數值分析Numerical Analysis

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聽眾Audience

- 碩一first year graduate
- 基礎課程 Basic curriculum
- 19 students

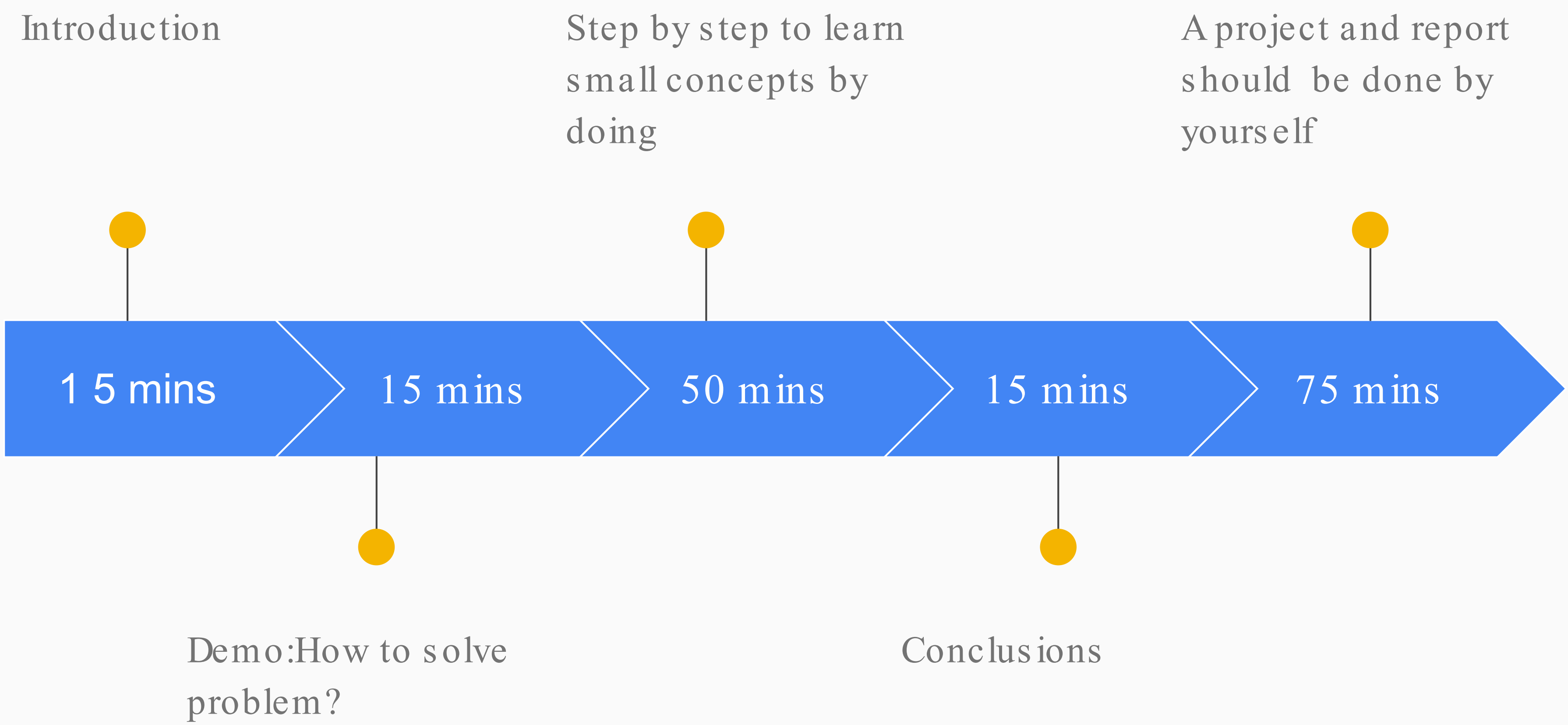
Objective

- 引導您了解如何自學。Guide you to know how to learn by yourself.
- 數值分析（NA）的歷史是什麼？
- What is the history of the Numerical Analysis (NA)?
    - Who are the important people NA? Newton, Lagrange, Gauss and Euler.
    - What are the important events of NA?
    - Where is the location of NA?
    - Which objects are related to NA?
    - When is NA popular?

材料Materials

- What is NA
- Python Language
- Tensor flow
- Project and report

程序Procedure


Introduction
1 5 mins
Demo:How to solve problem?
15 mins
Step by step to learn small concepts by doing
50 mins
Conclusions
15 mins
A project and report should be done by yourself
75 mins

家庭作業1/Homework1

Learning efficiency depends on motivation

1. Why do you choose this course?
2. What kind of jobs for NA?
3. What kind of jobs you want?

How to learn knowledge of NA efficiently?

1. Python programming
2. Mathematics
3. Tensorflow, scikit learn and, karas

Python programming

Learning topics

1. Using Python as a Calculator
    a. Arithmetic operations
2. Variables
3. if Statements
4. for Statements
5. The range() Function
6. Executing modules as scripts
7. Mathematics
8. Python Functions - W3Schools

Learning Resources

1. Welcome to Python.org
2. Download
3. Python For Beginners
4. The Python Tutorial
5. IntroductoryBooks
6. Python Functions - W3Schools
7. A Visual Introduction to Python
8. Try Jupyter

Python programming

Learning topics

1. Python Functions

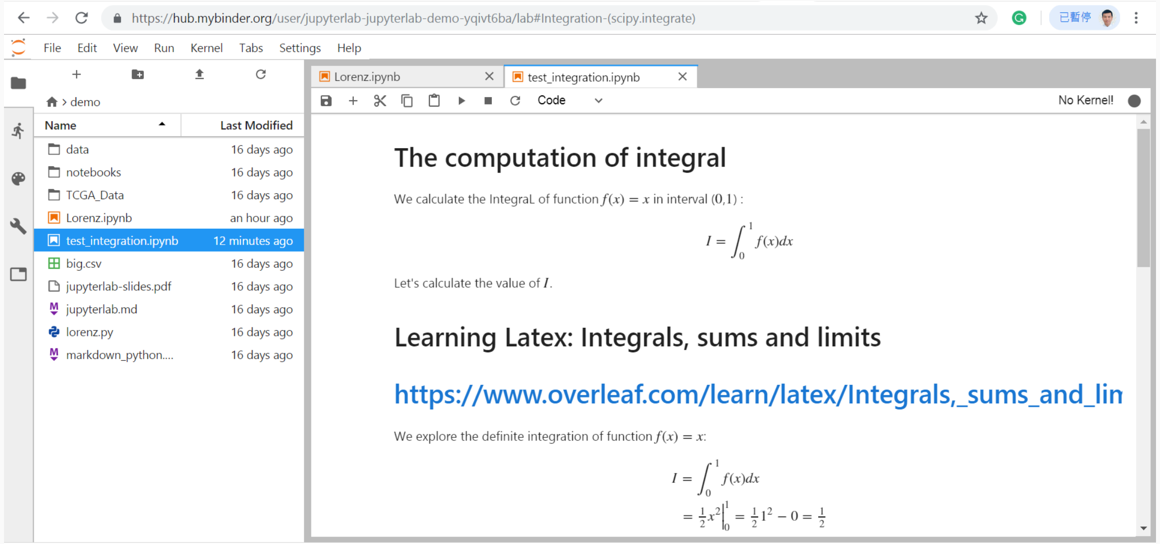
```
def my_function():
  print("Hello from a function")

my_function ()
```

Learning Resources

1. Python Functions - W3Schools
2. Run web python or Anaconda
3. http://jupyter.org/try
4. https://hub.mybinder.org/user/jupyterlab-jupyterlab-demo-yqivt6ba/lab#Integration-(scipy.integrate)

