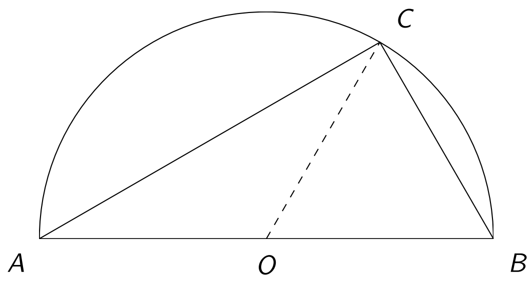
## P. 2-11




Since $OA = OB = OC$, we know $\angle OAC = \angle OCA$ and $\angle OBC = \angle OCB$. And since $ABC$ is an triangle, $\angle OAC + \angle OBC + \angle ACB = \pi$, we can simply find that $\angle ACB = \pi/2$, so that an angle inscribed in a semicircle is a right angle.

## P. 2-17

a)

$$|\mathbf{E}| = |\mathbf{a_R}(25/R^2)| = |\mathbf{a_R}| \cdot \frac{25}{3^2+4^2+5^2} = \frac{1}{2}.$$

$$E_x = |\mathbf{E}| \cdot \frac{x}{R} = \frac{1}{2} \cdot \frac{-3}{\sqrt{3^2+4^2+5^2}} = -\frac{3\sqrt{2}}{20}.$$

b)

$$\cos\theta = \frac{\mathbf{E}\cdot\mathbf{B}}{|\mathbf{E}||\mathbf{B}|} = \frac{-3\cdot 2 + 4 \cdot -2 - 5 \cdot 1}{\sqrt{3^2+4^2+5^2}\cdot\sqrt{2^2+2^2+1^1}} = -\frac{19\sqrt{2}}{30},$$

$$\theta = \arccos -\frac{19\sqrt{2}}{30} \approx 2.681\,\text{rad}.$$

## P. 2-21

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} (\mathbf{a_x}y + \mathbf{a_y}x)(\mathbf{a_x}dx + \mathbf{a_y}dy) = \int_{P_1}^{P_2} (ydx + xdy)$$

a)

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} (y \cdot 4ydy + 2y^2 \cdot dy) = \int_1^2 6y^2 dy = 14.$$

b)

$$x = 6y - 4,$$

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} [y \cdot 6dy + (6y-4)\cdot dy] = \int_1^2 (12y-4)dy = 14.$$

## P. 2-26

a)

$$\nabla \cdot f_1(\mathbf{R}) = \frac{1}{R^2} \cdot \frac{\partial}{\partial R}(R^n \cdot R^2) = (n+2)R^{n-1}.$$

b)

$$\nabla \cdot f_2(\mathbf{R}) = \frac{1}{R^2} \cdot \frac{\partial}{\partial R}(k/R^2 \cdot R^2) = 0.$$

## P. 2-29

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r^2 \cdot r) + \frac{\partial}{\partial z}(2z) = 3r + 2.$$

$$\int_V \nabla \cdot \mathbf{A} dV = \int_V (3r+2) dV = \int_0^4 \int_0^{2\pi} \int_0^5 (3r+2) r dr d\theta dz = 1200\pi.$$

$$\oint_S \mathbf{A} dS = \int_0^4 \int_0^{2\pi} (\mathbf{a_r} r^2) r d\theta dz \mathbf{a_r} + \int_0^{2\pi} \int_0^5 2z \mathbf{a_z} r dr d\theta \mathbf{a_z} = 1000\pi + 200\pi = 1200\pi.$$

So

$$\int_V \nabla \cdot \mathbf{A} dV = \oint_S \mathbf{A} dS.$$

## P. 2-33

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \frac{\partial}{\partial x}(E_y H_z - E_z H_y) + \frac{\partial}{\partial y}(E_z H_x - E_x H_z) + \frac{\partial}{\partial z}(E_x H_y - E_y H_x),$$

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = H_x \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + H_y \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + H_z \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right),$$

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = E_x \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + E_y \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + E_z \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right).$$

