VE230 — Electromagnetics I

Homework 8

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$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu \frac{\partial}{\partial t}\left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) = -\mu \frac{\partial \mathbf{J}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2},$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla \rho - \nabla^2 \mathbf{E},$$

$$\nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla \rho + \mu \frac{\partial \mathbf{J}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2},$$

$$\nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla \rho + \mu j\omega \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{E}.$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla \times \mathbf{J} + \varepsilon \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = \nabla \times \mathbf{J} - \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = -\nabla^2 \mathbf{H},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{H}.$$

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$$\begin{aligned}
\nabla \times \mathbf{E} &= \frac{1}{R^2 \sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & E_0 \sin\theta e^{-jkR} & 0 \end{vmatrix} \\
&= \frac{1}{R^2 \sin\theta}\left(\mathbf{a}_R \frac{\partial}{\partial \phi} E_0 \sin\theta e^{-jkR} - \mathbf{a}_\phi R\sin\theta \frac{\partial}{\partial R} E_0 \sin\theta e^{-jkR}\right) \\
&= \mathbf{a}_\phi \frac{-E_0 jk \sin\theta e^{-jkR}}{R},
\end{aligned}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} = -j\mu\omega \mathbf{H},$$

$$\mathbf{H} = \mathbf{a}_\phi \frac{E_0 k \sin\theta e^{-jkR}}{\mu\omega R}, \quad k = \omega\sqrt{\mu\varepsilon}.$$

$$\mathbf{H}(R) = \mathbf{a}_\phi \frac{E_0 \sqrt{\mu\varepsilon}\sin\theta e^{-j\omega\sqrt{\mu\varepsilon}R}}{\mu R},$$

$$\mathbf{H}(R,t) = \mathrm{Re}[\mathbf{H}(R)e^{j\omega t}] = \mathbf{a}_\phi \frac{E_0\sqrt{\mu\varepsilon}}{\mu R}\sin\theta\cos(\omega t - \omega\sqrt{\mu\varepsilon}R).$$

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$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H} = \omega^2\mu_0\varepsilon_0\nabla \times \pi_e,$$

$$\nabla \times (\mathbf{E} - \omega^2\mu_0\varepsilon_0\nabla \times \pi_e) = \mathbf{0},$$

$$\mathbf{E} = \omega^2\mu_0\varepsilon_0\nabla \times \pi_e + \mathbf{C}.$$

$$\nabla \times \mathbf{H} = j\omega\mathbf{D} = \omega^2\mu_0\varepsilon_0\nabla \times \pi_e,$$

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