

1. 2nd order.

$$\frac{\partial^2 \psi}{\partial x^2} = \beta^2 \psi \quad \psi = A_1 e^{\beta x} + A_2 e^{-\beta x};$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\beta^2 \psi \quad \psi = A_1 e^{i\beta x} + A_2 e^{-i\beta x} \quad \beta = \frac{2\pi}{\lambda} \quad \lambda: \text{wavelength.}$$

2. Wave Particle.

Matter Particle.

$$E = h\nu = \hbar\omega, \quad \omega = 2\pi\nu$$

$$P = \frac{h}{\lambda} = \hbar k \quad k = \beta = \frac{2\pi}{\lambda}$$

$$\hbar = 1.054 \times 10^{-34} \text{ J}\cdot\text{s}$$

Light is a wave and a particle.

3. Schrodinger Equation.

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = E\psi$$

Wave function  $\psi(x,t)$

$$|\psi(x,t)|^2 = \psi^* \psi$$

probability density function, independent of time

Boundary Condition.  $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

$\psi(x)$  and  $\frac{\partial \psi(x)}{\partial x}$  finite, single-valued, continuous.

4. Infinite Quantum Well.

$0 \leq x \leq a$   $V(x) = 0$   $\psi(x) = A e^{-ikx} + B e^{ikx}$

$x < 0$  or  $x > a$   $V(x) = \infty$   $\psi(x) = 0$

$$\int_0^a \psi^* \psi dx = 1 \quad \sin(ka) = 0 \quad k = \frac{n\pi}{a} \quad n = 0, \pm 1, \pm 2, \dots$$

$\psi(0) = A + B = 0 \Rightarrow$

$\psi(a) = 0 \Rightarrow$

$k_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$  if  $E < V_0$

$k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$  if  $E > V_0$

Finite Quantum Well.

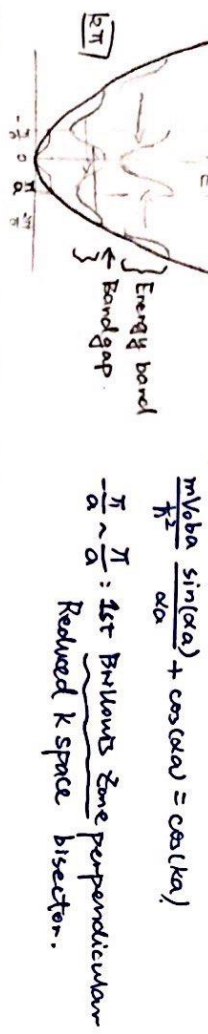
$x < 0$  or  $x > a$   $V(x) = V_0$   $\psi(x) = A e^{-k_1 x} + B e^{k_1 x}$

$0 \leq x \leq a$   $V(x) = 0$   $\psi(x) = C e^{-ik_2 x} + D e^{ik_2 x}$

$\psi(x)|_{x=0, a} \text{ continuous}$

$\int_{-\infty}^{\infty} \psi^* \psi dx = 1$

Periodic Finite Quantum Wells. Infinite number of  $E$  continuous.



5. Band structures of Metals, Semiconductors and Insulators

Conduction band	Valence band	$T = 0K$	$T = 300K$
Metal	Partially filled	Completely filled	conductive
			Electric field $\rightarrow$ $\frac{dc}{dt}$ propagating speed

Semi-conductors. Empty conductors. Completely filled. nonconductive. (large bandgap)

Insulator. Empty. Completely filled. nonconductive.

Doping in semiconductors.

①  $n$ -type.  $e$  right below conduction band

②  $p$ -type. hole right above valence band.

6. Crystal lattice.

Reciprocal lattice.  $a$  in real space  $\rightarrow \frac{2\pi}{a}$  in  $k$  space.

Muller index. intercept of the plane along 3 axes, ~~the~~ their reciprocals, int.

Any parallel plane is entirely equivalent to any other.

Location of origin is entirely arbitrary.

7. Direct Bandgap same  $k$ .  $G_{\text{Dir}}$

Indirect Bandgap. (Absorbing photon) \* (Phonon scattering)  $S_i, G_e$ .

