VE311 Homework 8

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Problem 1.

$$g_m V = \frac{V_I - V}{R_I + 1/C_1 s} - \frac{V}{R_S}$$

$$V = \frac{\dfrac{V_I}{R_I + 1/C_1 s}}{g_m + \dfrac{1}{R_I + 1/C_1 s} + \dfrac{1}{R_S}} = V_I \cdot \frac{R_S}{R_S + (g_m R_S + 1)(R_I + 1/C_1 s)}$$

$$V_O = g_m V \cdot \frac{R_D}{R_D + R_3 + 1/C_2 s} \cdot R_3$$

$$\begin{aligned}\frac{V_O}{V_I} &= g_m \cdot \frac{R_S}{R_S + (g_m R_S + 1)(R_I + 1/C_1 s)} \cdot \frac{R_D}{R_D + R_3 + 1/C_2 s} \cdot R_3 \\ &= \frac{g_m R_S R_D R_3 C_1 s C_2 s}{[C_1 s(R_S + R_I + g_m R_S R_I) + g_m R_S + 1][C_2 s(R_D + R_3) + 1]}\end{aligned}$$

$$\omega_{z_1} = \omega_{z_2} = 0$$

$$\omega_{p_1} = -\frac{g_m R_s + 1}{C_1(R_S + R_I + g_m R_S R_I)} \approx -569.91\,\mathrm{rad/s}$$

$$\omega_{p_2} = -\frac{1}{C_2(R_D + R_3)} \approx -9.59\,\mathrm{rad/s}$$

$$f_c = \frac{\omega_{p_1} + \omega_{p_2}}{2\pi} \approx 92.23\,\mathrm{Hz}$$

$$A_{mid} = \frac{g_m(R_D \parallel R_3)}{1 + g_m(R_I \parallel R_S)} \cdot \frac{R_S}{R_I + R_S} \approx 9.57$$

Problem 2.

$$g_m = \frac{I_C}{V_T} = \frac{1\,\mathrm{mA}}{0.025\,\mathrm{V}} = 40\,\mathrm{mS}$$

$$c_\pi = \frac{g_m}{2\pi f_T} = \frac{40\,\mathrm{mS}}{2\pi \cdot 500\,\mathrm{MHz}} \approx 12.73\,\mathrm{pF}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40\,\mathrm{mS}} = 2.5\,\mathrm{k\Omega}$$

$$r_{\pi_0} = (R_I \parallel R_B + r_x) \parallel r_\pi = [(1 \parallel 7.5 + 0.3) \parallel 2.5]\,\mathrm{k\Omega} \approx 802\,\Omega$$

$$R_L = R_C \parallel R_3 = 4.123\,\mathrm{k\Omega}$$

$$c_T = c_\pi + c_\mu \left(1 + g_m R_L + \frac{R_L}{r_{\pi_0}}\right) = 12.73\,\mathrm{pF} + 0.75\,\mathrm{pF}\left(1 + 40\,\mathrm{mS} \cdot 4.123\,\mathrm{k\Omega} + \frac{4.123\,\mathrm{k\Omega}}{802\,\Omega}\right) \approx 141.03\,\mathrm{pF}$$

$$f_{p_1} = \frac{1}{2\pi r_{\pi_0} c_T} = \frac{1}{2\pi \cdot 802\,\Omega \cdot 141.03\,\mathrm{pF}} \approx 1.41\,\mathrm{MHz}$$

$$f_{p_2} = \frac{g_m}{2\pi c_\pi} = \frac{40\,\mathrm{mS}}{2\pi \cdot 12.73\,\mathrm{pF}} \approx 500.09\,\mathrm{MHz}$$

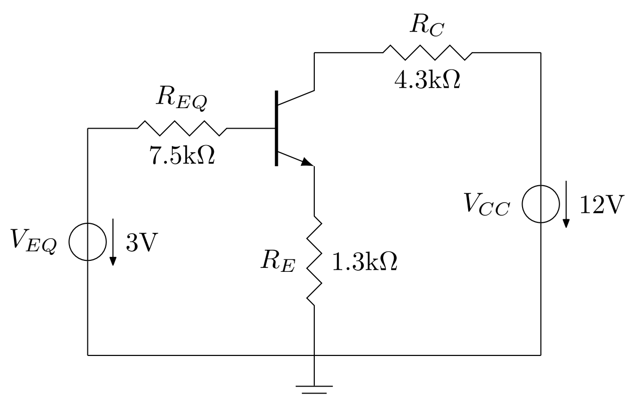
$$f_H = f_{p_1} = 1.41\,\mathrm{MHz}$$

$$A_{mid} = \frac{R_L[R_B \parallel (r_\pi + r_x)]}{R_I + [R_B \parallel (r_\pi + r_x)]} \cdot \frac{-g_m r_\pi}{r_\pi + r_x} \approx -98.79$$

$$GBW = |A_{mid}| f_H = 139.29\,\mathrm{MHz}$$

Problem 3.

The dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

$$I_C = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_0} + \dfrac{\beta_0 + 1}{\beta_0} R_E} = \frac{3\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{7.5\,\mathrm{k\Omega}}{100} + \dfrac{100+1}{100} \cdot 1.3\,\mathrm{k\Omega}} \approx 1.657\,\mathrm{mA}$$

$$I_E = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_0 + 1} + R_E} = \frac{3\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{7.5\,\mathrm{k\Omega}}{100+1} + 1.3\,\mathrm{k\Omega}} \approx 1.673\,\mathrm{mA}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 12\,\mathrm{V} - 1.657\,\mathrm{mA} \cdot 4.3\,\mathrm{k\Omega} - 1.673\,\mathrm{mA} \cdot 1.3\,\mathrm{k\Omega} = 2.7\,\mathrm{V}$$

So the Q point is $(1.657\,\mathrm{mA},\ 2.7\,\mathrm{V})$.

$$g_m = \frac{I_C}{V_T} = \frac{1.657\,\mathrm{mA}}{0.025\,\mathrm{V}} = 66.28\,\mathrm{mS}$$

$$r_\pi = \frac{\beta_0 V_T}{I_C} = \frac{100 \cdot 0.025\,\mathrm{V}}{1.657\,\mathrm{mA}} \approx 1.508\,\mathrm{k\Omega}$$

$$r_{\pi_0} = [(R_{EQ} \parallel R_I) + r_x] \parallel [r_\pi + (\beta_0 + 1)R_E] = [(7.5 \parallel 0.25 + 0.35) \parallel (1.508 + 101 \cdot 0.2)]\,\mathrm{k\Omega} \approx 576\,\Omega$$

$$R_L = R_C \parallel R_3 = (4.3 \parallel 47)\,\mathrm{k\Omega} = 3.94\,\mathrm{k\Omega}$$

$$c_\pi = \frac{g_m}{2\pi f_T} - c_\mu = \frac{66.28\,\mathrm{mS}}{2\pi \cdot 200\,\mathrm{MHz}} - 1\,\mathrm{pF} \approx 51.74\,\mathrm{pF}$$

$$c_T = \frac{c_\pi}{1 + g_m R_E} + c_\mu \left(1 + \frac{g_m R_L}{1 + g_m R_E +} + \frac{R_L}{r_{\pi_0}}\right) \approx 29.79\,\mathrm{pF}$$

For f_H,

$$f_{p_1} = \frac{1}{2\pi r_{\pi_0} c_T} = \frac{1}{2\pi \cdot 576\,\Omega \cdot 29.79\,\mathrm{pF}} \approx 9.275\,\mathrm{MHz}$$

$$f_{p_2} = \frac{g_m}{2\pi(1 + g_m R_E)c_\pi} = \frac{66.28\,\mathrm{mS}}{2\pi \cdot (1 + 66.28\,\mathrm{mS} \cdot 200\,\Omega) \cdot 51.74\,\mathrm{pF}} \approx 14.30\,\mathrm{MHz}$$

$$f_z = \frac{g_m}{2\pi(1 + g_m R_E)c_\mu} = \frac{66.28\,\mathrm{mS}}{2\pi \cdot (1 + 66.28\,\mathrm{mS} \cdot 200\,\Omega) \cdot 1\,\mathrm{pF}} \approx 739.95\,\mathrm{MHz}$$

$$f_H = \frac{1}{\sqrt{f_{p_1}^{-2} + f_{p_2}^{-2} - 2f_z^{-2}}} \approx 7.78\,\mathrm{MHz}$$

