

P. 4-1

Let x be the up direction.

$$D_1 = D_2,$$

$$\epsilon_0 E_1 = \epsilon_0 \epsilon_r E_2,$$

$$E_1 = 6E_2.$$

Let V_1 be the upper potential of the dielectric slab,

$$\begin{cases} V_0 = E_1 \cdot 0.2d + E_2 \cdot 0.8d \\ V_1 = E_2 \cdot 0.8d \end{cases} \Rightarrow V_1 = 0.4V_0.$$

Then

$$\begin{cases} V_0 = C_1 d + C_2 \\ V_1 = C_1 \cdot 0.8d + C_2 \\ V_1 = C_3 \cdot 0.8d + C_4 \\ 0 = C_3 \cdot 0 + C_4 \end{cases} \Rightarrow \begin{cases} C_1 = \frac{3V_0}{d} \\ C_2 = -2V_0 \\ C_3 = \frac{V_0}{2d} \\ C_4 = 0 \end{cases}.$$

a) In the dielectric slab, $0 \leq x \leq 0.8d$,

$$V = \frac{V_0}{2d}x, \quad \mathbf{E} = -\frac{V_0}{2d}\mathbf{a}_x.$$

b) In the air space between the dielectric slab and the upper plate, $0.8d \leq x \leq d$,

$$V = \frac{3V_0}{d}x - 2V_0, \quad \mathbf{E} = -\frac{3V_0}{d}\mathbf{a}_x.$$

c) On the upper plate,

$$\rho_s = \epsilon_0 |\mathbf{E}| = \frac{3V_0 \epsilon_0}{d}.$$

On the lower plate,

$$\rho_s = -\epsilon_0 \epsilon_r |\mathbf{E}| = -\frac{3V_0 \epsilon_0}{d}.$$

d) If there is no dielectric slab,

$$V = \frac{V_0}{d}x, \quad \mathbf{E} = -\frac{V_0}{d}\mathbf{a}_x.$$

P. 4-5

a)

$$R_+ = [x^2 + (y - d)^2 + z^2]^{1/2},$$

$$R_- = [x^2 + (y + d)^2 + z^2]^{1/2},$$

$$\frac{\partial R_+}{\partial x} = \frac{1}{2R_+} \cdot 2x = \frac{x}{R_+}, \quad \frac{\partial R_-}{\partial x} = \frac{1}{2R_-} \cdot 2x = \frac{x}{R_-},$$

$$\frac{\partial V}{\partial x} = \frac{Q}{4\pi\epsilon_0} \left(\frac{\partial R_+^{-1}}{\partial R_+} \cdot \frac{\partial R_+}{\partial x} - \frac{\partial R_+^{-1}}{\partial R_-} \cdot \frac{\partial R_-}{\partial x} \right) = \frac{Q}{4\pi\epsilon_0} (-R_+^{-3}x + R_-^{-3}x),$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{Q}{4\pi\epsilon_0} \left(-\frac{\partial R_+^{-3}x}{\partial R_+} \cdot \frac{\partial R_+}{\partial x} + \frac{\partial R_-^{-3}x}{\partial R_-} \cdot \frac{\partial R_-}{\partial x} \right) = \frac{Q}{4\pi\epsilon_0} (3R_+^{-5}x^2 - R_+^{-3} - 3R_-^{-5}x^2 + R_-^{-3}).$$

Similarly,

$$\frac{\partial^2 V}{\partial y^2} = \frac{Q}{4\pi\epsilon_0} [3R_+^{-5}(y - d)^2 - R_+^{-3} - 3R_-^{-5}(y + d)^2 + R_-^{-3}],$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{Q}{4\pi\epsilon_0} (3R_+^{-5}z^2 - R_+^{-3} - 3R_-^{-5}z^2 + R_-^{-3}),$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{Q}{4\pi\epsilon_0} \{3R_+^{-5}[x^2 + (y-d)^2 + z^2] - 3R_+^{-3} - 3R_-^{-5}[x^2 + (y+d)^2 + z^2] + 3R_-^{-3}\} = 0.$$

b)

$$V(x, 0, z) = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{(x^2 + d^2 + z^2)^{1/2}} - \frac{1}{(x^2 + d^2 + z^2)^{1/2}} \right] + k = V_0,$$

$$V(x, y, z) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_+} - \frac{1}{R_-} \right) + V_0.$$

c)

