UM–SJTU Joint Institute

Physics Laboratory
(Vp141)

Laboratory Report

Exercise 4
Measurement of the Speed of Sound

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1 Introduction

The objective of this exercise to study several methods of measuring the speed of sound in air: the resonance method, the phase comparison method, and the time difference method. In addition, you will get familiar with the successive difference method in measurement data processing.

1.1 Basic Quantitative Characteristics of Sound Waves

Sound is a mechanical wave that propagates through a compressible medium. It is a longitudinal wave because the direction of vibrations of the medium (here, change in the density or the pressure) is the same as the direction of propagation. The frequency of sound perceptible to a human ear ranges from about 20 Hz to 20 000 Hz. Sound with the frequency higher than 20 000 Hz is called ultrasound. In this experiment an ultrasonic wave is chosen as the signal source, because its wavelength is short enough to measure the speed of sound precisely.

The phase speed v, the frequency f and the length λ of a wave are related by the formula

$$v = \lambda f \tag{1}$$

For motion with constant speed v along a straight line, we have

$$v = \frac{L}{t} \tag{2}$$

where L is the distance traveled over time t. Hence, if the distance and the time a wavefront travels is known, the phase speed may be found.

2 Experimental setup

The experimental setup consists of a signal source, two piezoelectric transducers S_1 and S_2, and oscilloscope arranged as shown in Figure 1.

2.1 Resonance Method

The elements S_1 and S_2 are the wave source and the receiver (also reflector), respectively, placed a distance L from each other. If they are arranged parallel to each other, the sound wave is reflected. If

$$L = n\frac{\lambda}{2} \tag{3}$$

where $n = 1, 2, \ldots$, i.e. the distance is a multiple of half-wavelength, standing waves will form, and maximum output power will be observed in the oscillograph (Figure 2). The distance between two successive maxima ($L_{i+1} - L_i$) is always $\lambda/2$. After the position corresponding to each maximum is measured, it is easy to find the wavelength and then the speed of sound by using Eq. (1). The frequency f is displayed directly on the signal generator.

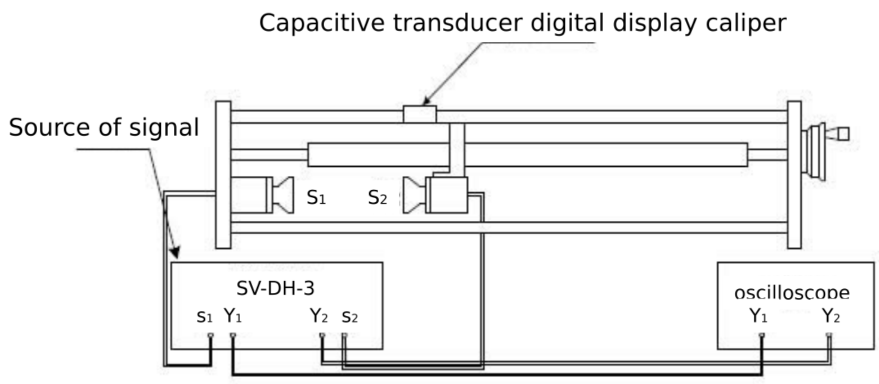

Figure 1: Experimental setup.

2.2 Phase-comparison method

If the phase of the wave at two points on the wave propagation direction is equal, then the distance between these points L has to be a multiple of the wavelength, *i.e.*

$$L = n\lambda$$

where $n = 1, 2, \ldots$. The experimental setup for the phase comparison method is the same as in the previous method (Figure 1). Lissajous figures are used to identify the values of L. Lissajous figures (or Lissajous curves) are trajectories of a particle that moves in a plane so that i.e. it moves in a harmonic motion independently along two perpendicular directions (for example the axes x and y of a Cartesian coordinate system), so that $r(t) = (Ax\cos(\omega xt + \varphi x), Ay\cos(\omega yt + \varphi y))$. When the two superimposed harmonic motions have identical frequency $\omega x = \omega y$ and phase difference $|\varphi x\varphi y| =$ n, where $n = 0, 1, 2, \ldots$, the Lissajous figure will show as a straight line. For other values of the phase difference the figures will have an elliptical shape.

2.3 Time-difference method

When an ultrasonic pulse signal emitted by S_1 arrives at S_2 , it is received and returned back to the processor. By contrasting the original signal with the received one, one can measure the time needed for the sound to travel from S_1 to S_2 over a distance of L. When the values of L and t are known, the phase speed of sound can be found from Eq. (2).

