Physics Laboratory
Vp141

Exercise 5

Damped and Driven Oscillations. Mechanical Resonance

This lab manual is based on the materials provided by the Department of Physics, Shanghai Jiaotong University. rev 4.1

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1 Pre-lab Reading

Chapter 14 (Young and Freedman); Volume 1, Chapters 21, 23, 24 (Feynman); mini-tutorial on rigid body dynamics (Sakai)

2 Objectives

The objective of this exercise is to study damped and driven oscillations in mechanical systems using the Pohl resonator. For driven oscillations, we will also observe and quantify the mechanical resonance phenomenon.

3 Theoretical Background

If a periodically varying external force is applied to a damped harmonic oscillator, the resulting motion is called forced (or driven) oscillations, and the external force is called the driving force. Assuming that the driving force is of the form

$$F = F_0(\sin \omega t + \delta),$$

with the amplitude F_0 and angular frequency ω, the resulting steady-state forced oscillations will be simple harmonic with the angular frequency equal to that of the driving force. The amplitude of these steady-state oscillations turns out to depend on the angular frequency of the the driving force, in particular on how far it is from the natural angular frequency, and the damping coefficient. The amplitude may become quite large, and this phenomenon is known as the mechanical resonance.

Another interesting property of driven steady-state oscillations is the fact that there is a phase lag between the driving force and the displacement from the equilibrium position of the oscillating particle. This phase lag reaches $\pi/2$ (a quarter of the cycle) when the system is driven at the natural angular frequency.

In this experiment, forced oscillation of a balance wheel will be studied. Damping in this system is provided by air drag and electromagnets inducing eddy currents in the wheel. It is a rotating system, hence the corresponding quantities (such as the force and the position) will be replaced by their angular counterparts.[1]

When the balance wheel is acted upon a periodic driving torque $\tau_{\mathrm{dr}} = \tau_0 \cos \omega t$ and a damping torque $\tau_f = -b\frac{d\theta}{dt}$, in addition to the restoring torque $\tau = -k\theta$, its equation of motion is of the form

$$I\frac{d^2\theta}{dt^2} = -k\theta - b\frac{d\theta}{dt} + \tau_0 \cos \omega t, \tag{1}$$

where I is the moment of inertia of the balance wheel, τ_0 is the amplitude of the driving torque, and ω is angular frequency of the driving torque. Introducing the symbols

$$\omega_0^2 = \frac{k}{I}, \qquad 2\beta = \frac{b}{I}, \qquad \mu = \frac{\tau_0}{I},$$

[1] Please read the mini-tutorial on rigid body dynamics to get familiar with basic concepts of rotational motion about a fixed axis.

Eq. (1) can be rewritten as

$$\frac{d^2\theta}{dt^2} + 2\beta\frac{d\theta}{dt} + \omega_0^2\theta = \mu\cos\omega t, \tag{2}$$

which, in the absence of the driving torque ($\mu = 0$), is the equation of motion for a damped harmonic oscillator. If, additionally, there is no damping in the system ($\beta = 0$), Eq. (2) describes a simple harmonic oscillator with the natural angular frequency ω_0.

The solution to Eq. (2), in the general case of a damped and driven system, is of the form

$$\theta(t) = \theta_{\mathrm{tr}}(t) + \theta_{\mathrm{st}}\cos(\omega t + \varphi) \tag{3}$$

where the former term θ_{tr} denotes the transient solution, that depends on the the initial conditions and vanishes exponentially as $t \to \infty$. The latter term describes steady-state oscillations, with the amplitude

$$\theta_{\mathrm{st}} = \frac{\mu}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}. \tag{4}$$

The phase shift φ gives information about to what extent the displacement from the equilibrium position lags behind the the driving force. It can be found as

$$\tan\varphi = \frac{2\beta\omega}{\omega^2 - \omega_0^2},$$

where $-\pi \leq \varphi < 0$. Note again that, the amplitude and the phase shift are determined by μ, ω, ω_0, and β, and but not the initial conditions.

By finding the maximum of θ_{st}, as a function of ω, we can find the *resonance angular frequency* $\omega = \omega_{\mathrm{res}} = \sqrt{\omega_0^2 - 2\beta^2}$, and the corresponding amplitude

$$\theta_{\mathrm{res}} = \theta_{\mathrm{st}}(\omega_{\mathrm{res}}) = \frac{\mu}{2\beta\sqrt{\omega_0^2 - \beta^2}}.$$

For small values of the damping coefficient β, the resonance angular frequency is close to the the natural angular frequency, and the amplitude of steady-state oscillations becomes large. The dependence of both the amplitude and the phase shift on the driving angular frequency are shown in the left and right Figure 1, respectively, for different values of the damping coefficient. Please note that with increasing damping, (1) the resonance frequency moves away from the natural frequency towards smaller values, (2) the amplitude of the steady-state oscillations decreases.

