UM–SJTU Joint Institute

Physics Laboratory
(VP241)

Laboratory Report

Exercise 2
The Hall Probe: Characteristics and Applications

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1 Objectives

In 1879 E.H. Hall observed that when an electric current passes through a sample placed in a magnetic field, electric potential difference proportional to the current and to the magnetic field appears across the material in the direction perpendicular to both the current and the magnetic field. This effect is known as the Hall effect, and since its discovery it has found many practical applications. The principle of the Hall effect is used in devices for magnetic field measurements as well as in position and motion detectors.

The objective of this exercise is to study the principle of the Hall effect and its applications by using a Hall probe. In particular, it will be verified that the Hall voltage is proportional to the magnetic field. Furthermore, the sensitivity of an integrated Hall probe will be studied by calculating the magnetic field at the center of a solenoid, and the magnetic field distribution along the axis of the solenoid will be measured and compared with the corresponding theoretical curve.

2 Theoretical Background

2.1 Hall Effect

Consider a conducting sheet (made of a metal or a semiconductor) placed in a magnetic field so that the plane of the sheet is perpendicular to the direction of the magnetic field B (Figure 1). When the electric current I passes through the sheet in the direction shown in Figure 1, an electric potential difference between the sides a and b of the sheet is generated. The corresponding electric field is perpendicular to both the direction of the current and the direction of the magnetic field. This effect is known as the Hall effect, and the electric potential difference is called the Hall voltage U_H.

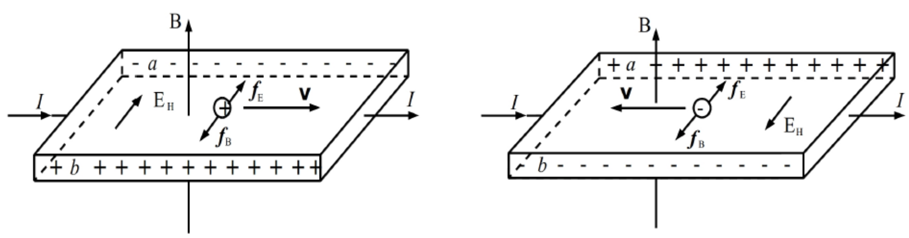

Figure 1: The principle of the Hall effect.

Microscopically, the Hall effect is caused by the Lorentz force, that is a force acting on charges moving in a magnetic field. The Lorentz force F_B leads to the deflection of the moving charges, and their accumulation on one side of the sheet, which in turn increases the magnitude of the transverse electric field E_H. Due to this field, there is an electric force F_E acting upon the charges, and since F_B and F_E act in opposite directions, a balance is eventually reached and U_H stabilizes.

When the external magnetic field is not too strong, the Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

where R_H is the so-called Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the so-called sensitivity of the Hall element.

2.2 Integrated Hall Probe

The magnitude of the magnetic field can be found by measuring the Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed. Since the Hall voltage is usually very small, it should be amplified before the measurement.

Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. Such a device is called an integrated Hall probe.

The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator (Figure 2). The output voltage U can be read ignoring the residual voltage. The working voltage $U_S = 5V$, and the output voltage U_0 is approximately $2.5V$ when the magnetic field is zero. The relation between the output voltage U and the magnitude of the magnetic field is

$$B = \frac{U - U_0}{K_H} \tag{2}$$

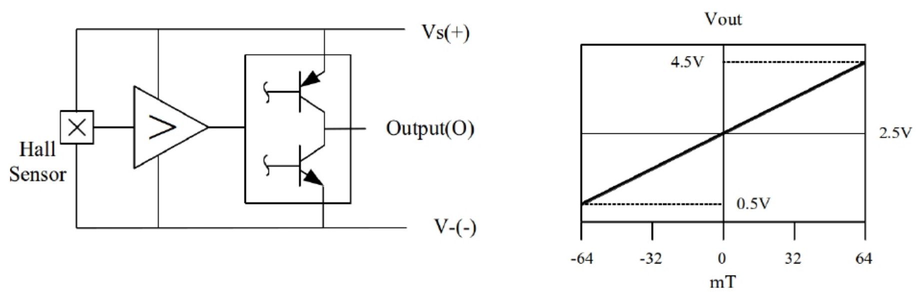

Figure 2: The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

2.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L+2x}{2[D^2 + (L+2x)^2]^{\frac{1}{2}}} + \frac{L-2x}{2[D^2 + (L-2x)^2]^{\frac{1}{2}}} \right\} = C(x) I_M \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7} H/m$