Python programming



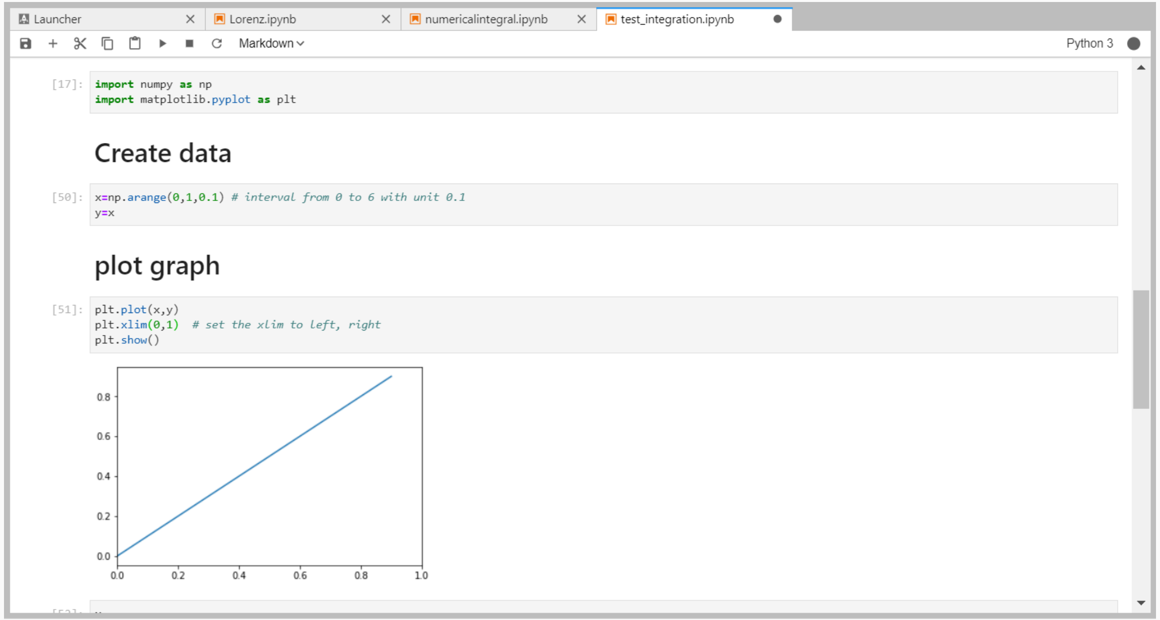
1. http://jupyter.org/try

Python programming

Plot graph curve y=f(x)=x

import numpy as np
import matplotlib.pyplot as plt

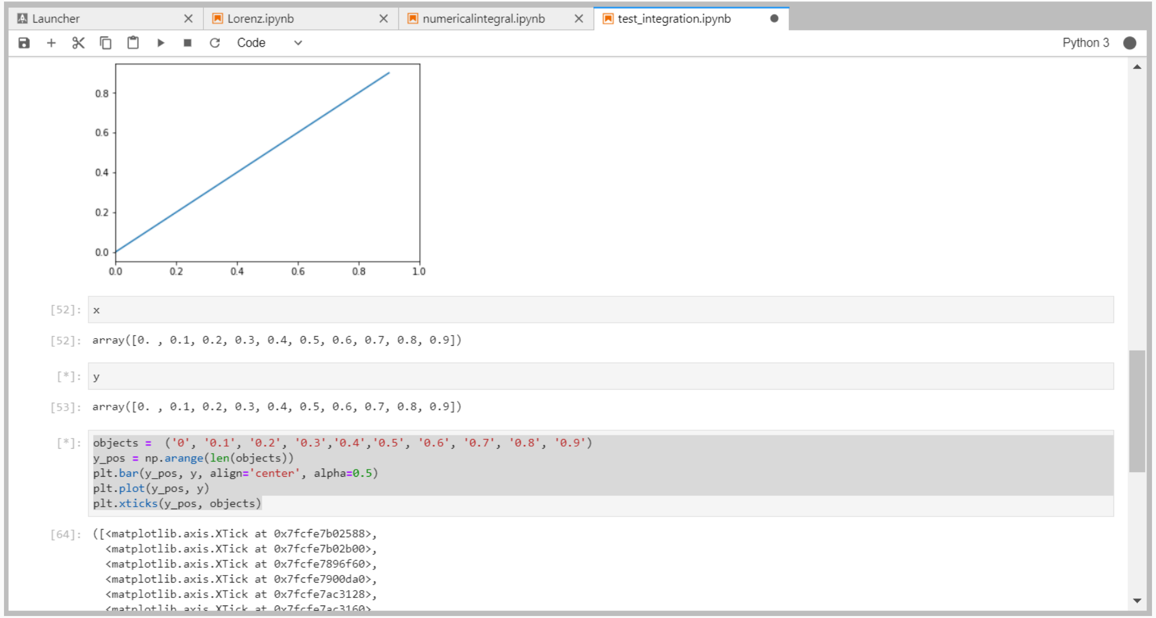
 # Create data
x=np.arange(0,1,0.1) # interval from 0 to 6 with unit 0.1
y=x
 # plot graph
plt.plot(x,y)
plt.xlim(0,1) # set the xlim to left, right
plt.show()



http://jupyter.org/try

程式設計 Python programming

Data
X
y

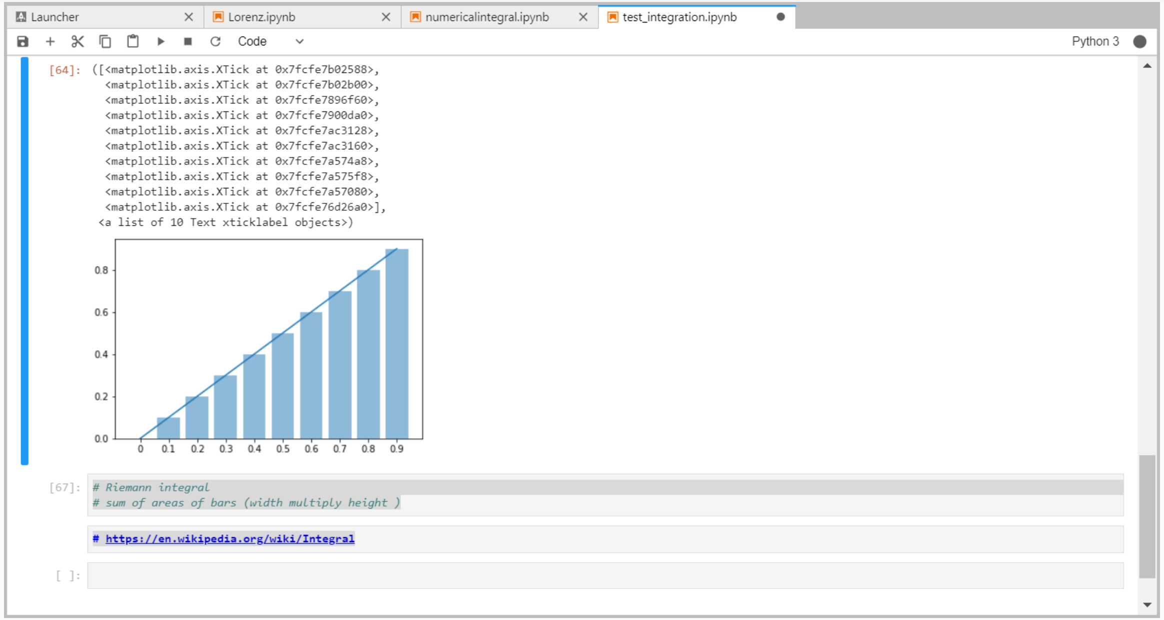


1. http://jupyter.org/try

程式設計 Python programming

Bar Plot
Riemann integral
sum of areas of bars (width multiply height)
https://en.wikipedia.org/wiki/Integral

objects = ('0', '0.1', '0.2', '0.3','0.4','0.5', '0.6', '0.7', '0.8', '0.9')
y_pos = np.arange(len(objects))
plt.bar(y_pos, y, align='center', alpha=0.5)
plt.plot(y_pos, y)
plt.xticks(y_pos, objects)

