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# SOME RECENT DEVELOPMENTS IN A CONCEPT OF CAUSALITY

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The paper considers three separate but related topics. (i) What is the relationship between causation and co-integration? If a pair of I(1) series are co-integration, there must be causation in at least one direction. An implication is that some tests of causation based on different series may have missed one source of causation. (ii) Is there a need for a definition of 'instantaneous causation' in a decision science? It is argued that no such definition is required. (iii) Can causality tests be used for policy evaluation? It is suggested that these tests are useful, but that they should be evaluated with care.

## 1. Introduction

Suppose that one is interested in the question of whether or not a vector of economic time series $y_t$ 'causes' another vector $x_t$. There will also exist a further vector of variables $w_t$ which provide a context within which the causality question is being asked. Two information sets are of interest:

$$J_t: \quad x_{t-j}, y_{t-j}, w_{t-j}, \qquad j \geq 0,$$

and

$$J_t': \quad x_{t-j}, w_{t-j}, \qquad j \geq 0,$$

so that $J_t$ uses all of the available information but $J_t'$ excludes the information in past and present $y_t$. It is important to assume that components of $y_t$ are not perfect functions of the other components of $J_t$, so that there does not exist a function $g(\ )$ such that $y_t = g(w_{t-j}, j \geq 0)$, for example. Let $f(x|J)$ be the conditional distribution of $x$ given $J$ and $\mathrm{E}[x|J]$ be the corresponding conditional mean, then the following definitions of causality and non-causality will be used in the following discussion:

(i) $y_t$ does not cause $x_{t+1}$ with respect to $J_t$ if

$$f(x_{t+1}|J_t) = f(x_{t+1}|J_t').$$

(ii) If

$$f(x_{t+1}|J_t) \neq f(x_{t+1}|J_t'),$$

then $y_t$ is a 'prima facie' cause of $x_{t+1}$ with respect to $J_t$.

(iii) If

$$\mathrm{E}[x_{t+1}|J_t] = \mathrm{E}[x_{t+1}|J_t'],$$

then $y_t$ does not cause $x_{t+1}$ *in mean*, with respect to $J_t$.

(iv) If

$$\mathrm{E}[x_{t+1}|J_t] \neq \mathrm{E}[x_{t+1}|J_t'],$$

then $y_t$ is a prima facie cause in mean of $x_{t+1}$ with respect to $J_t$.

The 'in mean' definitions were introduced in Granger (1963), based on a suggestion by Wiener (1956), and the general definition was discussed in Granger (1980) and elsewhere.

The definitions are based on two fundamental principles:

(a) The cause occurs before the effect.

(b) The causal series contains special information about the series being caused that is not available in the other available series, here $w_t$.

It follows immediately that there are forecasting implications of the definitions. The 'in mean' definition implies that, if $y_t$ causes $x_t$, then $x_{t+1}$ is better forecast if the information in $y_{t-j}$ is used than if it is not used, where 'better' means a smaller variance of forecast error, or the matrix equivalence of variance. The general definition (ii) implies that if one is trying to forecast any function $g(x_{t+1})$ of $x_{t+1}$, using any cost function, then one will be frequently better off using the information in $y_{t-j}$, $j \geq 0$, and never worse off. This has recently been proved formally by Granger and Thomson (1987) and indicates the considerably greater depth of the more general definition (ii) compared to (iv).

If $J_t$ contained all the information available in the universe at time $t$, then $y_t$ could be said to cause $x_{t+1}$. In practice $J_t$ will contain considerably less information and so the phase 'prima facie' has to be used. (ii) is a weaker definition than (i), but it is a definition of a type of causality which is given a specific name. The name is chosen to include the unstated assumption that possible causation is not considered for any arbitrarily selected group of variables, but only for variables for which the researcher has some prior belief that causation is, in some sense, likely. Because the question of possible causality is being asked, $y_t$ would have been considered a candidate for a cause before the definition was applied. Thus, one may start with a 'degree of belief' that $y_t$ causes $x_{t+1}$, measured as a probability, and after using a causality test based on these definitions, one's 'degree of belief' may change. For example, before the test the degree of belief could be 0.3 and after the test this could increase to 0.6. The extent to which the belief probability changes will depend on the perceived quality and quantity of the data, the size and relevance of $w_t$ and the perceived relevance, quality or power of the test. Naturally, using just statistical techniques, it is unlikely that the probability will go to one, or to zero, and if one does not like the definitions being used, then the tests are irrelevant and the degree of belief cannot change.

