Chapter 4

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Property	Aperiodic Signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time Shifting	$x(t - t_0$	$e^{-j\omega t_0} X(j\omega)$
Frequency Shifting	$e^{-j\omega t_0} x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time Reversal	$x(-t)$	$X(-j\omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{\vert a\vert} X(\frac{j\omega}{a})$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} X(j\omega)Y(j(\omega - \theta))d\theta$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int\limits_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathfrak{Re}\{X(j\omega)\} = \mathfrak{Re}\{X(-j\omega)\} \\ \mathfrak{Im}\{X(j\omega)\} = -\mathfrak{Im}\{X(-j\omega)\} \\ \vert X(j\omega)\vert = \vert X(-j\omega)\vert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and even	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathfrak{Ev}$ $[x(t)$ real$]$ $x_o(t) = \mathfrak{Od}$ $[x(t)$ real$]$	$\mathfrak{Re}\{X(j\omega)\}$ $j\mathfrak{Im}\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int\limits_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$