

# swyft: Truncated Marginal Neural Ratio Estimation in Python

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## Software

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## Summary

Parametric stochastic numerical simulators are ubiquitous in science. They model observed phenomena by mapping a parametric representation of simulation conditions to a hypothetical observation—effectively sampling from a probability distribution over observational data known as the likelihood. Simulators are advantageous because they easily encode relevant scientific knowledge. *Simulation-based inference* (SBI) is a machine learning technique which applies a simulator, a fitted statistical surrogate model, and a set of prior beliefs to estimate a probabilistic description of the parameters which plausibly generated some observational data. This description of parameters is known as the posterior and it is the end-product of Bayesian inference.

Our package *swyft* implements a specific, simulation-efficient SBI method called *Truncated Marginal Neural Ratio Estimation* (TMNRE) ([Miller et al., 2021](#)); it estimates the likelihood-to-evidence ratio to approximate the posterior, as in [Hermans et al. \(2020\)](#). *swyft* ([Miller et al., 2020](#)) provides a collection of tools to simulate and store data, locally or in a distributed computing setting, and perform (marginalized) simulation-based Bayesian inference. It produces ready-to-publish plots that demonstrate the calibration of the posterior estimate along with the posterior itself.

## Motivation

Estimating the posterior can be prohibitively expensive for complex data and slow simulators. Part of the reason is the sequential nature of likelihood-based Markov chain Monte-Carlo ([Hastings, 1970](#); [Metropolis et al., 1953](#)). In contrast, SBI parallelizes simulation in most circumstances, thereby reducing the practical waiting time for results. In pursuit of further simulation efficiency, [Miller et al. \(2021\)](#) argue that fitting the joint posterior for all parameters is unnecessary when a marginal estimate of the posterior will suffice. Some SBI methods are amortized, whereby the statistical model is fit to estimate posteriors for all possible observations simultaneously. While amortization enables necessary posterior calibration checks, like expected coverage probability ([Hermans et al., 2021](#); [Miller et al., 2021](#)), it is more efficient to fit the model on only a subset of the parameters that could have plausibly generated the observation.

*swyft* satisfies necessary requirements, like estimating the marginal posteriors of interest and enabling posterior calibration checks, while taking a lean approach to avoid all unnecessary simulation. In this pursuit, *swyft* truncates the prior to regions relevant for given observational data and reuses compatible existing simulations. *swyft* automates irksome matters like

distributed computing and data storage. `swyft` is designed to:

1. Estimate arbitrary marginal posteriors, i.e., the posterior over parameters of interest, marginalizing over nuisance parameters.
2. Perform targeted inference by truncating the prior distribution with an indicator function estimated in a sequence of inferences.
3. Estimate the expected coverage probability of fully amortized SBI posteriors and locally amortized posteriors that are limited to truncated regions.
4. Seamlessly reuse simulations from previous analyses by drawing already-simulated data first via a flexible storage solution.
5. Integrate advanced distribution and storage tools to simplify application of complex simulators.

Although there is a rich ecosystem of SBI implementations, TMNRE did not naturally fit in an existing framework since it requires parallel estimation of marginal posteriors and a truncated prior. `swyft` aims to meet the ever-increasing demand for efficient and testable Bayesian inference in fields like physics, cosmology, and astronomy by implementing TMNRE together with practical distributed computing and storage tools.

## Existing research with `swyft`

The software package has enabled inference on dark matter substructure in strongly lensed galaxies (Coogan et al., 2020), estimated cosmological parameters from cosmic microwave background simulation data (Cole et al., 2021), and was cited in a white paper laying out a vision for astroparticle physics research during the next decade (Batista et al., 2021). Ongoing work with `swyft` aims to reduce the response time to gravitational wave triggers from LIGO-Virgo by estimating the marginal posterior with unprecedented speed. There is an existing proof-of-concept by Delaunoy et al. (2020) although the `swyft` software package was not applied. Generally, speeding up gravitational wave inference using simulation-based inference is an active area of research (Chua & Vallisneri, 2020; Dax et al., 2021; Gabbard et al., 2022). In another work-in-progress, `swyft` helps to characterize the magnetohydrodynamics of binary neutron star mergers using multi-messenger gravitational and electrodynamic data where marginalization would be impossible with likelihood-based methods.