For a given $y_t$ and $J_t$, the definition in (ii) is a general one and not specific to a particular investigator. However, interpretations of tests based on the definition do depend on the degrees of belief of the investigator and so are specific. Further, going back a step, as the choice of variables to be considered in a causality analysis is in the hands of the investigator, the definition can also be thought of as being specific in this respect.

It would be interesting to try to give a more formal Bayesian viewpoint to these ideas, incorporating the dynamics of prior beliefs as new information becomes available, but I do not feel competent to undertake such an analysis.

There are many tests for causation that have been suggested and some are discussed in Geweke (1984) and will not be considered here. Many empirical papers in economics and in other fields have used definitions (iii) and (iv) although usually just with a pair of univariate series $x_t$, so that $w_t$ is empty. A few papers have considered wider information sets. The definitions have proved useful in various theoretical contexts, including rational expectations [Lucas and Sargent (1981)], exogeneity [Engle, Hendry and Richard (1983)] and econometric modelling strategy [Hendry and Richard (1983)]. There has also been some interest by philosophers in the definitions [Spohn (1984)]. Criticisms of the definitions have ranged from the inconsequential (the word causation cannot be used) to the more substantial. As examples of the latter, Zellner (1979) believes that causation cannot be securely established except in the context of a confirmed subject matter theory, and Holland (1986) believes that tests of causation can only be carried out within the context of controlled experiments. As I have attempted to answer various criticisms elsewhere [Granger (1980, 1986a)], I will not discuss them further here.

The present paper considers three separate but related questions:

(i) What is the effect of the relationship between the concepts of co-integration and causality on tests of causality?
(ii) Is there need for a definition of instantaneous causality in a decision science such as economics?
(iii) Can causality tests potentially be used for policy evaluation?

It should be noted that in the definitions only discrete time series are considered and that a time lag of one is involved. The size of this unit is not defined. It is merely assumed that a relevant, positive unit does exist for the definitions to hold. There is, of course, no reason for the available data to be measured on the same unit of time that the definitions would require for a proper test of causation. The data may be available monthly and the actual causal lag be only a couple of days, for example. The relevance of this difference in units is also discussed briefly in the consideration of the second of the above questions.

The paper is mostly concerned with bringing various results together for econometricians to see. Section 3 presents some new ideas on instantaneous causality and its interpretation, and section 4 is largely a summary of an unpublished paper.

## 2. Co-integration and causation

Define an I(0) series as one that has a spectrum that is bounded above and also is positive at all frequencies. If the first and second moments of the series are time-invariant, then $x_t$ will be second-order stationary and it can be assumed that the autocorrelations $\rho_k$ decline exponentially (in magnitude) for $k$ large. In practical terms, $x_t$ may be thought to have a generating mechanism that can be well approximated by stationary, invertible ARMA $(p, q)$ model, with finite $p$ and $q$. A series will be said to be integrated of order one, denoted I(1), if its changes are I(0). I(1) series are sometimes called 'non-stationary' because their variance increase linearly with time, provided they started a finite number of time units earlier. There is plenty of empirical evidence that macro-economic series often appear to be I(1). The causality definitions make no assumptions about whether the series being considered are I(0) or I(1), but if they are I(1), some care has to be taken with their empirical analysis.

