Lab 2 Report

Parametric Curves

CZ2003 Computer Graphics and Visualization

Nanyang Technological University

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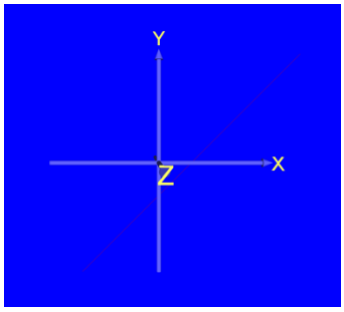
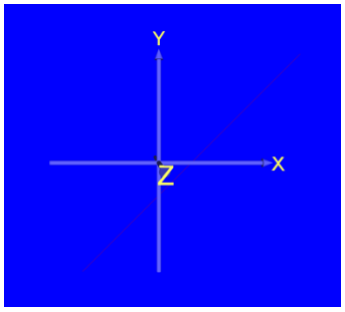
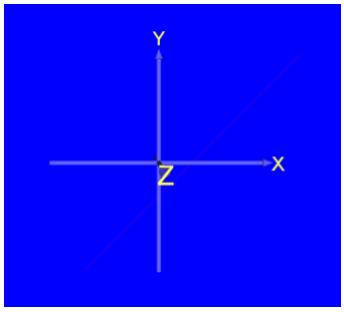
Straight Line Segment

Parametric equation of a straight line segment can be defined as

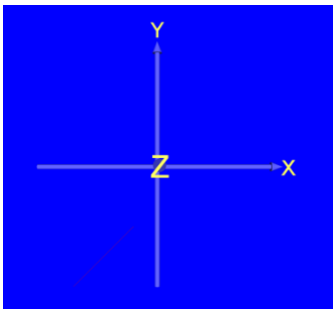
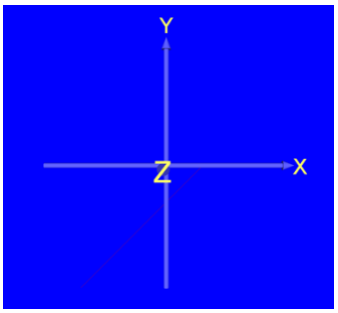
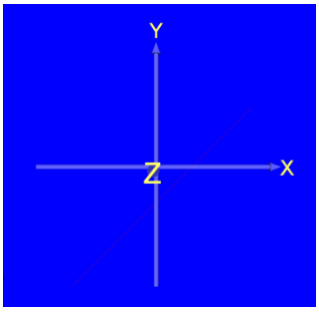
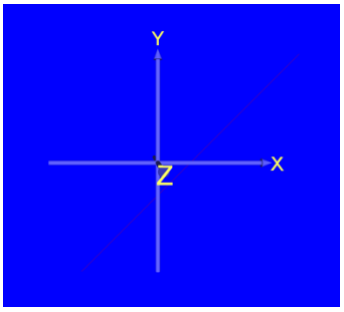
$$x = -0.7 + 2u$$

$$y = -1 + 2u$$

The resolution of a straight line segment do not affect its appearance as it is only made up of one line.

Resolution	100	10	2
Curve			

The parameter of a straight line segment will affect the range of values that the curve will be represented on. For example, with a parameter of [0 0.5], the line represented will be that of parameter *u* having values ranging from 0 to 0.5.

Parameter	[0 0.25]	[0 0.5]
Curve		
Parameter	[0 0.75]	[0 1]
Curve		

Circle and its Arc

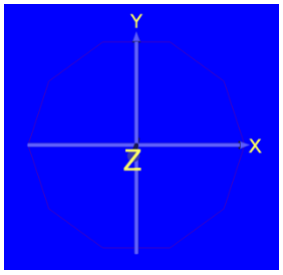
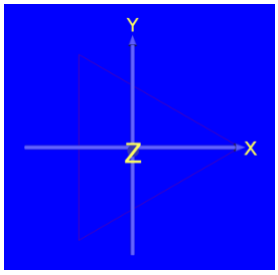
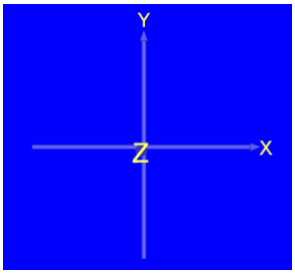
Parametric equation of a circle of radius 1 is defined as

