VE230 HW4

Due: Tuesday 25th June 2019

P.4–1 The upper and lower conducting plates of a large parallel-plate capacitor are separated by a distance d and maintained at potentials V_0 and 0, respectively. A dielectric slab of dielectric constant 6.0 and uniform thickness $0.8d$ is placed over the lower plate. Assuming negligible fringing effect, determine

a) the potential and electric field distribution in the dielectric slab,
b) the potential and electric field distribution in the air space between the dielectric slab and the upper plate,
c) the surface charge densities on the upper and lower plates.
d) Compare the results in part (b) with those without the dielectric slab.

P.4–5 Assume a point charge Q above an infinite conducting plane at $y = 0$.

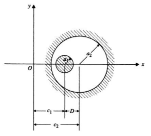
a) Prove that $V(x, y, z)$ in Eq. (4–37) satisfies Laplace's equation if the conducting plane is maintained at zero potential.
b) What should the expression for $V(x, y, z)$ be if the conducting plane has a nonzero potential V_0?
c) What is the electrostatic force of attraction between the charge Q and the conducting plane?

$$V(x, y, z) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right), \qquad (4\text{–}37)$$

P.4–11 A very long two-wire transmission line, each wire of radius a and separated by a distance d, is supported at a height h above a flat conducting ground. Assuming both d and h to be much larger than a, find the capacitance per unit length of the line.

P.4–14 A long wire of radius a_1 lies inside a conducting circular tunnel of radius a_2, as shown in Fig. 4–10(a). The distance between their axes is D.

a) Find the capacitance per unit length.
b) Determine the force per unit length on the wire if the wire and the tunnel carry equal and opposite line charges of magnitude ρ_ℓ.



(a) A cross-sectional view.

P.4–17 Two dielectric media with dielectric constants ϵ_1 and ϵ_2 are separated by a plane boundary at $x = 0$, as shown in Fig. 4–23. A point charge Q exists in medium 1 at distance d from the boundary.

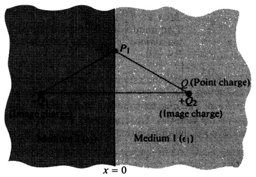


FIGURE 4–23
Image charges in dielectric media (Problem P.4–17).

P.4–23 Two infinite insulated conducting planes maintained at potentials 0 and V_0 form a wedge-shaped configuration, as shown in Fig. 4–24. Determine the potential distributions for the regions: (a) $0 < \phi < \alpha$, and (b) $\alpha < \phi < 2\pi$.

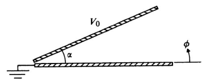


FIGURE 4–24
Two infinite insulated conducting planes maintained at constant potentials (Problem P.4–23).

P.4–28 Rework Example 4–10, assuming that $V(b, \theta) = V_0$ in Eq. (4–155a).

$$V(b, \theta) = 0^\dagger \qquad (4\text{–}155a)$$

Homework 5

You don't need to do P.5-6.

P.5–6 Lightning strikes a lossy dielectric sphere—$\epsilon = 1.2\,\epsilon_0$, $\sigma = 10$ (S/m)—of radius 0.1 (m) at time $t = 0$, depositing uniformly in the sphere a total charge 1 (mC). Determine, for all t,

a) the electric field intensity both inside and outside the sphere,
b) the current density in the sphere.

P.5–7 Refer to Problem P.5–6.

a) Calculate the time it takes for the charge density in the sphere to diminish to 1% of its initial value.
b) Calculate the change in the electrostatic energy stored in the sphere as the charge density diminishes from the initial value to 1% of its value. What happens to this energy?
c) Determine the electrostatic energy stored in the space outside the sphere. Does this energy change with time?

P.5–10 The space between two parallel conducting plates each having an area S is filled with an inhomogeneous ohmic medium whose conductivity varies linearly from σ_1 at one plate ($y = 0$) to σ_2 at the other plate ($y = d$). A d-c voltage V_0 is applied across the plates as in Fig. 5–11. Determine

a) the total resistance between the plates,
b) the surface charge densities on the plates,
c) the volume charge density and the total amount of charge between the plates.

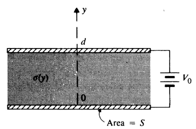


FIGURE 5–11
Inhomogeneous ohmic medium with conductivity $\sigma(y)$ (Problem P.5–10).

P.5–16 Determine the resistance between two concentric spherical surfaces of radii R_1 and R_2 ($R_1 < R_2$), assuming that a material of conductivity $\sigma = \sigma_0(1 + k/R)$ fills the space between them. (*Note:* Laplace's equation for V does not apply here.)

