PROBLEM SET 6

Due: 3 November 2016, 2 p.m.

Problem 1. A particle with mass m and charge q moves in mutually perpendicular electric and magnetic fields $\mathbf{E} = (0, 0, E_0)$ and $\mathbf{B} = (B_0, 0, 0)$, where E_0 and B_0 are positive constants. Find and sketch the trajectory of the particle if it starts at the origin with velocity

(a) $\mathbf{v}(0) = (E/B)\hat{n}_y$,
(b) $\mathbf{v}(0) = (E/2B)\hat{n}_y$,
(c) $\mathbf{v}(0) = (E/B)(\hat{n}_y + \hat{n}_z)$.

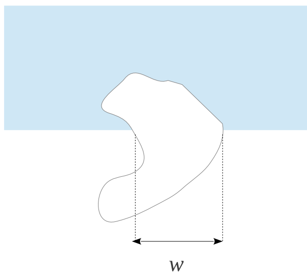
You are allowed to use all results we have derived in class.

(*3 × 3/2 marks*)

Problem 2. A particle with mass m and positive charge q moves in antiparallel electric and magnetic fields $\mathbf{E} = (-E_0, 0, 0)$ and $\mathbf{B} = (B_0, 0, 0)$, where E_0 and B_0 are positive constants. Assuming the initial conditions: $\mathbf{v}(0) = (v_{0x}, v_{0y}, 0)$ and $\mathbf{r}(0) = (0, 0, 0)$, find the velocity $\mathbf{v}(t)$ and position $\mathbf{r}(t)$ for $t > 0$.

(*6 marks*)

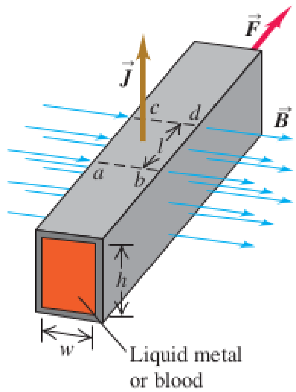
Problem 3. A plane wire loop of irregular shape is situated so that part of it is in a uniform magnetic field $\mathbf{B}$ (in the figure below the field occupies the shaded region and points perpendicular to the plane of the loop). The loop carries the current I. Show that the magnitude of the net magnetic force on the loop is $F = IBw$, where w is the chord subtended.



What is the direction of the force?

(*3 marks*)

Problem 4. Magnetic forces acting on conducting fluids provide a convenient means of pumping these fluids. For example, this method can be used to pump blood without the damage to the cells that can be caused by a mechanical pump. A horizontal tube with rectangular cross section (height h, width w) is placed at right angles to a uniform magnetic field with magnitude B so that a length l is in the field (see the figure below). The tube is filled with a conducting liquid, and an electric current of density J is maintained in the third mutually perpendicular direction.



(a) Show that the difference of pressure between a point in the liquid on a vertical plane through ab and a point in the liquid on another vertical plane through cd, under conditions in which the liquid is prevented from flowing, is $\Delta p = JlB$.

(b) What current density is needed to provide a pressure difference of 1 atm between these two points if $B = 2.2$ T and $l = 35$ mm?

(*2 + 1 marks*)

Problem 5. In class we derived an expression for the torque on a current loop assuming that the magnetic field $\mathbf{B}$ was uniform. But what if $\mathbf{B}$ is not uniform?

Assume we have a square loop of wire that lies in the xy–plane. The loop has corners at $(0,0)$, $(0,L)$, (L,L) and $(L,0)$ and carries a constant current I in the clockwise direction. The magnetic field $\mathbf{B} = (B_0 y/L, B_0 x/L, 0)$, where B_0 is a positive constant.

(a) Sketch the magnetic field lines in the xy–plane.

(b) Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop.

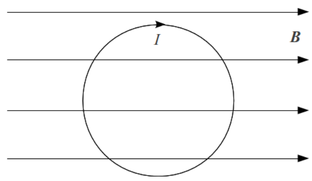
(c) If the loop is free to rotate about the x–axis, find the magnitude and direction of the magnetic torque on the loop.

(d) Repeat part (c) for the case in which the loop is free to rotate about the y–axis.

(e) Is equation $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ an appropriate description of the torque on this loop. Why or why not?

(1/2 + 1 + 2 + 2 + 1 marks)

Problem 6. A circular loop of radius R carries a clockwise electric current I. The loop is placed in a uniform magnetic field $\mathbf{B}$ (see the figure).



(a) What is the net force on the current loop?

(b) Find the torque on the current loop with respect to the axis of symmetry of the loop perpendicular to the vector $\mathbf{B}$.

(1 + 3 marks)

Problem 7. In a certain region of space, the magnetic field $\mathbf{B}$ is not uniform: it has both a z–component and a component that points radially away from or towards the z–axis. The z–component is given by $B_z(z) = \beta z$, where β is a positive constant. The radial component B_r depends only on r, the radial distance from the z–axis. (a) Use Gauss's law for magnetism, to find B_r as a function of r. (b) Sketch the magnetic field lines.

(2 + 1 marks)