Prüfung SoSe18
Date: 16th. April 2018
Examiner: Chih-Yu Hsu

OSTBAYERISCHE TECHNISCHE HOCHSCHULE REGENSBURG
INFORMATIK UND MATHEMATIK

Time: 90 minutes Maximum

score: 100 points

Lastname, Firstname: .....

Matr.-Nr.: .....

Major/Studiengang: .....

Seat/Platz: .....

Question	Maximum points	Achieved points
1	12	
2	8	
3	6	
4	6	
5	10	
6	8	
7	17	
8	21	
9	6	
10	6	
Σ	**100**	
Grade		

Remarks

1. This exam consists of **13 pages with 10 questions**. All questions need to be answered to attain all points. Please check whether your exam copy is complete!
2. **No additional material or resources are allowed**, except for a non-programmable calculator and the assignment printouts the examiner distributed.
3. Please **do not write with a pencil**! Please **do not remove the staples**!
4. You may answer in **English**.
5. Please **use the space below the questions to write your answers**. If – contrary to expectations - this space is not sufficient, please use the rear pages. If you use the rear pages please indicate clearly to which question your answer refers to!

Good luck!

Unit 1: What is Tensorflow and how to solve linear regression problem in Python and Tensorflow.

Question set 1

1.1 "Machine Learning" consists of six main steps and has tremendous impact on almost every industry. Name six main steps of the procedure of Machine Learning. (3 points)

Answer:

Six main steps

1. Get Data 2. Preprocessing Data 3. Select Algorithm/model

4. Training model 5. Evaluating model 6. Prediction

1.2 Explain linear regression problem in code by Tensorflow

- # Create input data using NumPy. y = x * 0.1 + 0.3 + noise

Code:

```
# Import tensorflow and other libraries.
import tensorflow as tf
import numpy as np
x_data = np.random.rand(100).astype(np.float32)
noise = np.random.normal(scale=0.01, size=len(x_data))
y_data = x_data * 0.1 + 0.3+noise
```

- # Buld inference graph.

```
# Create Variables W and b that compute y= W * x_data + b
```

Answer:

Code: (3 points)

```
W =tf.Variable(tf.random_uniform([1], 0.0, 1.0))
b =tf.Variable(tf.zeros([1]))
y=W * x_data + b
```

- # Build training graph.

Code: (4 points)

```
loss = tf.reduce_mean(tf.square(y - y_data))
# Create an operation that calculates loss.
optimizer = tf.train.GradientDescentOptimizer(0.5)
# Create an optimizer.
train = optimizer.minimize(loss)
# Create an operation that minimizes loss.
init = =tf.initialize_all_variables()
# Create an operation initializes all the variables.
```

- # Create a session and launch the graph.

Code: (2 points)

```
sess = = tf.Session()
sess.run(init)
y_initial_values = sess.run(y)
# Save initial values for plotting later
```

Unit 1: What is Tensorflow and how to solve linear regression problem in Python and Tensorflow.

Question set 2

2.1 Please use the mathematical expression to express the mean squared error function. (4 points)

Input data $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$

Linear Model $y = f(x) = mx + b,\ m:\ slope,\ b:\ intercept$

The Mean Squared Error (MSE) function with variables $m:\ slope,\ b:\ intercept$

$$MSE = \frac{1}{n}\{(y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 + \ldots + (y_n - f(x_n))^2\}$$
$$= \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x_i))^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - mx_i + b)^2$$

2.2 Please use one example to explain how to use the mean squared error function for the linear regression problem. (4 points)

- Example:

Input data $\{(0, 0.1), (1, 0.9), (2, 2.1)\}$

Linear Model $y = f(x) = mx + b,$ if $m = 1,\ b = 0,\ then\ y = f(x) = x$

$f(0) = 0,\ f(1) = 1,\ f(2) = 2$

$$MSE = \frac{1}{3}\{(0.1 - f(0))^2 + (0.9 - f(1))^2 + (2.1 - f(2))^2\}$$
$$= \frac{1}{3}\{(0.1 - 0)^2 + (0.9 - 1)^2 + (2.1 - 2)^2\}$$
$$= \frac{1}{3}\{(0.1)^2 + (-0.1)^2 + (0.1)^2\}$$
$$= \frac{1}{3}\{(0.01) + (0.01) + (0.01)\}$$
$$= 0.01$$

Unit 2: What is image classification and how to implement a solution?

