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Onset of Turbulence in a Rotating Fluid*

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(Received 17 July 1975)

Light-scattering measurements of the time-dependent local radial velocity in a rotating fluid reveal three distinct transitions as the Reynolds number is increased, each of which adds a new frequency to the velocity spectrum. At a higher, sharply defined Reynolds number all discrete spectral peaks suddenly disappear. Our observations disagree with the Landau picture of the onset of turbulence, but are perhaps consistent with proposals of Ruelle and Takens.

Thirty years ago, Landau proposed[1] that the turbulent state of a fluid results from a large number of discrete transitions or bifurcations, each of which causes the velocity field to oscillate with a different frequency f_i, until for sufficiently large i the motion appears chaotic, although the time correlation functions $C(\tau)$ of the velocity field do not strictly go to zero as $\tau\to\infty$. The Landau picture has been presumed applicable to a large class of systems, including the rotating fluid that we have studied. Systems in a second class (which we will not mention further) exhibit inverted bifurcations, where the transition to turbulence is hysteretic, and usually no periodic regime precedes the onset of chaotic behavior.

The Landau picture has been challenged by Ruelle and Takens,[2] who propose on the basis of abstract mathematical arguments that the motion should be aperiodic with exponentially damped correlation functions after three or four bifurcations to time-dependent states. Recently McLaughlin and Martin[3] have performed numerical calculations on a truncated set of equations applicable to Rayleigh-Bénard convection, and they found a sharp transition to aperiodic behavior following a periodic regime, in qualitative agreement with the arguments of Ruelle and Takens.

A great variety of periodic and chaotic states have been observed in past experiments on rotating[4] and convecting[5,6] systems. These experiments have not examined the onset of aperiodicity in sufficient detail to distinguish between what we term the Landau and Ruelle-Takens pictures. In contrast, Ahlers[7] has recently observed and characterized a sharp transition to aperiodic behavior in sensitive heat-flux measurements on convecting liquid helium; however, the periodic states which presumably precede the transition were not observed.

We present here the first detailed measurements of a *local* property that shows a sequence of periodic regimes followed by a sharp and reversible transition to an aperiodic state, as defined by the vanishing of all discrete spectral peaks (or equivalently, the decay of the time correlation functions). Specifically, we have studied the radial velocity in a fluid rotating between concentric cylinders. The observed behavior clearly contradicts the Landau model of the onset of turbulence.

In our experiments the fluid (water) was confined between an inner rotating stainless-steel cylinder of radius $r_1 = 2.224$ cm and a stationary precision-bore glass tube of inner radius $r_2 = 2.540$ cm. The gap d was uniform to within 1% over its entire length. The fluid height in the cell was 6.25 cm and the cell temperature was 27.5 ± 0.1°C. The ten-cycle average of the rotation period T was constant to within 0.3%.

The local radial velocity V_r was observed by an optical heterodyne technique using an optical arrangement described elsewhere.[8] The scattering volume was located at the center of the gap between the cylinders, and its largest dimension was 150 μm, about 0.05 of the gap. Thus the observations are essentially local measurements, and no significant spatial averaging is involved. The time-dependent frequency of the photocurrent oscillation, which is proportional to $V_r(t)$, was