$$x = \cos (2\Pi u)$$

$$y = \sin (2\Pi u)$$

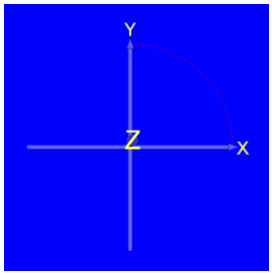
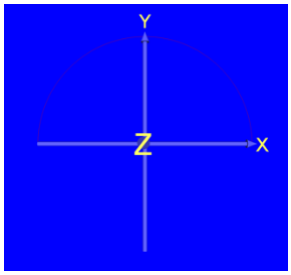
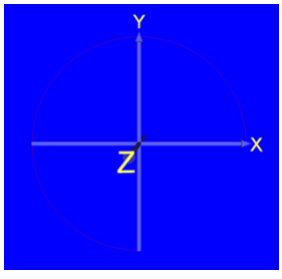
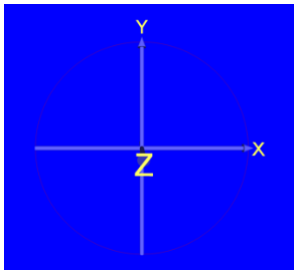
$$z = 0$$

The resolution of a circle affect how smooth its appearance will be. This is because the circle is formed by lines joining multiple points, whose number of points is dictated by the resolution value.

Parameter	[0 1]		
Resolution	10	3	2
Curve			

**Resolution = 2 : line is not visible as the two points will be on the x-axis, forming a straight line along the x-axis.*

The arc can be formed by varying the parameter value. Parameter [0 1], however, shows a full circle.

Parameter	[0 0.25]	[0 0.5]
Curve		
Parameter	[0 0.75]	[0 1]
Curve		

**Resolution = 100 for all parameters*

Alternatively, an arc formed by the previous circle with parameter [0 0.25] can be represented with the following parametric equation

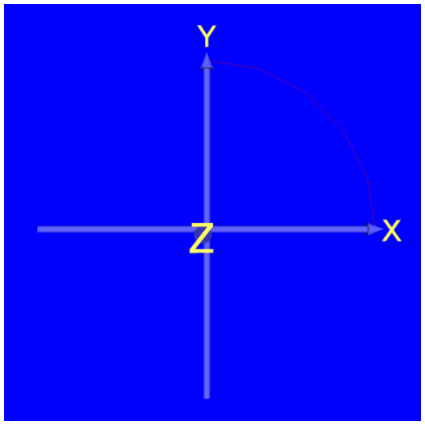
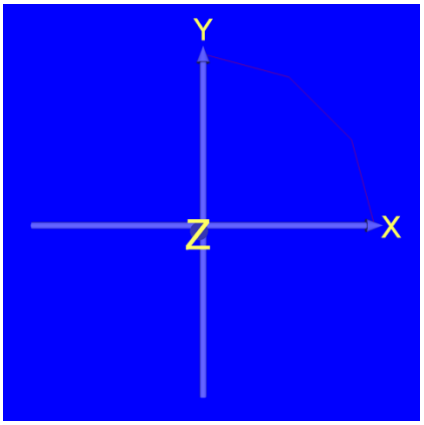
$$x = \cos\left(\frac{\Pi}{2}u\right)$$

$$y = \sin\left(\frac{\Pi}{2}u\right)$$

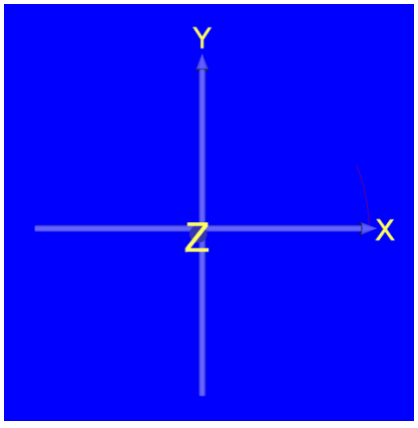
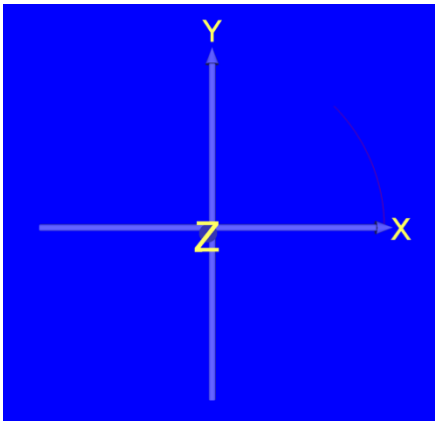
$$z = 0$$

where Parameter is set to [0 1].

