VE311 Homework 5

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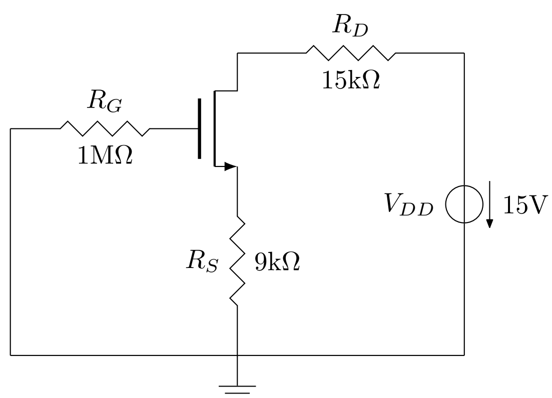
Problem 1.

First we should apply dc analysis to the circuit, it can be divided into three circuits.

(1) For M_1, the dc equivalent circuit is



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$
$$V_{GS} + I_D R_s = 0$$

We can get

$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = 0$$
$$V_{GS} = V_{TN} = \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)$$

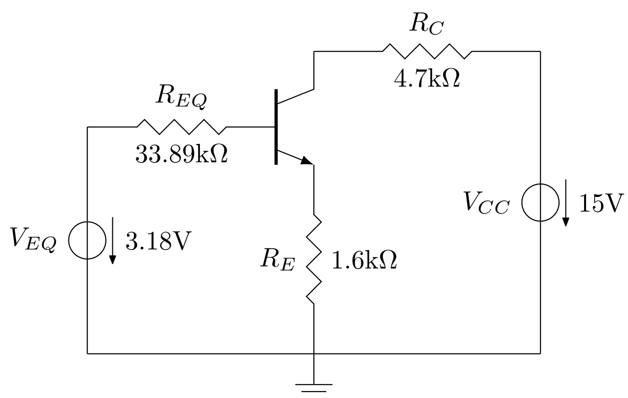
$$\begin{aligned} I_D &= \frac{1}{2K_n R_s^2}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)^2 \\ &= \frac{1}{2 \cdot 0.3\,\mathrm{A/V^2} \cdot (9\,\mathrm{k\Omega})^2}\left(\sqrt{1 - 2 \cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -3\,\mathrm{V}} - 1\right)^2 \\ &\approx 0.33\,\mathrm{mA} \end{aligned}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 15\,\mathrm{V} - 0.33\,\mathrm{mA} \cdot (15\,\mathrm{k\Omega} + 9\,\mathrm{k\Omega}) = 7.08\,\mathrm{V}$$

$$
\begin{aligned}
V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right) \\
&= \frac{1}{2 \cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega}}\left(\sqrt{1 - 2 \cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -3\,\mathrm{V}} - 1\right) \\
&= 0.023\,\mathrm{V} < V_{DS}
\end{aligned}
$$

So the Q point is $(0.33\,\mathrm{mA},\ 0.023\,\mathrm{V})$, and it is in the saturated region.

(2) For Q_1, the dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

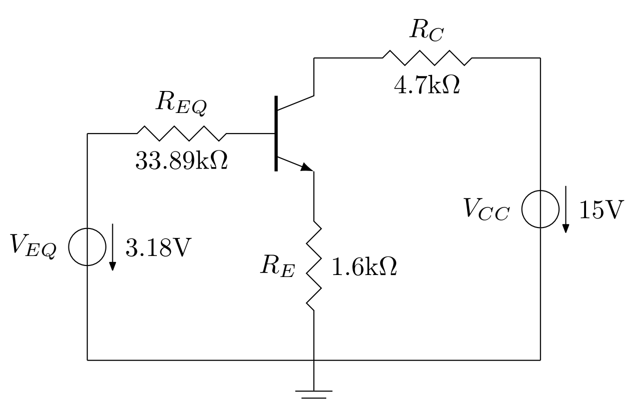
$$
I_C = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F} + \frac{\beta_F + 1}{\beta_F} R_E} = \frac{3.18\,\mathrm{V} - 0.7\,\mathrm{V}}{\frac{33.89\,\mathrm{k\Omega}}{17} + \frac{17+1}{17} \cdot 1.6\,\mathrm{k\Omega}} \approx 0.673\,\mathrm{mA}
$$

$$
I_E = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{3.18\,\mathrm{V} - 0.7\,\mathrm{V}}{\frac{33.89\,\mathrm{k\Omega}}{17+1} + 1.6\,\mathrm{k\Omega}} \approx 0.712\,\mathrm{mA}
$$

