Common

$$\operatorname{sinc}\theta = \frac{\sin \pi\theta}{\pi\theta}$$
$$\int e^{at}dt = \frac{1}{a}e^{at} + C$$
$$\int te^{at}dt = \frac{at-1}{t^2}e^{at} + C$$
$$\int t^n e^{at}dt = \frac{t^n}{a}e^{at} - \frac{n}{a}\int t^{n-1}e^{at}dt$$
$$\int e^{at}\sin bt dt = \frac{1}{a^2+b^2}e^{at}(a\sin bt - b\cos bt) + C$$
$$\int e^{at}\cos bt dt = \frac{1}{a^2+b^2}e^{at}(b\sin bt + a\cos bt) + C$$

Chapter 3

Continuous

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T))t}$$
$$a_k = \frac{1}{T}\int_T x(t)e^{-j\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-j(2\pi/T)t}dt$$
$$a_0 = \frac{1}{T}\int_T x(t)dt$$

Convergent

$$\int_T |x(t)|dt < \infty$$

Discrete

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$
$$a_k = \frac{1}{N}\sum_{k=<N>} x[n]e^{-jk\omega_0 n} = \frac{1}{N}\sum_{k=<N>} x[n]e^{-jk(2\pi/N)n}$$

Chapter 4

Continuous

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

Periodic

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$
$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Differential Equations

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$
$$H(j\omega) = \frac{(j\omega)+2}{(j\omega)^2 + 4(j\omega) + 3} = \frac{1/2}{j\omega+1} + \frac{1/2}{j\omega+3}$$
$$h(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$$

Chapter 5

Discrete

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}e^{j\omega n})d\omega$$
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Periodic

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$$

Differential Equations

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$
$$H(e^{j\omega}) = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}} = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$
$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

Chapter 6

Group Delay

$$\tau(\omega) = -\frac{d}{d\omega}\{\sphericalangle H_2 j\omega\}$$

Step response

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$
$$s[n] = \sum_{m=-\infty}^{n} h[m]$$

First Order Systems

$$\tau\frac{dy(t)}{dt} + y(t) = x(t)$$
$$H(j\omega) = \frac{1}{j\omega\tau + 1}$$
$$h(t) = \frac{1}{\tau}e^{-t/\tau}u(t)$$
$$s(t) = h(t) * u(t) = [1 - e^{-t/\tau}]u(t)$$

Second Order Systems

$$\frac{d^2y(t)}{dt^2} + 2\xi\omega_n^2\frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 x(t)$$

For $\xi \neq 1$,

$$H(j\omega) == \frac{M}{j\omega - c_1} - \frac{M}{j\omega - c_2}$$
$$c_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}, M = \frac{\omega_n}{2\sqrt{\xi^2-1}}$$
$$h(t) = M[e^{c_1 t} - e^{c_2 t}]u(t)$$
$$s(t) = h(t) * u(t) = \left\{1 + M\left[\frac{e^{c_1 t}}{c_1} - \frac{e^{c_2 t}}{c_2}\right]\right\}u(t)$$

For $\xi = 1$,

$$H(j\omega) = \frac{\omega_n^2}{(j\omega + \omega_n)^2}$$
$$h(t) = \omega_n^2 te^{-\omega_n t}u(t)$$
$$s(t) = [1 - e^{-\omega_n t} - \omega_n te^{-\omega_n t}]u(t)$$

For $0 < \xi < 1$,

$$H(j\omega) = \frac{1}{(j\omega/\omega_n)^2 + 2\xi(j\omega/\omega_n) + 1}$$
$$h(t) = \frac{\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[\sin(\omega_n\sqrt{1-\xi^2})t]u(t)$$

$0 < \xi < 1$	under damped
$\xi = 1$	critical damped
$\xi > 1$	over damped

$$\omega_{max} = \omega_n\sqrt{1-2\xi^2}$$
$$|H(j\omega_{max}) = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

7 Sampling

7.1 The Sampling Theorem

7.1.1 Impulse-Train Sampling

$$x_p(t) = x(t)p(t)$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

$$X_p(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)P(j(\omega-\theta))d\theta$$

$$P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty} \delta(\omega - k\omega_s)$$

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

Sampling Theorem:

Let $x(t)$ be a band-limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT)$, $n = 0, \pm 1, \pm 2, \ldots$, if

$$\omega_s > 2\omega_M$$

where

$$\omega_s = \frac{2\pi}{T}$$

Then we can reconstruct $x(t)$ with an ideal lowpass filter with gain T and cutoff frequency $\omega_M < \omega_c < \omega_s - \omega_M$.

