P. 7-2

$$\Phi = \int \mathbf{B} d\mathbf{S} = \int_0^{0.2}\int_0^{0.6} 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right) dxdy \cdot 10^{-6}$$
$$= \frac{9\times 10^{-7}}{\pi}[\sin(5\pi 10^7 t) + \sin(0.4\pi - 5\pi 10^7 t)]\text{Wb}.$$

$$V = -\frac{d\Phi}{dt} = -45[\cos(5\pi 10^7 t) - \cos(0.4\pi - 5\pi 10^7 t)],$$
$$i = \frac{V}{2R} = -1.5[\cos(5\pi 10^7 t) - \cos(0.4\pi - 5\pi 10^7 t)].$$

P. 7-6

a)

$$dR = \frac{2\pi r}{\sigma h dr},$$
$$V = \frac{d\Phi}{dt} = \frac{d(B_0 \sin\omega t \cdot \pi r^2)}{dt} = B_0 \omega \pi r^2 \cos\omega t,$$
$$dP = \frac{V^2}{dR} = \frac{B_0^2\omega^2\pi^2 r^4 \cos^2\omega t \cdot \sigma h dr}{2\pi r} = \frac{1}{2}B_0^2\omega^2\pi r^3 h\sigma \cos^2\omega t \cdot dr,$$
$$P = \int dP = \int_0^R \frac{1}{2}B_0^2\omega^2\pi r^3 h\sigma\cos^2\omega t \cdot dr = \frac{1}{8}B_0^2\omega^2\pi R^4 h\sigma \cos^2\omega t,$$
$$\overline{P} = \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{8}B_0^2\omega^2\pi R^4 h\sigma\cos^2\omega t dt = \frac{1}{16}B_0^2\omega^2\pi R^4 h\sigma.$$

b)

$$0.95\pi R^2 = N\cdot\pi R'^2,$$
$$R'^2 = \sqrt{\frac{0.95}{N}}R,$$
$$\overline{P'} = N\cdot\frac{1}{16}B_0^2\omega^2\pi R'^4 h\sigma = \frac{0.95^2}{16N}B_0^2\omega^2\pi R^4 h\sigma.$$

P. 7-11

$$\nabla\times\mathbf{E} = -\frac{\partial B}{\partial t},$$
$$\nabla\cdot(\nabla\times\mathbf{E}) = -\frac{\partial}{\partial t}(\nabla\cdot\mathbf{B}) = 0.$$

So $\nabla\cdot\mathbf{B}$ is a constant, and $\mathbf{B} = \mathbf{0}$ in infinite distance, which means $\nabla\cdot\mathbf{B} = 0$ at that point, so that $\nabla\cdot\mathbf{B} = 0$ always stands.

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t},$$
$$\nabla\cdot(\nabla\times\mathbf{H}) = \nabla\cdot\mathbf{J} + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{D}) = -\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{D}),$$
$$\nabla\cdot\mathbf{D} = \rho.$$

P. 7-12

$$\begin{cases}\nabla^2 V - \mu\varepsilon\dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\varepsilon}\\ \nabla^2\mathbf{A} - \mu\varepsilon\dfrac{\partial^2\mathbf{A}}{\partial t^2} = -\mu\mathbf{J}\end{cases} \Longrightarrow \begin{cases}\rho = \varepsilon\left(\mu\varepsilon\dfrac{\partial^2 V}{\partial t^2} - \nabla^2 V\right)\\ \mathbf{J} = \dfrac{1}{\mu}\left(\mu\varepsilon\dfrac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A}\right)\end{cases},$$

$$-\frac{\partial \rho}{\partial t} = -\varepsilon\left(\mu\varepsilon\frac{\partial^3 V}{\partial t^3} - \nabla^2\frac{\partial V}{\partial t}\right),$$

$$\nabla\cdot\mathbf{A} = -\mu\varepsilon\frac{\partial V}{\partial t},$$

$$\nabla\cdot\mathbf{J} = \frac{1}{\mu}\left(\mu\varepsilon\frac{\partial^2(\nabla\cdot\mathbf{A})}{\partial t^2} - \nabla^2(\nabla\cdot\mathbf{A})\right) = \varepsilon\left(\mu\varepsilon\frac{\partial^3 V}{\partial t^3} - \nabla^2\frac{\partial V}{\partial t}\right),$$