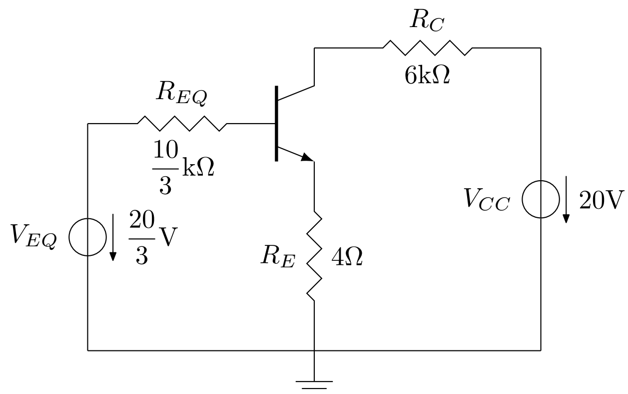
$$
V_{CE} = V_{CC} - I_C R_C - I_E R_E = 15\,\mathrm{V} - 0.673\,\mathrm{mA} \cdot 4.7\,\mathrm{k\Omega} - 0.712\,\mathrm{mA} \cdot 1.6\,\mathrm{k\Omega} \approx 10.70\,\mathrm{V}
$$

So the Q point is $(0.673\,\mathrm{mA},\ 10.70\,\mathrm{V})$.

(3) For Q_2, the dc equivalent circuit is



Problem 2.



Suppose $V_{BE} = 0.7V$,

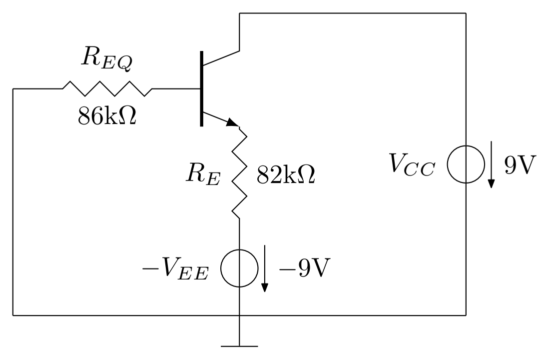
$$I_C = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F} + \frac{\beta_F + 1}{\beta_F} R_E} = \frac{20/3\,\mathrm{V} - 0.7\,\mathrm{V}}{\frac{10/3\,\mathrm{k\Omega}}{75} + \frac{75+1}{75} \cdot 4\,\mathrm{k\Omega}} \approx 1.456\,\mathrm{mA}$$

$$I_E = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{20/3\,\mathrm{V} - 0.7\,\mathrm{V}}{\frac{10/3\,\mathrm{k\Omega}}{75+1} + 4\,\mathrm{k\Omega}} \approx 1.475\,\mathrm{mA}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 20\,\mathrm{V} - 1.456\,\mathrm{mA} \cdot 6\,\mathrm{k\Omega} - 1.475\,\mathrm{mA} \cdot 4\,\mathrm{k\Omega} = 5.364\,\mathrm{V}$$

So the Q point is $(1.456\,\mathrm{mA},\ 5.364\,\mathrm{V})$.

Problem 3.



Suppose $V_{BE} = 0.7V$,

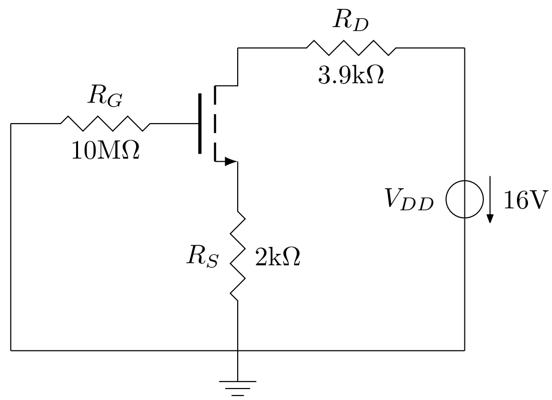
$$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{9\,\mathrm{V} - 0.7\,\mathrm{V}}{82\,\mathrm{k\Omega}} \approx 101\,\mu\mathrm{A}$$

$$I_C = \frac{\beta_F}{\beta_F + 1} I_E = \frac{100}{100+1} \cdot 100\,\mu\mathrm{A}$$

$$V_{CE} = V_{CC} - I_C R_C - (-V_{BE}) = 9\,\mathrm{V} + 0.7\,\mathrm{V} = 9.7\,\mathrm{V}$$

So the Q point is $(100\,\mu\mathrm{A},\ 9.7\,\mathrm{V})$.

