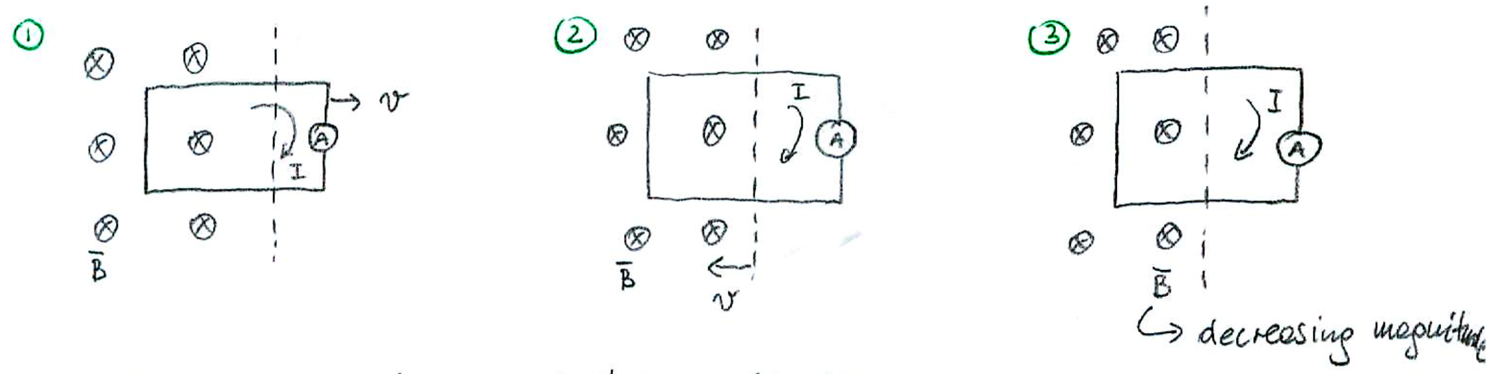
Electrodynamics. Motional emf and electromagnetic induction

Introduction : 3 experiments

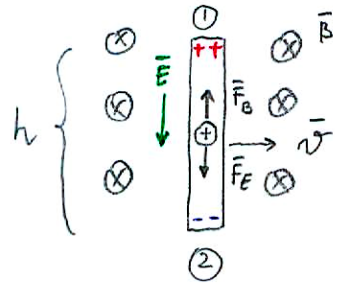


Common effect: current in clockwise direction

~ ∘ ~

Motional emf (experiment ①)

Consider a metal bar moving in a uniform magnetic field

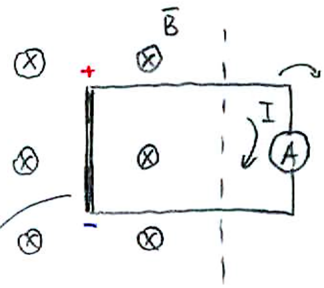


Potential difference

$$V_{12} = E \cdot h = v B h$$

$$(F_B = F_E)$$

After the circuit is closed ⇒ electric current



here charges flow from higher to lower potential

here charges flow from lower to higher potential

Compare with a battery

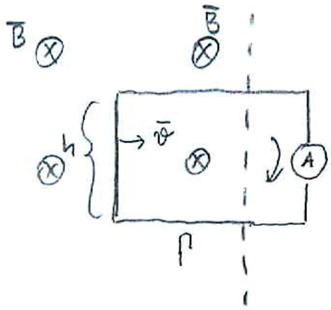
chemical processes

lower potential → higher potential

Here the role of "chemical processes" is played by the Lorentz force.

$$V_{12} = \mathcal{E} \quad \text{motional electromotive force}$$

Motional emf



elementary work per unit charge

$$\mathcal{E} = \oint_{\Gamma} (\bar{v} \times \bar{B}) \cdot d\bar{l}$$

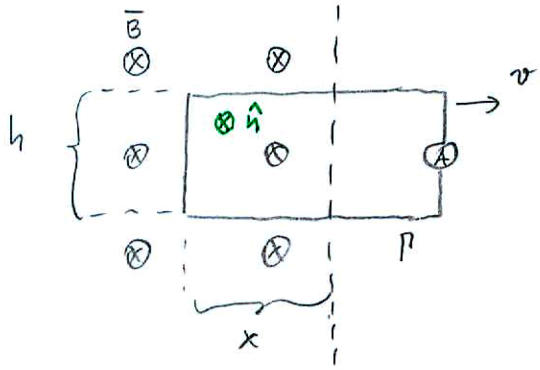
$\Gamma \rightarrow$ at an instant of time

In this case $\mathcal{E} = vB \cdot h$ (contribution only due to the bar)

