VE230 — Electromagnetics I

Homework 9

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a)

$$\mathbf{E}_i(x,z) = \mathbf{a}_y 10 e^{-j(6x+8z)} = \mathbf{a}_y E_0 e^{-jk_x x - jk_z z},$$

$$E_0 = 10 \quad k_x = 6, \quad k_z = 8,$$

$$k = \sqrt{k_x^2 + k_z^2} = 10,$$

$$\lambda = \frac{2\pi}{k} = 0.2\pi \text{ m} \approx 0.628\,\text{m},$$

$$f = \frac{c}{\lambda} = 4.777 \times 10^8 \text{ Hz}.$$

b)

$$\omega = 2\pi f = 3 \times 10^9 \,\text{rad}\,\text{s}^{-1},$$

$$\eta_0 = \frac{\omega\mu}{k} = 120\pi\ \Omega,$$

$$\mathbf{E}_i(x,z,t) = \text{Re}[\mathbf{E}_i(x,z)e^{j\omega t}] = \mathbf{a_y} 10\cos(3\times 10^9 t - 6x - 8z)\ \text{V}\,\text{m}^{-1}.$$

$$\mathbf{a}_{n_i} = \mathbf{a}_x \cdot \frac{k_x}{k} + \mathbf{a}_z \cdot \frac{k_z}{k},$$

$$\mathbf{H}_i(x,z) = \frac{1}{\eta_0}(\mathbf{a}_{n_i} \times E_i(x,z)) = \left(\frac{\mathbf{a}_z}{20\pi} - \frac{\mathbf{a}_x}{15\pi}\right) e^{-j(6x+8z)},$$

$$\mathbf{H}_i(x,z,t) = \text{Re}[\mathbf{H}_i(x,z)e^{j\omega t}] = \left(\frac{\mathbf{a}_z}{20\pi} - \frac{\mathbf{a}_x}{15\pi}\right) \cos(3\times 10^9 t - 6x - 8z)\ \text{A}\,\text{m}^{-1}.$$

c)

$$\theta_i = \arccos(\mathbf{a}_n \cdot \mathbf{a}_z) = \arccos 0.8 \approx 0.644 \text{ rad}.$$

d)

$$\mathbf{E}_r(x,z) = -\mathbf{E}_i(x,-z) = -\mathbf{a}_y 10 e^{-j(6x-8z)},$$

$$\mathbf{a}_{n_r} = \mathbf{a}_x \cdot \frac{k_x}{k} - \mathbf{a}_z \cdot \frac{k_z}{k}$$

$$\mathbf{H}_r(x,z) = \frac{1}{\eta_0}(\mathbf{a}_{n_r} \times E_r(x,z)) = -\left(\frac{\mathbf{a}_z}{20\pi} + \frac{\mathbf{a}_x}{15\pi}\right) e^{-j(6x-8z)}.$$

e)

$$\mathbf{E}_1(x,z) = \mathbf{E}_i(x,z) + \mathbf{E}_r(x,z) = -\mathbf{a}_y 20 j e^{-j6x} \sin 8z\ \text{V/m},$$

$$\mathbf{H}_1(x,z) = \mathbf{H}_i(x,z) + \mathbf{H}_r(x,z) = -\frac{\mathbf{a}_z}{10\pi} j \sin 8z - \frac{\mathbf{a_x}}{15\pi} 2\cos 8z.$$