VE311 Homework 5

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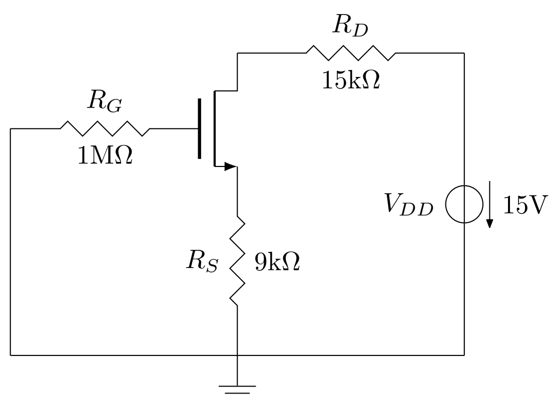
Problem 1.

First we should apply dc analysis to the circuit, it can be divided into three circuits.

(1) For M_1, the dc equivalent circuit is



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$
$$V_{GS} + I_D R_s = 0$$

We can get

$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = 0$$
$$V_{GS} = V_{TN} = \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)$$

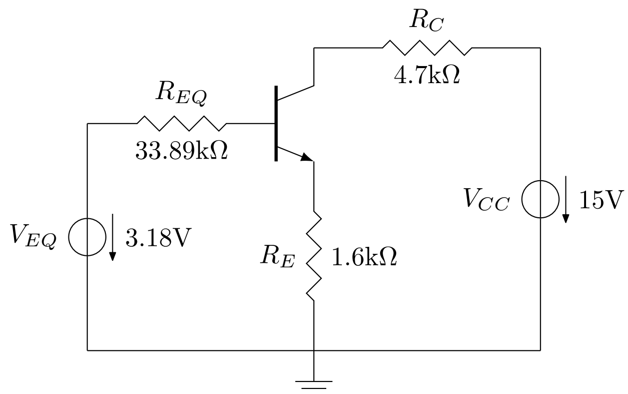
$$\begin{aligned} I_D &= \frac{1}{2K_n R_s^2}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)^2 \\ &= \frac{1}{2 \cdot 0.3\,\mathrm{A/V^2} \cdot (9\,\mathrm{k\Omega})^2}\left(\sqrt{1 - 2 \cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -3\,\mathrm{V}} - 1\right)^2 \\ &\approx 0.33\,\mathrm{mA} \end{aligned}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 15\,\mathrm{V} - 0.33\,\mathrm{mA} \cdot (15\,\mathrm{k\Omega} + 9\,\mathrm{k\Omega}) = 7.08\,\mathrm{V}$$

$$
\begin{aligned}
V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right) \\
&= \frac{1}{\cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega}}\left(\sqrt{1 - 2 \cdot 0.3\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -3\,\mathrm{V}} - 1\right) \\
&\approx 0.047\,\mathrm{V} < V_{DS}
\end{aligned}
$$

So the Q point is $(0.33\,\mathrm{mA},\ 7.08\,\mathrm{V})$, and it is in the saturated region.

(2) For Q_1, the dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

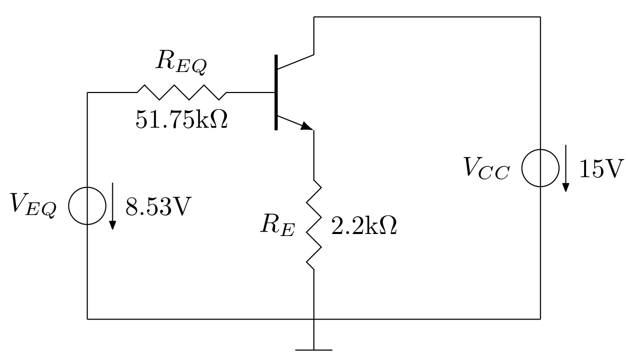
$$
I_C = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F} + \dfrac{\beta_F + 1}{\beta_F} R_E} = \frac{3.18\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{33.89\,\mathrm{k\Omega}}{17} + \dfrac{17+1}{17} \cdot 1.6\,\mathrm{k\Omega}} \approx 0.673\,\mathrm{mA}
$$

$$
I_E = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{3.18\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{33.89\,\mathrm{k\Omega}}{17+1} + 1.6\,\mathrm{k\Omega}} \approx 0.712\,\mathrm{mA}
$$

$$
V_{CE} = V_{CC} - I_C R_C - I_E R_E = 15\,\mathrm{V} - 0.673\,\mathrm{mA} \cdot 4.7\,\mathrm{k\Omega} - 0.712\,\mathrm{mA} \cdot 1.6\,\mathrm{k\Omega} \approx 10.70\,\mathrm{V}
$$

So the Q point is $(0.673\,\mathrm{mA},\ 10.70\,\mathrm{V})$.