What does work here? Pulling force. Note: Current flow in the direction to oppose the pulling force (magnetic force on the bar acts to the left)

~ o ~

Flux rule for motional emf



Flux through the surface Σ bounded by Γ

$$\Phi_B = \int_{\Sigma} \bar{B} \cdot d\bar{A} = B \times h$$

(positive orientation into the page)

Loop moving $\Rightarrow$ flux decreases

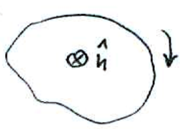
$$\frac{d\Phi_B}{dt} = Bh\frac{dx}{dt} = -\underbrace{Bhv}_{\mathcal{E}}$$

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

flux rule for motional emf

Comments: (1) Valid also for non-rectangular loops (loop may also change its shape), arbitrary direction of motion, non-uniform magnetic field

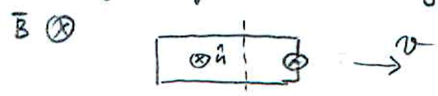
(2) sign of emf (current direction)



positive

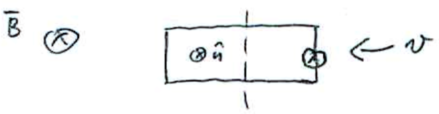
negative

(3) emf oposes change of the flux



$\Rightarrow$ Φ_B positive & decreasing $\Rightarrow \mathcal{E} > 0$

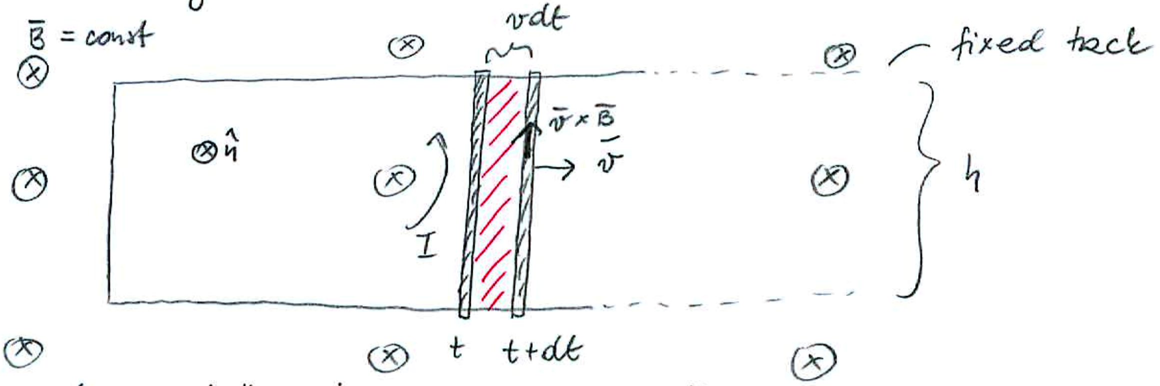
$\hookrightarrow$ generates $\bar{B}$ ⊗



$\Rightarrow$ Φ_B positive & increasing $\Rightarrow \mathcal{E} < 0$

generates $\bar{B}$ ⊙

Example. Slidewire generator – metal bar on a metal track



motional emf → only the moving parts (bar) contributes

$$\mathcal{E} = \oint_{\Gamma} (\bar{v}\times\bar{B})\, d\bar{l} =$$

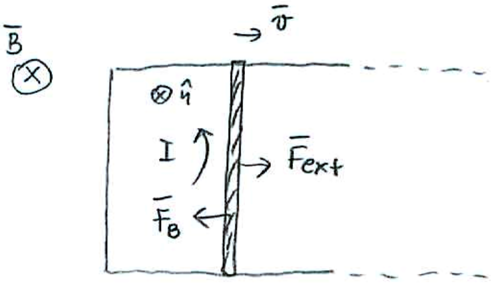
positive $\bar{v}\times\bar{B}$ ↓ ↑ orientation along the bar

$$= -vB\cdot h < 0 \Rightarrow \text{anticlockwise}$$

flux rule

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{Bhv\,dt}{dt} = -Bhv$$

Work & power



Force needed to move the bar with constant velocity

$$\bar{F}_{ext} = -\bar{F}_B = -I(\bar{l}\times\bar{B})$$

$$|F_{ext}| = IhB$$

Let R – resistance of the bar + u-shaped part of the circuit (assume const)

$$I = \frac{|\mathcal{E}|}{R}$$

$$F_{ext} = \frac{|\mathcal{E}|}{R} hB = \frac{B^2h^2v}{R}$$

Power provided

$$P_{ext} = F_{ext}\cdot v = \frac{B^2l^2v^2}{R}$$

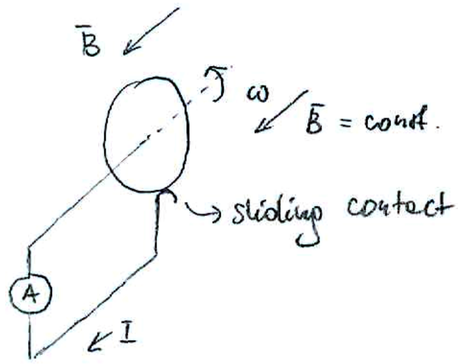
Power dissipated (due to resistance)