The solenoid used in this exercise has ten layers, and the magnetic field $B(x)$ for each layer can be calculated using Eq. (3). Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_M = 0.1A$ is given in Table 1.

x[cm]	B[mT]	x[cm]	B[mT]
±0.0	1.4366	±8.0	1.4057
±1.0	1.4363	±9.0	1.3856
±2.0	1.4356	±10.0	1.3478
±3.0	1.4343	±11.0	1.2685
±4.0	1.4323	±11.5	1.1963
±5.0	1.4292	±12.0	1.0863
±6.0	1.4245	±12.5	0.9261
±7.0	1.4173	±13.0	0.7233

Table 1: Theoretical value of the magnetic field inside the solenoid.

3 Measurement Setup and Procedure

3.1 Apparatus

The experimental setup shown in Figure 5 consists of an integrated Hall probe SS495A (see Figure 4) with $K_H = 31.25 \pm 1.25 V/T$ or $K_H = 3.125 \pm 0.125 mV/G$, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

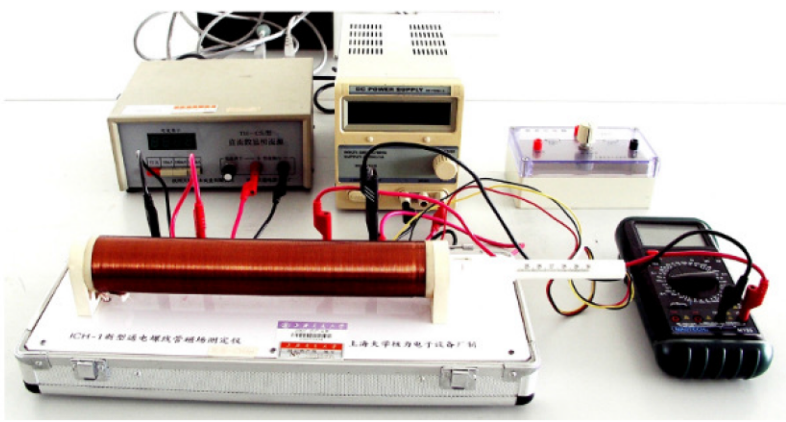

Figure 3: Measurement setup.

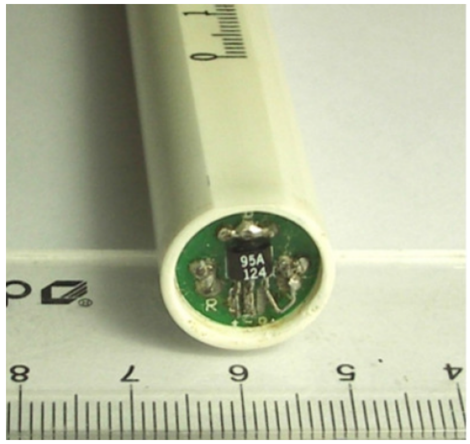

Figure 4: Integrated Hall probe SS495A.

3.2 Measurement Procedure

3.2.1 Relation Between Sensitivity K_H and Working Voltage U_S

1. Place the integrated Hall probe at the center of the solenoid. Set the working voltage at 5 V and measure the output voltage $U_0(I_M = 0)$ and $U(I_M = 250mA)$. Take the theoretical value of $B(x = 0)$ from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).

2. Measure K_H for different values of U_S (from $2.8V$ to $10V$). Calculate K_H/U_S and plot the curve $K_H/U_S vs. U_S$.

3.2.2 Relation Between Output Voltage U and Magnetic Field B

1. With $B = 0, U_S = 5V$, connect the $2.4 \sim 2.6V$ output terminal of the DC voltage divider and the negative port of the voltmeter. Adjust the voltage until $U_0 = 0$.

2. Place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to $500mA$, with intervals of $50mA$.

3. Explain the relation between $B(x = 0)$ and the Hall voltage U_H. Pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of $B(x = 0)$ can be found from Table 1.

