Introduction to Signals and Systems: V216

Lecture #1

Chapter 1: Signals and Systems

Signals

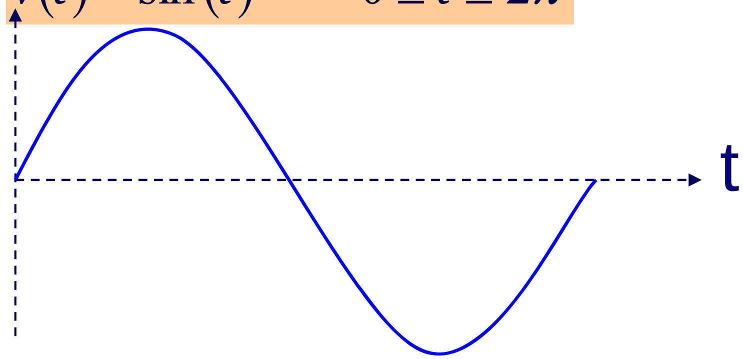
- Signals are function of independent variables that conveys information on the nature of a physical phenomenon.

- Signal may exist in many forms like acoustic, image, video, electrical, heat & light signal

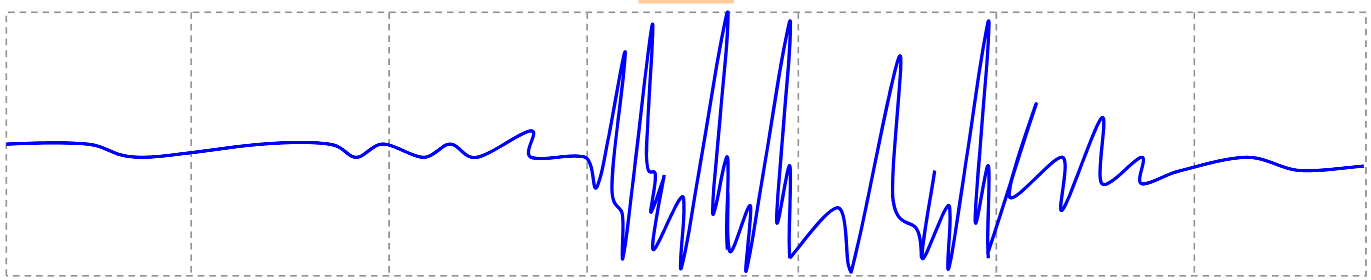
Signal

- A signal can be represented as a function of one or more independent variables
- Examples

$v(t) = \sin(t) \qquad 0 \le t \le 2\pi$

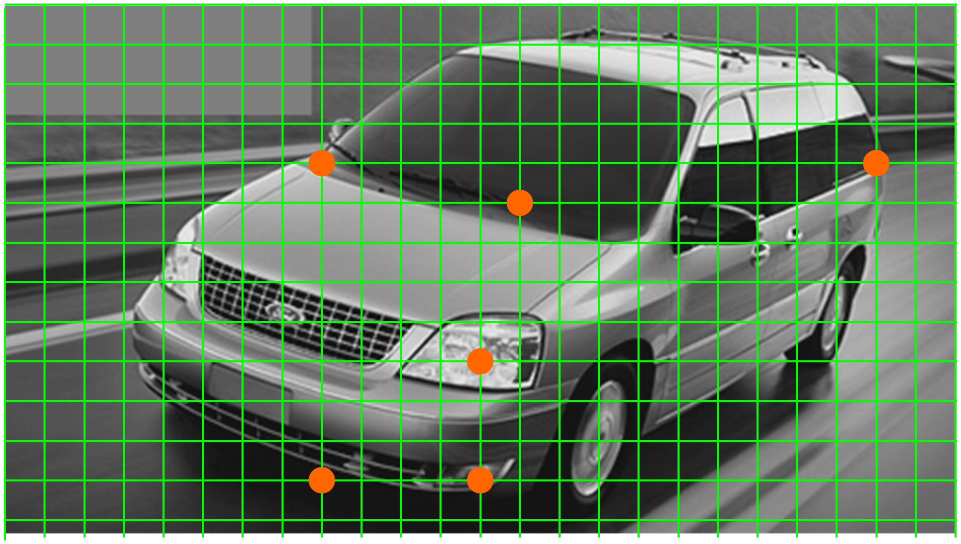


$s(t)$



Signal

- The image is a function of two spatial variables

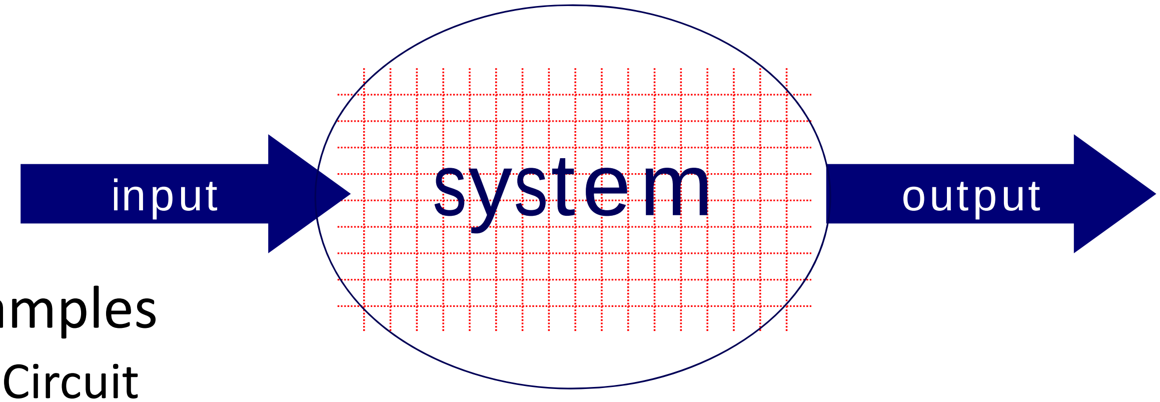


$$s(x, y)$$

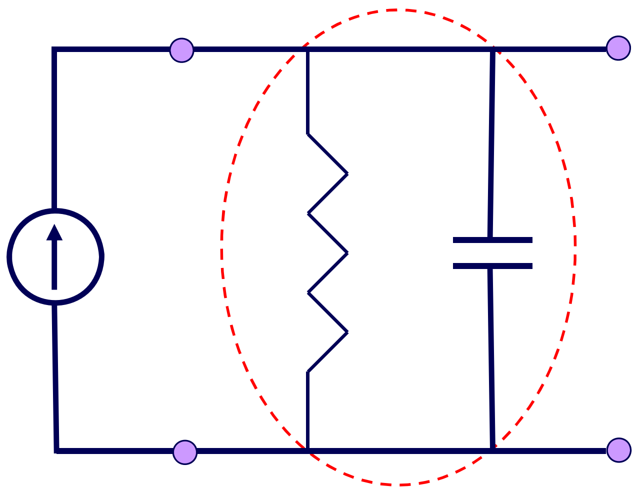
System

System is an entity response to input signals by producing other signals.

Example:- voltage & current are signals, Circuits are systems.



- Examples
  - Circuit



System

- Block Diagram representation of a system
  - Visual representation of a system

  - Shows inter-relations of many signals involved in the implementation of a complex system

Why Signals & Systems?

- Designing of systems to process signals in particular ways.
- Restoration of degraded or corrupted signals. E.g. Speech communication with background noise as in aircraft cockpit or car. Aim to retain the pilot voice and get rid of the engine noise.
- Restoration /enhancement of old recording/image.

Why Signals & Systems?

- Designing of signals with particular properties.
- In Communication - design signal to meet the constraints and requirement for successful transmission.
- To overcome distortion due to atmospheric effect and interference from other signals from other stations.

Classification of Signals

•Five methods of classifying signals, based on different features, are common:

–Continuous-time and discrete-time signals
–Even and odd signals
–Periodic and nonperiodic / aperiodic signals
–Deterministic and random signals
–Energy and power signals

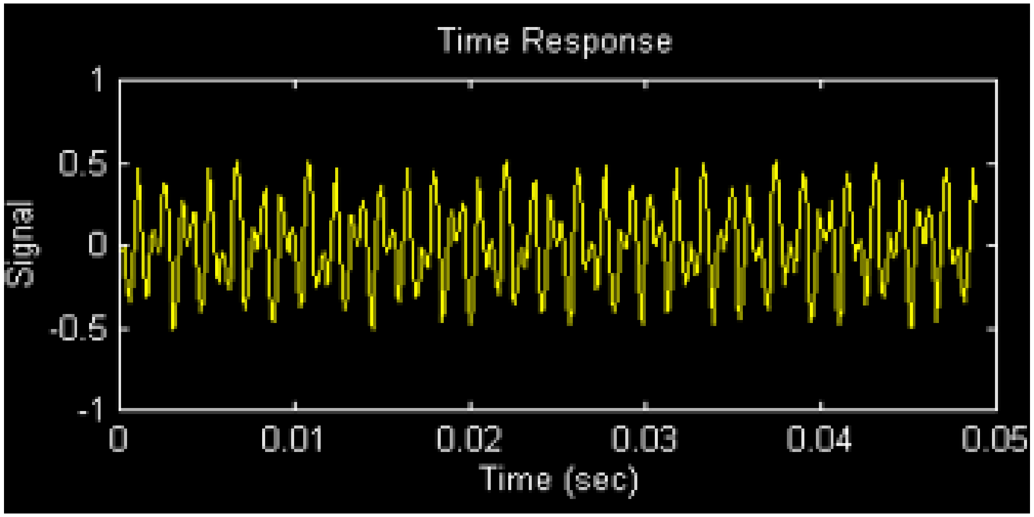
Continuous or Discrete

- Two types of signals are present naturally.
- Signals varying continuously with time or some other variable e.g. space or distance.
- Signals that exist only at discrete point of time e.g. daily closing stock market average or index, which are usually uniformly spaced.