4 Apparatus and Measurement Procedure

4.1 Apparatus

The BG-2 Pohl resonator consists of two main parts: a vibrometer and a control box. The vibrometer is shown in Figure 2. A copper balance wheel is mounted on a supporting

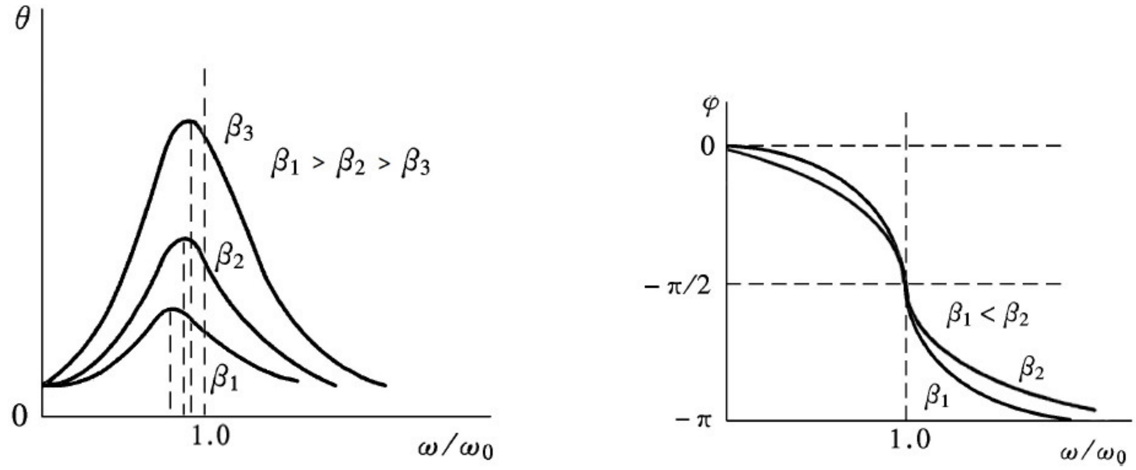


Figure 1. The dependence of the amplitude (left) and phase shift (right) of steady-state driven oscillations.

frame, and the axis of the balance wheel is attached to the supporting frame with a scroll spring. The spring provides an elastic restoring torque to the wheel, which makes the balance wheel rotating about an equilibrium position.

There are many notches on the edge of the balance wheel with one notch being much deeper than the others. A photoelectric detector is set above the deep notch. The detector is used to measure the amplitude and the period of oscillations, and it is connected to the electronic control box.

A pair of coils is placed at the bottom of the supporting frame, with the balance wheel fitting exactly into the gap between the two coils. Due to electromagnetic induction, the wheel will be acted upon an electromagnetic damping force when the coils are carrying current, and the magnitude of the damping force can be controlled by changing the current.

The device is equipped with a motor with an eccentric wheel and a rod used to drive the wheel.

There is a `Period Selection` switch and a `Period of Driving Force` knob on the electric control box, which allow to control the speed of the motor precisely. Another photoelectric detector is set above the turntable and connected to the control box to measure the period of driving force.

The phase shift can be measured using the glass turntable with an angle scale and a strobe light. The strobe is controlled by the photoelectric detector above the wheel. When the deep notch passes the equilibrium position, the detector sends a signal and the strobe flashes. In a steady state, a line on the angle scale will be highlighted by the flash of the strobe and the phase difference can be read from the angle scale directly.

The amplitude of oscillations is measured by counting the notches on the wheel, and this measurement is performed by a photoelectric detector with the result displayed on the electronic control box.

