VE475 Homework 5

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Ex. 1 — RSA setup

1. In the RSA encryption and decryption, we use

$$ed \equiv 1 \bmod \varphi(n)$$

$$m^{ed} \equiv m \bmod \varphi(n)$$

This is based on the Euler's theorem, which has a condition that m and n be two coprime integers. So it is likely for n to be coprime with m.

2. Suppose $k = a\varphi(n)$, $a \in N^*$, and $m < n$.

(a)

$$\begin{aligned} m^k &\equiv (m^{\varphi(n)})^a \bmod n \\ &\equiv 1^a \bmod n \\ &\equiv 1 \bmod n \end{aligned}$$

So

$$m^k \equiv 1 \bmod p \quad \text{and} \quad m^k \equiv 1 \bmod q$$

(b) First, if $\gcd(m, n) = 1$, according to (a), it's obvious that

$$m^{k+1} \equiv m \bmod p \quad \text{and} \quad m^{k+1} \equiv m \bmod q$$

Second, if $\gcd(m, n) = p$, so $\gcd(m/p, q) = 1$

$$\begin{aligned} m^{k+1} &\equiv p \left[\left(\frac{m}{p} \right)^{k+1} \bmod q \right] \bmod n \\ &\equiv p \left[\left(\frac{m}{p} \right)^{a(p-1)\varphi(q)+1} \bmod q \right] \bmod n \\ &\equiv p \cdot \frac{m}{p} \bmod n \\ &\equiv m \bmod n \end{aligned}$$

So

$$m^{k+1} \equiv m \bmod p \quad \text{and} \quad m^{k+1} \equiv m \bmod q$$

Third, if $\gcd(m, n) = q$, it is similar to the second case.
We can conclude that for any arbitrary m, $m^{k+1} \equiv m \bmod p$ and $\bmod q$.

3. (a) We know that $ed \equiv 1 \bmod \varphi(n)$, which means that $ed = k+1$ where k is a multiple of $\varphi(n)$. According to part 2(b), we know that for any arbitrary m, $m^{k+1} \equiv m \bmod p$ and mod q, or we can say $m^{k+1} \equiv m \bmod n$, so $m^{ed} \equiv m \bmod n$,

   (b) From the previous calculation, we can find that for all $m < n$, no matter m and n are coprime or not, we can both find that $m^{ed} \equiv m \bmod n$, so that the RSA encryption and decryption can be performed. So we can conclude that it is not necessary that $\gcd(m, n) = 1$.

Ex. 2 — RSA decryption

$$n = 11413 = 101 \times 113$$

So we can find that $p = 101$ and $q = 113$, and we should calculate d so that $ed \equiv 1 \bmod n$.

By applying the extended euclidean algorithm,

	q_i	r_i	s_i
0		7467	1
1		11413	0
2	$7467 \div 11413 = 0$	$7467 - 0 \times 11413 = 7467$	$1 - 0 \times 0 = 1$
3	$11413 \div 7467 = 1$	$11413 - 1 \times 7467 = 3946$	$0 - 1 \times 1 = -1$
4	$7467 \div 3946 = 1$	$7467 - 1 \times 3946 = 3521$	$1 - 1 \times -1 = 2$
5	$3946 \div 3521 = 1$	$3946 - 1 \times 3521 = 425$	$-1 - 1 \times 2 = -3$
6	$3521 \div 425 = 8$	$3521 - 8 \times 425 = 121$	$2 - 8 \times -3 = 26$
7	$425 \div 121 = 3$	$425 - 3 \times 121 = 62$	$-3 - 3 \times 26 = -81$
8	$121 \div 62 = 1$	$121 - 1 \times 62 = 59$	$26 - 1 \times -81 = 107$
9	$62 \div 59 = 1$	$62 - 1 \times 59 = 3$	$-81 - 1 \times 107 = -188$
10	$59 \div 3 = 19$	$59 - 19 \times 3 = 2$	$107 - 19 \times -188 = 3679$
11	$3 \div 2 = 1$	$3 - 1 \times 2 = 1$	$-188 - 1 \times 3679 = -3867$

$$e \cdot -3867 \equiv 1 \bmod n$$

$$e \cdot 7546 \equiv 1 \bmod n$$

So $d = 7546$, then we can apply modulo exponentiation to the equation

$$m \equiv c^d \bmod n$$

i	d_i	power mod 11413
12	1	$1^2 \cdot 5859 \equiv 5859$
11	1	$5859^2 \cdot 5859 \equiv 1415$
10	1	$1415^2 \cdot 5859 \equiv 1617$
9	0	$1617^2 \equiv 1112$
8	1	$1112^2 \cdot 5859 \equiv 7374$
7	0	$7374^2 \equiv 4344$
6	1	$4344^2 \cdot 5859 \equiv 6768$
5	1	$6768^2 \cdot 5859 \equiv 4445$
4	1	$4445^2 \cdot 5859 \equiv 4041$
3	1	$4041^2 \cdot 5859 \equiv 11111$
2	0	$11111^2 \equiv 11313$
1	1	$11313^2 \cdot 5859 \equiv 7071$
0	0	$7071^2 \equiv 10101$

So $m = 10101$.

Ex. 3 — Breaking RSA

Ex. 4 — Programming

Ex. 5 — Simple Questions

1.

2.

3.

4.

5.

$$(97-1) = 96 = 2^5 \times 3$$

So the generator x should satisfy that

$$x^{32} \neq 1 \bmod 97 \quad \text{and} \quad x^{48} \neq 1 \bmod 97$$

$$x^{16} \neq \pm 1, 35, 61 \bmod 97$$

We can find that

$$2^{16} \equiv 61 \bmod 97$$
$$3^{16} \equiv 61 \bmod 97$$
$$4^{16} \equiv 1 \bmod 97$$
$$5^{16} \equiv 36 \bmod 97$$

So the smallest generator of $U(Z/97Z)$ is 5.