VP260 PROBLEM SET 1

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Problem 1.

$$E = \frac{1}{4\pi\varepsilon_0}\int_{\theta_1}^{\theta_2}\frac{Q(\cos\theta\hat{n_x}+\sin\theta\hat{n_y})}{r^2(\theta_2-\theta_1)}d\theta = \frac{Q[(\sin\theta_2-\sin\theta_1)\hat{n_x}+(\cos\theta_1-\cos\theta_2)\hat{n_y}]}{4\pi\varepsilon_0 r^2(\theta_2-\theta_1)}$$

$$|E| = \frac{-Q}{4\pi\varepsilon_0 r^2(\theta_2-\theta_1)}\sqrt{2-2\sin\theta_1\sin\theta_2-2\cos\theta_1\cos\theta_2} = \frac{-Q}{2\pi\varepsilon_0 r^2}\cdot\frac{\sin\frac{\theta_1+\theta_2}{2}}{\theta_2-\theta_1}$$

$$|E_A| = \frac{-Q}{2\pi\varepsilon_0 r^2}\cdot\frac{2\sqrt{2}}{3\pi} \qquad |E_B| = \frac{-Q}{2\pi\varepsilon_0 r^2}\cdot\frac{2}{\pi}$$

$$|E_C| = \frac{-Q}{2\pi\varepsilon_0 r^2}\cdot\frac{2\sqrt{2}}{\pi} \qquad |E_D| = 0$$

$$|E_D| < |E_A| < |E_B| < |E_C|$$

Problem 2.

Suppose right to be the direction of x-axis and up to be the direction of the y-axis.

At point A,

$$E_1 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{4q}{a^2}\,\hat{n_y} \quad E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{8q}{a^2}\,\hat{n_y}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q}{5a^2}\left(\frac{2\sqrt{5}}{5}\hat{n_x}-\frac{\sqrt{5}}{5}\hat{n_y}\right)$$

$$E_A = \frac{q}{5\pi\varepsilon_0 a^2}\left(\frac{2\sqrt{5}}{5}\hat{n_x}+\frac{25-\sqrt{5}}{5}\hat{n_y}\right) = (6.43\times10^{20}\,\hat{n_x}+3.27\times10^{21}\,\hat{n_y})\,\mathrm{N/C}$$

At point B,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\,\hat{n_x} \quad E_2 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\,\hat{n_y}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\left(\frac{\sqrt{2}}{2}\hat{n_x}+\frac{\sqrt{2}}{2}\hat{n_y}\right)$$

$$E_B = \frac{q}{4\pi\varepsilon_0 a^2}\left(\frac{\sqrt{2}+2}{2}\hat{n_x}+\frac{\sqrt{2}-2}{2}\hat{n_y}\right) = (1.53\times10^{21}\,\hat{n_x}-2.63\times10^{21}\,\hat{n_y})\,\mathrm{N/C}$$

At point C,

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{a^2} \left(\frac{\sqrt{2}}{2}\hat{n_x} - \frac{\sqrt{2}}{2}\hat{n_y} \right)$$

$$E_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q}{a^2} \left(\frac{\sqrt{2}}{2}\hat{n_x} + \frac{\sqrt{2}}{2}\hat{n_y} \right)$$

$$E_C = \frac{\sqrt{2}q}{\pi\varepsilon_0 a^2}\,\hat{n_x} = 5.08 \times 10^{21}\,\hat{n_x}\,\mathrm{N/C}$$

Problem 3.

Suppose top to be the positive direction,

The two charges on the left will give a electric field

$$E_1 = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{(r+a/2)^2 + a^2/4} \cdot \frac{a/2}{\sqrt{(r+a/2)^2 + a^2/4}}$$

The two charges on the right will give a electric field

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{(r-a/2)^2 + a^2/4} \cdot \frac{a/2}{\sqrt{(r-a/2)^2 + a^2/4}}$$

As $r \gg a$,

$$\begin{aligned} E = E_1 + E_2 &\approx \frac{qa}{4\pi\varepsilon_0} \left(\frac{1}{(r-a/2)^3} - \frac{1}{(r+a/2)^3} \right) \\ &= \frac{qa}{4\pi\varepsilon_0} \cdot \frac{3r^2a + a^3/4}{(r^2 - a^2/4)^3} \approx \frac{3qa^2}{4\pi\varepsilon_0 r^4} \end{aligned}$$

Problem 4.

(a)

$$E = \int_0^l \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda}{(l+a-x)^2} dx = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{1}{l+a-x}\Bigg|_0^l = \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{l+a} \right) = \frac{\lambda l}{4\pi\varepsilon_0 a(l+a)}$$

(b)

$$E = \int_0^l \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ax}{(l+a-x)^2} dx = \frac{A}{4\pi\varepsilon_0} \left(\frac{l+a}{l+a-x} + ln(l+a-x) \right) \Bigg|_0^l = \frac{A}{4\pi\varepsilon_0} \left(\frac{l}{a} + ln\frac{a}{l+a} \right)$$

Problem 5.

(a)

$$E = \int_{R_1}^{R_2} \frac{1}{4\pi\varepsilon_0} \cdot \frac{x\sigma 2\pi r dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \int_{x^2+R_1^2}^{x^2+R_2^2} \frac{du}{2u^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}} \right)$$

(b) When $x \ll R_1 < R_2$,

$$E \approx \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

It is approximately proportional to the distance from the center.

$$mg = \frac{q\sigma x_0}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$F = \frac{q\sigma(x - x_0)}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$k = \frac{q\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2m\varepsilon_0 R_1 R_2}{q\sigma(R_2 - R_1)}}$$

Problem 6.

(a)

$$\begin{aligned}
F &= \frac{1}{4\pi\varepsilon_0}\int_0^l\int_0^l \frac{Q\frac{dx}{l}\cdot Q\frac{dy}{l}}{(a+x+y)^2} \\
&= \frac{Q^2}{4\pi\varepsilon_0 l^2}\int_0^l\int_0^l \frac{1}{(a+x+y)^2}dydx \\
&= \frac{Q^2}{4\pi\varepsilon_0 l^2}\int_0^l \left(\frac{1}{a+x} - \frac{1}{a+x+l}\right)dx \\
&= \frac{Q^2}{4\pi\varepsilon_0 l^2} ln\frac{a+x}{a+x+l}\Big|_0^l \\
&= \frac{Q^2}{4\pi\varepsilon_0 l^2} ln\left[\frac{(a+l)^2}{a(a+2l)}\right]
\end{aligned}$$

(b)

$$\lim_{x\to 0}\frac{ln(x+1)}{x} = 1$$

$$ln(x+1) \approx x \; when \; x \to 0$$

$$\frac{(a+l)^2}{a(a+2l)} = 1 + \frac{l^2}{a^2+2al}$$

If $a \gg l$,

$$ln\left[\frac{(a+l)^2}{a(a+2l)}\right] \approx \frac{l^2}{a^2+2al}$$

$$F \approx \frac{Q^2}{4\pi\varepsilon_0 l^2}\cdot\frac{l^2}{a^2+2al} = \frac{Q^2}{4\pi\varepsilon_0(a^2+2al)} \approx \frac{Q^2}{4\pi\varepsilon_0 a^2}$$