VE320 Homework3

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2018-06-13

Ex 3.1

(a)

$$n_i = 2.4 \times 10^{13}\,\mathrm{cm}^{-3}$$

(i)

$$\begin{aligned}
\frac{N_d - N_a}{2} &= \frac{2 \times 10^{15}\,\mathrm{cm}^{-3}}{2} = 10^{15}\,\mathrm{cm}^{-3} \\
n_0 &= \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \\
&= 10^{15}\,\mathrm{cm}^{-3} + \sqrt{(10^{15}\,\mathrm{cm}^{-3})^2 + (2.4 \times 10^{13}\,\mathrm{cm}^{-3})^2} \\
&= 2.000 \times 10^{15}\,\mathrm{cm}^{-3} \\
p_0 &= \frac{n_i^2}{n_0} = \frac{(2.4 \times 10^{13}\,\mathrm{cm}^{-3})^2}{2.000 \times 10^{15}\,\mathrm{cm}^{-3}} = 2.88 \times 10^{11}\,\mathrm{cm}^{-3}
\end{aligned}$$

(ii)

$$\begin{aligned}
\frac{N_a - N_d}{2} &= \frac{10^{16}\,\mathrm{cm}^{-3} - 7 \times 10^{15}\,\mathrm{cm}^{-3}}{2} = 1.5 \times 10^{15}\,\mathrm{cm}^{-3} \\
p_0 &= \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \\
&= 1.5 \times 10^{15}\,\mathrm{cm}^{-3} + \sqrt{(1.5 \times 10^{15}\,\mathrm{cm}^{-3})^2 + (2.4 \times 10^{13}\,\mathrm{cm}^{-3})^2} \\
&= 3.000 \times 10^{15}\,\mathrm{cm}^{-3} \\
n_0 &= \frac{n_i^2}{p_0} = \frac{(2.4 \times 10^{13}\,\mathrm{cm}^{-3})^2}{3.000 \times 10^{15}\,\mathrm{cm}^{-3}} = 1.92 \times 10^{11}\,\mathrm{cm}^{-3}
\end{aligned}$$

(b)

$$n_i = 1.8 \times 10^{6}\,\mathrm{cm}^{-3}$$

(i)

$$\begin{aligned}
\frac{N_d - N_a}{2} &= \frac{2\times 10^{15}\,\mathrm{cm^{-3}}}{2} = 10^{15}\,\mathrm{cm^{-3}} \\
n_0 &= \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \\
&= 10^{15}\,\mathrm{cm^{-3}} + \sqrt{(10^{15}\,\mathrm{cm^{-3}})^2 + (1.8\times 10^{6}\,\mathrm{cm^{-3}})^2} \\
&= 2.000\times 10^{15}\,\mathrm{cm^{-3}} \\
p_0 &= \frac{n_i^2}{n_0} = \frac{(1.8\times 10^{6}\,\mathrm{cm^{-3}})^2}{2.000\times 10^{15}\,\mathrm{cm^{-3}}} = 1.62\times 10^{-3}\,\mathrm{cm^{-3}}
\end{aligned}$$

(ii)

$$\begin{aligned}
\frac{N_a - N_d}{2} &= \frac{10^{16}\,\mathrm{cm^{-3}} - 7\times 10^{15}\,\mathrm{cm^{-3}}}{2} = 1.5\times 10^{15}\,\mathrm{cm^{-3}} \\
p_0 &= \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \\
&= 1.5\times 10^{15}\,\mathrm{cm^{-3}} + \sqrt{(1.5\times 10^{15}\,\mathrm{cm^{-3}})^2 + (1.8\times 10^{6}\,\mathrm{cm^{-3}})^2} \\
&= 3.000\times 10^{15}\,\mathrm{cm^{-3}} \\
n_0 &= \frac{n_i^2}{p_0} = \frac{(1.8\times 10^{6}\,\mathrm{cm^{-3}})^2}{3.000\times 10^{15}\,\mathrm{cm^{-3}}} = 1.08\times 10^{-3}\,\mathrm{cm^{-3}}
\end{aligned}$$

(c)

It means that there is about only one minor minority carrier every $10^3\,\mathrm{cm}^3$.

