VP260 PROBLEM SET 9

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Problem 1.

(a)

$$\overline{E} = 0$$

According to Maxwell's equations,

$$\text{rot } \overline{E} = -\frac{\partial \overline{B}}{\partial t} = 0$$

$$\frac{\partial \overline{B}}{\partial t} = 0$$

(b) According to Maxwell's equations,

$$\oint_r \overline{E} d\bar{l} = -\frac{d\Phi_B}{dt} = 0$$

So Φ_B is a constant.

Problem 2.

$$\overline{B} = 0$$

According to Maxwell's equations,

$$\oint_r \overline{B} d\bar{l} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = 0$$

Since $\overline{E} = 0$ according to Problem 1, we can simply get

$$i = 0$$

inside the conductor. So the electric current is confined to the surface.

Problem 3.

In picture (A),

$$\varepsilon_1 = \varepsilon_2 = \frac{\varepsilon}{2} = -\frac{1}{2}\frac{d\Phi_B}{dt}$$

In picture (B), in the loop containing the bulb in the bottom,

$$d\Phi_B = 0, \varepsilon_2 = 0$$

So the bulb in the bottom no longer glows.
In the loop containing the bulb above,

$$\varepsilon_1 = \varepsilon = -\frac{d\Phi_B}{dt}$$

Since $\varepsilon_{B1} = 2\varepsilon_{A1}$, the top bulb gets much brighter.

Problem 4.

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\alpha$$

$$I = \frac{\varepsilon}{R_1 + R_2} = -\frac{\alpha}{R_1 + R_2}$$

So the direction of I is clockwise. According to the connection direction of V_1 and V_2,

$$V_1 = -IR_1 = \frac{\alpha R_1}{R_1 + R_2}$$

$$V_2 = IR_2 = -\frac{\alpha R_2}{R_1 + R_2}$$

Problem 5.

$$\varepsilon_1 = -N_1 \frac{d\Phi_B}{dt}$$

$$\varepsilon_2 = -N_2 \frac{d\Phi_B}{dt}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}$$

Problem 6.

$$L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = U$$

$$L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} = U$$

$$\frac{dI_1}{dt} = \frac{U - M \frac{dI_2}{dt}}{L_1}$$

$$L_2 \frac{dI_2}{dt} + M \frac{U - M \frac{dI_2}{dt}}{L_1} = U$$

$$\frac{dI_2}{dt} = \frac{L_1 - M}{L_1 L_2 - M^2} U$$

$$\frac{dI_1}{dt} = \frac{L_2 - M}{L_1 L_2 - M^2} U$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} = \frac{L_1 + L_2 - 2M}{L_1 L_2 - M^2} U$$

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Problem 7.

(a)

$$L = \frac{d\Phi_B}{dt} = \mu_0 N \pi a^2 l$$

$$L_1 = \mu_0 N_1 \pi a^2 l, L_2 = \mu_0 N_2 \pi a^2 l$$

$$M = \mu_0 \pi a^2 N_1 N_2 l$$

$$M^2 = L_1 L_2$$

(b)

$$\varepsilon_2 = -M \frac{dI_2}{dt}$$

$$U_1 = -L_1 \frac{dI_1}{dt}$$

$$-M \frac{dI_2}{dt} - L_1 \frac{dI_1}{dt} + V_1 \cos \omega t = 0$$

$$M \frac{dI_2}{dt} + L_1 \frac{dI_1}{dt} = V_1 \cos \omega t$$

$$\varepsilon_2 = -M \frac{dI_1}{dt}$$

$$U_2 = -L_2 \frac{dI_2}{dt}$$

$$-M \frac{dI_1}{dt} - L_2 \frac{dI_2}{dt} - I_2 R = 0$$

$$M \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} = -I_2 R$$

(c)

$$\frac{dI_2}{dt} = -\frac{I_2 R + M \frac{dI_1}{dt}}{L_2}$$

$$-M \frac{I_2 R + M \frac{dI_1}{dt}}{L_2} + L_1 \frac{dI_1}{dt} = V_1 \cos \omega t$$

$$I_2 = -\frac{V_1 L_2 \cos \omega t}{MR}$$

$$L_1 I_1 + M I_2 = \int V_1 \cos \omega t$$

$$I_1 = \frac{V_1}{L_1 \omega} \sin \omega t + \frac{V_1 L_2}{L_1 R} \cos \omega t$$

(d)

$$V_{out} = I_2 R = -\frac{V_1 L_2 \cos \omega t}{M}$$

$$\frac{V_{out}}{V_{in}} = \frac{L_2}{M} = \frac{\sqrt{L_2}}{\sqrt{L_1}} = \frac{N_2}{N_1}$$

(e)

$$P_{in} = V_{in} I_1 = \frac{V_1^2}{L_1 \omega} \sin \omega t \cos \omega t + \frac{V_1^2 L_2}{L_1 R} \cos^2 \omega t$$

$$\overline{P_{in}} = \frac{1}{2\pi} \int_0^{2\pi} P_{in} = \frac{V_1^2 L_2}{2 L_1 R}$$

$$P_{out} = I_2^2 R = \frac{V_1^2 L_2^2 \cos^2 \omega t}{M^2 R}$$

$$\overline{P_{out}} = \frac{1}{2\pi} \int_0^{2\pi} P_{out} = \frac{V_1^2 L_2^2}{2 M^2 R}$$

$$\frac{V_1^2 L_2}{2 L_1 R} = \frac{V_1^2 L_2^2}{2 M^2 R}$$

So they are equal.

Problem 8.

(a) Since the current in the inductor can't change suddenly and the capacitor can be seen as a open circuit,

$$I_1 = I_3 = \frac{40}{50} = 0.8A, I_2 = I_4 = 0A$$
$$U_1 = 40V, U_2 = U_3 = U_4 = U_5 = 0V$$

(b) Since the inductor can be seen as a short circuit,

$$R = 50 + \frac{1}{1/50 + 1/100} = \frac{250}{3}$$

$$I_1 = \frac{40}{250/3} = 0.48A, I_2 = 0.48 \cdot \frac{50}{150} = 0.16A, I_3 = 0.32A, I_4 = 0A$$

$$U_1 = 40 \cdot \frac{50}{250/3} = 24V, U_2 = 0V, U_3 = U_4 = U_5 = 16V$$

(c)

$$Q = CV = 12 \times 10^{-6} \cdot 16 = 1.92 \times 10^{-4} C$$

(d)