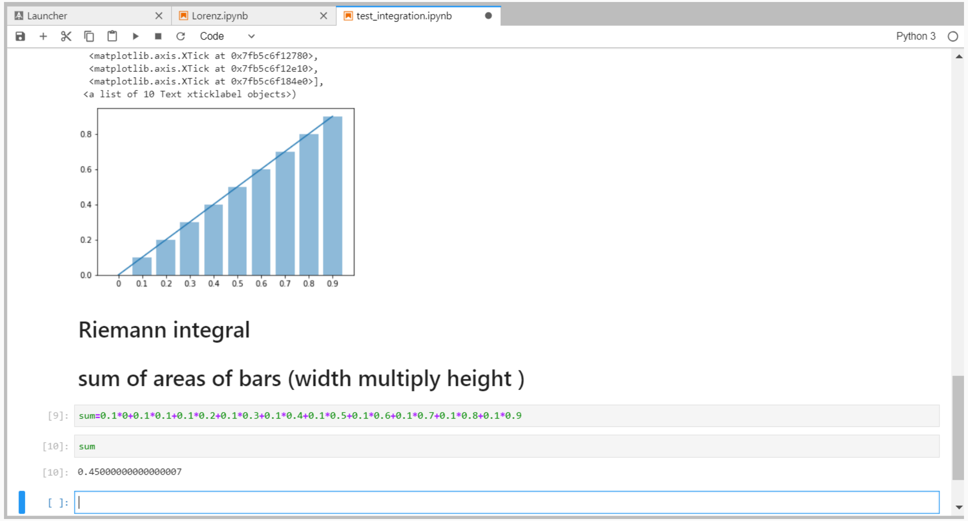


http://jupyter.org/try

Python programming

 # Riemann integral
 # sum of areas of bars (width multiply height)

 # https://en.wikipedia.org/wiki/Integral



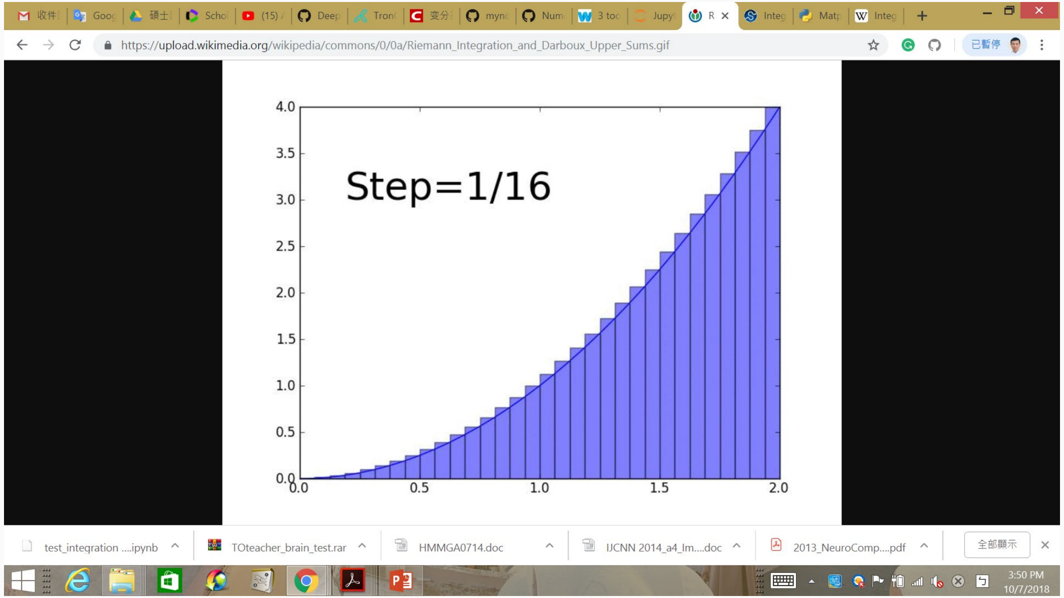
https://en.wikipedia.org/wiki/Numerical_integration

Python programming

Question:
Y=x**2   [0 1]
Summing the areas of these rectangles, we get a better approximation for the sought integral, namely

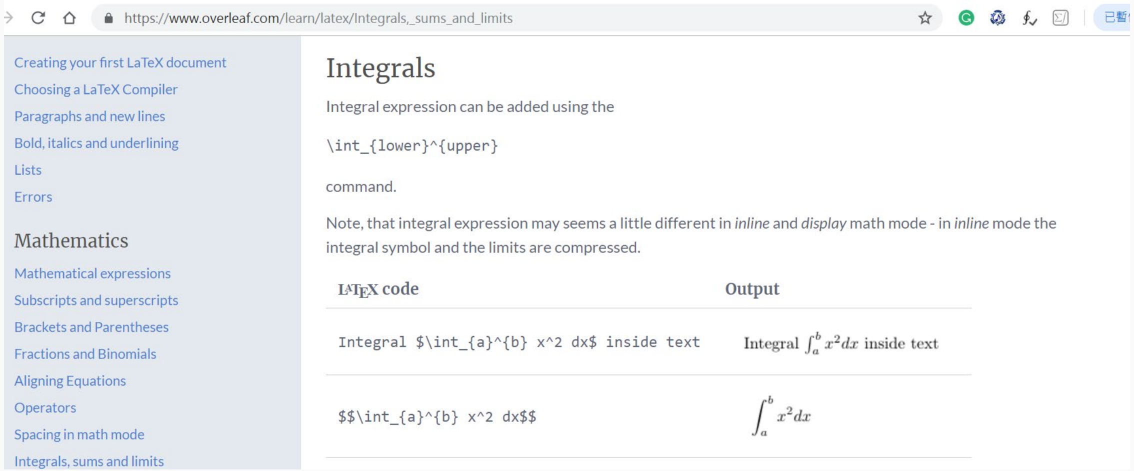
https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.quad.html

https://en.wikipedia.org/wiki/Integral



Latex programming

Learning to Latex to Integrals, sums and limits


https://www.overleaf.com/learn/latex/Integrals,_sums_and_limits

Integrals

Integral expression can be added using the

```
\int_{lower}^{upper}
```

command.

Note, that integral expression may seems a little different in *inline* and *display* math mode - in *inline* mode the integral symbol and the limits are compressed.

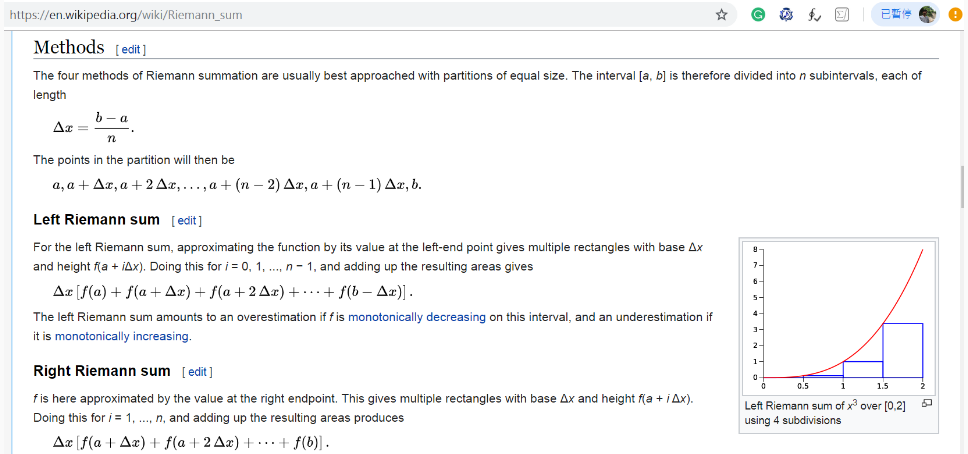
LaTeX code	Output
`Integral $\int_{a}^{b} x^2 dx$ inside text`	Integral $\int_a^b x^2 dx$ inside text
`$$\int_{a}^{b} x^2 dx$$`	$$\int_a^b x^2 dx$$

https://www.overleaf.com/learn/latex/Integrals,_sums_and_limits

Python programming

Learning to implement the algorithm on wiki

https://en.wikipedia.org/wiki/Riemann_sum



https://en.wikipedia.org/wiki/Riemann_sum

Methods [edit]

The four methods of Riemann summation are usually best approached with partitions of equal size. The interval [a, b] is therefore divided into n subintervals, each of length

$$\Delta x = \frac{b-a}{n}.$$

The points in the partition will then be

$$a, a+\Delta x, a+2\,\Delta x, \ldots, a+(n-2)\,\Delta x, a+(n-1)\,\Delta x, b.$$

Left Riemann sum [edit]

For the left Riemann sum, approximating the function by its value at the left-end point gives multiple rectangles with base Δx and height $f(a + i\Delta x)$. Doing this for $i = 0, 1, \ldots, n - 1$, and adding up the resulting areas gives

$$\Delta x\left[f(a) + f(a+\Delta x) + f(a+2\,\Delta x) + \cdots + f(b-\Delta x)\right].$$

The left Riemann sum amounts to an overestimation if f is monotonically decreasing on this interval, and an underestimation if it is monotonically increasing.

Left Riemann sum of x^3 over [0,2] using 4 subdivisions

Right Riemann sum [edit]

f is here approximated by the value at the right endpoint. This gives multiple rectangles with base Δx and height $f(a + i\,\Delta x)$. Doing this for $i = 1, \ldots, n$, and adding up the resulting areas produces

$$\Delta x\left[f(a+\Delta x) + f(a+2\,\Delta x) + \cdots + f(b)\right].$$

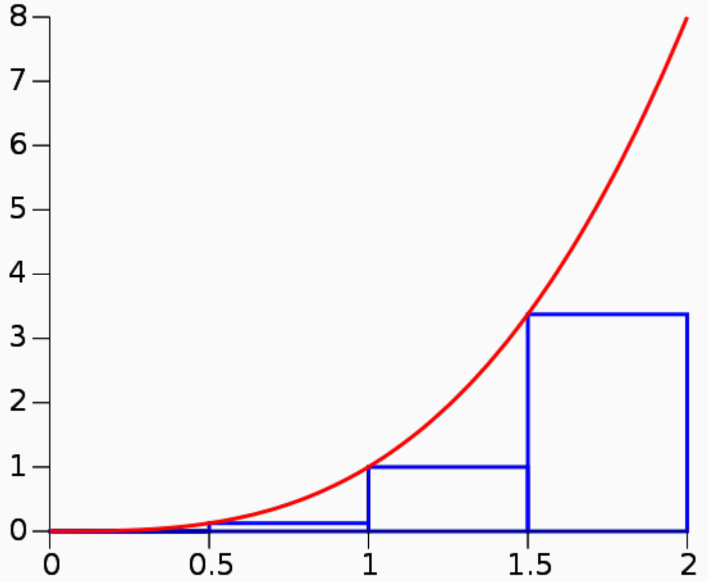
Python programming

Question:

Refer to:
https://github.com/tccnchsu/Numerical_Analysis/blob/master/NA_W4_Integration.ipynb

F(x)=x^2#

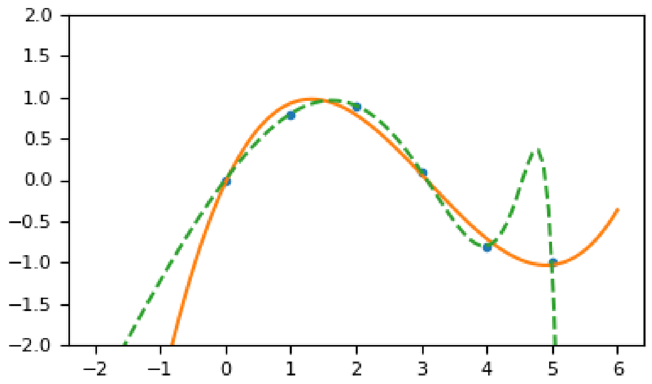
https://en.wikipedia.org/wiki/Riemann_sum



Python programming

polynomial interpolation

Two points <-> 2,3,4 degree polynomial
Three points <-> 2,3,4 degree polynomial
Four points <-> 2,3,4 degree polynomial



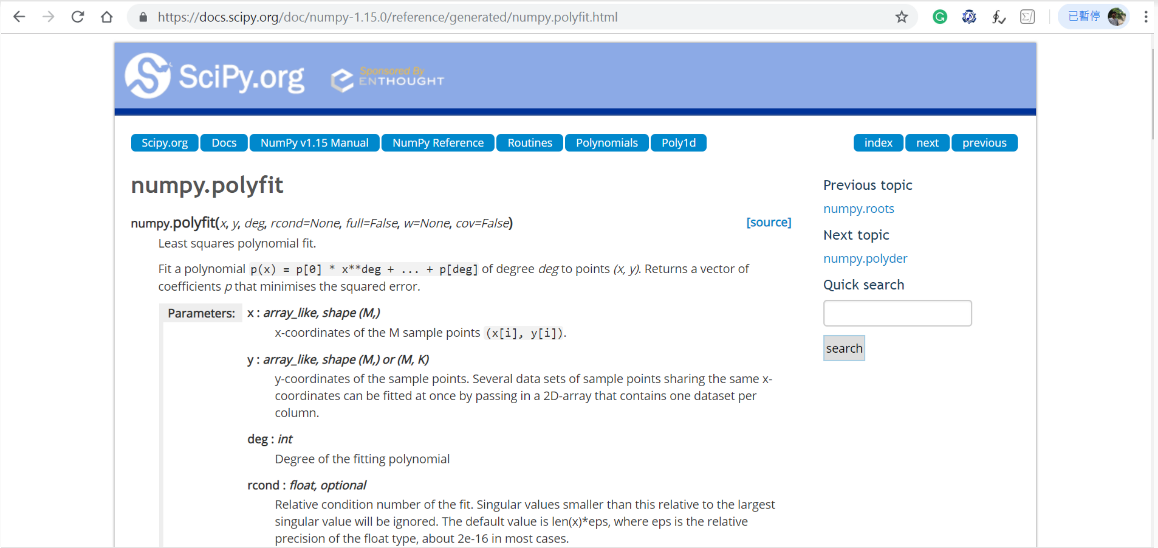
https://docs.scipy.org/doc/numpy-1.15.0/reference/generated/numpy.polyfit.html

Python programming

polynomial interpolation

Least squares polynomial fit.

numpy.polyfit



https://docs.scipy.org/doc/numpy-1.15.0/reference/generated/numpy.polyfit.html

Python programming

In Depth: Linear Regression

Open in Colab

https://jakevdp.github.io/PythonDataScienceHandbook/05.06-linear-regression.html

Python programming

In Depth: Linear Regression
線性迴歸


https://hk.saowen.com/a/be3251d506c792a17e28882445eb73a04fe79c650c5f0167ea54abf22b005705

掃文資訊

2017-07-02 / git.oschina.net

Python 數據科學手冊 5.6 線性迴歸

5.6 線性迴歸

原文： In Depth: Linear Regression

譯者： 飛龍

協議： CC BY-NC-SA 4.0

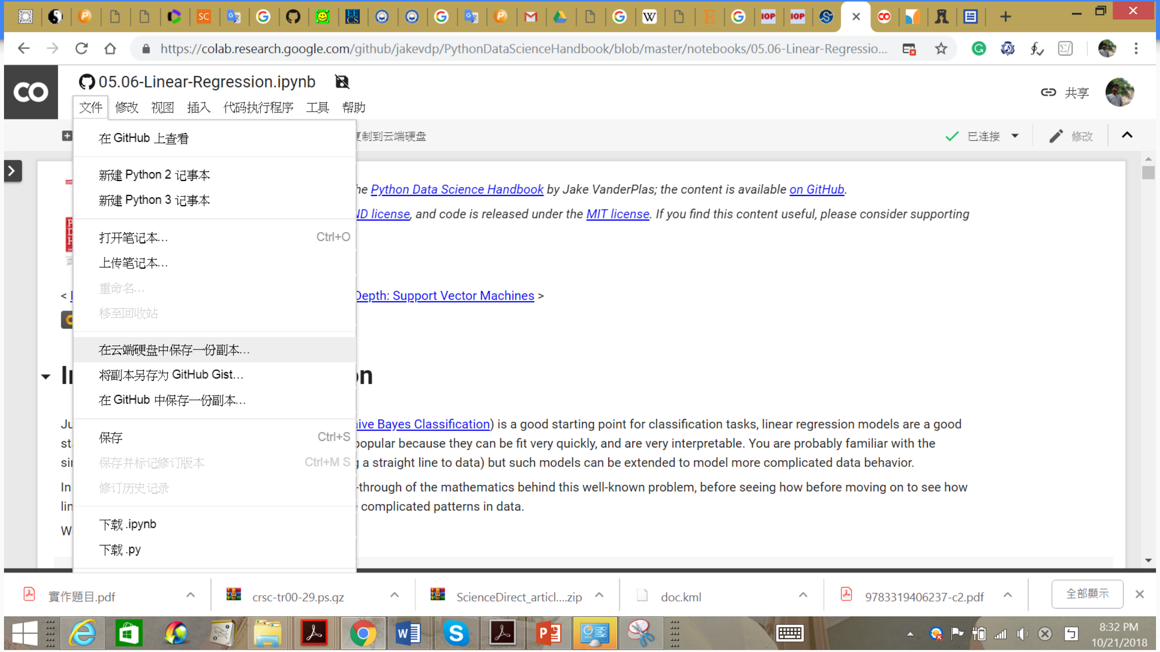
譯文沒有得到原作者授權，不保證與原文的意思嚴格一致。

https://hk.saowen.com/a/be3251d506c792a17e28882445eb73a04fe79c650c5f0167ea54abf22b005705

