JOINT INSTITUTE
交大密西根学院

HONORS PHYSICS I (Summer 2016)

Problem Set 4

Due: 14 June 2016, 10 a.m.

Problem 1. A ball with mass m is thrown vertically upwards with initial speed v_0. Assuming linear air drag write down and solve the equation of motion for the ball, i.e. find the time dependence of its position. Find the coordinate of the highest point of the trajectory as well the time needed to reach it.

(*4 marks*)

Problem 2. Acceleration due to gravity on a planet with the shape of a sphere with radius R depends on altitude h as $g(h) = g_0/(1 + h/R)^2$, where g_0 is a constant. A particle falls from a high altitude H with zero initial velocity. Neglecting air drag, find the time T when it hits the ground ($h = 0$) and the speed it has at this instant.

Hint. The chain rule.

(*6 marks*)

Problem 3. We have seen in class that the general solution of the equation of motion of a simple (neither driven, nor damped) harmonic oscillator can be expressed in the form $x(t) = A\cos(\omega_0 t + \varphi)$, where A is the amplitude and φ is the phase shift of the oscillations to be determined from the initial conditions.

(a) The linear combination $x(t) = B\cos\omega_0 t + C\sin\omega_0 t$, where B and C are real constants, is another possible representation of the general solution. Check that these two forms are equivalent, i.e. express B and C in terms of A and φ or *vice–versa*.

(b) Show that given the initial conditions $x(0) = x_0$ and $v(0) = v_0$ the amplitude and the phase shift can be found as $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$, and $\varphi = \arctan\left(-\frac{v_0}{\omega_0 x_0}\right)$. respectively.

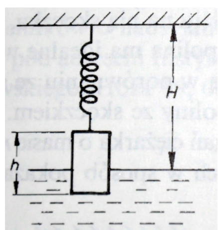
(*2 + 2 marks*)

Problem 4. A horizontal platform oscillates harmonically in the vertical direction with amplitude A. Find the maximum angular frequency of oscillations at which a block placed on the platform is still in contact with its surface.

Clearly indicate the frame of reference you are solving the problem in.

(*3 marks*)

Problem 5. A block, partially submerged in a liquid of density ϱ, is attached to a massless spring with spring constant k and equilibrium length l_0. The upper end of the spring is fixed to the ceiling at height H above the surface of the liquid. The block has mass m, height h and cross-sectional area S. Find the position of the center of the block y — measured from the ceiling — as a function of time. At the initial instant of time the center of the block is at $y(0) = y_0$ and its velocity $v_y(0) = 0$.

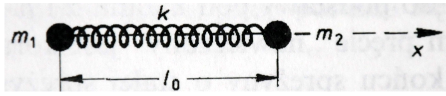


Neglect viscosity of the liquid and assume that the surface of the liquid remains at the same level while the block is in motion. Also, the initial displacement from the equilibrium position is small enough, so that the block always remains immersed in the liquid.

Hint. Archimedes' principle.

(*6 marks*)

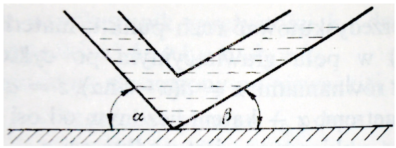
Problem 6. Show that a one-dimensional system of two point masses m_1 and m_2 connected by a massless spring with spring constant k and equilibrium length l_0 (see the figure) is a harmonic oscillator. Find its natural angular frequency.



(*5 marks*)

Problem 7. Find the natural angular frequency of small oscillations in a V-shaped tube of constant cross-sectional area and length of the liquid column l. The arms of the tube are inclined at angles α and β to the horizontal. Neglect viscosity.

(*6 marks*)



Problem 8. (*bonus problem*) A particle with mass m and positive charge q moves in uniform electric and magnetic fields $\mathbf{E}(\mathbf{r}) = (-E_0, 0, 0)$ and $\mathbf{B}(\mathbf{r}) = (B_0, 0, 0)$, where E_0 and B_0 are positive constants. For the initial conditions: $\mathbf{v}(0) = (v_{0x}, v_{0y}, 0)$ and $\mathbf{r}(0) = (0, 0, 0)$, find the velocity $\mathbf{v}(t)$ and the position $\mathbf{r}(t)$ of the particle for $t > 0$.

Assuming unit mass and unit charge of the particle, sketch the trajectories of the particle for a few values of E_0/B_0 (what are the units of this ratio?). Use a computer, attach the graphs to your answer.

(*4 bonus marks*)

Problem 9. For a critically damped harmonic oscillator show that the oscillating mass can pass through the equilibrium position at most once, regardless of initial conditions.

(*3 marks*)