VE311 Homework 4

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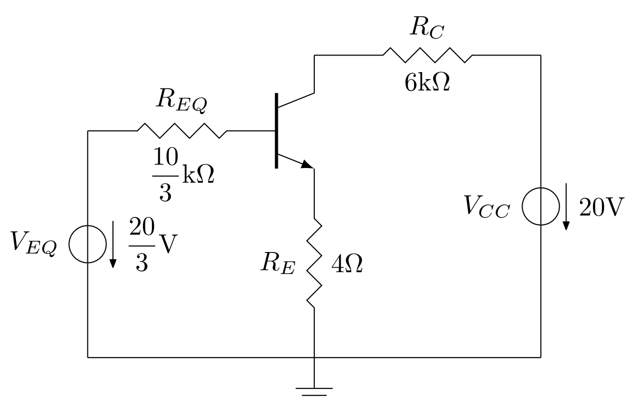
Problem 1.

(a) C_1 is a coupling capacitors which couples v_1, C_2 is a by-pass capacitor, C_3 is a coupling capacitors which couples v_0.

(b) The signal voltage at the emitter of Q_1 is zero.

(c) It is a PNP BJT transistor.

Problem 2.



Suppose $V_{BE} = 0.7V$,

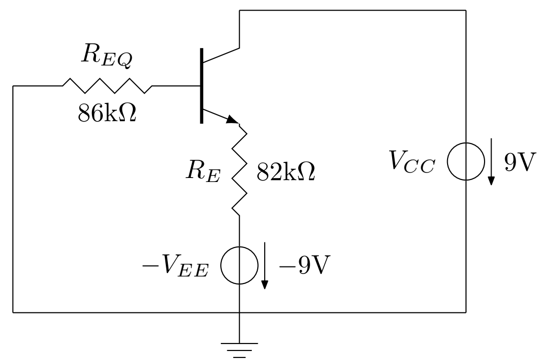
$$I_C = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F} + \frac{\beta_F + 1}{\beta_F} R_E} = \frac{20/3\,\text{V} - 0.7\,\text{V}}{\frac{10/3\,\text{k}\Omega}{75} + \frac{75+1}{75} \cdot 4\,\text{k}\Omega} \approx 1.456\,\text{mA}$$

$$I_E = \frac{V_{EQ} - V_{BE}}{\frac{R_{EQ}}{\beta_F + 1} + R_E} = \frac{20/3\,\text{V} - 0.7\,\text{V}}{\frac{10/3\,\text{k}\Omega}{75+1} + 4\,\text{k}\Omega} \approx 1.475\,\text{mA}$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 20\,\text{V} - 1.456\,\text{mA} \cdot 6\,\text{k}\Omega - 1.475\,\text{mA} \cdot 4\,\text{k}\Omega = 5.364\,\text{V}$$

So the Q point is $(1.456\,\text{mA},\ 5.364\,\text{V})$.

Problem 3.



Suppose $V_{BE} = 0.7V$,

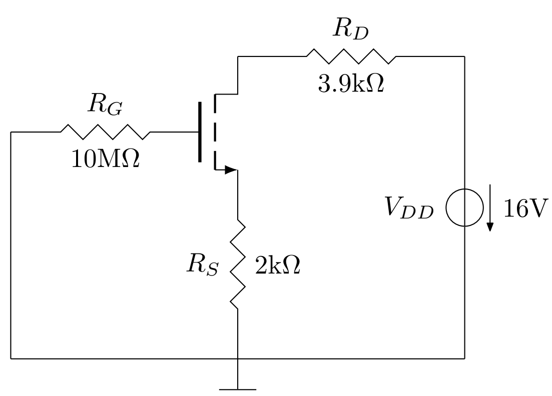
$$I_E = \frac{V_{EE} - V_{BE}}{R_E} = \frac{9\,\mathrm{V} - 0.7\,\mathrm{V}}{82\,\mathrm{k\Omega}} \approx 101\,\mu\mathrm{A}$$

$$I_C = \frac{\beta_F}{\beta_F + 1} I_E = \frac{100}{100+1} \cdot 100\,\mu\mathrm{A}$$

$$V_{CE} = V_{CC} - I_C R_C - (-V_{BE}) = 9\,\mathrm{V} + 0.7\,\mathrm{V} = 9.7\,\mathrm{V}$$

So the Q point is $(100\,\mu\mathrm{A},\ 9.7\,\mathrm{V})$.

Problem 4.