Continuous and discrete signals

- Examples of continuous signals
  - Speech, video, image

  - The variation of atmospheric pressure, wind speed and temperature with altitude



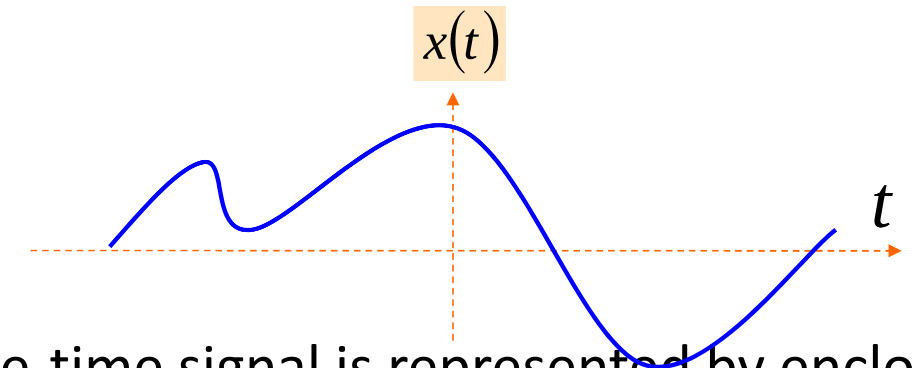
Continuous and discrete signals

- Examples of discrete signal

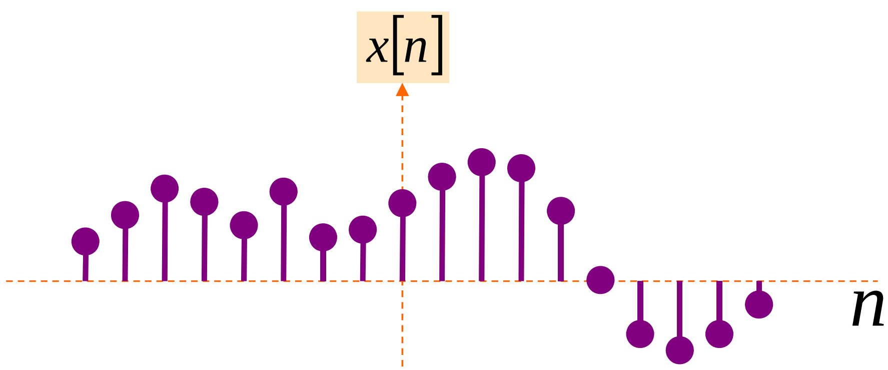
Figure 1.6 An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

Notation

- A continuous-time signal is represented by enclosing the independent variable (time) in parentheses ()



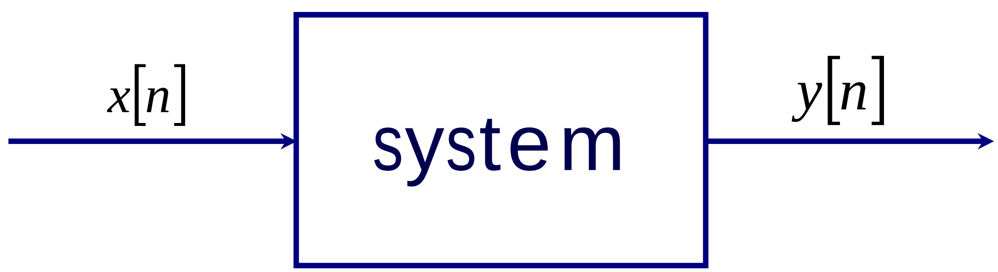
- A discrete-time signal is represented by enclosing the independent variable (index) in square brackets []



Continuous and discrete time system

- Like signals we have continuous and discrete-time systems as well

$$x(t) \mapsto y(t)$$



$$x[n] \mapsto y[n]$$

Continuous and discrete time system

- Examples of continuous and discrete-time systems

Squaring System

$$x(t) \mapsto [x(t)]^2$$

$$y(t) = [x(t)]^2$$

Differentiator System

$$y(t) = \frac{d}{dt} x(t)$$

Accumulator System

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

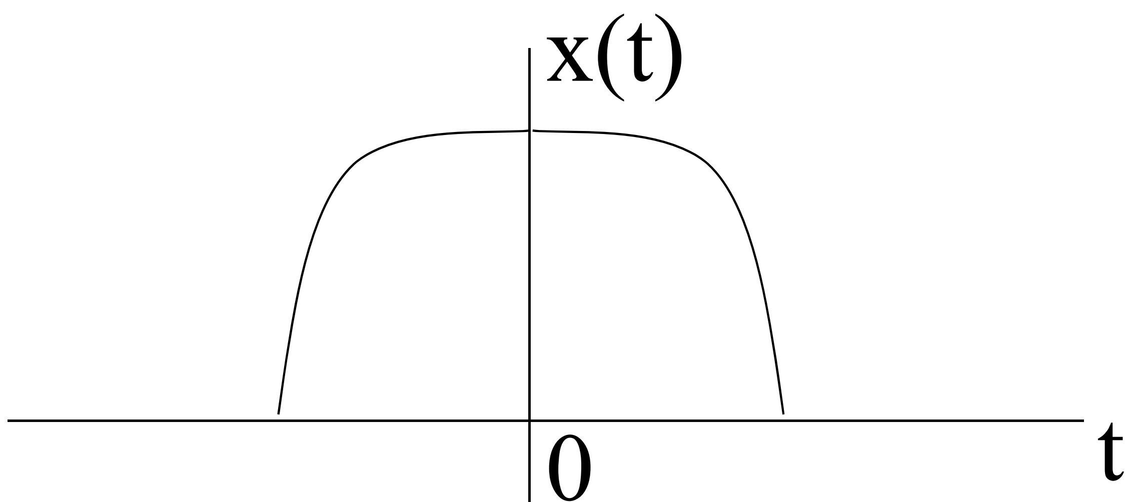
Even and odd Signals

Symmetry properties of signal:-

A signal $x(t)$, $x[n]$ is referred to as being **even signal** if it is identical to its time-reversed counter part i.e. with its reflection about the origin, $t=0$ or $n=0$.

$$x(t)=x(-t), \text{ or } x[n]=x[-n].$$

Graphical Representation of <u>Even Signals</u>.



Symmetry about t=0

Even and odd Signals

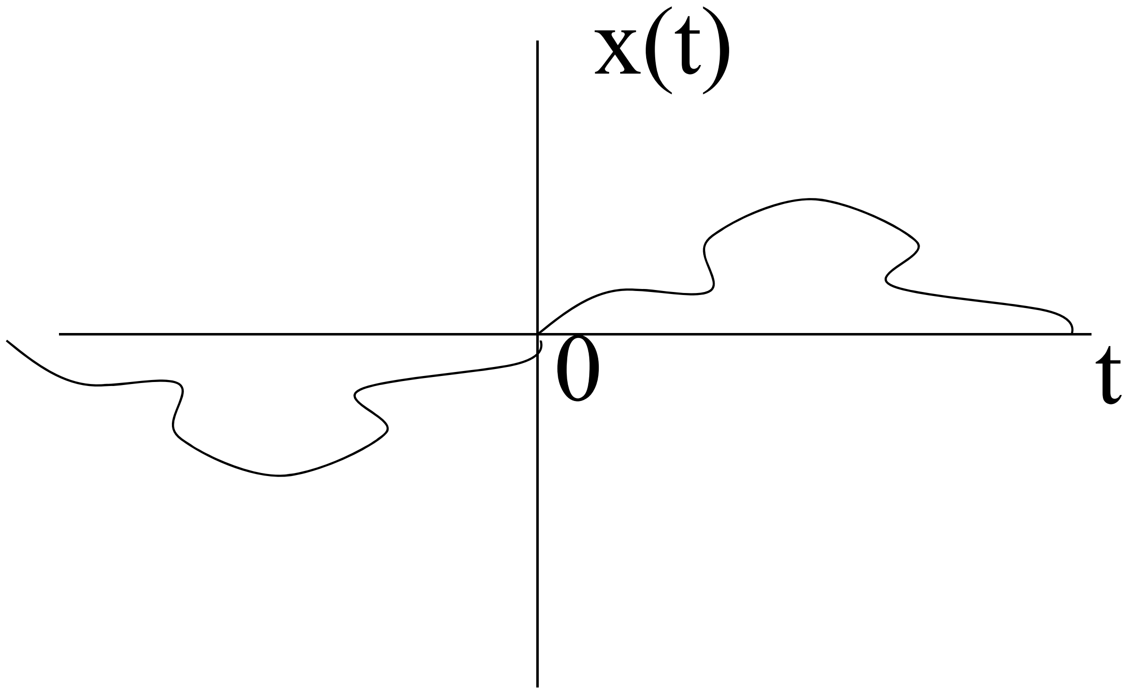
A signal is referred as being **odd** if it is asymmetry about the origin, t=0 or n=0.

$$x(t)=-x(-t), \text{ or } x[n]=-x[-n].$$

From this, all odd signals must necessarily be 0 at t=0 or n=0.