Question set 3

3.1 Please use the mathematical expression to express a discriminant function.

For example in two dimensional space (x1, x2), a discriminant function $f\left(x_1, x_2\right)$

(2 points)

Answer:

$f\left(x_1, x_2\right) = w_1 x_1 + w_2 x_2 + b,\ w_1, w_2$:weight, b:bias

3.2 Please use two points (0,0) and (1,1) to explain how to choose the weights and bias of a discriminant function to separate these two points.(4 points)

Answer:

$f\left(x_1, x_2\right) = w_1 x_1 + w_2 x_2 + b,\ w_1, w_2$:weight, b:bias

Choose $w_1 = -1, w_2 = -1, b = 0.5$

$f\left(x_1, x_2\right) = -1x_1 - 1x_2 + 0.5$

$f\left(0,0\right) = -1\times 0 - 1\times 0 + 0.5 = 0.5 > 0$

$f\left(1,1\right) = -1\times 1 - 1\times 1 + 0.5 = -1.5 < 0$

Unit 2: What is image classification and how to implement a solution?

Question 4

4.1 Please use the mathematical expression to express a decision boundary. (2 points)

For example in two dimensional space(x1, x2), a decision boundary is like as follow.

f(x1,x2)=0 (2 points)

Answer:

A decision boundary is an equation when the discriminant function equals to zero.
If the discriminant function $f\left(x_1, x_2\right) = w_1 x_1 + w_2 x_2 + b$, w_1, w_2:weight, b:bias is known then the decision boundery is $f\left(x_1, x_2\right) = 0$, that is, $w_1 x_1 + w_2 x_2 + b = 0$.

4.2 Please use two points (0, 0.5) and (0.5, 0) to explain how to choose the weights and bias to define a decision boundary passes through these two points but separates (0,0) and (1,1). (4 points)

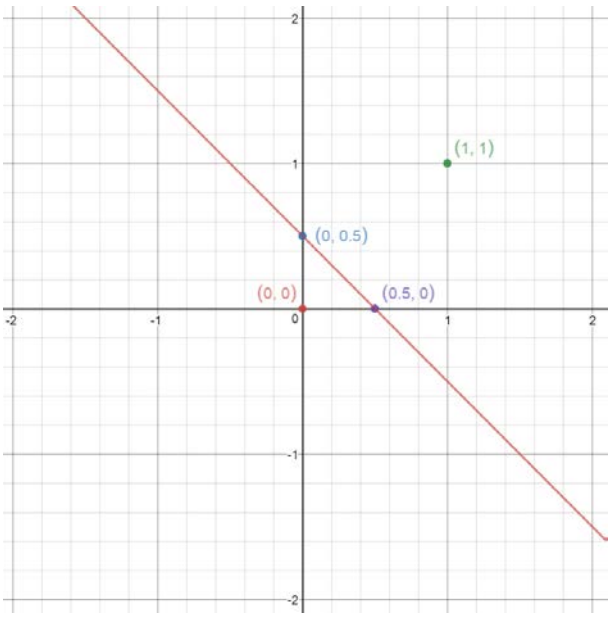
Answer:

A decision boundary is a line in two dimensional space(x1, x2).
A line (a decision boundary) passes through two points (0, 0.5) and (0.5, 0) can be represented by the Two Points Form.
The Two Point Form is

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}, \text{Slope } m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-0.5}{0.5-0} = -1,$$

$$\frac{y-0.5}{x-0} = -1 \Rightarrow y-0.5 = -1(x-0) \Rightarrow -x-y+0.5 = 0 \text{ (The General Form)}$$

change charater y with x_2 and x with x_1, the line equation is $-x_1 - x_2 + 0.5 = 0$



Unit 2: What is image classification and how to implement a solution?

Question 5

5.1 Please use the decision boundary in Q4/4.2 to define a discriminant function f(x1, x2) and calculate f(0,0) and f(1,1) . (2 points)

Answer:

The discriminant function is the left side of the line equation $-1x_1 - 1x_2 + 0.5 = 0.$

$f(x_1, x_2) = -1x_1 - 1x_2 + 0.5$ can separate the points (0,0) and (1,1).