Ex 3.2

(a)

$$N_a = 3\times 10^{16}\,\mathrm{cm^{-3}}, N_d = 1.5\times 10^{16}\,\mathrm{cm^{-3}}$$
$$n_i = 1.5\times 10^{10}\,\mathrm{cm^{-3}}$$

Since $N_a > N_d$, it is p-type. And $n_i \ll N_a - N_d$, so the majority carriers are holes, the concentration is

$$p_0 = N_a - N_d = 3\times 10^{16}\,\mathrm{cm^{-3}} - 1.5\times 10^{16}\,\mathrm{cm^{-3}} = 1.5\times 10^{16}\,\mathrm{cm^{-3}}$$

The minority carriers are electrons, the concentration is

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5\times 10^{10}\,\mathrm{cm^{-3}})^2}{1.5\times 10^{16}\,\mathrm{cm^{-3}}} = 1.5\times 10^{4}\,\mathrm{cm^{-3}}$$

(b)

$$\begin{aligned}
p_0 &= N_a + \Delta N_a - N_d \\
\Delta N_a' &= p_0 - (N_a - N_d) = 5\times 10^{16}\,\mathrm{cm^{-3}} - 1.5\times 10^{16}\,\mathrm{cm^{-3}} = 3.5\times 10^{16}\,\mathrm{cm^{-3}} \\
n_0 &= \frac{n_i^2}{p_0} = \frac{(1.5\times 10^{10}\,\mathrm{cm^{-3}})^2}{5\times 10^{16}\,\mathrm{cm^{-3}}} = 4.5\times 10^{3}\,\mathrm{cm^{-3}}
\end{aligned}$$

Ex 3.3

(a)

$$n_0 = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2} = \frac{N_d}{0.95} = 1.05 \times 10^{15}\,\mathrm{cm}^{-3}$$

$$n_i^2 = \left(\frac{N_d}{0.95} - \frac{N_d}{2}\right)^2 - \left(\frac{N_d}{2}\right)^2 = \left(\frac{10^{15}\,\mathrm{cm}^{-3}}{0.95} - \frac{10^{15}\,\mathrm{cm}^{-3}}{2}\right)^2 - \left(\frac{10^{15}\,\mathrm{cm}^{-3}}{2}\right)^2 = 5.25 \times 10^{28}\,\mathrm{cm}^{-6}$$

$$n_i^2 = N_c N_v \left(\frac{T}{T_0}\right)^3 \exp\left[\frac{-E_g}{kT}\right]$$

$$5.25 \times 10^{28}\,\mathrm{cm}^{-6} = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \cdot 1.04 \times 10^{19}\,\mathrm{cm}^{-3} \left(\frac{T}{300\,\mathrm{K}}\right)^3 \exp\left[\frac{-1.12\,\mathrm{eV}}{8.62 \times 10^{-5}\,\mathrm{eV/K} \cdot T}\right]$$

$$T = 537.25\,\mathrm{K}$$

(b)

For $T = 300K$,

$$E_c - E_F = kT \ln\left[\frac{N_c}{n_0}\right] = 0.0259\,\mathrm{eV} \ln\left[\frac{2.8 \times 10^{19}\,\mathrm{cm}^{-3}}{1.05 \times 10^{15}\,\mathrm{cm}^{-3}}\right] = 2.64 \times 10^{-1}\,\mathrm{eV}$$

