Problem Set 2

Due: 29 September 2016, 2 p.m.

Problem 1. A point charge q is placed at the point $\mathbf{r} = 0$.

(a) By direct calculation show that the divergence of the electric field $\operatorname{div} \mathbf{E}(\mathbf{r}) = 0$ for all $\mathbf{r} \neq 0$. What about the divergence at $\mathbf{r} = 0$?

(b) What is the condition $\operatorname{div} \mathbf{E}(\mathbf{r})$ should satisfy at $\mathbf{r} = 0$ for Gauss's law to be valid? *Hint.* Use Gauss's law in the integral form.

(*3/2 + 3/2 marks*)

Problem 2. Suppose the electric field in some region is found to be $\mathbf{E} = kr^3\hat{r}$, where k is a constant.

(a) Find the charge density.

(b) Find the total charge contained in a sphere of radius R, centered at the origin. (Do it in two different ways: (1) by integrating the charge density found in part (a) and (2) by using the integral form of Gauss's law.)

(*1 + 3 marks*)

Problem 3. In a certain region of space, the electric field $\mathbf{E}$ is uniform, *i.e.* is constant in both magnitude and direction.

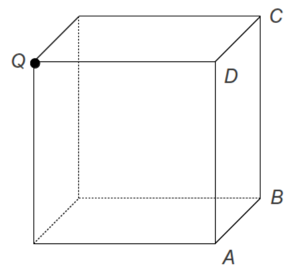
(a) Use Gauss's law to argue that this region of space must be electrically neutral, i.e., the volume charge density ρ must be zero.

(b) Is the converse true? That is, in a region of space where there is no charge, must $\mathbf{E}$ be uniform? Explain.

(*1 + 1 marks*)

Problem 4. A point charge $Q > 0$ is placed at a vertex of a cube as shown in the figure. Find the electric flux through the surface $ABCD$.

(*4 marks*)



Problem 5. A solid insulating ball with radius R is charged non-uniformly: the volume charge density $\rho = Ar/R$, where A is a positive constant, and r is the distance from the center of the ball

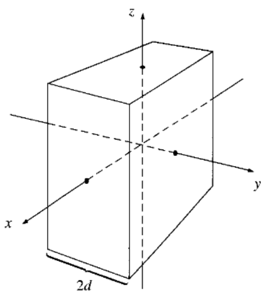
(a) Show that the total charge of the ball is $Q = \pi A R^3$.

(b) Find the electric field $\mathbf{E}(\mathbf{r})$ both inside and outside of the ball. Sketch $|\mathbf{E}|$ as a function of r.

(1 + 3 marks)

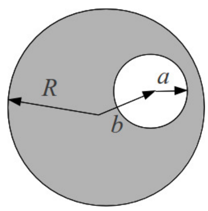
Problem 6. An infinite plane slab of thickness $2d$ (see the figure below) carries a uniform volume density ρ. Find the electric field as a function of y, where $y = 0$ at the center of the slab. Plot E versus y, calling E positive if it points in the $+y$ direction and negative if it points in the $-y$ direction.

(5 marks)



Problem 7. Consider an infinite solid insulating cylinder with radius R uniform charged with constant density $\rho < 0$. Imagine that a cylindrical hole with radius a has been bored along the entire length of the cylinder. The axis of the cavity is a distance b from the axis of the cylinder, where $a < b < R$.

Find the magnitude and the direction of the electric field $\mathbf{E}$ inside the cavity to show that $\mathbf{E}$ is uniform over the entire cavity.



(4 marks)

Problem 8. Two spherical cavities, of radii r_a and r_b, are hollowed out from the interior of a neutral conducting ball of radius R. At the center of each cavity a point charge is placed: q_a and q_b, respectively.

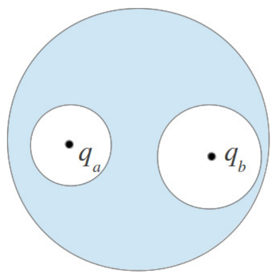
(a) Find the surface densities of charge σ_a, σ_b on the walls of the cavities as well as on the surface of the ball σ_R.

(b) What is the electric field outside of the conductor?

(c) What is the electric field within each cavity?

(d) What is the force on q_a and q_b?

(e) Which of these answers would change if a third charge q_c were brought near the conductor?



(cross-sectional sketch)

Explain your answers.

(5 × 2 marks)