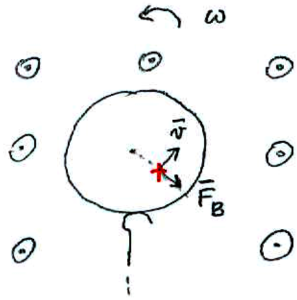
$$P_{diss} = I^2R = \frac{B^2h^2v^2}{R^2}\cdot R = \frac{B^2h^2v^2}{R}$$

Conclusions: (1) Mechanical energy (work done by external force pulling the bar) transformed into electrical energy

(2) Generated current opposes the process of its generation Lenz's Rule

Example Faraday disk dynamo – metal disk with radius R rotating with angular velocity ω; uniform magnetic field





Force per unit charge at a distance ℓ from the axis

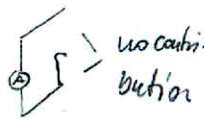
$$\frac{\bar{F}_B}{q} = \bar{v} \times \bar{B} = \omega \ell B \,\hat{\ell}$$

along radius

Motional emf

$$\mathcal{E} = \int_0^R \omega \ell B \, d\ell = \frac{1}{2} \omega B R^2$$

(no definite path the current flows along; non-zero contribution from the disk only



$$\boxed{\mathcal{E} = \frac{1}{2} \omega B R^2}$$

DC-generator

Example Alternator – AC generator

FiG

$$\Phi_B = \bar{B} \cdot \bar{A} = BA \cos\varphi = BA \cos \omega t$$

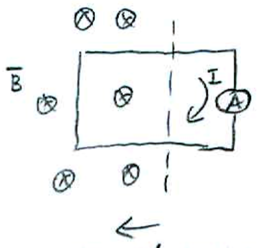
area of the loop

Flux rule

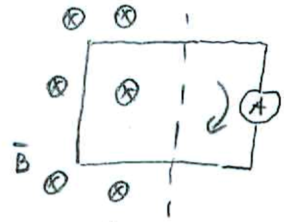
$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega BA \sin \omega t}$$

Electromagnetic induction, Faraday's law

Experiments ② and ③ – loop is stationary, magnetic field changes



moved away



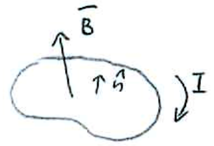
↳ decreasing magnitude

stationary loop!

Experimental observations – electromagnetic induction

$\bar{B}$ - uniform, Σ - plane surface bounded by a stationary loop (wire)

$|\bar{B}|$ - increasing

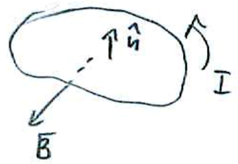


flux $\Phi_B > 0$

and $\frac{d\Phi_B}{dt} > 0$

$\Downarrow$

$\mathcal{E} < 0$ (clockwise)



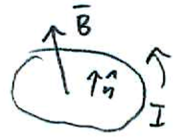
flux $\Phi_B < 0$

and $\frac{d\Phi_B}{dt} < 0$

$\Downarrow$

$\mathcal{E} > 0$ (anticlockwise)

$|\bar{B}|$ - decreasing



$\Phi_B > 0$

$\frac{d\Phi_B}{dt} < 0$

$\Downarrow$

$\mathcal{E} > 0$

(anticlockwise)

$\Phi_B < 0$

$\frac{d\Phi_B}{dt} > 0$

$\Downarrow$

$\mathcal{E} < 0$

(clockwise)

Faraday's Law

change of the magnetic flux

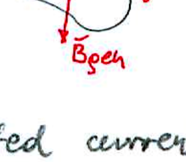
$\Downarrow$

emf

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

induced emf

Comment: Again, the generated current (emf) opposes the process of its generation.



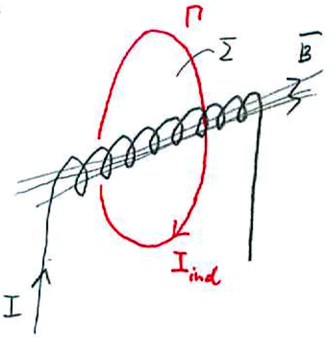
$|\bar{B}|$ - increasing

Lenz's Rule: the direction of the generated current (both for motional emf and induced emf) is such as to oppose the process of its generation

universal flux rule $\mathcal{E} = -\frac{d\Phi_B}{dt}$ → motional emf (moving loop) → induced emf (stationary loop)

Important question: What is driving the charges flowing along the loop?

