Problem Set 3

Due: 7 June 2016, 10 a.m.

Problem 1. A butterfly flies along a curved trajectory, so that the distance it travels is given by $s(t) = s_0 e^{ct}$, where s_0 and c are positive constants. Assuming that at any point of the trajectory the total acceleration vector $\mathbf{a}$ forms the angle $\phi_0 = \text{const}$ with the tangent to the trajectory at this point, find the following quantities characterizing butterfly's motion

(a) speed,

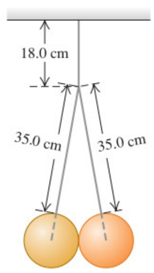
(b) magnitude of the tangential component of acceleration,

(c) magnitude of the normal component of acceleration,

(d) instantaneous radius of the trajectory's curvature.

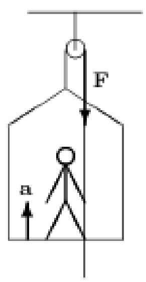
(*1/2 + 1 + 1 + 1/2 marks*)

Problem 2. Two identical balls, each with mass $m = 15$ kg, and $d = 25$ cm in diameter, are suspended by two wires with length $l_2 = 35$ cm as shown in figure. The entire apparatus is supported by a single wire with length $l_1 = 18$ cm, and the surfaces of the balls are perfectly smooth. (a) Find the tension in each of the three wires. (b) How hard does each ball push on the other one?



(*3/2 + 3/2 marks*)

Problem 3. A student of weight 320 N stands on a wooden bar of weight 160 N and pulls down the rope with force 250 N (see the left figure below). Find



(a) the acceleration of the student as he moves upward,

(b) the force he exerts on the bar.

(*3 + 1 marks*)

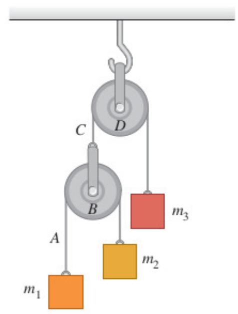
Problem 4. A container, partially filled with a liquid, slides down a plane inclined at an angle α to the horizontal. The coefficient of friction $\mu < \tan\alpha$. Find the angle that the surface of the liquid in the container forms with the inclined plane.

Solve the problem in an inertial frame of reference (clearly indicate the frame of reference you are solving the problem in). Sketch a free–body diagram illustrating your solution.

(*5 marks*)

Problem 5. For the system shown in the figure find, in terms of m_1, m_2, m_3, and g (a) the acceleration of block m_3; (b) the acceleration of pulley B; (c) the acceleration of block m_1; (d) the acceleration of block m_2; (e) the tension in string A; (f) the tension in string C? (g) What do your expressions give for the special case of $m_1 = m_2$ and $m_3 = m_1 + m_2$? Is this sensible?
Assume that all strings and pulleys are light and frictionless.

(5 × 3/2 + 1/2 marks)

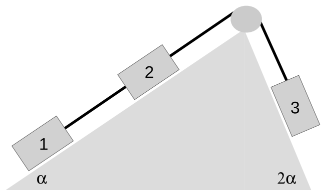


Problem 6. Three blocks with masses m_1, m_2, and m_3 are connected by massless strings and placed on planes inclined at the angles α and 2α, as shown in the figure below. The pulley on the top is frictionless, and the coefficients of kinetic friction between blocks 1 and 2 and the horizontal surface are equal to μ_1 and μ_2, respectively. There is no friction between block 3 and the incline.

(a) Assuming that the system moves so that block 3 slides downwards accelerating, find the acceleration of the blocks and the tensions in all strings.

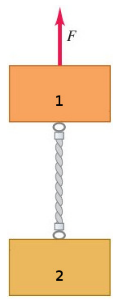
(b) What condition (relating the masses and the coefficients of friction) needs to be satisfied, if the blocks are to move as assumed in part (a)?

(4 + 1 marks)



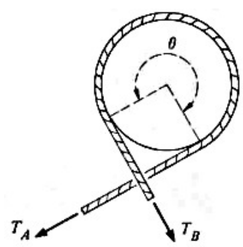
Problem 7. Two blocks with masses m_1 and m_2 are connected by a heavy uniform rope with mass m. An upward force of magnitude F is applied as shown. (a) Draw three free-body diagrams: one for block 1, one for the rope, and another one for block 2. For each force, indicate what body exerts that force. (b) What is the acceleration of the system? (c) What is the tension at the top of the heavy rope? (d) What is the tension at the midpoint of the rope?

(3/2 + 3/2 + 1 + 2 marks)



Problem 8. A device called *capstan* is used aboard ships in order to control a rope which is under great tension. The rope is wrapped around a fixed drum, usually for several turns (the figure shows about three–fourths turn as seen from overhead). The load on the rope pulls it with a force $\mathbf{T}_B$, and the sailor holds it with a much smaller force $\mathbf{T}_A$ Show that $T_A = T_B \exp(-\mu\theta)$, where μ is the coefficient of static friction and θ is the total angle subtended by the rope on the drum.
Hint. Start by considering an infinitesimal element of the rope.

(7 marks)



Problem 9. A straight uniform massive cord of length L is put on a table, so that at the initial instant of time 1/4 of its length hangs down over the edge of the table. Write down (do not attempt to solve!) the equation of motion for the cord in the case when (a) the table is perfectly smooth, (b) the table is rough with the kinetic coefficient of friction equal to μ_k (small enough, so that the cord moves after it is released from hold).

(3/2 + 3/2 marks)