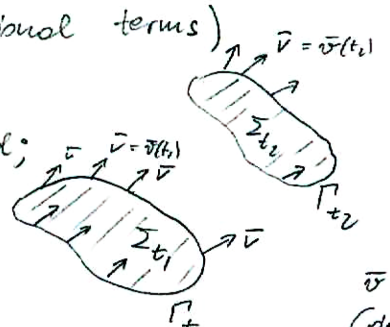
Appendix: EMF derivation (transformer & motional terms)

General situation: loop moves & may be deformed; $\bar B = \bar B(\bar r, t)$



$\bar v$ - field of velocities (describes movement/ deformation of loop Γ)

$$\mathcal{E} = \oint_{\Gamma_t} \frac{1}{q} \bar F d\bar l = \frac{1}{q} \oint_{\Gamma_t} (q\bar E + q\,\bar v \times \bar B)\, d\bar l =$$

$$= \oint_{\Gamma_t} (\bar E + \bar v \times \bar B)\, d\bar l$$

Universal flux rule (always valid) $\mathcal{E} = -\frac{d\Phi_B}{dt}$. We will show $\oint_{\Gamma_t} \bar E d\bar l = -\int_{\Sigma_t} \frac{\partial \bar B}{\partial t} d\bar S$ ($d\bar S$ – element of surface; $\int_{\Sigma_t} \frac{\partial \bar B}{\partial t} d\bar S$ → transformer emf)

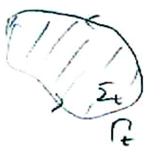
We will use the identity ($\bar w$ - any vector, see the materials on SAKAI for more mathematical details)

$$\frac{d}{dt} \int_{\Sigma_t} \text{rot}\, \bar w\, d\bar S = \oint_{\Gamma_t} \frac{\partial \bar w}{\partial t} d\bar l - \oint_{\Gamma_t} [\bar v \times \text{rot}\, \bar w]\, d\bar l =$$

depends on t!

$$\underset{\substack{\text{Stoke's thm}\\ \text{applied to}\\ \text{the former term}}}{=} \int_{\Sigma_t} \frac{\partial}{\partial t}(\text{rot}\, \bar w)\, d\bar S - \oint_{\Gamma_t} [\bar v \times \text{rot}\, \bar w]\, d\bar l \qquad (*)$$

Recall: Stoke's thm:



$$\oint_{\Gamma_t} \bar u\, d\bar l = \int_{\Sigma_t} \text{rot}\, \bar u\, d\bar S$$

Use the identity $(*)$ for $\bar w = \bar A$ (vector potential, i.e. $\bar B = \text{rot}\, \bar A$)

$$\frac{d}{dt} \int_{\Sigma_t} \underbrace{\text{rot}\, \bar A}_{=\bar B}\, d\bar S = \frac{d}{dt} \int_{\Sigma_t} \bar B\, d\bar S \overset{(*)}{=} \oint_{\Gamma_t} \frac{\partial \bar A}{\partial t} d\bar l - \oint_{\Gamma_t} (\bar v \times \overbrace{\text{rot}\, \bar A}^{=\bar B})\, d\bar l =$$

$$= \oint_{\Gamma_t} \Big(\underbrace{\frac{\partial \bar A}{\partial t} + \text{grad}\, V}_{= -\bar E} - \bar v \times \bar B \Big)\, d\bar l$$

Not. $\oint_{\Gamma_t} \text{grad}\, \varphi\, d\bar l = 0$ for any φ

$$= -\oint_{\Gamma_t} (\bar E + \bar v \times \bar B)\, d\bar l \qquad (**)$$

and (now use 2nd line of identity)

$$\frac{d}{dt} \int_{\Sigma_t} \bar B\, d\bar S = \int_{\Sigma_t} \frac{\partial}{\partial t} (\underbrace{\text{rot}\, \bar A}_{=\bar B})\, d\bar S - \oint_{\Gamma_t} (\bar v \times \bar B)\, d\bar l \qquad (*{*}*)$$

Compare first terms of rhs's of $(**)$ and $(***)$

$$\boxed{\oint_{\Gamma_t} \bar E\, d\bar l = -\int_{\Sigma_t} \frac{\partial \bar B}{\partial t} d\bar S}$$

→ transformer emf

Γ_t - any loop