

# Differential Amplifier

Mario Alberto García-Ramírez, PhD

MU-JTSU JI

June 19, 2017

## Intro

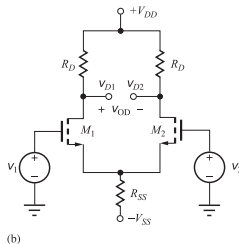
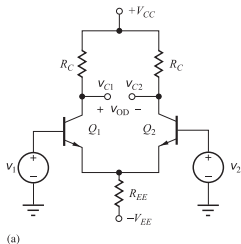
# Differential Amplifier

# Direct Coupled Amplifier

- ▶ Coupling capacitors limit the low-frequency of the amps and reduce its applications as dc amps
- ▶ For an amp to deliver gain at dc, capacitors connected in series must be removed
- ▶ Using a direct coupled design eliminates additional resistors that are needed to bias individual stages in an ac coupled amp
- ▶ dc-coupled differential amps are the most important basic building blocks for analog design
- ▶ Differential amp are like soja, it appear in every analog integrated circuit
- ▶ These circuits are key for operational amp as well as of most dc coupled analog circuits
- ▶ A differential amp has 2 BJT in symmetrical configuration, it usually is a single stage amp

# Direct Coupled Amplifier

- ▶ Figure shows a BJT & MOS version of a differential amp
- ▶ Each circuit has 2 input  $v_1$  &  $v_2$  and a differential output mode  $v_{OD}$
- ▶ Ground reference outputs are focused between collector and drain  $v_{C1}$ ,  $v_{C2}$ ,  $v_{D1}$ , or  $v_{D2}$  and ground
- ▶ The symmetrical nature of the amp gives useful dc and ac properties
- ▶ The diff. amp. behaves as an inverting or non-inverting amp for diff. input signals but tend to reject signals common to both inputs

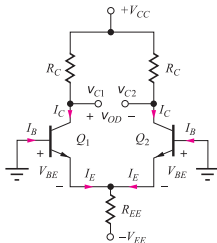


# Direct Coupled Amplifier

- ▶ Ideal performance is obtained from the differential amplifier only when it is perfectly symmetrical
- ▶ Best versions are built using IC technology in which the transistor characteristics can be closely matched
- ▶ Two transistors are said to be matched if they have identical characteristics and parameter values
- ▶ The parameter sets (  $I_S, \beta_{FO}, V_A$  ) or (  $K_n, V_{TN}$  and  $\lambda$  ), Q-points, and temperatures of the two transistors are identical

## dc Analysis of the BJT Diff. Amp.

- ▶ We, as always, start by finding the transistor operating point
- ▶ the quiescent operation points can be found by setting both input signals to zero as shown in fig.
- ▶ Both bases are grounded and the two emitters are connected together, ergo  $V_{BE1} = V_{BE2} = V_{BE}$
- ▶ If  $Q_1$  &  $Q_2$  match, the symmetry forces  $V_{C1} = V_{C2} = V_C$  and  $I_{S1} = I_{S2} = I_S$ ,  
 $I_{E1} = I_{E2} = I_E$



$$I_C = \alpha_F \frac{V_{EE} - V_{BE}}{2R_{EE}} \cong \frac{V_{EE} - V_{BE}}{2R_{EE}}$$

$$V_{CE} = V_{CC} + V_{BE} - I_C R_C$$

## dc Analysis of the BJT Diff. Amp.

- ▶ Emitter current according to  $Q_1$  is

$$V_{BE} + 2I_E R_{EE} - V_{EE} = 0$$

and

$$I_C = \alpha_F I_E = \alpha_F \frac{V_{EE} - V_{BE}}{2R_{EE}}$$

by considering

$$I_B = I_C / \beta_F$$

colector voltages are defined as:  $V_{C1} = V_{C2} = V_{CC} - I_C R_C$   
and  $V_{CE1} = V_{CE2} = V_{CC} + V_{BE} - I_C R_C$  For the symmetrical  
amplifier, the dc output is zero

$$V_{OD} = V_{C1} - V_{C2} = 0V$$

## Example 1

- Determine the Q-point for the emitter coupled pair by considering  $V_{CC} = V_{EE} = 15V$ ,  $R_{EE} = R_C = 75k\Omega$ ,  $\beta_F=100$

$$I_E = \frac{V_{EE} - V_{BE}}{2R_{EE}} = \frac{15 - 0.7}{2 \cdot 75} \approx 9.53 \times 10^{-2} mA$$

