VE230 — Electromagnetics I

Homework 7

Instructor: Sung-Liang Chen

Yihao Liu (515370910207)

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P. 7-2

$$\Phi = \int \mathbf{B} d\mathbf{S} = \int_0^{0.2} \int_0^{0.6} 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right) dxdy \cdot 10^{-6}$$
$$= \frac{9\times 10^{-7}}{\pi}[\sin(5\pi 10^7 t) + \sin(0.4\pi - 5\pi 10^7 t)]\text{Wb}.$$

$$V = -\frac{d\Phi}{dt} = -45[\cos(5\pi 10^7 t) - \cos(0.4\pi - 5\pi 10^7 t)],$$
$$i = \frac{V}{2R} = -1.5[\cos(5\pi 10^7 t) - \cos(0.4\pi - 5\pi 10^7 t)].$$

P. 7-6

a)

$$dR = \frac{2\pi r}{\sigma h dr},$$
$$V = \frac{d\Phi}{dt} = \frac{d(B_0 \sin\omega t \cdot \pi r^2)}{dt} = B_0 \omega \pi r^2 \cos\omega t,$$
$$dP = \frac{V^2}{dR} = \frac{B_0^2 \omega^2 \pi^2 r^4 \cos^2 \omega t \cdot \sigma h dr}{2\pi r} = \frac{1}{2} B_0^2 \omega^2 \pi r^3 h\sigma \cos^2 \omega t \cdot dr,$$
$$P = \int dP = \int_0^R \frac{1}{2} B_0^2 \omega^2 \pi r^3 h\sigma \cos^2 \omega t \cdot dr = \frac{1}{8} B_0^2 \omega^2 \pi R^4 h\sigma \cos^2 \omega t,$$
$$\overline{P} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{8} B_0^2 \omega^2 \pi R^4 h\sigma \cos^2 \omega t dt = \frac{1}{16} B_0^2 \omega^2 \pi R^4 h\sigma.$$

b)

$$0.95\pi R^2 = N \cdot \pi R'^2,$$
$$R'^2 = \sqrt{\frac{0.95}{N}} R,$$
$$\overline{P'} = N \cdot \frac{1}{16} B_0^2 \omega^2 \pi R'^4 h\sigma = \frac{0.95^2}{16N} B_0^2 \omega^2 \pi R^4 h\sigma.$$

P. 7-11

$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t},$$
$$\nabla \cdot (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \cdot \mathbf{B}) = 0.$$

So $\nabla \cdot \mathbf{B}$ is a constant, and $\mathbf{B} = \mathbf{0}$ in infinite distance, which means $\nabla \cdot \mathbf{B} = 0$ at that point, so that $\nabla \cdot \mathbf{B} = 0$ always stands.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t},$$

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J} + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = -\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}),$$

$$\nabla \cdot \mathbf{D} = \rho.$$

P. 7-12

$$\begin{cases} \nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \\ \nabla^2 \mathbf{A} - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J} \end{cases} \Longrightarrow \begin{cases} \rho = \varepsilon \left(\mu\varepsilon \frac{\partial^2 V}{\partial t^2} - \nabla^2 V \right) \\ \mathbf{J} = \frac{1}{\mu} \left(\mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} \right) \end{cases},$$

$$-\frac{\partial \rho}{\partial t} = -\varepsilon \left(\mu\varepsilon \frac{\partial^3 V}{\partial t^3} - \nabla^2 \frac{\partial V}{\partial t} \right),$$

$$\nabla \cdot \mathbf{A} = -\mu\varepsilon \frac{\partial V}{\partial t},$$

$$\nabla \cdot \mathbf{J} = \frac{1}{\mu} \left(\mu\varepsilon \frac{\partial^2 (\nabla \cdot \mathbf{A})}{\partial t^2} - \nabla^2 (\nabla \cdot \mathbf{A}) \right) = \varepsilon \left(\mu\varepsilon \frac{\partial^3 V}{\partial t^3} - \nabla^2 \frac{\partial V}{\partial t} \right),$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

P. 7-14

$$\begin{aligned} \mathbf{J} &= \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \\ &= \frac{1}{\mu} \nabla \times \mathbf{B} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} \\ &= \frac{1}{\mu} \nabla \times (\nabla \times \mathbf{A}) + \varepsilon \nabla \frac{\partial V}{\partial t} + \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} \\ &= \frac{1}{\mu} \nabla \times (\nabla \times \mathbf{A}) - \frac{1}{\mu} \nabla (\nabla \cdot \mathbf{A}) + \varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2}, \end{aligned}$$

$$\nabla^2 \mathbf{A} - \mu\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}.$$

$$\rho = \nabla \cdot \mathbf{D} = \varepsilon \nabla \cdot \mathbf{E} = -\varepsilon \nabla^2 V - \varepsilon \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) = -\varepsilon \nabla^2 V + \varepsilon \frac{\partial}{\partial t} \mu\varepsilon \frac{\partial V}{\partial t},$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}.$$

P. 7-17

a)

$$E_{1t} = E_{2t}, \quad B_{1n} = B_{2n}.$$

b)

$$D_{1n} = D_{2n}, \quad H_{1t} = H_{2t}.$$

P. 7-20

Let $u = t \pm R\sqrt{\mu\epsilon}$, $f(u) = U(R,t)$,

$$\left(\frac{\partial u}{\partial R}\right)^2 = \mu\varepsilon, \quad \left(\frac{\partial u}{\partial t}\right)^2 = 1.$$

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 f}{\partial u^2}\left(\frac{\partial u}{\partial R}\right)^2 - \mu\epsilon\frac{\partial^2 f}{\partial u^2}\left(\frac{\partial u}{\partial t}\right)^2 = 0.$$