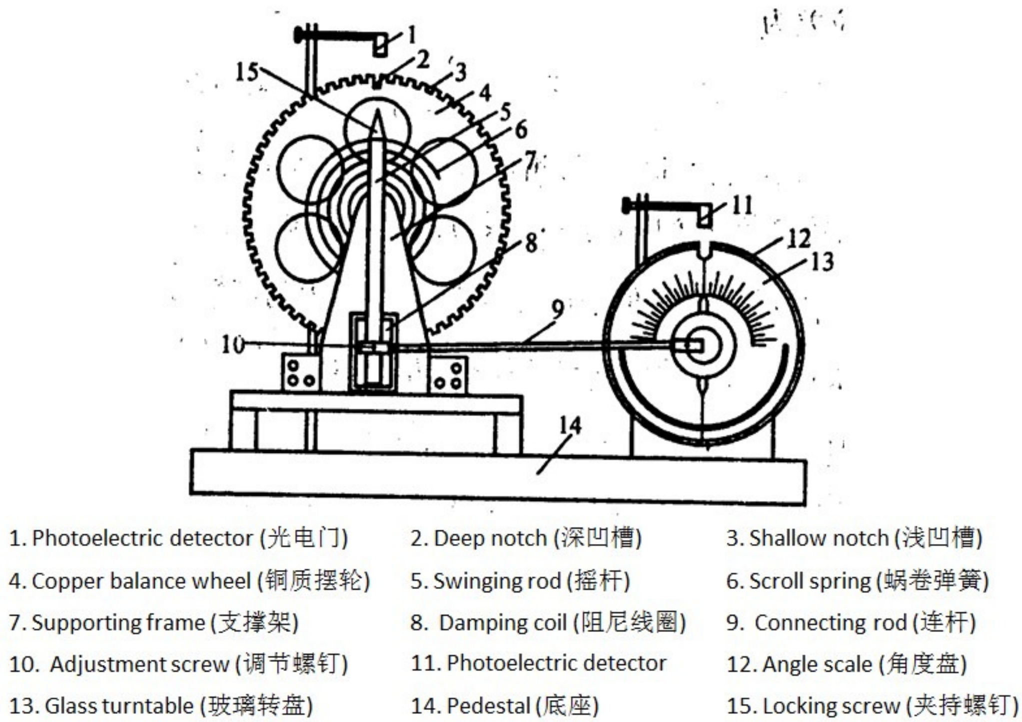


Figure 2. The vibrometer.

The front panel and the rear panel of the control box are shown in Figures 3 and 4, respectively.

The function `Amplitude Display` shows the oscillation amplitude of the balance wheel and `Period Display` shows the oscillation period in two modes. When the `Period Selection` switch is at position "1", a single oscillation period will be displayed; when the `Period Selection` switch is at "10", the time of 10 oscillation periods will be displayed. The reset button works only when the `Period Selection` button is at "10".

The period of the driving force can be changed precisely by using the `Period of the driving force` knob, but please pay attention that the scale on the knob is not very accurate.

The `Damping Selection` knob changes the damping force by adjusting the electric current through the coils at the bottom of the wheel. There are six options, ranging from "0" (no current) to "5" (current of about 0.6 A). You will use "2", "3" or "4" in this exercise.

The strobe generates a flash that allows you to read the phase difference from the angle scale directly. To protect the strobe, you should turn on the `Strobe` switch only when measuring the phase difference.

The `Motor Switch` is used to control the motor. You should turn the motor off when measuring the damping coefficient and the natural angular frequency.

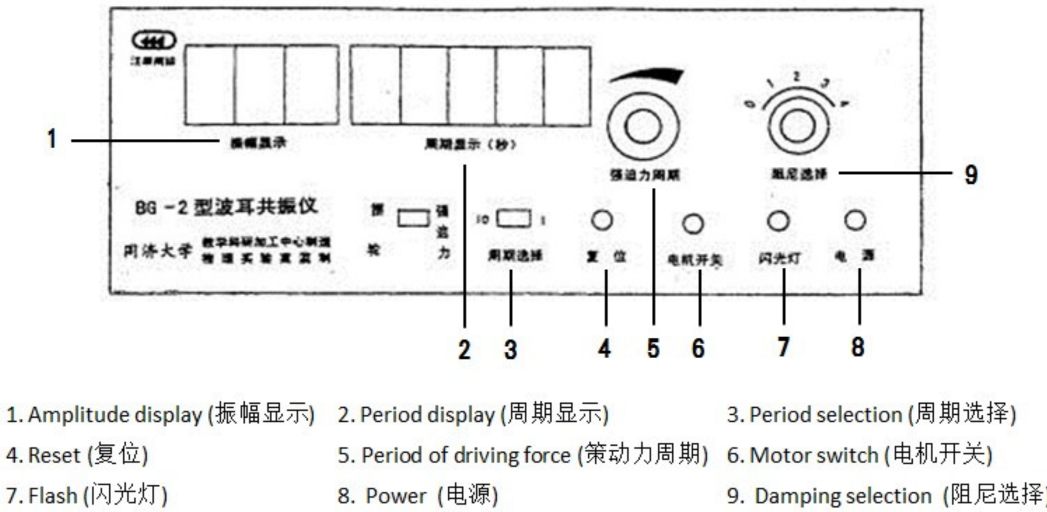


Figure 3. The front panel of the control box.

