Problem 1.

(a)

$$v(t) = s'(t) = cs_0e^{ct}$$

(b)

$$|\vec{a_t}| = |a|\cos\varphi_0 = v'(t) = c^2s_0e^{ct}$$

(c)

$$|\vec{a_n}| = |a|\sin\varphi_0 = c^2s_0e^{ct}\tan\varphi_0$$

(d)

$$R = \frac{v^2}{|\vec{a_n}|} = \frac{c^2s_0^2e^{2ct}}{c^2s_0e^{ct}\tan\varphi_0} = s_0e^{ct}\cot\varphi_0$$

Problem 2.

Let θ be the angle between one of the wires below and the horizontal plane, then $\theta = \arccos\frac{4\sqrt{21}}{19}$

(a) The tension in the two wires below:

$$T_1 = T_2 = \frac{mg}{\cos\theta} = \frac{15\times 9.8}{\frac{4\sqrt{21}}{19}} = 152.4N$$

The tension in the wire above:

$$T_3 = 2mg = 2\times 15\times 9.8 = 294N$$

(b)

$$N = mg\tan\theta = 15\times 9.8\times\frac{5}{4\sqrt{21}} \approx 40.1N$$

Problem 3.

Let the direction to the sky be the positive direction

(a)

$$\sum F = G_{man} + G_{bar} + 2T = -320 - 160 + 500 = 20N$$

$$a = \frac{\sum F}{m} = \frac{20}{(320+160)/9.8} = \frac{49}{120}m/s^2$$

(b)

$$\sum F_{man} = am_{man} = \frac{49}{120}\times\frac{320}{9.8} = \frac{40}{3}N$$

$$\sum F_{man} = G_{man} + T + N \Longrightarrow N = F_n - G_{man} - T = \frac{40}{3} + 320 - 250 = \frac{250}{3}N$$

Problem 4.

Suppose an inertial frame of reference whose x-axis is the perpendicular to the slide and y-axis is opposite to the motion direction.

$$g' = a = g\sin\alpha - \mu g\cos\theta$$

Obviously, in this frame of reference, the surface of the liquid in the container should be perpendicular to the direction of the acceleration.
On the direction of x-axis,

$$a_x = g' - g\sin\theta = -\mu g\cos\theta$$

On the direction of y-axis,

$$a_y = g\cos\theta$$

So the angle between a and y-axis equals $\arctan\mu$, which is the angle that the surface of the liquid in the container forms with the inclined plane.

Problem 5.

$$\begin{cases} m_3a_3 = T_3 - m_3g & (1) \\ 0 = T_3 - T_1 - T_2 & (2) \\ m_2a_2' = T_2 - m_2g + m_2a_3 & (3) \\ m_1a_1' = T_1 - m_1g + m_1a_3 & (4) \end{cases} \quad \& \quad \begin{cases} T_1 = T_2 \\ a_1' = -a_2' \\ a_b = -a_3 \end{cases}$$

(a)

$$(m_1+m_2)a_2' = (m_1-m_2)(g-a_3) \quad | \quad (3)-(4)$$

$$a_2' = \frac{(m_1-m_2)(g-a_3)}{m_1+m_2}$$

$$a_2' - a_3 = \frac{(m_1-m_2)(g-a_3)-(m_1+m_2)a_3}{m_1+m_2} = \frac{(m_1-m_2)g-2m_1a_3}{m_1+m_2}$$

$$m_3a_3 = 2T_2 - m_3g \quad | \quad (1)-(2)$$

$$m_3(a_3+g) = 2m_2(g+a_2'-a_3)$$

$$m_3(a_3+g) = \frac{2m_2[(m_1+m_2)g+(m_1-m_2)g-2m_1a_3]}{m_1+m_2} = \frac{4m_1m_2(g-a_3)}{m_1+m_2}$$

$$\left(\frac{4m_1m_2}{m_1+m_2}+m_3\right)a_3 = \left(\frac{4m_1m_2}{m_1+m_2}-m_3\right)g$$

$$a_3 = \frac{4m_1m_2-(m_1+m_2)m_3}{4m_1m_2+(m_1+m_2)m_3}g$$

(b)

$$a_b = -a_3 = -\frac{4m_1m_2 - (m_1+m_2)m_3}{4m_1m_2 + (m_1+m_2)m_3}g$$

(c)

$$a_2' = \frac{(m_1-m_2)(g-a_3)}{m_1+m_2} = \frac{2(m_1-m_2)m_3g}{4m_1m_2+(m_1+m_2)m_3}$$

$$a_1 = a_1' - a_3 = -a_2' - a_3 = -\frac{2(m_1-m_2)m_3g}{4m_1m_2+(m_1+m_2)m_3} - a_3 = \frac{-4m_1m_2 + (3m_2 - m_1)m_3}{4m_1m_2+(m_1+m_2)m_3}g$$

(d)

$$a_2 = a_2' - a_3 = \frac{2(m_1-m_2)m_3g}{4m_1m_2+(m_1+m_2)m_3} - a_3 = \frac{-4m_1m_2 + (3m_1 - m_2)m_3}{4m_1m_2+(m_1+m_2)m_3}g$$

(e)

$$T_a = \frac{1}{2}T_c = \frac{1}{2}m_3(a_3+g) = \frac{4m_1m_2m_3}{4m_1m_2+(m_1+m_2)m_3}g$$

(f)

$$T_c = \frac{8m_1m_2m_3}{4m_1m_2+(m_1+m_2)m_3}g$$

(g)

$$a_1' = a_2' = a_3 = 0,\ T_a = m_1g,\ T_c = 2m_1g$$

Obviously, this is sensible.