Even and odd Signals

Graphical Representation of Odd Signals.



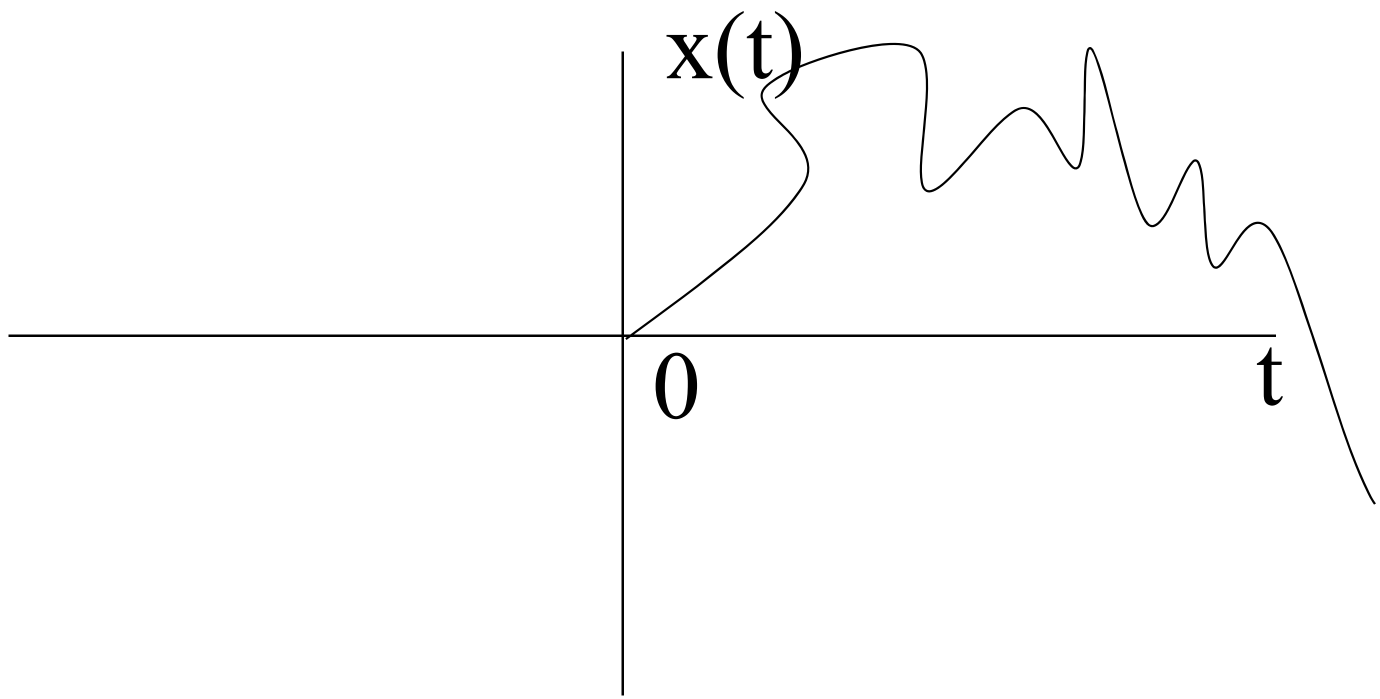
Asymmetry about t=0

Arbitrary Signal

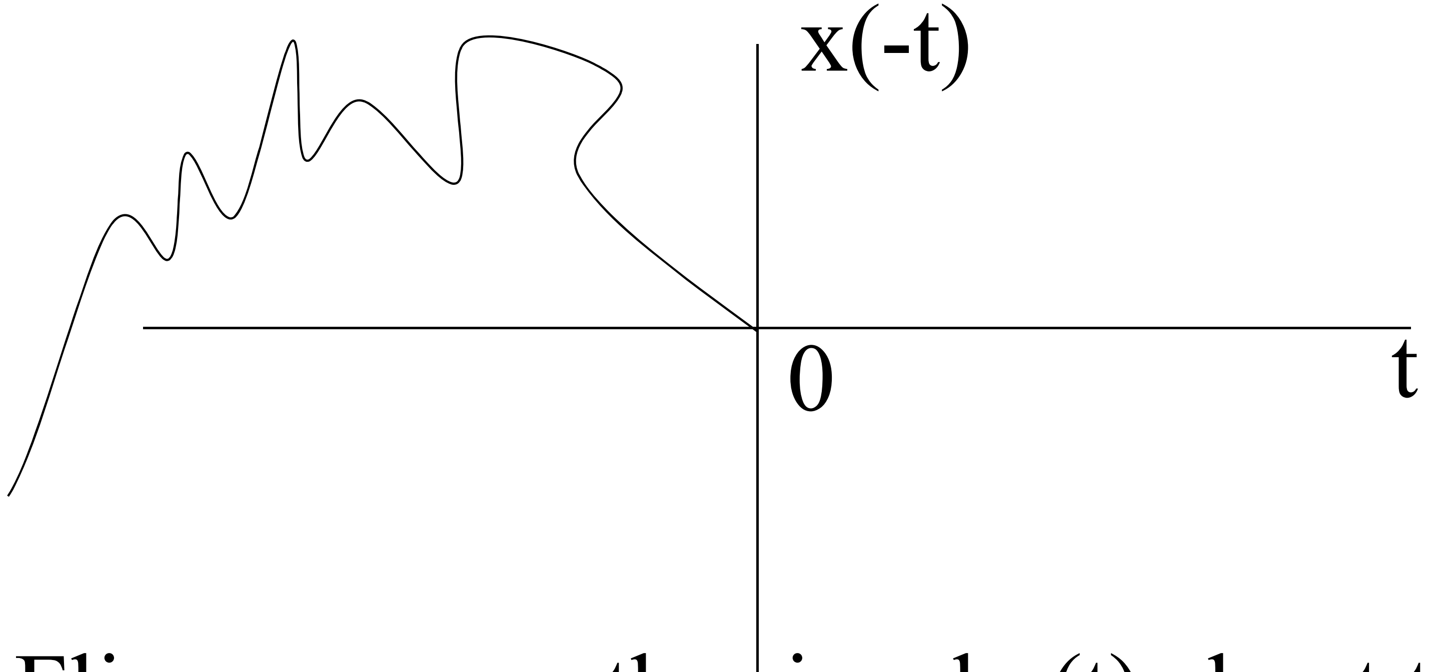
Any arbitrary signal can be broken up to an even part and and odd part.

$$x(t) = \frac{1}{2}\underbrace{[x(t) + x(-t)]}_{\text{even}} + \frac{1}{2}\underbrace{[x(t) - x(-t)]}_{\text{odd}}$$

Arbitrary signal.

