VE475
Introduction to Cryptography

Assignment 2 (06/06/2017)

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Non-programming exercises:
- Write in a neat and legible handwriting, or use LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb and object)

Progamming exercises:
- Write a README file for each program
- Upload an archive with all the programs onto Sakai

Ex. 1 — *Simple questions*

1. Find the inverse of 17 modulo 101
2. Find all the solutions to $12x \equiv 28 \bmod 236$.
3. Given a plaintext m modulo 31, its corresponding ciphertext is $c = m^7 \bmod 31$. Explain how to decrypt the message.
4. Factor 4883 and 4369 into a product of primes.
5. Find all the primes p such that $\begin{pmatrix} 3 & 5 \\ 7 & 3 \end{pmatrix} \bmod p$ is not invertible.
6. Let p be a prime and a and b be two integers such that $ab \equiv 0 \bmod p$. Show that either a or b is congruent to 0 mod p.
7. Compute 2^{2017} modulo 5, 13, and 31. What is $2^{2017} \bmod 2015$?

Ex. 2 — *Rabin cryptosystem*

1. Research and explain how the Rabin cryptosystem works.
2. To implement decryption for the Rabin cryptosystem on decides to build a machine that does the following. When the device is given a number x, it computes the square root of x mod n. Since there usually are more than one, it chooses one at random. If one get a meaningful message he assumes this is the correct result and otherwise input x again.
   a) Explain why a meaningful message can be expected fairly soon
   b) If Eve intercepts x, can she easily determine the original message?
   c) Eve has stolen the device and plans to run some attacks on it. What type of attack should she run to recover the factorization of n. Explain the process.

Note: the computation of square roots mod n will be covered later, so there is no need to detail this part.

Ex. 3 — *CRT*

A group prepares for a parade. If they arrange in rows of three, one person is left over. If they line up four to a row, two are left over and if they try rows of five, three are left over. What are the two smallest possible number of people in the group?