VE311 Homework 7

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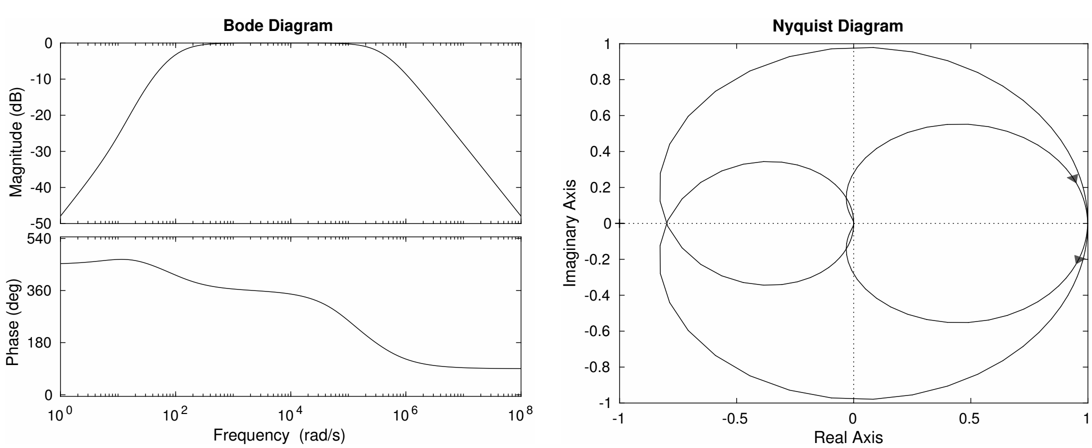
Problem 1.

$$A_v(s) = A_{mid}F_L(s)F_H(s) = A_{mid}\frac{s(s+10)\left(1-\frac{s}{10^5}\right)}{(s+100)(s+25)\left(1+\frac{s}{10000}\right)\left(1+\frac{s}{40000}\right)}$$

The poles are $\omega_{p_1} = -100\,\mathrm{rad/s}$, $\omega_{p_2} = -25\,\mathrm{rad/s}$, $\omega_{p_3} = -10000\,\mathrm{rad/s}$, $\omega_{p_4} = -40000\,\mathrm{rad/s}$, they are all negative, so the system is stable.

$$f_L = \frac{\omega_L}{2\pi} = \frac{\sqrt{\sum\omega_{p_n}^2 - 2\sum\omega_{z_n}^2}}{2\pi} = \frac{\sqrt{100^2+25^2-2\cdot 10^2}\,\mathrm{rad/s}}{2\pi} \approx 16.25\,\mathrm{Hz}$$

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi\sqrt{\sum 1/\omega_{p_n}^2 - 2\sum 1/\omega_{z_n}^2}} = \frac{1}{2\pi\sqrt{1/10000^2+1/40000^2-2/10^{10}}\,\mathrm{rad/s}} \approx 1559\,\mathrm{Hz}$$



(a) Bode plot (b) Nyquist plot

Figure 1: Bode and Nyquist plot under $A_{mid} = 1$.

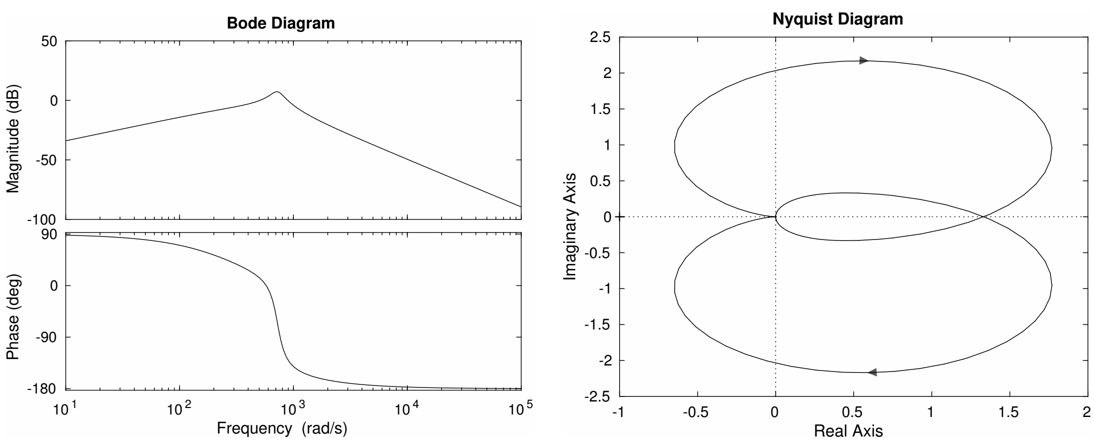
```
s=tf('s');
H=s*(s+10)*(1-s*1e-5)/(s+100)/(s+25)/(1+s*1e-5)/(1+s/4*1e-5);
bode(H,{0,1e8});
saveas(gcf,'p1_bode.eps');
nyquist(H);
saveas(gcf,'p1_nyquist.eps');
```

Problem 2.

