Robotics

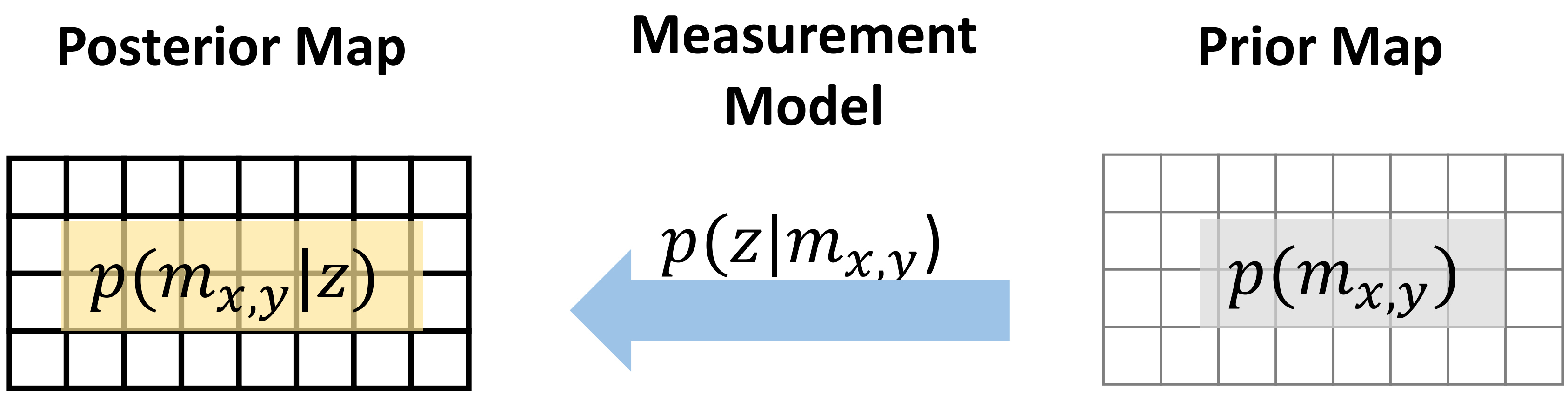
Estimation and Learning
with Dan Lee

Week 3.
Robotic Mapping

3.2 Occupancy Grid Mapping
3.2.2 Log-odd Update

Penn Engineering

Occupancy Grid Mapping



Bayes' Rule:

$$p(m_{x,y}|z) = \frac{p(z|m_{x,y})\,p(m_{x,y})}{p(z)}$$

Occupancy Grid Mapping

$$\boxed{Odd := \frac{(X\ happens)}{(X\ not\ happens)} = \frac{p(X)}{p(X^c)}}$$

- More convenient when we use “Odd”

$$Odd((m_{x,y} = 1)\ given\ z) = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)}$$

Occupancy Grid Mapping

- Odd

(Bayes' Rule)

$$p(m_{x,y} = 1|z) = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)}{p(z)}$$

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/p(z)}{p(m_{x,y} = 0|z)}$$

Occupancy Grid Mapping

- Odd

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/\cancel{p(z)}}{p(z|m_{x,y} = 0)p(m_{x,y} = 0)/\cancel{p(z)}}$$

$$p(m_{x,y} = 0|z) = \frac{p(z|m_{x,y} = 0)p(m_{x,y} = 0)}{p(z)}$$

(Bayes' Rule)

Occupancy Grid Mapping

- Take the log!

