Physics Laboratory
Vp141

Exercise 1

Measurements of the Moment of Inertia

This lab manual is based on the materials provided by the Department of Physics, Shanghai Jiaotong University.

Edited by: Qin Tian, Zeng Ming, Mateusz Krzyzosiak

rev 3.0

1 Pre-lab Reading

Chapters 9 and 10 (Young and Freedman)

2 Objectives

The objectives of this exercise are to get familiar with the constant-torque method for measuring the moment of inertia of a rigid body and study the dependence of the moment of inertia on the change of mass, mass distribution or axis of rotation. Moreover, in this exercise, the parallel axis theorem (Steiner's) theorem will be verified.

As a part of measurement technique skills, Learn to measure the time using the counter-type electronic timer.

3 Theoretical Background

Moment of inertia is a property of a rigid body that defines its resistance (inertia) to a change of angular velocity of rotation about an axis. This qualitative characteristics of an extended body constrained to rotate about an axis is determined by a combination of mass and its distribution. The moment of inertia of a rigid bodywith respect to a certain rotation axis can be calculated mathematically. However, if the body has relatively irregular shape or non-uniformly distributed mass, the calculation may be difficult. Experimental methods may be used in such cases.

3.1 Second Law of Dynamics for Rotational Motion

According to the second law of dynamics for rotational motion about a fixed axis

$$M = I\beta, \tag{1}$$

relates the component of the torque M about the axis of rotation with the moment of inertia about this axis, and angular acceleration component β. Therefore, the moment of inertia I can be found once the torque and the resultant angular acceleration are measured.

Moment of inertia is an additive quantity, *i.e.* if the moment of inertia of rigid body A about an axis is I_A and the moment of inertia of rigid body B about the same axis is I_B, then the moment of inertia of the combined rigid body AB composed of A and B is

$$I_{AB} = I_A + I_B.$$

3.2 Parallel Axis Theorem

If the moment of inertia of a rigid body with mass m about an axis through the center of mass is I_0. Then for any axis parallel to that axis, the moment of inertia is

$$I = I_0 + md^2, \tag{2}$$

where d is the distance between the axes. This result is known as the parallel axis theorem or Steiner's theorem.

3.3 Apparatus and Measurement Method

The experimental setup is shown in Figure 1. It consists of a turntable with an attached photo gate system used for time measurements.

Let us assume that, the empty turntable is initially rotating and its moment of inertia with respect to the rotation axis is I_1. Since the bearings are not frictionless, there will be a non-zero frictional torque M_μ causing the turntable to decelerate with angular acceleration β_1

$$M_\mu = -I_1\beta_1. \tag{3}$$

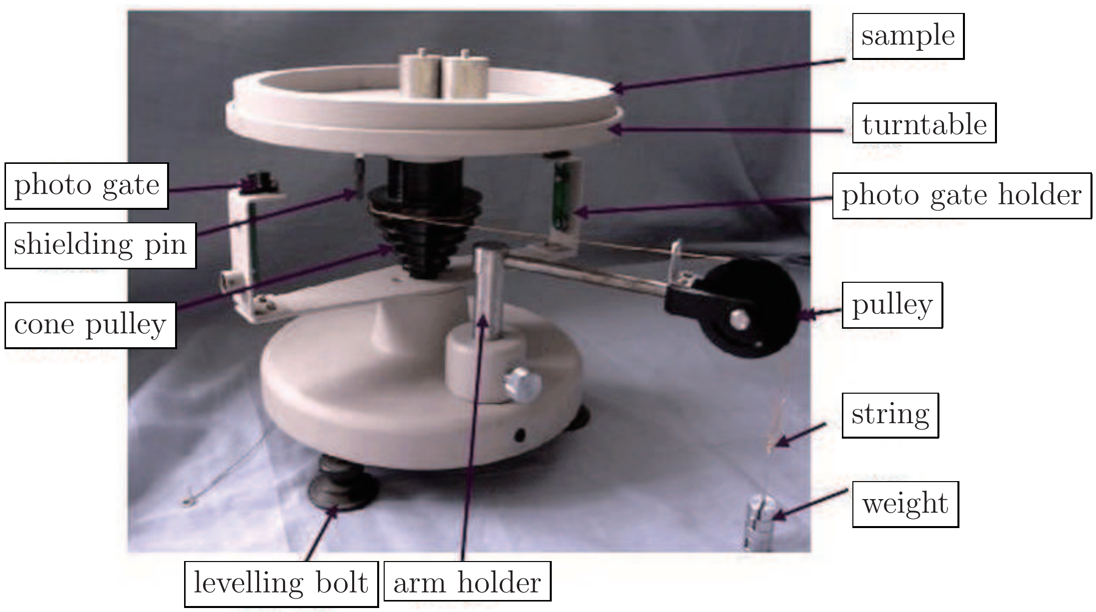

Figure 1. Experimental setup.