1. Let $s = j\omega = j277$,

$$\begin{aligned}
\frac{v_{L_2}(s)}{v_g(s)} &= \frac{L_2 s}{R + L_2 s} \cdot \frac{(R + L_2 s) \parallel (1/Cs)}{(R + L_2 s) \parallel (1/Cs) + L_1 s} \\
&= \frac{L_2 s}{L_1 L_2 C s^3 + R L_1 C s^2 + (L_1 + L_2) s + R} \\
&= \frac{4 \times 10^{-3} s}{1.2 \times 10^{-8} s^3 + 6 \times 10^{-6} s^2 + 7 \times 10^{-3} s + 2} \\
&\approx 0.3584 + j0.3277 \\
&\approx 0.4856 \angle 0.7407
\end{aligned}$$

$$v_{L_2}(t) = 0.4856\angle 0.7407 \cdot 10 \sin(277t) = 4.856 \sin(277t + 0.7407)$$



(a) Bode plot (b) Nyquist plot

Figure 2: Bode and Nyquist plot.

```
s=tf('s');
H=4e-3*s/(1.2e-8*s^3+6e-6*s^2+7e-3*s+2);
bode(H,{0,1e5});
saveas(gcf,'p2_1_bode.eps');
nyquist(H);
saveas(gcf,'p2_1_nyquist.eps');
```

So the system is stable.

2.

$$\begin{cases} -v_0 + i_1 R_1 + \dfrac{1}{C}\displaystyle\int (i_1 - i_L)dt = 0 \\ \dfrac{1}{C}\displaystyle\int (i_L - i_1)dt + L\dfrac{di_L}{dt} + i_L R_2 = 0 \end{cases}$$

$$-v_0 + i_1 R_1 + L\frac{di_L}{dt} + i_L R_2 = 0$$

$$i_1 = \frac{v_0}{R_1} - \frac{R_2}{R_1} i_L - \frac{L}{R_1}\frac{di_L}{dt}$$

$$\frac{i_L}{C} - \frac{v_0}{R_1 C} + \frac{R_2}{R_1 C} i_L + \frac{L}{R_1 C}\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2} + R_2\frac{di_L}{dt} = 0$$

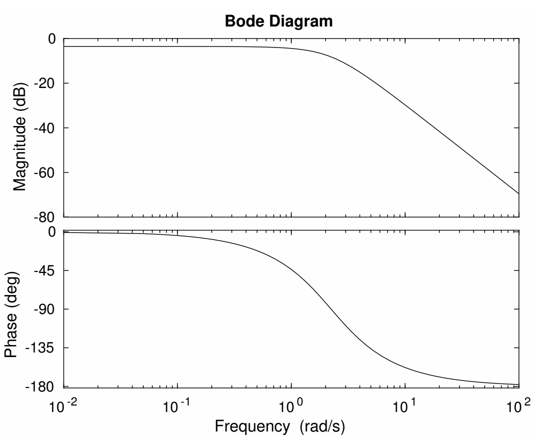
$$3\frac{d^2 i_L}{dt^2} + 11.5\frac{di_L}{dt} + 15 i_L = 10 v_0$$

$$3v_{R_2}s^2 + 11.5 v_{R_2}s + 15 v_{R_2} = 10 v_0$$

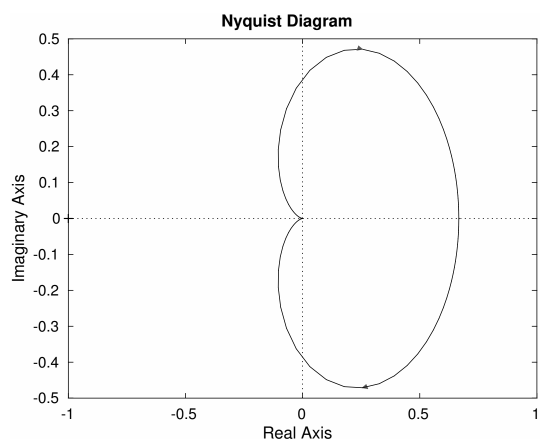
$$\frac{v_{R_2}(s)}{v_0(s)} = \frac{10}{3s^2 + 11.5s + 15}$$

$$v_{R_2}(s) = \frac{5\pi/s^2}{3s^2 + 11.5s + 15}$$

$$v_{R_2}(t) = \frac{\pi t}{3} - \frac{23\pi}{90} + \frac{23\pi}{90}\exp\left(-\frac{23t}{12}\right)\left[\cos\left(\frac{\sqrt{191}t}{12}\right) + \frac{169\sqrt{191}}{4393}\sin\left(\frac{\sqrt{191}t}{12}\right)\right]$$



(a) Bode plot



(b) Nyquist plot

Figure 3: Bode and Nyquist plot.

```
s=tf('s');
H=10/(3*s^2+11.5*s+15);
bode(H,{0,1e2});
saveas(gcf,'p2_2_bode.eps');
nyquist(H);
saveas(gcf,'p2_2_nyquist.eps');
```

So the system is stable.

