VP260 PROBLEM SET 6

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Problem 1.

From the class, we know

$$\omega = \frac{qB_0}{m}$$

$$x(t) = 0, v_x(t) = 0$$

$$y(t) = C_1 \sin \omega t - C_2 \cos \omega t + \frac{E_0}{B_0} t + C_3$$

$$v_y(t) = C_1 \omega \cos \omega t + C_2 \omega \sin \omega t + \frac{E_0}{B_0}$$

$$z(t) = C_1 \cos \omega t + C_2 \sin \omega t + C_4$$

$$v_z(t) = -C_1 \omega \sin \omega t + C_2 \omega \cos \omega t$$

We can get

$$v_y(0) = C_1 \omega + \frac{E_0}{B_0}$$

$$v_z(0) = C_2 \omega$$

And apply the initial condition, we obtain

$$-C_2 + C_3 = 0 \text{ and } \mathrm{C}_1 + \mathrm{C}_4 = 0$$

(a)

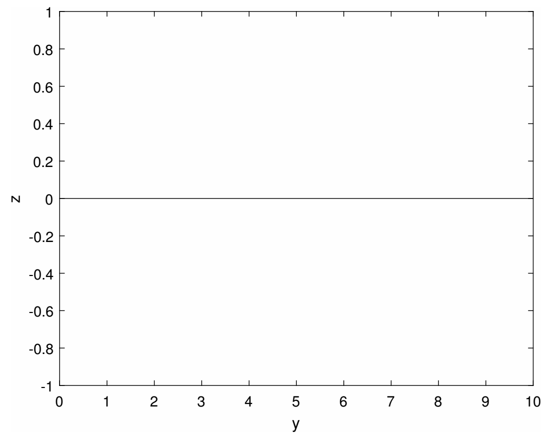
$$v_y(0) = \frac{E_0}{B_0}, C_1 = 0, C_4 = 0$$

$$v_z(0) = 0, C_2 = 0, C_3 = 0$$

$$y(t) = \frac{E_0}{B_0} t, z(t) = 0$$

The trajectory is

$$x = z = 0, y > 0$$

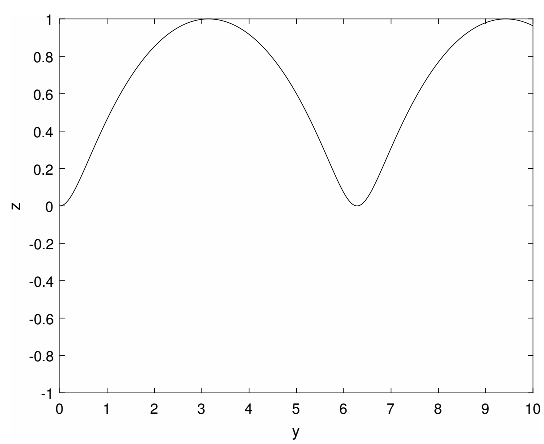


(b)

$$v_y(0) = \frac{E_0}{2B_0}, C_1 = -\frac{E_0}{2B_0\omega}, C_4 = \frac{E_0}{2B_0\omega}$$
$$v_z(0) = 0, C_2 = 0, C_3 = 0$$
$$y(t) = -\frac{E_0}{2B_0\omega}\sin\omega t + \frac{E_0}{B_0}t$$
$$z(t) = -\frac{E_0}{2B_0\omega}(\cos\omega t - 1)$$

The trajectory is

$$x = 0, \left(y - \frac{E_0}{B_0}t\right)^2 + \left(z - \frac{E_0}{2B_0\omega}\right)^2 = \left(\frac{E_0}{2B_0\omega}\right)^2$$



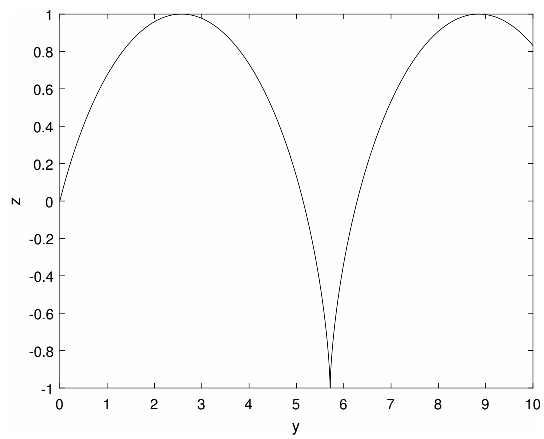
(c)

$$v_y(0) = \frac{E_0}{B_0}, C_1 = 0, C_4 = 0$$
$$v_z(0) = \frac{E_0}{B_0}, C_2 = \frac{E_0}{B_0\omega}, C_3 = \frac{E_0}{B_0\omega}$$

$$y(t) = -\frac{E_0}{B_0\omega}\cos\omega t + \frac{E_0}{B_0}t + \frac{E_0}{B_0\omega}$$
$$z(t) = \frac{E_0}{B_0\omega}\sin\omega t$$

