Optimal Control

Tommaso Caneva
PhD in Physics

Optimal Control

how to optimize tasks

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Speed

Optimal Control

Precision

Eff ciency

How?

Training

How?

Training

How?

Too long or too complex?

Training

How?

Too long or too complex?

Optimal control

Teach machines!

Framework: repeatable task

System

Control

Target

Framework: repeatable task

System

Control

Target

First time...

Framework: repeatable task

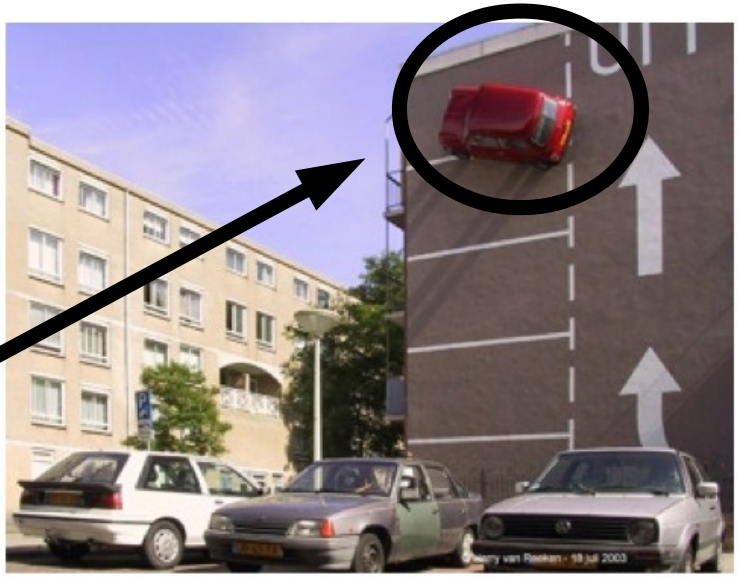
System

Control

Target

First time...

...but after years of driving experience...



Framework: repeatable task

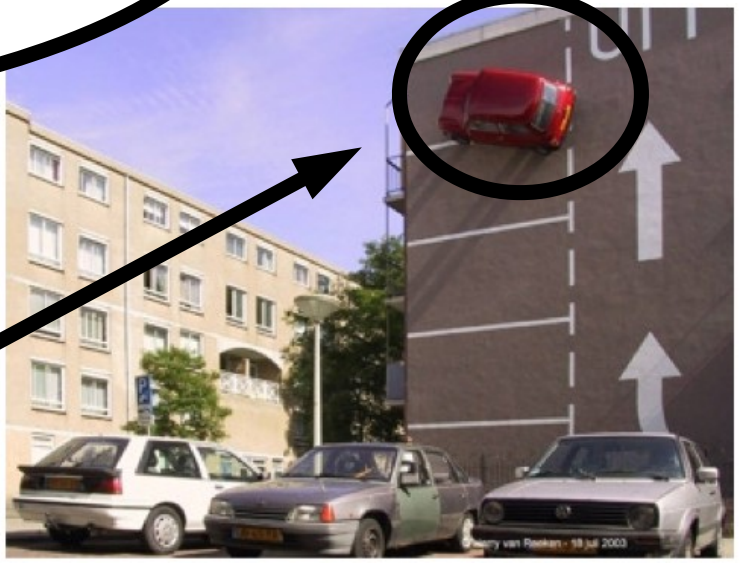
System

Control

Target

Repetita iuvant:
Learning from repetition of the task

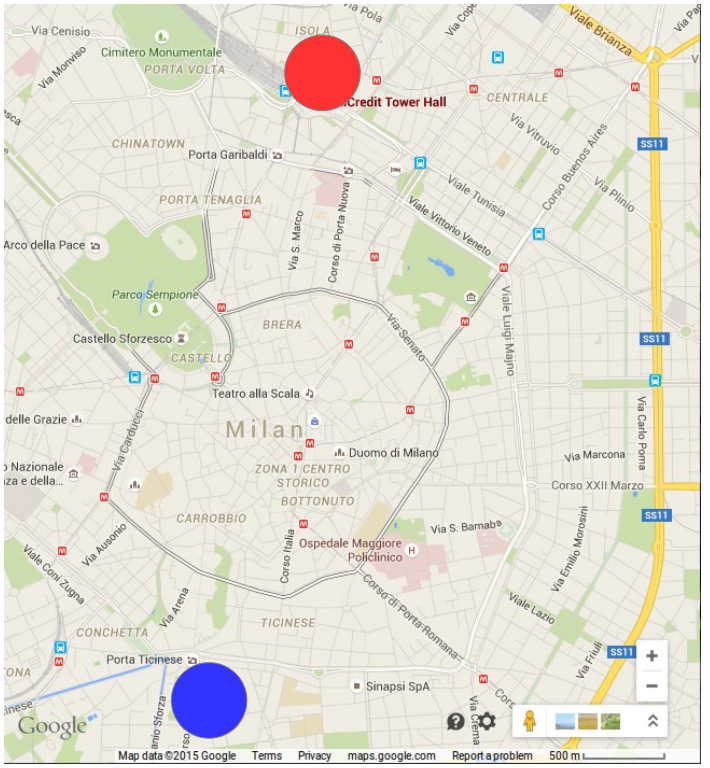
...but after years of driving experience...



