VE230 — Electromagnetics I

Homework 1

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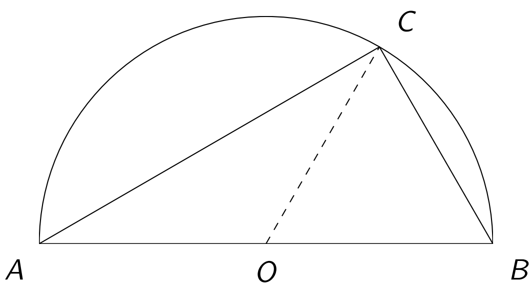
P. 2-11



Since $OA = OB = OC$, we know $\angle OAC = \angle OCA$ and $\angle OBC = \angle OCB$. And since ABC is an triangle, $\angle OAC + \angle OBC + \angle ACB = \pi$, we can simply find that $\angle ACB = \pi/2$, so that an angle inscribed in a semicircle is a right angle.

P. 2-17

a)

$$|\mathbf{E}| = |\mathbf{a_R}(25/R^2)| = |\mathbf{a_R}| \cdot \frac{25}{3^2+4^2+5^2} = \frac{1}{2}.$$

$$E_x = |\mathbf{E}| \cdot \frac{x}{R} = \frac{1}{2} \cdot \frac{-3}{\sqrt{3^2+4^2+5^2}} = -\frac{3\sqrt{2}}{20}.$$

b)

$$\cos\theta = \frac{\mathbf{E}\cdot\mathbf{B}}{|\mathbf{E}||\mathbf{B}|} = \frac{-3\cdot 2 + 4\cdot -2 - 5\cdot 1}{\sqrt{3^2+4^2+5^2}\cdot\sqrt{2^2+2^2+1^1}} = -\frac{19\sqrt{2}}{30},$$

$$\theta = \arccos -\frac{19\sqrt{2}}{30} \approx 2.681\,\text{rad}.$$

P. 2-21

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} (\mathbf{a_x}y + \mathbf{a_y}x)(\mathbf{a_x}dx + \mathbf{a_y}dy) = \int_{P_1}^{P_2}(ydx + xdy)$$

a)

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} (y\cdot 4ydy + 2y^2\cdot dy) = \int_1^2 6y^2dy = 14.$$

b)

$$x = 6y - 4,$$

$$\int_{P_1}^{P_2} \mathbf{E}\cdot d\mathbf{l} = \int_{P_1}^{P_2} [y\cdot 6dy + (6y-4)\cdot dy] = \int_1^2 (12y-4)dy = 14.$$

P. 2-26

a)

$$\nabla \cdot f_1(\mathbf{R}) = \frac{1}{R^2} \cdot \frac{\partial}{\partial R}(R^n \cdot R^2) = (n+2)R^{n-1}.$$

b)

$$\nabla \cdot f_2(\mathbf{R}) = \frac{1}{R^2} \cdot \frac{\partial}{\partial R}(k/R^2 \cdot R^2) = 0.$$

P. 2-29

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \cdot \frac{\partial}{\partial r}(r^2 \cdot r) + \frac{\partial}{\partial z}(2z) = 3r + 2.$$

$$\int_V \nabla \cdot \mathbf{A} dV = \int_V (3r+2) dV = \int_0^4 \int_0^{2\pi} \int_0^5 (3r+2) r dr d\theta dz = 1200\pi.$$

$$\oint_S \mathbf{A} dS = \int_0^4 \int_0^{2\pi} (\mathbf{a_r} r^2) r d\theta dz \mathbf{a_r} + \int_0^{2\pi} \int_0^5 2z\mathbf{a_z} r dr d\theta \mathbf{a_z} = 1000\pi + 200\pi = 1200\pi.$$

So

$$\int_V \nabla \cdot \mathbf{A} dV = \oint_S \mathbf{A} dS.$$

P. 2-33

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \frac{\partial}{\partial x}(E_y H_z - E_z H_y) + \frac{\partial}{\partial y}(E_z H_x - E_x H_z) + \frac{\partial}{\partial z}(E_x H_y - E_y H_x),$$

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = H_x \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + H_y \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + H_z \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right),$$

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = E_x \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + E_y \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + E_z \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right).$$

Since

$$\frac{\partial}{\partial x} AB = A\frac{\partial B}{\partial x} + B\frac{\partial A}{\partial x},$$

we can find that

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}).$$

P. 2-35

$$\Delta s_u = R^2 \sin\theta \Delta\theta \Delta\phi,$$

$$\begin{aligned}\oint_{C_u} \mathbf{A} \cdot d\mathbf{l} =& A_\theta \cdot (R, \theta, \phi - \Delta\phi/2) R\Delta\theta - A_\theta \cdot (R, \theta, \phi + \Delta\phi/2) R\Delta\theta + \\ & A_\phi \cdot (R, \theta + \Delta\theta/2, \phi) R\Delta\phi \sin(\theta + \Delta\theta/2) - A_\phi \cdot (R, \theta - \Delta\theta/2, \phi) R\Delta\phi \sin(\theta - \Delta\theta/2) \\ =& -\frac{\partial A_\theta}{\partial \phi} R\Delta\phi\Delta\theta + \frac{\partial A_\phi \sin\theta}{\partial \theta} R\Delta\phi\Delta\theta.\end{aligned}$$

$$\begin{aligned}(\nabla \times \mathbf{A})_u &= \lim_{\Delta s_u \to 0} \frac{1}{\Delta s_u} \oint_{C_u} \mathbf{A} \cdot d\mathbf{l} = \lim_{\Delta s_u \to 0} \frac{1}{R^2 \sin\theta \Delta\theta\Delta\phi} \cdot \left(-\frac{\partial A_\theta}{\partial \phi} + \frac{\partial A_\phi \sin\theta}{\partial \theta} \right) R\Delta\phi\Delta\theta \\ &= \frac{1}{R\sin\theta} \cdot \left(-\frac{\partial A_\theta}{\partial \phi} + \frac{\partial A_\phi \sin\theta}{\partial \theta} \right).\end{aligned}$$

P. 2-39

a)

$$\nabla \times \mathbf{F} = \mathbf{a_x}(c_3 + 3) + \mathbf{a_y}(c_1 - 1) + \mathbf{a_z} c_2 = \mathbf{0},$$

$$c_1 = 1, c_2 = 0, c_3 = -3.$$

b)

$$\nabla \cdot \mathbf{F} = 1 + 0 + c_4 = 0,$$

$$c_4 = -1.$$

c)

$$\mathbf{F} = -\nabla V = \mathbf{a_x}(x + z) + \mathbf{a_y}(-3z) + \mathbf{a_z}(x - 3y - z),$$

$$V = -\frac{1}{2}x^2 - xz + 3yz + \frac{1}{2}z^2 + C.$$