### VE311 Homework 3

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#### Problem 1.

(a) 
$$\phi_j = V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \,\mathrm{V} \cdot \ln \frac{10^{19} \,\mathrm{cm}^{-3} \cdot 10^{18} \,\mathrm{cm}^{-3}}{(10^{10} \,\mathrm{cm}^{-3})^2} \approx 0.979 \,\mathrm{V}$$
 
$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \phi_j}$$
 
$$= \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \times 10^{-14} \,\mathrm{F/cm}}{1.60 \times 10^{-19} \,\mathrm{C}} \left(\frac{1}{10^{19} \,\mathrm{cm}^{-3}} + \frac{1}{10^{18} \,\mathrm{cm}^{-3}}\right) 0.979 \,\mathrm{V}}$$
 
$$\approx 3.73 \times 10^{-2} \,\mu\mathrm{m}$$

$$\begin{split} x_p &= \frac{w_{do}}{1 + \frac{N_A}{N_D}} = \frac{3.73 \times 10^{-2} \, \mu\text{m}}{1 + \frac{10^{19} \, \text{cm}^{-3}}{10^{18} \, \text{cm}^{-3}}} \approx 3.39 \times 10^{-3} \, \mu\text{m} \\ x_n &= \frac{w_{do}}{1 + \frac{N_D}{N_A}} = \frac{3.73 \times 10^{-2} \, \mu\text{m}}{1 + \frac{10^{18} \, \text{cm}^{-3}}{10^{19} \, \text{cm}^{-3}}} \approx 3.39 \times 10^{-2} \, \mu\text{m} \end{split}$$

$$\phi_i = 0.979 \,\mathrm{V}$$

(d) 
$$E_{MAX} = \frac{qN_Ax_p}{\varepsilon_s} = \frac{1.60 \times 10^{-19} \,\mathrm{C} \cdot 10^{19} \,\mathrm{cm}^{-3} \cdot 3.39 \times 10^{-3} \,\mu\mathrm{m}}{11.7 \cdot 8.85 \times 10^{-14} \,\mathrm{F/cm}} \approx 523 \,\mathrm{kV/cm}$$

#### Problem 2.

(a) 
$$w_d = w_{do}\sqrt{1+\frac{v_R}{\phi_j}} = 3w_{do}$$
 
$$v_R = 8\phi_j = 8\cdot 0.85\,\mathrm{V} = 6.8\,\mathrm{V}$$

(b) 
$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 0.4 \, \mu \text{m} \cdot \sqrt{1 + \frac{7 \, \text{V}}{0.85 \, \text{V}}} \approx 1.22 \, \mu \text{m}$$

#### Problem 3.

$$j = \sigma E$$
 
$$E = \frac{j}{\sigma} = j\rho = 5000 \,\mathrm{A/cm^2} \cdot 2.5 \,\Omega \cdot \mathrm{cm} = 12.5 \,\mathrm{kV/cm}$$

#### Problem 4.

$$p(x) = N_A(x) = N_o \exp\left(-\frac{x}{L}\right)$$
$$j_p^T = qp\mu_p E - qD_p \frac{\partial p}{\partial x} = qp\mu_p \left(E - V_T \frac{1}{p} \frac{\partial p}{\partial x}\right) = 0$$
$$E(x) = V_T \frac{1}{p} \frac{\partial p}{\partial x} = \frac{V_T}{N_o \exp\left(-\frac{x}{L}\right)} \cdot -\frac{N_o}{L} \exp\left(-\frac{x}{L}\right) = -\frac{V_T}{L}$$

So a built-in electric field must exist.

$$E = -\frac{V_T}{L} = -\frac{0.025 \,\mathrm{V}}{1 \,\mu\mathrm{m}} = -0.25 \,\mathrm{kV/cm}$$

#### Problem 5.

$$nV_T = n\frac{kT}{q} = 1.05 \cdot \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 320 \text{ K}}{1.60 \times 10^{-19} \text{ C}} \approx 0.029 \text{ V}$$
If  $n = 1.00$ ,
$$nV_T = n\frac{kT}{q} = 0.029 \text{ V}$$

$$T = \frac{0.029 \text{ V} \cdot q}{k} = \frac{0.029 \text{ V} \cdot 1.60 \times 10^{-19} \text{ C}}{1.38 \times 10^{-23} \text{ J/K}} \approx 336 \text{ K}$$

#### Problem 6.

(a) 
$$V_D = nV_T \ln \left( 1 + \frac{I_D}{I_S} \right) = 1.07 \cdot 0.025 \,\text{V} \cdot \ln \left( 1 + \frac{70 \,\mu\text{A}}{10^{-17} \,\text{A}} \right) \approx 0.791 \,\text{V}$$

(b) 
$$V_D = nV_T \ln \left( 1 + \frac{I_D}{I_S} \right) = 1.07 \cdot 0.025 \,\text{V} \cdot \ln \left( 1 + \frac{5 \,\mu\text{A}}{10^{-17} \,\text{A}} \right) \approx 0.721 \,\text{V}$$