The explanation for how changes in resolution affect the circle applies the same way with the arc.

Parameter	[0 1]	
Resolution	5	3
Curve		

The parameter of the circle's arc will affect the range of values that the curve will be represented on. For example, with a parameter of [0 0.5], the curve represented will be that of parameter *u* having values ranging from 0 to 0.5.

Parameter	[0 0.25]	[0 0.5]
Curve		

**Resolution = 100 for all parameters*

Ellipse and its Arc

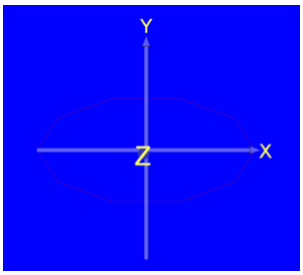
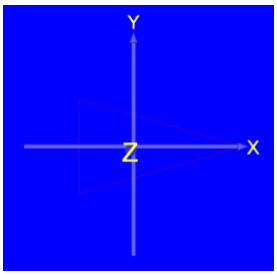
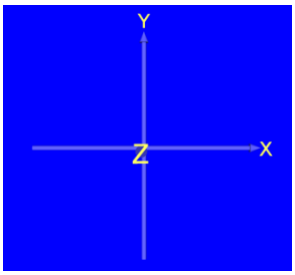
Parametric equation of an ellipse with a horizontal radius of 1 and a vertical radius of 0.5 is defined as

$$x = \cos (2\Pi u)$$

$$y = 0.5 \sin (2\Pi u)$$

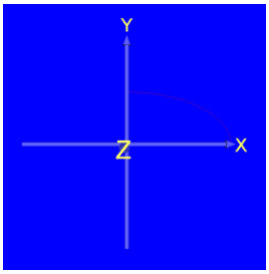
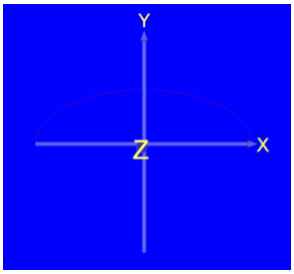
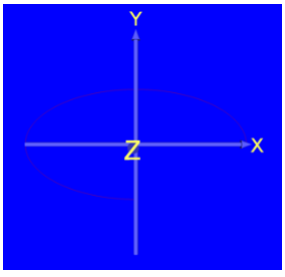
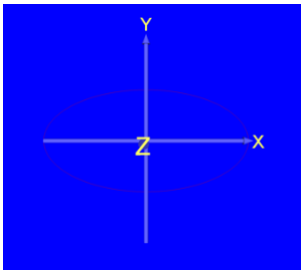
$$z = 0$$

Similar to the circle, the resolution of an ellipse affect how smooth its appearance will be. This is because the ellipse is formed by lines joining multiple points, whose number of points is dictated by the resolution value.

Parameter	[0 1]		
Resolution	10	3	2
Curve			

**Resolution = 2 : line is not visible as the two points will be on the x-axis, forming a straight line along the x-axis.*

The arc can be formed by varying the parameter value. Parameter [0 1], however, shows a full ellipse.

