

# VE311 Homework 3

Liu Yihao 515370910207

## Problem 1.

(a)

$$\phi_j = V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \text{ V} \cdot \ln \frac{10^{19} \text{ cm}^{-3} \cdot 10^{18} \text{ cm}^{-3}}{(10^{10} \text{ cm}^{-3})^2} \approx 0.979 \text{ V}$$

$$\begin{aligned} w_{do} &= \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \phi_j} \\ &= \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \times 10^{-14} \text{ F/cm}}{1.60 \times 10^{-19} \text{ C}} \left( \frac{1}{10^{19} \text{ cm}^{-3}} + \frac{1}{10^{18} \text{ cm}^{-3}} \right) 0.979 \text{ V}} \\ &\approx 3.73 \times 10^{-2} \mu\text{m} \end{aligned}$$

(b)

$$\begin{aligned} x_p &= \frac{w_{do}}{1 + \frac{N_A}{N_D}} = \frac{3.73 \times 10^{-2} \mu\text{m}}{1 + \frac{10^{19} \text{ cm}^{-3}}{10^{18} \text{ cm}^{-3}}} \approx 3.39 \times 10^{-3} \mu\text{m} \\ x_n &= \frac{w_{do}}{1 + \frac{N_D}{N_A}} = \frac{3.73 \times 10^{-2} \mu\text{m}}{1 + \frac{10^{18} \text{ cm}^{-3}}{10^{19} \text{ cm}^{-3}}} \approx 3.39 \times 10^{-2} \mu\text{m} \end{aligned}$$

(c)

$$\phi_j = 0.979 \text{ V}$$

(d)

$$E_{MAX} = \frac{q N_A x_p}{\varepsilon_s} = \frac{1.60 \times 10^{-19} \text{ C} \cdot 10^{19} \text{ cm}^{-3} \cdot 3.39 \times 10^{-3} \mu\text{m}}{11.7 \cdot 8.85 \times 10^{-14} \text{ F/cm}} \approx 523 \text{ kV/cm}$$

## Problem 2.

(a)

$$\begin{aligned} w_d &= w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 3w_{do} \\ v_R &= 8\phi_j = 8 \cdot 0.85 \text{ V} = 6.8 \text{ V} \end{aligned}$$

(b)

$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 0.4 \mu\text{m} \cdot \sqrt{1 + \frac{7 \text{ V}}{0.85 \text{ V}}} \approx 1.22 \mu\text{m}$$

### Problem 3.

$$j = \sigma E$$

$$E = \frac{j}{\sigma} = j\rho = 5000 \text{ A/cm}^2 \cdot 2.5 \Omega \cdot \text{cm} = 12.5 \text{ kV/cm}$$

### Problem 4.

$$p(x) = N_A(x) = N_o \exp\left(-\frac{x}{L}\right)$$

$$j_p^T = qp\mu_p E - qD_p \frac{\partial p}{\partial x} = qp\mu_p \left(E - V_T \frac{1}{p} \frac{\partial p}{\partial x}\right) = 0$$

$$E(x) = V_T \frac{1}{p} \frac{\partial p}{\partial x} = \frac{V_T}{N_o \exp\left(-\frac{x}{L}\right)} \cdot -\frac{N_o}{L} \exp\left(-\frac{x}{L}\right) = -\frac{V_T}{L}$$

So a built-in electric field must exist.

$$E = -\frac{V_T}{L} = -\frac{0.025 \text{ V}}{1 \mu\text{m}} = -0.25 \text{ kV/cm}$$

### Problem 5.

$$nV_T = n \frac{kT}{q} = 1.05 \cdot \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 320 \text{ K}}{1.60 \times 10^{-19} \text{ C}} \approx 0.029 \text{ V}$$

If  $n = 1.00$ ,

$$nV_T = n \frac{kT}{q} = 0.029 \text{ V}$$

$$T = \frac{0.029 \text{ V} \cdot q}{k} = \frac{0.029 \text{ V} \cdot 1.60 \times 10^{-19} \text{ C}}{1.38 \times 10^{-23} \text{ J/K}} \approx 336 \text{ K}$$