8. Effective mass of electrons.

In conduction band.

$$E = E_c + \frac{\hbar^2}{2m_n^*} k^2 \quad m_n^* > 0$$

In valence band.

$$E = E_v - \frac{\hbar^2}{2m_p^*} k^2 \quad m_p^* > 0$$

$E - E_c = C_1 k^2 \quad C_1 > 0$

$E - E_v = -C_2 k^2 \quad C_2 > 0$

$\frac{1}{m_n^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2}$

$\frac{1}{m_p^*} = -\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} = -\frac{2C_2}{\hbar^2}$

$m_n^* = \sqrt{\frac{\hbar^2}{2C_1}}$   $m_p^* = \sqrt{\frac{\hbar^2}{2C_2}}$

9. Density of States.

$$k = \frac{\sqrt{2m^*(E - E_c)}}{\hbar} \quad g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c}$$

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \quad (\text{real space})$$

10. Fermi Energy level and Fermi Dirac Distribution.

$$f(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$T = 0K$   $f(E > E_F) = 0$   $f(E < E_F) = 1$

$T > 0K$   $f(E = E_F) = \frac{1}{2}$

$T_2 > T_1$   $f(E > E_F): T_2 > T_1$   $f(E < E_F): T_2 < T_1$

Boltzmann distribution

when  $\exp(\frac{E - E_F}{kT}) \gg 1 \Rightarrow E - E_F \gg kT$   $f(E) \approx \exp(-\frac{E - E_F}{kT})$



# 10. Charge carrier concentration.

$$n_0 = \int_{-\infty}^{\infty} g_c(E) f(E) dE = 2 \frac{(2\pi m_e^* kT)^{3/2}}{h^3} \exp\left(\frac{E_F - E_c}{kT}\right) = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300 \text{ K. } k = 1.38 \times 10^{-23} \text{ J/K}$$

$$P_0 = \int_{-\infty}^{\infty} g_v(E) [1 - f(E)] dE = 2 \frac{(2\pi m_h^* kT)^{3/2}}{h^3} \exp\left(\frac{E_v - E_F}{kT}\right) = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$N_v = 1.04 \times 10^{19} \text{ cm}^{-3} \text{ at } T = 300 \text{ K. } kT = 0.0259 \text{ eV at } T = 300 \text{ K.}$$

## ⑩ Intrinsic.

$$E_F = E_i \quad n_0 = n_i = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \quad P_0 = P_i = n_i = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$n_0 P_0 = n_i^2 = N_c N_v \exp\left(-\frac{E_c - E_v}{kT}\right) \quad E_g = E_c - E_v = 1.12 \text{ eV for Si at } 300 \text{ K.}$$

$$n_i = n_0 = P_0 \quad E_F = \frac{E_c + E_v}{2} - kT \ln\left(\frac{N_c}{N_v}\right) = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right) = E_i$$

$$E_{midgap} \approx \frac{E_c + E_v}{2} = E_i$$

## ⑪ Extrinsic.

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \approx n_i \exp\left(\frac{E_F - E_i}{kT}\right) \approx N_D \quad E_F = E_c - kT \ln\left(\frac{N_c}{N_D}\right) = E_i + kT \ln\left(\frac{N_D}{N_c}\right)$$

$$P_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right) = n_i \exp\left(\frac{E_i - E_F}{kT}\right) \approx N_A \quad E_F = E_v + kT \ln\left(\frac{N_v}{N_A}\right) = E_i - kT \ln\left(\frac{N_A}{N_v}\right)$$

$$\begin{cases} n\text{-type } N_D > N_A & n_0 = N_D - N_A \\ p\text{-type } N_D < N_A & P_0 = N_A - N_D \end{cases}$$

## 11. Ionization of dopants.

$$\text{① Probability of electrons occupying dopant energy level. } f_D(E) = \frac{1}{1 + \exp\left(\frac{E_D - E_F}{kT}\right)}$$

$$\text{② Ionization rate } \gamma = f_D(E) = \frac{1}{1 + \exp\left(\frac{E_D - E_F}{kT}\right)} = \begin{cases} \frac{1}{3} & E_D = E_F \\ 1 & E_D < E_F \\ 0 & E_D > E_F \end{cases}$$

$$\text{③ Charge neutrality } n_0 + N_A^- = N_D^+ + P_0$$

$$n_0 = N_D^+ - N_A^- + P_0$$

$$n\text{-type doping: } n_0 = N_D^+ + P_0$$

$$\begin{cases} n_0 = N_D^+ + P_0 \\ n_0 P_0 = n_i^2 \end{cases}$$

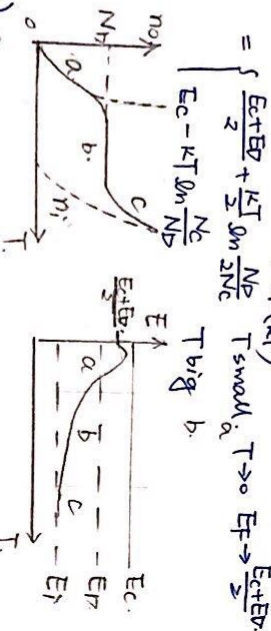
$$n_0 = \frac{N_D^+ + \sqrt{N_D^{+2} + 4n_i^2}}{2}$$

