PROBLEM SET 11

Due: 9 December 2016, 12 p.m.

Problem 1. A cylindrical conductor with radius a and resistivity ρ carries a constant current I.

(a) What are the magnitude and direction of the electric field vector $\mathbf{E}$ and the magnetic field vector $\mathbf{B}$ at a point just outside the wire and at a distance a from the axis of the cylinder?

(b) What are the magnitude and direction of the Poynting vector $\mathbf{S}$ at the same point?

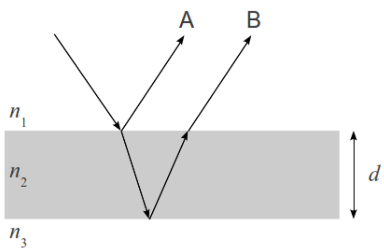
(c) Find the rate of flow of energy into the volume occupied by a length l of the conductor. *Hint.* What is the physical meaning of the Poynting vector integrated over a closed surface?

(d) How does your result compare to the rate of generation of thermal energy in the same volume? Why does the Poynting vector point in the direction found in (b)?

(*2 + 2 + 1/2 + 1/2 + 1 marks*)

Problem 2. Light traveling downward is incident on a horizontal film of thickness d as shown in the figure below. The incident ray splits into two rays, A and B. Ray A reflects from the top of the film. Ray B reflects form the bottom of the film and then refracts back into the material that is above the film. If the film has parallel faces, show that rays A and B end up parallel to each other.

(*3 marks*)



Problem 3. An inside corner of a cube is lined with mirrors to make a corner reflector. A ray of light is reflected successively from each of three mutually perpendicular mirrors. Show that its final direction is always exactly opposite to its initial direction.

(*4 marks*)

Problem 4. We want to rotate the direction of polarization of a beam of linearly polarized light by 90^o using one ore more polarizing sheets.

(a) What is the minimum number of sheets required?

(b) What is the minimum number of sheets required if the transmitted intensity is to be more than 60% of the original density.

(*1 + 3 marks*)

Problem 5. For interference of waves from two coherent sources show that the nodal lines and the antinodal lines are families of hyperbolas.

(*2 marks*)

Problem 6. Consider a two-slit interference experiment in which the two slits are of different widths. As measured on a distant screen, the amplitude of the wave from the first slit is E, while the amplitude of the wave from the second slit is $2E$.

(a) Show that the intensity at any point in the interference pattern is

$$I = I_{\max}\left(\frac{5}{9} + \frac{4}{9}\cos\phi\right),$$

where ϕ is the phase difference between the two waves as measured at a particular point of the screen and $I_{\max}$ is the maximum intensity in the pattern.

(b) Graph I versus ϕ. What is the minimum value of intensity, and for which values of ϕ does it occur?

(*3 + 1 marks*)

Problem 7. Consider a two-slit interference pattern, for which the intensity distribution is given by formula $I = I_{\max}\cos^2(\pi dy/\lambda l)$ (for the meaning of the symbols, see the lecture). Let θ_m be the angular position of the m-th bright fringe, where the intensity is $I_{\max}$. Assume that θ_m is small, so that $\sin\theta_m \approx \theta_m$. Let θ_m^+ and θ_m^- be the two angles on either side of θ_m for which $I = \frac{1}{2}I_{\max}$. The quantity $\Delta\theta_m = |\theta_m^+ - \theta_m^-|$ is the half-width if the m-th fringe. Calculate $\Delta\theta_m$. How does it depend on m?

(*4 marks*)

Problem 8. A compact disk (CD) is read from the bottom by a semiconductor laser with wavelength 790 nm passing through a plastic substrate of refractive index 1.8. When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region between the pits (see the figure), so these two beams interfere with each other. What must the minimum pit depth be so that the part of the beam reflected from a pit cancels the part of the beam reflected from the flat region? (It is this cancellation that allows the player to recognize the beginning and the end of a pit.)

(*4 marks*)