P.7–24 Derive the general wave equations for **E** and **H** in a nonconducting simple medium where a charge distribution ρ and a current distribution **J** exist. Convert the wave equations to Helmholtz's equations for sinusoidal time dependence. Write the general solutions for $\mathbf{E}(R, t)$ and $\mathbf{H}(R, t)$ in terms of ρ and **J**.

P.7–27 It is known that the electric field intensity of a spherical wave in free space is

$$\mathbf{E} = \mathbf{a}_\theta \frac{E_0}{R} \sin\theta \cos(\omega t - kR).$$

Determine the magnetic field intensity **H** and the value of k.

P.7–29 For a source-free polarized medium where $\rho = 0$, $\mathbf{J} = 0$, $\mu = \mu_0$, but where there is a volume density of polarization **P**, a single vector potential $\boldsymbol{\pi}_e$ may be defined such that

$$\mathbf{H} = j\omega\epsilon_0 \nabla \times \boldsymbol{\pi}_e. \tag{7–118}$$

a) Express electric field intensity **E** in terms of $\boldsymbol{\pi}_e$ and **P**.

b) Show that $\boldsymbol{\pi}_e$ satisfies the nonhomogeneous Helmholtz's equation

$$\nabla^2 \boldsymbol{\pi}_e + k_0^2 \boldsymbol{\pi}_e = -\frac{\mathbf{P}}{\epsilon_0}. \tag{7–119}$$

The quantity $\boldsymbol{\pi}_e$ is known as the ***electric Hertz potential.***

P.8–7 Show that a plane wave with an instantaneous expression for the electric field

$$\mathbf{E}(z, t) = \mathbf{a}_x E_{10} \sin(\omega t - kz) + \mathbf{a}_y E_{20} \sin(\omega t - kz + \psi)$$

is elliptically polarized. Find the polarization ellipse.

P.8–9 Derive the following general expressions of the attenuation and phase constants for conducting media:

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \quad \text{(Np/m)}.$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \quad \text{(rad/m)}.$$

P.8–14 Assume the ionosphere to be modeled by a plasma region with an electron density that increases with altitude from a low value at the lower boundary toward a value $N_{\max}$ and decreases again as the altitude gets higher. A plane electromagnetic wave impinges on the lower boundary at an angle θ_i with the normal. Determine the highest frequency of the wave that will be turned back toward the earth. (*Hint:* Imagine the ionosphere to be stratified into layers of successively decreasing constant permittivities until the layer containing $N_{\max}$. The frequency to be determined corresponds to that for an emerging angle of $\pi/2$.)

P.8–15 Prove the following relations between group velocity u_g and phase velocity u_p in a dispersive medium:

a) $u_g = u_p + \beta \dfrac{du_p}{d\beta}$ **b)** $u_g = u_p - \lambda \dfrac{du_p}{d\lambda}$.