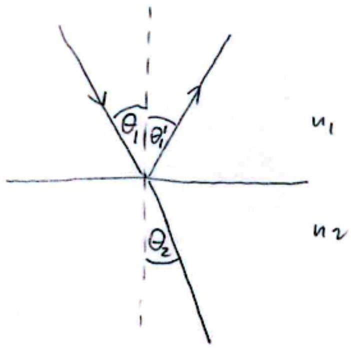
Dual nature of light: waves / particles behaviour

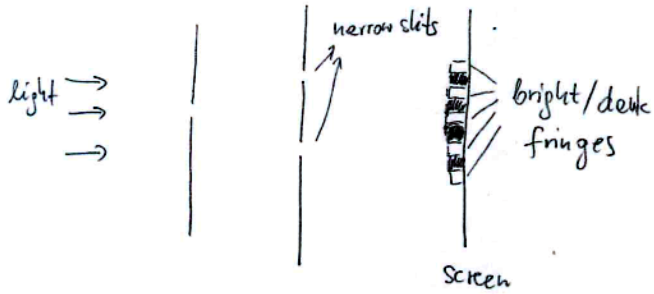
Geometric optics



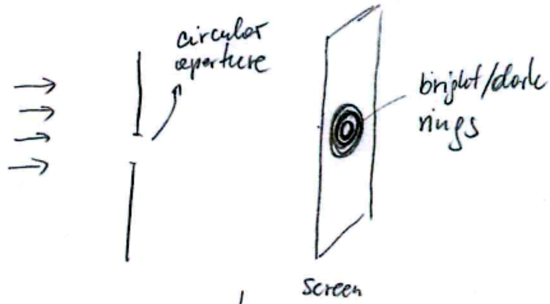
$$\begin{cases} \theta_1 = \theta_1' & \text{law of reflection} \\ \dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{n_2}{n_1} & \text{Snell's law} \end{cases}$$

↳ physics; the rest of geometric optics is geometry

Unable to explain:



↓ INTERFERENCE



↓ DIFFRACTION

Examples:

Colorful pattern on thin oil speck

FIG.

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Interference - overlapping of waves

SUPERPOSITION principle

if $\left.\begin{array}{l} \bar{E}_1 = \bar{E}_1(\bar{r},t) \\ \bar{E}_2 = \bar{E}_2(\bar{r},t) \end{array}\right\}$ satisfy wave eqn $\Rightarrow$ $\bar{E} = \alpha \bar{E}_1(\bar{r},t) + \beta E_2(\bar{r},t)$ also satisfies wave eqn

(α, β - any real numbers)

Wave eqns. in vacuum

$$\begin{cases} \nabla^2 \bar{E}_1(\bar{r},t) = \dfrac{1}{c^2} \dfrac{\partial^2 \bar{E}_1(\bar{r},t)}{\partial t^2} & /\cdot\alpha \\ \nabla^2 \bar{E}_2(\bar{r},t) = \dfrac{1}{c^2} \dfrac{\partial^2 \bar{E}_2(\bar{r},t)}{\partial t^2} & /\cdot\beta \end{cases}$$

+ analogously for $\bar{B}_1, \bar{B}_2$

⇓ linearity of the derivative

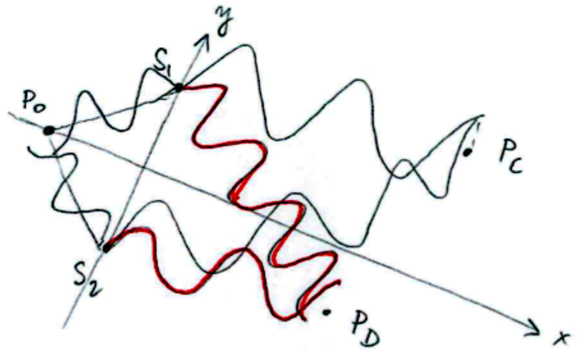
$$\nabla^2 \underbrace{\left(\alpha \bar{E}_1(\bar{r},t) + \beta \bar{E}_2(\bar{r},t)\right)}_{\bar{E}(\bar{r},t)} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \underbrace{\left(\alpha \bar{E}_1(\bar{r},t) + \beta \bar{E}_2(\bar{r},t)\right)}_{\bar{E}(r,t)} \Rightarrow \nabla^2 \bar{E}(\bar{r},t) = \frac{1}{c^2} \frac{\partial^2 \bar{E}(\bar{r},t)}{\partial t^2}$$

i.e. $\bar{E}$ is a solution of wave eq.

