UM–SJTU Joint Institute

Physics Laboratory
(VP241)

Laboratory Report

Exercise 3
Polarization of Light

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Contents

1 Theoretical Background

The objective of this exercise is to understand some properties of light, in particular to study the polarization phenomenon and verify Malus' law, the way half- and quarter-wave plates work in optical systems and generation and detection of elliptically and circularly polarized light.

Light can be described in terms of electromagnetic waves, with the plane of oscillations of the electric field vector (as well as the magnetic field vector) perpendicular to the direction of light propagation. Therefore, light is an example of a transverse wave. Natural light, also called unpolarized light is a random mixture of waves with the electric field vector oscillating in all possible transverse directions. This is due to the randomness of the radiation mechanism. For unpolarized light the distribution of the directions of the electric field vector, in the plane perpendicular to the direction of propagation, is uniform. If the distribution is not uniform, the light is said to be polarized. Studies of the polarization of light played an important role in the development of wave optics. They have resulted in a wide range of applications in numerous areas, such as optical measurement techniques, crystal structure research, and experimental stress analysis.

1.1 Polarization of Light

The electric field vector E, which in the context of electromagnetic waves corresponding to the visible part of the spectrum is sometimes referred to as the light vector describes a time-dependent, propagating electric field. In the plane perpendicular to the propagation direction of a light wave, the light vector may have different directions along which its magnitude oscillates. The light, for which the light vector maintains a certain oscillation direction, is called linearly polarized and the axis defining the direction is called the polarization axis (Figure 1). The light with the light vector direction rotating about the propagation direction, so that its endpoint traces a circle, is called circularly polarized light. If the vector traces an ellipse, the light said to be elliptically polarized (Figure 2).

Light emitted from ordinary light sources (natural light) is unpolarized. However it can be regarded as a statistical equal-weight mixture of linearly polarized waves with equal amplitudes. There the light may be also partially polarized, which means it can be regarded as a combination of a polarized and the natural (unpolarized) light. The direction corresponding to the maximum amplitude of the light vector of such partially polarized light is the oscillation direction of the polarized component.

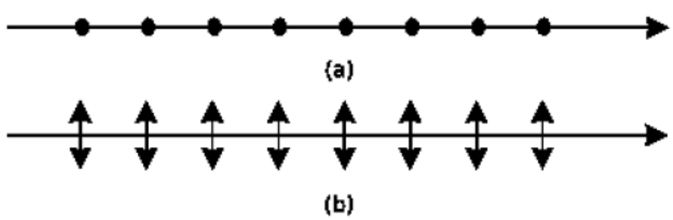

Figure 1: (a) Linearly polarized light with the polarization axis perpendicular to the page plane. (b) Linearly polarized light with the polarization axis parallel to the page plane.

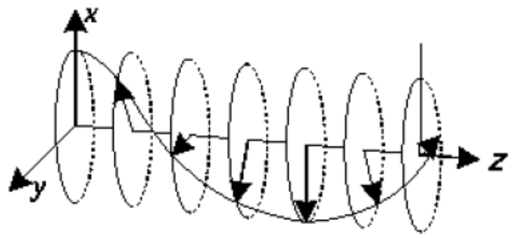

Figure 2: Elliptically polarized light propagating in the z direction. The light is polarized in the xy plane.

1.2 Polarizer

A device commonly used to produce polarized light is a polaroid (also called a po- larizer). It polarizes the light using the principle of dichroism: a selective absorptio mechanism tends to allow the light polarized in a certain direction (direction of the crystal alignment) to pass through the material, while the light polarized in all other directions is absorbed. This turns the incident natural light into linearly polarized.

A polarization device can not only change incident natural light to polarized light (it then acts as a polarizer), but may also be used to detect and analyze linearly polarized, natural, and partially polarized light (it is then called an analyzer).

1.3 Malus' law

A visible effect in the light coming out of a polarization device is a change of the light brightness.

Suppose that we have two polarizers arranged so that their planes are parallel- the left one plays the role of a polarizer, the other one is an analyzer (Figure 3). Let the angle between their transmission directions (polarization axes) be θ. The light is incident normally on the polarizer and then continues to the analizer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{light} = I_{light,0} cos^2\theta \tag{1}$$

where $I_{light,0}$ is the intensity of the linearly polarized light incident on the analyzer. Equation (1), named after Etienne-Louis Malus as the Malus' law, was derived in 1809.