Problem 3.

Let $s = j\omega$,

$$\frac{V_i}{R} = -\frac{V_1}{R \parallel (1/C_1 s)} = -V_1 \frac{R + 1/C_1 s}{R/C_1 s}$$

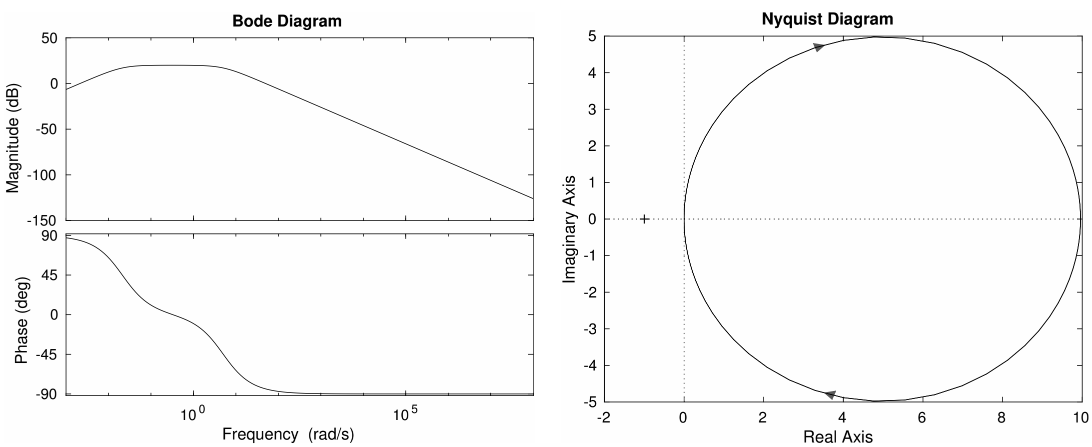
$$\frac{V_1}{V_i} = -\frac{1/C_1 s}{R + 1/C_1 s} = -\frac{1}{RC_1 s + 1}$$

$$\frac{V_2}{V_1} = -\frac{R}{R + 1/C_2 s} = -\frac{RC_2 s}{RC_2 s + 1}$$

$$\frac{V_o}{V_2} = -\frac{R_f}{R_1}$$

$$\frac{V_o}{V_i} = \frac{V_1}{V_i} \cdot \frac{V_2}{V_1} \cdot \frac{V_o}{V_2} = -\frac{RR_f C_2 s}{R_1(RC_1 s + 1)(RC_2 s + 1)} = -\frac{470s}{(47s + 1)(0.2s + 1)}$$

The poles are $\omega_{p_1} = -\dfrac{1}{47}$ rad/s and $\omega_{p_2} = -5$ rad/s. The zeros are $\omega_{z_1} = 0$.



(a) Bode plot (b) Nyquist plot

Figure 4: Bode and Nyquist plot.

```
s=tf('s');
H=470*s/(47*s+1)/(0.2*s+1);
bode(H,{0,1e8});
saveas(gcf,'p3_bode.eps');
nyquist(H);
saveas(gcf,'p3_nyquist.eps');
```

When $v_i = 100\cos(600t)$,

$$V_o = -\frac{470s}{(47s + 1)(0.2s + 1)} \cdot 100\cos(600t) \approx \frac{1}{12}\angle\frac{\pi}{2} \cdot 100\cos(600t) = \frac{25}{3}\sin(600t)$$

From the simulation, we found $v_o \approx 11.96982 \sin(600t)$.
The results are similar.

The SPICE code is

```
p3.cir
.TITLE Problem 3

Vi  8   0   SIN(0 100V 95.5HZ)
R0  8   1   10K
X1  1   0   2   OPAMP1
R1  1   2   10K
C1  1   2   20U
R2  2   3   10K
C2  3   4   4.7M
X2  4   0   5   OPAMP1
R3  4   5   10K
RI  5   6   10K
X3  6   0   7   OPAMP1
Rf  6   7   100K


.SUBCKT OPAMP1  1   2   6
RIN     1   2   10MEG
EGAIN   3   0   1   2   100K
RP1     3   4   1K
CP1     4   0   1.5915UF
EBUFFER 5   0   4   0   1
ROUT    5   6   10
.ENDS

.TRAN   1MS    10S
.MEASURE TRAN vo PP par('V(7)/2')

.PROBE
.END
```

The result is

```
Circuit: Problem 3

Doing analysis at TEMP = 27.000000 and TNOM = 27.000000

No. of Data Rows : 10016

  Measurements for Transient Analysis

vo                   =  1.196982e+01 from=  0.000000e+00 to=  0.000000e+00
```