## Related theoretical work

There is a long tradition of likelihood-free inference, also known as *Approximate Bayesian Computation* (ABC), going back to as early as the 1980s (Beaumont et al., 2009; Diggle & Gratton, 1984; Rubin, 1984; Tavaré et al., 1997; Toni et al., 2009). Traditional techniques use Monte-Carlo rejection sampling and are summarized by Sisson et al. (2018) and Karabatsos & Leisen (2018). We track the development of classifiers for the estimation of likelihood ratios to a few references. Cranmer et al. (2015) compared the ratio between the likelihood of a freely varying parameter and a fixed reference value for frequentist inference. Pham et al. (2014) estimated the ratio between likelihoods for Markov chain Monte-Carlo sampling. Thomas et al. (2016) and Gutmann et al. (2018) introduced the framework which allows for likelihood-to-evidence ratio estimation. Like `swyft`, Blum & François (2010) proposed to truncate the prior for sampling but do so within an ABC scheme.

Modern SBI is a quickly evolving field that has several techniques under development (Cranmer et al., 2020). Neural network-based methods are categorized according to the term they approximate in Bayes' formula. `swyft` is a method which approximates the likelihood-to-evidence ratio  $\frac{p(x|\theta)}{p(x)}$  where  $\theta$  are the parameters and  $x$  is the observational data. Works by Hermans et al. (2020), Durkan et al. (2020), and Rozet & Louppe (2021) are closely related to `swyft` as they also approximate the likelihood-to-evidence ratio. Like `swyft`, Rozet & Louppe (2021) estimate marginal posteriors, but unlike `swyft`, they attempt to amortize over all possible marginals with a single neural network. Other methods estimate the posterior

91 directly (Durkan et al., 2020; Greenberg et al., 2019; Lueckmann et al., 2017; Papamakarios  
92 & Murray, 2016) or the likelihood itself (Lueckmann et al., 2019; Papamakarios et al., 2019).

## 93 Related software

94 swyft is unique because it implements TMNRE and a method for simulation reuse. It also  
95 offers sophisticated distributed simulation and storage tools coupled directly to the software.  
96 We briefly discuss the alternatives in the thriving ecosystem of SBI software packages.

97 sbi (Tejero-Cantero et al., 2020) features a selection of modern neural SBI algorithms. It  
98 is accompanied by a benchmark sbibm (Lueckmann et al., 2021) which tests those methods  
99 against a set of tractable toy problems. pydelfi (Alsing, 2019) estimates the likelihood of a  
100 learned summary statistic (Alsing et al., 2018, 2019)—swyft users should pay special attention  
101 to this repository since it can also project out nuisance parameters (Alsing & Wandelt, 2019).  
102 carl (Louppe et al., 2016) uses a classifier to estimate the likelihood ratio as Cranmer et al.  
103 (2015) did and hypothesis (Hermans, 2019) includes several toy simulators.

104 Non-neural implementations for SBI also exist. elfi (Lintusaari et al., 2018) implements  
105 BOLFI, an algorithm based on Gaussian processes (Gutmann et al., 2016). pyabc (Klinger et  
106 al., 2018) and ABCpy (Dutta et al., 2017) are two suites of ABC algorithms.

## 107 Description of software

108 swyft implements *Marginal Neural Ratio Estimation* (MNRE), a method which trains an  
109 amortized likelihood-to-evidence ratio estimator for any marginal posterior of interest. swyft  
110 makes it easy to estimate a set of marginals in parallel, e.g., for a corner plot. To use swyft,  
111 the operator must provide a quantification of prior beliefs, a python-callable or bash-scriptable  
112 simulator, and an observation-of-interest.

113 Performing TMNRE with swyft, by restricting simulation to a truncated prior region, is simple  
114 and demonstrated in the documentation. Constructing these truncated regions can be done  
115 manually or based on a previous inference. Routines are provided for all necessary plots and for  
116 calculating the expected coverage probability of a given likelihood-to-evidence ratio estimator.  
117 This calculation is essential as a sanity check to determine whether the approximate posterior  
118 is calibrated.

119 The machine learning aspects of swyft are implemented in PyTorch (Paszke et al., 2019)  
120 while the truncated prior is implemented within numpy (Harris et al., 2020). Storing previously  
121 simulated data for reuse in later analyses is accomplished with zarr (Miles et al., 2021) and  
122 parallelization of simulation is achieved with dask (Dask Development Team, 2016). swyft has  
123 other important dependencies, namely scipy (Virtanen et al., 2020), seaborn (Waskom, 2021),  
124 matplotlib (Hunter, 2007), pandas (McKinney, 2010; Reback et al., 2021), and jupyter  
125 (Kluyver et al., 2016).

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