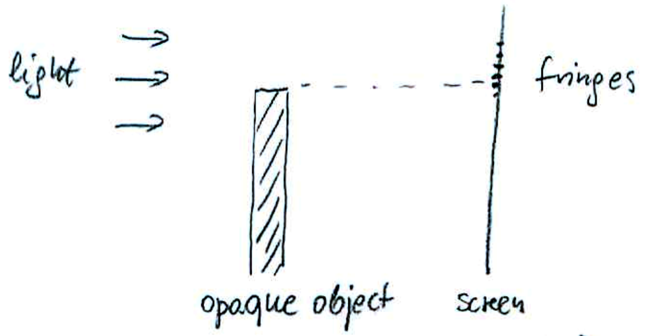
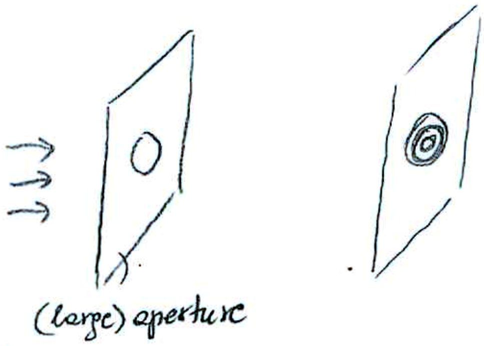
DIFFRACTION

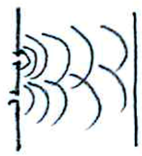




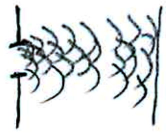
again, geometric optics fails

Interference vs Diffraction

few wavelets contribute

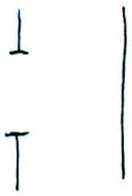


large number of wavelets contributing
(continuous distribution of point sources)



Two regimes of diffraction

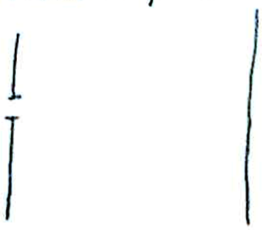
near-field
(Fresnel)



$F \gtrsim 1$

$$F = \frac{\text{size of aperture}}{\text{wavelength} \times \text{distance from aperture}}$$

far-field
(Fraunhofer)

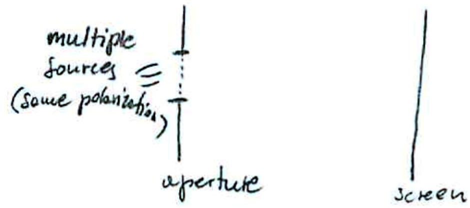


$F \gg 1$

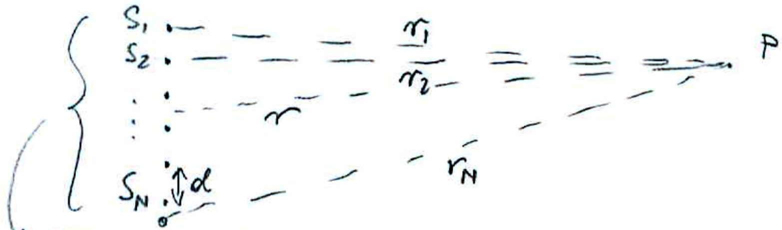
Digression: wavelength vs. diffraction effects

Why do radio waves propagate unhindered, whereas for visible light an opaque object creates optical shadow?

<u>Single-slit diffraction</u>
(Fraunhofer)



Interference from multiple discrete sources



distance small compared with wavelength

Note. Will use complex representation for amplitudes $\tilde{E}$ (physical meaning $E = \mathrm{Re}\,\tilde{E}$)

Amplitudes at P approximately equal $E_0(r_1) \approx E_0(r_2) \approx \ldots \approx E_0(r_N) \approx E_0(r)$

Net electric field at P (complex)

$$\tilde{E} = E_0(r)\, e^{i(kr_1 - \omega t)} + E_0(r)\, e^{i(kr_2 - \omega t)} + \ldots + E_0(r)\, e^{i(kr_N - \omega t)} =$$

$$= E_0(r)\, e^{i(kr_1 - \omega t)} \left[1 + e^{ik(r_2 - r_1)} + e^{ik(r_3 - r_1)} + \ldots + e^{ik(r_N - r_1)} \right]$$

