VP260 PROBLEM SET 4

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Problem 1.

(a)

$$\frac{V_{AB}}{d}q = \rho\frac{4}{3}\pi r^3 g$$

$$q = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$$

(b)

$$6\pi\eta r v_\infty = \rho\frac{4}{3}\pi r^3 g$$

$$r = 3\sqrt{\frac{\eta v_\infty}{2\rho g}}$$

$$q = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_\infty^3}{2\rho g}}$$

(c)

$$r = 3\sqrt{\frac{\eta v_\infty}{2\rho g}} = 5.066 \times 10^{-7} m$$

$$q = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_\infty^3}{2\rho g}} = 4.80 \times 10^{-19} C$$

There is about 3 electrons

Problem 2.

If we add a unit at the left, the equivalent capacitance won't change. Suppose the equivalent capacitance to be C'

$$\frac{1}{\frac{1}{C'+C} + \frac{1}{C} + \frac{1}{C}} = \frac{1}{C'}$$

$$2C'^2 + 2C'C - C^2 = 0$$

$$C' = \frac{\sqrt{3}-1}{2}C$$

Problem 3.

(a)

$$U = \frac{Q^2}{2C} = \frac{Q^2 x}{2\varepsilon_0 A}$$

(b)

$$U' = \frac{Q^2}{2C} = \frac{Q^2(x+dx)}{2\varepsilon_0 A}$$

$$\Delta U = U' - U = \frac{Q^2 dx}{2\varepsilon_0 A}$$

(c)

$$\frac{Q^2 dx}{2\varepsilon_0 A} = F dx$$

$$F = \frac{Q^2}{2\varepsilon_0 A}$$

(d)

$$E = \frac{Q}{\varepsilon_0 A}$$

$$F = \frac{1}{2} QE$$

Because the another $\frac{1}{2}QE$ is used to maintain the static plate.

Problem 4.

(a)

$$C = \frac{\varepsilon_0 L(L-x)}{D} + \frac{\varepsilon_r \varepsilon_0 L x}{D} = \frac{\varepsilon_0 L}{D}(L - x + \varepsilon_r x)$$

(b)

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 L V^2}{2D}(L - x + \varepsilon_r x)$$

$$U' = \frac{\varepsilon_0 L V^2}{2D}[L - (x+dx) + \varepsilon_r(x+dx)]$$

$$dU = U' - U = \frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D} dx$$

(c)

$$Q = CV = \frac{\varepsilon_0 L V}{D}(L - x + \varepsilon_r x)$$

$$Q' = -\frac{\varepsilon_0 L V}{D}(L - x + \varepsilon_r x)$$

$$U = \frac{Q^2}{2C} = \frac{Q^2 D}{2\varepsilon_0 L(L - x + \varepsilon_r x)}$$

$$dU = -\frac{Q^2 D}{2\varepsilon_0 L(L - x + \varepsilon_r x)^2}(\varepsilon_r - 1)dx = -\frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D} dx$$

(d)

$$F_b = -\frac{dU}{dx} = -\frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D}$$

So it pushes out the slab.

$$F_c = -\frac{dU}{dx} = \frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D}$$

So it pulls out the slab.

(e) The change of stored energy means that work should be done to the capacitor, the direction of the force is same to the change of stored energy, so it's wrong.

$$|F| = \frac{(\varepsilon_r - 1)\varepsilon_0 V^2 L}{2D}$$

Problem 5.

$$j(r) = \frac{I}{2\pi r a}$$

$$E(r) = \frac{j(r)}{\sigma} = \frac{I}{2\pi r\sigma a}$$

$$U = \int_{r_0}^{b-r_0} E dr = \int_{r_0}^{b-r_0} \frac{I}{2\pi r\sigma a} dr = \frac{I}{2\pi\sigma a} ln(\frac{b-r_0}{r_0})$$

$$R = 2\frac{U}{I} = \frac{1}{\pi\sigma a} ln(\frac{b-r_0}{r_0})$$

Problem 6.

(a)

$$j(r) = \frac{I}{4\pi r^2}$$

$$E(r) = \frac{j(r)}{\sigma} = \frac{I}{4\pi r^2\sigma}$$

$$V = \int_{r_a}^{r_b} E dr = \int_{r_a}^{r_b} \frac{I}{4\pi r^2\sigma} dr = \frac{I}{4\pi\sigma}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$I = \frac{4\pi\sigma r_a r_b V}{r_b - r_a}$$

(b)

$$R = \frac{V}{I} = \frac{r_b - r_a}{4\pi\sigma r_a r_b}$$

(c) When $r_b \gg r_a$,

$$R = \frac{1}{4\pi\sigma r_a}$$

So r_b can be ignored when $r_b \gg r_a$

$$I = \frac{V}{2R} = 8\pi\sigma r_a V$$

Problem 7.

$$R' = \frac{1}{\frac{1}{4+6} + \frac{1}{8} + \frac{1}{9}} = \frac{360}{121}\Omega$$

$$R = \frac{1}{\frac{1}{20} + \frac{1}{3+360/121}} = 4.6\Omega$$

Problem 8.

The voltages on the three points near point A are the same, so are the three points near point B.
So the three resistance between A and the three points near it is connected paralleled, and so are the three points near B and other six points.

$$R' = \frac{1}{3}R + \frac{1}{6}R + \frac{1}{3}R = \frac{5}{6}R$$

Problem 9.

Separate A and B, suppose the total current flowing from either point to be I, then the current flowing to four directions should be $\frac{I}{4}$.
For point A, a current of $\frac{I}{4}$ is flowing out for B.
For point B, a current of $\frac{I}{4}$ is flowing into B from A.
So the total current is $\frac{I}{2}$, and $R' = \frac{R}{2}$