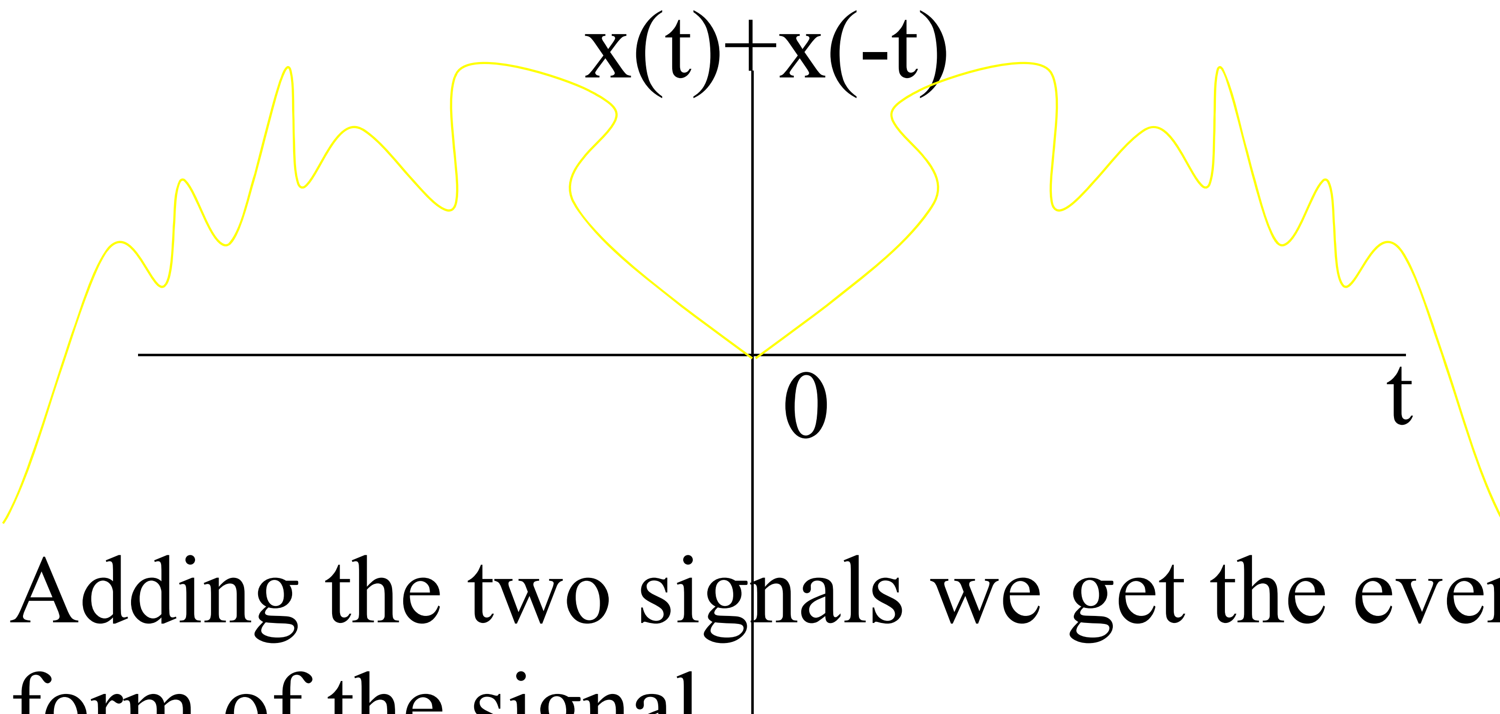


Flip the Arbitrary signal.



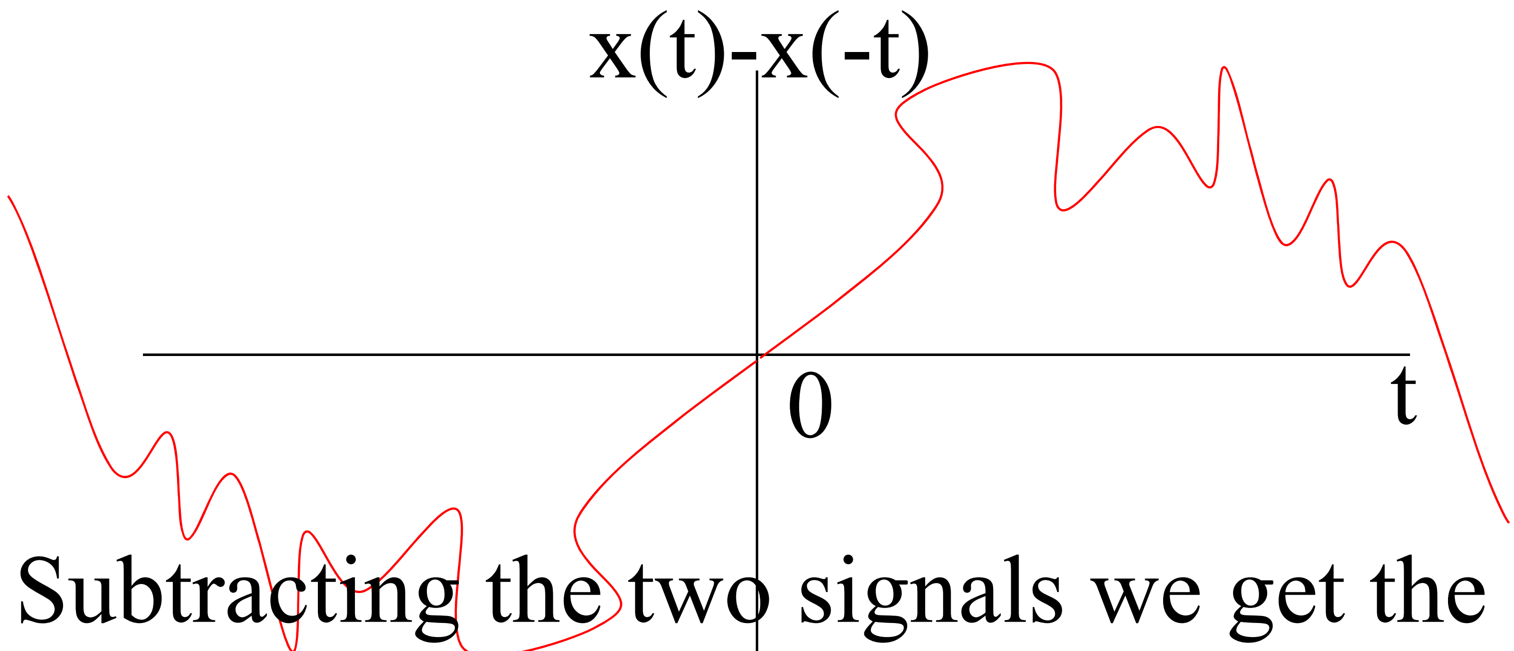
Flip or reverse the signal x(t) about t=0

Add the two previous signals:- x(t)+x(-t).



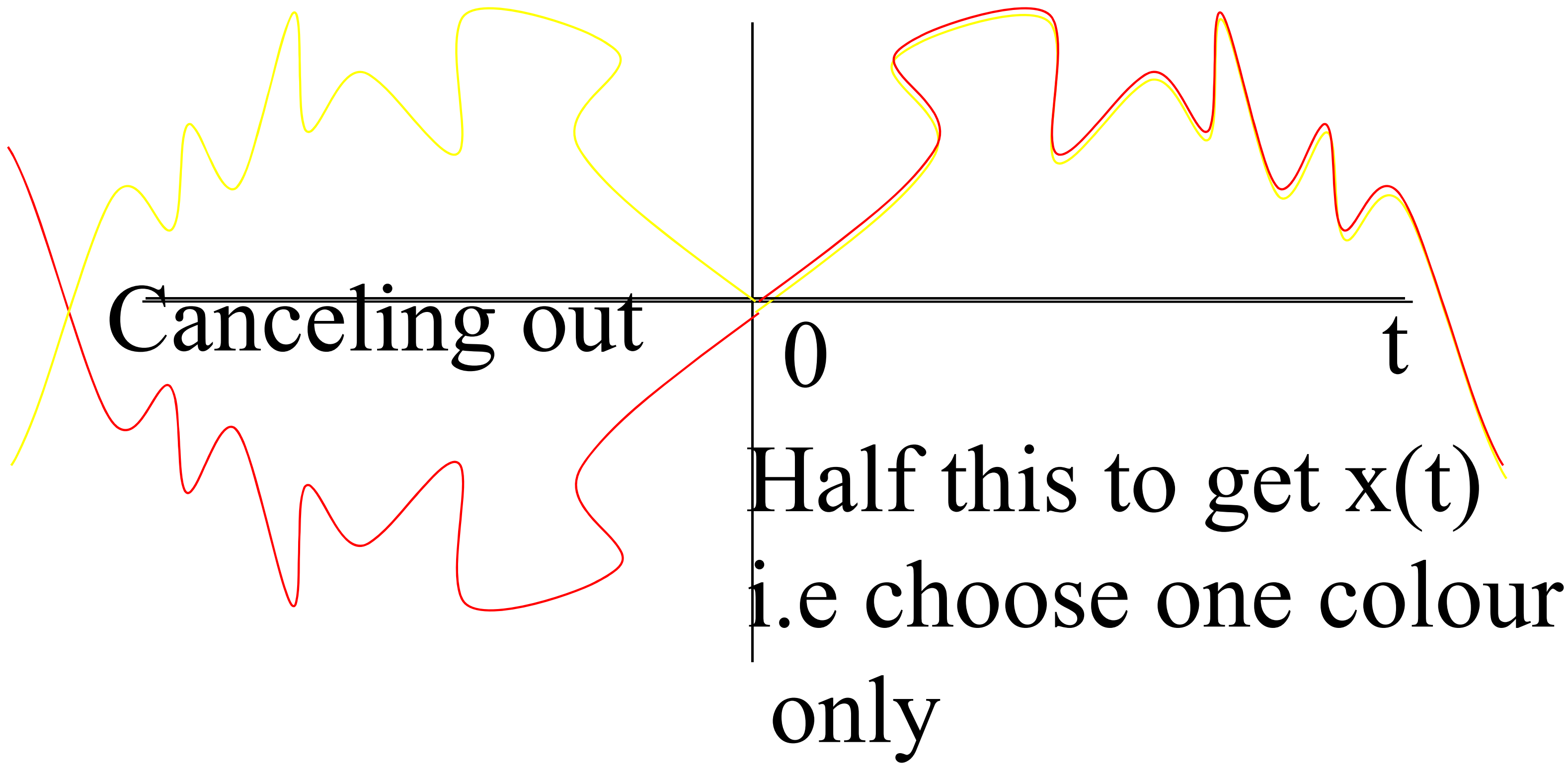
Adding the two signals we get the even form of the signal.