Interference of two sinusoidal waves

2D  circles

3D  spheres



cases/ 3 directions : toward P_0, P_C, P_D

S_1, S_2 – in phase, phase difference always constant

⇓

coherent sources

assume: same amplitude
same polarization

$\bar{E}_1$

$\bar{E}_2$ } only in one direction (can use scalars)

(a) P_0 – any point on x-axis

$|P_0 S_1| = |P_0 S_2|$ two waves in phase
(max + max)
min + min

⇓

amplitude of net wave is doubled

} $\boxed{\text{CONSTRUCTIVE INTERFERENCE}}$

(b) P_C – any point s.t.

$$\underbrace{|P_C S_2|}_{=r_2} - \underbrace{|P_C S_1|}_{=r_1} = m\lambda \qquad m = 0, \pm 1, \pm 2, \dots$$

$$E_1 = E_0 \cos(kr_1 - \omega t)$$

$$E_2 = E_0 \cos(kr_2 - \omega t) = E_0 \cos(k(r_1 + m\lambda) - \omega t) = E_0 \cos(kr_1 + \underbrace{km\lambda}_{= m \cdot 2\pi \text{ (because } k = \frac{2\pi}{\lambda})} - \omega t) =$$

$$= E_0 \cos(kr_1 - \omega t)$$

⇓

amplitude doubled as in a

(c) P_D – any point s.t.

$$\underbrace{|P_D S_2|}_{=r_2} - \underbrace{|P_D S_1|}_{=r_1} = (2m+1)\frac{\lambda}{2} \qquad m = 0, \pm 1, \pm 2, \dots$$

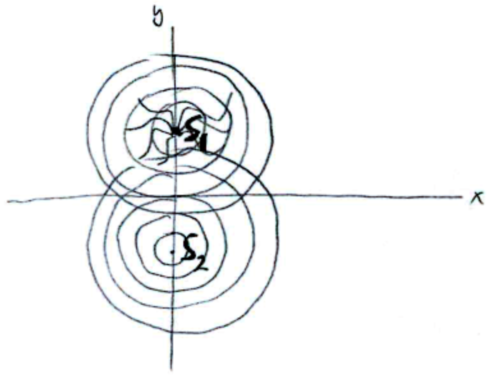
$$E_1 + E_2 = E_0 \cos(kr_1 - \omega t) + E_0 \cos(kr_1 + (2m+1)\pi - \omega t) = 0$$

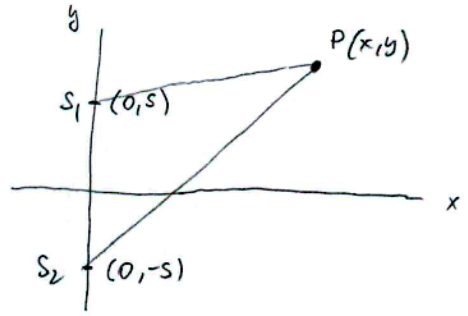
(equal amplitude ⇒ total cancellation)

$\boxed{\text{DESTRUCTIVE INTERFERENCE}}$

Nodal / antinodal lines

Antinodal lines — sets of points where waves interfere constructively
Nodal lines — " " " destructively





Antinodal lines: $\underbrace{\sqrt{x^2+(y-s)^2}}_{|S_1P|} = \underbrace{\sqrt{x^2+(y+s)^2}}_{|S_2P|} + m\lambda$

analogously nodal lines

} families of hyperbolas (cf. homework)

Comment: challenge in interference experiments: how to generate phase coherent light?