Problem 4.



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$V_{GS} + I_D R_s = 0$$

We can get

$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = 0$$

$$V_{GS} = V_{TN} = \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)$$

$$\begin{aligned} I_D &= \frac{1}{2K_n R_s^2}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)^2 \\ &= \frac{1}{2 \cdot 400\,\mu\text{A/V}^2 \cdot (2\,\text{k}\Omega)^2}\left(\sqrt{1 - 2 \cdot 400\,\mu\text{A/V}^2 \cdot 2\,\text{k}\Omega \cdot -5\,\text{V}} - 1\right)^2 \\ &= 1.25\,\text{mA} \end{aligned}$$

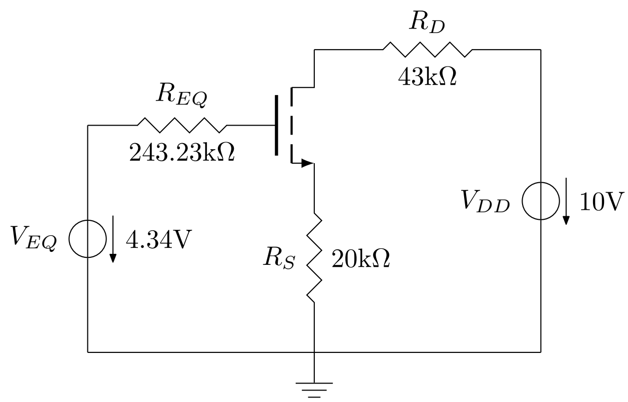
$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 16\,\text{V} - 1.25\,\text{mA} \cdot (3.9\,\text{k}\Omega + 2\,\text{k}\Omega) = 8.625\,\text{V}$$

$$\begin{aligned} V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right) \\ &= \frac{1}{2 \cdot 400\,\mu\text{A/V}^2 \cdot 2\,\text{k}\Omega}\left(\sqrt{1 - 2 \cdot 400\,\mu\text{A/V}^2 \cdot 2\,\text{k}\Omega \cdot -5\,\text{V}} - 1\right) \\ &= 1.25\,\text{V} < V_{DS} \end{aligned}$$

So the Q point is $(1.25\,\text{mA},\ 8.625\,\text{V})$, and it is in the saturated region.

Problem 5.

First, we draw the dc equivalent circuit:



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$
$$V_{GS} + I_D R_s = V_{EQ}$$

We can get

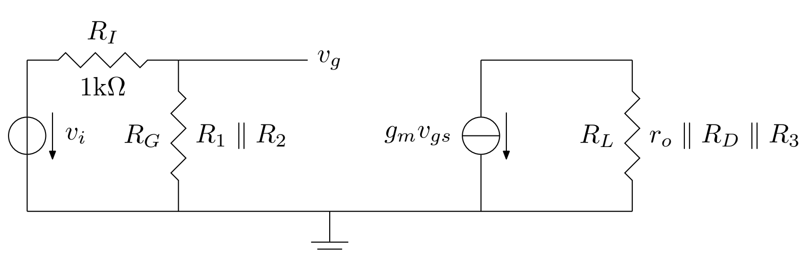
$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = V_{EQ}$$
$$V_{GS} = V_{TN} + \frac{1}{K_n R_s}\left(\sqrt{1 + 2K_n R_s(V_{EQ} - V_{TN})} - 1\right)$$

$$\begin{aligned}
I_D &= \frac{1}{2K_n R_s^2}\left(\sqrt{1 + 2K_n R_s(V_{EQ} - V_{TN})} - 1\right)^2 \\
&= \frac{1}{2 \cdot 0.5\,\mathrm{mA/V^2} \cdot (20\,\mathrm{k\Omega})^2}\left(\sqrt{1 + 2 \cdot 0.5\,\mathrm{mA/V^2} \cdot 20\,\mathrm{k\Omega} \cdot (4.34\,\mathrm{V} - 1\,\mathrm{V})} - 1\right)^2 \\
&\approx 131\,\mu\mathrm{A}
\end{aligned}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 10\,\mathrm{V} - 131\,\mu\mathrm{A} \cdot (43\,\mathrm{k\Omega} + 20\,\mathrm{k\Omega}) \approx 1.75\,\mathrm{V}$$

$$\begin{aligned}
V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 + 2K_n R_s(V_{EQ} - V_{TN})} - 1\right) \\
&= \frac{1}{0.5\,\mathrm{mA/V^2} \cdot 20\,\mathrm{k\Omega}}\left(\sqrt{1 + 2 \cdot 0.5\,\mathrm{mA/V^2} \cdot 20\,\mathrm{k\Omega} \cdot (4.34\,\mathrm{V} - 1\,\mathrm{V})} - 1\right) \\
&\approx 0.717\,\mathrm{V} < V_{DS}
\end{aligned}$$