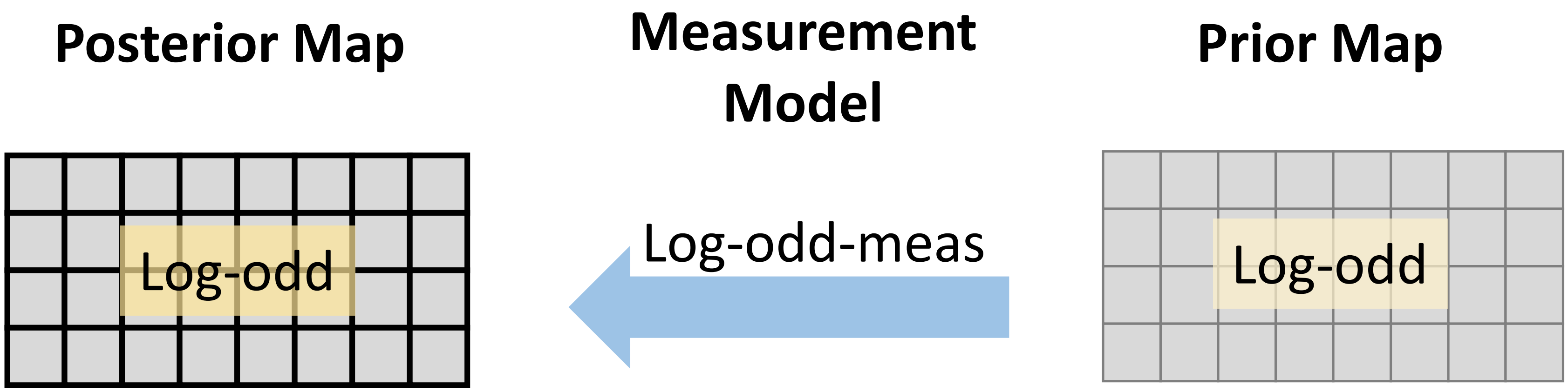
Odd:
$$\frac{p(m_{x,y}=1|z)}{p(m_{x,y}=0|z)}=\frac{p(z|m_{x,y}=1)p(m_{x,y}=1)}{p(z|m_{x,y}=0)p(m_{x,y}=0)}$$

Log-Odd:
$$\log\frac{p(m_{x,y}=1|z)}{p(m_{x,y}=0|z)}=\log\frac{p(z|m_{x,y}=1)p(m_{x,y}=1)}{p(z|m_{x,y}=0)p(m_{x,y}=0)}$$
$$=\log\frac{p\left(z|m_{x,y}=1\right)}{p\left(z|m_{x,y}=0\right)}+\log\frac{p\left(m_{x,y}=1\right)}{p\left(m_{x,y}=0\right)}$$