7.1.2 Sampling with a Zero-Order Hold

$$H_0(j\omega) = e^{-j\omega T/2}\left[\frac{2\sin(\omega T/2)}{\omega}\right]$$

$$H_r(j\omega) = \frac{e^{j\omega T/2}H(j\omega)}{\frac{2\sin(\omega T/2)}{\omega}}$$

7.2 Interpolation

$$x_r(t) = x_p(t) * h(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t - nT)$$

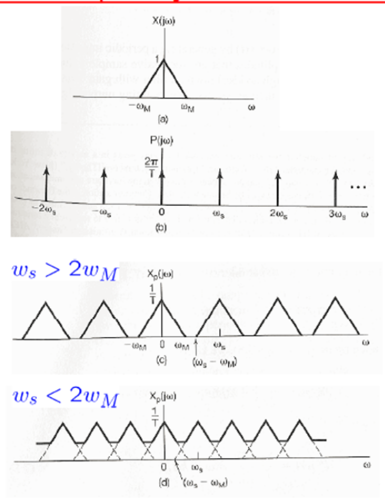
$$h(t) = \frac{\omega_c T\sin(\omega_c t)}{\pi\omega_c t}$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h\frac{\omega_c T\sin(\omega_c(t-nT))}{\pi\omega_c(t-nT)}$$

$$H(j\omega) = \frac{1}{T}\left[\frac{\sin(\omega T/2)}{\omega/2}\right]^2$$

7.3 Aliasing

Overlapping in Frequency-Domain: Aliasing



8 Communication Systems

8.1 Amplitude Modulation

$$y(t) = x(t)c(t)$$

8.1.1 Complex Exponential Carrier

$$c(t) = e^{j\omega_c t + \theta_c}$$

Choose $\theta_c = 0$,

$$y(t) = x(t) = e^{j\omega_c t}$$

$$Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)C(j(\omega-\theta))d\theta$$

$$C(j\omega) = 2\pi\delta(\omega - \omega_c)$$

$$Y(j\omega) = X(j\omega - j\omega_c)$$

$$x(t) = y(t)e^{-j\omega_c t}$$

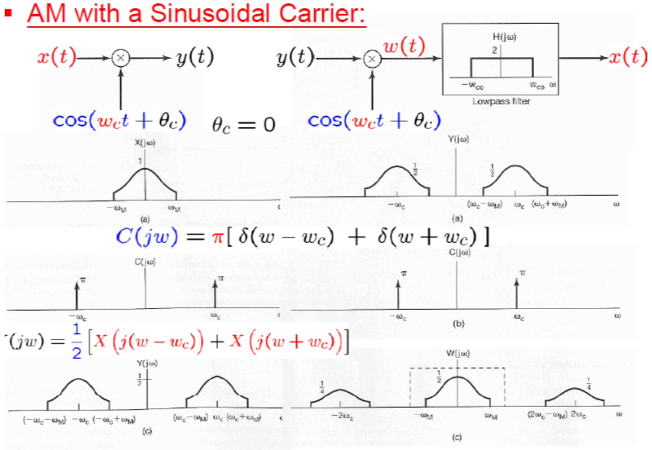
8.1.2 Sinusoidal Carrier

$$c(t) = \cos\omega_c t$$

$$C(j\omega) = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$

$$Y(j\omega) = \frac{1}{2}[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)]$$

- AM with a Sinusoidal Carrier:



8.2 Demodulation for Sinusoidal AM

8.2.1 Synchronous Demodulation

$$y(t) = x(t)\cos\omega_c t$$

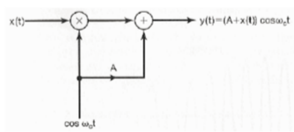
$$w(t) = y(t)\cos\omega_c t$$

$$w(t) = x(t)\cos^2\omega_c t = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t$$

8.2.2 Asynchronous Demodulation

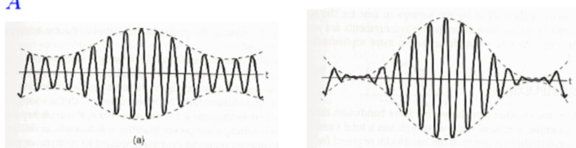
- Asynchronous Demodulation:
  - $w_c >> w_M$
  - $x(t) > 0, \ \ \forall t$