Problem 6.

(a)

$$\begin{cases} m_1a_1 &= T_1 - G_x - f_1 &= T_1 - m_1g\sin\alpha - \mu_1m_1g\cos\alpha & (1) \\ m_2a_2 &= T_2 - G_x - f_2 - T_1 &= T_2 - T_1 - m_2g\sin\alpha - \mu_2m_2g\cos\alpha & (2) \\ m_3a_3 &= G_x - T_3 &= m_3g\sin 2\alpha - T_2 & (3) \end{cases}$$

$$(a_1 = a_2 = a_3)$$

$$(m_1+m_2)a_3 = T_2 - (m_1+m_2)g\sin\alpha - (\mu_1m_1+\mu_2m_2)g\cos\alpha \quad | \quad (1)+(2)=(4)$$

$$(m_1+m_2+m_3)a_3 = m_3g\sin 2\alpha - (m_1+m_2)g\sin\alpha - (\mu_1m_1+\mu_2m_2)g\cos\alpha \quad | \quad (3)+(4)$$

$$\begin{aligned} a_3 &= \frac{m_3\sin 2\alpha - (m_1+m_2)\sin\alpha - (\mu_1m_1+\mu_2m_2)\cos\alpha}{m_1+m_2+m_3}g \\ T_3 = T_2 &= (m_1+m_2)a_3 + (m_1+m_2)g\sin\alpha + (\mu_1m_1+\mu_2m_2)g\cos\alpha \\ &= \frac{(m_1+m_2)(\sin 2\alpha + \sin\alpha) + (\mu_1m_1+\mu_2m_2)\cos\alpha}{m_1+m_2+m_3}m_3g \\ T_1 &= m_1a_3 + m_1g\sin\alpha + \mu_1m_1g\cos\alpha \\ &= \frac{m_3(\sin 2\alpha + \sin\alpha) + (\mu_1m_2 + \mu_1m_3 - \mu_2m_2)\cos\alpha}{m_1+m_2+m_3}m_1g \end{aligned}$$

(b)

$$a_3 > 0$$

$$m_3\sin 2\alpha - (m_1+m_2)\sin\alpha - (\mu_1m_1+\mu_2m_2)\cos\alpha > 0$$

Problem 7.

Suppose the direction to the sky to be the positive direction

(a)

(b)

$$a = \frac{F - (m_1 + m_2 + m)g}{m_1 + m_2 + m} = \frac{F}{m_1 + m_2 + m} - g$$

(c)

$$m_1 a_1 = F - m_1 g - T$$

$$T = \frac{m_2 + m}{m_1 + m_2 + m} F$$

(d)

$$\frac{1}{2} ma = T - T_m - \frac{1}{2} mg$$

$$T_m = \frac{2m_2 + m}{2(m_1 + m_2 + m)} F$$

Problem 8.

In a sector whose angle equals θ, Suppose the tangent line to be the x-axis and the radial line to be the y-axis.

$$\begin{cases} T \sin \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} = dN \\ T \cos \frac{d\theta}{2} + \mu dN = (T + dT) \cos \frac{d\theta}{2} \end{cases} \implies \begin{cases} dN = 2T \sin \frac{d\theta}{2} + dT \sin \frac{d\theta}{2} \\ \mu dN = dT \cos \frac{d\theta}{2} = dT \end{cases}$$

As $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$,

$$dN = T d\theta$$

$$\mu T d\theta = dT$$

$$\frac{dT}{T} = \mu d\theta$$

Do integral on both sides,

$$\int_{T_A}^{T_B} \frac{1}{T} dT = \mu \int_0^{\theta} d\theta$$

$$ln\left(\frac{T_B}{T_A}\right) = \mu\theta$$

$$\frac{T_B}{T_A} = e^{\mu\theta}$$

$$T_A = T_B e^{-\mu\theta}$$

Problem 9.

(a)

$$a = \frac{\delta m g}{m} = \frac{x + \frac{L}{4}}{L} g = \frac{4x + L}{4L} g$$

(b)

$$a = \frac{\delta m g - \mu_k(m - \delta m)g}{m} = \frac{x + \frac{L}{4} - \mu_k(\frac{3}{4}L - x)}{L} g = \frac{4x(1 + \mu_k) - L(1 + 3\mu_k)}{4L} g$$