VP260 PROBLEM SET 10

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Problem 1.

(a)

$$v_s = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} I$$

$$v_{hi} = \sqrt{R^2 + (\omega L)^2} I$$

$$\frac{v_{hi}}{v_s} = \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

When ω is small,

$$\frac{v_{hi}}{v_s} \approx \sqrt{\frac{R^2}{R^2 - 2LC + 1/\omega^2 C^2}} \approx \sqrt{R^2 \omega^2 C^2} = RC\omega$$

When ω is large,

$$\frac{v_{hi}}{v_s} \approx \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$$

(b)

$$v_{lo} = \frac{1}{\omega C} I$$

$$\frac{v_{lo}}{v_s} = \frac{1}{\omega C} \sqrt{\frac{1}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

When ω is large,

$$\frac{v_{hi}}{v_s} \approx \frac{1}{\omega C} \sqrt{\frac{1}{\omega^2 L^2}} = \frac{1}{LC} \omega^{-2}$$

When ω is small,

$$\frac{v_{hi}}{v_s} \approx \frac{1}{\omega C} \sqrt{\frac{1}{R^2 - 2LC + 1/\omega^2 C^2}} \approx \frac{1}{\omega C} \sqrt{\omega^2 C^2} = 1$$

Problem 2.

(a) According to KCL, $i = i_R + i_L + i_C$
According to KVL, $v = v_R + v_L + v_C$

(b)

$$i_R = \frac{V\angle 0}{R} = \frac{V}{R}\angle 0, \phi = 0$$
$$i_L = \frac{V\angle 0}{j\omega L} = \frac{V}{\omega L}\angle -\frac{1}{2}\pi, \phi = -\frac{1}{2}\pi$$
$$i_C = \frac{V\angle 0}{1/j\omega C} = \omega C V\angle\frac{1}{2}\pi, \phi = \frac{1}{2}\pi$$

(c)

$$I = I_R + I_L + I_C = V\left[\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right]$$
$$|I| = \sqrt{\frac{V^2}{R^2} + \left(\omega C V - \frac{V}{\omega L}\right)^2} = \sqrt{I_R^2 + (I_C - I_L)^2}$$
$$Z^{-1} = \frac{I}{V} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

(d) When $\omega = \frac{1}{\sqrt{LC}}$,

$$I_C - I_L = \frac{C}{\sqrt{LC}} - \frac{\sqrt{LC}}{L} = 0$$

Since $I = \sqrt{I_R^2 + (I_C - I_L)^2}$, $I_{min} = I_R$

$$P = \frac{V^2}{R} = \frac{V^2 \cos^2 \omega t}{R}$$
$$\overline{P} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{V^2 \cos^2 \omega t}{R} = \frac{V^2}{2R}$$

So the instant power is decided by t and the average power is constant. It is wrong to say that the power delivered to the resistor is also a minimum.

(e) In LRC parallel circuit,

$$I = V\left[\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right]$$
$$\phi = \arctan\left(\frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}}\right) = 0$$

In LRC series circuit,

$$I = \frac{V}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{V\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
$$\phi = \arctan\left(-\frac{\omega L - \frac{1}{\omega C}}{R}\right) = 0$$

Problem 3.

(a)

$$a = \frac{v^2}{r} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mr} = \frac{2E_k}{mr} = 1.533 \times 10^{15} m/s^2$$

$$\Delta E \approx \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} = 1.340 \times 10^{-23} J$$

$$\frac{\Delta E}{E} = 1.394 \times 10^{-11}$$

(b)

$$v = \sqrt{\frac{2E_k}{m_p}} = 3.390 \times 10^7 m/s$$

$$a = \frac{v^2}{r} = 1.533 \times 10^{15} m/s^2$$

$$\Delta E \approx \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} = 1.340 \times 10^{-23} J$$

$$\frac{\Delta E}{\frac{1}{2}m_e v^2} = 2.559 \times 10^{-8}$$

(c)

$$a = \frac{2E_k}{mr} = 4.834 \times 10^{19} m/s^2$$

$$\Delta E \approx \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} = 1.334 \times 10^{-14} J$$

$$\frac{\Delta E}{E} = 6.121 \times 10^3$$

So we can;t use classical physics in describing the atom within the model.

