VE216 Lab1

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1 Objectives

- To become familiar with the laboratory equipment: power supply, signal generator, digital oscilloscope, computer data acquisition system (Scope Connect).
- To review basic concepts of linear time-invariant systems.
- To illustrate several possible ways to determine the impulse response of a physical system from measured data.
- To use linearity, time-invariance and impulse response to compute the output of an LTI system when the input is a step, a pulse, or a more complicated signal. You will compare these calculations with actual measurements.
- To measure the frequency response of an LTI system and compare against theory.

2 Theoretical background

An RC circuit is used so that the computations are easy and physically meaningful. The same procedures can be applied to much more complicated systems.

2.1 RC circuit

The RC circuit shown below is an example of a simple LTI system. Of course, there are many other LTI systems that do not involve circuits at all.

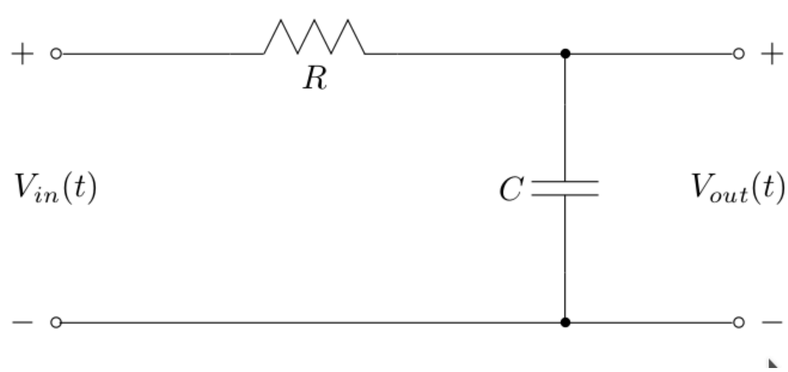


Figure 1: RC circuit

We will take the system input to be the voltage $V_{in}(t)$, while the system output is the voltage, $V_{out}(t)$, dropped across the capacitor. Notice that these voltages, in general, will be functions of time, t.

2.2 When is a linear circuit a linear system?

Using Kirchoffs current and voltage laws, one can easily derive a differential equation model of the RC-circuit in Figure 1, namely

$$RC\frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t) \tag{1}$$

Appealing to basic knowledge of ODEs from a sophomore level math course, the total solution is seen to be

$$V_{out}(t) = V_0 e^{-t/RC} + \int_0^t \frac{1}{RC} e^{-(t-\tau)/RC} V_{in}(\tau) d\tau, \quad t \geqslant 0 \tag{2}$$

where the initial condition at time zero is $V_{out}(0) = V_0$. It is very easy to verify that $V_{out}(t)$ is a linear function $V_{in}(t)$ if, and only if, $V_{out}(0) = V_0 = 0$, that is, the initial voltage on the capacitor has to be zero. This point is emphasized because you will have to assure this in the laboratory by either waiting for the charge to decay on the capacitor or by shorting the capacitor with a wire.

Reassuring remark: We will learn how to deal comfortably with nonzero initial conditions when we study the Laplace transform. For the time being, it is important to realize that we are assuming zero initial conditions when our models arise from a differential equation.

2.3 Impulse Response

The impulse response, $h(t)$, of an LTI system is, by definition, the output response when the input of the system is a delta function, $\delta(t)$. Of course, the delta function is a mathematical idealization. In practice, $h(t)$ can be well approximated by the response of the system when the input is a pulse of very short duration (compared with the response time of the system) and unit area, such as $p_\delta(t) = \frac{1}{\delta}(u(t) - u(t-\delta))$ for $\delta > 0$ sufficiently small.

Note that in order to keep the area of the pulse equal to unity, the amplitude has to increase as the pulse duration gets shorter. Often, this is a problem in a practical system as a large voltage pulse may fry an amplifier, for example. One way to get around this is to use linearity and realize that if the input is scaled by "b", then the output will be scaled by "b" as well. Consequently, if the measured response is divided by "b", an approximation of the impulse response is obtained.