### Problem 6.

(a)

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1.07 \cdot 0.025 \text{ V} \cdot \ln\left(1 + \frac{70 \mu\text{A}}{10^{-17} \text{ A}}\right) \approx 0.791 \text{ V}$$

(b)

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1.07 \cdot 0.025 \text{ V} \cdot \ln\left(1 + \frac{5 \mu\text{A}}{10^{-17} \text{ A}}\right) \approx 0.721 \text{ V}$$

(c)

$$I_D = I_S \left[ \exp\left(\frac{V_D}{nV_T}\right) - 1 \right] = 10^{-17} \text{ A} \cdot \left[ \exp\left(\frac{0}{1.07 \cdot 0.025 \text{ V}}\right) - 1 \right] = 0 \text{ A}$$

(d)

$$I_D = I_S \left[ \exp\left(\frac{V_D}{nV_T}\right) - 1 \right] = 10^{-17} \text{ A} \cdot \left[ \exp\left(\frac{-0.075 \text{ V}}{1.07 \cdot 0.025 \text{ V}}\right) - 1 \right] \approx -9.39 \times 10^{-18} \text{ A}$$

(e)

$$I_D = I_S \left[ \exp \left( \frac{V_D}{nV_T} \right) - 1 \right] = 10^{-17} \text{ A} \cdot \left[ \exp \left( \frac{-5 \text{ V}}{1.07 \cdot 0.025 \text{ V}} \right) - 1 \right] \approx -1.00 \times 10^{-17} \text{ A}$$

**Problem 7.**

$$\begin{aligned} V_D &= V_T \ln \left( 1 + \frac{I_D}{I_S} \right) \\ V_{D,MAX} &= V_T \ln \left( 1 + \frac{I_D}{I_{S,MIN}} \right) = 0.025 \text{ V} \ln \left( 1 + \frac{1 \text{ mA}}{10^{-14} \text{ A}} \right) \approx 0.633 \text{ V} \\ V_{D,MIN} &= V_T \ln \left( 1 + \frac{I_D}{I_{S,MAX}} \right) = 0.025 \text{ V} \ln \left( 1 + \frac{1 \text{ mA}}{10^{-12} \text{ A}} \right) \approx 0.518 \text{ V} \\ 0.518 \text{ V} &\leq V_D \leq 0.633 \text{ V} \end{aligned}$$

**Problem 8.**

(a)

$$\begin{aligned} V_D &= V_T \ln \left( 1 + \frac{I_D}{I_S} \right) = \frac{kT}{q} \ln \left( 1 + \frac{I_D}{I_S} \right) \\ &= \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 303 \text{ K}}{1.60 \times 10^{-19} \text{ C}} \ln \left( 1 + \frac{1 \text{ mA}}{2.5 \times 10^{-16} \text{ A}} \right) \\ &\approx 0.758 \text{ V} \end{aligned}$$

(b)

$$\begin{aligned} \frac{dV_D}{dT} &= \frac{v_D - V_{GO} - 3V_T}{T} = -2 \text{ mV/K} \\ V_D &= T \frac{dV_D}{dT} + V_{GO} + 3 \frac{kT}{q} \\ &= 323 \text{ K} \cdot -2 \text{ mV/K} + 1.12 \text{ V} + 3 \cdot \frac{1.38 \times 10^{-23} \text{ J/K} \cdot 323 \text{ K}}{1.60 \times 10^{-19} \text{ C}} \\ &\approx 0.558 \text{ V} \end{aligned}$$