Parameter	[0 0.25]	[0 0.5]
Curve		
Parameter	[0 0.75]	[0 1]
Curve		

**Resolution = 100 for all parameters*

Alternatively, an arc formed by the previous ellipse with parameter [0 0.25] can be represented with the following parametric equation

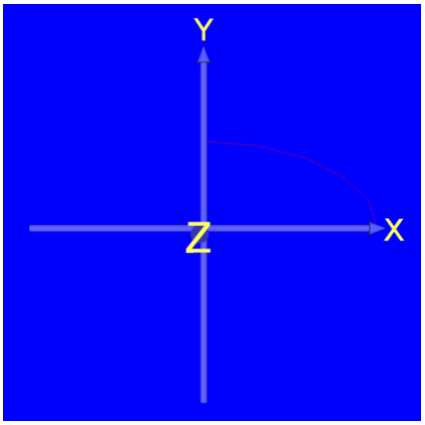
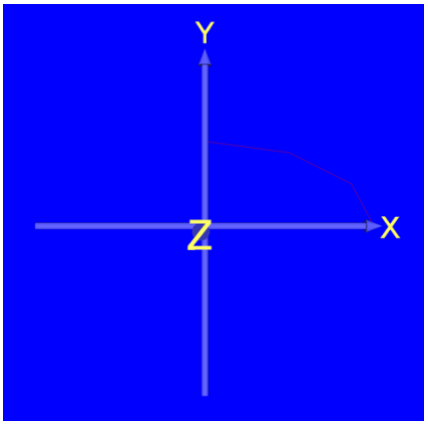
$$x = \cos\left(\frac{\Pi}{2}u\right)$$

$$y = 0.5 \sin\left(\frac{\Pi}{2}u\right)$$

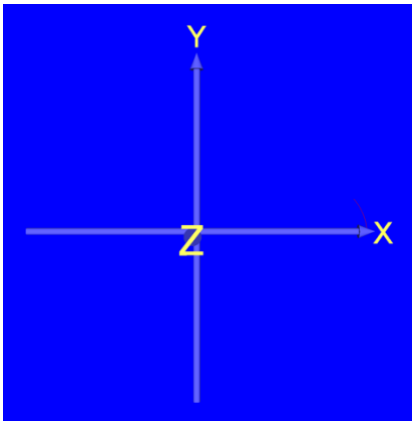
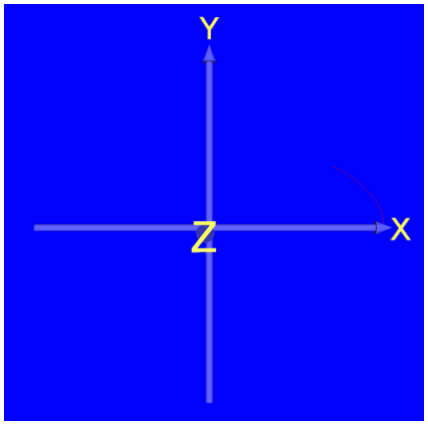
$$z = 0$$

where Parameter is set to [0 1].

The explanation for how changes in resolution affect the ellipse applies the same way with the arc.

Parameter	[0 1]	
Resolution	5	3
Curve		

The parameter of the ellipse's arc will affect the range of values that the curve will be represented on. For example, with a parameter of [0 0.5], the curve represented will be that of parameter *u* having values ranging from 0 to 0.5.

Parameter	[0 0.25]	[0 0.5]
Curve		

**Resolution = 100 for all parameters*

2D Spiral

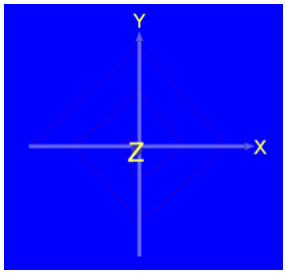
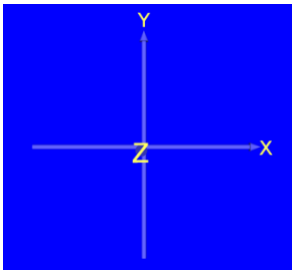
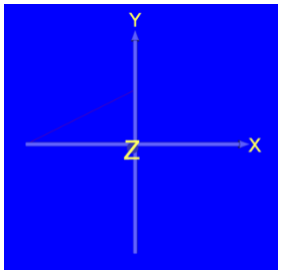
Parametric equation of a 2D spiral with 2 and a half revolutions is defined as