$$\boxed{\log odd^{+}=\log odd\ meas+\log odd^{-}}$$

Occupancy Grid Mapping

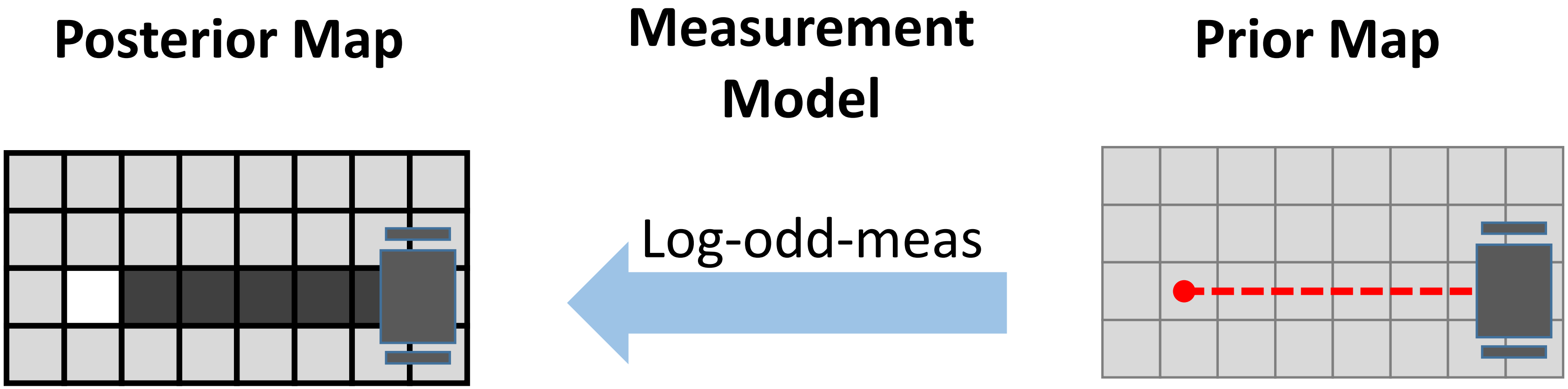
- Log-odd update



$$\log odd^{+} = \log odd\ meas + \log odd^{-}$$

Occupancy Grid Mapping

- Log-odd update



$$\log odd^{+} = \log odd\ meas + \log odd^{-}$$

Occupancy Grid Mapping

- Measurement model in log-odd form

$$\log \frac{p(z|m_{x,y}=1)}{p(z|m_{x,y}=0)}$$

- **Two possible measurement:**

Case I : cells with z=1

$$\log odd\_occ := \log \frac{p(z=1|m_{x,y}=1)}{p(z=1|m_{x,y}=0)}$$

Case II : cells with z=0

$$\log odd\_free := \log \frac{p(z=0|m_{x,y}=0)}{p(z=0|m_{x,y}=1)}$$

(Trivial Case : cells not measured)

Occupancy Grid Mapping

- Example

<u>Constant Measurement Model</u>

$\log odd\_occ := 0.9$

$\log odd\_free := 0.7$

<u>Initial Map:</u>

$\log odd = 0$ for all (x,y)

$p(m_{x,y} = 1) = p(m_{x,y} = 0) = 0.5$

Update Rule:

$\boxed{\log\ odd \mathrel{+}= \log odd\_meas}$

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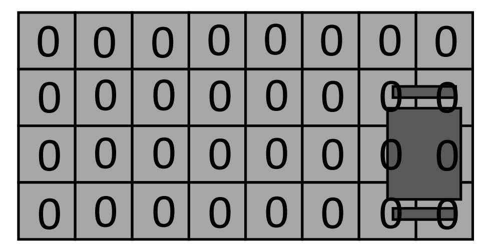


Occupancy Grid Mapping

- Example

Constant Measurement Model

$\log odd\_occ := 0.9$

$\log odd\_free := 0.7$

Update

- Case I : cells with z=1

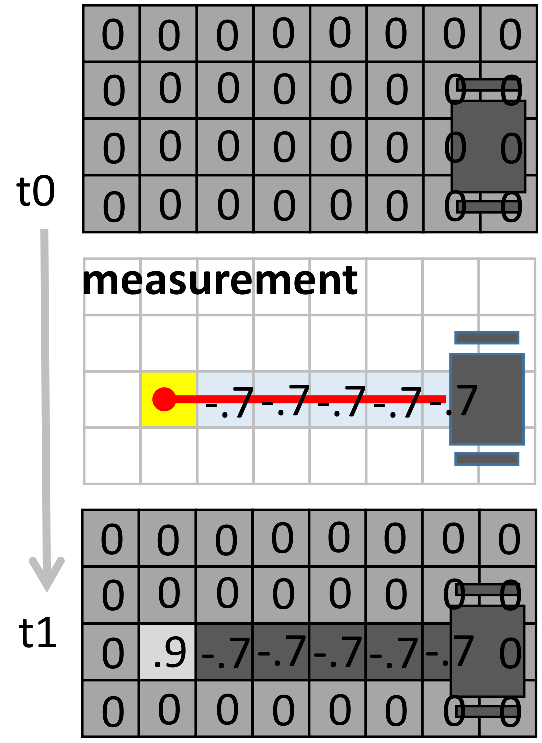
$\log odd \leftarrow 0 + \log odd\_occ$

- Case II : cells with z=0

$\log odd \leftarrow 0 - \log odd\_free$

Update Rule:

$\log\ odd \mathrel{+}= \log odd\_meas$



Occupancy Grid Mapping

- Example

<u>Constant Measurement Model</u>

$$\log odd\_occ := 0.9$$

$$\log odd\_free := 0.7$$

<u>Update</u>

- Case I : cells with z=1

$$\log odd \leftarrow 0 + \log odd\_occ$$

- Case II : cells with z=0

$$\log odd \leftarrow 0 - \log odd\_free$$

Update Rule:

$$\log odd \mathrel{+}= \log odd\_meas$$