**Problem 9.**

$$\begin{aligned} \phi_j &= V_T \ln \frac{N_A N_D}{n_i^2} = 0.025 \text{ V} \cdot \ln \frac{10^{18} \text{ cm}^{-3} \cdot 10^{20} \text{ cm}^{-3}}{(10^{10} \text{ cm}^{-3})^2} \approx 1.036 \text{ V} \\ w_{do} &= \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \phi_j} \\ &= \sqrt{\frac{2 \cdot 11.7 \cdot 8.85 \times 10^{-14} \text{ F/cm}}{1.60 \times 10^{-19} \text{ C}} \left( \frac{1}{10^{18} \text{ cm}^{-3}} + \frac{1}{10^{20} \text{ cm}^{-3}} \right) 1.036 \text{ V}} \\ &\approx 3.68 \times 10^{-2} \mu\text{m} \end{aligned}$$

When  $v_R = 5\text{ V}$ ,

$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 3.68 \times 10^{-2} \mu\text{m} \cdot \sqrt{1 + \frac{5\text{ V}}{1.036\text{ V}}} \approx 8.88 \times 10^{-2} \mu\text{m}$$

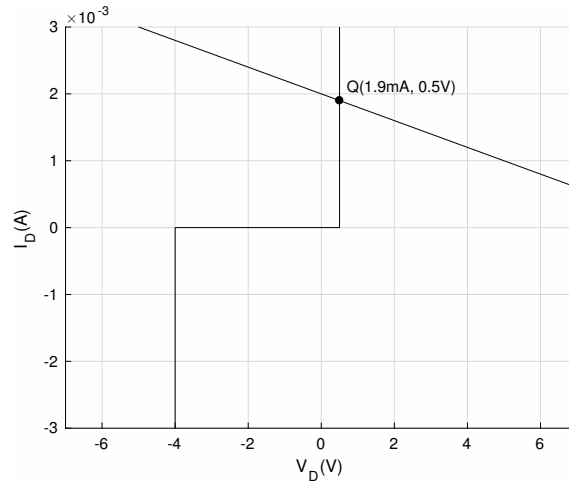
When  $v_R = 25\text{ V}$ ,

$$w_d = w_{do} \sqrt{1 + \frac{v_R}{\phi_j}} = 3.68 \times 10^{-2} \mu\text{m} \cdot \sqrt{1 + \frac{25\text{ V}}{1.036\text{ V}}} \approx 0.184 \mu\text{m}$$

## Problem 10.

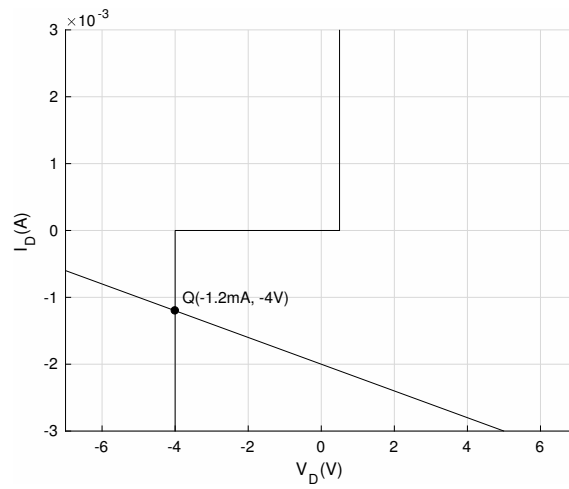
(a)

$$V_D = 0, I_D = 10\text{ V}/5\text{ k}\Omega = 2\text{ mA} \quad \text{and} \quad V_D = 10\text{ V}, I_D = 0$$



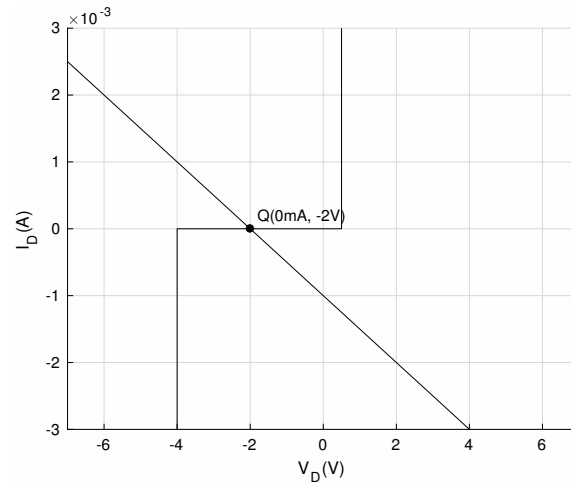
(b)

$$V_D = 0, I_D = -10\text{ V}/5\text{ k}\Omega = -2\text{ mA} \quad \text{and} \quad V_D = -10\text{ V}, I_D = 0$$