$f(0,0) = -1\times 0 - 1\times 0 + 0.5 = 0.5 > 0$

$f(1,1) = -1\times 1 - 1\times 1 + 0.5 = -1.5 < 0$

$f(x_1, x_2) = w_1 x_1 + w_2 x_2 + b,\ w_1, w_2$:weight, b:bias

Choose $w_1 = -1, w_2 = -1, b = 0.5$

$f(x_1, x_2) = -x_1 - x_2 + 0.5$

The other answer f(0,0)=-0.5, f(1,1)=1.5 is also correct when the discriminant function is by multiplied with -1, $f(x_1, x_2) = x_1 + x_2 - 0.5$

5.2 Please express the step function and sigmoid function in mathematical expression. (4 points)

Answer:

$$step(a) = \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}$$

$$sigmoid(a) = \frac{1}{1+e^{-a}}$$

5.3 Please calculate the following results.(4 points)

Answer:

If $a = f(0,0)$ then $step(0.5) = 1$

If $a = f(1,1)$ then $step(-1.5) = 0$

If $a = f(0,0)$ then $sigmoid(0.5) = \frac{1}{1+e^{-0.5}}$

If $a = f(1,1)$ then $sigmoid(-1.5) = \frac{1}{1+e^{1.5}}$

Unit 3: What is Neural Networks and how to apply on image classification?

Question 6

6.1 Please use the mathematical expression to explain what a Perception (Neuron) with two inputs is. (4 points)

Answer:

A Perception (Neuron) is a discriminant function with an activation function.

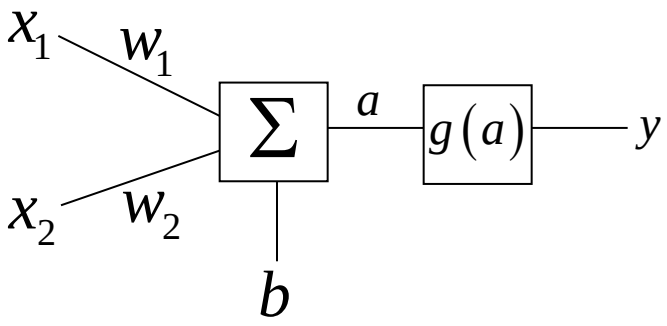
$$y = g\left(w_1 x_1 + w_2 x_2 + b\right),$$

where w_1, w_2 are weights, b is a bias

and $g(\cdot)$ *is an activation function.*

6.2: Draw a diagram to represent a Perception with two inputs. (4 points)

Answer:



Unit 3: What is Neural Networks and how to apply on image classification?

Question 7

7.1 Please write the truth table of AND Gate with input (x1, x2) and output y=x1^x2 . (4 points)

Answer:

Truth Table of AND Gate	
Input	Output
X1 x2	x1^x2
0 0	0
0 1	0
1 1	1
1 0	0

7.2: Please draw the diagram of a Single-layer Perception as a linear classifier for the AND Gate.

(a) Please give the weight parameters and bias (3 points)

$$f(x_1, x_2) = w_1 x_1 + w_2 x_2 + b,\ w_1, w_2 \text{:weight, b:bias}$$

$$Choose\ w_1 = 1, w_2 = 1, b = -1.5$$

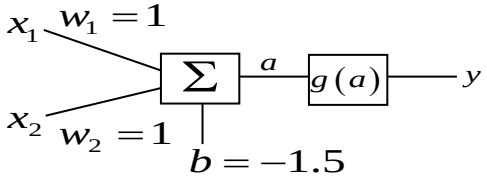
(b) Please give the discriminant function corresponding to the Perception (2 points)

$$f(x_1, x_2) = x_1 + x_2 - 1.5$$

(c) Please give the activation function (2 points)

$$step(a) = \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}$$

(d) Please draw the diagram of the Perception (2 points)



(e) Please show the input and the corresponding output is the same as truth table (4 points)

$$f(0,0) = 0 + 0 - 1.5 = -1.5,\ \text{step(-1.5)=0}$$

$$f(0,1) = 0 + 1 - 1.5 = -0.5,\ \text{step(-0.5)=0}$$

$$f(1,1) = 1 + 1 - 1.5 = 0.5,\ \text{step(0.5)=1}$$

$$f(1,0) = 1 + 0 - 1.5 = -0.5,\ \text{step(-0.5)=0}$$

Unit 3: What is Neural Networks and how to apply on image classification?

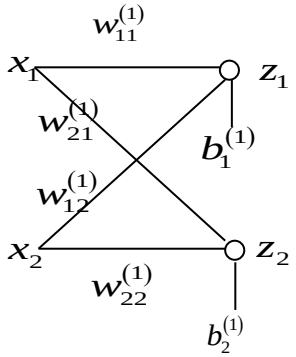
Question 8

8.1 Please write the truth table of XOR Gate with input (x1, x2) and output y= XOR(x1, x2). (4 points)

Truth Table of XOR Gate		
Input		Output
X1	x2	XOR (x1,x2)
0	0	0
0	1	1
1	1	0
1	0	1

8.2: Please draw the diagram of a Multi-layer Perception as a classifier for the XOR Gate.

