P. 4-1

Let x be the up direction.

$$D_1 = D_2,$$
$$\varepsilon_0 E_1 = \varepsilon_0 \varepsilon_r E_2,$$
$$E_1 = 6E_2.$$

Let V_1 be the upper potential of the dielectric slab,

$$\begin{cases} V_0 = E_1 \cdot 0.2d + E_2 \cdot 0.8d \\ V_1 = E_2 \cdot 0.8d \end{cases} \Longrightarrow V_1 = 0.4V_0.$$

Then

$$\begin{cases} V_0 = C_1 d + C_2 \\ V_1 = C_1 \cdot 0.8d + C_2 \\ V_1 = C_3 \cdot 0.8d + C_4 \\ \ 0 = C_3 \cdot 0 + C_4 \end{cases} \Longrightarrow \begin{cases} C_1 = \dfrac{3V_0}{d} \\ C_2 = -2V_0 \\ C_3 = \dfrac{V_0}{2d} \\ C_4 = 0 \end{cases}.$$

a) In the dielectric slab, $0 \leqslant x \leqslant 0.8d$,

$$V = \frac{V_0}{2d}x, \quad \mathbf{E} = -\frac{V_0}{2d}\mathbf{a_x}.$$

b) In the air space between the dielectric slab and the upper plate, $0.8d \leqslant x \leqslant d$,

$$V = \frac{3V_0}{d}x - 2V_0, \quad \mathbf{E} = -\frac{3V_0}{d}\mathbf{a_x}.$$

c) On the upper plate,

$$\rho_s = \varepsilon_0|\mathbf{E}| = \frac{3V_0\varepsilon_0}{d}.$$

On the lower plate,

$$\rho_s = -\varepsilon_0\varepsilon_r|\mathbf{E}| = -\frac{3V_0\varepsilon_0}{d}.$$

d) If there is no dielectric slab,

$$V = \frac{V_0}{d}x, \quad \mathbf{E} = -\frac{V_0}{d}\mathbf{a_x}.$$

P. 4-5

a)

$$R_+ = [x^2 + (y-d)^2 + z^2]^{1/2},$$
$$R_- = [x^2 + (y+d)^2 + z^2]^{1/2},$$
$$\frac{\partial R_+}{\partial x} = \frac{1}{2R_+} \cdot 2x = \frac{x}{R_+}, \frac{\partial R_-}{\partial x} = \frac{1}{2R_-} \cdot 2x = \frac{x}{R_-},$$
$$\frac{\partial V}{\partial x} = \frac{Q}{4\pi\epsilon_0}\left(\frac{\partial R_+^{-1}}{\partial R_+} \cdot \frac{\partial R_+}{\partial x} - \frac{\partial R_+^{-1}}{\partial R_-} \cdot \frac{\partial R_-}{\partial x}\right) = \frac{Q}{4\pi\epsilon_0}(-R_+^{-3}x + R_-^{-3}x),$$
$$\frac{\partial^2 V}{\partial x^2} = \frac{Q}{4\pi\epsilon_0}\left(-\frac{\partial R_+^{-3}x}{\partial R_+} \cdot \frac{\partial R_+}{\partial x} + \frac{\partial R_-^{-3}x}{\partial R_-} \cdot \frac{\partial R_-}{\partial x}\right) = \frac{Q}{4\pi\epsilon_0}(3R_+^{-5}x^2 - R_+^{-3} - 3R_-^{-5}x^2 + R_-^{-3}).$$

Similarly,

$$\frac{\partial^2 V}{\partial y^2} = \frac{Q}{4\pi\epsilon_0}[3R_+^{-5}(y-d)^2 - R_+^{-3} - 3R_-^{-5}(y+d)^2 + R_-^{-3}],$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{Q}{4\pi\epsilon_0}(3R_+^{-5}z^2 - R_+^{-3} - 3R_-^{-5}z^2 + R_-^{-3}),$$
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{Q}{4\pi\epsilon_0}\{3R_+^{-5}[x^2+(y-d)^2+z^2] - 3R_+^{-3} - 3R_-^{-5}[x^2+(y+d)^2+z^2] + 3R_-^{-3}\} = 0.$$

b)

$$V(x,0,z) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{(x^2+d^2+z^2)^{1/2}} - \frac{1}{(x^2+d^2+z^2)^{1/2}}\right] + k = V_0,$$
$$V(x,y,z) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right) + V_0.$$

c)