  If not, $x(t) \to x(t) + A \ > \ 0$



$$A \geq K, \ \ |x(t)| \leq K$$

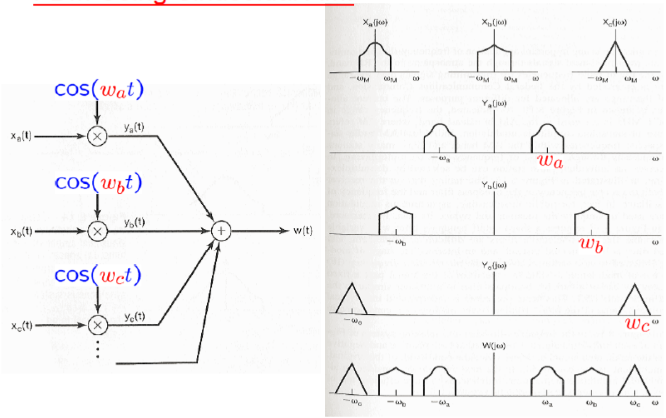
- $\frac{K}{A}$: modulation index m, in %



$m = 0.5$ $\quad$ $m = 1.0$

8.3 Frequency-Division Multiplexing

- FDM Using Sinusoidal AM:



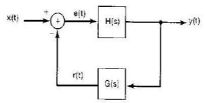
11 Linear Feedback System

Linear Feedback Systems

- From the diagram, we obtain the relation

$$Y(s) = H(s)E(s)$$
$$E(s) = X(s) - R(s)$$
$$R(s) = G(s)Y(s)$$

Continuous-Time LTI Feedback System



- These give the closed-loop system function is

$$Y(s) = H(s)E(s)$$
$$= H(s)[X(s) - R(s)]$$
$$= H(s)[X(s) - G(s)Y(s)]$$
$$\frac{Y(s)}{X(s)} = Q(s) = \frac{H(s)}{1 + G(s)H(s)}$$

Root-locus analysis of linear feedback systems

- The location of poles are the solutions of the following equations

Continuous-Time LTI Feedback System	Discrete-Time LTI Feedback System
$1 + KG(s)H(s) = 0$	$1 + KG(z)H(z) = 0$

- For a complex system, it is possible to sketch accurately the locus of the poles as the value of gain parameter K is varied from $-\infty$ to ∞, without actually solving for the location of poles for any specific value of the gain.

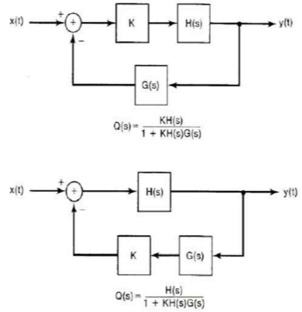
- Consider a modification of the basic feedback system where either $G(s)$ or $H(s)$ is cascaded with an adjustable gain K (real number).
- In either of these cases, the poles of the closed-loop system function is satisfied

$$1 + KG(s)H(s) = 0$$

- Rewrite the equation,

$$G(s)H(s) = \frac{-1}{K}$$

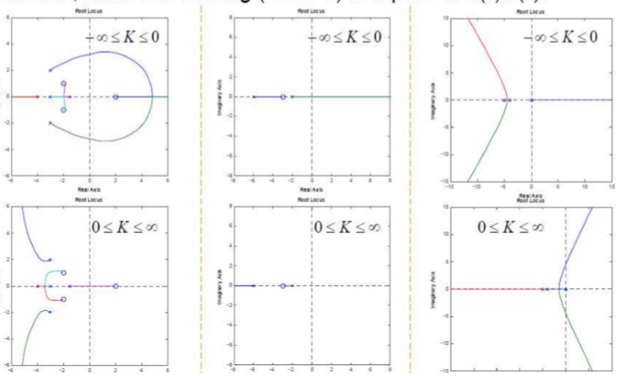
- The technique for plotting the root locus is based on the prosperities of this equation and its solutions.



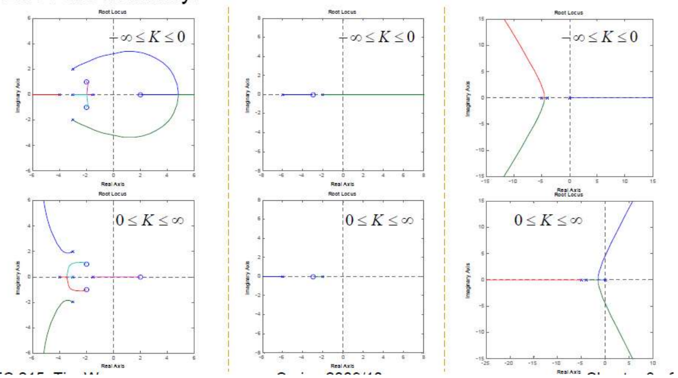
- Criteria 1: End points of the root locus
  - The closed-loop poles s_0 for K = 0, and $|K|$ = + ∞
  - For K = 0, $G(s_0)H(s_0) = \infty$, ➔ closed-loop poles s_0 = poles of $G(s)H(s)$
  - For $|K| = \infty$, $G(s_0)H(s_0) = 0$ ➔ closed-loop poles s_0 = zeroes of $G(s)H(s)$
  - If the order of the numerator of $G(s)H(s)$ is smaller than that of the denominator, then some of zeros, equal in number to the difference in order between the denominator and numerator, will be at infinity.