(3) For Q_2, the dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

$$I_C = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F} + \dfrac{\beta_F + 1}{\beta_F} R_E} = \frac{8.53\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{51.75\,\mathrm{k\Omega}}{23} + \dfrac{23+1}{23} \cdot 2.2\,\mathrm{k\Omega}} \approx 1.723\,\mathrm{mA}$$

$$I_E = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{8.53\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{51.75\,\mathrm{k\Omega}}{23+1} + 2.2\,\mathrm{k\Omega}} \approx 1.797\,\mathrm{mA}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 15\,\mathrm{V} - 1.723\,\mathrm{mA} \cdot 0\,\mathrm{k\Omega} - 1.797\,\mathrm{mA} \cdot 2.2\,\mathrm{k\Omega} \approx 11.05\,\mathrm{V}$$

So the Q point is $(1.723\,\mathrm{mA},\ 11.05\,\mathrm{V})$.

Then we should calculate the small signal parameters.
For M_1,

$$g_{m1} = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2 \cdot 0.33\,\mathrm{mA}}{0.047\,\mathrm{V}} \approx 0.014\,\mathrm{S}$$

$$r_{o1} = \frac{1/\lambda + V_{DS}}{I_D} = \frac{1/0.01 \times 10^{-2}\,\mathrm{V}^{-1} + 7.08\,\mathrm{V}}{0.33\,\mathrm{mA}} \approx 30.32\,\mathrm{M\Omega}$$

For Q_1,

$$g_{m2} = \frac{I_C}{V_T} = \frac{0.673\,\mathrm{mA}}{0.025\,\mathrm{V}} \approx 0.027\,\mathrm{S}$$

$$r_{\pi 2} = \frac{\beta_{F1}}{g_{m2}} = \frac{17}{0.027\,\mathrm{S}} \approx 629\,\Omega$$

$$r_{o2} = \frac{V_{A1} + V_{CE}}{I_C} = \frac{50\,\mathrm{V} + 10.70\,\mathrm{V}}{0.673\,\mathrm{mA}} \approx 90\,\mathrm{k\Omega}$$

For Q_2,

$$g_{m3} = \frac{I_C}{V_T} = \frac{1.723\,\mathrm{mA}}{0.025\,\mathrm{V}} \approx 0.069\,\mathrm{S}$$

$$r_{\pi 3} = \frac{\beta_{F2}}{g_{m3}} = \frac{23}{0.069\,\mathrm{S}} \approx 333\,\Omega$$

$$r_{o3} = \frac{V_{A2} + V_{CE}}{I_C} = \frac{35\,\mathrm{V} + 11.05\,\mathrm{V}}{1.723\,\mathrm{mA}} \approx 27\,\mathrm{k\Omega}$$

The voltage gain:

$$A_{vt1}^{CS} \approx -g_{m1}(R_{D1} \parallel R_{B2} \parallel r_{\pi 2}) = -0.014\,\mathrm{S} \cdot (15 \parallel (160 \parallel 43) \parallel 0.629)\,\mathrm{k\Omega} \approx -8.30$$

$$\begin{aligned} A_{vt2}^{CE} &\approx -g_{m2}(R_{C2} \parallel R_{B3} \parallel r_{\pi 3} + (1 + \beta_{F2})(R_{E3} \parallel R_L)) \\ &= -0.027\,\mathrm{S} \cdot (4.7 \parallel (91 \parallel 110) \parallel (24 \cdot (2.2 \parallel 0.25)))\,\mathrm{k\Omega} \\ &\approx -64.52 \end{aligned}$$

$$A_{vt3}^{CC} \approx \frac{g_{m3}(R_{E3} \parallel R_L)}{1 + g_{m3}(R_{E3} \parallel R_L)} = \frac{0.069\,\mathrm{S} \cdot (2.2 \parallel 0.25)\,\mathrm{k\Omega}}{1 + 0.069\,\mathrm{S} \cdot (2.2 \parallel 0.25)\,\mathrm{k\Omega}} \approx 0.94$$

$$A_v = A_{vt1}^{CS} \cdot A_{vt2}^{CE} \cdot A_{vt3}^{CC} \cdot \frac{R_G}{R_I + R_G} = -8.30 \cdot -64.52 \cdot 0.94 \cdot \frac{1\,\mathrm{m\Omega}}{10\,\mathrm{k\Omega} + 1\,\mathrm{m\Omega}} \approx 498$$