Obviously, for a single polarizer, if polarized light is incident on it, then the transmitted light intensity will change periodically when rotating the polarizer. If the incident light is partially or elliptically polarized, the minimum intensity will not be zero as there will be always some component of the light polarized in the transmission direction. The incident light must be natural or circularly polarized if the intensity does not change at all. Hence, by using a polarizer, one can distinguish linearly polarized light from the natural and circularly polarized light.

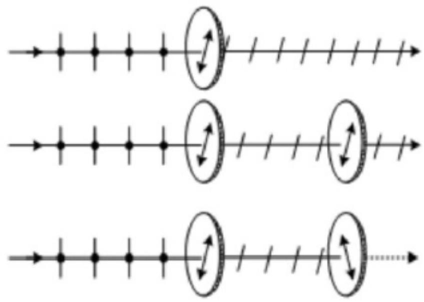

Figure 3: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

1.4 Generation of Elliptically and Circularly Polarized Light.Half-wave and Quarter-wave Plates

Suppose that a linearly polarized light is incident morally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α. Then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_e - n_o)d$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}(n_3 - n_o)d$$

where λ is the wavelength, ne is the refractive index for the extraordinary axis, and no is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.

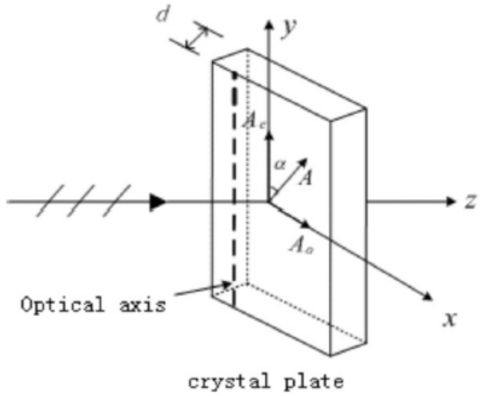

Figure 4: Linearly polarized light passing through a waveplate.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$E_x = A_o cos\omega t \qquad E_y = A_e cos(\omega t + \delta)$$

where $A_e = Acos\alpha, A_o = Asin\alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} - 2\frac{E_x E_y}{A_o A_e} cos\delta = sin^{\delta} \tag{2}$$

which is the equation of an ellipse. When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

- If $\Delta = k\lambda$, where $k = 0, 1, 2, \cdots$, the phase difference $\delta = 0$, and Eq.(2) reduces to

  $$E_y = \frac{A_e}{A_o} E_x$$

  which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a full-wave plate. The light goes through a full-wave plate without changing its polarization state.

- If $\Delta = (2k+1)\lambda/2$, where $k = 0, 1, 2, \cdots$, the phase difference $\delta = \pi$, and Eq. (2) simplifies to

  $$E_y = -\frac{A_e}{A_o} E_x$$

  The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α. A waveplate that satisfies the condition is called 1/2-wave plate or half-wave plate. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α. If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

- Finally, if $\Delta = (2k+1)\lambda/4$, where $k = 0, 1, 2, \cdots$, the phase difference $\delta = \pm\pi/2$. and Eq.(2) transforms into

$$\frac{E_x^2}{A_o^2} \pm \frac{E_y^2}{A_e^2} = 1$$

  The transmitted light is elliptically polarized withA waveplate that satisfies the above condition is called a 1/4-wave plate or a quarter-waveplate and is an important optical element in many polarization experiments.

If $A_e = A_o = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the o-wave and the e-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- if $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/4$,the transmitted light is circularly polarized;
- otherwise, the transmitted light is elliptically polarized.

2 Measurement Setup and Procedure

2.1 Apparatus

The main elements of the measurement setup are: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, two polarizers, a 1/2- wave plate, a 1/4-wave plate, a reflector and a stack of glass. The elements are placed on an optical bench.