Subtracting the two signals:- x(t)-x(-t).

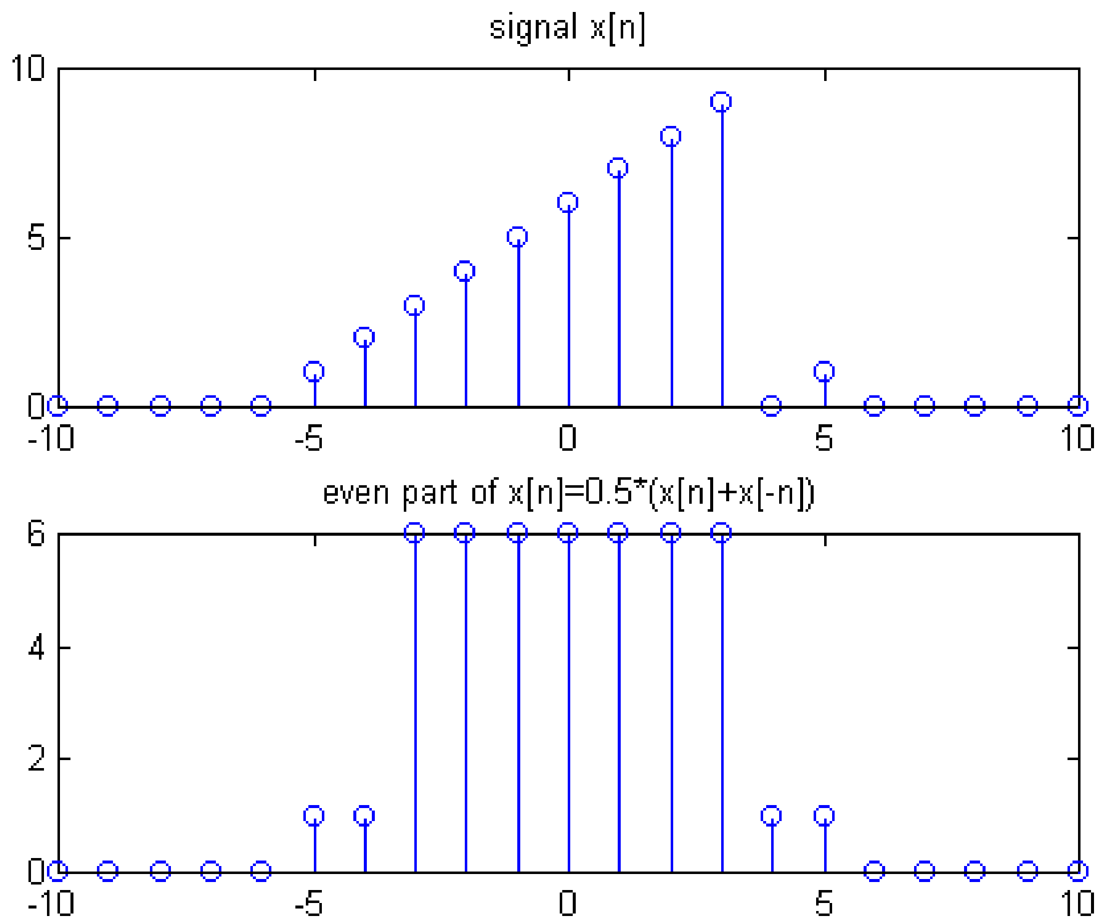


Subtracting the two signals we get the odd form of the signal.

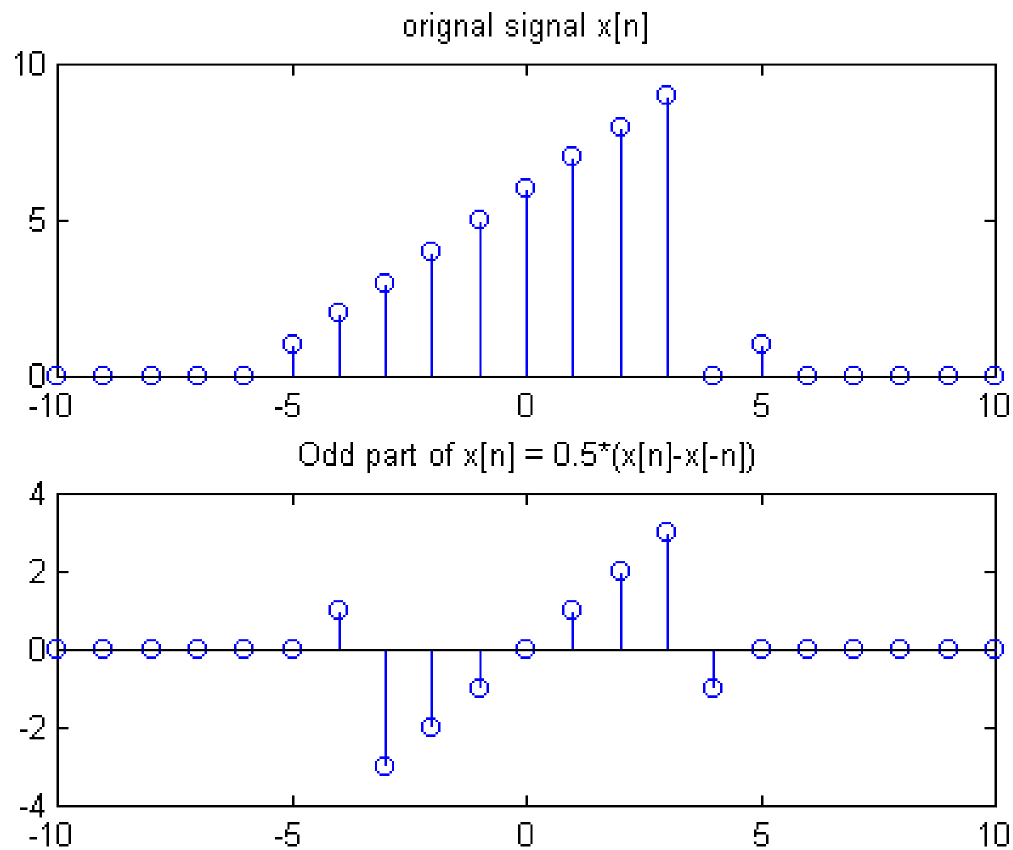
$x(t)+x(-t)+x(t)-x(-t)=2x(t).$



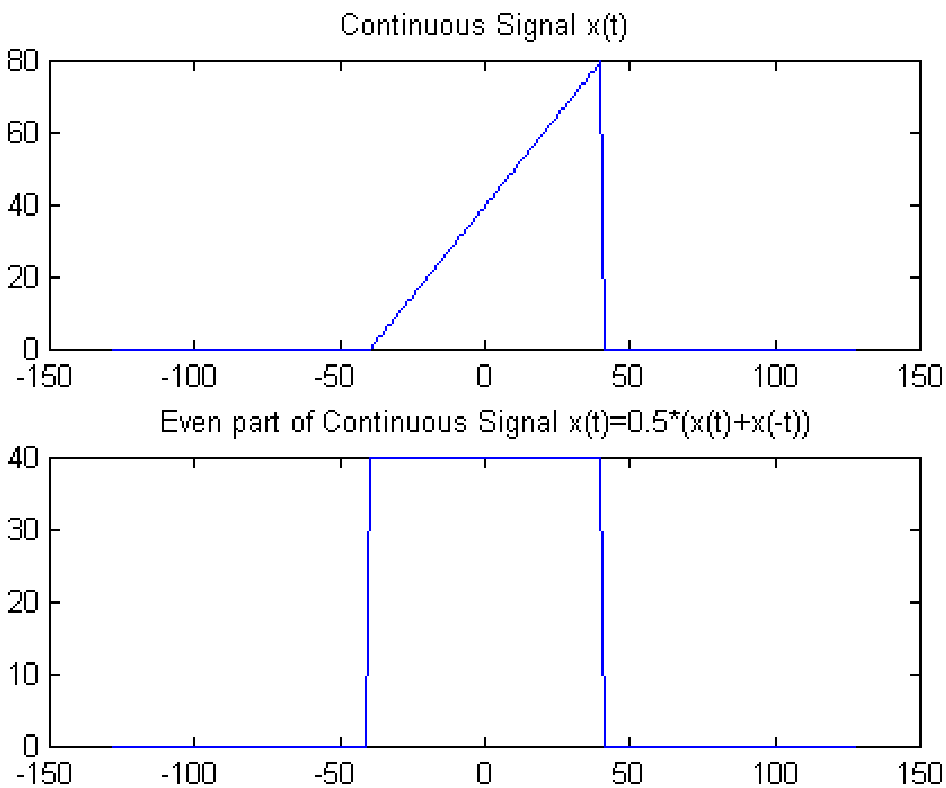
Even Part of Signal X[n]