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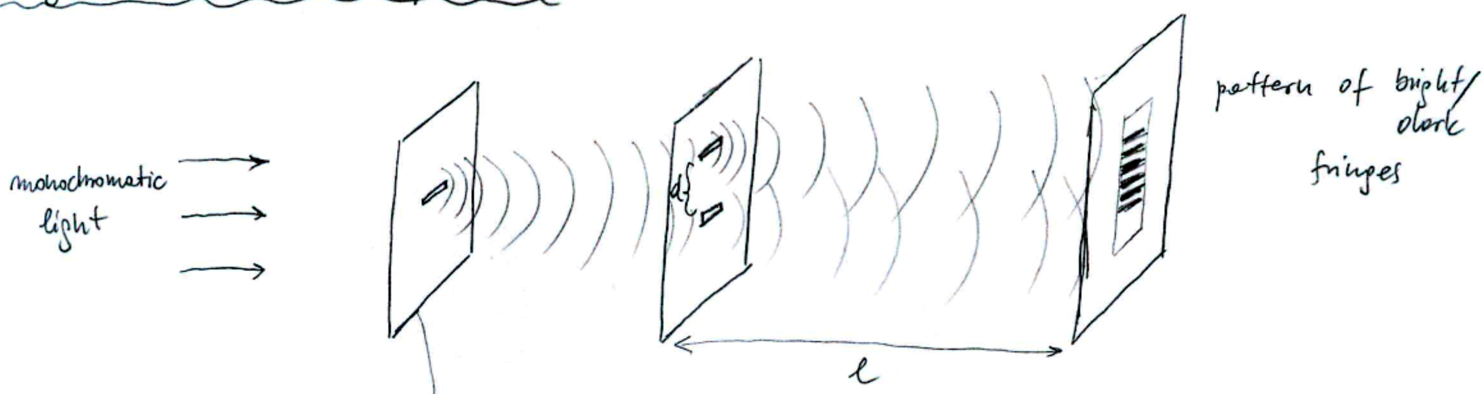
Young double-slit experiment

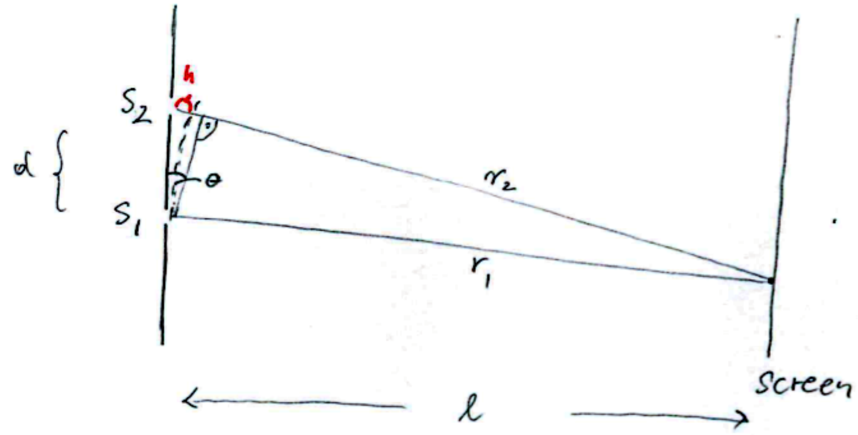


Note: Slits are very narrow, not far apart (in the figure the size/distance is exaggerated)

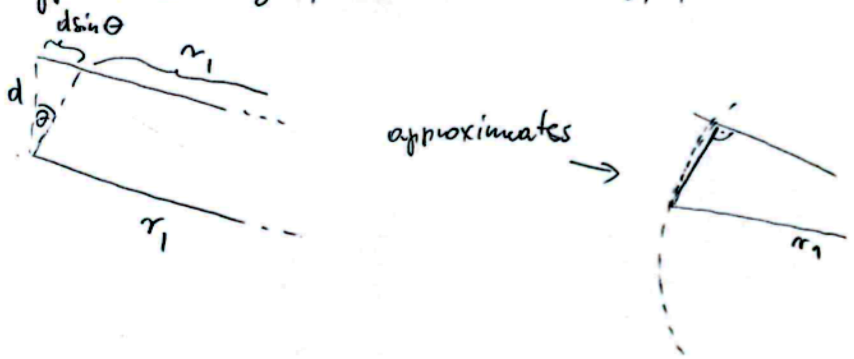
$d \ll \ell$

placed in order to have phase coherence at the two slits (can be replaced by a source of coherent light)

