Assignment 2

Wednesday, February 26, 2025 1:53 PM

pt. 1.2 → analyzing Blackbody

$B(T) = \int_0^\infty B_\lambda(\lambda, T)\, d\lambda$ prove it equals $\frac{\sigma T^4}{\pi}$

$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{(hc/\lambda kT)} - 1}$

$\frac{P}{A} = 2hc^2 \int \frac{1}{\lambda^5} \cdot \frac{1}{e^{(hc/\lambda kT)} - 1}$

$\frac{P}{A} = 2hc^2 \int \frac{1}{\lambda^5} \frac{1}{e^x - 1}$

$e^x = e^{hc/\lambda kT}$
where
$x = hc/\lambda kT$

$\lambda = v/c$?

$\frac{-hc}{\lambda^2 kT}\, d\lambda = dx$

$d\lambda = -\frac{\lambda^2 kT}{hc}\, dx$

$\frac{P}{A} = 2\pi \left(\frac{x\lambda Tk}{c}\right)\left(\frac{x\lambda kT}{h}\right)^2 \int_0^\infty \left[\frac{\left(-\frac{\lambda^2 kT}{hc}\right) dx}{e^x - 1}\right]$

$2\pi \left(\frac{\lambda^3 \lambda^5 k^4 T^4}{h^3 c^2 \lambda^5}\right) \int_0^\infty \left[\frac{dx}{e^x - 1}\right] dx$

$B_\lambda = \frac{2(k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{e^x - 1}\, dx$

$\frac{2(k_B T)^4}{h^3 c^2} \int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$

$\frac{2(k_B T)^4}{h^3 c^2} \cdot \frac{\pi^4}{15}$

$\frac{P}{A} = \frac{2(k_B T)^4 \, \pi^4}{h^3 c^2 \, 15}$

↓
σ

$F = \pi [B(T)] = \sigma T^4$

CODING 1?
import constants
"gen constant"

$h =$
$c =$
$k =$
$T =$
$x = \left(\frac{hc}{\lambda kT}\right)$

} constants

$B_\lambda = \left[\frac{2hc^2}{\lambda} \cdot \frac{1}{e(x) - 1}\right]$ ← int even if ∞ can be implemented

constant

int_ = np.linspace (0, ∞)

$\lambda =$

How can you upload imgs in markdown → yeah

can I use scipy?
↳ no

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pt 2) num int using a transform 2
handle $x \to \infty$

$B = \int_0^\infty$ planck_x(x) dx

planck − x $= \dfrac{2hc^2}{\lambda^5} \cdot \dfrac{1}{e^{(hc/\lambda kT)} - 1}$

$\sigma = \dfrac{2 k_B^4 \pi^5}{15 h^3 c^2}$

$I = \int_0^\infty f(x)\, dx$ ← WHATS the diff B/T this & top vers..!?

$x = a \dfrac{z}{1-z}$

$dx = a \cdot z(1-z)^{-1}$

use chain rule:

$-a \cdot z(1-z)^{-2}(-1)$

$I = a \int_0^\infty f\left(a \dfrac{z}{1-z}\right)$

↑ remember integrand $0 \to \infty$ change $0 \to 1$

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habitral zone

a) solve val of Teff 4 radius $d = 0.5$

tol $10^{-5} \to 10^{-11}$ w/ all root FINDERS.

$f(T_{eff}) = d(T_{eff}) - 0.5\ AU$

derivative

(1)

$\sqrt[4]{\dfrac{L}{L_\odot}} = \sqrt[4]{\left(\dfrac{T_{eff}}{5780}\right)^4}$

$5780 \times \sqrt[4]{\dfrac{L}{L_\odot}} = \dfrac{T_{eff}}{5780} \times 5780$

$T_{eff} = 5780 \sqrt[4]{\boxed{\dfrac{L}{L_\odot}}}$ → replace w/ dist eqn

WHERE

$(d)^2 = \left(\dfrac{\frac{L}{L_\odot}}{S_{eff}}\right)^{1/2}$ & $\dfrac{L}{L_\odot} = \left(\dfrac{T_{eff}}{5780K}\right)^4$

