VP260 PROBLEM SET 11

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Problem 1.

(a)

$$U = \bar{E}d = \bar{I}R = I\frac{\rho d}{\pi a^2}\hat{n_I}$$
$$\bar{E} = \frac{\rho I}{\pi a^2}\hat{n_I}$$
$$\bar{B} = \frac{\mu_0 I}{2\pi a}\hat{n_\theta}$$

(b)

$$S = \frac{1}{\mu_0}(\bar{E} \times \bar{B}) = -\frac{\rho I^2}{2\pi^2 a^3}\hat{n_r}$$

(c)

$$\frac{dE}{dt} = -S \cdot 2\pi a l = \frac{\rho l I^2}{\pi a^2}$$

(d)

$$\frac{dQ}{dt} = P = I^2 R = I^2 \frac{\rho l}{\pi a^2} = \frac{\rho l I^2}{\pi a^2}$$

So they are the same, because energy in the conductor is transformed into heat so that it should obtain energy from outside in preserve the constant current I.

Problem 2.

Suppose the incident angle of ray A is θ, then the reflection angle of A is θ and the refraction angle of A is $\frac{n_1}{n_2}\theta$. The reflection angle of the refraction ray is also $\frac{n_1}{n_2}\theta$. When the refraction ray refracts again, the refraction angle is $\frac{n_2}{n_1}\frac{n_1}{n_2}\theta = \theta$, which is same as the reflection angle of A, so they are parallel to each other.

Problem 3.

Suppose the initial direction is $x\hat{n_x} + y\hat{n_y} + z\hat{n_z}$.
If it reflects on the xy-plane, the direction of z-axis will become opposite.
If it reflects on the xz-plane, the direction of y-axis will become opposite.
If it reflects on the yz-plane, the direction of x-axis will become opposite.
So when the ray reflects on all of the three planes, the final direction is $-x\hat{n_x} - y\hat{n_y} - z\hat{n_z}$, which is opposite to the initial direction.

Problem 4.

(a) We can't directly rotate $90°$ since $I_0 \cos^2 90° = 0$, so at least two sheets are required.

(b)

$$I_0 \prod_{i=1}^{n} \cos^2 \theta_i \geqslant 0.6 I_0$$

$$\prod_{i=1}^{n} \cos \theta_i \geqslant \sqrt{0.6}$$

According to fundamental inequality,

$$\prod_{i=1}^{n} \cos \theta_i \leqslant \frac{1}{n} \left(\sum_{i=1}^{n} \cos \theta_i \right)^n$$

if and only if $\theta_1 = \theta_2 = \cdots = \theta_n$, it gets the maximum.
And we know $\sum_{i=1}^{n} \theta_i = 90°$, so $\theta_i = \frac{90}{n}°$

$$\cos^n \frac{90}{n}° \geqslant \sqrt{0.6}$$

$$n \geqslant 5$$

so at least five sheets are required.

Problem 5.

If $|PS_2| - |PS_1| = m\lambda, m \in Z \backslash \{0\}$, it is hyperbola.
If $|PS_2| - |PS_1| = \frac{2m+1}{2}\lambda, m \in Z \backslash \{0\}$, it is hyperbola.

Problem 6.

(a)

$$E_1 = E \cos(kr - \omega t)$$

$$E_2 = 2E \cos(kr - \omega t + \varphi)$$

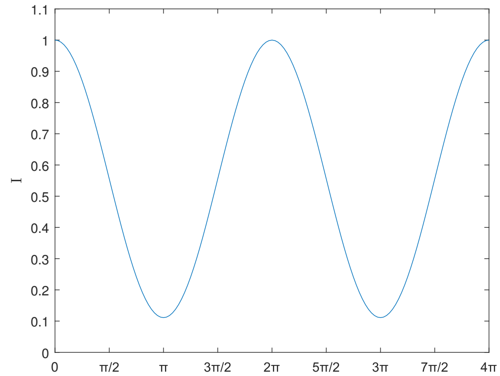
$$\begin{aligned} E_1 + E_2 &= E \cos(kr - \omega t)(2\cos\varphi + 1) - 2E \sin(kr - \omega t)\sin\varphi \\ &= E\sqrt{(2\cos\varphi + 1)^2 + (2\sin\varphi)^2} \sin\left(-\arctan\frac{2\cos\varphi + 1}{2\sin\varphi}\right) \\ &= E\sqrt{5 + 4\cos\varphi} \sin\left(-\arctan\frac{2\cos\varphi + 1}{2\sin\varphi}\right) \end{aligned}$$

$$I = \frac{E_0^2}{2\mu_0 c} = \frac{E^2}{2\mu_0 c}(5 + 4\cos\varphi)$$

$$I_{max} = \frac{9E^2}{2\mu_0 c}$$

$$I = I_{max}\left(\frac{5}{9} + \frac{4}{9}\cos\varphi\right)$$

(b)



$$I_{min} = \frac{1}{9} I_m ax$$

$$\varphi = (2k+1)\pi, k \in Z$$

Problem 7.

$$I_{max} \cos^2 \frac{\pi dy}{\lambda l} = \frac{1}{2} I_{max}$$

$$\cos \frac{\pi dy}{\lambda l} = \pm \frac{\sqrt{2}}{2}$$

$$\frac{\pi dy}{\lambda l} = \frac{(2k+1)\pi}{4}, k \in Z$$

$$y = \frac{(2k+1)\lambda l}{4d}$$

For every k,

$$\Delta\theta_m = \frac{\Delta y}{l} = \frac{\lambda}{2d}$$

It doesn't depend on m.

Problem 8.

$$r_2 - r_1 = \frac{\lambda}{2}$$

$$2nh = \frac{\lambda}{2}$$

$$h = \frac{\lambda}{4n} = 109.72\,\mathrm{nm}$$