$$G(s)H(s) = \frac{-1}{K}$$

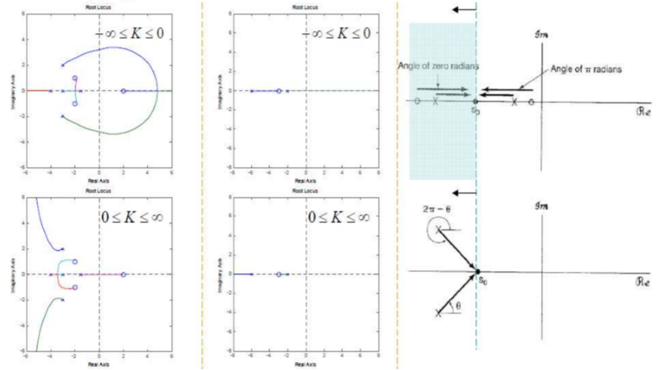
Property 1: For K = 0, the solution of $G(s)H(s)$ = 1/ K are the poles of $G(s)H(s)$. Since we are assuming n poles, the root locus has n branches, each one starting (for K=0) at a pole of $G(s)H(s)$.



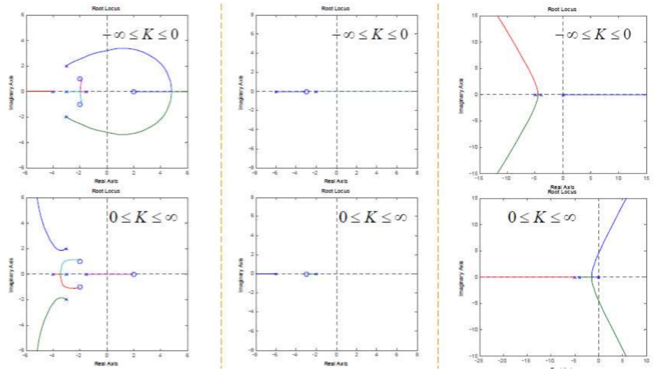
Property 2: As $|K| \rightarrow \infty$, each branch of the root locus approaches a zero of $G(s)H(s)$. Since we are assuming that $m \le n$, $n - m$ of these zeros are at infinity.



Property 3: Parts of the real s-axis that lie to the left of an odd number real poles and zeros of $G(s)H(s)$ are on the root locus for $K > 0$. Parts of the real s-axis that lie to the left of an even number (possibly zero) real poles and zeros of $G(s)H(s)$ are on the root locus for $K < 0$.



Property 4: Branches of the root locus between two real poles must break off into the complex plane for $|K|$ is large enough.



SOME LAPLACE TRANSFORM PAIRS

Signal	Transform	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} < 0$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} > -\alpha$
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} < -\alpha$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} > -\alpha$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} < -\alpha$
$\delta(t-T)$	e^{-sT}	All s
$[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\} > 0$
$[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathcal{Re}\{s\} > 0$
$[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
$[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$

PROPERTIES OF THE LAPLACE TRANSFORM

Property	Signal	Laplace Transform	ROC
	$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e., s is in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{Re}\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$

ZT Properties continued

Property	Signal	z-Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R, except for the possible addition or deletion of the origin
Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $\|a\|R$ = the set of points $\{\|a\|z\}$ for z in R)
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1}, where z is in R)
Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$, where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Initial Value Theorem
If $x[n] = 0$ for $n < 0$, then
$$x[0] = \lim_{z \to \infty} X(z)$$

Some common z-transform pairs

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\frac{1}{1 - z^{-1}}$	$\|z\| < 1$
4. $\delta[n - m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| > \|\alpha\|$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1 + \alpha z^{-1}}$	$\|z\| < \|\alpha\|$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
9. $[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2\cos \omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
10. $[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2\cos \omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
11. $[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r\cos \omega_0]z^{-1}}{1 - [2r\cos \omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$
12. $[r^n \sin \omega_0 n]u[n]$	$\frac{[r\sin \omega_0]z^{-1}}{1 - [2r\cos \omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$

Basic Continuous-Time Fourier Transform Pairs

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\,\delta(\omega)$	$a_0 = 1,\quad a_k = 0,\ k \neq 0$ $\left(\text{this is the Fourier series representation for any choice of } T > 0\right)$
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

DTFT Pairs

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]},\ k \neq 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N},\ k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi\delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!} a^n u[n], \quad \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—