H(j\omega) = \frac{\omega_n^2}{(j\omega - c_1)(j\omega - c_2)},$$

where

$$c_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1},$$
$$c_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}.$$

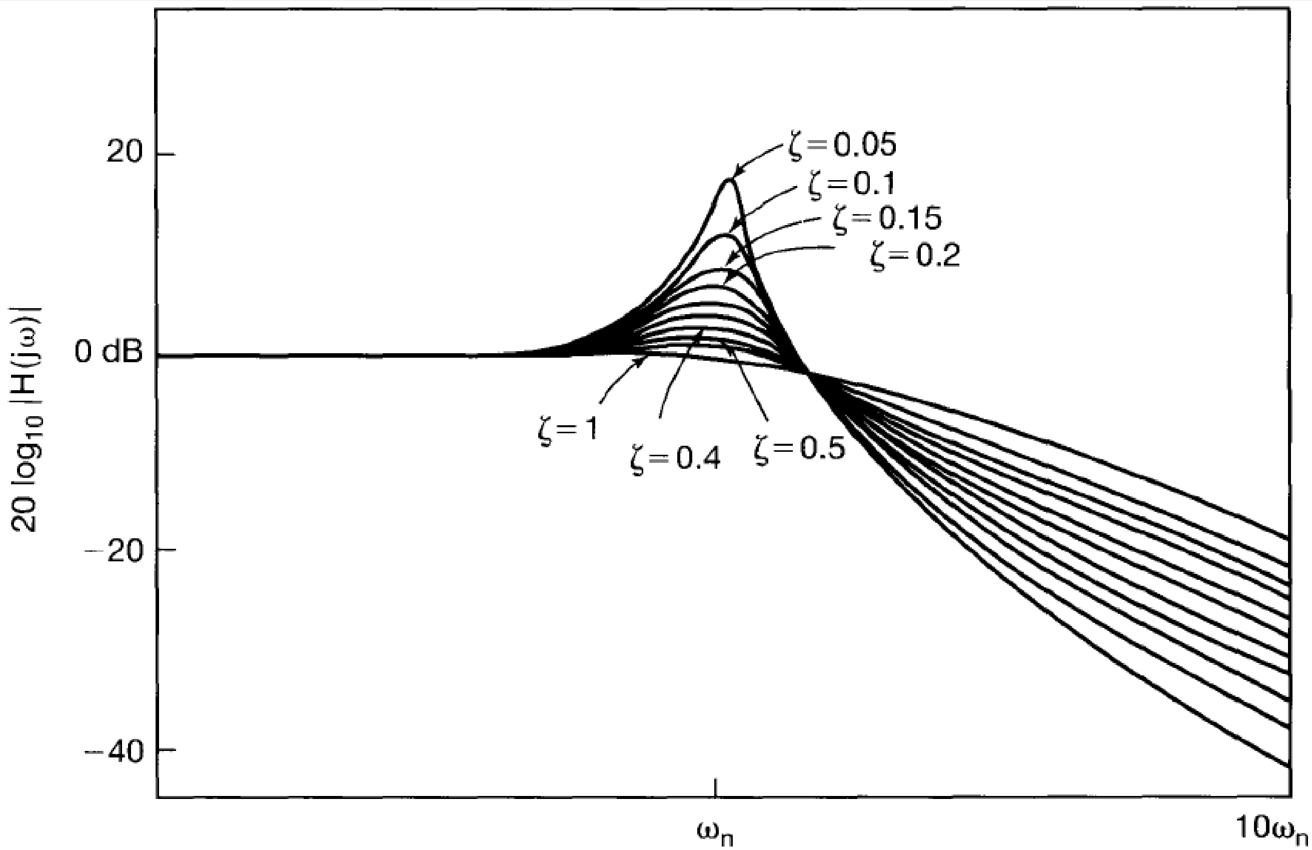
For $\zeta \neq 1$, c_1 and c_2 are unequal, and we can perform a partial-fractic
form

$$H(j\omega) = \frac{M}{j\omega - c_1} - \frac{M}{j\omega - c_2},$$

where

$$M = \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}.$$

Analysis of an Automobile Suspension System



$$\omega_{\max} = \omega_n \sqrt{1 - 2\zeta^2},$$

and the value at this maximum point is

$$|H(j\omega_{\max})| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}.$$

Analysis of an Automobile Suspension System

the corresponding impulse response for the system is

$$h(t) = M[e^{c_1 t} - e^{c_2 t}]u(t).$$

we take a more detailed look at the impulse response and the step response of a second-order system. First, from eq. (6.35), we see that for $0 < \zeta < 1$, c_1 and c_2 are complex, and we can rewrite the impulse response in eq. (6.37) in the form

$$\begin{aligned} h(t) &= \frac{\omega_n e^{-\zeta\omega_n t}}{2j\sqrt{1-\zeta^2}}\{\exp[j(\omega_n\sqrt{1-\zeta^2})t] - \exp[-j(\omega_n\sqrt{1-\zeta^2})t]\}u(t) \\ &= \frac{\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}[\sin(\omega_n\sqrt{1-\zeta^2})t]u(t). \end{aligned}$$

The step response of a second-order system can be calculated from $\zeta \neq 1$. This yields the expression

$$s(t) = h(t) * u(t) = \left\{1 + M\left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2}\right]\right\} u(t).$$