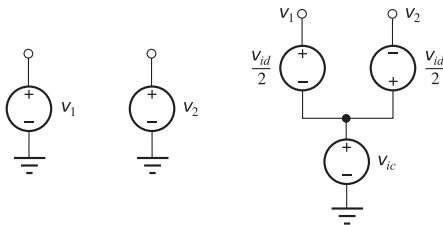
$$I_C = \alpha I_E = \frac{100}{100 + 1} \cdot 9.53 \times 10^{-2} = 9.44 \times 10^{-2} mA$$

$$V_{CE} = V_{CC} - I_C R_C + V_{BE} = 15 - 9.44 \times 10^{-2} \cdot 75 + 0.7 = 8.62V$$



## ac Analysis of the BJT Diff. Amp.

- ▶ Once we've got the Q-point info, we continue to small signal analysis to characterize the gain voltage as well as input and output resistance
- ▶ ac Analysis of the diff. amp. is simplified by breaking the input sources  $v_1$  &  $v_2$  into the equivalent differential-mode input ( $v_{id}$ ) and common-mode input ( $v_{ic}$ ),  $v_{id} = v_1 - v_2$  and  $v_{ic} = \frac{v_1 + v_2}{2}$
- ▶ The input voltage can be written in terms of  $v_{ic}$  and  $v_{id}$ , ergo  $v_1 = v_{ic} + \frac{v_{id}}{2}$  and  $v_2 = v_{ic} - \frac{v_{id}}{2}$



## ac Analysis of the BJT Diff. Amp.

- ▶ The differential-mode and common-mode output voltages,  $v_{od}$  and  $v_{oc}$ , are defined as  $v_{od} = v_{c1} - v_{c2}$  and  $v_{oc} = \frac{v_{c1} + v_{c2}}{2}$
- ▶ For the general case,  $v_{od}$  &  $v_{oc}$  are function of  $v_{id}$  and  $v_{ic}$  can be written as

$$M = \begin{bmatrix} v_{od} \\ v_{oc} \end{bmatrix} = \begin{bmatrix} A_{dd} & A_{cd} \\ A_{dc} & A_{cc} \end{bmatrix} = \begin{bmatrix} v_{id} \\ v_{ic} \end{bmatrix}$$

where:

$A_{dd}$  = differential-mode gain

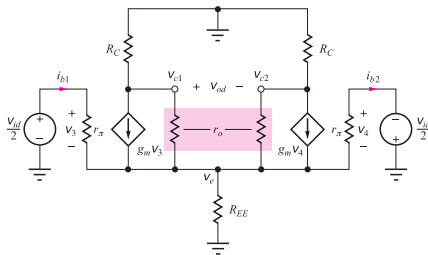
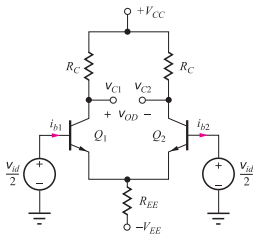
$A_{cd}$  = common-mode (to differential-mode) conversion gain

$A_{cc}$  = common-mode gain

$A_{dc}$  = differential-mode (to common-mode) conversion gain

# Differential mode gain and input resistance

- ▶ Purely differential-mode input signals are applied to the differential amplifier
- ▶ 2 transistors are replaced with their small-signal models



$$\frac{v_{be1}}{r_\pi} + g_m v_{ve1} + g_m v_{be2} + \frac{v_{be2}}{r_\pi} = \frac{v_e}{r_{EE}}$$

$$\frac{1}{r_\pi} + g_m (v_{be1} + v_{be2}) = \frac{v_e}{R_{EE}}$$

$$v_{be1} = \frac{v_{id}}{2} - v_e; \quad v_{be2} = -\frac{v_{id}}{2} - v_e; \quad v_{be1} + v_{be2} = -2v_e$$

$$(1/r_\pi + g_m)(-2v_e) = \frac{v_e}{R_{EE}} \quad v_e \left( \frac{1}{R_{EE}} + \frac{2}{r_\pi} + 2g_m \right) = 0 \rightarrow v_e = 0$$