P.5–22 Assume a rectangular conducting sheet of conductivity σ, width a, and height b. A potential difference V_0 is applied to the side edges, as shown in Fig. 5–14. Find

a) the potential distribution,
b) the current density everywhere within the sheet. (*Hint:* Solve Laplace's equation in Cartesian coordinates subject to appropriate boundary conditions.)

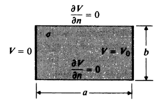


FIGURE 5–14
A conducting sheet (Problem P.5–22).

P.6–2 An electron is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into a region where both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ exist. Describe the motion of the electron if

a) $\mathbf{E} = \mathbf{a}_z E_0$ and $\mathbf{B} = \mathbf{a}_x B_0$,
b) $\mathbf{E} = -\mathbf{a}_z E_0$ and $\mathbf{B} = -\mathbf{a}_z B_0$.

Discuss the effect of the relative magnitudes of E_0 and B_0 on the electron paths in parts (a) and (b).

P.6–12 Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. 6–39. A current I flows in each coil in the same direction.

a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.
b) Show that dB_x/dx vanishes at the midpoint.
c) Find the relation between b and d such that d^2B_x/dx^2 also vanishes at the midpoint.

Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as ***Helmholtz coils***.

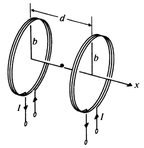


FIGURE 6–39
Helmholtz coils (Problems P.6–12).

P.6–24 Do the following by using Eq. (6–224):

a) Determine the scalar magnetic potential at a point on the axis of a circular loop having a radius b and carrying a current I.

b) Obtain the magnetic flux density $\mathbf{B}$ from $-\mu_0 \nabla V_m$, and compare the result with Eq. (6–38).

$$V_m = -\frac{I}{4\pi}\Omega, \qquad (6\text{–}224)$$

$$\boxed{\mathbf{B} = \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}} \quad \text{(T)}.} \qquad (6\text{–}38)$$

Homework 6

P.6–27 A toroidal iron core of relative permeability 3000 has a mean radius $R = 80$ (mm) and a circular cross section with radius $b = 25$ (mm). An air gap $\ell_g = 3$ (mm) exists, and a current I flows in a 500-turn winding to produce a magnetic flux of 10^{-5} (Wb). (See Fig. 6–44.) Neglecting flux leakage and using mean path length, find

a) the reluctances of the air gap and of the iron core,
b) $\mathbf{B}_g$ and $\mathbf{H}_g$ in the air gap, and $\mathbf{B}_c$ and $\mathbf{H}_c$ in the iron core,
c) the required current I.

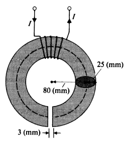


FIGURE 6–44
A toroidal iron core with air gap (Problem P.6–27).

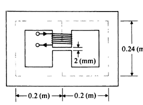


FIGURE 6–45
A magnetic circuit with air gap (Problem P.6–28).

P.6–28 Consider the magnetic circuit in Fig. 6–45. A current of 3 (A) flows through 200 turns of wire on the center leg. Assuming the core to have a constant cross-sectional area of 10^{-3} (m^2) and a relative permeability of 5000:

a) Determine the magnetic flux in each leg.
b) Determine the magnetic field intensity in each leg of the core and in the air gap.

P.6–31 What boundary conditions must the scalar magnetic potential V_m satisfy at an interface between two different magnetic media?

P.6–39 Determine the mutual inductance between a very long, straight wire and a conducting circular loop, as shown in Fig. 6–49.

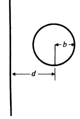


FIGURE 6–49
A long, straight wire and a conducting circular loop (Problem P.6–39).

P.6–43 The cross section of a long thin metal strip and a parallel wire is shown in Fig. 6–51. Equal and opposite currents I flow in the conductors. Find the force per unit length on the conductors.

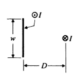


FIGURE 6–51
Cross section of parallel strip and wire conductor (Problem P.6–43).

P.6–48 One end of a long air-core coaxial transmission line having an inner conductor of radius a and an outer conductor of inner radius b is short-circuited by a thin, tight-fitting conducting washer. Find the magnitude and the direction of the magnetic force on the washer when a current I flows in the line.

P.6–53 A current I flows in a long solenoid with n closely wound coil-turns per unit length. The cross-sectional area of its iron core, which has permeability μ, is S. Determine the force acting on the core if it is withdrawn to the position shown in Fig. 6–55.

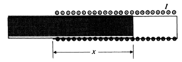


FIGURE 6–55
A long solenoid with iron core partially withdrawn (Problem P.6–53).