VE320 Homework2

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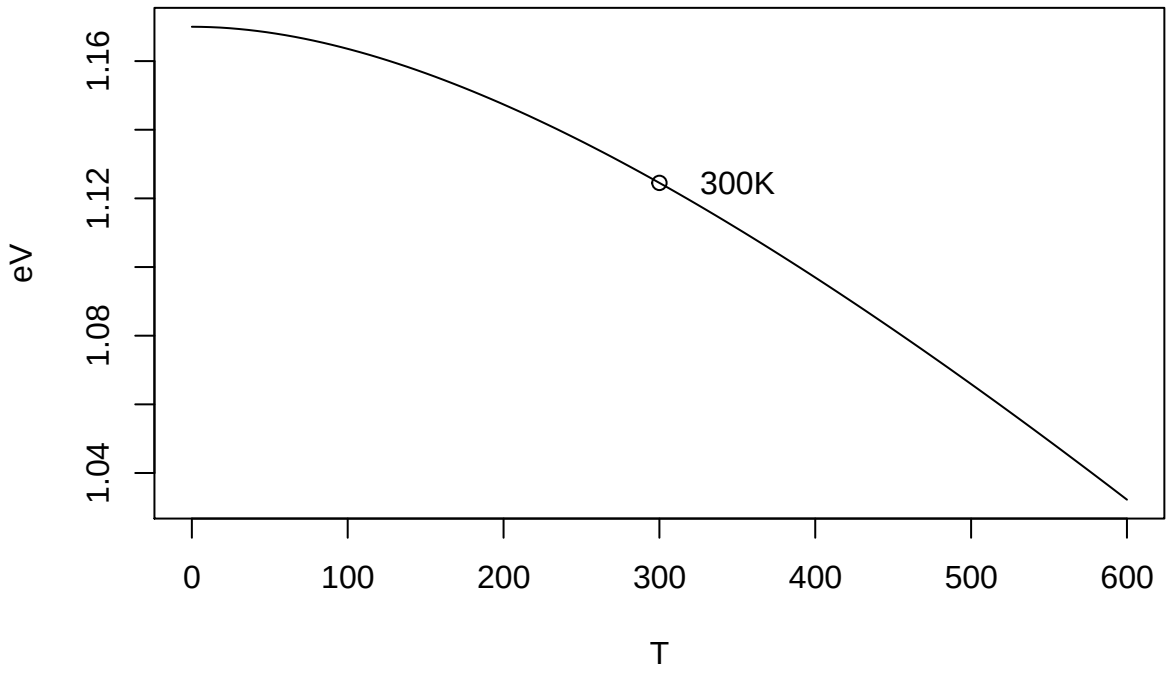
Ex 2.1

```
func <- function(x) {1.170-4.73e-4*x^2/(636+x)}
curve(func, from = 0, to = 600, xlab = 'T', ylab = 'eV')
points(300, func(300))
text(350, func(300), labels = '300K')
```



Ex 2.2

For A, $E - E_v = -C_2 k^2$,

$$0 - 0.025\,\text{eV} \cdot 1.6 \times 10^{-19} = -C_2 \cdot (0.08 \times 10^{10}\,\text{m}^{-1})^2$$

$$C_2 = 6.25 \times 10^{-39}\,\text{V/m}^2$$

$$m^* = \frac{\hbar^2}{-2C_2} = \frac{(1.054 \times 10^{-34}\,\text{J} \cdot \text{s})^2}{-2 \cdot 6.25 \times 10^{-39}\,\text{V/m}^2} = -8.89 \times 10^{-31}\,\text{kg} = -0.98 m_0$$

For B, $E - E_v = -C_2 k^2$,

$$0 - 0.3\,\text{eV} \cdot 1.6 \times 10^{-19} = -C_2 \cdot (0.08 \times 10^{10}\,\text{m}^{-1})^2$$
$$C_2 = 7.5 \times 10^{-38}\,\text{V/m}^2$$
$$m^* = \frac{\hbar^2}{-2C_2} = \frac{(1.054 \times 10^{-34}\,\text{J} \cdot \text{s})^2}{-2 \cdot 7.5 \times 10^{-38}\,\text{V/m}^2} = -7.41 \times 10^{-32}\,\text{kg} = -0.08 m_0$$