Cross-section



If $d \ll \ell$ approximate by parellel lines at S_2, S_1 : $h \approx d \sin\theta$



Interference

Constructive

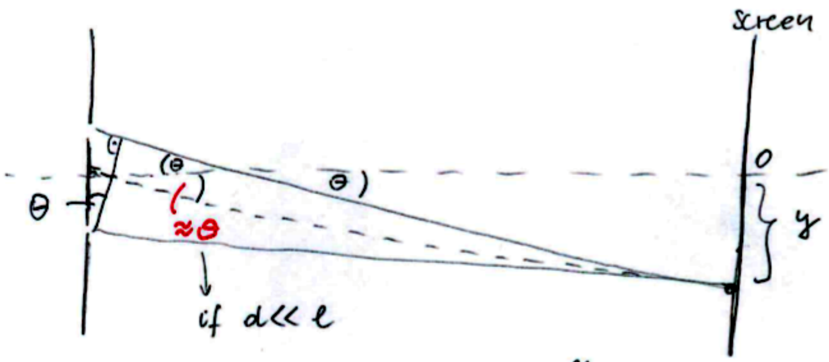
$r_2 - r_1 = d\sin\theta_m = m\lambda \quad (*)$

destructive

$r_2 - r_1 = d\sin\theta_m = (2m+1)\frac{\lambda}{2}$

$m = 0, \pm 1, \pm 2, \dots$

Position of bright fringes on the screen



$$\tan\theta = \frac{y}{\ell}$$

For small angles (fringes close to pnt 0) $\tan\theta \approx \sin\theta \approx \theta$

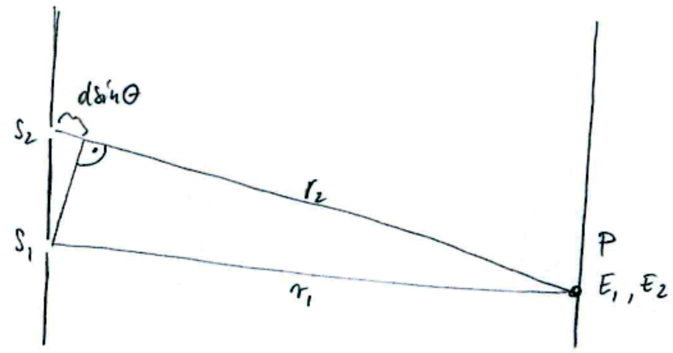
$$y = \ell \cdot \tan\theta \cong \ell \sin\theta \overset{(*)}{=} \ell \frac{m\lambda}{d}$$

Position of m-th bright fringe ($m=0$ central) $\boxed{y_m = \ell \frac{m\lambda}{d}}$

→ remember constraints due to approximations!

→ the closer the slits, the further the fringes

Interference fringes - intensity



Assume: 1) equal (and constant) amplitude ! → simplification

2) same polarization (can use scalars)

Recall:
$$I = \langle |\bar{S}| \rangle = \frac{E_0^2}{2\mu_0 c} \propto E_0^2$$

I ↓ intensity; $\langle|\bar{S}|\rangle$ ↓ Poynting vector

constant (simplification)

$$E_1 = E_0 \cos(kr_1 - \omega t)$$
$$E_2 = E_0 \cos(kr_2 - \omega t) = E_0 \cos(kr_1 + kd\sin\theta - \omega t)$$

Superposition at P

$$E = E_1 + E_2 = \underbrace{2E_0 \cos\left(\frac{kd\sin\theta}{2}\right)}_{\text{amplitude} = E_p} \cos\left(kr_1 - \omega t + \frac{kd\sin\theta}{2}\right)$$

Intensity

$$I_p = \frac{E_p^2}{2\mu_0 c} = \frac{1}{2}\varepsilon_0 c\, E_p^2 \qquad \left(c^2 = \frac{1}{\varepsilon_0\mu_0}\right)$$

$$I_p = \frac{1}{2}\varepsilon_0 c \left(2E_0 \cos\left(\frac{kd\sin\theta}{2}\right)\right)^2 =$$
$$= \underbrace{2\varepsilon_0 c E_0^2}_{I_{max}} \cos^2\left(\frac{kd\sin\theta}{2}\right)$$

