Problem 1.

(a)

$$W = -W_G = mg\Delta h = \frac{1}{2}mgl$$

(b)

$$\int_0^l \mu_0 x(l-x)dx = \mu_0(\frac{1}{2}l^3 - \frac{1}{3}l^3) = \frac{1}{6}\mu_0 l^3 = m$$

$$\mu_0 = \frac{6m}{l^3}$$

$$W = -W_G = \int_0^l \mu_0 x(l-x)^2 g dx = \mu_0 g(\frac{1}{2}l^4 - \frac{2}{3}l^4 + \frac{1}{4}l^4) = \frac{1}{2}mgl$$

Problem 2.

$$G = F_{buo0} = \rho g V = \frac{2}{3}\rho g \pi R^3$$

$$F = G - F_{buo} = \Delta F_{buo} = \rho g \Delta V = \rho g \int_0^r \pi(R^2 - r^2)dr = \rho g \pi(R^2 r - \frac{1}{3}r^3)$$

where r is the distance between the liquid and the medial surface of the ball

$$W = \int_0^R F dr = \int_0^R \rho g \pi(R^2 r - \frac{1}{3}r^3)dr = \rho g \pi(\frac{1}{2}R^4 - \frac{1}{12}R^4) = \frac{5}{12}\rho g \pi R^4$$

Problem 3.

(a)

$$W_G = mgx_m$$

$$W_F = \int_0^{x_m} -kx dx = -\frac{1}{2}kx_m^2$$

$$W_F = -W_G \Longrightarrow mgx_m = \frac{1}{2}kx_m^2$$

$$x_m = \frac{2mg}{k} \approx 0.0327m$$

$$|F_m| = kx_m = 98N$$

(b)

$$W_G = mgx_m$$

$$W_F = \int_0^{x_m} -k(x + 160x^3)dx = -\frac{1}{2}kx_m^2 - 40kx_m^4$$

$$W_F = -W_G \Longrightarrow mgx_m = \frac{1}{2}kx_m^2 + 40kx_m^4$$

$$x_m \approx 0.0304m$$

$$|F_m| = k(x + 160x^3) \approx 104.7N$$

Problem 4.

Suppose a very small distance dx on the x-axis, and the angle between the tangent line of the curve at that position to be θ, then

$$
\begin{aligned}
f_a &= \mu(mg\cos\theta + F_{ra}) \\
f_b &= \mu(mg\cos\theta) \\
f_c &= \mu(mg\cos\theta - F_{rc})
\end{aligned}
$$

where F_{ri} is the centripetal force at that point
And we can find the work done by the friction force in dx

$$
\begin{aligned}
dW_a &= -f_a\frac{dx}{\cos\theta} = -\mu mgdx - \frac{\mu F_{ra}}{\cos\theta}dx \\
dW_b &= -f_b\frac{dx}{\cos\theta} = -\mu mgdx \\
dW_c &= -f_c\frac{dx}{\cos\theta} = -\mu mgdx + \frac{\mu F_{rc}}{\cos\theta}dx
\end{aligned}
$$

Thus the work done by the friction force in the whole procedure is

$$
\begin{aligned}
W_a &= -\mu mgx - \int_0^x \frac{\mu F_{ra}}{\cos\theta}dx \\
W_b &= -\mu mgx \\
W_c &= -\mu mgx + \int_0^x \frac{\mu F_{rc}}{\cos\theta}dx
\end{aligned}
$$

which means W_c is smallest in magnitude
So path c gives the maximum speed at B.

Problem 5.

$$|F| = \frac{x}{L_0}\mu mg$$

$$W_F = -\int_0^{L_0/2} |F|dx = -\frac{L_0^2}{4}\frac{\mu mg}{2L_0} = -\frac{1}{8}\mu mgL_0$$

$$\Delta E_k = -W_F \Longrightarrow \frac{1}{2}mv^2 = \frac{1}{8}\mu mgL_0$$

$$v = \frac{1}{2}\sqrt{\mu gL_0}$$

Problem 6.

$$v' = \frac{vl}{L}$$

$$m' = \frac{Mdl}{l}$$

$$dE_k = \frac{1}{2}m'v'^2 = \frac{Mv^2l^2}{2L^3}dl$$

$$E_k = \int_0^L \frac{Mv^2l^2}{2L^3}dl = \frac{1}{3}L^3\frac{Mv^2l^2}{2L^3} = \frac{1}{6}Mv^2$$

Problem 7.

(a)

$$E_p = \frac{1}{2}kX^2$$

$$E_k = E_p = \frac{1}{2}mv^2 = \frac{1}{2}kX^2$$

$$v = X\sqrt{\frac{k}{m}}$$

(b)

$$E_k = E_p = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2 = \frac{1}{2}kX^2$$

$$v = X\sqrt{\frac{3k}{3m+M}}$$

(c)

$$v_a = X\sqrt{\frac{k}{m}} \approx 6.143m/s$$

$$v_b = X\sqrt{\frac{3k}{3m+M}} \approx 3.863m/s$$

Problem 8.