$S_{eff} = S_{eff\odot} + aT_\star + bT_\star^2$

$d = \left(\dfrac{\left(\frac{T_{eff}}{5780}\right)^4}{S_{eff\odot} + aT_\star + bT_\star^2}\right)$

$d = \left(\dfrac{\left(\frac{T_{eff}}{5780}\right)^4}{S_{eff\odot} + a[T_{eff} - 5780] + b[T_{eff} - 5780]^2}\right)^{1/2}$

$S_{eff} = S_{eff\odot} + aT_\star + bT_\star^2$

$T_\star = T_{eff} - 5780$

FIND EFFECTIVE temp where $r = 0.5\ AU$

☆ given

$T_{eff} = 5780$

$S_{eff} = 1.014$

$a = 8.177 \times 10^{-5}$

$b = 1.706 \times 10^{-9}$

CODE:

vars:

$S_{EFF\odot} = 1.014$
$a = 8.177 \times 10^{-5}$
$b = 1.706 \times 10^{-9}$
$T_{EFF} = 5780$
$d = 0.5$

~~tolerances = np.logspace [-5, -11, num = 6]~~ ← i want to change it to jump every 2...

res = np.zeros ((3, 1)) ?

```
def avg-dist [Teff, Seff :
        T* = Teff - 5780
      num = [Teff / 5780]** 4
      denom = S_EFF + a [ T* ] + b [T*]^2
      result = [num / denum]^1/2
      return result

  if [ value - d == 0 ] :
        print [f "the value is correct!", [value] ]

    for i, method in enumerate (rootfinder) :
        for j, toler in enumerate (tol) :
        print ( f" solve w/ tolerance : {toler} " )

        # BISECTION METHOD

              root = rootfinder ( f, start, end, tol )
              res = root . find ( method )
              print ( f" Result of [method] : ", res )
```

★ check if d = 0.5 = 0
if it does its correct ✓

problem 5: wien's law

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{(hc/\lambda kT)}-1}$$

$$\frac{\partial B}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left[\frac{2hc^2}{\lambda^5} \frac{1}{e^{(hc/\lambda kT)}-1}\right]$$

$$x = \frac{hc}{\lambda k T}$$

isolate λ to get $dx = d\lambda$

$$\lambda = \frac{hc}{x k T}$$

$$d[\lambda] = d\left[\frac{hc}{x k T}\right]$$

plug in our replacements

$$\frac{\partial B}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left[2 \frac{\cancel{hc^2}}{\left[\frac{\cancel{hc}}{[xkT]}\right]^{5\ 3}} \frac{1}{e^x - 1}\right]$$

$$= \frac{\partial}{\partial x}\left[2 \frac{(xkT)^5}{(hc)^3} \frac{1}{e^x - 1}\right]$$

$$\frac{2(kT)^5}{(hc^3)} \frac{\partial}{\partial x}\left[x^5 [e^x - 1]^{-1}\right]$$

Quotient rule:

$$\frac{\partial B}{\partial x} = \frac{(e^x - 1)\frac{d}{dx}[x^5] - x^5 \cdot \frac{d}{dx}[e^x - 1]}{(e^x - 1)^2}$$

Quotient rule:

$$\frac{\partial B}{\partial x} = \frac{(e^x-1)\frac{d}{dx}[x^5] - x^5 \cdot \frac{d}{dx}[e^x-1]}{(e^x-1)^2}$$

$$\frac{(e^x-1)(5x^4) - x^5 e^x}{(e^x-1)^2}$$

$$= \frac{5x^4 - x^5 e^x}{(e^x-1)^2}$$

$$= \frac{5x^4 [e^x-1] - x^5 e^x}{(e^x-1)^2}$$

$$0 = \frac{e^x[5x^4 - x^5] - 5x^4}{(e^x-1)^2}$$

$$\frac{0}{5x^4} = \frac{e^x[5x^4 - x^5] - 5x^4}{5x^4}$$

$$0 = e^x[5-x] - 5$$

$$\boxed{5 = e^x[5-x]}$$

FIND b numerically via root finding

$$\frac{dB}{dx} = 0$$

$$\frac{\partial B}{\partial x} = (5-x)e^x - 5 = 0$$