$$\text{④ } n_i \gg N_D^+ \Rightarrow T \text{ very high.}$$

$$n_0 = P_0 = n_i = \sqrt{N_D N_A} \exp\left(-\frac{E_g}{2kT}\right)$$

$$\text{⑤ } n_i \ll N_D^+ \Rightarrow T \text{ not very high.}$$

$$n_0 = N_D^+ = N_c \frac{1 + \sqrt{1 + \frac{N_D}{N_c} \exp\left(\frac{E_D - E_F}{kT}\right)}}{2} \approx \frac{N_D}{2} \exp\left(\frac{E_D - E_F}{kT}\right)$$



## 12. Carrier Transport

$$\text{Conductivity } \sigma = q(\mu_n n + \mu_p p)$$

$$\text{Resistivity } \rho = \frac{1}{\sigma}$$

## 13. Generation and Recombination

$$G = \text{generation rate, } R = \text{recombination rate, } \theta_0 = R = n_0 p_0 = n_i^2$$

$$\text{non-equilibrium } \text{① Light illumination } \text{② Current injection. } n = n_0 + \Delta n, p = p_0 + \Delta p$$

$$\text{Net recombination } U = R - G_0 = \tau_n (n_0 + p_0) + \tau_p (\Delta n)^2 \quad \text{neutrality } \Delta n \approx \Delta p$$

$$\text{Minority carrier lifetime } \tau = \frac{\Delta n}{U} = \frac{1}{\tau_n (n_0 + p_0) + \tau_p \Delta n}$$

$$\text{small injection condition } \Delta n \ll n_0 + p_0 \quad \tau = \frac{1}{\tau_n (n_0 + p_0)} = \text{constant}$$

$$\text{N-type } \tau = \frac{1}{\tau_n} \quad \text{P-type } \tau = \frac{1}{\tau_p}$$

$$\text{Direct recombination } U = \frac{1}{\tau} \frac{n - n_i}{[P + n_i \exp\left(\frac{E_F - E_i}{kT}\right)]}$$

$$\text{Bulk recombination (in a unit volume) } \tau_b = \frac{\Delta n}{U_b} \Rightarrow U_b = \Delta n \frac{1}{\tau_b}$$

$$\text{Surface recombination (in a unit surface area) } U_s = U_s(\Delta n) = \Delta n \frac{\Delta n}{\tau_s} = \Delta n \left(\frac{\Delta n}{\tau_s}\right)$$

$$\text{Total } U = U_b + U_s \Rightarrow \frac{1}{\tau_{eff}} = \frac{1}{\tau_b} + \frac{S}{\Delta n \tau_s} \Rightarrow \frac{1}{\tau_{eff}} = \frac{1}{\tau_b} + \frac{S}{\Delta n \tau_s}$$

$$\text{④ Carrier Concentration at non-equilibrium. time dependent } \frac{\partial \Delta p(t)}{\partial t} = -\frac{\Delta p(t)}{\tau}$$

$$\text{⑤ Quasi Fermi Level. } (n_0 + \Delta n) \times (P_0 + \Delta p) = n_i^2 \exp\left(\frac{E_F^n - E_F^p}{kT}\right)$$

$$n_0 + \Delta n = N_c \exp\left(\frac{E_F^n - E_c}{kT}\right) = n_i \exp\left(\frac{E_F^n - E_i}{kT}\right)$$

$$P_0 + \Delta p = N_v \exp\left(\frac{E_v - E_F^p}{kT}\right) = n_i \exp\left(\frac{E_i - E_F^p}{kT}\right)$$

$$\text{⑥ Carrier drift } E \neq 0, \text{ driving force. flux (\# charges) } J = qnv = qn\mu_n E, n = n_0 + \Delta n$$

$$\text{drift current } J_1 = J_2 = q(n_0 + \Delta n)\mu_n E$$

$$\text{Carrier diffusion. } E = 0, \text{ driving force: thermal dynamics. flux (\# carriers) } J_p = -D_p \frac{dp}{dx}$$