4. Plot the curve $U vs. B$ and find the sensitivity K_H by a linear fit (use a computer). Compare the value you obtained with the theoretical value in given in the Apparatus section.

3.2.3 Magnetic Field Distribution Inside the Solenoid

1. Measure the magnetic field distribution along the axis of the solenoid for $I_M = 250mA$, record the output voltage U and the corresponding position x. Then find $B = B(x)$. (Use the value of K_H found in the previous part of the experiment).

2. Use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. Use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

4 Results

4.1 Relation Between Sensitivity K_H and Working Voltage U_S

The measurement of U was shown in Table 3.

U_S [V] $\pm 0.5\%$ V	$U_0(I_M = 0)$ [V] $\pm 0.5\% + 6 \times 10^{-3}$ V	$U_0(I_M = 250mA)$ [V] $\pm 0.5\% + 6 \times 10^{-3}$ V
5.00	2.588	2.696

Table 2: Data for U_0 and U with $U_S = 5V$.

Then we can calculate K_H

$$K_H = \frac{U - U_0}{B} = \frac{2.696 - 2.588}{2.5 \cdot 1.4366 \times 10^{-3}} = 30.071 \pm 2.882\,\mathrm{V/T}$$

The measurement of different U was shown in Table 4.

	U_S [V] $\pm 0.5\%$ V	U_0 [V] $\pm 0.5\% + 6 \times 10^{-3}$ V	U_0 [V] $\pm 0.5\% + 6 \times 10^{-3}$ V
1	2.77	1.4958	1.5433
2	3.25	1.6613	1.7254
3	3.78	1.9408	2.0198
4	4.24	2.185	2.276
5	4.77	2.463	2.566
6	5.26	2.722	2.837
7	5.77	2.988	3.115
8	6.27	3.252	3.391
9	6.73	3.492	3.641
10	7.27	3.773	3.932
11	7.76	4.030	4.200
12	8.26	4.291	4.473
13	9.39	4.877	5.084
14	9.90	5.146	5.362

Table 3: Data for U_0 and U with $U_S = 5V$.

Then we can calculate K_H and K_H/U_S in in Table 5.
The curve U_S vs. K_H/U_S was plotted in Figure 5.

	K_H [V/T]	K_H/U_S [1/T]
1	13.226 ± 2.662	4.775 ± 0.961
2	17.848 ± 2.696	5.492 ± 0.830
3	21.996 ± 2.753	5.819 ± 0.728
4	25.338 ± 2.802	5.976 ± 0.661
5	28.679 ± 2.858	6.012 ± 0.599
6	32.020 ± 2.910	6.087 ± 0.553
7	35.361 ± 2.963	6.128 ± 0.514
8	38.702 ± 3.017	6.173 ± 0.481
9	41.487 ± 3.065	6.164 ± 0.455
10	44.271 ± 3.121	6.090 ± 0.429
11	47.334 ± 3.173	6.100 ± 0.409
12	50.675 ± 3.225	6.135 ± 0.390
13	57.636 ± 3.343	6.138 ± 0.356
14	60.142 ± 3.397	6.075 ± 0.343

Table 4: K_H and K_H/U_S.

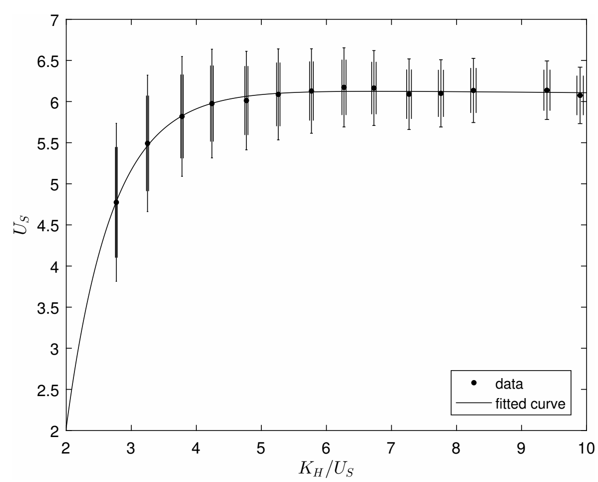

Figure 5: U_S vs. K_H/U_S.