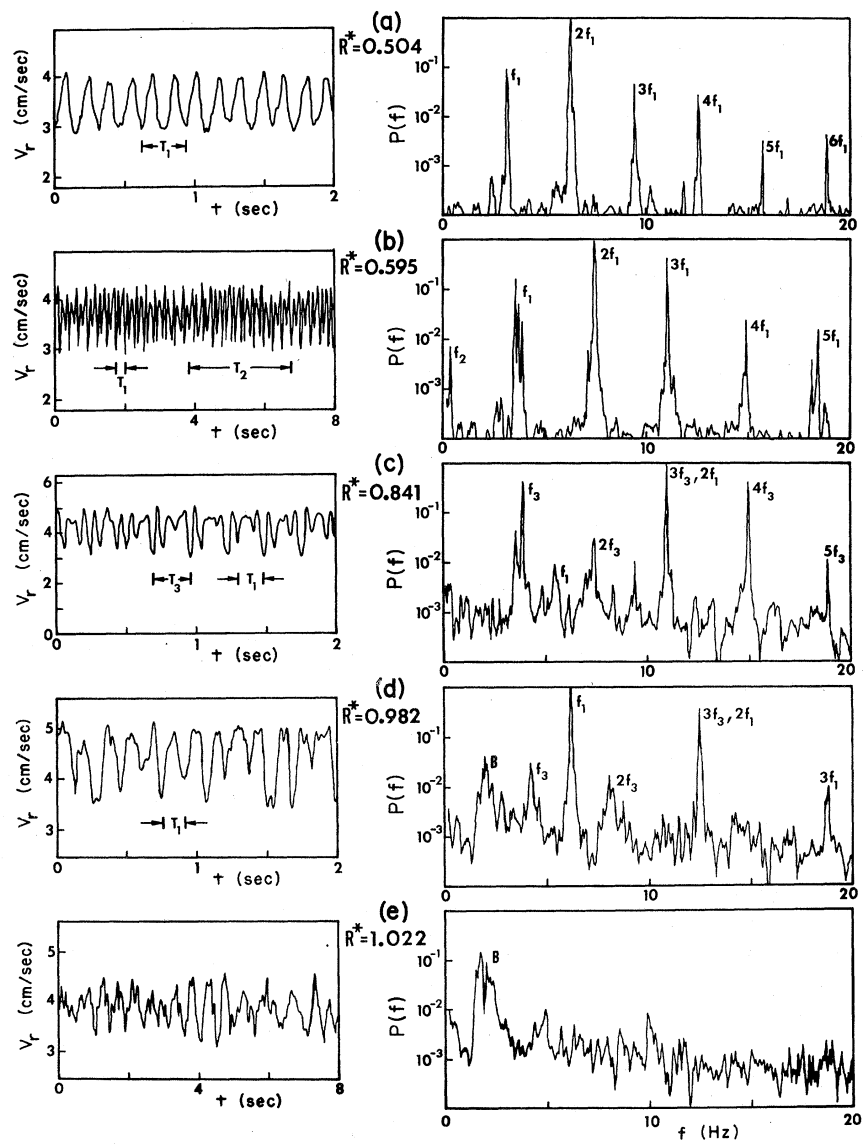

FIG. 1. Time dependence of the radial velocity and corresponding power spectra $P(f)$ [with units cm^2 sec^{-2} Hz^{-1}, normalized so that $\int_0^{25\,\mathrm{Hz}} P(f)\,df = \langle(\Delta V_r)^2\rangle$] for different reduced Reynolds numbers $R^* = R/R_T$.

measured for 1024 adjacent sampling intervals of 5×10^{-4} to 5×10^{-1} sec.

In the discussion to follow the rotation rate is expressed in terms of a reduced Reynolds number $R^*=R/R_T$, where $R=2\pi r_1 d/\nu T$ (ν is the kinematic viscosity) and $R_T=2501$ is the value of R at the onset of aperiodic motion.

We now describe the sequence of transitions which are observed reversibly as R is varied. The first instability (the Taylor instability) occurs at $R^*=0.051$, and results in a time-independent toroidal roll pattern that has been extensively studied.[4,8] The radial velocity is periodic in the axial coordinate z, with wavelength 0.79 cm. Our scattering volume was always positioned at or near one of the maxima in $V_r(z)$, and these locations persisted well into the aperiodic regime.

The first transition to a periodic state occurs at $R^*=0.064$, where transverse waves (with four wavelengths around the annulus) are superimposed on the toroidal vortices.[9] These waves, which have been previously observed visually,[4] manifest themselves as an oscillation at a frequency f_1 in our measurements, as shown in Fig. 1(a) for $R^*=0.504$. The frequency f_1 scales with $R^*\propto T^{-1}$, as Figs. 1(a)–1(d) illustrate; hence the dimensionless frequency $f_1^*\equiv f_1T$ is constant. The range in R^* of this and the subsequent time-dependent states is summarized in Fig. 2. The power spectrum $P(f)$ of the radial velocity, shown on a logarithmic scale in Fig. 1(a), contains strong peaks at the frequency f_1 and its harmonics nf_1, and the 0.05-Hz linewidth of these sharp peaks is determined only by the length of the data segment. The background noise in $P(f)$, which comprises only 2.3% of the total spectral power, is frequency independent to at least 100 Hz. This is mostly instrumental noise, with a magnitude of 10^{-4} cm^2 sec^{-2} Hz^{-1}, which corresponds to $\langle(\Delta V_r)^2\rangle^{1/2}=0.05$ cm/sec.

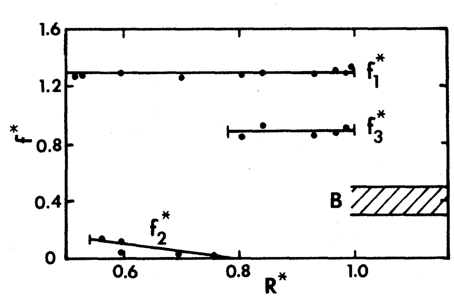

FIG. 2. The dimensionless frequencies $f_i^*=f_iT$ as a function of R^*. The solid lines are to guide the eye, and the vertical bars demarcate the regions in which the f_i are present (except that the lower bound for f_1 is $R^*=0.064$). The fact that $f_1^*=1.30$ and $f_3^*=0.87$ are constant indicates that f_1 and f_3 scale with rotation rate, whereas f_2 does not.