Ex 2.3

$$E = \frac{p^2}{2m} = \frac{k^2\hbar^2}{2m}$$
$$k = \sqrt{\frac{2mE}{\hbar^2}}$$
$$\frac{dk}{dE} = \sqrt{\frac{m}{2\hbar^2 E}}$$
$$g_T(k)dk = \frac{\pi k^2 dk}{\pi^3}$$
$$g(E)dE = \frac{2\pi mE}{\hbar^2\pi^3}\sqrt{\frac{m}{2\hbar^2 E}}dE = \frac{2\pi m^{3/2}\sqrt{E}}{\hbar^3\pi^3\sqrt{2}}dE = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}dE$$
$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$$

Ex 2.4

$$f_F = \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(E_c + kT - E_F)}{kT}\right]$$
$$1 - f_F = \exp\left[\frac{-(E_F - E_E)}{kT}\right] = \exp\left[\frac{-(E_F - (E_v - kT))}{kT}\right]$$
$$\exp\left[\frac{-(E_c + kT - E_F)}{kT}\right] = \exp\left[\frac{-(E_F - (E_v - kT))}{kT}\right]$$
$$E_c + kT - E_F = E_F - (E_v - kT)$$
$$E_F = \frac{E_c + E_v}{2}$$

Ex 2.5

(a)

$$kT = 8.62 \times 10^{-5}\,\text{eV/K} \cdot 200\,\text{K} = 1.724 \times 10^{-2}\,\text{eV}$$
$$f_F = 0.05 = \frac{1}{1 + \exp\left[\frac{E - E_F}{kT}\right]}$$
$$E - E_F = kT \ln 19 = 5.076 \times 10^{-2}\,\text{eV}$$
$$f_F = 0.95 = \frac{1}{1 + \exp\left[\frac{E - E_F}{kT}\right]}$$
$$E - E_F = kT \ln \frac{1}{19} = -5.076 \times 10^{-2}\,\text{eV}$$
$$\Delta E = 5.076 \times 10^{-2}\,\text{eV} \cdot 2 = 1.016 \times 10^{-1}\,\text{eV}$$

(b)

$$kT = 8.62 \times 10^{-5}\,\text{eV/K} \cdot 200\,\text{K} = 3.448 \times 10^{-2}\,\text{eV}$$
$$E - E_F = kT \ln 19 = 1.015 \times 10^{-1}\,\text{eV}$$
$$E - E_F = kT \ln \frac{1}{19} = -1.015 \times 10^{-1}\,\text{eV}$$
$$\Delta E = 1.015 \times 10^{-1}\,\text{eV} \cdot 2 = 2.03 \times 10^{-1}\,\text{eV}$$

Ex 2.6

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*}$$

For $T = 200K$, $kT = 1.724 \times 10^{-2}\,\text{eV}$,

$$E_{Fi} - E_{midgap} = \frac{3}{4} \cdot 1.724 \times 10^{-2}\,\text{eV} \cdot \ln \frac{0.56}{1.08} = -8.49 \times 10^{-3}\,\text{eV}$$

For $T = 400K$, $kT = 3.448 \times 10^{-2}\,\text{eV}$,

$$E_{Fi} - E_{midgap} = \frac{3}{4} \cdot 3.448 \times 10^{-2}\,\text{eV} \cdot \ln \frac{0.56}{1.08} = -1.70 \times 10^{-2}\,\text{eV}$$

For $T = 600K$, $kT = 5.172 \times 10^{-2}\,\text{eV}$,

$$E_{Fi} - E_{midgap} = \frac{3}{4} \cdot 5.172 \times 10^{-2}\,\text{eV} \cdot \ln \frac{0.56}{1.08} = -2.55 \times 10^{-2}\,\text{eV}$$

Ex 2.7

$$n_0 = \int g_c(E) f_F(E) dE = K \int f_F(E) dE$$
$$f_F(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{kT}\right]} \approx \exp\left[\frac{-(E - E_F)}{kT}\right]$$
$$\int_{E_c}^{\infty} \exp\left[\frac{-(E - E_F)}{kT}\right] = kT \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$
$$n_0 \approx KkT \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Ex 2.8

(a)

$$E_F - E_v = kT \ln \frac{N_v}{p_0} = 0.0259\,\text{eV} \cdot \ln \frac{1.04 \times 10^{19}\,\text{cm}^{-3}}{2 \times 10^{16}\,\text{cm}^3} = 1.62 \times 10^{-1}\,\text{eV}$$

(b)

$$E_c - E_F = E_g - (E_F - E_v) = 1.12\,\text{eV} - 1.62 \times 10^{-1}\,\text{eV} = 9.58 \times 10^{-1}\,\text{eV}$$

(c)