Since

$$\frac{\partial}{\partial x} AB = A \frac{\partial B}{\partial x} + B \frac{\partial A}{\partial x},$$

we can find that

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}).$$

## P. 2-35

$$\Delta s_u = R^2 \sin\theta \Delta\theta \Delta\phi,$$

$$\begin{aligned}\oint_{C_u} \mathbf{A} \cdot d\mathbf{l} =& A_\theta \cdot (R, \theta, \phi - \Delta\phi/2) R\Delta\theta - A_\theta \cdot (R, \theta, \phi + \Delta\phi/2) R\Delta\theta + \\ & A_\phi \cdot (R, \theta + \Delta\theta/2, \phi) R\Delta\phi \sin(\theta + \Delta\theta/2) - A_\phi \cdot (R, \theta - \Delta\theta/2, \phi) R\Delta\phi \sin(\theta - \Delta\theta/2) \\ =& -\frac{\partial A_\theta}{\partial \phi} R\Delta\phi\Delta\theta + \frac{\partial A_\phi \sin\theta}{\partial \theta} R\Delta\phi\Delta\theta.\end{aligned}$$

$$\begin{aligned}(\nabla \times \mathbf{A})_u &= \lim_{\Delta s_u \to 0} \frac{1}{\Delta s_u} \oint_{C_u} \mathbf{A} \cdot d\mathbf{l} = \lim_{\Delta s_u \to 0} \frac{1}{R^2 \sin\theta \Delta\theta \Delta\phi} \cdot \left( -\frac{\partial A_\theta}{\partial \phi} + \frac{\partial A_\phi \sin\theta}{\partial \theta} \right) R\Delta\phi\Delta\theta \\ &= \frac{1}{R\sin\theta} \cdot \left( -\frac{\partial A_\theta}{\partial \phi} + \frac{\partial A_\phi \sin\theta}{\partial \theta} \right).\end{aligned}$$

## P. 2-39

a)

$$\nabla \times \mathbf{F} = \mathbf{a_x}(c_3 + 3) + \mathbf{a_y}(c_1 - 1) + \mathbf{a_z} c_2 = \mathbf{0},$$

$$c_1 = 1, c_2 = 0, c_3 = -3.$$

b)

$$\nabla \cdot \mathbf{F} = 1 + 0 + c_4 = 0,$$

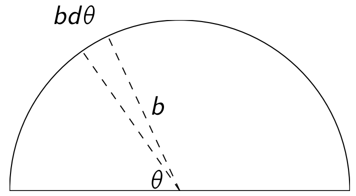
$$c_4 = -1.$$

c)

$$\mathbf{F} = -\nabla V = \mathbf{a_x}(x + z) + \mathbf{a_y}(-3z) + \mathbf{a_z}(x - 3y - z),$$

$$V = -\frac{1}{2}x^2 - xz + 3yz + \frac{1}{2}z^2 + C.$$

## P. 3-8



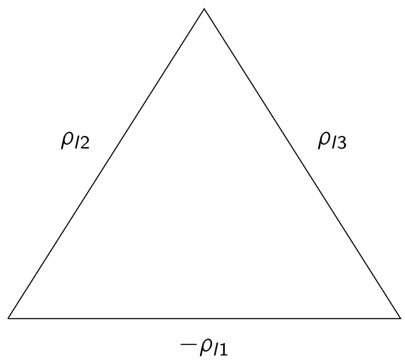

$$dQ = bd\theta\rho_l,$$

$$dE = \frac{dQ}{4\pi\epsilon_0 b^2} = \frac{\rho_l}{4\pi\epsilon_0 b}d\theta,$$

$$|\mathbf{E}| = \int_0^\pi dE\sin\theta = \frac{\rho_l}{4\pi\epsilon_0 b}\int_0^\pi \sin\theta d\theta = \frac{\rho_l}{2\pi\varepsilon_0 b}.$$

The direction is downwards.

## P. 3-9




$$dQ = \rho dl,$$

$$dE = \frac{dQ}{4\pi\varepsilon_0(l^2 + L^2/12)} = \frac{\rho}{4\pi\varepsilon_0}\cdot\frac{dl}{l^2 + L^2/12},$$

$$|\mathbf{E_1}| = \int_{-L/2}^{L/2} dE\cdot\sqrt{\frac{L^2/12}{l^2 + L^2/12}} = \frac{3\rho_{l1}}{2\pi\varepsilon_0 L},$$

$$|\mathbf{E_2}| = |\mathbf{E_3}| = \frac{1}{2}|\mathbf{E_1}| = \frac{3\rho_{l1}}{4\pi\varepsilon_0 L}.$$

$$|\mathbf{E}| = |\mathbf{E_1}| - \frac{1}{2}|\mathbf{E_2}| - \frac{1}{2}|\mathbf{E_3}| = \frac{3\rho_{l1}}{4\pi\varepsilon_0 L}.$$

The direction is upwards.