4.2 Measurement Procedure

4.2.1 Natural Angular Frequency

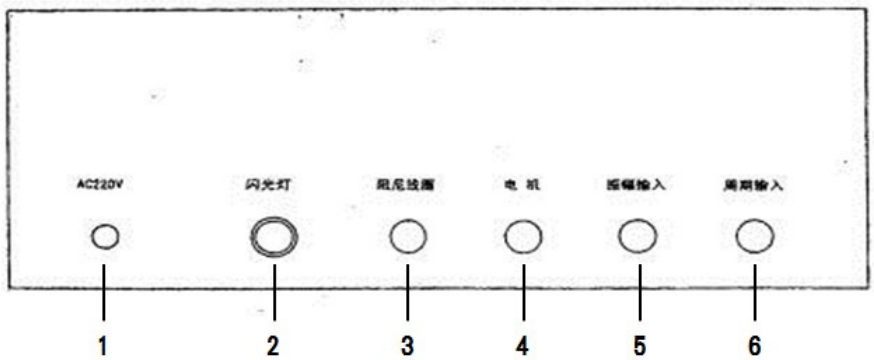
1. Turn the `Damping Selection` knob to "0".
2. Carefully rotate the balance wheel to the initial angular position $\theta_0 \approx 150°$ and release it. Record the time of 10 periods.
3. Repeat for four times and calculate the natural angular frequency ω_0.

4.2.2 Damping Coefficient

1. Turn the `Damping Selection` knob to "2", and the selection should not be changed during this part.
2. Carefully rotate the balance wheel to the initial amplitude of approximately 150° and release it. Record the amplitude of each period (start from the second amplitude after you release the wheel) and the time of 10 periods.

   Tip. In case you miss some readings, repeat the measurement recording it with your phone camera.
3. The solution to the homogeneous equation of motion (2), with the corresponding initial conditions, is $\theta(t) = \theta_0 e^{-\beta t}\cos(\omega_f t + \alpha)$. Hence $\theta_1 = \theta_0 e^{-\beta T}$, $\theta_2 = \theta_0 e^{-\beta(2T)}, \ldots, \theta_n = \theta_0 e^{-\beta(nT)}$. The damping coefficient β can then be calculated as

$$\ln\frac{\theta_i}{\theta_j} = \ln\frac{\theta_0 e^{-\beta(iT)}}{\theta_0 e^{-\beta(jT)}} = (j-i)\beta T.$$



1. 220V AC power (AC220V) 2. Flash (闪光灯) 3. Damping coil (阻尼线圈)
4. Motor (电机) 5. Amplitude input(振幅输入) 6. Period input (周期输入)

Figure 4. The rear panel of the control box.

4. The value of T should be the average period, and $\ln \frac{\theta_i}{\theta_{i+5}}$ should be obtained by the successive difference method as

$$\beta = \frac{1}{5T} \ln \frac{\theta_i}{\theta_{i+5}}.$$

4.2.3 θ_{st} vs. ω and φ vs. ω Characteristics of Forced Oscillations

1. Keep the `Damping Selection` at "2", and set the speed of the motor (record the position of the motor knob in case you need to repeat the measurement). Record the amplitude θ_{st}, the period T, and the phase shift φ when the oscillation reaches a steady state.

2. Repeat the steps above by changing the speed of the motor. It will result in a change of the phase shift φ (referred to as $\Delta\varphi$). To make your plots more accurate, you should collect more data when φ and θ_{st} change rapidly (*e.g.* near to the resonance point). At least 15 data should be collected for plotting.

3. Choose `Damping Selection` "1" or "3". Repeat the above steps.

4. Plot the $\theta_{\text{st}}(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and θ_{st} on the vertical axis. Two sets of data should be plotted on the same graph.
   Plot the $\varphi(\omega)$ characteristics, with ω/ω_0 on the horizontal axis and φ on the vertical axis. Two sets of data should be plotted on the same graph.

5 Caution

- ▶ Check the position of the photoelectric gate above the balanced wheel. Make sure there is enough space between the gate and the wheel.

- Pohl resonator is a very delicate device, you should operate the apparatus according to the manual and instructions of the teaching assistant.
- The motor must be turned off during steps 4.2.1 and 4.2.2.
- To ensure accuracy of the measurement, you should not change the `Damping Selection` until the entire measurement is completed.

6 Preview Questions

- What is the effect of damping on the amplitude and the frequency of non-driven oscillations?
- Sketch the time dependence of the displacement from the equilibrium position for a harmonic oscillator in the (a) underdamped, (b) overdamped regime.
- What is the phase difference between the displacement and a sinusoidal driving force if (a) the system is at resonance, (b) the frequency of the driving force is much larger than the natural frequency of the system.
- Is there any relation between the resonance frequency and the natural frequency? Explain briefly.
- Give an example of an application of mechanical resonance.
- Sketch the curves θ_{st} vs. ω for steady-state driven oscillations with two different damping coefficients $\beta_1 < \beta_2$. Label the curves with β_1 and β_2 accordingly.
- Sketch the curves φ vs. ω for steady-state driven oscillations with two different damping coefficients $\beta_1 < \beta_2$. Label the curves with β_1 and β_2 accordingly.