Intensity of single wave

$$I_0 = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{4} I_{max}$$

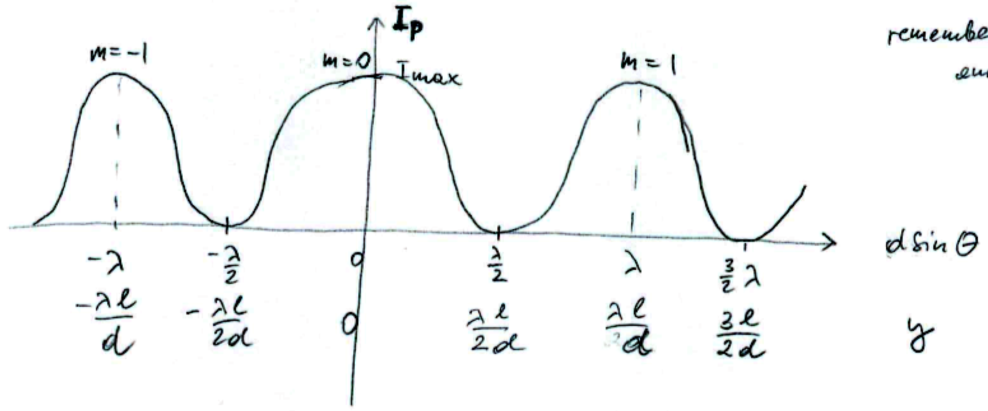
So

$$\boxed{I_p = \underbrace{4I_0}_{I_{max}} \cos^2\left(\frac{kd\sin\theta}{2}\right)}$$

Bright fringes $\quad I_P = I_{max} \cos^2\left(\frac{k d \sin\theta}{2}\right) = I_{max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$

$d \sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$

$$I_P = I_{max} \cos^2\left(\frac{\frac{2\pi}{\lambda} m\lambda}{2}\right) = I_{max} \cos^2(m\pi) = I_{max}$$



remember simplification: amplitudes constant

Recall: position on the screen (measured from the central fringe)

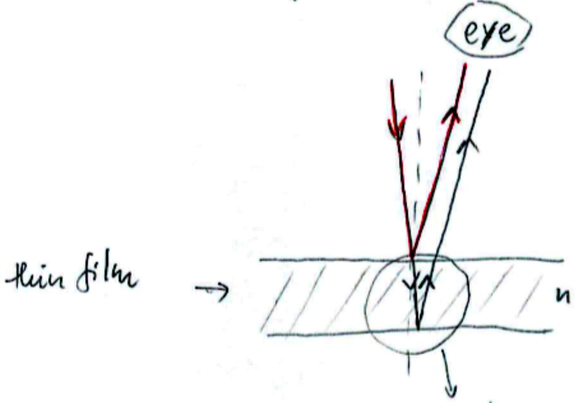
$$y = l \cdot \tan\theta \approx l \cdot \sin\theta = l \cdot \frac{d \sin\theta}{d}$$

~o~

Interference in thin films

FIG.
(oil speck)

Examples: soap bubbles, oil specks

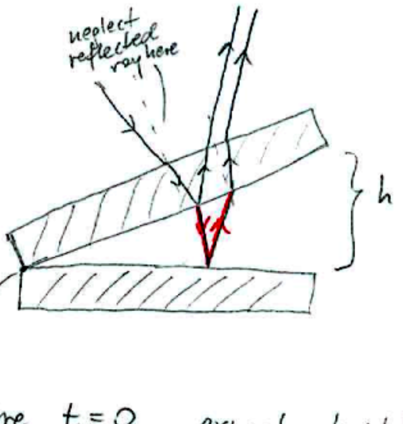


almost normal incidence

optical path difference ⇒ rays → and → arrive at eye with different phases ⇒ INTERFERENCE

different wavelengths ⇒ different colors interfere differently for the same optical path difference ⇒ some colors eliminated (destructive interference)

Consider (the angle in the sketch is exaggerated)



$\Rightarrow 2t$ path difference
$\downarrow$
not constant $\Rightarrow$ interference fringes

here $t=0$, expect bright fringe, but dark fringe observed

Observation: bright and dark fringes are interchanged!