## P. 3-12

a) For $0 < r < a$, $\mathbf{E} = \mathbf{0}$.
   For $a < r < b$,

$$2\pi aL\cdot\frac{\rho_{sa}}{\varepsilon_0} = 2\pi rL\cdot E,$$

$$\mathbf{E} = \frac{a\rho_{sa}}{\varepsilon_0 r}\mathbf{a_r}.$$

For $b < r$,

$$2\pi aL \cdot \frac{\rho_{sa}}{\varepsilon_0} + 2\pi bL \cdot \frac{\rho_{sb}}{\varepsilon_0} = 2\pi rL \cdot E,$$

$$\mathbf{E} = \frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}\mathbf{a_r}.$$

b)

$$\frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r} = 0,$$

$$a = -\frac{\rho_{sb}}{\rho_{sa}}b.$$

## P. 3-13

$$W = -\int \mathbf{E}qd\mathbf{l} = 2\mu \int (\mathbf{a_x}y + \mathbf{a_y}x)(\mathbf{a_x}dx + \mathbf{a_y}dy) = 2\mu \int ydx + xdy.$$

a)

$$W = 2\mu \int ydx + xdy = 2\mu C \int_2^8 4y^2 dy + 2y^2 dy = 28\mu.$$

b)

$$x = 6y - 4,$$

$$W = 2\mu \int ydx + xdy = 2\mu C \int_2^8 6ydy + (6y - 4)dy = 28\mu.$$

## P. 3-16

a)

$$dQ = \rho_l dl,$$

$$dV = \frac{dQ}{4\pi\varepsilon_0\sqrt{l^2 + y^2}} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{dl}{\sqrt{l^2 + y^2}},$$

$$V = \int_{-L/2}^{L/2} dV = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{1}{\sqrt{l^2 + y^2}}dl = \frac{\rho_l}{2\pi\varepsilon_0}\operatorname{arcsinh}\frac{L}{2y}.$$

b)

$$dE = \frac{dQ}{4\pi\varepsilon_0(l^2 + y^2)} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{dl}{l^2 + y^2},$$

$$\mathbf{E} = \mathbf{a_y}\int_{-L/2}^{L/2} dE \cdot \sqrt{\frac{y^2}{l^2 + y^2}} = \frac{\rho_l}{2\pi\varepsilon_0} \cdot \frac{y}{\sqrt{L^2 + 4y^2}}\mathbf{a_y}.$$

c)

$$-\nabla V = -\frac{dV}{dy}\mathbf{a_y} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{L}{\sqrt{L^2 + 4y^2}}\mathbf{a_y} = \mathbf{E}.$$

## P. 3-19

$$dQ = \frac{Q}{h}dz,$$

$$dV = \frac{dQ}{4\pi\varepsilon_0\sqrt{b^2 + z^2}} = \frac{Q}{4\pi\varepsilon_0 h\sqrt{b^2 + z^2}}dz.$$

a)

$$V = \int_{z-h}^{z} dV = \frac{Q}{4\pi\varepsilon_0 h}\int_{z-h}^{z} \frac{1}{\sqrt{b^2 + z^2}}dz = \frac{Q}{4\pi\varepsilon_0 h}\left[\operatorname{arcsinh}\frac{z}{b} - \operatorname{arcsinh}\frac{z-h}{b}\right].$$

$$\mathbf{E} = -\nabla V = -\frac{dV}{dz}\mathbf{a_z} = -\frac{Q}{4\pi\varepsilon_0 h}\left[\frac{1}{\sqrt{b^2 + z^2}} - \frac{1}{\sqrt{b^2 + (z-h)^2}}\right]\mathbf{a_z}.$$

b)

$$V = \int_0^z dV + \int_0^{h-z} dV = \frac{Q}{4\pi\varepsilon_0 h}\left[\operatorname{arcsinh}\frac{z}{b} + \operatorname{arcsinh}\frac{h-z}{b}\right].$$

$$\mathbf{E} = -\nabla V = -\frac{dV}{dz}\mathbf{a_z} = -\frac{Q}{4\pi\varepsilon_0 h}\left[\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{b^2+(h-z)^2}}\right]\mathbf{a_z}.$$