$$x = u \cos (5\Pi u)$$

$$y = u \sin (5\Pi u)$$

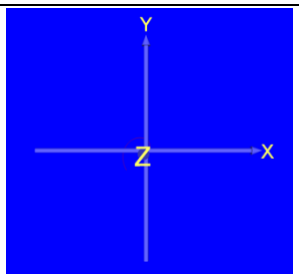
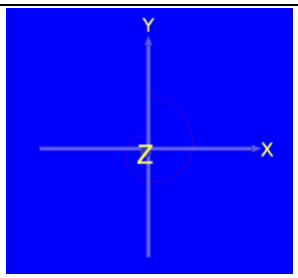
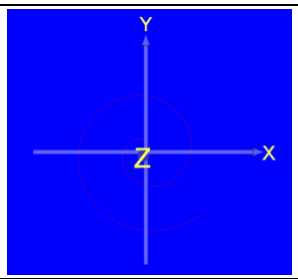
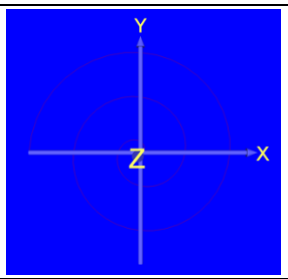
$$z = 0$$

The resolution of the spiral affect how smooth its appearance will be. This is because the spiral is formed by lines joining multiple points, whose number of points is dictated by the resolution value.

Parameter	[0 1]		
Resolution	10	5	2
Curve			

**Resolution = 5 : Since resolution = no. of revolutions*2, line is not visible as the points will be on the x-axis, forming a straight line along the x-axis.*

The parameter of the spiral will affect the range of values that the curve will be represented on. For example, with a parameter of [0 0.5], the curve represented will be that of parameter *u* having values ranging from 0 to 0.5.

Parameter	[0 0.25]	[0 0.5]
Curve		
Parameter	[0 0.75]	[0 1]
Curve		

**Resolution = 100 for all parameters*

3D Helix

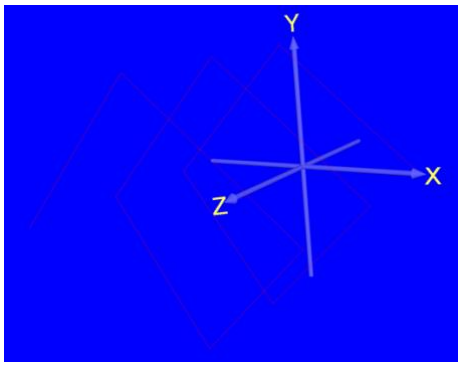
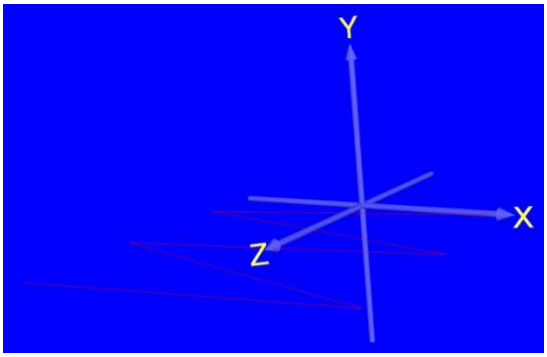
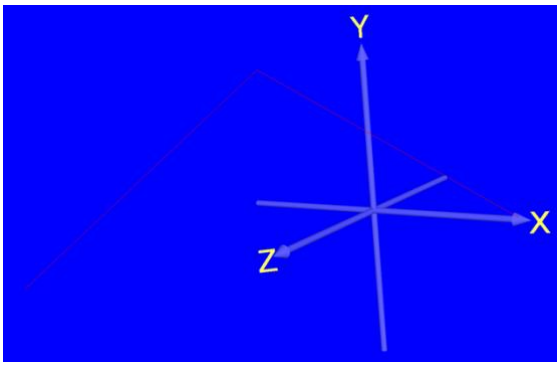
Parametric equation of a 3D helix with 2 and a half revolutions is defined as