For $\zeta = 1$, we can use eq. (6.39) to obtain

$$s(t) = [1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}]u(t).$$

Analysis of an Automobile Suspension System

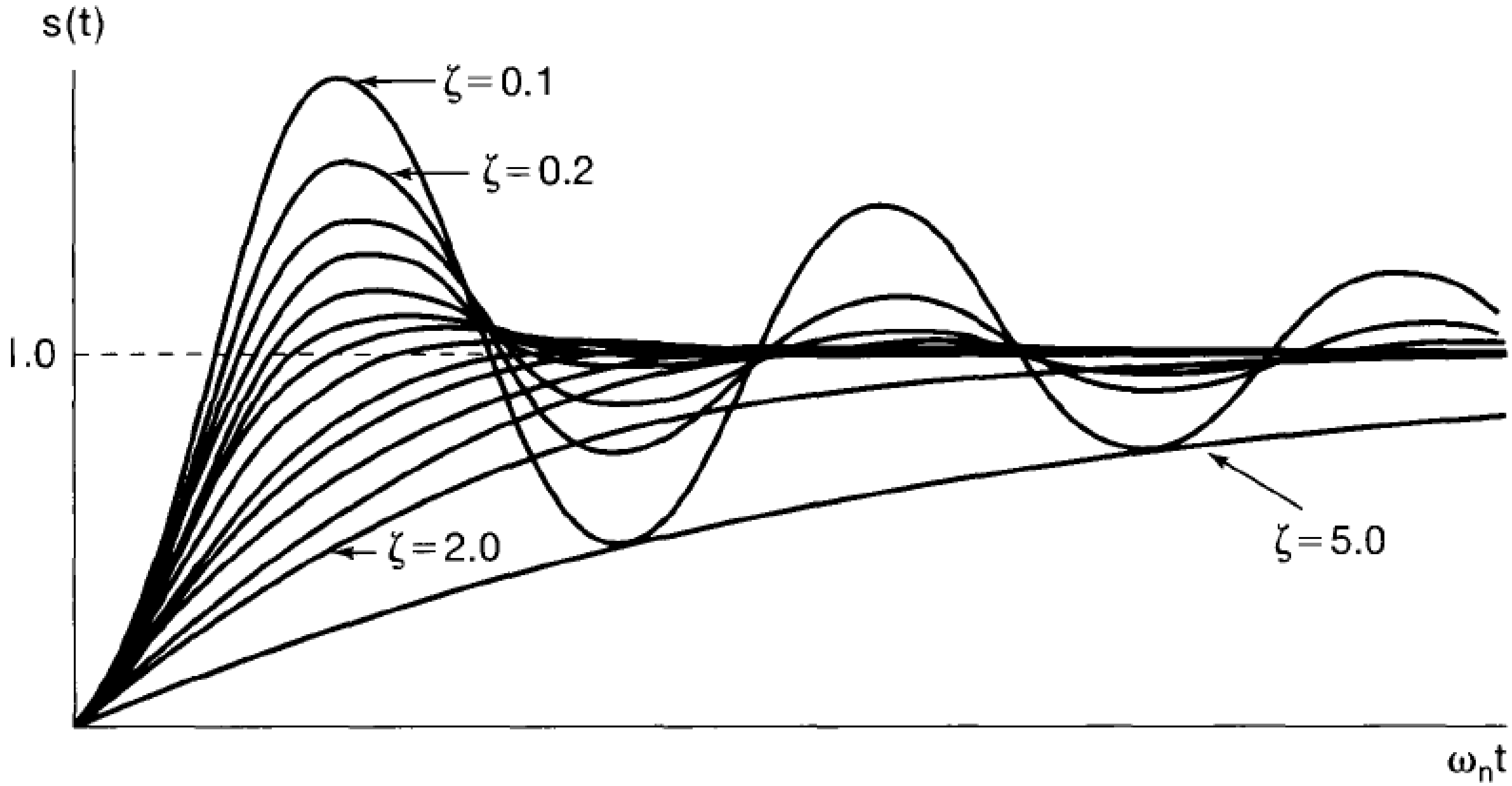


Figure 6.34 Step response of the automotive suspension system for various values of the damping ratio ($\zeta = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2, 1.5, 2.0, 5.0$).

Analysis of an Automobile Suspension System

-The filter cutoff frequency is controlled primarily through *wn* , or equivalently for a chassis with a fixed mass, by an appropriate choice of spring constant *k*. For a given *wn* , the damping ratio is then adjusted through the damping factor *b* associated with the shock absorbers.

- As the natural frequency *wn* is decreased, the suspension will tend to filter out slower road variations, thus providing a smoother ride. On the other hand, the rise time of the system increases, and thus the system will feel more slaw.

- On the one hand, it would be desirable to keep *wn* small to improve the low pass filtering; on the other hand, it would be desirable to have *Wn* large for a rapid time response.

- These, of course, are conflicting requirements and illustrate the need for a trade-off between time-domain and frequency-domain characteristics.

Analysis of an Automobile Suspension System

- Typically, a suspension system with a low value of *wn*, so that the rise time is long, is characterized as "soft" and one with a high value of *wn*, so that the rise time is short, is characterized as "hard."

- We observe also that, as the damping ratio decreases, the frequency response of the system cuts off more sharply, but the overshoot and ringing in the step response tend to increase, another trade-off between the time and frequency.

- Generally, the shock absorber damping is chosen to have a rapid rise time and yet avoid overshoot and ringing.

- This choice corresponds to the critically damped case, with = 1.0.