For f_L,

$$R_{iB} = r_\pi + r_x + (\beta_0 + 1)R_E = (1.508 + 0.35 + 101 \cdot 0.2)\,\mathrm{k\Omega} = 22.06\,\mathrm{k\Omega}$$

$$R_{1s} = R_I + R_{EQ} \parallel R_{iB} = (0.25 + 7.5 \parallel 22.06)\,\mathrm{k\Omega} \approx 5.85\,\mathrm{k\Omega}$$

$$R_{2s} = R_3 + R_C = (4.3 + 47)\,\mathrm{k\Omega} = 51.3\,\mathrm{k\Omega}$$

$$R_{3s} = R_{E2} \parallel \left[\frac{r_\pi + r_x + R_I \parallel R_{EQ}}{\beta_0 + 1} + R_{E1}\right] \approx 184\,\Omega$$

$$f_L = \frac{1}{2\pi(R_{1s}C_1)} + \frac{1}{2\pi(R_{2s}C_2)} + \frac{1}{2\pi(R_{3s}C_3} \approx 197\,\mathrm{Hz}$$

For A_{mid},

$$A_{mid} = \frac{-g_m R_L}{1 + g_m R_{E1}} \cdot \frac{R_{EQ} \parallel R_{iB}}{R_I + R_{EQ} \parallel R_{iB}} \cdot \frac{r_\pi + (\beta_0 + 1)R_{E1}}{r_\pi + r_x + (\beta_0 + 1)R_{E1}} \approx -17.26$$

Problem 4.

The SPICE code is

```
p4.cir
.TITLE Problem 4

Vi  12  0   AC  10V
RI  12  1   10K
C1  1   2   0.01U
RG  0   2   1MEG
RS1 0   3   200
C2  0   3   47U
```

```
RD1 4   5   620
R2  0   6   22K
C3  4   6   1U
R1  5   6   78K
RE2 0   7   1.5K
C4  0   7   22U
RC2 5   8   4.7K
R4  0   9   120K
C5  8   9   1U
R3  5   9   91K
RE3 0   10  3.3K
RL  0   11  250
C6  10  11  22U
VDD 5   0   DC  15V

M1  4   2   3   3   NFET
Q1  8   6   7   QMOD1
Q2  5   9   10  QMOD2

CA  2   3   5P
CB  2   4   1P

.MODEL NFET NMOS (LAMBDA=0.02 VTO=-2 KP=10M)
.MODEL QMOD1 NPN (BF=150 RB=250 VAF=80 TF=0.575N CJC=1.89P)
.MODEL QMOD2 NPN (BF=150 RB=250 VAF=80 TF=0.628N CJC=2.22P)

.AC DEC 100 1M 5G
.MEASURE AC vo1 FIND VM(11) AT=2M
.MEASURE AC vo2 FIND VM(11) AT=1
.MEASURE AC vo3 FIND VM(11) AT=50K
.MEASURE AC vo4 FIND VM(11) AT=2G
.MEASURE AC vmax MAX V(11)

.MEASURE AC f1 WHEN VM(11)='1.003815e+04/sqrt(2)' RISE=1
.MEASURE AC f2 WHEN VM(11)='1.003815e+04/sqrt(2)' FALL=1

.FUNC amp(x) {20*ln(x/10)/ln(10)}
.MEASURE AC a1 param='amp(vo1)'
.MEASURE AC a2 param='amp(vo2)'
.MEASURE AC a3 param='amp(vo3)'
.MEASURE AC a4 param='amp(vo4)'

.MEASURE AC p1 FIND VP(11) AT=2M
.MEASURE AC p2 FIND VP(11) AT=1
.MEASURE AC p3 FIND VP(11) AT=50K
.MEASURE AC p4 FIND VP(11) AT=2G

.PROBE
.END
```

The result is

```
No. of Data Rows : 1270
vo1                   =  6.593746e-14
vo2                   =  3.921827e-03
vo3                   =  1.001733e+04
vo4                   =  1.718621e-04
vmax                  =  1.003815e+04 at=  1.584893e+04
f1                    =   3.70283e+02
f2                    =   6.74390e+05
a1                    =  -2.83617e+02
a2                    =  -6.81302e+01
a3                    =  6.00150e+01
a4                    =  -9.52964e+01
p1                    =  -6.729757e-04
p2                    =  -3.238677e-01
p3                    =  -8.084412e-02
p4                    =  -1.020581e+00
```

So we can find the magnitude and phase of the circuit for different frequencies.

Frequency (Hz)	Magnitude (dB)	Phase (rad)
2m	-2.83617e+02	-6.729757e-04
1	-6.81302e+01	-3.238677e-01
50k	6.00150e+01	-8.084412e-02
2G	-9.52964e+01	-1.020581e+00

And the cut-off frequencies (Hz) are $f_1 = 3.70283e+02$ and $f_2 = 6.74390e+05$. When the input frequency is in the range of cut-off frequencies, the magnitude is high. Otherwise, the magnitude is very low.