$$\nabla\cdot\mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

P. 7-14

$$\begin{aligned}
\mathbf{J} &= \nabla\times\mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \\
&= \frac{1}{\mu}\nabla\times\mathbf{B} - \varepsilon\frac{\partial \mathbf{E}}{\partial t} \\
&= \frac{1}{\mu}\nabla\times(\nabla\times\mathbf{A}) + \varepsilon\nabla\frac{\partial V}{\partial t} + \varepsilon\frac{\partial^2\mathbf{A}}{\partial t^2} \\
&= \frac{1}{\mu}\nabla\times(\nabla\times\mathbf{A}) - \frac{1}{\mu}\nabla(\nabla\cdot\mathbf{A}) + \varepsilon\frac{\partial^2\mathbf{A}}{\partial t^2},
\end{aligned}$$

$$\nabla^2\mathbf{A} - \mu\varepsilon\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu\mathbf{J}.$$

$$\rho = \nabla\cdot\mathbf{D} = \varepsilon\nabla\cdot\mathbf{E} = -\varepsilon\nabla^2 V - \varepsilon\frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\varepsilon\nabla^2 V + \varepsilon\frac{\partial}{\partial t}\mu\varepsilon\frac{\partial V}{\partial t},$$

$$\nabla^2 V - \mu\varepsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}.$$

P. 7-17

a)

$$E_{1t} = E_{2t}, \quad B_{1n} = B_{2n}.$$

b)

$$D_{1n} = D_{2n}, \quad H_{1t} = H_{2t}.$$

P. 7-20

Let $u = t \pm R\sqrt{\mu\epsilon}$, $f(u) = U(R,t)$,

$$\left(\frac{\partial u}{\partial R}\right)^2 = \mu\varepsilon, \quad \left(\frac{\partial u}{\partial t}\right)^2 = 1.$$

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 f}{\partial u^2}\left(\frac{\partial u}{\partial R}\right)^2 - \mu\epsilon\frac{\partial^2 f}{\partial u^2}\left(\frac{\partial u}{\partial t}\right)^2 = 0.$$

P. 7-24

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu\frac{\partial \mathbf{H}}{\partial t},$$

$$\nabla\times(\nabla\times\mathbf{E}) = -\mu\frac{\partial}{\partial t}\left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) = -\mu\frac{\partial \mathbf{J}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2},$$

$$\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = \frac{1}{\varepsilon}\nabla\rho - \nabla^2\mathbf{E},$$

$$\nabla^2\mathbf{E} = \frac{1}{\varepsilon}\nabla\rho + \mu\frac{\partial \mathbf{J}}{\partial t} + \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2},$$

$$\nabla^2 \mathbf{E} = \frac{1}{\varepsilon}\nabla\rho + \mu j\omega \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{E}.$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla \times \mathbf{J} + \varepsilon\frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = \nabla \times \mathbf{J} - \mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla(\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = -\nabla^2 \mathbf{H},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon\frac{\partial^2 \mathbf{H}}{\partial t^2},$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J} + \mu\varepsilon\omega^2 \mathbf{H}.$$

P. 7-27

$$\begin{aligned}
\nabla \times \mathbf{E} &= \frac{1}{R^2 \sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & E_0 \sin\theta e^{-jkR} & 0 \end{vmatrix} \\
&= \frac{1}{R^2 \sin\theta}\left(\mathbf{a}_R \frac{\partial}{\partial\phi} E_0 \sin\theta e^{-jkR} - \mathbf{a}_\phi R\sin\theta \frac{\partial}{\partial R} E_0 \sin\theta e^{-jkR}\right) \\
&= \mathbf{a}_\phi \frac{-E_0 jk \sin\theta e^{-jkR}}{R},
\end{aligned}$$

$$\nabla \times \mathbf{E} = -\mu\frac{\partial \mathbf{H}}{\partial t} = -j\mu\omega \mathbf{H},$$

$$\mathbf{H} = \mathbf{a}_\phi \frac{E_0 k \sin\theta e^{-jkR}}{\mu\omega R}, \quad k = \omega\sqrt{\mu\varepsilon}.$$

$$\mathbf{H}(R) = \mathbf{a}_\phi \frac{E_0\sqrt{\mu\varepsilon}\sin\theta e^{-j\omega\sqrt{\mu\varepsilon}R}}{\mu R},$$

$$\mathbf{H}(R,t) = \mathrm{Re}[\mathbf{H}(R)e^{j\omega t}] = \mathbf{a}_\phi \frac{E_0\sqrt{\mu\varepsilon}}{\mu R}\sin\theta\cos(\omega t - \omega\sqrt{\mu\varepsilon}R).$$

