§Waves

A periodic wave in space characterized by the distance two corresponding values of the function and is called the wavelength λ. The speed of a wave is

$$v = f\lambda.$$

Any function of the form $f(x - vt)$ with finite support describes a traveling wave in space and time with the forms

$$\begin{aligned} y_{\text{pos-dir}} &= A_+ \cos(k_+ x - \omega_+ t + \phi_+) \\ y_{\text{neg-dir}} &= A_- \cos(k_- x + \omega_- t + \phi_-). \end{aligned}$$

The angular wavenumber k is defined as

$$k \equiv 2\pi/\lambda$$

which yields another relation for wave speed

$$\omega = kv.$$

Traveling waves can move in both positive and negative directions through time.

$$\begin{aligned} y_{j+} &= A_{j+} \cos(k_{j+} x - \omega_{j+} t + \phi_{j+}) \\ y_{j-} &= A_{j-} \cos(k_{j-} x + \omega_{j-} t + \phi_{j-}) \end{aligned}$$

In general, waves add at each point in space and time

$$\begin{aligned} y(x,t) = \quad & \sum_{j=1}^{N} A_{j+} \cos(k_{j+} x - \omega_{j+} t + \phi_{j+}) \\ & + \sum_{j=1}^{N} A_{j-} \cos(k_{j-} x + \omega_{j-} t + \phi_{j-}). \end{aligned}$$

When waves add interference occurs. For two identical waves save the phase difference

$$y(x,t) = 2A \cos\left(kx - \omega t + \tfrac{1}{2}\phi\right) \cos\left(\tfrac{1}{2}\phi\right)$$

leading to the notions of completely constructive, destructive, and simply reductive interference

$$\begin{aligned} \phi_{\text{constructive}} &= n\pi & n &= 0, \pm 1, \pm 2, \ldots \\ \phi_{\text{destructive}} &= (2n+1)\pi & n &= 0, \pm 1, \pm 2, \ldots \\ \phi_{\text{reductive}} &\neq (\phi_{\text{constr}}, \phi_{\text{destr}}) \end{aligned}$$

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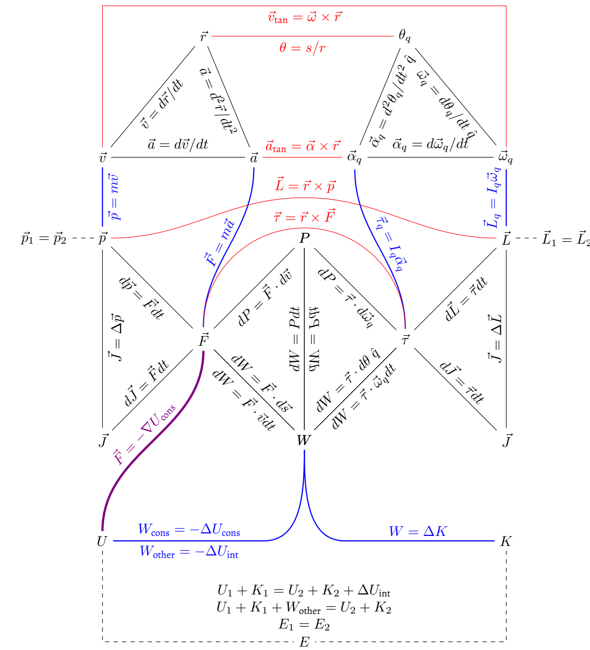

§Kinematics

Descriptors of linear motion begin with position $\vec{r}$ and of angular motion with θ_q

$$\begin{aligned} \vec{v} &= d\vec{r}/dt & \vec{\omega}_q &= d\theta_q/dt \\ \vec{a} &= d\vec{v}/dt & \vec{\alpha}_q &= d\vec{\omega}_q/dt. \end{aligned}$$

For constant acceleration

$$\begin{aligned} q &= q_0 + v_{q_0} t + \tfrac{1}{2} a_q t^2 & \theta_q &= \theta_{q_0} + \omega_{q_0} t + \tfrac{1}{2}\alpha_q t^2 \\ v_q &= v_{q_0} + a_q t & \omega_q &= \omega_{q_0} + \alpha_q t \\ v_q^2 &= v_{q_0}^2 + 2a_q(q - q_0) & \omega_q^2 &= \omega_{q_0}^2 + 2\alpha_q(\theta_q - \theta_{q_0}) \\ \Delta q &= \tfrac{1}{2}(v_{q_0} + v_q)t & \Delta\theta_q &= \tfrac{1}{2}(\omega_{q_0} + \omega_q)t. \end{aligned}$$

