Chapter 1

VE216 Mid1 Review

Transform of Variables

- $y(t)=x(at-b)=x(\frac{t-t_0}{w})$
- $y[n]=x[an-b]=x[\frac{n-n_0}{w}]$
  - first b then a
  - first w then n_0

Signal Characteristics

- periodic

continuous, x(t)	discrete, x[n]
$\exists T > 0, \quad x(t) = x(t+T) \forall \mathrm{t}$	$\exists N > 0, N \in \aleph \setminus \{0\},$ $x[n] = x[n+N] \forall n$

- sum of two periodic signals is periodic if the ratio of two signals' periods is rational

Signal Characteristics

- even & odd component

continuous, x(t)	discrete, x[n]
$x(t) = x_e(t) + x_o(t)$ $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$ $x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$	$x[n] = x_e[n] + x_o[n]$ $x_e[n] = \frac{1}{2}\left\{x[n] + x[-n]\right\}$ $x_o[n] = \frac{1}{2}\left\{x[n] - x[-n]\right\}$

Signal Characteristics

- energy & power

	average value	power	energy	power if periodic with period T/N
continuous, x(t)	$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x(t)dt$	$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\lvert x(t)\rvert^2 dt$	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2 dt$	$\frac{1}{T}\int_{0}^{T}\lvert x(t)\rvert^2 dt$
discrete, x[n]	$\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}x[n]$	$\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\lvert x[n]\rvert^2$	$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\frac{1}{\mathrm{N}}\sum_{n=0}^{N-1}\lvert x[n]\rvert^2$

Signal Characteristics

- energy & power
  - energy signal: $0 < E < \infty$
  - power signal: $0 < \mathrm{P} < \infty$

Sigularity Function

	discrete	continuous
unit impulse	$\delta[n]=\begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$	$\delta(t)=0 \quad t\neq 0$ $\int_{-\infty}^{\infty}\delta(t)dt=1$
unit step	$u[n]=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$	$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$
Relationship	$\delta[n]=u[n]-u[n-1]$ $u[n]=\sum_{k=0}^{\infty}\delta[n-k]$	$\delta(t)=\frac{du(t)}{dt}$ $u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau$

Sigularity Function

- sampling/shifting property of unit impulse

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

$$\sum_{n=-\infty}^{\infty} x[n]\delta[n-n_0] = x[n_0]$$

- other properties

$$\delta(at+b) = \frac{1}{|a|}\delta(t+\frac{b}{a}) \quad a \neq 0$$

$$u(\frac{t-t_0}{w}) = u(t-t_0) \quad w > 0$$

Classification of Systems

- linearity
- stability
- invertibility
- causality
- with/without memory
- time invariance

- While proving a proprety, strictly follow definition
- While denying a property, give a counterexample

Exercises

- Ex1:

$$y(t) = x(t-2) + \int_{t-1}^{t-3} e^{-(t-\tau)^2} x(\tau) d\tau$$

Exercises

- Ex1: $y(t) = x(t-2) + \int_{t-1}^{t-3} e^{-(t-\tau)^2} x(\tau) d\tau$

- linear
- stable
- with memory
- causal
- time invariant