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \exp\left[\frac{-9.58 \times 10^{-1}\,\mathrm{eV}}{0.0259\,\mathrm{eV}}\right] = 2.42 \times 10^{3}\,\mathrm{cm}^{-3}$$

(d)

$$n_i = \sqrt{p_0 n_0} = \sqrt{2 \times 10^{16}\,\mathrm{cm}^{-3} \cdot 2.42 \times 10^{3}\,\mathrm{cm}^{-3}} = 6.96 \times 10^{9}\,\mathrm{cm}^{-3}$$

$$E_{Fi} - E_F = kT \ln \frac{p_0}{n_i} = 0.0259\,\mathrm{eV} \cdot \ln \frac{2 \times 10^{16}\,\mathrm{cm}^{-3}}{6.96 \times 10^{9}\,\mathrm{cm}^{-3}} = 3.85 \times 10^{-1}\,\mathrm{eV}$$

Ex 2.9

(a)

$$E_F - E_v = kT \ln \frac{N_v}{p_0} = 0.0259\,\mathrm{eV} \cdot \ln \frac{1.04 \times 10^{19}\,\mathrm{cm}^{-3}}{5 \times 10^{15}\,\mathrm{cm}^{3}} = 1.98 \times 10^{-1}\,\mathrm{eV}$$

(b)

$$E_c - E_F = E_g - (E_F - E_v) = 1.12\,\mathrm{eV} - 1.98 \times 10^{-1}\,\mathrm{eV} = 9.22 \times 10^{-1}\,\mathrm{eV}$$

(c)

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \exp\left[\frac{-9.22 \times 10^{-1}\,\mathrm{eV}}{0.0259\,\mathrm{eV}}\right] = 9.70 \times 10^{3}\,\mathrm{cm}^{-3}$$

(d)

$p_0 > n_0$, holes.

(e)

$$n_i = \sqrt{p_0 n_0} = \sqrt{5 \times 10^{15}\,\mathrm{cm}^{-3} \cdot 9.70 \times 10^{3}\,\mathrm{cm}^{-3}} = 6.96 \times 10^{9}\,\mathrm{cm}^{-3}$$

$$E_{Fi} - E_F = kT \ln \frac{p_0}{n_i} = 0.0259\,\mathrm{eV} \cdot \ln \frac{5 \times 10^{15}\,\mathrm{cm}^{-3}}{6.96 \times 10^{9}\,\mathrm{cm}^{-3}} = 3.49 \times 10^{-1}\,\mathrm{eV}$$

Ex 2.10

(a)

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = 1.04 \times 10^{19}\,\mathrm{cm}^{-3} \exp\left[\frac{-0.25\,\mathrm{eV}}{0.0259\,\mathrm{eV}}\right] = 6.68 \times 10^{14}\,\mathrm{cm}^{-3}$$

$$E_c - E_F = E_g - (E_F - E_v) = 1.12\,\mathrm{eV} - 0.25\,\mathrm{eV} = 0.87\,\mathrm{eV}$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \exp\left[\frac{-0.87\,\mathrm{eV}}{0.0259\,\mathrm{eV}}\right] = 7.23 \times 10^{4}\,\mathrm{cm}^{-3}$$

(b)

$$N_v = 1.04 \times 10^{19}\,\mathrm{cm}^{-3} \cdot \left(\frac{4}{3}\right)^{3/2} = 1.60 \times 10^{19}\,\mathrm{cm}^{-3}$$

$$E_F - E_v = kT \ln \frac{N_v}{p_0} = 0.03448\,\mathrm{eV} \cdot \ln \frac{1.04 \times 10^{19}\,\mathrm{cm}^{-3}}{6.68 \times 10^{14}\,\mathrm{cm}^{-3}\,\mathrm{cm}^{3}} = 3.48 \times 10^{-1}\,\mathrm{eV}$$

$$E_c - E_F = E_g - (E_F - E_v) = 1.12\,\mathrm{eV} - 3.48 \times 10^{-1}\,\mathrm{eV} = 7.72 \times 10^{-1}\,\mathrm{eV}$$

$$N_c = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \cdot \left(\frac{4}{3}\right)^{3/2} = 4.31 \times 10^{19}\,\mathrm{cm}^{-3}$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 4.31 \times 10^{19}\,\mathrm{cm}^{-3} \exp\left[\frac{-7.72 \times 10^{-1}\,\mathrm{eV}}{0.03448\,\mathrm{eV}}\right] = 8.14 \times 10^{9}\,\mathrm{cm}^{-3}$$