Suppose that $x_t$, $y_t$ are both I(1), without trends in mean, so that their changes are both I(0) and with zero means. Then it will be typically true that any linear combination of $x_t$, $y_t$ will also be I(1). However, it is possible that there will exist a constant such that $z_t = x_t - Ay_t$ is I(0). This would happen for instance, if

$$x_t = Aq_t + z_{1t}, \tag{1a}$$

and

$$y_t = q_t + y_{1t}, \tag{1b}$$

where $q_t \sim \mathrm{I}(1)$ and $x_{1t}$, $y_{1t}$ are both I(0). When this occurs $x_t$, $y_t$ are said to be 'co-integrated'. Clearly, not all pairs of I(1) series have this property. It was shown in Granger (1983) that, if $x_t$, $y_t$ are both I(1) but are co-integrated, then they will be generated by an 'error-correction' model taking the form

$$\Delta x_t = \gamma_1 z_{t-1} + \text{lagged}\, \Delta x_t,\, \Delta y_t + \varepsilon_{1t},$$

and

$$\Delta y_t = \gamma_2 z_{t-1} + \text{lagged}\, \Delta x_t,\, \Delta y_t + \varepsilon_{2t},$$

where one of $\gamma_1, \gamma_2 \neq 0$ and $\varepsilon_{1t}$, $\varepsilon_{2t}$ are finite-order moving averages. Thus, changes in the variables $x_t$, $y_t$ are partly driven by the previous value of $z_t$. It can be shown that the line $x - Ay = 0$ can be considered to be an 'equilibrium' or 'attractor' for the system in the phase-space, where $x_t$ is plotted against $y_t$, so that $z_t$ can be interpreted as the extent to which the system is out of equilibrium. Further interpretations, methods of testing for and examples of co-integration can be found in the special issue of the *Oxford Bulletin of Economics and Statistics*, August 1986, which includes a survey article by Granger (1986b). A consequence of the error-correction model is that either $\Delta x_t$ or $\Delta y_t$ (or both) must be caused by $z_{t-1}$ which is itself a function of $x_{t-1}$, $y_{t-1}$. Thus, either $x_{t+1}$ is caused in means by $y_t$ or $y_{t+1}$ by $x_t$ if the two series are co-integrated. This is a somewhat surprising result, when taken at face value, as co-integration is concerned with the long run and equilibrium, whereas the causality in mean is concerned with short-run forecastability. However, what it essentially says is that for a pair of series to have an attainable equilibrium, there must be some causation between them to provide the necessary dynamics. The various concepts can be easily generalized to vectors of economic series, as in Granger (1986b). It is also possible to generalize the results to non-linear equilibria, as discussed in Granger (1986c). It should be noted that in the error-correction model, there are two possible sources of causation of $x_t$ by $y_{t-j}$, either through the $z_{t-1}$ term, if $\gamma_1 \neq 0$, or though the lagged $\Delta y_t$ terms, if they are present in the equation. To see what form the causation through $z_{t-1}$ takes, consider again the factor model (1) where, for simplicity, $q_t$ is taken to be a random walk, so that $\Delta q_t = a_t$, zero mean white noise, and $x_{1t}$, $y_{1t}$ are white noises. Then,

$$z_t = x_t - Ay_t = x_{1t} - Ay_{1t}.$$

Now consider $\Delta x_t$, which is given by

$$\Delta x_t = Aa_t + \Delta x_{1t} = -x_{1,t-1} + Aa_t + x_{1t}.$$

The first term is the forecastable part of $\Delta x_t$ and the final two terms will constitute the one-step forecast error of a forecast made at time $t-1$ based on $x_{t-j}$, $y_{t-j}$, $j \geq 1$. However, the forecast, $-x_{1,t-1}$, is not directly observable but is, generally, correlated to $z_{t-1}$, which results in the causation in mean. This causation will not occur only if $z_t$ has zero variance.