2.4 Step Response

For any LTI system the output can be expressed as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(t-\tau)h(\tau)d\tau \tag{3}$$

where $x(t)$ denotes the input and denotes the convolution operation. The output resulting when the input is a unit step function, $x(t) = u(t)$, is called the unit step response. Simple manipulation leads to

$$y_{step}(t) = u(t) * h(t) = \int_{-\infty}^{+\infty} h(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} h(t-\tau)u(\tau)d\tau + \int_{t}^{+\infty} h(t-\tau)u(\tau)d\tau \tag{4}$$

which, because the unit step function is equal to 1 for $t > 0$ and equal to 0 for $t < 0$ step response simplifies to

$$y_{step}(t) = \int_{-\infty}^{t} h(\tau)d\tau \tag{5}$$

By taking the derivative of $y_{step}(t)$ with respect to t, we obtain, by the fundamental theorem of calculus,

$$\frac{dy_{step}(t)}{dt} = \frac{d}{dt}\int_{-\infty}^{t} h(\tau)d\tau = h(t) \tag{6}$$

Thus the impulse response can be computed from the unit step response by calculating the derivative of the step response with respect to time. This is a useful observation because it is sometimes easier to apply a step input to a physical system than it is to apply (an approximation of) an impulse.

We now have two ways of determining the impulse response from data. A third way will be hinted at a little later.

3 Experiment procedures

Setup

- Function generator: Utility → Output Setup → Load → High Z
- Oscillator: Trigger Menu → $\begin{cases} \text{1.Trigger Mode} \rightarrow \text{Basic} \\ \text{2.Edge Trigger (Rising Edge)} \\ \text{3.Trigger Settings} \rightarrow \text{DC Coupling} \end{cases}$
- RC circuit: $R = 1K\Omega$, and $C = 1\mu F$

3.1 Step Response

- Function generator:
  Square wave Vpp: 1V frequency: 100Hz
- Oscillator:
  CH1: 200mV/div CH2: 200mV/div Time: 2ms
- Bonus: Compare your results with the ideal case

3.2 Pulse Response

- Function generator: Pulse frequency: 100Hz
  1. Width: 1ms A: 100mV
  2. Width: 0.5ms A: 200mV

3.3 Ramp Response

- Function generator:
  Ramp Vpp: 100mV frequency: 100Hz

3.4 Sine Response

- Function generator: 10 Vpp

Frequency (Hz)	Vout / Vin	Time Shift	Phase Shift
50			
500			
5k			

- Bonus: Compare your results with ideal case

4 Experimental results

4.1 Step Response

The figure was shown in Figure 2.

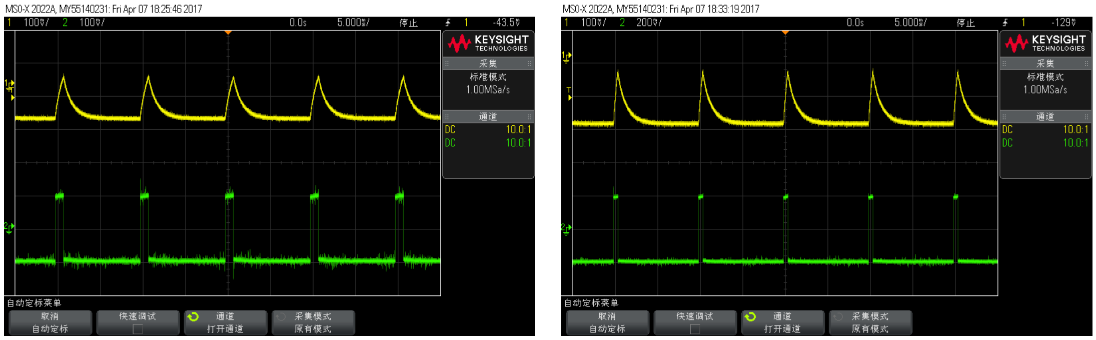
Figure 2: Step Response

4.2 Pulse Response

The figure was shown in Figure 3.

4.3 Sine Response

The figure was shown in Table 1.



(a) Width: 1ms, A: 100mV (b) Width: 0.5ms, A: 200mV

Figure 3: Pulse Response

Frequency (Hz)	Vout / Vin	Time Shift	Phase Shift
50			
500			
5k			

Table 1: Sine Response

5 Error analysis and discussion

5.1 Step Response

In ideal situation, we know

$$V_{in}(t) = u(t)$$
$$V_{out}(t) = y_{step}(t) = (1 - e^{-t/RC})u(t)$$

Then we can substitute $R = 1k\Omega$, $C = 1\mu F$ into $y_{step}(t)$ and plot the ideal graph when $t \in [0, 0.005)$ in MATLAB, as Figure **??**.

We can find that in Figure 2 (experimental), $V_{in}(t)$ was not a straight line, it is probably because of the inner resistance of the experiment instruments.

5.2 Pulse Response

In the experiment, we can only simulate a δ function since instantaneous time period doesn't exist in real world. However, the smaller time period (width) we get, the more accurate the experiment is. We know the equation of the pulse response is

$$h(t) = \frac{dy_{step}(t)}{dt} = \frac{1}{RC}e^{-t/RC}u(t)$$

5.3 Sine Response

6 Conclusion