VP260 PROBLEM SET 2

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Problem 1.

(a) When $r > 0$,

$$
\begin{aligned}
div\bar{E} = \nabla \cdot \bar{E} &= \frac{q}{4\pi\varepsilon_0}\left[\frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right)\right] \\
&= \frac{q}{4\pi\varepsilon_0} \cdot \frac{3(x^2+y^2+z^2)^{1.5} - 3x^2(x^2+y^2+z^2)^{0.5} - 3y^2(x^2+y^2+z^2)^{0.5} - 3z^2(x^2+y^2+z^2)^{0.5}}{(x^2+y^2+z^2)^3} \\
&= \frac{q}{4\pi\varepsilon_0} \cdot \frac{3(x^2+y^2+z^2)^{1.5} - 3(x^2+y^2+z^2)^{1.5}}{(x^2+y^2+z^2)^3} \\
&= 0
\end{aligned}
$$

When $r = 0$, $(x^2+y^2+z^2)^3 = 0$, so $div\bar{E} \to \infty$

(b)

$$\oint_\Sigma \bar{E}d\bar{A} = \int_\Omega div\bar{E}dV = \frac{1}{\varepsilon_0}\int_\Omega \rho(\bar{r})dV$$

Since $\rho(\bar{r}) \to \infty$ when $r = 0$, $div\bar{E} \to \infty$

Problem 2.

(a)

$$
\begin{aligned}
\rho(\bar{r}) = \varepsilon_0 div\bar{E} &= \varepsilon_0 k\left(\frac{\partial}{\partial x}xr^2 + \frac{\partial}{\partial y}yr^2 + \frac{\partial}{\partial z}zr^2\right) \\
&= \varepsilon_0 k(3r^2 + 2x^2 + 2y^2 + 2z^2) \\
&= 5\varepsilon_0 kr^2
\end{aligned}
$$

(b) (1)

$$q = \int_\Omega \rho(\bar{r})dV = \int_0^R \rho(\bar{r}) \cdot 4\pi r^2 dr = \int_0^R 20\varepsilon_0 k\pi r^4 dr = 4\varepsilon_0 k\pi r^5$$

(2)

$$\frac{q}{\varepsilon_0} = \oint_\Sigma \bar{E}d\bar{A} = ES = kr^3 \cdot 4\pi r^2$$

$$q = 4\varepsilon_0 k\pi r^5$$

Problem 3.

(a)

$$div\bar{E} = \frac{\rho(r)}{\varepsilon_0}$$

Suppose $E = A\hat{n_x} + B\hat{n_x} + C\hat{n_z}$,

$$div\bar{E} = 0$$

$$\rho(r) = 0$$

So this region of space must be electrically neutral.

(b) No. As is shown in Problem 1, $\rho(r) = 0$ when $r > 0$ around a point charge, but $\bar{E}$ is actually not uniform in this region.

Problem 4.

In the three surfaces which doesn't contain the point charge, since they are symmetric, and the cube is only $\frac{1}{8}$ of the complete cube around the point charge, each $\phi = \frac{1}{24}\Phi$

$$\Phi = \frac{Q}{\varepsilon_0}$$

$$\phi_{ABCD} = \frac{Q}{24\varepsilon_0}$$

Problem 5.

(a)

$$q = \int_\Omega \rho(\bar{r})dV = \int_0^R \rho(\bar{r}) \cdot 4\pi r^2 dr = \int_0^R \frac{4A\pi r^3}{R} dr = \pi AR^3$$

(b) $E(r)$ inside the ball when $r < R$,

$$q = \int_\Omega \rho(\bar{r})dV = \int_0^r \rho(\bar{r}) \cdot 4\pi r^2 dr = \int_0^r \frac{4A\pi r^3}{R} dr = \frac{\pi Ar^4}{R}$$

$$\frac{\pi Ar^4}{R\varepsilon_0} = E(r) \cdot 4\pi r^2$$

$$E(r) = \frac{Ar^2}{4\varepsilon_0 R}$$

$E(r)$ outside the ball when $r > R$,

$$\frac{\pi AR^3}{\varepsilon_0} = E(r) \cdot 4\pi r^2$$

$$E(r) = \frac{AR^3}{4\varepsilon_0 r^2}$$

Problem 6.

Suppose the surface of the plane is S
In the slab, where $|y| \leqslant d$

$$\frac{2\rho|y|S}{\varepsilon_0} = E(y) \cdot 2S$$

$$E(y) = \frac{\rho|y|}{\varepsilon_0}$$

Outside the slab, where $|y| > d$

$$\frac{2\rho dS}{\varepsilon_0} = E(y) \cdot 2S$$

$$E(y) = \frac{\rho d}{\varepsilon_0}$$

$$E(y) = \begin{cases} \frac{\rho|y|}{\varepsilon_0} & |y| \leqslant d \\ \frac{\rho d}{\varepsilon_0} & |y| > d \end{cases}$$

Problem 7.

Suppose we can fill the cylinder with charge of constant density ρ and $-\rho$, then the total charge in the cylinder won't change. Suppose there is a point A in the cavity, the center of the cavity is O_1 and the center of the cylinder is O_2

$$\frac{-\rho\pi O_1A^2h}{\varepsilon_0} = \overline{E_1} \cdot 2\pi\overline{O_1A}h$$

$$\overline{E_1} = -\frac{\rho}{2\varepsilon_0}\overline{O_1A}$$

$$\frac{\rho\pi O_2A^2h}{\varepsilon_0} = \overline{E_2} \cdot 2\pi\overline{O_2A}h$$

$$\overline{E_2} = \frac{\rho}{2\varepsilon_0}\overline{O_2A}$$

$$\bar{E} = \overline{E_1} + \overline{E_2} = \frac{\rho}{2\varepsilon_0}\overline{O_2O_1}$$

The magnitude is $\frac{\rho b}{2\varepsilon_0}$ and the direction is $\overline{O_1O_2}$ since $\rho < 0$

Problem 8.

(a)

$$\sigma_a = -\frac{q_a}{4\pi r_a^2}$$

$$\sigma_b = -\frac{q_b}{4\pi r_b^2}$$

They are uniform since they are symmetric.

$$\sigma_R = -\frac{q_a + q_b}{4\pi R^2}$$

It is uniform since electrostatic shield.

(b)

$$\frac{q_a + q_b}{\varepsilon_0} = E(r) \cdot 4\pi r^2$$

$$E(r) = \frac{q_a + q_b}{4\varepsilon_0 \pi r^2}$$

(c) In cavity a,

$$\frac{q_a}{\varepsilon_0} = E(r) \cdot 4\pi r^2$$

$$E_a(r) = \frac{q_a}{4\varepsilon_0 \pi r^2}$$

In cavity b,

$$\frac{q_b}{\varepsilon_0} = E(r) \cdot 4\pi r^2$$

$$E_b(r) = \frac{q_b}{4\varepsilon_0 \pi r^2}$$

(d) since electrostatic shield, there isn't force exerted on q_a and q_b. So $F_a = F_b = 0$

(e) σ_R and $E(r)$ will change because the electric field outside the ball is influenced by q_c. Others won't change because of electrostatic shield.