Why? There is a phase change by π at the boundary air-thin film

In reflection, the amplitudes

$$E_{reflected} = \frac{n_1 \cos\theta_{incident} - n_2 \cos\theta_{reflected}}{n_1 \cos\theta_{incident} + n_2 \cos\theta_{refracted}} E_{incident}$$

Normal incidence ($\theta_{incident} = \theta_{reflected} = \theta_{refracted} = 0$) (Fresnel equations)

$$E_{reflected} = \frac{n_1 - n_2}{n_1 + n_2} E_{incident}$$

$n_1 > n_2$: $E_{reflected}$, $E_{incident}$ have same sign

$n_1 = n_2$: no reflection

$n_1 < n_2$ (eg. from air to glass): $E_{reflected}$, $E_{incident}$ sign change (phase change by π)

Conditions for interference on thin films (normal incidence, thickness t)

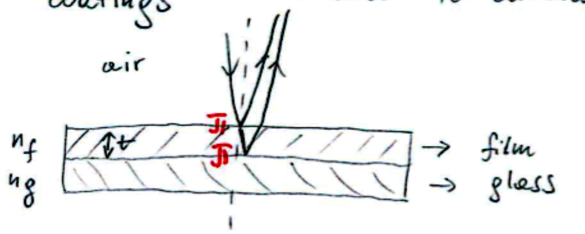
neither or both have phase shift		only one has phase shift
$2t = m\lambda$	constructive	$2t = (2m+1)\frac{\lambda}{2}$
$2t = (2m+1)\frac{\lambda}{2}$	destructive	$2t = m\lambda$

$m = 0, \pm 1, \pm 2$

Why thin? $\rightarrow$ to maintain phase coherence.

Examples :

(1) non - reflective coatings – we want to eliminate reflected ray of wavelength λ_{film} (in the film)



$n_f < n_g$

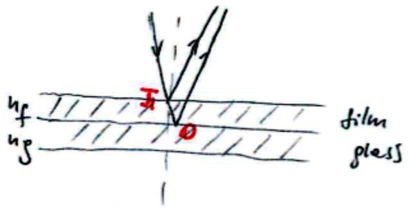
double phase change
⇓
destructive interference if $(m = 0)$

$$2t = \frac{\lambda_{film}}{2} \Rightarrow \boxed{t = \frac{\lambda_{film}}{4}} = \frac{\lambda_{vacuum}}{4n_f}$$

thickness of film (it eliminates one wavelength)

(2) reflective coatings

$n_f > n_g$

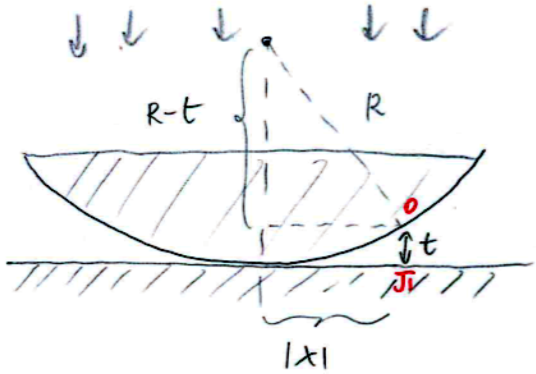


constructive interference $(m = 0)$

$$2t = \frac{\lambda_{film}}{4} \Rightarrow \boxed{t = \frac{\lambda_{f}}{4}} = \frac{\lambda_{vacuum}}{4n_f}$$

Application : Lenses, photographic filters, solar cells

(3) Newton rings



(exaggerated)

$$(R-t)^2 + x^2 = R^2 \Rightarrow 2Rt = t^2 + x^2$$

if $t \ll x$, then $x^2 \approx 2Rt$

Constructive interference if $2t = (2m+1)\frac{\lambda}{2}$

$$x \cong \sqrt{R\lambda\left(m + \frac{1}{2}\right)} \qquad m = 0, 1, 2, \ldots$$

↳ radius of Newton's rings