

### Problem 1.

$$F = ma = f$$

$$fR = I\varepsilon$$

$$I = \frac{2}{5}mR^2$$

$$\frac{\omega_0}{R\varepsilon} > \frac{v_0}{a}$$

$$\frac{\omega_0 I}{fR^2} > \frac{v_0 m}{f}$$

$$\omega_0 > \frac{5v_0}{2\omega_0}$$

### Problem 2.

$$F(x-d) = I \frac{d\omega}{dt}$$

Do integral on both sides,

$$\int_{t_1}^{t_2} F dt (x-d) = I\omega$$

and

$$J = \int_{t_1}^{t_2} F dt = mv$$

$$mv(x-d) = I\omega$$

$$x = \frac{I\omega}{mv} + d = \frac{I}{md} + d \approx 0.71m$$

### Problem 3.

$$\omega = \frac{19200}{60} \cdot 2\pi = 640\pi \text{ rad/s}$$

$$\Omega = \frac{\theta}{t} = \frac{1.1 \times 10^{-6}}{5 \times 3600} \cdot \frac{\pi}{180} \approx 1.07 \times 10^{-2} \text{ rad/s}$$

$$\tau = L\Omega = I\omega\Omega = \frac{1}{4}md^2\omega\Omega = 2.44 \times 10^{-12} \text{ kg} \cdot \text{m}^2/\text{s}^2$$

### Problem 4.

(a)

$$G_C L_C \cos 55^\circ + G_B L_B \cos 55^\circ = T L_T \sin 25^\circ$$

$$T = \frac{G_C L_C \cos 55^\circ + G_B L_B \cos 55^\circ}{L_T \sin 25^\circ} \approx 29336N$$

(b)

$$F_x = T \sin 60^\circ = 25406N \quad \text{Pointing to the right}$$

$$F_y = G_C + G_B + T \cos 60^\circ = 40668N \quad \text{Pointing to the sky}$$

## Problem 5.

(a)

$$N_B = 2mg \quad \text{Pointing to the sky}$$

$$N_C = mg \tan \theta = \frac{\sqrt{3}}{3} mg \quad \text{Pointing to the left}$$

$$N_A = N_C = \frac{\sqrt{3}}{3} mg \quad \text{Pointing to the right}$$

(b)

$$N = \frac{mg}{\cos \theta} = \frac{2\sqrt{3}}{3} mg$$

## Problem 6.

$$F = f + T \sin \theta$$

$$N = G + T \cos \theta = G + (F - f) \cot \theta$$

$$f = \mu_s N = \mu_s [G + (F - f) \cot \theta]$$

$$f = \frac{\mu_s (G + F \cot \theta)}{1 + \mu_s \cot \theta}$$

$$FHx = fH$$

where  $x = \frac{h}{X}$

$$Fx = \frac{\mu_s (G + F \cot \theta)}{1 + \mu_s \cot \theta}$$

$$F = \frac{\mu_s G}{(1 + \mu_s \cot \theta)x - \mu_s \cot \theta}$$

(a) When  $x = 0.5$ ,

$$F \approx 399.71N$$

(b) When  $x = 0.4$ ,

$$F \approx 748.77N$$

(c) When  $(1 + \mu_s \cot \theta)x - \mu_s \cot \theta < 0$ ,

F is negative so that the post can not slip.

$$x < \frac{\mu_s \cot \theta}{1 + \mu_s \cot \theta} \approx 0.285$$

So when the force is exerted less than 0.285h to the top, it won't slip.

### Problem 7.

(a)

$$\begin{aligned}P &= \frac{RT}{V} \\ \frac{dP}{dV} &= -\frac{RT}{V^2} \\ B &= -\frac{dP}{dV/V} = \frac{RT}{V} = P\end{aligned}$$

(b)

$$\begin{aligned}\frac{RT}{V} &= \frac{C}{V^\gamma} \\ T &= \frac{C}{V^{\gamma-1}R} \\ P &= \frac{C}{V^\gamma} \\ \frac{dP}{dV} &= -\frac{\gamma C}{V^{\gamma+1}} \\ B &= -\frac{dP}{dV/V} = \frac{\gamma C}{V^\gamma} = \gamma P\end{aligned}$$

### Problem 8.

$$\begin{aligned}P_C &= P_0 - \rho gh_2 \\ 2Sv_D &= Sv_C \\ P_0 + \rho gh_1 &= P_C + \frac{1}{2}\rho v_C^2 = P_0 + \frac{1}{2}\rho v_D^2 \\ \rho gh_1 &= \frac{1}{2}\rho v_D^2 = \frac{1}{8}\rho v_C^2 \\ \rho gh_1 &= \frac{1}{2}\rho v_C^2 - \rho gh_2 \\ h_2 &= 3h_1\end{aligned}$$

### Problem 9.

(a)

$$\begin{aligned}P_0 + \frac{1}{2}\rho v^2 &= P_0 + \rho gy \\ v &= \sqrt{2gy} \\ s = h - y &= \frac{1}{2}gt^2 \\ t &= \sqrt{\frac{2(h-y)}{g}} \\ D = vt &= \sqrt{4(h-y)y}\end{aligned}$$

(b)

$$D = vt = 2\sqrt{-\left(y - \frac{h}{2}\right)^2 + \frac{h^2}{4}}$$

When  $y = \frac{h}{2}$ , D gets its maximum value  $h$

### Problem 10.

(a)

$$P = P_0 + \rho gh \approx 1.025 \times 10^5 \text{ Pa}$$

(b)

$$V = \frac{1}{12}\pi \left(\frac{h}{H}D\right)^2 h$$

$$h = \sqrt[3]{\frac{12VH^2}{\pi D^2}}$$

$$P_0 + \frac{1}{2}\rho v^2 = P_0 + \rho gh$$

$$v = \sqrt{2gh} = \sqrt{2g} \sqrt[6]{\frac{12H^2}{\pi D^2}} V^{\frac{1}{6}}$$

$$dV = Sv dt = \frac{1}{4}\pi d^2 \sqrt{2g} \sqrt[6]{\frac{12H^2}{\pi D^2}} V^{\frac{1}{6}} dt$$

$$V^{-\frac{1}{6}} dV = \frac{1}{4}\pi d^2 \sqrt{2g} \sqrt[6]{\frac{12H^2}{\pi D^2}} dt$$

Do integral on both sides,

$$\int_{V_1}^{V_2} V^{-\frac{1}{6}} dV = \int_0^t \frac{1}{4}\pi d^2 \sqrt{2g} \sqrt[6]{\frac{12H^2}{\pi D^2}} dt$$

$$t = \frac{\frac{6}{5} V^{\frac{5}{6}} \Big|_{V_1}^{V_2}}{\frac{1}{4}\pi d^2 \sqrt{2g} \sqrt[6]{\frac{12H^2}{\pi D^2}}} = 102.86 \text{ s}$$