Python programming

In Depth: Linear Regression

File copy cloud disk

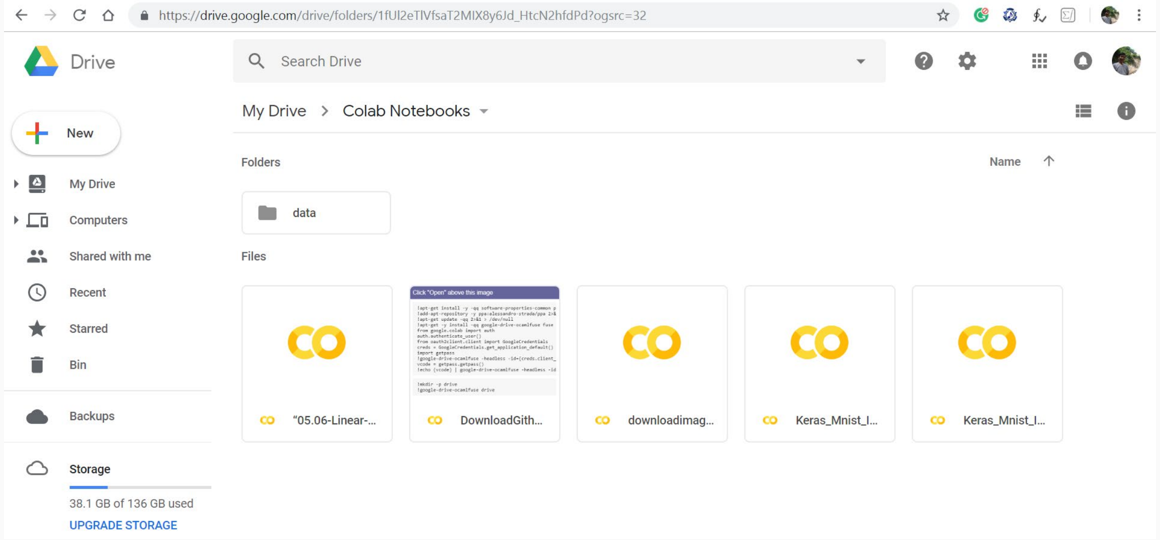


https://jakevdp.github.io/PythonDataScienceHandbook/05.06-linear-regression.html

Python programming

In Depth: Linear Regression

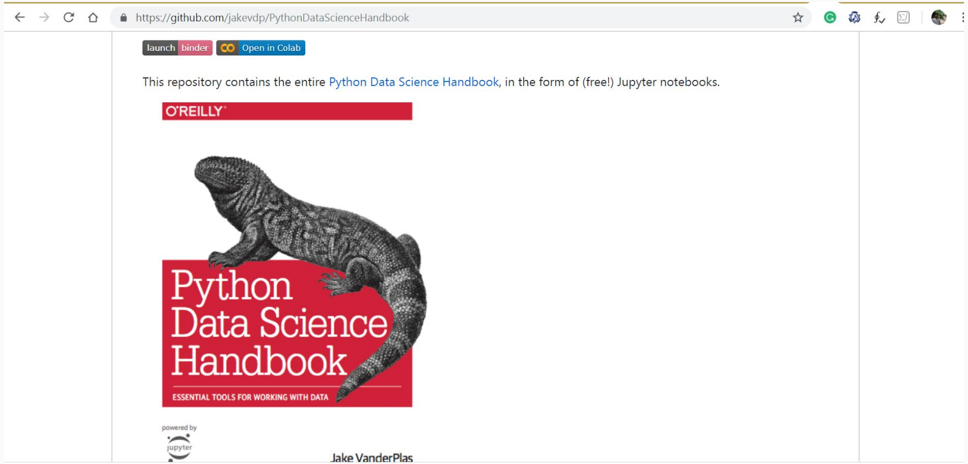
File copy cloud disk



https://jakevdp.github.io/PythonDataScienceHandbook/05.06-linear-regression.html

Python programming

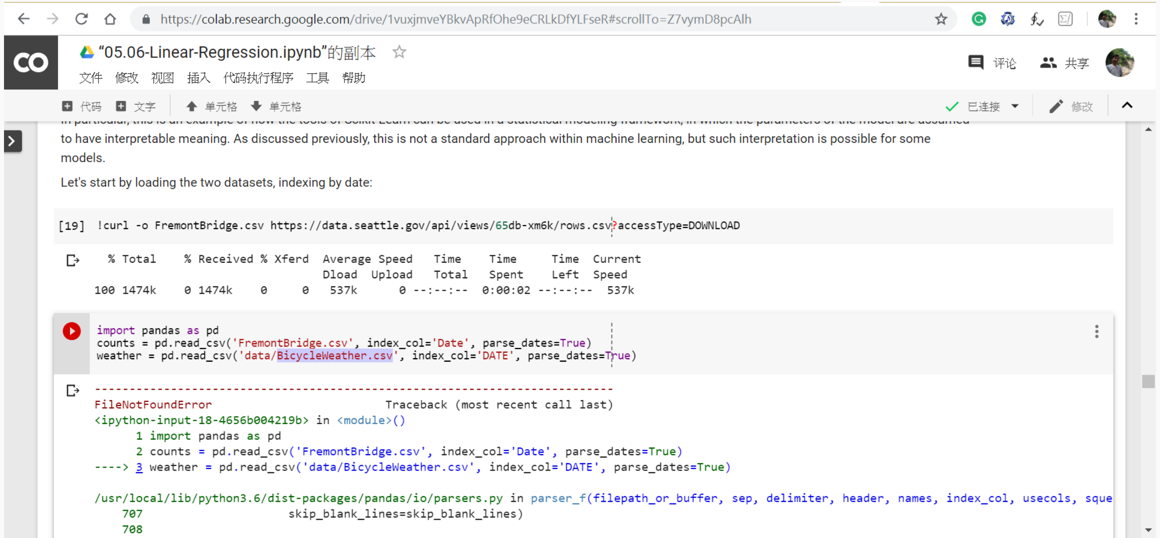
Python Data Science Handbook



https://github.com/jakevdp/PythonDataScienceHandbook

Python programming

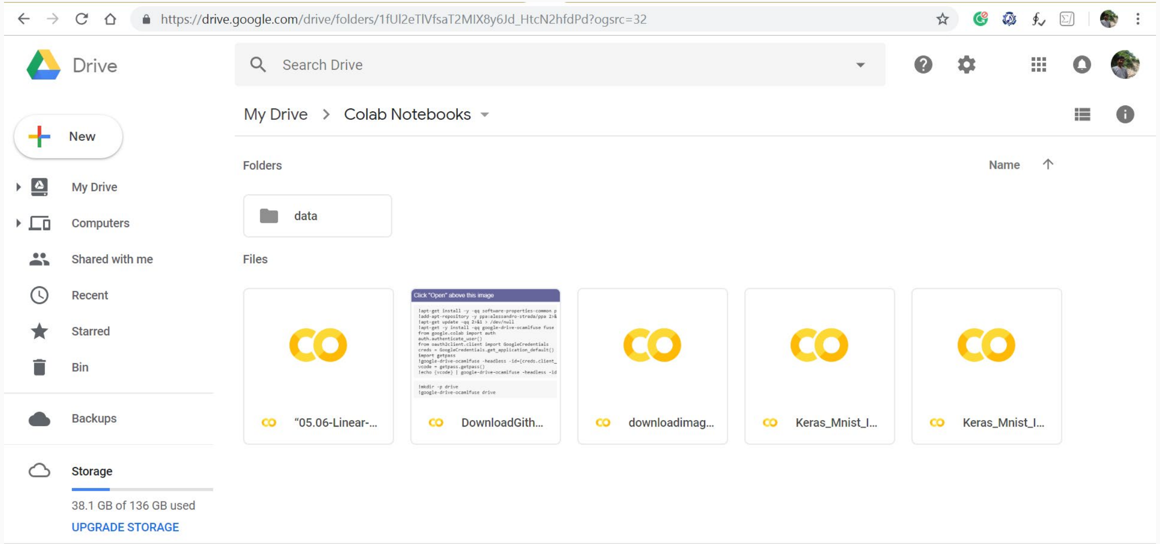
BicycleWeather.csv



https://github.com/jakevdp/PythonDataScienceHandbook

Python programming

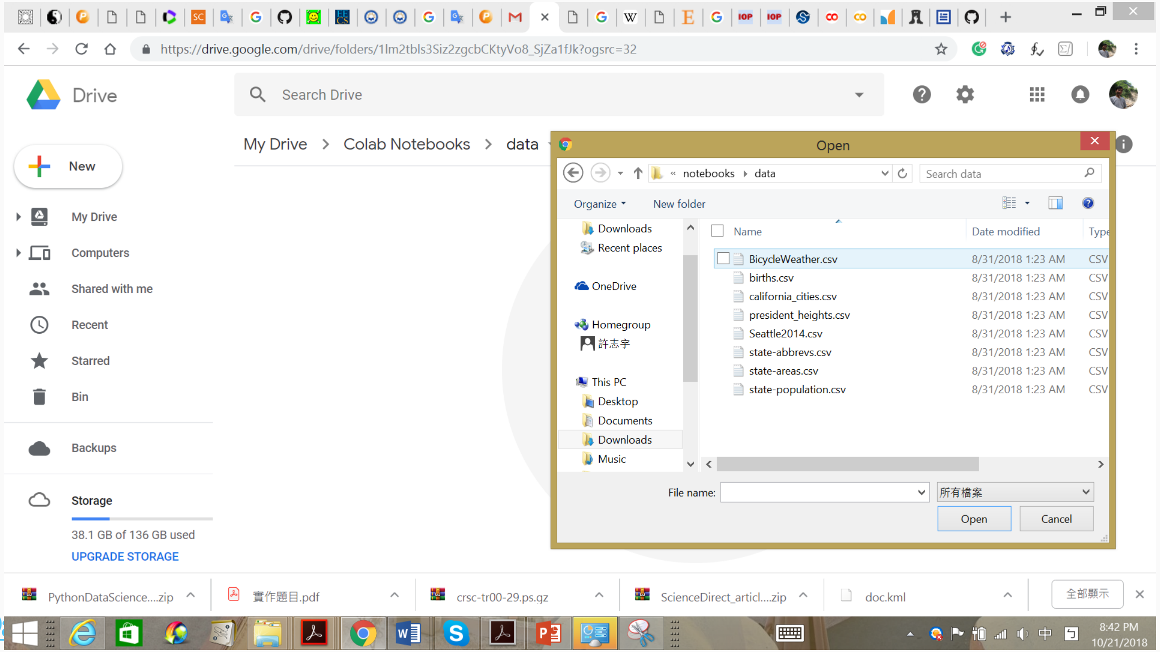
Make directory "data"