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{4d^2} = \frac{Q^2}{16\pi\epsilon_0 d^2}.$$

P. 4-11

P. 4-14

a)

$$C_1 = \frac{1}{2D}(a_2^2 - a_1^2 - D^2), \quad C_2 = \frac{1}{2D}(a^2 - a_1^2 + D^2), \quad b^2 = c_1^2 - a_1^2.$$
$$V_1 = \frac{\rho_l}{2\pi\epsilon_0}\ln\frac{b + (c_1 - a_1)}{b - (c_1 - a_1)},$$
$$V_2 = \frac{\rho_l}{2\pi\epsilon_0}\ln\frac{b + (c_2 - a_2)}{b - (c_2 - a_2)},$$
$$C = \frac{\rho_l}{V_1 - V_2} = 2\pi\epsilon_0 \ln\left[\frac{b + (c_1 - a_1)}{b - (c_1 - a_1)} \cdot \frac{b - (c_1 - a_1)}{b + (c_1 - a_1)}\right]^{-1}.$$

b)

$$F = \frac{1}{2\pi\epsilon_0} \cdot \frac{\rho_l^2}{4b^2}.$$

P. 4-17

a)

$$V_1(x,y,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R_+} - \frac{Q_1}{R_-}\right),$$
$$R_+ = [(x-d)^2 + y^2 + z^2]^{1/2},$$
$$R_- = [(x+d)^2 + y^2 + z^2]^{1/2}.$$

Similar to P. 4-5,

$$\nabla^2 V_1 = \frac{Q}{4\pi\epsilon_0}\{3R_+^{-5}[(x-d)^2 + y^2 + z^2] - 3R_+^{-3}\} - \frac{Q_1}{4\pi\epsilon_0}\{3R_-^{-5}[(x+d)^2 + y^2 + z^2] - 3R_-^{-3}\} = 0.$$

b)

$$V_2(x,y,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R_+} + \frac{Q_2}{R_+}\right)$$

Similar to P. 4-5,

$$\nabla^2 V_2 = \frac{Q + Q_2}{4\pi\epsilon_0}\{3R_+^{-5}[(x-d)^2 + y^2 + z^2] - 3R_+^{-3}\} = 0.$$

c) When $x = 0$, $R_+ = R_- = R = [d^2 + y^2 + z^2]^{1/2}$,

$$\frac{\partial V_1}{\partial x} = \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}(x-d) + Q_1R_-^{-3}(x+d)] = \frac{d}{4\pi\epsilon_0 R^3}(Q + Q_1),$$

$$\frac{\partial V_2}{\partial x} = \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}(x-d) - Q_2R_+^{-3}(x-d)] = \frac{d}{4\pi\epsilon_0 R^3}(Q + Q_2),$$
$$\frac{\partial V_1}{\partial y} = \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}y + Q_1R_-^{-3}y] = \frac{1}{4\pi\epsilon_0 R^3}(-Q + Q_1),$$
$$\frac{\partial V_2}{\partial y} = \frac{1}{4\pi\epsilon_0}[-QR_+^{-3}y - Q_2R_+^{-3}y] = \frac{1}{4\pi\epsilon_0 R^3}(-Q - Q_2).$$

Since

$$\frac{\partial V_1}{\partial x} = \frac{\partial V_2}{\partial x}, \quad \epsilon_1\frac{\partial V_1}{\partial y} = \epsilon_2\frac{\partial V_1}{\partial y},$$
$$Q_1 = Q_2 = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}Q.$$