For $T = 537.25\,\mathrm{K}$,

$$kT = 8.62 \times 10^{-5}\,\mathrm{eV/K} \cdot 537.25\,\mathrm{K} = 4.63 \times 10^{-2}\,\mathrm{eV}$$

$$N_c' = N_c \left(\frac{T}{T_0}\right)^{3/2} = 2.8 \times 10^{19}\,\mathrm{cm}^{-3} \cdot \left(\frac{537.25\,\mathrm{K}}{300\,\mathrm{K}}\right)^{3/2} = 6.71 \times 10^{19}\,\mathrm{cm}^{-3}$$

$$E_c - E_F = kT \ln\left[\frac{N_c}{n_0}\right] = 4.63 \times 10^{-2}\,\mathrm{eV} \ln\left[\frac{6.71 \times 10^{19}\,\mathrm{cm}^{-3}}{1.05 \times 10^{15}\,\mathrm{cm}^{-3}}\right] = 5.12 \times 10^{-1}\,\mathrm{eV}$$

So

$$\Delta(E_c - E_F) = 5.12 \times 10^{-1}\,\mathrm{eV} - 2.64 \times 10^{-1}\,\mathrm{eV} = 2.48 \times 10^{-1}\,\mathrm{eV}$$

(c)

It is closer to the intrinsic value at high temperature.

Ex 3.4

(a)

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left[\frac{m_p^*}{m_n^*}\right] = \frac{3}{4} \cdot 0.0259\,\mathrm{eV} \cdot \ln 10 = 4.47 \times 10^{-2}\,\mathrm{eV}$$

(b)

(i)

Since impurity atoms are added so that $E_{Fi} < E_{midgap}$, it is p-type, acceptors should be added.

(ii)

$$E_{Fi} - E_F = 0.45\,\text{eV} + 4.47 \times 10^{-2}\,\text{eV} = 0.494\,\text{eV}$$

$$p_0 = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = 10^5\,\text{cm}^{-3} \cdot \exp\left[\frac{0.494\,\text{eV}}{0.0259\,\text{eV}}\right] = 1.92 \times 10^{13}\,\text{cm}^{-3}$$

Ex 3.5

(a)

When replacing gallium, silcon is the donor,

$$N_d = 0.05 \cdot 7 \times 10^{15}\,\text{cm}^{-3} = 3.5 \times 10^{14}\,\text{cm}^{-3}$$

When replacing arsenic, silcon is the accepetor,

$$N_a = 0.95 \cdot 7 \times 10^{15}\,\text{cm}^{-3} = 6.65 \times 10^{15}\,\text{cm}^{-3}$$

(b)

Since $N_a > N_d$, it is p-type.

(c)

$$n_i = 1.8 \times 10^6\,\text{cm}^{-3}$$

Since $n_i \ll N_a - N_d$, so the majority carriers are holes, the concentration is

$$p_0 = N_a - N_d = 6.65 \times 10^{15}\,\text{cm}^{-3} - 3.5 \times 10^{14}\,\text{cm}^{-3} = 6.3 \times 10^{15}\,\text{cm}^{-3}$$

The minority carriers are electrons, the concentration is

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.8 \times 10^6\,\text{cm}^{-3})^2}{6.3 \times 10^{15}\,\text{cm}^{-3}} = 5.14 \times 10^{-4}\,\text{cm}^{-3}$$

(d)

$$E_{Fi} - E_F = kT \ln\left[\frac{p_0}{n_i}\right] = 0.0259\,\text{eV} \ln\left[\frac{6.3 \times 10^{15}\,\text{cm}^{-3}}{1.8 \times 10^6\,\text{cm}^{-3}}\right] = 5.69 \times 10^{-1}\,\text{eV}$$

Ex 3.6

(a)

$$R = \frac{V}{I} = \frac{10\,\text{V}}{100\,\text{mA}} = 100\,\Omega$$

(b)

$$\sigma = \frac{L}{RA} = \frac{10^{-3}\,\text{cm}}{100\Omega \cdot 0.001\,\text{cm}^2} = 0.01(\Omega \cdot cm)^{-1}$$

(c)

$$\sigma \approx e\mu_n N_d$$

$$N_d \approx \frac{\sigma}{e\mu_n} = \frac{0.01(\Omega \cdot cm)^{-1}}{1.6 \times 10^{-19}\,\mathrm{C} \cdot 1350\,\mathrm{cm^2/V \cdot s}} = 4.63 \times 10^{13}\,\mathrm{cm^{-3}}$$