(c) 
$$I_D = I_S \left[ \exp\left(\frac{V_D}{nV_T}\right) - 1 \right] = 10^{-17} \,\text{A} \cdot \left[ \exp\left(\frac{0}{1.07 \cdot 0.025 \,\text{V}}\right) - 1 \right] = 0 \,\text{A}$$

(d) 
$$I_D = I_S \left[ \exp\left(\frac{V_D}{nV_T}\right) - 1 \right] = 10^{-17} \,\text{A} \cdot \left[ \exp\left(\frac{-0.075 \,\text{V}}{1.07 \cdot 0.025 \,\text{V}}\right) - 1 \right] \approx -9.39 \times 10^{-18} \,\text{A}$$

(e) 
$$I_D = I_S \left[ \exp\left(\frac{V_D}{nV_T}\right) - 1 \right] = 10^{-17} \,\text{A} \cdot \left[ \exp\left(\frac{-5 \,\text{V}}{1.07 \cdot 0.025 \,\text{V}}\right) - 1 \right] \approx -1.00 \times 10^{-17} \,\text{A}$$

#### Problem 7.

$$\begin{split} V_D &= V_T \ln \left( 1 + \frac{I_D}{I_S} \right) \\ V_{D,MAX} &= V_T \ln \left( 1 + \frac{I_D}{I_{S,MIN}} \right) = 0.025 \, \mathrm{V} \ln \left( 1 + \frac{1 \, \mathrm{mA}}{10^{-14} \, \mathrm{A}} \right) \approx 0.633 \, \mathrm{V} \\ V_{D,MIN} &= V_T \ln \left( 1 + \frac{I_D}{I_{S,MAX}} \right) = 0.025 \, \mathrm{V} \ln \left( 1 + \frac{1 \, \mathrm{mA}}{10^{-12} \, \mathrm{A}} \right) \approx 0.518 \, \mathrm{V} \\ 0.518 \, \mathrm{V} &\leqslant V_D \leqslant 0.633 \, \mathrm{V} \end{split}$$

#### Problem 8.

(a)

$$V_D = V_T \ln \left( 1 + \frac{I_D}{I_S} \right) = \frac{kT}{q} \ln \left( 1 + \frac{I_D}{I_S} \right)$$

$$= \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 303 \text{ K}}{1.60 \times 10^{-19} \text{ C}} \ln \left( 1 + \frac{1 \text{ mA}}{2.5 \times 10^{-16} \text{ A}} \right)$$

$$\approx 0.758 \text{ V}$$

(b)

$$V'_D = V_D + \frac{dV_D}{dT} \cdot \Delta T$$
  
= 0.758 V + -2 mV/K · (50 °C - 30 °C)  
= 0.718 V

#### Problem 9.

$$\begin{split} \phi_j &= V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \, \mathrm{V} \cdot \ln \frac{10^{18} \, \mathrm{cm}^{-3} \cdot 10^{20} \, \mathrm{cm}^{-3}}{(10^{10} \, \mathrm{cm}^{-3})^2} \approx 1.036 \, \mathrm{V} \\ w_{do} &= \sqrt{\frac{2\varepsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \phi_j} \\ &= \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \times 10^{-14} \, \mathrm{F/cm}}{1.60 \times 10^{-19} \, \mathrm{C}} \left(\frac{1}{10^{18} \, \mathrm{cm}^{-3}} + \frac{1}{10^{20} \, \mathrm{cm}^{-3}}\right) 1.036 \, \mathrm{V}} \\ &\approx 3.68 \times 10^{-2} \, \mu \mathrm{m} \end{split}$$

When  $v_R = 5 \,\mathrm{V}$ ,

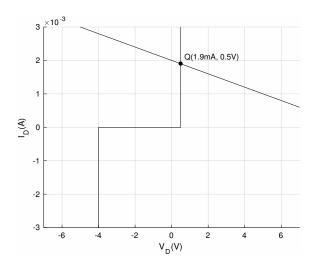
$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 3.68 \times 10^{-2} \,\mu\text{m} \cdot \sqrt{1 + \frac{5 \,\text{V}}{1.036 \,\text{V}}} \approx 8.88 \times 10^{-2} \,\mu\text{m}$$

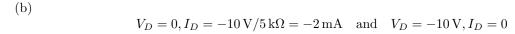
When  $v_R = 25 \,\mathrm{V}$ ,

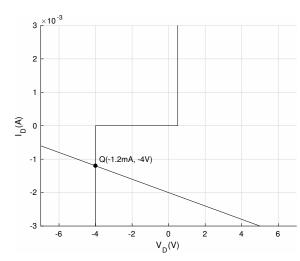
$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 3.68 \times 10^{-2} \,\mu\text{m} \cdot \sqrt{1 + \frac{25 \,\text{V}}{1.036 \,\text{V}}} \approx 0.184 \,\mu\text{m}$$