2.4 Successive Difference Method

The successive difference method is an effective method to increase the accuracy of the average value calculated from a series of measurement data. In this experiment, the usual method of calculating the average value, illustrated by the formula

$$\frac{\bar{\lambda}}{2} = \frac{[(L_1 - L_0) + (L_2 - L_1) + \cdots + (L_n - L_{n-1})]}{n} = \frac{L_n - L_0}{n} \tag{4}$$

will be modified, because as Eq. (4) shows, the average value of the wavelength is determined only by the first and the last value, L_0 and L_n .

A modification of the formula by rearranging terms as

$$n\frac{\bar{\lambda}}{2} = \frac{\sum_{i=1}^{n}(L_{n+i} - L_i)}{n} \tag{5}$$

produces more accurate results, as each value contributes to the final result.

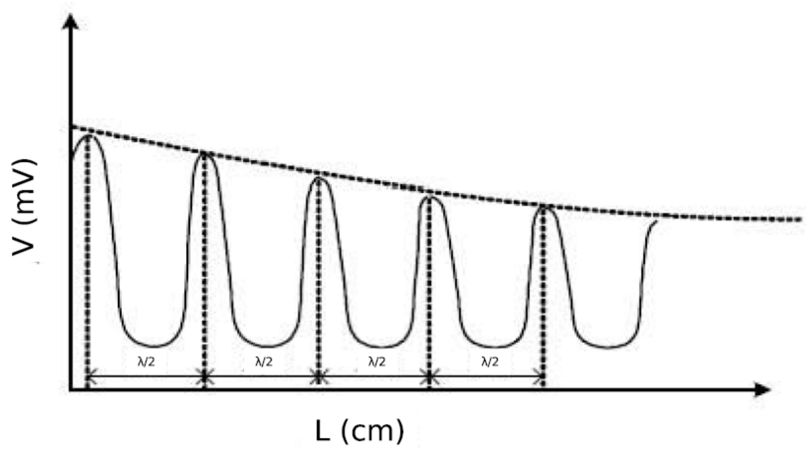

Figure 2: Relationship between the signal voltage and the distance between the transducers.

3 Measurement

3.1 Resonance Method

1. Set the initial distance between S_1 and S_2 at about 1 cm.
2. Turn on the signal source and the oscilloscope. Then set the following options on the panel of the signal source
   (1) Choose Continuous wave for Method.
   (2) Choose Air for Medium.
   (3) Adjust Signal Strength until a 10 V peak voltage is observed on the oscilloscope.

(4) Adjust Signal Frequency between 34.5 and 37.5 kHz until the peak-to-peak voltage reaches its maximum. Record the frequency.

3. Increase L gradually by moving S_2 , and observe the output voltage of S_2 on the oscilloscope. Record the position of S_2 as L_2 when the output voltage reaches an maximum.

4. Repeat step 3 to record 20 values of L_2 and calculate v.

3.2 Phase-comparison Method

1. Use Lissajous figures to observe the phase difference between the transmitted and the received signals. Move S_2 and record the position when the Lissajous figure becomes a straight line with the same slope.

2. Repeat step 1 to collect 12 sets of data. Use the successive difference method to process the data and calculate v.

3.3 Time-difference Method

Since the pulse wave causes damped oscillations at the receiver, there will be significant interference if S_1 and S_2 resonate. The resonance can be observed on the oscilloscope.

1. Choose Pulse Wave for Method and Air for Medium on the panel of the signal source.

2. Adjust the frequency to 25 Hz and the width to 500 s.

3. Record the distance L_1 and the time t_1 .

4. Move S_2 to another position and repeat step 3. Record L_i and t_i , i = 2, 3, 4, . . .

5. Repeat step 4 to collect 12 pairs of L_i and t_i . Plot the $L_i = L_i(t_i)$ graph and use computer software to find a linear fit to the data. The slope of the line is the speed v.

3.4 Time-difference Method in a Liquid

1. Change the medium to water.

2. Adjust the frequency to 100 Hz and the width to 500 s.

3. Use the cursor function of the oscilloscope to measure the time and the distance between the the starting points of neighboring periods. Record 12 pairs of data and calculate v_{water} .

4 Results

The frequency of the sound wave was measured as

$$f = 35.43 \pm 0.01\,\mathrm{kHz}$$

The temperature was measured as

$$T = 28 \pm 1\,^{\circ}\mathrm{C}$$

4.1 Resonance Method

The measurement of length L was shown in Table 1.

L_i	[mm] $\pm$ 0.01 [mm]	L_i	[mm] $\pm$ 0.01 [mm]	$L_{10+i} - L_i$	[mm]
1	14.66	11	64.60	1	49.94
2	19.56	12	69.63	2	50.07
3	24.69	13	74.47	3	49.78
4	29.73	14	79.56	4	49.83
5	34.73	15	84.52	5	49.79
6	39.63	16	89.43	6	49.80
7	44.74	17	94.43	7	49.69
8	49.63	18	99.34	8	49.71
9	54.69	19	104.37	9	49.68
10	59.65	20	109.22	10	49.57

Table 1: Data table for the resonance method.

$$v = \bar{\lambda} f = \frac{2f \sum_{i=1}^{10} (L_{10+i} - L_i)}{n^2} = 352.78 \pm 2\,\mathrm{m/s}$$

4.2 Phase-comparison Method

The measurement of length L was shown in Table 2.