(d)

$$\sigma \approx e\mu_p (N_a - N_d)$$

$$N_a \approx \frac{\sigma}{e\mu_p} + N_d = \frac{0.01(\Omega \cdot cm)^{-1}}{1.6 \times 10^{-19}\,\mathrm{C} \cdot 480\,\mathrm{cm^2/V \cdot s}} + 10^{15}\,\mathrm{cm^{-3}} = 1.13 \times 10^{15}\,\mathrm{cm^{-3}}$$

Ex 3.7

(a)

According to Figure 5.3, when $N_a = 2 \times 10^{16}\,\mathrm{cm^{-3}}$, $\mu_p = 400\,\mathrm{cm^2/V \cdot s}$.

$$R = \frac{L}{\sigma A} \approx \frac{L}{e\mu_p N_a A} = \frac{0.075cm}{1.6 \times 10^{-19}\,\mathrm{C} \cdot 400\,\mathrm{cm^2/V \cdot s} \cdot 2 \times 10^{16}\,\mathrm{cm^{-3}} \cdot 8.5 \times 10^{-4}\,\mathrm{cm^2}} = 68.93\,\Omega$$

$$I = \frac{V}{R} = \frac{2\,\mathrm{V}}{68.93\,\Omega} = 2.90 \times 10^{-2}\,\mathrm{A}$$

(b)

$$R' = 3R = 3 \times 68.93\,\Omega = 206.79\,\Omega$$

$$I = \frac{V}{R} = \frac{2\,\mathrm{V}}{206.79\,\Omega} = 9.67 \times 10^{-3}\,\mathrm{A}$$

(c)

$$J = ep_0 v_d$$

$$v_d = \frac{J}{ep_0} = \frac{I}{ep_0 A}$$

For (a),

$$v_d = \frac{I}{ep_0 A} = \frac{2.90 \times 10^{-2}\,\mathrm{A}}{1.6 \times 10^{-19}\,\mathrm{C} \cdot 2 \times 10^{16}\,\mathrm{cm^{-3}} \cdot 8.5 \times 10^{-4}\,\mathrm{cm^2}} = 1.07 \times 10^{4}\,\mathrm{cm/s}$$

For (b),

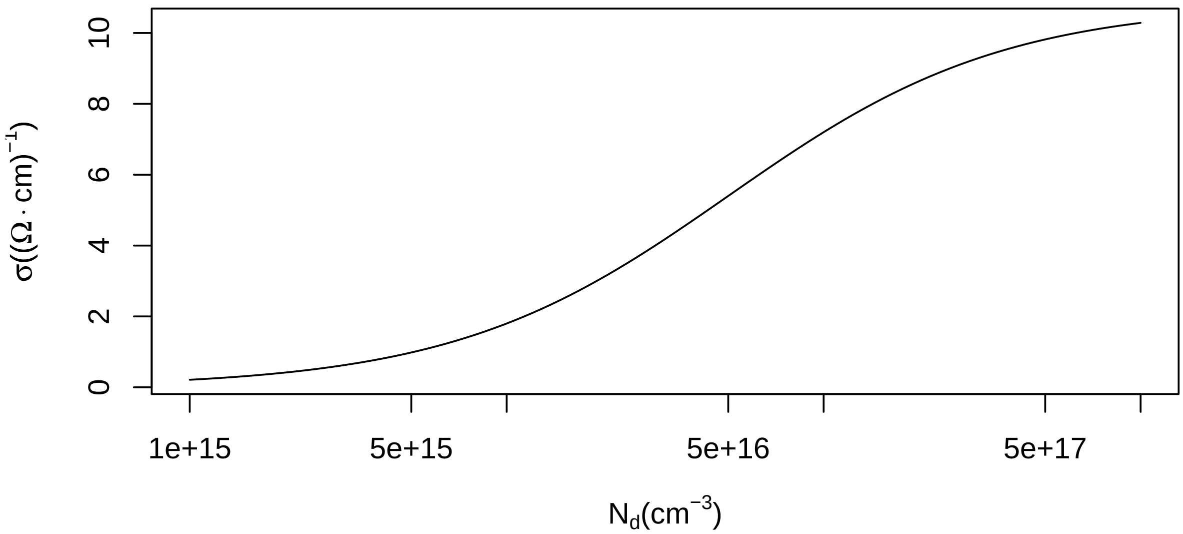
$$v_d = \frac{I}{ep_0 A} = \frac{9.67 \times 10^{-3}\,\mathrm{A}}{1.6 \times 10^{-19}\,\mathrm{C} \cdot 2 \times 10^{16}\,\mathrm{cm^{-3}} \cdot 8.5 \times 10^{-4}\,\mathrm{cm^2}} = 3.55 \times 10^{3}\,\mathrm{cm/s}$$