If $z_t$ is not used in the modelling, then $x_{1,t-1}$ will be related only to the sum of many lagged $\Delta x_t$, but this sum will also include the sum of $a_{t-j}$, which will give a 'noise' term of possibly large variance. Thus, $x_{1,t-1}$ will be little correlated to the sum of lagged $\Delta x_t$. If classical, multivariate time-series modelling techniques are used, as discussed in Box and Jenkins (1970) and in the first edition of Granger and Newbold (1977), then once it is realized that $x_t$, $y_t$ are I(1), then their changes will be modelled using a bivariate ARMA($p,q$) model, with finite $p,q$. Without $z_t$ being explicitly used, the model will be mis-specified and the possible value of lagged $y_t$ in forecasting $x_t$ will be missed. Thus, many of the papers discussing causality tests based on the traditional time-series modelling techniques could have missed some of the forecastability and hence reached incorrect conclusions about non-causality in mean. On some occasions, causation could be present but would not be detected by the testing procedures used. This problem only arises when the series are I(1) and co-integrated, but this could be a common situation when causality questions are asked. It does seem that many of the causality tests that have been conducted should be re-considered. It would also be interesting to try to relate the causal impact of the $z_{t-1}$ terms to the frequency-domain causation decompositions considered by Granger (1969) and by Geweke (1982). It is tempting to think that the main impact will be at very low frequencies, but this is not clear.

## 3. Instantaneous causality

One of the earliest, and most telling, criticisms of causality tests based on statistical techniques is that correlation cannot be equated to causality. A major difficulty with looking at a correlation is that it gives no indication about the direction of relationship. If $X, Y$ are correlated random variables, then $Y$ can be used to explain $X$ but $X$ can also be used to explain $Y$. For a definition of causation to be useful for statistical testing, it must contain an assumption on structure that allows such dual relationships to be disentangled.

In the definitions given in the first section this assumption is that the cause occurs before the effect and so the 'arrow of time' can be used to help distinguish between cause and effect. Other definitions use alternative methods of performing this distinction. Holland (1986), for instance, considers only situations in which the cause is the input to an experiment and the effect is found from the results of the experiment. Strict application of a time gap requirement means that the definitions given above can make no statements about instantaneous causality. Suppose that $x_t$, $y_t$ are a pair of series and let

$$e_{xt} = x_t - \mathrm{E}[x_t | J_{t-1}], \qquad e_{yt} = y_t - \mathrm{E}[y_t | J_{t-1}],$$

where $J_{t-1}$: $x_{t-j}$, $y_{t-j}$, $j \geq 1$, and suppose that $\rho = \mathrm{corr}(e_{xt}, e_{yt}) \neq 0$, then one may suppose that there is an apparent instantaneous causality in these series. At the very least, the question of whether or not a causal explanation can be given to this finding deserves consideration. Pierce and Haugh (1977) discuss whether $y_t$ causes $x_t$ instantaneously by using the information sets $J_t'$: $J_t$, $y_t$ and $J_t''$: $J_t$, $x_t$. If $x_t$ is better 'forecast' using $J_t'$ rather than $J_t$, one could say that $y_t$ instantaneously causes $x_t$ (in mean), a necessary condition being just $\rho \neq 0$. However, this same condition is necessary for the statement that $x_t$ instantaneously causes $y_t$. One is back to the symmetry problem and this definition of instantaneous causality is therefore unsatisfactory, as no direction of relationship can be deduced just from the data. It is possible, on occasions, to add further information and to reach a conclusion. If, for example, one 'knows' that $x_t$ cannot cause $y_t$ (instantaneously or otherwise), then the symmetry is broken. This extra 'knowledge' can come from some economic theory or a belief in exogeneity (the economy cannot cause weather) but the conclusion, or the change in the degree of belief about causation, will depend on the correctness of the extra knowledge.

Three possible explanations for the apparent instantaneous causality will be discussed:

(i) There is true instantaneous causality in an economic system so that some elements in the system react *without any measurable time delay* to changes in some other elements.
(ii) There is no true instantaneous causality, but the finite time delay between cause and effect is small compared to the time interval over which data is collected. Thus, the apparent causation is due to temporal aggregation.
(iii) There is a jointly causal variable $w_{t-1}$, that causes both $x_t$ and $y_t$ but is not included in the information set, possibly because it is not observed.