Phase difference for neighboring sources

e.g. $r_3 - r_1 = \underbrace{(r_3 - r_2)}_{d} + \underbrace{(r_2 - r_1)}_{d}$

Denote $kd\sin\theta = \delta$

$$\tilde{E} = E_0(r)\, e^{i(kr_1 - \omega t)} \left[1 + e^{i\delta} + (e^{i\delta})^2 + \ldots + (e^{i\delta})^{N-1} \right] =$$

$$= E_0(r)\, e^{i(kr_1 - \omega t)} \frac{1 - e^{i\delta N}}{1 - e^{i\delta}} = E_0(r)\, e^{i(kr_1 - \omega t)} \frac{e^{\frac{i\delta N}{2}}}{e^{i\delta/2}} \cdot \frac{e^{-\frac{i\delta N}{2}} - e^{\frac{i\delta N}{2}}}{e^{-\frac{i\delta}{2}} - e^{\frac{i\delta}{2}}} =$$

$$= E_0(r)\, e^{i(kR - \omega t)} \frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$

where $R = r_1 + \frac{1}{2}(N-1)\frac{\delta}{k}$
$= r_1 + \frac{1}{2}(N-1)d$

Intensity

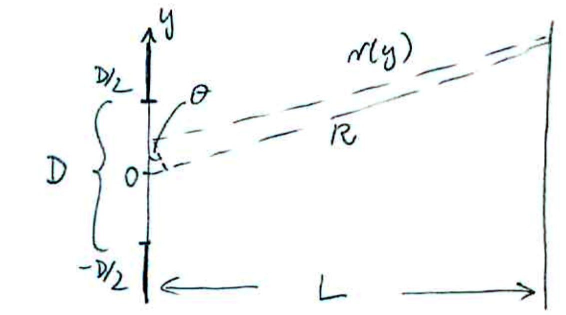
$$\boxed{I = I_0 \left(\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} \right)^2}$$

I_0 → single-source intensity

→ maximum if $\delta = 2\pi m$, $m = 0, \pm1, \pm2, \ldots$

$$\boxed{d\sin\theta_m = \lambda m}$$

Continuous distribution of sources over the aperture



$N \to \infty$ so that $\frac{N E_{0,N}}{D} \to \text{const} = E_A$

$$\tilde{E}(\tilde{r}) = \frac{E_A}{r} e^{i(kr - \omega t)} \quad r \approx R \quad \to \text{spherical wave}$$

$$r(y) \approx R - y \sin\theta$$

$$\tilde{E} \approx \frac{E_A}{R} \int_{-D/2}^{D/2} e^{i(k\, r(y) - \omega t)}\, dy = \frac{4}{L} \int_{-D/2}^{D/2} e^{i(k(R - y\sin\theta) - \omega t)}\, dy =$$

$$= \frac{E_A}{R} e^{i(kR - \omega t)} \int_{-D/2}^{D/2} e^{-iky\sin\theta}\, dy = \left\{ \beta = \frac{kD\sin\theta}{2} \right\}$$

$$= \frac{E_A}{R} e^{i(kR - \omega t)} \int_{-D/2}^{D/2} e^{-i\frac{2\beta}{D}y}\, dy =$$

$$= \frac{E_A}{R} e^{i(kR - \omega t)} \left(-\frac{D}{2\beta i}\right) \left[e^{-i\beta} - e^{i\beta} \right] =$$

$$= \frac{E_A D}{R} e^{i(kR - \omega t)} \frac{\sin\beta}{\beta}$$

Intensity

$$\boxed{I = I_{max} \left(\frac{\sin\beta}{\beta} \right)^2}$$

(single slit diffraction)

or

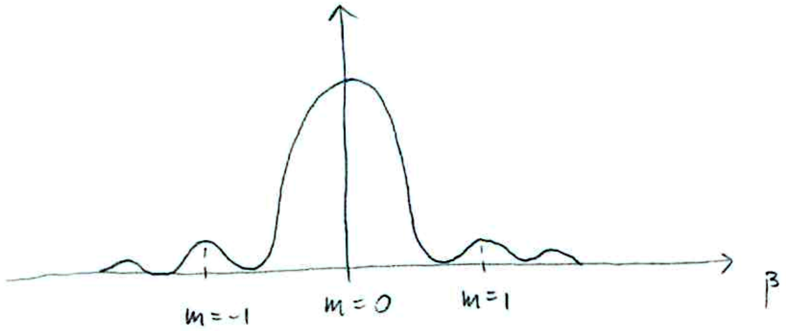
$$I = I_{max} \left(\frac{\sin \frac{\pi D \sin\theta}{\lambda}}{\frac{\pi D \sin\theta}{\lambda}} \right)^2$$