General formulation of the problem

Ingredients

- Initial state:
- Task: go to in 20 minutes
- Tunable sytem: car + indications, G



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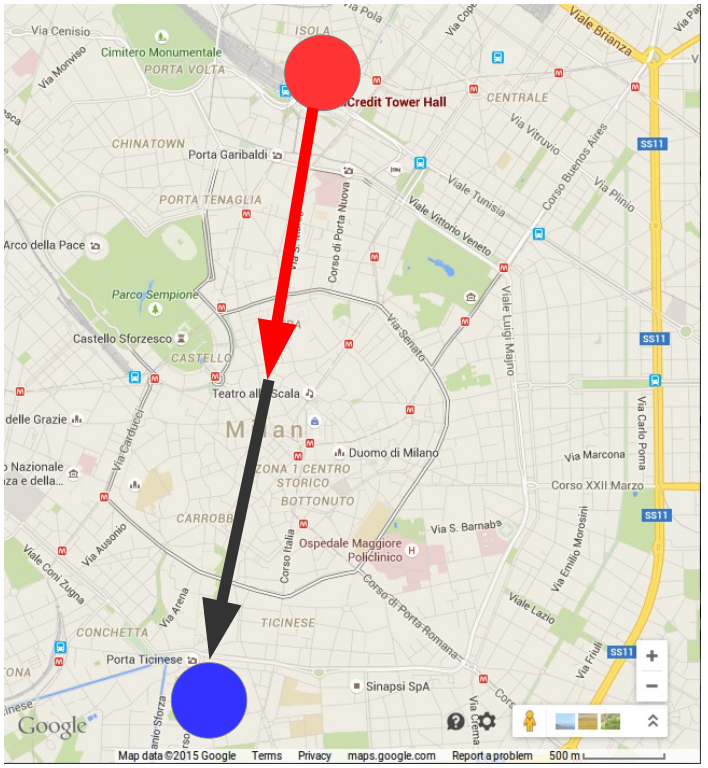
- Cost function: F(G)



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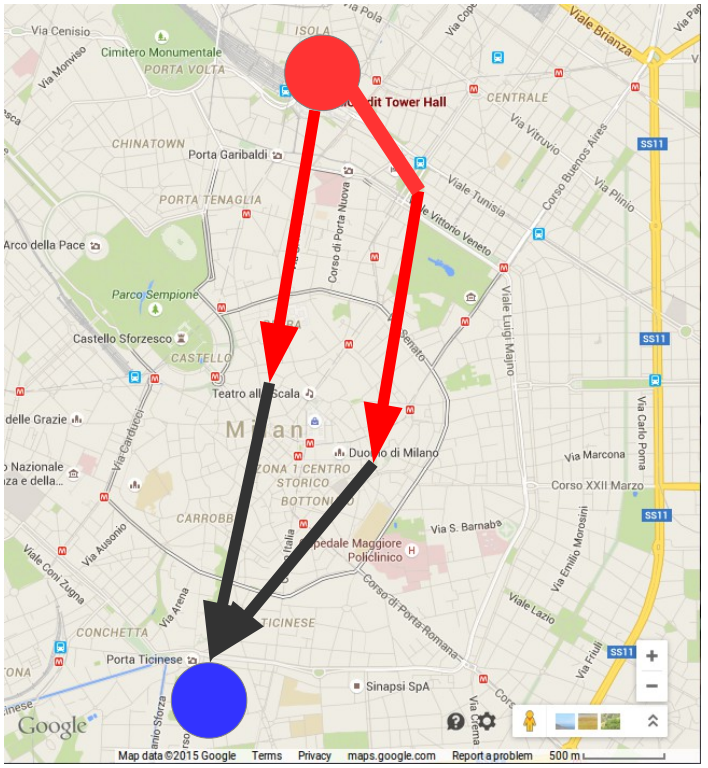

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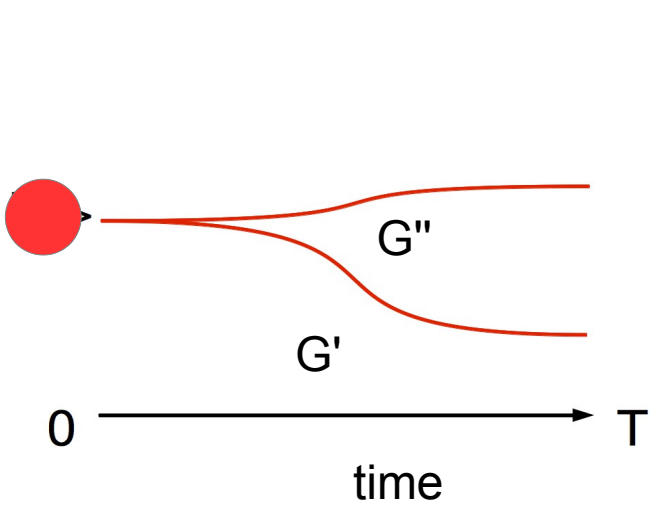