https://github.com/jakevdp/PythonDataScienceHandbook

Python programming

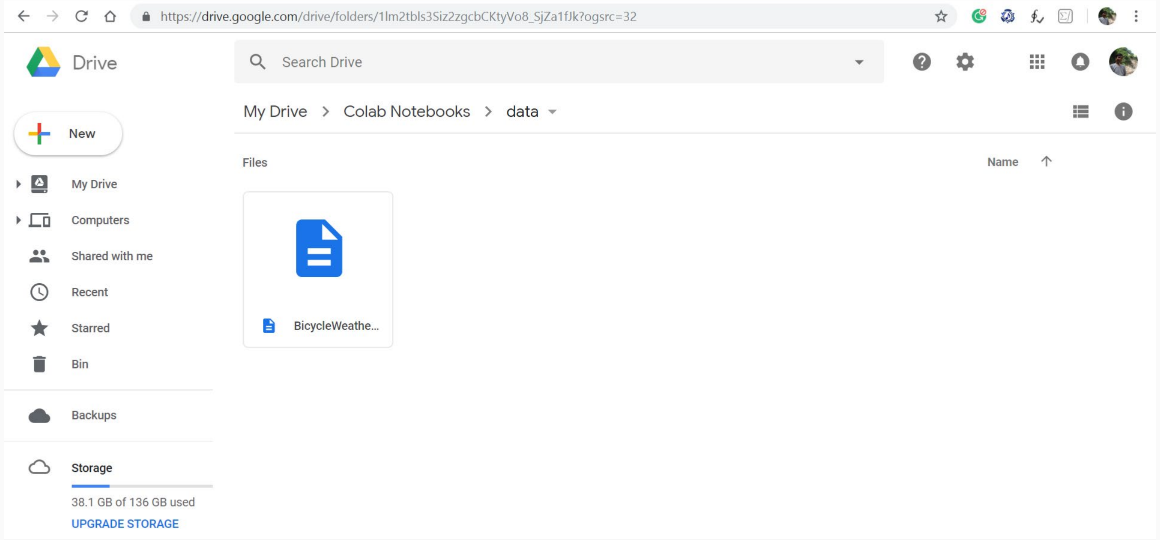
Upload BicycleWeather.csv



https://github.com/jakevdp/PythonD

Python programming

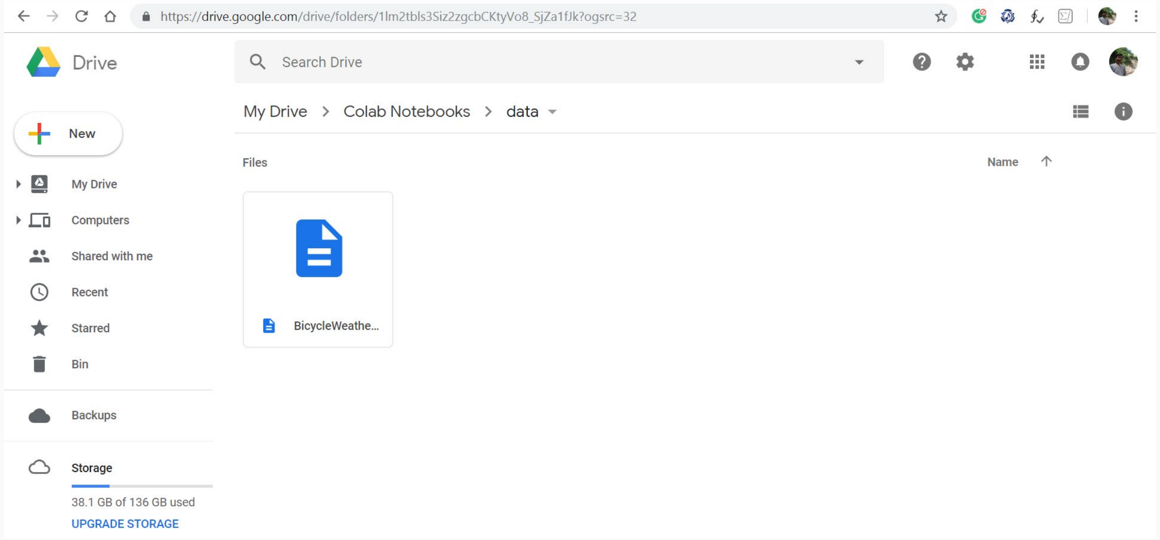
data/BicycleWeather.csv



https://github.com/jakevdp/PythonDataScienceHandbook

Python programming

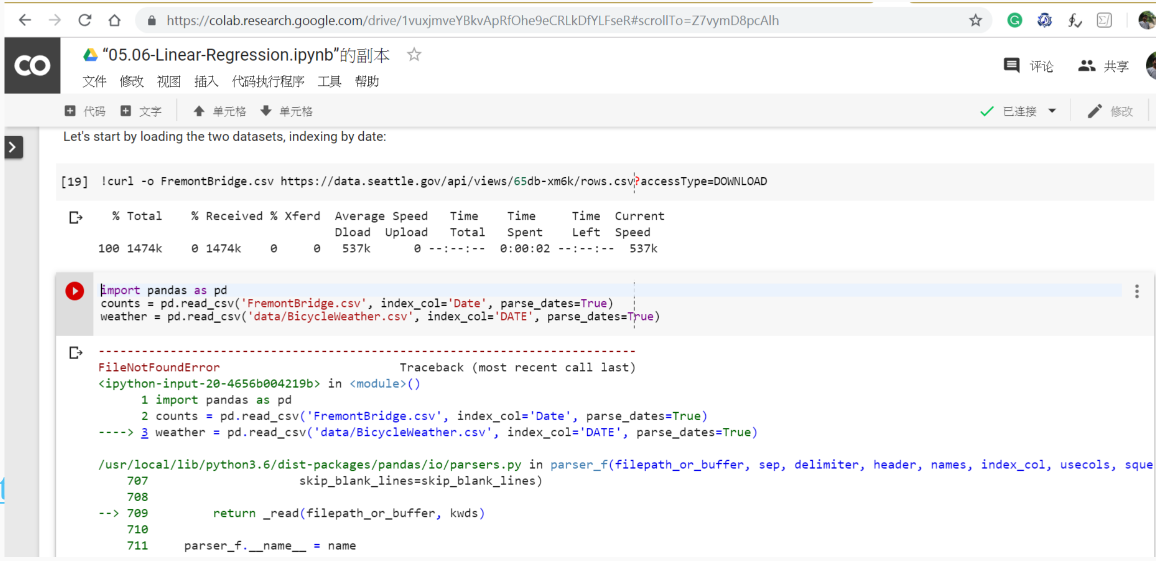
data/BicycleWeather.csv



https://github.com/jakevdp/PythonDataScienceHandbook

Python programming

Run “05.06-Linear-Regression.ipynb”的副本



https://github.com/jakevdp/PythonDat

Python programming

import os
print(os.getcwd())
os.chdir('..')
print(os.getcwd())
os.listdir()
os.chdir('content')
os.listdir()

```
[52] !curl -o FremontBridge.csv https://data.seattle.gov/api/views/65db-xm6k/rows.csv?accessType=DOWNLOAD
```

```
  % Total    % Received % Xferd  Average Speed   Time    Time     Time  Current
                                 Dload  Upload   Total   Spent    Left  Speed
100 1474k    0 1474k    0     0   495k      0 --:--:-- 0:00:02 --:--:--  495k
```