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{4d^2} = \frac{Q^2}{16\pi\epsilon_0 d^2}.$$

P. 4-11

P. 4-14

a)

$$C_1 = \frac{1}{2D}(a_2^2 - a_1^2 - D^2), \quad C_2 = \frac{1}{2D}(a^2 - a_1^2 + D^2), \quad b^2 = c_1^2 - a_1^2.$$

$$V_1 = \frac{\rho_l}{2\pi\epsilon_0} \ln \frac{b + (c_1 - a_1)}{b - (c_1 - a_1)},$$

$$V_2 = \frac{\rho_l}{2\pi\epsilon_0} \ln \frac{b + (c_2 - a_2)}{b - (c_2 - a_2)},$$

$$C = \frac{\rho_l}{V_1 - V_2} = 2\pi\epsilon_0 \ln \left[\frac{b + (c_1 - a_1)}{b - (c_1 - a_1)} \cdot \frac{b - (c_1 - a_1)}{b + (c_1 - a_1)} \right]^{-1}.$$

b)

$$F = \frac{1}{2\pi\epsilon_0} \cdot \frac{\rho_l^2}{4b^2}.$$

P. 4-17

a)

$$V_1(x, y, z) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R_+} - \frac{Q_1}{R_-} \right),$$

$$R_+ = [(x-d)^2 + y^2 + z^2]^{1/2},$$

$$R_- = [(x+d)^2 + y^2 + z^2]^{1/2}.$$

Similar to P. 4-5,

$$\nabla^2 V_1 = \frac{Q}{4\pi\epsilon_0} \{3R_+^{-5}[(x-d)^2 + y^2 + z^2] - 3R_+^{-3}\} - \frac{Q_1}{4\pi\epsilon_0} \{3R_-^{-5}[(x+d)^2 + y^2 + z^2] - 3R_-^{-3}\} = 0.$$

b)

$$V_2(x, y, z) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R_+} + \frac{Q_2}{R_+} \right)$$

Similar to P. 4-5,

$$\nabla^2 V_2 = \frac{Q + Q_2}{4\pi\epsilon_0} \{3R_+^{-5}[(x-d)^2 + y^2 + z^2] - 3R_+^{-3}\} = 0.$$

c) When $x = 0$, $R_+ = R_- = R = [d^2 + y^2 + z^2]^{1/2}$,

$$\frac{\partial V_1}{\partial x} = \frac{1}{4\pi\epsilon_0} [-QR_+^{-3}(x-d) + Q_1R_-^{-3}(x+d)] = \frac{d}{4\pi\epsilon_0 R^3} (Q + Q_1),$$

$$\begin{aligned}\frac{\partial V_2}{\partial x} &= \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}(x-d) - Q_2R_+^{-3}(x-d)] = \frac{d}{4\pi\epsilon_0R^3}(Q+Q_2), \\ \frac{\partial V_1}{\partial y} &= \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}y + Q_1R_-^{-3}y] = \frac{1}{4\pi\epsilon_0R^3}(-Q+Q_1), \\ \frac{\partial V_2}{\partial y} &= \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}y - Q_2R_+^{-3}y] = \frac{1}{4\pi\epsilon_0R^3}(-Q-Q_2).\end{aligned}$$

Since

$$\begin{aligned}\frac{\partial V_1}{\partial x} &= \frac{\partial V_2}{\partial x}, \quad \epsilon_1 \frac{\partial V_1}{\partial y} = \epsilon_2 \frac{\partial V_1}{\partial y}, \\ Q_1 &= Q_2 = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1} Q.\end{aligned}$$

