UM–SJTU Joint Institute

Physics Laboratory
(Vp141)

Laboratory Report

Exercise 3

Simple Harmonic Motion:
Oscillations in Mechanical Systems

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1 Introduction

The main objective of this exercise is to study simple harmonic oscillation. You will learn how to find the spring constant and effective mass of a spring, and how to use the air track. We will analyze the relationship between the oscillation period and the mass of the oscillator, check whether the oscillation period depends on the the amplitude, and examine the relationship between the maximum speed and the amplitude.

There are various kinds of periodic motion in nature, among which the simplest and the most fundamental one is the simple harmonic motion, where the restoring force is proportional to the displacement from the equilibrium position and as a result, the position of a particle depends on time as the sine (or cosine) function. Discussion of the simple harmonic motion is a basis for studying more complex situations.

1.1 Hookes Law

Within the elastic limit of deformation, the force F x needed to be applied in order to stretch or compress a spring by the distance x is proportional to that distance, i.e.,

$$F_x = kx \tag{1}$$

where k is a constant (called the spring constant) characterizing how easy it is to deform the spring. This constant will be found in the present exercise using a measurement device called the Jolly balance. The linear relation (1), between the force and the deformation, is known as the Hookes Law. According to Newtons third law of dynamics, the spring exerts a reaction force (called the elastic force) of the same magnitude but opposite direction. As this force tries to restore the system back to the equilibrium, it is known as the restoring force.

1.2 Equation of Motion of the Simple Harmonic Oscillator

As shown in Figure 1, an object with mass M is set on an air track with a spring attached to both of its sides. The purpose of using the air track is to eliminate frictional forces between moving surfaces. The other ends of the springs are fixed to the air track. The spring constants k_1 and k_2 are to be measured with the Jolly balance. The origin ($x = 0$) of the coordinate system is set at the equilibrium position of the mass M . Assuming that the masses of the springs can be ignored, and neglecting damping in the system, the elastic (restoring) forces of the springs are the only forces acting on mass M . According to Newtons second law of dynamics, the equation of motion of mass M is

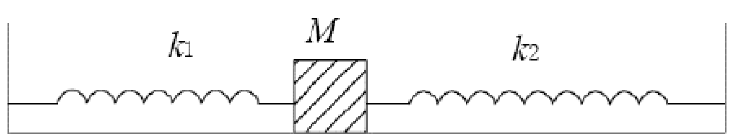


Figure 1: Mass-spring system.

$$M\frac{d^x}{dt^2} + (k_1 + k_2)x = 0 \tag{2}$$

The general solution to Eq. (2) is

$$x(t) = A\cos(\omega_0 t + \varphi_0) \tag{3}$$

where $\omega_0 = \sqrt{(k_1 + k_2)/M}$ is the natural angular frequency of the oscillations, A is their amplitude, and φ_0 is the initial phase. The natural angular frequency is determined by the parameters of the system itself, whereas the initial phase is determined by initial conditions. The natural period of oscillation is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k_1 + k_2}} \tag{4}$$

In this exercise, the relationship between the oscillation period and the mass of the oscillator will be studied.

1.3 Effect of the Mass of the Spring

Whenever the mass of the springs cannot be ignored, we take it into account in terms of the so-called effective mass. The effective mass of the oscillator is the sum of the mass of the object and the effective mass of the spring. When we take the effective mass of the spring into account, the angular frequency of the system can be expressed as

$$\omega_0 = \frac{k_1 + k_2}{M + m + 0} \tag{5}$$

where m_0 is the effective mass of the spring, which is 1/3 of the actual mass of the spring.

1.4 Mechanical Energy in Harmonic Motion

The elastic potential energy for a spring-mass system is $U = kx^2/2$ and the kinetic energy of an oscillating mass m is $K = mv^2/2$.