Ex 3.8

$$\sigma = e\mu_n N_d$$

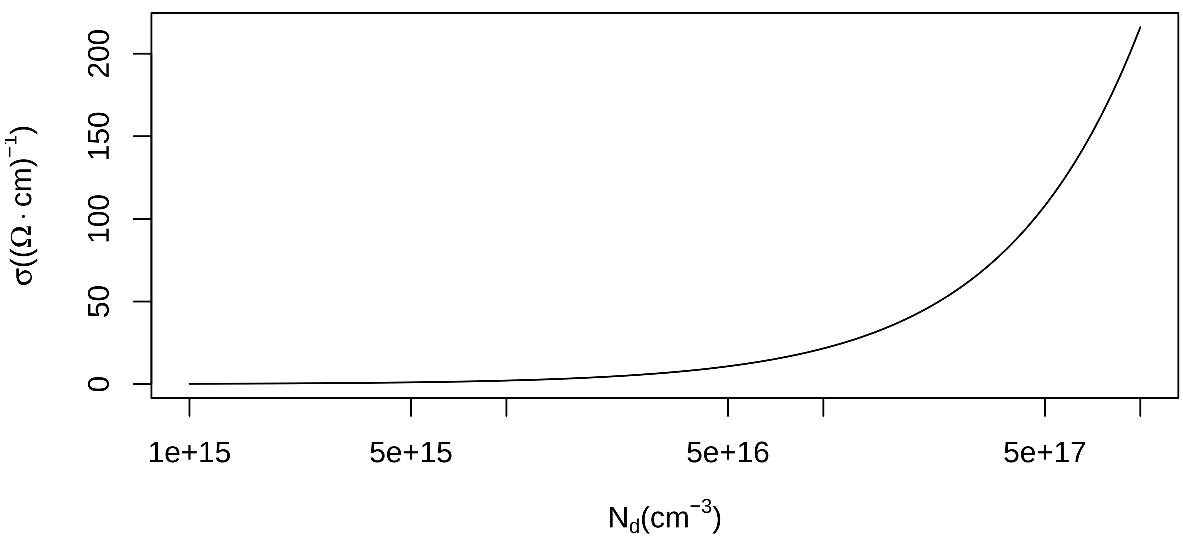
(a)

```r
func <- function(x) {1.6e-19*1350/(1+x/5e16)*x}
curve(func, from = 1e15, to = 1e18, xlab = TeX('$N_d(cm^{-3})$'),
      ylab = TeX('$\\sigma((\\Omega\\cdot cm)^{-1})$'), log = 'x')
```



(b)