$$x = \cos (5\Pi u)$$

$$y = \sin (5\Pi u)$$

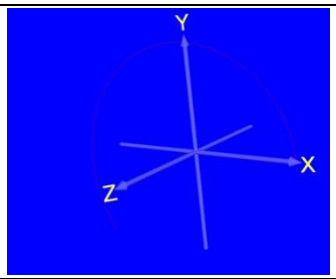
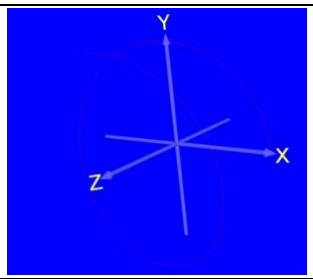
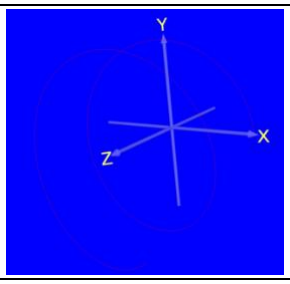
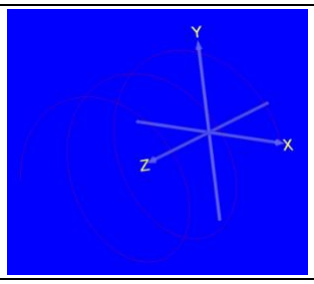
$$z = 2u$$

The resolution of the helix affect how smooth its appearance will be. This is because the helix is formed by lines joining multiple points, whose number of points is dictated by the resolution value.

Parameter	[0 1]	
Resolution	10	5
Curve		
Resolution	2	
Curve		

**Resolution = 5 : Since resolution = no. of revolutions*2, a zig-zag line along x-axis and z-axis is formed due to the sampling.*

The parameter of the spiral will affect the range of values that the curve will be represented on. For example, with a parameter of [0 0.5], the curve represented will be that of parameter *u* having values ranging from 0 to 0.5.

Parameter	[0 0.25]	[0 0.5]
Curve		
Parameter	[0 0.75]	[0 1]
Curve		

**Resolution = 100 for all parameters*

Conversion from Explicit to Parametric Representation

The explicitly define curve of equation

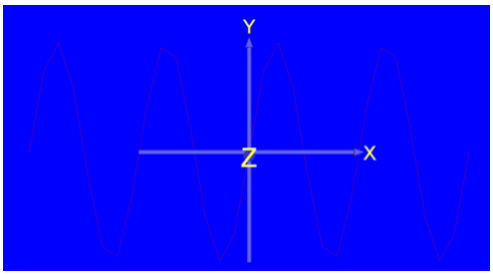
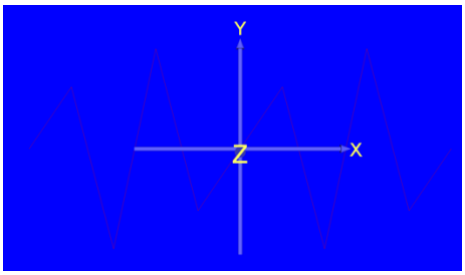
$$y = \sin(x)$$

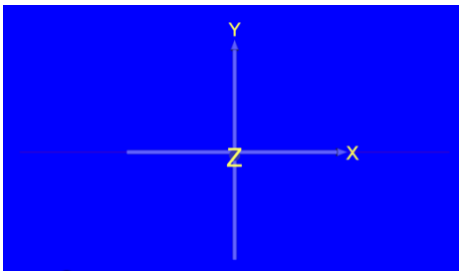
has the parametric representation of

$$x = u$$

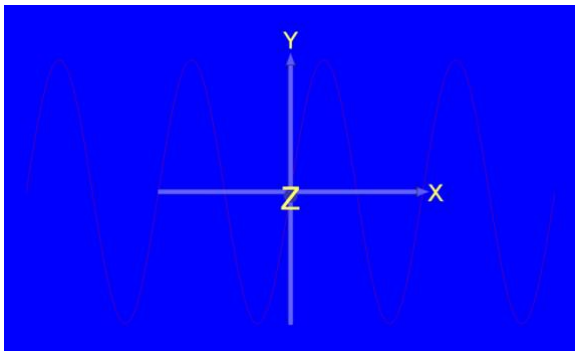
$$y = \sin(2\Pi u)$$

The resolution of the curve affect how smooth its appearance will be. This is because the curve is formed by lines joining multiple points, whose number of points is dictated by the resolution value.