## P. 3-22

a)

$$\rho_{ps} = \mathbf{P}\cdot\mathbf{a_n}|_{n=L/2} = \frac{1}{2}P_0 L.$$

$$\rho_p = -\nabla\cdot\mathbf{P} = -3P_0.$$

b)

$$Q_s = \oint_S \rho_{ps} dS = \frac{1}{2}P_0 L\cdot 6L^2 = 3P_0L^3,$$

$$Q_v = \int_v \rho_p dV = \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\rho_p dzdydx = -3P_0L^3,$$

$$Q = Q_s + Q_v = 0.$$

## P. 3-23

Let $\mathbf{P} = \mathbf{a_p}P_0$, $\theta = \langle \mathbf{P}, \mathbf{a_n}\rangle$,

$$\rho_{ps}(\theta) = \mathbf{P}\cdot\mathbf{a_n} = P_0\cos\theta,$$

$$dE_\theta = dv\cdot\frac{\rho_{ps}}{4\pi\varepsilon_0 R^2}\cdot\cos\theta = 2\pi R^2\sin\theta d\theta\cdot\frac{P_0\cos\theta}{4\pi\varepsilon_0 R^2}\cdot\cos\theta = \frac{P_0\sin\theta\cos\theta^2}{2\varepsilon_0}d\theta.$$

$$|\mathbf{E}| = \int dE_\theta = \int_0^\pi \frac{P_0\sin\theta\cos\theta^2}{2\varepsilon_0}d\theta = \frac{P_0}{3\varepsilon_0},$$

$$\mathbf{E} = \mathbf{a_p}\frac{P_0}{3\varepsilon_0} = \frac{\mathbf{P}}{3\varepsilon_0}.$$

## P. 3-25

$$E_{2t} = E_{1t} = \mathbf{a_x}2y - \mathbf{a_y}3x.$$

Since $\rho_s = 0$,

$$\varepsilon_{r1}E_{1n} = \varepsilon_{r2}E_{2n},$$

$$E_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}E_{1n} = \frac{2}{3}\cdot\mathbf{a_z}5 = \mathbf{a_z}\frac{10}{3}.$$

$$E_2 = E_{2t} + E_{2n} = \mathbf{a_x}2y - \mathbf{a_y}3x + \mathbf{a_z}\frac{10}{3}.$$

$$\mathbf{D_2} = \varepsilon_2\mathbf{E_2} = 3\varepsilon_0\left(\mathbf{a_x}2y - \mathbf{a_y}3x + \mathbf{a_z}\frac{10}{3}\right).$$

## P. 3-28

Obviously, $\mathbf{E_3}$ is parallel to $\mathbf{E_2}$, so we only need to find $\varepsilon_{r2}$ so that $\mathbf{E_2}$ is parallel to the x-axis.

$$E_{1t} = E_{2t} = -3.$$

$$\varepsilon_{r1}E_{1n} = \varepsilon_{r2}E_{2n},$$

$$E_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_{1n} = \frac{1}{\varepsilon_{r2}} \cdot 5 = \frac{5}{\varepsilon_{r2}}.$$

$$E_{2t} \cos\theta + E_{2n} \cos\theta = 0,$$

$$-3 + \frac{5}{\varepsilon_{r2}} = 0,$$

$$\varepsilon_{r2} = \frac{5}{3}.$$

## P. 3-32

$$C_1 = \frac{2\pi\varepsilon L}{\ln \frac{b}{r_i}} = \frac{2\pi\varepsilon_0\varepsilon_{r_1} L}{\ln \frac{b}{r_i}},$$

$$C_2 = \frac{2\pi\varepsilon L}{\ln \frac{r_o}{b}} = \frac{2\pi\varepsilon_0\varepsilon_{r_2} L}{\ln \frac{r_o}{b}},$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{2\pi\varepsilon_0 L}{\frac{1}{\varepsilon_{r_1}} \ln \frac{b}{r_i} + \frac{1}{\varepsilon_{r_3}} \ln \frac{r_o}{b}},$$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\frac{1}{\varepsilon_{r_1}} \ln \frac{b}{r_i} + \frac{1}{\varepsilon_{r_3}} \ln \frac{r_o}{b}}.$$

## P. 3-43

$$dW_e = V dq = \frac{q}{C} dq,$$

$$W_e = \int dW_e = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Since $Q = CV$, we can also get

$$W_e = \frac{1}{2} C V^2 = \frac{1}{2} Q V.$$