At the equilibrium position ($x = 0$), the speed of the mass is maximum $v = v_{max}$. At this point the total mechanical energy is equal to maximum kinetic energy K_{max} . On the other hand, at maximum displacement ($x = \pm A$) the mass is instantaneously at rest, i.e. $v = 0$, and the contribution to the total mechanical energy is due to the potential energy only, which is at its maximum U_{max} . In the absence of non-conservative forces (such as frictional forces or drag forces), the total mechanical energy is conserved and $K_{max} = U_{max}$, which implies

$$k = \frac{mv_{max}^2}{A^2} \tag{6}$$

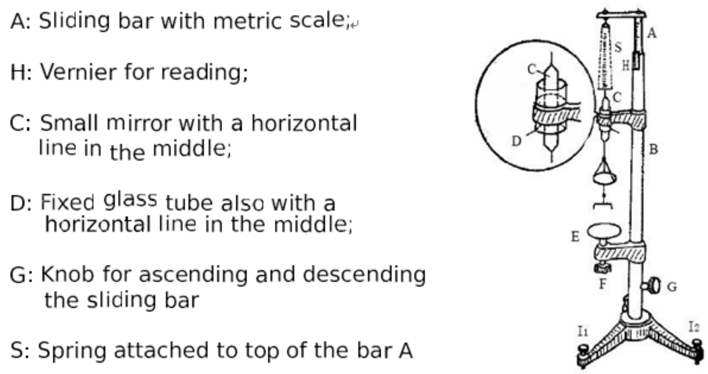


Figure 2: Jolly balance.

2 Experimental setup

The measurement equipment consists of the following elements: springs, Jolly balance, air track, electronic timer, electronic balance, and masses.

In order to measure the spring constant using the Jolly balance, we need to place the small mirror C (see Figure 2) in the tube D and make three lines coincide: the line on on the mirror, the line on the glass tube and its reflection in the mirror. First, without adding any weight on the bottom end of the spring, tune the knob G and make the three lines coincide. Then read the scale L_1 .

Second, add mass m to the bottom of the spring. The spring is stretched and the three lines no longer coincide. Tune knob G to make them into one line again and read the corresponding number on scale L_2 . The spring constant may be then found as

$$k = \frac{mg}{L_2 - L_1} \tag{7}$$

When you have a series of measurements, for different masses m, you may use the least square method to estimate the spring constant by finding a linear fit to the data.

A photoelectric measuring system consists of two photoelectric gates and an electronic timer. When a shutter placed on the object passes a gate, it blocks the light emitted from the light source at the top of the gate, and the receiver sends a signal to the electronic timer. Please note that for period measurements we use, we use the I-shape shutter.

When measuring the speed of the object, we use a U-shaped shutter (Figure 4), so that the light is blocked twice during a pass. The timer will then record the time interval Δt between the two generated signals. After the distance $\Delta x = \frac{1}{2}(x_{in} + x_{out})$ between the two arms of the U-shape shutter is measured, the speed of the object at the point of passing the gate is calculated

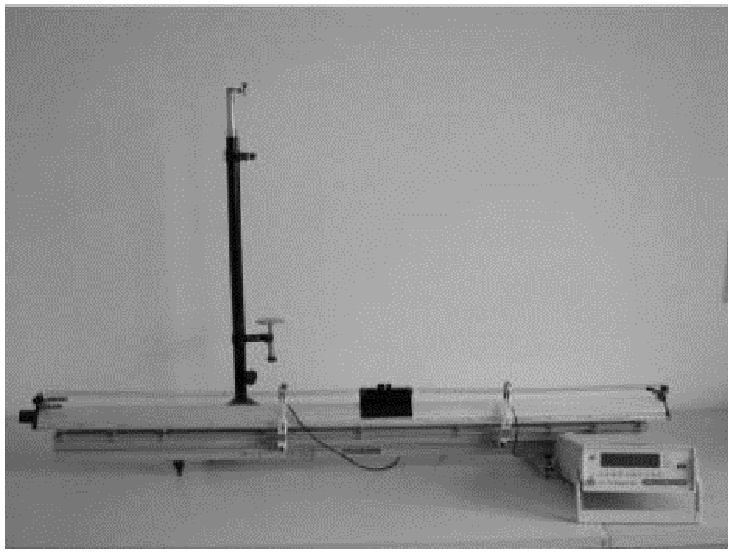


Figure 3: The experimental setup.

