PROBLEM SET 4

Due: 18 October 2016, 2 p.m.

Problem 1. The charge of the electron was first measured by the American physicist Robert Millikan. In his experiment, oil is sprayed in very fine drops (around 10^{-4} mm in diameter) into the space between two parallel horizontal plates separated by a distance d. A potential difference V_{AB} is maintained between the parallel plates, causing a downward electric field between them. Some of the oil drops acquire a negative charge because of frictional effects or because of ionization of the surrounding air by X-rays or radioactivity. The drops are observed through a microscope.

(a) Show that an oil drop of radius r at rest between the plates will remain at rest if the magnitude of its charge is

$$q = \frac{4\pi}{3}\frac{\rho r^3 g d}{V_{\mathrm{AB}}},$$

where ρ is the density of oil. (Ignore the buoyant force of the air.) By adjusting V_{AB} to keep a given drop at rest, the charge on that drop can be determined, provided its radius is known.

(b) Millikan's oil drops were much too small to measure their radii directly. Instead, Millikan determined r by cutting off the electric field and measuring the terminal speed v_∞ of the drop as it fell. The magnitude of the viscous force F on a sphere of radius r moving with speed v through a fluid with viscosity h is given by Stokes's law: $F = 6\pi\eta r v$. When the drop is falling at v_∞, the viscous force just balances the weight of the drop mg. Show that the magnitude of the charge on the drop is

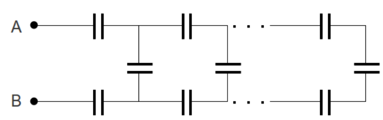
$$q = 18\pi\frac{d}{V_{\mathrm{AB}}}\sqrt{\frac{\eta^3 v_\infty^3}{2\rho g}}.$$

Within the limits of their experimental error, every one of the thousands of drops that Millikan measured had a charge equal to some small integer multiple of a basic charge e.

(c) A charged oil drop in a Millikan oil-drop apparatus is observed to fall 1.00 mm at constant speed in 39.3 s if $V_{\mathrm{AB}} = 0$. The same drop can be held at rest between two plates separated by 1.00 mm if $V_{\mathrm{AB}} = 9.16$ V. How many excess electrons has the drop acquired, and what is the radius of the drop? The viscosity of air is 1.81×10^{-5} N·s/m^2, and the density of the oil is 824 kg/m^3.

(*3 × 1 marks*)

Problem 2. For the infinite network of capacitors (each with capacitance C) shown in the figure, find the equivalent capacitance between points A and B.



Hint. Will the answer change if we attach one more module?

(*4 marks*)

Problem 3. A parallel-plate vacuum capacitor with plate area A and separation x has charges Q and $-Q$ on its plates. The capacitor is disconnected from the source of charge.

(a) What is the total energy stored in the capacitor?

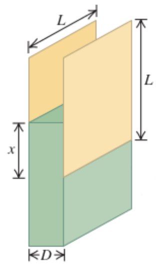
(b) The plates are pulled apart an additional distance $\mathrm{d}x$. What is the change in value of the stored energy?

(c) If F is the force with which the plates attract each other, then the change in the stored energy must be equal to the work $\delta W = F\mathrm{d}x$ done in pulling the plates apart. Find an expression for F.

(d) Explain why F is not equal to QE, where E is the electric field between the plates.

(*1/2 + 1 + 3/2 + 1 marks*)

Problem 4. Two square conducting plates with sides of length L are separated by a distance D. A dielectric slab with relative permittivity ε_r and dimensions $L \times L \times D$ is inserted a distance x into the space between the plates, as shown in the figure.



(a) Find the capacitance of this system.

(b) Suppose that the capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the dielectric slab is inserted an additional distance $\mathrm{d}x$ into the space between the plates, show that the change in stored energy is

$$\mathrm{d}U = \frac{(\varepsilon_\mathrm{r} - 1)\varepsilon_0 V^2 L}{2D}\,\mathrm{d}x.$$

(c) Suppose that before the slab is moved by $\mathrm{d}x$, the plates are disconnected from the battery, so that the charges on the plates remain constant. Determine the magnitude of the charge on each plate, and then show that when the slab is moved $\mathrm{d}x$ farther into the space between the plates, the stored energy changes by an amount that is the negative of the expression for $\mathrm{d}U$ given in part (b).

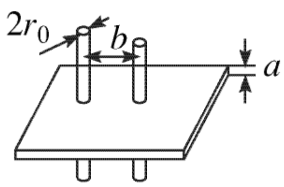
(d) If F is the force exerted on the slab by the charges on the plates, then $\mathrm{d}U$ should equal the work done against this force to move the slab a distance $\mathrm{d}x$. Thus $\mathrm{d}U = -F\mathrm{d}x$. Show that applying this expression to the result of part (b) suggests that the electric force on the slab pushes it out of the capacitor, while the result of part (c) suggests that the force pulls the slab into the capacitor.

(e) As we discussed in lecture, the force in fact pulls the slab into the capacitor. Explain why the result of part (b) gives an incorrect answer for the direction of this force, and calculate the magnitude of the force. (This method does not require knowledge of the nature of the fringing field.)

(*1 + 2 + 2 + 1 + 1 marks*)

Problem 5. Two thin conducing wires in the shape of cylinders with radii r_0 are attached to a large conducting slab of thickness a and separated by distance b from each other. Estimate the resistance between the wires, assuming that $a \ll r_0 \ll b$. Also, assume that the conductivity of the wires σ_0 is much larger than the conductivity of the slab σ.

(*7 marks*)



Problem 6. Two concentric spherical metal shells with radii r_a, r_b, where $r_a < r_b$, are separated by weakly conducting material with conductivity σ.

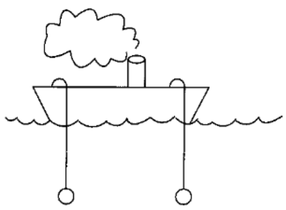
(a) If they are maintained at potential difference V, what current flows from one to each other?

Hint. Use the microscopic form of Ohm's law.

(b) What is the resistance between the shells?

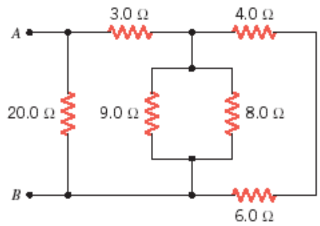
(c) Notice that if $r_b \gg r_a$, the outer radius r_b is irrelevant. How do you account for that? Exploit this observation to determine the current flowing between two metal spheres, each of radius r_a, immersed deep in the sea and held quite far apart, if the potential difference between them is V. (This arrangement can be used to measure the conductivity of sea water; see figure below.)

(*4 + 1 + 2 marks*)



Problem 7. Find the equivalent resistance between points A and B.

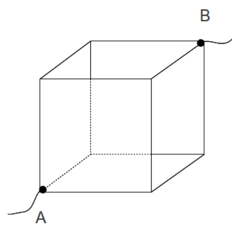
(*4 marks*)



Problem 8. Twelve identical resistors, each of resistance R, are connected to form a cube-shaped circuit (see the figure). Find the equivalent resistance between points A and B.

Hint. Use symmetry.

(*4 marks*)



Problem 9. Consider an infinite grid made of conducting wires arranged as in the figure below. The resistance of a single section of the wire in the grid is r. What is the effective resistance of the grid between two adjacent nodes, for example A and B?

Hint. Use symmetry.

(*4 marks*)