Problem 1.

(a)

$$rot\bar{F}_1 = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{n_x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{n_y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{n_z} = x^2\hat{n_y} - y\hat{n_z} \neq 0$$

So F_1 is not conservative.

$$rot\bar{F}_2 = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{n_x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{n_y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{n_z} = 0$$

So F_2 is conservative.

(b) Choose the path from $(-1,0,0)$ through $(-1,1,0)$, $(1,1,0)$ to $(1,0,0)$.

$$W = 0 + \int_{-1}^{1} -2x dx + 0 = 0$$

It is equal to the value obtained in my previous homework.

(c) Suppose the corresponding potential energy at point $(0,0,0)$ to be zero.

$$U_1 = -\int_{0}^{-1} -2x dx = 1$$

$$U_2 = -\int_{0}^{1} -2x dx = 1$$

$$U_1 = U_2$$

The corresponding potential energy at two points are the same.

Problem 2.

(a)

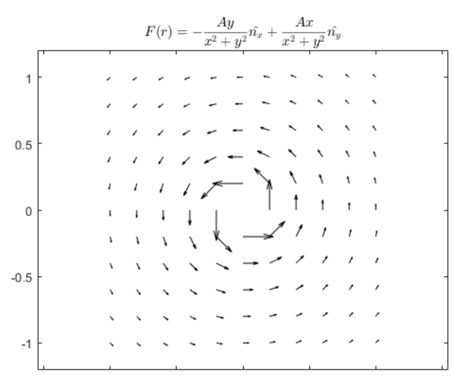


Figure 1: $F(r) = -\frac{Ay}{x^2+y^2}\hat{n_x} + \frac{Ax}{x^2+y^2}\hat{n_y}$

(b)

$$
\begin{aligned}
rot\bar{F} &= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{n_x} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{n_y} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{n_z} \\
&= \left[\frac{A(x^2+y^2) - 2Ax^2)}{(x^2+y^2)^2} + \frac{A(x^2+y^2) - 2Ay^2}{(x^2+y^2)^2}\right]\hat{n_z} \\
&= 0
\end{aligned}
$$

when $x^2 + y^2 \neq 0$ So the field has zero curl at every point of space except the z axis.

(c) Let $x = \cos\theta$, $y = \sin\theta$

$$F = -A\sin\theta\hat{n_x} + A\cos\theta\hat{n_y} = A\hat{n_\varphi}$$

where $\varphi = \theta + \frac{\pi}{2}$

$$W = \int_0^{2\pi} A\hat{n_\varphi} d\varphi \hat{n_\varphi} = 2\pi A$$

(d) No, although the result of (b) suggests that the force field is conservative while the result of (c) shows that the field did work when a particle moved a circle in it, they do not contradicts with each other. It is because the field is not continuous on the z axis, so that the curl can no longer judge whether the field is conservative. From the result of (c) we can know that the field is not conservative.

Problem 3.

Suppose the corresponding potential energy at point $(0, 0)$ to be zero.

(a)

$$F(r) = -\frac{\partial U}{\partial x}\hat{n_x} - \frac{\partial U}{\partial y}\hat{n_y} = -(y^2 + 2xy)\hat{n_x} - (x^2 + 2xy)\hat{n_y}$$

(b)

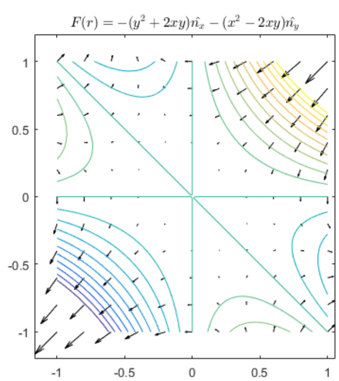


Figure 2: $F(r) = -(y^2 + 2xy)\hat{n_x} - (x^2 - 2xy)\hat{n_y}$

(c) The figure is symmetrical to the line $y = x$
The nearer to the origin, the sparser the equipotential lines.

Problem 4.

Suppose right to be the positive direction.

(a) Suppose the maximum speed occurs when the block is x cm right of the origin equilibrium point.

$$\Delta E_p = \frac{1}{2}k_2(x_0^2 - x^2) + \frac{1}{2}k_1(x_0^2 - x^2)$$

When $x = 0$,

$$\Delta E_{pmax} = \frac{1}{2}(k_2 + k_1)x_0^2 = 50.625J$$

$$\Delta E_{pmax} + \Delta E_{kmax} = 0 \Longrightarrow \frac{1}{2}mv_{max}^2 = 50.625$$

$$v_{max} = \frac{3\sqrt{15}}{2}m/s$$

The max speed occurs at the origin equilibrium point.