It can be argued that true instantaneous causality will never occur in economics, or any other decision science, and that the missing variable explanation is always a possible one, so that a definition of instantaneous causality is never actually needed. In this discussion, it is assumed that the cause can never occur after the effect, so that the causal lag is either zero, giving instantaneous causality, or positive, giving the causal definition used throughout this paper. It follows that one cannot have instantaneously causality between a pair of flow variables, such as imports and exports, or a pair of production series, as these variables are available only for discrete time, and part of one variable must almost inevitably occur after part of the other. Similarly, a stock variable, such as a price measured at time $t$ cannot instantaneously cause a flow variable, most of which occurs before $t$. This will be true however short the time interval used, provided it is finite and positive. Thus, instantaneous causality can only strictly be discussed for pairs or groups of stock variables. If one also believes that economic variables and the outcomes of large numbers of decisions made by economic agents or institutions, that each agent can only concentrate on a single decision at a time, so that their brains are single-track decision makers, and that there is always a delay in making a decision, as new information is assimilated, analyzed and a decision rule applied, and that there is then a further delay until the decision is implemented and becomes observable, then the presence of true instantaneous causality in economics becomes very unlikely. The true causal lag may be very small but never actually zero. The observed or apparent instantaneous causality can then be explained by either temporal aggregation or missing causal variables. Temporal aggregation is a realistic, plausible and well-known reason for observing apparent instantaneous causation and so needs no further discussion.

It is common practice in statistics in general, and in econometrics in particular, to discuss a pair of random variables, say $X$ and $Y$, that have a joint distribution function. The residuals $e_{xt}$, $e_{yt}$ introduced above provide an example. However, there is virtually no discussion about the mechanism that produces this joint distribution. How are the values of the variables $X$ and $Y$, observed at time $t$, say, actually generated such that they are also characterised by having a joint distribution? Clearly, these values have to be generated simultaneously. For example, if $X_t$, $Y_t$ are respectively stock market closing price indices from the Pacific Stock Exchange and the Sydney Stock Exchange, and suppose both exchanges close at the identical time, then, if $x_t$, $y_t$ have a joint distribution, a mechanism has to be described that can lead to this simultaneous generation of a pair of price indices at sites separated by several thousand miles. Of course, the physical locational difference is irrelevant as the 'electronic distance' is negligible, provided the members of one exchange pay very close or constant attention to what is happening at the other exchange. If the two variables are statistically independent, so that their joint distribution is the product of the two marginal distributions, the joint generation is easily understood as all that is needed is two generation mechanisms operating independently of each other. However, the concept of independence is not one that is always well understood, as it depends on the set of variables within which independence is being discussed. For example, if $X, Y, Z$ are three variables, then $X$ and $Y$ can be independent if only this pair is considered but $X|Z$ and $X|Z$ need not be independent, where $X|Z$ is the conditional variable $X$ given $Z$. This is easily seen by taking $X, Y, Z$ to be jointly Gaussian with a covariance matrix having $\operatorname{cov}(X, Y) = 0$ but other covariances non-zero.

A theorem can be proved that is the reverse of this example, so that if $X, Y$ are not independent, there always could exist another variable $Z$ such that $X|Z$, $Y|Z$ are independent. Thus, the apparent joint distribution between $X$ and $Y$ occurs because there are really three variables, $Z$ is affecting each of $X$ and $Y$ which are independent within the group of three variables, but as $Z$ is unobserved, and thus is marginalized out, the observed joint distribution occurs. Formally, the theorem takes the form: for any bivariate probability definity function (p.d.f.) $f(x, y)$, there exists a trivariate p.d.f. $\phi(x, y, z)$ such that

(i) $$f(x, y) = \int \phi(x, y, z)\,\mathrm{d}z,$$

and

(ii) $$\phi(x, y, z) = \phi_1(x|z)\phi_2(y|z)\phi_3(z).$$

A necessary and sufficient condition for (ii) is that

$$\phi(x|z) = \phi(x|y, z).$$

Here $\phi(x|z)$ is the conditional distribution of $X$ given $Z$, after $Y$ has been marginalized out.