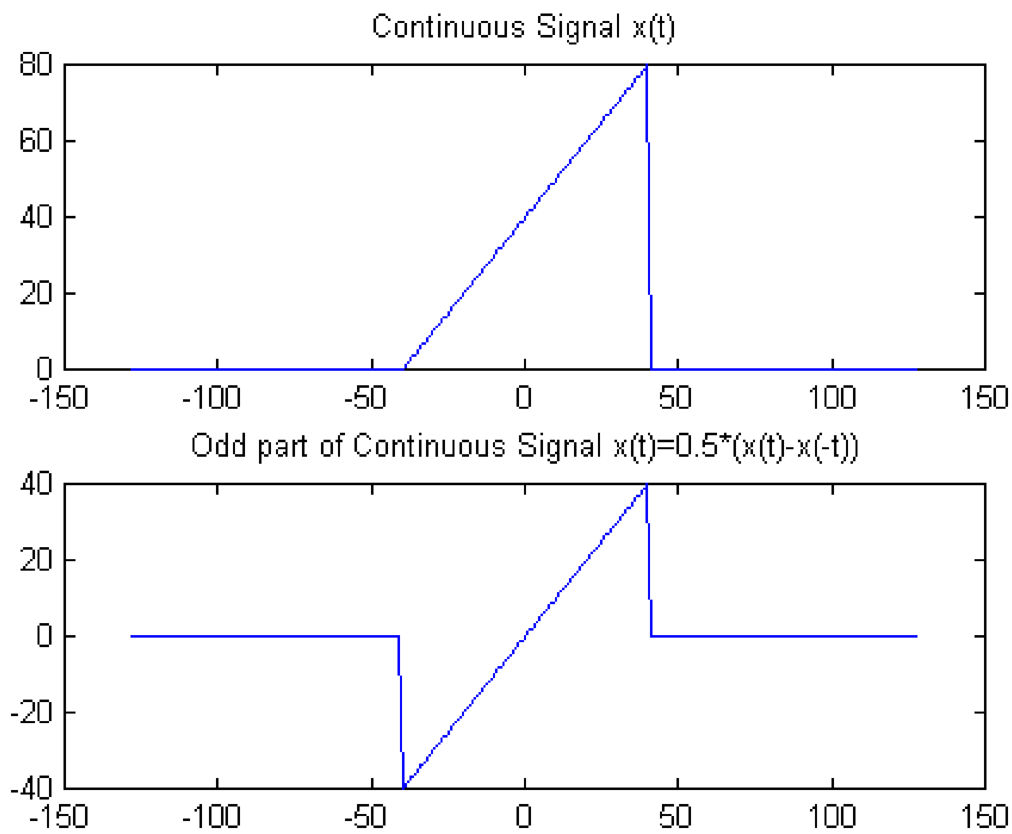
Odd Part of X[n]



Even Part of x(t)=0.5*(x(t)+x(-t))



Continuous signal and its odd part

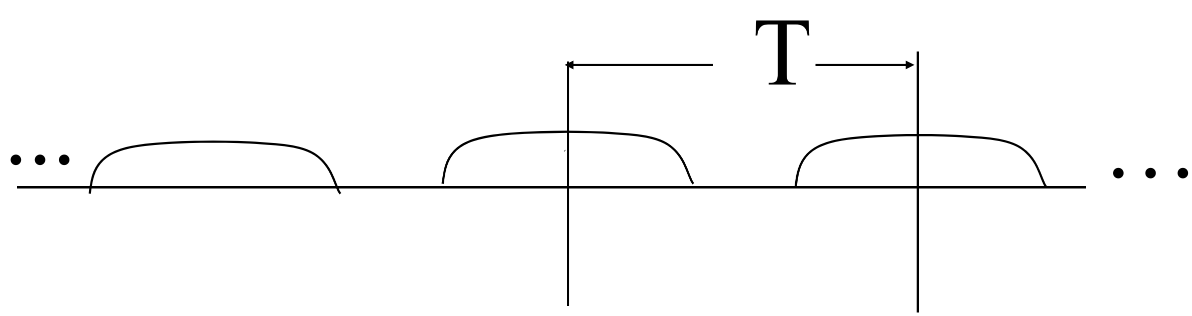


Periodic Signals

Condition for having a signal repeating itself after a certain duration called period (T):-

$x(t) = x(t+T)$ for all values of t.

x(t) is periodic with period T.



Periodic continuous-time signal

If $x(t)$ is periodic with period T:-

then $x(t)=x(t+mT)$ for all t

and for any integer m.

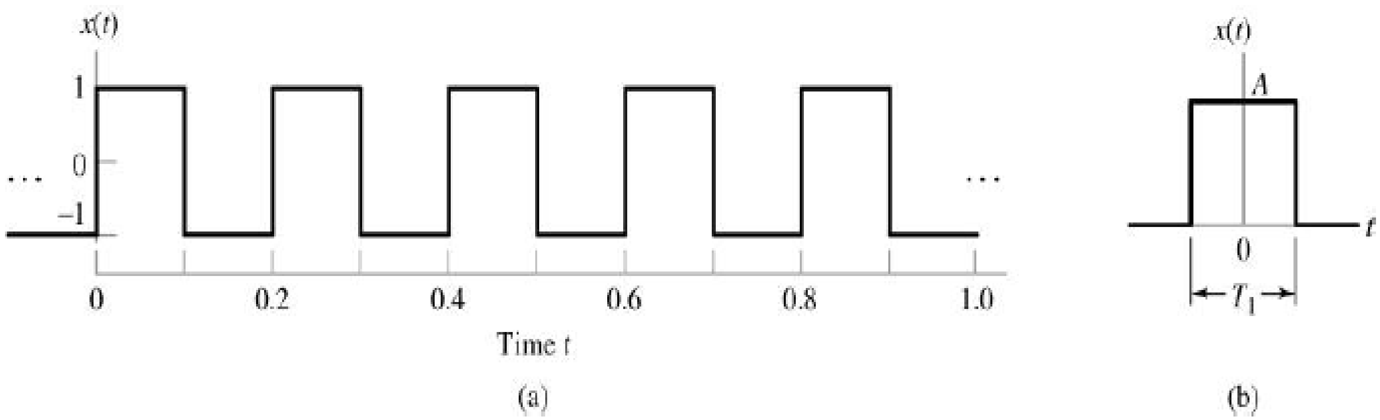
Thus, $x(t)$ is also periodic with period:-

2T,3T,4T,….

We define To the fundamental period of $x(t)$, is the smallest positive value of T_o such that $x(t)=x(t+T)$.

Periodic continuous-time signal

(a) Square wave with amplitude $A = 1$ and period $T = 0.2$s.
(b) Rectangular pulse of amplitude A and duration T_1.



Periodic discrete-time signal

A discrete-time signal x[n] is periodic with period N, where N is a positive integer,if it is unchanged by a time shift of N :-

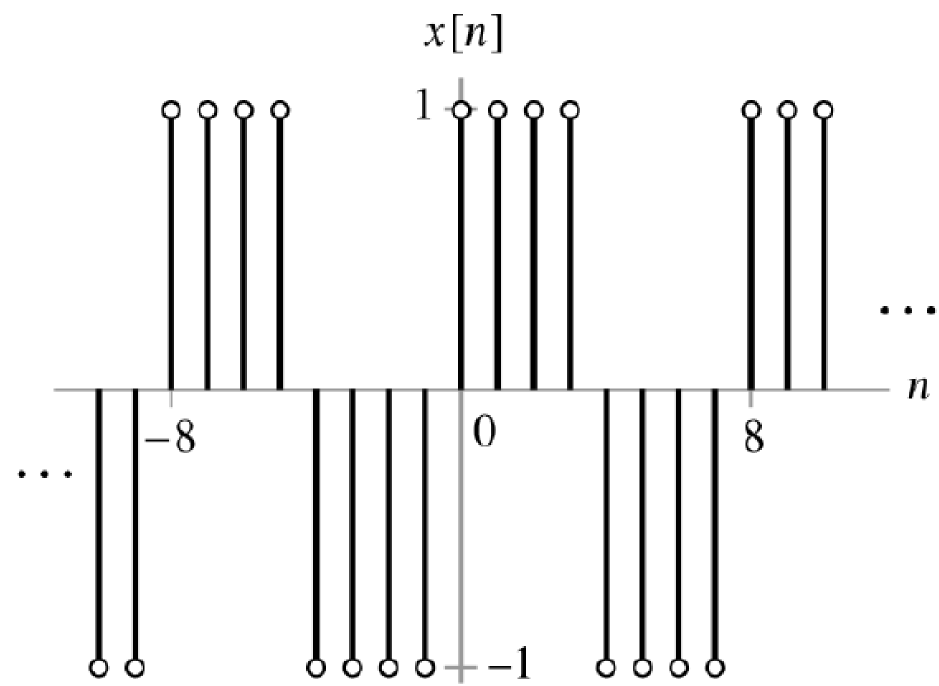
$$x[n]=x[n+N] \quad \text{for all values of n.}$$

x[n] is also periodic with period 2N,3N,4N….