(b)

$$E_{p0} = \frac{1}{2}(k_2 - k_1)x_0^2$$

$$E_p = \frac{1}{2}(k_2 - k_1)x^2$$

$$E_{p0} = E_p \Longrightarrow x = -15cm$$

$$F_m = k_1|x| = 375N$$

Problem 5.

(a)

$$E_p = mg\Delta h = \rho Shg\Delta h = 1000 \cdot 3.0 \times 10^6 \cdot 1 \cdot 9.8 \cdot 149.5 = 4.3953 \times 10^{12}J$$

(b)

$$W = 0.9\Delta E_p = 0.9mg\Delta h = 0.9\rho Shg(H - \frac{h}{2}) = 3.6 \times 10^9 J$$

$$-\frac{1}{2}h^2 + 150h - \frac{20}{147} = 0$$

$$h = 9.070^{-4}m$$

$$V = Sh = 2721.1m^2$$

(c)

$$E_p = mg\Delta h\rho Shg\Delta h = 1000 \cdot 3.0 \times 10^6 \cdot 150 \cdot 9.8 \cdot 7.5 = 3.3075 \times 10^{14}J = 9.1875 \times 10^7kWh$$

Problem 6.

(a)

$$F(r) = -\frac{dU}{dr} = U_0 \left[\frac{12}{r} \left(\frac{R_0}{r} \right)^{12} - \frac{12}{r} \left(\frac{R_0}{r} \right)^{6} \right]$$

The first term is responsible for repulsion and the second term is responsible for attraction.

(b) U_0 interprets the minimum potential energy associated with interaction between a pair of neutral atoms or molecules.

R_0 interprets the distance r between a pair of neutral atoms or molecules at the minimum potential energy associated with interaction between them.

(c)

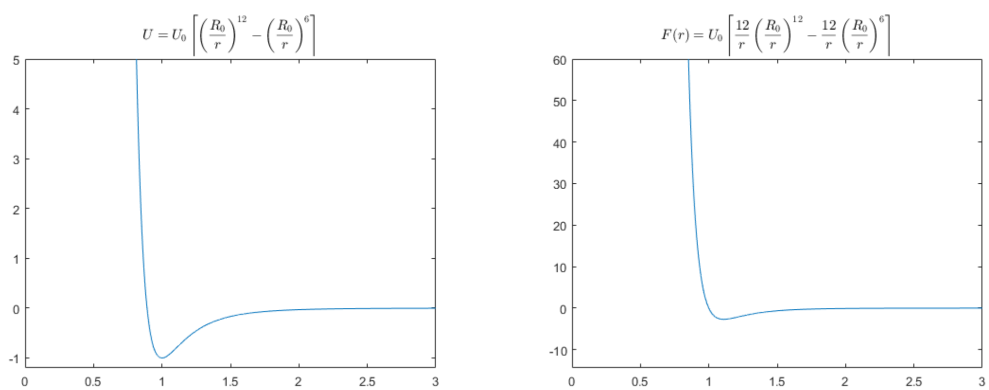
$$F(x) \approx -U''(x_0)(x - x_0) = -U_0 \left[\frac{156}{R_0^2} \left(\frac{R_0}{R_0} \right)^{12} - \frac{84}{R_0^2} \left(\frac{R_0}{R_0} \right)^{6} \right] (x - R_0) = -\frac{72U_0}{R_0^2}(x - R_0)$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{72U_0}{mR_0^2}}$$

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{mR_0^2}{72U_0}}$$

$$x(t) = R_0 + A\cos(\sqrt{\frac{72U_0}{mR_0^2}}t + \varphi)$$

(d) The oscillation near the point $x = R_0$ is a harmonic oscillation.



(a) $U(r)$ (b) $F(r)$

Figure 3: graphs of both the potential energy and the force

Problem 7.

$$\Delta E_k + \Delta E_p = 0 \Longrightarrow \frac{1}{2}mv^2 = E - U(x)$$

$$v = \sqrt{\frac{2[E-U(x)]}{m}} = \frac{dx}{dt}$$

$$dt = \sqrt{\frac{m}{2[E-U(x)]}}dx$$

Do integrals on both sides,

$$t = \int_{x_1}^{x_2} \sqrt{\frac{m}{2[E-U(x)]}}dx$$

$$T = 2t = 2\int_{x_1}^{x_2} \sqrt{\frac{m}{2[E-U(x)]}}dx$$

Problem 8.