P. 7-29

a)

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \mathbf{H} = \omega^2\mu_0\varepsilon_0 \nabla \times \pi_e,$$

$$\nabla \times (\mathbf{E} - \omega^2\mu_0\varepsilon_0\pi_e) = \mathbf{0},$$

$$\mathbf{E} = \omega^2\mu_0\varepsilon_0\pi_e + \mathbf{C}.$$

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} = \omega^2\mu_0\varepsilon_0 \nabla \times \pi_e,$$

$$\nabla \times (j\omega\varepsilon_0 \nabla \times \pi_e) = j\omega\varepsilon_0(\omega^2\mu_0\varepsilon_0\pi_e + \frac{\mathbf{P}}{\varepsilon_0} + \mathbf{C}),$$

$$\nabla \times (\nabla \times \pi_e) = \nabla(\nabla \cdot \pi_e) - \nabla^2\pi_e = \omega^2\mu_0\varepsilon_0\pi_e + \frac{\mathbf{P}}{\varepsilon_0} + \mathbf{C},$$

$$\nabla^2\pi_e + \omega^2\mu_0\varepsilon_0\pi_e = \nabla(\nabla \cdot \pi_e) - \frac{\mathbf{P}}{\varepsilon_0} - \mathbf{C},$$

$$\mathbf{C} = \nabla(\nabla \cdot \pi_e),$$

$$\mathbf{E} = \omega^2\mu_0\varepsilon_0\pi_e + \nabla(\nabla \cdot \pi_e).$$

b)

$$k_0^2 = \omega^2 \mu_0 \varepsilon_0,$$

$$\nabla^2 \pi_e + k_0^2 \pi_e = -\frac{\mathbf{P}}{\varepsilon_0}.$$

P. 8-7

Let $\phi = \omega_t - kz$,

$$\mathbf{E}(\phi) = \mathbf{a}_x E_{10} \sin\phi + \mathbf{a_y} E_{20} \sin(\phi + \psi),$$

$$\frac{E_x}{E_{10}} = \sin\phi,$$

$$\frac{E_y}{E_{20}} = \sin(\phi+\psi) = \sin\phi\cos\psi + \cos\phi\sin\psi = \frac{E_x}{E_{10}}\cos\psi + \sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi,$$

$$\left[\sqrt{1-\left(\frac{E_x}{E_{10}}\right)^2}\sin\psi\right]^2 = \left(\frac{E_y}{E_{20}}\right)^2 + \left(\frac{E_x}{E_{10}}\cos\psi\right)^2 - 2\left(\frac{E_y}{E_{20}}\right)\left(\frac{E_x}{E_{10}}\cos\psi\right),$$

$$\left(\frac{E_x}{E_{10}}\right)^2 + \left(\frac{E_y}{E_{20}}\right)^2 - 2\frac{E_x}{E_{10}}\frac{E_y}{E_{20}}\cos\psi = \sin^2\psi,$$

$$\left(\frac{E_x}{E_{10}\sin\psi}\right)^2 + \left(\frac{E_y}{E_{20}\sin\psi}\right)^2 - 2\frac{E_x}{E_{10}}\frac{E_y}{E_{20}}\frac{\cos\psi}{sin^2\psi} = 1.$$

P. 8-9

$$\nabla^2\mathbf{E} + k_c^2\mathbf{E} = 0,$$

$$k_c = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha,$$

$$\omega^2\mu\varepsilon = \omega^2\mu(\varepsilon - j\sigma/\omega) = \beta^2 - \alpha^2 - 2j\alpha\beta,$$

$$\begin{cases}\beta^2 + \alpha^2 = \omega^2\mu\sqrt{\varepsilon^2 + \sigma^2/\omega^2} \\ \beta^2 - \alpha^2 = \omega^2\mu\varepsilon\end{cases} \Longrightarrow \begin{cases}\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma^2}{\omega\varepsilon}\right)^2} - 1\right]^{1/2} \\ \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma^2}{\omega\varepsilon}\right)^2} + 1\right]^{1/2}\end{cases}.$$