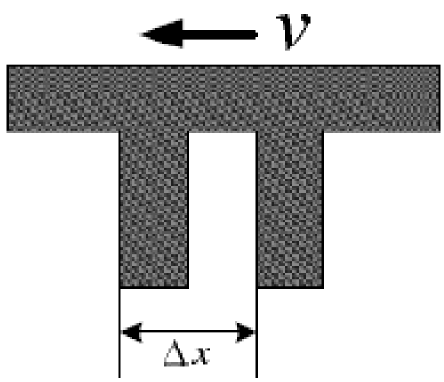


Figure 4: The U-shape shutter.

as $v = \Delta x/\Delta t$.

3 Measurement

3.1 Spring Constant

1. Adjust the Jolly balance to be vertical: Attach the spring and the mirror as shown in Figure 2. Add a 20 g preload and adjust knob I_1 and I_2 to make sure the mirror can move freely through the tube.

Check whether the Jolly balance is parallel to the spring, and adjust knobs if necessary. You should look at the balance from two orthogonal directions: from one direction, adjust the balance to be parallel to the spring; from the direction orthogonal to the previous one, check if the balance coincides with the spring.

2. Adjust knob G and make the three lines in the tube coincide. Adjust the position of the tube to set the initial position L_0 within 5.0 10.0 cm.

3. Record the reading L_0 on the scale, add mass m_1 and record L_1 .

4. Keep adding masses and take measurements for six different positions. The order of the masses should be recorded.

5. Estimate the spring constant k_1 using the least square method.

6. Replace spring 1 with spring 2, repeat the measurements and calculate k_2 .

7. Remove the preload and repeat the measurement for springs 1 and 2 connected in series. Calculate k_3 and compare it with the theoretical value.

3.2 Relation between the Oscillation Period T and the Mass of the Oscillator M

(a) Adjust the air track so that it is horizontal.

Caution: Do not place anything on the air track before you turn on the air pump.

1. Turn on the air pump and check if any of the holes on the air track are blocked. Call the instructor for help if you find blocked holes.
2. Place the object (cart) on the track without any initial velocity. Adjust the track until the object moves slowly back and forth in both directions.
   Adjustment method. The air track has three knobs at the bottom: two on one side and another one on the other side. You can only adjust that single knob.

(b) Horizontal air track

1. Attach springs to the sides of the cart, and set up the I-shape shutter. Make sure that the photoelectric gate is at the equilibrium position.
2. Add weight m_1 . Let the cart oscillate about the photoelectric gate. The amplitude of oscillations should be about 5 cm. Release the cart with a caliper or a ruler. Set the timer into the "T" mode. The timer in this mode will automatically record the time of 10 oscillation periods. Record the mass of the cart and the period.
3. Add weights to the object, repeat Step 2 and take measurements for 5 times.
4. Analyze the relation between M and T by plotting a graph.

(c) Inclined air track

1. Inclination of the air track is controlled with plastic plates. Place them under the air track, using three plates at a time.
2. Repeat the steps in (b) for two different inclinations (i.e., with 3 and 6 plates beneath the air track).
3. Discuss the relation between M and T by plotting a graph.

3.3 Relation Between the Oscillation Period T and the Amplitude A

1. Keep the mass of the cart unchanged and change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

2. Apply linear fit to the data and comment on the relation between the oscillation period T and the amplitude A based on the correlation coefficient γ.

3.4 Relation Between Maximum Speed and the Amplitude

1. Measure the outer distance x out and the inner distance x in of the U-shape shutter by a caliper. Calculate the distance $\Delta x = (x_{out} + x_{in})/2$.