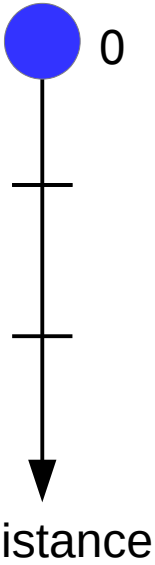

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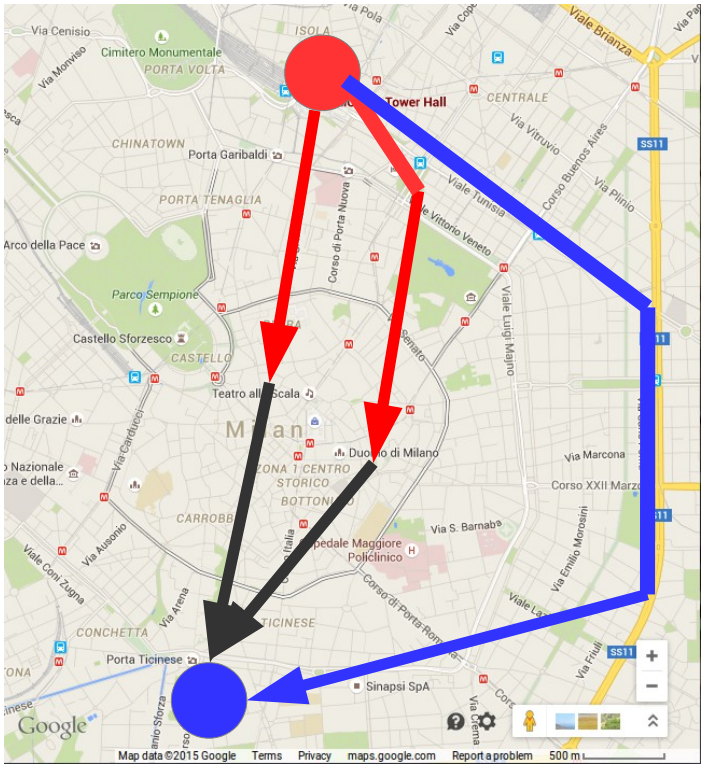

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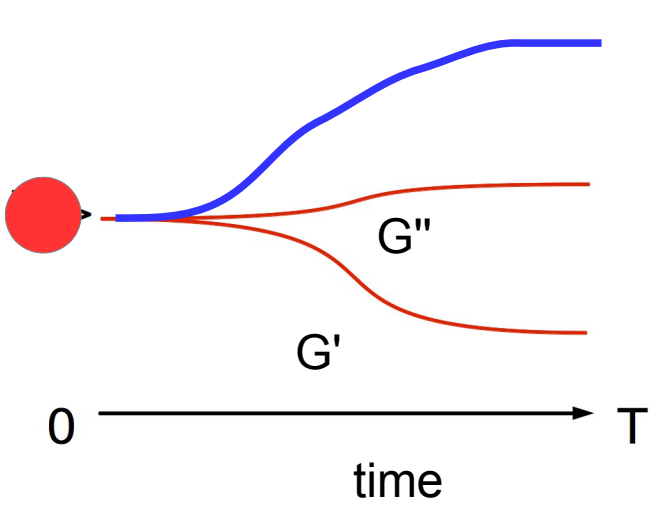
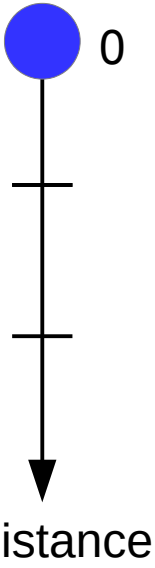