The theorem states that, if $X$ and $Y$ are a pair of continuous random variables, there potentially could exist a third variable $Z$ such that the joint distribution of $X, Y, Z$, $\phi(x, y, z)$, has the property

$$\phi(x, y, z) = \phi(x|z)\phi(y|z)\phi(z),$$

so that $X|Z$ and $Y|Z$ are independent. The result is given as Theorem 1 of Holland and Rosenbaum (1986) and originates with Suppes and Zanotti (1981). In private correspondence, Peter Thomson (Victoria University, New Zealand) proved the result for the case when $X, Y, Z$ are jointly Gaussian. In this case, Thomson shows that, if $X, Y, Z$ are Gaussian with zero means, unit variances and correlations

$$\rho = \operatorname{corr}(X, Y), \quad \rho_1 = \operatorname{corr}(X, Z), \quad \rho_2 = \operatorname{corr}(Y, Z),$$

then the joint distribution has the required property provided only that $\rho_1 \cdot \rho_2 = \rho$. The theorem can be expanded to any group of random variables $X_1, X_2, \ldots, X_N$ such that if they are conditional on $Z_1, Z_2, \ldots, Z_m$, $m \leq N-1$, they will be independent.

In the causality context, if the theorem is correct, then any apparent instantaneous causal relationship can be explained by the possible existence of an unobserved variable that causes both (or all) the variables of interest. As the missing variable is unobserved it could occur at an earlier time. It follows that the concept of (real) instantaneous causality is not required as the present definition of causation (with a lag between cause and effect) can be used to explain all joint distributions and thus any apparent instantaneous causality or joint distribution.

The question remains of how one can disentangle the actual causal structure between variables that have here been called apparently instantaneously causal. I suspect that this cannot be achieved by purely statistical means, although this important question deserves further consideration. One natural approach is to add extra structure, as mentioned above, such as suggested by theory, 'common sense' or by beliefs that 'small' cannot cause 'big', for example. The relevance of conclusions based on such ideas will depend on how correct are the assumptions made, the tests will be of 'conditional causality' and the interpretation of the test results will depend on the degree of belief that one has of the assumptions being made.

As it stands, the discussion in this section probably has no implications for practical econometrics but should have relevance for the interpretation of the results obtained from empirical work.

## 4. Causality and control variables

Although the definitions of causality discussed in the first section are very simple, there can be problems with their use and interpretation. Tests based on the definition can also give some apparently surprising results. Some of these questions can be illustrated with a discussion of the potential usefulness of causality tests on control variables. The illustration can be based on a very simple case. Suppose that $y_t$ is some economic variable which the government is trying to control. $y_t$ will be called the target series, $a_t$ will be the desired value for $y_t$ and the cost function is the expected square of the difference between $y_t$ and $a_t$. Let $x_t$ be the variable controlled by the government and suppose that $y_t$ is generated by what is called in the engineering literature the 'plant equation',

$$y_t = \alpha y_{t-1} + cx_t + u_t, \tag{2}$$

where $u_t$ is zero-mean, white noise. It is easily seen that optimum control is achieved by taking

$$x_t = -c^{-1}[\alpha y_{t-1} - a_t], \tag{3}$$

so that

$$y_t = a_t + u_t, \tag{4}$$

under the assumption that the specification and parameters of the plant equation are unchanged, however $x_t$ is generated. There is a difficulty with this specification as there is an apparent instantaneous causation in (2). However, this is largely illusory, as one can take the time interval in this generating process to be the decision lag (rather than the period between data observations), and then note that $a_t$ will be determined by the government at time $t-1$ as a desired value for $y_t$ to achieve during the period from $t-1$ to $t$. Thus, one can put $a_t = \bar{a}_{t-1}$, placing $a_t$ the time at which its value is determined. It is seen from (3) that $x_t$ is also determined at time $t-1$. Thus, the control variable may be denoted $w_{t-1} = x_t$ and is also associated with the time at which it is determined. The government will observe $w_{t-1}$ at time $t-1$. The public will observe $x_t$ at time $t$ but should equate it with $w_{t-1}$. It is important in causality discussions to associate a variable with the time at which it occurs, rather than when it is observed. This problem is discussed in Granger (1980), particularly concerning the temporal relationship between thunder and lightening.