Bright fringes

$$\frac{\pi D \sin\theta}{\lambda} = m\pi$$

$$\Downarrow$$

$$\sin\theta = \frac{m\lambda}{D} \qquad m = 0, \pm 1, \pm 2$$

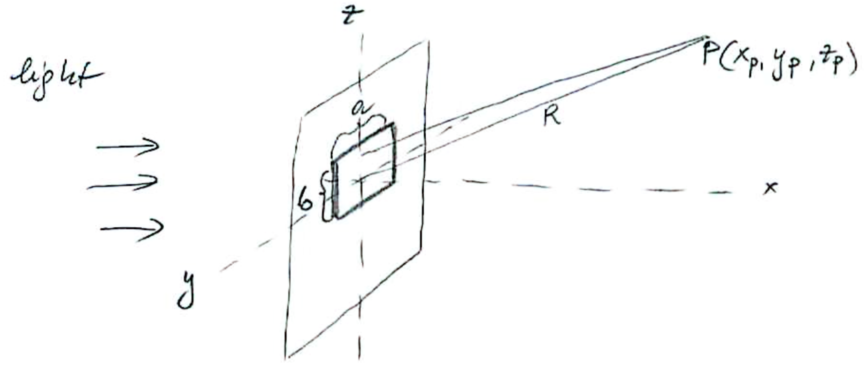


$\downarrow$

central maximum

D - larger $\Rightarrow$ pattern more "compressed"

Diffraction on a rectangular aperture



$$E_A = \lim_{N\to\infty} \frac{N \cdot E_{0,N}}{ab} = \text{const}$$

$$d\tilde{E} = \frac{E_A}{\underbrace{r}_{\approx R}} e^{i(kr-\omega t)} \, dA$$

where

$$r = \sqrt{x_P^2 + (y-y_P)^2 + (z-z_P)^2} = \sqrt{x_P^2 + y_P^2 + z_P^2 - 2(yy_P + zz_P) + y^2 + z^2} =$$

$$\approx \sqrt{R^2 - 2(yy_P + zz_P) + y^2 + z^2} = R\sqrt{1 - \frac{2}{R^2}(yy_P + zz_P) + \underbrace{\frac{y^2+z^2}{R^2}}_{\text{negligible}}}$$

$\sqrt{1-u} \approx 1 - \frac{u}{2}$

negligible $(R >> y^2+z^2)$ Fraunhofer limit

$$\approx R\left[1 - \frac{yy_P + zz_P}{R^2}\right]$$

Hence

$$\tilde{E} = \iint_{\text{aperture}} d\tilde{E} = \frac{E_A}{R} \iint_{\text{aperture}} e^{i\left(kR\left[1 - \frac{yy_P+zz_P}{R^2}\right] - \omega t\right)} \, dA =$$

$$= \frac{E_A}{R} e^{i(kR-\omega t)} \int_{-a/2}^{a/2} dy \int_{-b/2}^{b/2} dz \; e^{-ik\left(\frac{yy_P + zz_P}{R}\right)} =$$

$$= \frac{E_A}{R} e^{i(kR-\omega t)} \int_{-a/2}^{a/2} e^{-\frac{iky_P}{R}y} \, dy \int_{-b/2}^{b/2} e^{-\frac{ikz_P}{R}z} \, dz$$

Introduce $\alpha = \frac{kay_P}{2R}$, $\beta = \frac{kbz_P}{2R}$. Then

$$\tilde{E} = \frac{E_A}{R} e^{i(kR-\omega t)} \frac{a}{\alpha 2i}\left(e^{i\alpha} - e^{-i\alpha}\right) \frac{b}{\beta 2i}\left(e^{i\beta} - e^{-i\beta}\right) =$$

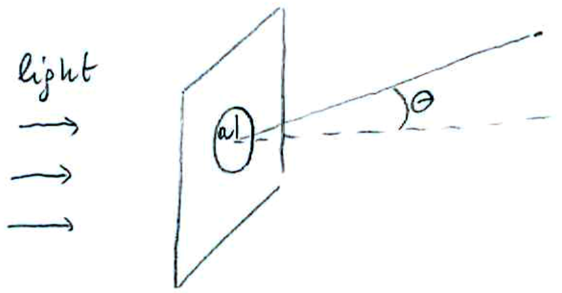
$$= \frac{E_A(ab)}{R} e^{i(kR-\omega t)} \left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin\beta}{\beta}\right)$$

Intensity

$$\boxed{I = I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2}$$

{FIG.}

Diffraction on a circular aperture – results



same method
(use spherical coordinates)

Intensity

$$I(\theta) = I_{max}\left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2$$

{FIG.}

Radii of the dark rings are given by zeros of the Bessel function of the 1st kind, order 1.

{FIG.}

Angular positions of first three dark fringes:

$$\sin\theta_1 = 1.22\frac{\lambda}{2a}$$

$$\sin\theta_2 = 2.23\frac{\lambda}{2a}$$

$$\sin\theta_3 = 3.24\frac{\lambda}{2a}$$

Why circular aperture is important?

→ human eyes – circular aperture

→ Rayleigh criterion: Two objects are said to be distinguishable, if the center of one diffraction pattern (corresponding to one of the objects) coincides with the first minimum of the other."

Hence, the angular separation of image centers is

$$\sin\Delta\theta = 1.22\frac{\lambda}{2a}$$

Question / exercise: A mobile phone camera's lens has a diameter of 2 mm. What is the maximum distance at which this camera can be used to resolve facial features?