When R^* is increased to 0.54 ± 0.01, a second time-dependent instability occurs, and a new frequency f_2 is visible as a low-frequency modulation of the radial velocity [Fig. 1(b)]. The corresponding power spectrum shows that f_2, though weaker than f_1, is still nearly 2 orders of magnitude above the noise level. The frequency f_2 is a transverse (i.e., axial) disturbance, as is f_1, but its precise nature is unknown. It does interact with the mode at f_1 to produce a splitting of some of the nf_1 lines. Further increase in rotation rate causes f_2 to decrease in frequency until it is no longer visible at $R^*=0.78\pm0.04$ (see Fig. 2). Associated with this decrease is a gradual increase in the background noise level to about 10^{-3} cm^2 sec^{-2} Hz^{-1} in the range $0<f<60$ Hz, as in Fig. 1(c). This background now represents real noise in the fluid, but the peaks, which still contain 90% of the power, remain sharp. A new frequency f_3 (and its harmonics) appears at $R^*=0.78\pm0.03$, and this is also visible in Fig. 1(c). Note that f_3 appears only after f_2 has disappeared. Since f_3 is two-thirds of f_1, the behavior of Fig. 1(c) is periodic except for the additive noise.

Figure 1(d) shows the behavior just below the transition at $R^*=1$. The qualitative features are unchanged, and the reduction in the amplitude of the f_3 peak is caused by a small change in the vertical position of the scattering volume. The peaks in $P(f)$ (which still contain 90% of the spectral power) remain sharp, and the corresponding correlation function $C(\tau)=\langle V_r(t)V_r(t+\tau)\rangle$ oscillates periodically without any detectable decay for time lags τ up to 10 sec.

At $R^*=1$ ($R=R_T=2501$) a dramatic change occurs, as shown in Fig. 1(e). The sharp peaks at nf_1 and nf_3 disappear completely, leaving a broad doublet B that contains 60% of the power. While B was incipient at $R^*=0.982$, it contained only 5% of the power there. Higher-resolution spectra confirm that B is in fact a broad doublet with a total linewidth of about 1 Hz. The disappearance of the sharp peaks is equally apparent in the correlation function, which now decays to zero in a few seconds. The velocity data of Fig. 1(e) show erratic and noisy behavior, but since there is also some noise in 1(d), one must examine $P(f)$ or $C(\tau)$ to see the qualitative distinction. A further increase in rotation rate to $R^* = 1.16$ produced no further qualitative changes, although $P(f)$ broadens substantially.

The transition at $R^* = 1$ is sharp, reversible, and nonhysteretic to within a resolution $\delta R^* = 0.01$. Although the behavior for $R^* > 1$ is independent of the total sample height L, there is some variation with L for $R^* < 1$. However, we always detect three basic frequencies (f_1^*, f_2^*, and f_3^*) followed by a sharp onset of aperiodicity.

Our observation of a sharply defined Reynolds number at which the correlation function $C(\tau)$ decays to zero and the discrete peaks in the power spectrum $P(f)$ disappear represents the first clear demonstration that the Landau picture of the onset of turbulence is wrong. The observed behavior seems to be of the general type described by Ruelle and Takens, in which a few nonlinearly coupled modes are sufficient to produce an aperiodic motion. However, there exists no specific theoretical model applicable to this experiment.

Many questions remain unanswered. The arguments of Ruelle and Takens are quite general, and seem to apply to all systems which exhibit normal bifurcations. For these systems how universal is the behavior we have observed? What physical assumptions are inherent in the arguments of Ruelle and Takens? Finally, what is the sequence of events describing the loss of *spatial* correlation of the velocity fluctuations?

It is a pleasure to acknowledge helpful discussions with H. Z. Cummins, W. Davidon, J. Gersten, J. B. McLaughlin, and W. A. Smith.

*Work supported by the National Science Foundation.

†Work performed while on leave from the Physics Department, Haverford College, Haverford, Pa. 19041.

‡J.P.G. gratefully acknowledges the support of the National Oceanic and Atmospheric Administration.

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Nonlinear Filamentation of Lower-Hybrid Cones*

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The nonlinear distortion of the propagation cones of lower-hybrid waves is shown to be governed by the modified Korteweg–de Vries equation. Since such an equation admits exact solutions of the multiple-soliton form, it is predicted that filamented cones should be formed when large-amplitude lower-hybrid waves are excited in a plasma.

At the present time it appears that rf heating in the frequency band near the lower-hybrid resonance is one of the promising methods for attaining fusion temperatures in magnetically confined plasmas. A central problem in this heating scheme consists of transporting the rf energy from the outer edge of the plasma to its interior, where it is hoped to be converted into kinetic energy of plasma ions. Various investigations[1-4] of the propagation of lower-hybrid waves have been made in which the linear response of a slightly nonuniform plasma has been emphasized. While this is a sensible method for understanding the basic physics of the problem, it must be realized that linear theory may provide a rather inadequate description of the plasma behavior under actual heating conditions, which by design will involve the application of large rf power lev-