VE320 Homework 1

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Ex. 1

$$E = hv = \frac{hc}{\lambda} \Longrightarrow \lambda = \frac{hc}{E}$$

For gold,

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s} \cdot 3 \times 10^{8}\,\mathrm{m/s}}{4.90\,\mathrm{eV}} = 2.54 \times 10^{-7}\,\mathrm{m}$$

For Cesium,

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s} \cdot 3 \times 10^{8}\,\mathrm{m/s}}{1.90\,\mathrm{eV}} = 6.54 \times 10^{-7}\,\mathrm{m}$$

Ex. 2

$$E_{avg} = \frac{3}{2}kT = 1.5 \cdot 0.0259\,\mathrm{eV} = 3.885 \times 10^{-2}\,\mathrm{eV}$$

$$p_{avg} = \sqrt{2mE_{avg}} = \sqrt{2 \cdot 9.11 \times 10^{-31}\,\mathrm{kg} \cdot 3.885 \times 10^{-2}\,\mathrm{eV}} = 1.064 \times 10^{-25}\,\mathrm{kg \cdot m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.064 \times 10^{-25}\,\mathrm{kg \cdot m/s}} = 6.23 \times 10^{-9}\,\mathrm{m} = 62.3\,\mathrm{\mathring{A}}$$

Ex. 3

(a)

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{8.5 \times 10^{-9}\,\mathrm{m}} = 7.80 \times 10^{-26}\,\mathrm{kg \cdot m/s}$$

$$v = \frac{p}{m} = \frac{7.80 \times 10^{-26}\,\mathrm{kg \cdot m/s}}{9.11 \times 10^{-31}\,\mathrm{kg}} = 8.56 \times 10^{4}\,\mathrm{m/s}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 9.11 \times 10^{-31}\,\mathrm{kg} \cdot (8.56 \times 10^{4}\,\mathrm{m/s})^2 = 3.34 \times 10^{-21}\,\mathrm{J} = 2.09 \times 10^{-2}\,\mathrm{eV}$$

(b)

$$v = 8 \times 10^{5}\,\mathrm{cm/s} = 8 \times 10^{2}\,\mathrm{m/s}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 9.11 \times 10^{-31}\,\mathrm{kg} \cdot (8 \times 10^{2}\,\mathrm{m/s})^2 = 2.92 \times 10^{-25}\,\mathrm{J} = 1.83 \times 10^{-4}\,\mathrm{eV}$$

$$p = mv = 9.11 \times 10^{-31}\,\mathrm{kg} \cdot 8 \times 10^{2}\,\mathrm{m/s} = 7.29 \times 10^{-27}\,\mathrm{kg \cdot m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{7.29 \times 10^{-27}\,\mathrm{kg \cdot m/s}} = 9.09 \times 10^{-8}\,\mathrm{m} = 909\,\mathrm{\mathring{A}}$$

Ex. 4

$$\int_{-1}^{+3} |\Phi(x,t)|^2 dx = \int_{-1}^{+3} A^2 \cos^2\left(\frac{\pi x}{2}\right) dx = A^2 \cdot \left(\frac{x}{2} + \frac{\sin(\pi x)}{2\pi}\right)\Big|_{-1}^{+3} = 2A^2 = 1$$
$$A = \pm\frac{\sqrt{2}}{2}$$

Ex. 5

(a) (i)

$$v_p = \frac{\omega}{k} = \frac{8\times 10^{12}\,\mathrm{rad/s}}{8\times 10^{8}\,\mathrm{m}^{-1}} = 10^4\,\mathrm{m/s}$$
$$\lambda = \frac{2\pi}{k} = \frac{2\cdot 3.14}{8\times 10^{8}\,\mathrm{m}^{-1}} = 7.85\times 10^{-9}\,\mathrm{m} = 78.5\,\text{Å}$$

(ii)

$$p = \hbar k = 1.05\times 10^{-34}\,\mathrm{J\cdot s}\cdot 8\times 10^{8}\,\mathrm{m}^{-1} = 8.40\times 10^{-26}\,\mathrm{kg\cdot m/s}$$
$$v = \frac{p}{m} = \frac{8.40\times 10^{-26}\,\mathrm{kg\cdot m/s}}{9.11\times 10^{-31}\,\mathrm{kg}} = 9.22\times 10^{4}\,\mathrm{m/s}$$
$$E = \frac{1}{2}mv^2 = \frac{1}{2}\cdot 9.11\times 10^{-31}\,\mathrm{kg}\cdot (9.22\times 10^{4}\,\mathrm{m/s})^2 = 3.87\times 10^{-21}\,\mathrm{J}$$

(b) (i)

$$v_p = \frac{\omega}{k} = \frac{1.5\times 10^{12}\,\mathrm{rad/s}}{-1.5\times 10^{9}\,\mathrm{m}^{-1}} = -10^4\,\mathrm{m/s}$$
$$\lambda = \frac{2\pi}{|k|} = \frac{2\cdot 3.14}{1.5\times 10^{9}\,\mathrm{m}^{-1}} = 4.19\times 10^{-9}\,\mathrm{m} = 41.9\,\text{Å}$$

