Problem Set 10

Due: 28 July 2016, 10 a.m.

Problem 1. Based on measurements of rotational energy levels of a HF molecule, its moment of inertia about the axis perpendicular to the line connecting both atoms, passing through the center of mass of the system, has been found equal to $1,37 \cdot 10^{-47}$ kg $\cdot$ m^2. Estimate the distance between H and F atoms if their masses are $m_{\mathrm{H}} = 1,67 \cdot 10^{-27}$ kg, $m_{\mathrm{F}} = 3,17 \cdot 10^{-26}$ kg, respectively.

(*3 marks*)

Problem 2. In each case find the moment of inertia of the object about the axis given:

(a) hollow cylinder of inner radius R_1, outer radius R_2, and height H, about the axis of symmetry. The bulk density of the material the cylinder is made of increases linearly as αr with the distance r from the axis of symmetry.

(b) triangle with surface density of mass σ, base a, and height h about the axis containing the base,

(c) the same triangle about the axis parallel to the base, through the third vertex,

(*2 + 2 + 2 marks*)

Problem 3. Prove the following statement: For a planar object, the moment of inertia about an axis perpendicular to the plane is the sum of the moments of inertia about two perpendicular axes through the same point in the plane of the object.

It is known as *the perpendicular axis theorem.*

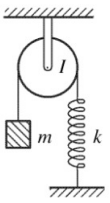
(*3 marks*)

Problem 4. A cylindrical container with radius R is filled with liquid and rotates with angular velocity ω about its axis of symmetry. Show that the free surface of the liquid assumes a shape of an paraboloid (give its equation) and find the total kinetic energy of the system.

The moment of inertia of an empty container I_0 about its axis of symmetry, density of the liquid ϱ, and volume of the liquid V_0 are also known.

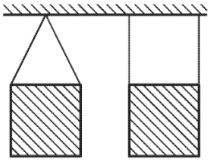
(*5 marks*)

Problem 5. A block with mass m is attached to one end of a light rope that runs over a pulley with the moment of inertia I and radius R. The other end of the rope is attached to a spring with spring constant k. Find the period of oscillations of this system.



(*3 marks*)

Problem 6. Two identical uniform squares are suspended on light inextensible cords in two different ways (see the figure). In both cases, the distance from the suspension point to the top side of the square is equal to the length of the side L.
The systems are made to oscillate in the vertical plane containing the squares. What are the periods of *small oscillations* of these physical pendula? Acceleration due to gravity is given.



(*5 marks*)

Problem 7. When an object is rolling without slipping, the rolling friction force is much less than the friction force when the object is sliding; a silver dollar will roll on its edge much farther than it will slide on its flat side. When an object is rolling without slipping on a horizontal surface, we can approximate the friction force to be zero, so that a_x and ε_z are approximately zero and v_x and ω_z are approximately constant. Rolling without slipping means $v_x = r\omega_z$ and $a_x = r\varepsilon_z$. If an object is set in motion on a surface without these equalities, sliding (kinetic) friction will act on the object as it slips until rolling without slipping is established.

A solid cylinder with mass M and radius R, rotating with angular speed ω_0 about an axis through its center, is set on a horizontal surface for which the kinetic friction coefficient is μ_k.

(a) Draw a free-body diagram for the cylinder on the surface. Think carefully about the direction of the kinetic friction force on the cylinder. Calculate the accelerations a_x of the center of mass and ε_z of rotation about the center of mass.

(b) The cylinder is initially slipping completely, so initially $\omega_z = \omega_0$ but $v_x = 0$. Rolling without slipping sets in when $v_x = R\omega_z$. Calculate the distance the cylinder rolls before slipping stops.

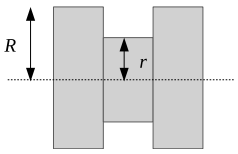
(c) Calculate the work done by the friction force on the cylinder as it moves from where it was set down to where it begins to roll without slipping.

(3 + 3 + 1 marks)

Problem 8. A uniform solid ball and a ring start rolling without slipping from the top of an incline with slope angle α. The initial velocity of the center of mass of the ball is zero. What should the initial linear velocity of the center of mass of the ring be, so that both objects travel the same distance over the same period of time t?

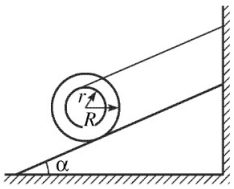
(4 marks)

Problem 9. A yo-yo is made of two identical cylinders of radius R permanently attached at their centers to a cylindrical axle, which has a smaller radius r. All three cylinders have the same height a, and the density of mass they are made of is ϱ. A string is attached to and wrapped around this axle. With the free end of the string held, the yo-yo is released. Find the acceleration of the center of mass of the yo-yo. Assume that the string does not slip while unwinding.



(4 marks)

Problem 10. A light inextensible string is winded on a reel with mass m, inner radius r, and outer radius R placed on a ramp inclined at an angle α to the horizontal. The free end of the string is attached to a wall, so that the string is parallel to the plane.
What is the acceleration of the center of mass of the reel?
The moment of inertia of the reel about the axis of symmetry is I_0 and the coefficient of friction is μ. The string does not slip on the reel.



(5 marks)