2.2 Measurement Procedure and Data Analysis

2.2.1 Demonstration of Malus' Law

1. Assemble the measurement setup as shown in Figure 5. Make sure that the laser ray passes through the polarizer to generate linearly polarized light before continuing to the analyzer and the silicon photo-cell.
2. Rotate the analyzer for 360° and observe a change in the light intensity to find the maximum electric current I_0.
3. Set the angle of analyzer to 90° and adjust the angle of the polarizer until the electric current measured by the multimeter reaches its minimum. At this point, the polarizing axes of the polarizer and the analyzer are perpendicular to each other.
4. Rotate the analyzer from 90° to 0° and record the magnitude of the current I every 5°. Record the values in a table and plot the graph I/I_0 vs. $cos^2\theta$. Perform linear fitting and compare the data with the theoretical result.

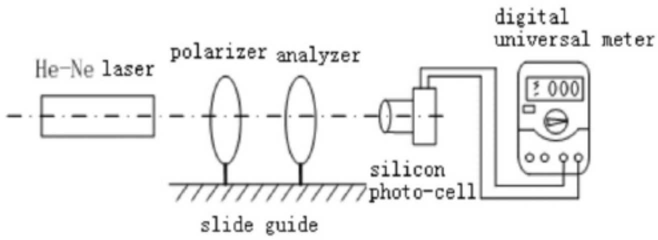

Figure 5: Experimental setup for a demonstration of Malus' law.

2.2.2 Linearly Polarized Light and the Half-wave Plate

1. Set up the equipment on the optical bench as shown in Figure 6. A is the analyzer and P is the polarizer. Set the polarizing axes of A and P perpendicular to each other before placing the 1/2-wave plate in the apparatus; extinction of the light can be observed on screen.

2. After inserting the 1/2-wave plate, rotate it to make the light extinction appear again and set this position as the initial position.

3. Rotate the 1/2-wave plate for $\alpha = 10°$ from the initial position and the light extinction will be broken. Then rotate A to make the light extinction appear again, record the angle of rotation $\Delta\theta$ in a table.

4. Rotate the 1/2-wave plate for 10° from the previous position (now $\alpha = 2°$) and repeat Step 3. Repeat this step (increase α) for 8 times. Plot the graph $\Delta\theta$ vs. θ.

5. After analyzing the data, the following questions will be answered:

   (a) How many times can the light extinction be observed when the 1/2-wave plate rotates for 360°?

   (b) How many times can the light extinction be observed when the analyzer rotates for 360°?

   (c) Explain the polarization state of linearly polarized light after passing through the 1/2-wave plate.

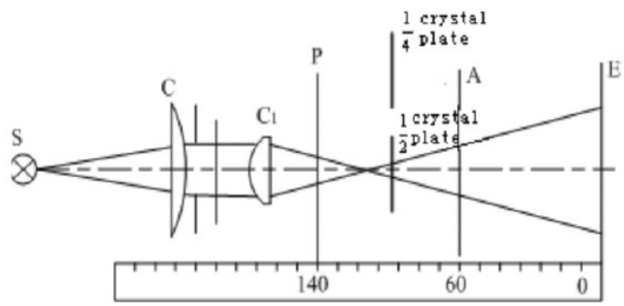

Figure 6: Experimental setup for the 1/2-wave plate.

2.2.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

1. Set up the equipment on the optical bench as shown in Figure 6. A is the analyzer and P is the polarizer. Set the polarizing axes of A and P perpendicular to each other before placing the 1/4-wave plate in the apparatus; extinction of the light can be observed on screen. At this point the angle $\theta = 90°$.

2. After inserting the 1/4-wave plate, rotate it to make the light extinction appear again and set this position as the initial position. At this point $\alpha = 0°$. Rotate the 1/4-wave plate and observe the change in the light intensity.

3. Rotate the analyzer for 360° and record the light intensity (which is indicated by the current I) for every 10°. Record the data in a table.

4. Rotate the 1/4-wave plate for 20°, repeat Step 3.

5. Rotate the 1/4-wave plate for 45°, repeat Step 3.

6. Rotate the 1/4-wave plate for 70°. Then rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.

7. Use a computer to plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 6 and compare it with the data recorded in Step 4.

   Pay attention to the fact that the light intensity is found indirectly by measuring the electric current, and the intensity is proportional to the amplitude squared. The current indicates the intensity, not the amplitude.

8. Compare the result of Step 5 with that for the circular polarization. Plot a linear fit to the data when the angle is 45°.

3 Results

Uncertainty of θ is $[2°]$

3.1 Demonstration of Malus' Law

The measurement of current I was shown in Table 1.