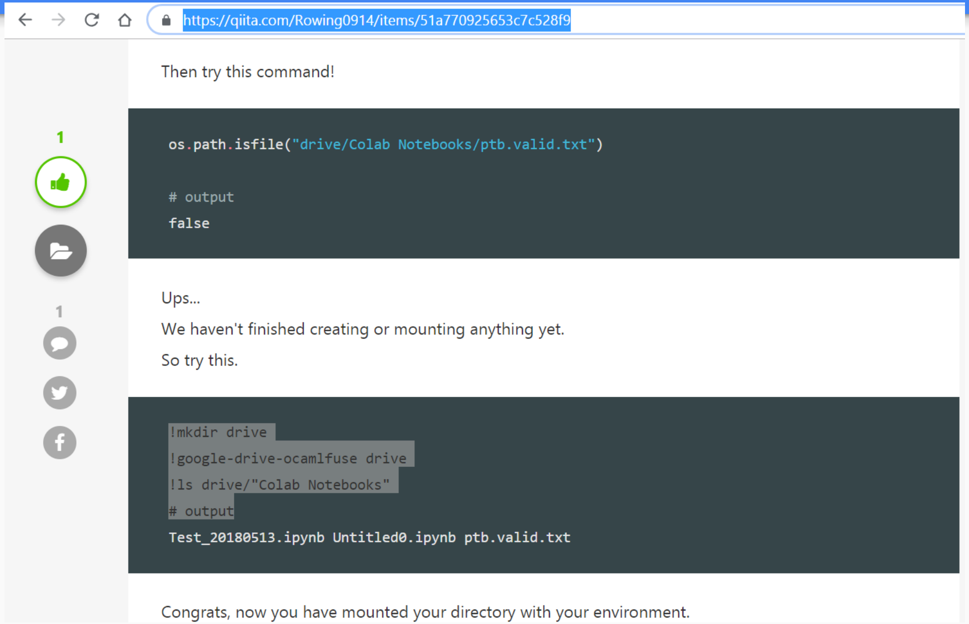
```
# https://www.programiz.com/python-programming/directory
import os
print(os.getcwd())
os.chdir('..')
print(os.getcwd())
os.listdir()
os.chdir('content')
os.listdir()
```

```
/content
/
/bin/bash: google-drive-ocamlfuse: command not found
ls: cannot access 'drive/Colab Notebooks': No such file or directory
```

https://www.programiz.com/python-programming/directory

Python programming

creating or mounting


https://qiita.com/Rowing0914/items/51a770925653c7c528f9

Then try this command!

```
os.path.isfile("drive/Colab Notebooks/ptb.valid.txt")

# output
false
```

Ups...

We haven't finished creating or mounting anything yet.

So try this.

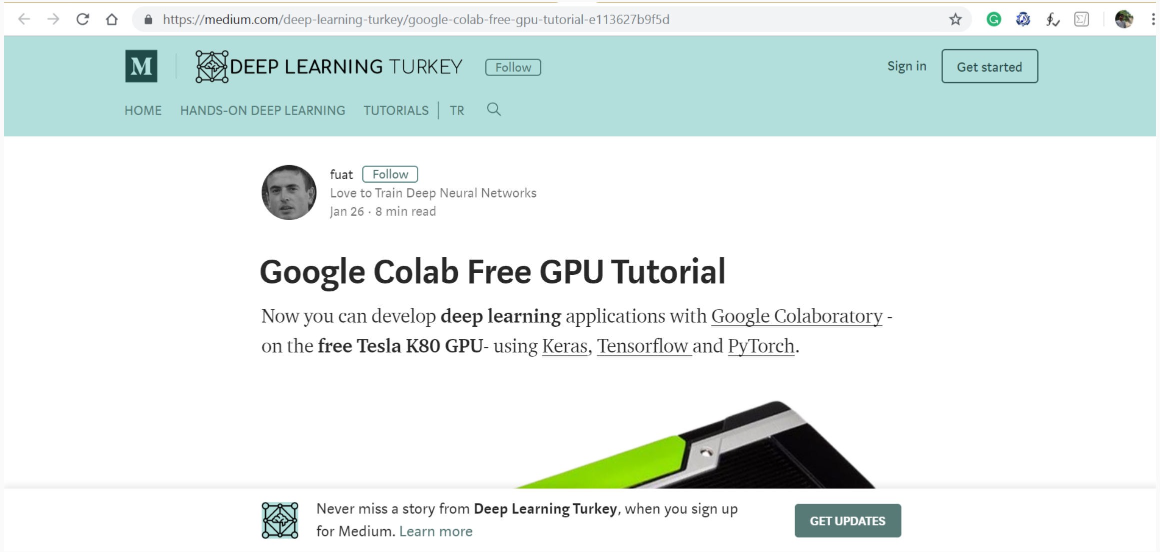
```
!mkdir drive
!google-drive-ocamlfuse drive
!ls drive/"Colab Notebooks"
# output
Test_20180513.ipynb Untitled0.ipynb ptb.valid.txt
```

Congrats, now you have mounted your directory with your environment.

https://qiita.com/Rowing0914/items/51a770925653c7c528f9

Python programming

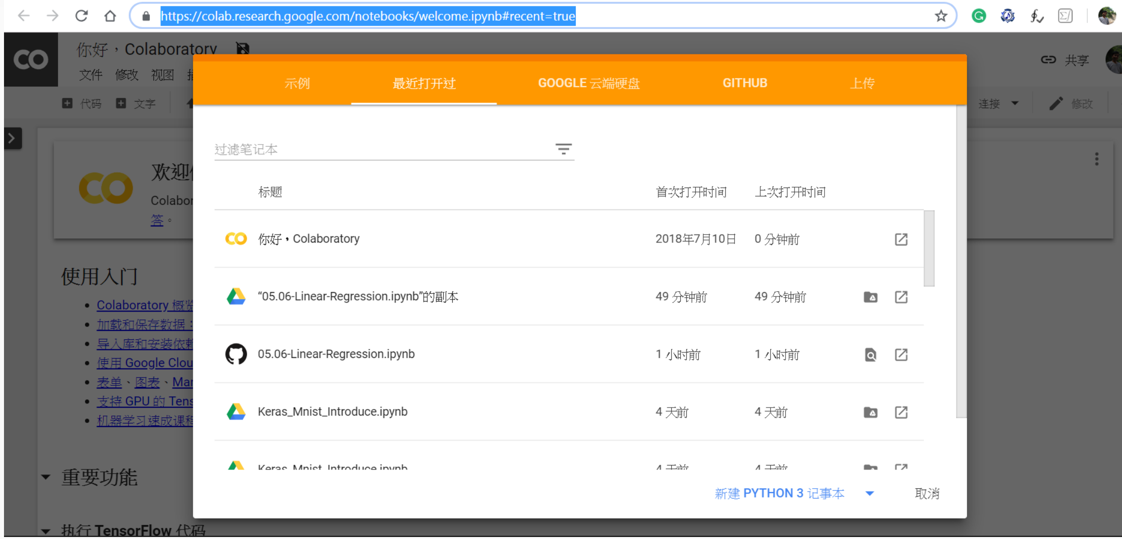
Google Colab Free GPU Tutorial



https://medium.com/deep-learning-turkey/google-colab-free-gpu-tutorial-e113627b9f5d

Python programming

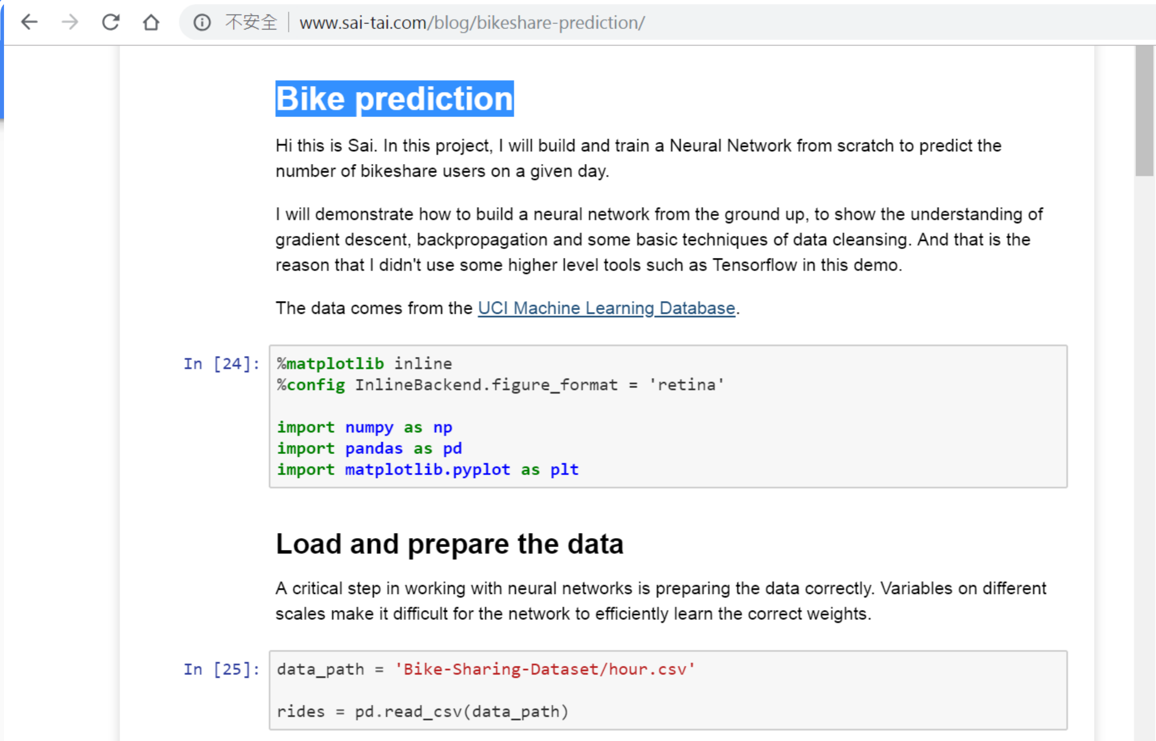
Open colab files



https://colab.research.google.com/notebooks/welcome.ipynb#recent=true

Python programming

Bike prediction


不安全 | www.sai-tai.com/blog/bikeshare-prediction/

Bike prediction

Hi this is Sai. In this project, I will build and train a Neural Network from scratch to predict the number of bikeshare users on a given day.

