Homework 7

P.7–2 The circuit in Fig. 7–10 is situated in a magnetic field

$$\mathbf{B} = \mathbf{a}_z 3 \cos(5\pi 10^7 t - \tfrac{2}{3}\pi x) \qquad (\mu\text{T}).$$

Assuming $R = 15\ (\Omega)$, find the current i.

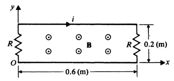
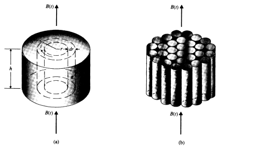


FIGURE 7–10
A circuit in a time-varying magnetic field (Problem P.7–2).



P.7–6 A suggested scheme for reducing eddy-current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig. 7–12, the section shown in part (a) is replaced by that in part (b). Assuming that $B(t) = B_0 \sin \omega t$ and that N filamentary areas fill 95% of the original cross-sectional area, find

a) the average eddy-current power loss in the section of core of height h in Fig. 7–12(a),
b) the total average eddy-current power loss in the N filamentary sections in Fig. 7–12(b).

The magnetic field due to eddy currents is assumed to be negligible. (*Hint:* First find the current and power dissipated in the differential circular ring section of height h and width dr at radius r.)

$$\boxed{\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}.} \qquad (7\text{–}63)$$

$$\boxed{\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon},} \qquad (7\text{–}65)$$

P.7–17 Discuss the relations

a) between the boundary conditions for the tangential components of **E** and those for the normal components of **B**,
b) between the boundary conditions for the normal components of **D** and those for the tangential components of **H**.

P.7–20 Prove by direct substitution that any twice differentiable function of $(t - R\sqrt{\mu\epsilon})$ or of $(t + R\sqrt{\mu\epsilon})$ is a solution of the homogeneous wave equation, Eq. (7–73).

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon \frac{\partial^2 U}{\partial t^2} = 0. \qquad (7\text{–}73)$$

P.7–11 Derive the two divergence equations, Eqs. (7–53c) and (7–53d), from the two curl equations, Eqs. (7–53a) and (7–53b), and the equation of continuity, Eq. (7–48).

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad (7\text{–}53a)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \qquad (7\text{–}53b)$$

$$\nabla \cdot \mathbf{D} = \rho, \qquad (7\text{–}53c)$$

$$\nabla \cdot \mathbf{B} = 0. \qquad (7\text{–}53d)$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \qquad (7\text{–}48)$$

P.7–12 Prove that the Lorentz condition for potentials as expressed in Eq. (7–62) is consistent with the equation of continuity.

$$\boxed{\nabla \cdot \mathbf{A} + \mu\epsilon \frac{\partial V}{\partial t} = 0,} \qquad (7\text{–}62)$$

P.7–14 Substitute Eqs. (7–55) and (7–57) in Maxwell's equations to obtain wave equations for scalar potential V and vector potential **A** for a linear, isotropic but inhomogeneous medium. Show that these wave equations reduce to Eqs. (7–65) and (7–63) for simple media. (*Hint:* Use the following gauge condition for potentials in an inhomogeneous medium:

$$\nabla \cdot (\epsilon \mathbf{A}) + \mu\epsilon^2 \frac{\partial V}{\partial t} = 0.) \qquad (7\text{–}117)$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} \qquad (\text{T}).} \qquad (7\text{–}55)$$

$$\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \qquad (\text{V/m}).} \qquad (7\text{–}57)$$

P.7–24 Derive the general wave equations for **E** and **H** in a nonconducting simple medium where a charge distribution ρ and a current distribution **J** exist. Convert the wave equations to Helmholtz's equations for sinusoidal time dependence. Write the general solutions for $\mathbf{E}(R, t)$ and $\mathbf{H}(R, t)$ in terms of ρ and **J**.

P.7–27 It is known that the electric field intensity of a spherical wave in free space is

$$\mathbf{E} = \mathbf{a}_\theta \frac{E_0}{R} \sin\theta \cos(\omega t - kR).$$

Determine the magnetic field intensity **H** and the value of k.

P.7–29 For a source-free polarized medium where $\rho = 0$, $\mathbf{J} = 0$, $\mu = \mu_0$, but where there is a volume density of polarization **P**, a single vector potential $\boldsymbol{\pi}_e$ may be defined such that

$$\mathbf{H} = j\omega\epsilon_0 \nabla \times \boldsymbol{\pi}_e. \qquad (7\text{–}118)$$

a) Express electric field intensity **E** in terms of $\boldsymbol{\pi}_e$ and **P**.

b) Show that $\boldsymbol{\pi}_e$ satisfies the nonhomogeneous Helmholtz's equation

$$\nabla^2 \boldsymbol{\pi}_e + k_0^2 \boldsymbol{\pi}_e = -\frac{\mathbf{P}}{\epsilon_0}. \qquad (7\text{–}119)$$

The quantity $\boldsymbol{\pi}_e$ is known as the ***electric Hertz potential***.

P.8–7 Show that a plane wave with an instantaneous expression for the electric field

$$\mathbf{E}(z, t) = \mathbf{a}_x E_{10} \sin(\omega t - kz) + \mathbf{a}_y E_{20} \sin(\omega t - kz + \psi)$$

is elliptically polarized. Find the polarization ellipse.

P.8–9 Derive the following general expressions of the attenuation and phase constants for conducting media:

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]^{1/2} \qquad (\text{Np/m}).$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right]^{1/2} \qquad (\text{rad/m}).$$

P.8–14 Assume the ionosphere to be modeled by a plasma region with an electron density that increases with altitude from a low value at the lower boundary toward a value $N_{\max}$ and decreases again as the altitude gets higher. A plane electromagnetic wave impinges on the lower boundary at an angle θ_i with the normal. Determine the highest frequency of the wave that will be turned back toward the earth. (*Hint:* Imagine the ionosphere to be stratified into layers of successively decreasing constant permittivities until the layer containing $N_{\max}$. The frequency to be determined corresponds to that for an emerging angle of $\pi/2$.)

P.8–15 Prove the following relations between group velocity u_g and phase velocity u_p in a dispersive medium:

a) $u_g = u_p + \beta \dfrac{du_p}{d\beta}$ **b)** $u_g = u_p - \lambda \dfrac{du_p}{d\lambda}$.