Maximum Electric Current I_0 6.852 ± 0.001 $[\mu A]$			
θ	$I[\mu A] \pm 0.01[\mu A]$	θ	$I[\mu A] \pm 0.01[\mu A]$
0°	6.728	50°	2.781
5°	6.667	55°	2.227
10°	6.519	60°	1.697
15°	6.303	65°	1.141
20°	5.966	70°	0.708
25°	5.542	75°	0.399
30°	5.078	80°	0.136
35°	4.536	85°	-0.012
40°	3.989	90°	-0.063
45°	3.343		

Table 1: Measurement data Malus's law demonstration.

The fitted curve was plotted in Figure 7.

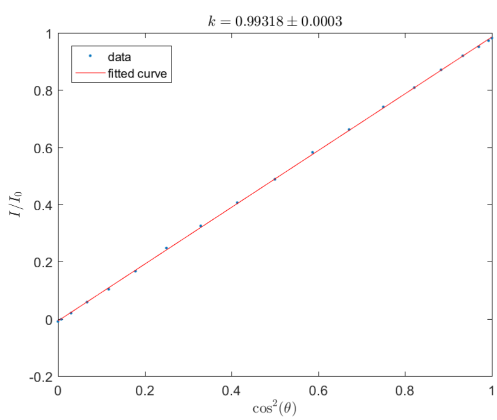

Figure 7: Fit graph for Table 1.

3.2 Linearly Polarized Light and the Half-wave Plate

The measurement data for the 1/2-wave plate was shown in Table 2.

Rotating angle of the 1/2-wave plate	Rotation angle of the analyzer [°] ± 2[°]
initial	0°
10°	20°
20°	40°
30°	60°
40°	80°
50°	100°
60°	118°
70°	138°
80°	156°
90°	174°

Table 2: Measurement data for the 1/2-wave plate.

The fitted curve was plotted in Figure 8.

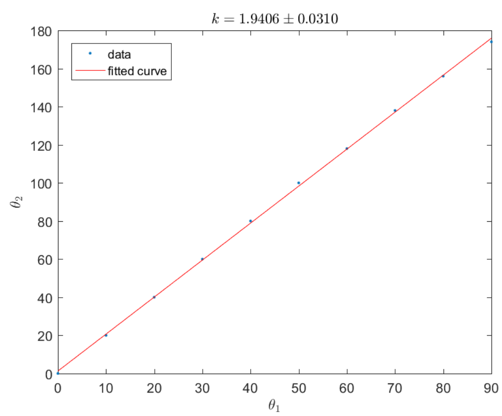

Figure 8: Fit graph for Table 2.

When the 1/2-wave plate rotates for 360°, 4 light extinctions can be observed.

When the analyzer rotates for 360°, 2 light extinctions can be observed.

When the 1/2-wave plate rotates for θ, the analyzer rotates for 2θ.

3.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

The measurement of current I was shown in Table 3.

Rotation angle of 1/4-wave plate: 0°					
Maximum Electric Current I_0 4.785 ± 0.001 $[\mu A]$					
θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0	θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0
0°	4.771	0.997 ± 0.0003	180°	4.781	0.999 ± 0.0003
10°	4.651	0.972 ± 0.0003	190°	4.677	0.977 ± 0.0003
20°	4.261	0.890 ± 0.0003	200°	4.300	0.899 ± 0.0003
30°	3.660	0.765 ± 0.0003	210°	3.725	0.778 ± 0.0003
40°	2.870	0.600 ± 0.0002	220°	2.955	0.618 ± 0.0002
50°	2.053	0.429 ± 0.0002	230°	2.112	0.441 ± 0.0002
60°	1.246	0.260 ± 0.0002	240°	1.300	0.272 ± 0.0002
70°	0.617	0.129 ± 0.0002	250°	0.635	0.133 ± 0.0002
80°	0.153	0.032 ± 0.0002	260°	0.179	0.037 ± 0.0002
90°	0.001	0.000 ± 0.0002	270°	0.000	0.000 ± 0.0002
100°	0.089	0.019 ± 0.0002	280°	0.103	0.022 ± 0.0002
110°	0.468	0.098 ± 0.0002	290°	0.479	0.100 ± 0.0002
120°	1.065	0.223 ± 0.0002	300°	1.094	0.229 ± 0.0002
130°	1.820	0.380 ± 0.0002	310°	1.863	0.389 ± 0.0002
140°	2.656	0.555 ± 0.0002	320°	2.737	0.572 ± 0.0002
150°	3.475	0.726 ± 0.0003	330°	3.498	0.731 ± 0.0003
160°	4.157	0.869 ± 0.0003	340°	4.159	0.869 ± 0.0003
170°	4.601	0.962 ± 0.0003	350°	4.615	0.964 ± 0.0003