P. 4-23

$$\nabla^2 V = \frac{1}{r^2} \cdot \frac{\partial^2 V}{\partial \phi^2} = 0.$$

a) When $\phi = 0$, $V = 0$; when $\phi = \alpha$, $V = V_0$,

$$V = C_1\phi + C_2 = \frac{V_0}{\alpha}\phi.$$

b) When $\phi = 2pi$, $V = 0$; when $\phi = \alpha$, $V = V_0$,

$$V = C_1\phi + C_2 = \frac{V_0}{\alpha - 2\pi}(\phi - 2\pi).$$

P. 4-28

a)

$$V(b,\theta) = V_0,$$
$$V(R,\theta) = B_0R^{-1} + (B_1R^{-2} - E_0R)\cos\theta + \sum_{n=2}^{\infty} B_nR^{-n-1}P_n\cos\theta, \quad R \geqslant b.$$

Since the sphere is charged, $B_0 = bV_0$, we can obtain $B_1 = E_0b^3$ and $B_n = 0$ for $n \geqslant 2$.

$$V(R,\theta) = \frac{bV_0}{R} - E_0\left[1 - \left(\frac{b}{R}\right)^3\right]R\cos\theta, \quad R \geqslant b.$$

b)

$$\mathbf{E}(R,\theta) = \mathbf{a}_R E_R + \mathbf{a}_\theta E_\theta,$$

where

$$E_R = -\frac{\partial V}{\partial R} = -\frac{bV_0}{R^2} + E_0\left[1 + 2\left(\frac{b}{R}\right)^3\right]\cos\theta, \quad R \geqslant b,$$
$$E_\theta = -\frac{\partial V}{R\partial\theta} = -E_0\left[1 - \left(\frac{b}{R}\right)^3\right]R\sin\theta, \quad R \geqslant b.$$

P. 5-7

a)

$$\exp\left[-\frac{\sigma}{\varepsilon}t\right] = 0.01,$$
$$t = -\frac{\varepsilon_0\varepsilon_r}{\sigma}\ln 0.01 \approx 4.89 \times 10^{-12}\,\text{s}.$$

b)

$$\Delta W = \exp\left[-\frac{\sigma}{\varepsilon}t\right]^2 = 1 \times 10^{-4}.$$

The energy are transformed into heat energy.

c)

$$\frac{Q}{\varepsilon_0} = E \cdot 4\pi r^2,$$
$$E = \frac{Q}{4\pi r^2\varepsilon_0},$$
$$W = \int_0^{Q_0} VdQ = \int_0^{Q_0}\int_R^{\infty} -EdrdQ = \int_0^{Q_0}\frac{Q}{4\pi R\varepsilon_0}dQ = \frac{Q_0^2}{8\pi R\varepsilon_0} \approx 4.494 \times 10^4\ \text{J}.$$

P. 5-10

a)

$$\sigma(y) = \frac{\sigma_2 - \sigma_1}{d}y + \sigma_1,$$
$$\frac{dy}{d\sigma} = \frac{d}{\sigma_2 - \sigma_1},$$
$$R = \int_0^d \frac{1}{\sigma S}dy = \frac{1}{S}\int_{\sigma_1}^{\sigma_2}\frac{1}{\sigma}d\sigma \cdot \frac{dy}{d\sigma} = \frac{d}{S(\sigma_2 - \sigma_1)}\int_{\sigma_1}^{\sigma_2}\frac{1}{\sigma}d\sigma = \frac{d}{S(\sigma_2 - \sigma_1)}\ln\frac{\sigma_2}{\sigma_1}.$$

b)

$$J = \frac{V}{RS} = \frac{V(\sigma_2 - \sigma_1)}{d\ln\frac{\sigma_2}{\sigma_1}},$$
$$\rho_b = -\varepsilon_0E(0) = -\varepsilon_0\frac{J}{\sigma_1} = -\frac{\varepsilon_0V(\sigma_2 - \sigma_1)}{\sigma_1 d\ln\frac{\sigma_2}{\sigma_1}},$$
$$\rho_t = \varepsilon_0E(d) = \varepsilon_0\frac{J}{\sigma_2} = \frac{\varepsilon_0V(\sigma_2 - \sigma_1)}{\sigma_2 d\ln\frac{\sigma_2}{\sigma_1}}.$$

c)

$$\rho = \frac{d}{dy}\varepsilon_0E = \frac{d}{dy}\left[-\varepsilon_0J \cdot \frac{d}{\sigma(y)}\right] = \frac{\varepsilon_0J(\sigma_2 - \sigma_1)d}{[(\sigma_2 - \sigma_1)y + d\sigma_1]^2} = \frac{\varepsilon_0V(\sigma_2 - \sigma_1)^2}{[(\sigma_2 - \sigma_1)y + d\sigma_1]^2\ln\frac{\sigma_2}{\sigma_1}}.$$
$$Q = (\rho_b + \rho_t)S = \frac{\varepsilon_0V(\sigma_2 - \sigma_1)S}{d\ln\frac{\sigma_2}{\sigma_1}}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right).$$