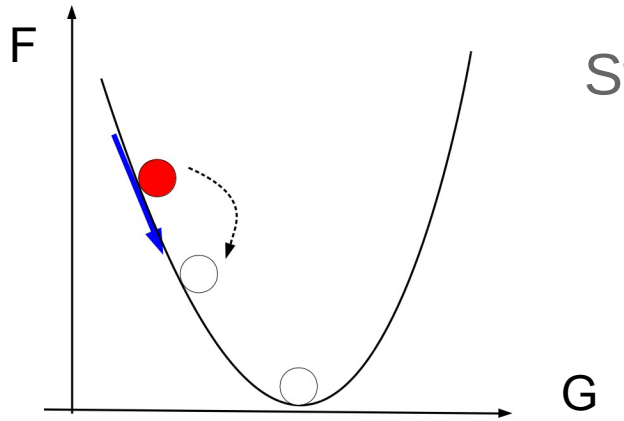
Optimal Control

$F(G) = 0$

minimization cost function

Recipe 1: gradient method

Single variable function minimization

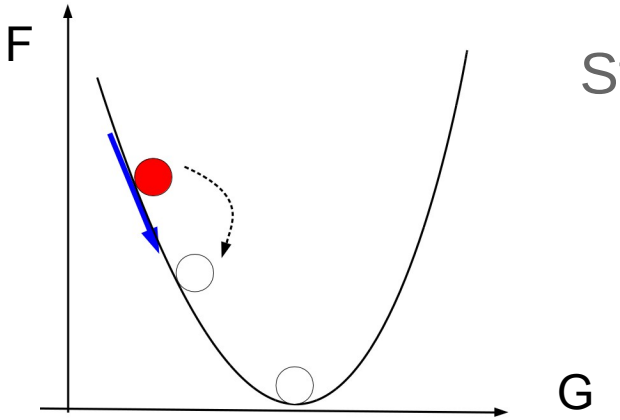


Steepest descent

gradient

Recipe 1: gradient method

Single variable function minimization



Steepest descent

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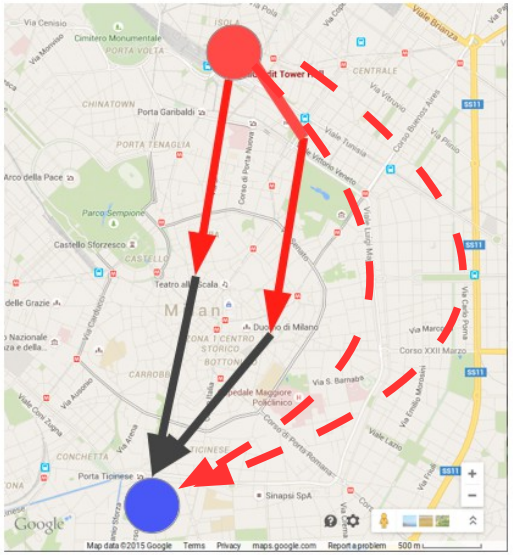
Many variable function minimization



- In principle inf nite paths

- Computationally slow
(~ Neural Network training)

- Maybe not necessary: close
to mimum is enough

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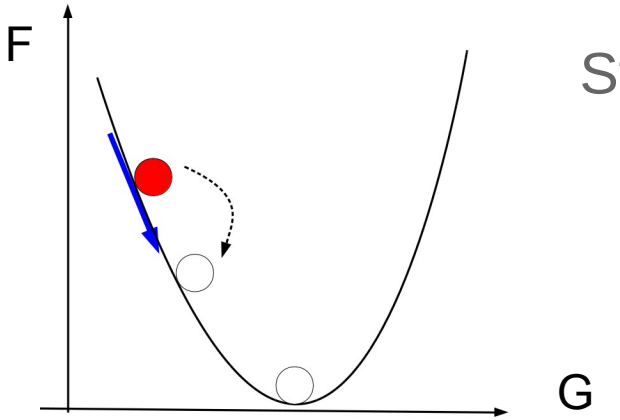

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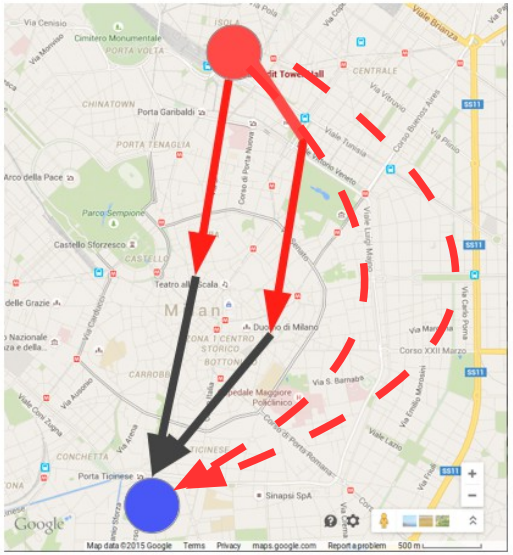

- In principle inf nite paths

- Computationally slow
(~ Neural Network training)

- Maybe not necessary: close
to mimum is enough

Alternatives?

Looking for alternatives: an important dinner

You have an important guest;

you decide to prepare an impressive dinner...

Looking for alternatives: an important dinner

You have an important guest;

you decide to prepare an impressive dinner...

That is what you f ȵd in your kitchen:

- a tomato

Looking for alternatives: an important dinner

You have an important guest;

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That is what you f ȵd in your kitchen:

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- a bottle of water

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You have an important guest;

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- a tomato
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- ...sorry, nothing else...