P. 4-23

$$\nabla^2 V = \frac{1}{r^2} \cdot \frac{\partial^2 V}{\partial \phi^2} = 0.$$

a) When $\phi = 0$, $V = 0$; when $\phi = \alpha$, $V = V_0$,

$$V = C_1\phi + C_2 = \frac{V_0}{\alpha}\phi.$$

b) When $\phi = 2\pi$, $V = 0$; when $\phi = \alpha$, $V = V_0$,

$$V = C_1\phi + C_2 = \frac{V_0}{\alpha - 2\pi}(\phi - 2\pi).$$

P. 4-28

a)

$$V(b, \theta) = V_0,$$

$$V(R, \theta) = B_0R^{-1} + (B_1R^{-2} - E_0R)\cos\theta + \sum_{n=2}^{\infty} B_nR^{-n-1}P_n\cos\theta, \quad R \geq b.$$

Since the sphere is charged, $B_0 = bV_0$, we can obtain $B_1 = E_0b^3$ and $B_n = 0$ for $n \geq 2$.

$$V(R, \theta) = \frac{bV_0}{R} - E_0 \left[1 - \left(\frac{b}{R} \right)^3 \right] R \cos\theta, \quad R \geq b.$$

b)

$$\mathbf{E}(R, \theta) = \mathbf{a}_R E_R + \mathbf{a}_\theta E_\theta,$$

where

$$E_R = -\frac{\partial V}{\partial R} = -\frac{bV_0}{R^2} + E_0 \left[1 + 2 \left(\frac{b}{R} \right)^3 \right] \cos\theta, \quad R \geq b,$$

$$E_\theta = -\frac{\partial V}{R\partial\theta} = -E_0 \left[1 - \left(\frac{b}{R} \right)^3 \right] R \sin\theta, \quad R \geq b.$$

P. 5-7

a)

$$\exp\left[-\frac{\sigma}{\epsilon}t\right] = 0.01,$$

$$t = -\frac{\epsilon_0\epsilon_r}{\sigma} \ln 0.01 \approx 4.89 \times 10^{-12} \text{ s}.$$

b)

$$\Delta W = \exp \left[-\frac{\sigma}{\varepsilon} t \right]^2 = 1 \times 10^{-4}.$$

The energy are transformed into heat energy.

c)

$$\frac{Q}{\varepsilon_0} = E \cdot 4\pi r^2,$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0},$$

$$W = \int_0^{Q_0} V dQ = \int_0^{Q_0} \int_R^\infty -E dr dQ = \int_0^{Q_0} \frac{Q}{4\pi R \varepsilon_0} dQ = \frac{Q_0^2}{8\pi R \varepsilon_0} \approx 4.494 \times 10^4 \text{ J}.$$

P. 5-10

a)

$$\sigma(y) = \frac{\sigma_2 - \sigma_1}{d} y + \sigma_1,$$

$$\frac{dy}{d\sigma} = \frac{d}{\sigma_2 - \sigma_1},$$

$$R = \int_0^d \frac{1}{\sigma S} dy = \frac{1}{S} \int_{\sigma_1}^{\sigma_2} \frac{1}{\sigma} d\sigma \cdot \frac{dy}{d\sigma} = \frac{d}{S(\sigma_2 - \sigma_1)} \int_{\sigma_1}^{\sigma_2} \frac{1}{\sigma} d\sigma = \frac{d}{S(\sigma_2 - \sigma_1)} \ln \frac{\sigma_2}{\sigma_1}.$$

b)

$$J = \frac{V}{RS} = \frac{V(\sigma_2 - \sigma_1)}{d \ln \frac{\sigma_2}{\sigma_1}},$$

$$\rho_b = -\varepsilon_0 E(0) = -\varepsilon_0 \frac{J}{\sigma_1} = -\frac{\varepsilon_0 V(\sigma_2 - \sigma_1)}{\sigma_1 d \ln \frac{\sigma_2}{\sigma_1}},$$

$$\rho_t = \varepsilon_0 E(d) = \varepsilon_0 \frac{J}{\sigma_2} = \frac{\varepsilon_0 V(\sigma_2 - \sigma_1)}{\sigma_2 d \ln \frac{\sigma_2}{\sigma_1}}.$$

c)

$$\rho = \frac{d}{dy} \varepsilon_0 E = \frac{d}{dy} \left[-\varepsilon_0 J \cdot \frac{d}{\sigma(y)} \right] = \frac{\varepsilon_0 J(\sigma_2 - \sigma_1) d}{[(\sigma_2 - \sigma_1)y + d\sigma_1]^2} = \frac{\varepsilon_0 V(\sigma_2 - \sigma_1)^2}{[(\sigma_2 - \sigma_1)y + d\sigma_1]^2 \ln \frac{\sigma_2}{\sigma_1}}.$$

$$Q = (\rho_b + \rho_t)S = \frac{\varepsilon_0 V(\sigma_2 - \sigma_1)S}{d \ln \frac{\sigma_2}{\sigma_1}} \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right).$$

