Homework 5

You don't need to do P.5-6.

P.5–6 Lightning strikes a lossy dielectric sphere— $\epsilon = 1.2 \epsilon_0$, $\sigma = 10 \text{ (S/m)}$ —of radius 0.1 (m) at time t = 0, depositing uniformly in the sphere a total charge 1 (mC). Determine, for all t,

- a) the electric field intensity both inside and outside the sphere,
- b) the current density in the sphere.
- P.5-7 Refer to Problem P.5-6.
 - a) Calculate the time it takes for the charge density in the sphere to diminish to 1% of its initial value.
 - b) Calculate the change in the electrostatic energy stored in the sphere as the charge density diminishes from the initial value to 1% of its value. What happens to this energy?
 - c) Determine the electrostatic energy stored in the space outside the sphere. Does this energy change with time?

P.5–10 The space between two parallel conducting plates each having an area S is filled with an inhomogeneous ohmic medium whose conductivity varies linearly from σ_1 at one plate (y = 0) to σ_2 at the other plate (y = d). A d-c voltage V_0 is applied across the plates as in Fig. 5–11. Determine

- a) the total resistance between the plates,
- b) the surface charge densities on the plates,
- c) the volume charge density and the total amount of charge between the plates.

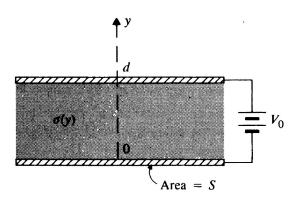


FIGURE 5-11 Inhomogeneous ohmic medium with conductivity $\sigma(y)$ (Problem P.5-10).

P.5–16 Determine the resistance between two concentric spherical surfaces of radii R_1 and R_2 ($R_1 < R_2$), assuming that a material of conductivity $\sigma = \sigma_0(1 + k/R)$ fills the space between them. (*Note:* Laplace's equation for V does not apply here.)

P.5–22 Assume a rectangular conducting sheet of conductivity σ , width a, and height b. A potential difference V_0 is applied to the side edges, as shown in Fig. 5–14. Find

- a) the potential distribution,
- b) the current density everywhere within the sheet. (*Hint*: Solve Laplace's equation in Cartesian coordinates subject to appropriate boundary conditions.)

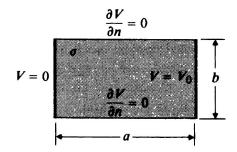


FIGURE 5-14 A conducting sheet (Problem P.5-22).

P.6-2 An electron is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into a region where both an electric field **E** and a magnetic field **B** exist. Describe the motion of the electron if

- a) $\mathbf{E} = \mathbf{a}_z E_0$ and $\mathbf{B} = \mathbf{a}_x B_0$,
- **b)** $\mathbf{E} = -\mathbf{a}_z E_0$ and $\mathbf{B} = -\mathbf{a}_z B_0$.

Discuss the effect of the relative magnitudes of E_0 and B_0 on the electron paths in parts (a) and (b).

P.6-12 Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. 6-39. A current I flows in each coil in the same direction.

- a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.
- **b)** Show that dB_x/dx vanishes at the midpoint.
- c) Find the relation between b and d such that d^2B_x/dx^2 also vanishes at the midpoint. Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as *Helmholtz coils*.

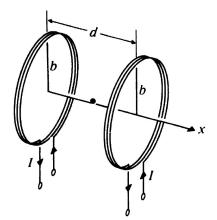


FIGURE 6-39
Helmholtz coils (Problems P.6-12).

P.6-24 Do the following by using Eq. (6-224):

- a) Determine the scalar magnetic potential at a point on the axis of a circular loop having a radius b and carrying a current I.
- **b)** Obtain the magnetic flux density **B** from $-\mu_0 \nabla V_m$, and compare the result with Eq. (6-38).

$$V_{m} = -\frac{I}{4\pi} \Omega, \qquad (6-224)$$

$$\mathbf{B} = \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}$$
 (T).