JOINT INSTITUTE
交大密西根学院

HONORS PHYSICS I (Summer 2016)

Problem Set 7
Due: 7 July 2016, 10 a.m.

Problem 1. Consider the two force fields $\mathbf{F}_1$ and $\mathbf{F}_2$ from Problem 8 of the previous problem set.

(a) Check whether they are conservative (i.e. potential).

(b) For those that are conservative: design your own path connecting the initial and final points (different from those already considered) to check that work done by the field on a particle moving along this path is equal to the value obtained in your previous homework,

(c) and find the corresponding potential energy. Finally, check that by subtracting the value of the potential energy at the initial and final points, you again obtain the same value of the work.

(1 + 2 + 3 marks)

Problem 2. Consider the following force field $\mathbf{F}(\mathbf{r}) = \left(-\frac{Ay}{x^2+y^2}, \frac{Ax}{x^2+y^2}, 0\right)$, where A is a positive constant with appropriate units.

(a) Sketch this field on the xy–plane (use a computer).

(b) Calculate $\operatorname{rot}\mathbf{F}$ to check that the field has zero curl at every point of space, except the z axis.

(c) Show that work done by this force field on a particle that makes one full turn along the unit circle Γ in the xy–plane in the positive (anti-clockwise) direction is equal to $2\pi A$. You may find polar coordinates useful.

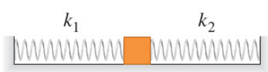
(d) Is there a contradiction between the results of (b) and (c)? Why or why not? Is this field conservative?

(1/2 + 3/2 + 2 + 1 marks)

Problem 3. Given potential energy $U(x, y) = xy^2 + yx^2$, (a) find the corresponding force field and visualize it (use a computer); (b) on the same graph sketch a few equipotential lines, i.e. lines defined by equation $U(x, y) = U_0$, for different values of U_0; (c) comment on the result.

(1 + 1/2 + 1 marks)

Problem 4. A block with mass $m = 3$ kg is connected to two ideal horizontal light springs having force constants $k_1 = 25$ N/cm and $k_2 = 20$ N/cm (see the figure). The system is initially in equilibrium on a horizontal, frictionless surface. The block is now pushed $l = 15$ cm to the right and released from rest. (a) What is the maximum speed of the block? Where in the motion does the maximum speed occur? (b) What is the maximum compression of spring 1?



(3/2 + 3/2 marks)

Problem 5. A hydroelectric dam holds back a lake of surface area 3.0×10^6 m^2 that has vertical sides below the water level. The water level in the lake is 150 m above the base of the dam. When the water passes through turbines at the base of the dam, its mechanical energy is converted to electrical energy with 90% efficiency. (a) If gravitational potential energy is taken to be zero at the base of the dam, how much energy is stored in the top meter of the water in the lake? The density of water is 1000 kg/m^3. (b) What volume of water must pass through the dam to produce 1000 kilowatt-hours of electrical energy? What distance does the level of water in the lake fall when this much water passes through the dam? (c) How much total mechanical energy is stored in the lake? As in part (a), take the gravitational potential energy to be zero at the base of the dam. Express your answer in joules and in kilowatt-hours. (*Hint.* Break the lake up into infinitesimal horizontal layers and integrate.)

(*1 + 3/2 + 2 marks*)

Problem 6. The Lennard–Jones potential energy $U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$, also called the L–J potential or 6–12 potential, is often used to approximate the potential energy associated with interaction between a pair of neutral atoms or molecules separated by distance r.

(a) Find the corresponding force (it is the force exerted by one atom/molecule on the other). Sketch the graphs of both the potential energy and the force as functions of r. Which term in the force is responsible for attraction and which for repulsion?

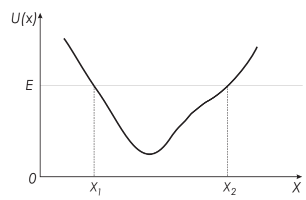
(b) What is the interpretation of the parameters U_0 and R_0 (both are positive)?

(c) Introducing a new variable $x = r - R_0$, find an approximate expression for the force in the regime $|x/R_0| \ll 1$. Interpret your result in terms of oscillations. Find their period.
Hint. The binomial theorem.

(d) What is oscillating here?

(*2 + 1 + 3 + 1 marks*)

Problem 7. A particle with mass m and energy E moves along the x axis in a potential well with $U = U(x)$. Find the formula for the period of particle's oscillations, i.e. the minimum time needed for the particle to return to the initial point having visited both turning points.
Hint. You are *not* allowed to assume that the oscillations are small.



(*5 marks*)

Problem 8. (a) Solve the previous problem for $U(x) = U_0 \tan^2 \alpha x$, where $U_0,\ \alpha$ are positive constants with appropriate units.

(b) What is the period of *small* oscillations about the equilibrium position? Compare it with the result obtained in (a).

(c) Sketch the graph of the force corresponding to this potential well for two sets of parameters U_0 and α. Use a computer, attach the graphs with the values of the parameters listed.

(*2 + 2 + 1 marks*)