4.2 Relation Between Output Voltage U and Magnetic Field B

The measurement of U was shown in Table 6.

	I_M [mA] $\pm$ 2% [mA]	U [V] $\pm 0.05\% + 6 \times 10^{-3}$ [V]
1	0.000	0.000
2	50.000	0.021
3	100.000	0.043
4	150.000	0.065
5	200.000	0.086
6	250.000	0.109
7	300.000	0.130
8	350.000	0.152
9	400.000	0.174
10	450.000	0.196
11	500.000	0.217

Table 5: Measurement data for the I_M vs. U relation.

$$B = \frac{U}{K_H} = \frac{kU_H}{K_H}$$

Then we can make a plot of U vs. B and find K_H, it was shown in Figure 6.

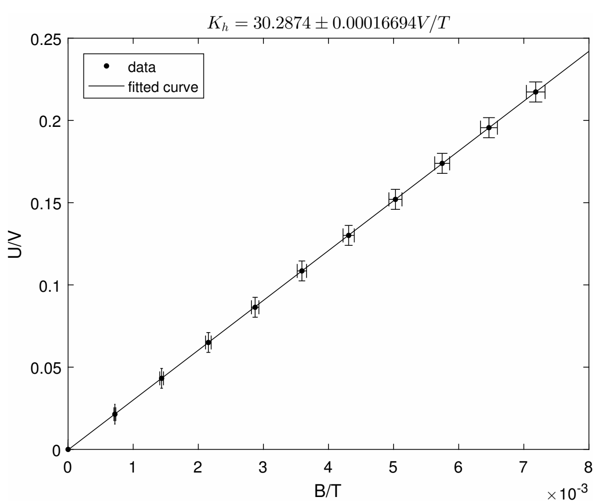

Figure 6: U vs. B.

So $K_H = 30.2874 \pm 0.00017\,\mathrm{V/T}$

4.3 Magnetic Field Distribution Inside the Solenoid

The measurement of U was shown in Table 7.

	x[cm]$\pm$0.05[cm]	U[V]$\pm 0.05\% + 6 \times 10^{-4}$[V]		x[cm]$\pm$0.05[cm]	U[V]$\pm 0.05\% + 6 \times 10^{-4}$[V]
1	0.00	1.7565	25	14.40	3.5890
2	0.60	2.3970	26	15.00	3.5890
3	1.20	2.8263	27	15.60	3.5856
4	1.80	3.0970	28	16.20	3.5823
5	2.40	3.2621	29	16.80	3.5790
6	3.00	3.3578	30	17.40	3.5790
7	3.60	3.4239	31	18.00	3.5724
8	4.20	3.4668	32	18.60	3.5658
9	4.80	3.4965	33	19.20	3.5559
10	5.40	3.5196	34	19.80	3.5460
11	6.00	3.5328	35	20.40	3.5262
12	6.60	3.5460	36	21.00	3.5097
13	7.20	3.5559	37	21.60	3.4899
14	7.80	3.5658	38	22.20	3.4536
15	8.40	3.5724	39	22.80	3.4008
16	9.00	3.5757	40	23.40	3.3248
17	9.60	3.5790	41	24.00	3.1927
18	10.20	3.5790	42	24.60	2.9715
19	10.80	3.5823	43	25.20	2.6282
20	11.40	3.5823	44	25.80	2.0768
21	12.00	3.5823	45	26.40	1.4461
22	12.60	3.5823	46	27.00	0.9311
23	13.20	3.5856	47	27.60	0.5976
24	13.80	3.5856			

Table 6: Data for he U vs. x relation.

The theoretical and the experimental curve showing the magnetic field distribution inside the solenoid was shown in Figure 7.