P. 5-16

$$\frac{1}{R^2(1 + k/R)} = \frac{1}{R(R+k)} = \frac{1}{k}\left(\frac{1}{R} - \frac{1}{R+k}\right),$$
$$R_0 = \int_{R_1}^{R_2}\frac{1}{\sigma \cdot 4\pi R^2}dR = \frac{1}{4\pi\sigma_0k}\int_{R_1}^{R_2}\left(\frac{1}{R} - \frac{1}{R+k}\right)dR = \frac{1}{4\pi\sigma_0k}\ln\frac{R}{R+k}\bigg|_{R_1}^{R_2} = \frac{1}{4\pi\sigma_0k}\ln\frac{R_2(R_1+k)}{R_1(R_2+k)}.$$

P. 5-22

a)

$$V = \frac{V_0}{a}x.$$

b)

$$\mathbf{E} = -\nabla V = -\frac{V_0}{a}\mathbf{a}_x.$$
$$\mathbf{J} = \sigma\mathbf{E} = -\frac{\sigma V_0}{a}\mathbf{a}_x.$$

P. 6-2

a)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -e(E_0 - u_0 B_0)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on -z and y direction;
If $E_0 < u_0 B_0$, the electron moves on z and y direction;
If $E_0 = u_0 B_0$, the electron moves on y direction.

b)

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = e(E_0 \mathbf{a}_z + u_0 B_0 \mathbf{a}_x)\mathbf{a}_z.$$

If $E_0 > u_0 B_0$, the electron moves on x-z direction, closer to z direction and y direction;
If $E_0 < u_0 B_0$, the electron moves on x-z direction, closer to x direction and y direction;
If $E_0 = u_0 B_0$, the electron moves on the midline of x-z direction and y direction.

P. 6-12

$$\mathbf{B} = \mathbf{a}_x \cdot \frac{\mu_0 I b^2 N}{2}\left\{[(x + d/2)^2 + b^2]^{-3/2} + [(x - d/2)^2 + b^2]^{-3/2}\right\}.$$

a)

$$\mathbf{B}(x = 0) = \mathbf{a}_x \cdot \mu_0 I b^2 N[(d/2)^2 + b^2]^{-3/2}.$$

b)

$$\frac{dB_x}{dx} = \frac{\mu_0 I b^2 N}{2}\left\{-\frac{3}{2}[(x + d/2)^2 + b^2]^{-5/2}(2x + d) - \frac{3}{2}[(x - d/2)^2 + b^2]^{-5/2}(2x - d)\right\},$$

$$\frac{dB_x(0)}{dx} = \frac{\mu_0 I b^2 N}{2}\left\{-\frac{3}{2}[(d/2)^2 + b^2]^{-5/2} d - \frac{3}{2}[(d/2)^2 + b^2]^{-5/2}(-d)\right\} = 0.$$

c)

$$\begin{aligned}\frac{d^2 B_x}{dx^2} = \frac{3\mu_0 I b^2 N}{4}&\left\{\frac{5}{2}[(x + d/2)^2 + b^2]^{-7/2}(2x + d)^2 - 2[(x + d/2)^2 + b^2]^{-5/2}\right.\\ &\left.+\frac{5}{2}[(x - d/2)^2 + b^2]^{-7/2}(2x - d)^2 - 2[(x - d/2)^2 + b^2]^{-5/2}\right\},\end{aligned}$$

$$\frac{d^2 B_x(0)}{dx^2} = \frac{3\mu_0 I b^2 N}{4}\left\{5d^2[(d/2)^2 + b^2]^{-7/2} - 4[(d/2)^2 + b^2]^{-5/2}\right\} = 0,$$