According to the equations,

$$I_D = \frac{K_n}{2}(V_{GS} - V_{TN})^2$$

$$V_{GS} + I_D R_s = 0$$

We can get

$$V_{GS} + \frac{K_n R_s}{2}(V_{GS} - V_{TN})^2 = 0$$

$$V_{GS} = V_{TN} = \frac{1}{K_n R_s}\left(\sqrt{1 - 2K_n R_s V_{TN}} - 1\right)$$

$$
\begin{aligned}
I_D &= \frac{1}{2K_nR_s^2}\left(\sqrt{1-2K_nR_sV_{TN}}-1\right)^2 \\
&= \frac{1}{2\cdot 400\,\mu\mathrm{A/V^2}\cdot(2\,\mathrm{k\Omega})^2}\left(\sqrt{1-2\cdot 400\,\mu\mathrm{A/V^2}\cdot 2\,\mathrm{k\Omega}\cdot -5\,\mathrm{V}}-1\right)^2 \\
&= 1.25\,\mathrm{mA}
\end{aligned}
$$

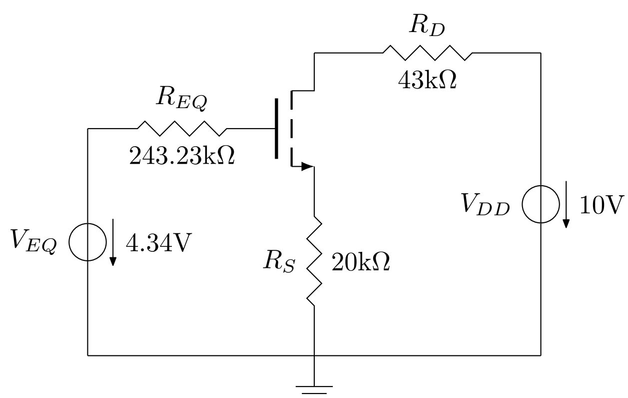
$$
V_{DS} = V_{DD} - I_D(R_D+R_S) = 16\,\mathrm{V} - 1.25\,\mathrm{mA}\cdot(3.9\,\mathrm{k\Omega}+2\,\mathrm{k\Omega}) = 8.625\,\mathrm{V}
$$

$$
\begin{aligned}
V_{GS}-V_{TN} &= \frac{1}{K_nR_s}\left(\sqrt{1-2K_nR_sV_{TN}}-1\right) \\
&= \frac{1}{2\cdot 400\,\mu\mathrm{A/V^2}\cdot 2\,\mathrm{k\Omega}}\left(\sqrt{1-2\cdot 400\,\mu\mathrm{A/V^2}\cdot 2\,\mathrm{k\Omega}\cdot -5\,\mathrm{V}}-1\right) \\
&= 1.25\,\mathrm{V} < V_{DS}
\end{aligned}
$$

So the Q point is $(1.25\,\mathrm{mA},\ 8.625\,\mathrm{V})$, and it is in the saturated region.

Problem 5.

First, we draw the dc equivalent circuit:



According to the equations,

$$
I_D = \frac{K_n}{2}(V_{GS}-V_{TN})^2
$$
$$
V_{GS} + I_DR_s = V_{EQ}
$$

We can get

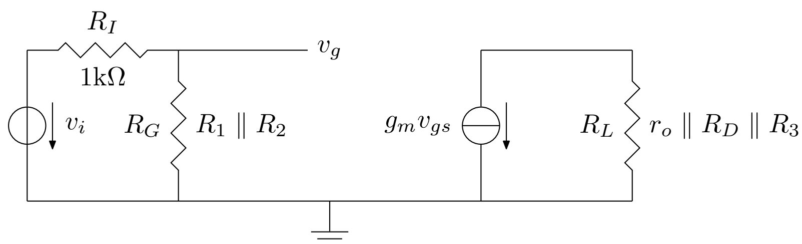
$$
V_{GS} + \frac{K_nR_s}{2}(V_{GS}-V_{TN})^2 = V_{EQ}
$$
$$
V_{GS} = V_{TN} + \frac{1}{K_nR_s}\left(\sqrt{1+2K_nR_s(V_{EQ}-V_{TN})}-1\right)
$$

$$
\begin{aligned}
I_D &= \frac{1}{2K_nR_s^2}\left(\sqrt{1+2K_nR_s(V_{EQ}-V_{TN})}-1\right)^2 \\
&= \frac{1}{2\cdot 0.5\,\mathrm{mA/V^2}\cdot(20\,\mathrm{k\Omega})^2}\left(\sqrt{1+2\cdot 0.5\,\mathrm{mA/V^2}\cdot 20\,\mathrm{k\Omega}\cdot(4.34\,\mathrm{V}-1\,\mathrm{V})}-1\right)^2 \\
&\approx 131\,\mu\mathrm{A}
\end{aligned}
$$

$$V_{DS} = V_{DD} - I_D(R_D + R_S) = 10\,\text{V} - 131\,\mu\text{A} \cdot (43\,\text{k}\Omega + 20\,\text{k}\Omega) \approx 1.75\,\text{V}$$

$$\begin{aligned} V_{GS} - V_{TN} &= \frac{1}{K_n R_s}\left(\sqrt{1 + 2K_n R_s (V_{EQ} - V_{TN})} - 1\right) \\ &= \frac{1}{0.5\,\text{mA/V}^2 \cdot 20\,\text{k}\Omega}\left(\sqrt{1 + 2 \cdot 0.5\,\text{mA/V}^2 \cdot 20\,\text{k}\Omega \cdot (4.34\,\text{V} - 1\,\text{V})} - 1\right) \\ &\approx 0.717\,\text{V} < V_{DS} \end{aligned}$$