P. 5-16

$$\frac{1}{R^2(1 + k/R)} = \frac{1}{R(R + k)} = \frac{1}{k} \left(\frac{1}{R} - \frac{1}{R + k} \right),$$

$$R_0 = \int_{R_1}^{R_2} \frac{1}{\sigma \cdot 4\pi R^2} dR = \frac{1}{4\pi\sigma_0 k} \int_{R_1}^{R_2} \left(\frac{1}{R} - \frac{1}{R + k} \right) dR = \frac{1}{4\pi\sigma_0 k} \ln \frac{R}{R + k} \Big|_{R_1}^{R_2} = \frac{1}{4\pi\sigma_0 k} \ln \frac{R_2(R_1 + k)}{R_1(R_2 + k)}.$$

P. 5-22

a)

$$V = \frac{V_0}{a} x.$$

b)

$$\mathbf{E} = -\nabla V = -\frac{V_0}{a} \mathbf{a}_x.$$

$$\mathbf{J} = \sigma \mathbf{E} = -\frac{\sigma V_0}{a} \mathbf{a}_x.$$

P. 6-2

a)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -e(E_0 - u_0 B_0)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on -z and y direction;

If $E_0 < u_0 B_0$, the electron moves on z and y direction;

If $E_0 = u_0 B_0$, the electron moves on y direction.

b)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = e(E_0 \mathbf{a}_z + u_0 B_0 \mathbf{a}_x)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on x-z direction, closer to z direction and y direction;

If $E_0 < u_0 B_0$, the electron moves on x-z direction, closer to x direction and y direction;

If $E_0 = u_0 B_0$, the electron moves on the midline of x-z direction and y direction.

P. 6-12

$$\mathbf{B} = \mathbf{a}_x \cdot \frac{\mu_0 I b^2 N}{2} \left\{ [(x + d/2)^2 + b^2]^{-3/2} + [(x - d/2)^2 + b^2]^{-3/2} \right\}.$$

a)

$$\mathbf{B}(x = 0) = \mathbf{a}_x \cdot \mu_0 I b^2 N [(d/2)^2 + b^2]^{-3/2}.$$

b)

$$\frac{dB_x}{dx} = \frac{\mu_0 I b^2 N}{2} \left\{ -\frac{3}{2} [(x + d/2)^2 + b^2]^{-5/2} (2x + d) - \frac{3}{2} [(x - d/2)^2 + b^2]^{-5/2} (2x - d) \right\},$$

$$\frac{dB_x(0)}{dx} = \frac{\mu_0 I b^2 N}{2} \left\{ -\frac{3}{2} [(d/2)^2 + b^2]^{-5/2} d - \frac{3}{2} [(d/2)^2 + b^2]^{-5/2} (-d) \right\} = 0.$$

c)

$$\frac{d^2 B_x}{dx^2} = \frac{3\mu_0 I b^2 N}{4} \left\{ \frac{5}{2} [(x + d/2)^2 + b^2]^{-7/2} (2x + d)^2 - 2 [(x + d/2)^2 + b^2]^{-5/2} \right. \\ \left. + \frac{5}{2} [(x - d/2)^2 + b^2]^{-7/2} (2x - d)^2 - 2 [(x - d/2)^2 + b^2]^{-5/2} \right\},$$

$$\frac{d^2 B_x(0)}{dx^2} = \frac{3\mu_0 I b^2 N}{4} \left\{ 5d^2 [(d/2)^2 + b^2]^{-7/2} - 4 [(d/2)^2 + b^2]^{-5/2} \right\} = 0,$$

$$5d^2 [(d/2)^2 + b^2]^{-1} - 4 = 0,$$

$$b = d.$$

P. 6-24

a)