$$D_p \frac{d^2 \Delta p}{dx^2} = \frac{\partial \Delta p}{\partial t} \quad \Delta p(x) = A e^{-x/L_p} + B e^{x/L_p}, L_p = \sqrt{D_p \tau_p}$$

$$\text{for holes. } J_p = -qD_p \frac{dp}{dx} \quad J_n = -qD_n \frac{dn}{dx}$$

$$\text{Diffusion Current Density } J_p = qD_p \frac{dp}{dx} \quad J_n = qD_n \frac{dn}{dx}$$

$$\text{Drift Current Density } J_p = q\mu_p p E \quad J_n = q\mu_n n E$$

$$\text{Current Density } J = J_p + J_n = q\mu_n n E + q\mu_p p E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

$$n\text{-type } I = \frac{\Delta \phi}{\Delta L} = \frac{qA \Delta \phi}{\Delta L} = qA \mu_n E = qA \mu_n \frac{V}{L}$$

$$\text{Conductivity } \sigma = \frac{I}{VA} = q\mu_n n = qA \mu_n E = \frac{V}{R}$$

$$\text{Resistivity } \rho = \frac{1}{\sigma} = \frac{L}{qA \mu_n n} = \frac{L}{qA \mu_n n}$$

$$\text{mobility } \mu = \frac{v_d}{E} = \frac{J_p}{q n E} = \frac{J_p}{q n E}$$

$$\text{neutrality } \Delta n \approx \Delta p$$

$$\text{N-type } \tau = \frac{1}{\tau_n} \quad \text{P-type } \tau = \frac{1}{\tau_p}$$

$$\text{small injection condition } \Delta n \ll n_0 + p_0 \quad \tau = \frac{1}{\tau_n (n_0 + p_0)} = \text{constant}$$

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$$n_0 + \Delta n = N_c \exp\left(\frac{E_F^n - E_c}{kT}\right) = n_i \exp\left(\frac{E_F^n - E_i}{kT}\right)$$

$$P_0 + \Delta p = N_v \exp\left(\frac{E_v - E_F^p}{kT}\right) = n_i \exp\left(\frac{E_i - E_F^p}{kT}\right)$$

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$$\text{Drift Current Density } J_p = q\mu_p p E \quad J_n = q\mu_n n E$$

$$\text{Current Density } J = J_p + J_n = q\mu_n n E + q\mu_p p E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

$$\text{N-type } \tau = \frac{1}{\tau_n} \quad \text{P-type } \tau = \frac{1}{\tau_p}$$

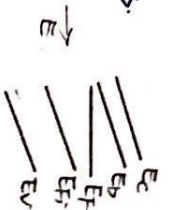


# 18. Induced Electric field.

nonuniformly doped with donor impurity atoms.

$$\phi = +e \left( E_F - E_F^0 \right) \quad E_F - E_F^0 = kT \ln \left( \frac{n}{n_0} \right)$$

$$E_x = - \frac{d\phi}{dx} = \frac{1}{e} \frac{dE_F}{dx} = - \frac{kT}{e} \frac{1}{n} \frac{dn}{dx}$$



# 19. Reflection Coefficient.

$$R = \frac{v_r \cdot B_r \cdot A_i^*}{v_i \cdot A_i \cdot A_i^*} \quad v_i: \text{incident} \quad [V = 0] \quad g_1(x) = A_1 e^{i k_1 x} + B_1 e^{-i k_1 x} \quad k_1 = \frac{m v_i}{\hbar}$$

$$v_r: \text{reflected} \quad v_t = v_r = \frac{\hbar}{m} k$$

# Transmission coefficient.

$$T = \frac{v_t \cdot A_t \cdot A_i^*}{v_i \cdot A_i \cdot A_i^*} \quad v_t: \text{transmitted} \quad [V = 0] \quad g_1(x) = A_1 e^{i k_1 x} + B_1 e^{-i k_1 x}$$

$$g_2(x) = A_2 e^{i k_2 x} + B_2 e^{-i k_2 x} \quad v_t = v_i$$

# 20. Continuity Equation.

Pure diffusion.  $D_p \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{\tau}$

$$D_p \frac{d^2 \Delta p}{dx^2} + D_p \frac{d^2 \Delta p}{dx^2} + \frac{d}{dx} (\mu_p p E) = - \frac{\Delta p}{\tau}$$

Drift + Diffusion + Light illumination

$$- D_p \frac{d^2 \Delta p}{dx^2} + \frac{d}{dx} (\mu_p p E) = - \frac{\Delta p}{\tau} + G_{ex}$$

Accumulation over time

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{d}{dx} (\mu_p p E) - \frac{\Delta p}{\tau} + G_{ex} \quad \Delta p = A e^{kx} + C$$

$$\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} + \eta \mu_n \frac{dE}{dx} + \mu_n E \frac{d n}{dx} - \frac{\Delta n}{\tau} + G_{ex}$$

\*. Steady state  $\frac{\partial n}{\partial t}, \frac{\partial p}{\partial t} = 0$ .