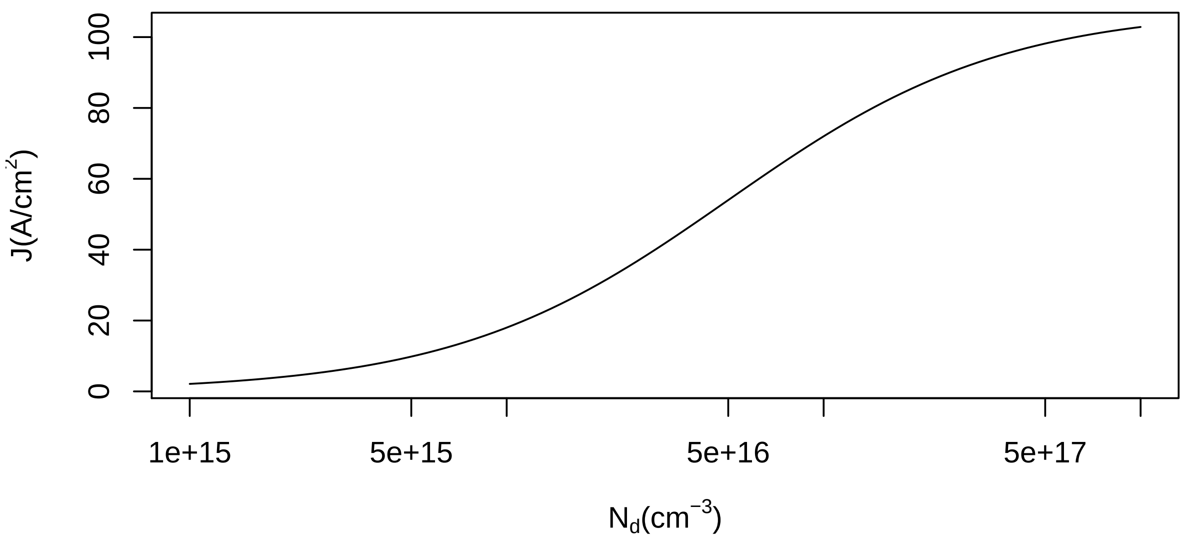
```r
func <- function(x) {1.6e-19*1350*x}
curve(func, from = 1e15, to = 1e18, xlab = TeX('$N_d(cm^{-3})$'),
      ylab = TeX('$\\sigma((\\Omega\\cdot cm)^{-1})$'), log = 'x')
```



$$J = \sigma E$$

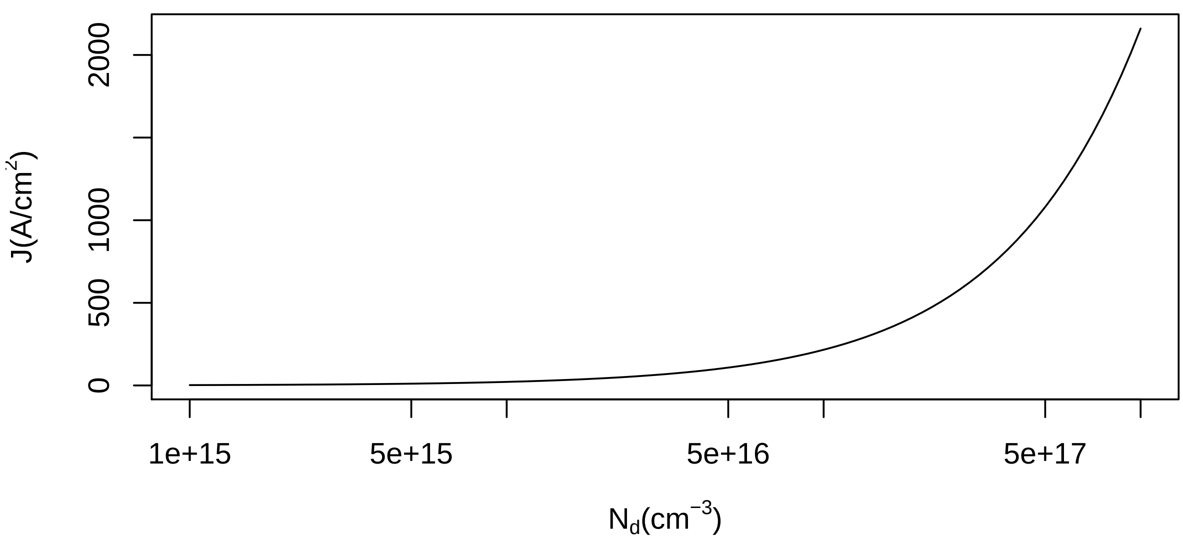
(c)

```
func <- function(x) {1.6e-19*1350/(1+x/5e16)*x*10}
curve(func, from = 1e15, to = 1e18, xlab = TeX('$N_d(cm^{-3})$'),
      ylab = TeX('$J(A/cm^2)$'), log = 'x')
```



