Triangle Scenario Manuscript Title

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This document is my current working draft of a paper to do with causal inference, inflation, incompatibility inequalities, hypergraph transversals and quantum correlations.

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I. INTRODUCTION

II. DEFINITIONS & NOTATION

III. CASUAL NETWORK INFLATION

IV. CASUAL NETWORK COMPATIBILITY

V. LOGICAL TAUTOLOGIES

A. Definitions

Blah blah blah [1]

B. Tautologies of The Marginal Problem

VI. HARDY TRANSVERSALS

VII. DERIVING SYMMETRIC INEQUALITIES

VIII. RESULTS

IX. CONCLUSIONS

Appendix A: Computationally Efficient Parametrization of the Unitary Group

Spengler, Huber and Hiesmayr [2] suggest the parameterization of the unitary group $\mathcal{U}(d)$ using a $d \times d$-matrix of real-valued parameters $\lambda_{n,m}$

$$U = \left[\prod_{m=1}^{d-1}\left(\prod_{n=m+1}^{d}\exp\left(iP_n\lambda_{n,m}\right)\exp\left(i\sigma_{m,n}\lambda_{m,n}\right)\right)\right]\cdot\left[\prod_{l=1}^{d}\exp\left(iP_l\lambda_{l,l}\right)\right] \tag{A1}$$

Where P_l are one-dimensional projective operators,

$$P_l = |l\rangle\langle l| \tag{A2}$$

and the $\sigma_{m,n}$ are generalized anti-symmetric σ-matrices,

$$\sigma_{m,n} = -i\,|m\rangle\langle n| + i\,|n\rangle\langle m|$$

Where $1 \leq m < n \leq d$.
For the sake of reference, let us label the matrix exponential terms in (A1) in a manner that corresponds to their affect on a orthonormal basis $\{|1\rangle, \ldots, |d\rangle\}$.

$$\begin{aligned} GP_l &= \exp\left(iP_l\lambda_{l,l}\right) \\ RP_{n,m} &= \exp\left(iP_n\lambda_{n,m}\right) \\ R_{n,m} &= \exp\left(i\sigma_{m,n}\lambda_{m,n}\right) \end{aligned} \tag{A3}$$

It is possible to remove the reliance on matrix exponential operations in (A1) by utilizing the explicit form of the exponential terms in (A3). As a first step, recognize the defining property of the projective operators (A2),

$$P_l^k = \left(|l\rangle\langle l|\right)^k = |l\rangle\langle l| = P_l$$

This greatly simplifies the global phase terms GP_l,

$$GP_l = \exp\left(iP_l\lambda_{l,l}\right) = \sum_{k=0}^{\infty} \frac{\left(iP_l\lambda_{l,l}\right)^k}{k!} = \mathbb{I} + \sum_{k=1}^{\infty} \frac{\left(i\lambda_{l,l}\right)^k}{k!} P_l^k = \mathbb{I} + P_l \left[\sum_{k=1}^{\infty} \frac{\left(i\lambda_{l,l}\right)^k}{k!}\right] = \mathbb{I} + P_l \left(e^{i\lambda_{l,l}} - 1\right)$$

Analogously for the relative phase terms $RP_{n,m}$,

$$RP_{n,m} = \cdots = \mathbb{I} + P_n \left(e^{i\lambda_{n,m}} - 1\right)$$

Finally, the rotation terms $R_{n,m}$ can also be simplified by examining powers of $i\sigma_{n,m}$,

$$R_{n,m} = \exp\left(i\sigma_{m,n}\lambda_{m,n}\right) = \sum_{k=0}^{\infty} \frac{\left(|m\rangle\langle n| - |n\rangle\langle m|\right)^k \lambda_{m,n}^k}{k!}$$

One can verify that the following properties hold,

$$\begin{aligned} \left(|m\rangle\langle n| - |n\rangle\langle m|\right)^0 &= \mathbb{I} \\ \forall k \in \mathbb{N}, k \neq 0 : \left(|m\rangle\langle n| - |n\rangle\langle m|\right)^{2k} &= (-1)^k \left(|m\rangle\langle m| + |n\rangle\langle n|\right) \\ \forall k \in \mathbb{N} : \left(|m\rangle\langle n| - |n\rangle\langle m|\right)^{2k+1} &= (-1)^k \left(|m\rangle\langle n| - |n\rangle\langle m|\right) \end{aligned}$$

Revealing the simplified form of $R_{n,m}$,

$$R_{n,m} = \mathbb{I} + \left(|m\rangle\langle m| + |n\rangle\langle n|\right) \sum_{j=1}^{\infty} (-1)^j \frac{\lambda_{n,m}^{2j}}{(2j)!} + \left(|m\rangle\langle n| - |n\rangle\langle m|\right) \sum_{j=0}^{\infty} (-1)^j \frac{\lambda_{n,m}^{2j+1}}{(2j+1)!}$$

$$R_{n,m} = \mathbb{I} + \left(|m\rangle\langle m| + |n\rangle\langle n|\right)\left(\cos\lambda_{n,m} - 1\right) + \left(|m\rangle\langle n| - |n\rangle\langle m|\right)\sin\lambda_{n,m}$$

TODO: Explanation of Computational Complexity $\mathcal{O}\left(d^3\right)$ vs. $\mathcal{O}\left(1\right)$ using [3]
TODO: Pre-Caching for Fixed Dimension d

Appendix B: Parametrization of Quantum States & Measurements

[1] Shane Mansfield and Tobias Fritz, "Hardy's non-locality paradox and possibilistic conditions for non-locality," Found. Phys. **42**, 709–719 (2012).

[2] Christoph Spengler, Marcus Huber, and Beatrix C. Hiesmayr, "A composite parameterization of unitary groups, density matrices and subspaces," (2010), 10.1088/1751-8113/43/38/385306, arXiv:1004.5252.

[3] Cleve Moler and Charles Van Loan, "Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later," SIAM Rev. **45**, 3–49 (2003).