(ii)

$$p = \hbar|k| = 1.05\times 10^{-34}\,\mathrm{J\cdot s}\cdot 1.5\times 10^{9}\,\mathrm{m}^{-1} = 1.58\times 10^{-25}\,\mathrm{kg\cdot m/s}$$
$$v = \frac{p}{m} = \frac{1.58\times 10^{-25}\,\mathrm{kg\cdot m/s}}{9.11\times 10^{-31}\,\mathrm{kg}} = 1.73\times 10^{5}\,\mathrm{m/s}$$
$$E = \frac{1}{2}mv^2 = \frac{1}{2}\cdot 9.11\times 10^{-31}\,\mathrm{kg}\cdot (1.73\times 10^{5}\,\mathrm{m/s})^2 = 1.36\times 10^{-20}\,\mathrm{J}$$

Ex. 6

(a)

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{h^2 n^2}{8ma^2} = n^2\cdot\frac{(6.63\times 10^{-43}\,\mathrm{J\cdot s})}{8\cdot 9.11\times 10^{-31}\,\mathrm{kg}\cdot (10^{-9}\,\mathrm{m})^2} = n^2\cdot 6.03\times 10^{-20}\,\mathrm{J} = 0.377n^2\,\mathrm{eV}$$
$$E_1 = 0.377\,\mathrm{eV}$$
$$E_2 = 1.508\,\mathrm{eV}$$
$$E_3 = 3.393\,\mathrm{eV}$$

(b)

$$\Delta E = 5\cdot 6.03\times 10^{-20}\,\mathrm{J} = 3.015\times 10^{-19}\,\mathrm{J}$$
$$\lambda = \frac{hc}{\Delta E} = \frac{6.63\times 10^{-34}\,\mathrm{J\cdot s}\cdot 3\times 10^{8}\,\mathrm{m/s}}{3.015\times 10^{-19}\,\mathrm{J}} = 6.60\times 10^{-7}\,\mathrm{m}$$

Ex. 7

$$\psi_1(x) = A_1 e^{jk_1x} + B_1 e^{-jk_1x}$$
$$\psi_2(x) = A_2 e^{jk_2x} + B_2 e^{-jk_2x}$$
$$\psi_3(x) = A_3 e^{jk_3x}$$

At $x = 0$, the function must be continuous, so $\psi_1(0) = \psi_2(0)$, $\frac{\partial \psi_1}{\partial x}(0) = \frac{\partial \psi_2}{\partial x}(0)$,

$$A_1 + B_1 = A_2 + B_2$$
$$k_1A_1 - k_1B_1 = k_2A_2 - k_2B_2$$

At $x = a$, the function must be continuous, so $\psi_2(a) = \psi_3(a)$, $\frac{\partial \psi_2}{\partial x}(a) = \frac{\partial \psi_3}{\partial x}(a)$,

$$A_2 e^{jk_2a} + B_2 e^{-jk_2a} = A_3 e^{jk_3a}$$
$$k_2A_2 e^{jk_2a} - k_2B_2 e^{-jk_2a} = k_3A_3 e^{jk_3a}$$

Since $k_2a = 2n\pi$, so $e^{jk_2a} = e^{-jk_2a} = 1$, we get

$$A_2 + B_2 = A_3 e^{jk_3a}$$
$$k_2A_2 - k_2B_2 = k_3A_3 e^{jk_3a}$$

Use the equations, we can get

$$A_1 + B_1 = A_3 e^{jk_3a}$$
$$A_1 - B_1 = \frac{k_3}{k_1} A_3 e^{jk_3a}$$
$$2A_1 = \frac{k_3 + k_1}{k_1} A_3 e^{jk_3a}$$
$$\frac{A_3 \cdot A_3^*}{A_1 \cdot A_1^*} = \frac{4k_1^2}{(k_1 + k_3)^2}$$

So the transmission coefficient is

$$T = \frac{v_t \cdot A_3 \cdot A_3^*}{v_i \cdot A_1 \cdot A_1^*} = \frac{k_3 \cdot A_3 \cdot A_3^*}{k_1 \cdot A_1 \cdot A_1^*} = \frac{4k_1k_3}{(k_1 + k_3)^2}$$

Ex. 8

(a) For Region I,

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_0 - E)}{\hbar^2}\psi_1(x) = 0$$
$$\psi_1(x) = A_1 e^{k_1x}, k_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

For Region II,

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$
$$\psi_2(x) = A_2 \sin(k_2x) + B_2 \cos(k_2x), k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

For Region III,

$$\psi_3(x) = 0$$

(b) At $x = 0$, the function must be continuous, so $\psi_1(0) = \psi_2(0)$, $\dfrac{\partial \psi_1}{\partial x}(0) = \dfrac{\partial \psi_2}{\partial x}(0)$,

$$A_1 = B_2$$

$$k_1 A_1 = k_2 A_2$$

which means

$$\frac{A_1}{A_2} = \frac{k_2}{k_1}$$

At $x = a$, the function must be continuous, so $\psi_2(a) = \psi_3(a)$, $\dfrac{\partial \psi_2}{\partial x}(a) = \dfrac{\partial \psi_3}{\partial x}(a)$,

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

$$A_2 \cos(k_2 a) - B_2 \sin(k_2 a) = 0$$

which means

$$\frac{B_2}{A_2} = -\tan(k_2 a)$$

(c)

$$\frac{A_1}{A_2} = \frac{k_2}{k_1} = -\tan(k_2 a)$$

$$\sqrt{\frac{2mE}{2m(V_0 - E)}} = -\tan\left(\sqrt{\frac{2mE}{\hbar^2}}\right)$$

This equation can be solved so that we can find the exact values of E, so the energy levels of the electrons are quantized.