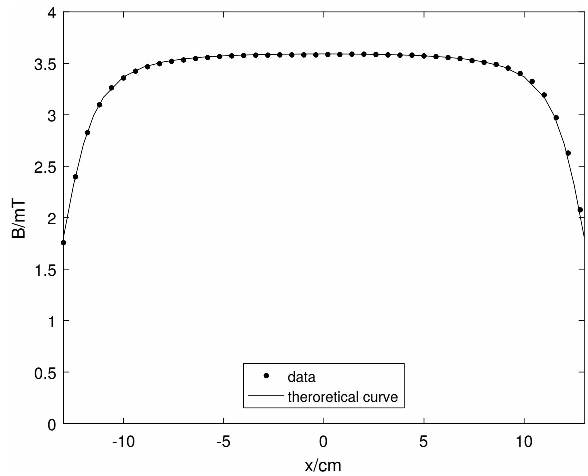

Figure 7: $B(x)$.

5 Measurement uncertainty analysis

5.1 Relation Between Sensitivity K_H and Working Voltage U_S

K_H can be calculated by the equation $K_H = \frac{U-U_0}{B}$. Therefore its uncertainty u_{K_H} is found by applying the uncertainty propagation formula

$$\begin{aligned} u_{K_H} &= \sqrt{\left(\frac{\partial K_H}{\partial U}\right)^2 u_U^2 + \left(\frac{\partial K_H}{\partial U_0}\right)^2 u_{U_0}^2} \\ &= \sqrt{\left(\frac{1}{B}\right)^2 u_U^2 + \left(\frac{1}{B}\right)^2 u_{U_0}^2} \\ &= 2.882\,\mathrm{V/T} \end{aligned}$$

So $K_H = 30.071 \pm 2.882\,\mathrm{V/T}$

The uncertainty u_{K_H/U_S} is found by applying the uncertainty propagation formula

$$\begin{aligned} u_{K_H/U_S} &= \sqrt{\left(\frac{\partial K_H/U_S}{\partial K_H}\right)^2 u_{K_H}^2 + \left(\frac{\partial K_H/U_S}{\partial U_S}\right)^2 u_{U_S}^2} \\ &= \sqrt{\left(\frac{1}{U_S}\right)^2 u_{K_H}^2 + \left(\frac{1}{U_S^2}\right)^2 u_{U_S}^2} \end{aligned}$$

The uncertainty of K_H and K_H/U_S was listed in the results above.

5.2 Relation Between Output Voltage U and Magnetic Field B

The uncertainty of the curve of the form rmse was calculated by MATLAB and was plotted in the figure.

5.3 Magnetic Field Distribution Inside the Solenoid

B can be calculated by the equation $B = \frac{U}{K_H}$. Therefore its uncertainty u_{K_H} is found by applying the uncertainty propagation formula

$$\begin{aligned} u_B &= \sqrt{\left(\frac{\partial B}{\partial U}\right)^2 u_U^2 + \left(\frac{\partial B}{\partial K_H}\right)^2 u_{K_H}^2} \\ &= \sqrt{\left(\frac{1}{K_H}\right)^2 u_U^2 + \left(\frac{1}{K_H^2}\right)^2 u_{K_H}^2} \end{aligned}$$

The uncertainty was calculated by MATLAB and was plotted in the figure.

6 Conclusion

I studied the principle of the Hall effect and its applications by using a Hall probe. In particular, it will be verified that the Hall voltage is proportional to the magnetic field. Furthermore, the sensitivity of an integrated Hall probe will be studied by calculating the magnetic field at the center of a solenoid, and the magnetic field distribution along the axis of the solenoid will be measured and compared with the corresponding theoretical curve.

The Hall Effect can be deduced into the formula

$$U_H = R_H \frac{IB}{d} = KIB$$

and

$$B = \frac{U - U_0}{K_H}$$

The magnetic field distribution inside a solenoid can be concluded in the the following formula

$$B(x) = \mu_0 \frac{N}{L} I_M \left\{ \frac{L + 2x}{2[D^2 + (L + 2x)^2]^{\frac{1}{2}}} + \frac{L - 2x}{2[D^2 + (L - 2x)^2]^{\frac{1}{2}}} \right\} = C(x) I_M$$

7 Reference

(a) Qin Tian, Wang Zhiyu, Mateusz Krzyzosiak, VP241 Exercise 2, The Hall Probe: Characteristics and Applications, based on materials provided by the Department of Physics, Shanghai Jiaotong University.

8 Data sheet

The Data sheet is attached at the end of the report.