VE230 — Electromagnetics I

Homework 5

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P. 5-7

a)

$$\exp\left[-\frac{\sigma}{\varepsilon}t\right] = 0.01,$$

$$t = -\frac{\varepsilon_0\varepsilon_r}{\sigma}\ln 0.01 \approx 4.89\times 10^{-12}\,\text{s}.$$

b)

$$\Delta W = \exp\left[-\frac{\sigma}{\varepsilon}t\right]^2 = 1\times 10^{-4}.$$

The energy are transformed into heat energy.

c)

$$\frac{Q}{\varepsilon_0} = E\cdot 4\pi r^2,$$

$$E = \frac{Q}{4\pi r^2\varepsilon_0},$$

$$W = \int_0^{Q_0} VdQ = \int_0^{Q_0}\int_R^{\infty} -EdrdQ = \int_0^{Q_0}\frac{Q}{4\pi R\varepsilon_0}dQ = \frac{Q_0^2}{8\pi R\varepsilon_0} \approx 4.494\times 10^4\,\text{J}.$$

P. 5-10

a)

$$\sigma(y) = \frac{\sigma_2-\sigma_1}{d}y + \sigma_1,$$

$$\frac{dy}{d\sigma} = \frac{d}{\sigma_2-\sigma_1},$$

$$R = \int_0^d \frac{1}{\sigma S}dy = \frac{1}{S}\int_{\sigma_1}^{\sigma_2}\frac{1}{\sigma}d\sigma\cdot\frac{dy}{d\sigma} = \frac{d}{S(\sigma_2-\sigma_1)}\int_{\sigma_1}^{\sigma_2}\frac{1}{\sigma}d\sigma = \frac{d}{S(\sigma_2-\sigma_1)}\ln\frac{\sigma_2}{\sigma_1}.$$

b)

$$J = \frac{V}{RS} = \frac{V(\sigma_2-\sigma_1)}{d\ln\frac{\sigma_2}{\sigma_1}},$$

$$\rho_b = -\varepsilon_0 E(0) = -\varepsilon_0\frac{J}{\sigma_1} = -\frac{\varepsilon_0 V(\sigma_2-\sigma_1)}{\sigma_1 d\ln\frac{\sigma_2}{\sigma_1}},$$

$$\rho_t = \varepsilon_0 E(d) = \varepsilon_0\frac{J}{\sigma_2} = \frac{\varepsilon_0 V(\sigma_2-\sigma_1)}{\sigma_2 d\ln\frac{\sigma_2}{\sigma_1}}.$$

c)

$$\rho = \frac{d}{dy}\varepsilon_0 E = \frac{d}{dy}\left[-\varepsilon_0 J\cdot\frac{d}{\sigma(y)}\right] = \frac{\varepsilon_0 J(\sigma_2-\sigma_1)d}{[(\sigma_2-\sigma_1)y+d\sigma_1]^2} = \frac{\varepsilon_0 V(\sigma_2-\sigma_1)^2}{[(\sigma_2-\sigma_1)y+d\sigma_1]^2\ln\frac{\sigma_2}{\sigma_1}}.$$

$$Q = (\rho_b + \rho_t)S = \frac{\varepsilon_0 V(\sigma_2 - \sigma_1)S}{d \ln \frac{\sigma_2}{\sigma_1}} \left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right).$$

P. 5-16

$$\frac{1}{R^2(1 + k/R)} = \frac{1}{R(R + k)} = \frac{1}{k} \left(\frac{1}{R} - \frac{1}{R + k} \right),$$

$$R_0 = \int_{R_1}^{R_2} \frac{1}{\sigma \cdot 4\pi R^2} dR = \frac{1}{4\pi\sigma_0 k} \int_{R_1}^{R_2} \left(\frac{1}{R} - \frac{1}{R + k} \right) dR = \frac{1}{4\pi\sigma_0 k} \ln \frac{R}{R + k} \bigg|_{R_1}^{R_2} = \frac{1}{4\pi\sigma_0 k} \ln \frac{R_2(R_1 + k)}{R_1(R_2 + k)}.$$