L_i	[mm] $\pm$ 0.01 [mm]	L_i	[mm] $\pm$ 0.01 [mm]	$L_{6+i} - L_i$	[mm]
1	14.53	11	74.45	1	59.92
2	24.70	12	84.39	2	59.96
3	34.49	13	94.33	3	59.84
4	44.66	14	104.23	4	59.57
5	54.44	15	114.16	5	59.72
6	64.54	16	124.09	6	59.55

Table 2: Data table for the phase comparison method.

$$v = \bar{\lambda} f = \frac{f \sum_{i=1}^{6} (L_{6+i} - L_i)}{n^2} = 352.56 \pm 2\,\mathrm{m/s}$$

4.3 Time-difference Method

The measurement of time t and length L was shown in Table 3.

	t_i [μs] $\pm$ 0.5 [μs]	L_i [mm] $\pm$ 0.01 [mm]
1	23.5	10.00
2	51.5	20.00
3	80.0	30.00
4	107.5	40.00
5	134.5	50.00
6	163.0	60.00
7	192.5	70.00
8	220.5	80.00
9	249.0	90.00
10	276.5	100.00
11	306.0	110.00
12	334.5	120.00

Table 3: Data table for the time difference method.

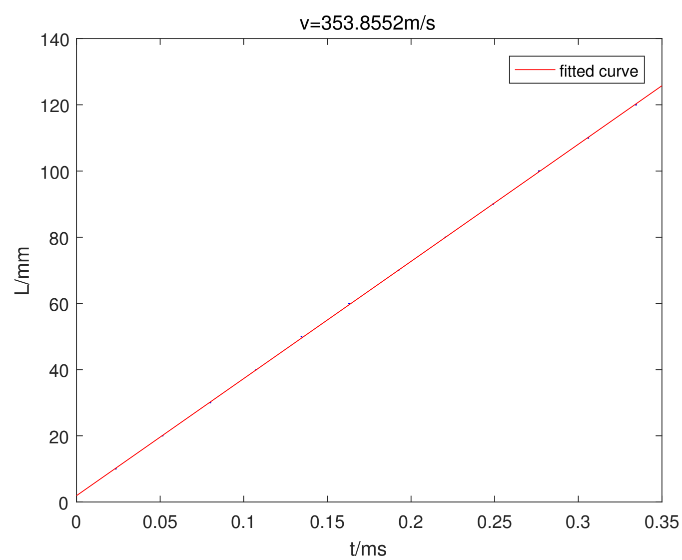

Figure 3: Relation ship between L_i and t_i.

$$v = 353.86 \pm 0.3\,\mathrm{m/s}$$

4.4 Time-difference Method in a Liquid

The measurement of time t and length L was shown in Table 4.

	t_i [μs] $\pm$ 0.2 [μs]	L_i [mm] $\pm$ 0.01 [mm]
1	32.2	10.00
2	37.6	20.00
3	43.6	30.00
4	49.4	40.00
5	55.4	50.00
6	62.6	60.00
7	69.0	70.00
8	75.6	80.00
9	82.0	90.00
10	89.0	100.00
11	95.6	110.00
12	102.0	120.00

Table 4: Data table for the time difference method in liquid.

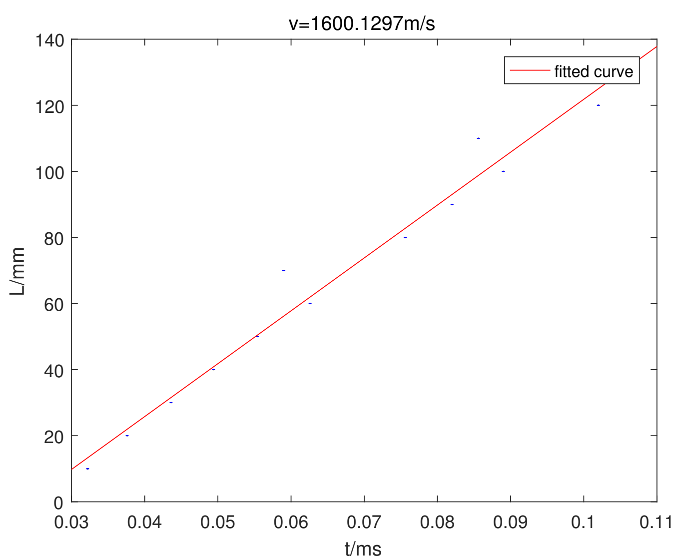

Figure 4: Relation ship between L_i and t_i in liquid.