Table 3: Measurement data for the 1/4-wave plate (rotation angle 0°).

The relation between rotation angle and light intensity was plotted in Figure 9.

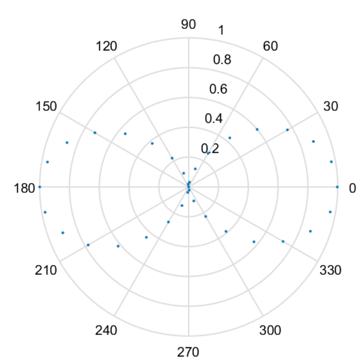

Figure 9: θ vs. I/I_0 graph.

The measurement of current I was shown in Table 4.

Rotation angle of 1/4-wave plate: 20°					
Maximum Electric Current I_0 4.169 ± 0.001 $[\mu A]$					
θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0	θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0
0°	3.641	0.873 ± 0.0003	180°	3.637	0.872 ± 0.0003
10°	4.010	0.962 ± 0.0003	190°	4.001	0.960 ± 0.0003
20°	4.163	0.999 ± 0.0003	200°	4.169	1.000 ± 0.0003
30°	4.087	0.980 ± 0.0003	210°	4.103	0.984 ± 0.0003
40°	3.797	0.911 ± 0.0003	220°	3.827	0.918 ± 0.0003
50°	3.313	0.795 ± 0.0003	230°	3.551	0.852 ± 0.0003
60°	2.700	0.648 ± 0.0003	240°	2.765	0.663 ± 0.0003
70°	2.056	0.493 ± 0.0003	250°	2.108	0.506 ± 0.0003
80°	1.433	0.344 ± 0.0003	260°	1.478	0.355 ± 0.0003
90°	0.981	0.235 ± 0.0002	270°	0.953	0.229 ± 0.0002
100°	0.626	0.150 ± 0.0002	280°	0.604	0.145 ± 0.0002
110°	0.461	0.111 ± 0.0002	290°	0.452	0.108 ± 0.0002
120°	0.518	0.124 ± 0.0002	300°	0.518	0.124 ± 0.0002
130°	0.791	0.190 ± 0.0002	310°	0.801	0.192 ± 0.0002
140°	1.261	0.302 ± 0.0003	320°	1.292	0.310 ± 0.0003
150°	1.828	0.438 ± 0.0003	330°	1.854	0.445 ± 0.0003
160°	2.482	0.595 ± 0.0003	340°	2.550	0.612 ± 0.0003
170°	3.117	0.748 ± 0.0003	350°	3.142	0.754 ± 0.0003

Table 4: Measurement data for the 1/4-wave plate (rotation angle 20°).

The relation between rotation angle and light intensity was plotted in Figure 10.

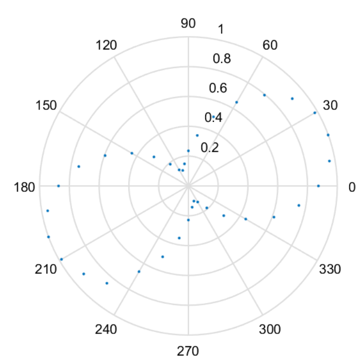

Figure 10: θ vs. I/I_0 graph.

The measurement of current I was shown in Table 5.

