

Homework 7

P.7-2 The circuit in Fig. 7-10 is situated in a magnetic field

$$\mathbf{B} = \mathbf{a}_z 3 \cos(5\pi 10^7 t - \frac{2}{3}\pi x) \quad (\mu\text{T}).$$

Assuming $R = 15\ (\Omega)$, find the current i .

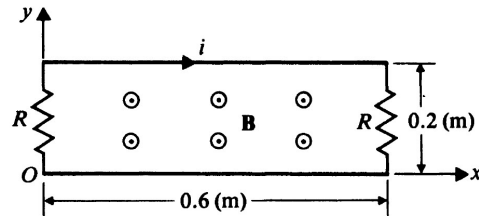
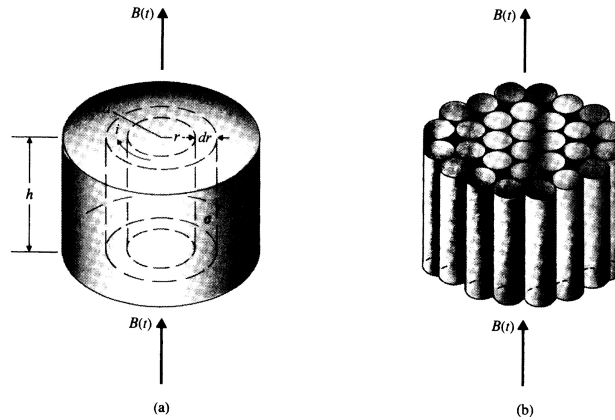


FIGURE 7-10
A circuit in a time-varying magnetic field
(Problem P.7-2).



P.7-6 A suggested scheme for reducing eddy-current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig. 7-12, the section shown in part (a) is replaced by that in part (b). Assuming that $B(t) = B_0 \sin \omega t$ and that N filamentary areas fill 95% of the original cross-sectional area, find

- the average eddy-current power loss in the section of core of height h in Fig. 7-12(a),
- the total average eddy-current power loss in the N filamentary sections in Fig. 7-12(b).

The magnetic field due to eddy currents is assumed to be negligible. (*Hint*: First find the current and power dissipated in the differential circular ring section of height h and width dr at radius r .)

P.7-11 Derive the two divergence equations, Eqs. (7-53c) and (7-53d), from the two curl equations, Eqs. (7-53a) and (7-53b), and the equation of continuity, Eq. (7-48).

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (7-53a)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (7-53b)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (7-53c)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (7-53d)$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \quad (7-48)$$

P.7-12 Prove that the Lorentz condition for potentials as expressed in Eq. (7-62) is consistent with the equation of continuity.

$$\nabla \cdot \mathbf{A} + \mu\epsilon \frac{\partial V}{\partial t} = 0, \quad (7-62)$$

P.7-14 Substitute Eqs. (7-55) and (7-57) in Maxwell's equations to obtain wave equations for scalar potential V and vector potential \mathbf{A} for a linear, isotropic but inhomogeneous medium. Show that these wave equations reduce to Eqs. (7-65) and (7-63) for simple media. (*Hint:* Use the following gauge condition for potentials in an inhomogeneous medium:

$$\nabla \cdot (\epsilon \mathbf{A}) + \mu\epsilon^2 \frac{\partial V}{\partial t} = 0. \quad (7-117)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (\text{T}). \quad (7-55)$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad (\text{V/m}). \quad (7-57)$$

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}. \quad (7-63)$$

$$\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}, \quad (7-65)$$

P.7-17 Discuss the relations

- a) between the boundary conditions for the tangential components of \mathbf{E} and those for the normal components of \mathbf{B} ,
- b) between the boundary conditions for the normal components of \mathbf{D} and those for the tangential components of \mathbf{H} .

P.7-20 Prove by direct substitution that any twice differentiable function of $(t - R\sqrt{\mu\epsilon})$ or of $(t + R\sqrt{\mu\epsilon})$ is a solution of the homogeneous wave equation, Eq. (7-73).

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon \frac{\partial^2 U}{\partial t^2} = 0. \quad (7-73)$$