Problem 4.

$$\frac{\partial^2 f_1}{\partial x^2} = Ak[(2x-2vt)^2-2]e^{-k(x-vt)^2}$$
$$\frac{\partial^2 f_1}{\partial t^2} = Akv^2[(2x-2vt)^2-2]e^{-k(x-vt)^2}$$
$$\frac{\partial^2 f_1}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}, \text{so it satisfies.}$$

$$\frac{\partial^2 f_2}{\partial x^2} = -Ak^2\sin[k(x-vt)]$$
$$\frac{\partial^2 f_2}{\partial t^2} = -Ak^2v^2\sin[k(x-vt)]$$
$$\frac{\partial^2 f_2}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}, \text{so it satisfies.}$$

$$\frac{\partial^2 f_3}{\partial x^2} = \frac{6Ak^2(x-vt)^2-2Ak}{[k(x-vt)^2+1]^3}$$
$$\frac{\partial^2 f_3}{\partial t^2} = \frac{6Ak^2v^2(x-vt)^2-2Akv^2}{[k(x-vt)^2+1]^3}$$
$$\frac{\partial^2 f_3}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f_3}{\partial t^2}, \text{so it satisfies.}$$

$$\frac{\partial^2 f_4}{\partial x^2} = Ak^2(4k^2x^2-2)e^{-k(kx^2+vt)}$$
$$\frac{\partial^2 f_4}{\partial t^2} = Ak^2v^2e^{-k(kx^2+vt)}$$
$$\frac{\partial^2 f_4}{\partial x^2} \neq \frac{1}{v^2}\frac{\partial^2 f_4}{\partial t^2}, \text{so it doesn't satisfy.}$$

$$\frac{\partial^2 f_5}{\partial x^2} = -Ak^2\sin(kx)\cos(kvt)^3$$
$$\frac{\partial^2 f_5}{\partial t^2} = -Ak^3v^3t\sin(kx)[6\sin(kvt)^3+9(kvt)^3\cos(kvt)^3]$$
$$\frac{\partial^2 f_5}{\partial x^2} \neq \frac{1}{v^2}\frac{\partial^2 f_5}{\partial t^2}, \text{so it doesn't satisfy.}$$

Problem 5.

(a)

$$\frac{\partial^2 \xi}{\partial x^2} = -Ak^2 \sin(kx)\cos(\omega t)$$
$$\frac{\partial^2 \xi}{\partial t^2} = -A\omega^2 \sin(kx)\cos(\omega t)$$
$$\frac{\partial^2 \xi}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 \xi}{\partial t^2}, \text{so it satisfies.}$$

(b)

$$\begin{aligned}\xi(x,t) &= \frac{1}{2}Asin(kx+\omega t) + \frac{1}{2}Asin(kx-\omega t)\\ &= \frac{1}{2}Asin(k(x+vt)) + \frac{1}{2}Asin(k(x-vt))\end{aligned}$$

Problem 6.

$$\frac{\partial \xi}{\partial x} = \frac{\partial \xi}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial x} + \frac{\partial \xi}{\partial \beta}\cdot\frac{\partial \beta}{\partial x} = \frac{\partial \xi}{\partial \alpha} + \frac{\partial \xi}{\partial \beta}$$
$$\frac{\partial^2 \xi}{\partial x^2} = \frac{\partial^2 \xi}{\partial \alpha^2} + 2\frac{\partial^2 \xi}{\partial \alpha \partial \beta} + \frac{\partial^2 \xi}{\partial \beta^2}$$
$$\frac{\partial \xi}{\partial t} = \frac{\partial \xi}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial t} + \frac{\partial \xi}{\partial \beta}\cdot\frac{\partial \beta}{\partial t} = v\left(\frac{\partial \xi}{\partial \alpha} - \frac{\partial \xi}{\partial \beta}\right)$$
$$\frac{\partial^2 \xi}{\partial t^2} = v^2\left(\frac{\partial^2 \xi}{\partial \alpha^2} - 2\frac{\partial^2 \xi}{\partial \alpha \partial \beta} + \frac{\partial^2 \xi}{\partial \beta^2}\right)$$
$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{v^2}\cdot\frac{\partial^2 \xi}{\partial t^2} = 4\frac{\partial^2 \xi}{\partial \alpha \partial \beta} = 0$$
$$\frac{\partial^2 \xi}{\partial \alpha \partial \beta} = 0$$

In the equation

$$\xi(x,t) = \xi_1(\alpha) + \xi_2(\beta)$$
$$\frac{\partial^2 \xi}{\partial \alpha \partial \beta} = 0 + 0 = 0$$

So it may be expressed.

Problem 7.

(a)

$$\frac{\partial^2 E_y}{\partial x^2} = -2E_0 k_C^2 e^{-k_C x} \cos(k_C x - \omega t) = -E_0 \omega \frac{\mu}{\rho} e^{-k_C x} \cos(k_C x - \omega t)$$

$$\frac{\partial E_y}{\partial t} = -E_0 \omega e^{-k_C x} \cos(k_C x - \omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu}{\rho} \frac{\partial E_y}{\partial t}$$

(b) The electric field decays because the energy is transformed into heat when it propagates. The transformation ratio is proportional to x, then

$$\frac{\partial E_y'}{\partial x} = -k_C x$$

$$E_y' = E_0 e^{-k_C x}$$

(c)

$$\frac{1}{k_C} = \sqrt{\frac{2\rho}{\omega\mu}} = 6.60 \times 10^{-5} m$$