vp260 RC5

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1 Maxwell's Equation

Gauss's law for E

$$\oint \overrightarrow{E} d\overrightarrow{A} = \frac{Q_{encl}}{\varepsilon_0} \tag{1}$$

Gauss's law for B

$$\oint \overrightarrow{B} d\overrightarrow{A} = 0 \tag{2}$$

Ampere's law

$$\oint \overrightarrow{B} d\overrightarrow{l} = \mu_0 (i + \varepsilon_0 \frac{d\phi_E}{dt})_{encl} \tag{3}$$

Faraday's law

$$\oint \overrightarrow{E} d\overrightarrow{l} = -\frac{d\phi_B}{dt} \tag{4}$$

The differential form is

$$div \overrightarrow{E} = \frac{\rho}{\varepsilon_0} \tag{5}$$

$$div \overrightarrow{B} = 0 \tag{6}$$

$$rot \overrightarrow{B} = \mu_0 \overrightarrow{j} + \mu_0 \varepsilon_0 \frac{\partial \overrightarrow{E}}{\partial t} \tag{7}$$

$$rot \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} \tag{8}$$

2 Displacement Current

We have seen the Ampere's law

$$\oint \overrightarrow{B} d\overrightarrow{l} = \mu_0 (i + \varepsilon_0 \frac{d\phi_E}{dt})_{encl} \tag{9}$$

We can also write it as

$$\oint \overrightarrow{B} d\overrightarrow{l} = \mu_0 (i + i_d) \tag{10}$$

Where i_d is called the displacement current. It is defined as

$$i_d = \frac{\partial}{\partial t}\int\int_s D \cdot dS = \int\int_s j_D \cdot dS \tag{11}$$

Where j_D is the density of the displacement current

3 Problem 1

A circular shape capacitor has radius R and the distance d between two boards ($d \ll R$). The capacitor is connected to the battery. Assuming the electric current is increasing from $t = 0$ with the speed I_0(unit:A/s), calculate

(1). The electric density on surface

(2). The magnetic field inside and outside the capacitor

Answer

Since

$$I = I_0 t$$

(1)

$$q = \int_0^t I dt = 0.5 I_0 t^2$$

So the surface density is

$$\sigma = q(t)/S = \frac{I_0 t^2}{2\pi R^2} \tag{12}$$

For $t = 0.2s$, $\sigma = I_0/100\pi R^2$ (2).The displacement current is given by

$$j_D = \partial D/\partial t = \partial\sigma/\partial t = I_0 t/\pi R^2 \tag{13}$$

For $t = 2s$, $j_D = I_0/5R^2\pi$ Using the Ampere's law

$$\oint B \cdot dl = \mu_0 \Sigma I_{self} + \mu_0 \int\int j_D \cdot dS \tag{14}$$

The outside is $j_D = 0$,$\Sigma_{self} = I_0 t$

So $B_{out} = \mu_0 I_0 t/2\pi r$

when$t = 0.2s$,$B_{out} = \mu_0 I_0/10\pi r$

The inside is given by

$$B_{in} = \mu_0 j_D \cdot \pi r^2/2\pi r = \mu_0 j_D r/2 = \mu_0 I_0 r/10\pi R^2$$