VE230 — Electromagnetics I

Homework 6

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a)

$$R_g = \frac{l_g}{\mu_0 A} = \frac{3\,\mathrm{mm}}{4\pi \times 10^{-7}\,\mathrm{H/m} \cdot \pi(25\,\mathrm{mm})^2} = 1.216 \times 10^6\,\mathrm{H^{-1}}.$$

$$R_c = \frac{l_c}{\mu_0 \mu_c A} = \frac{(2\pi \cdot 80 - 3)\mathrm{mm}}{4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 3000 \cdot \pi(25\,\mathrm{mm})^2} = 6.75 \times 10^4\,\mathrm{H^{-1}}.$$

b)

$$\mathbf{B}_g = \mathbf{B}_c = \mathbf{a}_\phi \frac{\Phi}{A} = \mathbf{a}_\phi \frac{1 \times 10^{-5}\,\mathrm{Wb}}{\pi(25\,\mathrm{mm})^2} = a_\phi 5.093 \times 10^{-3}\,\mathrm{T},$$

$$\mathbf{H}_g = \frac{\mathbf{B}_g}{\mu_0} = \frac{5.093 \times 10^{-3}\,\mathrm{T}}{4\pi \times 10^{-7}\,\mathrm{H/m}} = a_\phi 4.052 \times 10^3\,\mathrm{A/m},$$

$$\mathbf{H}_c = \frac{\mathbf{B}_c}{\mu_0 \mu_c} = \frac{5.093 \times 10^{-3}\,\mathrm{T}}{3000 \cdot 4\pi \times 10^{-7}\,\mathrm{H/m}} = a_\phi 1.351 \times 10^3\,\mathrm{A/m}.$$

c)

$$\Phi = \frac{NI_0}{R_g + R_c},$$

$$I_0 = \frac{\Phi(R_g + R_c)}{N} = \frac{1 \times 10^{-5}\,\mathrm{Wb} \cdot (1.216 \times 10^6\,\mathrm{H^{-1}} + 6.75 \times 10^4\,\mathrm{H^{-1}})}{500} = 2.567 \times 10^{-2}\,\mathrm{A}.$$

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Let L_1 be the left and right legs, L_2 be the core of the center leg, L_3 be the air gap.

$$R_1 = \frac{L_1}{\mu_0 \mu_c A} = \frac{(0.2 + 0.2 + 0.24)\mathrm{m}}{5000 \cdot 4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m^2}} = 1.019 \times 10^5\,\mathrm{H^{-1}},$$

$$R_2 = \frac{L_2}{\mu_0 \mu_c A} = \frac{(0.24 - 0.002)\mathrm{m}}{5000 \cdot 4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m^2}} = 3.788 \times 10^4\,\mathrm{H^{-1}},$$

$$R_3 = \frac{L_3}{\mu_0 A} = \frac{0.002\mathrm{m}}{4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m^2}} = 1.592 \times 10^6\,\mathrm{H^{-1}}.$$

a) In the center leg,

$$\Phi_2 = \frac{NI}{0.5R_1 + R_2 + R_3} = \frac{200 \cdot 3\,\mathrm{A}}{(0.5 \cdot 1.019 \times 10^5 + 3.788 \times 10^4 + 1.592 \times 10^6)\mathrm{H^{-1}}} = 3.570 \times 10^{-4}\,\mathrm{T}.$$

In the left and right leg,

$$\Phi_1 = 0.5\Phi_2 = 1.785 \times 10^{-4}\,\mathrm{T}.$$

b)

$$H_1 = \frac{\Phi_1}{\mu_0 \mu_c A} = \frac{1.785 \times 10^{-4}\,\mathrm{T}}{5000 \cdot 4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m}^2} = 2.841 \times 10^{1}\,\mathrm{A/m},$$

$$H_2 = \frac{\Phi_2}{\mu_0 \mu_c A} = \frac{3.570 \times 10^{-4}\,\mathrm{T}}{5000 \cdot 4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m}^2} = 5.682 \times 10^{1}\,\mathrm{A/m},$$

$$H_3 = \frac{\Phi_3}{\mu_0 A} = \frac{3.570 \times 10^{-4}\,\mathrm{T}}{4\pi \times 10^{-7}\,\mathrm{H/m} \cdot 10^{-3}\,\mathrm{m}^2} = 2.841 \times 10^{5}\,\mathrm{A/m}.$$

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$$B_{1n} = B_{2n}, \quad -\mu_1 \frac{\partial V_{m1}}{\partial n} = -\mu_2 \frac{\partial V_{m2}}{\partial n}.$$

$$H_{1t} = H_{2t}, \quad \frac{\partial V_{m1}}{\partial t} = \frac{\partial V_{m2}}{\partial t}.$$

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$$\begin{aligned} L &= \frac{N\Phi}{I} \\ &= \frac{1}{I} \int \mathbf{B} d\mathbf{S} \\ &= \frac{1}{I} \int_0^{2\pi} \int_0^b \frac{\mu_0 I}{2\pi(d + r\cos\theta)} r dr d\theta \\ &= -\mu_0 \int_0^b \frac{r}{\sqrt{d^2 - r^2}} dr \\ &= \mu_0 (d - \sqrt{d^2 - b^2}). \end{aligned}$$

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$$B = \frac{\mu_0 I}{2\pi r},$$

$$dB = \frac{\mu_0 I}{2\pi\sqrt{x^2 + D^2}} \cdot \frac{dx}{w},$$

$$dF = dB \cdot IL = \frac{\mu_0 I}{2\pi\sqrt{x^2 + D^2}} \cdot \frac{dx}{w} \cdot IL \frac{D}{\sqrt{x^2 + D^2}} = \frac{\mu_0 D I^2 L}{2\pi w (x^2 + d^2)} dx,$$

$$F = \int_{-w/2}^{w/2} \frac{\mu_0 D I^2 L}{2\pi w (x^2 + d^2)} dx = \frac{\mu_0 I^2 L}{\pi w} \arctan \frac{w}{2D},$$

$$\frac{F}{L} = \frac{\mu_0 I^2}{\pi w} \arctan \frac{w}{2D}.$$

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$$W_m = \frac{x}{2\mu_0} \int_a^b B^2 2\pi r dr = \frac{\mu_0 I^2 x}{4\pi} \int_a^b \frac{1}{r} r dr = \frac{\mu_0 I^2 x}{4\pi} \ln \frac{b}{a},$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \ln \frac{b}{a}.$$

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$$B_g = \frac{\mu_0 N I}{L} = \mu_0 n I,$$

$$B_c = \frac{\mu N I}{L} = \mu n I.$$

$$W_m = \frac{B_g^2}{2\mu_0} S(L - x) + \frac{B_c^2}{2\mu} Sx = \frac{1}{2} n^2 I^2 S[(\mu - \mu_0)x + \mu_0)],$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{1}{2} n^2 I^2 S(\mu - \mu_0).$$