I will demonstrate how to build a neural network from the ground up, to show the understanding of gradient descent, backpropagation and some basic techniques of data cleansing. And that is the reason that I didn't use some higher level tools such as Tensorflow in this demo.

The data comes from the UCI Machine Learning Database.

In [24]:
```
%matplotlib inline
%config InlineBackend.figure_format = 'retina'

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

Load and prepare the data

A critical step in working with neural networks is preparing the data correctly. Variables on different scales make it difficult for the network to efficiently learn the correct weights.

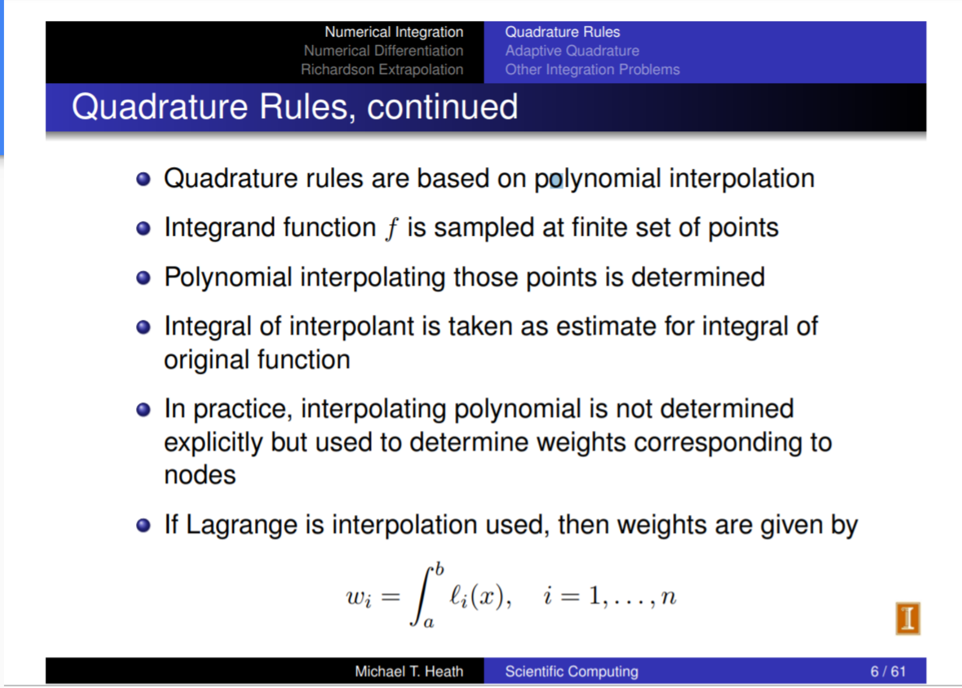
In [25]:
```
data_path = 'Bike-Sharing-Dataset/hour.csv'

rides = pd.read_csv(data_path)
```

http://www.sai-tai.com/blog/bikeshare-prediction/

Python programming

We will see two types of global (interpolatory) rules:
– Newton-Cotes — interpolatory on uniformly spaced nodes.
– Gauss rules — interpolatory on optimally chosen point sets.



https://relate.cs.illinois.edu/course/cs450-f17/file-version/f753d2d1a50ecbf2ca66202aa894e6d67c24efdb/lecture-notes/chap08_lec2.pdf

Python programming

5-point Gaussian quadrature rule

$$
\begin{aligned}
\int_a^b f(x)\,dx &= \int_{-1}^{1} f\left(\frac{b-a}{2}t + \frac{a+b}{2}\right)\frac{b-a}{2}\,dt \\
&\approx \sum_{i=1}^{5} f\left(\frac{b-a}{2}r_{5,i} + \frac{a+b}{2}\right)\frac{b-a}{2}c_{5,i} \\
&\approx \sum_{i=1}^{5} f(x_i)w_i,
\end{aligned}
$$

where

$$x_i = \frac{b-a}{2}r_{5,i} + \frac{a+b}{2}, \quad w_i = \frac{b-a}{2}c_{5,i}, \quad i = 1,\ldots,5.$$

In this example, $a = 0$ and $b = 1$, so the nodes and weights for a 5-point Gaussian quadrature rule for integrating over $[0,1]$ are given by

$$x_i = \frac{1}{2}r_{5,i} + \frac{1}{2}, \quad w_i = \frac{1}{2}c_{5,i}, \quad i = 1,\ldots,5,$$

which yields

i	Nodes x_i	Weights w_i
1	0.95308992295	0.11846344250
2	0.76923465505	0.23931433525
3	0.50000000000	0.28444444444
4	0.23076534495	0.23931433525
5	0.04691007705	0.11846344250

It follows that

$$
\begin{aligned}
\int_0^1 e^{-x^2}\,dx &\approx \sum_{i=1}^{5} e^{-x_i^2}w_i \\
&\approx 0.11846344250e^{-0.95308992295^2} + 0.23931433525e^{-0.76923465505^2} + \\
&\quad 0.28444444444e^{-0.5^2} + 0.23931433525e^{-0.23076534495^2} + \\
&\quad 0.11846344250e^{-0.04691007705^2} \\
&\approx 0.74682412673352.
\end{aligned}
$$

4

http://www.math.usm.edu/lambers/mat460/fall09/lecture31.pdf

Python programming

Gaussian Quadrature Weights and Abscissae

Weights and Abscissae Tables for n=2 to n=64

n = 2

jump to n = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64

i	weight - w_i	abscissa - x_i
1	1.0000000000000000	-0.5773502691896257
2	1.0000000000000000	0.5773502691896257

n = 3

jump to n = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64

i	weight - w_i	abscissa - x_i
1	0.8888888888888888	0.0000000000000000
2	0.5555555555555556	-0.7745966692414834
3	0.5555555555555556	0.7745966692414834

n = 4

jump to n = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64

i	weight - w_i	abscissa - x_i
1	0.6521451548625461	-0.3399810435848563
2	0.6521451548625461	0.3399810435848563
3	0.3478548451374538	-0.8611363115940526
4	0.3478548451374538	0.8611363115940526

https://pomax.github.io/bezierinfo/legendre-gauss.html

Python programming

Quadrature rules based on interpolating functions

https://en.wikipedia.org/wiki/Numerical_integration

Quadrature rules based on interpolating functions [edit]

A large class of quadrature rules can be derived by constructing interpolating functions that are easy to integrate. Typically these interpolating functions are polynomials. In practice, since polynomials of very high degree tend to oscillate wildly, only polynomials of low degree are used, typically linear and quadratic.

The simplest method of this type is to let the interpolating function be a constant function (a polynomial of degree zero) that passes through the point $\left(\frac{a+b}{2}, f\left(\frac{a+b}{2}\right)\right)$. This is called the *midpoint rule* or *rectangle rule*

$$\int_a^b f(x)\,dx \approx (b-a)\,f\left(\frac{a+b}{2}\right).$$

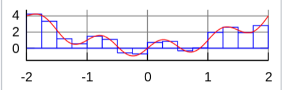


Illustration of the rectangle rule.

The interpolating function may be a straight line (an affine function, i.e. a polynomial of degree 1) passing through the points $(a, f(a))$ and $(b, f(b))$. This is called the *trapezoidal rule*

$$\int_a^b f(x)\,dx \approx (b-a)\left(\frac{f(a)+f(b)}{2}\right).$$

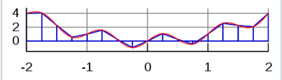


Illustration of the trapezoidal rule.

For either one of these rules, we can make a more accurate approximation by breaking up the interval $[a, b]$ into some number n of subintervals, computing an approximation for each subinterval, then adding up all the results. This is called a *composite rule*, *extended rule*, or *iterated rule*. For example, the composite trapezoidal rule can be stated as

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n}\left(\frac{f(a)}{2} + \sum_{k=1}^{n-1}\left(f\left(a+k\frac{b-a}{n}\right)\right) + \frac{f(b)}{2}\right),$$

where the subintervals have the form $[a+kh, a+(k+1)h] \subset [a, b]$, with $h = \frac{b-a}{n}$ and $k = 0, \ldots, n-1$. Here we used subintervals of the same length h but one could also use intervals of varying length $(h_k)_k$.

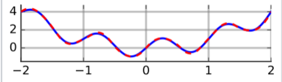


Illustration of Simpson's rule.

https://en.wikipedia.org/wiki/Numerical_integration