VE230 — Electromagnetics I

Homework 2

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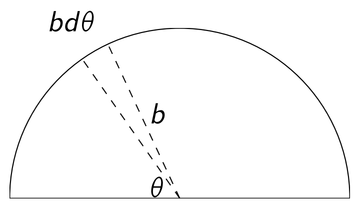
P. 3-8



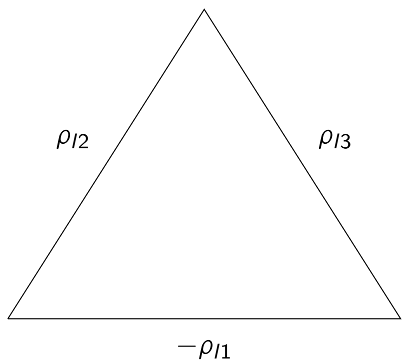
$$dQ = bd\theta\rho_l,$$

$$dE = \frac{dQ}{4\pi\epsilon_0 b^2} = \frac{\rho_l}{4\pi\epsilon_0 b} d\theta,$$

$$|\mathbf{E}| = \int_0^\pi dE \sin\theta = \frac{\rho_l}{4\pi\epsilon_0 b} \int_0^\pi \sin\theta d\theta = \frac{\rho_l}{2\pi\varepsilon_0 b}.$$

The direction is downwards.

P. 3-9



$$dQ = \rho dl,$$

$$dE = \frac{dQ}{4\pi\varepsilon_0(l^2 + L^2/12)} = \frac{\rho}{4\pi\varepsilon_0} \cdot \frac{dl}{l^2 + L^2/12},$$

$$|\mathbf{E_1}| = \int_{-L/2}^{L/2} dE \cdot \sqrt{\frac{L^2/12}{l^2 + L^2/12}} = \frac{3\rho_{l1}}{2\pi\varepsilon_0 L},$$

$$|\mathbf{E_2}| = |\mathbf{E_3}| = \frac{1}{2}|\mathbf{E_1}| = \frac{3\rho_{l1}}{4\pi\varepsilon_0 L}.$$

$$|\mathbf{E}| = |\mathbf{E_1}| - \frac{1}{2}|\mathbf{E_2}| - \frac{1}{2}|\mathbf{E_3}| = \frac{3\rho_{l1}}{4\pi\varepsilon_0 L}.$$

The direction is upwards.

P. 3-12

a) For $0 < r < a$, $\mathbf{E} = \mathbf{0}$.
For $a < r < b$,

$$2\pi aL \cdot \frac{\rho_{sa}}{\varepsilon_0} = 2\pi rL \cdot E,$$

$$\mathbf{E} = \frac{a\rho_{sa}}{\varepsilon_0 r}\mathbf{a_r}.$$

For $b < r$,

$$2\pi aL \cdot \frac{\rho_{sa}}{\varepsilon_0} + 2\pi bL \cdot \frac{\rho_{sb}}{\varepsilon_0} = 2\pi rL \cdot E,$$

$$\mathbf{E} = \frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}\mathbf{a_r}.$$

b)

$$\frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r} = 0,$$

$$a = -\frac{\rho_{sb}}{\rho_{sa}}b.$$

P. 3-13

$$W = -\int \mathbf{E}q d\mathbf{l} = 2\mu \int (\mathbf{a_x}y + \mathbf{a_y}x)(\mathbf{a_x}dx + \mathbf{a_y}dy) = 2\mu \int ydx + xdy.$$

a)

$$W = 2\mu \int ydx + xdy = 2\mu C \int_2^8 4y^2dy + 2y^2dy = 28\mu.$$

b)

$$x = 6y - 4,$$

$$W = 2\mu \int ydx + xdy = 2\mu C \int_2^8 6ydy + (6y-4)dy = 28\mu.$$

P. 3-16

a)

$$dQ = \rho_l dl,$$

$$dV = \frac{dQ}{4\pi\varepsilon_0\sqrt{l^2+y^2}} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{dl}{\sqrt{l^2+y^2}},$$

$$V = \int_{-L/2}^{L/2} dV = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{1}{\sqrt{l^2+y^2}}dl = \frac{\rho_l}{2\pi\varepsilon_0}\text{arcsinh}\frac{L}{2y}.$$

b)

$$dE = \frac{dQ}{4\pi\varepsilon_0(l^2+y^2)} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{dl}{l^2+y^2},$$

$$\mathbf{E} = \mathbf{a_y}\int_{-L/2}^{L/2} dE \cdot \sqrt{\frac{y^2}{l^2+y^2}} = \frac{\rho_l}{2\pi\varepsilon_0} \cdot \frac{y}{\sqrt{L^2+4y^2}}\mathbf{a_y}.$$

c)

$$-\nabla V = -\frac{dV}{dy}\mathbf{a_y} = \frac{\rho_l}{4\pi\varepsilon_0} \cdot \frac{L}{\sqrt{L^2 + 4y^2}}\mathbf{a_y} = \mathbf{E}.$$

P. 3-19

$$dQ = \frac{Q}{h}dz,$$

$$dV = \frac{dQ}{4\pi\varepsilon_0\sqrt{b^2 + z^2}} = \frac{Q}{4\pi\varepsilon_0 h\sqrt{b^2 + z^2}}dz.$$

a)

$$V = \int_{z-h}^{z} dV = \frac{Q}{4\pi\varepsilon_0 h}\int_{z-h}^{z} \frac{1}{\sqrt{b^2 + z^2}}dz = \frac{Q}{4\pi\varepsilon_0 h}\left[\text{arcsinh}\frac{z}{b} - \text{arcsinh}\frac{z-h}{b}\right].$$

$$\mathbf{E} = -\nabla V = -\frac{dV}{dz}\mathbf{a_z} = -\frac{Q}{4\pi\varepsilon_0 h}\left[\frac{1}{\sqrt{b^2 + z^2}} - \frac{1}{\sqrt{b^2 + (z-h)^2}}\right]\mathbf{a_z}.$$

b)

$$V = \int_{0}^{z} dV + \int_{0}^{h-z} dV = \frac{Q}{4\pi\varepsilon_0 h}\left[\text{arcsinh}\frac{z}{b} + \text{arcsinh}\frac{h-z}{b}\right].$$

$$\mathbf{E} = -\nabla V = -\frac{dV}{dz}\mathbf{a_z} = -\frac{Q}{4\pi\varepsilon_0 h}\left[\frac{1}{\sqrt{b^2 + z^2}} - \frac{1}{\sqrt{b^2 + (h-z)^2}}\right]\mathbf{a_z}.$$