## Differential mode gain and input resistance

- ▶ For a purely differential-mode input voltage, the voltage at the emitter node is identically zero
- ▶ The “virtual ground” at the emitter node causes the differential amplifier to behave as a common-emitter (or common-source) amplifier  $v_{be1} = \frac{v_{id}}{2}$ ;  $v_{be2} = -\frac{v_{id}}{2}$

$$v_{c1} = -(g_m v_{be1}) R_C$$

$$v_{c2} = -(g_m v_{be2}) R_C$$

$$v_{od} = -g_m R_C v_{id}$$

- ▶ The differential mode gain ( $A_{dd}$ ) for a balanced output,  $v_{od} = v_{c1} - v_{c2}$  is defined as

$$A_{dd} = \frac{v_{od}}{v_{id}} \Big|_{v_{ic}=0} = -g_m R_C$$

## Differential mode gain and input resistance

- ▶ If either  $v_{c1}$  or  $v_{c2}$  alone is used as the output, referred to as a single-ended (or ground-referenced) outputs

$$A_{dd1} = \frac{v_{c1}}{v_{id}} \Big|_{v_{ic}} = \frac{-g_m v_{be1} R_C}{2v_{be1}} = -\frac{g_m R_C}{2} = \frac{A_{dd}}{2}$$

$$A_{dd2} = \frac{v_{c2}}{v_{id}} \Big|_{v_{ic}} = \frac{g_m v_{be2} R_C}{2v_{be2}} = \frac{g_m R_C}{2} = -\frac{A_{dd}}{2}$$

- ▶ The differential mode input resistance

$$R_{id} = \frac{v_{id}}{i_{b1}} = 2r_{\pi}$$

- ▶ The differential-mode output resistance

$$R_{od} = 2R_C$$

- ▶ For single-ended outputs

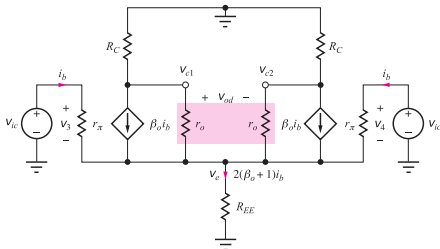
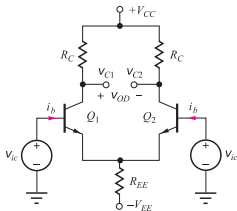
$$R_{out} = R_C$$

# Common mode gain and input resistance

- ▶ Both sides of the amplifier are completely symmetrical
- ▶ The two base, the emitter, the collector currents and the two collector voltages must be equal
- ▶ According to Figure, the small-signal model we've got

$$v_{ic} = i_b r_\pi + 2(\beta_0 + 1) i_b R_{EE} \quad i_b = \frac{v_{ic}}{r_\pi + 2(\beta_0 + 1) R_{EE}}$$

$$v_{c1} = v_{c2} = -(\beta_0 i_b) R_C$$



## Common mode gain and input resistance

$$A_{cc} = \frac{v_{oc}}{v_{ic}} \Big|_{v_{id}=0} = -\frac{\beta_0 R_C}{r_\pi + 2(\beta_0 + 1)R_{EE}} \approx -\frac{R_C}{2R_{EE}}$$

$$A_{cc} = -\frac{R_C}{2R_{EE}} = -\frac{C_{CC}}{2(V_{EE} - V_{BE})} \approx \frac{V_{CC}}{2V_{EE}}$$

the differential output voltage  $v_{od}$  is zero:  $v_{od} = v_{c1} - v_{c2} = 0$

$$A_{cd} = \frac{v_{od}}{v_{ic}} \Big|_{v_{id}=0} = 0$$

A more accurate expression for the common-mode gain is

$$A_{cc} = R_C \left( \frac{1}{\beta_0 r_0} - \frac{1}{2R_{EE}} \right)$$

For an infinite  $R_{EE}$ ,  $A_{cc}$  is limited to  $R_C/\beta_0 r_0 \approx V_{CC}/2\beta_0 V_A$