(c)

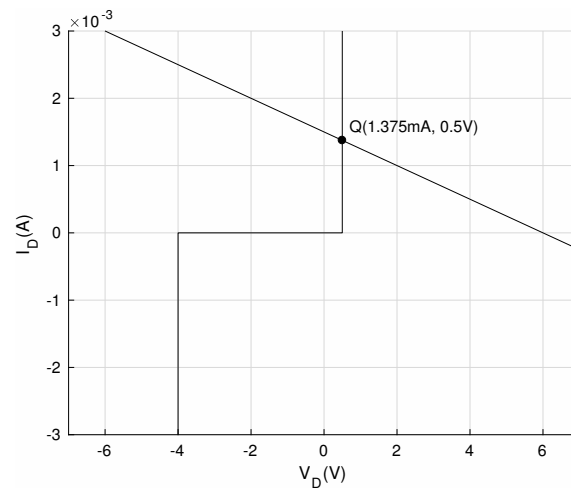
$$V_D = 0, I_D = -2\text{ V}/2\text{ k}\Omega = -1\text{ mA} \quad \text{and} \quad V_D = -2\text{ V}, I_D = 0$$



## Problem 11.

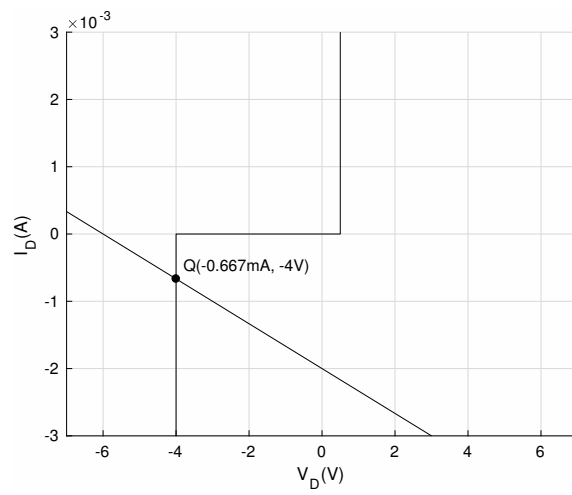
(a)

$$V_D = 0, I_D = 6\text{ V}/4\text{ k}\Omega = 1.5\text{ mA} \quad \text{and} \quad V_D = 6\text{ V}, I_D = 0$$



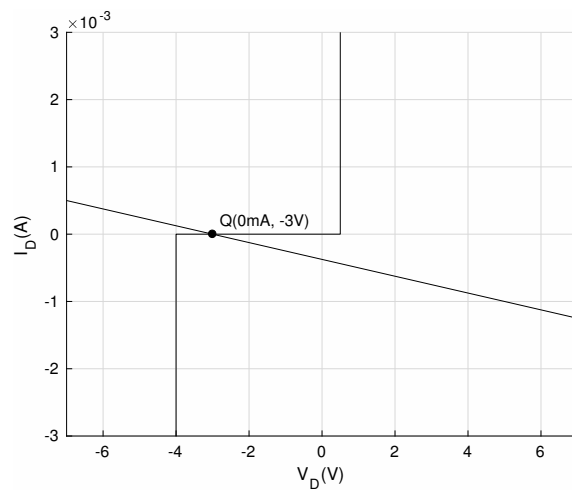
(b)

$$V_D = 0, I_D = -6\text{ V}/3\text{ k}\Omega = -2\text{ mA} \quad \text{and} \quad V_D = -6\text{ V}, I_D = 0$$



