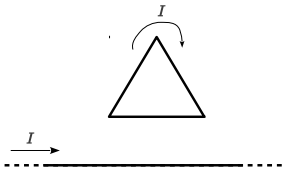
PROBLEM SET 7

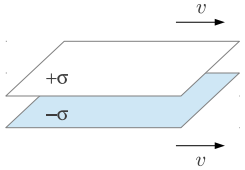
Due: 9 November 2016, 4 p.m.

Problem 1. What is the force on the equilateral triangle loop with side a placed a distance s from a long, straight–line wire? The electric current in both is I.



(3 marks)

Problem 2. A large parallel–plate capacitor with uniform surface charge of density σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v, as shown in the figure below.



(a) Find the magnetic field between the plates and also above and below them.

(b) Find the magnetic force per unit area on the upper plate, including its direction.

(c) At what speed v would the magnetic force balance the electric force? (See footnote 8 on p. 220 in Griffiths).

Hint. Attend recitation classes.

(3/2 + 3/2 + 1 marks)

Problem 3. A thin disk made of dielectric material with radius a has total charge $Q > 0$ distributed uniformly over its surface. It rotates n times per second about the axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

(4 marks)

Problem 4. An infinitely long conducting tape of width L and negligible thickness lies in a horizontal plane and carries a uniform current I (in the direction of the long dimension).

(a) Show that at on the axis of symmetry of the tape, at a distance y from its surface, the magnitude of the magnetic field is equal to $B(y) = (\mu_0 I/\pi L) \arctan L/2y$.

(b) Discuss the result in the limit $y \gg L$.

(3 + 1/2 marks)

Problem 5. A long, straight, solid cylinder of radius a, oriented with its axis in the z–direction, carries an electric current of density

$$\mathbf{J}(\mathbf{r}) = \begin{cases} \frac{b}{r} \exp\left(\frac{r-a}{\delta}\right) \hat{\mathbf{k}} & \text{for } r \leq a \\ 0 & \text{otherwise,} \end{cases}$$

where r is the radial distance from the axis of the cylinder and $a, b, \delta > 0$ are constants (what are their units?).

(a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of a, b, δ.

(b) Use Ampère's law to find the magnetic field $\mathbf{B}$ in the region $r > a$. Express your answer in terms of I_0 rather than b.

(c) Obtain an expression for the current I through a circular cross section of radius $r \leq a$ and centered at the cylinder axis. Express your answer in terms of I_0.

(d) Use Ampère's law to find the magnetic field $\mathbf{B}$ in the region $r \leq a$.

(*1/2 + 2 + 1 + 2 marks*)

Problem 6. Is Ampère's law consistent with the general rule that you know from calculus that divergence–of–curl is always zero? Show that Ampère's law *cannot* be valid, in general, outside magnetostatics.

Hint. The continuity equation.

(*3 marks*)