(d)

```
func <- function(x) {1.6e-19*1350*x*10}
curve(func, from = 1e15, to = 1e18, xlab = TeX('$N_d(cm^{-3})$'),
      ylab = TeX('$J(A/cm^2)$'), log = 'x')
```



Ex 3.9

(a)

$$J = e\mu_n n(x)E + eD_n \frac{dn(x)}{dx}$$

$$\frac{dn(x)}{dx} + \frac{\mu_n E}{D_n} n(x) = \frac{J}{eD_n}$$

$$p = p(x) = \frac{\mu_n E}{D_n}, q = q(x) = \frac{J}{eD_n}$$

$$n(x) = n_0 e^{-\int_{x_0}^{x} p(x)dx} + \int_{x_0}^{x} q(s) e^{-\int_s^x p(t)dt} ds$$

When $x_0 = 0$, $e\mu_n n_0 E = J/2$, $n_0 = \frac{J}{2e\mu_n E}$,

$$n(x) = \frac{J}{2e\mu_n E} e^{-px} + \int_0^x q e^{-p(x-s)} ds = \left(\frac{J}{2e\mu_n E} - \frac{q}{p} \right) e^{-px} + \frac{q}{p} = -\frac{J}{2e\mu_n E} \exp\left[-\frac{\mu_n E}{D_n} x \right] + \frac{J}{e\mu_n E}$$

$$\frac{J}{e\mu_n E} = \frac{100}{1.6 \times 10^{-19} \cdot 8000 \cdot 12} = 6.51 \times 10^{15}$$

$$\frac{\mu_n E}{D_n} = \frac{\mu_n E}{kT\mu_n} = \frac{E}{kT} = \frac{12}{0.0259} = 463.32$$

$$n(x) = -3.255 \times 10^{15} e^{-463.32x} + 6.51 \times 10^{15}\,\mathrm{cm}^{-3}$$

(b)

$$n(0) = 3.255 \times 10^{15}\,\mathrm{cm}^{-3}$$

$$n(50 \times 10^{-4}) = 6.189 \times 10^{15}\,\mathrm{cm}^{-3}$$

(c)

$$J_{drf} = e\mu_n n(50 \times 10^{-4})E = 1.6 \times 10^{-19} \cdot 8000\,\mathrm{cm}^2/\mathrm{V \cdot s} \cdot 6.189 \times 10^{15}\,\mathrm{cm}^{-3} \cdot 12\,\mathrm{V/cm} = 95.06\,\mathrm{A/cm}^2$$

$$J_{diff} = J - J_{drf} = 100\,\mathrm{A/cm}^2 - 95.06\,\mathrm{A/cm}^2 = 4.94\,\mathrm{A/cm}^2$$

Ex 3.10

(a)

$$E(x) = -\frac{kT}{e} \cdot \frac{1}{N_d} \cdot \frac{dN_d(x)}{dx} = \frac{kT}{e} \cdot \frac{\frac{1}{L} N_{d_0} e^{-x/L}}{N_{d_0} e^{-x/L}} = \frac{kT}{eL} = \frac{0.0259\,\mathrm{V}}{10 \times 10^{-4}\,\mathrm{cm}} = 25.9\,\mathrm{V/cm}$$

(b)

$$\Delta V = \int_L^0 E(x)dx = -25.9\,\mathrm{V/cm} \cdot 10 \times 10^{-4}\,\mathrm{cm} = -0.0259\,\mathrm{V}$$