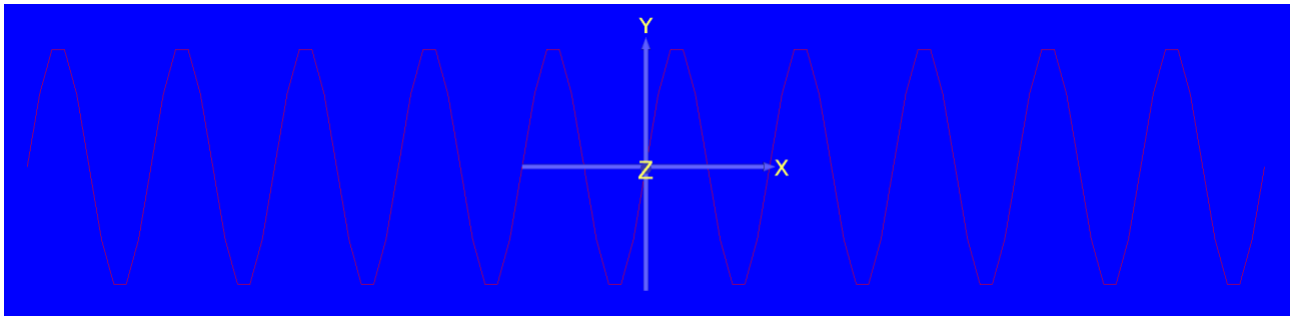
Parameter	[-2 2]	
Resolution	30	10
Curve		

Resolution	2
Curve	

The parameter of the curve will affect the range of values that it will be represented on. For example, below is the curve with a parameter of [-5 5], the curve represented will be that of parameter *u* having values ranging from -5 to 5. Although at a resolution of 100, the peaks and throughs of the curve can be seen to be less distinct as that of the curve with parameter [-2 2]. This is because more sampling is required to achieve the same quality of appearance when the parameter is increased.



Parameter [-2 2] and Resolution = 100



Parameter [-5 5] and Resolution = 100

Additional Patterns and Shapes Observed

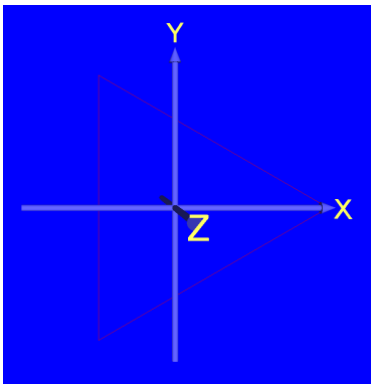
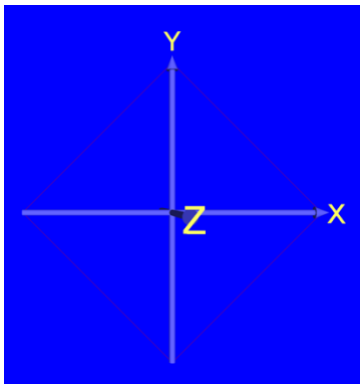
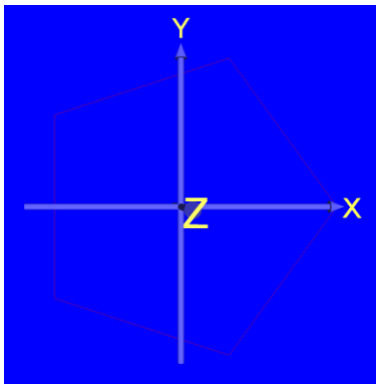
From the circle mentioned above (circle.wrl), a few shapes can be observed. With the parametric equation below, different shapes can be obtained by modifying the resolution.

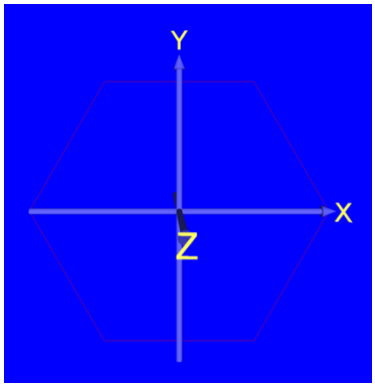
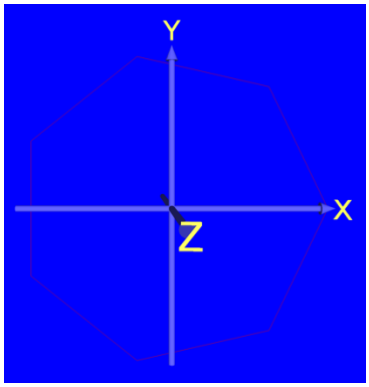
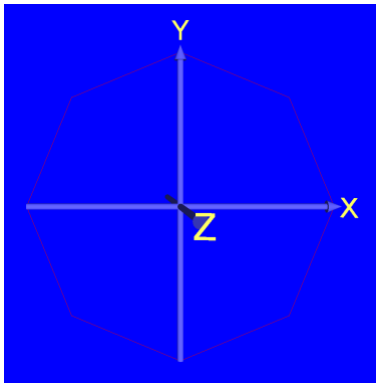
$$x = \cos (2\Pi u)$$