Momentum, impulse, Newton's first law, and Newton's second law are

$$\begin{aligned} \vec{p} &= m\vec{v} & \vec{L} &= I\vec{\omega} \\ \vec{J} &= \Delta\vec{p} & \vec{J} &= \Delta\vec{L} \\ \vec{F} &= d\vec{p}/dt & \vec{\tau} &= d\vec{L}/dt \\ \sum \vec{F} &= m\vec{a} & \sum \vec{\tau} &= I\vec{\alpha}. \end{aligned}$$

Symmetry of physical laws with respect to translation and rotation begets conservation of momentum

$$\vec{p}_1 = \vec{p}_2 \qquad \vec{L}_1 = \vec{L}_2.$$

Connections between linear and angular kinematics start with the definition of an angle as the ratio of arc length s to radius r

$$\begin{aligned} \theta &= s/r & & \\ \vec{v}_{\text{tan}} &= \vec{\omega} \times \vec{r} & \vec{L} &= \vec{r} \times \vec{p} \\ \vec{a}_{\text{tan}} &= \vec{\alpha} \times \vec{r} & \vec{\tau} &= \vec{r} \times \vec{F} \end{aligned}$$

Moment of inertia I is a measure of the resistance of a body to change its rotational motion and depends on the distribution of mass *about a particular axis*

$$\begin{aligned} I_{\text{discrete}} &= \textstyle\sum_j r_j^2 m_j \\ I_{\text{continuous}} &= \textstyle\int r^2 dm \end{aligned}$$

About a *parallel* axis the moment of inertia is related to the distance d from the center of mass

$$I = I_{\text{cm}} + Md^2$$

§Work and Energy

Work is the result of a motive cause acting over a path

$$\begin{aligned} dW &= \vec{F} \cdot d\vec{s} & dW &= \vec{\tau} \cdot d\theta\hat{q} \\ dW &= \vec{F} \cdot \vec{v}dt & dW &= \vec{\tau} \cdot \vec{\omega}dt. \end{aligned}$$

Kinetic energy expressions for translational and rotational motion are

$$K = \tfrac{1}{2}mv^2 \qquad K = \tfrac{1}{2}I\omega^2.$$

Two conservative potential energies of interest are gravitational and elastic

$$\begin{aligned} U_{\text{grav}} &= mgh \\ U_{\text{el}} &= \tfrac{1}{2}kx^2. \end{aligned}$$

Conservative potentials yield forces via the spatial gradient (derivative)

$$\vec{F} = -\nabla U_{\text{cons}}.$$

Symmetry of physical laws with respect to time begets conservation of energy

$$E_1 = E_2.$$

Conservation of energy can be written to highlight the work done or the internal energy change

$$\begin{aligned} U_1 + K_1 + W_{\text{other}} &= U_2 + K_2 \\ U_1 + K_1 &= U_2 + K_2 + \Delta U_{\text{int}}. \end{aligned}$$

The work-energy theorem always relates how much work is required to change the kinetic energy

$$W = \Delta K.$$

§Periodic Motion

A periodic wave in time is characterized by the time to complete one cycle called the period T, then linear and angular frequencies obtain

$$f = 1/T \qquad \omega = 2\pi f = 2\pi/T$$

A restoring cause that is proportional to displacement

$$F = -kx \qquad \tau = \kappa\theta$$

will yield simple harmonic motion with a solution

$$\begin{aligned} x &= A\cos(\omega t + \phi) & \theta &= A_\theta \cos(\omega t + \phi) \\ \omega &= \sqrt{k/m} & \omega &= \sqrt{\kappa/I}. \end{aligned}$$

Pendulums with small amplitudes approximate simple harmonic motion

$$\begin{aligned} k &= mg/\ell & \kappa &= mg\ell \\ \omega &= \sqrt{g/l} & \omega &= \sqrt{mg\ell/I} \end{aligned}$$