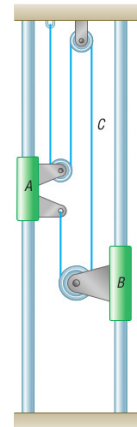**Problem Set 2**

Due: 2 June 2016, 10 a.m.

Problem 1. Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 24 cm/s, determine (a) the accelerations of A and B, (b) the velocity and the change in position of B after 6 s.

(3/2 + 3/2 marks)

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Problem 2. A particle is launched horizontally, close to the Earth's surface, with initial speed $v(0) = v_0$. Find both the normal and the tangential component of acceleration at instant $t > 0$. Neglect air drag.

Use a geometric argument to relate these components to the Cartesian components of velocity.

(4 marks)

Problem 3. A river flows eastwards, parallel to the banks, with the speed of the flow depending on the distance $0 \leq y \leq L$ from the southern bank as

$$v_2(y) = v_0 \sin \frac{\pi y}{L},$$

where v_0 is a positive constant and L is the river's width. A canoe is crossing the river with constant velocity $\mathbf{v}_1$ (with respect to the water), oriented perpendicularly to the direction of the stream. Find

(a) the velocity of the canoe with respect to the bank, and its magnitude,

(b) the canoe's trajectory,

(c) and the distance the canoe has drifted down the river.

(1 + 4 + 1 marks)

Problem 4. A wheel of radius R moves forward rolling (without sliding) on a flat surface with constant angular velocity ω. For a point P on the circumference of the wheel:

(a) Find parametric equations of the trajectory of P. What curve do they describe?

(b) Sketch the trajectory (use a computer, attach a printed graph).

(c) Find the velocity of point P,

(d) and the distance traveled by point P as a function of time.

$(3 + 1 + 1 + 2 \; marks)$

Problem 5. Planar motion of a particle is given by parametric equations in polar coordinates

$$r(t) = r_0(1 - ct), \qquad \varphi(t) = \frac{ct}{1 - ct},$$

where c and $r_0 > 0$ are constants (what are their units?). Find

(a) the implicit equations of the particle's trajectory (use a computer to plot it),

(b) velocity of the particle (transverse and radial components) and speed,

(c) as well as its acceleration (transverse and radial components).

(d) Comment on the motion in cases when $c > 0$ and $c < 0$.

$(1 + 3/2 + 3/2 + 1 \; marks)$

Problem 6. The pin at B is free to slide along the circular slot DE and along the rotating rod OC. Assuming that the rod OC rotates at a constant rate $\dot{\varphi}$, (a) show that the acceleration of pin B is of constant magnitude, (b) determine the direction of the acceleration of pin B.

$(2 + 3 \; marks)$
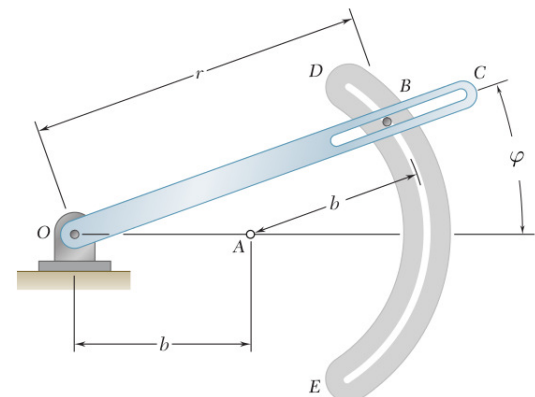
Problem 7. A particle moves in a plane so that the angle between the position vector $\mathbf{r}$ and the velocity vector $\mathbf{v}$ is constant and equal to α.

(a) Find the implicit equation of the trajectory in polar coordinates,

(b) and the total length of the trajectory.

(c) Sketch the trajectory.

(d) Discuss the solution with respect to the values of α.

Assume the initial conditions $\varphi(0) = 0$ and $r(0) = r_0$.

Hint for (a). Separation of variables.

$(3 + 2 + 1 + 1 \; marks)$

Problem 8. A pilot flies an ultrasonic aircraft with constant speed v so that it moves in a plane perpendicular to the ground. Find the trajectory he should follow if he wants his colleagues, standing at the airport's control tower (located in the plane of motion of the aircraft), to hear the accumulated sound from the whole flight at one instant of time? The speed of sound in air c is given.

Use polar coordinates. Assume that initially $r(0) = r_0$ and $\varphi(0) = 0$, where r_0 is the initial distance from the control tower.

$(8 \; marks)$