JOINT INSTITUTE
交大密西根学院

HONORS PHYSICS I (Summer 2016)

Problem Set 12

Due: 8 August 2016, 12 noon

Problem 1. A thin, uniform rod has length L and mass M. Calculate the magnitude of the gravitational force the rod exerts on a particle with mass m that is placed at a point along the axis of the rod a distance x from one end. Discuss and comment on the result if $x \gg L$.

(*2 marks*)

Problem 2. Consider a ring-shaped body and a particle with mass m is placed a distance x from the center of the ring, along the line through the center of the ring and perpendicular to its plane (see figure below).

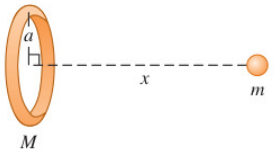
(a) Calculate the gravitational potential energy U of this system. Take the potential energy to be zero when the two objects are far apart.

(b) Show that your answer to part (a) reduces to the expected result when $x \gg a$.

(c) Find the magnitude and direction of the force on the particle, and show that your answer reduces to the expected result when $x \gg a$.

(d) What is the potential energy and the force when $x = 0$?

(*2 + 1/2 + 2 + 1/2 marks*)



Problem 3. The differential form of Gauss' law for gravitational field states that $\operatorname{div} \mathbf{E}_G(\mathbf{r}) = -4\pi G\varrho(\mathbf{r})$, where $\mathbf{E}_G$ is the gravitational field at point $\mathbf{r}$ due to mass distributed with spatial density ϱ, and G is the gravitational constant.

(a) Use the Gauss–Ostrogradsky theorem (also known as the divergence theorem) to rewrite the law in an integral form as $\iint\limits_{\Sigma} \mathbf{E}_G\, \hat{n}\, dS = -4\pi G M_\Sigma$, where M_Σ is the mass enclosed by the surface Σ, which is called the Gaussian surface.

(b) Use the integral form to find the gravitational field at a distance r from a point mass M.

(c) Find the gravitational field at a distance r from the center of a uniform solid ball with mass M and radius R. Consider both cases $r > R$ and $r < R$. Compare with the results discussed in class.

(d) Do the same in the case when the same amount of mass is distributed over the surface of the ball (the ball is empty inside).

Hint. Choose the Gaussian surface in a smart way, so that you will be able to calculate the oriented surface integrals in a simple way by using symmetry.

You will see similar calculations (for the electric field) in the fall semester.

(*1 + 2 + 3 + 2 marks*)

Problem 4. Suppose the you could drill a tunnel through a uniform planet with radius R and mass M, so that it passes through the planet's center (*i.e.* it is drilled along a diameter). Find the equation of motion of an object of mass m that is dropped into the tunnel (without initial velocity) and show that it describes a harmonic oscillator. Find the period of the oscillations and compare it with a period of a satellite orbiting around the planet close to its surface.

Will the answer change if the tunnel is drilled at an angle to the diameter, so that it does not pass through the center? Explain.

Hint. You may use the results of part (c) of the previous problem (or facts states in the lecture).

(*5 marks*)