$$5d^2[(d/2)^2 + b^2]^{-1} - 4 = 0,$$

$$b = d.$$

P. 6-24

a)

$$\begin{aligned}V_m &= \int \frac{I\mathbf{a}_n \cdot \mathbf{a}_R}{4\pi(z^2 + r^2)} dS\\ &= \int_0^{2\pi}\int_0^b \frac{I}{4\pi(z^2 + r^2)}\frac{z}{\sqrt{z^2 + r^2}} r dr d\theta\\ &= \frac{I}{8\pi}\int_0^{2\pi}\int_{z^2}^{z^2+b^2} z(z^2 + r^2)^{-3/2} d(z^2 + r^2) d\theta\\ &= \frac{I}{8\pi}\int_0^{2\pi} 2[1 - z(z^2 + b^2)^{-1/2}] d\theta\\ &= \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2 + b^2}}\right).\end{aligned}$$

b)

$$\begin{aligned}\mathbf{B} &= -\mu_0 \nabla V_m\\ &= -\mu_0 \cdot \mathbf{a}_z \frac{I}{2}\left(-\frac{\sqrt{z^2 + b^2} - 2z \cdot \frac{1}{2}/\sqrt{z^2 + b^2} \cdot z}{z^2 + b^2}\right)\\ &= \mathbf{a}_z \frac{\mu_0 I}{2}\left(\frac{z^2 + b^2 - z^2}{(z^2 + b^2)^{3/2}}\right)\\ &= \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}.\end{aligned}$$

P. 6-27

a)

$$R_g = \frac{l_g}{\mu_0 A} = \frac{3\,\text{mm}}{4\pi \times 10^{-7}\,\text{H/m} \cdot \pi(25\,\text{mm})^2} = 1.216 \times 10^6\,\text{H}^{-1}.$$

$$R_c = \frac{l_c}{\mu_0 \mu_c A} = \frac{(2\pi \cdot 80 - 3)\text{mm}}{4\pi \times 10^{-7}\,\text{H/m} \cdot 3000 \cdot \pi(25\,\text{mm})^2} = 6.75 \times 10^4\,\text{H}^{-1}.$$

b)

$$\mathbf{B}_g = \mathbf{B}_c = \mathbf{a}_\phi \frac{\Phi}{A} = \mathbf{a}_\phi \frac{1 \times 10^{-5}\,\text{Wb}}{\pi(25\,\text{mm})^2} = a_\phi 5.093 \times 10^{-3}\,\text{T},$$

$$\mathbf{H}_g = \frac{\mathbf{B}_g}{\mu_0} = \frac{5.093 \times 10^{-3}\,\text{T}}{4\pi \times 10^{-7}\,\text{H/m}} = a_\phi 4.052 \times 10^3\,\text{A/m},$$

$$\mathbf{H}_c = \frac{\mathbf{B}_c}{\mu_0 \mu_c} = \frac{5.093 \times 10^{-3}\,\text{T}}{3000 \cdot 4\pi \times 10^{-7}\,\text{H/m}} = a_\phi 1.351 \times 10^3\,\text{A/m}.$$

c)

$$\Phi = \frac{N I_0}{R_g + R_c},$$

$$I_0 = \frac{\Phi(R_g + R_c)}{N} = \frac{1 \times 10^{-5}\,\text{Wb} \cdot (1.216 \times 10^6\,\text{H}^{-1} + 6.75 \times 10^4\,\text{H}^{-1})}{500} = 2.567 \times 10^{-2}\,\text{A}.$$

P. 6-31

Let L_1 be the left and right legs, L_2 be the core of the center leg, L_3 be the air gap.

$$R_1 = \frac{L_1}{\mu_0 \mu_c A} = \frac{(0.2 + 0.2 + 0.24)\text{m}}{5000 \cdot 4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 1.019 \times 10^5\,\text{H}^{-1},$$

$$R_2 = \frac{L_2}{\mu_0 \mu_c A} = \frac{(0.24 - 0.002)\text{m}}{5000 \cdot 4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 3.788 \times 10^4\,\text{H}^{-1},$$

$$R_3 = \frac{L_3}{\mu_0 A} = \frac{0.002\text{m}}{4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 1.592 \times 10^6\,\text{H}^{-1}.$$

a) In the center leg,

$$\Phi_2 = \frac{NI}{0.5R_1 + R_2 + R_3} = \frac{200 \cdot 3\,\text{A}}{(0.5 \cdot 1.019 \times 10^5 + 3.788 \times 10^4 + 1.592 \times 10^6)\text{H}^{-1}} = 3.570 \times 10^{-4}\,\text{T}.$$