P. 5-22

a)

$$V = \frac{V_0}{a} x.$$

b)

$$\mathbf{E} = -\nabla V = -\frac{V_0}{a} \mathbf{a}_x.$$

$$\mathbf{J} = \sigma \mathbf{E} = -\frac{\sigma V_0}{a} \mathbf{a}_x.$$

P. 6-2

a)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -e(E_0 - u_0 B_0)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on -z and y direction;
If $E_0 < u_0 B_0$, the electron moves on z and y direction;
If $E_0 = u_0 B_0$, the electron moves on y direction.

b)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = e(E_0 \mathbf{a}_z + u_0 B_0 \mathbf{a}_x)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on x-z direction, closer to z direction and y direction;
If $E_0 < u_0 B_0$, the electron moves on x-z direction, closer to x direction and y direction;
If $E_0 = u_0 B_0$, the electron moves on the midline of x-z direction and y direction.

P. 6-12

$$\mathbf{B} = \mathbf{a}_x \cdot \frac{\mu_0 I b^2 N}{2} \left\{ [(x + d/2)^2 + b^2]^{-3/2} + [(x - d/2)^2 + b^2]^{-3/2} \right\}.$$

a)

$$\mathbf{B}(x = 0) = \mathbf{a}_x \cdot \mu_0 I b^2 N [(d/2)^2 + b^2]^{-3/2}.$$

b)

$$\frac{dB_x}{dx} = \frac{\mu_0 I b^2 N}{2} \left\{ -\frac{3}{2} [(x + d/2)^2 + b^2]^{-5/2} (2x + d) - \frac{3}{2} [(x - d/2)^2 + b^2]^{-5/2} (2x - d) \right\},$$

$$\frac{dB_x(0)}{dx} = \frac{\mu_0 I b^2 N}{2} \left\{ -\frac{3}{2} [(d/2)^2 + b^2]^{-5/2} d - \frac{3}{2} [(d/2)^2 + b^2]^{-5/2} (-d) \right\} = 0.$$

c)

$$\frac{d^2B_x}{dx^2} = \frac{3\mu_0 I b^2 N}{4}\left\{\frac{5}{2}[(x+d/2)^2+b^2]^{-7/2}(2x+d)^2 - 2[(x+d/2)^2+b^2]^{-5/2}\right.$$
$$\left. +\frac{5}{2}[(x-d/2)^2+b^2]^{-7/2}(2x-d)^2 - 2[(x-d/2)^2+b^2]^{-5/2}\right\},$$

$$\frac{d^2B_x(0)}{dx^2} = \frac{3\mu_0 I b^2 N}{4}\left\{5d^2[(d/2)^2+b^2]^{-7/2} - 4[(d/2)^2+b^2]^{-5/2}\right\} = 0,$$
$$5d^2[(d/2)^2+b^2]^{-1} - 4 = 0,$$
$$b = d.$$

P. 6-24

a)

$$\begin{aligned}
V_m &= \int \frac{I\mathbf{a}_n \cdot \mathbf{a}_R}{4\pi(z^2+r^2)}dS \\
&= \int_0^{2\pi}\int_0^b \frac{I}{4\pi(z^2+r^2)}\frac{z}{\sqrt{z^2+r^2}}rdrd\theta \\
&= \frac{I}{8\pi}\int_0^{2\pi}\int_{z^2}^{z^2+b^2} z(z^2+r^2)^{-3/2}d(z^2+r^2)d\theta \\
&= \frac{I}{8\pi}\int_0^{2\pi} 2[1 - z(z^2+b^2)^{-1/2}]d\theta \\
&= \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2+b^2}}\right).
\end{aligned}$$

b)

$$\begin{aligned}
\mathbf{B} &= -\mu_0 \nabla V_m \\
&= -\mu_0 \cdot \mathbf{a}_z \frac{I}{2}\left(-\frac{\sqrt{z^2+b^2} - 2z\cdot\frac{1}{2}/\sqrt{z^2+b^2}\cdot z}{z^2+b^2}\right) \\
&= \mathbf{a}_z \frac{\mu_0 I}{2}\left(\frac{z^2+b^2-z^2}{(z^2+b^2)^{3/2}}\right) \\
&= \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}.
\end{aligned}$$