The trajectory is

$$x = 0, \left(y - \frac{E_0}{B_0}t - \frac{E_0}{B_0\omega}\right)^2 + z^2 = \left(\frac{E_0}{B_0\omega}\right)^2$$



Problem 2.

$$F = q(E + v \times B)$$
$$a = \frac{F}{m} = -\frac{qE_0}{m}\hat{n_x} + \frac{qB_0}{m}v_z\hat{n_y} - \frac{qB_0}{m}v_y\hat{n_z}$$
$$\frac{a_y}{a_z} = \frac{\frac{dv_y}{dt}}{\frac{dv_z}{dt}} = \frac{dv_y}{dv_z} = -\frac{v_z}{v_y}$$
$$v_y dv_y = -v_z dv_z$$

Do integral on both side,

$$\int v_y dv_y = -\int v_z dv_z$$
$$\frac{1}{2}v_y^2 = -\frac{1}{2}v_z^2 + C$$
$$v_y^2 + v_z^2 = C = v_{0y}^2$$
$$v_y = v_{0y}\cos\frac{qB_0}{m}t,\ a_y = \frac{qB_0}{m}v_{0y}\sin\frac{qB_0}{m}t$$
$$v_z = -v_{0y}\sin\frac{qB_0}{m}t,\ a_z = -\frac{qB_0}{m}v_{0y}\cos\frac{qB_0}{m}t$$
$$v(t) = \left(v_{0x} - \frac{qE_0}{m}t\right)\hat{n_x} + \left(v_{0y}\cos\frac{qB_0}{m}t\right)\hat{n_y} - \left(v_{0y}\sin\frac{qB_0}{m}t\right)\hat{n_z}$$
$$r(t) = \left(v_{0x}t - \frac{qE_0}{2m}t^2\right)\hat{n_x} + \left(\frac{m}{qB_0}v_{0y}\sin\frac{qB_0}{m}t\right)\hat{n_y} + \left(\frac{m}{qB_0}v_{0y}(\cos\frac{qB_0}{m}t - 1)\right)\hat{n_z}$$

Problem 3.

$$
\begin{aligned}
F &= \int_{A\to B} I \cdot \overline{dl} \times \overline{B} \\
&= \int_{A\to B} I \cdot \overline{dx} \times \overline{B} + \int_{A\to B} I \cdot \overline{dy} \times \overline{B} \\
&= \int_{A\to B} I \cdot \overline{dx} \times \overline{B} \\
&= IBw
\end{aligned}
$$

Problem 4.

(a)

$$J = nqv$$

$$F = NqvB = qvB \cdot nV = BJ \cdot whl$$

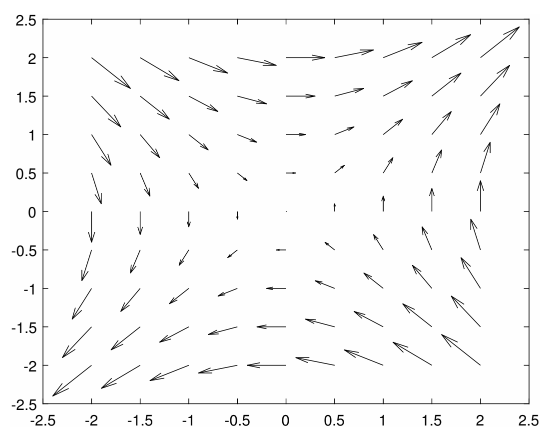
$$\Delta p = \frac{F}{S} = JlB$$

(b)

$$J = \frac{\Delta p}{lB} = \frac{1.013 \times 10^5}{3.5 \times 10^{-2} \cdot 2.2} = 1.316 \times 10^5 \text{ A/m}^2$$

Problem 5.

