VP260 PROBLEM SET 8

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Problem 1.

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$|U_{ab}| = \int_d^{d+L} \frac{\mu_0 I}{2\pi r} \cdot v dr = \frac{\mu_0 I v}{2\pi} \ln \frac{d+L}{d}$$

According to right-hand rule, point A is at higher potential.

Problem 2.

(a)

$$E = BLv\cos\varphi$$

$$F = BIL = \frac{BEL}{R} = \frac{B^2L^2v\cos\varphi}{R}$$

$$F\cos\varphi = mg\sin\varphi$$

$$v = \frac{mgR\sin\varphi}{B^2L^2v\cos^2\varphi}$$

(b) According to right-hand rule, the direction is from A to B.

(c)

$$BIL\cos\varphi = mg\sin\varphi$$

$$I = \frac{mg\tan\varphi}{BL}$$

(d)

$$P = I^2R = \frac{m^2g^2R\tan^2\varphi}{B^2L^2}$$

(e)

$$P' = mgv\sin\varphi = \frac{m^2g^2R\sin^2\varphi}{B^2L^2\cos^2\varphi} = \frac{m^2g^2R\tan^2\varphi}{B^2L^2}$$

The value of P is the same.

Problem 3.

$$\Phi = \int_0^a \int_0^a 4t^2 ydydx = 2a^3t^2$$

$$\varepsilon = -\frac{d\Phi}{dt} = -4a^3t$$

So the direction of I is clockwise.

Problem 4.

(a)

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$\Phi = \int_0^a \int_{d+vt}^{b+d+vt} \frac{\mu_0 I}{2\pi r} drdl = \frac{\mu_0 Ia}{2\pi} \ln\left(1 + \frac{b}{d+vt}\right)$$

(b)

$$I = \frac{\varepsilon}{R} = -\frac{d\Phi}{Rdt} = -\frac{\mu_0 Ia}{2\pi R} \frac{d+vt}{b+d+vt} \frac{-bv}{(d+vt)^2} = \frac{\mu_0 Iabv}{2\pi R(d+vt)(b+d+vt)}$$

So the direction of I is anti-clockwise.

Problem 5.

The flux in the square loop decreased, so the force pointed to outside the loop, according to left-hand rule, the direction of I is anti-clockwise.

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_0 = \int_0^a \int_s^{s+a} \frac{\mu_0 I}{2\pi r} drdx = \frac{\mu_0 Ia}{2\pi} \ln \frac{s+a}{s}$$

$$I = \frac{\varepsilon}{R} = -\frac{d\Phi}{Rdt}$$

$$Q = I \int Idt = -\frac{0 - \Phi_0}{R} = \frac{\Phi_0}{R} = \frac{\mu_0 Ia}{2\pi R} \ln \frac{s+a}{s}$$

Problem 6.

(a)

$$\frac{Q}{C} + R \cdot \frac{dQ}{dt} = 0$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}, \quad R = \frac{\rho d}{A}, \quad Q(0) = Q_0$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = 0$$

$$Q = Q_0 e^{-t/RC} = Q_0 e^{-t/\rho\varepsilon_0\varepsilon_r}$$

$$I = \frac{dQ}{dt} = -\frac{Q_0}{\rho\varepsilon_0\varepsilon_r} Q_0 e^{-t/\rho\varepsilon_0\varepsilon_r}$$

$$J_c = \frac{I}{A} = -\frac{Q_0}{\rho A \varepsilon_0 \varepsilon_r} e^{-t/\rho\varepsilon_0\varepsilon_r}$$

(b)

$$J_d = \frac{E}{\rho} = \frac{Q_0 d}{\rho A \varepsilon_0 \varepsilon_r} e^{-t/\rho\varepsilon_0\varepsilon_r}$$
$$J_c + J_d = 0$$

Problem 7.

In the side AB, we can find $U_{AB} = E \cdot AB$
In other three sides, we can simply find $U = 0$
So in the loop, we can find $\varepsilon = E \cdot AB$
Since the magnetic field is constant, $\frac{d\Phi}{dt} = 0$
And $\varepsilon = -\frac{d\Phi}{dt} = 0$, which reaches a contradiction.

Problem 8.

(a)

$$A(r) = \frac{1}{2}(B \times r) = \frac{1}{2} B\hat{n_z} \times (x\hat{n_x} + y\hat{n_y} + z\hat{n_z}) = \frac{1}{2}(-By\hat{n_x} + Bx\hat{n_y})$$

$$\begin{aligned} B = \nabla \times A &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{n_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{n_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{n_z} \\ &= \frac{1}{2}(B + B)\hat{n_z} \\ &= B\hat{n_z} \end{aligned}$$

$$div\ A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + 0 = 0$$

(b)

$$\begin{aligned} B = \nabla \times A &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{n_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{n_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{n_z} \\ &= (0-0)\hat{n_x} + (0-0)\hat{n_y} + (B-0)\hat{n_z} \\ &= B\hat{n_z} \end{aligned}$$

$$div\ A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0$$

Problem 9.

(a)

$$\nabla^2 A = \nabla(\nabla \cdot A) - \nabla \times (\nabla \times A) = 0 - \nabla \times B = -\mu_0 J$$

(b) The vector potential is defined as

$$A(r) = \frac{1}{4\pi} \int \frac{\nabla \times v(r')}{|r - r'|} d\tau'$$

where $v(r') = B(r')$ which is a solenoidal vector field.
Since $\nabla \times B(r') = \mu_0 J(r')$

$$A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r - r'|} d\tau'$$