2. Change the shutter from I- to U-shape. Set the timer into the "S_1" mode. Let the cart oscillate. Record the second readings of the time interval Δt only if the two subsequent readings show the same digits to the left of the decimal point.

3. Change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

4. Measure the maximum speed v_{max} for different values of the amplitude A. Obtain the spring constant from Eq. (7). Compare this result to that of the first part.

3.5 Mass measurement

1. Adjust the electronic balance every time before you use it. The level bubble should be in the center of the circle.

2. Add weights according to a fixed order. Weigh the cart with the I-shape shutter and with the U-shape shutter. Measure the mass of spring 1 and spring 2.

3. Record the data only after the circular symbol on the scales display disappears.

4 Results

4.1 Spring Constant

The measurement of length L was shown in Table 1.

Measurement	spring 1 [cm] $\pm$ 0.01[cm]	spring 2 [cm] $\pm$ 0.01[cm]	series [cm] $\pm$ 0.01[cm]
L_0	6.14	6.08	20.82
L_1	8.27	8.14	25.06
L_2	10.35	10.23	29.25
L_3	12.34	12.34	33.28
L_4	14.36	14.45	37.54
L_5	16.49	16.55	41.61
L_6	18.60	18.73	45.89

Table 1: Spring constant measurement data.

According to the mass data in Table 5, use MATLAB to plot the spring constant k with the least square method and the result was shown in Figure 6.

The spring constant of spring 1 is fit to be $k = 2.2581 \pm 6.1769 \times 10^{-4} N/m$

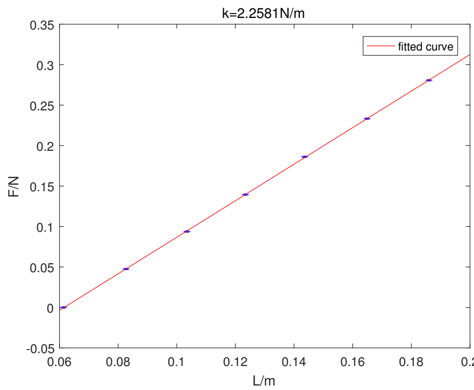
The spring constant of spring 2 is fit to be $k = 2.2138 \pm 7.8746 \times 10^{-4} N/m$

The spring constant of the series is fit to be $k = 1.12 \pm 5.4961 \times 10^{-4} N/m$

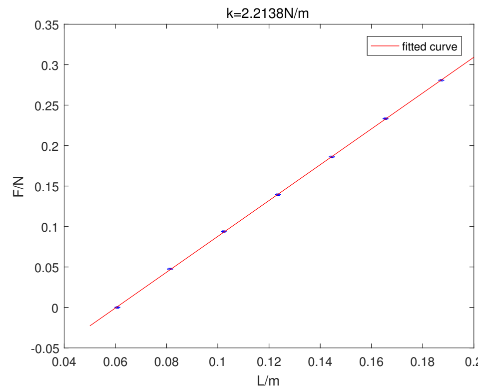
The theoretical value is

$$k = \frac{k_1 k_2}{k_1 + k_2} \approx 1.1178$$

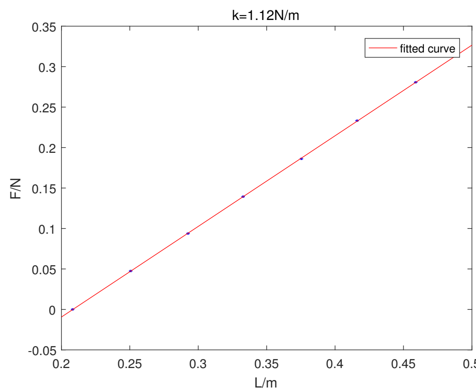
It is similar to the experimental value.



(a) k_1



(b) k_2



(c) series

Figure 5: Fit of spring constant.