On the other hand, assume that a light and inextensible string is wound on a cone pulley with radius R placed on the axis of rotation. Attached to the other end of the string passing through a disk pulley, there is a mass m hanging. After the mass is released the system moves (rotates) with a constant acceleration (angular acceleration) because of a constant net force (torque). If the mass moves downward with acceleration a, the tension in the string is $T = m(g-a)$. And if the turntable rotates with angular acceleration β_2 at the same time, we have $a = R\beta_2$ (we assume that the string does not slip on the pulleys). The torque the string exerts on the turntable is then $TR = m(g - R\beta_2)R$, and hence the equation of motion of the turntable reads

$$m(g - R\beta_2)R - M_\mu = I_1\beta_2. \tag{4}$$

Eliminating M_μ from Eqs. 3 and 4, one obtains

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1}. \tag{5}$$

Similarly, if a rigid body of unknown moment of inertia is placed on the turntable, we may find

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3}, \tag{6}$$

where β_3 is the magnitude of angular deceleration of the turntable with the body, and β_3 is its angular acceleration, when the mass m is released.

Using the fact that the moment of inertia is an additive quantity, the moment of inertia of the rigid object placed on the turntable may be found as the difference

$$I_3 = I_2 - I_1. \tag{7}$$

Measurement of the angular acceleration

In the experiment, two shielding pins are fixed at the edge of the turntable generating signals at the photo gate with the phase interval of π as the turntable rotates. A counter-type electronic timer is used to measure the number k and the time t of the photo gate signal. If $(k_m,\, t_m)$ and $(k_n,\, t_n)$ are two sets of data from the measurement, the angular displacements are

$$\theta_m = k_m\pi = \omega_0 t_m + \frac{1}{2}\beta t_m^2, \tag{8}$$

$$\theta_n = k_n\pi = \omega_0 t_n + \frac{1}{2}\beta t_n^2, \tag{9}$$

where ω_0 is the initial angular speed. After eliminating ω_0 in Eq. 8 and 9, the angular acceleration is obtained as

$$\beta = \frac{2\pi(k_n t_m - k_m t_n)}{t_n^2 t_m - t_m^2 t_n}. \tag{10}$$

4 Measurement Procedure

1. Measure the mass of the weight, hoop, disk, and cylinder, and the radius of the cone pulley and the cylinder (as requested by the instructor). Calculate the moment of inertia of the hoop and the disk mathematically. Find the local gravity acceleration on Internet.

2. Turn on the electronic timer and switch to mode 1-2 (single gate, multiple pulses).

3. Place the instrument close to the edge of the desk and stretch the pulley arm outside.

4. Level the turntable with the bubble level.
5. Make the turntable rotating and press the start button of the timer. After at least 8 signals are recorded, stop the turntable and write down the data.
6. Attach the weight to one end of the string. Place the string on the disk pulley, thread through the hole on the arm, and wind the string around the 3rd winder of the cone pulley. Adjust the arm holder so that the string is crossing the center of the hole.
7. Release the weight and start the timer. Stop the turntable when the weight hits the floor. Write down the recorded data.
8. The moment of inertia of the empty turntable is obtained by using the formulae in Section 3 and the data from step 5 and 7. Repeat the step $5 \sim 7$ with a rigid object placed on the turntable. Eq. 7 is used to find the moment of inertia of the rigid object.

In the experiment, each student should measure the moment of inertia of the empty turntable, the hoop, the disk and the cylinder, as well as verify the parallel-axis theorem by placing two cylinders off the axis while keeping their center of mass on axis in order to keep the rotation steady.

The distance of the holes to the center of the turntable are about 45, 60, 75, 90, 105 mm respectively. The timer resolution is 0.0001 s, and the error is 0.004%.

5 Cautions

- Stop the turntable as soon as the weight hits the ground in order to avoid the string on the cone pulley getting stuck.
- Press the timer start button after release the weight to assure a constant acceleration.
- Use the 3rd (or smaller) diameter of the cone pulley so that a sets of at least 8 data in one measurement can be obtained.
- The string must not overlap while being wound on the cone pulley.
- In each measurement or data obtained from anywhere else, the error should be written down together with the data. The error analysis should be included in the report.

6 Preview Questions

- Explain what a rigid body is.
- Is the angular momentum of a rigid body always parallel to the rotation axis? Give an example.
- When the torque of a force with respect to an axis is zero?
- Why should the turntable be levelled before measurements start?
- The method of calculating the angular acceleration from the experimental data that is used in this manual is called the successive differences method. Can you design another method to obtain the angular acceleration from the data? What are advantages and disadvantages of your method compared to the successive differences method?
- In order to verify Steiner's theorem, we use at least two cylinders placed at some distance from the center of the turntable, and keep their center of mass at the rotation axis. Why not use one cylinder only, or not keep their center of mass on the axis?
- Estimate the measuring range of this setup. What setup should be used if the sample has more or less moment of inertia than this range?
- Discuss other uncertainty factors that are not included in this manual and comment on their contribution to uncertainty of the final result.