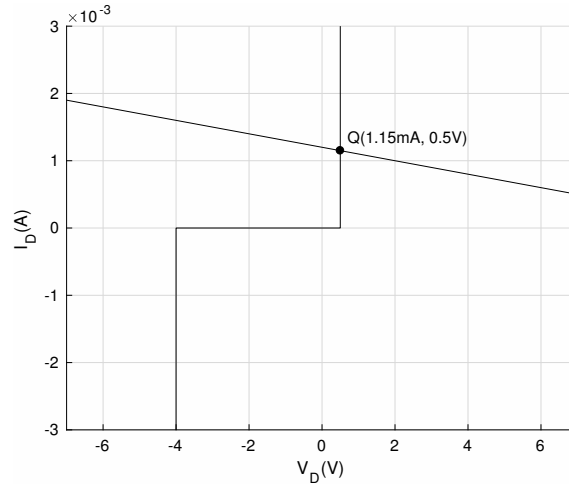
(c)

$$V_D = 0, I_D = -3V/3k\Omega = -1mA \quad \text{and} \quad V_D = -3V, I_D = 0$$



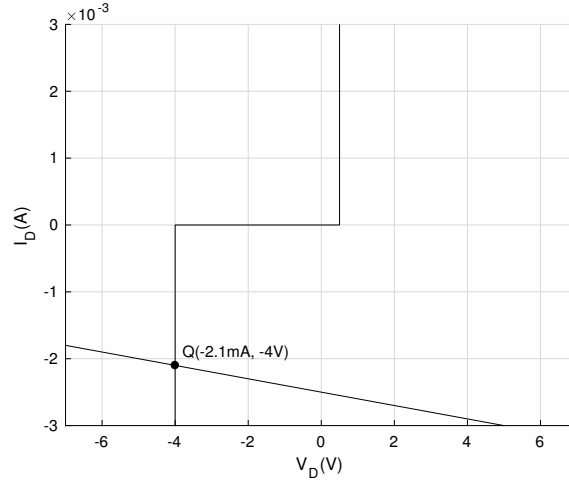
(d)

$$V_D = 0, I_D = 12V/8k\Omega = 1.5mA \quad \text{and} \quad V_D = 12V, I_D = 0$$



(e)

$$V_D = 0, I_D = -25 \text{ V} / 10 \text{ k}\Omega = -2.5 \text{ mA} \quad \text{and} \quad V_D = -25 \text{ V}, I_D = 0$$



## Problem 12.

When  $R_S = 0$ , assuming  $V_{on} = 1 \text{ V}$

$$V_{dc} = -(V_p - V_{on}) = -(10 \text{ V} - 1 \text{ V}) = -9 \text{ V}$$

$$\begin{cases} V_r = V_{dc} \left[ 1 - \exp\left(-\frac{T - \Delta T}{RC}\right) \right] \\ \Delta T = \frac{1}{\omega} \sqrt{\frac{2|V_r|}{V_p}} \end{cases} \Rightarrow \begin{cases} V_r = -6.01 \text{ V} \\ \Delta T = 2.91 \times 10^{-3} \text{ s} \end{cases}$$

$$I_P = I_{dc} \frac{2T}{\Delta T} = \frac{2|V_{dc}|T}{R\Delta T} = \frac{2 \cdot 9 \text{ V} \cdot \frac{2\pi}{120\pi} \text{ s}}{0.025 \Omega \cdot 2.91 \times 10^{-3} \text{ s}} \approx 4123 \text{ A}$$

### Problem 13.

assuming  $V_{on} = 1\text{ V}$ ,

$$V_1 = V_p - V_{on} = 40\text{ V} - 1\text{ V} = 39\text{ V}$$

$$V_2 = -(V_p - V_{on}) = -(40\text{ V} - 1\text{ V}) = -39\text{ V}$$