4.2 Relation between the Oscillation Period T and the Mass of the Oscillator M

ten period [ms] $\pm$ 0.1 [ms]			
Measurement	horizontal	incline 1	incline 2
m_1	12929.0	12925.7	12922.9
m_2	13084.7	13081.8	13077.7
m_3	13234.8	13230.4	13227.0
m_4	13388.8	13384.1	13377.8
m_5	13536.9	13532.3	13529.7
m_6	13687.0	13684.2	13679.4

Table 2: Measurement data for the T vs. M relation.

The measurement of ten periods $10T$ was shown in Table 2.

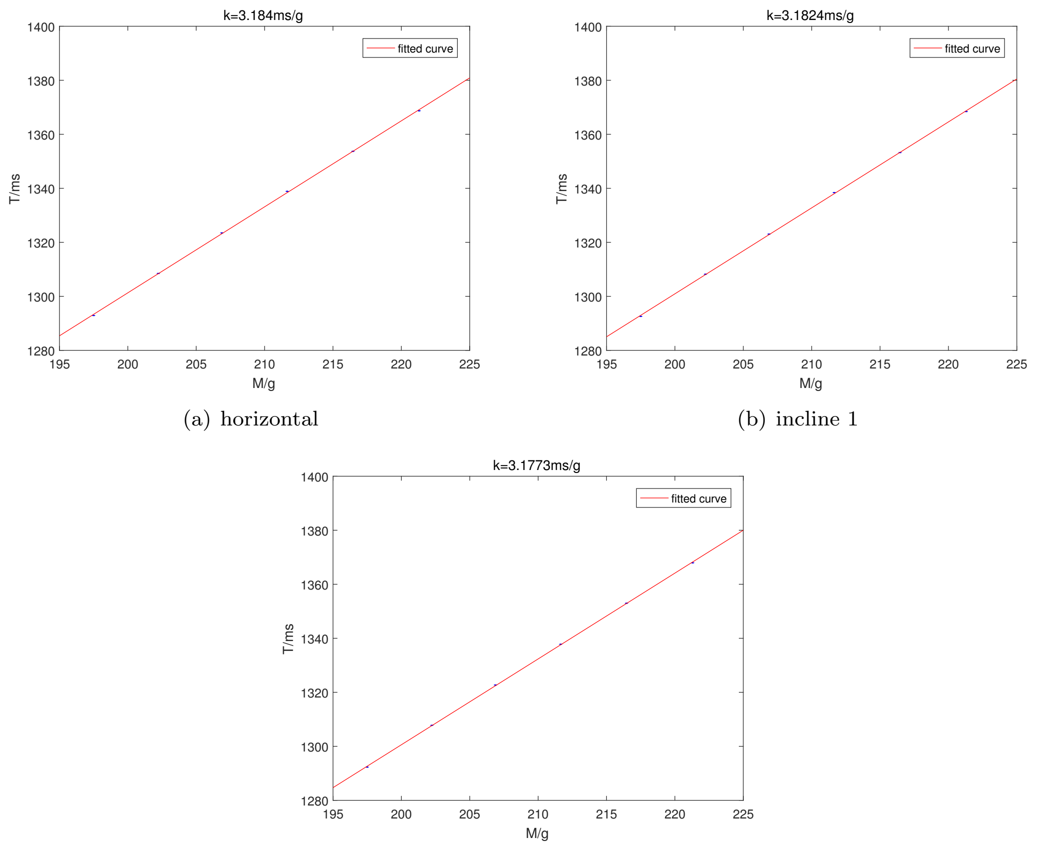
According to the mass data in Table 5, use MATLAB to plot T vs. M with the least square method and the result was shown in Figure 7.

The horizontal is fit to be $k = 3.184 \pm 0.4141 ms/g$

The incline 1 is fit to be $k = 3.1824 \pm 0.3503 ms/g$

The incline 2 is fit to be $k = 3.1773 \pm 0.2921 ms/g$

It can be concluded that T is in proportion to M and Then the incline became larger, the ratio became smaller.