Rotation angle of 1/4-wave plate: 45°					
Maximum Electric Current I_0 2.452 ± 0.001 $[\mu A]$					
θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0	θ	$I[\mu A] \pm 0.01[\mu A]$	I/I_0
0°	2.350	0.958 ± 0.0006	180°	2.349	0.958 ± 0.0006
10°	2.388	0.974 ± 0.0006	190°	2.390	0.975 ± 0.0006
20°	2.421	0.987 ± 0.0006	200°	2.431	0.991 ± 0.0006
30°	2.441	0.996 ± 0.0006	210°	2.444	0.997 ± 0.0006
40°	2.446	0.998 ± 0.0006	220°	2.451	1.000 ± 0.0006
50°	2.434	0.993 ± 0.0006	230°	2.441	0.996 ± 0.0006
60°	2.410	0.983 ± 0.0006	240°	2.420	0.987 ± 0.0006
70°	2.373	0.968 ± 0.0006	250°	2.382	0.971 ± 0.0006
80°	2.333	0.951 ± 0.0006	260°	2.333	0.951 ± 0.0006
90°	2.297	0.937 ± 0.0006	270°	2.295	0.936 ± 0.0006
100°	2.255	0.920 ± 0.0006	280°	2.251	0.918 ± 0.0006
110°	2.224	0.907 ± 0.0006	290°	2.222	0.906 ± 0.0006
120°	2.211	0.902 ± 0.0005	300°	2.199	0.897 ± 0.0005
130°	2.206	0.900 ± 0.0005	310°	2.192	0.894 ± 0.0005
140°	2.214	0.903 ± 0.0005	320°	2.204	0.899 ± 0.0005
150°	2.237	0.912 ± 0.0006	330°	2.232	0.910 ± 0.0006
160°	2.267	0.925 ± 0.0006	340°	2.267	0.925 ± 0.0006
170°	2.313	0.943 ± 0.0006	350°	2.303	0.939 ± 0.0006

Table 5: Measurement data for the 1/4-wave plate (rotation angle 45°).

The relation between rotation angle and light intensity was plotted in Figure 11.

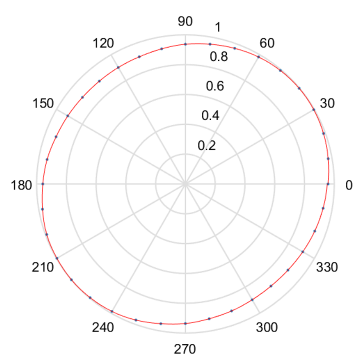

Figure 11: θ vs. I/I_0 graph.

The measurement of current I was shown in Table 6.

Rotating angle of the 1/2-wave plate: 70°	
$\theta[°] \pm 2[°]$	161°
$I[\mu A] \pm 0.01[\mu A]$	4.114

Table 6: Measurement data for the 1/2-wave plate (rotation angle 70°).

The maximum I when the angle is 20°, $\theta_1 \approx 180°$

The maximum I when the angle is 70°, $\theta_1 \approx 161°$

$$\theta_1 + \theta_2 = 341°$$

which proves that the theorem is true.

4 Measurement uncertainty analysis

The uncertainty of $\cos^2\theta$ can be found by applying the uncertainty propagation formula

$$u_{\cos^2\theta} = 2sin\theta \cos\theta u_\theta$$

The uncertainty of I/I_0 can be found by applying the uncertainty propagation formula

$$\begin{aligned} u_{I/I_0} &= \sqrt{\left(\frac{\partial I/I_0}{\partial I}\right)^2 u_I^2 + \left(\frac{\partial I/I_0}{\partial I_0}\right)^2 u_{I_0}^2} \\ &= \sqrt{\left(\frac{1}{I_0}\right)^2 u_I^2 + \left(\frac{I}{I_0^2}\right)^2 u_{I_0}^2} \end{aligned}$$

5 Conclusion

In this lab, we understand some properties of light, in particular to study the polarization phenomenon and verify Malus' law, the way half- and quarter-wave plates work in optical systems and generation and detection of elliptically and circularly polarized light.

A visible effect in the light coming out of a polarization device is a change of the light brightness.

$$I_{light} = I_{light,0}cos^2\theta$$

Since the amplitudes of the o-wave and the e-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

If $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;

If $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;

If $\alpha = \pi/4$,the transmitted light is circularly polarized;

Otherwise, the transmitted light is elliptically polarized.

6 Reference

(a) Qin Tian, Cao Jianjun, Mateusz Krzyzosiak, VP241 Exercise 3, Polarization of Light, based on materials provided by the Department of Physics, Shanghai Jiaotong University.

7 Data sheet

The Data sheet is attached at the end of the report.