## Problem 10.

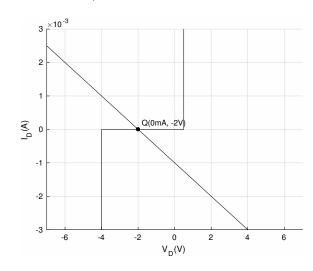
(a) 
$$V_D=0, I_D=10\,\mathrm{V}/5\,\mathrm{k}\Omega=2\,\mathrm{mA}\quad\mathrm{and}\quad V_D=10\,\mathrm{V}, I_D=0$$





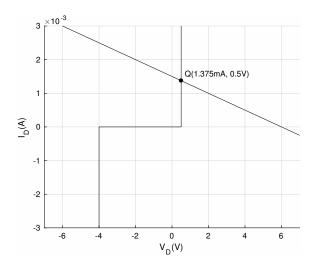


(c) 
$$V_D=0, I_D=-2\,{\rm V}/2\,{\rm k}\Omega=-1\,{\rm mA}\quad{\rm and}\quad V_D=-2\,{\rm V}, I_D=0$$

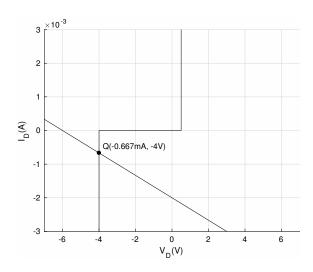


# Problem 11.

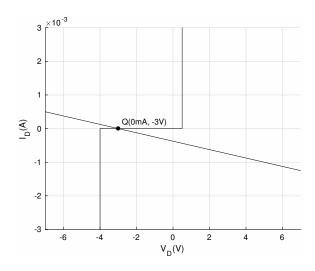
(a) 
$$V_D=0, I_D=6\,{\rm V}/4\,{\rm k}\Omega=1.5\,{\rm mA} \quad {\rm and} \quad V_D=6\,{\rm V}, I_D=0$$



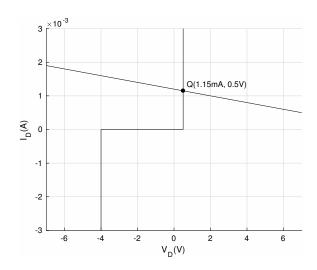
(b) 
$$V_D=0, I_D=-6\,{\rm V}/3\,{\rm k}\Omega=-2\,{\rm mA} \quad {\rm and} \quad V_D=-6\,{\rm V}, I_D=0$$



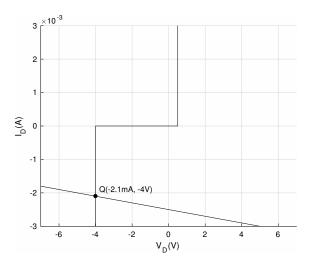
(c) 
$$V_D=0, I_D=-3\,{\rm V}/3\,{\rm k}\Omega=-1\,{\rm mA} \quad {\rm and} \quad V_D=-3\,{\rm V}, I_D=0$$



(d) 
$$V_D = 0, I_D = 12 \, {\rm V}/8 \, {\rm k}\Omega = 1.5 \, {\rm mA} \quad {\rm and} \quad V_D = 12 \, {\rm V}, I_D = 0$$



(e) 
$$V_D = 0, I_D = -25 \, \text{V}/10 \, \text{k} \Omega = -2.5 \, \text{mA} \quad \text{and} \quad V_D = -25 \, \text{V}, I_D = 0$$



## Problem 12.

When  $R_S = 0$ , assuming  $V_{on} = 1 \,\mathrm{V}$ 

$$V_{dc} = -(V_p - V_{on}) = -(10 \text{ V} - 1 \text{ V}) = -9 \text{ V}$$

$$\begin{cases} V_r = V_{dc} \left[ 1 - \exp\left( -\frac{T - \Delta T}{RC} \right) \right] \\ \Delta T = \frac{1}{\omega} \sqrt{\frac{2|V_r|}{V_p}} \end{cases} \Longrightarrow \begin{cases} V_r = -6.01 \text{ V} \\ \Delta T = 2.91 \times 10^{-3} \text{ s} \end{cases}$$

$$I_P = I_{dc} \frac{2T}{\Delta T} = \frac{2|V_{dc}|T}{R\Delta T} = \frac{2 \cdot 9 \,\text{V} \cdot \frac{2\pi}{120\pi} \,\text{s}}{0.025 \,\Omega \cdot 2.91 \times 10^{-3} \,\text{s}} \approx 4123 \,\text{A}$$

# Problem 13.

assuming 
$$V_{on} = 1 \,\mathrm{V},$$

$$V_1 = V_p - V_{on} = 40 \text{ V} - 1 \text{ V} = 39 \text{ V}$$
 
$$V_2 = -(V_p - V_{on}) = -(40 \text{ V} - 1 \text{ V}) = -39 \text{ V}$$