Experiment



$\frac{dI}{dt} \neq 0 \Rightarrow \bar{B}$ - changes

cross-sectional area of the solenoid: A

$$B = \mu_0 n I$$

$$\Phi_B = B \cdot A = \mu_0 n I A$$

↳ through Σ

$$-\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt} = \mathcal{E} \quad \left(\text{positive if } \frac{dI}{dt} < 0\right)$$

Can the magnetic force drive the charges? No, $\bar{B} = 0$ at Γ.

There has to be an induced electric field pushing the charges along the loop.

conservative? No! why no radial component?

Conclusion: time-dependent magnetic flux generates electric field

$$\mathcal{E} = \oint_\Gamma \bar{E}\, d\bar{l} \overset{\text{Stokes}}{=} \int_\Sigma (\nabla \times \bar{E})\, d\bar{A}$$

(stationary loop)

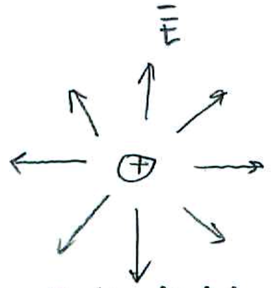
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_\Sigma \bar{B}\, d\bar{A} = \int_\Sigma \left(-\frac{\partial \bar{B}}{\partial t}\right) d\bar{A}$$

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

non-conservative in general ($\text{rot}\, \bar{E} \neq 0$)

CONSERVATIVE

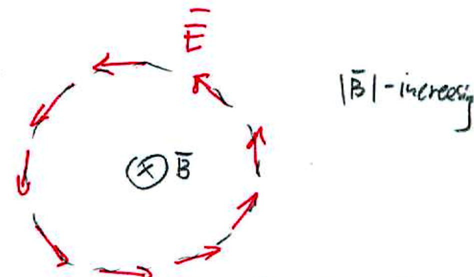
$\oint \bar{E}\, d\bar{l} = 0$



static field (source: static charge)

NON-CONSERVATIVE

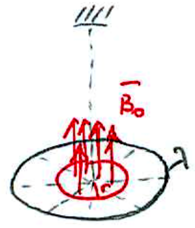
$\oint \bar{E} \cdot d\bar{l} \neq 0$



source: time-dependent magnetic flux

$$\mathcal{E} = \underbrace{\oint_\Gamma \bar{E} \cdot d\bar{l}}_{\text{"transformer emf" due to change in B}} + \underbrace{\oint_\Gamma (\bar{v} \times \bar{B})\, d\bar{l}}_{\text{"motional emf" due to motion of the loop (zero if stationary loop)}} = -\frac{d\Phi_B}{dt}$$

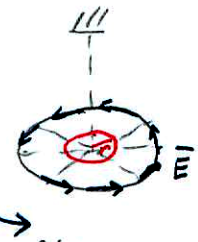
Example



charge of linear density λ "glued" onto the rim of a wheel with radius R; magnetic field $\bar{B}_0$ in the central region (radius r)

What happens when the field is turned off?

Change in the magnetic field $\Rightarrow$ induced electric field $\Rightarrow$ wheel rotates ? Lenz's rule for direction (rotating charge $\equiv$ current loop)



tries to keep the flux as $|\bar{B}|$ decreases

Qualitative analysis:

$$\oint_\Gamma \bar{E}\, d\bar{l} = -\frac{d\Phi_B}{dt} = -\pi r^2 \frac{dB}{dt}$$

FARADAY'S LAW

Torque on an infinitesimal element of the wheel $\bar{R} \times \underbrace{\lambda\, dl}_{dq} \bar{E}$ pointing upwards $\bar{\tau}$

Total torque (magnitude)

$$\tau = \oint_\Gamma |\bar{R} \times \lambda\, dl\, \bar{E}| = R\lambda \oint_\Gamma E\, dl = -\pi R r^2 \lambda \frac{dB}{dt}$$

Angular momentum acquired by the wheel after field drops to zero (magnitude)

$$L_0 = \int_{t_0}^{t_f} \tau\, dt = -\pi R^2 r \lambda \int_{t_0}^{t_f} \frac{dB}{dt}\, dt = -\pi R^2 r \lambda \int_{B_0}^{0} dB =$$

$$= \underline{\underline{\pi R^2 r \lambda B_0}}$$

Where does this angular momentum come from?