(A) (a)

$$y = z = 0$$

$$W = \int_{-1}^{1} F_x dx = \int_{1}^{1} x^2 z dx = 0$$

(b)

$$y = \sqrt{1-x^2} \quad z = 0$$

$$W = \int_a^b (F_x dx + F_y dy) = \int_a^b -xydy = \int_a^b -x\sqrt{1-x^2}d\sqrt{1-x^2} = \int_{-1}^{1} -x^2 dx = \frac{2}{3}$$

(c)

$$W = \int_a^b (F_x dx + F_z dz) = \int_a^b [t^2(t^2-1)dt + 5d(t^2-1)] = \int_{-1}^{1} t^4 - t^2 + 10tdt = -\frac{4}{15}$$

(B) (a)

$$y = z = 0$$

$$W = \int_{-1}^{1} F_x dx = \int_{1}^{1} -2xdx = 0$$

(b)

$$y = \sqrt{1-x^2} \quad z = 0$$

$$W = \int_{-1}^{1} F_x dx = \int_{1}^{1} -2xdx = 0$$

(c)

$$W = \int_a^b (F_x dx + F_z dz) = \int_{-1}^1 -2t dt = 0$$

Problem 9.

(a)

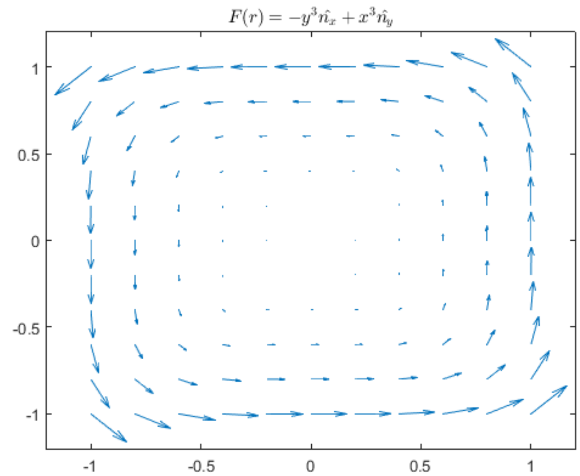


Figure 1: $F(r) = -y^3\hat{n_x} + x^3\hat{n_y}$

(b)

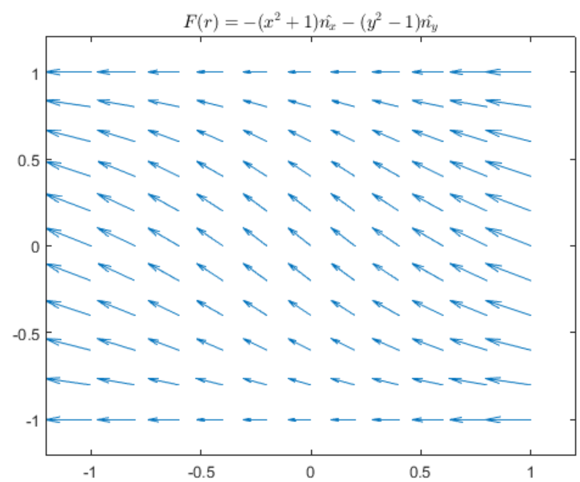
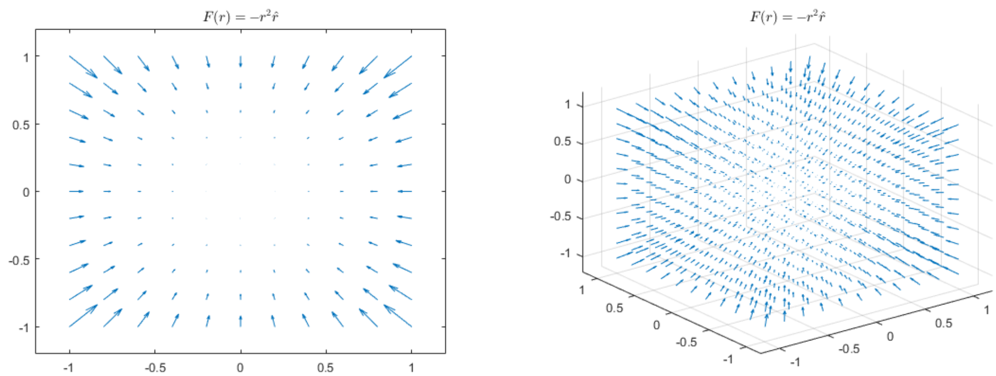


Figure 2: $F(r) = -(x^2+1)\hat{n_x} - (y^2-1)\hat{n_y}$

(c)



(a) 2D (b) 3D

Figure 3: $F(r) = -r^2\hat{r}$

(d)

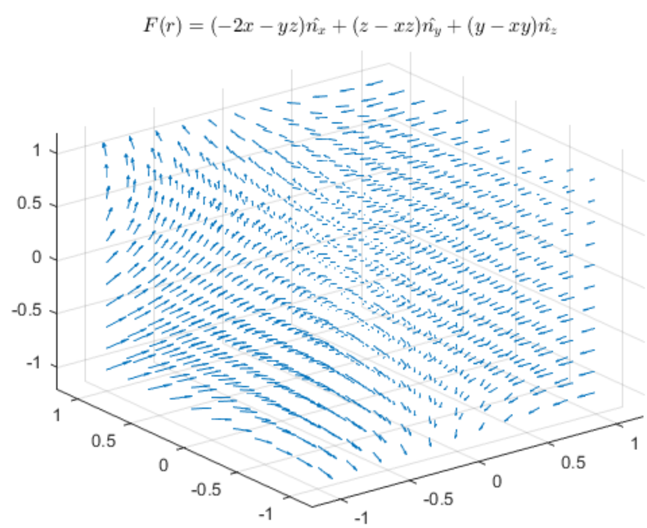


Figure 4: $F(r) = (-2x - yz)\hat{n_x} + (z - xz)\hat{n_y} + (y - xy)\hat{n_z}$