Looking for alternatives: an important dinner

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That is what you f ɲd in your kitchen:

- a tomato
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What are you going to do?

Looking for alternatives: an important dinner

You have an important guest;
you decide to prepare an impressive dinner...

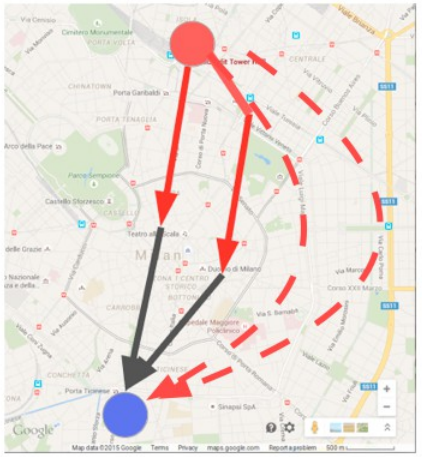
That is what you find in your kitchen:
- a tomato
- a bottle of water
- ...sorry, nothing else...

What are you going to do?

Try to get the best
of what you have at disposal!

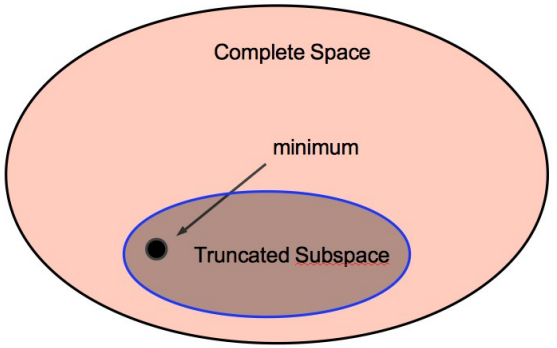
Recipe 2: approximate method

Initial state  → Dynamics G(t) → target state  , cost F



Problem: space too big

Solution: truncated space



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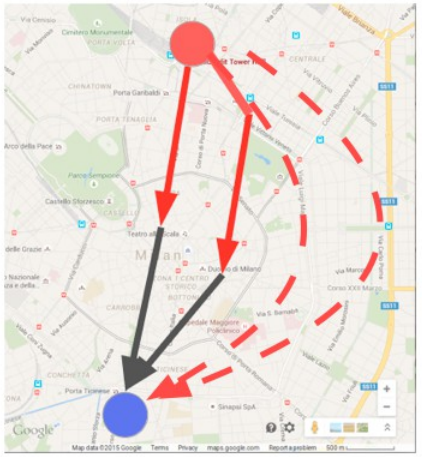

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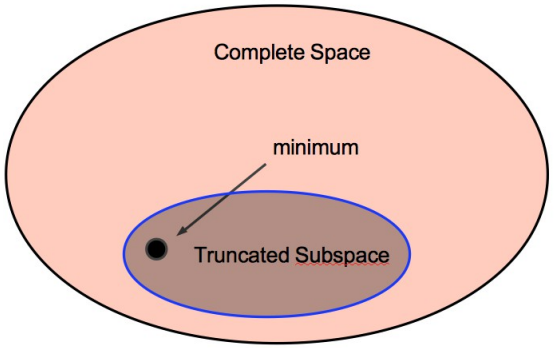
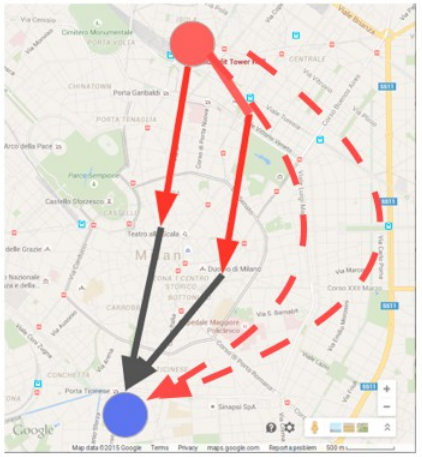


Example:

Draw a man: 

Recipe 2: approximate method

Initial state  → Dynamics G(t) → target state  , cost F



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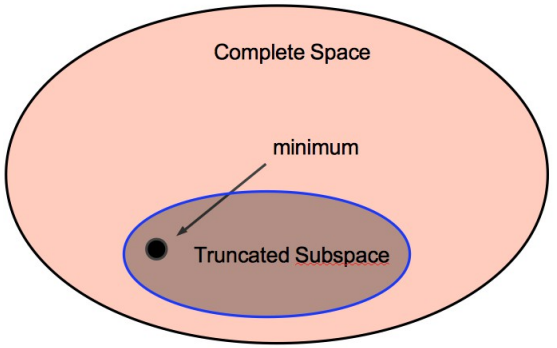
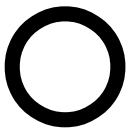
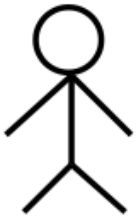