Input signal range:

$$v_i \leqslant \frac{0.2 \cdot (V_{GS1} - V_{TN})}{0.990} \approx 9.49\,\mathrm{mV}$$

$$v_i \leqslant \frac{5\,\mathrm{mV}}{|A_{vt1}| \cdot 0.990} = \frac{5\,\mathrm{mV}}{8.30 \cdot 0.990} \approx 0.61\,\mathrm{mV}$$

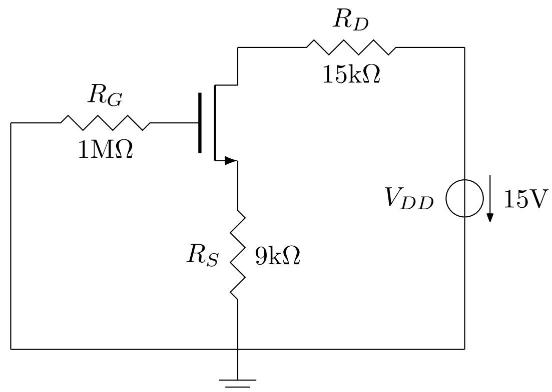
$$v_i \leqslant \frac{(1 + g_{m3} R_{L3}) \cdot 5\,\mathrm{mV}}{A_{vt1} A_{vt2} \cdot 0.990} = \frac{(1 + 0.069\,\mathrm{S} \cdot 224.5\,\Omega) \cdot 5\,\mathrm{mV}}{8.30 \cdot 64.52 \cdot 0.990} \approx 0.156\,\mathrm{mV}$$

So

$$v_i \leqslant 0.156\,\mathrm{mV}$$

Problem 2.

First we should apply dc analysis to the circuit, it can be divided into three circuits. In the first circuit, the dc equivalent circuit is



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$V_{GS} + I_D R_s = 0$$

We can get

$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = 0$$

$$V_{GS} = V_{TN} = \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)$$

$$\begin{aligned} I_D &= \frac{1}{2K_n R_s^2}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)^2 \\ &= \frac{1}{2 \cdot 0.9\,\mathrm{A/V^2} \cdot (9\,\mathrm{k\Omega})^2}\left(\sqrt{1 - 2 \cdot 0.9\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -12\,\mathrm{V}} - 1\right)^2 \\ &\approx 1.33\,\mathrm{mA} \end{aligned}$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 15\,\mathrm{V} - 1.327\,\mathrm{mA} \cdot (15\,\mathrm{k\Omega} + 9\,\mathrm{k\Omega}) = -16.92\,\mathrm{V}$$

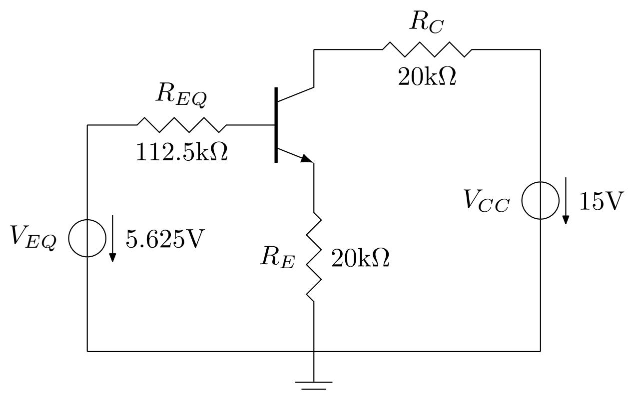
$$
\begin{aligned}
V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right) \\
&= \frac{1}{\cdot 0.9\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega}}\left(\sqrt{1 - 2 \cdot 0.9\,\mathrm{A/V^2} \cdot 9\,\mathrm{k\Omega} \cdot -12\,\mathrm{V}} - 1\right) \\
&\approx 0.054\,\mathrm{V} > V_{DS}
\end{aligned}
$$

So it is not in the saturated region, this problem can't be solved.

Problem 3.

First we should apply dc analysis to the circuit, it can be divided into two circuits.