(a)



(b)

$$F = \int I \cdot \overline{dl} \times \overline{B}$$

For the left side, $\overline{L} = L\hat{n_y}$, $\overline{B} = \frac{B_0 y}{L}\hat{n_x}$

$$F = \int_0^l I \cdot \frac{B_0 l}{L} dl(\hat{n_y} \times \hat{n_x}) = -\frac{1}{2} I B_0 L \hat{n_z}$$

For the right side, $\overline{L} = -L\hat{n_y}$, $\overline{B} = \frac{B_0 y}{L}\hat{n_x} + B_0\hat{n_y}$

$$F = \int_0^l I \cdot \frac{B_0 l}{L} dl(-\hat{n_y} \times \hat{n_x}) = \frac{1}{2} I B_0 L \hat{n_z}$$

For the bottom side, $\overline{L} = -L\hat{n_x}$, $\overline{B} = \frac{B_0 x}{L}\hat{n_y}$

$$F = \int_0^l I \cdot \frac{B_0 l}{L} dl(-\hat{n_x} \times \hat{n_y}) = -\frac{1}{2} I B_0 L \hat{n_z}$$

For the top side, $\overline{L} = L\hat{n_x}$, $\overline{B} = B_0\hat{n_x} + \frac{B_0 x}{L}\hat{n_y}$

$$F = \int_0^l I \cdot \frac{B_0 l}{L} dl(\hat{n_x} \times \hat{n_y}) = \frac{1}{2} I B_0 L \hat{n_z}$$

(c)

$$\begin{aligned}
\tau &= \int \overline{r} \times \overline{F} \\
&= \int_0^l I \cdot l \frac{B_0 l}{L} dl[\hat{n_y} \times (\hat{n_y} \times \hat{n_x})] + \int_0^l I \cdot l \frac{B_0 l}{L} dl[\hat{n_y} \times (-\hat{n_y} \times \hat{n_x})] + \int_0^l I \cdot L \frac{B_0 l}{L} dl[\hat{n_y} \times (\hat{n_x} \times \hat{n_y})] \\
&= \int_0^l I \cdot L \frac{B_0 l}{L} dl \hat{n_x} \\
&= \frac{1}{2} I B_0 L^2 \hat{n_x}
\end{aligned}$$

(d)

$$\begin{aligned}
\tau &= \int \overline{r} \times \overline{F} \\
&= \int_0^l I \cdot l \frac{B_0 l}{L} dl[\hat{n_x} \times (\hat{n_x} \times \hat{n_y})] + \int_0^l I \cdot l \frac{B_0 l}{L} dl[\hat{n_x} \times (-\hat{n_x} \times \hat{n_y})] + \int_0^l I \cdot L \frac{B_0 l}{L} dl[\hat{n_x} \times (-\hat{n_y} \times \hat{n_x})] \\
&= \int_0^l I \cdot L \frac{B_0 l}{L} dl(-\hat{n_y}) \\
&= -\frac{1}{2} I B_0 L^2 \hat{n_y}
\end{aligned}$$

(e) When it is rotating on one of its edge, the total τ of two edges perpendicular to the rotating axis is 0, and the τ on another edge is

$$\tau = L \sin\theta \int_0^l I \cdot \frac{B_0 l}{L} dl \hat{n_L} = -\frac{1}{2} I B_0 L^2 \sin\theta \hat{n_L}$$

Let $|\mu| = IL^2$, $|B| = \frac{1}{2}B_0$ and $\mu \perp B, B \perp L, L \perp \mu$, we obtain

$$\tau = \mu \times B$$

Problem 6.

(a)

$$
\begin{aligned}
F &= \int I \cdot \overline{dl} \times \overline{B} \\
&= \int_0^{2\pi} I \cdot Rd\theta \cdot |B| \cos\theta \\
&= I|B|R \int_0^{2\pi} \cos\theta d\theta \\
&= 0
\end{aligned}
$$

(b)

$$
\begin{aligned}
\tau &= \int \overline{r} \times \overline{F} \\
&= \int_0^{2\pi} I \cdot Rd\theta \cdot |B| \hat{n_x} \cos\theta \cdot R\cos\theta \\
&= I|B|\hat{n_x} R^2 \int_0^{2\pi} \cos^2\theta d\theta \\
&= I\overline{B}\pi R^2
\end{aligned}
$$

Problem 7.

$$B_r(r) \cdot 2\pi rh + [B_z(z+h) - B_z(z)] \cdot \pi r^2 = 0$$

$$B_r(r) = \frac{-\beta h \pi r^2}{2\pi rh} = -\frac{\beta r}{2}$$