Problem Set 1

Due: 27 May 2016, 10 a.m.

Problem 1. Given the Dirac's constant $\hbar = h/2\pi$ (where h is the Planck's constant), the gravitational constant G, and the speed of light in vacuum c, use dimensional analysis to construct the so-called *Planck's units* of time t_{P}, length l_{P}, and mass m_{P}.

Find their numerical values in the SI units. How do they compare to the time, distance, and mass that we are able to measure nowadays?

(*3 marks*)

Problem 2. In Wagner's opera *Das Rheingold*, the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight. Estimate the monetary value of this pile. The density of gold is 19.3 $\mathrm{g/cm^3}$. For the current value of gold, please look up a financial service (remember to cite the source).

(*3 marks*)

Problem 3. In the methane molecule, CH_4, each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the center. In coordinates where one of the C–H bonds is in the direction of $\hat{n}_x + \hat{n}_y + \hat{n}_z$, an adjacent C–H bond is in the $\hat{n}_x - \hat{n}_y - \hat{n}_z$ direction. Calculate the angle between these two bonds.

(*2 marks*)

Problem 4. The positions of two particles at an instant of time are given by the vectors $\mathbf{r}_1 = 4\hat{n}_x + 3\hat{n}_y + 8\hat{n}_z$ and $\mathbf{r}_2 = 2\hat{n}_x + 10\hat{n}_y + 5\hat{n}_z$, respectively (in the units of m/s). Find the

(a) magnitudes of these vectors,

(b) displacement vector $\mathbf{r}_{12}$ from the position of particle 1 to the position of particle 2 and the unit vector in the direction of vector $\mathbf{r}_{12}$,

(c) angles between all pairs of vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_{12}$ (express the angles both in radians and degrees),

(d) orthogonal projection of the vector $\mathbf{r}_2$ onto the vector $\mathbf{r}_1$,

(e) cross product $\mathbf{r}_2 \times \mathbf{r}_{12}$,

(f) cylindrical coordinates of the point defined by the position vector $\mathbf{r}_1$ (use a calculator).

(*1/2 + 1/2 + 1 + 1 + 1 + 1 marks*)

Problem 5. Recall that $\mathbf{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z$ is the position vector pointing from the origin $(0, 0, 0)$ to a point in space with the Cartesian coordinates (x, y, z). Use what you know about vectors to show the following: All points (x, y, z) that satisfy the equation $Ax + By + Cz = 0$, where A, B, and C are constants, lie in a plane that passes through the origin and that is perpendicular to the vector $A\hat{n}_x + B\hat{n}_y + C\hat{n}_z$.

(*4 marks*)

Problem 6. A vector $\mathbf{n} = \mathbf{n}(t)$ has unit length for any instant of time t.

(a) Is the magnitude of $\dot{\mathbf{n}}$ equal to one, too? If so, prove it, otherwise find a counterexample.

(b) Show that, for any t, the vectors $\mathbf{n}$ and $\dot{\mathbf{n}}$ are perpendicular perpendicular to each other.

(*3 + 2 marks*)

Problem 7. A particle moves with velocity $v_x(t) = At \sin Bt$, where A and B are constants with appropriate units. Find its (a) acceleration and (b) position at any instant of time t.

(*1/2 + 1 marks*)

Problem 8. A particle moves along x-axis in the positive direction with acceleration $a_x = -\alpha v_x$, where α is a positive constant (what are the units of α?). At the initial instant of time ($t = 0$ s), the particle is at the origin, and has velocity $v_x(0) = v_0$.

(a) Find the time after which the particle stops.

(b) What is the distance the particle travels from $t = 0$ s until it stops?

(c) Find the time needed for the particle to travel the distance s_1 (less than the distance found in the previous question).

(d) Sketch the graphs $x(t)$, $v_x(t)$, and $a_x(t)$.

(e) Suggest a situation this model can be applied to.

(*2 + 2 + 1 + 3/2 + 1 marks*)

Problem 9. Show that in one-dimensional motion $a_x\,\mathrm{d}x = v_x\,\mathrm{d}v_x$. *Hint*: The chain rule.

(*2 marks*)

Problem 10. A particle is moving in the positive direction of the x–axis so, that its speed measured at position x turns out to be $v_x(x) = \sqrt{px}$, where $p > 0$ is constant with appropriate units. Assuming that the particle starts (at $t = 0$ s) from the origin

(a) find its velocity and acceleration as functions of time,

(b) calculate its average velocity from $t = 0$ s to the time when it travels distance s.

(c) What kind of motion is it?

(*3/2 + 3/2 + 1 marks*)

Problem 11. Criticize the following statement: A fleet-of-foot Achilles is unable to catch a plodding tortoise which has been given a head start, since during the time it takes Achilles to catch up to a given position, the tortoise has moved forward some distance.

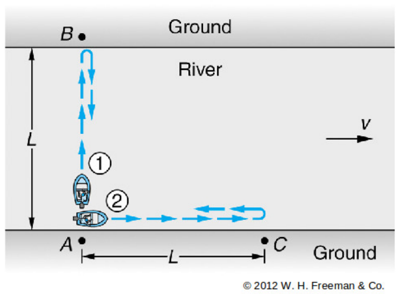
(*4 marks*)

Problem 12. A spare paddle drops from a fisherman's canoe traveling up the river exactly at the moment as it passes under a bridge. After one hour of paddling the fisherman realizes that the paddle is missing He turns around and paddles his canoe back to find the paddle 6 km down the bridge. Find the speed of the river's current.

Assume that the fisherman paddles always with the same speed with respect to the river. Clearly indicate the frame of reference the problem is being solved in.

(*3 marks*)

Problem 13. Two equally matched rowers race each other over courses as shown in figure below. Each oarsman rows at speed c in still water, the current in the river moves at speed $v < c$. Boat 1 goes from A to B, a distance L, and back. Boat 2 goes from A to C, also a distance L, and back. A, B, and C are marks on the riverbank. Which boat wins the race, or is it a tie?



Hint: The binomial expansion may be useful at some point.

(*4 marks*)