The fundamental period N_o is the smallest positive value of N for $x[n]=x[n+N]$.

Periodic discrete-time signal

Discrete-time square wave alternative between –1 and +1.



Periodic Complex Exponential Signal

$$e^{j\omega_o t} = e^{j\omega_o (t+T)}$$

$$\text{since} \quad e^{j\omega_o (t+T)} = e^{j\omega_o t} e^{j\omega_o T}$$

and

$$e^{j\omega_o T} = 1 \text{ if for } \omega_\mathrm{o} \neq 0, \quad \mathrm{T} = T_o = \frac{2\pi}{|\omega_o|}$$

where for periodicity $\mathrm{T_0}$ (Fundamental Period) is the smallest positive value of T

Sinusoidal Signal

$$x(t) = e^{j\omega_0 t} = Cos\omega_0 t + j \sin \omega_0 t$$

A signal that is closely related to the

real part of the exponential signal is the sinusoidal signal

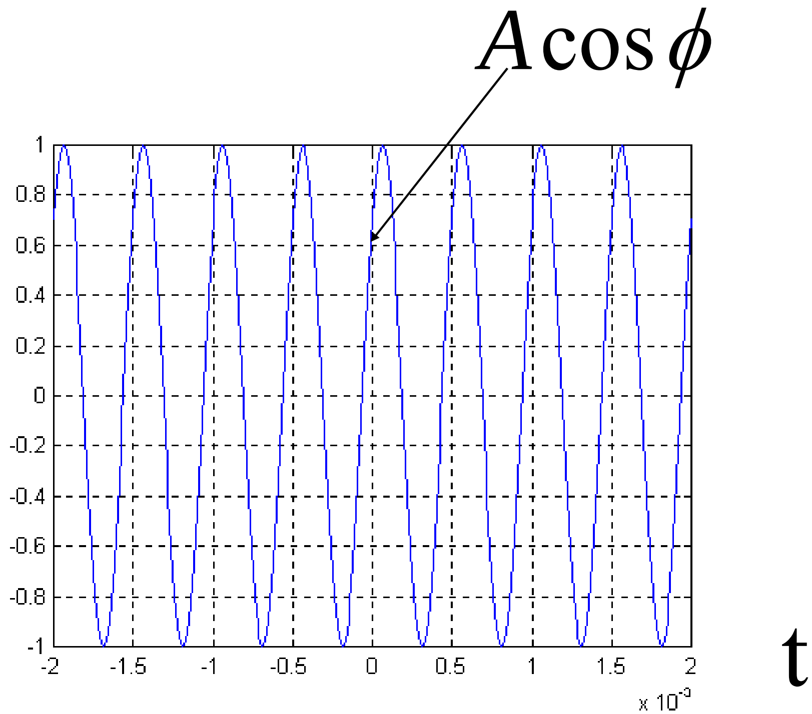
$x(t)=A\cos(\omega_o t+\varphi)$

where t is in seconds, φ in radians,

ω_o in radians per second.

$$x(t) = A\cos(\omega_o t + \phi)$$

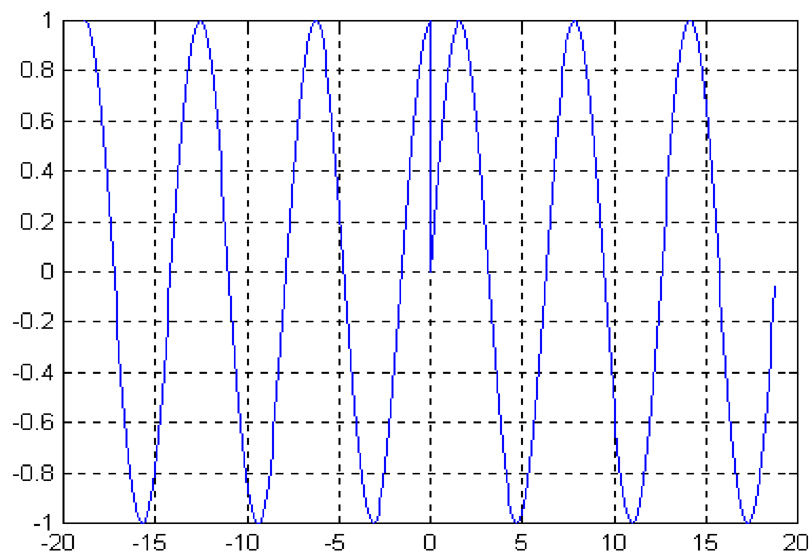
$$\omega_o = 2\pi f_o$$



$$T_o = \frac{2\pi}{\omega_o} = 0.5mS$$

X(t) = x(t+0.005) for all t.

Example 1.4. Is this signal periodic?



X(t)=cos(t) if t<0 , x(t)=sin(t) t>=0

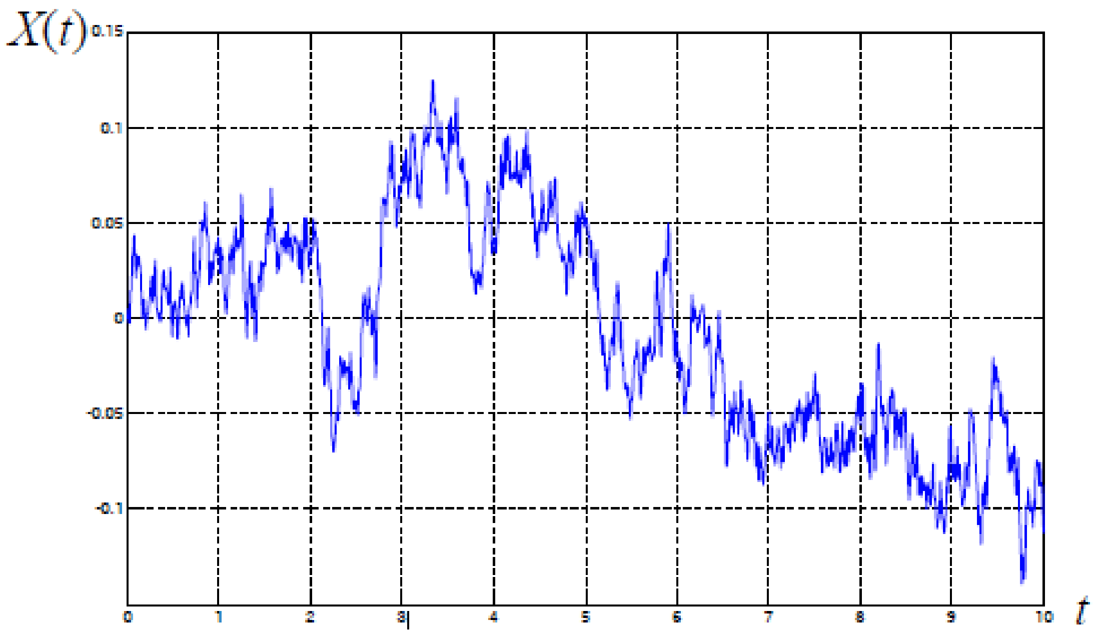
X(t) not equal x(t+T) for all t.

Deterministic and Random Signals

- A deterministic signal is a signal about which there is no uncertainty with respect to its value at any time. Deterministic signals may be modeled as completely specified functions of time.
- A random signal is a signal about which there is uncertainty before it occurs. A random signal may be viewed as belonging to an ensemble, or a group, of signals, with each signal in the ensemble has a certain probability of occurrence.

Deterministic and Random Signals

Example: No deterministic signal.



Signal Energy & Power

Instantaneous Power Dissipated By Resistor R ohms.

$$\boxed{p(t) = i(t) * v(t) = \frac{1}{R} * v^2(t).}$$

Total energy expanded over time interval between t1 and t2 is :-

$$\boxed{energy = \int_{t1}^{t2} p(t)dt.}$$

Signal Energy & Power

$$total \quad energy \quad = \quad \int_{t1}^{t2} p(t)dt.$$

$$average \quad power \quad \frac{1}{t2 - t1} \int_{t1}^{t2} p(t)dt$$

Signal Energy & Power

$$For\ \ Continuous\ \ complex\ \ signal\ \ x(t):-$$

$$Total\ \ energy\ = \int_{t1}^{t2} |\, x(t)\,|^2\ dt$$

$$For\ \ discrete\ \ complex\ \ signal\ \ x[n]:-$$

$$Total\ \ energy\ = \sum_{n=n1}^{n2} |\, x[n]\,|^2$$

Signal Energy & Power

- If the signal is periodic, its average power is defined by

$$P = \frac{1}{T}\int_0^T |x(t)|^2\, dt \text{ or } P = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2$$

- The square root of the average power is called the root mean square (rms) value of the periodic signal.