So the Q point is $(131\,\mu\mathrm{A},\ 1.75\,\mathrm{V})$, and it is in the saturated region.
Then we can draw the ac equivalent circuit:



$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2 \cdot 131\,\mu\text{A}}{0.717\,\text{V}} \approx 3.65 \times 10^{-4}\,\Omega^{-1}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{\frac{1}{0.0133\,\text{V}^{-1}} + 1.75\,\text{V}}{131\,\mu\text{A}} \approx 587\,\text{k}\Omega$$

$$R_G = R_1 \parallel R_2 \approx 243.23\,\text{k}\Omega$$

$$R_L = r_o \parallel R_D \parallel R_3 \approx 28.6\,\text{k}\Omega$$

The voltage gain is

$$\begin{aligned} A_v^{CS} &= -g_m R_L \left(\frac{R_G}{R_G + R_I} \right) \\ &= -3.65 \times 10^{-4}\,\Omega^{-1} \cdot 28.6\,\text{k}\Omega \cdot \left(\frac{243.23\,\text{k}\Omega}{243.23\,\text{k}\Omega + 1\,\text{k}\Omega} \right) \\ &\approx -10.4\,\text{dB} \end{aligned}$$

Problem 6.

(a)

$$g_d = \frac{I_S}{V_T} \exp\left(\frac{V_D}{V_T}\right) = \frac{8\,\text{fA}}{0.025\,\text{V}} \exp\left(\frac{0.6\,\text{V}}{0.025\,\text{V}}\right) \approx 8.48 \times 10^{-3}\,\Omega^{-1}$$

$$r_d = \frac{1}{g_d} = \frac{1}{8.48 \times 10^{-3}\,\Omega^{-1}} \approx 118\,\Omega$$

(b)

$$g_d = \frac{I_S}{V_T} \exp\left(\frac{V_D}{V_T}\right) = \frac{8\,\text{fA}}{0.025\,\text{V}} \exp\left(\frac{0\,\text{V}}{0.025\,\text{V}}\right) \approx 3.2 \times 10^{-13}\,\Omega^{-1}$$

$$r_d = \frac{1}{g_d} = \frac{1}{3.2 \times 10^{-13}\,\Omega^{-1}} \approx 3.125 \times 10^{12}\,\Omega$$

(c)

$$g_d = \frac{I_S}{V_T} \exp\left(\frac{V_D}{V_T}\right) = \frac{1}{r_d}$$

$$V_D = V_T \ln\left(\frac{V_T}{I_S r_d}\right) = 0.025\,\text{V} \ln\left(\frac{0.025\,\text{V}}{8\,\text{fA} \cdot 10^{12}\,\Omega}\right) \approx 2.85 \times 10^{-2}\,\text{V}$$

Problem 7.

$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{100 \cdot 0.025\,\text{V}}{40\,\mu\text{A}} = 62.5\,\text{k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{40\,\mu\text{A}}{0.025} = 1.6 \times 10^{-3}\,\Omega^{-1}$$

$$r_o = \frac{1}{g_o} = \frac{V_A + V_{CE}}{I_C} = \frac{75\,\text{V} + 10\,\text{V}}{40\,\mu\text{A}} = 2125\,\text{k}\Omega$$

$$R_L = r_o \parallel R_C \parallel R_3 \approx 48.85\,\mathrm{k\Omega}$$

The input resistance is

$$R_B \parallel r_\pi \approx 38.46\,\mathrm{k\Omega}$$

The voltage gain is

$$\begin{aligned} A_v^{CE} &= -g_m R_L \left[\frac{R_B \parallel r_\pi}{R_I + (R_B \parallel r_\pi)} \right] \\ &= -1.6 \times 10^{-3}\,\Omega^{-1} \cdot 48.85\,\mathrm{k\Omega} \cdot \left(\frac{38.46\,\mathrm{k\Omega}}{750\,\Omega 38.46\,\mathrm{k\Omega}} \right) \\ &\approx 76.7\,\mathrm{dB} \end{aligned}$$