PROBLEM SET 9

Due: 24 November 2016, 2 p.m.

Problem 1. In a *perfect conductor*, the conductivity is infinite, so $\mathbf{E} = 0$, and any net charge resides on the surface (just as it does for an imperfect conductor in *electrostatics*). (a) Show that the magnetic field is constant, *i.e.* $\frac{\partial \mathbf{B}}{\partial t} = 0$, inside a perfect conductor. (b) Show that the magnetic flux through a perfectly conducting loop is constant.

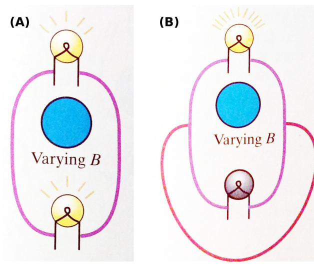
(*2 + 2 marks*)

Problem 2. A *superconductor* is a perfect conductor with the additional property that the (constant) magnetic field $\mathbf{B}$ inside is in fact zero. (This "flux exclusion", known as the Meissner effect, was shown in one of the videos we watched in class.)

Show that the electric current in a superconductor is confined to the surface.

(*2 marks*)

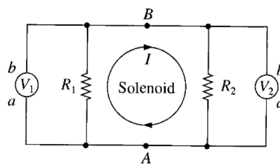
Problem 3. Consider two light bulbs connected in series around a solenoid (see figure (A)) producing a sinusoidally varying magnetic field. If we alter the circuit by connecting a thick copper wire across the circuit (see figure (B)), we find that the top bulb gets much brighter and the bottom one no longer glows.



Why does this happen?

(*2 marks*)

Problem 4. Consider a circuit shown in the figure below. The current in a long solenoid piercing the plane of the circuit is increasing linearly with time, so that the magnetic flux through the cross-section of the solenoid $\Phi_B = \alpha t$. Two voltmeters are connected to diametrically opposite points A and B, together with resistors R_1 and R_2.

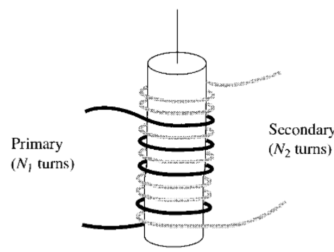


Show that the readings on voltmeters are $V_1 = \frac{\alpha R_1}{R_1+R_2}$ and $V_2 = -\frac{\alpha R_2}{R_1+R_2}$, respectively, *i.e.* $V_1 \neq V_2$, even though they are connected to the same points.

Assume that these are ideal voltmeters that draw negligible current (*i.e.* have huge resistance), and that a voltmeter registers $\int_a^b \mathbf{E} \cdot d\mathbf{l}$ between the terminals and through the meter.

(*3 marks*)

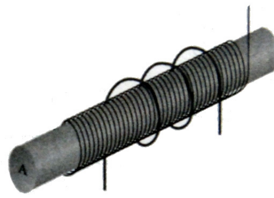
Problem 5. Two coils are wrapped around a cylindrical form in such a way that the same flux passes through every turn of both coils. (In practice this is achieved by inserting an iron core through the cylinder; this has the effect of concentrating the flux.) The "primary" coil has N_1 turns and the "secondary" has N_2. If the current I in the primary is changing, show that the emf in the secondary is given by $\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1$, where $\mathcal{E}_1$ is the (back emf) of the primary.



This is a primitive transformer — a device for raising or lowering the emf of an alternating current source. By choosing the appropriate number of turns, any desired secondary emf can be achieved.

(*2 marks*)

Problem 6. Two coils are wrapped around each other as shown in the figure below. The current travels in the same sense around each coil. One coil has self-inductance L_1, and the other coil has self-inductance L_2. The mutual inductance of the two coils is M. Show that if the two coils are connected in parallel, the equivalent inductance of the combination is $L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$.



(*3 marks*)

Problem 7. A transformer takes an input AC voltage of amplitude V_1, and delivers an output voltage of amplitude V_2, which is determined by the turns ratio, $V_2/V_1 = N_2/N_1$. If $N_2 > N_1$ the output voltage is greater than the input voltage. Why doesn't this violate conservation of energy? *Answer*: Power is the product of voltage and current; evidently if the voltage goes up, the current must come down.

The purpose of this problem is to see exactly how this works out, in a simplified model.

(a) In an ideal transformer the same flux passes through all turns of the primary and of the secondary. Show that in this case $M^2 = L_1 L_2$, where M is the mutual inductance of the coils and L_1, L_2 are there individual self-inductances.

(b) Suppose the primary is driven with AC voltage $V_{\text{in}} = V_1 \cos\omega t$, and the secondary connected to a resistor with resistance R. Show that the two currents satisfy the relations

$$L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} = V_1 \cos\omega t, \qquad L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt} = -I_2 R.$$

(c) Using the result in (a) solve the equations for I_1 and I_2. (Assume that I_1 has no DC component.)

(d) Show that the output voltage $V_{\text{out}} = I_2 R$ divided by the input voltage V_{in} is equal to the turns ratio, *i.e.* $V_{\text{out}}/V_{\text{in}} = N_2/N_1$.

(e) Calculate the input power $P_{\text{in}} = V_{\text{in}} I_1$ and the output power $P_{\text{out}} = V_{\text{out}} I_2$, and show that their averages over a full cycle are equal.

(*3/2 + 1 + 2 + 1 + 3/2 marks*)

Problem 8. In the circuit shown in the figure below, switch S is closed at time $t = 0$ with no charge initially on the capacitor.

(a) Find the reading of each ammeter and each voltmeter just after S is closed.

(b) Find the reading of each meter after a long time has elapsed.

(c) Find the maximum charge on the capacitor.

(d) Sketch a qualitative graph of the reading of voltmeter V_2 as a function of time.

(*4 × 1 marks*)