(a) Please draw a diagram of the first layer Perception (2 points)



(b)Please give the weight parameters and bias (3 points)

$$w_{11}^{(1)} = 1, w_{12}^{(1)} = 1, b_1^{(1)} = -\frac{1}{2}$$

$$w_{21}^{(1)} = 1, w_{22}^{(1)} = 1, b_2^{(1)} = -\frac{3}{2}$$

(c)Please give the activation function (2 points)

$$step(a) = \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}$$

(d)Please give the coordinate transformation from (x1, x2) to (z1, z2) (2 points)

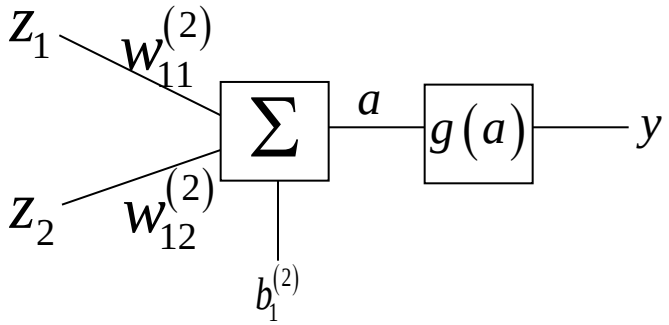
$$z_1 = step\left(w_{11}^{(1)}x_1 + w_{12}^{(1)}x_2 + b_1^{(1)}\right), w_{11}^{(1)} = 1, w_{12}^{(1)} = 1, b_1^{(1)} = -\frac{1}{2}, z_1 = step\left(x_1 + x_2 - \frac{1}{2}\right)$$

$$z_2 = step\left(w_{21}^{(1)}x_1 + w_{22}^{(1)}x_2 + b_2^{(1)}\right), w_{21}^{(1)} = 1, w_{22}^{(1)} = 1, b_2^{(1)} = -\frac{3}{2}, z_2 = step\left(x_1 + x_2 - \frac{3}{2}\right)$$

coordinate transformation			
Input		hidden	
X1	x2	z1	z2
0	0	0	0
0	1	1	0
1	1	1	1
1	0	1	0

$$x_1 = 0, x_2 = 0, z_1 = step\left(0+0-\frac{1}{2}\right) = 0, z_2 = step\left(0+0-\frac{3}{2}\right) = 0.$$
$$x_1 = 0, x_2 = 1, z_1 = step\left(0+1-\frac{1}{2}\right) = 1, z_2 = step\left(0+1-\frac{3}{2}\right) = 0.$$
$$x_1 = 1, x_2 = 1, z_1 = step\left(1+1-\frac{1}{2}\right) = 1, z_2 = step\left(1+1-\frac{3}{2}\right) = 1.$$
$$x_1 = 1, x_2 = 0, z_1 = step\left(1+0-\frac{1}{2}\right) = 1, z_2 = step\left(1+0-\frac{3}{2}\right) = 0.$$

(e) Please draw a diagram of the second layer Perception (2 points)



(f) Please give the discriminant function corresponding to the second layer Perception (2 points)

$$f(z_1, z_2) = w_{11}^{(2)} z_1 + w_{12}^{(2)} z_2 + b_1^{(2)},$$
$$Choose\ w_{11}^{(2)} = 1, w_{12}^{(2)} = -1, b_1^{(2)} = -0.5$$
$$f(z_1, z_2) = z_1 - z_2 - 0.5$$

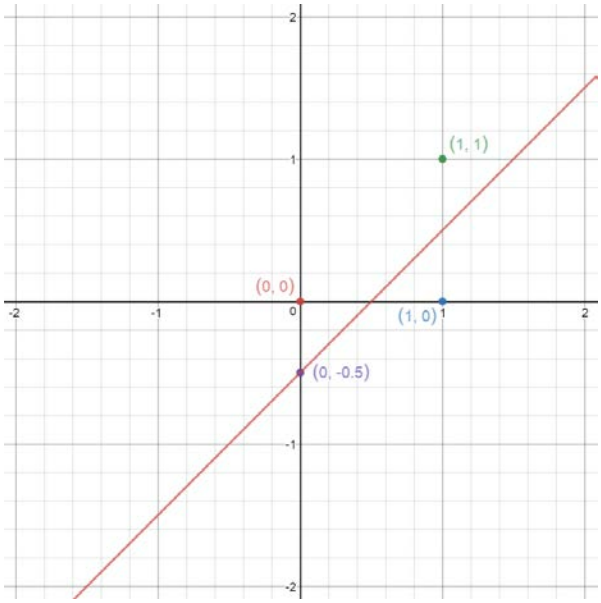
(g) Please give the activation function (2 points)

$$step(a)=\begin{cases}1 & a\geq 0\\ 0 & a<0\end{cases}$$

(h) Please show the input and the corresponding output is the same as truth table (2 points)

$$z_1=0, z_2=0, y_1=step(0+0-0.5)=0.$$
$$z_1=1, z_2=0, y_1=step(1-0-0.5)=1.$$
$$z_1=1, z_2=1, y_1=step(1-1-0.5)=0.$$