Signal Energy & Power

Let us examine energy over the time interval or number of sample that is infinite:-

$$-\infty < t < +\infty \quad or \; -\infty < n < +\infty$$

$$Total\ energy = \mathrm{E}_{\infty} \Rightarrow \lim_{T \to \infty} \int_{-T}^{T} |\, x(t)\,|^2 \; dt = \int_{-\infty}^{\infty} |\, x(t)\,|^2 \; dt.$$

$$for\ discrete\ signal :-$$

$$Total\ energy = \mathrm{E}_{\infty} \Rightarrow \lim_{N \to \infty} \sum_{n=-N}^{+N} |\, x[n]\,|^2 = \sum_{n=-\infty}^{+\infty} |\, x[n]\,|^2$$

Time-average power over infinite interval

$$\boxed{\begin{aligned} P_\infty &\Rightarrow \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |\,x(t)\,|^2\, dt \\ P_\infty &\Rightarrow \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{+N} |\,x[n]\,|^2 \end{aligned}}$$

Energy Signals and Power Signals

With this 3 classes of signals can be identified:-

1) Finite total energy, $0 < \mathrm{E}_\infty < \infty$

2) Finite average power, $0 < P_\infty < \infty, \mathrm{E}_\infty = \infty$

3) Neither power nor energy are finite, $P_\infty = \infty, \mathrm{E}_\infty = \infty$

Time-average power over infinite interval

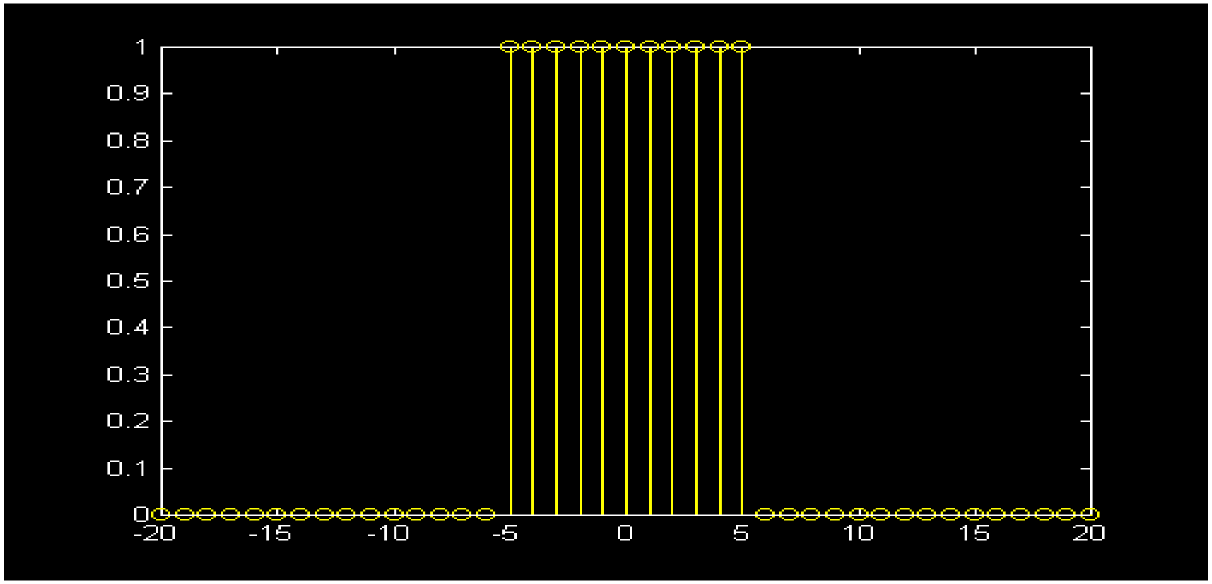
- A signal is referred to as an energy signal if and only if

$$0 < E_\infty < \infty$$

- A signal is referred to as an power signal if and only if

$$0 < P_\infty < \infty$$

Finite total energy signal.

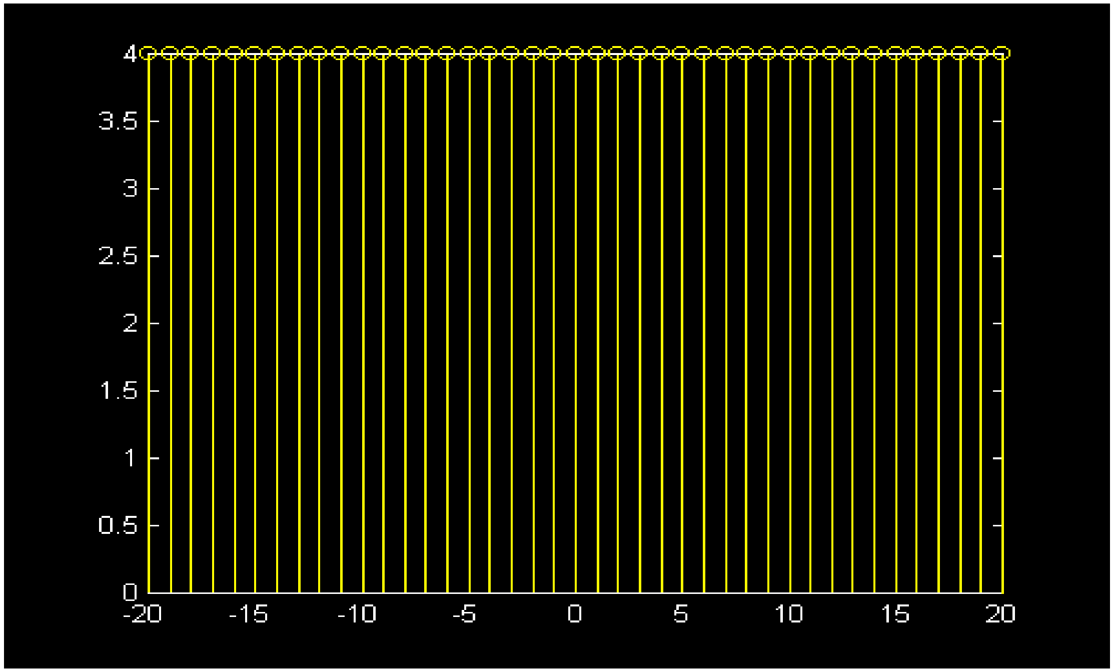


-5 < n <+5, x[n] = 1, otherwise x[n]=0.

ENERGY $= \sum_{n=n1}^{n2} |x[n]|^2 \quad =11.$

$$E_\infty < \infty, P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$$

Finite average power

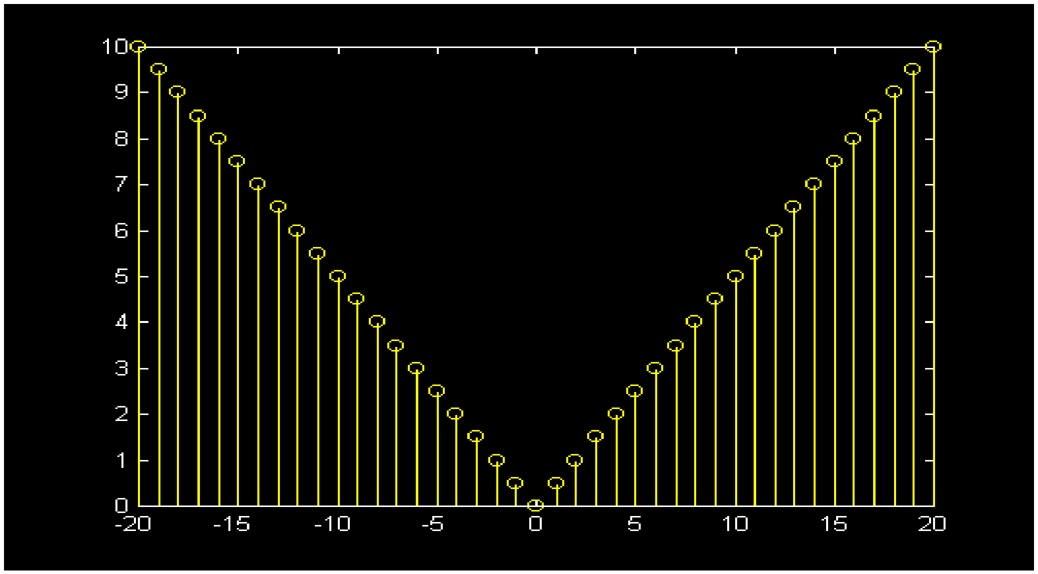


x[n] = 4, for all n.

ENERGY =infinite $\quad E_{\infty} = \infty$

$$P_{\infty} \Rightarrow \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 \quad = 16$$

Neither power nor energy are finite



X[n]=0.5n, for all n.

$$E_\infty = \infty$$

$$P_\infty \Rightarrow \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 \qquad P_\infty = \infty$$

Home Work

Chapter 1 problems: 1.6, 1.14, 1.24, 1.25, 1.26, 1.27, 1.28,1.31, 1.32, 1.46, 1.47

Book: Signals & Systems. by Alan V. Oppenheim, Alan S. Willsky with S. Hamid Nawab. Prentice-Hall, Second Edition, 1997.