So the Q point is $(131\,\mu\text{A},\ 1.75\,\text{V})$, and it is in the saturated region.
Then we can draw the ac equivalent circuit:



$$g_m = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2 \cdot 131\,\mu\text{A}}{0.717\,\text{V}} \approx 3.65 \times 10^{-4}\,\Omega^{-1}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{\frac{1}{0.0133\,\text{V}^{-1}} + 1.75\,\text{V}}{131\,\mu\text{A}} \approx 587\,\text{k}\Omega$$

$$R_G = R_1 \parallel R_2 \approx 243.23\,\text{k}\Omega$$

$$R_L = r_o \parallel R_D \parallel R_3 \approx 28.6\,\text{k}\Omega$$

The voltage gain is

$$\begin{aligned} A_v^{CS} &= -g_m R_L \left(\frac{R_G}{R_G + R_I}\right) \\ &= -3.65 \times 10^{-4}\,\Omega^{-1} \cdot 28.6\,\text{k}\Omega \cdot \left(\frac{243.23\,\text{k}\Omega}{243.23\,\text{k}\Omega + 1\,\text{k}\Omega}\right) \\ &\approx -10.4 \end{aligned}$$

Problem 6.

(a)

$$g_d = \frac{I_S}{V_T}\exp\left(\frac{V_D}{V_T}\right) = \frac{8\,\text{fA}}{0.025\,\text{V}}\exp\left(\frac{0.6\,\text{V}}{0.025\,\text{V}}\right) \approx 8.48 \times 10^{-3}\,\Omega^{-1}$$

$$r_d = \frac{1}{g_d} = \frac{1}{8.48 \times 10^{-3}\,\Omega^{-1}} \approx 118\,\Omega$$

(b)

$$g_d = \frac{I_S}{V_T}\exp\left(\frac{V_D}{V_T}\right) = \frac{8\,\text{fA}}{0.025\,\text{V}}\exp\left(\frac{0\,\text{V}}{0.025\,\text{V}}\right) \approx 3.2 \times 10^{-13}\,\Omega^{-1}$$

$$r_d = \frac{1}{g_d} = \frac{1}{3.2 \times 10^{-13}\,\Omega^{-1}} \approx 3.125 \times 10^{12}\,\Omega$$

(c)

$$g_d = \frac{I_S}{V_T} \exp\left(\frac{V_D}{V_T}\right) = \frac{1}{r_d}$$

$$V_D = V_T \ln\left(\frac{V_T}{I_S r_d}\right) = 0.025\,\mathrm{V} \ln\left(\frac{0.025\,\mathrm{V}}{8\,\mathrm{fA} \cdot 10^{12}\,\Omega}\right) \approx 2.85 \times 10^{-2}\,\mathrm{V}$$

Problem 7.

$$r_\pi = \frac{\beta_o V_T}{I_C} = \frac{100 \cdot 0.025\,\mathrm{V}}{40\,\mu\mathrm{A}} = 62.5\,\mathrm{k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{40\,\mu\mathrm{A}}{0.025} = 1.6 \times 10^{-3}\,\Omega^{-1}$$

$$r_o = \frac{1}{g_o} = \frac{V_A + V_{CE}}{I_C} = \frac{75\,\mathrm{V} + 10\,\mathrm{V}}{40\,\mu\mathrm{A}} = 2125\,\mathrm{k\Omega}$$

$$R_L = r_o \parallel R_C \parallel R_3 \approx 48.85\,\mathrm{k\Omega}$$

The input resistance is

$$R_B \parallel r_\pi \approx 38.46\,\mathrm{k\Omega}$$

The voltage gain is

$$\begin{aligned} A_v^{CE} &= -g_m R_L \left[\frac{R_B \parallel r_\pi}{R_I + (R_B \parallel r_\pi)}\right] \\ &= -1.6 \times 10^{-3}\,\Omega^{-1} \cdot 48.85\,\mathrm{k\Omega} \cdot \left(\frac{38.46\,\mathrm{k\Omega}}{750\,\Omega \cdot 38.46\,\mathrm{k\Omega}}\right) \\ &\approx -76.7 \end{aligned}$$