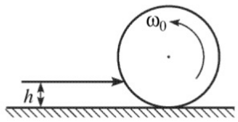
Problem Set 11

Due: 4 August 2016, 10 a.m.

Problem 1. A ball with radius R rotates about the axis of symmetry horizontal to the surface with angular speed ω_0. At an instant of time, the rotating ball is placed on a rough surface and gets a push at height $h < R$ above the surface (see the figure), so that its center of mass starts moving with the linear velocity $\mathbf{v_0}$ to the right.

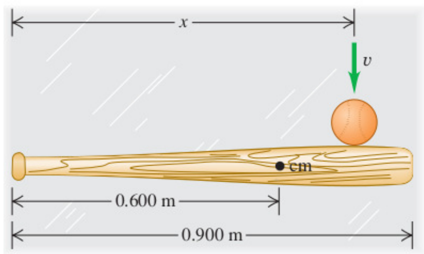


What should the initial angular speed ω_0 be, so that after some time the center of mass of the ball will start moving in the opposite direction?

(*6 marks*)

Problem 2. A baseball bat rests on a frictionless, horizontal surface (see the figure below). The bat has a length of $l = 0.9$ m, a mass of $m = 0.8$ kg, and its center of mass is $d = 0.6$ m from the handle end of the bat (see figure below). The moment of inertia of the bat about the axis (perpendicular to the surface) through the center of mass is $I_{\rm cm} = 0.0530$ kg·m^2. The bat is struck by a baseball traveling perpendicular to the bat. The impact applies an impulse of magnitude $J = \int_{t_1}^{t_2} F\,dt$ at a point a distance x from the handle end of the bat. What must x be so that the handle end of the bat remains at rest as the bat begins to move?

The point on the bat you will locate is called the center of percussion. Hitting a pitched ball at the center of percussion of the bat minimizes the "sting" the batter experiences on the hands.



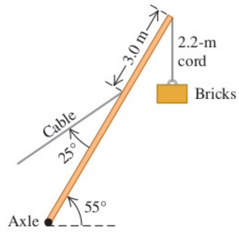
Hint. Consider the motion of the center of mass and the rotation about the center of mass. Find x so that these two motions combine to give $v = 0$ for the end of the bat just after the collision.

(*5 marks*)

Problem 3. The Hubble Space Telescope is stabilized to within an angle of about 2-millionths of a degree by means of a series of gyroscopes that spin at 19 200 rpm. Although the structure of these gyroscopes is actually quite complex, we can model each of the gyroscopes as a thin-walled cylinder of mass 2.0 kg and diameter 5.0 cm, spinning about its central axis. How large a torque would it take to cause these gyroscopes to precess through an angle of $1.0 \cdot 10{-}6$ degree during a 5.0-hour exposure of a galaxy?

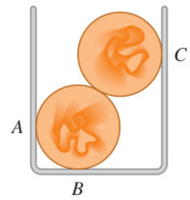
(*2 marks*)

Problem 4. A 15 000-N crane Figure pivots around a friction-free axle at its base and is supported by a cable making a 25^o angle 2.2-m cord with the crane The crane is 16 m long and is not uniform, its center of gravity being 7.0 m from the axle as measured along the crane. The cable is attached 3.0 m from the upper end of the crane. When the crane is raised to 55^o above the horizontal holding an 11 000-N pallet of bricks by a 2.2-m, very light cord, find (a) the tension in the cable and (b) the horizontal and vertical components of the force that the axle exerts on the crane. Start with a free-body diagram of the crane.



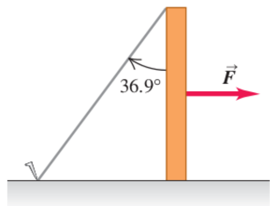
(3/2 + 3/2 marks)

Problem 5. Two uniform marbles, each with mass m and diameter $2r$, are stacked as shown in in a container that is $3r$ wide. (a) Find the force that the container exerts on the marbles at the points of contact A, B, and C. (b) What force does each marble exert on B the other?



(2 + 2 marks)

Problem 6. One end of a post weighing 400 N and with height h rests on a rough horizontal surface with $\mu_s = 0.3$. The upper end is held by a rope fastened to the surface and making an angle of 36.9^o with the post A horizontal force **F** is exerted on the post as shown. (a) If the force F is applied at the midpoint of the post, what is the largest value it can have without causing the post to slip? (b) How large can the force be without causing the post to slip if its point of application is 6/10 of the way from the ground to the top of the post? (c) Show that if the point of application of the force is too high, the post cannot be made to slip, no matter how great the force. Find the critical height for the point of application.



(3/2 + 3/2 + 2 marks)

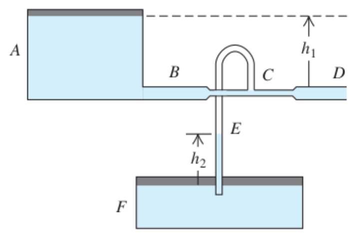
Problem 7. The equation of state (relating pressure, volume, and temperature) for one mole of an ideal gas is $pV = RT$, where R is a constants.

(a) Show that if the gas is compressed while the temperature T is held constant, the bulk modulus is equal to the pressure.

(b) When an ideal gas is compressed without the transfer of any heat into or out of it (adiabatic compression), the pressure and volume are related by $pV^\gamma =$ const, where γ is a constant having different values for different gases. Show that, in this case, the bulk modulus is given by $B = \gamma p$.

(2 + 3 marks)

Problem 8. Two very large open tanks A and F contain the same liquid. A horizontal pipe BCD, having a constriction at C and open to the air at D, leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross-sectional area at C is one-half the area at D and if D is a distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E? Express your answer in terms of h_1.



(4 marks)

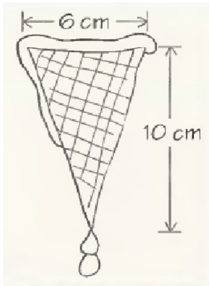
Problem 9. An open water tank stands on a plane surface. The water surface in the tank is a height h above the plane. A small hole is opened up at a depth y below the surface of the water

(a) Show that the jet of water will hit the plane surface a distance D from the tank, where $D = \sqrt{4y(h-y)}$.

(b) Show that the hole should be placed at a depth $y = h/2$ for the jet to cover a maximum horizontal distance.

(2 + 2 marks)

Problem 10. An ice-cream cone is filled with melted ice-cream of density $\varrho = 1.2\ \mathrm{g/cm^3}$. The cone has a diameter $D = 6$ cm at the larger end and is $l = 10$ cm long. Find the pressure at the bottom of the cone.
If a small hole of diameter $d = 1$ mm is opened at the bottom, the ice cream starts to run out. Ignoring the viscosity of the melted ice-cream, find the amount of time it takes the ice cream to run out. Assume that the fluid speed is zero at the top.



(5 marks)