Example:

Draw a man: 1x  + 5x  = 

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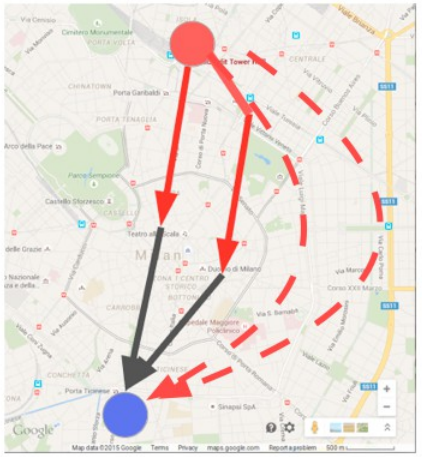

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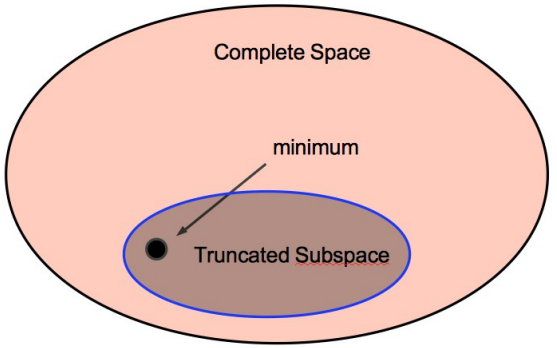

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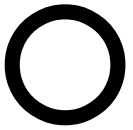
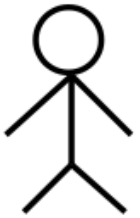
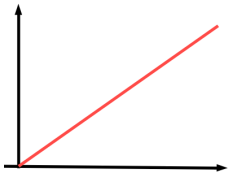
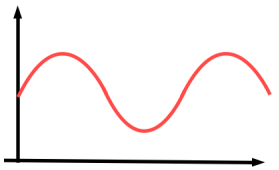
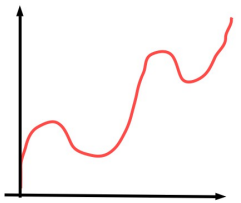
Draw a man: 1x  + 5x  = 

Draw Opt. path: 1x  + 5x  = 

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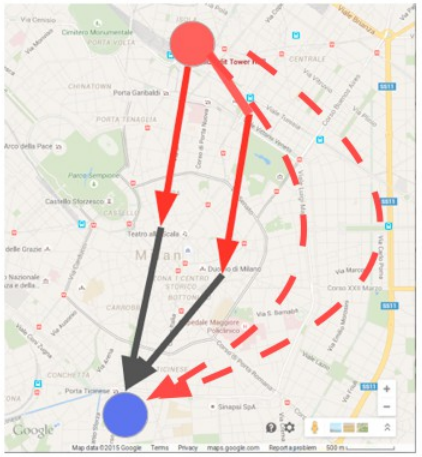

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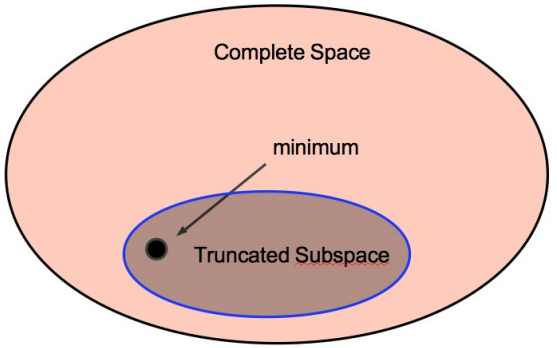


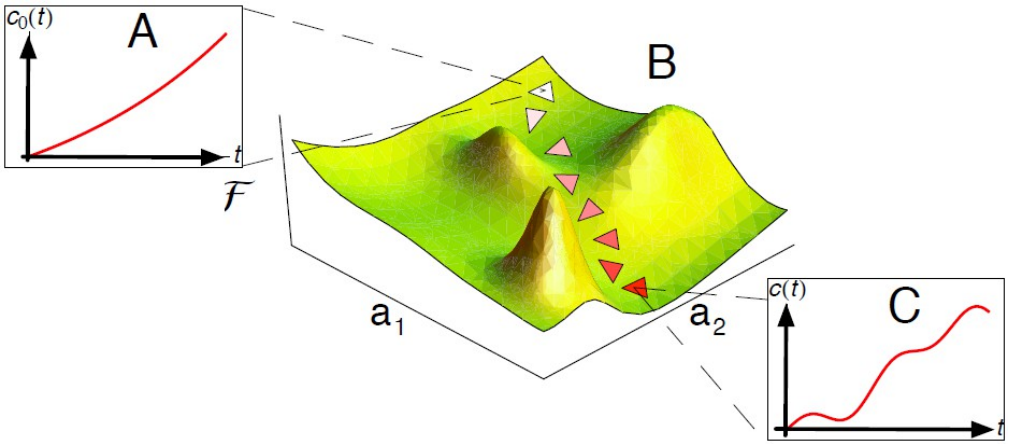
Algorithm CRAB:

- G(t) in truncated basis of functions:

$$G(t) = \sum_i w_i b_i(t)$$

- Cost: few variable function of weigths W

$$F[G(t)] = F(\vec{W})$$



P. Doria, T. Calarco and S. Montangero, PRL (2011); T. Caneva, T. Calarco and S. Montangero, PRA (2011)

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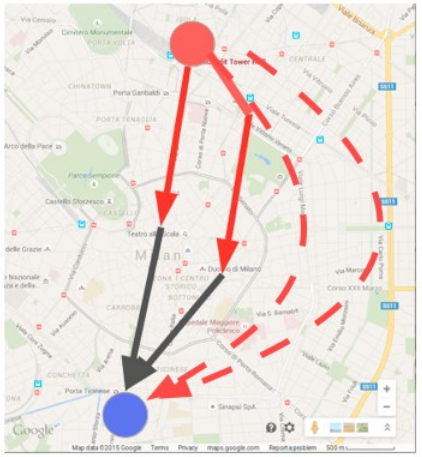

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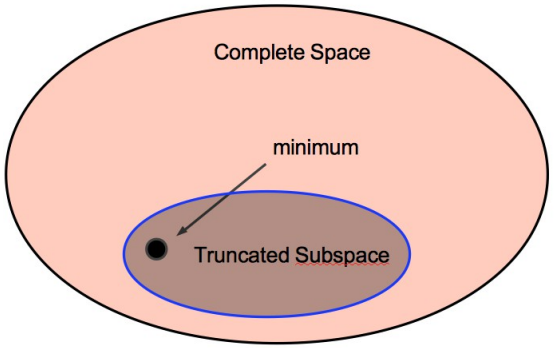

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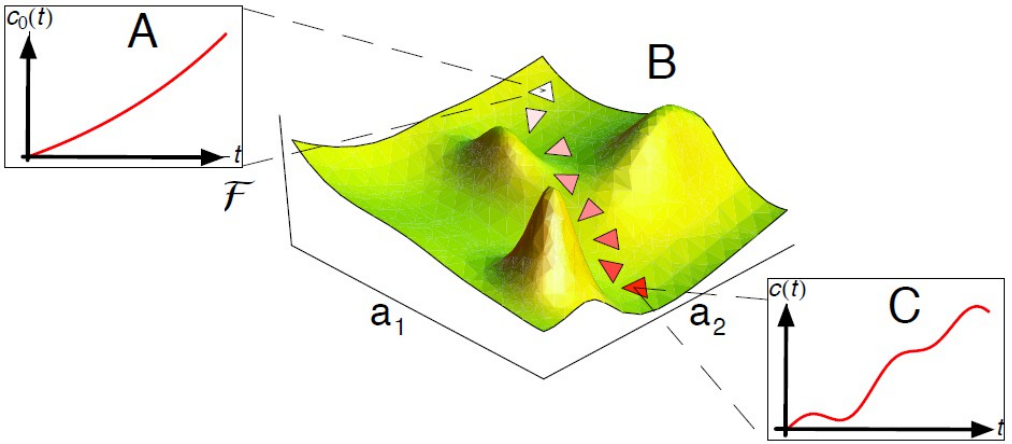

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Application: reversibility problem

?

- Is it possible to revert dynamics?
- Is the protocol robust?

Application: reversibility problem

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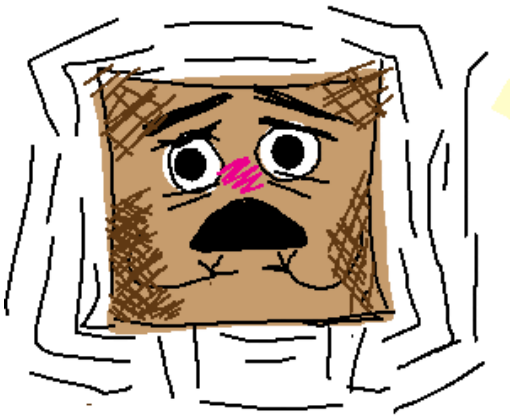
- Is it possible to revert dynamics?
- Is the protocol robust?