$$\begin{aligned} V_m &= \int \frac{I \mathbf{a}_n \cdot \mathbf{a}_R}{4\pi(z^2 + r^2)} dS \\ &= \int_0^{2\pi} \int_0^b \frac{I}{4\pi(z^2 + r^2)} \frac{z}{\sqrt{z^2 + r^2}} r dr d\theta \\ &= \frac{I}{8\pi} \int_0^{2\pi} \int_{z^2}^{z^2 + b^2} z(z^2 + r^2)^{-3/2} d(z^2 + r^2) d\theta \\ &= \frac{I}{8\pi} \int_0^{2\pi} 2[1 - z(z^2 + b^2)^{-1/2}] d\theta \\ &= \frac{I}{2} \left(1 - \frac{z}{\sqrt{z^2 + b^2}} \right). \end{aligned}$$

b)

$$\begin{aligned}
 \mathbf{B} &= -\mu_0 \nabla V_m \\
 &= -\mu_0 \cdot \mathbf{a}_z \frac{I}{2} \left(-\frac{\sqrt{z^2 + b^2} - 2z \cdot \frac{1}{2} / \sqrt{z^2 + b^2} \cdot z}{z^2 + b^2} \right) \\
 &= \mathbf{a}_z \frac{\mu_0 I}{2} \left(\frac{z^2 + b^2 - z^2}{(z^2 + b^2)^{3/2}} \right) \\
 &= \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}.
 \end{aligned}$$

P. 6-27

a)

$$\begin{aligned}
 R_g &= \frac{l_g}{\mu_0 A} = \frac{3 \text{ mm}}{4\pi \times 10^{-7} \text{ H/m} \cdot \pi (25 \text{ mm})^2} = 1.216 \times 10^6 \text{ H}^{-1}. \\
 R_c &= \frac{l_c}{\mu_0 \mu_c A} = \frac{(2\pi \cdot 80 - 3) \text{ mm}}{4\pi \times 10^{-7} \text{ H/m} \cdot 3000 \cdot \pi (25 \text{ mm})^2} = 6.75 \times 10^4 \text{ H}^{-1}.
 \end{aligned}$$

b)

$$\begin{aligned}
 \mathbf{B}_g &= \mathbf{B}_c = \mathbf{a}_\phi \frac{\Phi}{A} = \mathbf{a}_\phi \frac{1 \times 10^{-5} \text{ Wb}}{\pi (25 \text{ mm})^2} = \mathbf{a}_\phi 5.093 \times 10^{-3} \text{ T}, \\
 \mathbf{H}_g &= \frac{\mathbf{B}_g}{\mu_0} = \frac{5.093 \times 10^{-3} \text{ T}}{4\pi \times 10^{-7} \text{ H/m}} = \mathbf{a}_\phi 4.052 \times 10^3 \text{ A/m}, \\
 \mathbf{H}_c &= \frac{\mathbf{B}_c}{\mu_0 \mu_c} = \frac{5.093 \times 10^{-3} \text{ T}}{3000 \cdot 4\pi \times 10^{-7} \text{ H/m}} = \mathbf{a}_\phi 1.351 \times 10^3 \text{ A/m}.
 \end{aligned}$$

c)

$$\begin{aligned}
 \Phi &= \frac{NI_0}{R_g + R_c}, \\
 I_0 &= \frac{\Phi(R_g + R_c)}{N} = \frac{1 \times 10^{-5} \text{ Wb} \cdot (1.216 \times 10^6 \text{ H}^{-1} + 6.75 \times 10^4 \text{ H}^{-1})}{500} = 2.567 \times 10^{-2} \text{ A}.
 \end{aligned}$$

P. 6-31

Let L_1 be the left and right legs, L_2 be the core of the center leg, L_3 be the air gap.