In the left and right leg,

$$\Phi_1 = 0.5\Phi_2 = 1.785 \times 10^{-4}\,\text{T}.$$

b)

$$H_1 = \frac{\Phi_1}{\mu_0 \mu_c A} = \frac{1.785 \times 10^{-4}\,\text{T}}{5000 \cdot 4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 2.841 \times 10^1\,\text{A/m},$$

$$H_2 = \frac{\Phi_2}{\mu_0 \mu_c A} = \frac{3.570 \times 10^{-4}\,\text{T}}{5000 \cdot 4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 5.682 \times 10^1\,\text{A/m},$$

$$H_3 = \frac{\Phi_3}{\mu_0 A} = \frac{3.570 \times 10^{-4}\,\text{T}}{4\pi \times 10^{-7}\,\text{H/m} \cdot 10^{-3}\,\text{m}^2} = 2.841 \times 10^5\,\text{A/m}.$$

P. 6-31

$$B_{1n} = B_{2n}, \quad -\mu_1 \frac{\partial V_{m1}}{\partial n} = -\mu_2 \frac{\partial V_{m2}}{\partial n}.$$

$$H_{1t} = H_{2t}, \quad \frac{\partial V_{m1}}{\partial t} = \frac{\partial V_{m2}}{\partial t}.$$

P. 6-39

$$\begin{aligned}L &= \frac{N\Phi}{I}\\ &= \frac{1}{I}\int \mathbf{B} d\mathbf{S}\\ &= \frac{1}{I}\int_0^{2\pi}\int_0^b \frac{\mu_0 I}{2\pi(d + r\cos\theta)} r dr d\theta\\ &= -\mu_0 \int_0^b \frac{r}{\sqrt{d^2 - r^2}} dr\\ &= \mu_0(d - \sqrt{d^2 - b^2}).\end{aligned}$$

P. 6-43

$$B = \frac{\mu_0 I}{2\pi r},$$

$$dB = \frac{\mu_0 I}{2\pi\sqrt{x^2 + D^2}} \cdot \frac{dx}{w},$$

$$dF = dB \cdot IL = \frac{\mu_0 I}{2\pi\sqrt{x^2 + D^2}} \cdot \frac{dx}{w} \cdot IL\frac{D}{\sqrt{x^2 + D^2}} = \frac{\mu_0 D I^2 L}{2\pi w(x^2 + d^2)} dx,$$

$$F = \int_{-w/2}^{w/2} \frac{\mu_0 D I^2 L}{2\pi w(x^2 + d^2)} dx = \frac{\mu_0 I^2 L}{\pi w} \arctan\frac{w}{2D},$$

$$\frac{F}{L} = \frac{\mu_0 I^2}{\pi w} \arctan\frac{w}{2D}.$$

P. 6-48

$$W_m = \frac{x}{2\mu_0}\int_a^b B^2 2\pi r dr = \frac{\mu_0 I^2 x}{4\pi}\int_a^b \frac{1}{r} r dr = \frac{\mu_0 I^2 x}{4\pi} \ln\frac{b}{a},$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{\mu_0 I^2}{4\pi} \ln\frac{b}{a}.$$

P. 6-53

$$B_g = \frac{\mu_0 N I}{L} = \mu_0 n I,$$

$$B_c = \frac{\mu N I}{L} = \mu n I.$$

$$W_m = \frac{B_g^2}{2\mu_0} S(L - x) + \frac{B_c^2}{2\mu} S x = \frac{1}{2} n^2 I^2 S[(\mu - \mu_0)x + \mu_0)],$$

$$\mathbf{F} = -\nabla W_m = -\mathbf{a}_x \frac{1}{2} n^2 I^2 S(\mu - \mu_0).$$