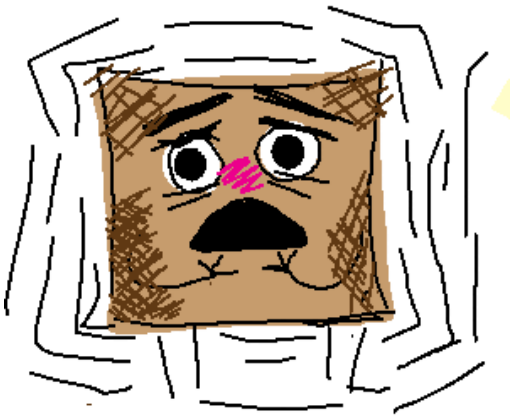
✔ Of course, use OC!
✔ Extremely robust

Application: reversibility problem



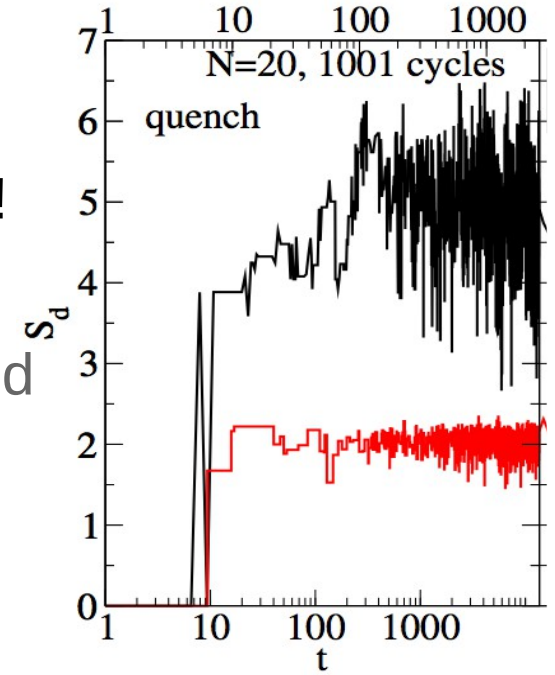
Shake it randomly!

Application: reversibility problem



Shake it randomly!

Entropy grows and saturates



T. Caneva et al, PRA (2014)

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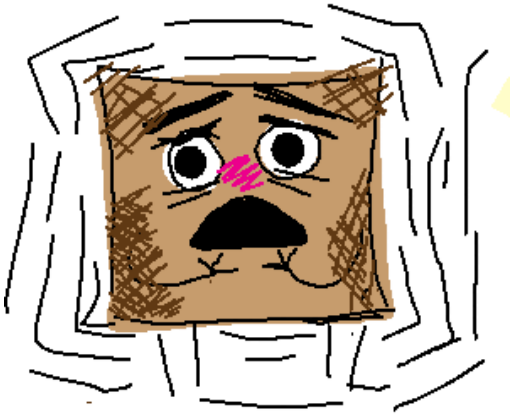

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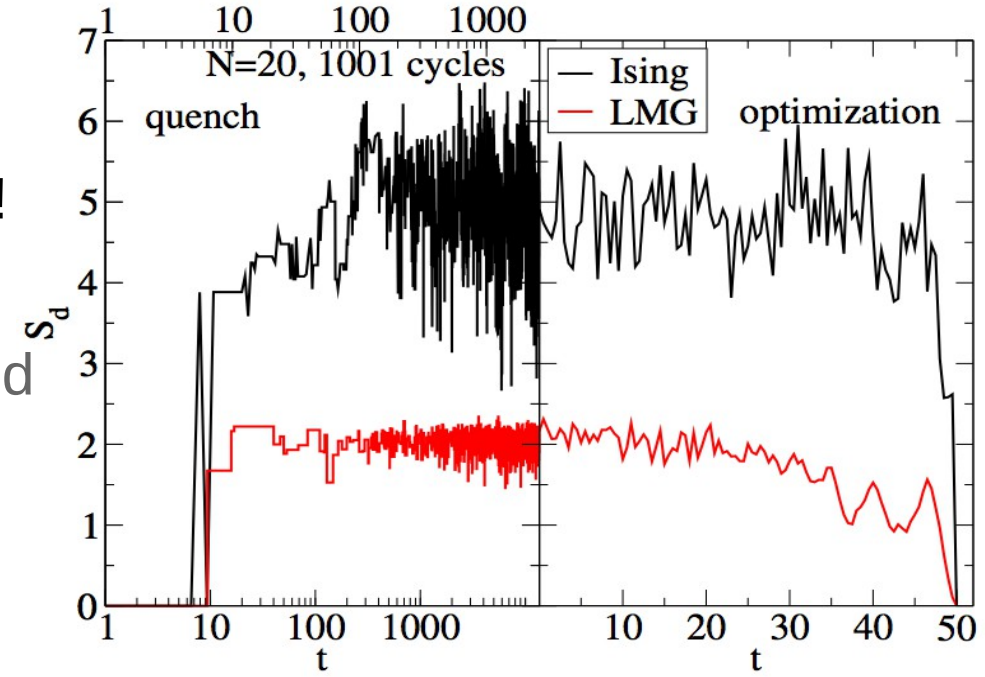

Shake it with OC!

Entropy decreases, system goes back!

T. Caneva et al, PRA (2014)

Conclusions...

how to optimize tasks

Speed

Optimal Control

Precision

Eff ciency