$$\begin{aligned}
 R_1 &= \frac{L_1}{\mu_0 \mu_c A} = \frac{(0.2 + 0.2 + 0.24) \text{ m}}{5000 \cdot 4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 1.019 \times 10^5 \text{ H}^{-1}, \\
 R_2 &= \frac{L_2}{\mu_0 \mu_c A} = \frac{(0.24 - 0.002) \text{ m}}{5000 \cdot 4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 3.788 \times 10^4 \text{ H}^{-1}, \\
 R_3 &= \frac{L_3}{\mu_0 A} = \frac{0.002 \text{ m}}{4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 1.592 \times 10^6 \text{ H}^{-1}.
 \end{aligned}$$

a) In the center leg,

$$\Phi_2 = \frac{NI}{0.5R_1 + R_2 + R_3} = \frac{200 \cdot 3 \text{ A}}{(0.5 \cdot 1.019 \times 10^5 + 3.788 \times 10^4 + 1.592 \times 10^6) \text{ H}^{-1}} = 3.570 \times 10^{-4} \text{ T}.$$

In the left and right leg,

$$\Phi_1 = 0.5\Phi_2 = 1.785 \times 10^{-4} \text{ T}.$$

b)

$$H_1 = \frac{\Phi_1}{\mu_0 \mu_c A} = \frac{1.785 \times 10^{-4} \text{ T}}{5000 \cdot 4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 2.841 \times 10^1 \text{ A/m},$$

$$H_2 = \frac{\Phi_2}{\mu_0 \mu_c A} = \frac{3.570 \times 10^{-4} \text{ T}}{5000 \cdot 4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 5.682 \times 10^1 \text{ A/m},$$

$$H_3 = \frac{\Phi_3}{\mu_0 A} = \frac{3.570 \times 10^{-4} \text{ T}}{4\pi \times 10^{-7} \text{ H/m} \cdot 10^{-3} \text{ m}^2} = 2.841 \times 10^5 \text{ A/m}.$$

P. 6-31

$$B_{1n} = B_{2n}, \quad -\mu_1 \frac{\partial V_{m1}}{\partial n} = -\mu_2 \frac{\partial V_{m2}}{\partial n}.$$

$$H_{1t} = H_{2t}, \quad \frac{\partial V_{m1}}{\partial t} = \frac{\partial V_{m2}}{\partial t}.$$

P. 6-39

$$L = \frac{N\Phi}{I}$$

$$= \frac{1}{I} \int \mathbf{B} d\mathbf{S}$$

$$= \frac{1}{I} \int_0^{2\pi} \int_0^b \frac{\mu_0 I}{2\pi(d + r \cos \theta)} r dr d\theta$$

$$= -\mu_0 \int_0^b \frac{r}{\sqrt{d^2 - r^2}} dr$$

$$= \mu_0(d - \sqrt{d^2 - b^2}).$$

P. 6-43

$$B = \frac{\mu_0 I}{2\pi r},$$

$$dB = \frac{\mu_0 I}{2\pi \sqrt{x^2 + D^2}} \cdot \frac{dx}{w},$$

$$dF = dB \cdot IL = \frac{\mu_0 I}{2\pi \sqrt{x^2 + D^2}} \cdot \frac{dx}{w} \cdot IL \frac{D}{\sqrt{x^2 + D^2}} = \frac{\mu_0 D I^2 L}{2\pi w(x^2 + D^2)} dx,$$

$$F = \int_{-w/2}^{w/2} \frac{\mu_0 D I^2 L}{2\pi w(x^2 + D^2)} dx = \frac{\mu_0 I^2 L}{\pi w} \arctan \frac{w}{2D},$$

$$\frac{F}{L} = \frac{\mu_0 I^2}{\pi w} \arctan \frac{w}{2D}.$$

P. 6-48

$$W_m = \frac{x}{2\mu_0} \int_a^b B^2 2\pi r dr = \frac{\mu_0 I^2 x}{4\pi} \int_a^b \frac{1}{r} r dr = \frac{\mu_0 I^2 x}{4\pi} \ln \frac{b}{a},$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \ln \frac{b}{a}.$$

P. 6-53

$$B_g = \frac{\mu_0 NI}{L} = \mu_0 n I,$$

$$B_c = \frac{\mu NI}{L} = \mu n I.$$

$$W_m = \frac{B_g^2}{2\mu_0} S(L - x) + \frac{B_c^2}{2\mu} Sx = \frac{1}{2} n^2 I^2 S[(\mu - \mu_0)x + \mu_0 L],$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{1}{2} n^2 I^2 S(\mu - \mu_0).$$