$$y = \sin (2\Pi u)$$

$$z = 0$$

$$u \in [0 \ 1]$$

Resolution	Shape	Appearance
3	Equilateral Triangle	
4	Square	
5	Pentagon	

6	Hexagon	
7	Heptagon	
8	Octagon	

It is observed that having a resolution of *n* results in a *n*-sided regular polygon.

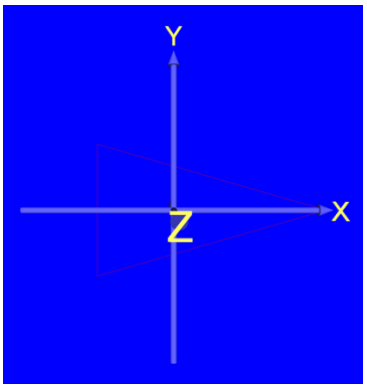
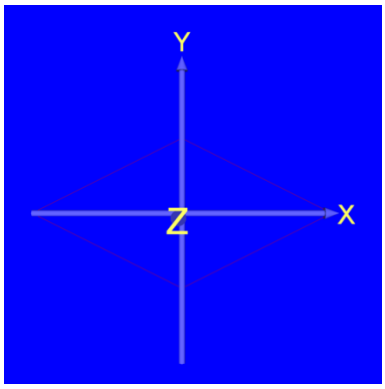
From the ellipse mentioned above (ellipse.wrl), a few shapes can be observed. With the parametric equation below, different shapes can be obtained by modifying the resolution.

$$x = \cos (2\Pi u)$$

$$y = 0.5 \sin (2\Pi u)$$

$$z = 0$$

$$u \in [0 \ 1]$$

Resolution	Shape	Appearance
3	Isosceles Triangle	
4	Rhombus	

It is observed that having a resolution of *n* results in a *n*-sided polygon.

Description of Files

File Name	Equation	Description
2D spiral.wrl	$x = u \cos(5\Pi u)$ $y = u \sin(5\Pi u)$ $z = 0$ $u \in [0\ 1]$	Shows 2D spiral with 2 and a half revolutions
3D helix.wrl	$x = \cos(5\Pi u)$ $y = \sin(5\Pi u)$ $z = 2u$ $u \in [0\ 1]$	Shows a 3D helix with 2 and a half revolutions
circle arc.wrl	$x = \cos(\frac{\Pi}{2} u)$ $y = \sin(\frac{\Pi}{2} u)$ $z = 0$ $u \in [0\ 1]$	Shows a circle's arc
circle.wrl	$x = \cos(2\Pi u)$ $y = \sin(2\Pi u)$ $z = 0$ $u \in [0\ 1]$	Shows a circle with radius of 1
ellipse arc.wrl	$x = \cos(\frac{\Pi}{2} u)$ $y = 0.5 \sin(\frac{\Pi}{2} u)$ $z = 0$ $u \in [0\ 1]$	Shows an ellipse's arc
ellipse.wrl	$x = \cos(2\Pi u)$ $y = 0.5 \sin(2\Pi u)$ $z = 0$ $u \in [0\ 1]$	Shows an ellipse with a horizontal radius of 1 and a vertical radius of 0.5
straight line segment.wrl	$x = -0.7 + 2u$ $y = -1 + 2u$ $u \in [0\ 1]$	Shows straight line segment
y=sin(x).wrl	$x = u$ $y = \sin(2\Pi u)$ $u \in [0\ 1]$	Shows sine curve represented in parametric form