P. 8-14

$$f_{max} = \frac{1}{2\pi}\sqrt{\frac{N_{max}e^2}{m\varepsilon_0}},$$

$$\varepsilon_{min} = \varepsilon_0\left[1 - \left(\frac{f_{max}}{f}\right)^2\right],$$

$$\frac{1}{\sin\theta_i} = \sqrt{\frac{\varepsilon_0}{\varepsilon_{min}}},$$

$$\varepsilon_{min} = \varepsilon_0\sin^2\theta_i = \varepsilon_0\left[1 - \left(\frac{f_{max}}{f}\right)^2\right],$$

$$f = \frac{f_{max}}{\cos\theta_i} = \frac{1}{2\pi\cos\theta_i}\sqrt{\frac{N_{max}e^2}{m\varepsilon_0}}.$$

P. 8-15

a)

$$u_g = \frac{dw}{d\beta} = \frac{d}{d\beta}(\beta u_p) = u_p + \beta\frac{du_p}{d\beta}.$$

b)

$$\lambda = \frac{2\pi}{\beta},$$

$$\frac{d\lambda}{d\beta} = \frac{d}{d\beta}\left(\frac{2\pi}{\beta}\right) = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta},$$

$$u_g = u_p + \beta\frac{du_p}{d\beta} = u_p + \beta\frac{du_p}{d\lambda}\cdot\frac{d\lambda}{d\beta} = u_p - \lambda\frac{du_p}{d\lambda}.$$

P. 8-22

a)

$$\mathbf{E}_i(x,z) = \mathbf{a}_y 10 e^{-j(6x+8z)} = \mathbf{a}_y E_0 e^{-jk_x x - jk_z z},$$

$$E_0 = 10 \quad k_x = 6, \quad k_z = 8,$$

$$k = \sqrt{k_x^2 + k_z^2} = 10,$$

$$\lambda = \frac{2\pi}{k} = 0.2\pi \text{ m} \approx 0.628\,\text{m},$$

$$f = \frac{c}{\lambda} = 4.777 \times 10^8\,\text{Hz}.$$

b)

$$\omega = 2\pi f = 3 \times 10^9\,\text{rad}\,\text{s}^{-1},$$

$$\eta_0 = \frac{\omega\mu}{k} = 120\pi\ \Omega,$$

$$\mathbf{E}_i(x,z,t) = \text{Re}[\mathbf{E}_i(x,z)e^{j\omega t}] = \mathbf{a_y}10\cos(3\times 10^9 t - 6x - 8z)\ \text{V}\,\text{m}^{-1}.$$

$$\mathbf{a}_{n_i} = \mathbf{a}_x \cdot \frac{k_x}{k} + \mathbf{a}_z \cdot \frac{k_z}{k},$$

$$\mathbf{H}_i(x,z) = \frac{1}{\eta_0}(\mathbf{a}_{n_i} \times E_i(x,z)) = \left(\frac{\mathbf{a}_z}{20\pi} - \frac{\mathbf{a}_x}{15\pi}\right)e^{-j(6x+8z)},$$

$$\mathbf{H}_i(x,z,t) = \text{Re}[\mathbf{H}_i(x,z)e^{j\omega t}] = \left(\frac{\mathbf{a}_z}{20\pi} - \frac{\mathbf{a}_x}{15\pi}\right)\cos(3\times 10^9 t - 6x - 8z)\ \text{A}\,\text{m}^{-1}.$$

c)

$$\theta_i = \arccos(\mathbf{a}_n \cdot \mathbf{a}_z) = \arccos 0.8 \approx 0.644\ \text{rad}.$$

d)

$$\mathbf{E}_r(x,z) = -\mathbf{E}_i(x,-z) = -\mathbf{a}_y 10 e^{-j(6x-8z)},$$

$$\mathbf{a}_{n_r} = \mathbf{a}_x \cdot \frac{k_x}{k} - \mathbf{a}_z \cdot \frac{k_z}{k}$$

$$\mathbf{H}_r(x,z) = \frac{1}{\eta_0}(\mathbf{a}_{n_r} \times E_r(x,z)) = -\left(\frac{\mathbf{a}_z}{20\pi} + \frac{\mathbf{a}_x}{15\pi}\right)e^{-j(6x-8z)}.$$

e)

$$\mathbf{E}_1(x,z) = \mathbf{E}_i(x,z) + \mathbf{E}_r(x,z) = -\mathbf{a}_y 20j e^{-j6x}\sin 8z\ \text{V/m},$$

$$\mathbf{H}_1(x,z) = \mathbf{H}_i(x,z) + \mathbf{H}_r(x,z) = -\frac{\mathbf{a}_z}{10\pi}j\sin 8z - \frac{\mathbf{a_x}}{15\pi}2\cos 8z.$$