Uniform distribution of excess carrier  $D_p \frac{d^2 \Delta p}{dx^2}, D_n \frac{d^2 \Delta n}{dx^2} = 0$ .

Zero electric field  $E = 0$ . constant  $E = \frac{dE}{dx} = 0$ .

No excess carrier generation  $G_{ex} = 0$ .

No excess carrier recombination  $\frac{\Delta n}{\tau_n}, \frac{\Delta p}{\tau_p} = 0 \quad (T_n, T_p \rightarrow \infty)$

\*. Centrifugal force  $F = m^* \omega^2 r \quad \omega = \frac{v}{r} = 2\pi f$

Magnetic force  $F = e v B$ .

\*. Conductivity  $\sigma = q(\mu_{nn} + \mu_{pp}) = \{ g_{np} p = g_{np} N_A \quad p\text{-type}$

$g_{nn} n = g_{nn} N_D \quad n\text{-type}$

\*. Ambipolar mobility  $\mu' = \frac{\mu_{np} \mu_p (p - n)}{\mu_{nn} n + \mu_{pp} p}$

$$k = 1.38 \times 10^{-23} \text{ J/K} \quad \hbar = 1.6 \times 10^{-19} \text{ C} \quad m_{0.9} = 9.1 \times 10^{-31} \text{ kg} \quad kT = 0.0259 \text{ eV}$$

	$N_c (cm^{-3})$	$N_v$	$m_n^*/m_0$	$m_p^*/m_0$	$\eta$	$\mu_n = 1350 \text{ cm}^2/Vs$	$\mu_p = 480 \text{ cm}^2/Vs$
Si	$2.8 \times 10^{19}$	$1.04 \times 10^{19}$	1.08	0.56	1.5	$1.5 \times 10^{10}$	
GaAs	$4.7 \times 10^{17}$	$7 \times 10^{18}$	0.067	0.48	1.8	$8 \times 10^6$	
Ge	$1.04 \times 10^{19}$	$6 \times 10^{18}$	0.55	0.3	2.4	$2.4 \times 10^{13}$	

# 21. Continuity

$$10. \frac{dS}{dx} = - \frac{\Delta p}{\tau} \Rightarrow D_p \frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{\tau} \quad \Delta p(x) = A e^{-x/L_p} + B e^{x/L_p} \quad L_p = \sqrt{D_p \tau}$$

②  $\Delta p$  uniform, no electric field, light is cut off when  $t = 0, \Delta p(0) = \Delta p_0$

$$\frac{\partial \Delta p}{\partial t} = - \frac{\Delta p}{\tau} \quad \Delta p = \Delta p_0 e^{-t/\tau}$$

③ At equilibrium, no electric field.  $x \in (0, \infty)$ . At  $x = 0$ , surface velocity  $S$ .

$$D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau} + g = 0 \Rightarrow \Delta p(x) = A e^{-x/L_p} + B e^{x/L_p} + C \quad C = g \cdot \tau \quad B = 0$$

$x = 0$ .  $-S_p = g \Delta p(0) \Rightarrow D_p \frac{d \Delta p}{dx} = g \Delta p(0) \Rightarrow A = - \frac{g \cdot \tau \cdot S}{S + D_p/L_p}$

$$\text{④ At equilibrium, constant } E. \text{ no } g. \Delta p(0) = \Delta p_0$$

$$D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p p E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau} = 0 \Rightarrow \Delta p = \Delta p_0 \exp \left[ \frac{L_p(E) - (L_p^2(E) + 4 L_p^2)}{2 L_p^2} x \right]$$

$$L_p = \sqrt{D_p \tau} \quad L_p(E) = \mu_p \cdot \tau \cdot E$$

$$5. \frac{\partial \Delta p}{\partial t} = G_{ex} - \frac{\partial S}{\partial x} - \frac{\Delta p}{\tau}$$

gradient

\*. n-type. P. As. Te.

P-type. B. Ga

