Problem Set 6

Due: 30 June 2016, 10 a.m.

Problem 1. A chain with length l and mass m rests on a flat surface. Find minimum work needed to lift it, by holding one of its ends, in the case when

(a) the chain is uniform,

(b) the (linear) mass density of the chain is $\lambda(x) = \lambda_0 x(l - x)$ (what is the value of λ_0?), where x is the distance, measured along the chain, from the end that is being lifted.

(*1 + 2 marks*)

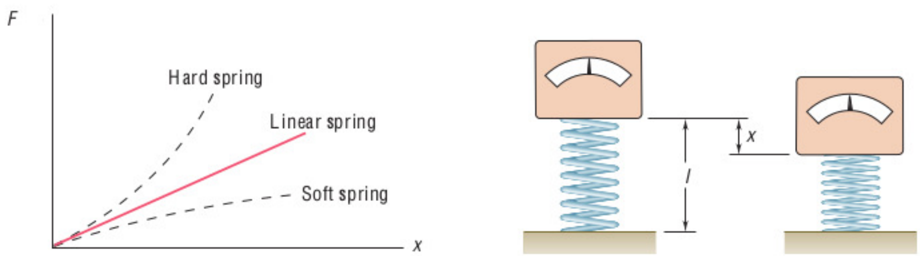
Problem 2. A uniform ball with radius R is floating half-submerged in a liquid of density ρ filling a large container. What is minimum work needed to pull the ball above the liquid?

Neglect viscosity of the liquid.

(*5 marks*)

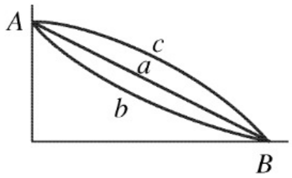
Problem 3. Nonlinear springs are classified as hard or soft, depending upon the curvature of their force-deflection curve (see figure). If a delicate instrument having a mass of $m = 5$ kg is placed on a spring of length l so that its base is just touching the undeformed spring and then inadvertently released from that position, determine the maximum deflection x_m of the spring and the maximum force F_m exerted by the spring, assuming (a) a linear spring of constant $k = 3$ kN/m, (b) a hard, nonlinear spring, for which $F = (3\ [\text{kN/m}])(x + 160x^3)$.

(*1 + 2 marks*)

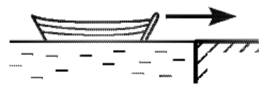


Problem 4. A sledge can slide down a hill along three paths AaB, AbB, and AB (see the figure below). Which path gives the maximum speed at B? Assume that in all three cases the magnitude of the frictional force is proportional to the magnitude of the normal force exerted by the sledge on the hill with the same coefficient of kinetic friction.

(*3 marks*)



Problem 5. A boat of length L_0, with its engine turned off, reaches the beach and stops with half of its length on the sand. The kinetic coefficient of friction between the boat and the sand is μ. What was the initial speed of the boat?



(*3 marks*)

Problem 6. We usually ignore the kinetic energy of the moving coils of a spring, but lets try to get a reasonable approximation to this. Consider a spring of mass M, equilibrium length L_0, and spring constant k. The work done to stretch or compress the spring to the length L is $kX^2/2$, where $X = L - L_0$. Consider a spring, as described above, that has one end fixed and the other end moving with speed v. Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring.

Calculate the kinetic energy of the spring in terms of M and v.

Hint. Divide the spring into infinitesimal pieces of length dl; find the speed of each piece in terms of l, v, and L; find the mass of each piece in terms of dl, M, and L; and integrate from 0 to L.

(*6 marks*)

Problem 7. In a spring gun, a spring of mass M and force constant k is compressed by distance X from its unstretched length. When the trigger is pulled, the spring pushes horizontally on a ball with mass m. The work done by friction is negligible. Calculate the ball's speed when the spring reaches its uncompressed length

(a) ignoring the mass of the spring,
(b) including the mass of the spring (use the results derived in the previous problem). What is the final kinetic energy of the ball and of the spring?
(c) Find the numerical values of your answers for $M = 0.243$ kg, $k = 3200$ N/m, $X = 2.50$ cm, $m = 0.053$ kg.

(*1 + 3 + 1 marks*)

Problem 8. Calculate work done by the force fields

(A) $\mathbf{F}_1(\mathbf{r}) = (x^2 z, -xy, 5)$, (B) $\mathbf{F}_2(\mathbf{r}) = (-2x - yz, z - xz, y - xy)$,

acting upon a particle that moves from $(-1, 0, 0)$ to $(1, 0, 0)$ along

(a) the x axis,
(b) an arc of the circle $x^2 + y^2 = 1$, so that $y \geq 0$,
(c) the curve Γ defined by parametric equations $x(t) = t$, $y(t) \equiv 0$, $z(t) = t^2 - 1$, where $-1 \leq t \leq 1$.

$2 \times$ (*1 + 3/2 + 3/2 marks*)

Problem 9. Use a computer to visualize the following force fields, attach the graphs to your homework (e.g. in *Wolfram's Mathematica* the relevant commands are `VectorPlot` and `VectorPlot3D`).

(a) $\mathbf{F}(\mathbf{r}) = -y^3 \hat{n}_x + x^3 \hat{n}_y$ in 2D,
(b) $\mathbf{F}(\mathbf{r}) = -(x^2 + 1)\hat{n}_x - (y^2 - 1)\hat{n}_y$ in 2D,
(c) $\mathbf{F}(\mathbf{r}) = -r^2\, \mathbf{r}$ in 2D and 3D,
(d) one of the force fields from the previous problem.

(*1/2 + 1/2 + 1 + 1 marks*)