Truth Table of XOR Gate					
coordinate transformation					
Input		hidden		output	model output
x1	x2	z1	z2	y	XOR (x1,x2)
0	0	0	0	0	0
0	1	1	0	1	1
1	1	1	1	0	0
1	0	1	0	1	1



Unit 3: What is Neural Networks and how to apply on image classification?

Question 9

9.1 Please using a mathematical expression to express the softmax activation function. (1 points)

$$\text{softmax}(x)_i = \frac{\exp(x_i)}{\sum_j \exp(x_j)}$$

9.2: Explain softmax regression in code by Tensorflow

- # MNIST For ML Beginners

Code: (5 points)

```
# Import tensorflow and other libraries.
import tensorflow as tf
import numpy as np
from tensorflow.examples.tutorials.mnist import input_data
mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
# The MNIST Data
```

Answer:

```
#x isn't a specific value. It's a placeholder, a value that we'll input
x =tf.placeholder(tf.float32, [None, 784])

#the model parameters are Variable
W =tf.Variable(tf.zeros([784, 10]))
b =tf.Variable(tf.zeros([10]))

# implement our model
y =..tf.matmul(x, W) + b

#Training #a new placeholder to input the correct answers
y_=tf.placeholder(tf.int64, [None]).....

#implement the cross-entropy function
cross_entropy = tf.reduce_mean(-tf.reduce_sum(y_ * tf.log(y), reduction_indices=[1]))

#choice of optimization algorithm
train_step =tf.train.GradientDescentOptimizer(0.5).minimize(cross_entropy)
sess = tf.InteractiveSession()
tf.global_variables_initializer().run()
```

Unit 5: What is image segmentation and how to implement a solution?

Question set 10

10.1 Please use the mathematical expression to express Dice coefficient (index)? (2 points)

Answer:

$$DC = \frac{2|A \cap B|}{|A| + |B|} \quad \text{or} \quad DC = \frac{2TP}{2TP + FP + FN}$$

10.2 There are two sets A and B. If A∩B=A and |A|=|B| , please calculate Dice coefficient (index)? (2 points)

Answer:

$$\because A \cap B = A \Rightarrow \therefore |A \cap B| = |A|$$

$$\because |A| = |B|$$

$$\therefore DC = \frac{2|A \cap B|}{|A| + |B|} = \frac{2|A|}{|A| + |A|} = \frac{2|A|}{2|A|} = 1$$

10.3 There are two sets A and B. If A = {[0,0,0,1,1,0]} and B = {[0,0,0,0,1,0]} , If each element of one array is a pixel, then A and B are binary image vectors, please calculate Dice coefficient (index)? (2 points)

Answer:

For segmentation problem, only consider region of interest (ROI), where those pixels' values are true.

$$\because A = \{[0,0,0,1,1,0]\} \Rightarrow \therefore |A| = 2,$$

and

$$\because B = \{[0,0,0,0,1,0]\} \Rightarrow \therefore |B| = 1.$$

$|A \cap B| = 1$ only the fifth pixel 1(true) is the same by comparing A and B

because in segemtation results only consider the region of interest (ROI)

$$\therefore DC = \frac{2|A \cap B|}{|A| + |B|} = \frac{2 \times 1}{2 + 1} = \frac{2}{3}$$

If A is ground true, TP=1,FP=0,FN=1

$$DC = \frac{2TP}{2TP + FP + FN} = \frac{2 \times 1}{2 \times 1 + 0 + 1} = \frac{2}{3}$$

If B is ground true, TP=1,FP=1,FN=0

$$DC = \frac{2TP}{2TP + FP + FN} = \frac{2 \times 1}{2 \times 1 + 1 + 0} = \frac{2}{3}$$

Everyone gets two points because of different definitions will get different results. If A is ground true, TP=1, FP=0,FN=1, P= TP+ FN=2.

$$DC = \frac{TP}{P + FP} = \frac{1}{2 + 1} = \frac{1}{3}$$ Please refer web site.