$$v_{water} = 1600.13 \pm 6\,\mathrm{m/s}$$

5 Measurement uncertainty analysis

5.1 Resonance Method

For a single measurement of $\Delta L = L_{n+i} - L_i$, its uncertainty (of type B) is $\Delta_{\Delta LB} = \Delta_{dev} = 0.01mm$. In the experiment, $\overline{\Delta L}$ is found by taking the average of 10 measurements. In order to estimate type-A uncertainty of the time, the standard deviation of the average value is calculated as

$$S_{\Delta L} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\Delta L - \overline{\Delta L})^2}$$

Using the data from Table.1 one obtains $S_{\Delta L} = 0.141mm$. Taking into account that $t_{0.95} = 2.26$ for $n = 10$, the Type-A uncertainty is estimated as $\Delta_{\Delta LA} = 2.26 \cdot 0.141 = 0.32mm$

Hence the combined uncertainty

$$u_{\Delta L} = \sqrt{\Delta_{\Delta LA}^2 + \Delta_{\Delta LB}^2} = \sqrt{0.32^2 + 0.01^2} = 0.32mm$$

v can be calculated by the equation $v = \frac{2f\Delta L}{n}$. Therefore its uncertainty u_v is found by applying the uncertainty propagation formula

$$\begin{aligned} u_v &= \sqrt{\left(\frac{\partial v}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial v}{\partial \Delta L}\right)^2 u_{\Delta L}^2} \\ &= \sqrt{\left(\frac{2\Delta L}{n}\right)^2 u_f^2 + \left(\frac{2f}{n}\right)^2 u_{\Delta L}^2} \\ &= 2.27\,\mathrm{m/s} \end{aligned}$$

Hence the density of the balls found from the measurement of the fixed mass and diameter is

$$v = 352.78 \pm 2\,\mathrm{m/s}$$

with relative uncertainty 0.57%

5.2 Phase-comparison Method

For a single measurement of $\Delta L = L_{n+i} - L_i$, its uncertainty (of type B) is $\Delta_{\Delta LB} = \Delta_{dev} = 0.01mm$. In the experiment, $\overline{\Delta L}$ is found by taking the average of 10 measurements. In order to estimate type-A uncertainty of the time, the standard deviation of the average value is calculated as

$$S_{\Delta L} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(\Delta L - \overline{\Delta L})^2}$$

Using the data from Table.1 one obtains $S_{\Delta L} = 0.15mm$. Taking into account that $t_{0.95} = 2.57$ for $n = 6$, the Type-A uncertainty is estimated as $\Delta_{\Delta LA} = 2.57 \cdot 0.15 = 0.39mm$

Hence the combined uncertainty

$$u_{\Delta L} = \sqrt{\Delta^2_{\Delta LA} + \Delta^2_{\Delta LB}} = \sqrt{0.39^2 + 0.01^2} = 0.39mm$$

v can be calculated by the equation $v = \frac{f\Delta L}{n}$. Therefore its uncertainty u_v is found by applying the uncertainty propagation formula

$$\begin{aligned} u_v &= \sqrt{\left(\frac{\partial v}{\partial f}\right)^2 u_f^2 + \left(\frac{\partial v}{\partial \Delta L}\right)^2 u_{\Delta L}^2} \\ &= \sqrt{\left(\frac{\Delta L}{n}\right)^2 u_f^2 + \left(\frac{f}{n}\right)^2 u_{\Delta L}^2} \\ &= 2.31\,\mathrm{m/s} \end{aligned}$$

Hence the density of the balls found from the measurement of the fixed mass and diameter is

$$v = 352.56 \pm 2\,\mathrm{m/s}$$

with relative uncertainty 0.57%

5.3 Time-difference Method

According to MATLAB,

$$v = 353.86 \pm 0.3\,\mathrm{m/s}$$

with relative uncertainty 0.08%

$$v_{water} = 1600.13 \pm 6\,\mathrm{m/s}$$

with relative uncertainty 0.46%

6 Conclusion

In this experiment, we measured the speed of sound in three different methods.

According to the data, we can conclude that the speed of sound in the air is about 350 m/s and the speed of sound in a certain liquid is 1600 m/s, which is similar to data from the Internet.

The uncertainty of the data is quite small, which suggests that the experiment is accurate.

7 Reference

(a) Qin Tian, Cao Jianjun, Yi Hankun, Mateusz Krzyzosiak, VP141 Exercise 4, Measurement of the Speed of Sound, based on materials provided by the Department of Physics, Shanghai Jiaotong University.