(a)

$$\begin{aligned}
T &= 2\int_{x_1}^{x_2} \sqrt{\frac{m}{2(E-U_0\tan^2\alpha x)}}dx \\
&= \sqrt{2m}\int_{x_1}^{x_2} \cos\alpha x\sqrt{\frac{1}{E\cos^2\alpha x - U_0\sin^2\alpha x}}dx \\
&= \sqrt{2m}\frac{1}{\alpha}\int_{\sin\alpha x_1}^{\sin\alpha x_2} \sqrt{\frac{1}{E-(E+U_0)u^2}}du \quad (u=\sin\alpha x) \\
&= \frac{1}{\alpha}\sqrt{\frac{2m}{E+U_0}}\int_{\sin\alpha x_1}^{\sin\alpha x_2} \sqrt{\frac{1}{\frac{E}{E+U_0}-u^2}}du \\
&= \frac{1}{\alpha}\sqrt{\frac{2m}{E+U_0}}\left[\arcsin\frac{u}{\sqrt{\frac{E}{E+U_0}}}\right]_{\sin\alpha x_1}^{\sin\alpha x_2} \\
&= \frac{1}{\alpha}\sqrt{\frac{2m}{E+U_0}}\left(\arcsin\sqrt{\frac{E+U_0}{E}}\sin\alpha x_2 - \arcsin\sqrt{\frac{E+U_0}{E}}\sin\alpha x_1\right)
\end{aligned}$$

(b)

$$F(x) = -U'(x) = -2U_0\tan\alpha x\frac{1}{\cos^2\alpha x}\alpha = -2U_0\alpha\frac{\sin\alpha x}{\cos^3\alpha x}$$

When $F(x)=0$, $x_0=0$

$$U''(x_0) = 2U_0\alpha\frac{\alpha\cos^4\alpha x_0 + \sin\alpha x_0(3\alpha\cos^2\alpha x_0\sin\alpha x_0)}{\cos^6\alpha x_0} = 2U_0\alpha^2\frac{\cos^2\alpha x_0 + 3\sin^2\alpha x_0}{\cos^4\alpha x_0} = 2U_0\alpha^2$$

$$F(x) \approx -U''(x_0)(x-x_0) = -2U_0\alpha^2 x$$

$$T' = 2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{\alpha}\sqrt{\frac{2m}{U_0}}$$

When $E \to U_0 \tan^2 \alpha x$,

$$\arcsin \sqrt{\frac{E+U_0}{E}} \sin \alpha x \to \arcsin \sqrt{\frac{1+tan^2\alpha x}{tan^2\alpha x}} \sin \alpha x = \arcsin \frac{\sin \alpha x}{|\sin \alpha x|}$$

When $x_2 \to 0^+$,

$$\arcsin \frac{\sin \alpha x}{|\sin \alpha x|} \to \frac{\pi}{2}$$

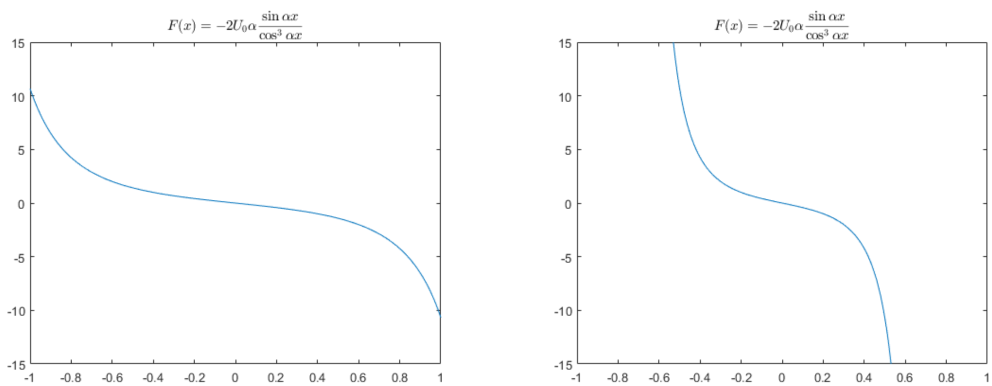
When $x_1 \to 0^-$,

$$\arcsin \frac{\sin \alpha x}{|\sin \alpha x|} \to -\frac{\pi}{2}$$

$$T = \frac{1}{\alpha}\sqrt{\frac{2m}{E+U_0}}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) \to \frac{\pi}{\alpha}\sqrt{\frac{2m}{U_0(1+\tan^2\alpha x)}} = \frac{\pi}{\alpha}\sqrt{\frac{2m}{U_0}}$$

So we can concluded that in small oscillations, the approx of T is accurate.

(c)



(a) $U_0 = 1, \alpha = 1$

(b) $U_0 = 0.5, \alpha = 0.5$

Figure 4: $F(x) = -2U_0\alpha\frac{\sin \alpha x}{\cos^3 \alpha x}$