VP260 PROBLEM SET 7

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Problem 1.

Let up to be the direction of y-axis and right to the direction of x-axis.
In the bottom side,

$$B \cdot 2\pi s = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi s}$$

$$F = -LIB = -\frac{\mu_0 I^2 L}{2\pi s}\hat{n_y}$$

In the left and right sides,

$$B \cdot 2\pi x = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi x}$$

$$F_{ly} = F_{ry} = \int_s^{s+\frac{\sqrt{3}}{2}a} \frac{\mu_0 I^2 dl}{2\pi s} = \frac{\mu_0 I^2}{2\pi} \ln \frac{s + \frac{\sqrt{3}}{2}a}{s}\hat{n_y}$$

$$F_x = F_{lx} + F_{rx} = 0$$

$$F = \frac{\mu_0 I^2}{2\pi}\left(2\ln \frac{s+\frac{\sqrt{3}}{2}a}{s} - \frac{L}{s}\right)\hat{n_y}$$

Problem 2.

Suppose the direction of v to be the x-axis and a side pointing outside the paper to be the y-axis.

(a) If we split the plate to infinite wires, and the width of each of the wires is dy,
then $q = \sigma dy \cdot x$ on each wire.

$$dI = \frac{q}{t} = \frac{\sigma dy \cdot x}{x/v} = \sigma v dy$$

Since it is symmetric, B_z on the z-axis formed by the wires are cancelled, and B_y on the y-axis is $B\cos\theta$, suppose the distance between a point and the plate is r,

$$y = r\tan\theta$$

$$dy = \frac{r}{\cos^2\theta}d\theta$$

$$B = \int_{-y/2}^{y/2} \frac{\mu_0 \sigma v \cos\theta}{2\pi\sqrt{r^2+y^2}}dy = \int_{-\pi/2}^{\pi/2} \frac{\mu_0\sigma v\cos\theta \frac{r}{\cos^2\theta}}{2\pi\frac{r}{\cos\theta}}d\theta = \frac{1}{2}\mu_0\sigma v$$

Above and below the plates, B is cancelled, so $B = 0$ Between the plates, B is added, so $B = \mu_0\sigma v$, opposite the y-axis.

(b)

$$dF = LIB = x \cdot \sigma v dy \cdot \frac{1}{2}\mu_0 \sigma v = \frac{1}{2} x \mu_0 \sigma^2 v^2 dy$$

$$F = \int_0^y dF = \frac{1}{2} xy\mu_0 \sigma^2 v^2$$

$$F_u = \frac{F}{xy} = \frac{1}{2}\mu_0 \sigma^2 v^2, \text{ pointing to the top}$$

(c)

$$\mu_0 \sigma v^2 = 2 \cdot \frac{\sigma}{2\varepsilon_0}$$

$$v = \frac{1}{\mu_0 \varepsilon_0}$$

Problem 3.

$$B \cdot 2\pi r = \mu_0 \int dI$$

$$dI = \frac{dQ}{t} = \frac{Q \cdot 2\pi r dr}{\pi R^2} n$$

$$B = \int_0^a \frac{n\mu_0 Q}{\pi R^2} dr = \frac{n\mu_0 Q a}{\pi R^2}$$

Problem 4.

Similar to Problem 2,

$$y = r \tan\theta$$

$$dy = \frac{r}{\cos^2\theta} d\theta$$

$$B = \int_{-L/2}^{L/2} \frac{\mu_0 I \frac{dy}{L} \cos\theta}{2\pi\sqrt{r^2 + y^2}} = \int_{-\arctan L/2y}^{\arctan L/2y} \frac{\mu_0 I}{2\pi L} d\theta = \frac{\mu_0 I}{\pi L} \arctan\frac{L}{2y}$$

Problem 5.

(a)

$$I_0 = \int J(r) dS = \int_0^a \frac{b}{r} e^{\frac{r-a}{\delta}} 2\pi r dr = 2\pi b\delta e^{\frac{r-a}{\delta}}\Big|_0^a = 2\pi b\delta \left(1 - e^{-\frac{a}{\delta}}\right)$$

(b)

$$B(r) \cdot 2\pi r = \mu_0 I_0$$

$$B(r) = \frac{\mu_0 I_0}{2\pi r}$$

(c)

$$I = 2\pi b\delta e^{\frac{r-a}{\delta}}\Big|_0^r = 2\pi b\delta \left(e^{\frac{r-a}{\delta}} - e^{-\frac{a}{\delta}}\right) = \frac{e^{r/\delta} - 1}{e^{a/\delta} - 1} I_0$$

(d)

$$B(r) \cdot 2\pi r = \mu_0 I$$

$$B(r) = \frac{e^{r/\delta} - 1}{e^{a/\delta} - 1} \cdot \frac{\mu_0 I_0}{2\pi r}$$

Problem 6.

The identity for the divergence of a curl states that a vector field's curl divergence must always be zero.

$$\nabla \cdot (\nabla \times B) = 0$$

In the Ampere's law, we can find

$$\nabla \cdot (\nabla \times B) = \nabla \cdot J = -\frac{\partial \rho}{\partial t} \neq 0$$

So Ampere's law cannot be valid, in general, outside magnetostatics.