The equations now are

$$y_t = \alpha y_{t-1} + c w_{t-1} + u_t, \tag{5}$$

$$w_t = -c^{-1}(\alpha y_t - \bar{a}_t), \tag{6}$$

$$y_t = \bar{a}_{t-1} + u_t. \tag{7}$$

If an economic theory gives the plant equation (5), it may appear that $w_{t-1}$ should cause $y_t$, but this would be an incorrect interpretation as the whole system has to be considered jointly rather than one equation at a time. From the government's perspective, the question asked is: is $y_t$ better forecast using the information set $J_{t-1}$: $y_{t-j}, w_{t-j}, \bar{a}_{t-j}$, $j \geq 0$, rather than the information set $J'_{t-1}$: $y_{t-j}, \bar{a}_{t-j}$, $j \geq 0$? However, from (6), clearly these information sets contain the same information as $w_t$ is exactly explained by $y_t$ and $\bar{a}_t$. Thus, the government would not find $w_{t-1}$ causing $y_t$ in this case. This result was proved by Sargent (1976). The same conclusion would hold if $w_t$ were selected sub-optimally, but still exactly a function of other variables, such as

$$w_t = g_1 y_t + g_2 \bar{a}_t, \tag{8}$$

as pointed out by Buiter (1984).

The situation for the public is somewhat different if $\bar{a}_t$ is not publically announced and is also stochastic, such as if $\bar{a}_t$ is generated by

$$\bar{a}_t = \beta y_t + \gamma \bar{a}_{t-1} + e_t. \tag{9}$$

Now, $w_{t-1}$ would seem to cause $y_t$ in that $y_t$ is better forecast by $Y_{t-j}, w_{t-j}$, $j \geq 0$, than by $y_{t-j}$, $j \geq 0$, alone.

The Sargent and Buiter results are not very robust if the very stringent conditions of the model are relaxed. For example, if a white-noise error term, $v_t$, is added to (6) the government no longer is able to perfectly control its variable, as is surely the case in most instances. The vector autoregressive or reduced form representation for the structural system (5), (6) and (9) is

$$y_t = \alpha y_{t-1} + c w_{t-1} + u_t,$$

$$w_t = c^{-1}[\alpha(\beta - \alpha) y_{t-1} - \alpha c w_{t-1} + \gamma a_{t-1}] + v_t + c^{-1}[e_t - \alpha u_t],$$

$$\bar{a}_t = \alpha\beta y_{t-1} + \gamma \bar{a}_{t-1} + \beta c w_{t-1} + e_t + \beta u_t.$$

In general, it is easy to use such a VAR model to asked causality questions. Any left-hand-side variable is caused by any right-hand-side variable having a non-zero coefficient. Thus, for example, $w_{t-1}$ will cause $\bar{a}_t$ if $\beta c \neq 0$. However, the results above indicate that this type of result only holds true if there is no linear combination of the residuals to the various equations that have zero variance. No such linear combination exists if $u_t$, $v_t$ and $e_t$ all have positive variances, but is not true if $v_t = 0$, all $t$, as assumed by Sargent and Buiter.

There is one other case where the public finds $w_{t-1}$ causing $y_t$, but the government will find no causation. If $v_t = 0$ at $t$, but the plant equation includes a stochastic variable $z_t$,

$$y_t = \alpha y_{t-1} + c w_{t-1} + d z_{t-1} + u_t,$$

but $z_t$ is observed by the government and not by the public. The optimal value of the control variable will then be

$$w_t = -c^{-1}[\alpha y_t + d z_{t-1} - \bar{a}_{t-1}].$$

However, as the public does not observe $z_t$, but does observe $w_t$ which is related to it, again the public will find the control variable causing the target variable.

It is thus seen that the public and the government, if performing causality tests of $y_t$ by $w_{t-1}$, can reach different conclusions, depending on who is doing the test and what information set is available. The timing of variables is also clearly important. Some care has to be taken in interpreting causality tests as this exercise clearly shows. These questions are discussed in more detail in Granger (1987) where the I(1) case and co-integration aspects are also considered.

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