(a) horizontal

(b) incline 1

(c) incline 2

Figure 6: Fit of T vs. M.

4.3 Relation Between the Oscillation Period T and the Amplitude A

The measurement of ten periods $10T$ was shown in Table 3.

Measurement	A [cm] $\pm$ 0.1 [cm]	ten period [ms] $\pm$ 0.1 [ms]
1	5.0	12770.7
2	10.0	12759.3
3	15.0	12754.3
4	20.0	12751.0
5	25.0	12752.4
6	30.0	12782.1

Table 3: Data for the T vs. A relation.

We can find that there is no relationship between T and A according to Figure 7

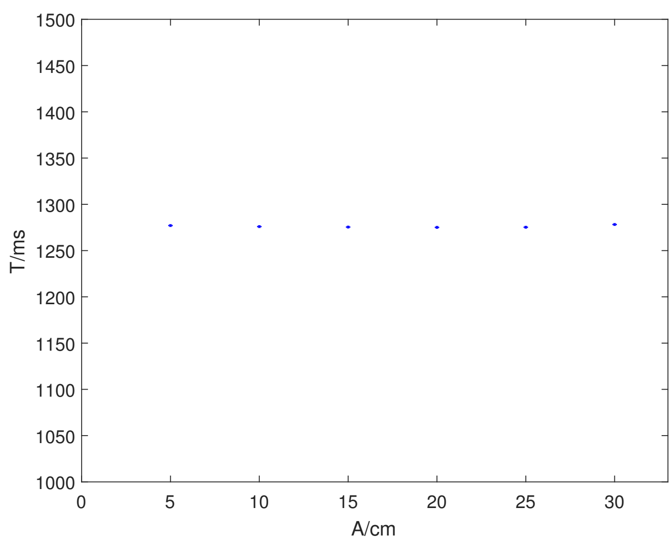


Figure 7: T vs. A relation.

4.4 Relation Between Maximum Speed and the Amplitude

The measurement of Δt was shown in Table 4.

We can first calculate Δx by the formula

$$\Delta x = \frac{1}{2}(x_{out} + x_{in}) = \frac{1}{6}\sum_{i=1}^{3}(x_{out} + x_{in}) = 0.997 \pm 0.002cm$$

Measurement	A [cm] $\pm$ 0.1 [cm]	Δt [ms] $\pm$ 0.01 [ms]
1	5.0	40.19
2	10.0	20.12
3	15.0	13.51
4	20.0	10.18
5	25.0	8.15
6	30.0	6.88

Measurement	x_{in} [cm] $\pm$ 0.002 [cm]	x_{out} [cm] $\pm$ 0.002 [cm]
1	0.450	1.538
2	0.456	1.540
3	0.452	1.546

Table 4: Data for the v_{max}^2 vs. A relation.

Use MATLAB to plot v_{max}^2 vs. A with the least square method and the result was shown in Figure 8.

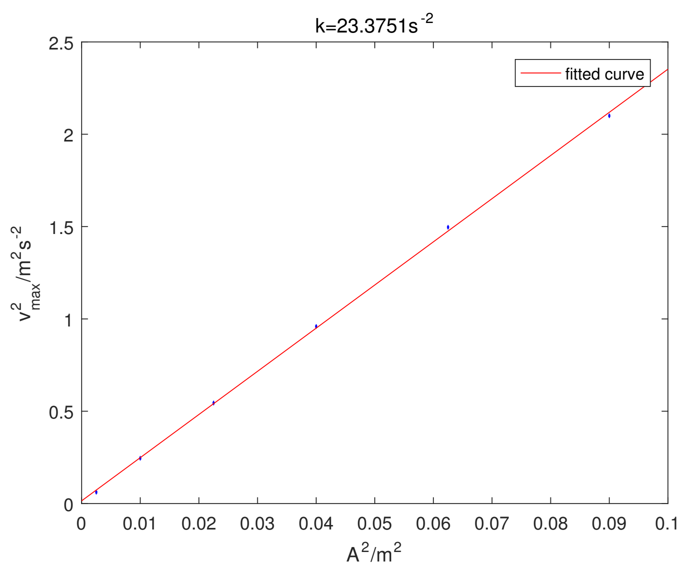


Figure 8: v_{max}^2 vs. A relation.

where

$$k = 23.3751 \pm 0.0160 s^{-2}$$

4.5 Mass measurement

The measurement of mass was shown in Table 5 and Table 6.