It is determined by the total signal current ( $2i_b$ ) being supplied from the common-mode source

$$R_{ic} = \frac{v_{ic}}{2i_b} = \frac{r_\pi + 2(\beta_0 + 1)R_{EE}}{2} = \frac{r_\pi}{2} + (\beta_0 + 1)R_{EE}$$

# Common mode rejection ratio (CMRR)

- ▶ CMRR, characterizes the ability of an amplifier to amplify the desired differential-mode input signal and reject the undesired common-mode input signal
- ▶ It is defined as

$$CMRR = \left| \frac{A_{dm}}{A_{cm}} \right|$$

where  $A_{dm}$  and  $A_{cm}$  are the overall differential-mode and common-mode gains

- ▶ For a differential output  $v_{od}$ , the common-mode gain of the balanced amplifier is zero, and the CMRR is infinite and the output is taken from each collector as

$$v_{c1} = v_{oc} + \frac{v_{od}}{2} = A_{cc} v_{ic} + \frac{A_{dd}}{2} v_{id} \text{ and}$$

$$v_{c2} = v_{oc} + \frac{v_{od}}{2} = A_{cc} v_{ic} + \frac{A_{dd}}{2} v_{id}$$



# Common mode rejection ratio (CMRR)

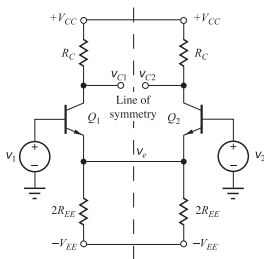
- ▶ CMRR is based as

$$\frac{A_{dm}}{a_{cm}} = \frac{\frac{A_{dd}}{2}}{A_{cc}} = \frac{\frac{g_m R_C}{2}}{R_C \left( \frac{1}{\beta_0 r_0} - \frac{1}{2R_{EE}} \right)} = \frac{1}{2 \left( \frac{1}{\beta_0 r_0} - \frac{1}{2g_m R_{EE}} \right)}$$

- ▶ For infinite  $R_{EE}$ ,  $CMRR \approx \frac{\beta_0 \mu_f}{2}$
- ▶ If the term containing  $R_{EE}$  is dominant  $CMRR \approx g_m R_{EE} = 40I_C R_{EE} = 20(2I_E R_{EE}) = 20(V_{EE} - V_{BE}) \approx 20V_{EE}$

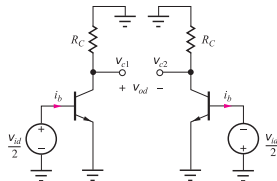
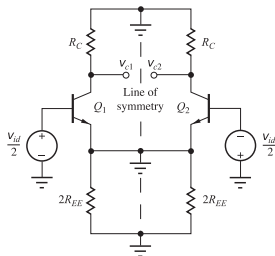
# Rules to design differential half-circuits

- ▶ We noted that the differential amplifier behaves much as the single-transistor common-emitter amplifier
- ▶ Differential-mode signals, Points on the line of symmetry represent virtual grounds and can be connected to ground for ac analysis
- ▶ Common-mode signals, Points on the line of symmetry can be replaced by open circuits

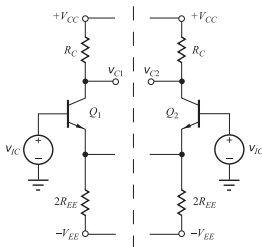


# Common-mode half circuits

- Emitters are in short

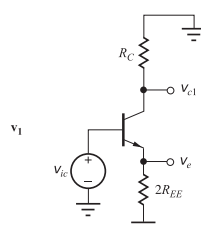
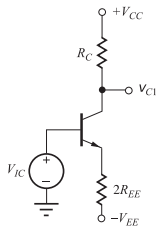
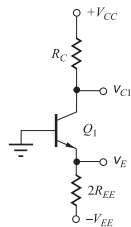


- we've got



# Common-mode half circuits

- Emitters are in short



- To be continue...