VE230 — Electromagnetics I

Homework 3

Instructor: Sung-Liang Chen

Yihao Liu (515370910207)

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P. 3-22

a)

$$\rho_{ps} = \mathbf{P} \cdot \mathbf{a_n}|_{n=L/2} = \frac{1}{2}P_0 L.$$

$$\rho_p = -\nabla \cdot \mathbf{P} = -3P_0.$$

b)

$$Q_s = \oint_S \rho_{ps} dS = \frac{1}{2}P_0 L \cdot 6L^2 = 3P_0 L^3,$$

$$Q_v = \int_v \rho_p dV = \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \rho_p dzdydx = -3P_0 L^3,$$

$$Q = Q_s + Q_v = 0.$$

P. 3-23

Let $\mathbf{P} = \mathbf{a_p} P_0$, $\theta = \langle \mathbf{P}, \mathbf{a_n} \rangle$,

$$\rho_{ps}(\theta) = \mathbf{P} \cdot \mathbf{a_n} = P_0 \cos\theta,$$

$$dE_\theta = dv \cdot \frac{\rho_{ps}}{4\pi\varepsilon_0 R^2} \cdot \cos\theta = 2\pi R^2 \sin\theta d\theta \cdot \frac{P_0 \cos\theta}{4\pi\varepsilon_0 R^2} \cdot \cos\theta = \frac{P_0 \sin\theta \cos\theta^2}{2\varepsilon_0} d\theta.$$

$$|\mathbf{E}| = \int dE_\theta = \int_0^\pi \frac{P_0 \sin\theta \cos\theta^2}{2\varepsilon_0} d\theta = \frac{P_0}{3\varepsilon_0},$$

$$\mathbf{E} = \mathbf{a_p} \frac{P_0}{3\varepsilon_0} = \frac{\mathbf{P}}{3\varepsilon_0}.$$

P. 3-25

$$E_{2t} = E_{1t} = \mathbf{a_x} 2y - \mathbf{a_y} 3x.$$

Since $\rho_s = 0$,

$$\varepsilon_{r1} E_{1n} = \varepsilon_{r2} E_{2n},$$

$$E_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_{1n} = \frac{2}{3} \cdot \mathbf{a_z} 5 = \mathbf{a_z} \frac{10}{3}.$$

$$E_2 = E_{2t} + E_{2n} = \mathbf{a_x} 2y - \mathbf{a_y} 3x + \mathbf{a_z} \frac{10}{3}.$$

$$\mathbf{D_2} = \varepsilon_2 \mathbf{E_2} = 3\varepsilon_0 \left(\mathbf{a_x} 2y - \mathbf{a_y} 3x + \mathbf{a_z} \frac{10}{3} \right).$$

P. 3-28

Obviously, $\mathbf{E_3}$ is parallel to $\mathbf{E_2}$, so we only need to find ε_{r2} so that $\mathbf{E_2}$ is parallel to the x-axis.

$$E_{1t} = E_{2t} = -3.$$

$$\varepsilon_{r1} E_{1n} = \varepsilon_{r2} E_{2n},$$
$$E_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} E_{1n} = \frac{1}{\varepsilon_{r2}} \cdot 5 = \frac{5}{\varepsilon_{r2}}.$$
$$E_{2t} \cos\theta + E_{2n} \cos\theta = 0,$$
$$-3 + \frac{5}{\varepsilon_{r2}} = 0,$$
$$\varepsilon_{r2} = \frac{5}{3}.$$

P. 3-32

$$C_1 = \frac{2\pi\varepsilon L}{\ln \frac{b}{r_i}} = \frac{2\pi\varepsilon_0\varepsilon_{r_1} L}{\ln \frac{b}{r_i}},$$
$$C_2 = \frac{2\pi\varepsilon L}{\ln \frac{r_o}{b}} = \frac{2\pi\varepsilon_0\varepsilon_{r_2} L}{\ln \frac{r_o}{b}},$$
$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{2\pi\varepsilon_0 L}{\frac{1}{\varepsilon_{r_1}} \ln \frac{b}{r_i} + \frac{1}{\varepsilon_{r_3}} \ln \frac{r_o}{b}},$$
$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\frac{1}{\varepsilon_{r_1}} \ln \frac{b}{r_i} + \frac{1}{\varepsilon_{r_3}} \ln \frac{r_o}{b}}.$$

P. 3-43

$$dW_e = Vdq = \frac{q}{C} dq,$$
$$W_e = \int dW_e = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Since $Q = CV$, we can also get

$$W_e = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$