Measurement	m [g] $\pm$ 0.01 [g]
1	4.84
2	9.57
3	14.22
4	18.99
5	23.80
6	28.64

Table 5: Weight measurement data.

object with I-shape	m_{obj} [g] $\pm$ 0.01[g]
185.54	
object with U-shape	m_{obj} [g] $\pm$ 0.01[g]
186.14	
mass of spring 1	m_{spr1} [g] $\pm$ 0.01[g]
10.63	
mass of spring 2	m_{spr2} [g] $\pm$ 0.01[g]
10.74	
equivalent mass	$M_0 = m_{obj} + \frac{1}{2}m_{spr1} + \frac{1}{2}m_{spr2}$ [g]
192.66	
193.26	

Table 6: Mass measurement data.

5 Measurement uncertainty analysis

5.1 Uncertainty of Spring Constant

MATLAB gives the uncertainty in the form of RMSE.

$$u_{k1} = 6.1769 \times 10^{-4} N/m$$
$$u_{k2} = 7.8746 \times 10^{-4} N/m$$
$$u_{series} = 5.4961 \times 10^{-4} N/m$$

5.2 Uncertainty of Relation between T and M

MATLAB gives the uncertainty in the form of RMSE.

$$u_{horizontal} = 0.4141 ms/g$$
$$u_{incline_1} = 0.3503 ms/g$$
$$u_{incline_2} = 0.2921 ms/g$$

5.3 Uncertainty of Relation between v_{max}^2 and A^2

MATLAB gives the uncertainty in the form of RMSE.

$$u_k = 0.0160 s^{-2}$$

The measurement of Δx was type-A uncertainty, so

$$u_{\Delta x} = 0.002 cm$$

The uncertainty of v_{max}^2 is found by applying the uncertainty propagation formula

$$u_{v_{max}^2} = \sqrt{\left(\frac{\partial v_{max}^2}{\partial \Delta x}\right)^2 u_{\Delta x}^2 + \left(\frac{\partial v_{max}^2}{\partial \Delta t}\right)^2 u_{\Delta t}^2} = \sqrt{\frac{4\Delta x^2 u_{\Delta x}^2}{\Delta t^4} + \frac{4\Delta x^4 u_{\Delta t}^2}{\Delta t^6}}$$

$$u_1 = 0.0004$$
$$u_2 = 0.0026$$
$$u_3 = 0.0084$$
$$u_4 = 0.0192$$
$$u_5 = 0.0372$$
$$u_6 = 0.0616$$

6 Conclusion

We study simple harmonic oscillation in this experiment.

We learned how to find the spring constant and effective mass of a spring, and how to use the air track. We also analyzed the relationship between the oscillation period and the mass of the oscillator, check whether the oscillation period depends on the the amplitude, and examine the relationship between the maximum speed and the amplitude.

We can simply find the formula $F = kx$ according to the results in part (a), and when two springs are connected the result was found to be

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Then we can also find that the period of the oscillator has to connection with the amplitude, but is proportion to the mass. And when the plane was inclined, the ratio would become smaller

The square of the maximum speed of oscillator is proportion to the amplitude because of the energy rule.

7 Reference

(a) Qin Tian, Zheng Huan, Li Yingyu, Mateusz Krzyzosiak, VP141 Exercise 3, Simple Harmonic Motion: Oscillations in Mechanical Systems, based on materials provided by the Department of Physics, Shanghai Jiaotong University.