# VE230 — Electromagnetics I

*Homework 8*

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## P. 7-24

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu \frac{\partial}{\partial t}\left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) = -\mu \frac{\partial \mathbf{J}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2},$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla\rho - \nabla^2 \mathbf{E},$$

$$\nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla\rho + \mu\frac{\partial \mathbf{J}}{\partial t} + \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2},$$

$$\nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla\rho + \mu j\omega \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{E}.$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla \times \mathbf{J} + \varepsilon\frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = \nabla \times \mathbf{J} - \mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = -\nabla^2 \mathbf{H},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{H}.$$

## P. 7-27

$$\begin{aligned}
\nabla \times \mathbf{E} &= \frac{1}{R^2 \sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & E_0 \sin\theta e^{-jkR} & 0 \end{vmatrix} \\
&= \frac{1}{R^2 \sin\theta}\left(\mathbf{a}_R \frac{\partial}{\partial \phi} E_0 \sin\theta e^{-jkR} - \mathbf{a}_\phi R\sin\theta \frac{\partial}{\partial R} E_0 \sin\theta e^{-jkR}\right) \\
&= \mathbf{a}_\phi \frac{-E_0 jk \sin\theta e^{-jkR}}{R},
\end{aligned}$$

$$\nabla \times \mathbf{E} = -\mu\frac{\partial \mathbf{H}}{\partial t} = -j\mu\omega\mathbf{H},$$

$$\mathbf{H} = \mathbf{a}_\phi \frac{E_0 k \sin\theta e^{-jkR}}{\mu\omega R}, \quad k = \omega\sqrt{\mu\varepsilon}.$$

$$\mathbf{H}(R) = \mathbf{a}_\phi \frac{E_0\sqrt{\mu\varepsilon}\sin\theta e^{-j\omega\sqrt{\mu\varepsilon}R}}{\mu R},$$

$$\mathbf{H}(R,t) = \text{Re}[\mathbf{H}(R)e^{j\omega t}] = \mathbf{a}_\phi \frac{E_0\sqrt{\mu\varepsilon}}{\mu R}\sin\theta\cos(\omega t - \omega\sqrt{\mu\varepsilon}R).$$

## P. 7-29

a)

$$\nabla\times\mathbf{E} = -j\omega\mu_0\mathbf{H} = \omega^2\mu_0\varepsilon_0\nabla\times\pi_e,$$

$$\nabla\times(\mathbf{E} - \omega^2\mu_0\varepsilon_0\pi_e) = \mathbf{0},$$

$$\mathbf{E} = \omega^2\mu_0\varepsilon_0\pi_e + \mathbf{C}.$$

$$\nabla\times\mathbf{H} = j\omega\mathbf{D} = \omega^2\mu_0\varepsilon_0\nabla\times\pi_e,$$

$$\nabla\times(j\omega\varepsilon_0\nabla\times\pi_e) = j\omega\varepsilon_0(\omega^2\mu_0\varepsilon_0\pi_e + \frac{\mathbf{P}}{\varepsilon_0} + \mathbf{C}),$$

$$\nabla\times(\nabla\times\pi_e) = \nabla(\nabla\cdot\pi_e) - \nabla^2\pi_e = \omega^2\mu_0\varepsilon_0\pi_e + \frac{\mathbf{P}}{\varepsilon_0} + \mathbf{C},$$

$$\nabla^2\pi_e + \omega^2\mu_0\varepsilon_0\pi_e = \nabla(\nabla\cdot\pi_e) - \frac{\mathbf{P}}{\varepsilon_0} - \mathbf{C},$$

$$\mathbf{C} = \nabla(\nabla\cdot\pi_e),$$

$$\mathbf{E} = \omega^2\mu_0\varepsilon_0\pi_e + \nabla(\nabla\cdot\pi_e).$$

b)

$$k_0^2 = \omega^2\mu_0\varepsilon_0,$$

$$\nabla^2\pi_e + k_0^2\pi_e = -\frac{\mathbf{P}}{\varepsilon_0}.$$

## P. 8-7

Let $\phi = \omega_t - kz$,

$$\mathbf{E}(\phi) = \mathbf{a}_x E_{10}\sin\phi + \mathbf{a_y}E_{20}\sin(\phi+\psi),$$

$$\frac{E_x}{E_{10}} = \sin\phi,$$

$$\frac{E_y}{E_{20}} = \sin(\phi+\psi) = \sin\phi\cos\psi + \cos\phi\sin\psi = \frac{E_x}{E_{10}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi,$$

$$\left[\sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi\right]^2 = \left(\frac{E_y}{E_{20}}\right)^2 + \left(\frac{E_x}{E_{10}}\cos\psi\right)^2 - 2\left(\frac{E_y}{E_{20}}\right)\left(\frac{E_x}{E_{10}}\cos\psi\right),$$

$$\left(\frac{E_x}{E_{10}}\right)^2 + \left(\frac{E_y}{E_{20}}\right)^2 - 2\frac{E_x}{E_{10}}\frac{E_y}{E_{20}}\cos\psi = \sin^2\psi,$$

$$\left(\frac{E_x}{E_{10}\sin\psi}\right)^2 + \left(\frac{E_y}{E_{20}\sin\psi}\right)^2 - 2\frac{E_x}{E_{10}}\frac{E_y}{E_{20}}\frac{\cos\psi}{sin^2\psi} = 1.$$

## P. 8-9

$$\nabla^2\mathbf{E} + k_c^2\mathbf{E} = 0,$$

$$k_c = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha,$$

$$\omega^2\mu\varepsilon = \omega^2\mu(\varepsilon - j\sigma/\omega) = \beta^2 - \alpha^2 - 2j\alpha\beta,$$

$$\begin{cases} \beta^2 + \alpha^2 = \omega^2\mu\sqrt{\varepsilon^2 + \sigma^2/\omega^2} \\ \beta^2 - \alpha^2 = \omega^2\mu\varepsilon \end{cases} \Longrightarrow \begin{cases} \alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1 + \left(\dfrac{\sigma^2}{\omega\varepsilon}\right)^2} - 1\right]^{1/2} \\ \beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}}\left[\sqrt{1 + \left(\dfrac{\sigma^2}{\omega\varepsilon}\right)^2} + 1\right]^{1/2} \end{cases}.$$