(1) For Q_1, the dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

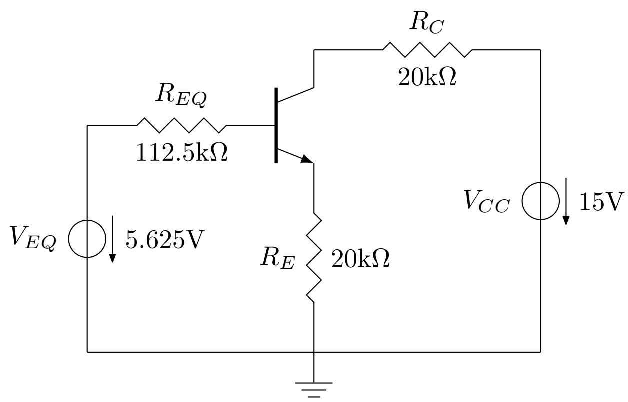
$$
I_C = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F} + \dfrac{\beta_F + 1}{\beta_F} R_E} = \frac{5.625\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{112.5\,\mathrm{k\Omega}}{300} + \dfrac{300 + 1}{300} \cdot 20\,\mathrm{k\Omega}} \approx 0.241\,\mathrm{mA}
$$

$$
I_E = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{5.625\,\mathrm{V} - 0.7\,\mathrm{V}}{\dfrac{112.5\,\mathrm{k\Omega}}{300 + 1} + 20\,\mathrm{k\Omega}} \approx 0.242\,\mathrm{mA}
$$

$$
V_{CE} = V_{CC} - I_C R_C - I_E R_E = 15\,\mathrm{V} - 0.241\,\mathrm{mA} \cdot 20\,\mathrm{k\Omega} - 0.242\,\mathrm{mA} \cdot 20\,\mathrm{k\Omega} = 5.34\,\mathrm{V}
$$

So the Q point is $(0.241\,\mathrm{mA},\ 5.34\,\mathrm{V})$.

(2) For Q_2, the dc equivalent circuit is



Suppose $V_{BE} = 0.7V$,

$$I_C = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F} + \dfrac{\beta_F + 1}{\beta_F} R_E} = \frac{5.625\,\text{V} - 0.7\,\text{V}}{\dfrac{112.5\,\text{k}\Omega}{42} + \dfrac{42+1}{42} \cdot 20\,\text{k}\Omega} \approx 0.213\,\text{mA}$$

$$I_E = \frac{V_{EQ} - V_{BE}}{\dfrac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{5.625\,\text{V} - 0.7\,\text{V}}{\dfrac{112.5\,\text{k}\Omega}{42+1} + 20\,\text{k}\Omega} \approx 0.217\,\text{mA}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 15\,\text{V} - 0.213\,\text{mA} \cdot 20\,\text{k}\Omega - 0.217\,\text{mA} \cdot 20\,\text{k}\Omega = 6.4\,\text{V}$$

So the Q point is $(0.213\,\text{mA},\ 6.4\,\text{V})$.

Then we should calculate the small signal parameters.
For Q_1,

$$g_{m1} = \frac{I_C}{V_T} = \frac{0.241\,\text{mA}}{0.025\,\text{V}} = 9.64\,\text{mS}$$

$$r_{\pi 1} = \frac{\beta_{F1}}{g_{m1}} = \frac{300}{9.64\,\text{mS}} \approx 31.12\,\text{k}\Omega$$

For Q_2,

$$g_{m2} = \frac{I_C}{V_T} = \frac{0.213\,\text{mA}}{0.025\,\text{V}} = 8.52\,\text{mS}$$

$$r_{\pi 2} = \frac{\beta_{F2}}{g_{m2}} = \frac{42}{8.52\,\text{mS}} \approx 4.93\,\text{k}\Omega$$

Input resistance:

$$R_{in} = r_{\pi 1} \parallel R_{B1} = (41.12 \parallel 180)\,\text{k}\Omega \approx 33.47\,\text{k}\Omega$$

Output resistance:

$$R_{out} = R_L \parallel \left(\frac{1}{g_{m2}} + \frac{R_{C1} \parallel R_{EQ2}}{\beta_{F2}} \right) = 100\,\text{k}\Omega \parallel \left(\frac{1}{8.52\,\text{ms}} + \frac{16.98\,\text{k}\Omega}{42} \right) \approx 519\,\Omega$$

Midband voltage gain:

$$A_{vt1}^{CE} \approx \frac{-g_{m1}(R_C \parallel R_{EQ} \parallel r_{\pi 2})}{1 + R_{E1} \cdot g_{m1}} = \frac{-9.64\,\text{mS} \cdot (20 \parallel 112.5 \parallel 4.93)\,\text{k}\Omega}{1 + 2\,\text{k}\Omega \cdot 9.64\,\text{mS}} = -1.814$$

$$A_{vt2}^{CE} \approx -g_{m2} R_L = -8.52\,\text{mS} \cdot (20 \parallel 100)\,\text{k}\Omega = -142$$

$$A_v = A_{vt1}^{CE} \cdot A_{vt2}^{CE} \cdot \frac{R_{in}}{R_I + R_{in}} = -1.814 \cdot -142 \cdot \frac{33.47\,\text{k}\Omega}{(2 + 33.47)\,\text{k}\Omega} \approx 243$$